

Quiz

1. Using a simple 3X3 matrix A , show that A and $A^T A$ have the same rank. Note, this is not a formal proof. I just need you to work through the example. [1 mark]
2. Prove that $f(\theta_1, \theta_2) = \theta_1^2 + \theta_2^2$ is a convex function [1 mark]
3. Weighted linear regression
 - a. We want to weigh the error for each sample differently. Say, the i^{th} sample has an error weight of w_i . Thus, the weighted error for the i^{th} sample denoted by ϵ_i is $w_i(y_i - \hat{y}_i)$. Find the closed form solution (similar to normal equation method) for θ for linear regression when the cost function is $\sum_{i=1}^N \epsilon_i^2$ [3 marks]
 - b. In your estimate for θ , show that when $w_i = 1 \forall i$, we get the same solution as the normal equation. [0.5 marks]
4. The objective for ridge regression is: $\min_{\theta} (y - X\theta)^T (y - X\theta) + \mu \theta^T \theta$. For two different values of μ , we observed. How can θ_0 increase when we increase μ from 1 to 2 [0.5 marks]

MU	Theta_0	Theta_1
1	5	5
2	5.3	4

5. Plot the contour plot of $z = f(x, y) = |x + y|$ [1 mark]
6. Show the first iteration of gradient descent when you are trying to minimise the objective function $f(x, y) = x^2 + y$. Start with initial x and y as 2 and 2. Use a learning rate of 0.5. Also show that the new solution gives a lower objective (lesser $f(x, y)$) value compared to original x and y (2, 2) [1 mark]