

Next Token Prediction

Building ChatGPT from Scratch (Conceptually)

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What We'll Learn Today

The Journey to GPT

Level	Topic	Key Concept
1	The Intuition	What does "predicting the next word" really mean?
2	The Counting Era	Bigrams: Count letter pairs
3	Representing Meaning	Embeddings: Words as vectors in space
4	Learning Patterns	Neural networks for next-token prediction
5	The Context Problem	Why we need to remember more
6	The Revolution	Attention and Transformers
7	From Theory to ChatGPT	Scaling up to billions of parameters

Level 1: The Intuition

What Are We Really Doing?

The Core Problem

Every language model answers **one simple question**:

"Given what I have seen so far, what word comes next?"

Example: "The capital of France is __" → "Paris"

That's it. **Predict the next word. Repeat until done.**

You Already Know This!

You've been using next-word prediction your whole life:

Application	You type...	Suggestions
Phone Keyboard	"I'm running __"	[late] [out] [away]
Google Search	"how to make __"	money, pancakes, friends
Gmail Smart Compose	"Thanks for the __"	<i>quick response!</i>

All of these are next-word prediction models!

Let's Play: The Autocomplete Game

Round 1: "The Eiffel Tower is located in __"

Your brain: **Paris** (very confident!)

Round 2: "I want to eat __"

Your brain: **pizza? pasta? nothing?** (uncertain!)

Round 3: "Once upon a __"

Your brain: **time** (almost certain!)

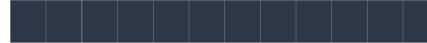
Round 4: "To be or not to __"

Your brain: **be** (Shakespeare hardcoded in culture!)

Your brain assigns **probabilities** to each possible next word. Some contexts have obvious answers, others don't!

The Mathematical View

When you read "The capital of France is __", your brain computes:

Word	$P(\text{word} \mid \text{context})$	Confidence
Paris	0.85	
the	0.02	
London	0.01	
beautiful	0.01	
...	0.11	

All probabilities sum to 1.0

This is called a **PROBABILITY DISTRIBUTION** over the vocabulary. Language models learn to produce these distributions!

It's JUST Prediction

You might think ChatGPT "understands" physics or history.
But all it does is predict the next word.

Prompt	Prediction	Domain
"F = m"	"a"	Newton's Law
"To be or not to"	"be"	Shakespeare
"E = mc"	" ² "	Einstein
"print('Hello"	")"	Python syntax
"2 + 2 ="	"4"	Math
"The mitochondria is"	"the"	Biology meme

If you predict well enough, you ****appear**** to understand everything. The model has compressed patterns from human knowledge into its weights.

The Shocking Simplicity

The One Algorithm

```
for token in generate(prompt):
    probabilities = model(all_tokens_so_far)
    next_token = sample(probabilities)
    output(next_token)
```

That's literally it. ChatGPT is this loop run millions of times with a really good model.

The Magic of "Just Prediction"

Q: "What is $17 + 28$?"

The model has seen **thousands** of math problems in training:

- " $2 + 2 = 4$ "
- " $15 + 10 = 25$ "
- " $17 + 28 = 45$ " ← Saw this pattern!

So when asked " $17 + 28 =$ ", it predicts "45" — not because it "knows" math, but because that **pattern exists** in training data!

This is why LLMs can make math mistakes — they're pattern matching, not actually computing! Try asking "What is 4738×2951 ?" and you'll see errors.

Emergent Behaviors

As models get bigger, surprising abilities **emerge**:

Model Size	Emergent Capabilities
Small (100M params)	Complete simple sentences, basic grammar
Medium (1B params)	Answer factual questions, simple reasoning
Large (100B+ params)	Complex reasoning, code generation, creative writing, multi-step problem solving, "understanding" context

All from the same objective: ****predict the next token!****

Level 2: The Counting Era

Bigrams: The Simplest Language Model

The Simplest Possible Model

Idea: Just count what letter usually follows each letter.

Training data: Names like `aabid`, `zeel`, `priya`, `nipun`

Count transitions:

- After `a`: saw `a` (1 time), `b` (1 time)
- After `z`: saw `e` (1 time)
- After `e`: saw `e` (1 time), `l` (1 time)
- After `n`: saw `i` (1 time) in "nipun"

This is called a **Bigram** model (looks at pairs of 2 characters).

Let's Build It Step by Step

Training Data: "aabid", "priya", "zeel", "nipun"

Step 1: Add special tokens

".aabid.", ".priya.", ".zeel.", ".nipun."
(. marks beginning and end)

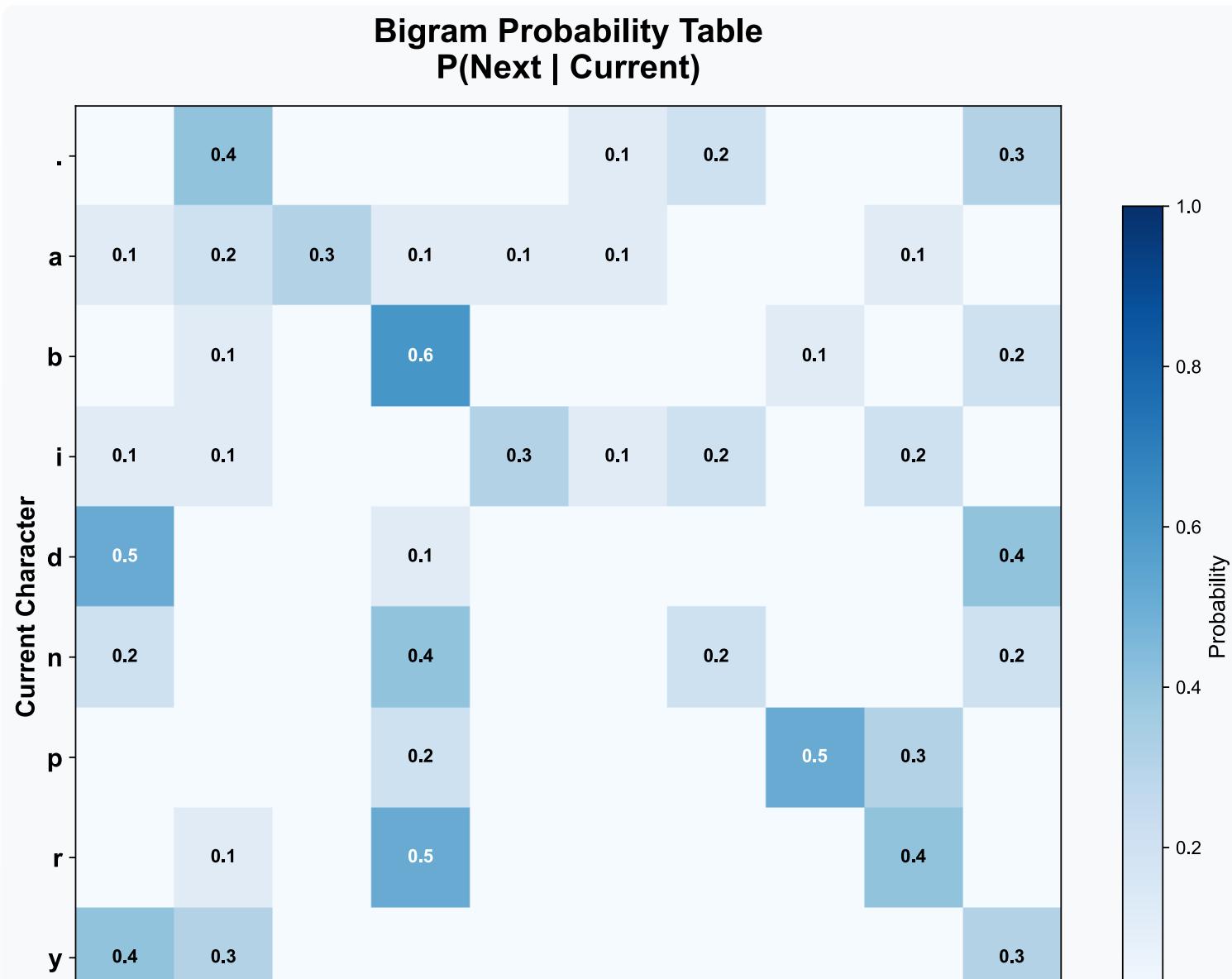
Step 2: Count all pairs

".a" appears 2 times (from aabid, priya doesn't start with 'a')
"aa" appears 1 time
"ab" appears 1 time
"bi" appears 1 time
"id" appears 1 time
"d." appears 1 time
... and so on

Step 3: Convert counts to probabilities

$P(\text{next} = 'a' \mid \text{current} = '.') = \text{Count}(\text{".a"}) / \text{Total pairs starting with } '.'$
 $P(\text{next} = 'a' \mid \text{current} = '.') = 2 / 4 = 0.50$

Bigram: The Counting Table



Generating Names with Bigrams

Step 1: Start with "." (beginning token)

Look up row "." → High prob for 'a', 's', 'm'

Sample → Got 'a'

Step 2: Current = 'a'

Look up row "a" → Moderate prob for 'a', 'b', 'n'

Sample → Got 'b'

Step 3: Current = 'b'

Look up row "b" → High prob for 'i', 'a', 'r'

Sample → Got 'i'

Step 4: Current = 'i'

Look up row "i" → High prob for 'd', 'n', 'a'

Sample → Got 'd'

Step 5: Current = 'd'

Look up row "d" → High prob for "." (end token)

Sample → Got "." (DONE!)

Result: "abid" ← Looks like a real name!

Interactive Example: Generating from Bigrams

Step	Current	Top Options	Roll	Selected
1	.	a(0.25), m(0.20), s(0.15)	0.18	m
2	m	a(0.40), i(0.25), o(0.15)	0.32	a
3	a	n(0.20), r(0.18), l(0.15)	0.45	r
4	r	i(0.25), a(0.20), y(0.18)	0.52	y
5	y	.(0.45), a(0.15), i(0.10)	0.30	.

Generated: "mary" ← A real name!

Why Bigrams Fail: The Memory Problem

The Problem:

Sentence: "The quick brown
fox jumps over
the lazy dog."

Question: After "dog",
what comes next?

Bigram sees: "dog" → ?
(forgot everything
before "dog"!)

Context is Lost:

With context:

"The cat sat on the ___"
→ Probably "mat"

Without context:

"the ___"
→ Could be anything!

Bigram only sees 1 char!

Bigrams have **no memory**. They forget everything except the last character!

A Concrete Example: The Context Problem

Sentence 1: "I love eating pizza with extra cheese"

Sentence 2: "I love eating pizza with my friends"

After "with", what comes next?

Model	What it sees	Problem
Bigram	Just "h" → ?	Doesn't even know it's in "with"!
Smarter model	Full context	Can reason about toppings vs companions

A smarter model would know:

- "pizza with" → usually followed by toppings or people
- "eating with" → suggests companions
- "love eating" → suggests food context

We need to see **MORE context!**

The Curse of Dimensionality

Why not just count longer patterns?

N-gram	Entries (27 chars)	Feasibility
1-gram	27	Fits in memory
2-gram	$27^2 = 729$	Still fine
3-gram	$27^3 = 19,683$	OK
4-gram	$27^4 = 531,441$	Getting big
5-gram	$27^5 = 14,348,907$	Very big
10-gram	$27^{10} \approx 205 \text{ TRILLION}$	Impossible!

For words (50,000 vocabulary): 2-gram = 2.5B, 3-gram = 125T entries!

We can't just count longer patterns — we need to **generalize**. This is where neural networks come in!

Bigrams: Summary

Aspect	Bigrams
What it does	Counts $P(\text{next char} \mid \text{current char})$
Memory	1 character only
Size	$27 \times 27 = 729$ numbers
Speed	Instant (just table lookup)
Quality	Poor (no context)
Training	Just counting

Key insight: The model is just a lookup table. No learning, no generalization.

Level 3: Representing Meaning

Embeddings: Words as Vectors

How Do Computers Read?

Computers only understand numbers. How do we convert letters?

Option A: One-Hot Encoding

```
'a' = [1, 0, 0, 0, ..., 0]      (27 dimensions for letters)  
'b' = [0, 1, 0, 0, ..., 0]  
'c' = [0, 0, 1, 0, ..., 0]  
...  
'z' = [0, 0, 0, 0, ..., 1]
```

Problem: These vectors are **orthogonal** (dot product = 0).

The computer thinks 'a' and 'b' are completely unrelated!

The Problem with One-Hot

Distance between letters: $\text{Distance}(a, b) = \text{Distance}(a, z) = \sqrt{2}$

Every letter is equally far from every other letter!

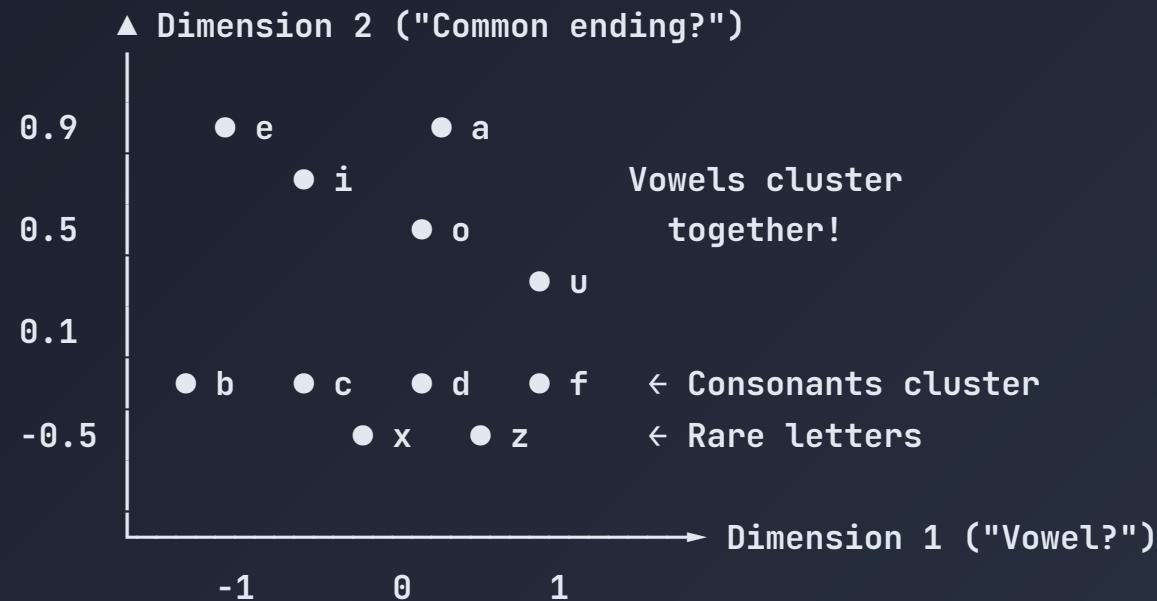
But we **KNOW**:

- 'a' and 'e' are both vowels (similar!)
- 'a' and 'x' have nothing in common (different!)
- 'p' and 'b' look similar (related!)

We need a smarter representation where **similar things are close**.

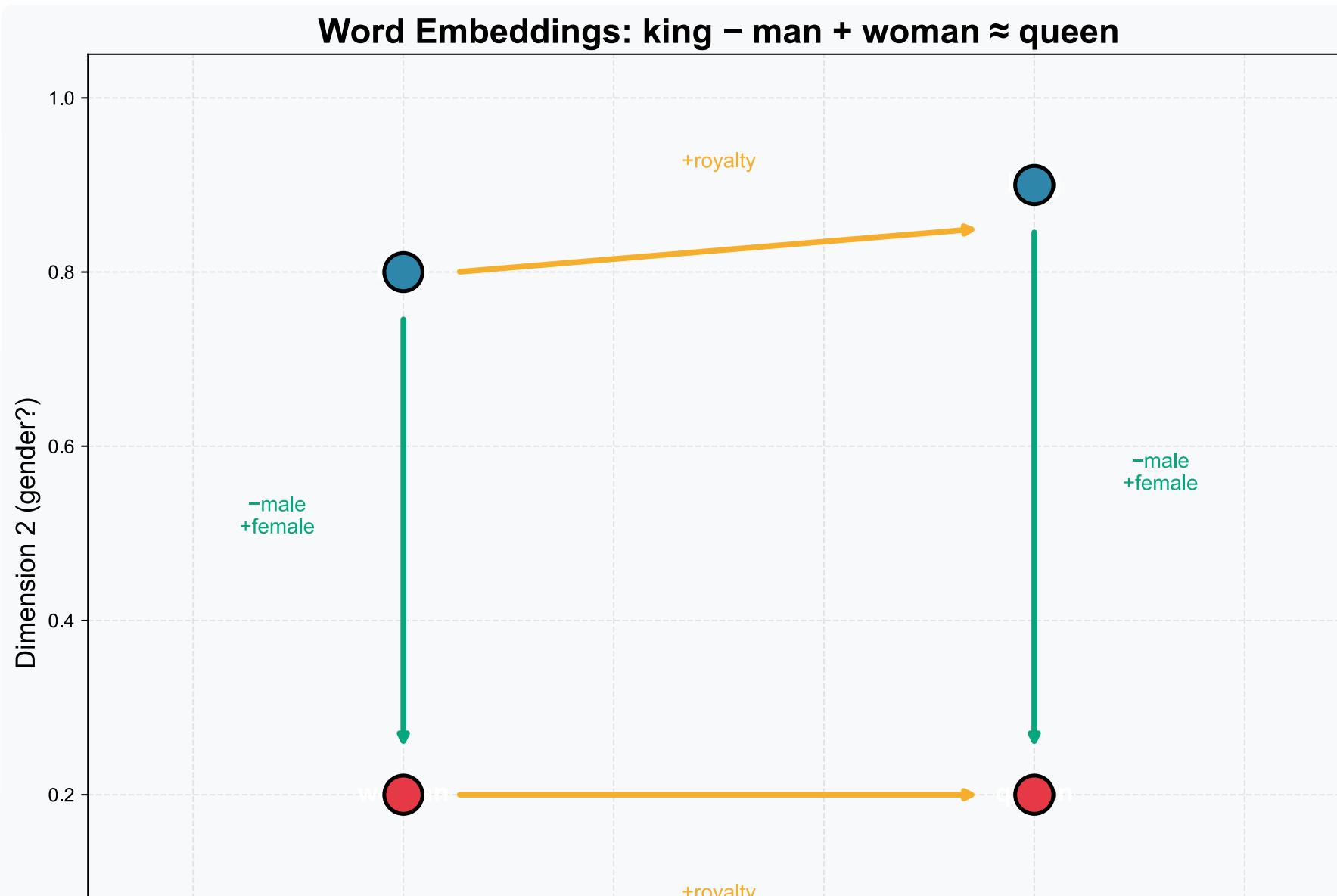
Dense Embeddings: Meaning as Coordinates

Idea: Represent each character as a point in space where **similar things are close**.



Now **a** and **e** are **mathematically close!**

Word Embeddings: The Famous Example



The King - Man + Woman = Queen Example

Word Arithmetic

Word	Vector	Meaning
king	[0.8, 0.3, 0.9, ...]	royalty, male, power
man	[0.1, 0.3, 0.5, ...]	person, male, average
woman	[0.1, 0.9, 0.5, ...]	person, female, average

Calculation: king – man + woman =?

$$[0.8, 0.3, 0.9] - [0.1, 0.3, 0.5] + [0.1, 0.9, 0.5] = [0.8, 0.9, 0.9]$$

Nearest word to [0.8, 0.9, 0.9]: "**queen**"!

The model learned: "The relationship between king and man is the same as the relationship between queen and woman"

More Word Analogies

Analogy	Answer
France : Paris :: Japan : ?	Tokyo
good : better :: bad : ?	worse
walking : walked :: swimming : ?	swam
Einstein : physicist :: Picasso : ?	painter

The embeddings capture ****RELATIONSHIPS**** automatically! No one told the model about capitals or verb tenses!

How Embeddings Are Learned

Start: Random vectors for each word

Training on: "The cat sat on the mat"

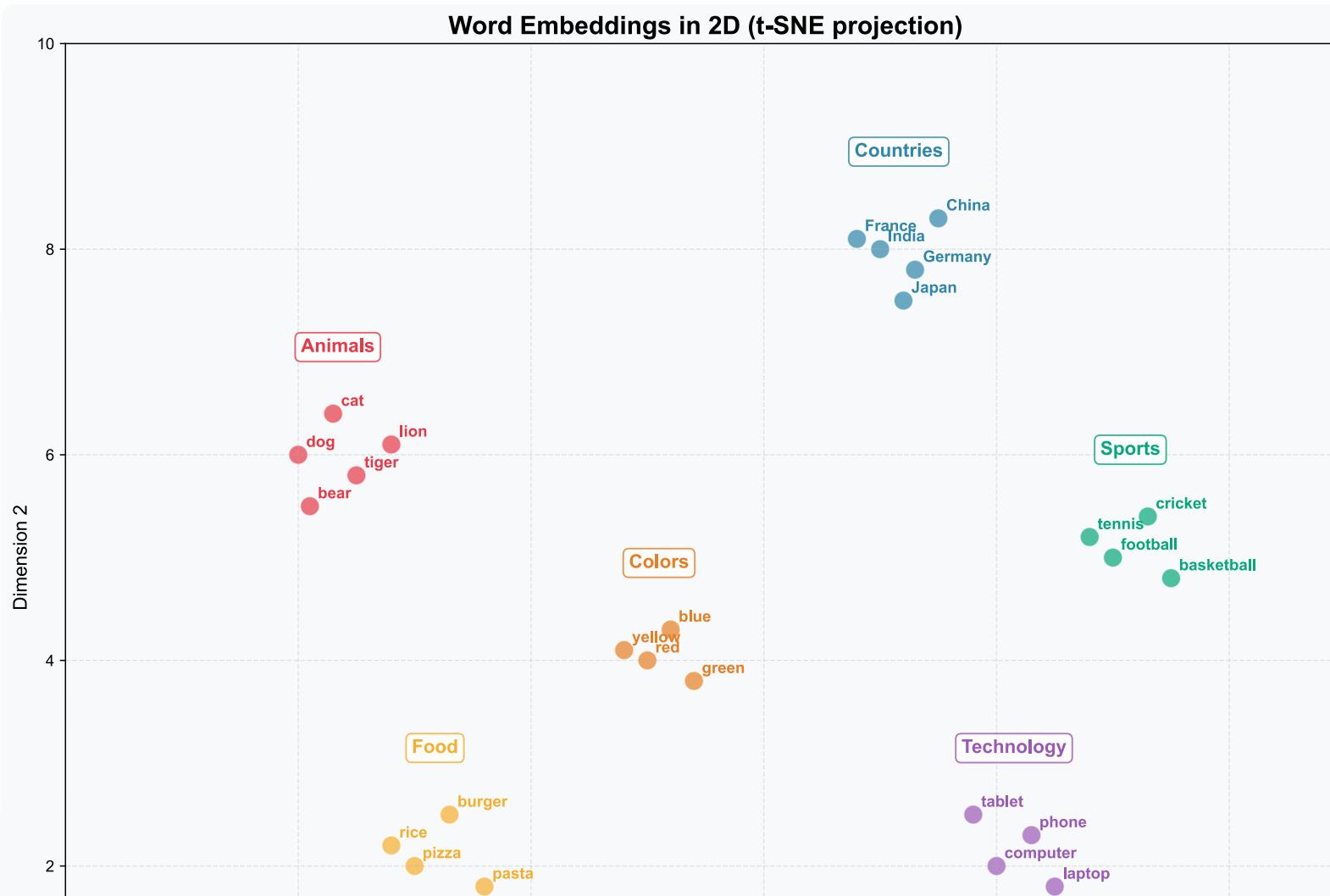
Observation	Action
"cat" often appears near "sat", "dog", "pet"	Push embeddings closer
"cat" rarely appears near "quantum", "fiscal"	Push embeddings apart

After billions of examples:

- Similar words → Similar vectors
- Related concepts → Close in space

Visualizing Embeddings

Real word embeddings projected to 2D (using t-SNE):



Embedding Dimensions

What Do Dimensions Mean?

We use 256-4096 dimensions in practice, but imagine 4:

Dim	Meaning
1	Is it alive?
2	Is it a person?
3	Concrete vs abstract?
4	Positive/negative sentiment?

Word	Vector	Interpretation
dog	[0.9, 0.1, 0.8, 0.7]	alive, not person, concrete, +
cat	[0.9, 0.1, 0.8, 0.6]	very similar to dog!
love	[0.2, 0.3, -0.8, 0.9]	abstract, positive
hate	[0.2, 0.3, -0.8, -0.9]	abstract, negative

In reality, dimensions are learned and not so interpretable!

Level 4: Learning Patterns

Neural Networks for Next-Token Prediction

From Counting to Learning

BIGRAM (Counting)

$$P(b|a) = \frac{\text{Count}(b|a)}{\text{Count}(a)}$$

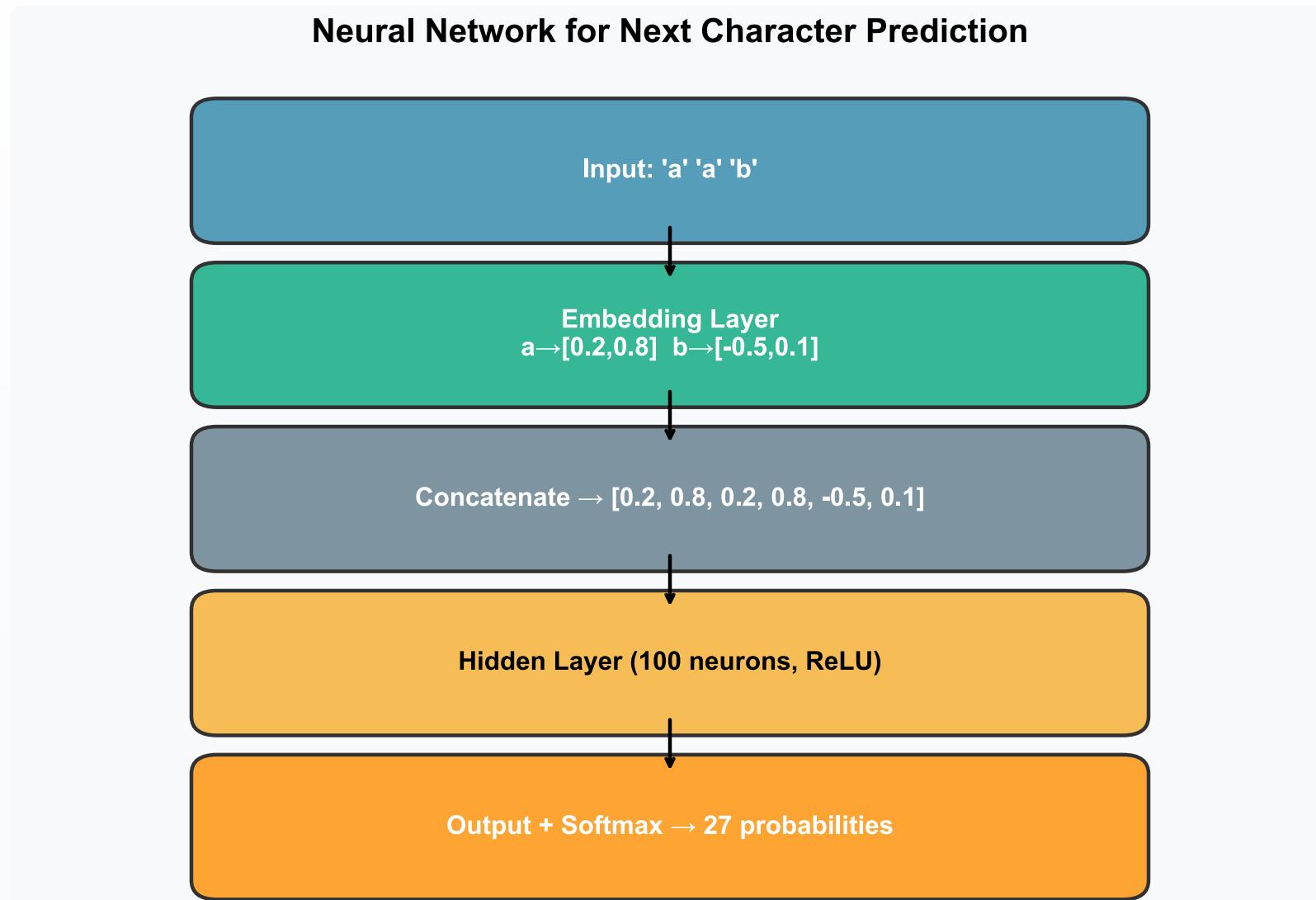
- Fixed lookup table
- Only memorizes exact patterns

NEURAL NETWORK (Learning)

$$P(\text{next}|a) = f(\text{embed}(a); \theta)$$

- Learned weights θ
- Can **generalize** to unseen patterns!

The Neural Network Architecture



The Architecture Unpacked

MLP Language Model — Input: Last 3 characters [a, a, b]

Step	Operation	Example
1. Embed	Look up each character	a → [0.2, 0.5], b → [0.8, -0.3]
2. Concatenate	Join all embeddings	[0.2, 0.5, 0.2, 0.5, 0.8, -0.3]
3. Hidden Layer	$\mathbf{h} = \text{ReLU}(W_1 \cdot \text{concat} + b_1)$	[0.7, 0.1, 0.9, 0.3]
4. Output Layer	$\text{logits} = W_2 \cdot \mathbf{h} + b_2$	P(a)=0.1, P(b)=0.05, ..., P(i)=0.6

The softmax converts logits to probabilities that sum to 1.

Creating Training Data: The Sliding Window

Text: "aabid" — Create (context → target) pairs by sliding a window:



Training: Learning from Mistakes

Step	Action	Example
1. Forward Pass	Input: $[a, a, b] \rightarrow$ Network predicts	$P(i)=0.10, P(z)=0.30, P(a)=0.20$
2. Compute Loss	$\text{Loss} = -\log(P(\text{correct}))$	$-\log(0.10) = 2.3$ (high = bad!)
3. Backpropagation	Compute gradients for all weights	"How should each weight change?"
4. Update Weights	Adjust to make $P(i)$ higher	Repeat millions of times!

Actual answer: 'i' — The model was only 10% confident, so it gets a high loss and learns to do better!

The Loss Function: Cross-Entropy

$$\text{Loss} = -\log(P(\text{correct answer}))$$

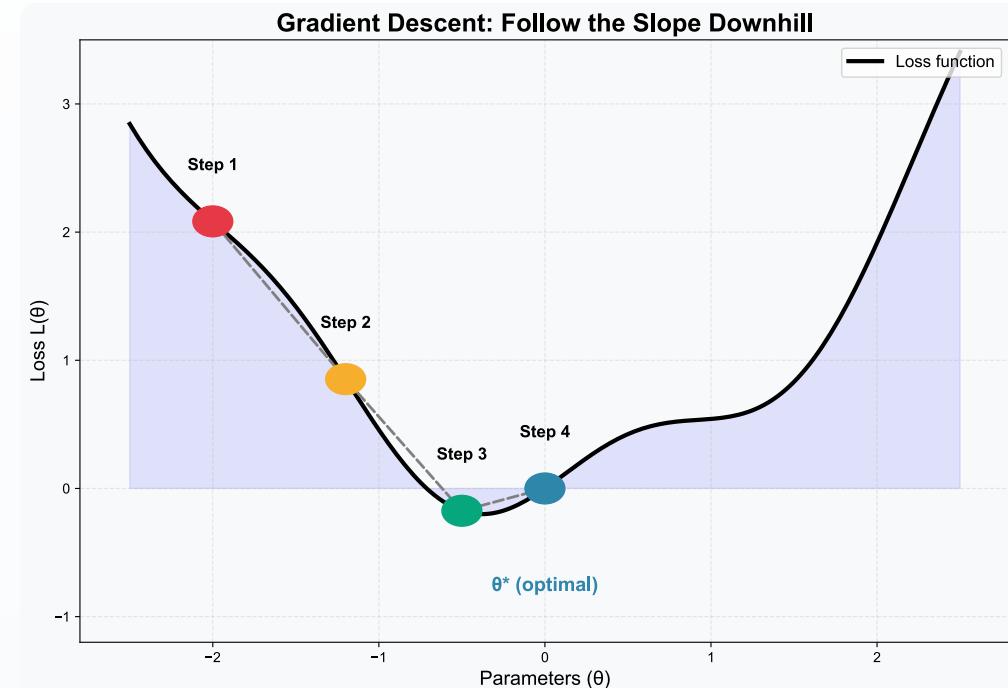
Scenario	P(correct)	Loss	Interpretation
Confident and RIGHT	0.95	$-\log(0.95) = 0.05$	Low loss ✓
Uncertain	0.50	$-\log(0.50) = 0.69$	Medium loss
Confident and WRONG	0.01	$-\log(0.01) = 4.6$	High loss ✗

The model gets heavily penalized for confident wrong answers! This encourages well-calibrated uncertainty.

Gradient Descent: Finding the Best Weights

Analogy: You're blindfolded on a mountain. Goal: reach the lowest point.

Step	Action
1	Feel slope $\rightarrow \nabla L$
2	Step downhill $\rightarrow \theta - \alpha \nabla L$
3	Repeat until minimum



Learning rate α controls step size: too big = overshoot, too small = slow.

Level 5: The Context Problem

Why Fixed Windows Aren't Enough

The Fatal Flaw

Our neural network has a **fixed context window** (e.g., 3 characters).

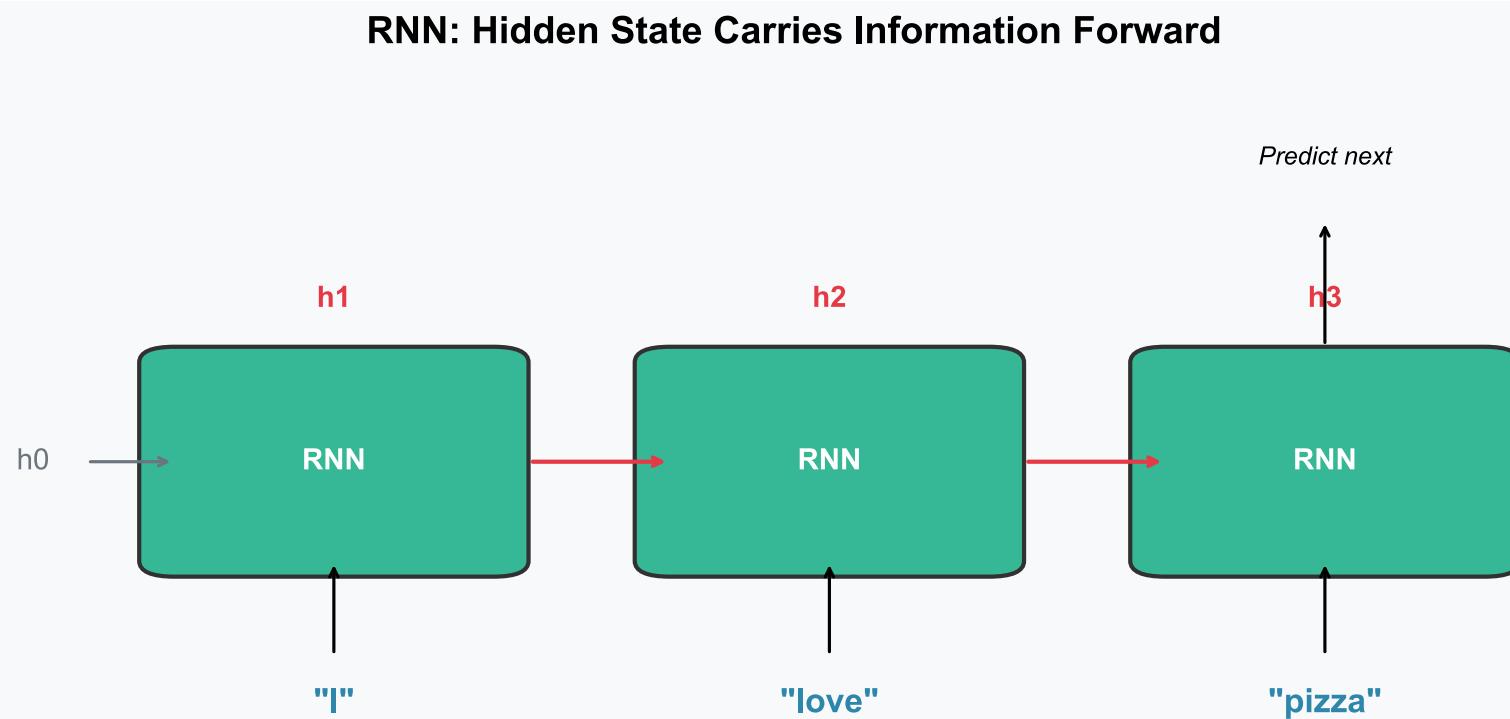
Story: "Alice picked up the golden key. She walked to the door and tried to open it with the __"

Viewer	What They See	Problem
Human	"golden key" (earlier in story)	Full context
Model	"with the" (only last 3 words!)	"key" is outside the window!

The model forgot the key! Fixed windows lose important information from earlier context.

The Solution: RNNs (The Relay Race)

Idea: Pass information forward like a **baton in a relay race**.



RNN Intuition: The Telephone Game

How it works:

- Each word updates the hidden state
- Hidden state = "memory" of what came before
- Pass memory to next step

Message Length	Quality
10 words	Clear
50 words	Fuzzy
100 words	Lost!

The Problem:

- Like a game of telephone!
- Message gets corrupted over time

RNNs forget old information — the "vanishing gradient problem".

Further reading: LSTM/GRU cells help but don't fully solve this.

Level 6: The Revolution

Attention: "Just Look Back!"

The Brilliant Idea

What if, instead of compressing everything into a hidden state...

We could just **look back** at everything directly?

Text: "Alice picked up the golden key. She walked to the door and tried to open it with the __"

Approach	What It Sees	Limitation
Fixed Window	"with the"	Can only see last few words
RNN	Blurry summary	Memory degrades over time
ATTENTION	Any word directly!	"Let me check... 'key' was mentioned!"

Attention is like having a ****searchable index**** over the entire text!

Attention: The Library Analogy

You're at the library looking for information.

Your Question (Query):

"What opens doors?"

You scan the shelves (Keys):

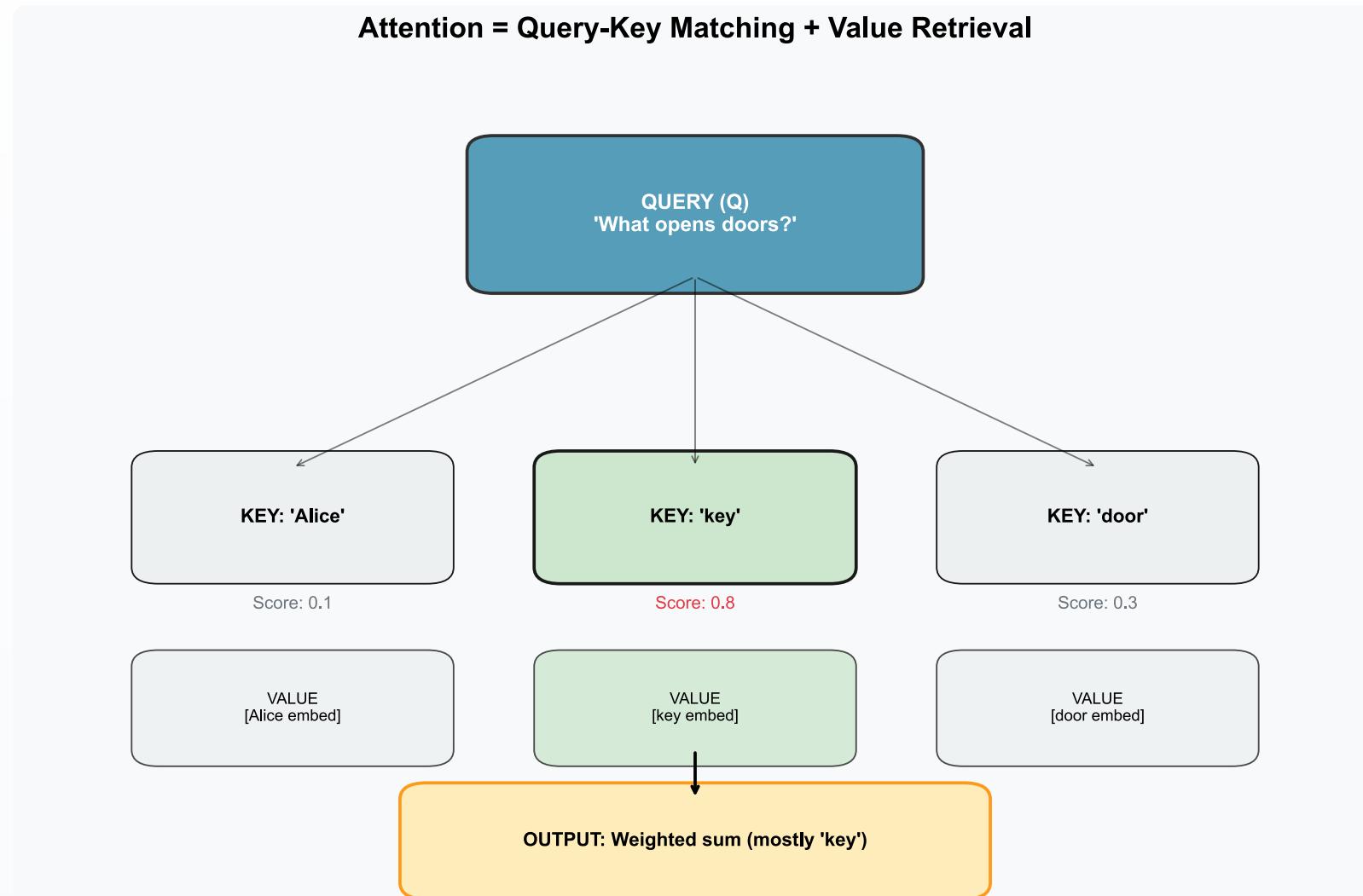
- "key" → Looks relevant!
- "door" → Somewhat related
- "Alice" → Not relevant

Book (Key)	Match Score
"key"	0.8
"door"	0.3
"Alice"	0.1

You read (Values) mostly from "key"!

Attention = Query → match with Keys → weighted sum of Values

Attention in Action



Why Attention is Powerful

Example: "The animal didn't cross the street because **it** was too __"

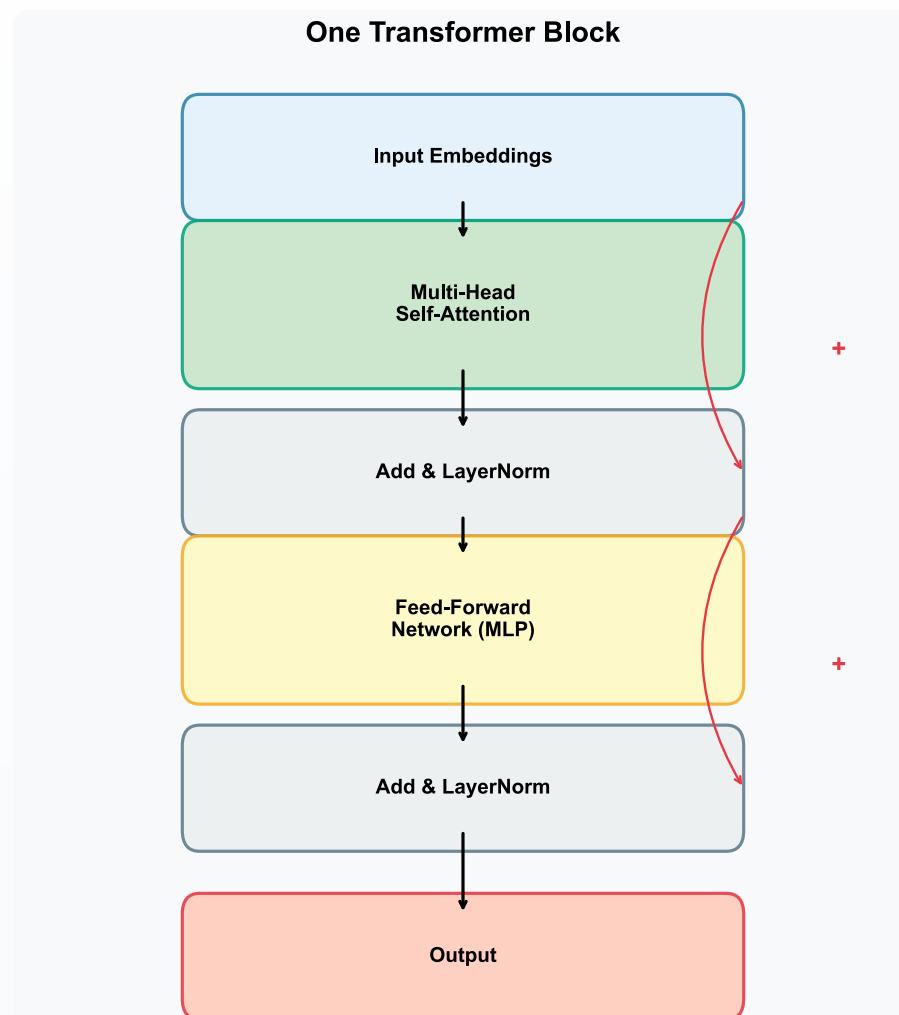
What does "it" refer to?

Word	animal	street
Attention Score	0.75	0.15

The model learns to connect "it" to "animal" — this is called **coreference resolution**, learned automatically!

Further reading: Multi-head attention, self-attention matrices, positional encoding details

The Transformer: Putting It Together



The Two Key Components:

1. Self-Attention

- "Who should I pay attention to?"
- Every word looks at every word

2. Feed-Forward Network

- Process the gathered information
- Make predictions

Stack 96+ of these blocks!

Level 7: From Theory to ChatGPT

Scaling Up

Our Toy Model vs ChatGPT

Feature	Our Toy Model	ChatGPT
Vocabulary	27 (letters)	100,000 (tokens)
Embedding Size	2 dimensions	12,288 dims
Layers	1 layer	96 layers
Attention Heads	1 head	96 heads
Parameters	~1,000	175 BILLION
Training Data	1,000 names	500B+ tokens
Context Window	3 chars	128K tokens
Training Time	1 minute	Months on 1000s of GPUs

Same core algorithm. Just ****much, much bigger****.

Tokenization: Not Characters, Not Words

LLMs use **TOKENS** — subword units (BPE algorithm):

Example: "unhappiness" → ["un", "happiness"]

Approach	Problem	Vocabulary Size
Characters	Too slow (many steps per word)	~100
Words	Too many unique words	Millions!
Tokens	Best of both worlds	~50,000-100,000

Text: "ChatGPT is amazing!"

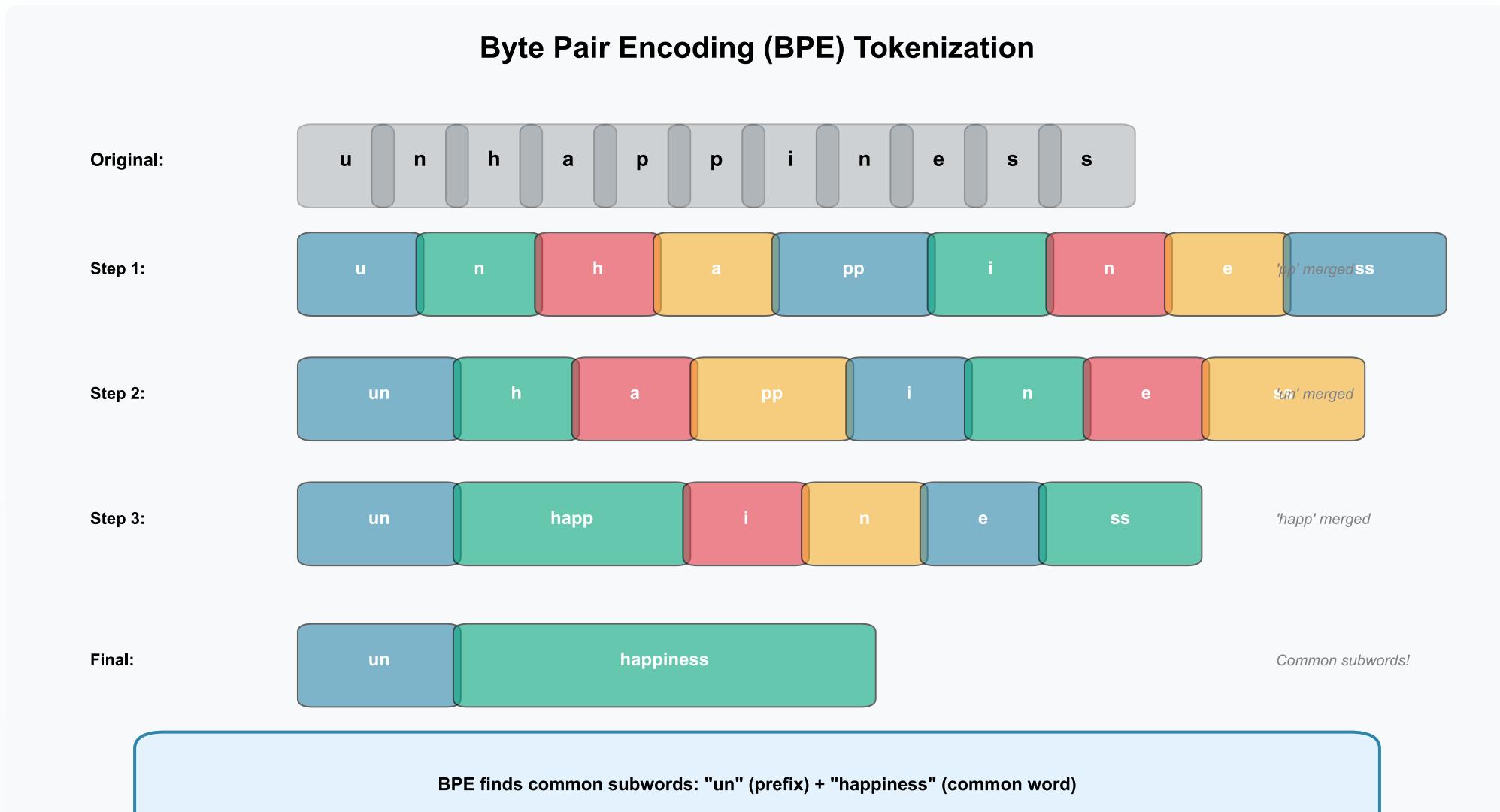
Tokens: ["Chat", "G", "PT", "is", "amazing", "!"]

Token IDs: [15496, 38, 2898, 318, 4998, 0]

Note: Spaces are often part of tokens (" is" not "is")

How BPE Tokenization Works

Byte Pair Encoding (BPE) — Start with characters, merge common pairs:



Tokenization Quirks

Why LLMs struggle with certain tasks:

Problem	Example	Why It's Hard
Counting letters	"strawberry" → ["str", "aw", "berry"]	'r' split across tokens!
Non-English	"Hello" → 1 token, "नमस्ते" → 6 tokens	Same meaning, 6x cost!
Numbers	"1234" = 1 token, "12345" = 2 tokens	Math becomes inconsistent
Code indentation	" " (4 spaces) = 1 token	" " (3 spaces) = 3 tokens

Tokenization artifacts explain many LLM failure modes — they don't "see" characters, they see tokens!

Positional Encoding

How does the model know word ORDER?

Problem: Attention is permutation-invariant — "Dog bites man" and "Man bites dog" look the same!

Solution: Add position information to each embedding.

Component	Value	Example
token_embedding("cat")	[0.5, 0.3, 0.8, ...]	Word meaning
position_encoding(pos=3)	[0.1, -0.2, 0.4, ...]	Position info
final_embedding	[0.6, 0.1, 1.2, ...]	Sum of both

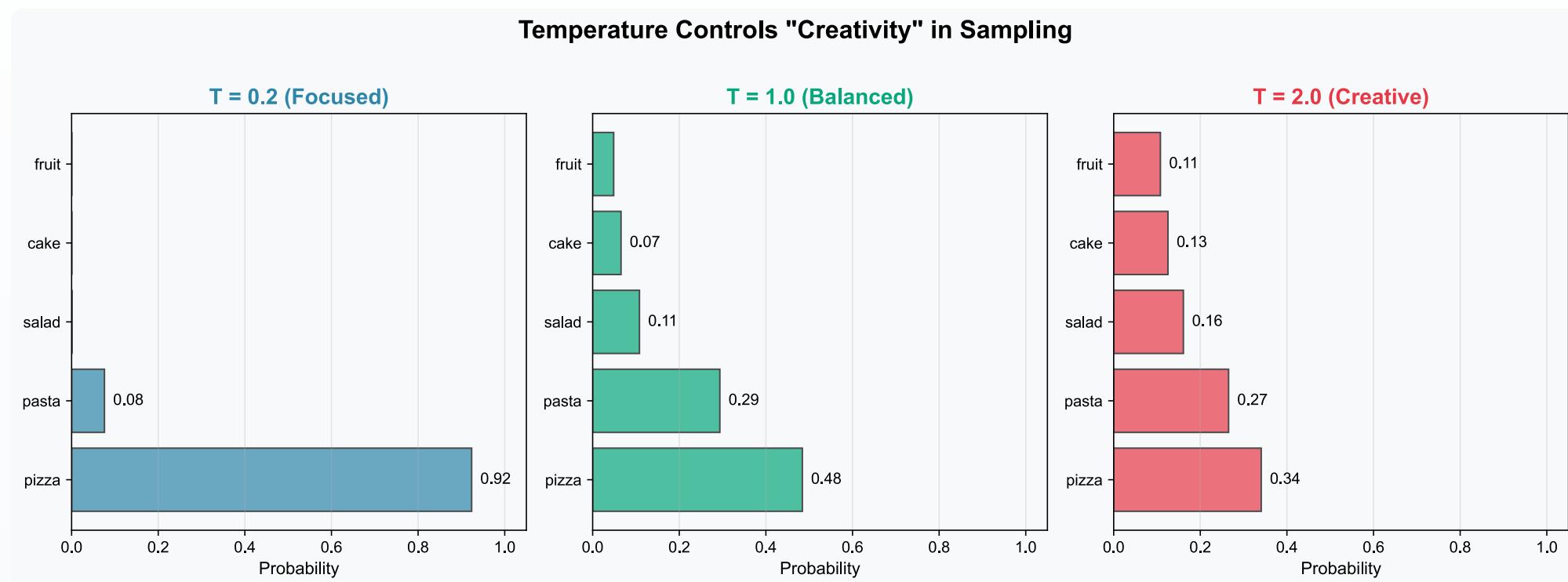
Now "cat" at position 3 \neq "cat" at position 10!

Original formula: $PE_{(pos,2i)} = \sin(pos/10000^{2i/d})$, $PE_{(pos,2i+1)} = \cos(pos/10000^{2i/d})$

Modern models: **Learn** position embeddings!

Temperature: The Creativity Knob

When sampling the next token, we can adjust **temperature**:



- **Low temp** → Always picks the most likely (boring but safe)
- **High temp** → Spreads probability more evenly (creative but risky)

Temperature: The Math

$$\text{probs} = \text{softmax}(\text{logits}/T)$$

Example logits: [2.0, 1.0, 0.5, 0.1]

Temperature	Probabilities	Effect
T = 1.0 (normal)	[0.43, 0.26, 0.19, 0.12]	Balanced
T = 0.1 (cold)	[0.99, 0.01, 0.00, 0.00]	Almost deterministic
T = 2.0 (hot)	[0.32, 0.27, 0.22, 0.19]	More uniform/random
T = 0 (greedy)	argmax	Always pick highest

Lower temperature → more focused; Higher temperature → more creative/random

Top-k and Top-p Sampling

Top-K Sampling

Only consider the top K most likely tokens.

All tokens	Top - 3 (renormalized)
------------	------------------------

Problem: K is fixed — sometimes 3 options make sense, sometimes 10.

Top-P (Nucleus) Sampling

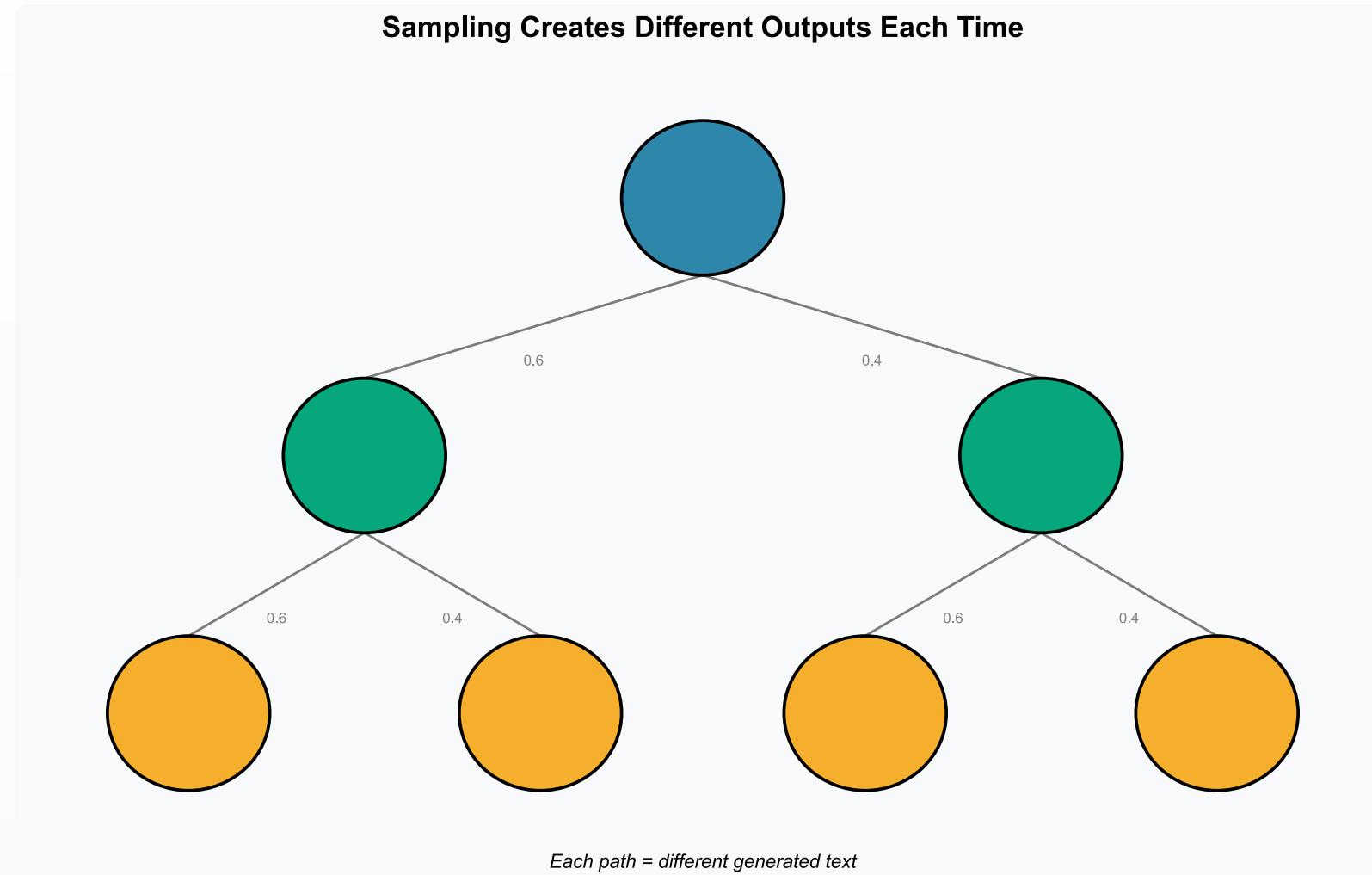
Include tokens until cumulative probability > P.

Token	Prob	Cumulative	Include? (P=0.9)
pizza	0.40	0.40	✓
pasta	0.30	0.70	✓
shoes	0.10	0.80	✓

Top-P is **adaptive**: narrow when confident, wide when uncertain!

The Sampling Tree

Because we sample probabilistically, each generation is different!



Training at Scale

GPT-3 Training

Data:

Source	Tokens
Common Crawl	410B
Books	67B
Wikipedia	3B
Total	~500B

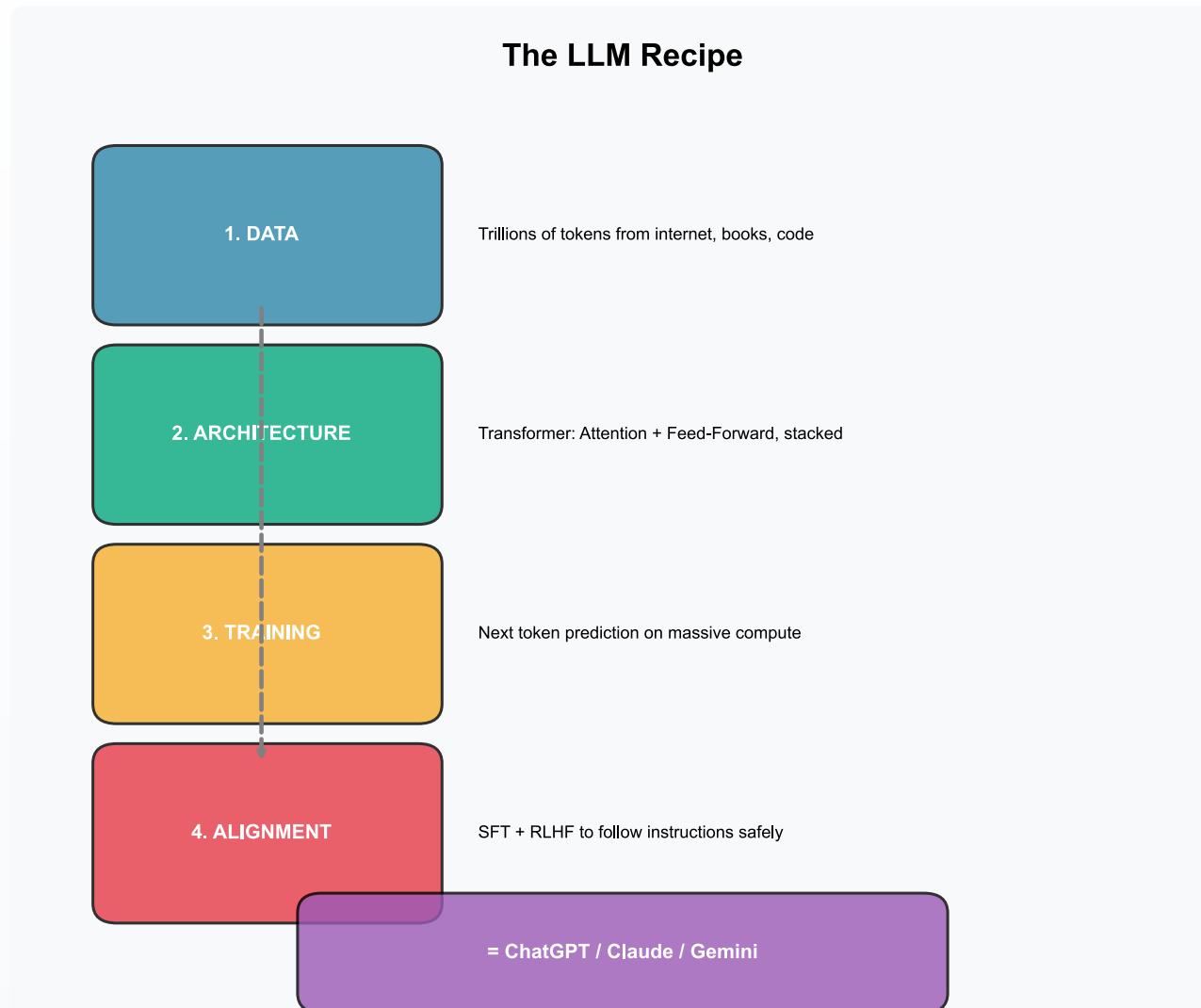
Compute:

- 10,000 GPUs
- Training time: ~1 month
- Cost: ~\$4.6 million (electricity!)

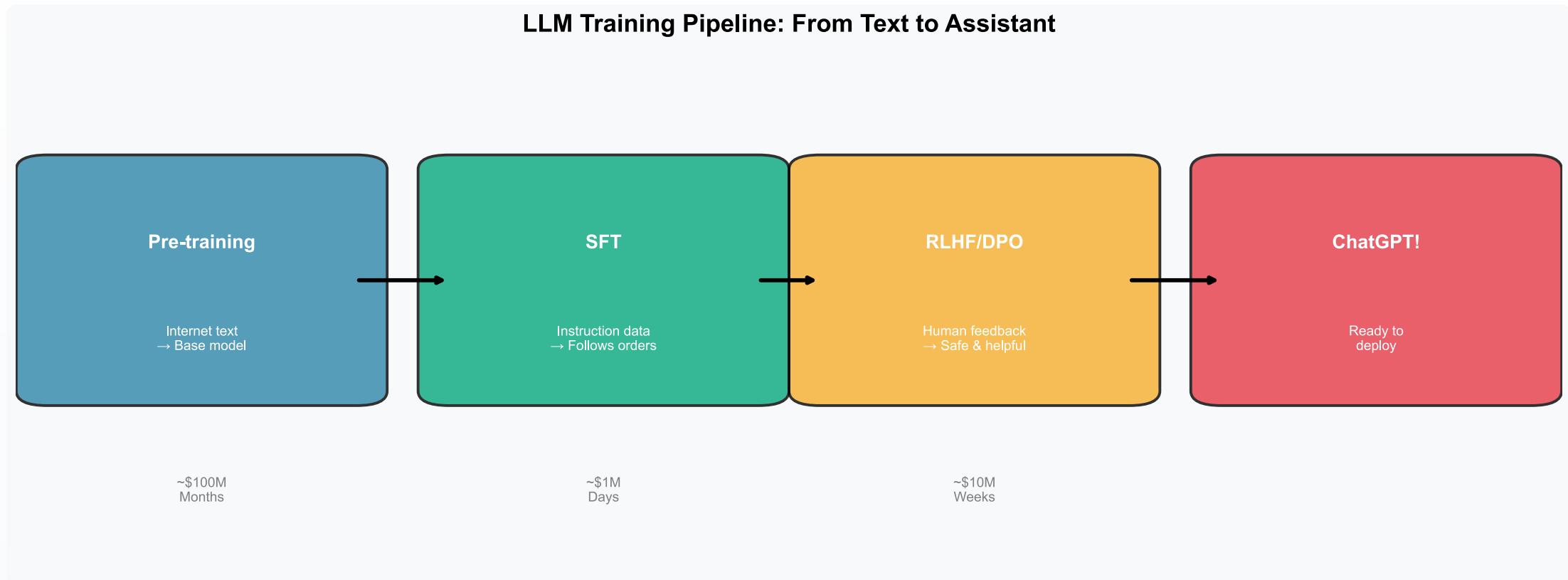
Model:

- 175 billion parameters
- 96 layers, 96 attention heads
- 12,288 embedding dimensions

The Complete Recipe



From GPT to ChatGPT: The Full Training Pipeline



Stage 1: Pre-Training

Goal: Learn language from massive text data

Aspect	Details
Data	Internet, books, Wikipedia (~trillions of tokens)
Objective	Next token prediction: $P(\text{next} \mid \text{context})$
Compute	1000s of GPU-hours
Result	Base model - can complete text but not helpful

This is the most expensive step! OpenAI, Anthropic, Google spend \$10M-\$100M+ here.

Stage 2: Supervised Fine-Tuning (SFT)

Goal: Learn to follow instructions

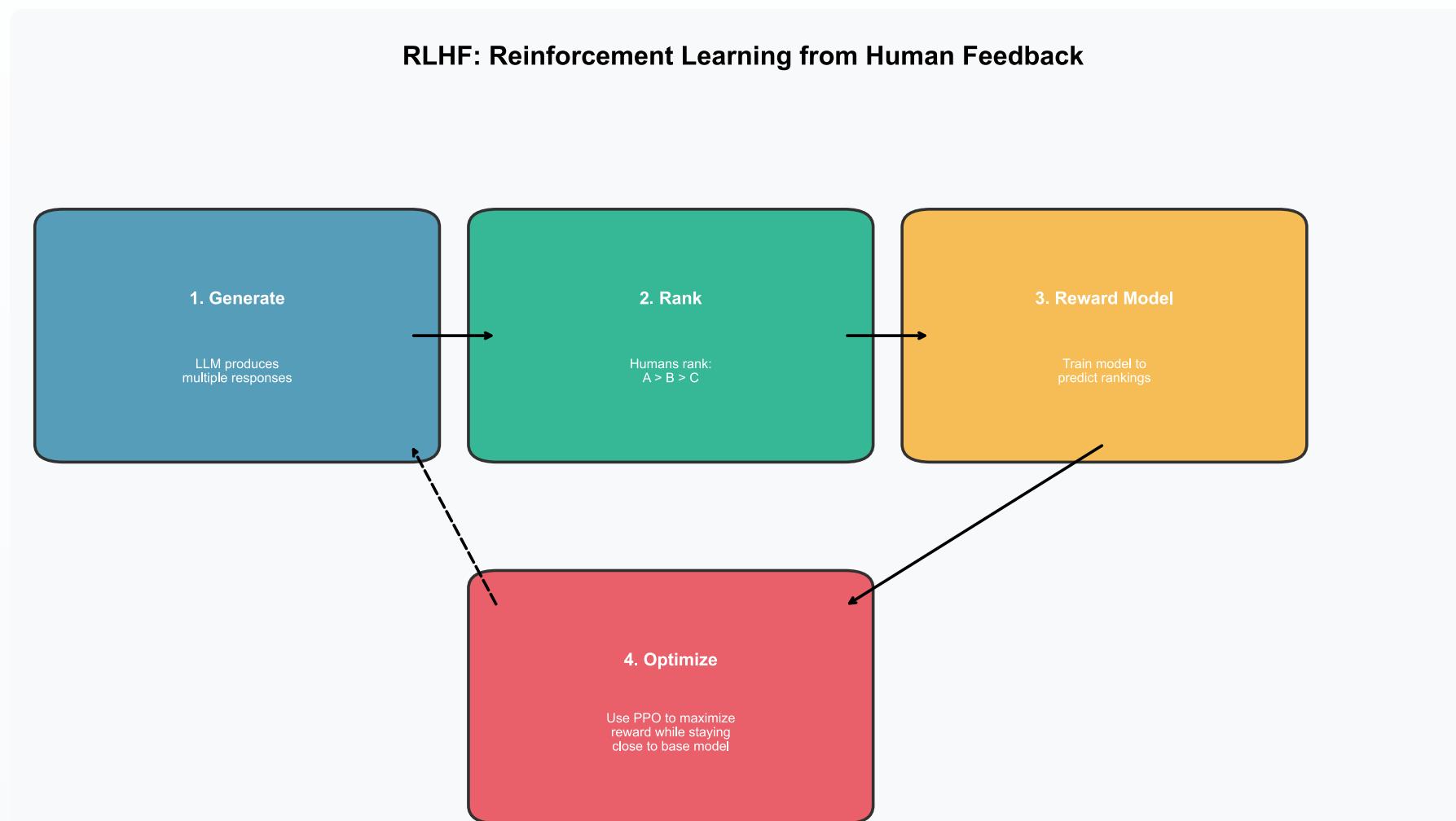
Aspect	Details
Data	Human-written (instruction, response) pairs (~100K)
Objective	Imitate high-quality responses
Compute	10s of GPU-hours
Result	Instruction-tuned - follows directions

Example training data:

- **User:** "Explain photosynthesis to a 5-year-old"
- **Assistant:** "Plants eat sunlight! They use it to make food from air and water..."

Stage 3: RLHF (Alignment)

Goal: Learn human values and preferences



RLHF Details

Why RLHF? SFT models can still be:

- Harmful (follow dangerous instructions)
- Dishonest (make up facts confidently)
- Unhelpful (technically correct but useless)

The Solution:

1. Generate multiple responses to each prompt
2. Have humans rank them (which is better?)
3. Train a reward model to predict human preferences
4. Use RL (PPO) to optimize the LLM for high reward

Result: ChatGPT = GPT + SFT + RLHF

Alternative: DPO (Direct Preference Optimization)

New approach (2023): Skip the reward model!

Method	Steps	Complexity
RLHF	Reward model + PPO	High
DPO	Direct optimization	Lower

DPO trains directly on preference data:

- Input: (prompt, chosen_response, rejected_response)
- Output: Model that prefers good responses

Used by: Llama 2, many open-source models

Summary: The Full Stack

Layer	Component	Purpose
0	The Task	Predict $P(\text{next} \mid \text{context})$
1	Representation	Tokens → Embeddings (meaning as vectors)
2	Context	Self-Attention (look at relevant past tokens)
3	Computation	Feed-Forward layers (process information)
4	Stacking	Repeat attention+FFN 96 times for depth
5	Training	Next token prediction on internet-scale data
6	Alignment	Instruction tuning + RLHF for helpfulness

Key Takeaways

The 5 Big Ideas

#	Idea	Key Insight
1	Prediction is All You Need	Just predicting the next token gives emergent abilities
2	Embeddings Capture Meaning	Similar words → Similar vectors
3	Attention Enables Long-Range Context	Every token can look at every other token
4	Scale Matters	Bigger models + more data = better capabilities
5	Alignment is Crucial	Raw prediction → helpful assistant through RLHF

Resources to Learn More

Videos:

1. [Andrej Karpathy](#) - "Neural Networks: Zero to Hero"

- Builds GPT from scratch

2. [3Blue1Brown](#) - "Attention in Transformers"

- Beautiful animations

Code & Blogs:

1. [NanoGPT](#) - Karpathy's GitHub

- Full GPT in ~300 lines

2. [Jay Alammar](#) - "The Illustrated Transformer"

- Best visualizations

3. [HuggingFace Course](#)

- Practical transformer tutorials

What's Next?

In the Labs:

- Lab 4: Build bigram & neural LM
- Lab 5: Deploy with Gradio
- Generate names, explore temperature

Beyond:

- Fine-tune a real LLM
- Build RAG applications
- Explore multimodal models

The same simple idea — predicting the next token — powers everything from autocomplete to ChatGPT to Claude. Now you understand how!

Thank You!

"The best way to predict the future is to create it."

The same simple idea — predicting the next token — powers everything from autocomplete to ChatGPT to Claude.

Questions?

