

Next Token Prediction

Building ChatGPT from Scratch (Conceptually)

Nipun Batra · IIT Gandhinagar

What We'll Learn Today

THE JOURNEY TO GPT

Level 1: The Intuition

└─ What does "predicting the next word" really mean?

Level 2: The Counting Era

└─ Bigrams: Count letter pairs

Level 3: Representing Meaning

└─ Embeddings: Words as vectors in space

Level 4: Learning Patterns

└─ Neural networks for next-token prediction

Level 5: The Context Problem

└─ Why we need to remember more

Level 6: The Revolution

└─ Attention and Transformers

Level 7: From Theory to ChatGPT

└─ Scaling up to billions of parameters

The Journey Ahead



Level 1: The Intuition

What Are We Really Doing?

The Core Problem

Every language model answers **one simple question**:

"Given what I have seen so far, what word comes next?"

The capital of France is ___



"Paris"

That's it. **Predict the next word. Repeat until done.**

You Already Know This!

You've been using next-word prediction your whole life:

Your Phone's Keyboard:

I'm running ___
[late] [out] [away]

Google Search:

how to make ___
• how to make money
• how to make pancakes
• how to make friends

Gmail Smart Compose:

Thanks for the ___
quick response! ← Suggested

Let's Play: The Autocomplete Game

Round 1: "The Eiffel Tower is located in ____"

Your brain: **Paris** (very confident!)

Round 2: "I want to eat ____"

Your brain: **pizza? pasta? nothing?** (uncertain!)

Round 3: "Once upon a ____"

Your brain: **time** (almost certain!)

Round 4: "To be or not to ____"

Your brain: **be** (Shakespeare hardcoded in culture!)

Your brain assigns **probabilities** to each possible next word. Some contexts have obvious answers, others don't!

The Mathematical View

When you read "The capital of France is ___", your brain computes:

```
P("Paris" | "The capital of France is") = 0.85   
P("the" | "The capital of France is") = 0.02   
P("London" | "The capital of France is") = 0.01   
P("beautiful" | "The capital of France is") = 0.01   
P("..." | "The capital of France is") = 0.11   
  
All probabilities sum to 1.0
```

This is called a PROBABILITY DISTRIBUTION over the vocabulary.
Language models learn to produce these distributions!

It's JUST Prediction

You might think ChatGPT "understands" physics or history.

But all it does is predict the next word.

Prompt: "F = m"
Prompt: "To be or not to"
Prompt: "E = mc"
Prompt: "print('Hello"
Prompt: "2 + 2 ="
Prompt: "The mitochondria is"

Prediction: "a"
Prediction: "be"
Prediction: "z"
Prediction: ")"
Prediction: "4"
Prediction: "the"

← Newton's Law!
← Shakespeare!
← Einstein!
← Python syntax!
← Math!
← Biology meme!

If you predict well enough, you **appear** to understand everything. The model has compressed patterns from human knowledge into its weights.

The Shocking Simplicity

THE ONE ALGORITHM

```
for token in generate(prompt):
    probabilities = model(all_tokens_so_far)
    next_token = sample(probabilities)
    output(next_token)
```

That's literally it. ChatGPT is this loop run millions of times with a really good model.

The Magic of "Just Prediction"

| Q: "What is $17 + 28$?"

| The model has seen THOUSANDS of math problems in training:

| "2 + 2 = 4"

| "15 + 10 = 25"

| "17 + 28 = 45" ← Saw this pattern!

| So when asked "17 + 28 =", it predicts "45"

| Not because it "knows" math, but because that pattern exists!

This is why LLMs can make math mistakes — they're pattern matching, not actually computing! Try asking "What is 4738×2951 ?" and you'll see errors.

Emergent Behaviors

As models get bigger, surprising abilities **emerge**:

WHAT EMERGES FROM PREDICTION

Small Model (100M parameters):

- Complete simple sentences
- Basic grammar

Medium Model (1B parameters):

- Answer factual questions
- Simple reasoning

Large Model (100B+ parameters):

- Complex reasoning
- Code generation
- Creative writing
- Multi-step problem solving
- "Understanding" context and nuance

All from the same objective: predict the next token!

Level 2: The Counting Era

Bigrams: The Simplest Language Model

The Simplest Possible Model

Idea: Just count what letter usually follows each letter.

Training data: Names like `aabid`, `zeel`, `priya`, `nipun`

Count transitions:

- After `a`: saw `a` (1 time), `b` (1 time)
- After `z`: saw `e` (1 time)
- After `e`: saw `e` (1 time), `l` (1 time)
- After `n`: saw `i` (1 time) in "nipun"

This is called a **Bigram** model (looks at pairs of 2 characters).

Let's Build It Step by Step

Training Data: "aabid", "priya", "zeel", "nipun"

Step 1: Add special tokens

".aabid.", ".priya.", ".zeel.", ".nipun."
(. marks beginning and end)

Step 2: Count all pairs

".a" appears 2 times (from aabid, priya doesn't start with 'a')
"aa" appears 1 time
"ab" appears 1 time
"bi" appears 1 time
"id" appears 1 time
"d." appears 1 time
... and so on

Step 3: Convert counts to probabilities

$P(\text{next} = 'a' \mid \text{current} = '.') = \text{Count}('.a') / \text{Total pairs starting with } .$
 $P(\text{next} = 'a' \mid \text{current} = '.') = 2 / 4 = 0.50$

Bigram: The Counting Table

		Next Character →					
		a	b	e	i	l	...
C u r r ↓	a	0.3	0.2	0.1	0.2	0.1	...
	b	0.1	0.0	0.1	0.5	0.0	...
	e	0.2	0.0	0.3	0.1	0.2	...
	i	0.4	0.1	0.1	0.0	0.1	...
	↓

(probabilities)

Each row sums to 1.0 (it's a probability distribution!)

To generate: Look up current letter → Sample from that row

This table IS the model. No neural network needed!

Generating Names with Bigrams

Step 1: Start with "." (beginning token)

Look up row "." → High prob for 'a', 's', 'm'

Sample → Got 'a'

Step 2: Current = 'a'

Look up row "a" → Moderate prob for 'a', 'b', 'n'

Sample → Got 'b'

Step 3: Current = 'b'

Look up row "b" → High prob for 'i', 'a', 'r'

Sample → Got 'i'

Step 4: Current = 'i'

Look up row "i" → High prob for 'd', 'n', 'a'

Sample → Got 'd'

Step 5: Current = 'd'

Look up row "d" → High prob for "." (end token)

Sample → Got "." (DONE!)

Result: "abid" ← Looks like a real name!

Interactive Example: Generating from Bigrams

BIGRAM GENERATION DEMO

```
Current: .
Options: a(0.25), m(0.20), s(0.15), j(0.10), r(0.08), ...
Rolled: 0.18 → Selected 'm'
```

```
Current: m
Options: a(0.40), i(0.25), o(0.15), u(0.10), ...
Rolled: 0.32 → Selected 'a'
```

```
Current: a
Options: n(0.20), r(0.18), l(0.15), y(0.12), .(0.10), ...
Rolled: 0.45 → Selected 'r'
```

```
Current: r
Options: i(0.25), a(0.20), y(0.18), .(0.15), ...
Rolled: 0.52 → Selected 'y'
```

```
Current: y
Options: .(0.45), a(0.15), i(0.10), ...
Rolled: 0.30 → Selected .'
```

```
Generated: "mary" ← A real name!
```

Why Bigrams Fail: The Memory Problem

The Problem:

Sentence: "The quick brown
fox jumps over
the lazy dog."

Question: After "dog",
what comes next?

Bigram sees: "dog" → ?
(forgot everything
before "dog"!)

Context is Lost:

With context:
"The cat sat on the ___"
→ Probably "mat"

Without context:
"the ___"
→ Could be anything!

Bigram only sees 1 char!

Bigrams have **no memory**. They forget everything except the last character!

A Concrete Example

THE CONTEXT PROBLEM

Sentence 1: "I love eating pizza with extra cheese"

Sentence 2: "I love eating pizza with my friends"

After "with", what comes next?

Bigram's view: "h" → ?

(It doesn't even know it's in "with"!)

A smarter model would know:

- "pizza with" usually followed by toppings or people
- "eating with" suggests companions
- "love eating" suggests food context

We need to see MORE context!

The Curse of Dimensionality

Why not just count longer patterns?

1-gram (Unigram):	27 entries	← Fits in memory
2-gram (Bigram):	$27^2 = 729$	← Still fine
3-gram (Trigram):	$27^3 = 19,683$	← OK
4-gram:	$27^4 = 531,441$	← Getting big
5-gram:	$27^5 = 14,348,907$	← Very big
10-gram:	$27^{10} \approx 205$ TRILLION	← Impossible!

For words (50,000 vocabulary):

2-gram:	$50,000^2 = 2.5$ BILLION
3-gram:	$50,000^3 = 125$ TRILLION

We can't just count longer patterns — we need to **generalize**. This is where neural networks come in!

Bigrams: Summary

Aspect	Bigrams
What it does	Counts $P(\text{next char} \mid \text{current char})$
Memory	1 character only
Size	$27 \times 27 = 729$ numbers
Speed	Instant (just table lookup)
Quality	Poor (no context)
Training	Just counting

Key insight: The model is just a lookup table. No learning, no generalization.

Level 3: Representing Meaning

Embeddings: Words as Vectors

How Do Computers Read?

Computers only understand numbers. How do we convert letters?

Option A: One-Hot Encoding

```
'a' = [1, 0, 0, 0, ..., 0]      (27 dimensions for letters)  
'b' = [0, 1, 0, 0, ..., 0]  
'c' = [0, 0, 1, 0, ..., 0]  
...  
'z' = [0, 0, 0, 0, ..., 1]
```

Problem: These vectors are **orthogonal** (dot product = 0).

The computer thinks 'a' and 'b' are completely unrelated!

The Problem with One-Hot

Distance between letters:

'a' ● 'b' ●

$$\text{Distance}(a, b) = \text{Distance}(a, z) = \sqrt{2}$$

Every letter is equally far from every other letter!

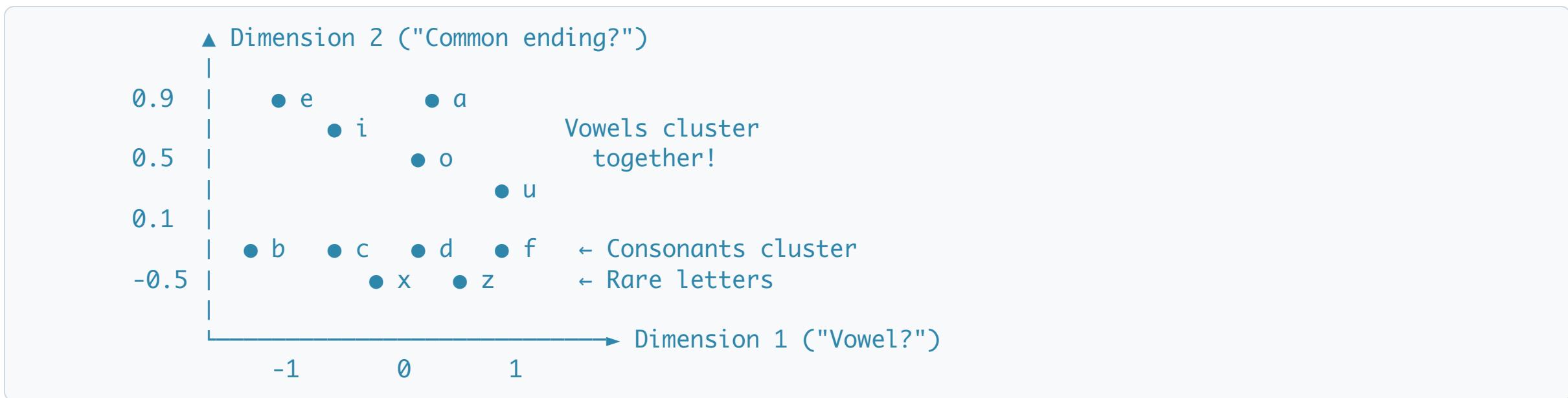
But we KNOW:

- 'a' and 'e' are both vowels (similar!)
- 'a' and 'x' have nothing in common (different!)
- 'p' and 'b' look similar (related!)

We need a smarter representation where **similar things are close**.

Dense Embeddings: Meaning as Coordinates

Idea: Represent each character as a point in space where **similar things are close**.



Now **a** and **e** are **mathematically close!**

Word Embeddings: The Famous Example



The King - Man + Woman = Queen Example

WORD ARITHMETIC

```
king  = [0.8, 0.3, 0.9, ...] (royalty, male, power)
man   = [0.1, 0.3, 0.5, ...] (person, male, average)
woman = [0.1, 0.9, 0.5, ...] (person, female, average)
```

```
king - man + woman = ?
```

```
[0.8, 0.3, 0.9] - [0.1, 0.3, 0.5] + [0.1, 0.9, 0.5]
= [0.8, 0.9, 0.9]
```

```
Nearest word to [0.8, 0.9, 0.9]: "queen"!
```

The model learned that:

"The relationship between king and man is the same as
the relationship between queen and woman"

More Word Analogies

EMBEDDING ANALOGIES

France : Paris :: Japan : ?
→ Tokyo

good : better :: bad : ?
→ worse

walking : walked :: swimming : ?
→ swam

Einstein : physicist :: Picasso : ?
→ painter

The embeddings capture RELATIONSHIPS automatically!
No one told the model about capitals or verb tenses!

How Embeddings Are Learned

Start: Random vectors for each word

Training:

"The cat sat on the mat"

"cat" often appears near "sat", "dog", "pet"

→ Push these embeddings closer together

"cat" rarely appears near "quantum", "fiscal"

→ Push these embeddings apart

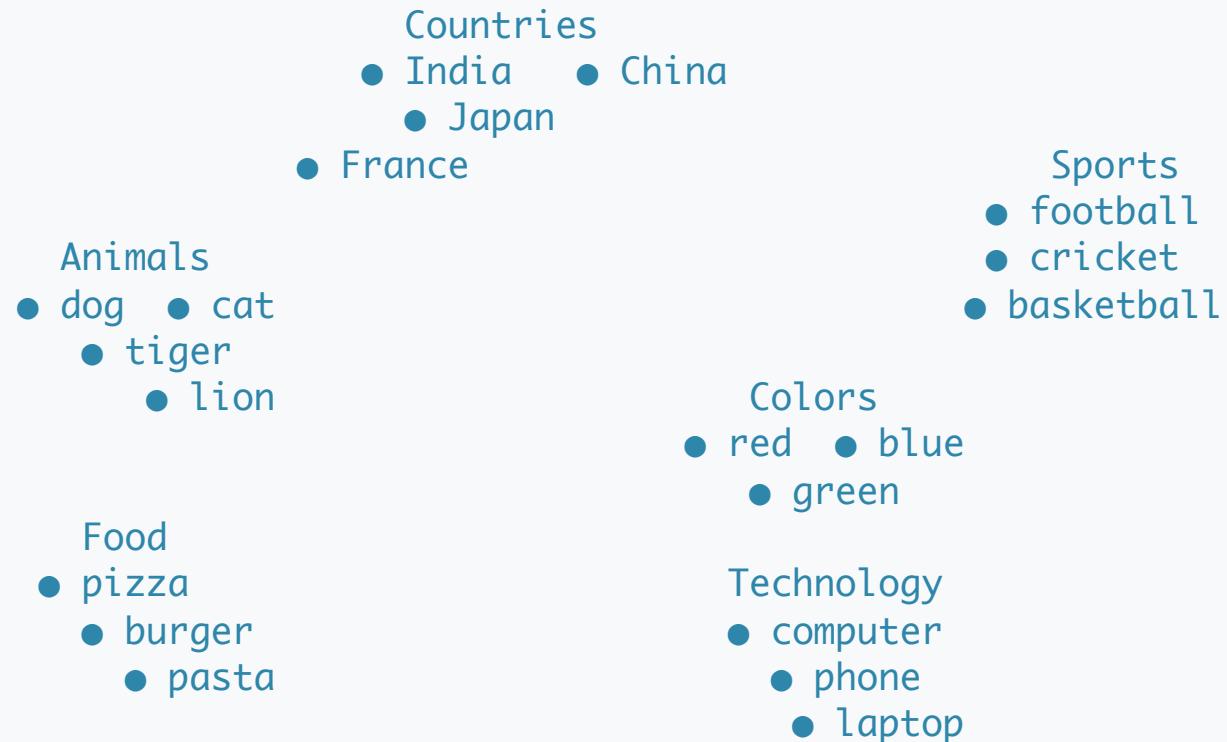
After billions of examples:

Similar words → Similar vectors

Related concepts → Close in space

Visualizing Embeddings

Real word embeddings projected to 2D (using t-SNE):



Similar things automatically cluster together!

Embedding Dimensions

WHAT DO DIMENSIONS MEAN?

We use 256-4096 dimensions in practice, but imagine 4:

Dimension 1: "Is it alive?"

Dimension 2: "Is it a person?"

Dimension 3: "Is it concrete vs abstract?"

Dimension 4: "Positive or negative sentiment?"

"dog" = [0.9, 0.1, 0.8, 0.7] (alive, not person, concrete, +)

"cat" = [0.9, 0.1, 0.8, 0.6] (very similar to dog!)

"love" = [0.2, 0.3, -0.8, 0.9] (abstract, positive)

"hate" = [0.2, 0.3, -0.8, -0.9] (abstract, negative)

"table" = [0.0, 0.0, 0.9, 0.0] (not alive, concrete, neutral)

In reality, dimensions are learned and not so interpretable!

Level 4: Learning Patterns

Neural Networks for Next-Token Prediction

From Counting to Learning

BIGRAM (Counting):

Count(b|a)

Count(a)

(Fixed table)

NEURAL NETWORK (Learning):

$f(\text{embed}(a); \theta)$

θ = learned
weights
(Flexible function)

Only memorizes
exact patterns

Can GENERALIZE to
unseen patterns!

The Neural Network Architecture



The Architecture Unpacked

MLP LANGUAGE MODEL

Input: Last 3 characters [a, a, b]

Step 1: EMBED each character

$$a \rightarrow [0.2, 0.5]$$

$$a \rightarrow [0.2, 0.5]$$

$$b \rightarrow [0.8, -0.3]$$

Step 2: CONCATENATE embeddings

[0.2, 0.5, 0.2, 0.5, 0.8, -0.3] (6 numbers)

Step 3: HIDDEN LAYER (learn patterns)

$$h = \text{ReLU}(W_1 \cdot \text{concat} + b_1)$$

$$h = [0.7, 0.1, 0.9, 0.3] \text{ (example)}$$

Step 4: OUTPUT LAYER (predict next char)

$$\text{logits} = W_2 \cdot h + b_2$$

$$\text{probs} = \text{softmax}(\text{logits})$$

$$[P(a)=0.1, P(b)=0.05, \dots, P(i)=0.6, \dots]$$

Creating Training Data: The Sliding Window

Text: "aabid"

Create (context → target) pairs by sliding a window:

Position 0:	[., ., .] → 'a'	"What comes first?"
Position 1:	[., ., a] → 'a'	"After nothing+a?"
Position 2:	[., a, a] → 'b'	"After a, a?"
Position 3:	[a, a, b] → 'i'	"After a, a, b?"
Position 4:	[a, b, i] → 'd'	"After a, b, i?"
Position 5:	[b, i, d] → .'	"After b, i, d?"



→ Target: 'b' (what we want to predict)

Training: Learning from Mistakes

Step 1: Forward Pass

```
| Input: [a, a, b]  
| ↓  
| Network predicts:  
| P(i)=0.10 ← Wrong!  
| P(z)=0.30 ← Very wrong  
| P(a)=0.20
```

Actual answer: 'i'

Step 2: Compute Loss

$\text{Loss} = -\log(P(\text{correct answer}))$
 $\text{Loss} = -\log(0.10) = 2.3 \leftarrow \text{High loss} = \text{bad prediction}$

Step 3: Backpropagation

Compute gradients: "How should each weight change?"
Adjust weights to make $P(i)$ higher next time

Step 4: Repeat millions of times

→ Network learns to predict well!

The Loss Function: Cross-Entropy

CROSS-ENTROPY LOSS

$\text{Loss} = -\log(P(\text{correct_answer}))$

Examples:

If model is confident and RIGHT:

$P(\text{correct}) = 0.95 \rightarrow \text{Loss} = -\log(0.95) = 0.05 \leftarrow \text{Low loss!}$

If model is uncertain:

$P(\text{correct}) = 0.50 \rightarrow \text{Loss} = -\log(0.50) = 0.69 \leftarrow \text{Medium loss}$

If model is confident and WRONG:

$P(\text{correct}) = 0.01 \rightarrow \text{Loss} = -\log(0.01) = 4.6 \leftarrow \text{High loss!}$

The model gets heavily penalized for confident wrong answers!

Gradient Descent: Finding the Best Weights



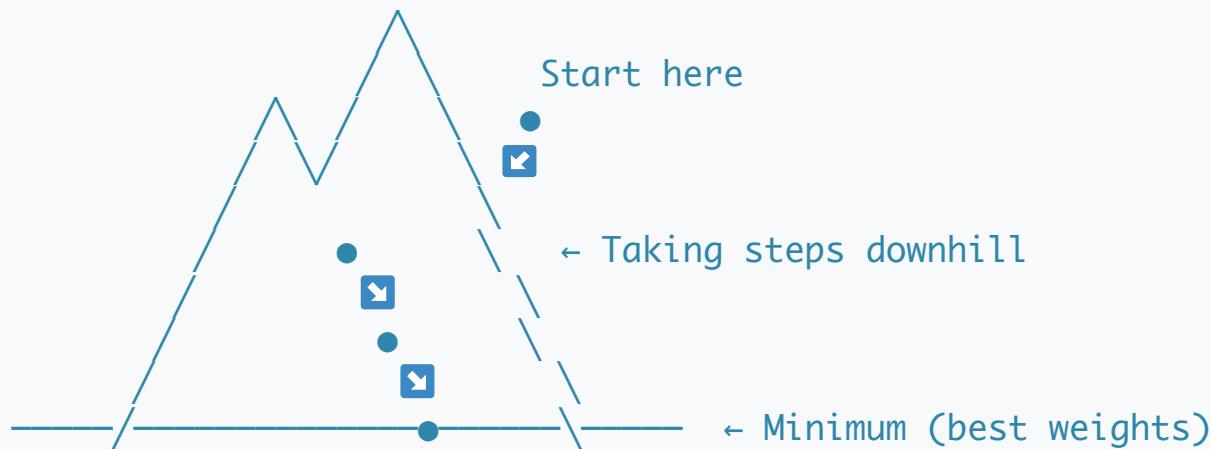
Gradient Descent Intuition

FINDING THE VALLEY

Imagine you're blindfolded on a mountain. Goal: reach the lowest point (minimum loss).

Strategy:

1. Feel the ground slope with your feet (compute gradient)
2. Take a step downhill (update weights)
3. Repeat until you can't go lower



Level 5: The Context Problem

Why Fixed Windows Aren't Enough

The Fatal Flaw

Our neural network has a **fixed context window** (e.g., 3 characters).

"Alice picked up the golden key. She walked to the door
and tried to open it with the ___"

What the human sees: "golden key" (earlier in story)
What the model sees: "with the" (only last 3 words!)

Model's window:

| ... with the ___



"key" is outside the window! The model forgot it!

Why Is This a Problem?

CONTEXT EXAMPLES

Example 1: Pronouns

"John gave Mary a book. She thanked ___"

Answer: "him" (need to remember "John")

Example 2: Callbacks

"In Chapter 1, we introduced X. Now, let's explore ___ further."

Answer: "X" (need to remember earlier topic)

Example 3: Long dependencies

"The cat, which was sleeping on the mat that grandmother
made last winter, suddenly ___"

Answer: verb about "cat" (need to skip 15 words!)

Example 4: Instructions

"I want you to translate to French: Hello"

Need to remember "translate to French" while processing "Hello"

Attempted Solution: RNNs

Recurrent Neural Networks maintain a "memory" that carries forward.



How RNNs Work

RNN: PASSING THE BATON

Processing: "I love pizza"

Step 1: "I"

$h_1 = f("I", h_0)$

h_1 encodes: "Someone is speaking"

Step 2: "love"

$h_2 = f("love", h_1)$

h_2 encodes: "Someone loves something"

Step 3: "pizza"

$h_3 = f("pizza", h_2)$

h_3 encodes: "Someone loves pizza"

The hidden state h carries information forward!

Why RNNs Still Fail

The Telephone Game Problem:

Start: "Alice has a key"

|

▼ (pass through 50 words)

|

▼ (memory gets compressed, some info lost)

|

▼ (pass through 50 more words)

|

▼ (even more degraded)

|

End: "She opened the door with the ___"

By now, "key" has been corrupted or forgotten!

This is the "Vanishing Gradient" problem:

Gradients become tiny → Old info can't influence predictions

The Vanishing Gradient Problem

SIGNAL DEGRADATION

Original message: "THE CAT HAS A KEY"

After 10 steps:	"The cat has a key"	(still clear)
After 50 steps:	"Something about a cat"	(getting fuzzy)
After 100 steps:	"Animal? Object?"	(very unclear)
After 200 steps:	"...???"	(information lost)

Like whispering a message through 100 people:

"The cat has a key" → "The hat has a tree" → "???"

Mathematical cause:

$$\text{gradient} = \text{gradient} \times \text{weight} \times \text{weight} \times \dots \times \text{weight}$$

If weight < 1: gradient $\rightarrow 0$ (vanishes)

If weight > 1: gradient $\rightarrow \infty$ (explodes)

LSTM: A Better RNN

Long Short-Term Memory cells have "gates" that control information flow.

LSTM: THE MEMORY CELL

Three gates control information:

FORGET GATE: "Should I forget old stuff?"

Example: When starting a new sentence, forget the old one

INPUT GATE: "Should I remember this new thing?"

Example: "key" is important, remember it!

OUTPUT GATE: "What should I output now?"

Example: Output information relevant to current prediction

Result: Information can "skip" through time without degradation!

But LSTMs are still slow (sequential) and still struggle with very long contexts.

Level 6: The Revolution

Attention: "Just Look Back!"

The Brilliant Idea

What if, instead of compressing everything into a hidden state...

We could just **look back** at everything directly?

"Alice picked up the golden key. She walked to the door
and tried to open it with the ___"

Fixed Window: Can only see "with the"
RNN: Remembers a blurry summary
ATTENTION: Can look at ANY word!

↑
└ "Let me check... 'key' was mentioned!"

Attention: The Searchlight Analogy

Reading Methods Compared:

FIXED WINDOW (MLP):

Reading with tunnel vision



← Can only see this tiny part

RNN:

Reading while trying to remember everything

"I think there was a key... or was it a lock?"

ATTENTION:

Reading with a highlighter and search engine!

"Let me search for 'object that opens doors'..."

Found: "key" at position 7!

— Spotlight on relevant words!

How Attention Works: The Library Analogy

THE LIBRARY OF WORDS

You're at the library, looking for information.

YOUR QUESTION (Query): "What opens doors?"

BOOK LABELS (Keys): BOOK CONTENTS (Values):

- └─ "key"
- └─ "door"
- └─ "Alice"
- └─ "walked"

- └─ Info about keys
- └─ Info about doors
- └─ Info about Alice
- └─ Info about walking

You compare your Query to each Key:

- Query • "key" = HIGH match! (0.8)
- Query • "door" = Medium match (0.5)
- Query • "Alice" = Low match (0.1)
- Query • "walked" = Low match (0.1)

You read mostly from "key" book, a little from "door" book.

How Attention Works: Q, K, V

Think of it like a **database lookup**:



The Math of Attention

ATTENTION COMPUTATION

Query (Q): What am I looking for? (from current position)

Key (K): What does each position offer? (from all positions)

Value (V): What is the content? (from all positions)

Step 1: Compute similarity scores

$$\text{scores} = Q \cdot K^T$$

Step 2: Normalize with softmax

$$\text{attention_weights} = \text{softmax}(\text{scores} / \sqrt{d})$$

Step 3: Weighted sum of values

$$\text{output} = \text{attention_weights} \cdot V$$

$$\text{Formula: } \text{Attention}(Q, K, V) = \text{softmax}(QK^T / \sqrt{d}) \cdot V$$

Attention Scores Visualized

Predicting: "The animal didn't cross the street because it was too ___"

"it" refers to what?

Attention Scores (what "it" is looking at):

The animal didn't cross the street because it
0.02 0.75 0.01 0.02 0.01 0.15 0.02 ---
^^^^^ ^^^^^
High attention! Some attention

The model figures out "it" = "animal" (not "street")!

This is learned automatically from data.

Why "Wide" Beats "Narrow"

COMPARISON

Predicting the word after "The quick brown fox jumps over
the lazy dog and then runs to the ___"

Bigram: sees only "the" → could predict anything!

3-char window: sees "the " → still very ambiguous

RNN: has a blurry memory of earlier words
"something about a fox... and running?"

Attention: can directly look at "fox", "runs", "dog"
Weights: fox(0.3), runs(0.2), lazy(0.1), ...
"The fox is running, so it might go to a... den? forest?"

Self-Attention: Every Word Looks at Every Word

Sentence: "The cat sat on the mat"

	The	cat	sat	on	the	mat
The	0.5	0.2	0.1	0.05	0.1	0.05
cat	0.1	0.5	0.2	0.05	0.1	0.05
sat	0.1	0.3	0.3	0.1	0.1	0.1
on	0.1	0.1	0.2	0.3	0.2	0.1
the	0.2	0.1	0.1	0.1	0.3	0.2
mat	0.1	0.2	0.1	0.05	0.2	0.35

Each row: Where does this word look for context?

Each row sums to 1.0 (softmax)

Computed in parallel (not sequential like RNN)!

Multi-Head Attention

Idea: Different "heads" can look for different things.

MULTI-HEAD ATTENTION

Head 1: "What is the subject?"
"The cat sat on the mat"
 ^
 ^
 ^

Head 2: "What is the action?"
"The cat sat on the mat"
 ^
 ^
 ^

Head 3: "What is the location?"
"The cat sat on the mat"
 ^
 ^
 ^

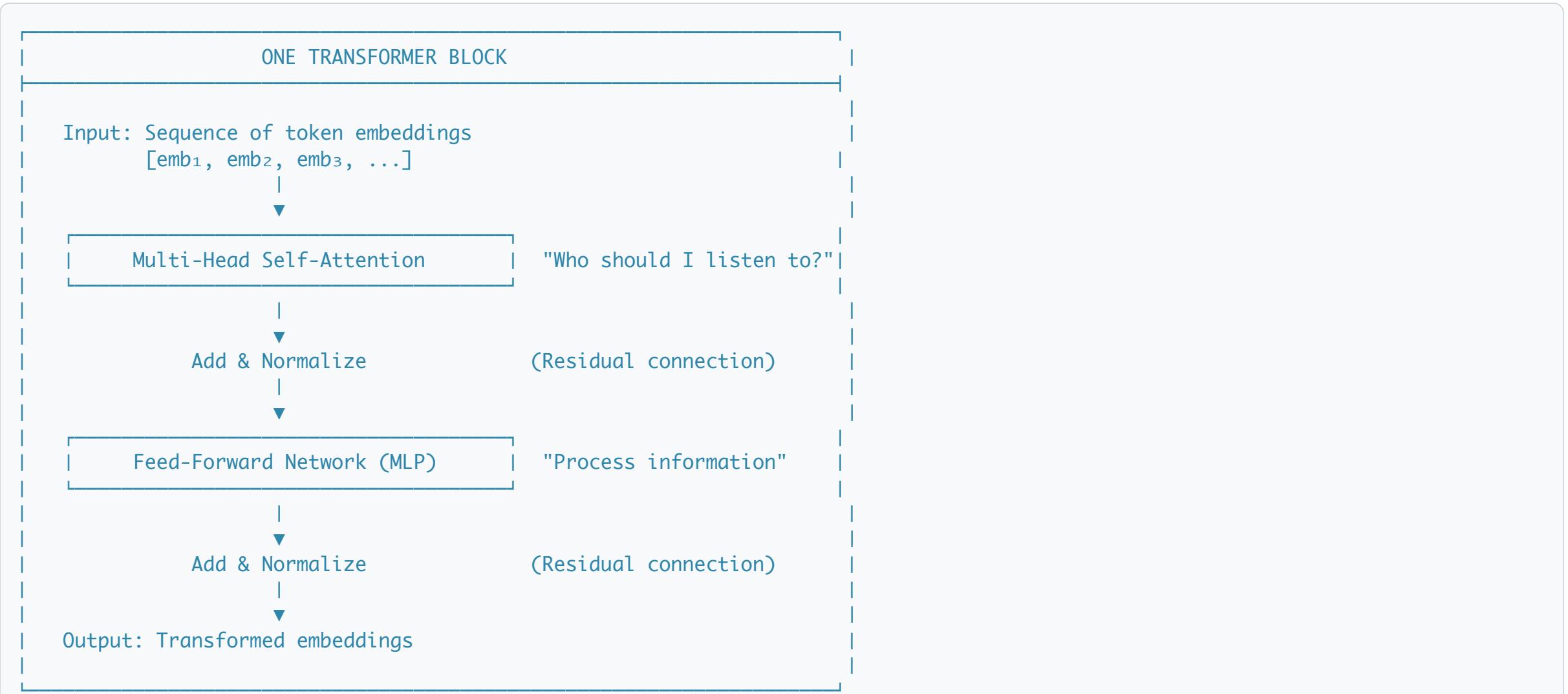
Head 4: "What came before?"
Looks at previous sentence...

Combine all heads → Rich understanding!

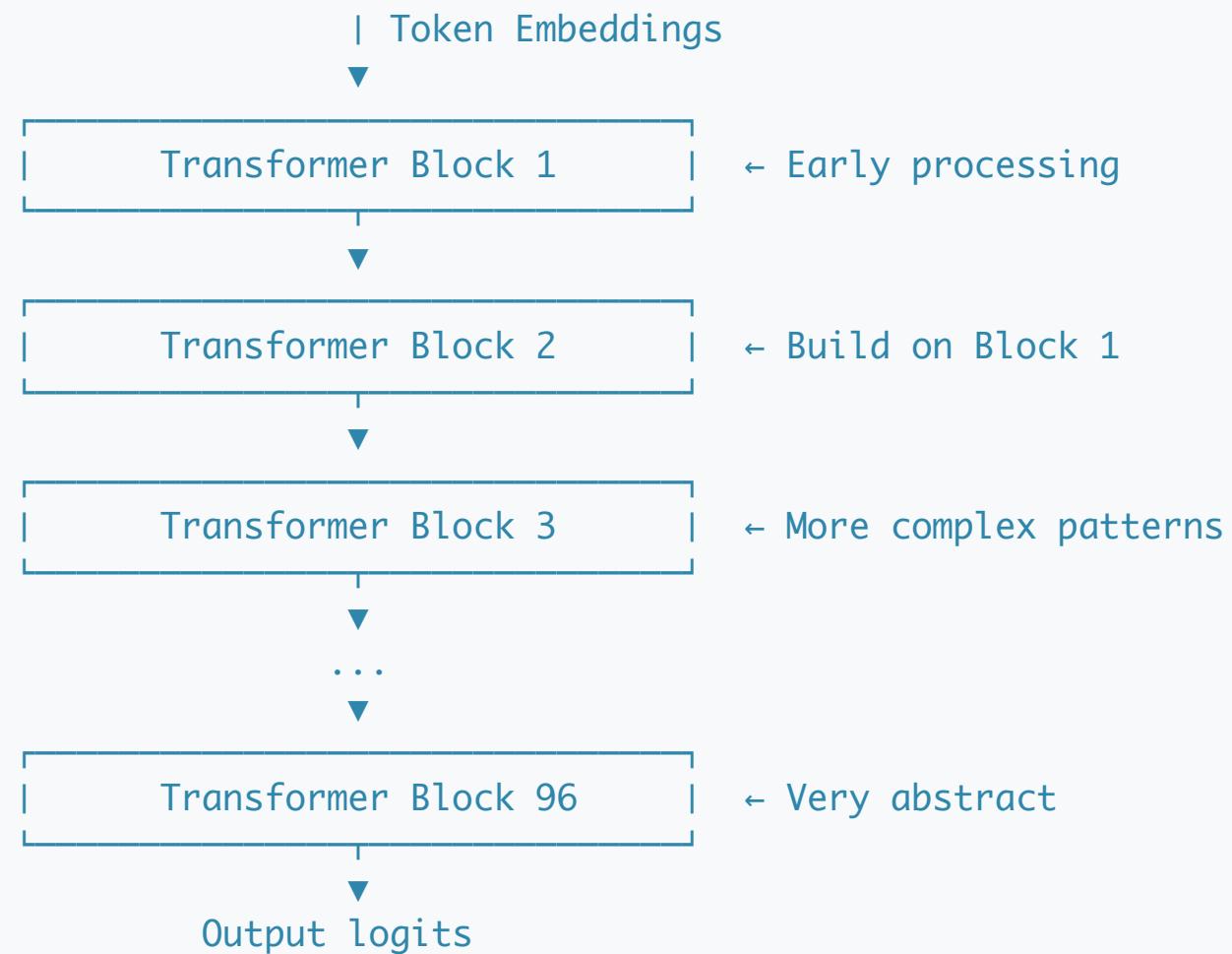
The Transformer Architecture



The Transformer Block



Stacking Transformer Blocks



GPT-4 has ~120 layers! Each layer refines understanding.

Level 7: From Theory to ChatGPT

Scaling Up

Our Toy Model vs ChatGPT

Feature	Our Toy Model	ChatGPT
Vocabulary	27 (letters)	100,000 (tokens)
Embedding Size	2 dimensions	12,288 dims
Layers	1 layer	96 layers
Attention Heads	1 head	96 heads
Parameters	~1,000	175 BILLION
Training Data	1,000 names	500B+ tokens
Context Window	3 chars	128K tokens
Training Time	1 minute	Months on 1000s of GPUs

Same core algorithm. Just **much, much bigger.**

Tokenization: Not Characters, Not Words

LLMs use "TOKENS" – subword units (BPE algorithm):

"unhappiness" → ["un", "happiness"] or ["un", "hap", "piness"]

Why?

- Characters: Too slow (many steps to generate a word)
- Words: Too many unique words (millions!)
- Tokens: Best of both worlds (~50,000-100,000 tokens)

Example:

```
| Text: "ChatGPT is amazing!"  
|  
| Tokens: ["Chat", "G", "PT", "is", "amazing", "!"]  
| [15496, 38, 2898, 318, 4998, 0] ← Token IDs
```

Note: Spaces are often part of tokens (" is" not "is")

How BPE Tokenization Works

BYTE PAIR ENCODING (BPE)

Start with character vocabulary: {a, b, c, ..., z, space, ...}

Algorithm:

1. Count all adjacent pairs in corpus
2. Most common pair: "th" (appears 10,000 times)
3. Merge: create new token "th"
4. Repeat until vocabulary reaches target size

After many merges:

"the" → single token
"ing" → single token
"tion" → single token
"unhappiness" → ["un", "happiness"]

Common words = 1 token, rare words = multiple tokens

Tokenization Quirks

TOKENIZATION FUN FACTS

Why LLMs struggle with:

1. Counting letters:

"strawberry" → ["str", "aw", "berry"] ← 'r' split across!
Model doesn't "see" individual letters easily

2. Non-English languages:

"Hello" → 1 token
"नमस्ते" (Hindi) → 6 tokens ← Same meaning, 6x tokens!

3. Numbers:

"1234" might be 1 token
"12345" might be 2 tokens ["1234", "5"]
Math becomes harder!

4. Code:

" " (4 spaces) might be 1 token
" " (3 spaces) might be 3 tokens

Positional Encoding

How does the model know word ORDER?

POSITIONAL ENCODING

Problem: Attention is permutation-invariant!

"Dog bites man" and "Man bites dog" look the same to attention.

Solution: Add position information to each embedding.

`token_embedding("cat") = [0.5, 0.3, 0.8, ...]`

`position_encoding(pos=3) = [0.1, -0.2, 0.4, ...]`

`final_embedding = [0.6, 0.1, 1.2, ...]`

Now "cat" at position 3 is different from "cat" at position 10!

Original paper uses sinusoidal patterns:

$\text{PE}(\text{pos}, 2i) = \sin(\text{pos} / 10000^{(2i/d)})$

$\text{PE}(\text{pos}, 2i+1) = \cos(\text{pos} / 10000^{(2i/d)})$

Modern models: Learn position embeddings!

Temperature: The Creativity Knob

When sampling the next token, we can adjust **temperature**:

Probabilities for next word after "I love to eat":

	Low Temp (0.1) (Conservative)	High Temp (2.0) (Creative)
pizza	0.80	0.25
pasta	0.15	0.20
shoes	0.01	0.15
clouds	0.001	0.12
dreams	0.0001	0.10

Low temp → Always picks "pizza" (boring but safe)

High temp → Might pick "clouds" (creative but weird)

Temperature: The Math

HOW TEMPERATURE WORKS

`softmax(logits / temperature)`

Example logits: [2.0, 1.0, 0.5, 0.1]

Temperature = 1.0 (normal):

probs = [0.43, 0.26, 0.19, 0.12]

Temperature = 0.1 (cold/greedy):

probs = [0.99, 0.01, 0.00, 0.00] ← Almost deterministic

Temperature = 2.0 (hot/random):

probs = [0.32, 0.27, 0.22, 0.19] ← More uniform

Temperature = 0 (greedy):

Always pick highest probability (argmax)

Top-k and Top-p Sampling

OTHER SAMPLING METHODS

TOP-K SAMPLING:

Only consider the top K most likely tokens.

All tokens: pizza(0.4), pasta(0.3), shoes(0.1), clouds(0.05)...

Top-3: pizza(0.53), pasta(0.40), shoes(0.07)

Problem: K is fixed. Sometimes 3 options make sense, sometimes 10.!

TOP-P (NUCLEUS) SAMPLING:

Include tokens until cumulative probability > P.

P = 0.9:

Include pizza(0.4) + pasta(0.3) + shoes(0.1) = 0.8 < 0.9

Include clouds(0.05) → 0.85 < 0.9

Include dreams(0.04) → 0.89 < 0.9

Include hope(0.02) → 0.91 > 0.9 → Stop!

Adaptive: Narrow when confident, wide when uncertain!

The Sampling Tree

Because we sample probabilistically, each generation is different!



Training at Scale

GPT-3 TRAINING

Data:

- Common Crawl (web pages): 410B tokens
- Books: 67B tokens
- Wikipedia: 3B tokens
- Total: ~500B tokens

Compute:

- 10,000 GPUs
- Training time: ~1 month
- Cost: ~\$4.6 million (just electricity!)

Model:

- 175 billion parameters
- 96 layers
- 96 attention heads
- 12,288 embedding dimensions

The Complete Recipe



From GPT to ChatGPT

PRE-TRAINING → FINE-TUNING

Step 1: PRE-TRAINING (GPT)

- Train on internet text (next token prediction)
- Model learns language patterns, facts, reasoning
- But it's just an autocomplete engine!

Step 2: SUPERVISED FINE-TUNING (SFT)

- Train on (instruction, response) pairs
- Humans write example responses
- Model learns to follow instructions

Step 3: RLHF (Reinforcement Learning from Human Feedback)

- Humans rank model responses
- Train a reward model on these rankings
- Use RL to optimize for human preferences
- Makes responses helpful, harmless, honest

Result: ChatGPT = GPT + SFT + RLHF

Summary: The Full Stack

Layer 0: THE TASK

Predict $P(\text{next_token} \mid \text{all_previous_tokens})$

Layer 1: REPRESENTATION

Tokens \rightarrow Embeddings (meaning as vectors)

Layer 2: CONTEXT

Self-Attention (look at relevant past tokens)

Layer 3: COMPUTATION

Feed-Forward layers (process information)

Layer 4: STACKING

Repeat attention+FFN 96 times for depth

Layer 5: TRAINING

Next token prediction on internet-scale data

Layer 6: ALIGNMENT

Instruction tuning + RLHF for helpfulness

Key Takeaways

THE 5 BIG IDEAS

1. PREDICTION IS ALL YOU NEED
Just predicting the next token gives emergent abilities
2. EMBEDDINGS CAPTURE MEANING
Similar words → Similar vectors
3. ATTENTION ENABLES LONG-RANGE CONTEXT
Every token can look at every other token
4. SCALE MATTERS
Bigger models + more data = better capabilities
5. ALIGNMENT IS CRUCIAL
Raw prediction → helpful assistant through RLHF

Resources to Learn More

Videos:

1. **Andrej Karpathy** - "Neural Networks: Zero to Hero"
 - Builds GPT from scratch
2. **3Blue1Brown** - "Attention in Transformers"
 - Beautiful animations

Code & Blogs:

1. **NanoGPT** - Karpathy's GitHub
 - Full GPT in ~300 lines
2. **Jay Alammar** - "The Illustrated Transformer"
 - Best visualizations
3. **HuggingFace Course**
 - Practical transformer tutorials

What's Next?

In the Labs:

- Lab 4: Build bigram & neural LM
- Lab 5: Deploy with Gradio
- Generate names, explore temperature

Beyond:

- Fine-tune a real LLM
- Build RAG applications
- Explore multimodal models

The same simple idea — predicting the next token — powers everything from autocomplete to ChatGPT to Claude.
Now you understand how!

Thank You!

"The best way to predict the future is to create it."

The same simple idea — predicting the next token — powers everything from autocomplete to ChatGPT to Claude.

Questions?

