Neural Networks Foundation

From Perceptrons to Deep Learning

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Learning Goals

By the end of this lecture, you will:

Goal	What You'll Learn
Understand	Why we need neural networks
Build	Neurons, layers, and architectures
Choose	Activation functions (ReLU, sigmoid)
Train	Gradient descent and backpropagation
Implement	PyTorch fundamentals

The Journey So Far

Week	Model	What It Does	Limitation
3	Linear Regression	Fit a line	Only linear patterns
3	Logistic Regression	Binary classification	Only linear boundaries
3	Decision Trees	If-then rules	Can overfit, unstable
4	Random Forest	Many trees	Hard to interpret
5	Neural Networks	Learn any pattern!	Need lots of data

The Big Picture

```
Traditional ML:                Deep Learning:

Features ──→ Model ──→ Output   Raw Data ──→ [Learn Features] ──→ Output
    │                                               │
    │                                        Neural Network
 You design                                learns automatically!
 features
```

Neural networks learn **both** features **and** the model!

Part 1: Why Neural Networks?

The Limits of Linear Models

A Problem Linear Models Can't Solve

The XOR Function:

Input A	Input B	Output (A XOR B)
0	0	0
0	1	1
1	0	1
1	1	0

"Output is 1 if inputs are different"

XOR: Visualized

```
    Input B
       │
     1 ├──○──────────●──    ○ = class 0 (same)
       │                     ● = class 1 (different)
       │
     0 ├──●──────────○──
       │
       └──┼──────────┼───
    ▶️
     Input A
          0          1
```

Can you draw ONE line to separate ● from ○?

No Line Works!

```
         Try 1:              Try 2:              Try 3:
      │  ○    ●        │  ○    ●        │  ○    ●
      │      /         │ \              │    │
      │    /           │   \            │    │
      │  /             │     \          │    │
      ├──●────○──      ├──●────○──      ├──●────○──


❌
 Fails
❌
 Fails
❌
 Fails
```

XOR is NOT linearly separable!

The Historical Impact

1969: Minsky & Papert proved perceptrons can't solve XOR.

Before 1969	After 1969
"Neural networks will solve everything!"	"Neural networks are useless"
Lots of funding	"AI Winter" - funding cut
Active research	Field nearly died

Solution discovered: Multi-layer networks! (But took until 1986)

The Solution: Multiple Layers

Key insight: Transform the space so XOR becomes separable!

```
Original Space:                After Hidden Layer:
    │  ○    ●                      │  ●    ●
    │                              │      /
    │                 TRANSFORM    │    /
    │                   →          │  /
    ├──●────○──                    ├──○────○──

    Can't separate                    Now we CAN draw a line!
```

Inspiration from Biology

The brain has ~86 billion neurons, each connected to ~10,000 others.

Biological Neuron	Artificial Neuron
Receives signals via dendrites	Receives inputs (x_1, x_2, ...)
Processes in cell body	Computes weighted sum
Fires if threshold exceeded	Applies activation function
Sends signal via axon	Produces output
Connections strengthen with use	Weights updated during training

The Artificial Neuron

```
Inputs        Weights       Sum          Activation    Output
  x₁ ─────────[w₁]────\
                        \
  x₂ ─────────[w₂]──────(Σ)───────[f]──────────
 ▶️
 y
                        /
  x₃ ─────────[w₃]────/
                       |
                      [b] (bias)

          z = w₁x₁ + w₂x₂ + w₃x₃ + b
          y = f(z)
```

Part 2: The Perceptron

The Simplest Neural Network

The Perceptron (Frank Rosenblatt, 1958)

The first artificial neural network!

$$z = \sum_{i=1}^{n} w_i x_i + b = \mathbf{w}^T \mathbf{x} + b$$

$$y = \text{activation}(z)$$

Component	Symbol	Meaning
Inputs	$x_1, x_2, \ldots$	Features
Weights	$w_1, w_2, \ldots$	Importance of each input
Bias	b	Threshold adjustment
Activation	f	Non-linear function

Perceptron: A Concrete Example

Task: Classify emails as spam (1) or not spam (0)

Input	Meaning	Value
x_1	Has "FREE"	1
x_2	Has attachment	0
x_3	From known contact	0

Weights (learned): $w_1 = 2.0$, $w_2 = 0.5$, $w_3 = -1.5$, $b = -0.5$

$$z = 2.0(1) + 0.5(0) + (-1.5)(0) + (-0.5) = 1.5$$

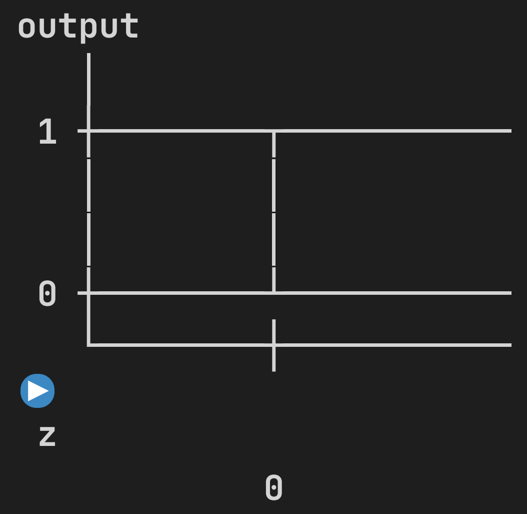
If z > 0 → Spam! ✓

Perceptron in Python

```python
import numpy as np

def perceptron(x, w, b):
    """Single perceptron with step activation."""
    z = np.dot(w, x) + b     # Weighted sum
    return 1 if z > 0 else 0  # Step activation

# Example
x = np.array([1, 0, 0])        # Has FREE, no attachment, unknown sender
w = np.array([2.0, 0.5, -1.5]) # Learned weights
b = -0.5                        # Learned bias

output = perceptron(x, w, b)    # → 1 (spam)
```

The Step Activation Function

$$f(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{otherwise} \end{cases}$$



Problem: Not differentiable at z=0 (can't use calculus!)

What Can a Single Perceptron Learn?

Function	Can Learn?	Why
AND	✅ Yes	Linearly separable
OR	✅ Yes	Linearly separable
NOT	✅ Yes	Linearly separable
XOR	❌ No	Not linearly separable

Single perceptron = Linear decision boundary

Perceptron Learning (AND Gate)

Goal: Output 1 only when BOTH inputs are 1

```
      x₂
      │
  1   ├──○─────●──     ○ = 0, ● = 1
      │   \
      │     \   ← Decision boundary
  0   ├──○─────○──
      └──┼─────┼──
x₁
         0     1
```

Perceptron CAN learn this line!

Weights: $w_1=1$, $w_2=1$, $b=-1.5$

$z = x_1 + x_2 - 1.5 > 0$ only when both are 1

Part 3: Activation Functions

The Secret to Non-Linearity

Why Do We Need Activation Functions?

Without activation, stacking layers is useless!

$$\text{Layer 1: } h = W_1 x + b_1$$

$$\text{Layer 2: } y = W_2 h + b_2$$

$$\text{Combined: } y = W_2(W_1 x + b_1) + b_2 = \underbrace{(W_2 W_1)}_{W'} x + \underbrace{(W_2 b_1 + b_2)}_{b'}$$

Still linear! Multiple layers collapse to one.

The Magic of Non-Linearity

With activation:

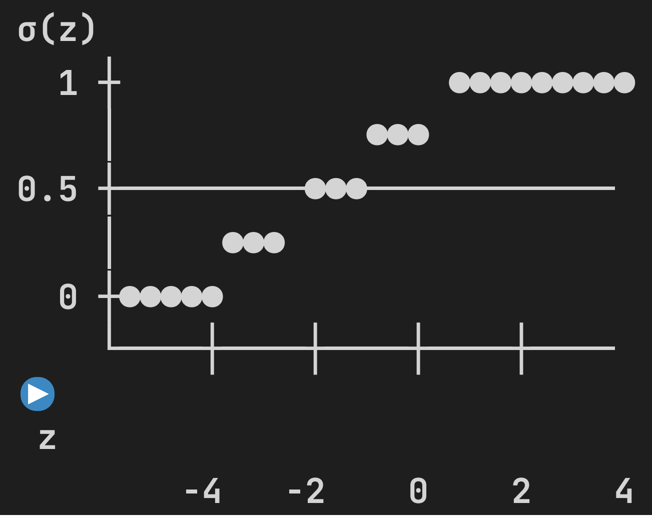
$$h = \mathrm{ReLU}(W_1 x + b_1)$$
$$y = W_2 h + b_2$$

Cannot simplify! The ReLU breaks the linearity.

> Activation functions allow networks to learn **curves**, not just lines!

Sigmoid Activation

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



Property	Value
Output range	(0, 1)
At z=0	0.5
As z→+∞	→ 1
As z→-∞	→ 0

Sigmoid: Pros and Cons

Pros	Cons
Output is probability-like (0-1)	**Vanishing gradient** (saturates)
Smooth, differentiable	Outputs not centered at 0
Historically important	Computationally expensive (exp)

Vanishing gradient: When z is very large or small, gradient ≈ 0
→ Network stops learning!

Tanh Activation

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Property	Value
Output range	(-1, 1)
Zero-centered	✅ Yes
Vanishing gradient	Still a problem

ReLU: The Modern Choice

$$\mathrm{ReLU}(z) = \max(0, z)$$

Property	Value
Output range	[0, ∞)
Gradient (z>0)	1 (constant!)
Gradient (z<0)	0

ReLU: Pros and Cons

Pros	Cons
No vanishing gradient (z>0)	"Dead neurons" (z<0 forever)
Very fast (just max)	Not zero-centered
Sparse activation (many zeros)	Unbounded output
Works extremely well in practice	

Use ReLU by default! It's the standard for hidden layers.

ReLU Variants

Variant	Formula	Fixes
Leaky ReLU	$\max(0.01z, z)$	Dead neurons
ELU	z if $z>0$, else $\alpha(e^z-1)$	Smoothness
GELU	$z \cdot \Phi(z)$	Used in transformers
Swish	$z \cdot \sigma(z)$	Used in EfficientNet

```python
# In PyTorch
F.relu(x)
F.leaky_relu(x, negative_slope=0.01)
F.gelu(x)
```

Activation Function Summary

Function	Use Case	Range
ReLU	Hidden layers (default)	$[0, \infty)$
Sigmoid	Binary classification output	$(0, 1)$
Tanh	RNNs, specific architectures	$(-1, 1)$
Softmax	Multi-class output	$(0, 1)$, sums to 1
Linear	Regression output	$(-\infty, \infty)$

Part 4: Multi-Layer Networks

Going Deep

The Multi-Layer Perceptron (MLP)

```
  Input Layer      Hidden Layer       Output Layer
     (3)              (4)                (2)

     o ─────────\
                ─o────────\
     o ─────────X       o────
   ▶️
   output 1
                ─o────────/
     o ─────────/       o────
   ▶️
   output 2
                ─o───────/

    Features       Learned          Predictions
                  features
```

How Information Flows

```
# Input: x (shape: 3)
# Hidden: 4 neurons
# Output: 2 classes

# Layer 1: Linear + Activation
z1 = W1 @ x + b1      # Shape: (4,)  - Linear transform
h1 = relu(z1)          # Shape: (4,)  - Non-linearity

# Layer 2: Linear + Activation
z2 = W2 @ h1 + b2     # Shape: (2,)  - Linear transform
output = softmax(z2)   # Shape: (2,)  - Probabilities
```

Weight Matrix Dimensions

From	To	Weight Shape	Bias Shape
3 inputs	4 hidden	(4, 3)	(4,)
4 hidden	2 outputs	(2, 4)	(2,)

```
# Total parameters:
# W1: 4 × 3 = 12
# b1: 4
# W2: 2 × 4 = 8
# b2: 2
# Total: 26 parameters to learn!
```

Why Multiple Layers Work

Layer 1: Learn simple features (edges, colors)
Layer 2: Combine into patterns (shapes, textures)
Layer 3: Combine into objects (faces, cars)

```
Raw Pixels → [Edges] → [Shapes] → [Objects] → Class
                ↑          ↑           ↑
             Layer 1    Layer 2     Layer 3
```

Each layer builds on the previous!

Universal Approximation Theorem

Theorem (Cybenko, 1989):

A neural network with a single hidden layer can approximate any continuous function to arbitrary accuracy.

Catch	Reality
May need infinitely many neurons	Deep > Wide in practice
Doesn't say how to find weights	Training is still hard
Doesn't guarantee generalization	More depth helps

Deep Networks: Why More Layers?

Layers	What It Can Learn	Examples
1	Linear functions	Linear regression
2	Simple curves	XOR, simple patterns
3-10	Complex functions	MNIST, tabular data
10-100	Hierarchical patterns	ImageNet, NLP
100+	Very complex	GPT, state-of-the-art

Solving XOR with 2 Layers

```
# Architecture: 2 → 2 → 1
# Input: [x₁, x₂]
# Hidden: 2 neurons with ReLU
# Output: 1 neuron with sigmoid

# Hidden layer transforms space:
# (0,0) → hidden representation
# (0,1) → hidden representation
# (1,0) → hidden representation
# (1,1) → hidden representation

# Output layer draws a line in the NEW space!
```

Softmax: Multi-Class Output

Problem: Network outputs raw scores (logits). How to get probabilities?

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$

Logits	Softmax
[2.0, 1.0, 0.1]	[0.66, 0.24, 0.10]

Properties:

- All outputs between 0 and 1
- All outputs sum to 1
- Largest logit → highest probability

Common Architectures

Task	Input	Hidden	Output Activation
Binary Classification	Features	ReLU layers	Sigmoid (1 neuron)
Multi-class (10 classes)	Features	ReLU layers	Softmax (10 neurons)
Regression	Features	ReLU layers	Linear (1 neuron)

Part 5: Training Neural Networks

How Do Networks Learn?

The Training Process

```
┌─────────────────────────────────────────────────────────┐
│                     TRAINING LOOP                       │
│                                                         │
│   1. FORWARD PASS   │  Compute prediction from input    │
│          ↓          │                                   │
│   2. COMPUTE LOSS   │  How wrong are we?                │
│          ↓          │                                   │
│   3. BACKWARD PASS  │  Which weights caused the error?  │
│          ↓          │                                   │
│   4. UPDATE WEIGHTS │  Adjust to reduce error           │
│          ↓          │                                   │
│   5. REPEAT         │  Until loss is small enough       │
└─────────────────────────────────────────────────────────┘
```

Loss Functions: Measuring Error

Task	Loss Function	Formula
Regression	MSE	$\frac{1}{n}\sum(y-\hat{y})^2$
Binary Class.	Binary Cross-Entropy	$-[y\log\hat{y}+(1-y)\log(1-\hat{y})]$
Multi-class	Cross-Entropy	$-\sum_c y_c\log(\hat{y}_c)$

Lower loss = Better predictions!

Cross-Entropy Loss Intuition

If true label is class 0:

Prediction for class 0	Loss
0.99 (confident, correct)	0.01
0.50 (unsure)	0.69
0.01 (confident, WRONG)	4.60

Severely punishes confident wrong predictions!

Gradient Descent: The Key Idea

Goal: Find weights that minimize loss.

Method: Follow the slope downhill.

$$w_{\text{new}} = w_{\text{old}} - \eta \cdot \frac{\partial \text{Loss}}{\partial w}$$

Symbol	Meaning
w	Weight to update
η	Learning rate (step size)
$\frac{\partial \text{Loss}}{\partial w}$	Gradient (slope)

Gradient Descent Visualization

```
Loss
  │
  │\
  │ \   ← Start here (random weights)
  │  \
  │   ●→→→→→●
  │          \
  │           \
  │            ●   ← End here (optimal weights)
  └───────────
▶️
 W
```

Each step moves toward the minimum!

Learning Rate: Crucial Choice

```
    Too Small (η=0.0001)    Just Right (η=0.01)    Too Large (η=1.0)
       │●                       │●                       │●      ●
       │
  ↘
                  │
  ↘
                 │
  ↘
  ↗
       │ ●                      │   ●                    │   ●
       │
  ↘
                │
  ↘
              │
       │  ●                     │    ●                   │
       │
  ↘
              │                       │
       │    ...                 │                        │
       │     (slow!)            │    (converges)         │  (diverges!)
```

Learning Rate Guidelines

Learning Rate	Effect
0.1 - 1.0	Usually too large
0.01 - 0.001	Good starting point
0.0001 - 0.00001	Fine-tuning, large models

Common practice: Start high, decrease over time (learning rate schedule)

Backpropagation: Computing Gradients

Problem: Network has millions of weights. How to compute all gradients?

Solution: Chain rule applied backward!

$$\frac{\partial \text{Loss}}{\partial w_1} = \frac{\partial \text{Loss}}{\partial y} \cdot \frac{\partial y}{\partial h} \cdot \frac{\partial h}{\partial w_1}$$

Backprop: Step by Step

```
Forward pass:
    x ──[W₁]──
▶️
 h ──[W₂]──
▶️
 y ──
▶️
 Loss = 2.5
        ↓           ↓
       h=3         y=1.2

Backward pass (compute gradients):
    ∂L/∂W₁
◀️
── ∂L/∂h
◀️
── ∂L/∂y
◀️
── ∂L/∂Loss = 1
        │         │         │
       0.4       0.3       0.5

Then update:
    W₁ = W₁ - η × 0.4
    W₂ = W₂ - η × 0.3
```

Stochastic Gradient Descent (SGD)

Full Batch GD: Use ALL data to compute gradient (slow!)

SGD: Use small random batch each step

Approach	Batch Size	Pros	Cons
Full Batch	All data	Stable	Very slow, memory
Mini-Batch	32-256	Fast, stable	Good default
Pure SGD	1	Very fast	Noisy, unstable

Epochs and Batches

Epoch: One pass through entire dataset

Batch: Subset of data used for one gradient update

```
Dataset: 10,000 samples
Batch size: 100

Batches per epoch = 10,000 / 100 = 100 updates

5 epochs = 500 total updates
```

Better Optimizers Than SGD

Optimizer	Key Idea	When to Use
SGD	Basic gradient descent	Simple, stable baseline
Momentum	Use past gradients	Faster convergence
Adam	Adaptive learning rate	Default choice
AdamW	Adam + weight decay	State-of-the-art

```python
# In PyTorch:
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)
```

Adam: Why It Works

Adam combines:

1. **Momentum:** Remember past gradients (don't zigzag)
2. **RMSProp:** Adapt learning rate per parameter

Result: Works well with minimal tuning!

```
# Adam is the default choice
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)
```

Part 6: PyTorch Basics

From Theory to Code

Why PyTorch?

Feature	Benefit
Dynamic graphs	Debug like normal Python
Autograd	Automatic gradient computation
GPU support	10-100x faster training
Rich ecosystem	Torchvision, HuggingFace, etc.
Industry standard	Tesla, Meta, OpenAI

```python
import torch
import torch.nn as nn
```

Tensors: PyTorch's Arrays

```
import torch

# Create tensors
x = torch.tensor([1.0, 2.0, 3.0])          # From list
y = torch.zeros(3, 4)                        # 3×4 of zeros
z = torch.randn(3, 4)                        # Random normal

# Operations (like numpy!)
a = x + 1                                    # Add scalar
b = x @ y                                    # Matrix multiply
c = x.sum()                                  # Reduce

# Move to GPU
x_gpu = x.to('cuda')  # or x.cuda()
```

Tensor Properties

```
x = torch.randn(3, 4, 5)

print(x.shape)         # torch.Size([3, 4, 5])
print(x.dtype)         # torch.float32
print(x.device)        # cpu (or cuda:0)
print(x.requires_grad)  # False by default
```

Automatic Differentiation

PyTorch computes gradients automatically!

```python
# Create tensor that tracks gradients
x = torch.tensor([2.0], requires_grad=True)

# Forward pass: compute y = x² + 3x + 1
y = x**2 + 3*x + 1

# Backward pass: compute dy/dx
y.backward()

# Gradient is stored in x.grad
print(x.grad)  # tensor([7.])
# Because dy/dx = 2x + 3 = 2(2) + 3 = 7
```

Building Networks: nn.Module

```python
import torch.nn as nn

class SimpleNetwork(nn.Module):
    def __init__(self):
        super().__init__()
        self.fc1 = nn.Linear(784, 256)   # Input → Hidden
        self.fc2 = nn.Linear(256, 10)    # Hidden → Output

    def forward(self, x):
        x = torch.relu(self.fc1(x))      # Hidden + ReLU
        x = self.fc2(x)                   # Output (logits)
        return x

model = SimpleNetwork()
```

nn.Linear: The Linear Layer

```python
# nn.Linear(in_features, out_features)
layer = nn.Linear(3, 4)

# Has learnable parameters:
print(layer.weight.shape)  # (4, 3)
print(layer.bias.shape)    # (4,)

# Forward pass:
x = torch.randn(2, 3)  # Batch of 2, 3 features each
y = layer(x)            # → shape (2, 4)
```

nn.Sequential: Quick Networks

```python
# Same network, less code:
model = nn.Sequential(
    nn.Linear(784, 256),
    nn.ReLU(),
    nn.Linear(256, 128),
    nn.ReLU(),
    nn.Linear(128, 10)
)

# Forward pass
x = torch.randn(32, 784)  # Batch of 32 images
output = model(x)          # → shape (32, 10)
```

The Training Loop

```
# 1. Create model, loss, optimizer
model = SimpleNetwork()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)

# 2. Training loop
for epoch in range(num_epochs):
    for batch_x, batch_y in dataloader:
        # Forward pass
        outputs = model(batch_x)
        loss = criterion(outputs, batch_y)

        # Backward pass
        optimizer.zero_grad()   # Clear old gradients
        loss.backward()         # Compute new gradients
        optimizer.step()        # Update weights
```

Understanding the Training Loop

Line	What It Does
`outputs = model(batch_x)`	Forward pass: input → prediction
`loss = criterion(outputs, batch_y)`	Compute how wrong we are
`optimizer.zero_grad()`	Reset gradients to zero
`loss.backward()`	Backprop: compute all gradients
`optimizer.step()`	Update all weights

Loading Data

```python
from torch.utils.data import DataLoader
from torchvision import datasets, transforms

# Define preprocessing
transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.1307,), (0.3081,))
])

# Load MNIST
train_data = datasets.MNIST('data', train=True,
                            download=True, transform=transform)
train_loader = DataLoader(train_data, batch_size=64, shuffle=True)

# Iterate
for images, labels in train_loader:
    print(images.shape)  # (64, 1, 28, 28)
    print(labels.shape)  # (64,)
```

Complete MNIST Example

```python
# Model
model = nn.Sequential(
    nn.Flatten(),                 # (batch, 1, 28, 28) → (batch, 784)
    nn.Linear(784, 256),
    nn.ReLU(),
    nn.Linear(256, 10)
)

# Training
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)

for epoch in range(5):
    for images, labels in train_loader:
        outputs = model(images)
        loss = criterion(outputs, labels)

        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

    print(f"Epoch {epoch+1}, Loss: {loss.item():.4f}")
```

Evaluation

```python
model.eval()  # Switch to evaluation mode (disables dropout, etc.)
correct = 0
total = 0

with torch.no_grad():  # Don't track gradients (saves memory)
    for images, labels in test_loader:
        outputs = model(images)
        _, predicted = outputs.max(1)  # Get class with highest score
        total += labels.size(0)
        correct += (predicted == labels).sum().item()

accuracy = 100 * correct / total
print(f"Test Accuracy: {accuracy:.2f}%")  # ~97-98%
```

Common Pitfalls

Mistake	Symptom	Fix
Forgot `zero_grad()`	Loss doesn't decrease	Add `optimizer.zero_grad()`
Wrong input shape	Shape error	Check tensor dimensions
Training with `no_grad()`	Loss stays constant	Remove `torch.no_grad()`
Not calling `model.eval()`	Bad test accuracy	Add `model.eval()`
Learning rate too high	Loss explodes (NaN)	Reduce learning rate

Key Takeaways

1. **Perceptron** = Linear model + activation (can't solve XOR alone)
2. **Activation functions** add non-linearity (use **ReLU** by default)
3. **Multiple layers** can approximate any function
4. **Backpropagation** efficiently computes all gradients
5. **PyTorch training loop:**
    - Forward → Loss → zero_grad → Backward → Step

Summary: Neural Network Recipe

```python
# 1. Define architecture
model = nn.Sequential(
    nn.Linear(input_size, hidden_size),
    nn.ReLU(),
    nn.Linear(hidden_size, output_size)
)

# 2. Define loss and optimizer
criterion = nn.CrossEntropyLoss()   # For classification
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)

# 3. Training loop
for epoch in range(num_epochs):
    for x, y in dataloader:
        loss = criterion(model(x), y)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
```

Welcome to Deep Learning!

Next: Computer Vision - How Machines See

Lab: Build and train a neural network from scratch

"What I cannot create, I do not understand."
— Richard Feynman

Questions?