

Expectation, Higher Moments

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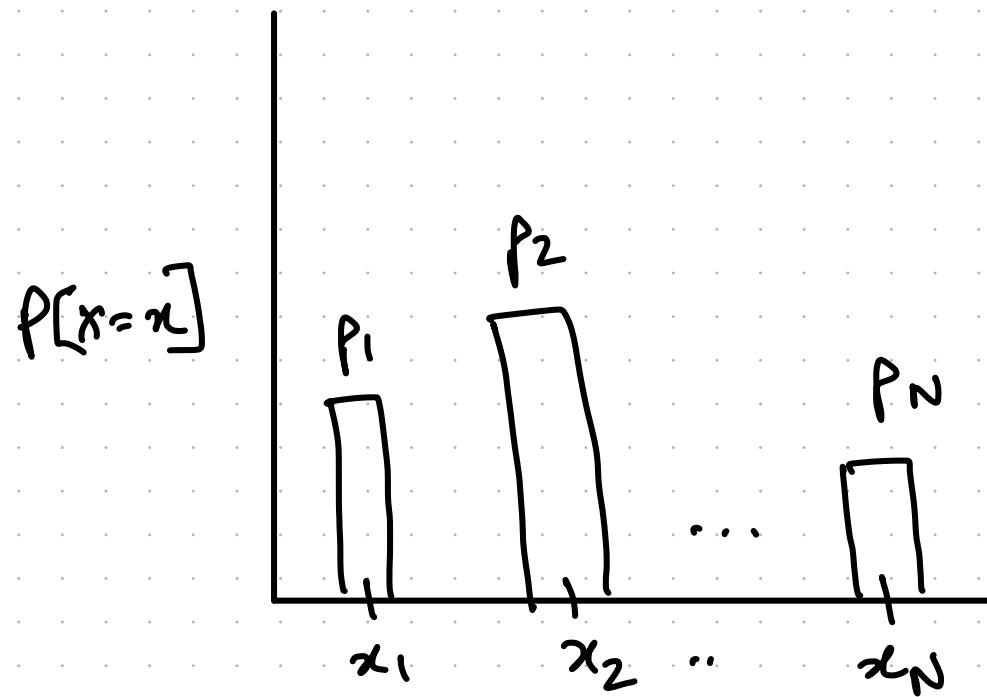
NO!

In expectation, you will lose 50 paise

$$E[X] = \sum_{x \in X(\Omega)} x p_X(x)$$

where

$$p_X(x) = P[X=x]$$



$$E[x] = p_1 x_1 + p_2 x_2 + \dots + p_N x_N$$

Q) COIN TOSS

- 2 coin toss
- $X$  is. r.v. # H.

$\omega_i$	$X(\omega_i)$
TT	0
TH	1
HT	1
HH	2

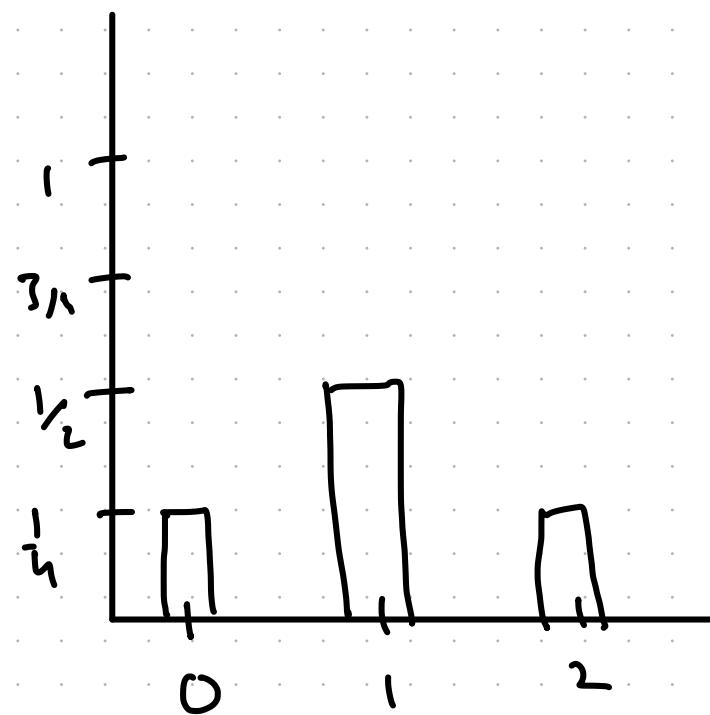
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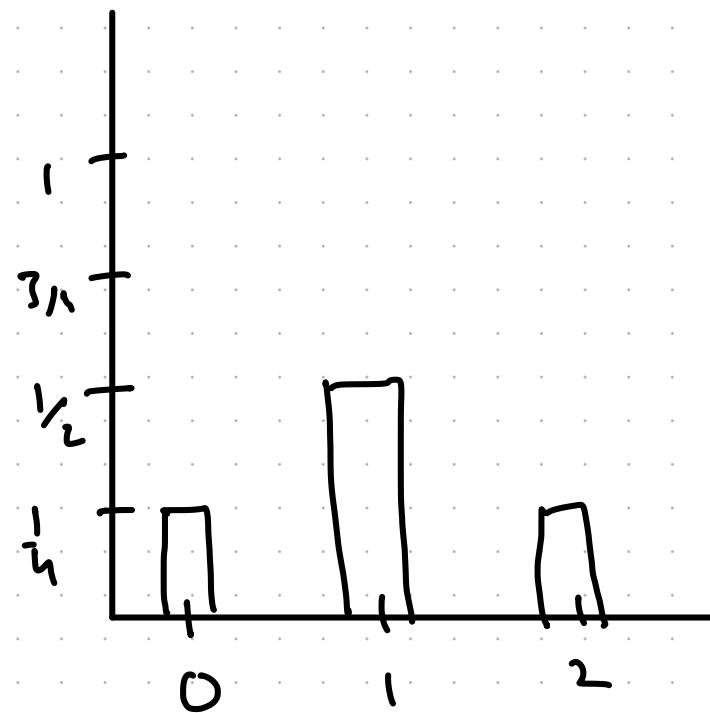


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	$X(\omega)$
$\omega_1$	0
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$$E[X] = ?$$



$$E[X] = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$= 1$$

Q)  $x \sim \text{Bernoulli}(p)$

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$$\begin{aligned} E[X] &= \sum_{x \in \{0, 1\}} x P_X(x) = 0 * (1-p) + 1 * p \\ &= p \end{aligned}$$

Q)  $X \sim \text{Binomial}(n, p)$

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HINT: what is  $\sum_{k=0}^n P_X(k)$ ?

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HINT: what is  $\sum_{k=0}^n P_X(k)$ ? 1 (as PMF)

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$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = \sum_{k=0}^n k * \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^n k * \frac{n!}{k! (n-k)!} p^k (1-p)^{n-k} \quad \left[ \text{As } 0 \times \dots = 0 \right]$$

So start from  
 $k=1$

$$= n \sum_{k=1}^n \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

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$$E[X] = ?$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = n \sum_{k=1}^n \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

$$= np \sum_{k=1}^n \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}$$

$$= np \sum_{m=0}^{n-1} \binom{n-1}{m} p^m (1-p)^{n-1-m}$$

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$$E[X] = np \sum_{m=0}^{n-1} \binom{n-1}{m} p^m (1-p)^{n-1-m}$$

$$= np \sum P_{Y(m)}$$

where  $Y \sim \text{Binomial}(n-1, m)$

$$= np \cdot 1$$

$$= np$$

Q)  $X \sim \text{Poisson}(\lambda)$

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$$= \lambda \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda} = \lambda \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} e^{-\lambda}$$

$$= \lambda \cdot 1 = \lambda$$

# Properties of Expectation

i) Function of  $X$

For any function  $g$ :

$$E[g(x)] = \sum_x g(x) p_x(x)$$

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Q). you toss two coins with  $p = .5$

0 heads = 0 Rupees

1 heads = 10 Rupees

2 heads = 25 Rupees

game playing = 20 Rupees

On average what is your return?

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on average what is your return?  $p_x(2) = \frac{1}{4}$

$$X \sim \text{Binomial}(2, 0.5)$$

$$P[X=0] = p_x(0) = \binom{2}{0} (0.5)^0 (0.5)^2 = \frac{1}{4}$$

$$p_x(1) = r_2$$

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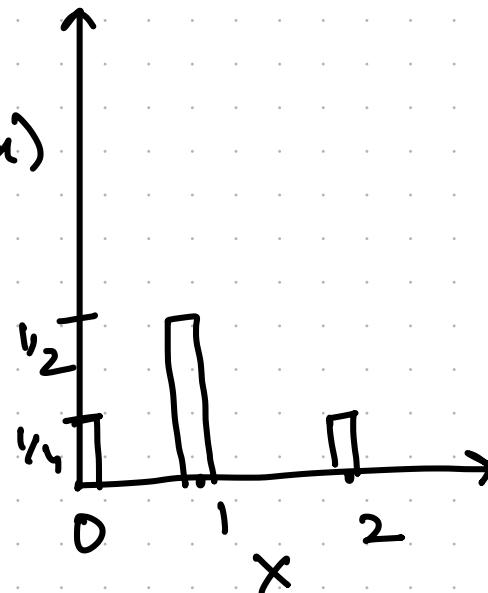
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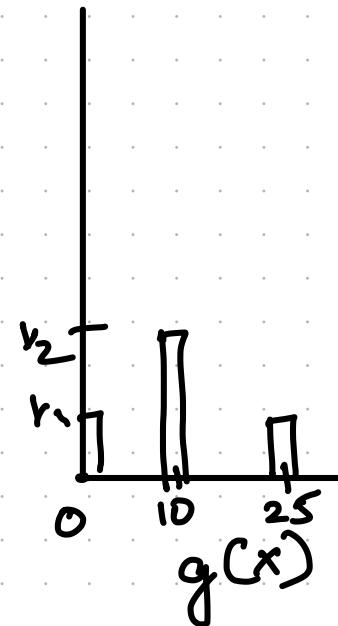
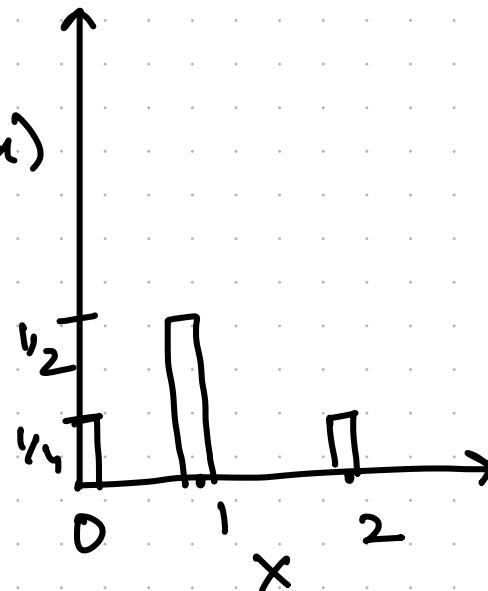
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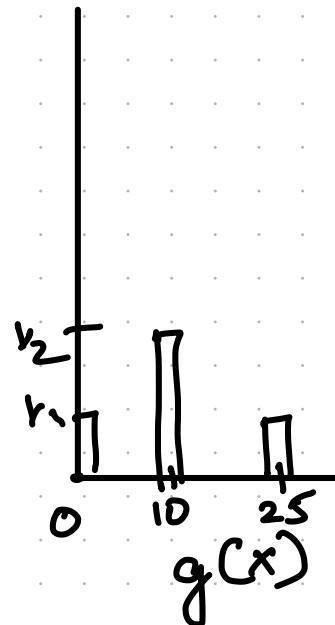
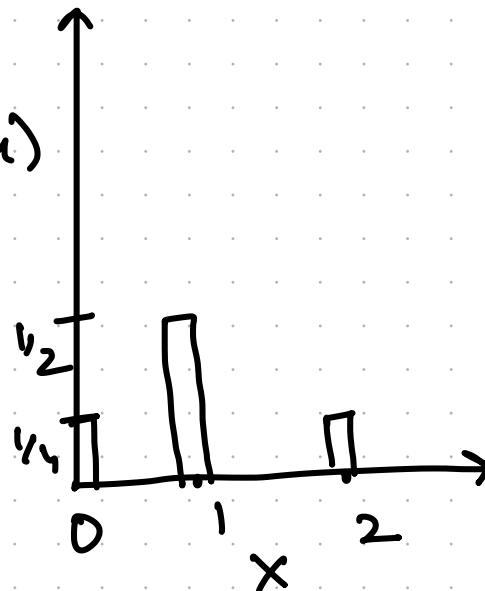
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$$\begin{aligned} E[g(x)] &= 0 \times \frac{1}{4} + 10 \times \frac{1}{2} + 25 \times \frac{1}{4} \\ &= 5 + 6 \cdot 25 = 11.25 \end{aligned}$$

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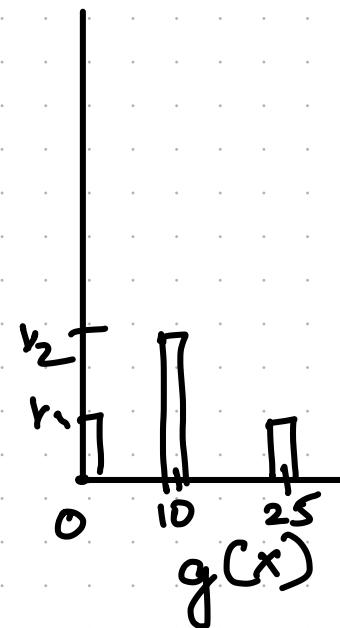
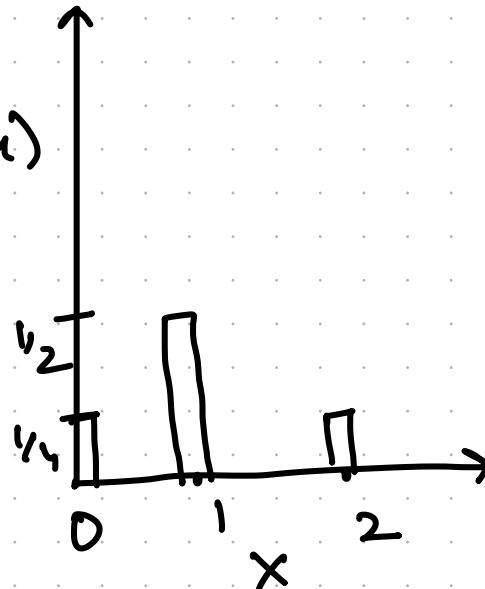
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$$E[g(x)] = 11.25$$

$$\text{Expected gain} = E[g(x)] - 20 = -8.75$$



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$$E[cx] = \sum_x cx p_x(x)$$

$$= c \sum_x x p_x(x)$$

$$= c E[x]$$

## Properties of Expectation

1) Scale  $E[cx] = cE[x]$

Q) Assume  $X \sim \text{Poisson}(\lambda=5)$  = # orders you sell per day

For each sale you get 1000 INR:

What is Expected money earned daily

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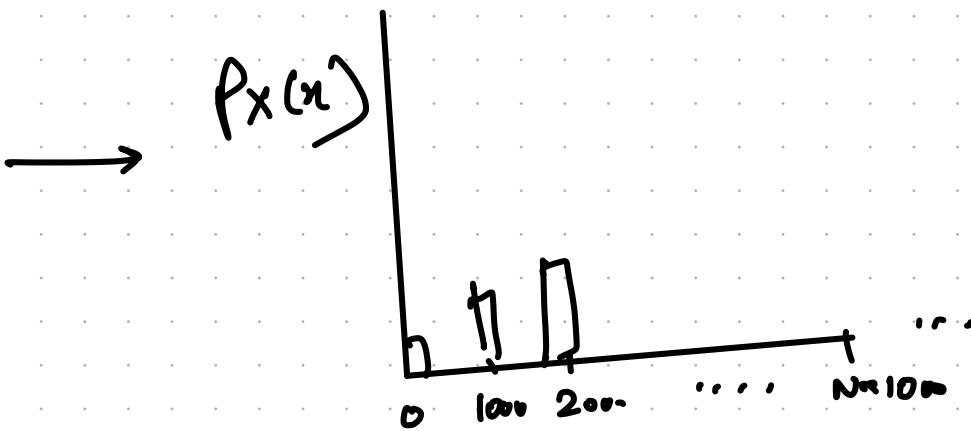
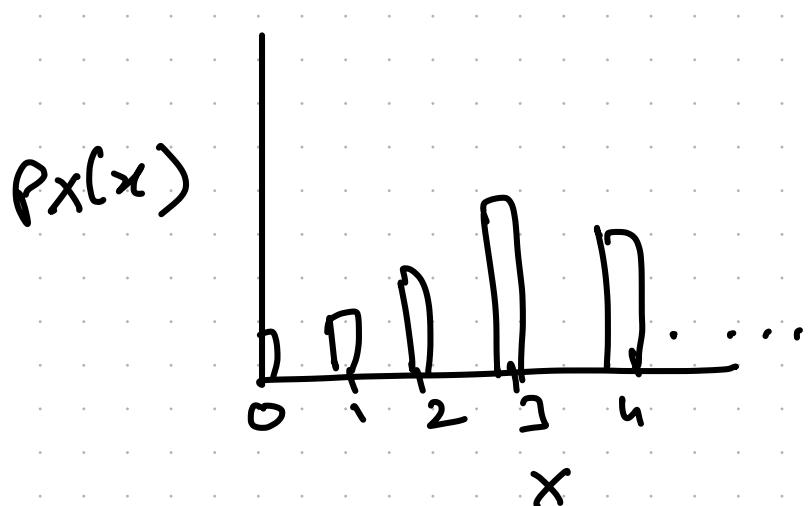
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$$= \sum_x (g(x) + h(x)) p_x(x)$$

$$= \sum_x g(x) p_x(x) + \sum_x h(x) p_x(x)$$

$$= E[g(x)] + E[h(x)]$$

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Q).  $X \sim \text{Poisson}(3)$  denote r.v. for # daily sales.

$$g(x) = 1000x \quad (\text{Commission per sale})$$

$$h(x) = \begin{cases} -2000 &; 0 \text{ sale} \\ -500 &; 1 \text{ sale} \\ 0 &; \geq 2 \text{ sales} \end{cases} \quad [\text{Penalty}]$$

What is expected amt. you take home

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$$= 1000 \times 3 + \frac{(-2000) \times e^{-3} 3^0}{0!} + \frac{(-500) \times e^{-3} 3^1}{1!}$$

$$= 3000 + (-99) + (-75) \approx 3000$$

# Properties of Expectation

IV) DC shift

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$$= \sum_x (x+c) p_x(x)$$

$$= \sum_x x p_x(x) + c \sum_x p_x(x)$$

$$= E[x] + c^{\star 1}$$

$$= E[x] + c$$

(Q) you have option to play 2 games

G<sub>1</sub>: Heads +2000Rs

Tails -2000Rs

G<sub>2</sub>: Heads +200Rs

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What is expected return for both games?

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What is expected return for both games?



BUT G<sub>1</sub> is riskier : larger variance!

## Moments

$k^{\text{th}}$  moment of r.v.  $X$  is

$$E[X^k] = \sum_x x^k p_x(x)$$

## Variance

Variance of r.v. showing spread is

$$\text{VAR}[X] = E[(X - \mu_X)^2]$$

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$$\begin{aligned} \text{VAR}[X] &= E[(X - E[X])^2] \\ &= E[X^2 - 2X E[X] + E[X]^2] \\ &= E[X^2] - 2 E[X] \cdot E[X] + E[X]^2 \\ &= E[X^2] - E[X]^2 \end{aligned}$$

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 &= E[X^2] - 2 E[X] \cdot E[X] + E[X]^2 \\
 &= E[X^2] - E[X]^2
 \end{aligned}$$

$X$  is r.v.  
 $E[X]$  is a constant once computed.

$$\begin{aligned}
 \therefore E[E[X]] &= E[X]
 \end{aligned}$$

$$(\because E[c] = c)$$

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Find 2<sup>nd</sup>, 3<sup>rd</sup> moment and variance.

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$$= p$$

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$$\text{VAR}[X] = E[X^2] - E[X]^2$$
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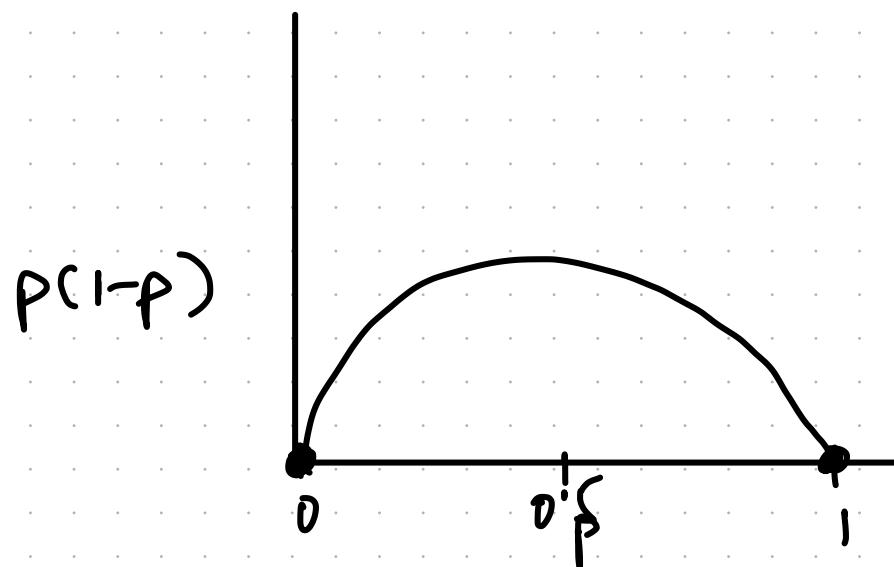
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$$= \sum_{k=1}^n \frac{k^2 + n!}{(n-k)! k!(k-1)!} \cdot p^k (1-p)^{n-k}$$

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$$= \sum_{k=1}^n \frac{k(k-1) n! p^k (1-p)^{n-k}}{(n-k)! k!(k-1)!} + \sum_{k=1}^n \frac{n! p^k (1-p)^{n-k}}{(n-k)! k!(k-1)!}$$

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$$= \sum_{k=2}^n \frac{n! p^k (1-p)^{n-k}}{(n-k)! (k-2)!} + E[X]$$

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$$\text{Now, } E[X^2] = \sum_{k=2}^n \frac{k(k-1) n! p^k (1-p)^{n-k}}{(n-k)! k(k-1)!} + \sum_{k=1}^n k \frac{n! p^k (1-p)^{n-k}}{(n-k)! k!}$$

$$= \sum_{k=2}^n \frac{n! p^k (1-p)^{n-k}}{(n-k)! (k-2)!} + E[X]$$

$$= \sum_{k=2}^n \frac{n(n-1)(n-2)! p^k (1-p)^{n-k}}{(n-k)! (k-2)!} + E[X]$$

$$(Q) \quad X \sim \text{Binomial}(n, p)$$

$$\text{Now, } E[X^2] = \sum_{k=2}^n n(n-1) \frac{(n-2)! p^k (1-p)^{n-k}}{(n-k)! (k-2)!} + E[X]$$

$$= \sum_{k=2}^n n(n-1)(n-2)! p^{k-2} \cdot (1-p)^{n-k} \cdot p^2 + E[X]$$

$$\frac{(n-k)! (k-2)!}{(n-k)! (k-2)!}$$

$$= p^2 n(n-1) \sum_{k=2}^n \frac{(n-2)! p^{k-2} (1-p)^{n-k}}{(n-k)! (k-2)!} + E[X]$$

$$\text{let } l = k-2$$

$$k_{\min} = 2$$

$$l_{\min} = 0$$

$$k_{\max} = n$$

$$l_{\max} = n-2 = m$$

$$Q) \quad X \sim \text{Binomial}(n, p)$$

$$\text{Now, } E[X^2] = p^2 n(n-1) \sum_{l=0}^m \frac{m! p^l (1-p)^{m-l}}{(m-l)! l!} + E[X]$$

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$$\text{VAR}(X) = E[X^2] - E[X]^2$$

$$= p^2 n(n-1) + np - (np)^2$$

$$= p^2 n^2 - p^2 n + np - n^2 p^2$$

$$\text{VAR}(X) = np(1-p)$$

# Properties of Variance

i) Scale

$$\text{VAR}[cx] = c^2 \text{VAR}[x]$$

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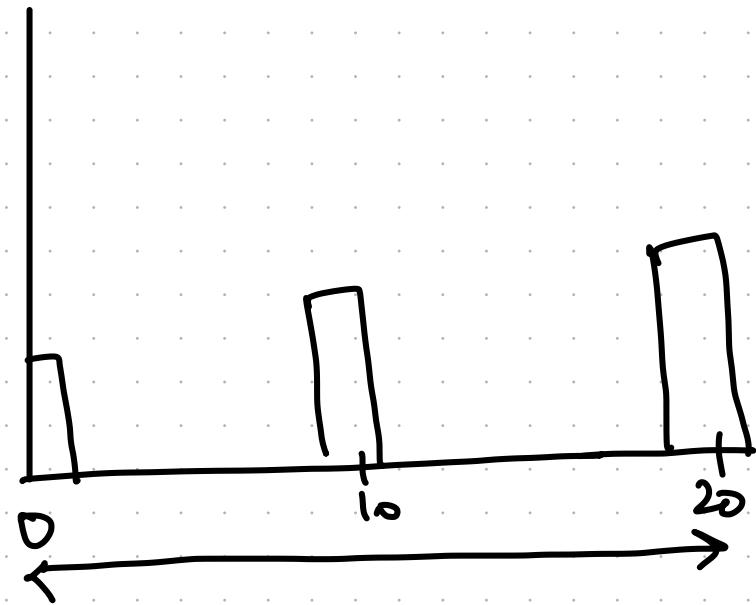
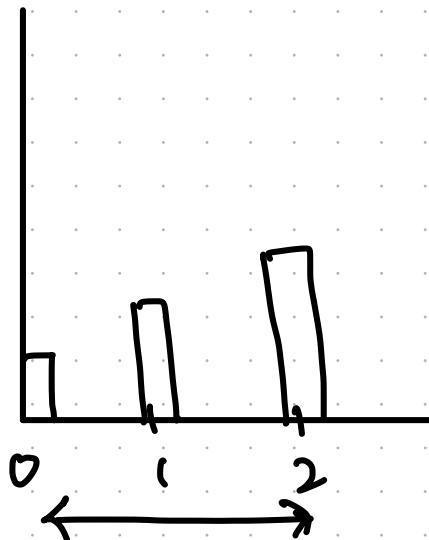
# Properties of Variance

i) Scale

$$\text{VAR}[cx] = c^2 \text{VAR}[x]$$

$$\begin{aligned}\text{VAR}[cx] &= E[c^2 x^2] - E[cx]^2 \\ &= c^2 E[x^2] - c^2 E[x]^2\end{aligned}$$

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II) DC shift

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