
Problem 1: Use quantifiers to express the below statements, and then derive the negation of the statement. Make sure that the no negation lies in the left of the quantifier. Finally, express the obtained negation in English text.

(a) All cats have parasites. (take it easy! I don't mean it actually.)
Approach 1: $\forall x(C(x) \to P(x)) \equiv \forall x(\neg C(x) \lor F(x))$
Negation: $\neg\forall x(\neg C(x) \lor F(x)) \equiv \exists x(C(x) \land \neg F(x))$.

Approach 2: Considering cats as the domain, we obtain $\forall x(C(x))$ and the negation of it as $\neg(\forall x(C(x))) \equiv \neg(\exists x(D(x)))$. However, we don't follow this approach.

English: There's a cat that does not have parasite

(b) There is a cow that can add two numbers.
Solution: $\exists x(C(x) \land A(x))$. Negation of it becomes $\neg\exists x(C(x) \land A(x)) \equiv \forall x\neg(C(x) \land A(x))$
English: No cow can add two numbers.

(c) Every monkey you encounter can climb.
$\forall x(M(x) \to Climb(x)) \equiv \forall x(M(x) \lor \neg Climb(x))$
Negation: $\neg\forall x(M(x) \lor \neg Climb(x)) \equiv \exists x(\neg M(x) \land Climb(x))$
English: There is a monkey you encounter cannot climb.

(d) There is a fish that can speak Bengali.
$\exists x(F(x) \land Bengali(x))$
Negation:$\neg\exists x(F(x) \land Bengali(x) \equiv \forall x\neg(F(x) \land Bengali(x) \equiv \forall x(\neg F(x) \lor \neg Bengali(x))$
English: No fish can speak Bengali.

(e) There exists a horse that can fly and catch bird as needed.
As the horse that exists does fly and catch bird, conjunction operator is needed here:
$\exists x(H(x) \land F(x) \land B(x))$
Negation: $\neg\exists x(H(x) \land F(x) \land B(x)) \equiv \exists x\neg(H(x) \land F(x) \land B(x)) \equiv \forall x(\neg H(x) \lor \neg F(x) \lor \neg B(x))$
English: Every horse either can not fly or it can not catch bird.

Problem 2: Assume $Q(x, y)$ as the statement saying *student x in CSE173 class is a contestant on TV reality show*. Express the below sentences using $Q(x, y)$, and other logical connectives. Consider all students in CSE 173 class as the domain for x and all TV reality shows as the domain for y.

(a) There is a student at CSE 173 who is a contestant on a TV reality show.
As it mentions about at least one student, we have to use existential quantifier, and the expression is follows:
$\exists x\exists yQ(x, y)$

(b) No student at CSE 173 has ever been a contestant on a TV reality show.
It is exactly the negation of previous answer. That is,
$\neg(\exists x\exists yQ(x, y) \equiv \forall x\forall y\neg(Q(x, y))$

(c) There is a student at CSE 173 who is a contestant on Close-up and Bangladeshi Idol.
$\exists x\exists yQ(x, \text{Close-up}) \land Q(x, \text{Bangladeshi Idol})$

(d) Every TV reality show aired so far, had a student from CSE 173 as a contestant.
"Every TV reality show" requires y to be quantified by universal quantifier, and the presence of a (which sounds similar here as at least a) CSE 173 contestant refers to an existential quantifier.
$\forall y\exists xQ(x, y)$

(e) At least two students from CSE 173 are the contestants on Bangladeshi Idol.
Here, at least two students x, z (where $x \neq z$) participating in Bangladeshi Idol. Exact expression is as follows:
$\exists x\exists z(x \neq z)Q(x, \text{Bangladeshi Idol}) \land Q(z, \text{Bangladeshi Idol})$

Problem 3: Derive the negation of the below logical expressions; use logical equivalences and move the negation operator onto the smallest element possible. For instance, negation of $\forall x[P(x) \to Q(x)]$ is obtained as per the critera stated as follows: $\neg\forall x[P(x) \to Q(x)]$, convert this to $\exists x[\neg P(x) \to Q(x)]$, and finally to $\exists x[P(x) \wedge \neg Q(x)]$

(a) $\forall x[P(x) \vee Q(x)]$
$\neg\forall x[P(x) \vee Q(x)] \equiv \exists x\neg[P(x) \vee Q(x)] \equiv \exists x[\neg P(x) \wedge \neg Q(x)]$

(b) $\exists y[P(y) \vee (Q(y) \vee R(y))]$
$\neg\exists y[P(y) \vee (Q(y) \vee R(y))] \equiv \forall y\neg[P(y) \vee (Q(y) \vee R(y))] \equiv \forall y[\neg P(y) \wedge \neg(Q(y) \wedge \neg R(y))]$

(c) $\exists x[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x)]$
$\neg\exists x[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x)] \equiv \forall x\neg[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x)] \equiv \forall x[\neg(P(x) \wedge Q(x)) \wedge \neg(Q(x) \wedge \neg P(x)] \equiv \forall x[(\neg P(x) \vee \neg Q(x)) \wedge (\neg Q(x) \vee P(x)]$

Problem 4: Use predicates, quantifiers, logical connectives and mathematical operators to express the below mathematical statements. Consider all integers as the domain.

(a) If m and n both are negative, their product is always positive.
As there's no restriction on the total number of possible of m and n (any negative m and n), it requires a universal quantifier. The exact expression is as follows: $\forall m\forall n(((m < 0) \wedge (n < 0)) \to mn > 0)$

(b) Assume m and n are postive, then average of m and n positive.
Again, m and n are any positive numebr. Hence, it requires a universal quantifier. The exact expression is as follows: $\forall m\forall n(((m > 0) \wedge (n > 0))) \to (m+n)/2 > 0)$

(c) If m and n are negative, $m - n$ is not necessarily negative.
It suggests that $m - n$ is not always negative, but it is negative for some cases. This refers to an existential quantifier, and the exact expression for the quantification process goes as follows: $\exists m\exists n(((m < 0) \wedge (n < 0)) \wedge \neg(m - n) > 0)$