

1. Express each of these statements using predicates, quantifiers, logical connectives, and mathematical operators, where the domain consists of all integers.

- (a) The product of two negative integers is positive.
- (b) The average of two positive integers is positive.
- (c) The difference of two negative numbers is not necessarily negative.
- (d) The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.

SOLUTION

- (a) $\forall x \forall y ((x < 0) \wedge (y < 0)) \rightarrow (xy > 0)$
- (b) $\forall x \forall y ((x > 0) \wedge (y > 0)) \rightarrow ((x + y)/2 > 0)$
- (c) $\exists x \exists y ((x < 0) \wedge (y < 0)) \wedge ((x - y) \geq 0)$
- (d) $\forall x \forall y (|x + y| \leq |x| + |y|)$