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H.W-2

Problem 1

a) All cats have parasites.

here,

$C(x)$: 'x' is a cat.

$P(x)$: 'x' have parasite.

Logical expression form: $\forall x\,(C(x) \rightarrow P(x))$

$\equiv \forall x\,(\neg C(x) \vee P(x))$

$\equiv \neg \forall x\,(\neg C(x) \vee P(x))$; [Negation fom]

$\equiv \cancel{\forall} \exists x \neg (\neg C(x) \vee P(x))$ [Demorgan's Law]

$\equiv \exists x\,(C(x) \wedge \neg P(x))$ [demorgan's Law]

English text: There exist a cat that don't have parasite.

b) There is a cow that can add two numbers.

here,

$C(x)$: 'x' is a cow

$T(x)$: 'x' can add two numbers.

Logical expression form: $\cancel{\forall}\ \cancel{\exists}$

$\exists x\,(C(x) \wedge T(x))$

$\equiv \neg \exists x\,(C(x) \wedge T(x))$; [Negation form]

$\equiv \forall x \neg (C(x) \wedge T(x))$; [Demorgan's Law]

$\cancel{\equiv \forall x\,(\neg C(x))}$

English text: No cow can add two numbers.

c) Every monkey you encounter can climb.

here,

$M(x)$: 'x' is a monkey.

$C(x)$: 'x' can climb.

Logical expression form: $\forall x(M(x) \rightarrow C(x))$

$\equiv \neg \forall x(\neg M(x) \vee C(x))$; ~~[Negation form]~~

$\equiv \neg \forall x(\neg M(x) \vee C(x))$; [Negation form]

$\equiv \exists x \neg(\neg M(x) \vee C(x))$; [Demorgan's Law]

$\equiv \exists x (M(x) \wedge \neg C(x))$; [Demorgan Law]

English text: there exist a monkey ~~that~~ you encounter that can't climb.

d) There is a fish that can speak Bengali.

here,

$F(x)$: 'x' is a fish.

$B(x)$: 'x' can speak Bengali.

Logical expression form:

$$\exists x\,(F(x) \wedge B(x))$$

$$\equiv \neg\exists x\,(F(x) \wedge B(x))\text{ ; [Negation form]}$$

$$\equiv \forall x\,\neg\,(F(x) \wedge B(x))\text{ ; [Demorgan's Law]}$$

$$\equiv \forall x\,(\neg F(x) \vee \neg B(x))\text{ ; [Demorgan's Law]}$$

English text: No fish can speak in Bengali.

e) There exist a horse that can fly and catch bird as needed.

$H(x)$: x is a horse.

$F(x)$: x can fly as needed

$C(x)$: x can catch bird as needed.

Logical expression form:

$$\exists x\,(H(x) \wedge F(x) \wedge C(x))$$

$$\equiv \neg\exists x\,(H(x) \wedge F(x) \wedge C(x))\text{ ; [Negation form]}$$

$$\equiv \forall x\,\neg\,(H(x) \wedge F(x) \wedge C(x))\text{ ; [Demorgan's Law]}$$

$$\equiv \forall x\,(\neg H(x) \vee \neg F(x) \vee \neg C(x))\text{ [Demorgan's Law]}$$

English text: Every horse either cannot fly or it cannot catch bird as needed.

Problem-21

$Q(x,y)$: Student 'x' in ~~class~~ CSE173 class is a contestant on TV reality show.

domain for x: All students in CSE173 class

domain for y: All TV reality show.

a) There is a student in CSE173 who is a contestant on a TV reality show.

ans: Here the statement says there is a student, so we have to use existential quantifier, and the expression follows,

$$\exists x \exists y \; Q(x,y)$$

b) No student at CSE173 has ever been a contestant on a TV reality show.

ans: Here, no student, so we have to use universal quantifier and the expression follows

$$\forall x \forall y \; \neg Q(x,y)$$

c) There is a student at CSE173 who is a contestant on Close-up and Bangladeshi Idol.

Here, there is a student, we have to use existential quantification so the expression follows,

$$\exists x \exists y\, Q(x, \text{Close-up}) \land Q(x, \text{Bangladesh Idol})$$

d) Every TV reality show aired so far, had a student from CSE173 as a contestant.

Here, Every tv reality show, we have to use universal quantifiers for y and A student so we have to use existential quantifiers for x, So the expression follows,

$$\forall y \exists x\, Q(x, y)$$

e) At least two students from CSE173 are the contestants on Bangladeshi Idol.

Here, At least two students, so we have to use existential quantifier, the expression follows

$$\exists x \exists z\, (x \neq z)\, Q(x, \text{Bangladeshi Idol}) \land Q(z, \text{Bangladeshi Idol})$$

Problem-3

a) $\forall x [P(x) \vee Q(x)]$

$\equiv \neg \forall x [P(x) \vee Q(x)]$; [Negation form]

$\equiv \exists x \neg [P(x) \vee Q(x)]$; [Demorgan's Law]

$\equiv \exists x [\neg P(x) \wedge \neg Q(x)]$; [Demorgan's Law]

b) $\exists y [P(y) \vee Q(y) \vee R(y)]$

$\equiv \neg \exists y [P(y) \vee Q(y) \vee R(y)]$; [Negation form]

$\equiv \forall x \neg [P(y) \vee Q(y) \vee R(y)]$; [Demorgan's Law]

$\equiv \forall x [\neg P(y) \wedge \neg Q(y) \wedge \neg R(y)]$; [Demorgan's Law]

c) $\exists x [(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$

$\equiv \neg \exists x [(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$; [Negation form]

$\equiv \forall x \neg [(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$; [Demorgan's Law]

$\equiv \forall x [\neg (P(x) \wedge Q(x)) \wedge \neg (Q(x) \wedge \neg P(x))]$; [Demorgn's Law]

$\equiv \forall x\,[(\neg P(x) \vee \neg Q(x)) \wedge (\neg Q(x) \vee \neg(\neg P(x)))]$; [Demorgan's Law]

$\equiv \forall x\,[(\neg P(x) \vee \neg Q(x)) \wedge (\neg Q(x) \vee P(x))]$; [Double negation]

Problem 4|

a) If m and n both are negetive, their product is always positive.

ans:
As there is no restriction for m an n. It requires a ~~a~~ universal quantifier. The expression is as follows,

~~$\forall m \forall n\ (m<0$ and $n<0$~~

$\forall m \forall n\ (((m<0) \wedge (n<0)) \rightarrow mn>0)$

b) Assume m and n are positive, then average of m and n positive.

ans: As ~~of~~ m and n is positive, it requires a universal quantifiers the exact expression follow

~~$\forall m \forall n\ (m<0) \wedge n$~~

$\forall m \forall n\ (((m>0) \wedge (n>0)) \rightarrow \frac{(m+n)}{2} > 0)$

c) If m and n are negetive, m-n is not necessarily negative.

Here, (m-n) not nessisarily negetive. ~~But~~ This refers to an existential quantifier. It's expression:

$$\exists m \exists n \left(\left((m<0) \wedge (m<0) \right) \wedge \neg (m-n) > 0 \right)$$

H.W-3

Question 1 | Given the equation $2x^2+5y^2=14$ prove that, no integer solution exists for x and y.

Ans:

To prove it, we need to check a few possible solutions. For $|x|\geq 3$, $2x^2>14$ and hence we know that any possible solution has $|x|\leq 2$. Furthermore, if $|y|\geq 2$, $5y^2>14$ and hence we know that any possible solutions has $|y|\leq 1$. As we are squaring both ~~sides~~ x any, if (x,y) is a solution, then the four possible combination of signs with $(\pm x, \pm y)$ also accurs as solution. Thus we need to check with $0\leq x\leq 2$ and $0\leq y\leq 1$, possible cases are,

case 1: $x=0$, $y=0$

so, $2x^2+5y^2=2\times 0^2+5\times 0^2=0\neq 14$; [No solution]

case 2: $x=0$, $y=1$

so, $2x^2+5y^2=2\times 0^2+5\times 1^2=5\neq 14$, [No solution]

case 3: $x=1$, $y=0$

so, $2x^2+5y^2=2\times 1^2+5\times 0^2=2\neq 14$ [No solution] case 4] $x=1, y=1$

so, $2x^2+5y^2 = 2\times1^2+5\times1^2 = 2+5 = 7 \neq 14$ [No solution]

case 5] $x=2, y=0$

so, $2x^2+5y^2 = 2\times2^2+5\times0^2 = 8 \neq 14$ [No solution]

case 6] $x=2, y=1$

so, $2x^2+5y^2 = 2\times2^2+5\times1^2 = 8+5 \neq 13$ [No solution]

This cover all the cases, and we have proved that $2x^2+5y^2 = 14$ has no integer solutions. by cases.

Question 02 | Prove that if x^2-2a+7 is even, then x is odd.

Here, $p \to q$

p: ~~xxx~~ x^2-2a+7 is even

q: x is odd.

Contrapositive form of $p \to q$ is $\neg q \to \neg p$

$\neg q$: x is not odd $\equiv$ x is even

$\neg p$: x^2-2a+7 is not even $\equiv$ x^2-2a+7 is odd

So, we have to show that if x is even then x^2-2a+7 is odd.

As, x is even,

Let, $x = 2j$; j is an integer

Now, $x^2-2a+7 = (2j)^2 - 2a+7$

$= 4j^2 - 2a + 6+1$

$= 2(2j^2-a+3)+1$; ~~a~~ $(2j^2-a+3=K=$ integer$)$

$= 2 \times K+1$

$=$ odd

$\therefore$ the statement is proved by contrapositive sense.

Question-03| Use contrapositive method to prove the statement: If $3n^2+4n+1$ is even, then $3n+1$ is even or $n+1$ is even.

Here, $p \to (q \vee r)$

p: $3n^2+4n+1$ is even

q: $3n+1$ is even

r: $n+1$ is even

Contrapositive form of $p \to (q \vee r)$ is $\neg (q \vee r) \to \neg p$

$\neg p$: $3n^2+4n+1$ is not even $\equiv$ $3n^2+4n+1$ is odd.

$\neg q$: $3n+1$ is not even $\equiv$ $3n+1$ is odd

$\neg r$: $n+1$ is not even $\equiv$ $n+1$ is odd.

Now, we need to show that, If $3n+1$ is odd or $n+1$ is odd, then $3n^2+4n+1$ is odd.

Let,

$3n+1 = 2k+1$; k is an integer —①

$n+1 = 2l+1$; l is an integer —②

Now, $3n^2+4n+1 = 3n^2+3n+n+1$

$= 3n(n+1)+1(n+1)$

$= (3n+1)(n+1)$

$= (2k+1)(2l+1)$ from ① and ②

$= 4kl + 2k + 2l + 1$

$= 2(2kl + k + l) + 1$

$= 2j + 1$; $(2kl + k + l = k = \text{integer})$

$= \text{odd}$

∴ The statement is proved by contrapositive sence.

Question-4| Prove that, no natural number n both even and odd.

Here, we need to apply proof by contradiction.

∴ Negation form of the statement: There exist a natural number n ~~which~~ such that 'n' is both even and odd.

$n = 2k$; $k = \text{an integer}$; [even]

$n = 2l + 1$; $l = \text{an integer}$; [odd]

So, therefore,

$$2k = 2l + 1$$

$$\Rightarrow 2k - 2l = 1$$

$$\Rightarrow 2(k-l) = 1$$

$\Rightarrow 2j = 1$; $[(k-l) = j$ = an integer$]$

$\Rightarrow$ even = odd ; [$2j$ = even and 1 = odd]

[Not possible]

Thus, "There exist a natural numbe 'n' such that 'n' is both ~~odd~~ even and odd." the wrong statement."

$\therefore$ No natural number n is both even and odd is proved by contradiction.

Question.5a) $1+3+5+\ldots+(2n-1)=n^2; \forall n \geq 1$

Basic step:

Let; $n=1$

$L.H.S = 2n-1$

$= 2\times 1 - 1 = 1$

$R.H.S = n^2 = 1^2 = 1$

$\because L.H.S = R.H.S$

$\therefore$ the statement is true.

Inductive step:

Let's assume for $n=k$ the statement is true.

$\therefore 1+3+5+\ldots+(2k-1) = k^2$ —— (1) (Inductive hypothesis)

Now, we have to show that, for $n=k+1$ the statement is true.

$1+3+5+\ldots+\{(2(k+1)-1\} = (k+1)^2$

$\Rightarrow 1+3+5+\ldots+(2k+1) = (k+1)^2$

$\Rightarrow 1+3+5+\ldots+(2k-1)+(2k+1) = (k+1)^2$

$\Rightarrow k^2+(2k+1) = (k+1)^2$; [from (1)]

$= (k+1)^2 = (k+1)^2$

$\because L.H.S = R.H.S$

∴ The statement is true, proved by mathematical induction process.

5b) $4^n - 1$ is a multiple of 3; $\forall n \geq 1$

Basic step:

Let; $n = 1$

So, $4^n - 1$

$= 4^1 - 1$

$= 3$

here, 3 is divisible by 3

∴ $4^n - 1$ is a multiple by 3

∴ the statement is true.

Inductive step:

Let's assume that for $n = k$ the statement is true.

∴ $4^k - 1$ is a multiple by 3.

∴ $4^k - 1 = 3m$; m is an integer (Inductive hypothesis) — ①

Now have to show that for $n=k+1$

the statement is true.

$$4^{k+1}-1 = 4^k.4^1-1$$
$$= 4^k(3+1)-1$$
$$= 3.4^k+4^k-1$$
$$= 3.4^k+3m \text{; from } 1$$
$$= 3(4^k+m)$$
$$= 3l \quad (4^k+m) = l = \text{integer})$$

= is multiple by 3

The statement is true proved by mathematical induction process.

5. c) $2+4+6+\dots+2n = n(n+1);\ \forall n\geq 1$

Basis step:

Let $n=1$

$$L.H.S = 2n = 2\times 1 = 2$$

$$R.H.S = n(n+1) = 1\times(1+1)$$
$$= 1\times 2$$
$$= 2$$

$\because L.H.S = R.H.S$

$\therefore$ The statement is true.

Inductive step:

Let's assume that for $n=k$ the statement is true.

$$2+4+6+8\ldots\ldots+2k = k(k+1) \quad \text{①} \text{(Inductive hypothesis)}$$

Now we have to show that for $n=k+1$ the statement is true.

$$2+4+6+\ldots\ldots+2(k+1) = (k+1)(k+1+1)$$

$$\Rightarrow 2+4+6+\ldots\ldots+(2k+2) = (k+1)(k+2)$$

$$\Rightarrow 2+4+6+\ldots\ldots+2k+(2k+2) = (k+1)(k+2)$$

$$\Rightarrow \cancel{2+4+6+} \, k(k+1)+(2k+2) = (k+1)(k+2) \text{ from ①}$$

$$\Rightarrow k(k+1)+2(k+1) = (k+1)(k+2)$$

$$\Rightarrow (k+1)(k+2) = (k+1)(k+2)$$

$\because$ L.H.S = R.H.S

$\therefore$ The statement is true proved by

$\therefore$ ~~the~~ mathematical induction process.

5] d) $-1+2+5+8+\ldots+(3n-4) = (n/2)(3n-5)$; $\forall n \geq 1$

<u>Basic step:</u>

Let, $n = 1$

$L.H.S = 3n-4$
$= 3\times 1 - 4$
$= -1$

$R.H.S = (n/2).(3n-5)$
$= (1/2)\times(3\times 1-5)$
$= 1/2 \times (-2)$
$= -1$

~~R.T~~ $\because$ L.H.S = R.H.S

$\therefore$ The statement is true.

<u>Inductive step:</u>

Let's assume that for $n=k$ the statement is true

$-1+2+5+8+\ldots+(3k-4) = (k/2)(3k-5)$ — (1) (Inductive hypothesis)

Now, we need to show that for $n=k+1$ the statement is true.

$-1+2+5+8+ \dots + [3(k+1)-4] = \frac{(k+1)}{2} \times [3(k+1)-5]$

$\Rightarrow -1+2+5+8+ \dots + [3k+3-4] = \frac{(k+1)}{2} \times [3k-2]$

$\Rightarrow -1+2+5+8+ \dots + (3k-1) = \frac{(k+1)}{2} \times (3k-2)$

$\Rightarrow -1+2+5+8+ \dots + (3k-4)+(3k-1) = \frac{(k+1)}{2} \times (3k-2)$

$\Rightarrow (k/2) \times (3k-5) + (3k-1) = \frac{(k+1)}{2} \times (3k-2)$, from (1)

$\Rightarrow \frac{k(3k-5)+2(3k-1)}{2} = \frac{(k+1)(3k-2)}{2}$

$\Rightarrow k(3k-5)+2(3k-1) = (k+1)(3k-2)$

$\Rightarrow 3k^2-5k+6k-2 = (k+1)(3k-2)$

$\Rightarrow 3k^2+k-2 = (k+1)(3k-2)$

$\Rightarrow 3k^2+3k-k-2 = (k+1)(3k-2)$

$\Rightarrow 3k(k+1)-2(k+1) = (k+1)(3k-2)$

$\Rightarrow (k+1)(3k-2) = (k+1)(3k-2)$

$\because$ L.H.S = R.H.S

$\therefore$ The statement is true proved by the mathematical induction process.