Problem 1: Determine if the below conditional statements are True or False.

(a) If $2 + 7 = 6$, then crocodiles can fly.
Here, p is False, q is False. So, from the truth table of $p \to q$ suggests that the statement is TRUE.

(b) If $5 + 5 = 10$, then dogs can talk like humans.
Here, p is True, q is False. So, from the truth table of $p \to q$ suggests that the statement is FALSE.

(c) If -3 is a negative number, then birds can fly.
Here, p is True, q is True. So, from the truth table of $p \to q$ suggests that the statement is TRUE.

(d) If $1 + 1 = 2$, then $5 + 7 = 12$.
Here, p is True, q is True. So, from the truth table of $p \to q$ suggests that the statement is TRUE.

Problem 2: Let us assume that p and q are two propositions. Using p and q you are asked to form a number of compound proposition as shown in Table 2. Fill up the Truth Table for all the propostions.

p	q	$\neg(\neg p \wedge q)$	$(p \vee q) \leftrightarrow \neg(\neg p \wedge q)$	$\neg(p \wedge \neg q)$	$(p \to q) \leftrightarrow \neg(p \wedge \neg q)$
T	T	T	T	T	T
T	F	T	T	F	T
F	T	T	T	T	T
F	F	F	T	T	T

Table 1: Problem 2

Problem 3: Translate the bellow English sentences into propositional logics, making the propositional variables as clear as possible.

(a) Neither the thunderstorm nor the heavy rain did any damage to the house.
Solution: Assume p and q as the two propositions stating " p:*Thunderstorm did damage to the house*" and "q:*The heavy rain did damage to the house*". As stated, both thunderstorm and heavy rain can damage the house, therefore, $p \vee q$ denotes the combined possibility of damage. Finally, "Neither ... nor" requires a negation of $p \vee q$: That is, $\neg(p \vee q) \equiv \neg(p) \wedge \neg(q)$.

(b) If global warming is not controlled, low-lying land will go under water within the next few decades.
Solution: Here, p: "*Global warming isn't controlled*", and q stands for: "*low-lying land will go under water within the next few decades*". The compound propostion is in "if p, then q" form, which suggests the expression to be $p \to q \equiv \neg p \vee q$

(c) Uber/Pathao driver should not drive more than 60 miles per hour nor violate traffic signals, or they will be penalized.
Solution: A Uber/Pathao driver can have two different faults, and each fault should be treated as separate propostion. That is, p: *Uber driver drives more than 60 miles per hour* and q: *Uber driver violates traffic signals* are the propostions, and they should be combined using $\vee$ operation. So, $\neg(p \vee q)$ realizes all the possible dirver's fault. Let's define another propostion, stating p:*Drivers will be penalized.* Finally, disjuction (as we have "or") of r with $\neg(p \vee q)$ provides us the exact expression $\neg(p \vee q) \vee r \equiv (p \vee q) \to r$.

Problem 4: Show if the compound proposition $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology or a contradiction.
Solution: Develop the Truth-Table for the expression provided. It turns out as a Tautology– that is, all the outcomes have True (T) values regardless of the combinations of p, q, r; leaving the details of the truth table for your practice

Problem 5: Suppose that you are doing a part-time job at the library at NSU. Students visited the library often leave books on tables. Librarians asked you to check the condition of all the books, and the below statements are made available for you. Use Predicate, Quantifiers, and Logical Operators to represent the statements made by the librarians:
We define: i) $R(x)$: Book x is at the right place, ii) $E(x)$: Book x is in excellent condition

(a) Some books are not at the right place.
Because of "some", we use Existential Quantifier ($\exists$). The expression goes as follows: $\exists x(\neg(C(x)))$; this reads as, there exists at least one x book that is not at the right place.

(b) All books are at the right place and are in excellent condition.
$\forall x(R(x) \wedge E(x))$

(c) Everybook is in the right place and is in excellent condition.
$\forall x(R(x) \wedge E(x))$. Number b and c are shown similar here. However, if we assume a domain that includes all things, we would have different expression for this two problems. Again, domain may decide the types of operators we need to connect the propositions of interests.

(d) Nothing in the library is at the right place and is in excellent condition.
This is exactly the opposite of *Everybook is in the right place and is in excellent condition.* So, we take the negation ($\neg$) of " right place and is in excellent condition", and we obtain:
$\forall x \neg(R(x) \wedge E(x))$

(e) One of the books is not in the right place, but it is in excellent condition.
$\exists x(\neg R(x) \wedge E(x))$

Problem 6: Write a compound proposition involving the propositional variables p, q and r that is true when p or q are true and r is false; the proposition is false otherwise.
Solution: p or q true requires $p \vee q$ to be true. As it requires "and r is false", we need conjunction ($\wedge$) of negation of r with the preceding criteria for p and q. Together, we need $(p \vee q) \wedge \neg(r)$ to obtain the desired compound proposition.