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Section: 02

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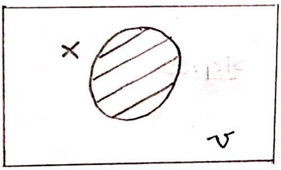
H.W-03

Answer to the que No - 1

Venn Diagrams:

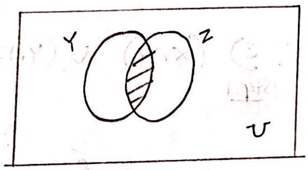
a) $X \cap (Y \cap Z)$

Step: 01



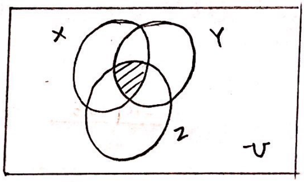
X

Step: 02



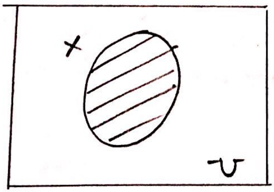
$Y \cap Z$

Step: 03

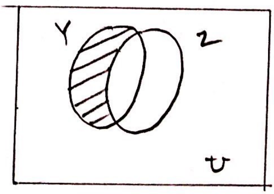


$X \cap (Y \cap Z)$

b) $X \cap (Y - Z)$

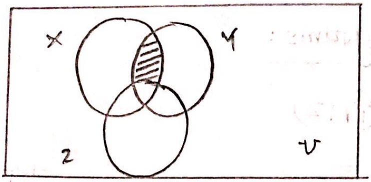


X.



$Y - Z$

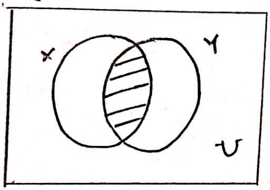
Step:03



$X \cap (Y - Z)$

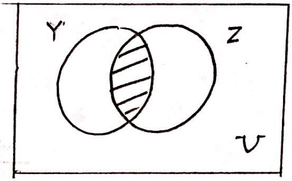
c) $(X \cap Y) \cup (Y \cap Z)$

Step:1



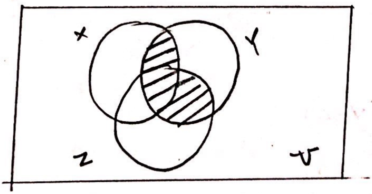
$X \cap Y$

Step:02



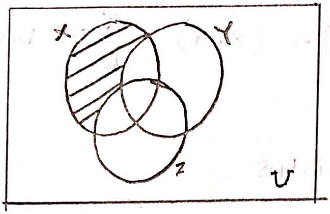
$Y \cap Z$

Step:03



$(X \cap Y) \cup (Y \cap Z)$

d) $(X-Y-Z)$



$(X-Y-Z)$

Ans to the que NO- 2

a) Using set buider notation to prove,

$X \oplus Y = (X \cup Y) - (X \cap Y)$

L.H.S $= X \oplus Y$

$\equiv \{A \mid A \in X \oplus Y\}$; [Set builder notation]

$\equiv \{A \mid (A \in X \text{ or } A \in Y) \text{ and } (A \notin X \text{ and } A \notin Y)\}$;
[defination of '$\oplus$']

$\equiv \{A \mid (A \in X \cup Y) \text{ and} \neg (A \in X \text{ and } A \in Y)\}$;
[definatiof of '$\cup$' and defination of '$\notin$']

$\equiv \{A \mid (A \in X \cup Y) \cap \neg (A \in X \cap Y)\}$;
[defination of '$\cap$']

$\equiv \{A \mid (A \in X \cup Y) \setminus (A \in X \cap Y)\}$; [defination of '$\setminus$']

$\equiv (X \cup Y) - (X \cap Y)$; [set builder notation]

$\equiv$ R.H.S

$\therefore$ L.H.S = R.H.S

$\therefore X \oplus Y = (X \cup Y) - (X \cap Y)$ [proved]

b) prove, $(Y-X) \cup (Z-X) = (Y \cup Z) - X$ by using set builder notation.

L.H.S = $(Y-X) \cup (Z-X)$

$\equiv \{A \mid A \in (Y-X) \cup (Z-X)\}$; [set builder notation]

$\equiv \{A \mid (A \in Y \text{ and } A \notin X) \text{ or } (A \in Z \text{ and } A \notin X)\}$;

[defination of '$\setminus$']

$\equiv \{A \mid (A \in Y \text{ or } A \in Z) \text{ and } (A \notin X \text{ or } \cancel{\text{and}}\ A \notin X)\}$;

[distribution Law]

$\equiv \{A \mid (A \in Y \cup Z) \text{ and } (A \notin X)\}$; [defination of $\cup$ and Idempotent Law of set]

$\equiv \{A \mid (A \in Y \cup Z) \cap \neg (A \in X)\}$; [defination of '∩' and defination of '∉']

$\equiv \{A \mid (A \in Y \cup Z) \setminus (A \in X)\}$; [defination of '∖']

$\equiv \cancel{\{A \mid A \in} (Y \cup Z) - X$; [set builder notation]

$\equiv$ R.H.S

$\because$ L.H.S = R.H.S

$\therefore \cancel{(Y \cup Z)}\ (Y - X) \cup (Z - X) = (Y \cup Z) - X$

[proved]

Ans to the que NO-3

Given $f: \mathbb{Z} \to \mathbb{Z}$ ($\mathbb{Z}$ = integer number set)

and $f(x) = 6x$

Domain x	Co-domain $f(x)=6x$
⋮	⋮
-5	-30
-4	-24
-3	-18
-2	-12
-1	-6
0	0
1	6
2	12
3	18
4	24
5	30
⋮	⋮

unused ~~doma~~ co-domain elements
⋮
-5
-4
-3
-2
-1
1
2
3
4
5
⋮

∴ Domain: Integer number set $\mathbb{Z}$

∴ Codomain: Interger set multiple of 6 ($6\mathbb{Z}$)

$\Rightarrow \{\ldots -30, -24, -18, -12, -6, 0, 6, 12, 18, 24, 30, \ldots\ldots\}$

One-to-one function: As ~~Every~~ each and every element of domain have a single distinct image in co domain. ~~Therefor it is~~ So, $f(x) = 6x$ is a one-to-one function.

Onto function: As All the co domain elements ~~are not~~ don't have a pre-image in domain. So, $f(x) = 6x$ is not a onto function.

One-to-one and Onto both/ Bijection function: As it satisfy the criteria of one-to-one but not onto. So it is not a bijection function.

Therefore, ~~f(x)~~ $f: Z \to Z$; $f(x) = 6x$ is a One-to-one function but not onto and bijection function.

Answer: Domain: set of integers (Z)

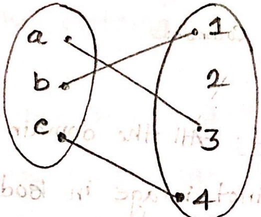
Co-domain: multiple of 6 integers set ($6Z$)

$\{ \ldots -30, -24, -18, -12, -6, 0, 6, 12, 18, 24, 30, \ldots \}$

$f: Z \to Z : f(x) = 6x$ is one-to-one function but not onto and bijection function.

Ans to the que NO - 4

i) An example of one-to-one but not-Onto function:



Domain: A   co-domain B

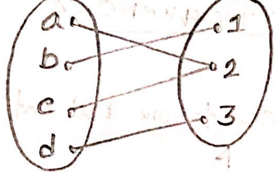
Explanation:

* One-to-one: All the domain elements of A have a distinct image in co-domain B. So it ~~is a~~ satisfy one-to-one function. criteria.

* Not onto~~-one~~: Here, each an every element of codomain B ~~is~~ are not the image of domain A elements. Exception is 2. It is not an image of any domain elements. so it is not satisfy the criteria of Onto function.

∴ It is a One-to-one function but Onto function.

(ii) An example of Onto but not One-to-One function:



Domain: A Co-domain: B

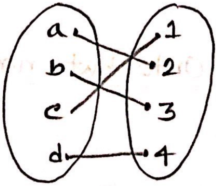
Explanation:

* **Not one-to-one:** All the domain elements of A don't have distinct image in codomain elements in B. Here, a and c have a same image 2. so it is not satisfy ~~a~~ one-to-one function. criteria.

* **Onto function:** Here each an every elements of codomain B are the image of domain elements of A. So, it is satisfy ~~a~~ Onto function. criteria.

So. the example is Onto but not one-to-one function.

(iii) Both One-to-one and ~~on-to~~ Onto function example:



A B

Domain co-domain

Explanation:

*One-to-one: Here, all the domain elements of A have ~~an~~ a distinct image in codomain B. So it satisfy the criteria of one-to-one function.

*Onto function: Here, all the codomain elements of B ~~have~~ is a the ~~img~~ image of domain elements of A. so it satisfy the criteria of Onto function.

∴ The example is ~~Ont~~ Both One-to-one and Onto function.

Answer to the question-5

Given $f(x) = x^2+1$

$g(x) = x+2$

$\therefore fog(x) = f(g(x))$

$= f(x+2)$

$= (x+2)^2+1$

$= x^2+4x+4+1$

$= x^2+4x+5$

Now,

$gof(x) = g(f(x))$

$= g(x^2+1)$

$= x^2+1+2$

$= x^2+3$

$\because fog(x) \neq gof(x)$

$\therefore$ $fog(x)$ and $gof(x)$ are not equal for the given $f(x)$ and $g(x)$

Answer: $fog(x) = x^2+4x+5$

$gof(x) = x^2+3$

$fog(x)$ and $gof(x)$ are not equal

Answer to the que No-6

Given,

$$f(x) = \frac{x+4}{2x-5}$$

Now,

$$y = \frac{x+4}{2x-5}$$

$$\Rightarrow y(2x-5) = x+4$$

$$\Rightarrow 2xy - 5y = x+4$$

$$\Rightarrow 2xy - x = 5y+4$$

$$\Rightarrow x(2y-1) = 5y+4$$

$$\Rightarrow x = \frac{5y+4}{2y-1}$$

$\Rightarrow$ Now, replace x by y and y by x,

$$\Rightarrow y = \frac{5x+4}{2x-1}$$

$$\therefore f^{-1}(x) = \frac{5x+4}{2x-1}$$

Answer: Inverse function $f^{-1}(x) = \frac{5x+4}{2x-1}$

Answer to the que NO-7

Given,

$$H(m) = 3m-2$$

Now,

$$n = 3m-2$$

$$\Rightarrow 3m = n+2$$

$$\Rightarrow m = \frac{n+2}{3}$$

Now, replace m by n and n by m.

$$\Rightarrow n = \frac{m+2}{3}$$

$$\therefore H^{-1}(m) = \frac{m+2}{3}$$

L.H.S =

$$\therefore HoH^{-1}(m) = H\left(H^{-1}(m)\right)$$

$$\Rightarrow H\left(\frac{m+2}{3}\right)$$

$$\Rightarrow 3\times\left(\frac{m+2}{3}\right) - 2$$

$$\Rightarrow m+2-2$$

$$\Rightarrow m = \text{R.H.S}$$

$\therefore$ L.H.S = R.H.S

$\therefore$ $HoH^{-1}(m) = m$ [Showed]

Answer: $H^{-1}(m) = \frac{m+2}{3}$

and, $HoH^{-1}(m) = m$ [Showed]

Functions (2.3)

21 a) $f: \mathbb{Z} \to R$

$f(n) = \pm n$

Let, $n = -2$, $f(-2) = \pm 2$

$n = -1$, $f(-1) = \pm 1$

$n = 0$, $f(0) = 0$

$n = 1$, $f(1) = \pm 1$

$n = 2$, $f(2) = \pm 2$

- - - ——— - - -

Here we can see, single elements in domain have multiple images in codomain. so it is not a function.

Answer: $f: \mathbb{Z} \to \mathbb{R}$

$f(n) = \pm n$ is not a function.

2) $f: \mathbb{Z} \rightarrow \mathbb{R}$

$f(n) = \sqrt{n^2+1}$

Let,

$\ldots \ldots \ldots$

$n = -2$, $f(-2) = \sqrt{(-2)^2+1} = \sqrt{5}$

$n = -1$; $f(-1) = \sqrt{(-1)^2+1} = \sqrt{2}$

$n = 0$; $f(0) = \sqrt{0^2+1} = \sqrt{1} = 1$

$n = 1$; $f(1) = \sqrt{1^2+1} = \sqrt{2}$

$n = 2$; $f(2) = \sqrt{2^2+1} = \sqrt{5}$

$\ldots \ldots \ldots$

$\ldots \ldots \ldots$

$\because$ Here we can see every domain elements have a single image in codomain.

$\therefore$ $f: \mathbb{Z} \rightarrow \mathbb{R}$ is a function.

$f(n) = \sqrt{n^2+1}$

Answer: $f: \mathbb{Z} \rightarrow \mathbb{R}$; $f(n) = \sqrt{n^2+1}$ is a function.

12) $f: \mathbb{Z} \to \mathbb{Z}$

a) $f(n) = n-1$

Let,

- - - - - - - - -

- - - - - - - - -

$n=-2$; $f(-2) = -2-1 = -3$

$n=-1$; $f(-1) = -1-1 = -2$

$n=0$; $f(0) = 0-1 = -1$

$n=1$; $f(1) = 1-1 = 0$

$n=2$; $f(2) = 2-1 = 1$

- - - - - - - - - -

- - - - - - - - - -

here we can see the every domain elements has single distinct image in codomain. So it is an one-to-one function.

$f: \mathbb{Z} \to \mathbb{Z}$

$\therefore$ $f(n) = n-1$ is one-to-one function.

b) $f(n) = n^2+1$

Let,

- - - - - - - - - -

- - - - - - - - - -

$n=-2$; $f(-2) = (-2)^2+1 = 5$

$n=-1$; $f(-1) = (-1)^2+1 = 2$

$n=0$; $f(0) = 0^2+1 = 1$

$n = 1;\ f(1) = 1^2 + 1 = 2$

$n = 2;\ f(2) = 2^2 + 1 = 5$

- - - - - . - - - - - - -

Here, we can see the every domain elements have a multiple imagesin codomain. So it is not a one-to-one function.

$f: \mathbb{Z} - \mathbb{Z}$

$\therefore f(n) = n^2 + 1$ is not one-to-one function.

~~c) $f(n) = n^3$~~

13| a) $f: \mathbb{Z} \to \mathbb{Z}$

$f(n) = n - 1$

we can see from 12a) that every codomain element have an pre-image in domain. So it is a onto function.

13|b| $f: \mathbb{Z} \to \mathbb{Z}$

$f(n) = n^2 + 1$

we can see from 12b) that here codomain elements don't have a distinct pre-image in domain. $\therefore f(n) = n^2 + 1$ is not a onto function.

15| $f: Z \times Z \to Z$

a) $f(m,n) = m+n$

Let,

$m=0, n=-1 \therefore f(0,-1) = 0+(-1) = -1$

$m=0, n=1 \therefore f(0,1) = 0+1 = 1$

$m=0; n=-2; f(0,-2) = 0+(-2) = -2$

$m=0; n=2 \quad f(0,2) = 0+2 = 2$

- - - - - - - - - -

$\therefore$ for any integer n, we have $f(0,n)$, so the function is Onto.

b) $f(m,n) = m^2 - n^2$

Let,

$m=0; n=-1 \quad f(0,-1) = 0^2 + (-1)^2 = 1$

$m=0, n=1 \quad f(0,1) = 0^2 + 1^2 = 1$

$m=0, n=-2 \quad f(0,-2) = 0^2 + (-2)^2 = 4$

$m=0, n=2 \quad f(0,2) = 0^2 + 2^2 = 4$

$\therefore$ Clearly the rang contains no negative integers.

$\therefore$ rang $\neq$ codomain

$\therefore$ This is not Onto function.

21| $f: \mathbb{Z} \to \mathbb{Z}^+$

b) ~~One to One~~ Onto but not one-to-one

We can take $f(x) = |x| + 1$

here,

$x = -2$; $f(-2) = |-2| + 1 = 3$

$x = -1$; $f(-1) = |-1| + 1 = 2$

$x = 0$; $f(0) = |0| + 1 = 1$

$x = 1$; $f(1) = |1| + 1 = 2$

$x = 2$; $f(2) = 2 + 1 = 3$

here, Every codomain ~~domain~~ elements have multiple ~~images~~ pre-image in domain so it is an onto function.

But, the domain elements don't have single distinct image in codomain.

$\therefore f: \mathbb{Z} \to \mathbb{Z}^+$; $f(x) = |x| + 1$ is

Onto but not One to One.

d) neither one to one nor onto.

$f: \mathbb{Z} \to \mathbb{Z}^+$

here we can use a trivial example $f(n) = 17$

or a simple non ~~trivial~~ trivial on like ~~x^2+1~~ $f(x) = x^2+1$

here, - - - - - - - - - -

$n = -1 \; f(-1) = (-1)^2 + 1 = 2$

$n = -2 = f(-2) = (-2)^2 + 1 = 5$

$n = 0 \Rightarrow f(0) = 0^2 + 1 = 1$

$n = 1 \; ; f(1) = 1^2 + 1 = 2$

$n = 2; \; f(2) = 2^2 + 1 = 5$

- - - - - - - - - -

here, domain elements don't have single element in codomain so it is not on-to. Again, Every codomain element don't have preimage in codomin.

$\therefore f: \mathbb{Z} \to \mathbb{Z}^+$

$f(x) = x^2+1$ is either one-to-one nor onto.

23) $f: \mathbb{R} \to \mathbb{R}$

a) $f(x) = 2x+1$

Let,

- - - - - - - - - - -

$n=-2$; $f(-2) = 2\times(-2)+1 = -4+1 = -3$

$n=-1$; $f(-1) = 2\times(-1)+1 = -2+1 = -1$

$n=0$; $f(0) = 2\times0+1 = 1$

$n=1$; $f(1) = 2\times1+1 = 3$

$n=2$; $f(2) = 2\times2+1 = 5$

- - - - - - - - - - -

Here, we can see, ~~for~~ every domain element has a single distinct image in codomain. So it satisfy the criteria of one-to-one function.

Again, Every codomain elements has a pre-imain in domain. So it satisfy the criteria onto function.

As it satisfy the criteria of both one-to-one ~~function~~ and Onto function. So ~~it is an~~ this function is a bijection ~~fo~~ from $\mathbb{R}$ to $\mathbb{R}$

(c) $f(x) = x^3 \quad f: \mathbb{R} \to \mathbb{R}$

Let, $n = -2$; $f(-2) = (-2)^3 = -8$

$n = -1$; $f(-1) = (-1)^3 = -1$

$n = 0$; $f(0) = 0^3 = 0$

$n = 1$; $f(1) = 1^3 = 1$.

$n = 2$; $f(2) = 2^3 = 8$

We can see, every domain elements ~~n~~ have a single distinct image in codomain. so it satisfy the criteria of one-to-one function.

Every Codomain elements have an pre-image in domain. so it also satisfy the criteria of Onto function.

$\therefore$ As it satisfy both one-to-one and onto function criteria. $f(x) = x^3$ is a bijection from R to R.

25] $f: \mathbb{R} \to \mathbb{R}$

Let, $f(x) > 0$ for all $x \in \mathbb{R}$

The key here is that larger denominator make smaller fractions. and smaller denominator make larger fractions. We have two things to prove since this is an "if and only if" statement:

first suppose that f is strictly decreasing.

This means that $f(x) > f(y)$ whenever $x < y$.

To show that g is strictly increasing, suppose that $x < y$. Then $g(x) = \frac{1}{f(x)} < \frac{1}{f(y)} = g(y)$

Conversely, suppose that g is strictly increasing.

This means that $g(x) < g(y)$ whenever ~~φ~~ $x < y$.

To show f strictly increasing, ~~this mean~~ suppose $x < y$. Then $f(x) = \frac{1}{g(x)} > \frac{1}{g(y)} = f(y)$

30| Let, $S = \{-1, 0, 2, 4, 7\}$

a) $f(x) = 1$

$\therefore f(S) = \{1\}$

for all the elements of domain in set S

f is a constant function.

here, $f(-1) = 1$

$f(0) = 1$

$f(2) = 1$

$f(\cancel{7}4) = 1$

$f(7) = 1$

$\therefore f(x) = 1$; $f(S) = \{1\}$ constant function.

~~Answer~~: $f(S) = \{1\}$

b| $f(x) = 2x+1$

for set S,

$f(S) = f(-1) = 2\times(-1)+1 = -1$

$f(0) = 2\times 0 + 1 = 1$

$f(2) = 2\times 2+1 = 5$

$f(4) = 2\times 4+1 = 9$

$f(7) = 2\times 7+1 = 15$

$\therefore f(S) = \{-1, 1, 5, 9, 15\}$

Answer: $\{-1, 1, 5, 9, 15\}$

36) Given $f(x) = x^2+1$ $\quad f: \mathbb{R} \to \mathbb{R}$

$g(x) = x+2$

$$\therefore fog(x) = f(g(x))$$
$$= f(x+2)$$
$$= (x+2)^2+1$$
$$= x^2+2x+4+1$$
$$= x^2+2x+5$$

$$gof(x) = g(f(x))$$
$$= g(x^2+1)$$
$$\Rightarrow x^2+1+2$$
$$= x^2+3$$

<u>Answer:</u> $fog(x) = x^2+2x+5$

$gof(x) = x^2+3$

38) Let, $f(x) = ax+b$

$g(x) = cx+d$

$$\therefore fog = f(g(x)) = f(cx+d)$$
$$\Rightarrow a(cx+d)+b$$
$$\Rightarrow cax+ad+b$$

$$g o f = g(f(x))$$
$$= g(ax+b)$$
$$= c(ax+b)+d$$
$$= cax+bc+d$$

As,

$$f o g = g o f$$

$\therefore$ $cax+ad+b = cax+bc+d$

$\Rightarrow$ $\therefore$ $ad+b = bc+d$

$\therefore$ The necessary and sufficient condition on constants a, b, c and d is $ad+b = bc+d$.

As f or g is a idendity ~~dn~~ function,

ie $a=1$ and $b=0$ and $f(x)=x$

$$ad+b = bc+d$$
$$\Rightarrow ad-d = bc-b$$
$$\Rightarrow d(a-1) = b(c-1)$$
$$\therefore \frac{d}{c-1} = \frac{b}{a-1}$$

$\therefore$ conditions $\frac{d}{c-1} = \frac{b}{a-1}$; where $a=1$ and $b=0$ and $f(x)=x$

Ans.

39) $f(x) = ax+b \qquad f: \mathbb{R} \to \mathbb{R}$

$\Rightarrow y = ax+b$

$\Rightarrow ax = y-b$

$\Rightarrow x = \dfrac{y-b}{a} \quad ; \ a \neq 0$

Now replace x by y and y by x

$\Rightarrow y = \dfrac{x-b}{a}$

$\therefore f^{-1}(x) = \dfrac{x-b}{a}$

$\therefore$ inverse function, $f^{-1}(x) = \dfrac{x-b}{a}$

To check it is correct or not we need to show that: $f \circ f^{-1}(x) = x$ for all $x \in \mathbb{R}$

and also $f^{-1} \circ f(x) = x$ for all $x \in \mathbb{R}$

$\therefore f \circ f^{-1}(x) = f\left(\dfrac{x-b}{a}\right)$

$= a\left(\dfrac{x-b}{a}\right) + b$

$\Rightarrow x-b+b$

$\Rightarrow x$

$f^{-1} \circ f(x) = f^{-1}\left(\dfrac{x-b}{a}\right)$

$= f^{-1}(ax+b)$

$= \dfrac{ax+b-b}{a}$

$\Rightarrow \dfrac{ax}{a}$

$\Rightarrow x$

$\therefore f \circ f^{-1}(x) = x = f^{-1} \circ f$

$\therefore$ It is inverse of $f(x)$.

Chapter: 24

Sequence and summation

6) a) the sequence obtained by starting with and obtaining each term by subtracting 3 from the previous term.

$a_0 = 10$

$a_1 = 10 - 3 = 7$

$a_2 = a_1 - 3 = 4$

$a_3 = a_2 - 3 = 1$

$a_4 = a_3 - 3 = 1 - 3 = -2$

$a_5 = a_4 - 3 = -2 - 3 = -5$

$a_6 = a_5 - 3 = -5 - 3 = -8$

$a_7 = a_6 - 3 = -8 - 3 = -11$

$a_8 = a_7 - 3 = -11 - 3 = -14$

$a_9 = a_8 - 3 = -14 - 3 = -17$

$\therefore$ first 10 terms are, $10, 7, 1, -2, 5, -8, -11, -14, -17$

Answer: $10, 7, 1, -2, -5, -8, -11, -14, -17$

e) the sequence whose first 2 terms are 1 and 5 each succeeding term is the sum of the previous terms.

$a_0 = 1$

$a_1 = 5$

$a_2 = a_0 + a_1 = 1 + 5 = 6$

$a_3 = a_1 + a_2 = 5 + 6 = 11$

$a_4 = a_2 + a_3 = 6 + 11 = 17$

$a_5 = a_3 + a_4 = 11 + 17 = 28$

$a_6 = a_4 + a_5 = 17 + 28 = 45$

$a_7 = a_5 + a_6 = 45 + 28 = 73$

$a_8 = a_6 + a_7 = 73 + 45 = 118$

$a_9 = \cancel{a_5 + a_8}\ a_7 + a_8 = 73 + 118 = 191$

$\therefore$ first 10 terms of the sequence, 1, 5, 6, 11, 17, 28, 45, 73, 118, 191

Answer: 1, 5, 6, 11, 17, 28, 45, 73, 118, 191

9 a) $a_n = 6a_{n-1}$, $a_0 = 2$

$$a_1 = 6a_{1-1}$$

$$a_1 = 6a_0 = 6\times 2 = 12$$

$$a_2 = 6a_1 = 6\times 12 = \cancel{24}\ 72$$

$$a_3 = 6\times a_2 = 6\times \overset{72}{\cancel{24}} = \cancel{144}\ 432$$

$$a_4 = 6\times a_3 = 6\times \overset{432}{\cancel{144}} = \cancel{864}\ 2592$$

Answer: 2, 12, 72, 432, 2592

b) $a_n = a_{n-1}^2$, $a_1 = 2$

$$a_2 = a_1^2 = 2^2 = 4$$

$$a_3 = a_2^2 = 4^2 = 16$$

$$a_4 = a_3^2 = 16^2 = 256$$

$$a_5 = a_4^2 = 256^2 = 65536$$

Answer: 2, 4, 16, 256, 65536

~~12) $a_n = -3a_{n-1} + 4a_{n-2}$~~

~~a) $a_n = 0$~~

~~R.H.S. $= -3a_{n-1} + 4a_{n-2} = 0$~~

~~$\Rightarrow 3a_{n-1} = 4a_{n-2}$~~

$=$

[12] a) $a_n = 0$

$R.H.S = -3a_{n-1} + 4a_{n-2}$

$= -3 \times 0 + 4 \times 0$

$= 0 = L.H.S$

~~Answer~~: The sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if $a_n = 0$ [Showed]

b) $a_n = 1$

$R.H.S = \cancel{3a_{n-1}} -3a_{n-1} + 4a_{n-2}$

$= -3 \times 1 + 4 \times 1$

$= -3 + 4 = 1 = L.H.S$

$\therefore$ The sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if $a_n = 1$.

[Showed]

(d) 15 a) Given, $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$

$$a_{n-1} = -(n-1)+2 = -n+3$$

$$a_{n-2} = -(n-2)+2 = -n+4$$

$$a_{n-1} + 2a_{n-2} + 2n - 9 = -n+3-2n+8+2n-9$$
$$= -n+2$$
$$= a_n$$

[showed]

b) Given, $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$

$$a_{n-1} = 5(-1)^{n-1} - (n-1)+2$$
$$= -5(-1)^n - n + 3$$

$$a_{n-2} = 5(-1)^{n-2} - (n-2) + 2$$
$$= 5(-1)^n - n + 4$$

$$a_{n-1} + 2a_{n-2} + 2n - 9 = -5(-1)^n - n + 3 + 2 \cdot 5(-1)^n - 2n + 8$$
$$+ 2n - 9 = 5(-1)^n - n + 2$$
$$= a_n$$

[showed]

26| a) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102 ....

The pattern is $a_n = n^2 + 2$

∴ the next three terms are,

~~$a_{10} = 10^2 + 2 =$~~

$a_{11} = 11^2 + 2 = 121 + 2 = 123$

$a_{12} = 12^2 + 2 = 144 + 2 = 146$

$a_{13} = 13^2 + 2 = 169 + 2 = 171$

Answer: 123, 146, 171

b| 7, 11, 15, 19, 23, 27, 31, 35, 39, 43

The pattern is $a_n = 4n + 3$

$a_{11} = 4 \times 11 + 3 = 47$

$a_{12} = 4 \times 12 + 3 = 51$

$a_{13} = 4 \times 13 + 3 = 55$

Answer: 47, 51, 55

29) a) $\sum_{k=1}^{5} (k+1)$

$\Rightarrow 2+3+4+5+6 = 20$ Answer: 20

b) $\sum_{j=0}^{4} (-2)^j = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4$

$= 1-2+4-8+16$

$= 11$ Answer: 11

30) S= $\{1,3,5,7\}$

a) $\sum_{j\in S} j = 1+3+5+7 = 16$

Answer: 16

b) $\sum_{j\in S} j^2 = 1^2+3^2+5^2+7^2 = 84$

Answer: 84

33) a) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$

$\Rightarrow \sum_{j=1}^{3} (1+j) + (2+j)$

$\Rightarrow (1+1)+(2+1)+(1+2)+(2+2)+(1+3)+(2+3)$

$= 2+3+3+4+4+5 = 21$ Answer: 21

b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

$\Rightarrow \sum_{j=0}^{3} (2\times 0+3j)+(2\times 1+3\times j)+(2\times 2+3j)$

$= \sum_{j=0}^{3} 3j+(2+3j)+(4+3j)$

$\Rightarrow \sum_{j=0}^{3} (6+9j)$

$\Rightarrow (6+9\times 0)+(6+9\times 1)+(6+9\times 2)+(6+9\times 3)$

$= 6+6+9+6+18+6+27$

$=78$

Answer: 78

34) a) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i-j)$

$\Rightarrow \sum_{j=1}^{2} (1-j)+(2-j)+(3-j)$

$\Rightarrow \sum_{j=1}^{2} 6-3j$

$\Rightarrow (6-3\times 1)+(6-3\times 2)$

$\Rightarrow (6-3)+(6-6)$

$\Rightarrow 3+0$

$\Rightarrow 3$

Answer: 3

b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i+2j)$

$\Rightarrow \sum_{j=0}^{2} (3\times0+2j) + (3\times1+2j) + (3\times2+2j) + (3\times3+2j$

$\Rightarrow \sum_{j=0}^{2} 2j + 3 + 2j + 6 + 2j + 9 + 2j$

$\Rightarrow \sum_{j=0}^{2} \cancel{9+8j}\ 18+8j$

$\Rightarrow \cancel{(9+6\times0)+(9+6\times1)+(9+}$

$\Rightarrow (18+8\times0)+(18+8\times1)+(18+8\times2)$

$= 18+26+34$

$= 78$

Answer: 78

39) $\sum_{k=100}^{200} k = \sum_{k=1}^{200} k - \sum_{k=1}^{99}$

$\sum_{k=1}^{200}$ $= \frac{200\cdot201}{2} - \frac{99\cdot100}{2}$

$= 20100 - 4950$

$\Rightarrow 15150$

Answer: 15150

40] $\sum_{k=99}^{200} k^3 = \sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3$

$$= \frac{200^2(200+1)^2}{4} - \frac{98^2(98+1)^2}{4}$$

$$= 404010000 - 23532201$$

$$= 380477799$$

Answer: ~~308 380 47799~~ 380477799

43] a) $\prod_{i=0}^{10} i = 0$

(anything times 0) Answer: 0

b) $\prod_{i=5}^{8} i$

$\Rightarrow$ ~~5×1~~ $5 \times 6 \times 7 \times 8 = 1680$ Answer: 1680

Induction (5.1)

3] a) Plugging, $n=1$

$\therefore$ P(1) statement is $1^2 = \frac{1\times(1+1)\times(2\times1+1)}{6}$

$\Rightarrow \quad = \frac{1\times2\times3}{6} = 1$

Ans: 1

b) Basic step:

Let, $n=1$

$L.H.S = n^2 = 1^2 = 1$

$R.H.S = \frac{n(n+1)(2n+1)}{6}$

$= \frac{1\times(1+1)\times(2\times1+1)}{6}$

$= \frac{1\times2\times3}{6} = 1$

$\because L.H.S = R.H.S$

$\therefore$ The statement P(1) is true.

[Showed]

4] a) Pluggin, $n=1$

$\therefore$ P(1) statement is $1^3 = \left\{\frac{1\times(1+1)}{2}\right\}^2$

$= \left\{\frac{2}{2}\right\}^2 = 1$

Ans: 1

b] basic step:

Let, $n=1$

$L.H.S = n^3$

$= 1^3 = 1$

$R.H.S = \left\{\frac{n(n+1)}{2}\right\}^2$

$= \left\{\frac{1\times(1+1)}{2}\right\}$

$\Rightarrow \left\{\frac{2}{2}\right\}^2 = 1$

$\because L.H.S = R.H.S$

$\therefore$ P(1) the statement is true.

[showed]

5] Prove that; $1^2+3^2+5^2+\ldots\ldots+(2n+1)^2=\frac{(n+1)(2n+1)(2n+3)}{3}$

whenever, n is a non-negetive integer.

Basic step:

Let; ~~n = (2n+1)~~ n=0

$$L.H.S = (2n+1) = (2\times0+1)^2 = (1)^2 = 1$$

$$R.H.S = \frac{(n+1)(2n+1)(2n+3)}{3} = \frac{(0+1)(2\times0+1)(2\times0+3)}{3}$$

$$= \frac{1\times1\times3}{3}$$

$$= 1$$

$\because L.H.S = R.H.S$

$\therefore$ The statement is true.

Inductive step:

Let's assume that for n=k the statement is true.

$$\therefore 1^2+3^2+5^2+\ldots\ldots+(2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} \quad -(1)$$

[Inductive hypothesis]

$\Rightarrow$ Now we need to show that for n=k+1 the ~~thate~~ statement is true.

So,

$$1^2+3^2+5^2+\ldots\ldots+\{2(k+1)+1\}^2=\frac{(k+1+1)(2(k+1)+1)(2(k+1)+3)}{3}$$

$$\Rightarrow 1^2+3^2+5^2+\ldots+(2k+1)^2+(2k+3)^2=\frac{(k+2)(2k+3)(2k+5)}{3}$$

$$\Rightarrow \frac{(k+1)(2k+1)(2k+3)}{3}+(2k+3)^2=\frac{(k+2)(2k+3)(2k+5)}{3};\ \boxed{\text{from }1}$$

$$\Rightarrow (k+1)(2k+1)(2k+3)+3(2k+3)^2=(k+2)(2k+3)(2k+5)$$

$$\Rightarrow (k+1)(2k+1)+3(2k+3)=(k+2)(2k+5)$$

$$\Rightarrow 2k^2+k+2k+1+6k+9=(k+2)(2k+5)$$

$$\Rightarrow 2k^2+9k+10=(k+2)(2k+5)$$

$$\Rightarrow 2k^2+5k+4k+10=(k+2)(2k+5)$$

$$\Rightarrow k(2k+5)+2(k+2)=(k+2)(2k+5)$$

$$\Rightarrow (k+2)(2k+5)=(k+2)(2k+5)$$

$\therefore$ L.H.S = R.H.S

$\therefore$ The statement is true proved by mathematical induction.

7] $3+3\cdot5+3\cdot5^2+\dots+3\cdot5^n = \frac{3(5^{n+1}-1)}{4}$

whenever, n is a non-negetive integer.

Basic step:

Let, $n=0$

$L.H.S = 3\cdot5^0 = 3$

$$R.H.S = \frac{3\times(5^{n+1}-1)}{4}$$

$$\Rightarrow \frac{3\times 5^1-1}{4}$$

$$\Rightarrow \frac{3\times4}{4} = 3$$

$\because L.H.S = R.H.S$

$\therefore$ The statement is true.

Inductive step:

Let's assume that for $n=k$ the statement is true.

$$\therefore 3+3\cdot5+3\cdot5^2+\dots+3\cdot5^k = \frac{3(5^{k+1}-1)}{4} \quad -①$$

[Inductive hypothesis]

Now we need to show that, for $n=k+1$ the statement is true.

$$3+3\cdot5+3\cdot5^2+\dots+3\cdot5^{k+1}=\frac{3(5^{k+1+1}-1)}{4}$$

$$\Rightarrow 3+3\cdot5+3\cdot5^2+\dots+3\cdot5^{k+1}=\frac{3(5^{k+2}-1)}{4}$$

$$\Rightarrow 3+3\cdot5+3\cdot5^2+\dots+3\cdot5^k+3\cdot5^{k+1}=\frac{3(5^{k+2}-1)}{4}$$

$$\Rightarrow \frac{3\cdot(5^{k+1}-1)}{4}+3\cdot5^{k+1}=\frac{3(5^{k+2}-1)}{4}$$

$$\Rightarrow 3(5^{k+1}-1)+4\cdot3\cdot5^{k+1}=3(5^{k+2}-1)$$

$$\Rightarrow 5^{k+1}-1+4\cdot5^{k+1}=5^{k+2}-1$$

$$\Rightarrow 5^k\cdot5^1+4\cdot5^k\cdot5^1-1=5^{k+2}-1$$

$$\Rightarrow 5^k(5+20)-1=5^{k+2}-1$$

$$\Rightarrow 5^k\cdot25-1=5^{k+2}-1$$

$$\Rightarrow 5^k\cdot5^2-1=5^{k+2}-1$$

$$\Rightarrow 5^{k+2}-1=5^{k+2}-1$$

$\therefore$ The statement is true proved by mathematical induction.

16] Prove that for every positive integer n,

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Basic step:

Let, $n = 1$

$L.H.S = n(n+1)(n+2)$

$= 1 \times (1+1) \times (1+2)$

$= 1 \times 2 \times 3 = 6$

$R.H.S = \frac{n(n+1)(n+2)(n+3)}{4}$

$= \frac{1 \times (1+1) \times (1+2) \times (1+3)}{4}$

$= \frac{1 \times 2 \times 3 \times 4}{4}$

$\Rightarrow 6$

$\because L.H.S = R.H.S$

$\therefore$ the the statement is true.

Inductive step:

Let's assume that for $n = k$ the statement is true.

$$\therefore 1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4} \quad -(1)$$

[inductive hypothesis]

Now we need to assume that for $n = k+1$ the statement is true.

$$1\cdot2\cdot3+2\cdot3\cdot4+\ldots+(k+1)(k+2)(k+3)=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$\Rightarrow 1\cdot2\cdot3+2\cdot3\cdot4+\ldots+k(k+1)(k+2)+(k+1)(k+2)(k+3)$$

$$=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$\Rightarrow \frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$\Rightarrow k(k+1)(k+2)(k+3)+4\{(k+1)(k+2)(k+3)\}=(k+1)(k+2)(k+3)(k+4)$$

$$\Rightarrow (k+1)(k+2)(k+3)(k+4)=(k+1)(k+2)(k+3)(k+4)$$

∵ L.H.S = R.H.S

∴ The statement is true proved by mathematical induction.

31| Prove that, 2 divides n^2+n whenever n is a positive integer.

Basic step: Let, $n = 1$

L.H.S $= n^2+n$

$= 1^2+1 = 2 =$ 2 is divisible by 2

$=$ R.H.S

∴ The statement is true.

Inductive step:

Let's assume that for $n=k$ the statement is true.

$\therefore$ k^2+k is divisible by 2 —①

[Inductive hypothesis]

Now, we need to show that, for $n = k+1$ the statement is true.

$(k+1)^2+(k+1)$ = divisible by 2.

$$L.H.S = (k+1)^2 + (k+1)$$

$$= k^2+2k+1 + k+1$$

$$= (k^2+k) + (2k+2)$$

$$\Rightarrow (k^2+k) + 2(k+1)$$

here, k^2+k is divisible by 2. by inductive hypothesis. and $2(k+1)$ is divisible by 2 by defination. So the sum of two multiples of 2 must be divisible by 2

$\therefore$ The statement is true proved by mathematical induction.

Recursion (5.3)

3] (a) $f(n+1) = f(n) + 3f(n-1)$; $f(0) = -1$, $f(1) = 2$

(a) $f(2) = f(1) + 3f(1-1)$

$f(2) = f(1) + 3 \times f(0)$

$= 2 + 3 \times (-1)$

$= 2 - 3 = -1$

$f(3) = f(2) + 3f(2-1)$

$= f(2) + 3 \times f(1)$

$= -1 + 3 \times 2$

$= -1 + 6 = 5$

$f(4) = f(3) + 3 \times f(3-1)$

$= f(3) + 3 \times f(2)$

$= 5 + 3 \times -1$

$= 2$

$f(5) = f(4) + 3 \times f(4-1)$

$= f(4) + 3 \times f(3)$

$= 2 + \cancel{5}\, 3 \times 5$

$= 17$

Ans: $f(2) = -1$, $f(3) = 5$, $f(4) = 2$, $f(5) = 17$

3) b) $f(n+1) = f(n)^2 \cdot f(n-1)$ $\quad f(0) = -1,\ f(1) = 2$

$f(2) = f(1)^2 \cdot f(1-1)$

$= f(1)^2 \cdot f(0)$

$= 2^2 \times -1 = -4$

$f(3) = \cancel{f(3)} \cancel{f(} f(2)^2 \cdot f(2-1)$

$\Rightarrow (-4)^2 \cdot 2$

$= 16 \times 2 = 32$

$f(4) = f(3)^2 \cdot f(4-1)$

$= (-32)^2 \cdot (-4)$

$\Rightarrow \cancel{264 \times 4} - 4096$

$f(5) = f(4)^2 \cdot f(4-1)$

$\Rightarrow (-4096)^2 \cdot 32$

$\Rightarrow 536,870,912$

<u>Answer:</u> $f(2) = -4$, $f(3) = 32$, $f(4) = -4096$

$f(5) = 536,870,912$

8) $\{a_n\}$, $n = 1, 2, 3,$

a) $a_n = 4n - 2$

$a_1 = 4 \times 1 - 2 = 4 - 2 = 2$

$a_2 = 4 \times 2 - 2 = 8 - 2 = 6$

$a_3 = 4 \times 3 - 2 = 12 - 2 = 10$

here

$\therefore a_{n-1} = \cancel{4(n+1)-2}\ 4(n-1) - 2$

$= 4n - 4 - 2$

$= (4n - 6) + 4$

$= a_{n-1} + 4$ for, $n \geq 1$ and $a_1 = 2$

$= a_n$

$\therefore$ a_n is defined by a_n it self. so it is a recursion.

Ans: $a_n = a_{n-1} + 4$

b) $a_n = 1 + (-1)^n$

$a_1 = 1 + (-1)^1 = 1 - 1 = 0$

$a_2 = 1 + (-1)^2 = 1 + 1 = 2$

$a_3 = 1 + (-1)^3 = 1 - 1 = 0$

ধরা $a_{n-1} = 1 + (-1)^{n-1}$

$$a_n = 1 + (-1)^n$$

$$= 1 + (-1)^{(n-1)+1}$$

$$= 1 + (-1)^{(n-1)}.(-1)$$

$$= 1 - (-1)^{n-1}$$

$$= 1 - [1 + (-1)^{n-1} - 1]$$

$$= 1 - (a_{n-1} - 1) = 2 - a_{n-1} \quad \text{for } n \geq 1$$

and $a_1 = 0$

$\therefore$ a_n is defined by a_n it self. So it is a recursion.

~~Ans:~~

Ans: $a_n = 2 - a_{n-1}$

12) prove that, $f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.

$f_0 = 0$, $f_1 = 1$, $f_2 = 1$

Basic Step:

Let, $n = 1$

$$L.H.S = f_1^2 = 1 \times 1 = 1$$

$$R.H.S = f_n . f_{n+1} = f_1 . f_{1+1} = f_1 . f_2 = 1 \times 1 = 1$$

L.H.S = R.H.S

$\therefore$ The statement is true.

Inductive step:

Let's assume that, for $n=k$ the statement is true.

$$f_1^2 + f_2^2 + \ldots\ldots + f_k^2 = f_k \cdot f_{k+1} \quad \text{--- (1)}$$

[Inductive hypothesis]

Now we need to prove that for $n=k+1$ the statement is true.

$$\therefore f_1^2 + f_2^2 + \cancel{f} \ldots\ldots + f_{k+1}^2 = f_{k+1} \cdot f_{k+2}$$

$$\Rightarrow \cancel{f_k \cdot f_{k+1}} + f_1^2 + f_2^2 + \ldots f_k^2 + f_{k+1}^2 = f_{k+1} \cdot f_{k+2}$$

$$\Rightarrow f_k \cdot f_{(k+1)} + f_{(k+1)}^2 = f_{(k+1)} \cdot f_{(k+2)} \; ; [\text{from } (1)]$$

$$\Rightarrow f_{(k+1)} \left(f_k + f_{k+1} \right) = f_{(k+1)} \cdot f_{(k+2)}$$

$$\Rightarrow f_{(k+2)} + f_{(k+2)} = f_{(k+1)} \cdot f_{(k+2)}$$; using recursive defination

$\because$ L.H.S = R.H.S

$\therefore$ The statement is true proved by mathematical induction.

23| We can define set, $S = \{x \mid x$ is a positive integer and x is a multiple of $5\}$

$S = \{5, 10, 15, 20, 25, 30, 35, 40 \ldots\}$

Basic step:

$5 \in S, \quad S = \{5\}$

Recursive step: If $x \in S$, $y \in S$, then $x+y \in S$

Itaration: 1

$S = \{5\}$

$x \in S \Rightarrow x = 5$

$y \in S \Rightarrow y = 5$

$\therefore x+y \in S$

$5+5 \in S$

$\Rightarrow 10 \in S$

$\therefore S = \{5\} \cup \{10\}$

$\therefore S = \{5, 10\}$

Itaration: 2

$x \in S, \; x = 5, 10$

$y \in S, \; y = 5, 10$

x/y	5	10
5	10	15
10	15	20

$\therefore S = \{5,10\} \cup \{15,20\}$

$= \{5,10,15,20\}$

Itavation 3:

$x \in S$; $x = 5, 10, 15, 20$

$y \in S$, $y = 5, 10, 15, 20$

x/y	5	10	15	20
5	10	15	20	25
10	~~20~~ 15	20	25	30
15	20	25	30	35
20	25	30	35	40

$\therefore S = \{5,10,15,20\} \cup \{\cancel{5}, \cancel{10}, \cancel{15}, 25, 30, 35, 40\}$

$= \{5, 10, 15, 20, 25, 30, 35, 40\}$

$\therefore$ multiple of 5 recarsively defined.

24 a) Odd positive integers set

$S = \{1, 3, 5, 7\}$

Basic step:

$1 \in S, \ S = \{1\}$

Recursive step: If $x \in S$, then $x+2 \in S$

Itaration 1:

$S = \{1\}$

$x \in S; \ x = 1$

$x+2 \in S$

$1+2 \in S$

$3 \in S$

$\therefore S = \{1\} \cup \{3\}$

$= \{1, 3\}$

Itaration 2:

$S = \{1, 3\}$

$x \in S, \ x = 3$

$x+2 \in S$

$\Rightarrow 3+2 \in S$

$5 \in S$

$\therefore S = \{1, 3\} \cup \{5\}$

$= \{1, 3, 5\}$

Itarations 3:

$S = \{1, 3, 5\}$

$x \in S$; $x = 5$

$x + 2 \in S$

$5 + 2 \in S$

$7 \in S$

$\therefore S = \{1, 3, 5\} \cup \{7\}$

$= \{1, 3, 5, 7\}$

$\therefore$ the set of odd positive integers ~~are~~ is recursivly defined.

b) The set of positive integer of powers of 3

$S = \{3, 9, 27, 81\}$

$\cancel{S = \{3, 6, 9, 12\}}$

Basic step: $3 \in S$, $S = \{3\}$

Recursive step: If $x \in S$, then $3x \in S$

Itaration of 1:

$S = \{3\}$

$x \in S$, $x = 3$

$3x \in S$

$3 \times 3 = 9 \in S$

$\therefore S = \{3\} \cup \{9\}$

$= \cancel{\{3\}} \{3, 9\}$

Itaration 2

$S = \{3, 9\}$

$x \in S,\ x = 9$ $\therefore S = \{3, 9\} \cup \{27\}$

$3x \in S$ $= \{3, 9, 27\}$

$3 \times 3 \in S$

$27 \in S$

Itaration 3

$S = \{3, 9, 27\}$

$x \in S,\ x = 27$

$3x \in S$ $\therefore S = \{3, 9, 27\} \cup \{81\}$

$3 \times 27 \in S$ $= \{3, 9, 27, 81\}$

$81 \in S$

$\therefore$ the set of positive integer powers of 3 recursively defined.

27] Recursive step: If $(a,b) \in S$, then $(a, b+1) \in S$, ~~(a+1)~~. $(a+1, b+1) \in S$ and ~~(a+2)~~ $(a+2, b+1) \in S$

a) Iteration 1:

$(0,1)$, $(1,1)$, and $(2,1)$

Iteration 2:

$(0,2)$, $(1,2)$, $(2,2)$, $(3,2)$, and $(4,2)$

Iteration 3:

$(0,3)$, $(1,3)$, $(3,3)$, $(4,3)$, $(5,3)$, and $(6,3)$

Iteration 4:

$(0,4)$, $(1,4)$, $(2,4)$, $(3,4)$, $(4,4)$, $(5,4)$, $(6,4)$, $(7,4)$, and $(8,4)$

c) This hold for basic step: since $0 \le 0$. If this holds for (a,b), then it also hold for the elements obtained from (a,b) in recursive step. since adding $0 \le 2$, $1 \le 2$ and $2 \le 2$, repectively to $a \le 2b$ yields $a \le 2(b+1)$, $(a+1) \le 2(b+1)$ and $(a+2) \le 2(b+1)$