

# Package ‘fTB’

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**Type** Package

**Title** Functional Tolerance Bands

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**Description** Construct tolerance bands for functional data

**License** GPL-3

**LazyLoad** tab yes

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fTB-package

*Functional Tolerance Bands*

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## Description

Methods to construct tolerance bands for functional data.

## Author(s)

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## References

Rathnayake, L. N. and Choudhary, P. K. (2015) Tolerance bands for functional data , Biometrics, doi: 10.1111/biom.12434.

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<code>boot.tolfactor</code>	<i>bootstrap tolerance factor</i>
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### Description

Calculate tolerance factor using bootstraps

### Usage

```
boot.tolfactor(n.boot, y, y.mean, y.sigma, y.G, root.int, prob, alpha, simul,
I, D, argvals)
```

### Arguments

<code>n.boot</code>	number of bootstrap samples
<code>y</code>	sample from original data
<code>y.mean</code>	mean of the sample
<code>y.sigma</code>	standard deviation of the sample
<code>y.G</code>	standard deviation of true values
<code>root.int</code>	Interval for root finding
<code>prob</code>	content probability of the tolerance interval
<code>alpha</code>	
<code>simul</code>	TRUE (if simultaneous tolerance factor need to be computed)
<code>I</code>	number of subjects
<code>D</code>	number of points in the union grid
<code>argvals</code>	arguments values for gam model

### Value

list of point wise critical points and simultaneous critical points for onesided upper TI .

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<code>onesided.tolfactor</code>	<i>Onesided tolerance factor</i>
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### Description

Calculate onesided tolerance factor for functional data

### Usage

```
onesided.tolfactor(prob, alpha, y.mean, y.sigma, y.G, ystar.mean, ystar.sigma,
ystar.G, root.int, simul, I, D, n.boot)
```

**Arguments**

prob	content probability of tolerance intervals
alpha	$1-\alpha$ confidence level
y.mean	mean of the sample
y.sigma	standard deviation of the sample
y.G	standard deviation of the true values
ystar.mean	mean of bootstrap
ystar.sigma	standard deviation of the sample
ystar.G	standard deviation of the true values
root.int	Interval for root finding
simul	TRUE (if simultaneous tolerance factor is computed)
I	number of subject
D	number of points in the union grid
n.boot	number of bootstraps

**Value**

pwise.obs  
 pointwise tolerance factor for observed curves  
  
 sim.obs  
 simultaneous tolerance factor for observed curves  
 pwise.true pointwise tolerance factor for true curves  
 sim.true simultaneous tolerance factor for true curves

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param.estimate	<i>Estimate parameters</i>
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**Description**

Function for estimate parameters ( $\mu$  and  $\sigma^2$  and  $G^2$  and  $\tau^2$ )

**Usage**

```
param.estimate(Y, I, D, argvals, nbasis = 10, pve = 0.99)
```

**Arguments**

Y	data
I	number of subjects
D	number of points in the union grid
argvals	argument values for the gam model
nbasis	number of basis functions for the gam model
pve	proportion of variance explained by the extracted principal components

**Value**

mean.hat	estimated mean
sigma.sq.hat	estimated sigma squared
G.sq.hat	estimated G squared

**Note**

This function is a modified version of ccb.fpc function in the refund package.

**Examples**

```
##---- Should be DIRECTLY executable !! ----
##-- ==> Define data, use random,
##--or do help(data=index) for the standard data sets.

## The function is currently defined as
function (Y, I, D, argvals, nbasis = 10, pve = 0.99)
{
  library(mgcv)
  d.vec <- rep(argvals, each = I)
  gam0 <- gam(as.vector(Y) ~ s(d.vec, k = nbasis)), method = "REML")
  mu <- predict(gam0, newdata = data.frame(d.vec = argvals))
  Y.tilde <- Y - matrix(mu, I, D, byrow = TRUE)
  cov.sum <- cov.count <- cov.mean <- matrix(0, D, D)
  for (i in 1:I) {
    obs.points <- which(!is.na(Y[i, ]))
    cov.count[obs.points, obs.points] <- cov.count[obs.points,
      obs.points] + 1
    cov.sum[obs.points, obs.points] <- cov.sum[obs.points,
      obs.points] + tcrossprod(Y.tilde[i, obs.points])
  }
  G.0 <- ifelse(cov.count == 0, NA, cov.sum/cov.count)
  diag.G0 <- diag(G.0)
  diag(G.0) <- NA
  row.vec <- rep(argvals, each = D)
  col.vec <- rep(argvals, D)
  npc.0 <- matrix(predict(gam(as.vector(G.0) ~ te(row.vec,
    col.vec, k = nbasis), method = "REML", weights = as.vector(cov.count)),
    newdata = data.frame(row.vec = row.vec, col.vec = col.vec)),
    D, D)
  npc.0 <- (npc.0 + t(npc.0))/2
  evalues <- eigen(npc.0, symmetric = TRUE, only.values = TRUE)$values
  evalues <- replace(evalues, which(evalues <= 0), 0)
  npc <- min(which(cumsum(evalues)/sum(evalues) > pve))
  efunctions <- matrix(eigen(npc.0, symmetric = TRUE)$vectors[,
    seq(len = npc)], nrow = D, ncol = npc)
  evalues <- eigen(npc.0, symmetric = TRUE, only.values = TRUE)$values[1:npc]
  cov.hat <- efunctions %*% tcrossprod(diag(evalues, nrow = npc,
    ncol = npc), efunctions)
  DIAG <- (diag.G0 - diag(cov.hat))[floor(D * 0.2):ceiling(D *
    0.8)]
  tau.sq <- max(mean(DIAG, na.rm = TRUE), 0)
  sigma.sq <- diag(cov.hat) + tau.sq
  return.obj <- list(mean.hat = mu, sigma.sq.hat = sigma.sq,
    G.sq.hat = diag(cov.hat))
  return(return.obj)
```

}

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solve.prob.eqn	<i>solution of a probability equation</i>
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**Description**

Function to find the solution of a probability equation

**Usage**

```
solve.prob.eqn(guess.root, prob.eqn, step.size = 0.2, delta = 0.1, grid.size = 0
```

**Arguments**

guess.root	temporary root for guessing an initial interval
prob.eqn	probability equation
step.size	size of the grid where the search interval is partitioned
delta	increment/decrement size for finding a temporary interval
grid.size	size of the grid where the search interval is partitioned

**Value**

solution of the probability equation

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TB.calc	<i>Calculation of tolerance bands</i>
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**Description**

This function calculate tolerance bands for given functional data. It include poinwise, simultaneous bands for both true and observed curves.

**Usage**

```
TB.calc(y, alpha = 0.05, prob, simul = TRUE, n.boot = 500)
```

**Arguments**

y	functional data that we need to construct tolerance bands for
alpha	parameter for confidence level such that 1-alpha = confidence level (default = .05).
prob	content probability
simul	logical argument to indicate simultaneous tolerance bands is constructed or not.
n.boot	number of bootstrap

**Value**

Return pointwise or simultaneous tolerance intervals depending upon the arguments

```
onesided.pwise TB
    Onesided pointwise tolerance band
twosided.pwise TB
    Twosided pointwise tolerance band
onesided.sim TB
    Onesided simultaneous tolerance bands
twosided.sim TB
    Twosided simultaneous tolerance bands
estimated parameters
```

**Note**

further notes

**Examples**

```
##----- Should be DIRECTLY executable !! -----
##-- ==> Define data, use random,
##--or do help(data=index) for the standard data sets.

## The function is currently defined as
function (y, alpha = 0.05, prob = TRUE, n.boot = 500)
{
  I <- dim(y)[1]
  D <- dim(y)[2]
  col.names <- as.integer(colnames(y))
  arg <- seq(col.names[1], col.names[length(col.names)], 1)
  root.int <- c(-20, 20)
  set.seed(1234)
  require(mgcv)
  y.param <- param.estimate(Y = y, I = I, D = D, argvals = arg)
  y.mean <- y.param$mean.hat
  y.sigma <- sqrt(y.param$sigma.sq.hat)
  y.G <- sqrt(y.param$G.sq.hat)
  k.boot <- boot.tolfactor(n.boot = n.boot, y = y, y.mean = y.mean,
    y.sigma = y.sigma, y.G = y.G, root.int = root.int, prob = prob,
    alpha = alpha, simul = simul, I = I, D = D, argvals = arg)
  onesided.boot.obs.u <- y.mean + k.boot[[1]]$pwise.obs * y.sigma
  onesided.boot.true.u <- y.mean + k.boot[[1]]$pwise.true *
    y.G
  onesided.pwise <- cbind(onesided.boot.obs.u, onesided.boot.true.u)
  if (simul == T) {
    onesided.boot.obs.sim <- y.mean + k.boot[[1]]$sim.obs *
      y.sigma
    onesided.boot.true.sim <- y.mean + k.boot[[1]]$sim.true *
      y.G
    onesided.sim <- cbind(onesided.boot.obs.sim, onesided.boot.true.sim)
  }
  twosided.boot.obs.u <- y.mean + k.boot[[2]]$pwise.obs * y.sigma
  twosided.boot.true.u <- y.mean + k.boot[[2]]$pwise.true *
    y.G
```

```

twosided.boot.obs.l <- y.mean - k.boot[[2]]$pwise.obs * y.sigma
twosided.boot.true.l <- y.mean - k.boot[[2]]$pwise.true *
  y.G
twosided.pwise <- cbind(twosided.boot.obs.l, twosided.boot.obs.u,
  twosided.boot.true.l, twosided.boot.true.u)
if (simul == T) {
  twosided.boot.obs.sim.u <- y.mean + k.boot[[2]]$sim.obs *
    y.sigma
  twosided.boot.obs.sim.l <- y.mean - k.boot[[2]]$sim.obs *
    y.sigma
  twosided.boot.true.sim.u <- y.mean + k.boot[[2]]$sim.true *
    y.G
  twosided.boot.true.sim.l <- y.mean - k.boot[[2]]$sim.true *
    y.G
  twosided.sim <- cbind(twosided.boot.obs.sim.l, twosided.boot.obs.sim.u,
    twosided.boot.true.sim.l, twosided.boot.true.sim.u)
}
if (simul == T) {
  result <- list(onesided.pwise, onesided.sim, twosided.pwise,
    twosided.sim, cbind(y.mean, y.sigma, y.G))
  names(result) <- c("onesided.pwise TB", "onesided.sim TB",
    "twosided.pwise TB", "twosided.sim TB", "estimated parameters")
}
else {
  result <- list(onesided.pwise, twosided.pwise, cbind(y.mean,
    y.sigma, y.G))
  names(result) <- c("onesided.pwise TB", "twosided.pwise TB",
    "estimated parameters")
}
return(result)
}

```

---

twosided.tolfactor *twosided tolerance factor*


---

## Description

Calculate the twosided tolerance factor

## Usage

```
twosided.tolfactor(prob, alpha, y.mean, y.sigma, y.G, ystar.mean, ystar.sigma,
  ystar.G, root.int, simul, I, D, n.boot)
```

## Arguments

prob	content probability
alpha	parameter for confidence level such that $1 - \alpha =$ confidence level (default = .05)
y.mean	mean of the sample
y.sigma	standard deviation of the sample
y.G	standard deviation of true curves

<code>ystar.mean</code>	bootstrap mean
<code>ystar.sigma</code>	bootstrap standard deviation
<code>ystar.G</code>	bootstrap standard deviation of true curves
<code>root.int</code>	interval for root finding
<code>simul</code>	TRUE (if simultaneous tolerance factor is computed)
<code>I</code>	number of subjects
<code>D</code>	number of points in the union grid
<code>n.boot</code>	number of bootstraps

**Value**

<code>pwise.obs</code>	pointwise tolerance factors for observed curves
<code>sim.obs</code>	simultaneous tolerance factors for observed curves
<code>pwise.true</code>	pointwise tolerance factors for true curves
<code>sim.true</code>	simultaneous tolerance factors for true curves