**INTRODUCTION**

Animals identify and discriminate odors using olfactory receptors (Ors) expressed in olfactory receptor neurons (ORNs)~\cite{Or\_ORNs\_maps, buck1991novel}. In both insects and vertebrates, distinct ORNs, which typically express a single Or, respond broadly to many distinct volatile odorants~\cite{malnic1999combinatorial, mosquito\_combinatorial\_coding, hildebrand1997mechanisms, hallem\_carlson, debryune\_odor\_coding, friedrich1997combinatorial}. Likewise, any given odorant may incite strong responses in a number of ORNs. ORN tuning curves are therefore broad and overlapping, implying that odors are encoded combinatorially -- by the particular combination of responses they elicit in the ORN repertoire, patterns which are eventually decoded downstream into behavioral response~\cite{early\_olfactory\_processing}.

Odor signals in nature are uniquely complex in composition, space, and time, and like other sensory modalities, olfactory systems must tune their response to the character of input stimuli~\cite{murlis\_odor\_plumes, fluid\_dynamics\_chemosensory, celani, carde\_navigation, srinivas\_elife}. Various features of natural odor signals are power-law distributed, producing whiffs of widely varying duration and intensity~\cite{celani}. Sensitivity to low concentrations could thus lead to saturation at high intensities, confounding combinatorial representations of odor identity. Further, ethologically-relevant odors are often mixed together with nuisance odors; due to the non-specificity of ORN response, distinct odors could simultaneously activate overlapping ORN subsets, preventing accurate disambiguation~\cite{odor\_backgrounds}.

What invariances and adaptive mechanisms might help preserve combinatorial odor representations in natural environments? In \textit{Drosophila} larvae, dose responses of 324 OR-odorant combinations obey a single activation function shape, with similar Hill coefficients but different activation thresholds, which are distributed as a power law~\cite{si2017invariances}. \textit{Drosophila} ORNs also exhibit a large degree of similarity in their temporal integration of fluctuating stimuli. While for some Or-odor combinations, ORN response can exhibit large differences, such as super-sustained responses~\cite{montague2011similar}, deconvolution of stimulus dynamics from neuron responses produces stereotyped filters that differ little among ORNs~\cite{martelli}.

Beyond these invariances are adaptive mechanisms which can preserve odor representations amid environmental changes. In \textit{Drosophila}, antennal lobe glomeruli receiving connections from ORNs send mutually inhibitory connections, effectively normalizing incoming ORN signals before they are projected to the lateral horn and mushroom body~\cite{lateral\_inh\_asahina, divisive\_normalization}. In the mushroom body, inhibition from giant interneuron upon Kenyon cells further acts to suppress patterns of activity elicited by these odors~\cite{giant\_inhibitory\_neuron}. Further, upstream of these connectivity-mediated mechanisms, individual ORNs themselves adapt in time an apparently universal way: ORN gain varies inversely with mean odor concentration according to Weber's Law of psychophysics~\cite{weber1996eh}, while maintaining response time independent of odor intensity~\cite{srinivas\_elife,cafaro\_WL, cao\_WL}. Weber's Law has been observed in several OR-odorant combinations, and is traced to feedback mechanisms operating at the level of odor transduction, upstream of ORN firing machinery~\cite{srinivas\_elife}. This points to a mechanistic origin involving modification of phosphorlyation sites on the universal \textit{Drosophila} co-receptor Orco. Other phosphorylation sites on Orco have been implicated in desensitization to odors over longer timescales as well~\cite{Guo\_Smith\_review,Guo\_Smith}.

While in a single channel system such as \textit{E. coli}, adaptive feedback via Weber's Law is known to robustly maintain sensitivity in response to mean concentration changes, the implications for a multiple-channel system -- which combines information from several sensors with overlapping receptive fields -- is less straightforward. Here we combine a biophysical model of universal ORN adaptive response and neural firing with various sparse signal decoding frameworks to explore whether and how ORN adaptation maintains accurate combinatorial coding of odor signals that span varying degrees of intensity, molecular complexity, and temporal structure. We find that front-end gain modulation helps preserve coding capacity within the non-specifically sensing ORN repertoire, and maintains abstract representations of odor identity across different intensities. As such, this adaptive mechanism promotes the accurate discrimination of weak odor signals from strong backgrounds of varying molecular complexity, both in static odor environments and in fluctuating ones. We also investigate the predictions of our model for the \textit{primacy coding} hypothesis -- that odors are encoded entirely by the subset of a few earliest responding ORNs~\cite{primacy\_coding, primacy\_math}. Our results agree with primacy coding when odor signals are sufficiently simple, though signals composed of more molecular constituents require the recruitment of the full ORN repertoire. Finally, we show that front-end adaptation acts in concert with divisive normalization in the antennal lobe to maintain the invariance of activity patterns in the mushroom body. Together, our results suggest that despite the broad overlap of ORN tuning curves, a universal mechanism of front-end adaptation may play a vital role in preserving representations of odor identity in naturalistic odor landscapes.

**RESULTS**

**Model of ORN sensing repertoire**

Odor identification consists of encoding in the sensing periphery followed by decoding in higher-level processing centers of the olfactory circuit. We first examine how front-end adaptation can maintain odor encoding capacity, by drawing upon a model of odor-to-ORN firing recently shown to reproduce experimental findings: Weber-Fechner scaling, signal transduction kinetics, and firing rate dynamics of individual \textit{Drosophila} ORNs to fluctuating stimuli ~\cite{srinivas\_elife}. Here we generalize this model to a repertoire of $M=50$ ORN types. ORNs house olfactory receptor complexes $\textup{C}\_a$, $a=1,...,M$, each consisting of an ORN-specific OR and the universally-expressed olfactory co-receptor Orco, which mediates odor transduction through dendritic localization of and heteromerization with ORs (Fig.~\ref{fig:tuning\_curves\_a}). These odorant-response functional units interact with odor mixtures, each of which is composed of some combination of $N$ odorant molecules with time-dependent concentrations $s\_i(t)$, $i=1,...,N$ (Fig.~\ref{fig:tuning\_curves\_b}). We choose $N=150$, as this number is sufficiently larger than the size of the sensing repertoire~$M$. Functionally, $\textup{C}\_a$ forms a non-selective cation channel whose current is mediated by the strength and nature of bound ligands. We thus model a given complex as stochastically switching between active (channels open) and inactive states, while also being bound or unbound with odorant $i$. The active conformation binds odorant $i$ with higher affinity than the inactive conformation, resulting in distinct dissociation constants, $K^\*\_{ia}$ and $K\_{ia}$, respectively. In steady state, the active fraction $A\_a$ of Or/Orco complexes in ORN $a$ can be solved for as (see Methods):

\begin{align}

A\_a(t) &= \left(1 + e^{\epsilon\_a(t)}\right)^{-1} \nonumber \\

\epsilon\_a(t) &= \epsilon\_{a, \textup{act}}(t) + \epsilon\_{a, \textup{ligand}}(t) \nonumber \\

\epsilon\_{a, \textup{ligand}}(t) &= \frac{1 + \sum\_i^N s\_i(t)/K\_{ai}}{1 + \sum\_i^N s\_i(t)/K^\*\_{ai}},

\label{eq:steady\_state\_act\_OR}

\end{align}

where $\epsilon\_{a, \textup{act}}(t)$ is the free energy cost of $\textup{C}\_a$ activation.

Inward currents elicited by activation of the Or/Orco receptor complexes then incite firing activity in ORNs. Following~\cite{srinivas\_elife}, we model the Or/Orco-to-ORN transformation with a temporal filter followed by rectifying nonlinearity $f$ (see Methods):

\begin{align}

r\_a(t) = f\left(\int h(\tau - t)A\_a(t) d\tau\right).

\label{eq:steady\_state\_firing}

\end{align}

At the single ORN level, this nonlinear-linear-nonlinear framework (Or/Orco activation $\rightarrow$ temporal filter $\rightarrow$ nonlinear rectifier) reproduces Weber Law gain adaptation and signal transduction kinetics, notably the temporal slowdown of the local field potential upon adaptation, along with a complementary speed-up in the firing machinery.

Here $\epsilon\_a(t)$ represents the free energy change due to modifications of the Or/Orco complexes by adaptation. Opening of the channels causes an inward current that eventually results in a negative feedback onto $A(t)$. This is modeled minimally by:

\begin{align}

\frac{d\epsilon\_a(t)}{dt} = \frac{{A}\_{a0} - A\_a(t)}{\tau\_a}

\label{eq:adaptation\_dynamics}

\end{align}

within the finite range $\epsilon\_{\textup{L}, a} < \epsilon\_a(t) < \epsilon\_{\textup{H}, a}$. It has been shown that when properly deconvolved from the stimulus dynamics, the shape of temporal kernels in adult \textit{Drosophila} ORNs is largely odor-independent, though may differ by brief ($\sim$10 ms) odor-dependent delays~\cite{martelli}. Accordingly, we model $h(t)$ by an ORN- and odor-independent double-exponential function, with parameters matched to experiment~\cite{martelli}. We assume that the lower cutoffs $\epsilon\_{\textup{L}, a}$ are receptor-dependent and choose them from a normal distribution. This variability ensures that ORNs are activated above quiescence (around 5 Hz) at distinct stimulus levels~\cite{srinivas\_elife}.

Diversity among maximal odor-ORN response arises from the distribution of chemical affinities, encapsulated in $K^\*\_{ai}$. We choose these from a power law distribution ($\alpha = 0.35$), as was recently found across ORN-odor pairs in \textit{Drosophila} larvae (Fig.~\ref{fig:tuning\_curves\_c}). To mimic the presence of private odors relevant to innate responses, we manually add a high responder ($K^\*\_{ai} \sim $ small) to a handful of ORNs; the addition of these private odors did not affect the general findings. Together, the power-law distributed $K^\*\_{ai}$, receptor-dependent $\epsilon\_{\textup{L}, a}$, and invariance in temporal filters, when incorporated into the steady-state model responses Eq.~\ref{eq:steady\_state\_act\_OR}, produce tuning curves mimicking the maximal \textit{Drosophila} ORN responses to many individual odorants~\cite{hallem\_carlson} (Fig.~\ref{fig:tuning\_curves\_d}-\ref{fig:tuning\_curves\_e}).

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**Concentration-invariant preservation of coding capacity and abstract representations of odor identity**

To investigate the dependence of encoding capacity on odor concentration, we calculate the mutual information (MI) between odor signals $\mathbf s$ and responses $\mathbf A$ in two sensing systems, one with ORN adaptive feedback (via Eq.~\ref{eq:adaptation\_dynamics}) and one without. We consider a simple situation, in which a step of odor A, $\textbf{s}\_A$, turns on at time~$t\_1$, persists for some time, and then odor B, $\mathbf s\_B$ (a distinct identity) turns on at some later time~$t\_2$. For simplicity, we assume that both odors have similar intensities $s\_0 = \langle s\_i \rangle$ and calculate the MI between the ORN responses $\mathbf {r}\_a$ and signals $\mathbf s\_A + \mathbf s\_B$ at various times after $t\_1$, as a function of $s\_0$. In the non-adaptive case, MI peaks around the region of maximum sensitivity ($\sim 10^2$ a.u.) after $t\_1$ (Fig.~\ref{fig:coding\_a}. ORNs are firing at an elevated rate, however, more susceptible to saturation with further odor onsets. Thus, following $t\_2$, the maximum shifts leftward as odors of high intensities have saturated the system and cannot pass any more information.

The adaptive system mimics the non-adaptive system at $t\_1$, before adaptation has kicked in (Fig.~\ref{fig:coding\_a}). As the activity feeds back onto $\epsilon\_a$, the response to higher concentrations passes through the regime of high sensitivity, and the MI peak shifts rightward. Over time, the responses for all signals have reached the baseline firing rate, and the mutual information is mostly eliminated since the firing rate is independent of odor identity. However, having now adjusted its regime of maximum sensitivity to the presence of odor A, the system can respond appropriately to odor B: the MI at $t\_2$ is nearly 6 bits across 3 decades of concentration, in contrast to the non-adaptive case. These results suggest that in this multi-channel compressive system, a simple mechanism of universal integral feedback can help maintain sensitivity in changing environments.

We expect that this preservation of information capacity might therefore help maintain abstract representations of odor identity. To examine such representations, we project the ORN response repertoire to a lower dimensional space using t-distributed stochastic neighbor embedding (t-SNE), a nonlinear, local dimensionality reduction technique~\cite{tsne}. Here, each data point is a 50-dimensional vector representing the ORN firing responses to a given odor of a given intensity. Each of these data points is then reduced by t-SNE to two dimensions (Fig.~\ref{fig:coding\_b}). %In Fig. 2B, colors represent odors of distinct identities; sizes represent odors of distinct concentrations.

Testing this both for a smaller odor repertoire (10 odor identities) and a larger one (60 identities), we find that odors separate by identity in the adaptive system, while in the unadaptive system, representations mix among their identity and concentration (Fig.~\ref{fig:coding\_b}). Together, these results suggest that at the level of ORN response, front-end adaptation helps maintain representations of odor identity across changes in odor intensity.

**Front-end adaptation promotes odor decoding and discrimination accuracy amid potential confounds**

Next, we ask how front-end adaptive mechanisms can aid in accurate signal reconstruction from a repertoire of ORN response. One potentially complicating factor in signal reconstruction is the disparity between measurement dimension and stimulus dimension: while \textit{Drosophila} only express $\sim 60$ olfactory receptor genes~\cite{olfactory\_sensory\_map}, the space of aromatic odorants is far greater~\cite{vijay\_1}. However, many naturally-occurring odors are comprised of only a small subset of these volatile compounds -- they are sparse in the space of odorants~\cite{vijay\_1}. This is suggestive as mathematical results in compressed sensing guarantee the reconstruction of these sparse signals, assuming a sufficiently random response~\cite{CS\_donoho, CS\_tao, CS\_ganguli}. We emphasize that while at this stage of the analysis we use a compressed sensing framework to decode these sparse signals, there is no evidence that this algorithm is being implemented in the \textit{Drosophila} olfactory circuit~\cite{chlovskii\_pevlavan}. Later, we show that similar conclusions follow for odor categorization via binary and multi-class classification.

To incorporate the linear framework of compressed sensing into our nonlinear encoding model, we treat the odor encoding process exactly, while approximating the decoding to first order. Specifically, we represent the nonzero components $s\_i$ of a sparse odor mixture as $s\_i = s\_0 + \Delta s\_i$, where $s\_0$ is the center of the linearization. The target of the decoding process are the `excess' odor signal components $\Delta s\_i$, which are determined by enforcing signal sparsity and the measured ORN responses through constrained optimization. The cost function to be optimized and the linear constraints are:

\begin{align}

\hat {\mathbf s} &= \text{arg min} \sum |s\_i| \\

\Delta r\_a &= {f}\left(\int h(\tau - t)\frac{dA\_a}{ds\_i}d \tau\right) \bigg |\_{s\_0} \Delta s\_i

\end{align}

To assess the decoding performance, we denote an odor signal as accurately decoded if (i) the sparse odorant components are all estimated to within 25\% of their correct value and (ii) the components absent in the original signal (``zero" components) are all estimated as less than 10\% of the mean excess concentration, $\hat s\_i \le \langle \Delta s\_j \rangle$.

We apply this scheme to the ORN system described above, consisting of 50 Or/Orco complexes interacting with a 150-dimensional odorant space. We assume that number of nonzero odorants comprising the odor, $K$, is small. Note, however, that this still allows for a huge number of distinct odors, e.g. nearly 1 billion for $K= 7$. In the absence of ORN adaptation, signals are still correctly inferred in a particular regime of mean odor concentration (Fig.~\ref{fig:decoding\_b}), corresponding to that of higher coding capacity in Fig.~\ref{fig:coding\_a}. Elsewhere, decoding accuracy is low. Conversely, enforcing Weber-Fechner scaling within the thresholds $\epsilon\_{\textup{L}, a}$ and $\epsilon\_{\textup{H}, a}$, coding fidelity is maintained over a several-fold change in odor intensity (Fig.~\ref{fig:decoding\_b}).

In natural odor environments, accurate olfactory sensing relies on the ability to discriminate multiple odors, which may differ in chemical makeup and intensity. Even if adaptation could preserve decoding accuracy of a single odor amid intensity changes, it is conceivable that a system which adapts to average concentrations alone may well fail for multiple odors of widely differing concentrations. Accordingly, we next consider two sparse odors, which we call the ``foreground" and ``background", and ask how well foreground odors can be decoded in the presence of backgrounds of a given intensity and molecularly complexity. In the unadaptive system, decoding accuracy in the regime of maximum sensitivity is maintained if the background concentration is low enough, but is compromised as concentration increases (Fig.~\ref{fig:decoding\_c}). For higher background concentrations, molecular complexity also has a more damaging effect on decoding accuracy. Finally, for sufficiently strong and complex background odors, the foreground is virtually undetectable (top right plot in Fig.~\ref{fig:decoding\_c}). The adaptive system is substantially more robust to backgrounds (Fig.~\ref{fig:decoding\_d}), although the minimum detectable concentration increases with background intensity. Taken together, these results indicate a universal mechanism of adaptive feedback operating on the activity of Or/Orco complexes promotes odor identification amid potential confounds in both identity and intensity.

**Odor decoding in fluctuating odor environments**

So far, we have assumed that odor signals are static in time, and that adaptation from the neural circuitry feeds back onto the receptor sensitivity instantly and perfectly. But realistic odor environments are highly intermittent and widely fluctuating, with odor concentrations that can span several orders~\cite{celani}. Further, metabolic constraints can limit adaptation speed and accuracy~\cite{ESA}. To account for temporal aspects in both the odor environment and sensing periphery, we set the timescale of adaptation in Eq.~\ref{eq:adaptation\_dynamics} at $\tau\_A = 250$ ms, in accordance with experimental estimates~\cite{srinivas\_elife}. For simplicity, we assume that while the sensory response modulates in time, the decoding process itself is instantaneous. We also mimic a finite length of short-term memory, such that only changes in detected odor signal are remembered, and only for up to $\tau\_{\textup {F}}$ seconds in the past. If the signal is static, it will be decoded optimally between $\sim\tau\_A$ and $\sim\tau\_{\textup{F}}$. For fluctuating environments, we expect that $\tau\_{\textup{F}}$ a few times as large as $\tau\_A$ should be sufficient in detecting of whiffs of novel odors amid slowly fluctuating or static backgrounds.

We first consider the simple case of step stimuli. For shallow steps, odors are rapidly decoded, though slightly more quickly for smaller $\tau\_A$ (Fig.~\ref{fig:temporal\_coding\_a}). This is attributed the recruitment of a sufficient number of ORNs beneath the point of response saturation, such that response adaptation has little effect. For larger steps, decoding accuracy improves gradually as the system adapts at its characteristic timescale. In all cases, the accuracy drops to zero after the forgetting time $\tau\_{\textup{F}}$ (here set to $4\tau\_A$).

We next considered a naturalistic plume, using a recorded time trace from a photo-ionization detector placed downwind of an odor source (Fig.~\ref{fig:temporal\_coding\_b}). The signal magnitude was scaled linearly to values applicable to our model framework (a.u.), and we verified that the plume statistics agree with theoretical predictions {~\color{blue} TODO}. %The signal is composed of a series of intermittent whiffs, defined as contiguous regions at which the concentration is above a given value.

This signal serves as the intensity of the odor, to which we randomly assign a sparse identity from the $N$-dimensional odor space. To investigate discrimination on potentially confounding backgrounds, we add to this signal a static background odor of varying intensities and complexities, and calculate the percentage of correctly decoded odor whiffs. Without ORN adaptation, sufficiently strong backgrounds eliminate whiff-detection ability, irrespective of the complexity of either the foreground or background odor (Fig.~\ref{fig:temporal\_coding\_c}, green lines). In the adaptive system, this is substantially mitigated, (red lines in Fig.~\ref{fig:temporal\_coding\_c}) although accuracy does decrease in general with increasing background complexity. Further, accurate whiff detection requires $\tau\_{\textup {F}}$ only on the order of $\tau\_A$ (darker red lines). Together, this indicates that ORN adaptation acting at measured timescales aids the detection of fluctuating odor signals amidst confounding backgrounds.

**Relationship to primacy coding hypothesis**

An intriguing hypothesis emerging from recent experiments in vertebrates, ``primacy coding," posits that odor identity is encoded entirely by the set (but not temporal order) of the $p$ earliest responding glomeruli, known as ``primacy sets" of order $p$ (Fig.~\ref{fig:primacy\_coding\_a}). Such primacy sets would in principle comprise a concentration-invariant representation of odor identity. In our framework, odors are decoded via information passed simultaneously from all 50 ORNs. However, some of this information may be redundant, whereby a set of earliest active ORNs are sufficient for odor recognition; if so, our theory would generate predictions in agreement with primacy coding. To test this, we consider a steep sigmoidal stimulus with half-max slope of $1/50$ ms$^{-1}$, as in Fig.~\ref{fig:primacy\_coding\_a}. We calculate the decoding accuracy as a function of time, and plot in Fig.~\ref{fig:primacy\_coding\_b} the accuracy as a function of number of active ORNs, which increases monotonically as the signal rises (Fig.~\ref{fig:primacy\_coding\_a}). To illustrate how the recruitment of ORNs incrementally improves odor signal recognition, we allow for partial accuracy by calculating the percentage of correctly decoded individual odor components.

For sufficiently simple odors, our results are indeed in accordance with primacy coding: the set of earliest responding neurons fully account for the odor identity ($K=1, 3, 5$ plots in Fig.~\ref{fig:primacy\_coding\_b}). Though all ORNs eventually activate as the stimulus increases, the latter responders confer no further information in odor recognition. As expected, the active ORN subset comprising the primacy set is distinct for each odor, {\color{blue} Show that recruited ORNs are all distinct}. We do find, however, that for more complex odor mixtures, the full ORN repertoire must be active for accurate decoding ($K=7, 9$ plots in Fig.~\ref{fig:primacy\_coding\_b}), a result that holds across odor odor concentrations (Fig.~\ref{fig:primacy\_coding\_c}). In this regime, there can be no primacy code, since a primacy set consisting of the full ORN repertoire would encode only a single odor. Conversely, our framework can decode the odor for a maximal primacy set since it utilizes not just the identity of the active ORNs, but also their individual firing responses. Primacy coding also predicts that for stronger stimuli, behavioral responses shift earlier in time, since the primacy set is activated quicker. We calculate this time shift and find that it rises monotonically with odor intensity over a decade of concentrations.

Primacy sets are inherently concentration-invariant. But to what extent are they conserved among varying environmental conditions, such as persistent background odors? ORNs that have adapted their gain in response to a background odor could in principle be pushed out of the primacy sets of a novel odor due to reduced sensitivity. To test this, we calculated the primacy sets for 1000 sparse odor mixtures atop a static background of a low and high intensity, comparing their primacy sets for each mixture. Primacy sets of sufficient size ($p \gtrapprox 8$) are almost entirely preserved across odor concentration, indicating that primacy coding, if true, would benefit from universal ORN adaptation in maintaining concentration-invariant odor representations in confounding environments. (Figure forthcoming)

Together, these results suggest that front-end adaptation, in concert with the compressed sensing paradigm, are largely in agreement with the predictions of the primacy coding hypotheses. Our framework also provides the testable prediction that primacy coding may break down for more complex odor mixtures.

**Cooperative effects of ORN adaptation and downstream normalization**

Lateral inhibition among antennal lobe glomeruli normalizes ORN responses prior to their projections to the mushroom body~\cite{lateral\_inh, lateral\_inh\_asahina}. This inhibition obeys a type of divisive gain control, normalizing each input by the sum of ORN activity~\cite{divisive\_normalization}. To what extent does Weber Law adaptation in the olfactory periphery act in tandem with (or counteract) this downstream normalization to maintain odor representations? To investigate this, we extended our ORN encoding model by adding uniglomerular connections from ORNs to the antennal lobe, followed by sparse, divergent connections to 2500 Kenyon cells (KCs) in the mushroom body~\cite{memory\_review, litwinkumar, abbott\_axel} (Fig.~\ref{fig:downstream\_a}). Divisive normalization in the AL was modeled via~\cite{divisive\_normalization}:

\begin{align}

r\_{\textup{PN}, a}(t) = \frac{r\_a(t)^{1.5}}{\sigma^{1.5} + r\_a(t)^{1.5} + (\gamma\sum\_b^Mr\_b(t))^{1.5}}

\end{align}

We then quantified decoding accuracy by training and testing a binary linear classifier on the KC activity output of many sparse odors of distinct intensity and identity, each randomly categorized as appetitive or aversive. Odor signals of the same identity but differing intensity were assigned the same valence (Fig.~\ref{fig:downstream\_a}). We trained the classifier on $p$ sparse odor identities at intensities chosen randomly over 4 orders of magnitude, then tested the classifier accuracy on the same odor identities but of differing concentrations.

Classification accuracy degrades to chance level as the number of distinct odor identities $N\_{\text ID}$ becomes very high (Fig.~\ref{fig:downstream}). When acting alone, either divisive normalization or ORN adaptation can mitigate this, although the effect of ORN adaptation is stronger (Fig.~\ref{fig:downstream\_b}). When both are active, accuracy improves further, suggesting that these distinct adaptive transformations may act jointly at different stages of neural processing to preserve representations of odor identity. If odors are instead chosen from a narrower intensity range, performance improves without either adaptive mechanism at play~(Fig.~\ref{fig:downstream\_c}). Interestingly, if we train the classifier to distinguish odors by their distinct identity using multiclass categorization (Fig.~\ref{fig:downstream\_a}), we find that the benefits conferred by divisive normalization do not appear until $p$ is substantial, with accuracy below $65\%$ for $N\_{\text ID} > 50$ (Fig.~\ref{fig:downstream\_d}). On the other hand, with ORN adaptation accuracy remains above $85\%$ for $N\_{\text ID}$ as high as 1000. Together, this indicates that front-end adaptation plays a key role in maintaining odor identity representations, before they are further normalized and diverged in downstream processing. In some categorization tasks, subsequent normalization can further preserve these representations.

**Discussion**

We investigate, through theory and computations, the implications of recent experimental evidence for a universal mechanism of input gain control in \textit{Drosophila} olfactory receptor neurons. We argue that this mechanism, Weber's Law of psychophysics, plays a key role in preserving neural representations of odor identity at the front-end of the olfactory pathway, prior to further transformations downstream. Tnput gain control also acts jointly with normalization in the antennal lobe, implicating the importance of signal transformations at multiple steps in the circuit. We support our conclusions using both a decoding scheme that fully reconstructs odor signals from neural response, and a classification scheme that categorizes odors by identity or valence. We find that input gain control is especially central to the discrimination of fluctuating odor signals from potentially saturating odor backgrounds. For single odorants or simple odor mixtures, our results are also consistent with the primacy coding hypothesis, that signals may be fully encoded by the primacy set of earliest activating glomeruli.

**Universal front-end adaptation in multi-channel sensory systems**

In living systems, sensory adaptation ensures that responses remain in regimes of maximum sensitivity, increasing their effective dynamic range~\cite{adaptation\_fairhall, adaptation\_nagel, laughlin, deweese\_adaptation}. Viewing sensory systems as input/output machines, the role of adaptation is therefore to maintain information capacity in dynamic environments ~\cite{information\_theory\_adaptation}. Doing so requires matching sensory response to attributes of the environment, either by adapting to specific stimuli or to stimuli statistics~\cite{adaptation\_fairhall}. In a single-channel system such as bacterial chemotaxis, information capacity is increased by matching the midpoint of the nonlinear dose-response curve, where sensitivity is highest, to mean ligand concentration~\cite{information\_theory\_adaptation}. This is enacted in a robust and dynamic way, through a feedback loop from the activity output of the pathway onto proteins dictating receptor sensitivity~\cite{robustness\_barkai, robustness\_alon}. It is hypothesized that an analogous feedback loop exists in olfactory receptor neurons, from Orco-mediated Or channel activity onto the free energy of receptor activation~\cite{srinivas\_elife}. This mechanism appears to act identically across ORNs, and because olfactory receptive fields are highly overlapping, raises questions about its efficacy in complex environments: adaptation to one odor could adversely affect identification of a new odor if the latter excites some but not all of the same ORNs. Our results show that this universal mechanism of front-end gain control does help to preserve combinatorial representations of odor identity, despite these broad overlaps.

While Weber-Fechner gain control operates at the level of individual ORNs, lateral inhibition in antennal lobe glomeruli mixes signals among all ORNs~\cite{divisive\_normalization}. We find that olfactory circuits exhibiting both of these adaptive mechanisms outperform systems containing one alone.

Combinatorial coding, therefore, benefits both from the separate adaptation of individual single sensory neurons, as well as the normalization of aggregated response. It is notable that both Weber-Fechner adaptation and divisive normalization modulate the location of maximal sensitivity rather than the scale of absolute activity -- they move dose-response curves horizontally, rather than stretching them vertically. They are both mechanisms of input gain control rather than response gain control.

Other mechanisms in the sensory periphery likely play a role in maintaining neural representations of odor identity. ORNs that contain both excitatory and inhibitory responses to odorants can increase information capacity by exploiting bidirectionality of response~\cite{Cao\_Tu\_WL}. Antagonism among odorants, in which multiple odorants compete for common ORN binding sites, could help maintain the sparsity of glomerular activation, provided that binding and activation strengths are uncorrelated~\cite{reddy2017antagonism}. This mechanism requires odor-specific activation processes, i.e. G protein-coupled, cAMP-mediated transduction as in mammalian olfaction (Spehr, Munger 2009). We rely instead on experimental evidence for non-ORN-specific and non-odor-specific dynamic gain control acting via the universally expressed co-receptor Orco (Or83b)~\cite{srinivas\_elife}. %it is plausible that ion channel dynamics carry odor-specific dependencies we have not accounted for here. {\color {blue} blah..}.

**Odor identification in natural odor environments**

Our results are relevant for understanding combinatorial odor coding in naturalistic odor environments. Dynamic gain control acting on a universal timescale of $\sim 250$ ms allows the of determination of odor identity from single whiffs, particularly when these whiffs are mixed among static backgrounds. The gains afforded by rapid ORN adaptation increase with the strength and complexity of the background odor, suggesting the involvement of ORN adaptation in odor discrimination.Our results indicate that despite the generality in the adaptive mechanism (which also acts on a universal timescale), odor coding capacity and decoding fidelity are greatly enhanced.

**Spatiotemporal odor coding and the primacy hypothesis**

Previous studies implicate not only the combination of active ORNs but also their distinct temporal patterns as signatures of odor identity~\cite{stopfer\_temporal\_model, multiple\_timescales\_stopfer, stopfer\_nat\_neuro, stopfer\_temporal\_channel}. ORNs modeled on observed features of these temporal patterns form distinct trajectories in low-dimensional projections, projections which cluster by odor identity, much as we have found here (Fig.~\ref{fig:coding\_b}). Though we do explicitly utilize the temporal history of neural firing in our decoding schemes, the transmission of information over time is implicit in this framework. Because the strength of ORN feedback onto receptor complex activation depends on each ORN's unique tuning curve, odor signals are naturally formatted into temporal patterns that are both odor- and ORN-specific. The response repertoire at a given time is shaped by response history via integral feedback, and the short forgetting timescales, $\tau\_{\textup F} \sim \tau\_{\textup A} ~ \sim 250$ ms, suggest that only information in brief temporal windows is required for accurate odor identification, consistent with previous findings~\cite{stopfer\_nat\_neuro}. On the other hand, the classification scheme we employ here (Figs.~\ref{fig:downstream}) could ...

Primacy coding also exploits the temporal sequence of glomeruli activation, but in a coarser sense: while the order of ORN activation defines the primacy set, within this set the order is immaterial. We too find that information contained in a primacy subset of the full ORN repertoire can be sufficient for accurate reconstruction of simple odor mixtures. An extension we find, in support of the primacy coding hypothesis, is that primacy sets are also preserved even in the presence of potential confounds such as background odors.