UNIVERSITY PARTNER

<u>4MM013 - Computational Mathematics</u>

Mathematics Assignment-2

Full Marks: 20

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1. Using Cramer's rule obtain the solutions to the following set of equations:

$$2\,x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

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Q.NO:- 1

Soln.

The given equation is

$$2x_1 + x_2 - x_3 = 0$$
$$x_1 + x_3 = 4$$
$$x_1 + x_2 + x_3 = 0$$

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= 2|0-1| - 1|1-1| - 1|1-0|$$

$$= -2 - 0 - 1$$
$$= -3$$

$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 0 - 1|4-0| - 1|4-0|$$

$$= -4 - 4 = -8$$

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$$D_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2|4-0| - 0 - 1|0-4|$$

$$= 8 + 4 = 12$$

$$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= 2|-4| - 1|0-4| + 0$$

$$= -8 + 4 = -4$$

$$x = \frac{D_1}{D} = \frac{\cancel{-}8}{\cancel{-}3} = \frac{8}{3}$$

$$y = \frac{D_2}{D} = \frac{12}{-3} = -4$$

$$z = \frac{D_3}{D} = \frac{\cancel{-}4}{\cancel{-}3} = \frac{4}{3}$$

(4)

2.

a) Solve the following using Gauss elimination:

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

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Q.NO :- 2a

The given equation is

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 3$$
$$x_1 - 2x_2 - x_3 = 1$$

Writing in the matrix form is

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & -2 & -1 & 1 \end{array}\right|$$

$R_2 \to R_2 - 2R_1$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 1 & -2 & -1 & 1 \end{array}\right|$$

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$R_3 \to R_3 - R_1$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right|$$

$R_3 \to R_3 + 3R_2$

$$\left|\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right|$$

Writing in the equation form is

$x_1 + x_2 + x_3 = 2$ — (i)
$x_2 + 2x_3 = -1$ — (ii)
$4x_3 = -4$ — (iii)

Now,
Eqn (iii)

$4x_3 = -4$
$x_3 = -1$

Putting x_3 in eqn (ii)

$x_2 + 2x_3 = -1$
$x_2 + 2 \times (-1) = -1$
$x_2 - 2 = -1$
$x_2 = -1 + 2$
$x_2 = 1$

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Putting x_2, x_3 in eqn ①

$$x_1+x_2+x_3=2$$
$$x_1+\cancel{1}-\cancel{1}=2$$
$$x_1=2$$

$\therefore x_1=2$, $x_2=1$, $x_3=-1$

(4)

b) Find the inverse of the matrix from (a) using elimination.

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Q NO:- 2b

$$= \left| \begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array} \right|$$

= $R_2 \to R_2 - 2R_1$

$$\left| \begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array} \right|$$

$R_3 \to R_3 - R_1$

$$\left| \begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array} \right|$$

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$R_3 \to R_3 + 3R_2$

$$\left|\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right|$$

$R_2 \to R_2 \times 2 - R_3$

$$\left|\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right|$$

$R_1 \to 2R_1 - R_2$

$$\left|\begin{array}{ccc:ccc} 2 & 0 & 2 & -1 & 1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right|$$

$R_1 \to 2R_1 - R_3$

$$\left|\begin{array}{ccc:ccc} 4 & 0 & 0 & 5 & -1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right|$$

$R_1 \to R_1/4$

$$\left|\begin{array}{ccc:ccc} 1 & 0 & 0 & 5/4 & -1/4 & 1/4 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right|$$

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$R_2 \to R_2/2$, $R_3 \to R_3/4$

$$\left|\begin{array}{ccc:ccc} 1 & 0 & 0 & 5/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right|$$

(4)

3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

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Q.NO:- 3
soln.

let,
$t_n = a_n$

$a_n = \frac{n+1}{n^2+3}$

$= \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2+3}$

$= \lim_{n\to\infty} \frac{\frac{n+1}{n^2}}{\frac{n^2+3}{n^2}}$

$= \lim_{n\to\infty} \frac{\frac{1}{n}+\frac{1}{n^2}}{1+\frac{3}{n^2}}$

$= \frac{0+0}{1+0} = 0$

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$\therefore$ since, $\lim_{n\to\infty} a_n$ = Real number, the sequence -e convergence.
Therefore, the sequence is convergent.

4. Find the Maclaurin series expansion of **Sinx,** also calculate the radius of convergence. (4).

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Q.No:- 4

Soln.

$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$
$f'''(x) = -\cos x$
$f^{iv}(x) = +\sin x$

Now,
$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$

$f'''(0) = -\cos 0 = -1$

$f^{iv}(0) = \sin 0 = 0$

The formula for maclaurin series is

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0)$$

$$= 0 + x + x^2 . 0 - \frac{x^3}{3!} + \frac{x^4}{4!} . 0 + \frac{x^5}{5!} + \dots$$

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$$f(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

Now,

Radius of convergence (R) $= \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$

$$a_n = (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$a_{n+1} = (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!}$$

$$\lim_{n \to \infty} \frac{x^{2n+3}}{(2n+3)!} \times \frac{(2n+1)!}{x^{2n+1}}$$

$$= \lim_{n \to \infty} x^{2n+3-2n-1} \times \frac{(2n+1)!}{(2n+3)(2n+2)(2n+1)!}$$

$$= \lim_{n \to \infty} x^2 \times \frac{1}{(2n+3)(2n+2)}$$

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$$= |x|^2 \times \infty$$

$$= \infty$$

$$R = \infty$$

$\therefore$ This series converges for all values of x, so its radius of convergence 'R' is infinite i.e. $R = \infty$

The End