

UNIVERSITY PARTNER



## **4MM013 - Computational Mathematics**

### Mathematics Assignment-2

Full Marks: 20

University ID	: 2332917
Submitted by	: Niraj Chaudhary
Submitted on	: Subash Khatiwada

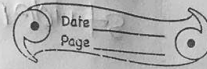
1. Using Cramer's rule obtain the solutions to the following set of equations:

$$2x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

Name: Niraj Chaudhary  
Student ID: 2332917



Q.No:-1

Soln.

The given equation is

$$2x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= 2(0 - 1) - 1(1 - 1) - 1(1 - 0)$$

$$= -2 - 0 - 1$$

$$= -3$$

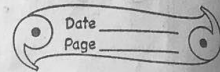
$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 0 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 0 - 1(4 - 0) - 1(4 - 0)$$

$$= -4 - 4 = -8$$

Name: Nitya chandhary  
Student Id: 2332917



$$D_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2(4 - 0) - 0 - 1(1 - 4)$$

$$= 8 + 4 = 12$$

$$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 1 & 10 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 2(1 - 1) - 1(0 - 1) + 0$$

$$= -8 + 4 = -4$$

$$x = \frac{D_1}{D} = \frac{+8}{+3} = \frac{8}{3}$$

$$y = \frac{D_2}{D} = \frac{+12}{+3} = +4$$

$$z = \frac{D_3}{D} = \frac{+4}{+3} = \frac{4}{3}$$

(4)

2.

a) Solve the following using Gauss elimination:

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

Name:- Niraj chaudhary  
Student Id:- 2332917

Q.No :- 29

The given equation is

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

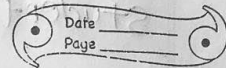
Writing in the matrix form is

$$\begin{vmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 4 & : & 3 \\ 1 & -2 & -1 & : & 1 \end{vmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{vmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 1 & 2 & : & -1 \\ 1 & -2 & -1 & : & 1 \end{vmatrix}$$

Name:- Nitya chaudhary  
Student Id:- 2332917



$$R_3 \rightarrow R_3 - R_1$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array} \right|$$

$$R_3 \rightarrow R_3 + 3R_2$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array} \right|$$

Writing in the equation form is

$$x_1 + x_2 + x_3 = 2 \quad \text{--- (i)}$$

$$x_2 + 2x_3 = -1 \quad \text{--- (ii)}$$

$$4x_3 = -4 \quad \text{--- (iii)}$$

Now,

Eqn (iii)

$$4x_3 = -4$$

$$x_3 = -1$$

Putting  $x_3$  in eqn (ii)

$$x_2 + 2x_3 = -1$$

$$x_2 + 2 \times (-1) = -1$$

$$x_2 - 2 = -1$$

$$x_2 = -1 + 2$$

$$x_2 = 1$$



Name: Nirag chandhary  
Student Id: 2332917



Putting  $x_2, x_3$  in eq<sup>n</sup> ①

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + 1 - 1 = 2$$

$$x_1 = 2$$

$$\therefore x_1 = 2, \quad x_2 = 1, \quad x_3 = -1$$

(4)

b) Find the inverse of the matrix from (a) using elimination.

Name: Nitya chandhary  
Student Id: 2332917

Q. NO: 2b

$$= \left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array} \right|$$

$$= R_2 \rightarrow R_2 - 2R_1$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & -2 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array} \right|$$

$$R_3 \rightarrow R_3 - R_1$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & -2 & 1 & 0 \\ 0 & -2 & -2 & -1 & 0 & 1 \end{array} \right|$$



Name: - Nivag chandhary  
 student id: 2332917

Date \_\_\_\_\_  
 Page \_\_\_\_\_

$$R_3 \rightarrow R_3 + 3R_2$$

$$\left| \begin{array}{cccccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 & 0 & 0 & 0 \end{array} \right|$$

$$R_2 \rightarrow R_2 \times 2 - R_3$$

$$\left| \begin{array}{cccccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 4 & -7 & 3 & 1 & 0 & 0 & 0 \end{array} \right|$$

$$R_1 \rightarrow 2R_1 - R_2$$

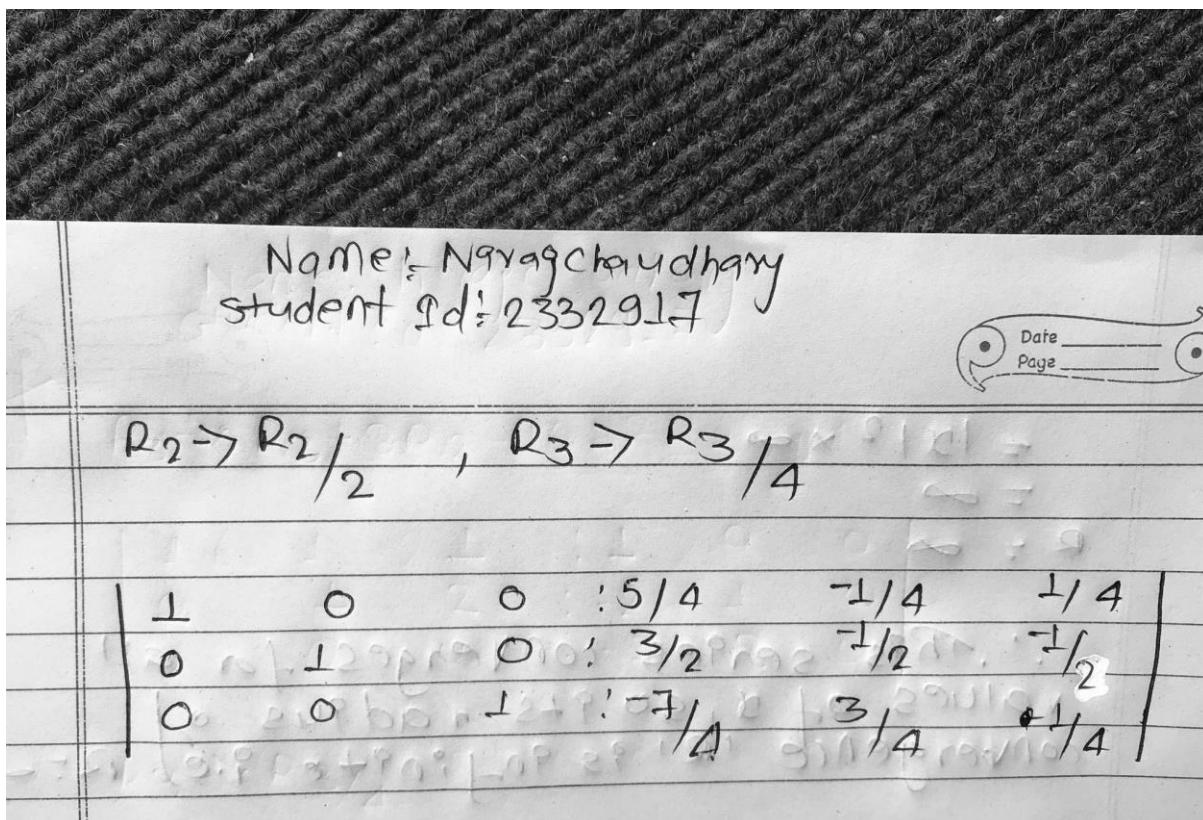
$$\left| \begin{array}{cccccc|ccc} 2 & 0 & 2 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 4 & -7 & 3 & 1 & 0 & 0 & 0 \end{array} \right|$$

$$R_1 \rightarrow 2R_1 - R_3$$

$$\left| \begin{array}{cccccc|ccc} 4 & 0 & 0 & 5 & -1 & 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 4 & -7 & 3 & 1 & 0 & 0 & 0 \end{array} \right|$$

$$R_1 \rightarrow R_1 / 4$$

$$\left| \begin{array}{cccccc|ccc} 1 & 0 & 0 & 5/4 & -1/4 & 0 & 1/4 & 1/4 & 1/4 \\ 0 & 2 & 0 & 3 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 4 & -7 & 3 & 1 & 0 & 0 & 0 \end{array} \right|$$



(4)

3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1} \frac{n+1}{n^2+3}$$

Name: Nitya chandhary

Student Id: 2332917

Q. No: 3

Soln.

let,

$$t_n = a_n$$

$$a_n = \frac{n+1}{n^2+3}$$

$$= \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n+1}{n^2+3}$$

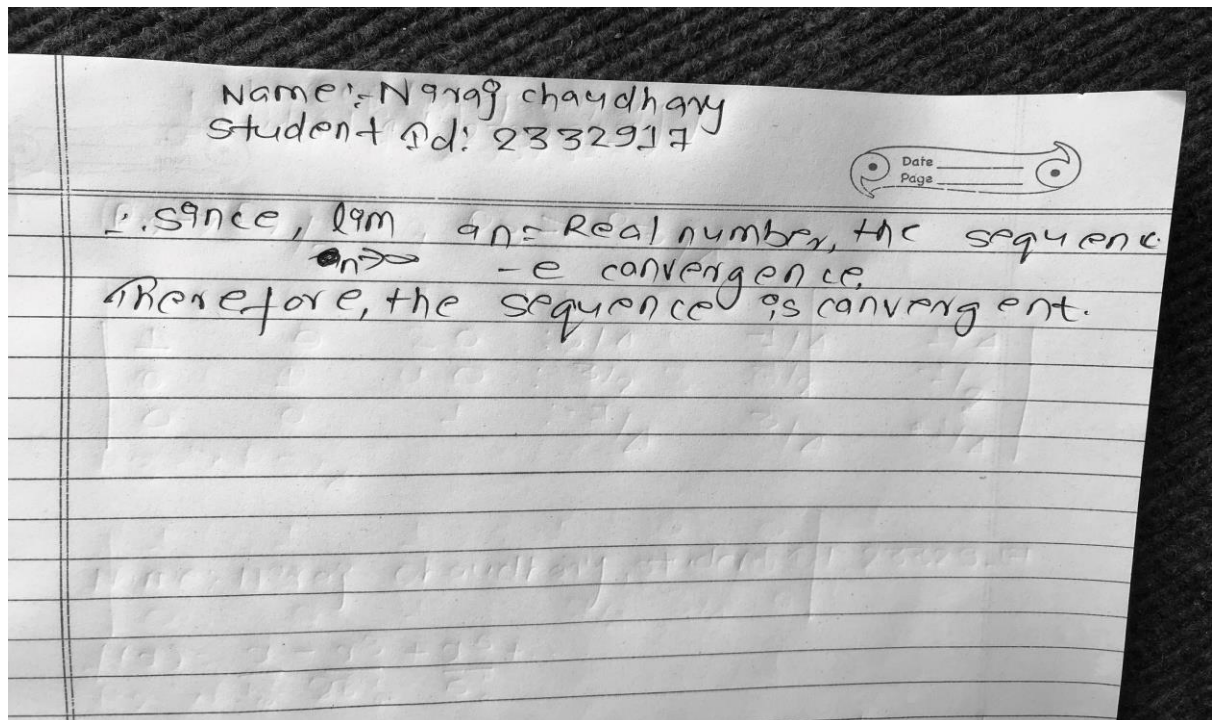
$$= \lim_{n \rightarrow \infty} \frac{n+1}{n^2}$$

$$\frac{n^2+3}{n^2+3}$$

$$= \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{1 + \frac{3}{n^2}}$$

$$= \frac{0+0}{1+0}$$

$$= 0$$



4. Find the Maclaurin series expansion of **Sinx**, also calculate the radius of convergence. (4).

Name:- Nitya chandhary

Student Id: 2332914

Q.No:-4

Q.1.

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

$$f'''(x) = -\cos x$$

$$f^{(4)}(x) = +\sin x$$

Now,

$$f(0) = \sin 0 = 0$$

$$f'(0) = \cos 0 = 1$$

$$f''(0) = -\sin 0 = 0$$

$$f'''(0) = -\cos 0 = -1$$

$$f^{(4)}(0) = \sin 0 = 0$$

The formula for Maclaurin series is

$$\begin{aligned} f(x) &= f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{(4)}(0) \\ &= 0 + x + \frac{x^2}{2!} \cdot 0 - \frac{x^3}{3!} + \frac{x^4}{4!} \cdot 0 + \frac{x^5}{5!} + \dots \end{aligned}$$



Name: Nityaj Chaudhary, student Id: 2332917

$$f(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

Now,

$$\text{Radius of convergence } (R) = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$a_n = (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$a_{n+1} = (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!}$$

$$\lim_{n \rightarrow \infty} \frac{x^{2n+3}}{(2n+3)!} \times \frac{(2n+1)!}{x^{2n+1}}$$

$$= \lim_{n \rightarrow \infty} x^{2n+3-2n-1} \times \frac{(2n+1)!}{(2n+3)(2n+2)(2n+1)!}$$

$$= \lim_{n \rightarrow \infty} \frac{x^2 \times 1}{(2n+3)(2n+2)}$$

$$= |x|^2 \times \infty$$

$$= \infty$$

$$R = \infty$$

∴ This series converges for all values of  $x$ , so its radius of convergence ' $R$ ' is infinite i.e.  $R = \infty$

The End

