Total No. of Printed Pages:2

SUBJECT CODE:- 253

FACULTY OF ENGINEERING AND TECHNOLOGY

F. E.(ALL) Examination Nov/Dec 2015

Engineering Mathematics - I

(Revised)

[Time: Three Hours] **[Max. Marks: 80]**

"Please check whether you have got the right question paper."

N.B i) Q.No.1 and Q.No.6 are compulsory.
ii) Solve any two questions from Q.2, Q.3, Q.4 and Q.5.
iii) Solve any two questions from Q.7, Q.8, Q.9 and Q.10.
iv) Figures to the right indicate full marks.
v) Assume suitable data, if necessary.

Section- A

Q.1 Solve any five questions from the following. 10

a) Find the locus represented by $|z-3|+|z+3|=0$
b) Find tanhx, if $\sin hx - \cos hx = 5$
c) Find the n[th] derivative of $e^{5x}\cos(3x+2)$
d) State the Ratio test of series
e) Derive the series for f(x)= sinhx using Maclaurin's theorem.
f) Evaluate $\lim_{n\to\infty} \frac{\log x}{x^n}$, if n>0
g) Verify the exactness of differential equation.
$[1+2xy\,cosx^2-2xy]dx+[sinx^2-x^2]dy=0$
h) Reduce the Bernoulli's differential equation : $\frac{dy}{dx}+xsin2y=x^3\cos^2 y$ to linear differential equation.

Q.2
a) Prove that $(x+iy)^{\frac{m}{n}}+(x-iy)^{\frac{m}{n}}=2(x^2+y^2)^{\frac{m}{2n}}\cos(\frac{m}{n}\tan^{-1}\frac{y}{n})$ 04
b) Find the n[th] derivative of $\frac{x^4}{(x-1)(x-2)}$ 06
c) Solve: $y^2dx+(3xy-1)dy=0$ 05

Q.3
a) Simplify :- $\left[\frac{1+cos\frac{\pi}{9}+i\sin\frac{\pi}{9}}{1+cos\frac{\pi}{9}-i\sin\frac{\pi}{9}}\right]^{18}$ 05
b) Prove that $\tan^{-1}[\frac{x\sin\theta}{1-x\cos\theta}]=x\sin\theta+\frac{x^2}{2}\sin2\theta+\frac{x^3}{3}sin3\theta+\cdots\ldots$ 05
c) Solve : $\frac{dy}{dx}=\frac{y-2x}{2y-x}, y(1)=2$ 05

Q.4
a) If $\sin 6\theta = a\cos^5\theta\sin\theta+b\cos^3\theta\sin^3\theta+c\cos\theta\sin^5\theta$, find the value of a, b, c. 05
b) Prove that: $\lim_{n\to\infty}\left[\frac{a^{\frac{1}{x}}+b^{\frac{1}{x}}+c^{\frac{1}{x}}+d^{\frac{1}{x}}}{4}\right]^x=(abcd)^{\frac{1}{4}}$ 05
c) Show that $\frac{g}{n^2}\log(\cos hnt)$ is the distance passed over by a body falling vertically from rest, assuming that the resistance of air is $\frac{n^2}{g}$ times the square of the velocity. 05

Q.5

a) Considering the principle value, express $\left(\sqrt{i}\right)^{\sqrt{i}}$ in the form $a+ib$ 05

b) Test the convergence of the series $\sum_{n=1}^{\infty} sin^{\frac{1}{n}}$ 06

c) Find the orthogonal trajectories of the family of curve $x^2+cy^2=1$ 04

Section- B

Q.6 Solve any five questions from the following. 10

a) Find the equation of asymptote to the curve $y=\frac{x}{1+x^2}$

b) Write the symmetry of the curve $x=a(\theta-\sin\theta), y=a(1-cos\theta)$

c) Find the equations of tangent at pole to the curve $r=2a\sin\theta$

d) The length of curve $r=f(\theta)$ from the lines $\theta=a\ to\ \theta=\beta$ is given by the formula…….

e) $if=\log\left[\frac{\sqrt{x^2+y^2}}{x+y}\right]$ find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$

f) If $u=lx+my, v=mx-ly$ then show that $\left(\frac{\partial u}{\partial x}\right)_y\left(\frac{\partial x}{\partial u}\right)_v=\frac{l^2}{l^2+m^2}$

g) Find the stationary points of the function $f(x,y)=x^2+y^2+6x+12$

h) If $\gamma=\sqrt{x^2+y^2}, =\tan^{-1}\frac{y}{x}, find\ \frac{\partial(\gamma,\theta)}{\partial(x,y)}$

Q.7

a) Trace the curve $y^2(a-x)=x^2(a+x)$ with full justification. 05

b) If $z=x^y+y^x$then show that $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$ 05

c) If $u=\frac{2yz}{x}, v=\frac{3zx}{y}, w=\frac{4xy}{z}$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ 05

Q.8

a) Trace the curve $r^2=a^2cos2\theta$ with full justification. 05

b) Verify Euler's theorem for the function $u=\sin^{-1}(\frac{x}{y})+\tan^{-1}\left(\frac{y}{x}\right)$ 05

c) Find the length of the curve $x=a(\cos\theta-\theta sin\theta), y=a(\sin\theta-\theta cos\theta)$ from $\theta=0\ to\ \theta=2\pi$ 05

Q.9

a) Trace the curve $x=a\cos^3 t, y=b\sin^3 t$ with full justification. 05

b) If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$ 05

c) Find the extreme value of $u=x^3+3xy^2-3x^2-3y^2+7$ 05

Q.10

a) Find total length of perimeter of cardioids $r=a(1+\cos\theta)$ 05

b) Find the point on the plane $ax+by+cz=p$ at which the function $f=x^2+y^2+z^2$has a minimum value and find this minimum f. 05

c) Find the length of the loop the curve $9y^2=(x+7)(x+4)^2$ 05