Midterm Solutions - Probability #1650

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2 hours ago in Midterm – Probability

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Please find below the solutions to the **Probability** section of the midterm exam.

Question 1

1. C (U' = f(U, m, n) = (n-m)*U + m)
   To obtain uniform random numbers in a new interval [m,n], we can warp the interval from [0,1] to [m,n] using the function f(x) = (n-m)*x + m

2. 0.5
   Since U' generates random numbers uniformly in the interval [-1,1], the probability of generating random numbers in the interval [-0.5,0.5] will be length of this interval divided by the length of the total interval, i.e. ½ = 0.5

3. 0.25
   We need to compute the joint probability of U' generating two numbers in the intervals [-0.5,0.5], which will be 0.5*0.5 = 0.25

4. A (Yes)
   Since the probability of generating random numbers in a certain sub-region is proportional to the size (area) of that sub-region, F(U') is a uniform random number generator.

Question 2

1. 0.5
   The probability of lying on the right semi-circle is equal to the probability of the rotation angle being sampled in [0, pi/2] and [3*pi/2, 2*pi], which is equal to the size of this interval divided by the total interval, i.e. (pi/2 + pi/2)/(2*pi) = ½ = 0.5, multiplied by the probability of the radius lying anywhere in [0,1], i.e. probability 1. Hence, the total probability is 0.5*1 = 0.5

2. 0.5
   For the point to lie within a circle of radius 0.5, the radius should lie in the interval [0,0.5], i.e. probability 0.5, and the sampled angle can be anything in the region [0,2*pi], i.e. probability 1. Hence the total probability is 1*0.5 = 0.5

3. No

In 2a, the probability of generating random numbers in a region that was ½ the size of the total circle was 0.5. However, in 2b, we saw that the probability of generating numbers within a region that was ¼ the size of the total circle was still 0.5. Since the probability of generating random numbers in a certain sub-region is NOT proportional to the size (area) of that sub-region, G(U) is NOT a uniform random number generator.

M Michael Joseph Matsako 2h

Safe to assume rounded to 6 decimals is perfectly valid?

0.500000
0.250000

Otherwise answers look correct.

♡ 3 Reply ···

 Harrison Takuya Ooi 2h

it said to round to 6 decimal places so we should be fine.

♡ Reply ···

P Pardeep Singh 2h

Yeah, Usually the auto-graders use a value with a margin of error. So 0.5 - 0.500000 should be 0 and thus valid

♡ Reply ···

 Roshni Nita Mahtani 1h

I simulated these probabilities, so mine are close but not the exact values, so I hope they pass with that margin like eg: 0.500118, 0.249948

♡ Reply ···

 Dana Tareq Alnabulsi 2h

I think question 2.2 is slightly off. Even if the coordinates generated are for example (0.9, 0.9), pushing it outside the circle of radius 0.5, multiplying it by 0.5 as a value for a would push it back in the circle as it becomes (0.45, 0.45). I used code to run this probability test and consistently came to the probability 0.561096.

cos(b) has a 50% chance of generating a number between -0.5 and 0.5, with sin(b) having the same probability. But multiplying it with a gives us a slightly higher probability to be pushed into the circle, hence my above probability.

♡ 2 Reply ···

P Piyush Mishra 2h

My experiment came close to 0.5. The probability of sine is affected by the probability of cosine I think since they are dependent.

♡ 1 Reply ···

Dana Tareq Alnabulsi 2h

Yes they are dependent in that they use the same value for a and b. However the value of x and y are independent given the values of a and b.

♡ Reply ···

Z

Zhi Yao Tee 2h

I think (0.45,0.45) is outside the circle. sqrt(0.45^2 + 0.45^2) =0.636396

♡ 1 Reply ···

P

Pardeep Singh 2h

essentially generating these a and b values is generating polar coordinates and then translating them to cartesian. Since a is generated from U(0,1) that used as radius. So probability of being inside a circle of radius 0.5 is dependent purely on the value we choose for a, thus the 0.5

♡ 11 Reply ···

Dana Tareq Alnabulsi 2h

This explanation makes sense, thank you 😃

♡ 1 Reply ···

Benjamin Snider 34m

Well shoot, I didn't think to study trig for an AI exam. Thanks for the explanation though.

♡ 1 Reply ···

Michael David Logan 2h

I did it my hand and verified with code. Maybe my explanation will help.

Let d = sqrt(x^2, y^2), we want P(d < 0.5)...

--> 0.5 > sqrt(x^2 + y^2)

--> 0.25 > x^2 + y^2

Subbing in x = a*cos(b), y = a*sin(b)

--> 0.25 > a^2 * cos(b)^2 + a^2 * sin(b)^2

--> 0.25 > (a^2) * (cos(b)^2 + sin(b)^2)

--> 0.25 > a^2 * 1

IDENTITY: cos(b)^2 + sin(b)^2 = 1

--> 0.5 > a

a is uniformly distributed in [0, 1] so P(a < 0.5) would be 0.5

♡ 4 Reply ···

A Anh Hong Nguyen 2h

I also simulated this distribution several times with 1,000,000 samples each time. My results came out to be very close to 0.5 every time.

♡ Reply ···

J Jun Zhu 2h

The function basically just generates a random angle from 0-2pi then projects a vector of random length [0,1] in the direction of that angle.

♡ Reply ···

 Dana Tareq Alnabulsi 2h

I haven't done trigonometry in a long time, so I didn't consider the logic whatsoever

♡ 1 Reply ···

J Jun Zhu 1h

Replying to Dana Tareq Alnabulsi

It was tricky for sure. I have a game dev background and this is actually the same function you use to translate controller coordinates to movement. Even then I had to pause and stare at it a bit.

♡ Reply ···

 Dana Tareq Alnabulsi 1h

Replying to Jun Zhu

Oh man that's nice though that you ran into this function in actual use

♡ 1 Reply ···