CS 6601 Artificial Intelligence
Fall Semester 2022
Midterm Examination Paper

Duration of Exam: 10 Oct 2022, 8:00 AM (EDT) - 17 Oct 2022, 8:00 AM (EDT)

Weight: 15% **No. pages:** 35 **No. Questions:** 6 **Total marks:** 103

Instructions:

- Before solving the exam, you should read the Ed post titled **"Midterm Exam Next Week"**.
- You should fill out this PDF and submit it on **BOTH Gradescope and Canvas**. We reserve the right to deduct points if you don't submit on both platforms.
- You have an unlimited number of submissions until the deadline. You can either type directly into the PDF or you can print, hand-write, and scan your solutions. If you decide to type into the PDF, **we strongly recommend using Adobe Acrobat Reader DC**. If you are on MacOS, **please do not use Preview**, as we have seen major issues with it in the past. Other programs may not save your answers. Thus, **always make sure to keep a backup of your answers**.
- You should **submit only a single PDF**. Also, **make sure your typed answers appear clearly in the Gradescope Preview**. After each question, a blank page is provided if you wish to highlight some of your calculations. However, you should not expect any points to be awarded for your calculations (only your final answers will be considered). The TAs reserve the right to assign partial credits based on your calculations. Make sure to submit ALL the pages of the exam, not only the completed ones.
- You must **fill out the checklist at the end** of the exam. The exam may not be graded if it is left blank.
- The exam is open-book, open-note, and open video lectures, with no time limit aside from the duration of the exam. No internet use is allowed, except for e-text versions of the textbook, this semester's CS6601 course materials, Ed, and any links provided in the PDF itself. No resources outside this semester's 6601 class should be used. Do not discuss the exam on Ed, Slack, or any other platform. In particular, do not post publicly about the exam. If there is a question for the teaching staff, please make it private on Ed and tag it as Midterm Exam with the question number in the subject line (for example, a question on Search would be "Midterm Exam #2"). Please make different posts for different questions.
- **Please round all your final answers to 6 decimal places**. Don't round intermediate results. You can use round(answer, 6) function in Python for help. You may not receive full credit if your answers are not given to the specified precision.
- Points breakdown is provided below.

Question No.	1	2	3	4	5	6	7
Points	12	20	20	12	18	18	3

Problem 1: Game Playing [12 points]

For this question we will look at a variant of Isolation, which takes place on a 3x3 board. Player 1 controls piece ‘X’, while player 2 controls piece ‘O’. Squares which are no longer visited are grayed out. Both players have a single piece, and this piece can move to adjacent squares, but cannot move diagonally. For example, on player 1’s turn, with the board shown below in Figure 1, they have 4 valid moves:

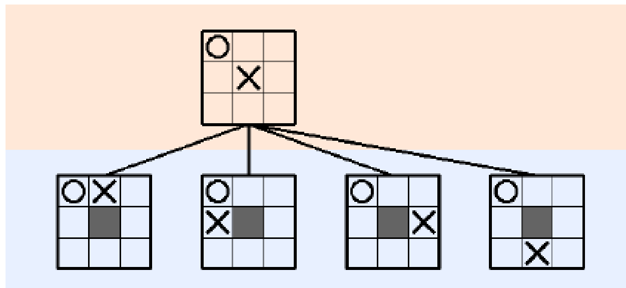

Figure 1: Board Diagram

On the first turn, player 1 can place their piece in multiple locations. For this question, we will only look at the subtree where player 1 places their piece in the middle of the board. Because the rules for this variant of Isolation exhibit reflectional and rotational symmetry, we can greatly simplify the tree by removing duplicates:

X win = 1
X loss = -1
[-1, 1]

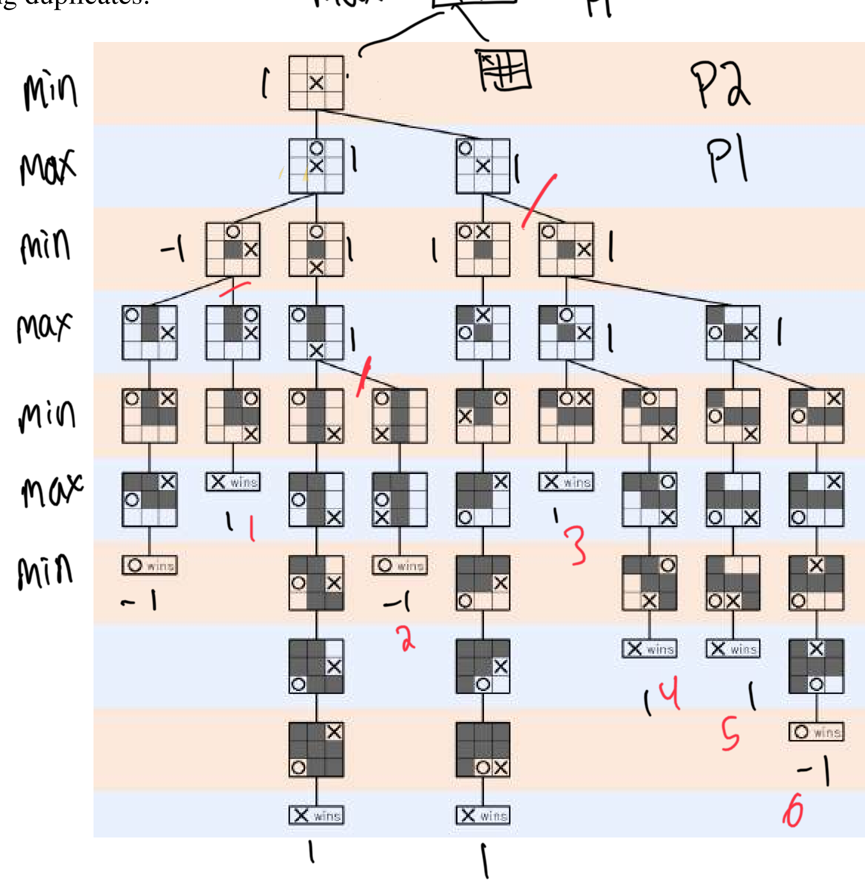

Figure 2: The Game Tree

Minimax & Alpha-Beta Pruning:
Take a couple minutes to study this game tree in Figure 2 and get a feel for how it works. Use Figure 2 to answer the following two questions:

(a) (2 points) Use the Minimax algorithm to determine the value of this first move for Player 1. Does Player 1 have a guaranteed win?

Yes ☐ No ☐

(b) (4 points) Apply alpha-beta pruning to the tree (traversing branches from left to right). How many leaves are pruned?

Type Your Answer Here: ☐

Evaluation Function:
Rather than searching the full tree, let's limit our search to a fixed depth. In order to do this, we need a function to evaluate a non-terminal board state. Let's use a simple function. Let A be the number of empty squares adjacent to player 1, and let B be the number of empty squares adjacent to player 2. Our evaluation function will simply be: **A - B**
For example, in the board shown below in Figure 3, player 1 has 2 adjacent empty squares, and player 2 has 1 empty adjacent square, so the heuristic function will evaluate to 2 - 1 = 1. Player 1 seeks to maximize this function, while player 2 seeks to minimize this function.

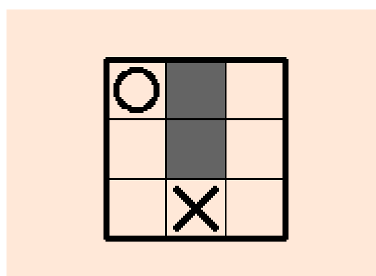

Figure 3: Example Board

We will also add one exception to this function – if it is player 1's turn and player 1 has 0 adjacent squares, the function will evaluate to negative infinity, and if it is player 2's turn and player 2 has 0 adjacent squares, it will evaluate to positive infinity. This ensures that win / loss conditions outweigh our adjacency heuristic.
Use this heuristic function to evaluate the board to a depth of 6, as shown below in Figure 4:

(c) (2 points) Use the minimax algorithm to evaluate the truncated tree in Figure 4. Apply alpha-beta pruning to the tree (traversing branches from left to right). How many leaves are pruned?

Type Your Answer Here: ☐

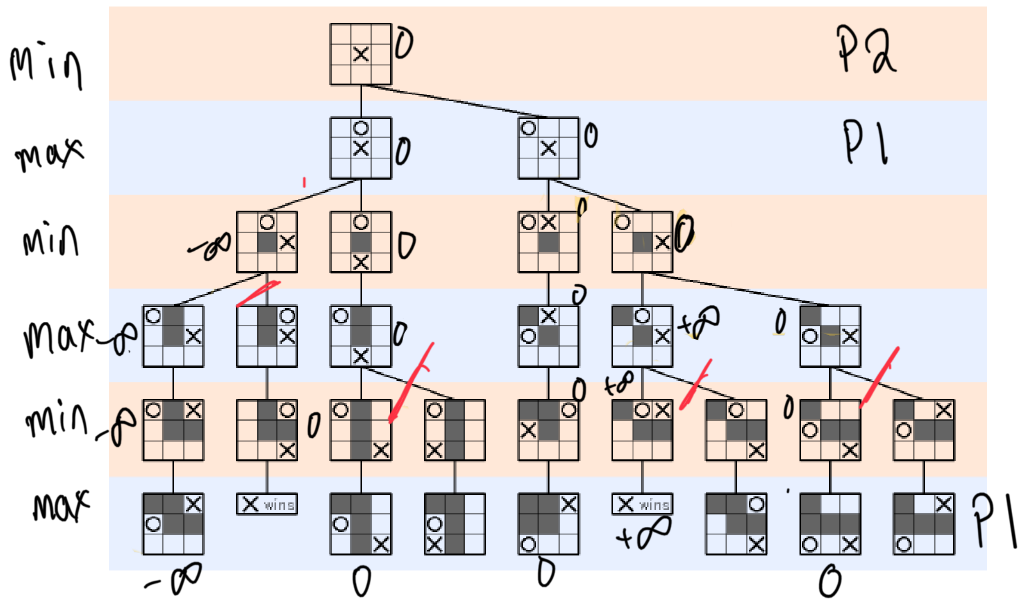

Figure 4: Game Tree to a Depth of 6

For the next questions, you may use the trees shown in Figures 5, 6, 7, 8, 9 for computations.

Let's say player 1 and player 2 use different algorithms to decide their moves. Player 1 uses the depth-limited (to depth 6) minimax search to pick their moves, while player 2 has the luxury of an infinite depth search (the search from the first part of this problem). Player 2 also knows that player 1 will only search the board to a depth of 6, and uses this knowledge in their minimax algorithm.

(d) (2 points) Given this first move, and assuming that both players strictly follow their algorithms, does player 2 win the game?

Yes ☐ No ☐

(e) (4 points) Apply alpha-beta pruning to player 2's searches (traversing branches from left to right). How many leaves are pruned? *(Hint: you will have to repeat player 1's depth-limited searches as they proceed deeper into the game)*

Type Your Answer Here: ☐

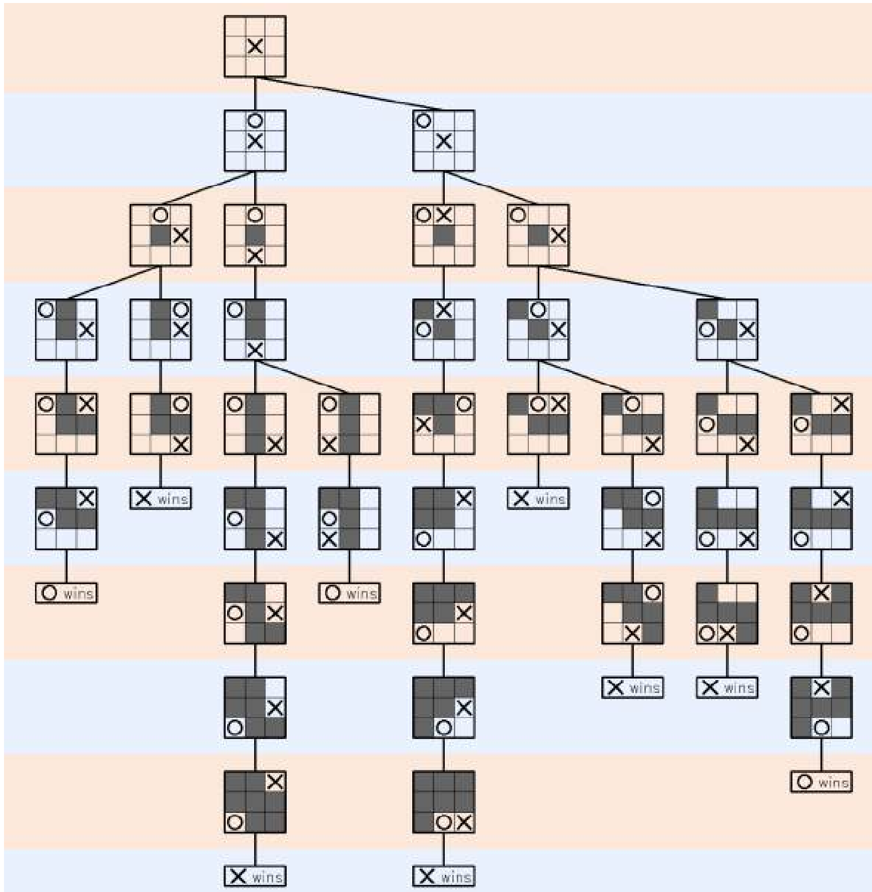

Figure 5: Game Tree

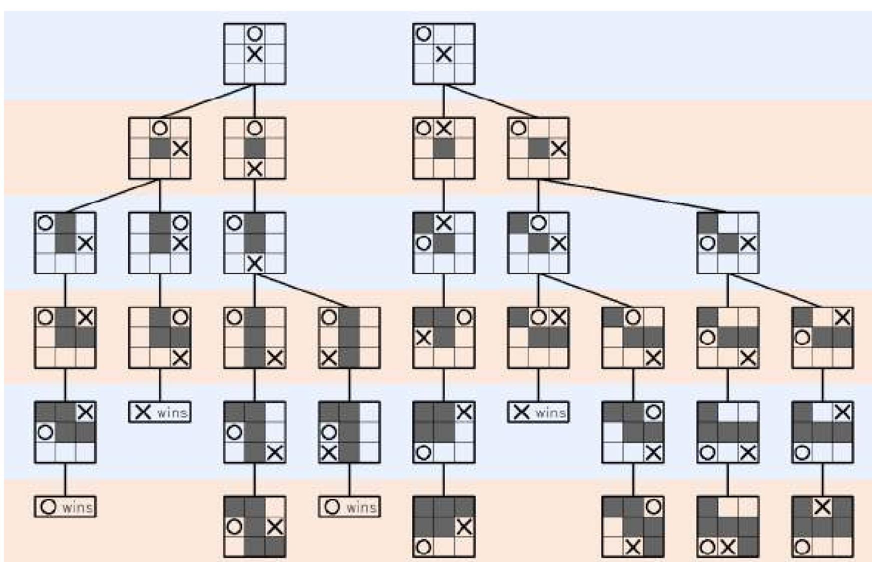

Figure 6: Game Tree

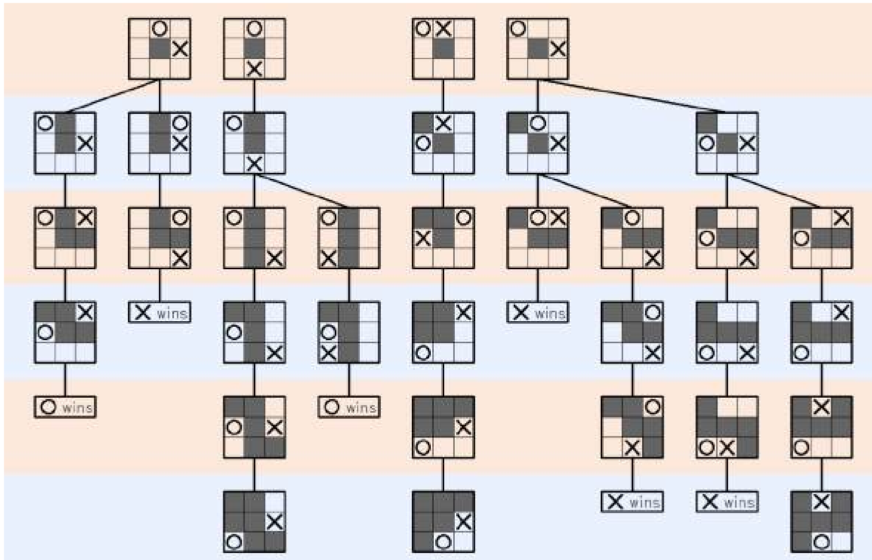

Figure 7: Game Tree

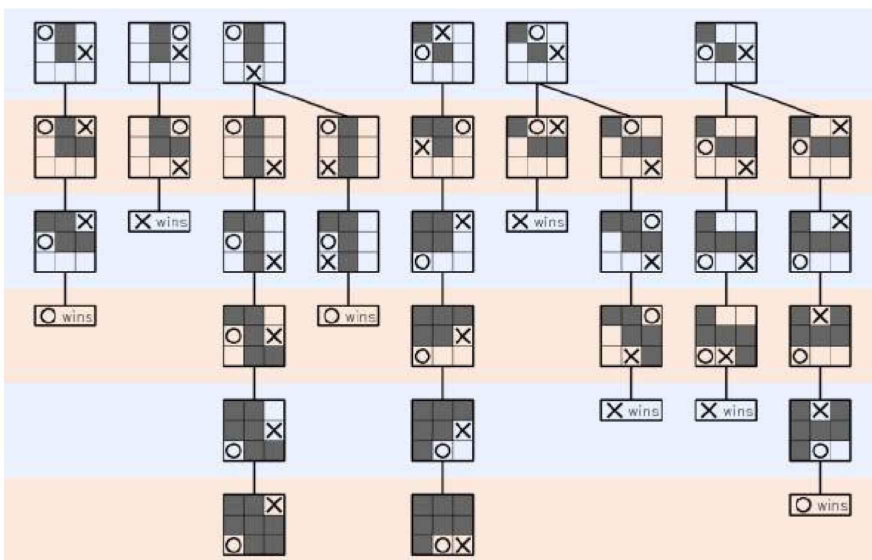

Figure 8: Game Tree

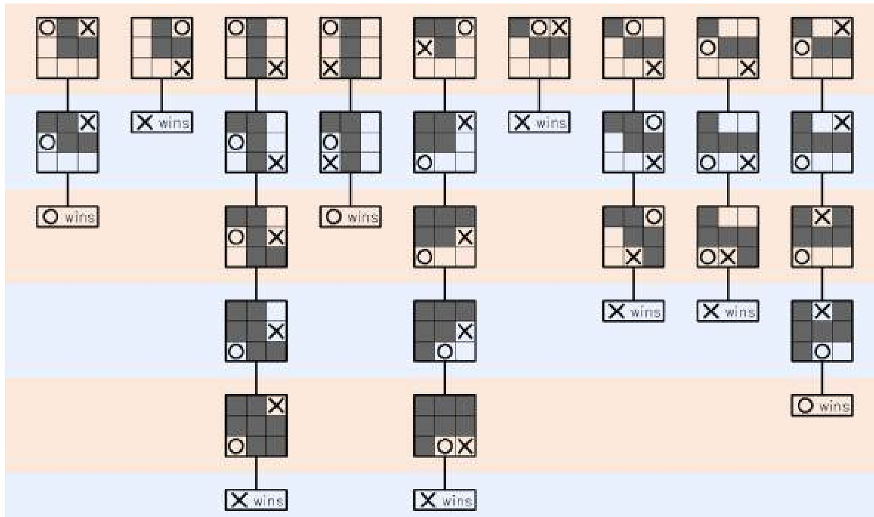

Figure 9: Game Tree

You can add your notes on this page (if any)

Problem 2: Search [20 points]

Part 1: Search Knowledge

You are given an admissible heuristic $h_1(n)$ for A^*, and that $h_2(n) = 2h_1(n)$. Please answer the following questions:

$A \to B = 6$

$h_1() = 5$

$h_2() = 10$

(a) (1 point) Would $h_2(n)$ be always admissible?

Yes ☐ No ☐

(b) (2 points) A^* graph-search with $h_2(n)$ would provide a guaranteed optimal solution.

True ☐ $\boxed{\text{False}}$ ☐

(c) (2 points) An A^* tree-search solution with $h_2(n)$ has a guaranteed cost $\leq$ 2(true cost of A^* tree-search with $h_1(n)$)?

$\boxed{\text{True}}$ ☐ False ☐

h_2

A — G

$h_1 \leq \overline{AG}$

Now, you are given two different admissible A^* heuristics, $a(n)$ and $b(n)$. Please answer the following questions:

(d) (2 points) Which one of the functions given below will combine the two heuristics into a single heuristic, which guarantees admissibility and results in A* expanding the fewest number of nodes?

1. $F_1 = min(a(n), b(n))$
2. $F_2 = max(a(n), b(n))$
3. $F_3 = avg(a(n), b(n))$
4. $F_4 = ln(a(n), b(n))$

admissible

F_1 ☐ $\boxed{F_2}$ ☐ F_3 ☐ F_4 ☐

(e) (3 points) Let $h^*(n)$ be the true cost function, what are the relationships between $h^*(n)$ to $a(n)$ and $h^*(n)$ to $b(n)$? How can you use these relationships to arrive at a "Conclusion", which is a relationship between $a(n)$, $b(n)$, and $h^*(n)$ that guarantees admissibility and expansion of fewest nodes for your function above? E.g.$(=, \leq, \geq, a + b)$

Relationship between $a(n)$ and $h^*(n)$: ☐

Relationship between $b(n)$ and $h^*(n)$: ☐

Conclusion: ☐

Part 2: Robotic Assembly

As a lead engineer at Ploetz-Tharner, you are selected to design the new robotic assembly program. A 5-part sequencing will be performed by a robotics system with 3DOF (3 degrees of freedom). The robot can move in the x,y,z space, and the parts are arranged in a grid on a conveyor. The robot will optimally move the parts to the target positions.

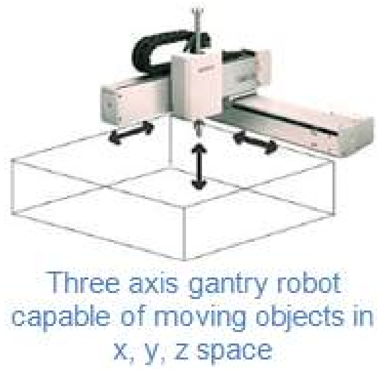

Figure 10: Robot with 3DOF

The following (Figure 11) is a relaxed grid representation that shows the parts on the conveyor and the goal location. Assume that you have programmed the robot to lift, lower, grasp and release the parts. This allows you to write a program that can ignore the robot movement which will discretize the search space and avoid solving for continuous movement. Although you are concerned over producing efficient algorithms, and must optimize your steps, there must be a way to limit the complexity. You've decided to limit the ability of the robot to move according to a compass directions of N, NE, E, SE, S, SW, W, and NW. Based on that, answer the following question:

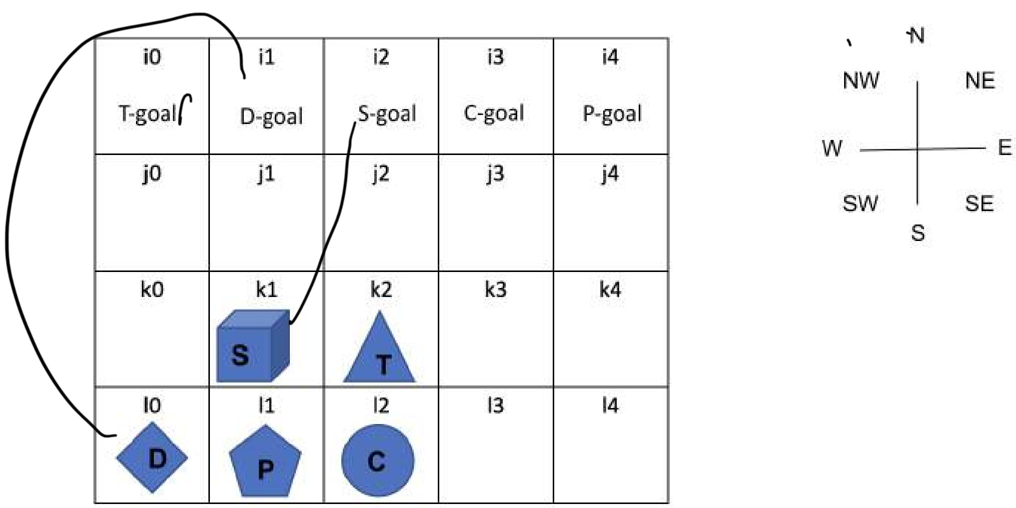

Figure 11: Grid of the Conveyor

(f) (2 points) What is the branching factor for this space?

Type Your Answer Here:

That seems like a challenge, but you consider what you have learned in AI@GT, and decide you should be able to limit this to the expected branching factor you should consider.

(g) (2 points) What number of states would you expect to consider at depth k, given the rules below?

Type Your Answer Here: [ ]

The rules of the assembly are as follows:
The parts must be moved one at a time toward their prescribed goal position aligning the parts for assembly. Only one part can occupy a cell, and your robot can sense what cell a part is in. Your robot can perform the following actions:

1. Ready (positions the robot at i0) no cost
   (a) Only available for step 0, result returns i0
2. Move (direction, # of cells) (Result returns current cell)
   (a) Moving N, S, W, E has a cost of 1 per cell moved.
   (b) Diagonal movement (i.e. NW, NE, SW, SE) has a cost of 1 per cell moved.

Answer the following questions given the problem formulation:

(h) (1 point) For memory space optimality which type of search would you use?

Tree Search [ ] Graph Search [ ]

(i) (1 point) What is the state space assuming each part originates in its cell as shown in the grid above?

Type Your Answer Here: [ ]

(j) (1 point) What is the state space assuming each part originates randomly in the grid?

Type Your Answer Here: [ ]

(k) (1 point) What are the admissible heuristics for this problem? (multiple-option correct MCQ)

- [ ] Manhattan Distance
- [ ] $\sqrt{((x_2 - x_2)^2 + (y_2 - y_2)^2)}$
- [ ] Max(steps horizontally, vertically or diagonally)
- [ ] Number of misplaced parts
- [ ] |Part distance to current robot discrete position|+|Part distance to goal|

(l) (1 point) What would be the most effective strategy to transform the search to solve for optimal sequence and path concurrently? (multiple-option correct MCQ)

- [ ] Use a priority queue that kept paths and costs from init to end-goals
- [ ] Add a final goal to the end of the search
- [ ] Set all of the g-values to infinity
- [ ] Add linking nodes between the end-goals and initial locations

(m) (1 point) Enter your choice for the best informed search algorithm to use for this problem that will return an optimal solution. It must minimize the cost of your movements individually and determine the optimal ordering of the parts moved. [Note: ID refers to Iterative-Deepening]

A^* ☐ ID-DFS ☐ Backtracking DFS ☐ ID-A^* ☐ Multi-goal A^* ☐

You can add your notes on this page (if any)

Problem 3: Optimization Algorithms [20 points]

We have introduced hill climbing in the lectures. Note that hill climbing is an analogy for optimizing a general function, that is, to find its minima and maxima. Let's investigate a concrete algorithm, and its useful variants.

First we introduce our general problem formulation. For $x \in X$, where X can be some arbitrary set, define $f(x) : X \to R$ a function takes elements from X and produces a real number so that $f(x) \in R$. Our goal is to find a minimum solution $f(x^*)$ over the entire set X. Assume the derivative of f at any point x exists and denoted $f'(x)$. Recall that we learned hill climbing is essentially performing the following Gradient Descent iterations: $x_{k+1} = x_k - \eta_k f'(x_k)$, where x_k is the current estimate of the optimal solution, and x_{k+1} is the next estimate. η_k is the step size, which is a real number. Notice that in general, if X is n-dimensional, i.e., $X \subseteq R^n$, then we define the gradient of the function f at x to be:

$$\nabla f(x) = [\underbrace{\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, ..., \frac{\partial f}{\partial x_n}}]^T.$$

As iterations progress, with proper step size, $f(x_k)$ would decrease. Hence the name Gradient Descent.

Part 1: Gradient Descent (GD)

Now generally attributed its invention to Cauchy and Hadamard in the 1800s, gradient descent has been one of the most influential and useful methods in modern mathematical computation. In the era of machine learning, gradient descent has been one of the driving forces behind computational models, including but not restricted to deep neural networks, support vector machines, and maximum likelihood estimation. In this subsection, we first look at two examples of gradient descent, followed by some conceptual questions.

$4x - 4$

(a) (1 point) Let $f(x) = 2x^2 - 4x + 5$, $f : R \to R$. Suppose $x_k = 3$, and we set constant step size $\eta_k = \frac{1}{4}, \forall k = 1, 2, 3, ...$ What is the value of x_{k+1}?

3 ☐ 2 ☐ 1 ☐ 0 ☐

$x_{k+1} = x_k - n_k f'(x_k)$

$x_{k+1} = x_k - (x_k - 1)$ $\quad x_{k+1} = 1$

(b) (1 point) True or False: for an arbitrary function that does not go to negative infinity, you can always find a global optimal solution by applying gradient descent one time with proper step sizes.

True ☐ False ☐

(c) (1 point) True or False: for an arbitrary function that does not go to negative infinity, you can always find a local optimal solution by applying gradient descent with proper step sizes (without restart).

True ☐ False ☐

It is known that if Gradient Descent converges, it would converge in sub-linear speed for arbitrary functions, that is: $f(x_k) - f(x^*) \leq \frac{C}{k}, \forall k = 1, 2, 3, ...$ where C is a problem dependent constant. It takes more effort to improve precision in later iterations (notice that the ratio $k/(k+1)$ approaches 1 as k grows). We wish to improve this dreadful result, which follows two variants of Gradient Descent.

Part 2: Line Search

Let X be a subset of R^n. $f : X \to R$ is a real-valued function. Consider the GD iteration update rule defined as: $x_{k+1} = x_k - \eta_k \nabla f(x_k)$. Now we wish to know: what is the optimal step size for this particular iteration? The answer can be ~~formulated~~ as another optimization problem: find $t' \in R$ such that

$f(x_k - t'\nabla f(x_k)) = \min_{t \in R} f(x_k - t\nabla f(x_k))$

The optimal step size should decrease the function value the furthest along the direction $f'(x_k)$. Hence this method is termed line search.

Let $f(x) = 2x^2 - 4x + 5$, $f : R \to R$. Suppose that $x_1 = 3$. Answer the following questions.

(d) (2 points) What is the value for the optimal step size for $k = 2$?

$f(x_2 - t\nabla f(x_2))$

Type Your Answer Here:

$\nabla f = 4x-4$

$x_2 = x_1 - t\nabla f(x_1)$

(e) (2 points) Using the step size you found above, what is value of x_2?

$x_2 = 3 - t8$

Type Your Answer Here:

$f(3-8t - t\nabla f(3-t8))$

$f(3-8t - t(12-32t-4))$

Part 3: Acceleration Scheme

First proposed in the 60s by Polyak, the Heavy Ball method is inspired by physics. Imagine you are rolling a ball down a hill. The ball would accelerate due to gravity, so that towards the bottom of the hill, velocity would gradually increase. Reflected in the algorithm, the current estimate of the solution would get to the optimal point faster in later iterations. Now consider the following update rule for minimizing a real-valued function f defined over subsets of R^n:

$f(3-8t-8t-32t^2)$

$x_{k+1} = x_k - \alpha_k \nabla f(x_k) + \beta_k(x_k - x_{k-1})$

$f(-32t^2-16t-3)$

where α_k, β_k are real numbers, usually $\alpha_k, \beta_k \in (0, 1)$. The additional parameter takes inspiration from physics. If we image our current iterate x_k as an object with mass, then adding this additional term causes acceleration toward the bottom of the hill.

$-128t^2-64t-12-4$

Now, consider the following problem in R^2 and answer the following questions.

$-128t^2-64t-16$

$\min_{x \in R^2} f(x) = x^T Ax - b^T x$

$t = -0.25$ or 0.25

where the matrix $A = [a_1 \ \ a_2] \in R^{2x2}$, having the first column $a_1 = [1 \ \ 3]^T$, and the second column $a_2 = [2 \ \ 2]^T$. Let $b = [3 \ \ 5]^T$. Suppose $\alpha_k = 1/4, \beta_k = 1/2$, and $x_1 = [4 \ \ 4]^T$, $x_0 = [0 \ \ 1]^T$.

$x = [a, b]$

(f) (1 point) Compute $f(x_1)$

$f(x) = x^T A x - b^T x$

Type Your Answer Here:

$[a \ b]\begin{bmatrix}1 & 2\\3 & 2\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} - [3 \ 5]\begin{bmatrix}a\\b\end{bmatrix}$

(g) (2 points) What is the value of x_2? Say $x_2 = [a \ \ b]^T$, then fill up the following blanks:

$3(a^2-a)+5(b^2-b)$

$[a+3b \ \ 2a+2b]\begin{bmatrix}a\\b\end{bmatrix} - [3a+5b]$

a:

$[3(2a-1), \ 5(2b-1)]$

$[a^2+3ab+2ab+2b^2]$

b:

$[21, 35]$

$[a^2+5ab+2b^2]$

$x_2 = x_1 - \alpha_1 \nabla f(x_1) + \beta_1(x_1 - x_0)$

(h) (2 points) Compute $f(x_2)$

$\begin{bmatrix}4\\4\end{bmatrix} - \frac{1}{4}\begin{bmatrix}25\\31\end{bmatrix} + \frac{1}{2}\left(\begin{bmatrix}4\\4\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix}\right)$

2,3

Type Your Answer Here:

$[a^2+2b^2+5ab-3a-5b]$

$\begin{bmatrix}4\\4\end{bmatrix} + \begin{bmatrix}-6.25\\-7.75\end{bmatrix} + \begin{bmatrix}2\\1.5\end{bmatrix} = \begin{bmatrix}-0.25\\-2.25\end{bmatrix}$

$$[2a+5b-3,\ 4b+5a-5]$$

$x_k = [\ ]$

$f'(x) = 4x-4$

(i) (3 points) Let $f(x) = 2x^2 - 4x + 5$, $f : R \to R$. Suppose that $x_k = 3$, $x_{k-1} = 4$ and we set constant step size $\alpha_k = 1/4, \forall k = 1, 2, 3, ...$ and $\beta_k = 1/3$. What is the value of x_{k+1}?

$x_{k+1} = x_k - \alpha_k \nabla f(x_k) + \beta_k(x_k - x_{k-1})$

$x_{k+1} = x_k - \frac{1}{4}(4x_k - 4) + \frac{1}{3}(x_k - x_{k-1})$

Type Your Answer Here:

In his seminal work, Nestrov proposed the Accelerated Gradient Descent (AGD) method, which uses the following update:

$x_{k+1} = x_k - x_k + 1 + \frac{1}{3}(x_k - x_{k-1})$

$x_{k+1} = \frac{1}{3}(x_k - x_{k-1}) + 1$

$y_k = x_k + \beta_k(x_k - x_{k-1})$
$x_{k+1} = y_k - \alpha_k \nabla f(y_k)$

In general, both Polayk's Heavy Ball method and Nestrov's Accelerated Gradient Descent method will be faster than vanilla gradient descent, i.e., they require fewer iterations to reach the same level of precision to the optimal solution. Such results would be reflected on convergence speed, often pushing the convergence rate to linear, that is, one requires as many iterations to reach the next digit precision as in pushing previous digit precisions. For example, if we need 3 iterations to drive $f(x_k - f_{x^*}) \leq 10^{-1}$ to 10^{-2}, then we only need roughly three iterations to push from 10^{-6} to 10^{-7} when using an optimization method with linear convergence speed. This is a significant improvement from a sub-linear rate (hence the name "sub"-linear).

Using the Nestrov's AGD method defined above, solve the following question.

Consider the same problem in R^2

$$\min_{x \in R^2} f(x) = x^T A x - b^T x$$

$x = \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

where the matrix $A = [a_1 \ \ a_2] \in R^{2x2}$, having the first column $a_1 = [1 \ \ 3]^T$, and the second column $a_2 = [2 \ \ 2]^T$. Let $b = [3 \ \ 5]^T$. Suppose $\alpha_k = 1/5, \beta_k = 1/4$, and $x_k = [2 \ \ 3]^T$, $x_{k-1} = [1 \ \ 0]^T$.

(j) (1 point) Compute $f(x_k)$

$f(x) = c^2 + 2d^2 + 5cd - 3c - 5d$

$\nabla f(x) = [2c + 5d - 3,\ 4d + 5c - 5]$

Type Your Answer Here:

(k) (2 points) What is the value of x_{k+1}? Say $x_{k+1} = [a \ \ b]^T$, then fill up the following blanks:

$x_{k+1} = y_k - \alpha \nabla f(y_k)$

a:

$x_{k+1} = x_k + \beta_k(x_k - x_{k-1}) - \alpha \nabla f(y_k)$

b:

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \frac{1}{4}\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) - \frac{1}{5}\nabla f\left(\begin{bmatrix} 2.25 \\ 3.75 \end{bmatrix}\right)$

(l) (2 points) Compute $f(x_{k+1})$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$

Type Your Answer Here:

$\begin{bmatrix} 2.25 \\ 3.75 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 20.25 \\ 21.25 \end{bmatrix} = \begin{bmatrix} -1.8 \\ -0.5 \end{bmatrix}$

Hint: To compute the gradient of $x^T A x - b^T x$, notice that this is a quadratic function plus a linear function. Results follow similar differentiation rules in R, for example, $\nabla(x^2 + tx) = 2x + t$.

You can add your notes on this page (if any)

Problem 4: Constraint Satisfaction Problem [10 points]

The Ploetzobots and Starnticons are two factions in the world of the Transformers who fight for power of Sourishetron, their home planet. The Ploetzobots are led by Opthomas Prime and the Starnticons are led by Thadotron.

Figure 12: Ploetzobots vs. Starnticons

In one of the final battles, Opthomas Prime and Thadotron have engaged in a final battle, whereas Opthomas Prime's team was in charge of defeating giant Trypticon. His team decided to use the Ploetzobots spaceship, the Ark, to fight Trypticon.

Figure 13: Trypticon

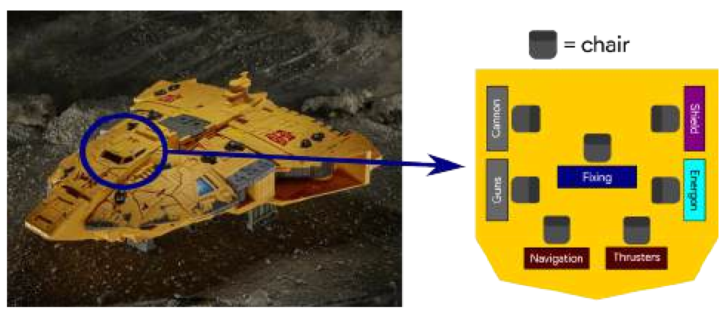

Figure 14: The Ark

However, in order to operate the Ark, his team needs to figure out which part of the Ark they should operate. There are seven parts of the ship to operate:

1. The **cannons**
2. The **laser guns**
3. To **fix** the broken parts of the ship
4. The **shield**
5. The **energon** battery cells, which ensure power to all parts of the ship
6. The **navigation**, essentially being the pilot of the ship
7. The **thrusters** of the ship

There are a few constraints of the ship to consider.

1. Whoever works in the **cannons** and in the **laser guns** need to be great **shooters**
2. Whoever works in the **navigation** needs to have very good **reflexes** in order to avoid all of the attacks
3. Whoever is in charge of **fixing** the ship needs to know how to properly operate the **Teletraan II**, which is the computer which will help fix different parts of the ship.
4. The following pairing constraints need to be addressed for easy communication, since they sit next to each other.
   (a) The bots who take care of the **cannons** and the **laser guns** need to be good buddies
   (b) The bots who take care of the **shields** and the **energon battery cells** need to be good buddies
   (c) The bots who take care of the **navigation** and the **thrusters** need to be good buddies

Additionally, the Ploetzobots needed to attend the bot academy before becoming soldiers. For this, they needed to study well or improve their skills well. Next we will cover the Ploetzobots during their time in their academy:

1. Bumblebee has great **reflexes**, but it was never good at **shooting** nor was it good at understanding the technical side of their central computer, the **Teletraan II**. It was great friends with Jazz, Buckhead, and Arcee.
2. Wheeljack didn't have great **reflexes**, but he was a great **shooter** and pretty much invented the **Teletraan II**. But it didn't really interact with many of his peers, so it was pretty much a great partner with Ratchet and Arcee.
3. Ratchet was the medic, so it spent a lot of time understanding how to repair its peers using the **Teletraan II**. Since it was the medic, it took an oath to not engage in battle, so it refuses to use any of the **shooters** (i.e. cannons or laser guns) and it didn't have very good **reflexes**. It was really nice friends with Arcee, Wheeljack, and Jazz, but found Bumblebee's immaturity hard to deal with, and Blurr spoke too fast for him to understand.
4. Blurr is a speedster, so he has amazing **reflexes**. Because of that, it was also a great **shooter**, but it didn't really enjoy studying the **Teletraan II**, so it pretty much doesn't know much about it. Because it speaks very quickly, not many people could understand him, so pretty much only Arcee learned to work well with him.
5. Arcee is friendly bot. It worked well with everyone; literally anyone and everyone. Above that, it was top of its class in **shooting**, although it **reflexes** weren't great, and its knowledge in the **Teletraan II** wasn't great either.
6. Jazz was a slick fellow. It was great at **shooting**, with very good **reflexes**, but it wasn't great at operating the **Teletraan II**. It was a great friend of Bumblebee, Arcee, and Ratchet.
7. Finally, Buckhead was a very big and friendly fellow, but rather timid. It was great friends with Bumblebee and Arcee. Although he didn't have the capabilities of having good **reflexes** or knowledge of the **Teletraan II**, he was an amazing fighter and great **shooter**.

The information about friendships and skills can also be found in the tables shown in Figure 15.

	Arcee	Blurr	Buckhead	Bumblebee	Jazz	Ratchet	Wheeljack
Arcee	X	:)	:)	:)	:)	:)	:)
Blurr	:)	X					
Buckhead	:)		X	:)			
Bumblebee	:)		:)	X	:)		
Jazz	:)			:)	X	:)	
Ratchet	:)				:)	X	:)
Wheeljack	:)					:)	X

Names	Good Reflex	Good Shooter	Good operating Teletraan II
Arcee		Yes	
Blurr	Yes	Yes	
Buckhead		Yes	
Bumblebee	Yes		
Jazz	Yes	Yes	
Ratchet			Yes
Wheeljack		Yes	Yes

Figure 15: Friendship and Skill Tables

(a) (0.5 points) Given the listed constraints and the characteristics of each of the Ploetzobots, is it possible to find a solution that satisfies all of the constraints?

Yes No

(b) (3.5 points) If yes, what are the possible bots that can seat in each of the listed roles, **listed in alphabetical order and separated by a comma and a space (e.g. Arcee, Bumblebee, Jazz)**? If not, leave the fields blank

Cannon:

↕ shooters, buddies

Laser Gun:

OT Fixing:

Shield:

↕ buddies

Energon:

reflexes

Navigation:

buddies

Thrusters:

After a lot of fighting, the shields were heavily damaged. Due to that, greater communication was needed between the fixer and the shield user. In other words, they now need to be able to work well together. All of the previous constraints are to remain.

(c) (0.5 points) Given the listed constraints and the characteristics of each of the Ploetzobots, is it possible to find a solution that satisfies all of the constraints?

Yes ☐ No ☐

(d) (3.5 points) If yes, what are the possible bots that can seat in each of the listed roles, **listed in alphabetical order and separated by a comma and a space (e.g. <u>Arcee, Bumblebee, Jazz</u>)**? If not, leave the fields blank

Cannon:

Laser Gun:

Fixing:

Shield:

Energon:

Navigation:

Thrusters:

In the battle, the laser gun is hit, and now, it needs to be fixed. But it also needs to be closely monitored by the fixer, so the fixer and the laser gun user need to be able to work well together.

(e) (0.5 points) Given the listed constraints and the characteristics of each of the Ploetzobots, is it possible to find a solution that satisfies all of the constraints?

Yes ☐ No ☐

(f) (3.5 points) If yes, what are the possible bots that can seat in each of the listed roles, **listed in alphabetical order and separated by a comma and a space (e.g. <u>Arcee, Bumblebee, Jazz</u>)**? If not, leave the fields blank

Cannon:

Laser Gun:

Fixing:

Shield:

Energon:

Navigation:

Thrusters:

You can add your notes on this page (if any)

Problem 5: Probability: Some memories are best forgotten! [18 points]

Leonard has a rare condition, which prevents him from forming new memories. Because of this, he tries to remind himself of what to do by writing himself notes. Additionally, as if his memory impairment weren't challenging enough, he's also trying to solve a murder mystery!

Each day after sleuthing, he arranges a plan for the following day: he either asks an informant to meet him at the City Grill restaurant, or he invites the informant to visit him in the hotel parking lot at the Discount Inn, where he stays. In either case, the meeting time is at noon each day. The problem is, once he wakes up the following day, he can't remember the plan on his own; he can only remember it if he left himself a note the night before. But he doesn't always remember to write the notes!

If Leonard wakes up without a note for himself, he goes to the City Grill 80% of the time, because he doesn't like being alone in the parking lot. For the other times he wakes up without a note, he stays in the hotel parking lot because he has a hunch that is where he is supposed to be. Unfortunately, his hunches are independent of whether he actually asked that day's informant to meet him at the parking lot or not.

He has a tattoo of the meeting time and two possible locations in case he forgets to make notes. So, if he guesses the location correctly on days without notes, he will meet them there at the correct time. If he goes to the wrong location, he does not meet his informant that day.

When he does leave a note for himself the night before, he always successfully meets the informant at the arranged location.

It turns out there is a pattern to when he leaves a note for himself. When he goes to the City Grill, there's an 80% chance he'll remember to leave a note for himself that night about the next day's plans! But if he goes to the hotel parking lot during the day, there's only a 30% chance he remembers to write the note. These probabilities hold whether or not he met his informant successfully earlier in the day.

Because there are suspects who don't like Leonard snooping around, Leonard wants to avoid being too predictable. When he invites an informant to a location, he flips a fair coin (a fair coin has 0.5 probability of landing heads, and 0.5 probability landing tails). If it lands heads, he invites the informant to the restaurant for the following day; otherwise, to the hotel parking lot. The informants always go to the correct location that they were told to go to.

Earlier today (a Monday), he successfully met an informant at the City Grill after leaving himself a note Sunday evening.

For each answer, use only the information provided in the scenario above and follow the rounding rules for the exam.

(a) (2 points) Consider the table shown in Figure 16. Fill in the values for conditional probabilities of Leonard's location each day:

Type the value of A here:

Type the value of B here:

Type the value of C here:

Type the value of D here:

Type the value of E here:

Type the value of F here:

Type the value of G here:

Type the value of H here:

Left note for himself the night before?	Informant Location	Probability of Leonard's Location	
		City Grill	Hotel Parking Lot
True	City Grill	A	B
True	Hotel Parking Lot	C	D
False	City Grill	E	F
False	Hotel Parking Lot	G	H

Figure 16: Table for Leonard's Location

(b) (1 point) What is the probability of Leonard meeting the informant on any given day in which Leonard wakes up without a note from the previous night?

Type Your Answer here:

(c) (1.5 points) What is the probability that Leonard goes to the City Grill on Tuesday?

Type Your Answer here:

(d) (1.5 points) What is the probability Leonard goes to a different location than his informant on Tuesday?

Type Your Answer here:

(e) (1.5 points) What is the probability Leonard goes to the hotel parking lot and meets his informant on Tuesday?

Type Your Answer here:

(f) (2.5 points) What is the probability Leonard remembers to write a note for himself Tuesday night for Wednesday's plan?

Type Your Answer here:

(g) (3 points) What is the probability Leonard goes to the City Grill restaurant on Wednesday?

Type Your Answer here: ☐

(h) (3 points) What is the probability Leonard successfully meets his informant on Wednesday?

Type Your Answer here: ☐

Now, Natalie, whom Leonard previously met, has some important evidence on the murder suspect she needs to share with Leonard! Leonard will not try to coordinate a meeting with her this week, but she knows that Leonard flips a coin to decide where to direct each informant and that the meeting time is noon each day. So, she also plans to flip a fair coin each day to decide which meeting spot to try to catch him. She intends to do this for up to two days, on Tuesday and Wednesday of the same week as described earlier in the problem.

(i) (1 point) What is the probability Natalie will successfully find Leonard at least once on Tuesday or Wednesday?

Type Your Answer here: ☐

(j) (1 point) Given all the information, which of the following strategies would give Natalie the highest probability of finding Leonard at one of his two meeting spots by the end of Wednesday? (Single-Option Correct MCQ)

1. **Strategy A**: The stated strategy (flipping a fair coin each day to choose where to go)
2. **Strategy B**: Always going to the Discount Inn parking lot
3. **Strategy C**: Always going to the City Grill

Strategy A ☐ Strategy B ☐ Strategy C ☐

You can add your notes on this page (if any)

Problem 6: Bayes Nets: There are no accidents! [18 points]

As foretold by master Oogway, Shifu meets his destiny on the road he takes to avoid it. As a result of his own actions, Tai Lung has escaped the Chorh-Gom prison, and he is on his way toward the Jade palace to steal the dragon scroll. However, Shifu can barely concentrate now as he has to deal with a lot of things!

He has just witnessed the death of master Oogway, who requested him to trust Po as the Dragon Warrior. Po on the other hand is clueless about Kung Fu and Shifu needs to train him starting from level zero, and is out of time! To achieve this near-impossible feat, Shifu has to think of something clever: perhaps using delicious food as positive reinforcement in Po's Kung Fu training (?). However, Shifu is also worried about his beloved students, the Furious 5, who are on their way to fight Tai Lung. Deep down, Shifu knows that they are no match for Tai Lung's aggressive Kung Fu style and the best they can do is stall him and buy Shifu some more time to train Po. So, it is all up to Po to take a stand against Tai Lung and save the Valley of Peace. Despite being a novice, Po has access to the secrets of the dragon scroll. But to use that, he has to truly understand what the "blank" dragon scroll really means in order to equip himself for a legendary fight against Tai Lung!

In these challenging times, you are required to step up and help Shifu to make rational decisions. Thus, you decide to analyze the situation and come up with a list of factors influencing the outcome of the fight between Po and Tai Lung. Below are the factors that you came up with:

1. Shifu's faith in Po (F): {High, Medium, Low}
2. Deliciousness of the food used by Shifu in Po's training (D): {Tasty, Edible, Bland}
3. Tai Lung's Kung Fu skills (TK): {High, Medium, Low}
4. Furious 5's Kung Fu skills (FK): {High, Medium, Low}
5. Tai Lung's speed in his march to Jade palace (TS): {High, Medium, Low}
6. Po's ability to quickly grasp Kung Fu (PK): {High, Medium, Low}
7. Time available for Po to learn Kung Fu (PT): {7, 5, 3}
8. Po's perception of the message conveyed by the "blank" dragon scroll (PD): {Aware, Indifferent, Clueless}
9. Tai Lung's condition before the battle (TC): {Good, Normal, Bad}
10. Po's condition before the battle (PC): {Good, Normal, Bad}

The outcomes of the fight between Po and Tai Lung with respect to Po (O) can be: {Win, Draw, Lose}.

(a) (1 point) Now, you decide to construct the joint probability distribution table for these random variables involved. How many *parameters* would be required to construct this joint probability distribution table without any assumptions about the independence of random variables?

Type Your Answer Here: $\boxed{}$

That's a lot of parameters! So, you decide to construct a Bayes Net using these random variables. After giving it a serious thought, you have come up with the Bayes Net shown below in Figure 17:

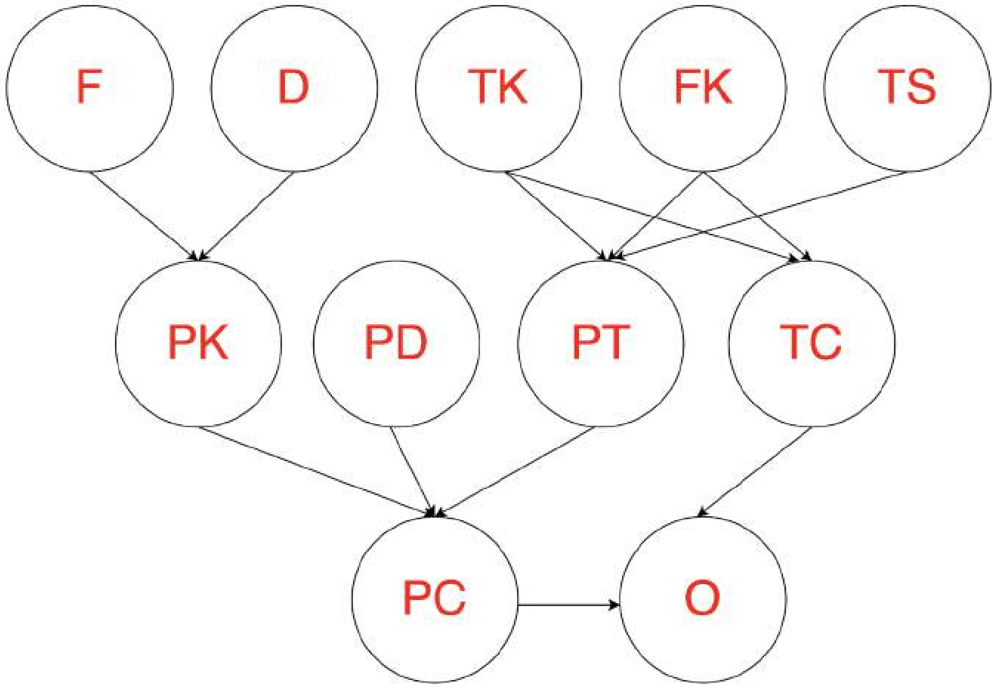

Figure 17: Bayes Net

(b) (3 points) Based on the independence relations originated only by the structure of the Bayes net, how many parameters are needed to specify this Bayes net?

Type Your Answer Here: ☐

(c) (2 points) Now, consider a scoring system, where a node gets a score of k if it belongs in the Markov blanket of k nodes. If this scoring system is applied to the Bayes net shown above in Figure 17, which node has the highest score?

Type Your Answer Here: ☐

(d) (1.25 points) What is the value of this highest score from (c)?

Type Your Answer Here: ☐

(e) (2.5 points) In this Bayes net shown above in Figure 17, F is independent of which of the following variables? (assuming only the independence relations originated by the structure of the Bayes net) (multiple-options correct MCQ)

☐ D ☐ TK ☐ FK ☐ TS ☐ PK ☐ PD ☐ TC ☐ PT ☐ PC ☐ O

(f) (2.25 points) In this Bayes net shown above in Figure 17, given PK, F is independent of which of the following variables? (assuming only the independence relations originated by the structure of the Bayes net) (multiple-options correct MCQ)

☐ D ☐ TK ☐ FK ☐ TS ☐ PD ☐ TC ☐ PT ☐ PC ☐ O

To help Shifu make rational decisions, you decide to run a few simulations. Below, in Figure 18, are the results of your 10 simulations. [Note: H = High, M = Medium, L = Low, T = Tasty, E = Edible, B = Bland (for node D) or Bad (for nodes TC and PC), A = Aware, I = Indifferent, C = Clueless, G = Good, N = Normal, W = Win, L = Lose]

	F	D	TK	FK	TS	PK	PD	TC	PT	PC	O
1	L	T	H	M	H	M	A	G	3	G	W
2	H	T	M	M	H	H	A	G	5	G	W
3	H	E	H	M	H	H	A	N	5	G	W
4	M	T	H	M	H	M	I	G	5	G	W
5	H	E	H	H	H	L	A	B	7	N	W
6	M	E	H	M	H	L	A	G	3	G	W
7	M	T	H	M	M	H	A	G	7	G	W
8	H	T	M	L	H	M	C	N	5	G	L
9	L	T	H	M	H	H	A	G	5	N	W
10	H	B	H	M	H	H	A	G	3	B	L

Figure 18: Results of 10 Simulations

(g) (1 point) If this data is generated using Prior Sampling, estimate P(O = L | D = T)

Type Your Answer Here:

(h) (1 point) If this data is generated using Prior Sampling, estimate P(O = W | PD = A)

Type Your Answer Here:

(i) (1 point) Now, suppose you are using rejection sampling. Assuming that the 10 samples were generated using Prior Sampling (i.e. the rejection stage hasn't been performed yet). Then, estimate P(O = W | TK = H, FK = M)

Type Your Answer Here:

(j) (1 point) How many samples did you reject in the previous question?

Type Your Answer Here:

After witnessing the progress that you've made, Shifu is impressed with your analysis. So far so good! However, it has already been more than 1 day since the Furious 5 left Jade palace to fight Tai Lung. They must be battling their hearts out stop Tai Lung! So, Shifu must prepare Po to the best of his abilities. To do that, he really wants to have an estimate of P(O = W | PC = G, PT = 5). To help Shifu, you decide to run 5 more simulations. Below, in Figure 19 are the results of these additional 5 simulations.

	F	D	TK	FK	TS	PK	PD	TC	PT	PC	O
1	H	T	H	M	H	H	A	G	5	G	W
2	H	T	H	M	M	H	A	G	5	G	W
3	H	T	H	M	M	H	A	G	5	G	W
4	H	B	H	M	M	H	A	G	5	G	L
5	H	B	M	M	M	H	A	G	5	G	W

Figure 19: Results of the next 5 Simulations

(k) (2 points) Using which sampling method(s) it is possible to generate these additional 5 samples? (multiple-options correct MCQ)

- ☐ Rejection Sampling
- ☐ Gibbs Sampling
- ☐ MH Sampling

You can add your notes on this page (if any)

Problem 7: Extra Credits: Bonus Question [3 points]

SURPRISE!

This is another opportunity to gain extra credits in this course. Although the first page of this PDF mentions total marks as 103, the last 3 points will be considered as bonus points and your exam grade will be considered out of 100. (If you solve the following question correctly, 3 points will be added to your total grade. So, the maximum you can score in this exam is 103 out of 100)

Here is the bonus question:

After cycling continuously for 5 hours and winning the "Georgia State Cycling Championship 2022", Thomas needs instant hydration. Thus, he decides to play a lemonade drinking game. The game goes as follows:

The game starts with 6 empty glasses on the table, labeled G1, G2, G3, G4, G5, and G6. Thomas has to make "moves" in order to play this game. Each move is defined like this: "Select a glass at random (probability of selecting each glass is equal). If the selected glass is empty, then fill it up with lemonade. In case the selected glass is already filled with lemonade, then drink it completely to make it empty again". The game ends when ALL the 6 glasses are filled with lemonade at the same time (In that case, Thomas will drink all 6 glasses of lemonade to make them empty again, and the game ends there).

(a) (3 points) At the start of the game, let X be the random variable representing the number of moves required for Thomas to complete this game. What is the value of E[X]?

Type Your Answer Here: $\boxed{}$

You can add your notes on this page (if any)

Checklist

Mark all the boxes from the checklist below to make sure you have taken care of each of the points listed:

1. I have read the pinned Ed post with the title "Midterm Exam Clarifications Thread" and I am familiar with all of the clarifications made by the Teaching staff.

   I Confirm ☐

2. All answers with more than 6 digits after the decimal point have been rounded to 6 decimal places.

   I Confirm ☐

3. All the pages in my submission PDF are in the correct order in which they were presented to me and none of the pages are missing/removed.

   I Confirm ☐

4. Any extra pages (excluding the pages provided after each question) are only attached at the END of this exam, after page number 35, with clear pointers to wherever the actual answer is in the PDF (references are made properly).

   I Confirm ☐

5. I am submitting only one PDF and nothing else (no docx, doc, etc.).

   I Confirm ☐

6. The PDF that I am submitting is not blank (unless I want it to be blank).

   I Confirm ☐

7. I will go over the uploaded PDF on the Gradescope and make sure that all the answers are clearly visible. I acknowledge that I am aware that dull / illegible / uneven scans will not be graded.

   I Confirm ☐

8. I have submitted a copy of my submission PDF to the Canvas.

   I Confirm ☐