MATH 2551, Written Assignment 1

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1. (a) To find the point of intersection, set the parametric equations equal:

$$\begin{aligned} -2 + 2u &= 6 - 2t \\ 5 + 3u &= -1 + 3t \\ 7 - 5u &= 5 - t \end{aligned}$$

From the first equation:

$$\begin{aligned} 2u + 2t &= 8 \\ u + t &= 4 \quad (1) \end{aligned}$$

From the second equation:

$$\begin{aligned} 3u - 3t &= -6 \\ u - t &= -2 \quad (2) \end{aligned}$$

Adding equations (1) and (2):

$$\begin{aligned} 2u &= 2 \\ u &= 1 \end{aligned}$$

Substituting $u = 1$ in equation (1):

$$\begin{aligned} 1 + t &= 4 \\ t &= 3 \end{aligned}$$

Thus, the point of intersection is:

$$(-2 + 2(1), 5 + 3(1), 7 - 5(1)) = (0, 8, 2)$$

(b) The plane passing through the lines L_1 and L_2 has a normal vector $\vec{n}$ given by the cross product of the direction vectors of L_1 and L_2:

$$\begin{aligned} \vec{d_1} &= \langle 2, 3, -5 \rangle \\ \vec{d_2} &= \langle -2, 3, -1 \rangle \\ \vec{n} = \vec{d_1} \times \vec{d_2} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -5 \\ -2 & 3 & -1 \end{vmatrix} = \langle 12, 12, 12 \rangle = 12\langle 1, 1, 1 \rangle \end{aligned}$$

The plane equation is given by:

$$x + y + z = 10$$

where $(0, 8, 2)$ is the point of intersection.

(c) The distance d from the point $S(1,6,5)$ to the line L_1 is given by:

$$d = \frac{|\vec{PS} \times \vec{d_1}|}{|\vec{d_1}|}$$

where $\vec{P}$ is a point on L_1, say $(-2,5,7)$, and $\vec{PS} = \langle 1-(-2), 6-5, 5-7\rangle = \langle 3,1,-2\rangle$.

$$\begin{aligned}
\vec{d_1} &= \langle 2,3,-5\rangle \\
\vec{PS} \times \vec{d_1} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & -2 \\ 2 & 3 & -5 \end{vmatrix} = \langle 1, 11, 7\rangle \\
|\vec{PS} \times \vec{d_1}| &= \sqrt{1^2 + 11^2 + 7^2} = \sqrt{171} \\
|\vec{d_1}| &= \sqrt{2^2 + 3^2 + (-5)^2} = \sqrt{38} \\
d &= \frac{\sqrt{171}}{\sqrt{38}} = \sqrt{\frac{171}{38}} = \sqrt{\frac{171}{38}} \approx 2.11
\end{aligned}$$

2. (a) The unit tangent vector $\mathbf{T}(t)$ is given by:

$$\mathbf{T}(t) = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|}$$

where $\vec{r}(t) = t\cos t\mathbf{i} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\mathbf{j} + t\sin t\mathbf{k}$.

$$\begin{aligned}
\frac{d\vec{r}}{dt} &= \cos t\mathbf{i} + t(-\sin t)\mathbf{i} + \frac{d}{dt}\left(\frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\right)\mathbf{j} + \sin t\mathbf{k} + t\cos t\mathbf{k} \\
&= (\cos t - t\sin t)\mathbf{i} + \frac{2\sqrt{2}}{2}t^{\frac{1}{2}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k} \\
&= (\cos t - t\sin t)\mathbf{i} + \sqrt{2}t^{\frac{1}{2}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k} \\
\left|\frac{d\vec{r}}{dt}\right| &= \sqrt{(\cos t - t\sin t)^2 + (\sqrt{2}t^{\frac{1}{2}})^2 + (\sin t + t\cos t)^2}
\end{aligned}$$

(b) The arc length parameter $s(t)$ is given by:

$$s(t) = \int_{t_0}^{t} \left|\frac{d\vec{r}}{dt}\right| dt$$

(c) To find the length of the curve from $t_0 = 0$ to $t = 4$, evaluate:

$$s(t) = \int_0^4 \sqrt{(\cos t - t\sin t)^2 + (\sqrt{2}t^{\frac{1}{2}})^2 + (\sin t + t\cos t)^2}\, dt$$

(d) Given $P_0(4\cos 4, \frac{16\sqrt{2}}{3}, 4\sin 4)$, if the bee travels a distance of 12 units along the curve, solve for t from:

$$s(t) = \int_4^t \sqrt{(\cos t - t\sin t)^2 + (\sqrt{2}t^{\frac{1}{2}})^2 + (\sin t + t\cos t)^2}\, dt = 12$$

The coordinates of the bee will be:

$$\vec{r}(t) = t\cos t\mathbf{i} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\mathbf{j} + t\sin t\mathbf{k}$$