MATH 1554 QH, Written Assignment 1

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1. (4 points) Consider the data in the table below.

x	-1	0	1	2
$y(x)$	-1	1.5	2	-2.5

(a) From the table above, we can construct a system of linear equation; 4 equations to be exact.

$$y(-1) = a_0 + (-1)a_1 + (-1)^2 a_2 + (-1)^3 a_3 = -1$$
$$y(0) = a_0 + (0)a_1 + (0)^2 a_2 + (0)^3 a_3 = 1.5$$
$$y(1) = a_0 + (1)a_1 + (1)^2 a_2 + (1)^3 a_3 = 2$$
$$y(2) = a_0 + (2)a_1 + (2)^2 a_2 + (2)^3 a_3 = -2.5$$

The augmented matrix for this system is:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1.5 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 8 & -2.5 \end{bmatrix}$$

(b) We can apply a series of row manipulations to effectively row reduce our matrix. Here are the key steps:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1.5 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 8 & -2.5 \end{bmatrix} \xrightarrow{R_2-R_1, R_3-R_1, R_4-R_1} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & 2.5 \\ 0 & 2 & 0 & 2 & 3 \\ 0 & 3 & 3 & 9 & -1.5 \end{bmatrix}$$

$$\xrightarrow{R_3-2R_2, R_4-3R_2} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & 2.5 \\ 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 6 & 6 & -9 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_3, R_4-3R_3} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & 2.5 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 6 & -3 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{6}R_4} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & 2.5 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -0.5 \end{bmatrix}$$

$$\xrightarrow{R_1+R_4,R_2-R_4} \begin{bmatrix} 1 & -1 & 1 & 0 & -1.5 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -0.5 \end{bmatrix}$$

$$\xrightarrow{R_1+R_3,R_2+R_3} \begin{bmatrix} 1 & -1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -0.5 \end{bmatrix}$$

$$\xrightarrow{R_1+R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -0.5 \end{bmatrix}$$

(c) From the RREF matrix, we can see the values:

$$a_0 = 1.5$$
$$a_1 = 2$$
$$a_2 = -1$$
$$a_3 = -0.5$$

Therefore, the polynomial that passes through the given points is:

$$y(x) = 1.5 + 2x - x^2 - 0.5x^3$$

2. (5 points) Consider the linear system of equations below.

$$x_1 + x_2 + 2x_3 + 7x_5 = 20 \tag{1}$$
$$x_2 + 3x_5 = 1 \tag{2}$$
$$x_3 + 5x_5 = 2 \tag{3}$$

The variables in the system are x_1, x_2, x_3, x_4, and x_5.

(a) The 3×6 augmented matrix of the system is:

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 7 & 20 \\ 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 5 & 2 \end{bmatrix}$$

(b) Let's perform the row reduction to bring the matrix into RREF. Here are the steps:

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 7 & 20 \\ 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 5 & 2 \end{bmatrix} \xrightarrow{R_1 - 2R_3} \begin{bmatrix} 1 & 1 & 0 & 0 & -3 & 16 \\ 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 5 & 2 \end{bmatrix}$$

$$\xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & -6 & 15 \\ 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 5 & 2 \end{bmatrix}$$

Thus, the matrix in RREF is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -6 & 15 \\ 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 5 & 2 \end{bmatrix}$$

(c) Express the solution set of the linear system in parametric vector form.

From the RREF matrix, we can see that x_1, x_2, and x_3 are basic variables, while x_4 and x_5 are free variables. The solution set can be written as:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 15 \\ 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -6 \\ 3 \\ 5 \\ 0 \\ 1 \end{pmatrix}$$

3. (1 point) There are two parts to this question

   (a)

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   (b) Please confirm that you have followed all submission guidelines:

   1. Your work is legible in the files you uploaded.
   2. Questions are answered in the order in which they were given.
   3. Each question is answered on its own page (or pages).
   4. Your work is submitted as a single PDF file.
   5. You uploaded your work to the correct location in Gradescope.
   6. During the upload process, you indicated which pages correspond to which question.
   7. None of your pages are upside down or sideways.

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