Home...

$\cdot X_1, X_2, \ldots X_n$: independent events

1. $cdf = F_X(x)$ and pdf $f_X(x) = F_X'(x)$

Let $Y_1 = \max(X_1, X_2, \ldots X_n)$ and $Y_2 = \min(X_1, X_2, \ldots X_n)$

For, Y_1,

$$P(Y_1 \le y) = P(\max(X_1, X_2, \ldots X_n) \le y)$$
$$= P(X_1 \le y, X_2 \le y, \ldots X_n \le y)$$

Now, as the variables $X_1, X_2 \ldots X_n$ are independent,

$$P(Y_1 \le y) = \prod_{i=1}^{n} P(X_i \le y)$$

$$\Rightarrow P(Y_1 \le y) = \underbrace{F_X(y) \cdot F_X(y) \ldots F_X(y)}_{n \text{ times}}$$

$$F_{Y_1}(y) = [F_X(y)]^n$$

Hence,
$$f_{Y_1}(y) = \frac{d F_{Y_1}(y)}{dy} = n[F_X(y)]^{n-1} f_X(y)$$
$$= n[F_X(y)]^{n-1} F_X'(y)$$

For, Y_2,

$$P(Y_2 > y) = P(\min(X_1, X_2, \ldots, X_n) > y)$$
$$= P(X_1 > y, X_2 > y, \ldots X_n > y)$$

As the variables are independent,
$$1 - P(Y_2 \le y) = \prod_{i=1}^{n} P(X_i > y)$$
$$= (1 - F_X(y))^n$$

$$F_{Y_2}(y) = 1 - [1 - F_X(y)]^n$$

Hence,
$$f_{Y_2}(y) = n[1 - F_X(y)]^{n-1} f_X(y)$$
$$= n[1 - F_X(y)]^{n-1} \cdot F_X'(y)$$