Question 2

2.1 : Part A

Given a Mean Vector, and the Covariance Matrix of a Multivariate Gaussian, we need to draw variables from it.
We have :

$$\mathbf{X_{2x1}} = \boldsymbol{\mu}_{\mathbf{2x1}} + \mathbf{A_{2x2}}.\mathbf{W_{2x1}}$$

where the covariance matrix $= \mathbf{C} = \mathbf{A}.\mathbf{A^T}$
Now since the Covariance Matrix is Symmetric and SPD, we have

$$\mathbf{C} = \mathbf{U}.\mathbf{S}.\mathbf{U^T}$$

where, $\mathbf{U} = \begin{bmatrix}\mathbf{V_1} & \mathbf{V_2} & ... & \mathbf{V_n}\end{bmatrix}$ and $\mathbf{S}$ is $\text{diag}(\lambda_1, \lambda_2 ... \lambda_n)$
where $\mathbf{V_i}$ are eigenvectors and λ_i are the corresponding eigenvalues

Now, consider a symmetric matrix $\mathbf{S_1} = \mathbf{S_1^T} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2} ... \sqrt{\lambda_n})$.
We have

$$\mathbf{C} = \mathbf{A}.\mathbf{A^T} = \mathbf{C} = \mathbf{U}.\mathbf{S}.\mathbf{U^T} = \mathbf{U}.\mathbf{S_1}.\mathbf{S_1^T}.\mathbf{U^T} = (\mathbf{U}.\mathbf{S_1})(\mathbf{U}.\mathbf{S_1})^\mathbf{T}$$

Hence, $\mathbf{A} = \mathbf{U}.\mathbf{S_1}$

Now that we have $\mathbf{A}$, we can easily draw random variables in the following manner:

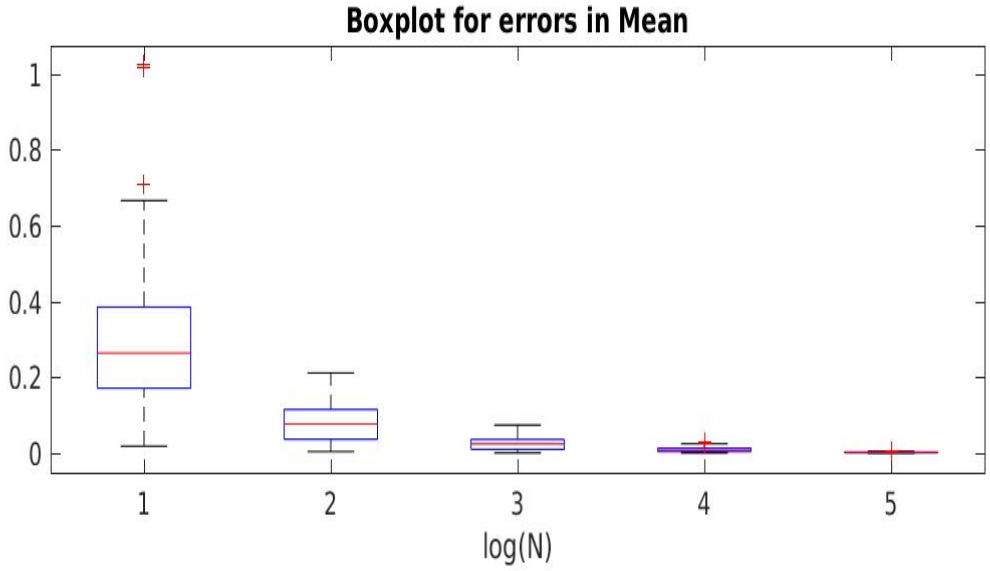
- We can draw 2 Normal Gaussian Random Variables into a 2x1 matrix $\mathbf{W}$.
- Since we have our $\mathbf{A}$, which is a 2x2 matrix, we can compute a MVG by the following equation

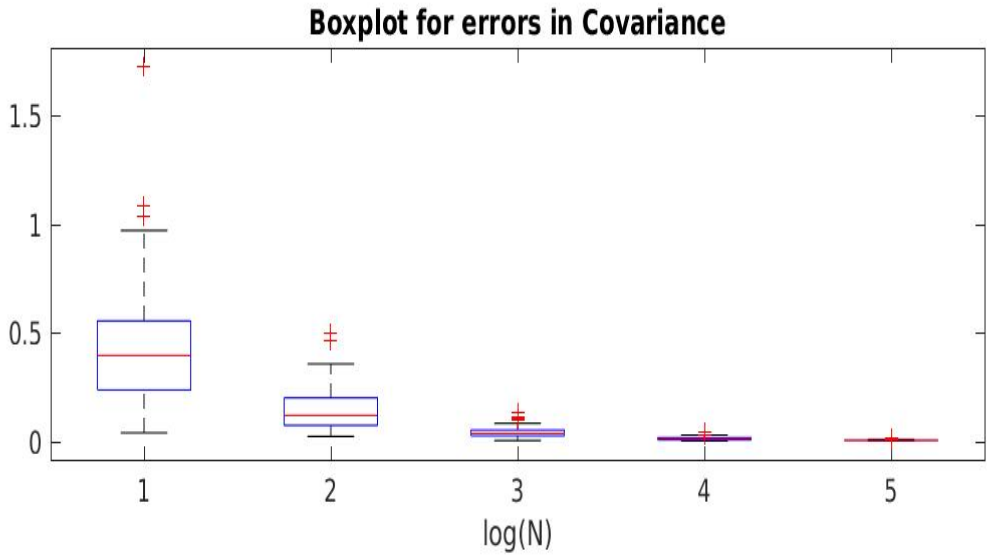
  $$\mathbf{X_{2x1}} = \boldsymbol{\mu}_{\mathbf{2x1}} + \mathbf{A_{2x2}}.\mathbf{W_{2x1}}$$

  This is the process to draw RV from a MVG with just mean and covariance given!

2.2, 2.3 : Part B, Part C

For every value of N, 100 readings of errors in Mean and Covariance were computed and the respective plots are as follows:

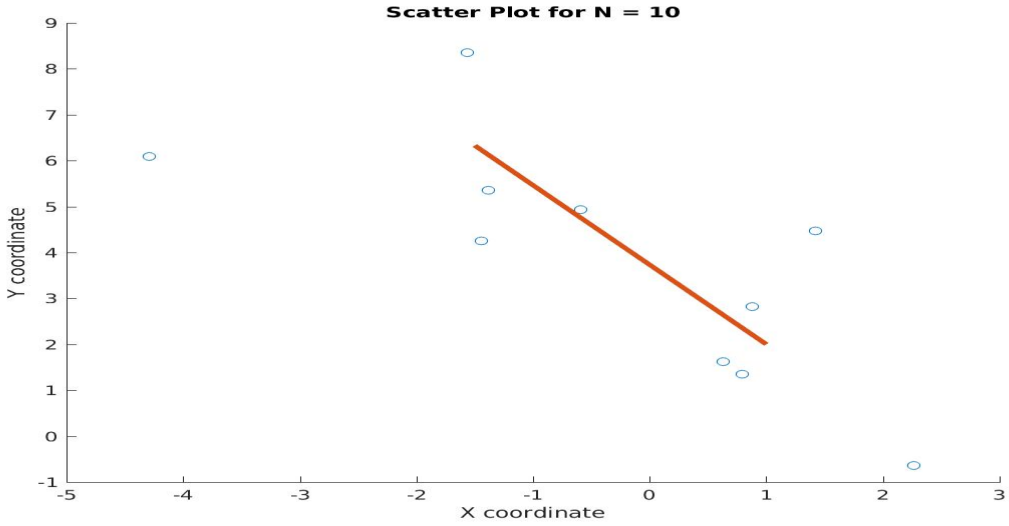


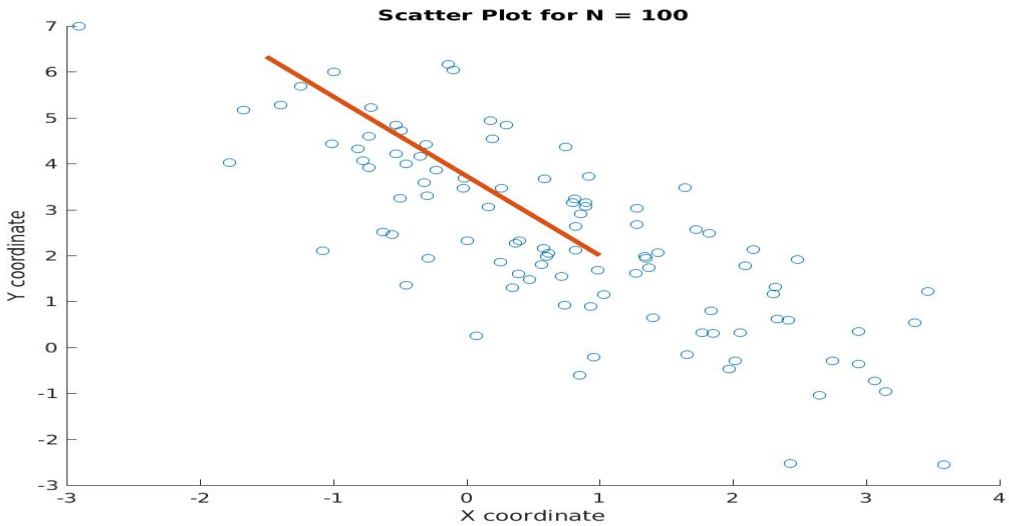


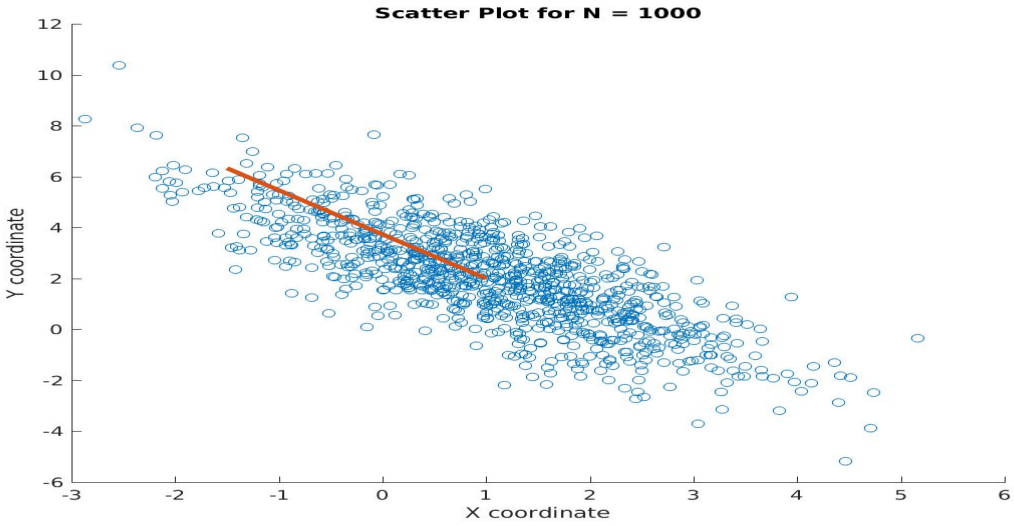
2.4 : Part D

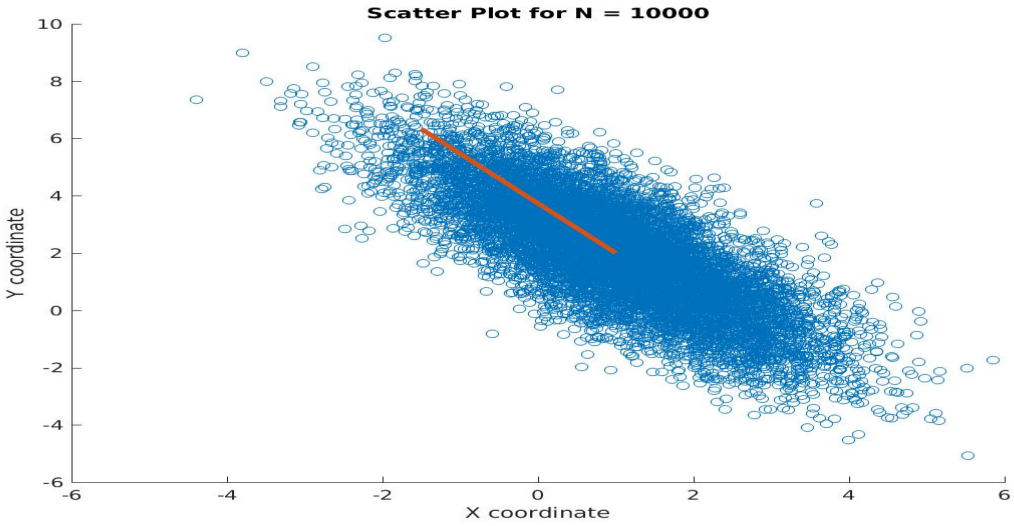
For each N, for every data sample, the scatter plots of the generated data are plotted along with the principal modes of variation of the data. The plots follow:

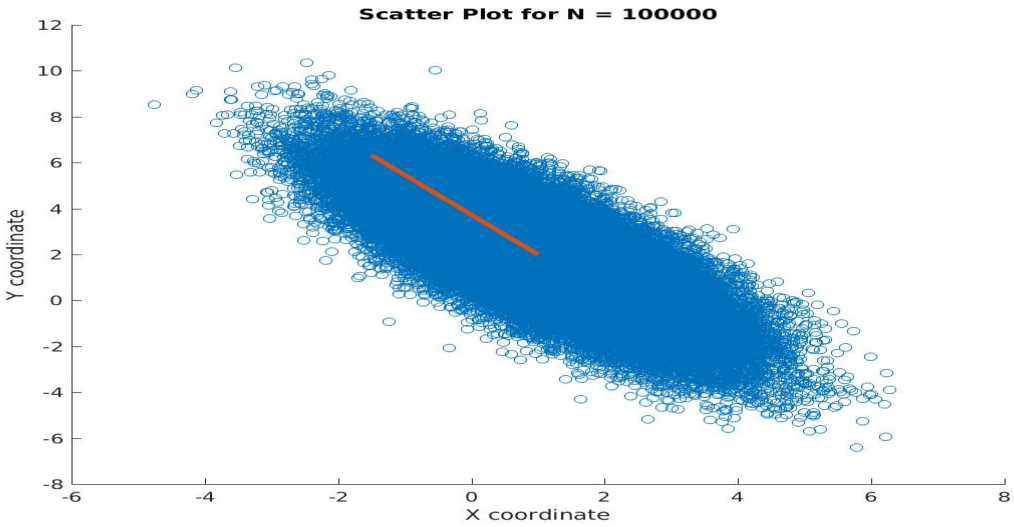
The plots of the comparison follow:






Scatter Plot for N = 1000
Y coordinate
X coordinate


Scatter Plot for N = 10000
Y coordinate
X coordinate


Scatter Plot for N = 100000
Y coordinate
X coordinate

2.3 : Usage of Code

The following are the instructions for the usage of the code:

- Load the code present in 'submission/code/q2/q2.m ' .
- In the same directory are functions implemented like myMean, myCov which return the mean and covariance of appropriate matrices.
- Simply run the code in 'q2.m ' and this wil automatically create the required plots.
- Lines 55, 70 (commented by default) hava a code to save jpg files of the respective plots. Comment/Uncomment these lines appropriately according to need.