Q-2.1]. Given $K(x, x')$ is valid kernel.

Then there has to exist $\phi : R^m \to H$ s.t

$$K(x, x') = \phi(x)^T \phi(x') \quad - (1)$$

(i) Consider a function $\phi_{new} : R^m \to H$.

s.t

$$\phi_{new}(x) = \phi(g(x)).$$

$$\therefore \phi_{new}(x)^T \phi_{new}(y) = \phi(g(x))^T \phi(g(y)).$$

Using eq^n - (1)

$$= K(g(x), g(y)).$$

Hence, $K(g(x), g(y))$ can be represented in terms of inner space product of some ϕ_{new}.

Hence $K_{new}(x, y) = K(g(x), g(y))$ is a valid kernel.

ii) Let $q(x) = \sum_{i=0}^{n} a_i x^i$

Property-1: If K_1, K_2 are valid kernels, then $\alpha_1 K_1 + \alpha_2 K_2$ are valid kernels $\forall \alpha_1, \alpha_2 \geq 0$

[Proved in class]