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Date
Page

# CS-335 Assignment 0

1.1] a). Given $X \sim F_x$
$Y \sim Unif(0,1)$

Let $X = g(Y)$ $\forall$ $g$ is monotonically increasing.
$g: [0,1] \to R$

Using mass conservation,

$P(X < g(y)) = P(Y < y)$ where y is drawn from Y

$F_x(g(y)) = F_y(y)$ ~~Since~~ ~~...~~ ~~if~~

→ Now, since g is defined on the domain $[0,1]$ we can safely assume $1 \geq y \geq 0$

→ Also, for $Y \sim Unif[0,1]$, $F_y(y) = y \quad \forall\, 1 \geq y \geq 0$

Hence $F_x(g(y)) = F_y(y) = y$

$$\boxed{g(y) = F_x^{-1}(y)} \Rightarrow \boxed{X = F_x^{-1}(Y)}$$

Hence we have expressed X in terms of Y.

1.1] b). given $X \sim exp(\lambda)$.

$$f_x = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

$$F_x = \begin{cases} \int_{-\infty}^{x} \lambda e^{-\lambda y} dy & ; x \geq 0 \\ 0 & ; x < 0. \end{cases} = \begin{cases} 1 - e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}.$$

As in the earlier part, $g(\cdot)$ is defined as $F_x^{-1}$

$$F_x^{-1}(x) = \frac{\ln(1-x)}{-\lambda}$$

$\therefore X = F^{-1}(Y)$.

$$\boxed{X = \frac{\ln(1-Y)}{-\lambda}.}$$

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Q2]

a) $\vec{x}, \vec{y} \in R^d$

$d(x, y) = |\vec{x} - \vec{y}|^2$ $\forall$ $|\vec{A}|$ is the $L_2$ norm of $\vec{A}$

$d(x,y) = \sum_{i=0}^{d} (x_i - y_i)^2 =$

Q-2] a) $\vec{x}, \vec{y} \in R$.

$$d(x,y) = \sum_{i=0}^{d} (x_i - y_i)^2$$

$$= \sum_{i=0}^{d} (x_i^2 + y_i^2 - 2x_i \cdot y_i).$$

$$= \sum_{i=0}^{d} x_i^2 + \sum_{i=0}^{d} y_i^2 - 2 \cdot \sum_{i=0}^{d} x_i \cdot y_i.$$

$$= \bar{x} \cdot \bar{x} + \bar{y} \cdot \bar{y} - 2\, \bar{x} \cdot \bar{y}$$

b, c) ~~Done~~ [Code]

d) With a change in the dimensionality of the data, we do not expect a huge change in the execution time. This is because this depends on the way 'd' is computed, which is quite efficient, since we are using vectorization to get the dot products.

On the other hand, varying 'n' will impact the execution time greatly and we ~~will have to~~ can expect a huge increase in the same. This is because vectorizatⁿ hardly depends on the input size but loops will definitely take $O(n^2)$ time.

- Plots attached at the end. [Thanks for your co-operation!]

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Date
Page

3) $P(H) = 0.75$ Let $p$.
$P(T) = 0.25$ Let $(1-p)$.

$\therefore$ Let the expected number of tosses be $x$.

Case - 1

Initial toss is tails.
↳ Since each toss is independent, the process has basically reset. So we again will require $x$ tosses.

Answer $= (x+1)$

Case 2.1

Initial toss is heads, second toss is heads.
↳ We are done!
Answer is $\underline{2}$.

Case - 2.2

Initial toss is heads, second toss is tails,
↳ Like Case-1, we have reset again. But this time, answer will be $\underline{x+2}$.

$\Rightarrow$ $P(\text{Case } 1) = (1-p)$
$P(\text{Case-}2) = p^2$.
$P(\text{Case-}3) = p(1-p)$

$\Rightarrow$ Expected ~~valyee~~ $= x = \sum$ Expected Value in case $i$ $\cdot P(\text{case } i)$

$$x = (1-p)\cdot(x+1) + p^2 \cdot 2 + p(1-p)\cdot x+2.$$

$\therefore x = 0.25(x+1) + 2\cdot(0.75)^2 + 0.75\cdot0.25(x+2)$

$\therefore x = \frac{(x+1)}{4} + 2\cdot\frac{9}{16} + \frac{3}{16}(x+2).$

$\therefore 16x = 4x + 4 + 18 + 3x + 6.$

$\therefore 9x = 28$

$\therefore \boxed{x = \frac{28}{9} = 3.\overline{111}}$

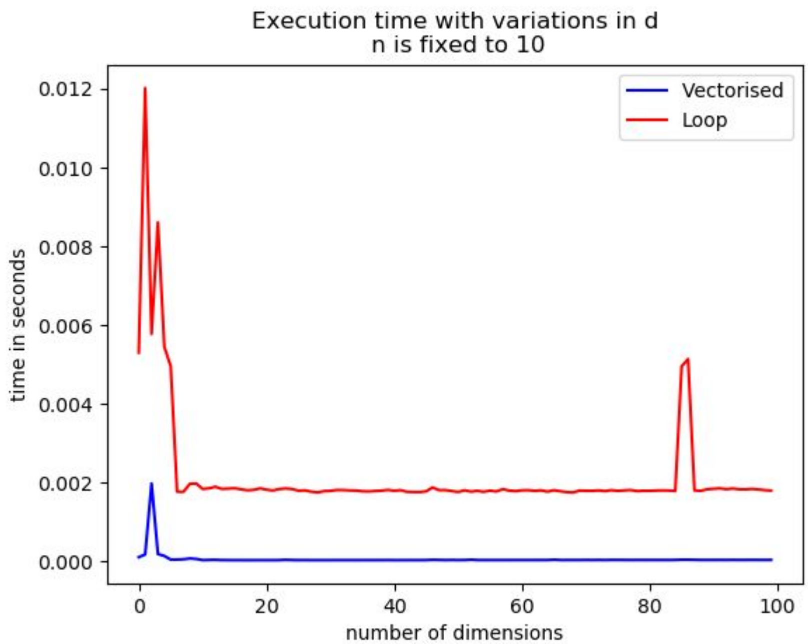
Hence the expected value required is $3.\overline{1}$

3). b) <u>Observation</u>

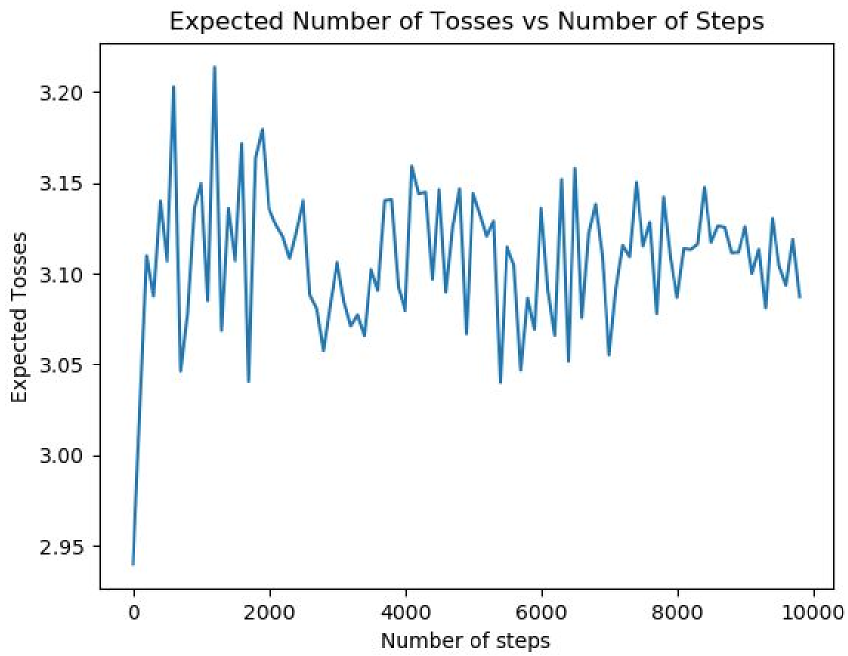
The observed expected value draws closer to our $3.\overline{11}$ value as the number of steps increases.

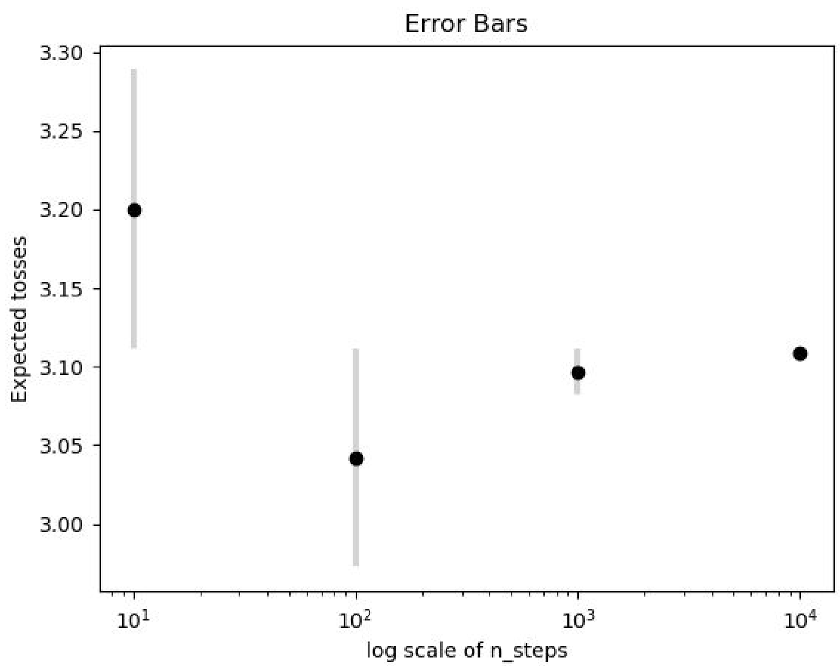
# Assignment 0 Graphs : Niraj Mahajan : 180050069

Q2)d)




Q3) b)


Expected Number of Tosses vs Number of Steps
Expected Tosses
3.20
3.15
3.10
3.05
3.00
2.95
0
2000
4000
6000
8000
10000
Number of steps


Error Bars
Expected tosses
3.30
3.25
3.20
3.15
3.10
3.05
3.00
$10^1$
$10^2$
$10^3$
$10^4$
log scale of n_steps