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Question 2

1D image

$$g(x) = (h * f)(x)$$

where g is the gradient image (in 1D), h is the convolution kernel to represent the gradient operation, and f is the original 1D image

Using convolution theorem in 1D

$$G(u) = H(u)F(u)$$

where G represents the Fourier Transform of g, F is the Fourier Transform of f and H is the Fourier Transform of h

g and h are known $\Rightarrow$ G and H are known

$$F(u) = \frac{G(u)}{H(u)}$$

i.e. We can get F by point-wise division of G by H
Once we get F, we can use inverse Fourier Transform to get f
Fundamental difficulties:

- **Division by zero**: H can be zero for some u, which will result in a 0/0 indeterminate form.
- **Uniqueness** of f: Adding a constant to f will not affect g, thus for same G and H we can have different f. But above approach will always result in only one F and f.
- **Different dimensions** of h and f matrix: We need G, F and H to be of the same dimension. g and f are of the same dimension but h can have different dimension. So we need to zero-pad h and g appropriately before calculating their Fourier transform.

2D image

$$g(x, y) = (h * f)(x, y)$$

where g is the gradient image (in 2D), h is the convolution kernel to represent the gradient operation, and f is the original 2D image

Using convolution theorem in 2D

$$G(u, v) = H(u, v)F(u, v)$$

where G represents the Fourier Transform of g, F is the Fourier Transform of f and H is the Fourier Transform of h

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$$F(u,v) = \frac{G(u,v)}{H(u,v)}$$

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