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Question 3

Part A (8 points)

After performing histogram equalization, the resultant histogram follows the pdf of a Uniform Distribution. After equalizationof the histograms h_1 and h_2, let the transformed histograms be q_1 and q_2 in the regions [0,a] and [a,1] respectively.

Using mass conservation, the transformed histograms can be obtained as:

$$q_1(I) = \frac{\alpha}{a} \qquad ; \quad I \,\epsilon\, [0,a] \tag{1}$$

$$q_2(I) = \frac{1-\alpha}{1-a} \qquad ; \quad I \,\epsilon\, [a,1] \tag{2}$$

$$where\ \alpha = \int_0^a h(I).dI$$

For these transformed histograms, the mean intensity is given by:

$$Mean\ Intensity = \int_0^a Iq_1(I).dI + \int_a^1 Iq_2(I).dI$$

Substituting Equations (1), (2)

$$\begin{aligned} Mean\ Intensity &= \frac{\alpha}{a}\int_0^a IdI + \frac{1-\alpha}{1-a}\int_a^1 IdI \\ &= \frac{\alpha}{a}\left(\frac{I^2}{2}\right)\Bigg|_0^a + \frac{1-\alpha}{1-a}\left(\frac{I^2}{2}\right)\Bigg|_a^1 \\ &= \frac{\alpha}{a}\left(\frac{a^2}{2}\right) + \frac{1-\alpha}{1-a}\left(\frac{1-a^2}{2}\right) \\ &= \frac{\alpha.a}{2} + \frac{(1-\alpha)(1+a)}{2} \end{aligned}$$

$$\boxed{Mean\ Intensity = \frac{1-\alpha+a}{2}} \tag{3}$$

Part B (2 points)

Now, since the chosen intensity a is the median intensity, this means that the weight is divided exactly into two equal parts at a. But since the weight upto a was α, we have $\alpha = \frac{1}{2}$

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Substituting this value of α in Eqn (3), we get:

$$\begin{aligned} Mean\ Intensity &= \frac{1-\alpha+a}{2} \\ &= \frac{1-\frac{1}{2}+a}{2} \end{aligned}$$

$$\boxed{Mean\ Intensity = \frac{0.5+a}{2}} \tag{4}$$

Part C (5 points)

Consider an image (grayscale for simplicity) which has a lot of pixels concentrated within a certain intensity range. Say an image has a lot of pixels with low intensity values.

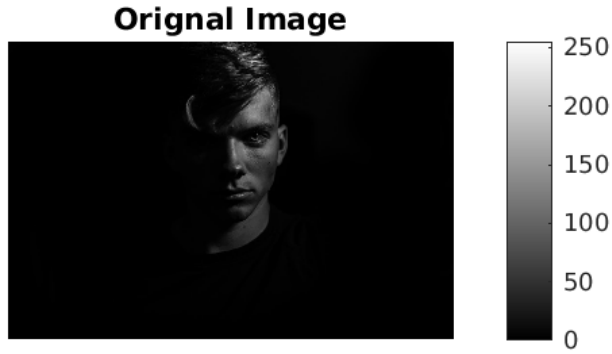
Simple Histogram Equalization (performed globally) tries to adjust these intensity values such that the transformed values are drawn from an Uniform Distribution, keeping the total weight constant. If we apply this method in the case mentioned in the earlier paragraph, it will result in an undesirable stretching of the low intensity pixels towards higher intensities.

On the other hand, when we divide the histogram into two components according to the median intensity, we basically are isolating the low intensity pixels in our image and reducing the dominating effect they have while equalizing the histograms. This preserves the brightness of the orignal image to a greater extent, while ensuring overall image enhancement.

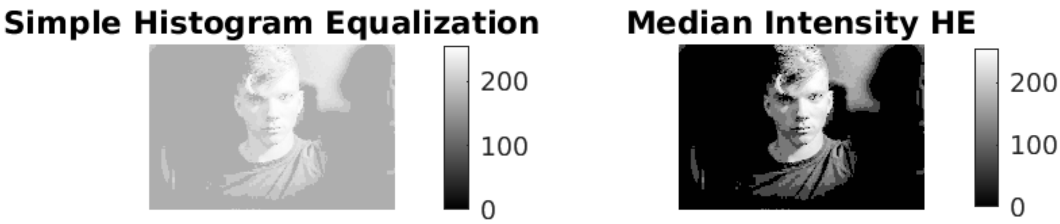
Part D (10 points)

Below are the results obtained using the two methods discussed, in two scenarios. As we can see, when the image has a lot of pixels with low intensity, the method with two separate histogram equalizations greatly outperforms the simple histogram equalization.

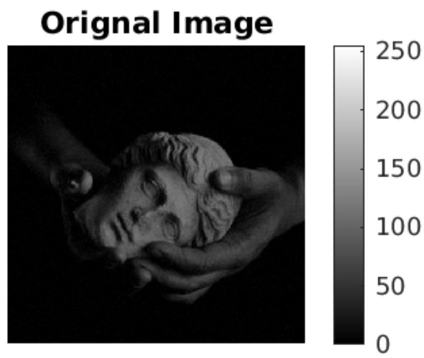
Please Turn Over for the images



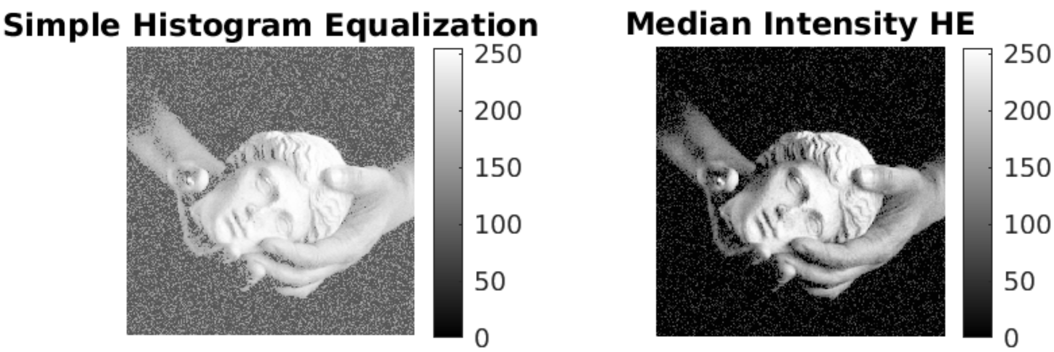
Orignal Image
250
200
150
100
50
0



Simple Histogram Equalization
200
100
0
Median Intensity HE
200
100
0



Orignal Image
250
200
150
100
50
0



Simple Histogram Equalization
250
200
150
100
50
0
Median Intensity HE
250
200
150
100
50
0

Part E - Usage of Code

- The main file **myMainScript.m** simple has function calls to **myplot.m** for both two test images.
- The file **myplot.m** calls the two different methods to perform histogram equalization from **histogram_equalization.m** and **half_histogram_equalization.m** respectively.