CS 663 - Assignment 1

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Question 1

Part A

Moire Patterns occur when we undersample an image. Below you can find the striking difference between original image and the undersampled images. Here undersampling factor is denoted by d.

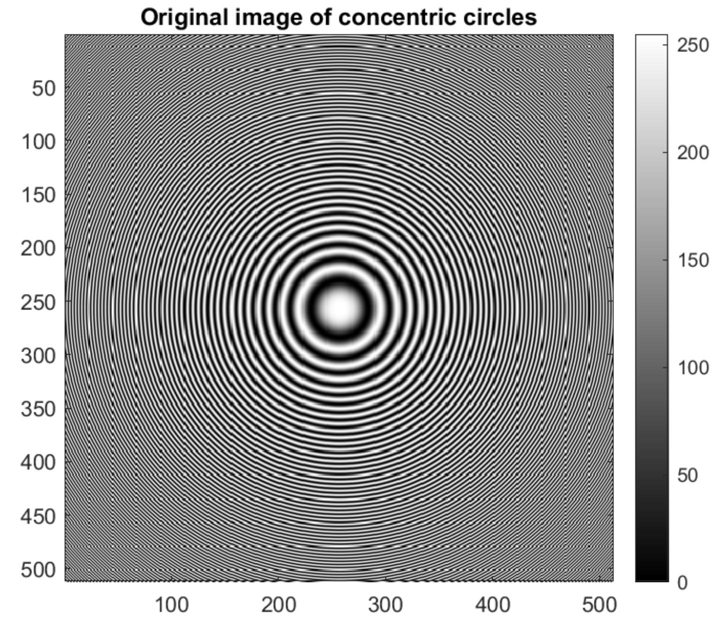

Figure 1: Concentric Circles

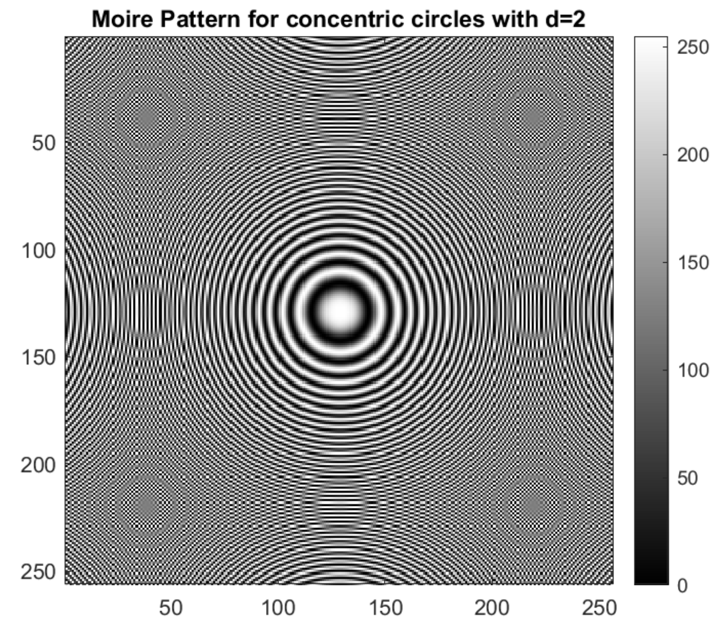

Figure 2

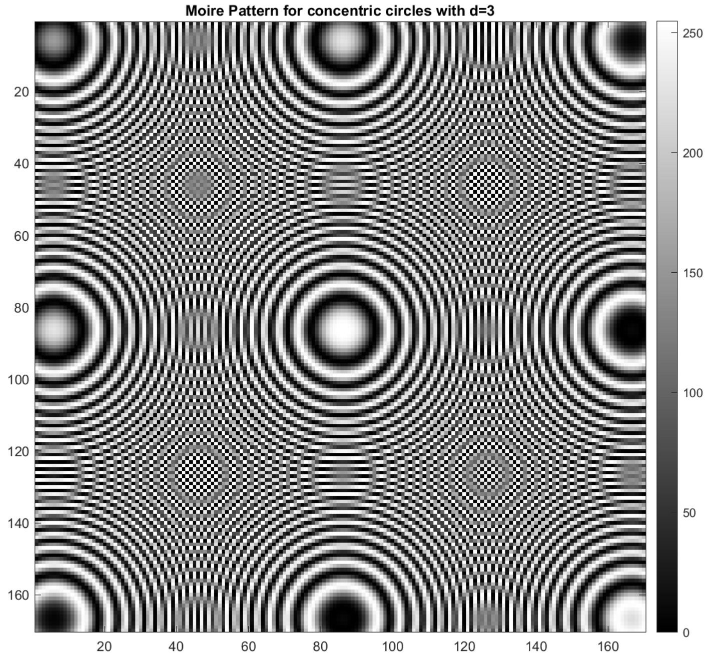

Figure 3

Part B

Here the original image is an N*N pixel dimensional image, where N=103. We want to enlarge the image by increasing the image pixel dimensions. The increment is as follows:

$$new\_rows = 3 * M - 2,\ where\ M = number\ of\ rows$$

$$new\_cols = 2 * M - 1,\ where\ N = number\ of\ columns$$

Here M=N.
We used different interpolation techniques. This part is for Bilinear Interpolation. Results are shown in next page:

Part C

This part has has the same requirements as needed previously only we interpolated the function using nearest neighbor.Results are shown in next page:

Figure 4: Bilinear Interpolation

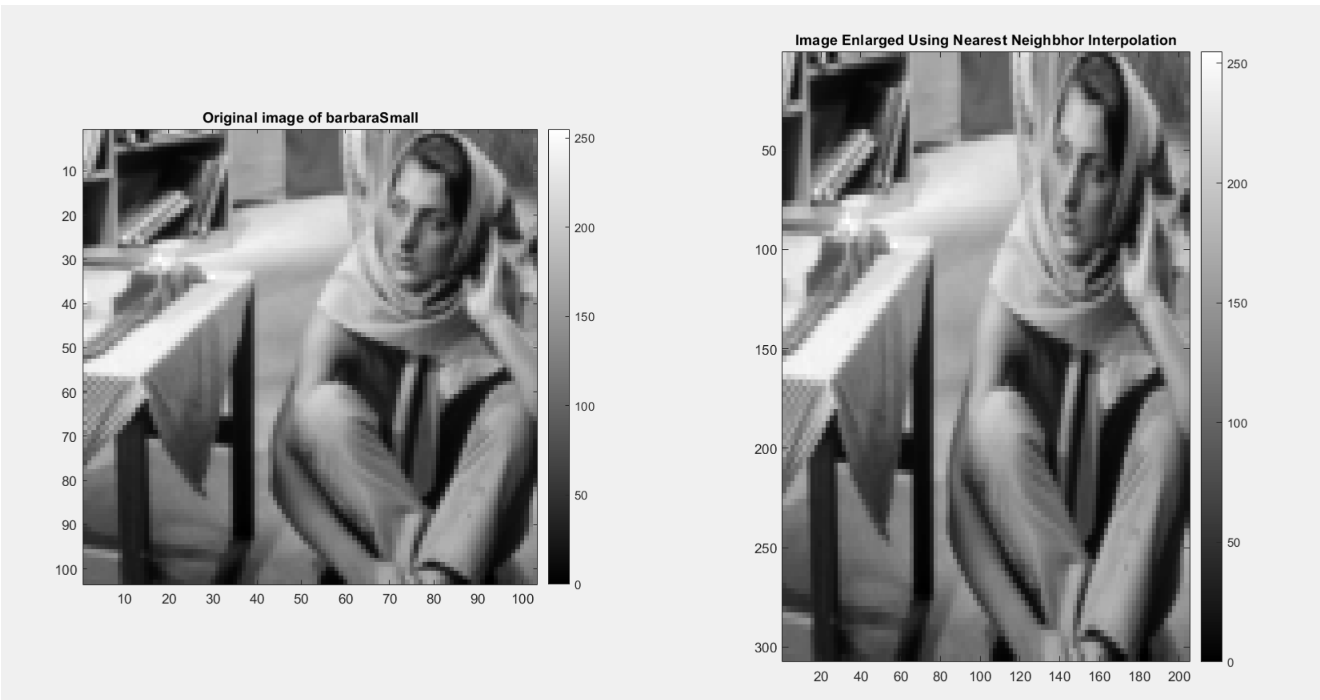

Figure 5: Nearest Neighbor Interpolation

Part D

Again, this part also has has the same requirements as needed previously only we interpolated the function using bicubic interpolation.Results are shown below:

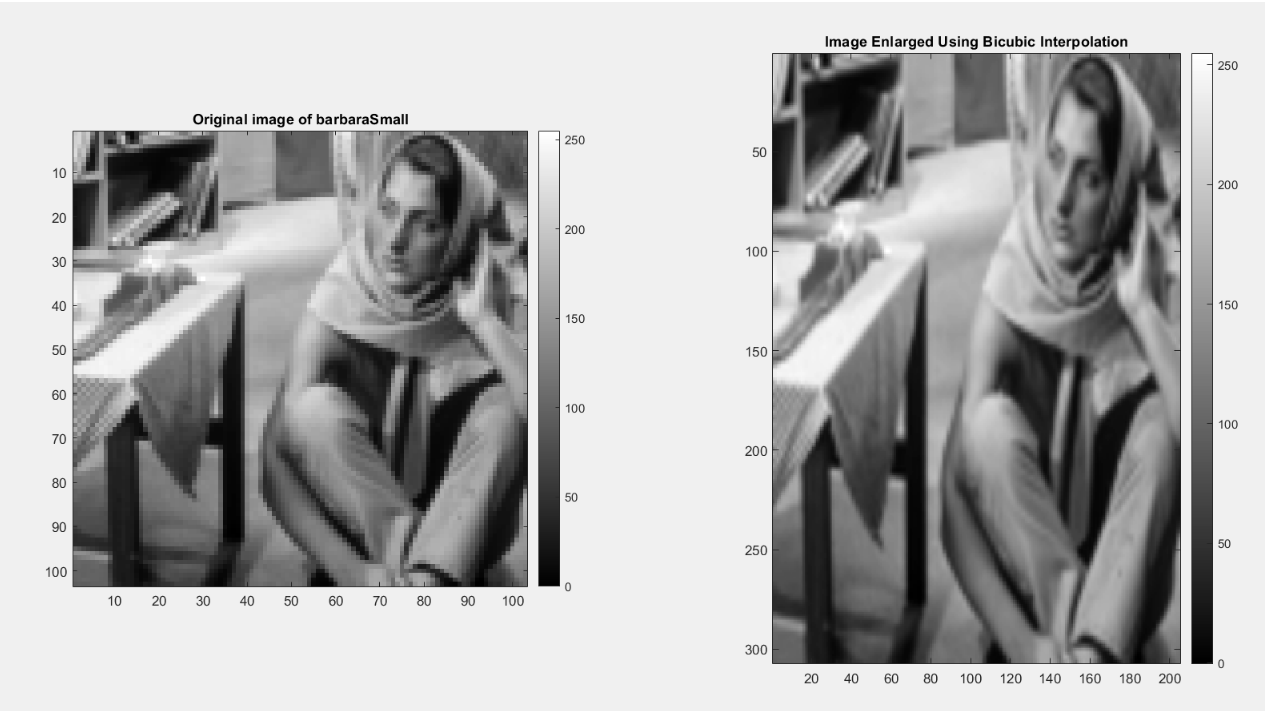

Figure 6: Bicubic Interpolation

Part E

The coarsest image obtained was from nearest neighbor interpolation. This interpolation method just looks at the nearest function value from a required point and assigns it the same value thus leads to very pixelated image.
Bilinear interpolation approximates the function using linear approximation thus leads to better image. Though the function value is continuous everywhere the derivative is discontinuous at given function points.
Bicubic interpolation approximates the function using cubic approximation thus leads to even better image. The interpolated function generated is continuous as well as its first derivative is also continuos a every point. The image is smoothest generated by this method.

We took a small patch of image and used colormap to generate the differences. Though bilinear and bicubic give almost same results but we find smoother transition from darker to lighter pixel intensities in bicubic technique (especially in red and blue regions). in the image are Results are shown in next page.

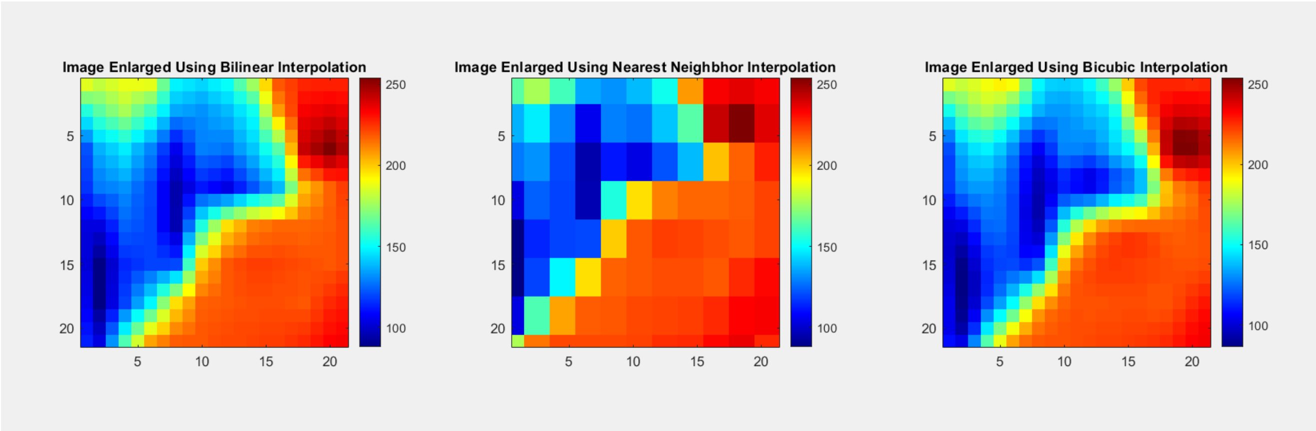

Figure 7: Three interpolation methods

Part F

Our last task in this question was to rotate the image by 30° in the clockwise direction. Here we employed the bilinear interpolation method to find function values at the new image grid. Since we generated the image having same size as the original image edges were cut off. Result is shown below.

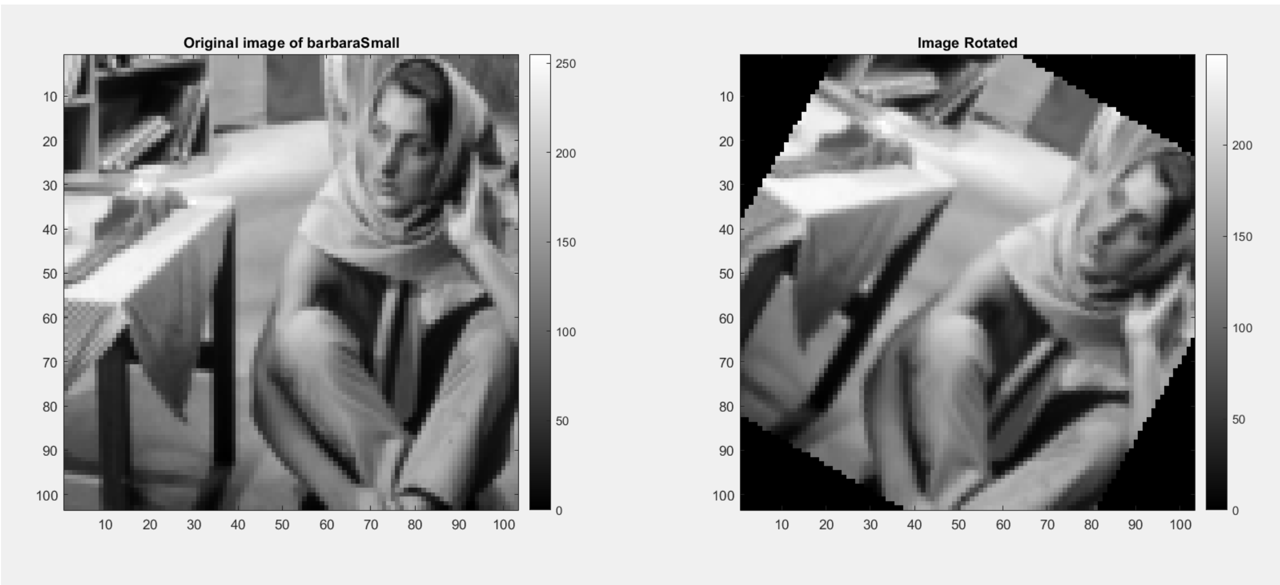

Figure 8: Image Rotation by 30°

Part G

The main file myMainScript.m is divided into 6 parts.

- Part A (Image Shrinking)- It contains a simple function, myShrinkImageByFactorD, which just takes the function (image matrix) and shrinking factor d. We shrunk the image size by a factor of d along each dimension using image sub-sampling by sampling / selecting every d-th pixel along the rows and columns.
- Part B (Bilinear Interpolation)- It contains the function myBilinearInterpolation which takes four original image points, a point and area and then calculates the function at the given point using the image points.
- Part C (Nearest Neighbor Interpolation)- It contains the function myNearestNeighborInterpolation which takes four original image points, a point and then calculates the image point nearest to the given point and assigns it this value.
- Part D (Bicubic Interpolation)- We first calculate the first derivatives w.r.t. x and y and mixed second derivative for the image grid using difference method. Now we pass these matrices along with the image value, the point at which function value is required and the matrix A_inv(a constant matrix). It then calculates the value at the given point using the bicubic formula.
- Part E (Interpolation Comparison)- It uses the previously defined function to generate a small patch of image to compare the differences in the interpolation techniques.
- Part F (Image Rotation)- It contains the function myImageRotation which takes four original image points, a point and then bilinearly interpolates to get the new image grid.

Question 2

Part A : Foreground Mask (2 points)

Figure 1: Transformation on Image(7)

Part B : Linear Contrast Stretching (3 points)

Formula used :-

$$F(x) = a + ((x - a)/(b - a)) * 255$$

where,

- a is the intensity with 5 percentile (w.r.t. intensities sorted in ascending order)
- b is the intensity with 95 percentile (w.r.t. intensities sorted in ascending order)

Observations after applying on Image(5) :-
Application of Contrast stretching on image(5) doesn't create any significant visible change in the image.
Reason :-
The image already had a complete intensity range, so linear contrast stretching won't be effective as according the formula (b-a) tends to 255 so the intensities will be mapped to the same value.

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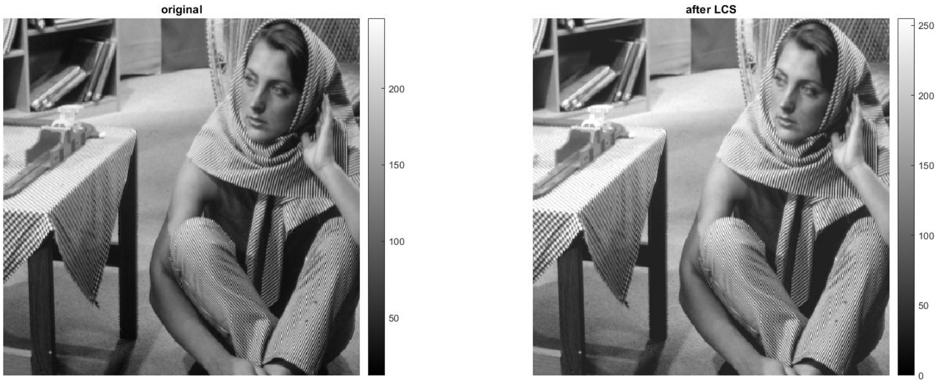

Figure 2.1: Transformation on Image(1)

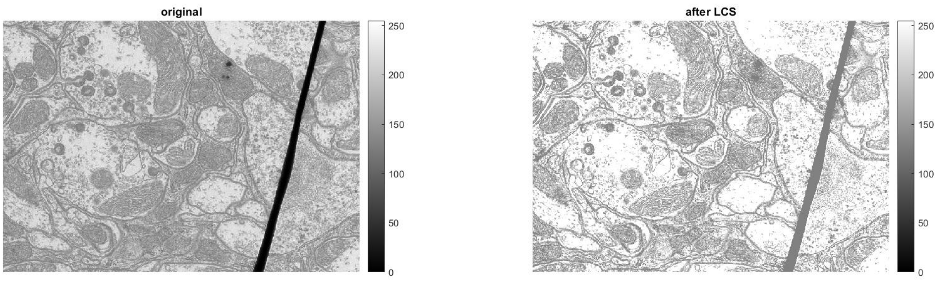

Figure 2.2: Transformation on Image(2)

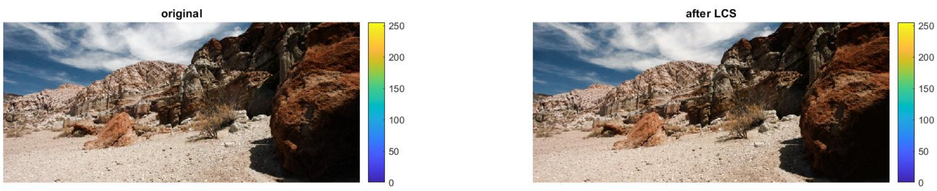

Figure 2.3: Transformation on Image(3)

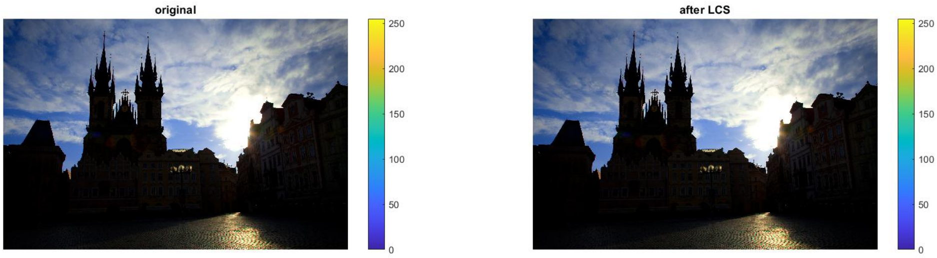

Figure 2.5: Transformation on Image(5)

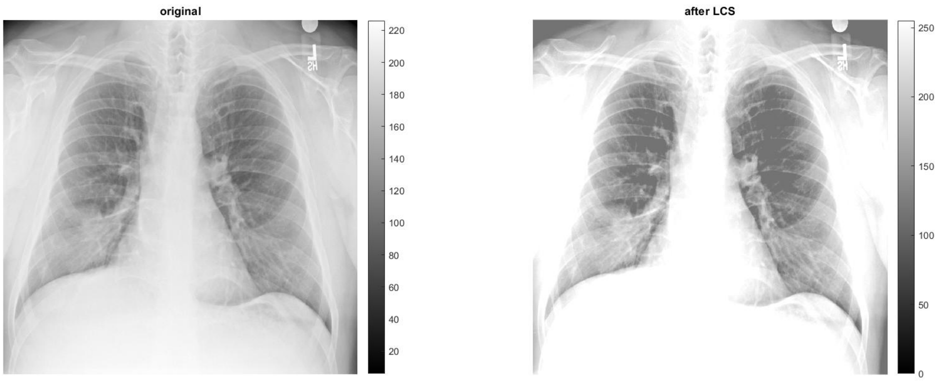

Figure 2.6: Transformation on Image(6)

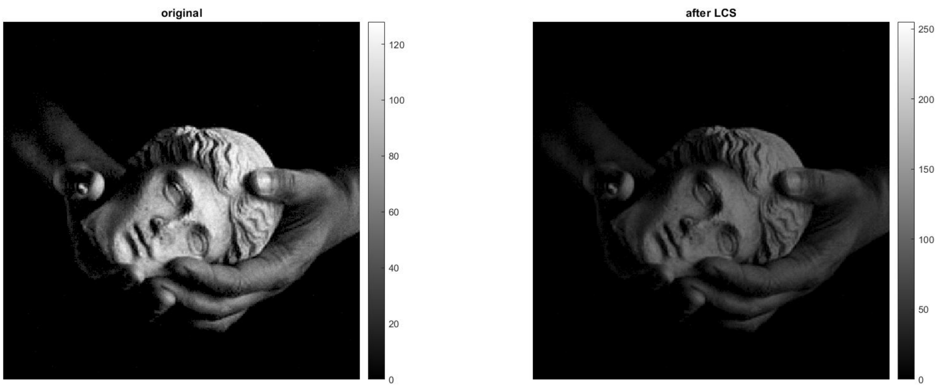

Figure 2.7: Transformation on Image(7)
Please note the image on the left is after LCS, while the one on the right is orignal

Part C : Histogram Equalization (HE) (5 points)

Observations after applying on Image(5) :-
Application of Histogram Equalization on Image(5) results in a significant improvement in the image quality. Thus, we should prefer HE over linear contrast stretching.
Reason :-
The Image(5) had a higher frequency of lower intensities, and after HE, the equalized intensities resulted in a much better image.

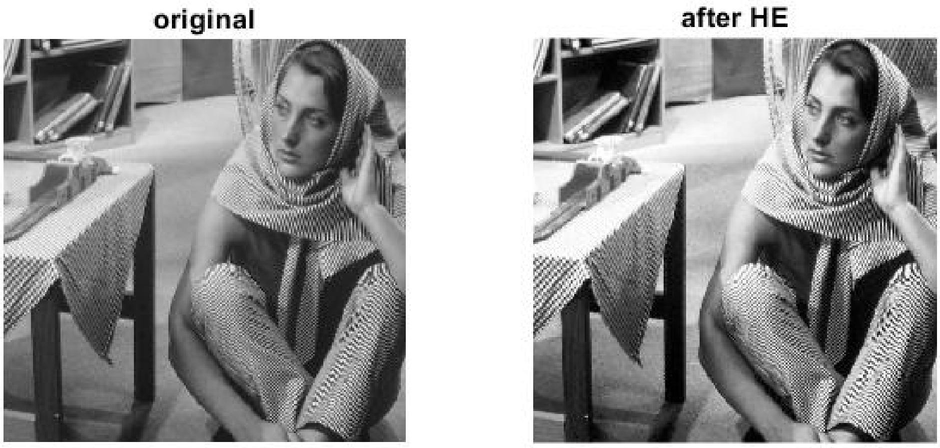

Figure 3.1: Transformation on Image(1)

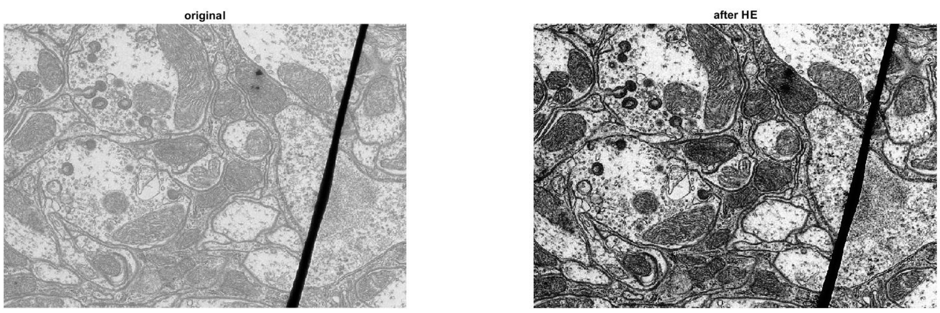

Figure 3.2: Transformation on Image(2)

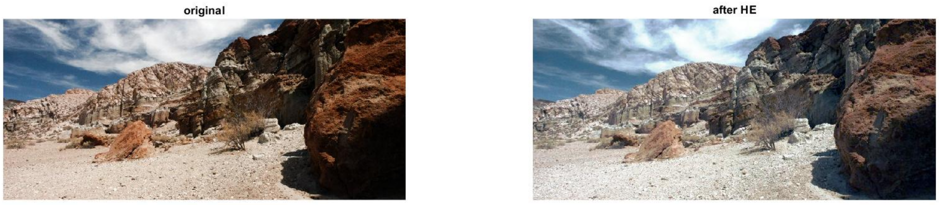

Figure 3.3: Transformation on Image(3)

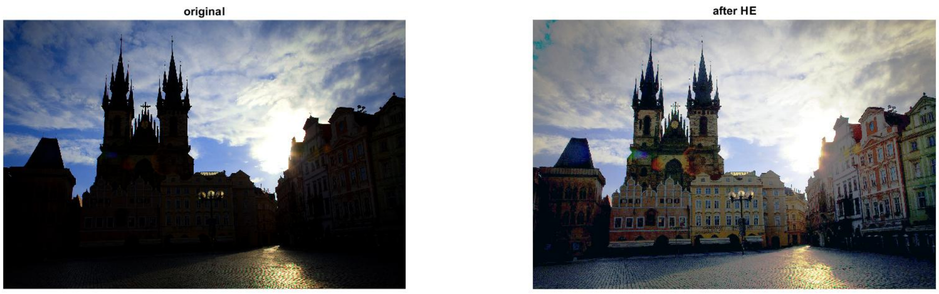

Figure 3.5: Transformation on Image(5)

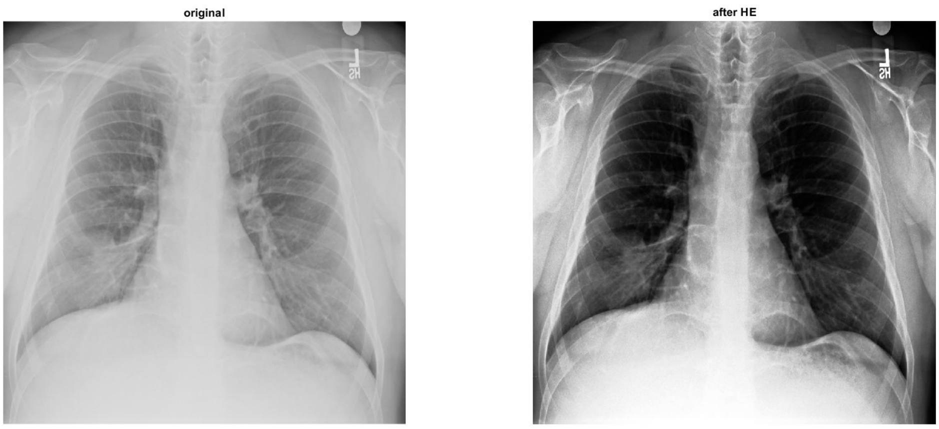

Figure 3.6: Transformation on Image(6)

Figure 3.7: Transformation on Image(7)

Part D : Histogram Matching (HM) (15 points)

Observations :- It can be observed from the above 3 images that the output of Histogram Matching is quite similar to the input image (the orange tint), while the output of HE has far more blood vessels highlighted and clearly visible.
Reason :- Histogram Matching is basically "enhancing" the input image while constraining the features of the output to a particular reference image. Hence, although the HE output has a lot more blood vessels visible, the HM output is quite similar in appearance to the input image, and hence more realistic. This is particulary useful when we want to enhnace the features in an image, while making it look like similar to a reference image.

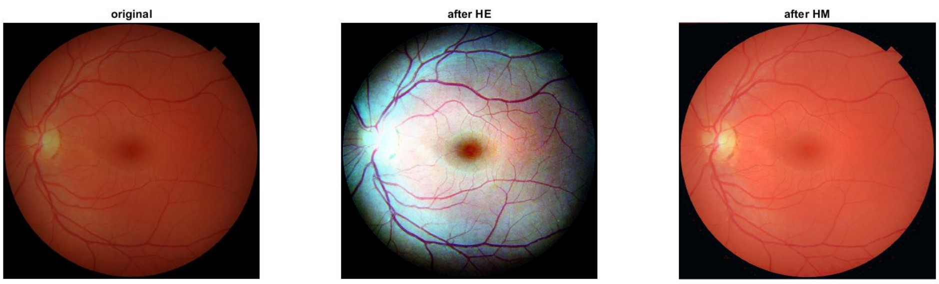

Figure 4.1: Transformation on Image(4)

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Part E : Contrast-Limited Adaptive Histogram Equalization (CLAHE) (30 points)

Figure 5.11: Transformation 1 on Image(1)

Figure 5.12: Transformation 2 on Image(1)

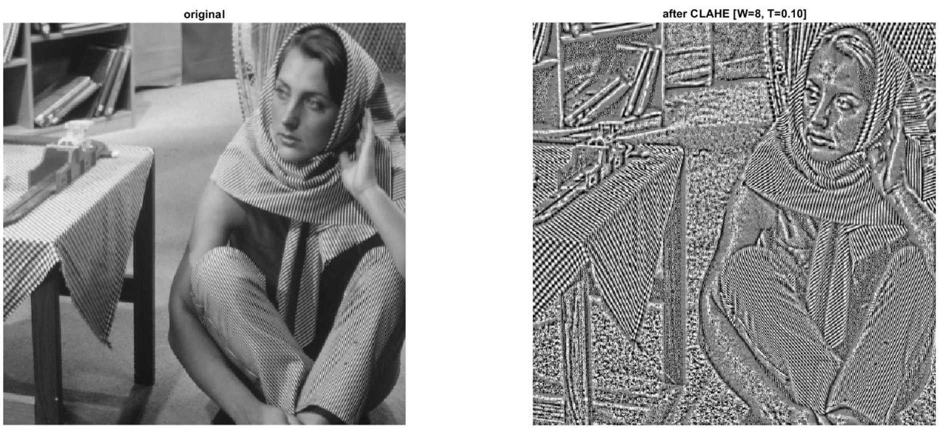

Figure 5.13: Transformation 3 on Image(1)

Figure 5.14: Transformation 4 on Image(1)

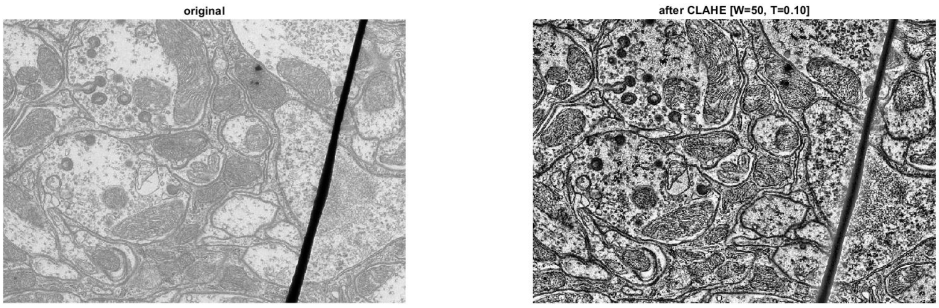

Figure 5.21: Transformation 1 on Image(2)

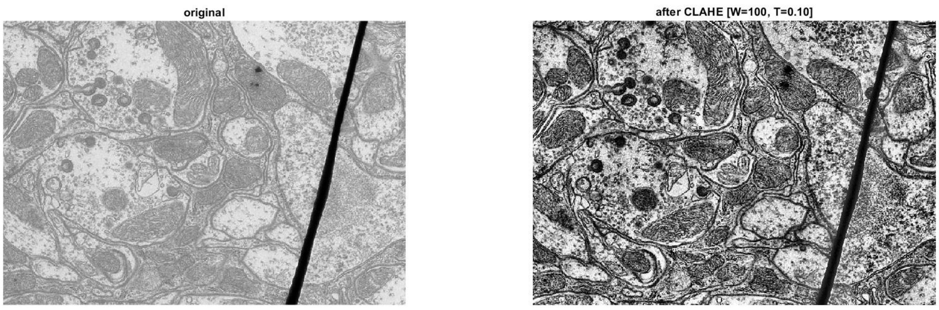

Figure 5.22: Transformation 2 on Image(2)

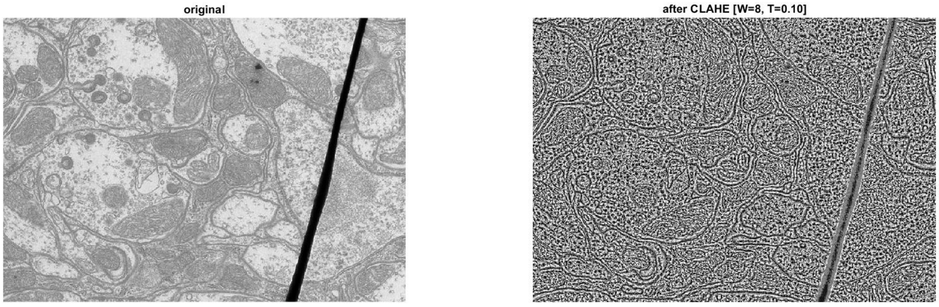

Figure 5.23: Transformation 3 on Image(2)

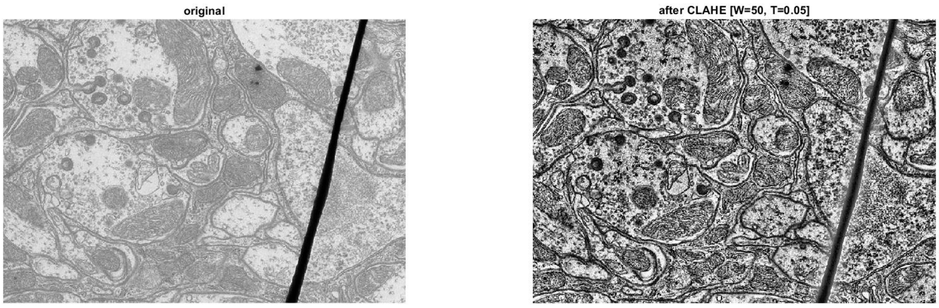

Figure 5.24: Transformation 4 on Image(2)

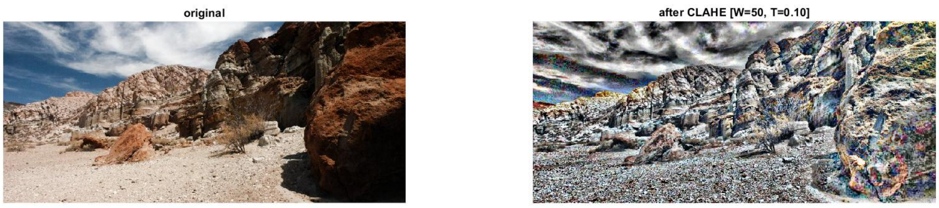

Figure 5.31: Transformation 1 on Image(3)

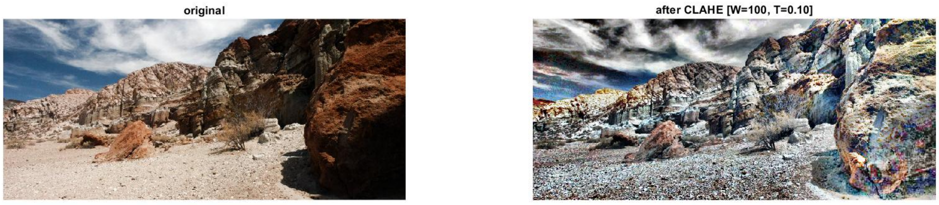

Figure 5.32: Transformation 2 on Image(3)

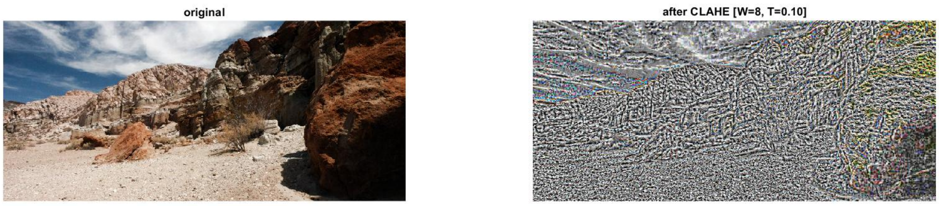

Figure 5.33: Transformation 3 on Image(3)

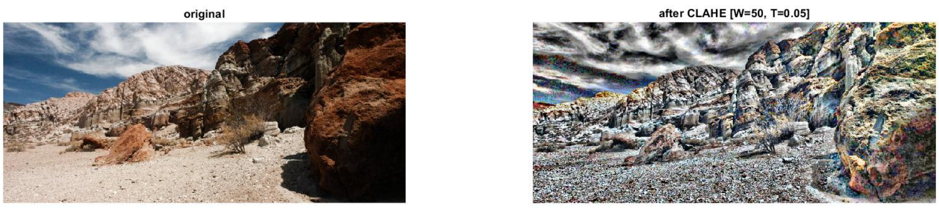

Figure 5.34: Transformation 4 on Image(3)

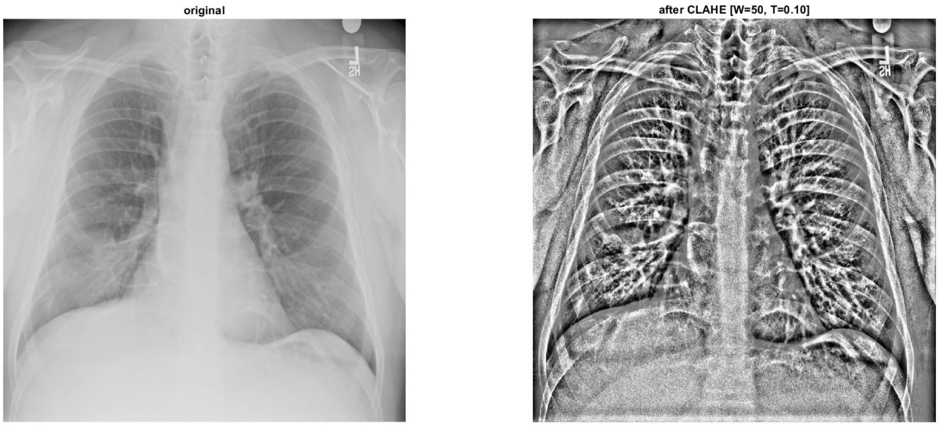

Figure 5.61: Transformation 1 on Image(6)

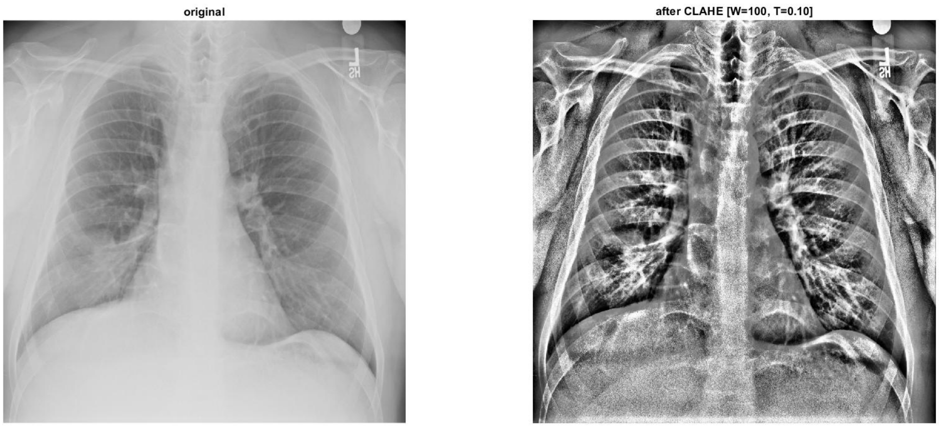

Figure 5.62: Transformation 2 on Image(6)

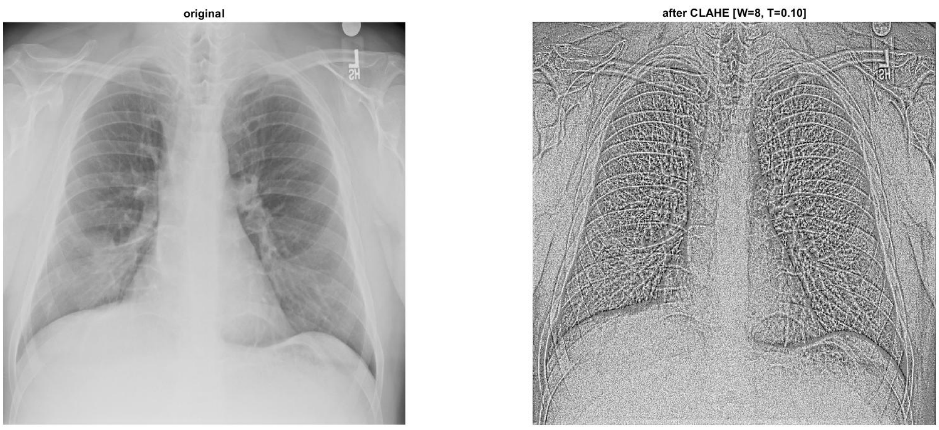

Figure 5.63: Transformation 3 on Image(6)

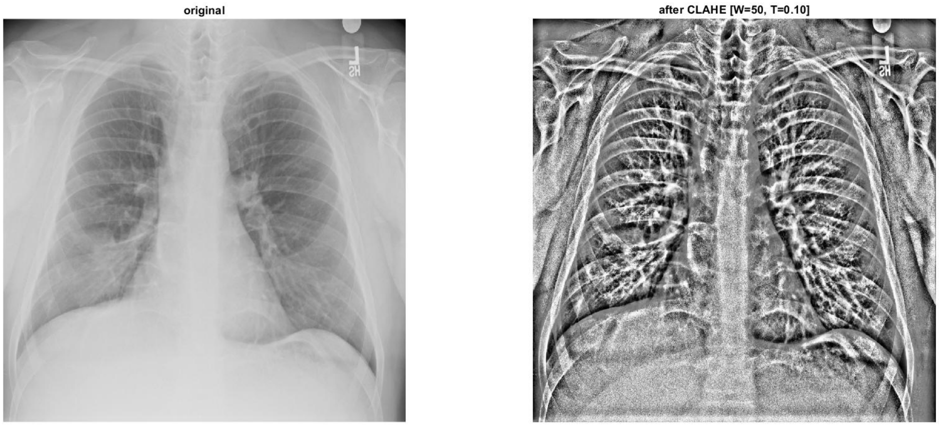

Figure 5.64: Transformation 4 on Image(6)

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Question 3

Part A (8 points)

After performing histogram equalization, the resultant histogram follows the pdf of a Uniform Distribution. After equalizationof the histograms h_1 and h_2, let the transformed histograms be q_1 and q_2 in the regions [0,a] and [a,1] respectively.

Using mass conservation, the transformed histograms can be obtained as:

$$q_1(I) = \frac{\alpha}{a} \qquad ; \quad I \,\epsilon\, [0, a] \tag{1}$$

$$q_2(I) = \frac{1-\alpha}{1-a} \qquad ; \quad I \,\epsilon\, [a, 1] \tag{2}$$

$$where\ \alpha = \int_0^a h(I).dI$$

For these transformed histograms, the mean intensity is given by:

$$Mean\ Intensity = \int_0^a Iq_1(I).dI + \int_a^1 Iq_2(I).dI$$

Substituting Equations (1), (2)

$$\begin{aligned} Mean\ Intensity &= \frac{\alpha}{a}\int_0^a IdI + \frac{1-\alpha}{1-a}\int_a^1 IdI \\ &= \frac{\alpha}{a}\left(\frac{I^2}{2}\right)\Bigg|_0^a + \frac{1-\alpha}{1-a}\left(\frac{I^2}{2}\right)\Bigg|_a^1 \\ &= \frac{\alpha}{a}\left(\frac{a^2}{2}\right) + \frac{1-\alpha}{1-a}\left(\frac{1-a^2}{2}\right) \\ &= \frac{\alpha.a}{2} + \frac{(1-\alpha)(1+a)}{2} \end{aligned}$$

$$\boxed{Mean\ Intensity = \frac{1-\alpha+a}{2}} \tag{3}$$

Part B (2 points)

Now, since the chosen intensity a is the median intensity, this means that the weight is divided exactly into two equal parts at a. But since the weight upto a was α, we have $\alpha = \frac{1}{2}$

Substituting this value of α in Eqn (3), we get:

$$\begin{aligned} Mean\ Intensity &= \frac{1-\alpha+a}{2} \\ &= \frac{1-\frac{1}{2}+a}{2} \end{aligned}$$

$$\boxed{Mean\ Intensity = \frac{0.5+a}{2}} \tag{4}$$

Part C (5 points)

Consider an image (grayscale for simplicity) which has a lot of pixels concentrated within a certain intensity range. Say an image has a lot of pixels with low intensity values.

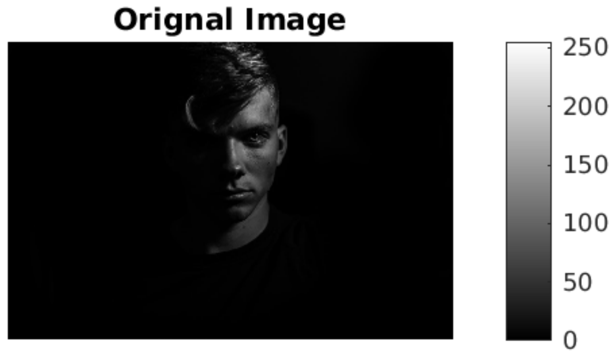
Simple Histogram Equalization (performed globally) tries to adjust these intensity values such that the transformed values are drawn from an Uniform Distribution, keeping the total weight constant. If we apply this method in the case mentioned in the earlier paragraph, it will result in an undesirable stretching of the low intensity pixels towards higher intensities.

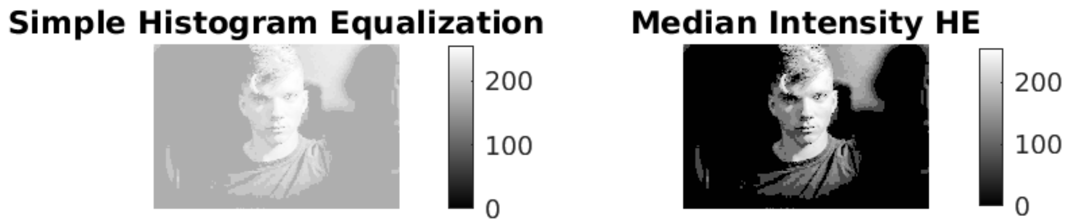
On the other hand, when we divide the histogram into two components according to the median intensity, we basically are isolating the low intensity pixels in our image and reducing the dominating effect they have while equalizing the histograms. This preserves the brightness of the orignal image to a greater extent, while ensuring overall image enhancement.

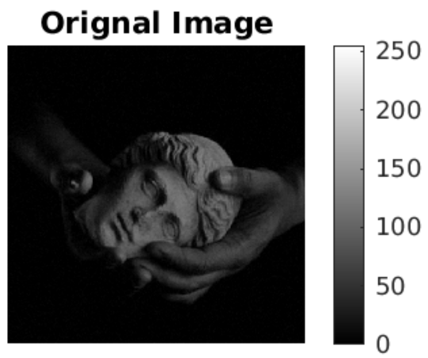
Part D (10 points)

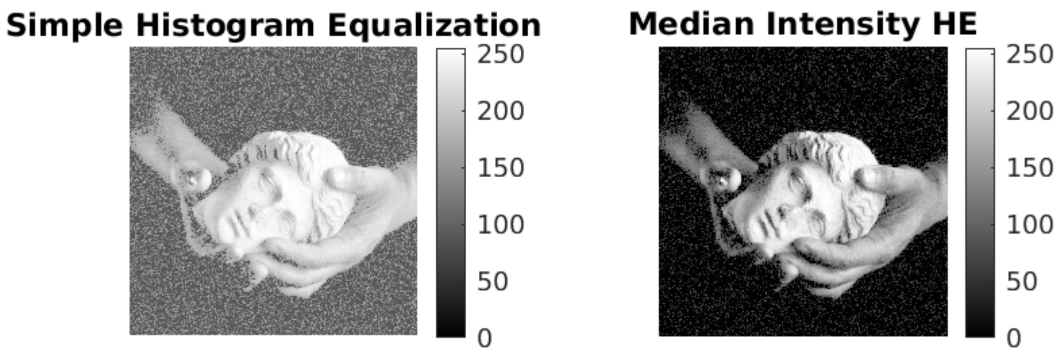
Below are the results obtained using the two methods discussed, in two scenarios. As we can see, when the image has a lot of pixels with low intensity, the method with two separate histogram equalizations greatly outperforms the simple histogram equalization.

Please Turn Over for the images


Orignal Image
250
200
150
100
50
0


Simple Histogram Equalization
200
100
0
Median Intensity HE
200
100
0


Orignal Image
250
200
150
100
50
0


Simple Histogram Equalization
250
200
150
100
50
0
Median Intensity HE
250
200
150
100
50
0

Part E - Usage of Code

- The main file **myMainScript.m** simple has function calls to **myplot.m** for both two test images.
- The file **myplot.m** calls the two different methods to perform histogram equalization from **histogram_equalization.m** and **half_histogram_equalization.m** respectively.