

1 Ideal LPF on the image

Original Image and its Fourier Transform are shown in fig. below.

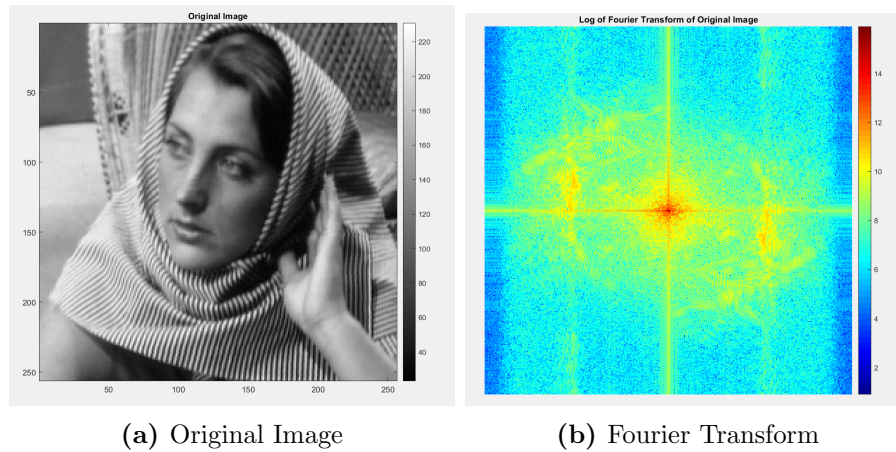


Figure 1

We want to pass this image through an ideal lpf and see its effect on the image. Ideal low pass filter is defined by the following equation:

$$H(u, v) = 1 ; \text{ if } u^2 + v^2 \leq D^2$$

$$H(u, v) = 0; \text{ otherwise}$$

1.1 Cutoff frequency $|D| = 40$

The frequency response corresponding the filter is shown below.

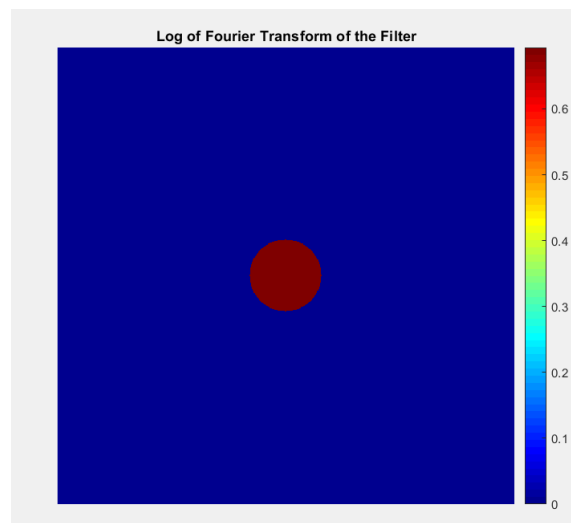


Figure 2: Frequency response of an ideal LPF with cutoff frequency $|\omega| = 40$

Image and the corresponding Fourier Transform are shown in fig. given below.

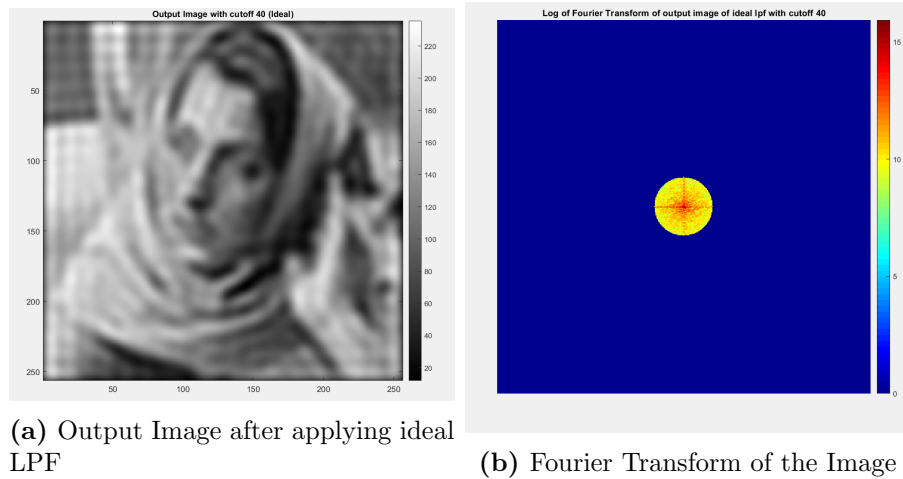


Figure 3

1.2 Cutoff frequency $|D| = 80$

The frequency response corresponding the filter is shown below.

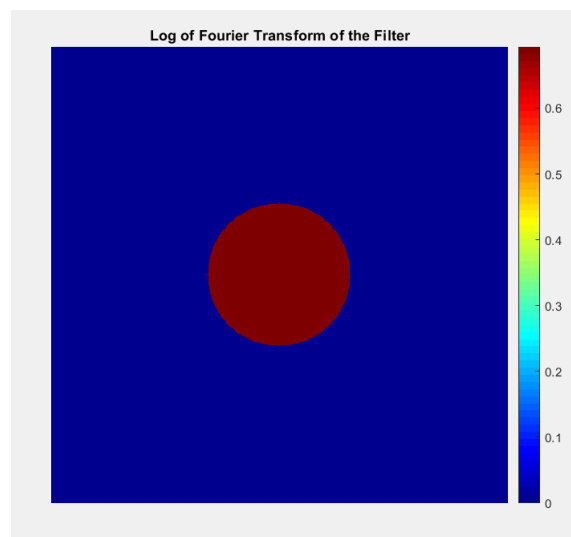


Figure 4: Frequency response of an ideal LPF with cutoff frequency $|\omega| = 80$

Image and the corresponding Fourier Transform are shown in fig. given below.

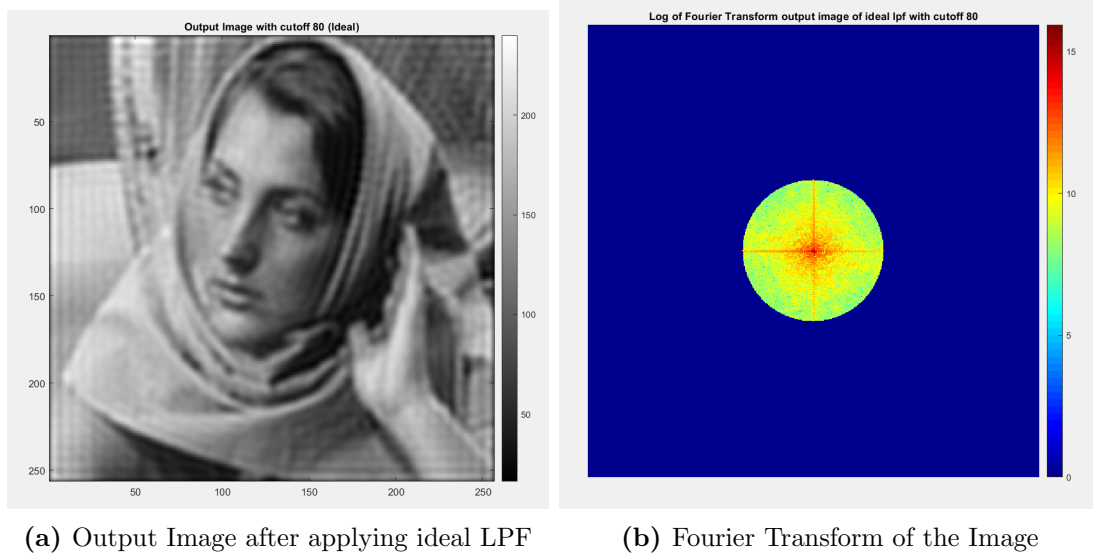


Figure 5

2 Gaussian low pass filter

2.1 Standard Deviation $\sigma = 40$

The frequency response corresponding the filter is shown below.

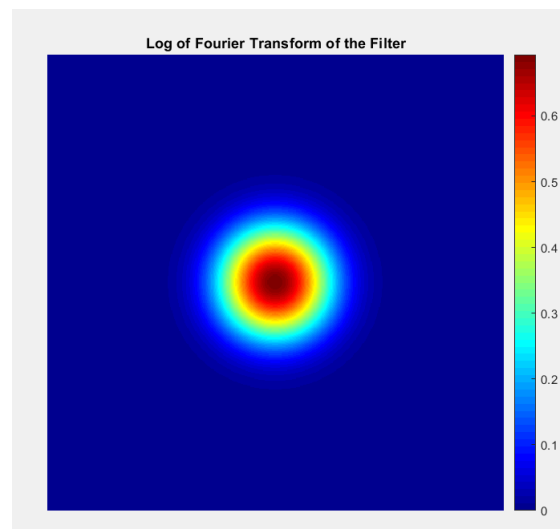


Figure 6: Frequency response of an Gaussian LPF with standard deviation $|\sigma| = 40$

Image and the corresponding Fourier Transform are shown in fig. given below.

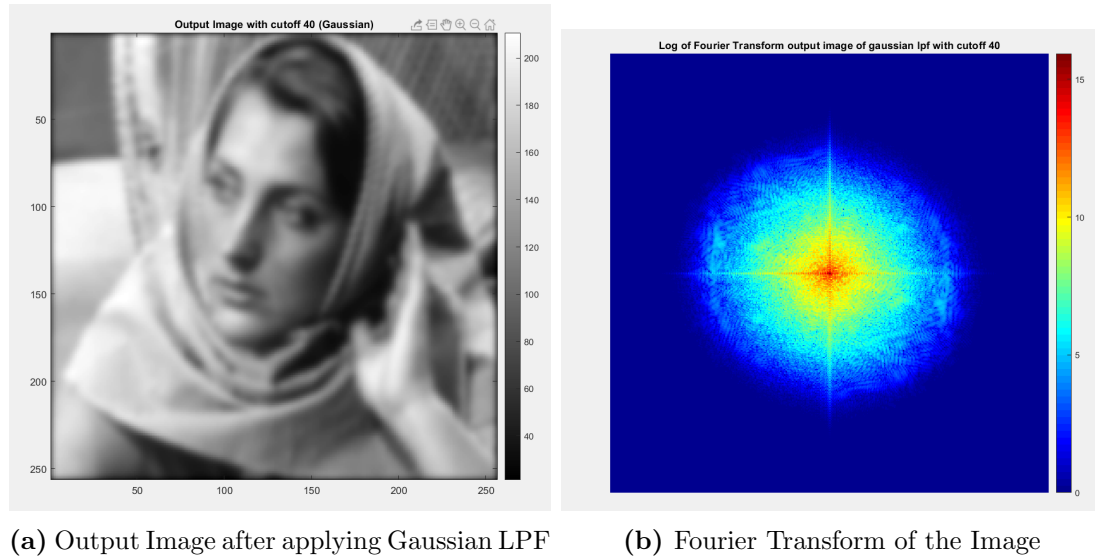


Figure 7

2.2 Standard Deviation $\sigma = 80$

The frequency response corresponding the filter is shown below.

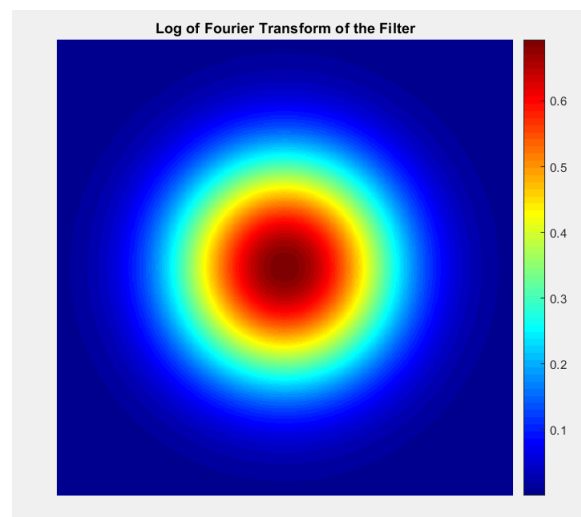
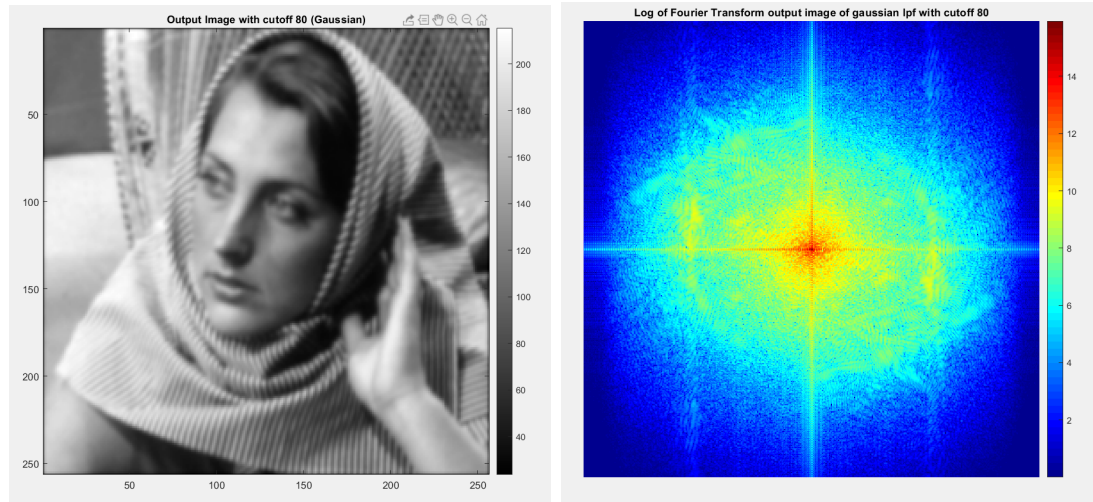


Figure 8: Frequency response of an Gaussian LPF with standard deviation $|\sigma| = 80$

Image and the corresponding Fourier Transform are shown in fig. given below.



(a) Output Image after applying Gaussian LPF (b) Fourier Transform of the Image

Figure 9

3 Explanation

We can see a stark difference in the output image which was expected also. The ideal LPF corresponds to a sinc function in the spatial domain which gives rippling or ringing effect in the image. As the size of the LPF is increased the ringing effect also decreases. The Fourier Transform of a Gaussian Filter gives a Gaussian kernel in the spatial domain also which is essentially Gaussian smoothing of the image.

4 Code Usage

The code is divided into three parts.

- Part 1 is just extracts the image from the folder and displays it and its Fourier Transform.
- Part 2 corresponds to an LTI system which is an ideal LPF with cutoff frequencies, $f \in \{40, 80\}$.
- Part 3 corresponds to an LTI system which is a Gaussian LPF with standard deviation, $\sigma \in \{40, 80\}$.