

## Question 6

### Formula:

$$F(u, v) = \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x, y) \exp(-j2\pi(\frac{ux}{W_1} + \frac{vy}{W_2}))$$

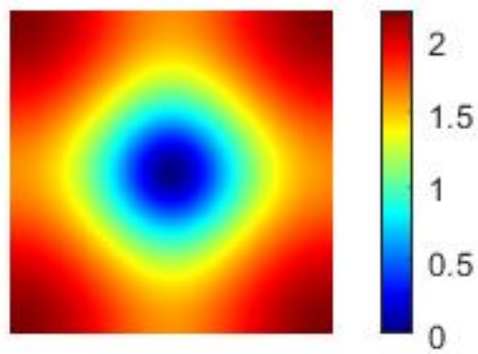
$$F_{k1}(u, v) = -6 \exp(-j2\pi \frac{\lfloor N/2 \rfloor u + \lfloor N/2 \rfloor v}{N}) + \sum_{x=\lfloor N/2 \rfloor-1}^{\lfloor N/2 \rfloor+1} \exp(-j2\pi(\frac{ux + v(\lfloor N/2 \rfloor)}{N})) + \sum_{y=\lfloor N/2 \rfloor-1}^{\lfloor N/2 \rfloor+1} \exp(-j2\pi(\frac{u(\lfloor N/2 \rfloor) + vy}{N}))$$

$$F_{k2}(u, v) = 9 \exp(-j2\pi \frac{\lfloor N/2 \rfloor u + \lfloor N/2 \rfloor v}{N}) - \sum_{x=\lfloor N/2 \rfloor-1}^{\lfloor N/2 \rfloor+1} \sum_{y=\lfloor N/2 \rfloor-1}^{\lfloor N/2 \rfloor+1} \exp(-j2\pi(\frac{ux + vy}{N}))$$

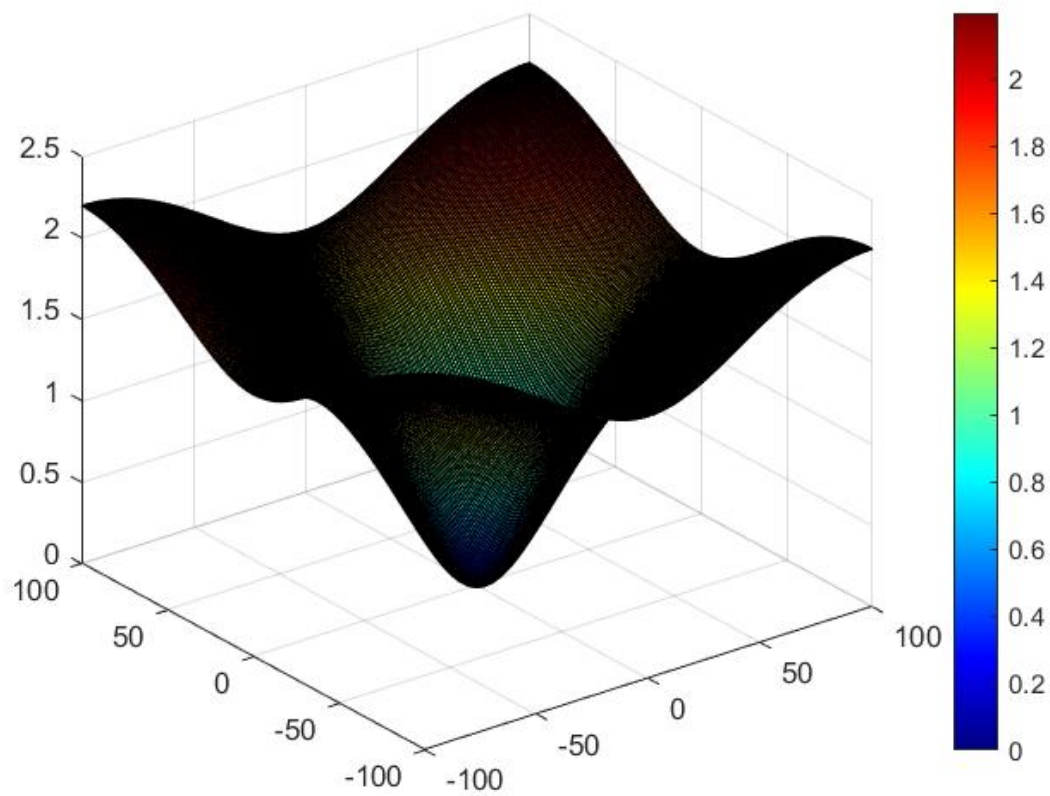
### Code Snippet:

```
N = 201;
F1 = zeros(N,N);
F2 = zeros(N,N);
for u = 1:201
    for v = 1:201
        for x = 99:101
            for y = 99:101
                if x==y && x==100
                    F1(u,v) = F1(u,v)-4*exp(-1i*2*pi*((u-1-100)*x/N+(v-1-100)*y/N));
                    F2(u,v) = F2(u,v)+8*exp(-1i*2*pi*((u-1-100)*x/N+(v-1-100)*y/N));
                else
                    if x~=y && (x+y)~=(N-1)
                        F1(u,v)= F1(u,v)+exp(-1i*2*pi*((u-1-100)*x/N+(v-1-100)*y/N));
                    end
                    F2(u,v) = F2(u,v)-exp(-1i*2*pi*((u-1-100)*x/N+(v-1-100)*y/N));
                end
            end
        end
    end
end
[U,V]=meshgrid(-100:100,-100:100);
lF1 = log(abs(F1)+1);
figure('Name','k1-2d');imshow(lF1,[min(lF1(:)) max(lF1(:))]);colormap('jet');colorbar
figure('Name','k1-3d'); surf(U,V,lF1); colormap('jet'); colorbar;
lF2 = log(abs(F2)+1);
figure('Name','k2-2d');imshow(lF2,[min(lF2(:)) max(lF2(:))]);colormap('jet');colorbar
figure('Name','k2-3d'); surf(U,V,lF2); colormap('jet'); colorbar;
```

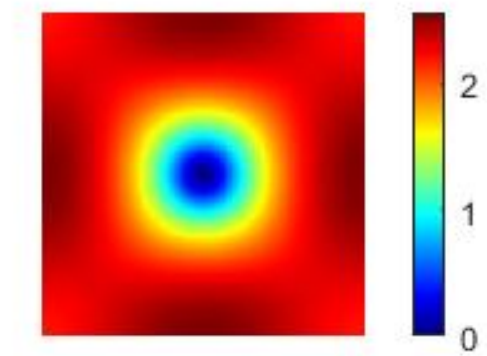
## Results:



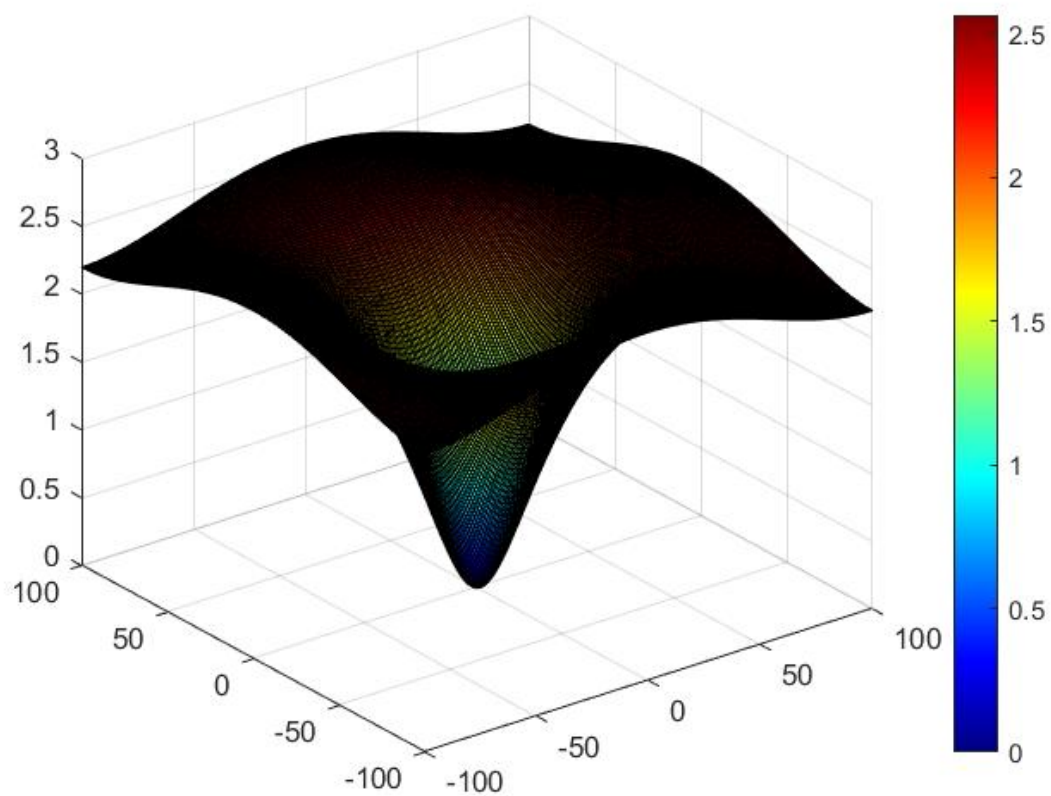
**Figure 6.1:** N,N-point DFT on  $k_1$  (imshow)



**Figure 6.2:** N,N-point DFT on  $k_1$  (surf)



**Figure 6.3:** N,N-point DFT on  $k_2$  (imshow)



**Figure 6.4:** N,N-point DFT on  $k_2$  (imshow)

## Difference in Fourier Transforms of $k_1$ and $k_2$

Contours for  $k_2$  are roughly square (parallel to  $u$  and  $v$  axes) whereas for  $k_1$  they are roughly rotated squares. This can be inferred from the structure of kernels  $k_1$  and  $k_2$ . In  $k_2$  we can visualise a square formed by connecting coordinates with value -1. We know that horizontal lines in  $x$ - $y$  space result in vertical lines in  $u$ - $v$  space on Fourier transformation and similarly vertical lines are translated to horizontal lines. Thus the Fourier transform will result in some square shaped contours parallel to  $u$  and  $v$  axes. In  $k_1$  we can visualise a rotated square formed by connecting coordinates with value 1. Using rotation transformation in DFT we know that the Fourier transform will result in some rotated square shaped contours.