FischerFaces vs EigenFaces

CS 663 Course Project

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EigenFaces Algorithm

- Exploits the fact that most natural face images have a high variance in a lower dimensional subspace.
- We use PCA to determine this lower dimensional subspace.
- Compute the PCA coefficients of every training image using this PCA.
- For a test image compute the coefficients again using the same PCA
- Classify the image to a class using the nearest neighbor algorithm

EigenFaces with varying lighting

- Eigenfaces fails on images with a variation in lighting
- Variation in lighting dominates the variation in the facial expression, pose and the identity, of a person and hence pollutes the PCA eigenvectors
- We can remove the top 3 eigenvectors to get better results.

Why top 3 eigenvectors?

- Images of a person with the same pose and expression form a rank 3 subspace under varied lighting conditions.

$$I = L\rho(x,y)\boldsymbol{l}^T\boldsymbol{N}$$

- Here I = scene radiance, L = lighting Intensity, $\boldsymbol{l}$ = lighting direction, N = unit surface normal, ρ = surface reflectivity

Why top 3 eigenvectors? (ctd)

- $I_k(x,y) = L\rho(x,y)\boldsymbol{N}(x,y).\boldsymbol{l}_k$
- $(\mathbf{I}_1\ \mathbf{I}_2 \dots\ \mathbf{I}_m) = L\rho\boldsymbol{N}(\boldsymbol{l}_1\ \boldsymbol{l}_2 \dots\ \boldsymbol{l}_m)$
- $\mathbf{I} = \mathbf{ND}$, $\mathbf{I} \in \mathbf{R}^{Mxm}$, $\mathbf{N} \in \mathbf{R}^{Mx3}$ has the unit normals at (x,y) times $L\rho(x,y)$ $\mathbf{D} \in \mathbf{R}^{3xm}$
- Using property $\text{Rank}(A_{mxk}B_{kxn}) \leq \min\{\text{Rank}(A_{mxk}), \text{Rank}(B_{kxn})\}$.

Fischer’s Linear Discriminant

- Maximise the interclass mean distance
- Minimise the intraclass scatter
- Let the between-class scatter matrix be defined as

$$S_B = \sum_{i=1}^{c} N_i(\mu - \mu_i)(\mu - \mu_i)^T$$

and the within-class scatter matrix be defined as

$$S_W = \sum_{i=1}^{c} \sum_{x_k \in X_i} (x_k - \mu_i)(x_k - \mu_i)^T$$

Fischer’s Linear Discriminant (ctd)

- If S_W is nonsingular, the optimal projection W_{opt} is chosen as the matrix with orthonormal columns which maximizes the ratio of the determinant of the between-class scatter matrix of the projected samples to the determinant of the within-class scatter matrix of the projected samples
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$$W_{opt} = \underset{W}{\operatorname{argmax}} \frac{|W^T S_B W|}{|W^T S_W W|} = [\boldsymbol{w}_1 \boldsymbol{w}_2 \ldots \boldsymbol{w}_m]$$

where $\{w_i \mid i = 1, 2, \ldots m\}$ is the set of generalized eigenvectors of S_B and S_W corresponding to the m largest generalized eigenvalues $\{\lambda_i \mid i = 1, 2, \ldots m\}$, i.e.

$$S_B \boldsymbol{w}_i = \lambda_i S_w \boldsymbol{w}_i$$

FischerFaces

- Note that S_W has rank N-c, and hence is always singular.
- We first perform PCA to reduce dimensionality to N-c and then perform a Fischer Linear Discriminant on the transformation
- Hence W_{opt} is given by

$$W_{opt}^T = W_{fld}^T W_{pca}^T$$

where

$$W_{pca} = \underset{W}{\arg\max} |W^T S_T W|$$

$$W_{fld} = \underset{W}{\arg\max} \frac{|W^T W_{pca}^T S_B W_{pca} W|}{|W^T W_{pca^T} S_W W_{pca} W|}$$

Results - Face Recognition - YaleA

Method	Error Rate (in %)		
	CMU Face	YaleA	YaleB
Eigen Faces	0.91	16.67	45.85
Eigen Faces (w/o top 3 eigvectors)	1.82	13.33	20.24
Fischer Faces	38.64	1.67	5.87

- The yale A dataset contains 165 images of 15 individuals.
- Each person has images with different facial expressions, configurations and lighting.
- Varying lighting -> Fischerfaces outperform eigenfaces

Results - Face Recognition - YaleB

Method	Error Rate (in %)		
	CMU Face	YaleA	YaleB
Eigen Faces	0.91	16.67	45.85
Eigen Faces (w/o top 3 eigvectors)	1.82	13.33	20.24
Fischer Faces	38.64	1.67	5.87

- The Yale Face Database B contains 2415 images of 38 human subjects under varying 64 illumination conditions.
- Varying lighting -> Fischerfaces outperform eigenfaces

Results - Face Recognition - CMU Face

Method	Error Rate (in %)		
	CMU Face	YaleA	YaleB
Eigen Faces	0.91	16.67	45.85
Eigen Faces (w/o top 3 eigvectors)	1.82	13.33	20.24
Fischer Faces	38.64	1.67	5.87

- The CMU database consists of images of 20 individuals in different orientations, expressions, but with the same illumination
- EigenFaces outperforms FischerFaces!

Results - Face Recognition - CMU Face (ctd)

- **Change in Face Alignment** - Linear Discriminant Analysis ignores highly changing features of a face. Changing alignment induces excessive variation in face leading the algorithm to neglect whole face.
- **Non-cropped Images** - Even when only frontal images are used results for EigenFaces are better than Fischer Faces because Fischer Face neglects features of an image which vary a lot (such as eyes, mouth, face alignment, etc) and gives a lot of weightage to background due to its non variability and thus loses important information regarding the face.

Results - Glasses Recognition - YaleA

Method	Error Rate (in %) for YaleA
Eigen Faces	40.00
Eigen Faces (w/o top 3 eigvectors)	53.33
Fischer Faces	33.33

- The yale A dataset contains images of 15 individuals, each with and without glasses. (No variation in lighting)
- Removing the top 3 eigenvectors will lead to loss of vital data, and this too will decrease the accuracy.
- Small dataset - leads to high error even for fischerfaces

Conclusions

- We thus conclude that the Fisherface method appears to be the best at extrapolating and interpolating over variation in lighting.
- Removing the largest three principal components does improve the performance of the Eigenface method in the presence of lighting variation, but harms the accuracy if lighting variations are not present.
- The Fisherface method appears to be the best at simultaneously handling variation in lighting and expression. Also, the fischerface method tries to ignore the features within the same class that have high variances.

References

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