**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

* **(B) 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

* **(A)**We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question
* a) Probability of employees greater than age of 44= P (X>44)
* P (X > 44) = 1 – P (X ≤ 44).
* Z = (X -38)/6
* Thus, the question can be answered by using the normal table to find
* P (X ≤ 44) = P (Z ≤ (44 -38)/6) = P (Z ≤ 1) = 84.1345%
* Probability that the employee will be greater than age of 44 =

1. - 84.1345=15.86%

* So, the probability of number of employees between 38 - 44 years of age Pr(X<44) - 0.5 = 84.1345 - 0.5= 34.1345%.
* Therefore, the statement that, more employees at the processing centre are older than 44 than between 38 and 44 is TRUE.
* **(B)** Probability of employees less than age of 30 = P(X<30). Z = (30 -38)/6.
* Thus, the question can be answered by using the normal table to find
* P (X ≤ 30) = P (Z ≤ (30 38)/6) = P (Z ≤ -1.333) = 9.12%.
* So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(or 36 employees).
* Therefore, the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

* As we know that if X ∼N (μ1, σ1^2), and Y ∼N (μ2, σ2^2) are two independent random
* Variables then X + Y ∼N (μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼N (μ1 − μ2, σ1^2 + σ2^2).
* Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y, then Z ∼N (aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).
* Therefore, in the question
* 2X1~ N (2u,4 σ^2) and
* X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)
* 2X1-(X1+X2) = N (4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

* **(D) 48.5, 151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

* **(A)** 95% of the **probability** **lies**between 1.96 **standard deviations**of the **mean**.
* (12 – 1.96\*5, 12 + 1.96\*5)
* ($2.2M, $22.8M)
* Thus, Range = (Rs.99M, Rs.1026M)
* **(B)** Fifth percentile is calculated as:
* From p values of z score table, we get:

= -1.644

P = 12 – 8.22 = 3.78

* Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.
* Or 5th percentile of profit is Rs. 170.1M.
* **(C)** First **division**of**company**, thus have **larger probability**of making a loss in a given year.