Week 1

S1 - Circuit Elements

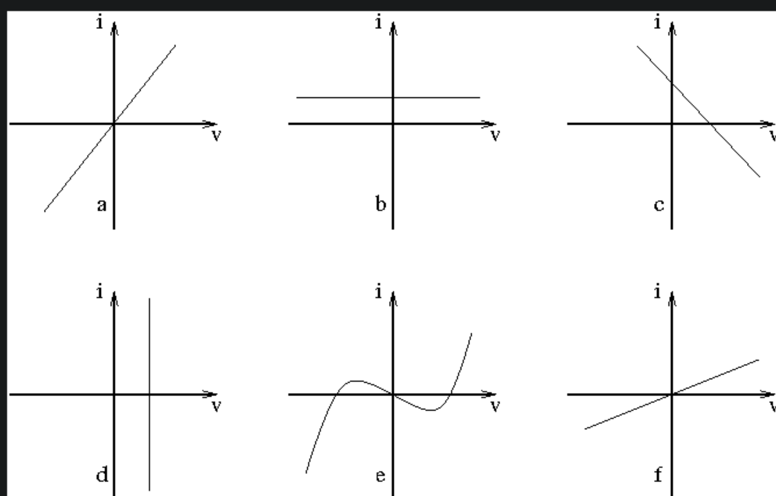
S2 - Circuit Analysis Toolchest

Lectures

S1E1: Various V-I characteristics

0 points possible (ungraded)

The figure below shows a variety of possible V-I characteristics for a two terminal device.



For each of the following circuit elements enter the label of an appropriate V-I characteristic in the slot provided. Note, there may be more than one correct answer to each question, but please only provide one answer.

A linear resistor:

a ✔ **Answer:** either a or f

An independent voltage source:

d ✔ **Answer:** d

An independent current source:

b ✔ **Answer:** b

Submit

Show answer

S1E1.5: Simple Power

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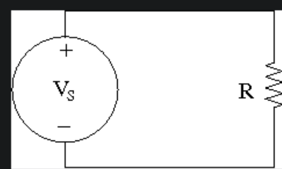
S1E1.5: Simple Power

0 points possible (ungraded)

In the circuit shown below,



the strength of the source is $V_S = 10$ V, and the resistance of the resistor is $R = 50\Omega$.

What is the power dissipated in the resistor (in Watts)?

2 ✔

What is the power entering the source (in Watts)?

-2 ✔

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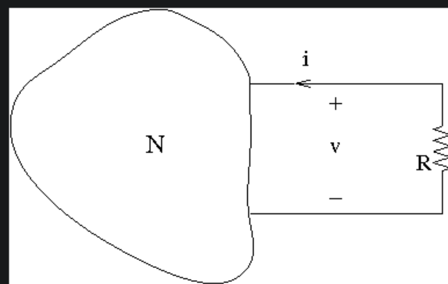
In[∘]:=
```
V = 10; R = 50;
P = V^2 / R
```

Out[∘]= 2

S1E2: Power

0 points possible (ungraded)

The picture shows a resistor connected to some unknown network N. The resistor is immersed in an isolated water bath, and its temperature is observed and recorded. The resistor has resistance $R = 8.0\Omega$.



By observing the rate of increase of the temperature in the water bath, it is determined that the power dissipated in the resistor is 11.0W.

Assuming that the voltage across the resistor is constant, what is the voltage v (in Volts) across the resistor?

9.38083 ✔ **Answer:** 9.38 V

What is the current i (in Amperes) entering the network N from the resistor?

-1.1726 ✔ **Answer:** -1.17 A

Submit | Show answer

In[ ]:=
```
R = 8; P = 11;
Solve[P == V^2 / R, V] // N
Solve[P == i^2 R, i] // N
```

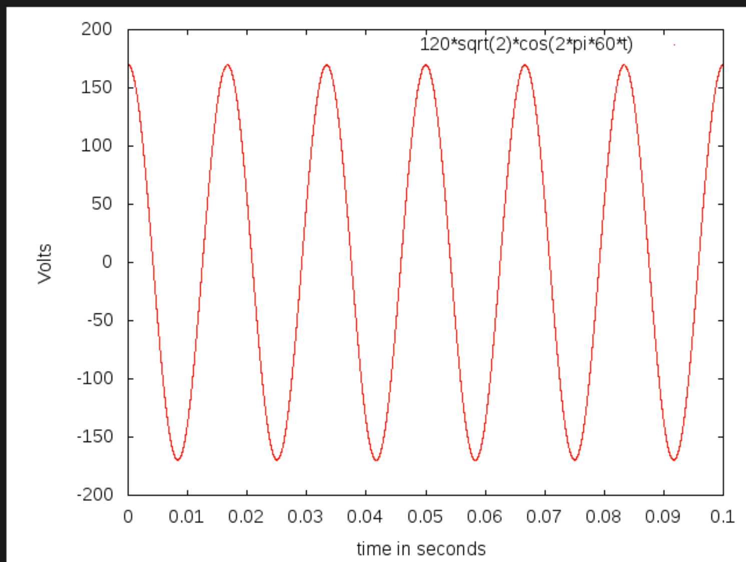
Out[ ]=
```
{{V → -9.38083}, {V → 9.38083}}
```

Out[ ]=
```
{{i → -1.1726}, {i → 1.1726}}
```

S1E3: AC power

0 points possible (ungraded)

The plot shows 1/10 second of the voltage waveform of a 120V 60Hz AC (Alternating Current) power circuit, like that delivered to residences in the United States.



The actual voltage is $120 \cdot \sqrt{2} \cdot \cos\left(2\pi \cdot 60 \cdot t\right)$ Volts. If we apply this voltage across a resistor of resistance 110.0Ω the resistor will dissipate a time-varying power. What is the peak power (in Watts) dissipated by the resistor?

261.818 ✔ **Answer:** 261.81818181818187

What is the average power (in Watts) dissipated by the resistor? (Hint: you compute the average power by integrating the instantaneous power over one cycle of the waveform and dividing the result by the length of the cycle.)

130.909 ✔ **Answer:** 130.9090909090909

What would be the power (in Watts) dissipated by the resistor if the voltage was a constant value of 120V?

130.909 ✔ **Answer:** 130.9090909090909

If a time-varying, AC voltage dissipates the same power in a resistor as a constant voltage would dissipate, we say that the time-varying voltage has a root-mean-square (RMS) value that is equal to the constant voltage.

Submit

Show answer

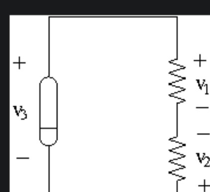
```
In[]:= ω = 2 π / T;
V[t_] := 120 √2 Cos[ω t]; R = 110;
Vpeak = V[0];
Ppeak = Vpeak^2 / R // N

Pavg = Integrate[V[t]^2 / R, {t, 0, T}] / T // N

Vrms = Vpeak / √2;
Pavg = Vrms^2 / R // N

Vconst = 120;
P = Vconst^2 / R // N
```

```
Out[]= 261.818

Out[]= 130.909

Out[]= 130.909

Out[]= 130.909
```

S1E5: KVL-0

0 points possible (ungraded)

Joe was debugging part of an experimental apparatus, probing around with his voltmeter. Part of the apparatus had two obvious resistors in series with an unknown element, as shown in the diagram below:



The unknown element is hard to reach, so Joe put the negative (black) probe of his voltmeter at the interconnection of the two obvious resistors and then put the positive (red) probe at the other end of each resistor, measuring $v_1 = 1.4$V and $v_2 = 0.8999999999999999$V.

What is the voltage (in Volts) v_3 measured across the unknown element?

0.5 ✔ **Answer:** 0.5

Submit

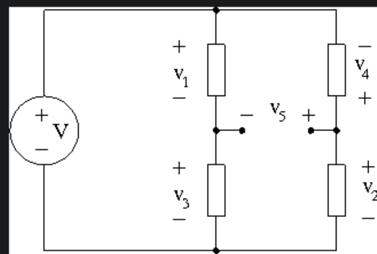
Show answer

ℹ Answers are displayed within the problem

S1E6: KVL

0 points possible (ungraded)

In the circuit shown there are four unknown elements and an independent voltage source.



The strength of the source V is given. Also, there are two known branch voltages: we know v_1 and v_2.

In terms of the known voltages, write an algebraic expression for the branch voltage v_3. Be careful, algebraic expressions are case sensitive, and remember to use the format "vXYZ" to specify v_{XYZ} when you have subscripts in the answer boxes.

V-v1

✔ **Answer:** V - v1

$V - v_1$

Write an algebraic expression for the branch voltage v_4:

v2-V

✔ **Answer:** v2 - V

$v_2 - V$

Write an algebraic expression for the branch voltage v_5:

v1+v2-V

✔ **Answer:** v1 + v2 - V

$v_1 + v_2 - V$

Submit

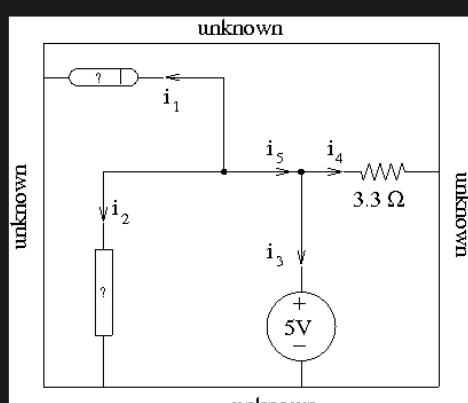
Show answer

Answers are displayed within the problem

S1E7: KCL-0

0 points possible (ungraded)

While further poking around in his experiment, Joe found an exposed interconnect. It connected the positive output of the 5 V power supply, a 3.3Ω resistor, and two unknown 2-terminal elements, as shown:



He couldn't trace where the other ends of the elements went. However, he had a nifty (very expensive!) Hall-effect clamp-on ammeter that he used to measure the three of the four currents entering the elements. He didn't have enough space to measure the current entering the fourth element. He found that $i_1 = -0.7$A, $i_3 = 3.0$A, $i_4 = 1.3$A.

What was the current (in Amperes) i_2 into the fourth element?

-3.6 ✔ **Answer:** -3.5999999999999996

Joe managed to get his ammeter probe around a wire and measured i_5. What current (in Amperes) did he measure?

4.3 ✔ **Answer:** 4.3

Submit

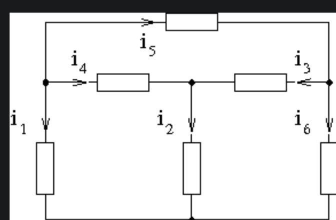
Show answer

Answers are displayed within the problem

S1E8: KCL

0 points possible (ungraded)

In the circuit shown there are six unknown elements.



By measurements we have values for three of the branch currents: i_1, i_2, and i_3.

In terms of the known currents, write an algebraic expression for the current i_4?

i2-i3 ✔ **Answer:** i2 - i3

$i_2 - i_3$

Write an algebraic expression for the current i_5?

i3-i1-i2 ✔ **Answer:** i3 - (i1 + i2)

$i_3 - i_1 - i_2$

Write an algebraic expression for the current i_6?

-i1-i2 ✔ **Answer:** -(i1 + i2)

$-i_1 - i_2$

Submit

Show answer

In[∘]:=
```
v1 = 1.4; v2 = 0.8999999999999999;
Solve[v1 - v2 - v3 == 0, v3]
```

Out[∘]= {{v3 → 0.5}}

```
In[]:= Solve[v1 + v3 - V == 0, v3]
       Solve[-v4 + v2 - V == 0, v4]
       Solve[v4 + v1 - v5 == 0, v5] /. {v4 → -V + v2}
Out[]= {{v3 → V - v1}}
Out[]= {{v4 → -V + v2}}
Out[]= {{v5 → -V + v1 + v2}}
```

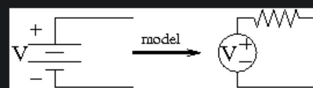
```
In[]:= i1 = -0.7; i3 = 3; i4 = 1.3;
       R = 3.3; V = 5;
       Solve[i1 + i2 + i5 == 0, i2] /. {i5 → i4 + i3}
       i5 = i4 + i3
Out[]= {{i2 → -3.6}}
Out[]= 4.3
```

Skip S1E8: KCL

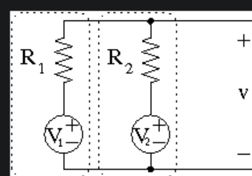
S1E9: Battery Model

0 points possible (ungraded)

A real battery is not an ideal independent voltage source. A voltage source is an appropriate idealization of the battery if the load on the battery is negligible. A better model for a battery is a voltage source in series with an ideal linear resistor whose resistance varies with temperature. Sometimes even better models are required - this Energizer technical bulletin gives more information. However, let's use the simple model of a linear resistor in series with an ideal independent voltage source, as in the figure.



It is suggested in section 1.5.1 of the textbook that to increase the current-capacity of a battery without increasing the voltage at the terminals we can connect batteries of the same voltage in parallel. Let's examine this using our model.



Let's assume that both component batteries have the same voltage $V_1 = V_2 = 1.5$. The internal resistances of small batteries are about 0.2Ω, but they vary a bit. Let's assume that $R_1 = 0.25\Omega$ and $R_2 = 0.32\Omega$. What is the open-circuit voltage (in Volts) V of the combination?

1.5 ✔ **Answer:** 1.5

Now, suppose we short-circuit the compound battery. (This is very dangerous. NEVER do this to a large battery, such as a lead-acid battery in a car, or to a lithium-ion battery from your laptop. You MAY live to regret it, but you may not.) What is the current (in Amperes) you should expect to go through the short circuit?

10.7 ✔ **Answer:** 10.6875

We can think of this combination as a bigger battery of the same voltage as the two component batteries. What is the equivalent resistance (in Ohms) of the compound battery? (Hint: you have the voltage with nothing connected and the current when shorted out.) (note: Don't assume the short-circuit situation in question 2 above.)

0.14 ✔ **Answer:** 0.14035087719298245

Now, suppose that the voltages of the two component batteries are not quite the same. For example, suppose that $V_2 = 1.6$. Then when we hook the two batteries together current will flow and the higher voltage battery will charge the lower voltage one. What is the current (in Amperes) that will flow?

0.18 ✔ **Answer:** 0.17543859649122806

Submit

Show answer

For those who are stuck!

discussion posted 2 years ago by **apsinghbeast**

Let's assume that both component batteries have the same voltage . The internal resistances of small batteries are about , but they vary a bit. Let's assume that and . What is the open-circuit voltage (in Volts) of the combination?

The voltages V1 and V2 are defined as 1.5; the only thing that might affect it would be a voltage drop across the resistors R1 and R2. So let's go to Ohm's Law: V= IR

Vdrop = I * R

Since the open circuit has zero current flowing through it, I = 0

Vdrop = 0 * R = 0 V

Thus the open-circuit voltage must be 1.5 V.

Now, suppose we short-circuit the compound battery. (This is very dangerous. NEVER do this to a large battery, such as a lead-acid battery in a car, or to a lithium-ion battery from your laptop. You MAY live to regret it, but you may not.) What is the current (in Amperes) you should expect to go through the short circuit?

We're looking for the current going through the short circuit. We'll use KCL to discover the currents entering the node above R2. (Ohm's Law in terms of current is: I = V/R)

i1 = V1 / R1

i2 = V2 / R2

i3 = ?

(+i1) + (+i2) + (-i3) = 0

(-i3) = (-i1) + (-i2)

i3 = i1 + i2

i3 = (V1 / R1) + (V2 / R2)

i3 = (1.5 / 0.25) + (1.5 / 0.32) = 6 + 4.7 = 10.7 A

We can think of this combination as a bigger battery of the same voltage as the two component batteries. What is the equivalent resistance (in Ohms) of the compound battery? (Hint: you have the voltage with nothing connected and the current when shorted out.)

Let's use Ohm's Law in terms of resistance to figure it out: R = V / I

R = 1.5 / 10.7 = 0.14 Ohms

Now, suppose that the voltages of the two component batteries are not quite the same. For example, suppose that . Then when we hook the two batteries together current will flow and the higher voltage battery will charge the lower voltage one. What is the current (in Amperes) that will flow?

We choose the node above R2 and formulate a KCL equation:

The current (i2) on the branch that includes V2 and R2 will be pointing at our chosen node. The current (i1) on the branch that includes V1 and R1 will be pointing away from our chosen node. So by our convention, the equation is:

(+i2) + (-i1) = 0 i2 = i1

This shows there will be one current. That current will be given by Ohm's Law: I = V / R

So the question is .. what is V and what is R?

V will be the difference between the two votages: 1.6 - 1.5 = 0.1 Volts R will be the sum of the resistors: 0.25 + 0.32 = 0.57 Ohm

I = 0.1 / 0.57 = 0.18 A

YOU CAN THANK ME IF IT WAS HELPFUL!

In[ ]:=
```
v1 = v2 = 1.5;
r1 = .25; r2 = .32;
i1 = v1 / r1; i2 = v2 / r2;
v1
Solve[i1 + i2 - i3 == 0 && i3 - i1 - i2 == 0, i3]
```

Out[ ]= 1.5

Out[ ]= {{i3 → 10.6875}}

In[ ]:=
```
Rp[r_List] := 1 / Total[1 / r];
Rp[{r1, r2}]
R = v1 / i3 /. {i3 → 10.6875`}
```

Out[ ]= 0.140351

Out[ ]= 0.140351

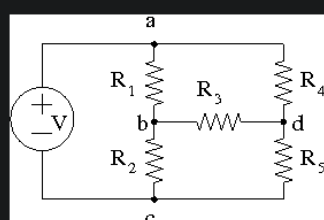
In[ ]:=
```
v = 1.6 - 1.5;
i = v / (r1 + r2)
```

Out[ ]= 0.175439

S2E1: Circuit Topology

0 points possible (ungraded)

In the bridge network shown below:



How many nodes are there?

4 ✔ **Answer:** 4

How many KCL equations are independent?

3 ✔ **Answer:** 3

How many loops are there?

7 ✔ **Answer:** 7

How many KVL equations are independent?

3 ✔ **Answer:** 3

Notice that in any circuit there is always one more node than there are independent KCL equations. If two nodes share a branch, the current entering one node from that branch is the negative of the current entering the other node from that branch, the current entering one node from that branch is the negative of the current entering the other node from that branch. So the sum of the currents entering two nodes does not count current going from one to the other. As a consequence, the sum of all the currents entering all but one of the nodes is the same as the current entering the remaining node. So the KCL equation for that node is the sum of the KCL equations for all the other nodes.

Also, in a circuit with more than one loop there are always more loops than KVL equations. The argument is analogous with the argument for nodes: if two loops share a branch, and we count the voltages counterclockwise in each loop, then the KVL equation for the branches containing both subloops is the sum of the KVL equations for each subloop. This bigger equation does not count the current in the shared branch, since it is of opposite sign in the two subloops.

A thought: if you have take a class in multivariate calculus, these arguments should remind you of the proofs of the divergence theorem and of Stokes's theorem.

Detailed solution below provided by Community TA Irrational_Kongt:

All loops moving clockwise (listed in order of increasing size):

a d b a,

b d c b,

a b c a,

a d c b a,

a d b c a,

a b d c a,

a d c a.

Here are all KCL and KVL equations for these nodes and loops. Let I denote the current flowing up into the - terminal of V, i_1, i_2, i_4 and i_5 denote currents flowing down through R_1, R_2, R_4 and R_5 respectively, and i_3 denote the current flowing to the right through R_3. Voltage references polarities go from positive to negative with the current, except for V.

Applying KCL, taking current to be positive when leaving a node (negative when entering), we then get 4 equations as below:

a: $-I + i_1 + i_4 = 0$

b: $i_2 + i_3 - i_1 = 0$

c: $I - i_2 - i_5 = 0$

d: $-i_3 - i_4 + i_5 = 0$

Note that by summing equations a, b and c you get equation d. Alternatively, by subtracting equations a and b from d you get c. A system of equations is *dependent* if any one of them may be described as a linear sum (sum with multipliers) of the others. In a dependent system, at least one equation may be removed without losing information the other equations already provide.

Applying KVL, moving clockwise around a loop and using the first voltage reference sign encountered:

a d b a: $v_4 - v_3 - v_1 = 0$

b d c b: $v_3 + v_5 - v_2 = 0$

a b c a: $v_1 + v_2 - V = 0$

a d c b a: $v_4 + v_5 - v_2 - v_1 = 0$

a d b c a: $v_4 - v_3 + v_2 - V = 0$

a b d c a: $v_1 + v_3 + v_5 - V = 0$

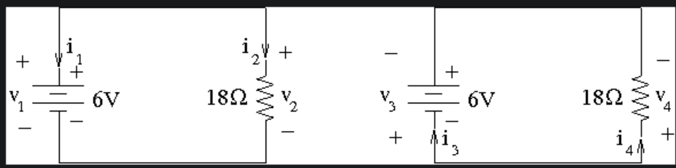
a d c a: $v_4 + v_5 - V = 0$

Let (a b c a) = L1, (a d b a) = L2, and (b d c b) = L3. Check that equation (a d c b a) is just the sum of equations L2 and L3, equation (a d b c a) = L2 + L1, equation (a b d c a) = L3 + L1, and equation (a d c a) = L2 + L3 + L1.

S2E2: Associated Reference Directions

0 points possible (ungraded)

The figure below shows two identical circuits connecting a 6V battery to an 18Ω resistor. The difference is that we chose to measure the voltages and currents in the two circuits differently: we used a different coordinate system of voltages and currents in our measurements.



You are to determine the voltages and currents indicated and compute the powers entering the elements.

What is the voltage (in Volts) v_1 measured across the battery?

6 ✔ **Answer:** 6.0

What is the voltage (in Volts) v_2 measured across the resistor?

6 ✔ **Answer:** 6.0

What is the current (in Amperes) i_1 measured entering the battery?

-.33 ✔ **Answer:** -0.3333333333

What is the current (in Amperes) i_2 measured entering the resistor?

0.33 ✔ **Answer:** +0.3333333333

What is the power (in Watts) $P_1 = v_1 \cdot i_1$ entering the voltage source?

-2 ✔ **Answer:** -2.0

What is the power (in Watts) $P_2 = v_2 \cdot i_2$ entering the resistor?

2 ✔ **Answer:** +2.0

What is the voltage (in Volts) v_3 measured across the battery?

-6 ✔ **Answer:** -6.0

What is the voltage (in Volts) v_4 measured across the resistor?

-6 ✔ **Answer:** -6.0

What is the current (in Amperes) i_3 measured entering the battery?

.33 ✔ **Answer:** 0.3333333333

What is the current (in Amperes) i_4 measured entering the resistor?

-.33 ✔ **Answer:** -0.3333333333

What is the power (in Watts) $P_3 = v_3 \cdot i_3$ entering the voltage source?

-2 ✔ **Answer:** -2.0

What is the power (in Watts) $P_4 = v_4 \cdot i_4$ entering the resistor?

2 ✔ **Answer:** +2.0

Notice that the powers are the same in the two circuits. That is physical reality: power moves from the battery to the resistor, independent of the coordinate systems we use to measure the voltage and the current, so long as we use associated reference directions. We must always measure current into the terminal that we put the + sign of the voltage measurement on. Think of this as the red probe of the voltmeter.

Explanation
The following explanation was contributed as a forum post by Grove in a previous run of the course. Many students found this to be extremely helpful.
Circuit analysis needs to be done in an organised and ordered way. That is not to say that there is only one method of doing the analysis but the course team has chosen to use *Associated Reference Directions* as described very well both in the lectures and in the course textbook.
However at the start there can be some confusion particularly when a profusion of positive and negative signs suddenly appear as in example S2E2.

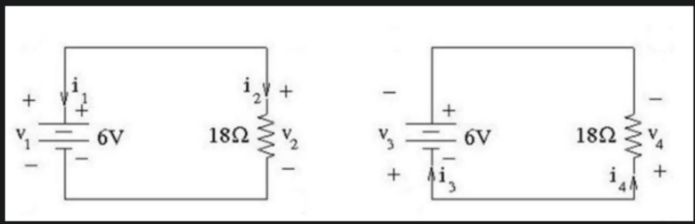


Figure 1

So it is best if one first identifies those labels which are not arbitrarily chosen as shown in red in the circuit diagram below?

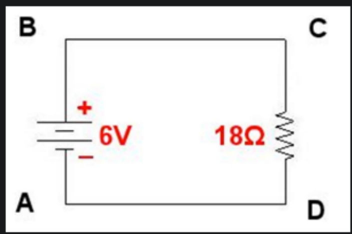
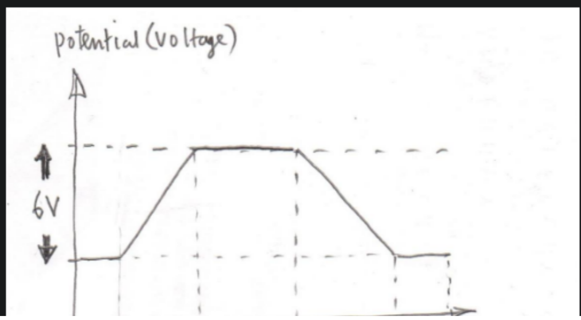


Figure 2

Notice there is only one diagram as both circuits shown in the example are identical.
There is a symbol for a resistor which has a resistance of 18Ω. The resistor is in series with a battery with a potential difference (voltage) across its terminals of $6V$. Then addition vital information is duplicated:

- By convention the top long line of the symbol indicates that that terminal is at a higher potential by 6 volts than that of the bottom short line of the symbol.
- The red plus sign indicates that that terminal is at a higher potential by 6 volts than that of the bottom terminal labelled with a minus sign.

None of the red labelling can be changed, otherwise you will be considering a different circuit.
The schematic potential (voltage) diagram for this complete circuit is as follows.



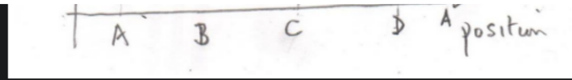


Figure 3

Note that there is an increase in potential of 6 volts in going from terminal A to terminal B and then a decrease in potential of 6 volts in going from terminal C to terminal D.
The net change of potential in going round the complete circuit is zero which is KVL.
Now the (black) plus and (black) minus signs with the appropriate voltage labels v_1, v_2, v_3, v_4 can be added arbitrarily with the labels indicating the voltage of the terminal labelled (black) plus **relative to** the terminal labelled (black) minus.

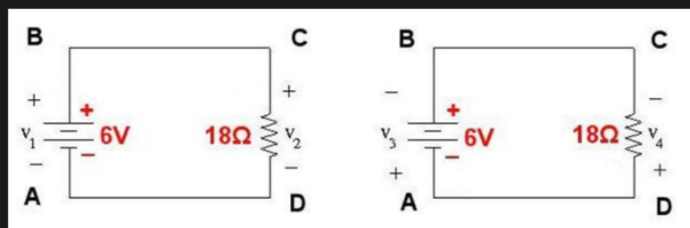


Figure 4

Finally *associated reference directions* requires the currents to be shown as entering in at a (black) plus label or out at a (black) minus sign.

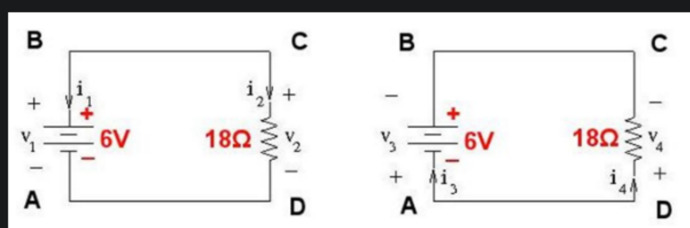


Figure 5

Remembering that the arbitrary voltage labels are the voltages of the arbitrarily chosen (black) plus terminals relative to the corresponding (black) minus terminals these voltage values are found to be as follows.

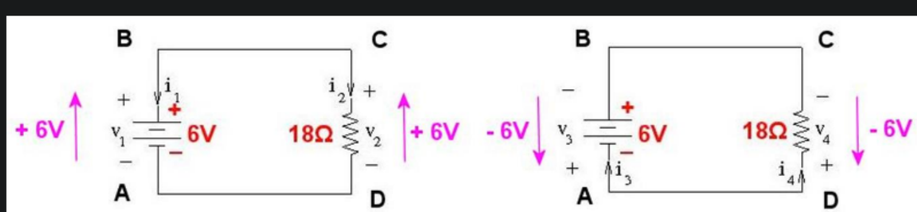


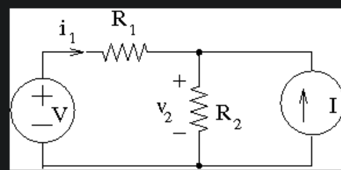
Figure 6

So hopefully where the -6 comes from is no longer a mystery.

S2E3: Using KVL, KCL, and VI constraints

0 points possible (ungraded)

In the network shown you are given that $V = 2.0$ V, $I = 3.0$ A, $R_1 = 4.0\Omega$, and $R_2 = 5.0\Omega$.



What is the voltage (in Volts) v_2 across the resistor with resistance R_2?

7.78 ✔ **Answer:** 7.777777777777779

What is the power (in Watts) dissipated by the resistor with resistance R_2?

12.09 ✔ **Answer:** 12.098765432098768

What is the current (in Amperes) i_1 through the resistor with resistance R_1?

-1.44 ✔ **Answer:** -1.4444444444444446

What is the power (in Watts) dissipated by the resistor with resistance R_1?

8.34 ✔ **Answer:** 8.34567901234568

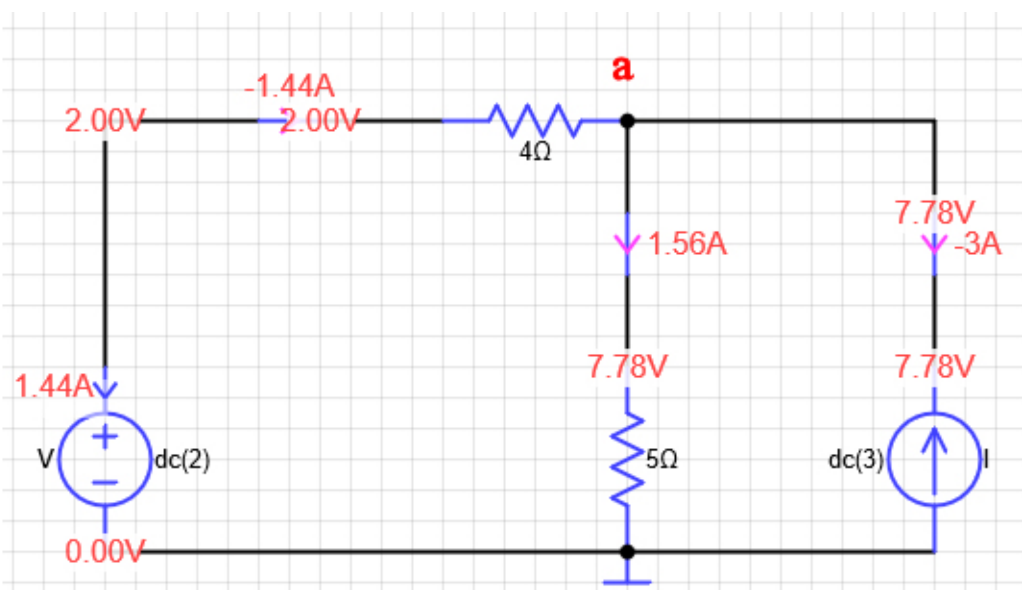
What is the power (in Watts) supplied by the voltage source?

-2.88 ✔ **Answer:** -2.8888888888888893

What is the power (in Watts) supplied by the current source?

23.33 ✔ **Answer:** 23.333333333333336

You should observe that the sum of the power supplied by the sources is the sum of the power dissipated by the resistors. If this is not true you have done something wrong.



In[ ]:=
```
V = 2; i = 3; r1 = 4; r2 = 5;
(*Node analysis using KCL with current going into the node marked a above*)
Solve[-v2/r2 + (V - v2)/r1 + i == 0, v2] // N
P2 = v2^2 / r2 /. {v2 → 7.777777777777778`}
i1 = (V - v2)/r1 /. {v2 → 7.777777777777778`}
i2 = v2/r2 /. {v2 → 7.777777777777778`};
P1 = i1^2 r1
P2 = i2^2 r2;
(*When calculating power for source elements, don't consider resistance,
simply use P=VI. For voltage source consider current in/out of the
  source. For current source consider voltage across the source*)
Pv = V * i1 // N
PI = v2 * i /. {v2 → 7.777777777777778`}
P1 + P2 == Pv + PI
```

Out[ ]= {{v2 → 7.77778}}

Out[ ]= 12.0988

Out[ ]= -1.44444

Out[ ]= 8.34568

Out[ ]= -2.88889

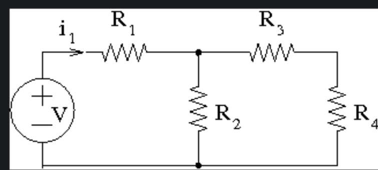
Out[ ]= 23.3333

Out[ ]= True

S2E4: Series and Parallel

0 points possible (ungraded)

In the network shown you are given that $V = 2.0$V and that the resistors have the resistances $R_1 = 4.0\Omega$, $R_2 = 4.0\Omega$, $R_3 = 2.0\Omega$, and $R_4 = 2.0\Omega$.



What is the equivalent resistance (in Ohms) of the series combination of R_3 and R_4?

4 ✔ **Answer:** 4.0

What is the equivalent resistance (in Ohms) of the parallel combination of R_2 and the series combination of R_3 and R_4?

2 ✔ **Answer:** 2.0

What is the equivalent resistance (in Ohms) of the series combination of R_1 and the parallel combination of R_2 and the series combination of R_3 and R_4?

6 ✔ **Answer:** 6.0

What is the current (in Amperes) i_1?

0.33 ✔ **Answer:** 0.3333333333333333

What is the voltage (in Volts) across the resistor R_2?

0.66 ✔ **Answer:** 0.6666666666666666

What is the voltage (in Volts) across the resistor R_4?

0.33 ✔ **Answer:** 0.3333333333333333

This is an important circuit structure called a ladder. Many filters are built with ladders of capacitors and inductors. We will see this kind of circuit later.

In[∘]:=
```
Rs[r_List] := Total[r];
Rp[r_List] := 1/Total[1/r];
V = 2; r1 = 4; r2 = 4; r3 = 2; r4 = 2;
Rs[{r3, r4}]
Rp[{r2, Rs[{r3, r4}]}]
Rs[{r1, Rp[{r2, Rs[{r3, r4}]}]}]
Solve[(V - e)/r1 - e/r2 - e/(r3 + r4) == 0, e]
i1 = (V - e)/r1 /. {e -> 2/3} // N
(e * r3)/(r3 + r4) /. {e -> 2/3} // N
```

Out[∘]= 4

Out[∘]= 2

Out[∘]= 6

Out[∘]= $\left\{\left\{e \to \frac{2}{3}\right\}\right\}$

Out[∘]= 0.333333

Out[∘]= 0.333333

Node analysis practice, part 1

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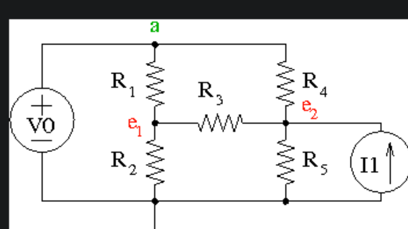
Node analysis practice, part 1

0 points possible (ungraded)



In the circuit shown, what is the node potential of the node labeled "a" relative to the ground indicated? Write the expression in the box provided. Note, this is case sensitive.

V0 ✔ **Answer:** V0

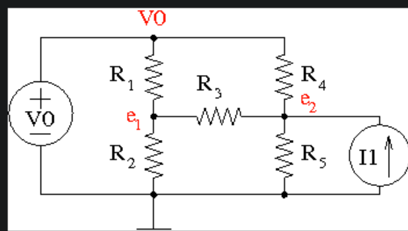
V_0

Submit

Show answer

Node analysis practice, part 2

0 points possible (ungraded)



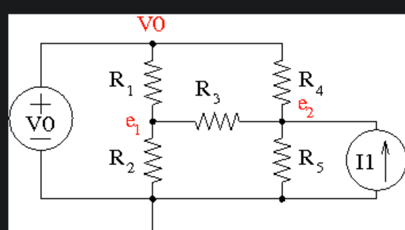
In the circuit shown, what is the current going up through the resistor with resistance R_1 in terms of the node potentials? Write the expression in the box:

(e1-V0)/R1

$$\frac{e_1 - V_0}{R_1}$$

Node analysis practice, part 3

0 points possible (ungraded)



Write the expression that represents the sum of the currents leaving the node with potential e_2 in the circuit given. Express your answer in terms of the resistances, not the conductances.

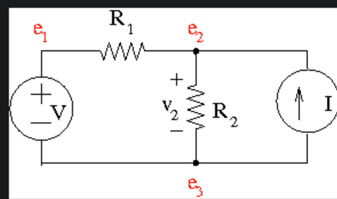
(e2-e1)/R3+(e2-V0)/R4+e2/R5-I1 **Answer:** (e2-e1)/R3 + (e2-V0)/R4 + e2/R5 - I1

$$\frac{e_2 - e_1}{R_3} + \frac{e_2 - V_0}{R_4} + \frac{e_2}{R_5} - I_1$$

S2E5: Node Method

0 points possible (ungraded)

In the network shown you are given that $V = 5.0$V and $I = 3.0$, and that the resistors have the resistances $R_1 = 3.0\Omega$ and $R_2 = 5.0\Omega$.



There are three node potentials labeled in this network e_1, e_2, and e_3. We can choose any node to be the reference node (the ground) from which voltages are measured, setting the corresponding node potential to zero. (Only voltages, which are differences of node potentials, are physically meaningful.) Let's start by choosing the ground at the negative terminal of the voltage source, so $e_3 = 0$.

What is the value (in Volts) of the node potential e_1?

5 ✔ **Answer:** 5.0

Now there is one remaining unknown node, with potential e_2. Write a node equation for that node. What is the value (in Volts) of e_2?

8.75 ✔ **Answer:** 8.75

What is the voltage (in Volts) v_2 across the resistor with resistance R_2?

8.75 ✔ **Answer:** 8.75

Now, let's start again. Suppose we chose a different node as the ground reference. Let's choose the top terminal of the current source, where it connects to the two resistors. So now $e_2 = 0$, and there are two remaining nodes. But if we know e_3 then we know that $e_1 = e_3 + V$, so we still have only one unknown. What is the value (in Volts) of the node potential e_3?

-8.75 ✔ **Answer:** -8.75

Now, what is the voltage (in volts) v_2 across the resistor with resistance R_2?

8.75 ✔ **Answer:** 8.75

Notice that the voltage across the resistor is the same, independent of the choice of the node chosen for the reference potential. Indeed, the physics does not care about the potentials, only the voltages that are their differences.

In[∘]:=
```
V = 5; i = 3; r1 = 3; r2 = 5;
V
(*Current entering e2*)
Solve[(V - e2)/r1 - e2/r2 + i == 0, e2] // N
(*Current leaving e3*)
Solve[(e3 + V)/r1 + e3/r2 + i == 0, e3] // N
v2 = -e3 /. {e3 → -8.75`}
```

Out[∘]= 5

Out[∘]= {{e2 → 8.75}}

Out[∘]= {{e3 → -8.75}}

Out[∘]= 8.75

S2E6: Modeling

0 points possible (ungraded)

Joe has a barn that is 113.0 feet from his house. He needs to supply 1000 Watts at 240V to a resistive load at his barn from the 60Hz power line at his house. Note that the circuit from the house to the barn requires two lengths of the interconnecting wire. He proposes to use number 12 AWG wire to connect his house to his barn. (AWG is American Wire Gauge: a specification of the size of the wire. For more information see.) Number 12 AWG copper wire has a resistance of 1.588Ω per 1000 feet.

What is the total resistance (in Ohms) of the transmission line?

0.358 ✔ **Answer:** 0.35888800000000004

What is the resistance (in Ohms) of Joe's load at his barn?

57.59 ✔ **Answer:** 57.599999999999994

What is the voltage drop (in Volts) from the house to the load at the barn and back to the house due to the resistance in Joe's transmission line, assuming that Joe is able to get 240V across the load at the barn?

1.49 ✔ **Answer:** 1.495366666666667

The total resistance can be computed based on the length of wire and the resistance per length:

$R_{line} = (2 \cdot 113 ft) \cdot \frac{1.588\Omega}{1000 ft} = 0.35888800000000004\Omega$

The resistance of Joe's load can be computed based on the power dissipated and the voltage across the load:

$P_{load} = \frac{V_{load}^2}{R_{load}}$

$R_{load} = \frac{V_{load}^2}{P_{load}} = \frac{(240V)^2}{1000W} = 57.599999999999994\Omega$

By Ohm's Law,

$I_{load} = \frac{V_{load}}{R_{load}} = \frac{240V}{57.599999999999994\Omega} = 4.166666666666667A$

Since the currents through the house, transmission line, and load are equal,

$V_{line} = I_{line} \cdot R_{line} = I_{load} \cdot R_{line} = (4.166666666666667A) \cdot (0.35888800000000004\Omega) = 1.495366666666667V$

Homework

MITx 6.002.1x
Circuits and Electronics 1: Basic Circuit Analysis

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Course / Week 1 / Homework 1

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Homework 1

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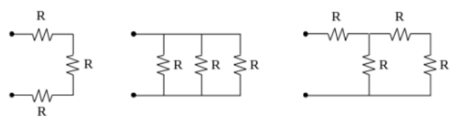
Homework due Jul 19, 2022 18:12 +04 Completed

H1P1: Resistor Combinations

6.0/6.0 points (graded)

This problem investigates how resistors combine. Consider the three resistor networks shown below:



Network A Network B Network C

What is the equivalent resistance as an algebraic expression (in terms of R) of network A as viewed from its port?

3*R ✔ **Answer:** 3*R

$3 \cdot R$

What is the equivalent resistance as an algebraic expression (in terms of R) of network B as viewed from its port?

R/3 ✔ **Answer:** R/3

$\frac{R}{3}$

What is the equivalent resistance as an algebraic expression (in terms of R) of network as C viewed from its port?

5*R/3 ✔ **Answer:** 5*R/3

$5 \cdot \frac{R}{3}$

You are given three resistors: two 4Ω resistors and one 6Ω resistor.
What is the value in Ohms (Ω) of the largest-valued resistor that can be fabricated by combining these three resistors?

14 ✔ **Answer:** 14

What is the value in Ohms (Ω) of the smallest-valued resistor that can be fabricated by combining these three resistors?

1.5 ✔ **Answer:** 1.5

Given that each individual resistor can dissipate up to 1 watt of power before burning up, how much total power in watts (W) can the smallest-valued composite resistor dissipate before burning up?

2.6667 ✔ **Answer:** 2.67

Explanation:
For Network A, the three resistors are in series and the equivalent resistance is therefore $R + R + R = 3R$.
For Network B, the three resistors are in parallel and the equivalent resistance is therefore $R \parallel R \parallel R = \frac{R}{3}$. (Recall that $R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$.)
For Network C, the equivalent resistance can be modeled as $R + R \parallel (R + R) = \frac{5R}{3}$
The maximum resistance that can be made from these resistors is a composite resistor containing all three given resistors in series. Using $R_1 = R_2 = 4\Omega$, $R_3 = 6\Omega$, we get $4 + 4 + 6 = 14\Omega$
The minimum resistance that can be made from these resistors is a composite resistor containing all three given resistors in parallel. $4 \parallel 4 \parallel 6 = 1.5\Omega$.
The total power P in Watts that can be consumed by the smallest-valued composite resistor is found by calculating the voltage at which the first of the three resistors burns up. As the composite resistor consists of three resistors in parallel, all three resistors share the same potential V. It follows that the voltage at which the smallest resistor burns up is the maximum voltage one can apply to the resistor network. Given that the smallest resistor is $R_{min} = 4\Omega$, we can find the maximum

voltage one can apply to the resistor network. Given that the smallest resistor is $R_{min} = 4\Omega$, we can find the maximum voltage the network can sustain before burning up:

Using $P = \frac{V^2}{R_{min}} = 1\,W$, the maximum voltage is $V = 2$ V.

At a potential of $2V$ both 4Ω resistors dissipate $1W$. The 6Ω resistor dissipates:

$$\frac{(2^2)}{6} = \frac{2}{3}W$$

The total power consumed is therefore $1W + 1W + \frac{2}{3}W = 2.6667W$

Submit You have used 4 of 25 attempts

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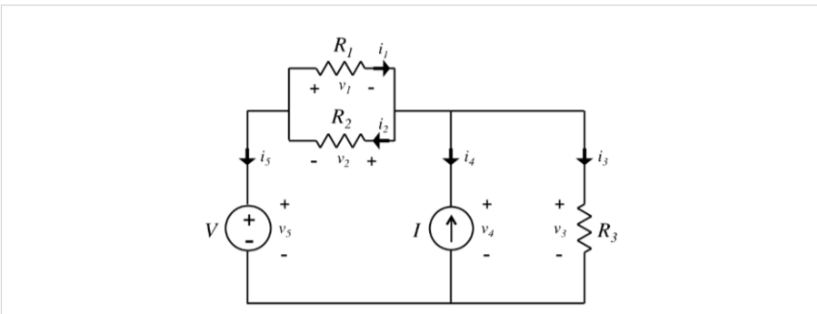
Answers are displayed within the problem

H1P2: KCL-KVL vs. Node Method

12.0/12.0 points (graded)

This problem explores the difference between solving a circuit using the KCL/KVL method and the node method. The circuit shown below has five elements: three resistors, a current source and a voltage source. The resistance of the resistors and the strengths of the sources are all given below the image. The five branch currents (i_1 to i_5) and the five branch voltages (v_1 to v_5) are also defined in the circuit using the associated variables convention. Recall that solving a circuit means solving for these branch variables (currents and voltages).



$V = 6V, I = 11A, R_1 = 2\Omega, R_2 = 6\Omega$, and $R_3 = 6\Omega$

How many nodes are there in the circuit?

3 ✔ **Answer:** 3

A KCL equation can be written at each of these nodes. How many of these KCL equations are independent?

2 ✔ **Answer:** 2

How many loops are there in the circuit?

6 ✔ **Answer:** 6

A KVL equation can be written around each of these loops. How many of the KVL equations are independent?

3 ✔ **Answer:** 3

How many additional independent equations do we need to solve this circuit?

5 ✔ **Answer:** 5

We can get these additional equations from the constitutive relations (or element laws) of the five elements. The total number of independent equations needed to solve a circuit using the KCL/KVL method is twice the number of elements in the circuit (ten in this example) - matching the number of unknown branch variables.

On the other hand, the node method can require far fewer equations. In this example, if the ground node is appropriately chosen there is only one unknown node voltage. Hence, only one node equation is needed.

Using either the KCL/KVL method or the node method, solve this circuit.

The value (in Volts) of v_3 is:

18 ✔ **Answer:** 18.0

The value (in Amperes) of i_5 is:

8 ✔ **Answer:** 8.0

The power (in watts) coming out of the current source:

198 ✔ **Answer:** 198.0

The power (in watts) coming out of the voltage source:

-48 ✔ **Answer:** -48.0

The power (in watts) dissipated in R_1 resistor:

72 ✔ **Answer:** 72.0

The power (in watts) dissipated in R_2 resistor:

24 ✔ **Answer:** 24.0

The power (in watts) dissipated in R_3 resistor:

54 ✔ **Answer:** 54.0

By conservation of power, the net power coming out of the two sources must equal the total power dissipated in the three resistors. If this is not true, then you made a mistake.

Explanation:
There are three nodes in this circuit: The intersection of both R_1, R_2, and the positive terminal of V, the intersection of R_1, R_2, R_3, and I, and the intersection at the negative terminal of V, I, and R_3.
If we write the KCL equations for this circuit, we get:

$$i_1 = i_2 - i_5$$

$$i_1 = i_2 + i_3 + i_4$$

$$0 = i_3 + i_4 + i_5$$

If we subtract the third equation from the second one, we get the first, so only two KCL equations are independent.
The number of loops in the circuit is six. The loops are described by the KVL equations below:

$$V + v_2 = v_4$$

$$V - v_1 = v_4$$

$$V + v_2 = v_3$$

$$V - v_1 = v_3$$

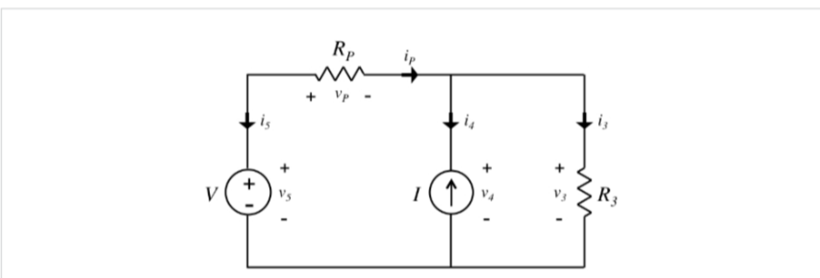
$$v_3 = v_4$$

$$v_1 = -v_2$$

If we subtract equation 3 from 1 we get equation 5. If we subtract equation 5 from 2 we get equation 4. If we substract equation 2 from equation 1 we get equation 6. Therefore there are 3 independent KVL equations.
The number of additional independent equations needed to solve the circuit is five, to give a total of ten independent equations. The general rule is the number of equations equals two times the number of circuit elements.
To solve for our element voltages and currents, we can start by simplifying the circuit and combining R_1 and R_2 into the equivalent resistor R_P.

$$R_P = \frac{R_1 * R_2}{R_1 + R_2} = \frac{2*6}{2+6} = 1.5$$

We can label the current, i_P, and voltage, v_P, on R_P in the figure below:



We can then write KCL at the intersection between R_P, R_3, and I:

$$i_P = i_3 - I$$

Using Ohm's Law we rewrite i_P and i_3:

$$\frac{v_P}{R_P} = \frac{v_3}{R_3} - I$$

Looking at the KVL loop through the voltage source, R_P and R_3:

$$V = v_p + v_3 => v_p = V - v_3$$

Therefore,

$$\frac{V - v_3}{R_P} = \frac{v_3}{R_3} - I$$

After substituing our known variable we get $v_3 = 18.0$, which implies $i_3 = 3.0$. From this solution and all the KCL/KVL equations listed above, you can solve for all element currents and voltages.
The power coming out of the current source can be found by multiplying the magnitude of the source, I, by v_3 since

$v_3 = v_4$:

$$P = 11A \times 18V = 198W$$

The power coming out of the voltage source can be found by multiplying the current $-(i_5)$ by $6V$:

$$P = -8A \times 6V = -48W$$

The power dissipated by R_1 is:

$$P = \frac{v_1^2}{R} = \frac{(6V - 18V)^2}{2\Omega} = 72W$$

The power dissipated by R_2 is:

$$P = \frac{(v_1 - v_2)^2}{6\Omega} = 24W$$

The power dissipated by R_3 is:

$$P = \frac{(v_3)^2}{6\Omega} = 54W$$

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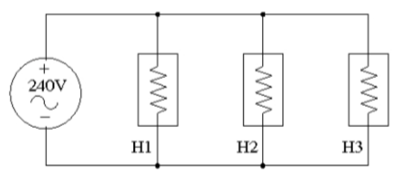
Answers are displayed within the problem

H1P3: Poor Workmanship

5.0/5.0 points (graded)

Joe wants to heat his 12'X20' workshop with electric heat. He has hired the HACME electrician company to build the system. They propose to use three 900.0W 240V baseboard heaters to provide a total heating capacity of 2700.0W. (A heater is basically a resistor. This is not quite true, because there is a thermostatic switch incorporated into the heater and because the resistance of a heater varies a bit with its temperature. But we will use a linear resistor as a model of a heater.)

In the proposed system the heaters are connected in parallel with the 240V 60Hz AC power line (modeled by a voltage source) as shown in the diagram:



Remember (from Exercise S1E3: AC power) that AC power-line voltages and currents are specified as RMS values. So 120V AC heats a given resistance exactly as much as 120V DC would heat that same resistance.

How much current is expected to be drawn from the power line by this heating system when all three heaters are on?

11.25 ✔ **Answer:** 11.25

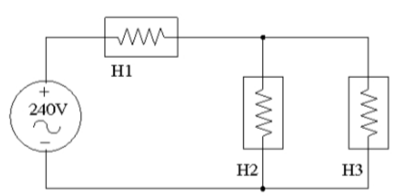
If instead, HACME chose to implement the system with 120V heaters, how much current would have been needed?

22.5 ✔ **Answer:** 22.5

Notice that this would require much heavier and more expensive wire to distribute the power.

Back to the original plan with 240V power.

Unfortunately, Sparky, who works for HACME, was a little sleepy that day. He accidentally connected the heaters as shown below:



As a consequence, Joe found his workshop too cold. H1 was weak; H2 and H3 barely warmed up.
What power was being dissipated in H1?

400 ✔ **Answer:** 400.0

What power was being dissipated in H2 (or in H3)?

100 ✔ **Answer:** 100.0

So the total heating power in Joe's shop was:

600 ✔ **Answer:** 600.0

No wonder Joe was cold.

Explanation:
The current drawn from the power line when all three heaters are on is calculated by dividing the total power consumed by the voltage supplied.

$$P = VI = 2700.0W \rightarrow \frac{VI}{V} = I = \frac{2700.0}{240} = 11.25\ Amps$$

The current drawn from the power line when the magnitude of the voltage source is decreased by a factor of two must be twice the initial current if the same power is to be delivered.

$$\therefore I = 22.5\ Amps.$$

The power being dissipated in H1 is calculated by first calculating the resistance of a single heater:

$$R = \frac{V^2}{P} = \frac{(240^2)}{900.0} = 64.0\Omega$$

The equivalent resistance of the new arrangement of resistors is:

$$R_{eq} = R + R \parallel R = 1.5R = 96.0\Omega$$

We now calculate the current flowing through H1:

$$I_{H1} = \frac{V}{R_{eq}} = \frac{240}{96.0} = 2.5A$$

Plugging this current into the equation for power, we get:

$$P_{H1} = I^2 R = 2.5^2\,(64.0) = 400.0W$$

To calculate P_{H2} (Which is the same as P_{H3}), we know that the current I_{H1} will divide evenly between H2 and H3 because both H2 and H3 have the same resistance and are in parallel with each other.

$$\therefore P_{H2} = P_{H3} = (\frac{2.5}{2})^2 R = 100.0W$$

Finally we calculate the total power being consumed by adding the power consumed by each individual resistor.

$$P_{total} = P_{H1} + P_{H2} + P_{H3} = 400.0 + 100.0 + 100.0 = 600.0W$$

Submit You have used 4 of 25 attempts Save Show answer

Answers are displayed within the problem

Discussion

Topic: Week 1 / Homework 1

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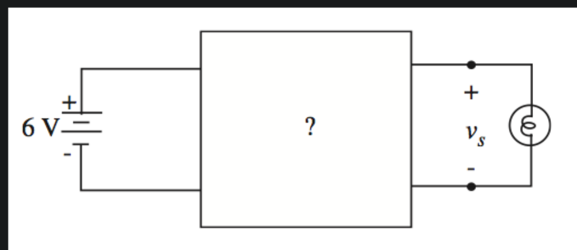
Lab

Lab due Jul 19, 2022 18:12 +04 Completed

Lab 1

2.0/2.0 points (graded)

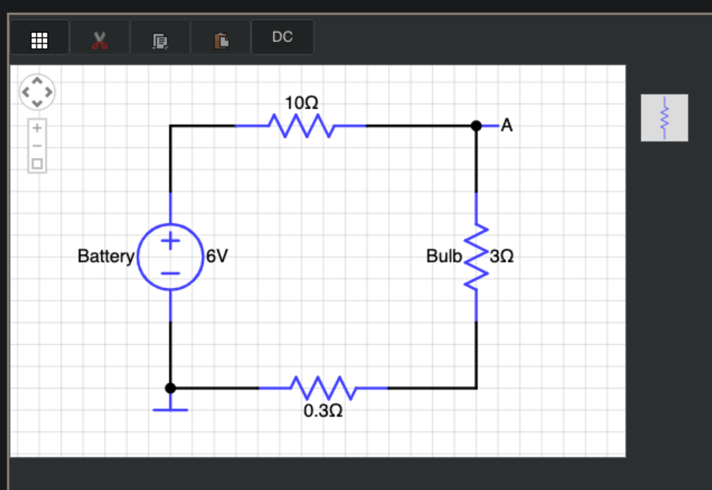
You have a 6-volt battery (assumed ideal) and a 1.5-volt flashlight bulb, which is known to draw $0.5A$ when the bulb voltage is $1.5V$ (see figure below). Design a network of resistors to go between the battery and the bulb to give $v_s = 1.5V$ when the bulb is connected, yet ensures that v_s does not rise above $2V$ when the bulb is disconnected.



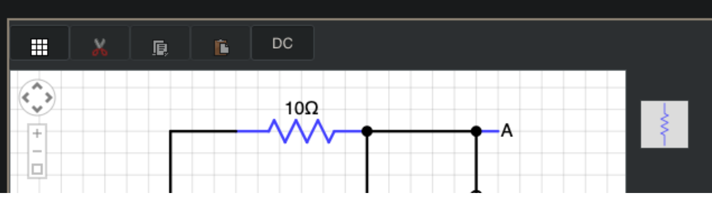
There are two schematic diagrams below. Please enter the network of resistors you've designed into both diagrams. The top diagram is the model when the bulb is connected; the bottom diagram is the model when the bulb is disconnected.

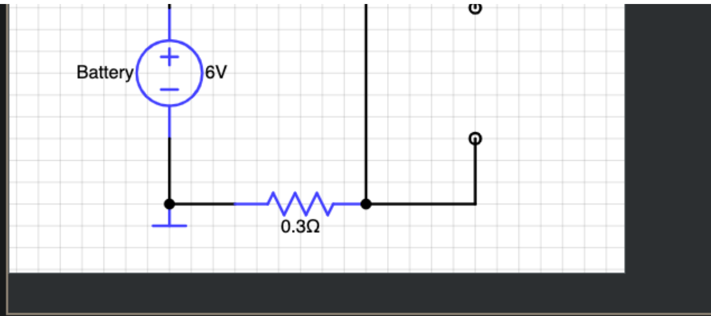
Run a DC analysis on both diagrams to show that the node labeled "A" has a voltage of approximately $1.5V$ in the top diagram and less than or equal to $2V$ in the bottom diagram. Please submit your results *after* the DC analyses have been run so that the results of the analyses will also be submitted. Because we will be checking the voltage at node A, you should have an assigned voltage at node A after the DC analysis; otherwise, your submission will be deemed incorrect.

Schematic model when bulb is connected:



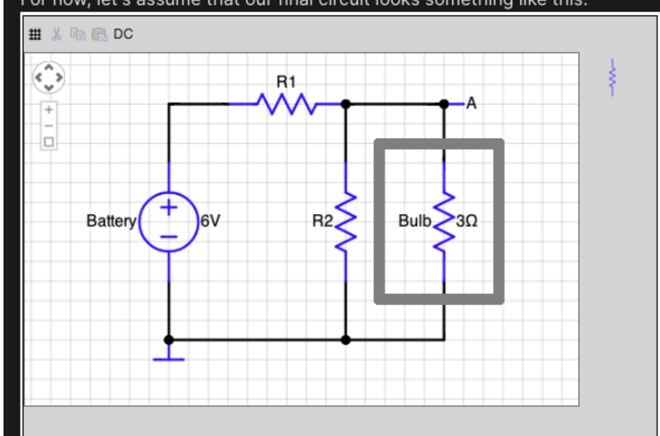
Schematic model when bulb is disconnected:





Hint: use a two-resistor voltage divider to create the voltage for node A. You'll have two unknowns (R_1 and R_2) which can be determined by solving the two equations for v_s derived from the constraints above: one involving R_1, R_2 and R_{bulb} where $v_s = 1.5$, and one involving R_1 and R_2 where $v_s = 2$.

Explanation:
The voltage v_s has two requirements, it must be at least 1.5V with the light bulb inserted and no greater than 2.0V when the lightbulb is not part of the circuit. The presence or absence of the lightbulb is the only thing that should differentiate these two circuits - everything else should be the same. Therefore, we can try to express in math what constraints these requirements put on our answer, and then solve for the appropriate unknowns.
First let's consider what our two constraints have in common. They both want a fraction of the input voltage at the output. This means that both circuits should be voltage dividers. The two different circuits want two different voltages at the output, though, which is the tricky part. Let's consider what we have to work with.
The resistance of the light bulb is fixed at R_{bulb}, but we can add as many resistors of whatever resistance we want to this circuit. All that we will probably need, though, is one resistor in series and one resistor in parallel with the lightbulb. That seems like a reasonable place to start, since series and parallel are the two operations that we know how to do right now.
For now, let's assume that our final circuit looks something like this:



Assuming that we have the right circuit skeleton for the solution, then we can start writing our constraints in terms of equations in order to figure out what the value of those resistors should be.
On one hand, without the light bulb, we know that $(R_1 + R_2) \cdot I_{overall} = 6V$ from KVL, and $R_2 \cdot I_{circuit} = 2V$ from the constraint on the solution. After some algebra, we can see that $R_1 = 2R_2$ in this first circuit.
When we take the second circuit into account, we again use KVL: $(R_1 + R_2 \parallel R_{bulb}) \cdot I_{overall} = 6V$, and then apply the constraint: $(R_2 \parallel R_{bulb}) \cdot I_{overall} = 1.5V$. Taking R_{bulb} to be3Ω and the equations from the lightbulb-free circuit, we get that $R_1 = 3\Omega$ and $R_2 = 1.5\Omega$. If we run a DC analysis on the circuit, we will see that we get the voltages that the problem asked for, and we can submit our problem for grading.