Exam 3

Practice Problems

Q1

0 points possible (ungraded)

The impulse response of a circuit is its response to a unit impulse, $\delta(t)$. Knowing the impulse response of a linear circuit is extremely valuable as we can figure out the circuit's response to an arbitrary input from it. In this problem you will find the response of a circuit when it is driven by a unit impulse.

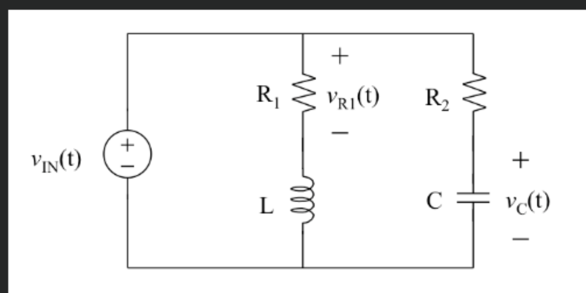


Figure 1-1

Consider the circuit shown above in which $v_{IN}(t) = 1\delta(t)$ volt-seconds. That is, v_{IN} is a unit impulse at time t = 0. In the circuit, $L = 5mH$, $R_1 = 10\Omega$, $C = 33nF$ and $R_2 = 10k\Omega$. Note that since this circuit is driven by an impulse and there is no other source of energy, the capacitor voltage and inductor current at t = 0 will be zero.

(a) What is the value of $v_{R_1}(t)$ at $t = 0^-$ in Volts (V)?

0 ✔ **Answer:** 0

(b) What is the value of $v_{R_1}(t)$ at $t = 0^+$ in volts (V)? Hint: Recall that that $\int_{0^-}^{0^+} \delta(t)\, dt = 1$

2000 ✔ **Answer:** 2000

(c) What is the value for $v_{R_1}(t)$ at $t = 1ms$ in volts(V)? (Hint: The L-R_1 and C-R_2 branches of the circuit are decoupled.)

270.671 ✔ **Answer:** 270.6

(d) What is the value of $v_C(t)$ at $t = 0^+$ in volts (V)?

3030.3 ✔ **Answer:** 3030.3

(e) What is the value for $v_C(t)$ at $t = 1ms$ in volts (V)?

146.367 ✔ **Answer:** 146.36

Explanation:

1. The Votlage across resistor R_1 at $t = 0^-$ is calculated as $v_{r1} = R_1 \cdot i_L$, where i_L is the current through the

inductor. From the problem description, we know that there are no other sources in the circuit apart from the voltage impulse source and therefore the current through the inductor before the impulse is zero. From that, we can calculate $v_{r1} = 0$ at $t = 0^-$

2. After the impulse, some energy has been transferred into the circuit. Following the analysis done during lecture sequece 15, we know that the flux linkage of the voltage source $V \cdot T = \Lambda$ is immediatly transferred to the inductor. Since $\Lambda = LI$, we can calculate the current through the inductor immediatly after the impulse to be $I = \frac{\Lambda}{L}$. Using the fact that $\int_{-\inf}^{inf} v_{IN} dt = 1$, then $\Lambda = 1$ and $i_L = \frac{1}{5 \cdot 10^{-3}} = 200A$. Then:

$$v_{r1} = R_1 \cdot i_L = (10\Omega) \cdot (200A) = 2000V$$

3. After the impulse, the v_{IN} is zero (a short circuit), and the inductor will dissipate its energy throught R_1. The equation describing this energy dissipation was studied before and is:

$$i_L = \frac{\Lambda}{L} \cdot e^{-t \cdot \frac{R}{L}} = (200A) \cdot e^{-1 \cdot 10^{-3} s \cdot \frac{10\Omega}{5 \cdot 10^{-3} H}} = 27.06A$$

Voltage across the resistor is then:

$$v_{r1} = R_1 \cdot i_L = (10\Omega) \cdot (27.06A) = 270.6V$$

4. To analyze the capacitor branch it is easier to transform the voltage source in series with R_2 (remember that both branches are decoupled), to its Norton equivalent using the equations: $R_{No} = R_{Th}$ and $I_{No} = \frac{V_{th}}{R_{No}}$. The new circuit is a current impulse source with $i_{IN} = \frac{1}{R_2} v_{IN}$.

   As studied in lecture sequence 15, some charge will be transferred immediatly from the current impulse source to the capacitor. This charge is simply the area under the curve of the current source or, mathematically, $Q = \int_{-\inf}^{inf} i_{IN} dt = \frac{1}{R_2} = 1 \cdot 10^{-4} C$. The voltage in the capacitor after the impulse is then calculated as:

$$v_C = \frac{Q}{C} = \frac{1 \cdot 10^{-4} C}{33 \cdot 10^{-9} F} = 3030.3V$$

5. Finally, after the impulse, the current i_{IN} is zero (open circuit) and the capacitor discharges through R_2. The expression for this energy dissipation is:

$$v_C = \frac{Q}{C} \cdot e^{\frac{-t}{RC}} = (3030.3V) \cdot e^{-\frac{1 \cdot 10^{-3} s}{(10k\Omega)(33 \cdot 10-9)}} = 146.36V$$

In[∘]:=
```
l = 5*^-3; r1 = 10; c = 33*^-9; r2 = 10*^3;

Λ = 1;
il = Λ/l;
vr1 = il r1

t = 1*^-3;
il = Λ/l Exp[-t*r1/l];
vr1 = il r1 // N

Q = 1/r2;
vc = Q/c // N

vc = Q/c Exp[-t/(r2 c)] // N
```

Out[∘]= 2000

Out[∘]= 270.671

Out[∘]= 3030.3

Out[∘]= 146.367

Q2

0 points possible (ungraded)

In this problem we investigate the time response of the circuit shown below which contains two switches S_1 and S_2.

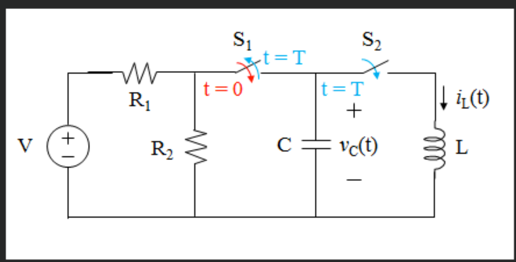


Figure 2-1

Until time $t = 0$, both switches are open and the circuit is initially at rest, i.e., $v_C\,(t = 0^-) = 0$ and $i_L\,(t = 0^-) = 0$. At $t = 0$, switch S_1 is closed but S_2 stays open. After a VERY LONG time T, S_1 is opened and S_2 is simultaneously closed.

Answer the following questions in terms of the parameters: V, R_1 R_2, C, L, T and t, as applicable.

What is the expression for $v_C\,(t)$ during the time interval $0 < t < T$?

((V*R2)/(R1+R2))*(1-e^((-t*(R1+R2))/(R1*R2*C))) ✔ **Answer:** ((V*R2)/(R1+R2))*(1-e^((-t*(R1+R2))/(R1*R2*C)))

$$\left(\frac{V \cdot R_2}{R_1 + R_2}\right) \cdot \left(1 - e^{\frac{-t \cdot (R_1 + R_2)}{R_1 \cdot R_2 \cdot C}}\right)$$

What is the expression for $v_C(t)$ during the time interval $t > T$?

(V*R2)/(R1+R2)*cos((t-T)/sqrt(L*C)) ✔ **Answer:** (V*R2)/(R1+R2)*cos(((1/(L*C))^(0.5)*(t-T)))

$$\frac{V \cdot R_2}{R_1 + R_2} \cdot \cos\left(\frac{t-T}{\sqrt{L \cdot C}}\right)$$

What is the expression for $i_L(t)$ during the time interval $t > T$?

sqrt(C/L)*((V*R2)/(R1+R2))*sin((t-T)/sqrt(L*C)) ✔ **Answer:** ((C/L)^(0.5))*((V*R2)/(R1+R2))*sin(sqrt(1/(L*C))*(t-T))

$$\sqrt{\frac{C}{L}} \cdot \left(\frac{V \cdot R_2}{R_1 + R_2}\right) \cdot \sin\left(\frac{t-T}{\sqrt{L \cdot C}}\right)$$

Explanation:
(a) For time $0 < t < T$, we can ignore the inductor because it is disconnected from the circuit. Thus, we have a first-order system: a charging RC circuit.

To find $v_c(t)$, we first derive the differential equation governing this variable by writing KCL at the node intersecting the R_1, R_2 and C branches. Recalling that the current through a capacitor is equal to $C \cdot \frac{dv}{dt}$, we end up with:

$$\frac{V - v_C(t)}{R_1} = \frac{v_C(t)}{R_2} + C \cdot \frac{dv_C(t)}{dt}$$

And with a little algebra to get the equation in standard form:

$$\frac{dv_C(t)}{dt} + \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C} \cdot v_C(t) = \frac{V}{R_1 \cdot C}$$

This is a linear, first-order differential equation with constant coefficients. This can be solved one of two ways: we can do it the standard drawn-out way, using integrating factors, or we can instead use intuition and what we know about the circuit to simply "write down" the answer. Either approach will result in the correct answer; the latter approach is expanded upon below.

First, we consider our initial and final states in the time interval $0 < t < T$. Initially, we are told $v_C(0) = 0$, so that covers the initial state. What about after a long time, when $t = T$? We can safely assume that the capacitor has charged up fully to its final value because a long time has passed. Therefore, that branch of the circuit is an open circuit, and all we are left with is a simple voltage divider between R_1 and R_2. So $v_C(t = T) = \frac{V \cdot R_2}{R_1 + R_2}$, which covers our final state.

But what happens in between? Because we know this is a first order RC circuit, just like all other first-order RC circuits we've studied in this class, we know the capacitor voltage will undergo some sort of exponential behavior- in this case, an exponential rise from its initial value (zero) to its final value.

All of this information leads us to know our solution will be of this form:

$$\alpha\left(1 - e^{\frac{-t}{\tau}}\right)$$

Where α is our final value, and τ is the time constant. Note how at $t = 0$, out trial solution evaluates to 0, which makes sense and is a good sanity check.

We already reasoned what α is: it's just the result of the voltage divider, $\frac{V \cdot R_2}{R_1 + R_2}$. But what is the time constant, τ? The answer is actually staring at us in the differential equation. Because it is in standard form for a first order system, recall that the coefficient in front of $v_C(t)$ is actually equal to $\frac{1}{\tau}$! Therefore,

$$\tau = \frac{R_1 \cdot R_2 \cdot C}{}$$

$$R_1 + R_2$$

Alternatively, we can find τ by remembering this fact, valid for first-order RC circuits: $\tau = R_{eq} \cdot C$, where R_{eq} is the thevenin equivalent resistance of the circuit looking through the terminals of the capacitor. It should be clear that R_{eq} is the parallel combination of R_1 and R_2, so again,

$$\tau = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot C$$

This gives us our final answer, which is identical to what we would have gotten if we solved the above differential equation by hand:

$$v_C(t) = \frac{V \cdot R_2}{R_1 + R_2}\left(1 - e^{-t \cdot \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C}}\right)$$

(b) For the time $t > T$, the entire left half of the circuit (the resistors and voltage source) is disconnected, and we are dealing purely with a second-order LC circuit. Again, we start by deriving the differential equation for $v_C(t)$.

A preliminary note: to account for the fact that the switching event happened at time $t = T$, we can analyze the circuit and get the answer as if we were starting at $t = 0$, just like in part (a), but in our final solution, we "time-shift" by T- that is, we substitute all instances of t with $t - T$.

First, by KCL, we have:

$$C \cdot v'_C(t) = -i_L(t)$$

where $v'_C(t)$ is shorthand for the first time derivative of $v_C(t)$. Note the negative sign - this is due to the defined direction of $i_L(t)$, going OUT of the positive terminal of C, is in the opposite direction as $i_C(t)$ (if it was going in, we would make it positive, because that would mean $i_C(t) = i_L(t)$).

Second, by KVL, remembering the expression for the voltage drop across the inductor, we have:

$$L \cdot i'_L(t) = v_C(t)$$

But we want one equation in terms of $v_C(t)$, so we proceed as follows. First, take the derivative of both sides of the KCL equation:

$$C \cdot v''_C(t) = -i'_L(t)$$

And second, substitute in for $i'_L(t)$ using the expressin derived with the KVL equation:

$$v''_C(t) = -\frac{i'_L(t)}{C} = -\frac{v_C(t)}{LC}$$

You might recognize the form of this equation - it is a simple harmonic oscillator, identical to the equation of motion for a mass on a spring. The magnitude of the coefficient in front of the term on the right hand side ($\frac{1}{LC}$) is equal to the angular frequency squared.

Solving this is the same as solving any other simple harmonic oscillator - we guess a general solution in the form of a linear combination of sines and cosines:

$$v_C(t) = \alpha \cdot sin(\omega \cdot t) + \beta \cdot cos(\omega \cdot t)$$

where $\omega = \frac{1}{\sqrt{LC}}$. Then we apply the initial condition:

$$v_C(0) = \frac{V \cdot R_2}{R_1 + R_2}$$

the final value from the "charging-up" phase of the capacitor in part (a).

Our final answer should be:

$$v_C(t) = \frac{V \cdot R_2}{R_1 + R_2} cos\left(\frac{1}{\sqrt{LC}} \cdot (t - T)\right)$$

(c) We already have $v_C(t)$, as well as the relation between it and $i_L(t)$, which is again:

$$i_L(t) = -C \cdot v'_C(t)$$

Therefore our answer is:

$$i_L(t) = \sqrt{\frac{C}{L}} \cdot \frac{V \cdot R_2}{R_1 + R_2} \cdot sin\left(\frac{1}{\sqrt{LC}}(t - T)\right)$$

Q3

0 points possible (ungraded)

The circuits in this problem are driven by sinusoidal sources and are in the steady state. For each circuit, enter the letter corresponding to its magnitude and phase plot for the transfer funrction $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ from the magnitude (A-F) and phase (G-L) plots sketched in the figures below. The magnitude plots are on a log-log scale and the phase plots are on a linear-log scale. Note these are sketches, look for correct approximate shape.

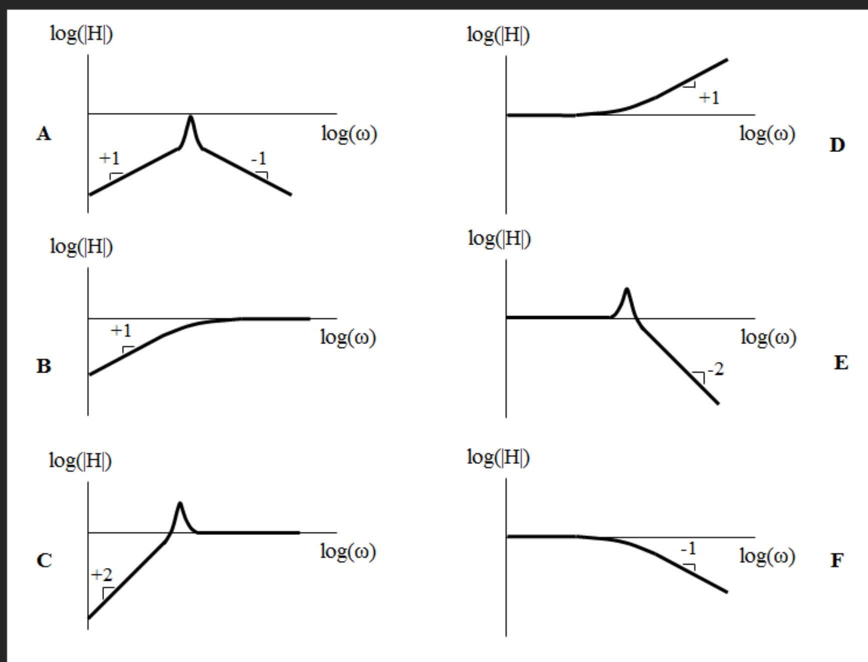
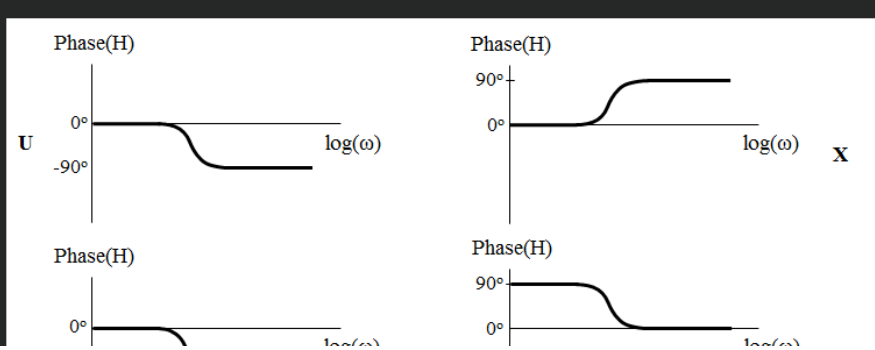


Figure 3-1

Transfer function $H(j\omega)$ magnitude plots



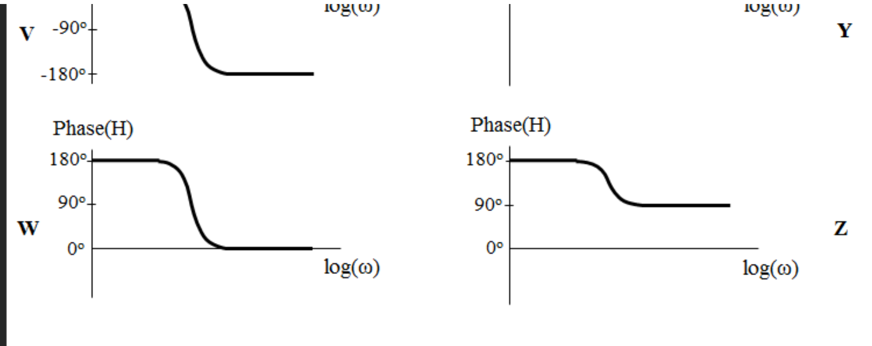


Figure 3-2

Transfer function $H(j\omega)$ phase plots

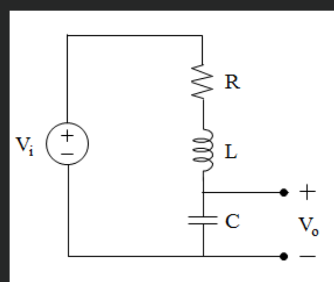


Figure 3-3

Which plot (A-F) corresponds to the magnitude of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

E

Answer: E

E

Which plot (U-Z) corresponds to the phase of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

V

Answer: V

V

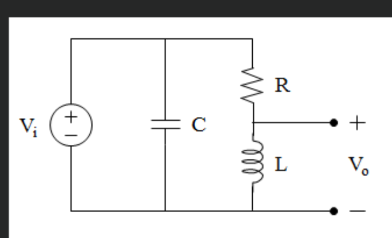


Figure 3-4

Which plot (A-F) corresponds to the magnitude of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

B

Answer: B

B

Which plot (U-Z) corresponds to the phase of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

Y

✔ **Answer:** Y

Y

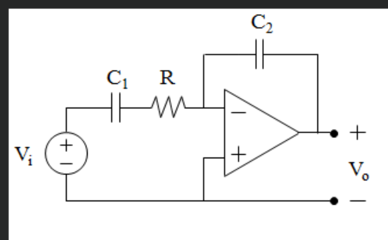


Figure 3-5

Which plot (A-F) corresponds to the magnitude of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

F

✔ **Answer:** F

F

Which plot (U-Z) corresponds to the phase of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

Z

✔ **Answer:** Z

Z

Explanation:
(a), (b) The transfer function for the circuit is:

$$H(s) = \frac{\frac{1}{s \cdot C}}{R + s \cdot L + \frac{1}{s \cdot C}} = \frac{1}{s^2 \cdot L \cdot C + s \cdot R \cdot C + 1}$$

First of all, we can see this is a second order system (because of the quadratic s^2 term in the denominator). This will do two things: it will cause a "peak" or a maximum in the magnitude bode plot, and it will make the difference between the high and low frequency asymptotes 180 degrees (as opposed to 90 degrees in first order systems).

We can elinimate half the magnitude and phase graphs based on these stipulations. To find which of the remaining graphs actually match, we first note that the low frequency magnitude asymptote is unity- only one second-order magnitude graph matches; E. Doing the same for the phase, the low frequency asymptote ends up being 0, which is only matched by V. So the answers are E and V.
(c), (d) The transfer function for the circuit is:

$$H(s) = \frac{s \cdot L}{R + s \cdot L}$$

Note that the capacitor doesn't matter; this system is actually a first-order system. We should expect no magnitude peaking, and a 90 degree phase difference between low and high frequency asymptotes.

Looking at the high frequency magnitude asymptote we see that it's unity: that matches graph B. The high-frequency phase asymptote is zero; the only graph that does this and is first-order is Y. So the answers are B and Y.

(e), (f) The transfer function for the circuit is:

$$H(s) = \frac{\frac{\cdots}{s\cdot C_2}}{\frac{1}{s\cdot C_1}+R} = \frac{\frac{\cdots}{C_2}}{s\cdot C_1\cdot R+1}$$

Again, we have a first order system. The negative sign will only change the "location" of the phase curve; it will not change the fact that it has a 90 degree difference between high and low frequency asymptotes.

First, we see that we have a constant gain at low frequencies and one that goes to zero at high frequencies; the only magnitude graph matching this is F. As for the phase, it would be the same as it normally is in a capacitive low-pass filter, but the negative sign will contribute a 180 degree phase shift (you can remember this by remembering that $-1 = e^{j\pi}$ in polar form, with π being 180 degrees in radians). At low frequency, the phase shift would normally be 0, but now it's 180, and will asymptotically approach 90 at high frequencies. The graph matching this is Z. So the answers are F and Z.

Q4

0 points possible (ungraded)

The circuit shown below is intended to implement C as a logic function of A and B.

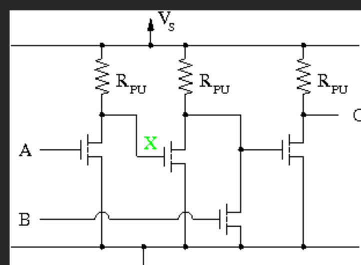


Figure 4-1

Three of the four following logic diagrams compute the same logic function as the circuit above.

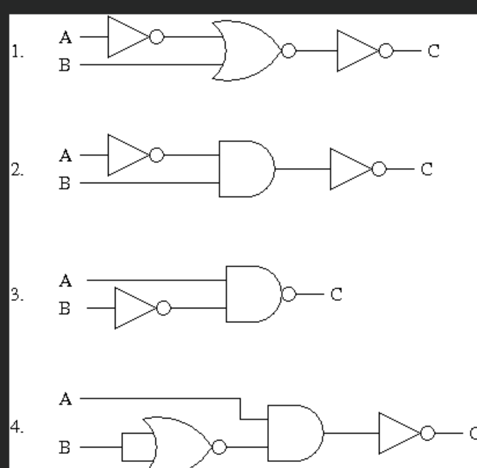


Figure 4-2

Which diagram computes something different?
Write the number of the incorrect diagram in the box below:

2 ✔ **Answer:** 2

The circuit is intended to obey the following static discipline:

$V_S = 5.0$V, $V_{OH} = 4.0$V, $V_{IH} = 2.8$V, $V_{IL} = 2.2$V, $V_{OL} = 1.4$V.

What is the low-level noise margin? (Express your answer in Volts.)

0.8 ✔ **Answer:** 0.8000000000000003

What is the high-level noise margin? (Express your answer in Volts.)

1.2 ✔ **Answer:** 1.2000000000000002

How big (in Volts) is the forbidden region?

0.6 ✔ **Answer:** 0.5999999999999996

Assume that $R_{ON} = 5.0\text{k}\Omega$ for our MOSFETS. What is the minimum value that R_{PU} can have, in kilOhms, and still satisfy the static discipline?

12.8571 ✔ **Answer:** 12.857142857142856

We now have $R_{ON} = 5.0\text{k}\Omega$. Let's choose $R_{PU} = 14\text{k}\Omega$. What is the maximum amount of power, in milliWatts, that is consumed by this circuit for legitimate logic levels applied at A and B?

2.6316 ✔ **Answer:** 2.631578947368421

Given our choices of R_{ON} and R_{PU}, assume that the gate-to-source capacitance $C_{GS} = 50.0\text{fF}$. What is the rising edge time constant, in nanoseconds, for the signal at the node labeled X?

0.7 ✔ **Answer:** 0.7000000000000001

What is the falling edge time constant, in nanoseconds, for the signal at the node labeled X?

0.1842 ✔ **Answer:** 0.1842105263157895

Q5

0 points possible (ungraded)

The circuits in this problem are driven by sinusoidal sources and are in the steady state. For each circuit, enter the letter corresponding to its magnitude and phase plot for the transfer funrction $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ from the magnitude (A-F) and phase (G-L) plots sketched in the figures below. The magnitude plots are on a log-log scale and the phase plots are on a linear-log scale. Note these are sketches, look for correct approximate shape.

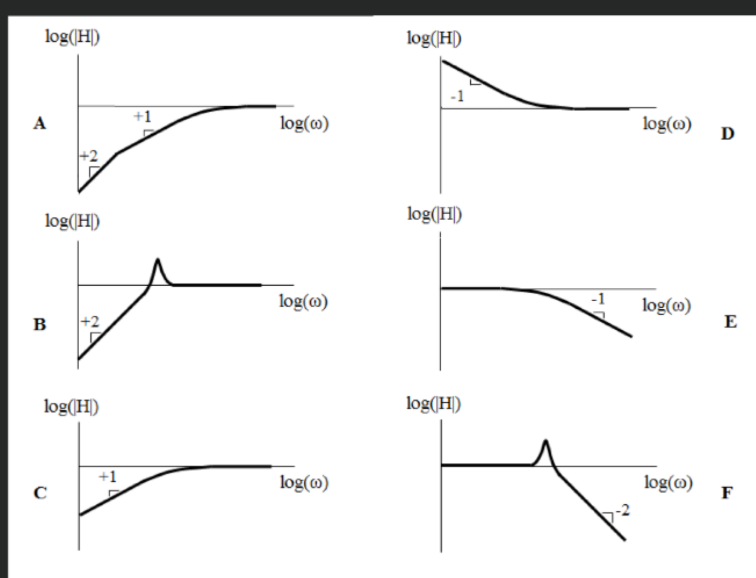


Figure 5-1

Transfer function $H(j\omega)$ magnitude plots

Transfer function $H\,(j\omega)$ magnitude plots

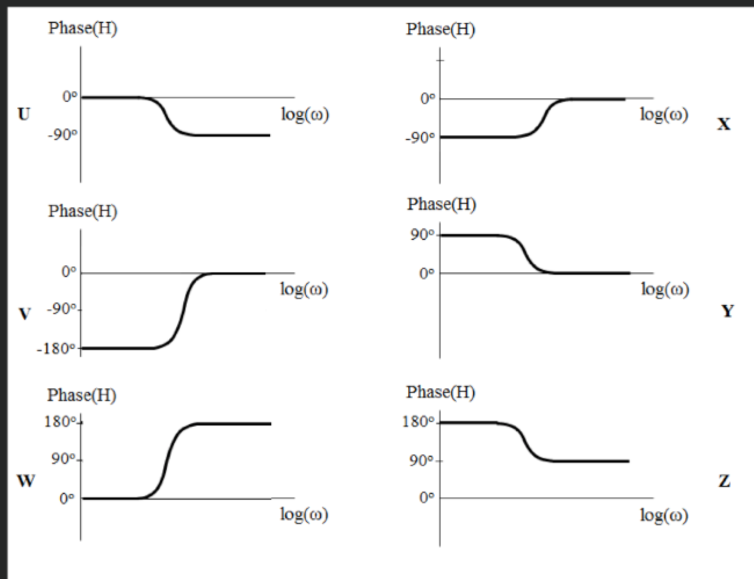


Figure 5-2

Transfer function $H\,(j\omega)$ phase plots

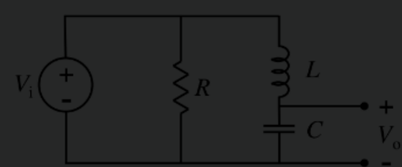


Figure 5-3

Which plot (A-F) corresponds to the magnitude of $H\,(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

F ✔ **Answer:** F

Which plot (U-Z) corresponds to the phase of $H\,(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

W ✔ **Answer:** W

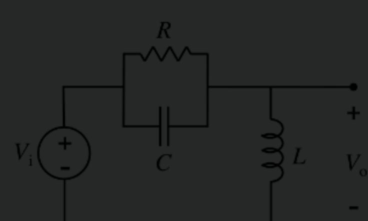


Figure 5-4

Which plot (A-F) corresponds to the magnitude of $H\,(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

C ✔ **Answer:** C

C ✔ **Answer:** C

Which plot (U-Z) corresponds to the phase of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

Y ✔ **Answer:** Y

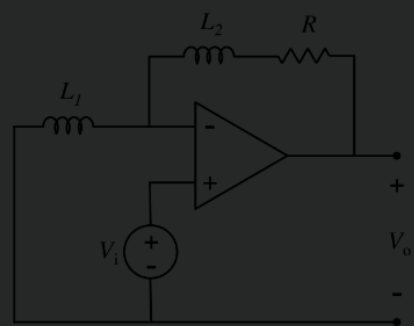


Figure 5-5

Which plot (A-F) corresponds to the magnitude of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

D ✔ **Answer:** D

Which plot (U-Z) corresponds to the phase of $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ in the above circuit?

X ✔ **Answer:** X

Explanation:
(a), (b) The transfer function for the circuit is:

$$H(s) = \frac{\frac{1}{s\cdot C}}{s\cdot L + \frac{1}{s\cdot C}} = \frac{1}{s^2\cdot L\cdot C + 1}$$

First of all, we can see this is a second order system (because of the quadratic s^2 term in the denominator). This will do two things: it will cause a "peak" or a maximum in the magnitude bode plot, and it will make the difference between the high and low frequency asymptotes 180 degrees (as opposed to 90 degrees in first order systems).

We can elinimate half the magnitude and phase graphs based on these stipulations. To find which of the remaining graphs actually match, we first note that the low frequency magnitude asymptote is unity - only one second-order magnitude graph matches; F. Doing the same for the phase, the low frequency asymptote ends up being 0, which is only matched by W. So the answers are F and W.

Another way to see that the phase graph would jump from 0 degrees to 180 degrees is that $H(j\omega)$ is a positive real value for low frequencies and a negative real value for high frequencies.

(c), (d) The transfer function for the circuit is:

$$H(s) = \frac{s\cdot L}{s\cdot L + R||\frac{1}{s\cdot C}} = \frac{s\cdot L}{s\cdot L + \frac{R\cdot\frac{1}{s\cdot C}}{R+\frac{1}{s\cdot C}}} = \frac{s\cdot L}{s\cdot L + \frac{R}{s\cdot R\cdot C+1}} = \frac{s\cdot L\cdot(1+s\cdot R\cdot C)}{R+s\cdot L+s^2\cdot R\cdot L\cdot C}$$

Looking at the high frequency magnitude asymptote we see that it's unity and at zero frequency the magnitude is 0 (which is $-\infty$ in dB). We also note that for low frequencies this transfer function behaves like a first-order system with an increasing magnitude so our slope will be $+1$ for low frequencies: that matches graph C. The high frequency phase asymptote is zero and at zero frequency the phase is *positive* 90 degrees; the only graph that does this is Y. So the answers are C and Y.

(e), (f) The transfer function for the circuit is:

$$H(s) = 1 + \frac{s\cdot L_2 + R}{\ldots} = \frac{s\cdot L_1 + s\cdot L_2 + R}{\ldots} = \frac{R + s\cdot(L_1+L_2)}{\ldots}$$

$$s \cdot L_1 \qquad s \cdot L_1 \qquad s \cdot L_1$$

Looking at the high frequency magnitude asymptote we see that it's unity and at zero frequency the magnitude asymptote is $\frac{L_1+L_2}{L_1}$ (which is some positive number in dB). We also note that because this is a first-order system and our magnitude is decreasing our slope will be -1: that matches graph D. The high frequency phase asymptote is 0 and at zero frequency the phase asymptote is *negative* 90 degrees; the only graph that does this is X. So the answers are D and X.

(*Check below, for the expression that is not rendered correctly*)

Q6

0 points possible (ungraded)

In this problem we investigate the time response of the circuit shown in Figure 6-1 which contains one switch. We are given that $V_0 = 40V$, $R_1 = 100\Omega$, $R_2 = 100\Omega$, $R_3 = 200\Omega$, $L = 1H$, and $C = 100\mu F$.

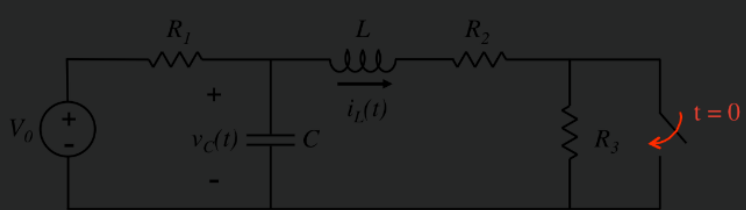


Figure 6-1

Until time $t = 0$, the switch is open and the circuit is initially at steady-state. For $t < 0$ what is the voltage across the capacitor, $v_C\,(t < 0)$, and the current through the inductor, $i_L\,(t < 0)$?

$v_C\,(t < 0)$ (in Volts)

30 ✔ **Answer:** 30.0

$i_L\,(t < 0)$ (in milliamps)

100 ✔ **Answer:** 100.0

At $t = 0$, the switch is closed and after a LONG LONG time reaches steady-state again. What is the voltage across the capacitor and the current through the inductor after the switch is closed for a LONG LONG time?

$v_C\,(t >> 0)$ (in Volts)

20 ✔ **Answer:** 20.0

$i_L\,(t >> 0)$ (in milliamps)

200 ✔ **Answer:** 200.0

For $t > 0$, we can write a second order differential equation for the current through the inductor of the form:

$$V_0 = X\frac{d^2 i_L\,(t)}{dt^2} + Y\frac{di_L\,(t)}{dt} + Z i_L\,(t)$$

Write your answers for X, Y and Z below in terms of R_1, R_2, R_3, C, and L.

X

R1*L*C

Answer: R1*L*C

$R_1 \cdot L \cdot C$

Y

R1*R2*C + L

Answer: R1*R2*C + L

$R_1 \cdot R_2 \cdot C + L$

Z

R1+R2

Answer: R1+R2

$R_1 + R_2$

Explanation:
When the switch is open and the circuit is in steady state, the inductor has no voltage across it and the capacitor has no current through it. To get the current through the inductor, we can look at the voltage loop consisting of V_0, R_1, L, R_2, and R_3. Since the voltage across the inductor is 0, we can write KVL:

$$V_0 = i_L \cdot R_1 + i_L \cdot R_2 + i_L \cdot R_3 i_L = \frac{V_0}{R_1 + R_2 + R_3} i_L = 100.0mA$$

For the voltage across the capacitor we can look at R_1 and write:

$$V_0 - v_C = i_L \cdot R_1 v_C = V_0 - i_L \cdot R_1 v_C = 30.0V$$

When the switch is closed and the circuit is in steady state, the same concepts hold and the inductor has no voltage across it and the capacitor has no current through it. The switch closed provides a current path to ground bypassing R_3, so to get the current through the inductor, we can look at the voltage loop consisting of V_0, R_1, L, and R_2. Since the voltage across the inductor is 0, we can write KVL:

$$V_0 = i_L \cdot R_1 + i_L \cdot R_2 i_L = \frac{V_0}{R_1 + R_2} i_L = 200.0mA$$

For the voltage across the capacitor we can again look at R_1 and write:

$$V_0 - v_C = i_L \cdot R_1 v_C = V_0 - i_L \cdot R_1 v_C = 20.0V$$

For the differential equation, we can use KCL at the node intersecting the capacitor and inductor:

$$i_{R1} = i_C + i_L$$

where i_{R1} is the current through R_1 flowing to the right.

and KVL for two loops:

$$V_0 = i_{R1} \cdot R_1 + v_C v_C = v_L + i_L \cdot R_2$$

We take the derivative of our last equation and get:

$$\frac{dv_C}{dt} = \frac{dv_L}{dt} + R_2 \cdot \frac{di_L}{dt}$$

With these equations and the identity equations:

$$i_C = C \cdot \frac{dv_C}{dt} v_L = L \cdot \frac{di_L}{dt}$$

We can substitute these expressions into the expression for V_0:

$$V_0 = i_{R1} \cdot R_1 + v_C = (i_C + i_L) \cdot R_1 + (v_L + i_L \cdot R_2) V_0 = R_1 \cdot \left(C \cdot \frac{dv_C}{dt} + i_L\right) + L \cdot \frac{di_L}{dt} + i_L \cdot R_2 V_0 = R_1 \cdot$$

we get the following differential equation:

$$V_0 = (R_1 \cdot L \cdot C)\,\frac{d^2 i_L}{dt^2} + (R_1 \cdot R_2 \cdot C + L)\,\frac{di_L}{dt} + (R_1 + R_2) \cdot i_L$$

```
(*The second last expression after
 "We can substitute these expressions into the expression for V₀:"
 is rendered again here:*)
```

$$\begin{aligned}
V_0 &= i_{R1} \cdot R_1 + v_C = (i_C + i_L) \cdot R_1 + (v_L + i_L \cdot R_2) \\
&= R_1 \cdot \left(C \cdot \frac{dv_C}{dt} + i_L\right) + L \cdot \frac{di_L}{dt} + i_L \cdot R_2 \\
&= R_1 \cdot \left(C \cdot \left(\frac{dv_L}{dt} + R_2 \cdot \frac{di_L}{dt}\right) + i_L\right) + L \cdot \frac{di_L}{dt} + i_L \cdot R_2 \\
&= R_1 \cdot C \cdot \frac{dv_L}{dt} + R_1 \cdot R_2 \cdot C \cdot \frac{di_L}{dt} + R_1 \cdot i_L + L \cdot \frac{di_L}{dt} + i_L \cdot R_2 \\
&= R_1 \cdot C \cdot \frac{d}{dt}\left(L \cdot \frac{di_L}{dt}\right) + (R_1 \cdot R_2 \cdot C + L) \cdot \frac{di_L}{dt} + (R_1 + R_2) \cdot i_L
\end{aligned}$$