

# Week 2

S3 - Linearity and Superposition

S4 - Static Discipline and Boolean Logic

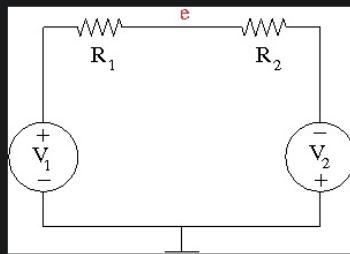
## Lectures

### S3E1: Node Equation Review

0 points possible (ungraded)

This is just a quick review of the node equation idea.

In the following circuit there is one unknown node potential, labeled  $e$ . The device parameters are:  $R_1 = 6800.0\Omega$ ,  $R_2 = 5600.0\Omega$ ,  $V_1 = 5.0V$ ,  $V_2 = -7.2V$ .



What is the unknown potential  $e$  (in Volts)?

6.20645

✓ Answer: 6.2064516129032254

### Explanation

The following explanation was contributed as a forum post by Grove in a previous run of the course. Many students found this to be extremely helpful. The instructors also found it to be potentially funny.

The term voltage can be used to mean two things - potential and potential difference.

Below are three diagrams showing voltage sources.

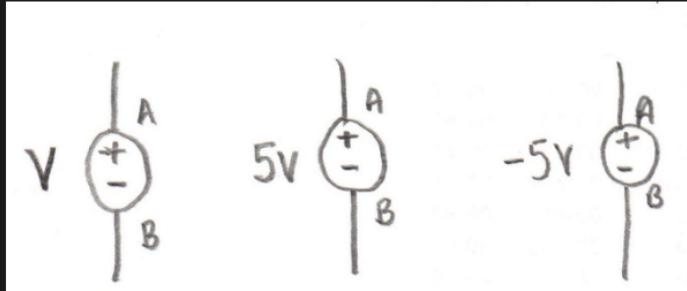


Figure 1

By convention the diagram on the left is to be interpreted as follows:

The potential (voltage) of terminal  $A$  (the terminal labelled positive) relative to terminal  $B$  (the terminal labelled negative) is  $V$  volts.

So moving from the negative terminal  $B$  to the positive terminal  $A$  there is a change of potential of  $V$  volts.

So in the middle diagram terminal  $A$  has a potential that is +5 volts relative to that of terminal  $B$ . Going from terminal  $B$  to

terminal  $A$  the voltage rises by 5 volts.

For the right hand diagram terminal  $A$  has a potential that is  $-5$  volts relative to that of terminal  $B$ . Going from terminal  $B$  to terminal  $A$  the voltage falls by 5 volts.

Note that all of the above statements are to do with potential difference and there has been no mention of reference node or zero of potential.

Now when dealing with circuits rather than having to keep saying the potential (voltage) of  $A$  relative to the potential of  $B$  one chooses a node which is at a potential of zero volts. This is called the reference node.

The diagrams below show how a choice of reference node influences various situations. (Note that the left hand and middle diagrams in Figure 2 represent the same situation as the corresponding diagrams in Figure 1, while the right hand diagram in Figure 2 represents a different situation than the right hand diagram in Figure 1.)

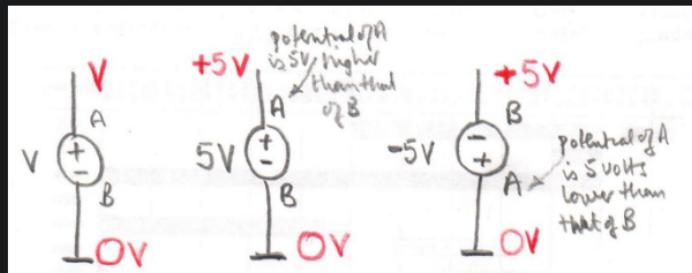


Figure 2

The middle diagram is easy to interpret since the label  $+5V$  indicates that the potential of  $A$  is  $+5$  volts above that of the negative terminal  $B$  which in turn has been chosen to be at zero volts. So the voltage of terminal  $A$  (relative to the reference node) is  $+5$  volts.

The potentially (excuse the pun) confusing diagram is the right hand one.

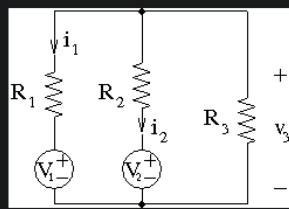
Note that in that diagram the label  $-5$  volts means that the potential of terminal  $A$  is  $5$  volts **below** that of terminal  $B$ .

Put another way the potential of terminal  $B$  is  $5$  volts **above** that of terminal  $A$ . But terminal  $A$  is at zero volts and so terminal  $B$  is at a voltage (relative to the reference node) of  $+5$  volts.

### S3E2: Circuit Voltages and Currents are Linear Combinations of Source Strengths

0 points possible (ungraded)

Consider the following network containing two voltage sources and three resistors. The device parameters are  $V_1$ ,  $V_2$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



The voltage  $v$  across resistor  $R_3$  and the currents  $i_1$  and  $i_2$  can be expressed as a linear combinations of  $V_1$  and  $V_2$ :

$$v_3 = a_1 * V_1 + a_2 * V_2$$

$$i_1 = b_1 * V_1 + b_2 * V_2$$

$$i_2 = c_1 * V_1 + c_2 * V_2$$

In each of the following, write algebraic expressions for the coefficients in terms of  $R_1$ ,  $R_2$ , and  $R_3$ .

$$a_1 =$$

$$R_2 * R_3 / (R_1 * R_2 + R_1 * R_3 + R_2 * R_3)$$

**Answer:**  $R_2 * R_3 / (R_1 * R_2 + R_1 * R_3 + R_2 * R_3)$

$a_2 =$	$R1*R3/(R1*R2+R1*R3+R2*R3)$	✓ Answer: $R1*R3/(R1*R2+R1*R3+R2*R3)$
$b_1 =$	$-(R2+R3)/(R1*R2+R1*R3+R2*R3)$	✓ Answer: $-(R2+R3)/(R1*R2+R1*R3+R2*R3)$
$b_2 =$	$R3/(R1*R2+R1*R3+R2*R3)$	✓ Answer: $R3/(R1*R2+R1*R3+R2*R3)$
$c_1 =$	$R3/(R1*R2+R1*R3+R2*R3)$	✓ Answer: $R3/(R1*R2+R1*R3+R2*R3)$
$c_2 =$	$-(R1+R3)/(R1*R2+R1*R3+R2*R3)$	✓ Answer: $-(R1+R3)/(R1*R2+R1*R3+R2*R3)$

**Explanation**

We first note that the voltage across all three branches must be equal, so we can write the following two expressions for  $v_3$  using Ohm's Law and KVL:

$$v_3 = V_1 + i_1 \cdot R_1$$

$$v_3 = V_2 + i_2 \cdot R_2$$

Next we focus on the top node and note that by KCL the sum of all currents leaving must be equal to zero. Then we can write the following:

$$v_3 = -(i_1 + i_2) \cdot R_3$$

Rewriting the first two equations to get expressions for  $i_1$  and  $i_2$ , we get:

$$i_1 = \frac{v_3 - V_1}{R_1}$$

$$i_2 = \frac{v_3 - V_2}{R_2}$$

Substituting these expressions into the third equation, we get:

$$v_3 = -\left(\frac{v_3 - V_1}{R_1} + \frac{v_3 - V_2}{R_2}\right) \cdot R_3$$

$$v_3 = \left(\frac{V_1 - v_3}{R_1}\right) \cdot R_3 + \left(\frac{V_2 - v_3}{R_2}\right) \cdot R_3$$

$$v_3 \cdot R_1 \cdot R_2 = (V_1 - v_3) \cdot R_2 \cdot R_3 + (V_2 - v_3) \cdot R_1 \cdot R_3$$

Collecting all terms containing  $v_3$  to the left hand side:

$$v_3 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3) = V_1 \cdot R_2 \cdot R_3 + V_2 \cdot R_1 \cdot R_3$$

$$v_3 = V_1 \cdot \frac{R_2 \cdot R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \frac{R_1 \cdot R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}$$

We can then substitute this expression for  $v_3$  into the first equation to get the expression for  $i_1$ :

$$i_1 = \frac{v_3}{R_1} - \frac{V_1}{R_1}$$

$$i_1 = V_1 \cdot \frac{R_2 \cdot R_3}{R_1 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} + V_2 \cdot \frac{R_1 \cdot R_3}{R_1 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} - \frac{V_1}{R_1}$$

$$i_1 = V_1 \cdot \left(\frac{R_2 \cdot R_3}{R_1 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} - \frac{1}{R_1}\right) + V_2 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}$$

$$R_1 \cdot R_3 = (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3) \cdot R_1$$

$$\begin{aligned}
i_1 &= V_1 \cdot \frac{-e_2 - e_3 - (e_1 - e_2 + e_1 - e_3 + e_2 - e_3)}{R_1 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} + V_2 \cdot \frac{-e_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} \\
i_1 &= V_1 \cdot \frac{-R_1 \cdot (R_2 + R_3)}{R_1 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} + V_2 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} \\
i_1 &= V_1 \cdot \frac{-(R_2 + R_3)}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}
\end{aligned}$$

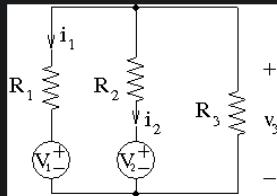
The expression for  $i_2$  can likewise be found by substitution:

$$\begin{aligned}
i_2 &= \frac{v_3}{R_2} - \frac{V_2}{R_2} \\
i_2 &= V_1 \cdot \frac{R_2 \cdot R_3}{R_2 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} + V_2 \cdot \frac{R_1 \cdot R_3}{R_2 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} - \frac{V_2}{R_2} \\
i_2 &= V_1 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \left( \frac{R_1 \cdot R_3}{R_2 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} - \frac{1}{R_2} \right) \\
i_2 &= V_1 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \frac{R_1 \cdot R_3 - (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)}{R_2 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} \\
i_2 &= V_1 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \frac{-R_2 \cdot (R_1 + R_3)}{R_2 \cdot (R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} \\
i_2 &= V_1 \cdot \frac{R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} + V_2 \cdot \frac{-(R_1 + R_3)}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}
\end{aligned}$$

### S3E3: Circuit Variables are Superpositions of values due to each source separately

0 points possible (ungraded)

Consider the familiar network containing two voltage sources and three resistors. You are given that  $R_1 = 7.0\Omega$ ,  $R_2 = 3.0\Omega$ ,  $R_3 = 5.0\Omega$ ,  $V_1 = 2.0V$ , and  $V_2 = 8.0V$ .



The voltage  $v_3$  across resistor  $R_3$  can be expressed as the sum of the voltage  $x_1$  due to  $V_1$  acting alone and the voltage  $x_2$  due to  $V_2$  acting alone.

The value (in Volts) of  $x_1$  is:

✓ Answer: 0.4225352112676056

The value (in Volts) of  $x_2$  is:

✓ Answer: 3.943661971830986

Similarly, the current  $i_1$  into the resistor  $R_1$  can be expressed as the sum of the current  $y_1$  due to  $V_1$  acting alone and the current  $y_2$  due to  $V_2$  acting alone. (NOTE:  $y_1$  and  $y_2$  are not labeled in the diagram, but  $i_1 = y_1 + y_2$ .)

The value (in Amperes) of  $y_1$  is:

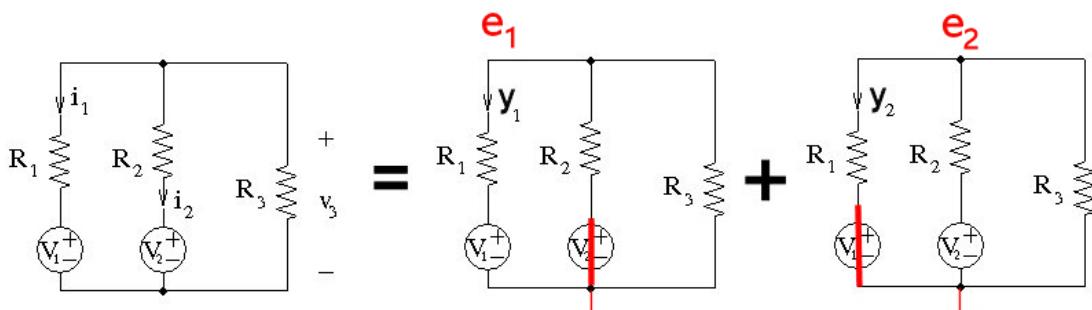
✓ Answer: -0.22535211267605634

The value (in Amperes) of  $y_2$  is:

✓ Answer: 0.5633802816901409

Surely this is true of every other circuit voltage and current, such as  $i_2$  (but NOT device parameters).

We will not plague you with working them all out. :-)



```

In[6]:= Rs[r_List] := Total[r];
Rp[r_List] := 1/Total[1/r];
r1 = 7; r2 = 3; r3 = 5; v1 = 2; v2 = 8;
Solve[ $\frac{x_1 - v_1}{r_1} + \frac{x_1}{r_2} + \frac{x_1}{r_3} = 0, x_1$ ] // N
Solve[ $\frac{x_2}{r_1} + \frac{x_2 - v_2}{r_2} + \frac{x_2}{r_3} = 0, x_2$ ] // N
y1 =  $\frac{x_1 - v_1}{r_1}$  /. {x1 → 0.4225352112676056`}
y2 =  $\frac{x_2}{r_1}$  /. {x2 → 3.943661971830986`}
Out[6]= { {x1 → 0.422535} }

Out[6]= { {x2 → 3.94366} }

Out[6]= -0.225352

Out[6]= 0.56338

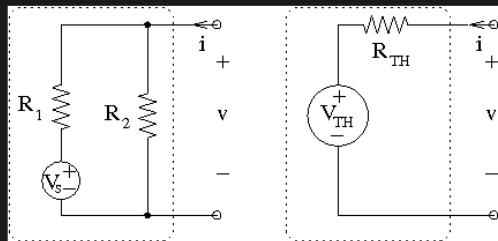
```

### S3E4: Simple Thevenin

0 points possible (ungraded)

Given the circuit below with the indicated terminals, you will construct a Thevenin equivalent circuit.

The device parameters of the circuit to be modeled are  $V_S = 5.0V$ ,  $R_1 = 56000.0\Omega$ , and  $R_2 = 18000.0\Omega$ .



What is the Thevenin open-circuit voltage  $V_{TH}$  (in Volts)?

1.21622

✓ Answer: 1.2162162162162162

What is the Thevenin equivalent resistance  $R_{TH}$  (in Ohms)?

13621.6

✓ Answer: 13621.621621621622

$V_S = 5; r1 = 56000; r2 = 18000;$

(\*Here VTH is actually the voltage portion in  $R2=V_S * (R2 / (R1+r2)) . . .$ \*)

$Vth = V_S * (r2 / (r1 + r2)) // N$

$rth = Rp[{r1, r2}] // N$

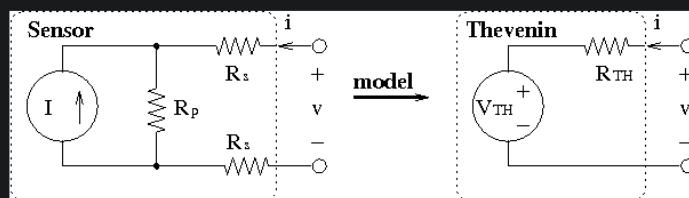
Out[6]= 1.21622

Out[6]= 13621.6

### S3E5: Thevenin Model

0 points possible (ungraded)

A certain light sensor is modeled as a current source that produces a current proportional to the intensity of light falling on it. There is leakage through the sensor, modeled by a resistor  $R_p = 1.8\text{M}\Omega$  in parallel with the current source. There is resistance in the contacts to the sensor, modeled by the series resistances  $R_s = 17.0\text{k}\Omega$ . In this experiment the light produces a current of  $I = 1.430000000000002\mu\text{A}$ .



It is useful to summarize this model by constructing a Thevenin equivalent circuit for it.

What is the Thevenin voltage  $V_{TH}$  of the sensor (in Volts)?

2.574

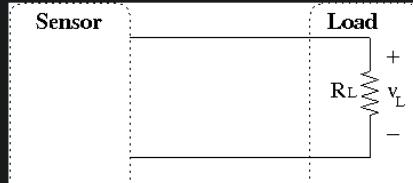
✓ Answer: 2.5740000000000003

What is the Thevenin resistance  $R_{TH}$  of the sensor (in  $\text{M}\Omega$ )?

1.836

✓ Answer: 1.834

The sensor is intended to be used in a light meter. In this application the output of the sensor is connected to a system that amplifies the signal and displays it, scaled to appropriate units.



The amplifier is designed to present a resistance of  $R_L = 100.0\text{k}\Omega$ . So, from the point of view of the sensor we can model it as just a resistor.

What is the voltage  $V_L$  presented to the amplifier (in Volts)?

0.133092

✓ Answer: 0.1330920372285419

Note that the circuit connecting the sensor to the amplifier and display does not care about the insides of either the sensor or of the the amplifier and display. So we don't have to consider those complications to compute the properties of their interconnection. This is the modeling power of the Thevenin idea!

```
In[]:= rp = 1.8*^6; rs = 17*^3; i = 1.430000000000002*^-6;
(*When you calculate VTH,note that you need to assume an "open circuit".Thus
there will be NO current in the two resistors (Rs).Therefore,VTH=I*Rp*)
vth = i * rp
rth = rp + 2 rs
rl = 100*^3;
(*There is a series circuit consisting of a voltage source Vth
and two resistors Rth and RL. Find the current in this circuit
and hence VL or treat the circuit as a potential divider.*)
Icircuit = vth / (rth + rl);
vl = Icircuit * rl

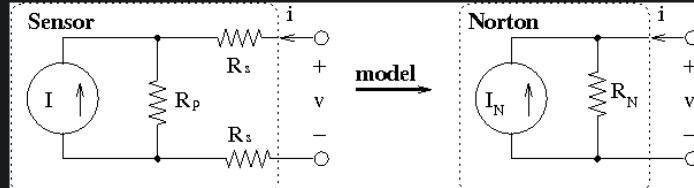
Out[]= 2.574

Out[=] 1.834 × 106

Out[=] 0.133092
```

## S3E6: Norton Model

0 points possible (ungraded)

This is the same circuit as in the Thevenin exercise, with  $R_p = 1.8\text{M}\Omega$ ,  $R_s = 17.0\text{k}\Omega$ , and  $I = 1.4300000000000002\mu\text{A}$ .

In this case, we want to summarize the circuit with a Norton model.

What is the Norton current  $I_N$  of the sensor (in  $\mu\text{A}$ )?

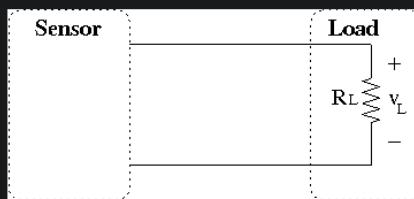
1.40349

✓ Answer: 1.4034896401308616

What is the Norton resistance  $R_N$  of the sensor (in  $\text{M}\Omega$ )?

1.834

✓ Answer: 1.834

Again, we present a load as a resistance  $R_L = 100.0\text{k}\Omega$ .What is the voltage  $V_L$  presented to the amplifier (in Volts)?

0.133092

✓ Answer: 0.1330920372285419

 $rp = 1.8 * ^6; rs = 17 * ^3; i = 1.4300000000000002 * ^{-6};$  $vth = i * rp;$  $rth = rp + 2 * rs;$  $in = vth / rth$  $rth$ 

(\*Potential divider\*)

$$vl = \frac{vth * rl}{rl + rth}$$

 $Out[=] = 1.40349 \times 10^{-6}$  $Out[=] = 1.834 \times 10^6$  $Out[=] = 0.133092$

## S4E2: Boolean Functions

0 points possible (ungraded)

How many distinct values can be represented with two boolean-valued signals?

4

✓ Answer: 4

How many distinct boolean-valued functions are there of two boolean-valued signals?

16

✓ Answer: 16

How many distinct values can be represented with three boolean-valued signals?

8

✓ Answer: 8

How many distinct boolean-valued functions are there of three boolean-valued signals?

256

✓ Answer: 256

```
In[]:= (*Number of bits*)
n = 2;
(*Number of outputs*)
2^n
(*Number of boolean functions mapping input→output*)
2^(2^n)
```

```
n = 3;
2^n
2^(2^n)
```

Out[]:= 4

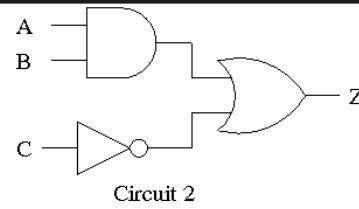
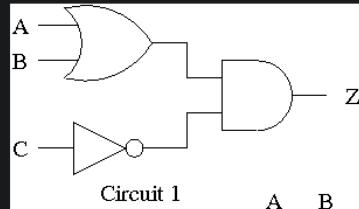
Out[]:= 16

Out[]:= 8

Out[]:= 256

## S4E3: Truth Table

0 points possible (ungraded)



A	B	C	Z
0	0	0	1
0	0	1	a
0	1	0	b
0	1	1	c
1	0	0	d
1	0	1	e
1	1	0	f
1	1	1	g

In the figure above there are two circuits and a partially filled in truth table. From the information in the truth table you can decide which circuit is described by that truth table. Enter the number of the chosen circuit:

 ✓

What is the entry in the box labeled *a*?

 ✓

What is the entry in the box labeled *f*?

 ✓

What is the entry in the box labeled *d*?

 ✓

What is the entry in the box labeled *b*?

 ✓

```
In[=]:= c1 = Boole[BooleanTable[{a, b, c, (a || b) && ! c}, {a, b, c}]] // TableForm
c2 = Boole[BooleanTable[{a, b, c, (a && b) || ! c}, {a, b, c}]] // TableForm
Out[=]/TableForm=
1 1 1 0
1 1 0 1
1 0 1 0
1 0 0 1
0 1 1 0
0 1 0 1
0 0 1 0
0 0 0 0

Out[=]/TableForm=
1 1 1 1
1 1 0 1
1 0 1 0
1 0 0 1
0 1 1 0
0 1 0 1
0 0 1 0
0 0 0 1
```

## Homework

 MITx 6.002.1x  
Circuits and Electronics 1: Basic Circuit Analysis

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### Homework 2

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Homework due Jul 22, 2022 22:34 +04 Completed

#### H2P1: Current Divider

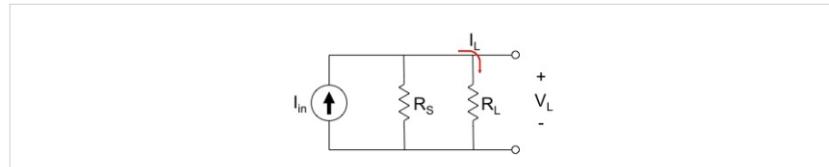
4/4 points (graded)

The resistances of commercially-available discrete resistors are restricted to particular sets. For example, the available values of resistors with 10% tolerance are selections from the *E12* set multiplied by a power of ten from  $10^0$  through  $10^5$ . The *E12* set is:

$$E12 = \{10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82\}$$

Thus, you can buy 10% resistors with a nominal resistance of  $330\Omega$  or  $33k\Omega$ , but not  $350\Omega$ . Furthermore, the "tolerance" means that if you buy a 10%  $390\Omega$  resistor you can be sure that its resistance is between  $351\Omega$  and  $429\Omega$ .

In this problem we need to choose 10% resistors to make a current divider that meets a given specification.



We are given an input current  $I_{in} = 100\mu A$  and a linear resistive load represented by the resistor  $R_L$ . Our resistive load is

not regulated for the high current provided by our current source, so we add a resistor  $R_S$  in parallel to divide the current such that  $I_L \approx 40\mu A$ . An additional requirement is that the Thevenin resistance as seen from the load terminals (with both  $R_S$  and  $R_L$  connected) is between  $60k\Omega$  and  $80k\Omega$ . Assume first that the resistors have their nominal resistance. Come up with resistors  $R_S$  and  $R_L$  such that the divider ratio  $I_{L,nom}/I_m$  is within 10% of the requirement, where  $I_{L,nom}$  is the load current using the nominal resistance values.

Of course, the resistances you chose are just nominal. Given that they are only guaranteed to have resistances within 10% of the nominal value, what is the largest and smallest value that  $I_L$  may have?

Enter your values below. (NOTE: Both resistor values are needed for a correct answer.)

$R_S$  (in Ohms):

 ✓

$R_L$  (in Ohms):

 ✓

$I_{Lmax}$  (in Amps):

 ✓

$I_{Lmin}$  (in Amps):

 ✓

#### Explanation:

For this solution, we are given that  $I_{in} = 100\mu A$ ,  $I_L = 40\mu A$ ,  $60k\Omega \leq R_{TH} \leq 80k\Omega$ .

We apply the current divider equation:

$$I_L = \frac{I_{in}R_S}{R_S + R_L} \rightarrow 40\mu A = 100\mu A \frac{R_S}{R_S + R_L}$$

$$\therefore \frac{100R_S}{40} = R_S + R_L, \text{ or } 3R_S = 2R_L$$

Since we know that the chosen resistors must obey the ratio 3:2, we choose resistances from the list of nominal resistances. Choosing  $R_S = 120k\Omega$  and  $R_L = 180k\Omega$ , we approximately observe this resistor ratio.

To verify that the equivalent thevenin resistance is within the required limit, we simply calculate the equivalent resistance of these two resistors as if they were in parallel, which is given by  $R_{TH} = \frac{R_S R_L}{R_S + R_L} = 72k\Omega$

Finally, to calculate the maximum and minimum voltages, we apply the following formula:

$$I_{L,max} = \frac{I_{in} \times (1.1)(R_S)}{(0.9)(R_L) + (1.1)(R_S)} = 44.8\mu A$$

$$I_{L,min} = \frac{I_{in} \times (.9)(R_S)}{(1.1)(R_L) + (.9)(R_S)} = 35.2\mu A$$

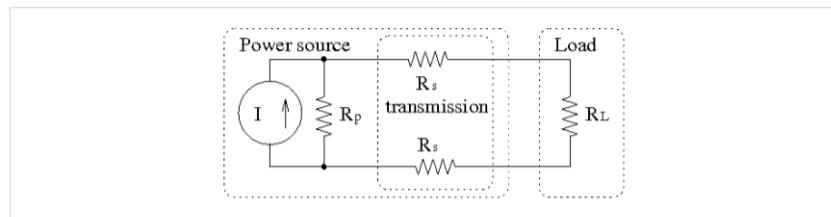
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## H2P2: Solar Power

3/3 points (graded)

A simple model of a photovoltaic solar cell is a current source, with the current proportional to the amount of sunlight falling on it. A more accurate model includes a diode (a nonlinear element we will see later). There is some leakage current that we can model with a parallel resistor, and there is a voltage drop in the interconnect that we can model with series resistances connecting to the load resistor. So a crude model of a complete system might be the circuit shown below.



In this system we have  $I = 0.2A$ ,  $R_p = 3.0\Omega$ , and  $R_s = 1.8\Omega$ .

You are to determine the load resistance,  $R_L$ , for which the maximum power is transferred to the load. (Hint: remember your calculus!)

What is this optimum load resistance (in Ohms)?

✓ Answer: 6.6

What is the power (in Watts) that is delivered to this best load resistance?

✓ Answer: 0.01364

What is the Thevenin equivalent resistance (in Ohms) of the power source as seen by the load resistance?

✓ Answer: 6.6

Hmmmmm.

**Explanation:**

The optimum load resistance occurs when the maximum power is transferred. To find that point, we take the derivative of an expression for power with respect to the load resistance.

But first, we can convert the Norton sub-circuit on the left side of the circuit diagram to its Thevenin equivalent, which gives us a voltage  $V_{Th} = (0.2)(3.0) = 0.6 \text{ V}$ . The  $R_{Th\text{new}}$  of the entire new circuit will be:  $R_s + R_p + R_s + R_L$ . The equation for power consumed by the load is  $P = I^2 R_L$ , where:

$$I = \frac{V}{R_{Th\text{new}}} = \frac{0.6}{R_s + R_p + R_s + R_L}$$

Setting the derivative of the expression for power equal to zero and solving for the load resistance will give us the optimal load resistance for maximum power transfer.

$$\frac{dP}{dR_L} = \frac{d\left(\left[\frac{0.6}{R_s + R_p + R_s + R_L}\right]^2 \times R_L\right)}{dR_L} = 0$$

Simplifying the power expression by substituting known variables and re-naming  $R_L$  as  $x$ :

$$\begin{aligned} \frac{d}{dx} \left( \frac{0.36x}{(6.6+x)^2} \right) &= 0 \\ = 0.36 \left( \frac{d}{dx} \left[ \frac{x}{(x+6.6)^2} \right] \right) \end{aligned}$$

Applying product rule:

$$\begin{aligned} &= 0.36 \left( x \left( \frac{d}{dx} \left[ \frac{1}{(x+6.6)^2} \right] \right) + \frac{\frac{d}{dx}(x)}{(x+6.6)^2} \right) \\ &= 0.36 \left( x \left( \frac{d}{dx} \left[ \frac{1}{(x+6.6)^2} \right] \right) + \frac{1}{(x+6.6)^2} \right) \\ &= 0.36 \left( x \left[ \frac{-2}{(x+6.6)^3} \right] + \frac{1}{(x+6.6)^2} \right) \\ &= 0.36 \left( \frac{-2x}{(x+6.6)^3} + \frac{1}{(x+6.6)^2} \right) \end{aligned}$$

Equating the expression to zero, we solve for  $x$ :

$$\begin{aligned} 0 &= \frac{-2x}{(x+6.6)^3} + \frac{1}{(x+6.6)^2} \\ \therefore \frac{1}{(x+6.6)^2} &= \frac{2x}{(x+6.6)^3} \\ \therefore x+6.6 &= 2x \\ \therefore x &= R_L = 6.6\Omega \end{aligned}$$

To solve for power consumed, we simply use the following expression:

$$\begin{aligned} P &= I^2 R_L \rightarrow I = \frac{0.6V}{13.2\Omega} \text{ and } R_L = 6.6\Omega \\ &= 0.01364 W \end{aligned}$$

$R_{TH}$  seen by the load resistance is:  $R_s + R_p + R_s = 6.6\Omega$

Note that the optimum load resistance is the same as the thevenin equivalent resistance.

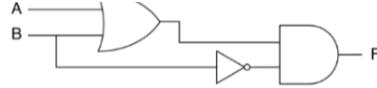
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## H2P3: Logic Circuits

5/5 points (graded)



Circuit 1

In the figure above, Circuit 1 is a two-input single-output logic circuit. The truth table for a two-input function has four rows as shown below:

A	B	F
0	0	w
0	1	x
1	0	y
1	1	z

Fill in the values of F in the truth table for Circuit 1.  
What is the entry in the box labeled w?

0

✓ Answer: 0

What is the entry in the box labeled x?

0

✓ Answer: 0

What is the entry in the box labeled y?

1

✓ Answer: 1

What is the entry in the box labeled z?

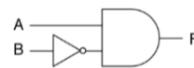
0

✓ Answer: 0

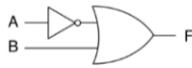
The figure below shows four additional two-input single-output logic circuits. One of these circuits is equivalent to Circuit 1; i.e., they compute the same logic function.



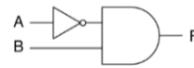
Circuit 2



Circuit 3



Circuit 4



Circuit 5

Which of these circuits is equivalent to Circuit 1? (Enter the number of the circuit)

3

✓ Answer: 3

We see there are many different logic circuits that can implement the same functionality. Using equations instead of truth tables help us simplify logic circuits. You can write an equation from Circuit 1 and simplify the equation to obtain the simpler equivalent logic circuit found in Circuits 2-5.

#### Explanation:

Circuit 1 displays a circuit which evaluates the boolean expression  $F = (A + B) \cdot \bar{B}$

Let's evaluate this boolean expression using a truth table in steps:

1. evaluate  $(A + B)$
2. evaluate  $\bar{B}$
3. evaluate  $(A + B) \cdot \bar{B}$

A	B	$(A + B)$	$\bar{B}$	$(A + B) \cdot \bar{B}$
0	0	0	1	0
0	1	1	0	0
1	0	1	1	1
1	1	1	0	0

Thus, we find that  $w = 0$ ,  $x = 0$ ,  $y = 1$ ,  $z = 0$

Now we determine which of the remaining four circuits is equivalent to circuit 1. In circuits 2 and 4, the signal 'F' corresponds to the output of an 'or' gate, while 'F' in circuits 3 and 5 correspond to the output of an 'and' gate. We can rule out 2 and 4 because the desired F only has one '1' value, which corresponds to the output of an 'and' gate.

In order to pick from circuits 3 and 5, let's evaluate the boolean expression for each:

- Circuit 3:  $F = A \cdot \bar{B}$
- Circuit 5:  $F = \bar{A} \cdot B$

We can simplify our original expression from circuit 1 using boolean logic:

$$\begin{aligned}
 & (A + B) \cdot \bar{B} \\
 & A \cdot \bar{B} + B \cdot \bar{B} \text{ (distributive property)} \\
 & A \cdot \bar{B} + 0 \text{ (complement law - i.e. } A \cdot \bar{A} = 0\text{)} \\
 & A \cdot \bar{B} \text{ (identity law - i.e. } A + 0 = A\text{)}
 \end{aligned}$$

Thus, we see the boolean expression of circuit 1 simplifies to the boolean expression of circuit 3. They must be equivalent circuits.

As an alternative way to see the equivalence, let's make a truth table for circuit 3:

A	B	$\bar{B}$	$A \cdot \bar{B}$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

We see that Circuit 3 evaluates to the same final column (the signal F) in the truth table, and thus is an equivalent circuit

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- H2P2 13
  - ? H2P2 I can get VTH= \*\*xxxx\*\* V, RTH=\*\*xxxx\*\*, IL= \*\*xxxx\*\*, but I don't know how to use Thevenin Method to determine RL, I'm stuck. \*\*Please do... 2
  - ? H2P1: Current Divider Sorry I dont get it: First I made a Norton circuit And I Use RN the values 60kohm and 80 Kohm and I get 2 values of VN, with that values I get 2 v... 3
  - ? Need advise for this problem. Need help with implementation of variables. Thank you. 2
  - ? ILmax vs ILmin I got ILmax and ILmin correctly after I switched them. My ILmax was accepted correctly in the ILmin box and vice versa. I don't know why this is ... 2
  - ? H2P3 F= \*\*xxxx\*\* but the answer was \*\*[xxxx]\*\*? \*\*Please do not post answers.\*\* 4
  - ? PRACTICE QUESTIONS Are there similar practice questions with answers we can practice before we attempt the homework??, I struggling to find the answers and I'm bur... 2
  - ? H2P2 resistor Not to sure how to get RL in this question, is it okay if someone could help? 1
  - ? H2P1 via Voltage Hi all! I got the right answers for IL max and min, but I am wondering if there is another way to do it. I set up a Norton equation with IN=Iin and R... 3
  - ? H2P@ I really tried a lot in this question. but still didn't get ? What I am suppose to do 2
  - ? [STAFF] H2P2 Question Why can't I use P=V\_L^2/R\_L to determine maximum power, where R\_L is optimum load resistance? 9
  - ? [Staff] Can the course director provide explanations for each question? I would really like to know the detailed answer and explanation for each question. It would be helpful for me to understand which step I've done ... 2
  - ? Forgot about calculus and couldn't get the MAX power. Couldn't see the answers either Can anyone gives a hint about how to calculate dp/drL=0? 1

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