Week 5

S9 - MOSFETs: Large Signals

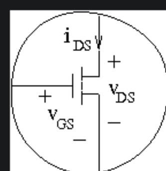
S10 - MOSFET Amplifiers: Small Signal Model

Lectures

S9E1: MOSFET model

0 points possible (ungraded)

Joe found the following n-channel enhancement-mode MOSFET in a piece of apparatus he was working on. The markings on it indicated that it was a TN0205A, which has a threshold voltage $V_T \approx 1.0$V and $K \approx 0.4$A/V^2, using the SCS model.



In operation, Joe measured the gate-to-source voltage $v_{GS} = 1.2$V and the drain-to-source voltage $v_{DS} = 2.5$V. What region is the MOSFET operating in? Your answer should be one of "cutoff", "saturated", or "triode":

saturated ✔ **Answer:** saturated

What is the approximate drain current i_{DS} (in mA) that Joe should expect?

8 ✔ **Answer:** 7.9999999999999964

What is the minimum value that v_{DS} may have (in Volts) for the MOSFET to be operating in saturation?

0.2 ✔ **Answer:** 0.19999999999999996

`(*From textbook A&L*)`

The constraint curve separating the triode and saturation regions in Figure 7.16 given by

$$v_{DS} = v_{GS} - V_T \tag{7.10}$$

can also be rewritten in terms of i_{DS} and v_{DS} by substituting $v_{DS} = (v_{GS} - V_T)$ in Equation 7.8 as follows:

$$i_{DS} = \frac{K}{2} v_{DS}^2. \qquad (7.11)$$

The following is a summary of the SCS model of the MOSFET in algebraic form. The model applies only in the saturation region of MOSFET operation, that is, when $v_{DS} \geq v_{GS} - V_T$.

$$i_{DS} = \begin{cases} \dfrac{K(v_{GS} - V_T)^2}{2} & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \\ 0 & \text{for } v_{GS} < V_T. \end{cases} \qquad (7.12)$$

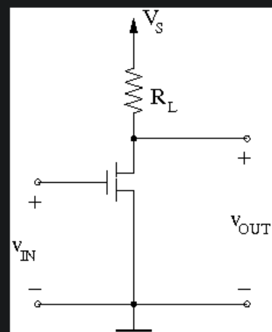
```
vgs = 1.2; vds = 2.5; vt = 1; k = 0.4;
(*The If statement doesnt quite do what we want,
but I'm showing the options for completeness*)
reg = If[vds > vgs - vt, "saturated", "triode", "cutoff"]
(*SCS model of the MOSFET, valid for saturation region*)
ids = k (vgs - vt)^2 / 2
(*Cutoff voltage V_DS*)
vgs - vt
```

```
Out[•]= saturated
Out[•]= 0.008
Out[•]= 0.2
```

S9E2: Amplifier 1

0 points possible (ungraded)

Consider the simple common-source MOSFET amplifier:



In the spaces provided enter the algebraic expression for your answer.
What is the minimum value that v_{IN} can have for the MOSFET to operate in saturation? Express your answer in terms of one or more of the circuit parameters V_S, V_T and R_L.

Be careful, algebraic expressions are case sensitive, and remember to use the format "vXYZ" to specify v_{XYZ} when you have subscripts in the answer boxes.

VT ✔

V_T

What is the minimum value that v_{OUT} can have for the MOSFET to operate in saturation? Express your answer in terms of one or more of the circuit parameters V_S, v_{IN}, V_T and R_L. Remember that algebraic expressions are case sensitive.

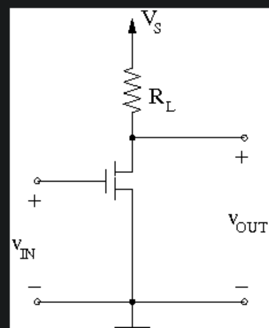
vIN-VT ✔

$v_{\text{IN}} - V_T$

S9E3: MOSFET Amplifier 2

0 points possible (ungraded)

Consider the simple common-source MOSFET amplifier again:



In the spaces provided enter the algebraic expression for your answer.
What is the maximum value that v_{IN} can have for the MOSFET to operate in saturation? Express your answer in terms of the device parameters K, V_T, R_L, and V_S.
And remember, algebraic expressions are case sensitive, and use the format "vXYZ" to specify v_{XYZ} when you have subscripts in the answer boxes.

(sqrt(1+2*K*RL*VS)-1)/(K*RL)+VT ✔

$$\frac{\sqrt{1+2\cdot K\cdot R_L\cdot V_S}-1}{K\cdot R_L}+V_T$$

Summarizing, the maximum valid input voltage range is

$$V_T \to \frac{-1+\sqrt{1+2V_S R_L K}}{R_L K}+V_T$$

and the maximum valid output voltage range is

$$V_S \to \frac{-1+\sqrt{1+2V_S R_L K}}{R_L K}.$$

S10E1: Incremental Voltage

0 points possible (ungraded)

For the simple common-source amplifier the lecturer just derived expressions for the operating point (bias) output voltage V_O as

$$V_O = V_S - \frac{R_L \cdot K}{2} \cdot (V_I - V_T)^2$$

and for the total output voltage v_O, including the incremental output voltage v_o

$$v_O = V_O + v_o = V_S - \frac{R_L \cdot K}{2} \cdot \left[(V_I - V_T)^2 + 2 \cdot (V_I - V_T) \cdot v_i + v_i^2\right]$$

In the space provided below write an algebraic expression for the best linear approximation to the incremental output voltage v_o, assuming that the incremental input voltage v_i is very small. Express your answer in terms of the device parameters K, V_T, R_L, the bias voltages V_S and V_I, and the incremental input voltage v_i. Be careful, algebraic expressions involving incremental quantities are case sensitive.

-RL*K*(VI-VT)*vi ✔ **Answer:** -RL*K*(VI-VT)*vi

$-R_L \cdot K \cdot (V_I - V_T) \cdot v_i$

Explanation:

$$\begin{aligned} v_O = V_O + v_o &= V_S - \frac{1}{2} \cdot R_L \cdot K \cdot ((V_I + v_i) - V_T)^2 \\ &= V_S - \frac{1}{2} \cdot R_L \cdot K \cdot \left((V_I - V_T)^2 + 2 \cdot (V_I - V_T) \cdot v_i + v_i^2\right) \end{aligned}$$

$V_O = V_S - \frac{1}{2} \cdot R_L \cdot K \cdot (V_I - V_T)^2$ and v_i^2 is negligible.
Therefore, we have $v_o \approx -R_L \cdot K \cdot (V_I - V_T) \cdot v_i$.

$$\begin{aligned} v_O &= f(v_I) \\ &= V_S - K\frac{(v_I - V_T)^2}{2} R_L. \end{aligned}$$

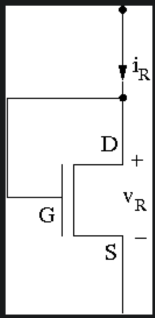
As before, let $v_i = \Delta v_I$ denote a small change in the input voltage, and let $v_o = \Delta v_O$ denote the corresponding change in the output voltage. Then,

$$\begin{aligned} v_o &= \left.\frac{df(v_I)}{dv_I}\right|_{v_I = V_I} v_i \\ &= -K(v_I - V_T)R_L\Big|_{v_I = V_I} v_i \\ &= -K(V_I - V_T)R_L v_i \\ &= -g_m R_L v_i. \end{aligned}$$

S10E2: Two terminal connection

0 points possible (ungraded)

Consider a two-terminal device formed by a MOSFET with its gate tied to its drain, as illustrated in the figure below.



The MOSFET is characterized by parameters V_T and K, and its drain-to-source voltage and drain current are denoted as v_R and i_R, respectively. You can assume the following parameters: $V_T = 0.5$V and $K = 90.0\frac{mA}{V^2}$.

If $v_R = 2.7$V, what is i_R (in mA)?

217.8 ✔ **Answer:** 217.8

Develop a small-signal model for this device around the operating point $V_R = 2.7$, describing the relationship between v_r and i_r.

Using this model, find v_r, if $i_r = 6.0$mA. What is v_r (in mV)?

30.303 ✔ **Answer:** 30.3030303030303

Grove (Community TA)
2 years ago - endorsed 2 years ago by **MIT_Lover_UA** (Staff)

From the diagram $v_{\mathrm{GS}} = v_{\mathrm{DS}} = v_{\mathrm{R}}$ and you have the relationship $i_{\mathrm{DS}} = \frac{K}{2}(v_{\mathrm{GS}} - V_{\mathrm{T}})^2 \Rightarrow i_{\mathrm{R}} = \frac{K}{2}(v_{\mathrm{R}} - V_{\mathrm{T}})^2$ in this problem.

In question 1 you are asked, for a given v_{R} find i_{R}.
These are the operating voltage and current.

For question 2 you are asked to find the change in v_{R}, which is $\Delta v_{\mathrm{R}} = v_{\mathrm{r}}$, if the current i_{R} changes, $\Delta i_R = i_{\mathrm{r}}$, by a given amount.

Two ways to do Question 2.

- For the given change in current i_{r} use the equation $i_{\mathrm{R}} \pm i_{\mathrm{r}} = \frac{K}{2}(v_{\mathrm{R}} \pm v_r - V_{\mathrm{T}})^2$ to find the corresponding change in voltage v_{r}. A solution to a quadratic equation will be involved.
- Use the equation $i_{\mathrm{R}} = \frac{K}{2}(v_{\mathrm{R}} - V_{\mathrm{T}})^2$ and find $\frac{di_{\mathrm{R}}}{dv_{\mathrm{R}}} = \frac{i_{\mathrm{r}}}{v_{\mathrm{r}}}$ from which having been given i_{r} you can find v_r in **milli** volts.

In[∘]:=
```
vt = 0.5; k = 90;(*Leave K in mA units*)
ir[vr_] := k/2 (vr - vt)^2;
ir[2.7]
Solve[ir'[2.7] == 6/vr, vr]
(*Must convert v_r from V to mV*)
```

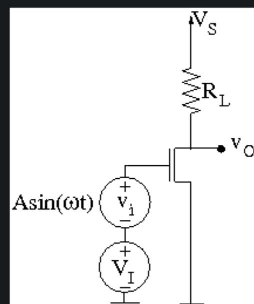
Out[∘]= 217.8

Out[∘]= {{vr → 0.030303}}

S10E3: Small Signal Amplifier

0 points possible (ungraded)

Consider the amplifier shown below:



The MOSFET operates in its saturation region and is characterized by the parameters $V_T = 0.5$V and $K = 20.0\frac{mA}{V^2}$. The input voltage v_I is the sum of a DC bias voltage V_I and a sinusoid of the form $v_i = A\sin(\omega t)$, where A is very small compared to V_I. The output voltage v_O is the sum of a DC bias term V_O and a small-signal response term v_o.

The supply voltage $V_S = 22.0$V and the resistance of the drain resistor $R_L = 140.0\Omega$. The input bias voltage is $V_I = 2.7$ V.

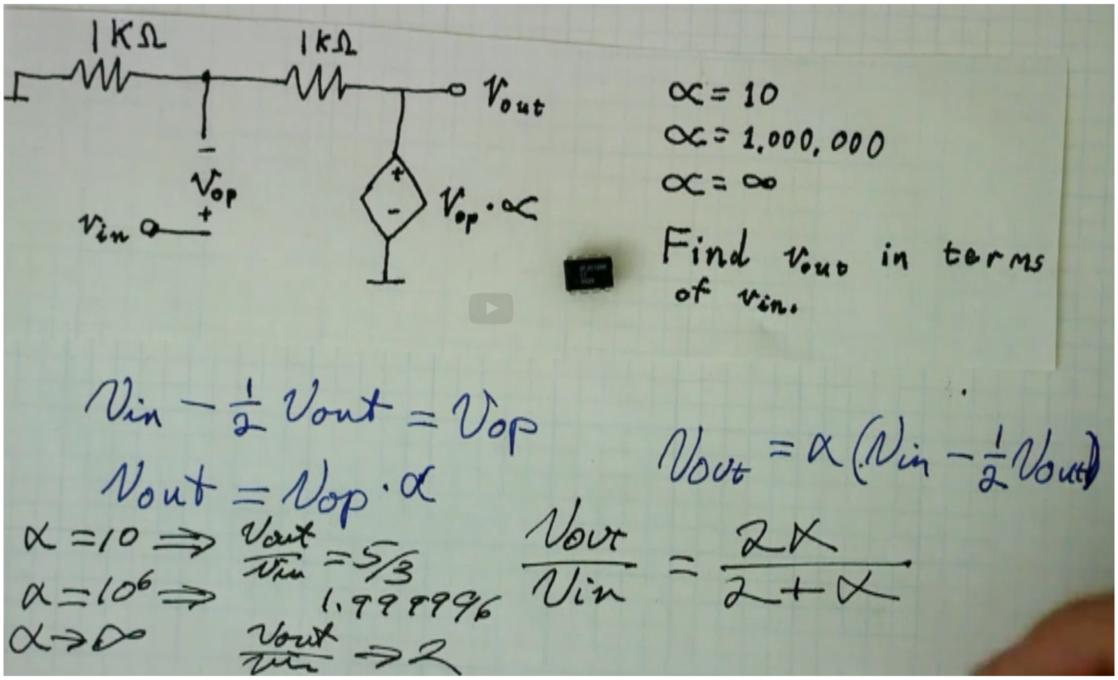
What is the output operating-point bias voltage V_O (in Volts)?

15.224 ✔ **Answer:** 15.224

What is the small-signal gain v_o/v_i of the amplifier?

-6.16 ✔ **Answer:** -6.160000000000001

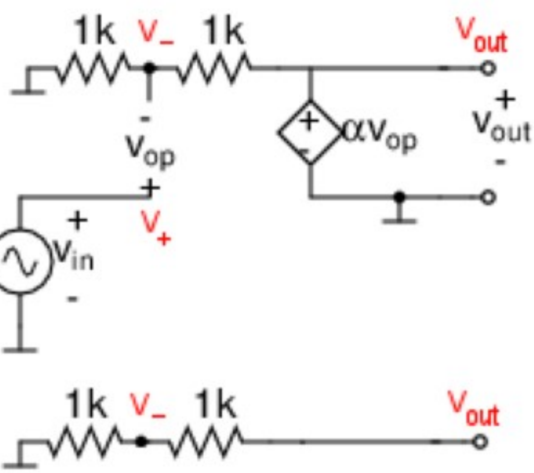
Explanation:
The current through the MOSFET can be found as:
$I_D = \frac{1}{2} \cdot K \cdot (V_{GS} - V_T)^2 = \frac{1}{2} \cdot K \cdot (V_I - V_T)^2 = \frac{1}{2} \cdot (20mA/V^2) \cdot (2.7V - 0.5V)^2 = 48.4mA$
By KCL, the current through the resistor R_L must be equal to the current through the MOSFET:
$V_O = V_S - I_D \cdot R_L = (22.0V) - (48.4mA) \cdot (140.0\Omega) = 15.224V$
The small-signal gain of the amplifier can be found by taking the deriviative of the expression for $v_O = V_O + v_o$ with respect to $v_I = V_I + v_i$:
$v_O = V_S - i_D \cdot R_L = V_S - R_L \cdot \left(\frac{1}{2} \cdot K \cdot (v_{GS} - V_T)^2\right)$
$v_O = V_S - R_L \cdot \left(\frac{1}{2} \cdot K \cdot (v_I - V_T)^2\right) = V_S - R_L \cdot (\frac{1}{2} \cdot K \cdot (v_I^2 - 2 \cdot V_T \cdot v_I + V_T^2))$
$\frac{dv_O}{dv_I}|_{v_I = V_I} = -\frac{1}{2} \cdot K \cdot R_L \cdot (2 \cdot V_I - 2 \cdot V_T) = -K \cdot R_L \cdot (V_I - V_T)$
$\frac{v_o}{v_i} = -(20.0\frac{mA}{V^2}) \cdot (140.0\Omega) \cdot (2.7V - 0.5V) = -6.160000000000001$



Grove (Community TA)

2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)

In essence the circuit boils down to be a potential divider (bottom circuit) with $v_- = \frac{1k}{1k+1k} v_{\text{out}} = \frac{1}{2} v_{\text{out}}$



For an op amp with a large gain so $v_- \approx v_+$ and in this circuit $v_+ = v_{\text{in}}$ so $v_{\text{in}} = \frac{1}{2} v_{\text{out}}$

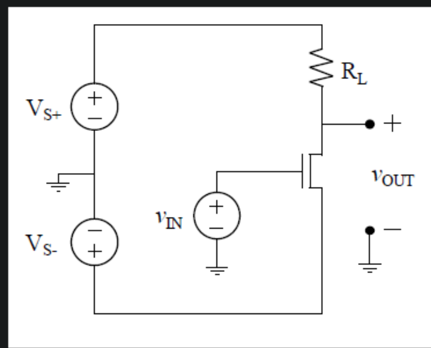
Homework

Homework due Aug 9, 2022 15:35 +04

H1P1: Zero Offset Amplifier

3/3 points (graded)

In many amplifiers, we use dual power supplies so we can obtain a 0V offset at the output, that is, the DC operating point at the output is 0V. An example amplifier circuit that can achieve this is shown below.



In this circuit, $V_{S+} = 1.0V$, $V_{S-} = -1.0V$, and the MOSFET parameters are $K = \frac{1\ \text{mA}}{V^2}$ and $V_T = 0.5V$.

What is the minimum value of input voltage V_{IN} in volts for the MOSFET to be operating in saturation region?

-0.5 ✔ **Answer:** -0.5

What must be the value of R_L in Ohms such that it achieves $V_{OUT} = 0V$ when $V_{IN} = 0V$? Assume that the MOSFET is operating in saturation.

8000 ✔ **Answer:** 8000

As V_{IN} is increased, the output voltage V_{OUT} decreases and the MOSFET goes out of saturation. For the value or R_L found above, what is the maximum input voltage V_{IN} in volts such that the MOSFET will remain in the saturation region?

0.093 ✔ **Answer:** 0.093

Explanation:
(a) A MOSFET turns on when the voltage across its gate and source V_{GS} exceeds the threshold voltage V_T of the MOSFET. This MOSFET is said to be in its saturation state when the voltage across its drain and source V_{DS} exceeds $V_{GS} - V_T$. Here, $V_{GS} = V_{IN} - V_{S-}$, $V_T = 0.5V$, and the MOSFET turns on at $V_{IN} - 0.5V$. Since the voltage drop across the resistor is zero when the MOSFET is just turning on:

$$V_{DS} \approxeq V_{S+} - V_{S-} = 2V$$

$$2V > V_{GS} - V_T$$

and the MOSFET is in saturation. Thus the minimum voltage for V_{IN} to turn the MOSFET on and put it in saturation is $V_{IN} = -0.5V$.
(b) The current conducted by a MOSFET is written as:

$$I_{DS} = \frac{K}{2}(V_{GS} - V_T)^2$$

This current conducts through R_L with a voltage drop of:

$$V_R = V_{S+} - V_{out}$$

Equating the current conducted by the MOSFET with the current through R_L using Ohm's law, we write:

$$\frac{V_{S+} - V_{out}}{R_L} = \frac{K}{2}(V_{IN} - V_{S-} - V_T)^2$$

Substituting $V_{IN} = V_{OUT} = 0$ we get $R_L = 8k\Omega$
(c) The MOSFET leaves saturation when:

$$V_{DS} < V_{GS} - V_T$$

The condition $V_{DS} = V_{GS} - V_T$ is the threshold for saturation. In this state, we can write the output voltage in terms of V_{DS}:

$$\begin{aligned} V_{OUT} &= V_{S-} + V_{DS} \\ &= V_{S-} + (V_{GS} - V_T) \\ &= V_{S-} + [(V_{IN} - V_{S-}) - V_T] \\ V_{OUT} &= V_{IN} - V_T \end{aligned}$$

Equating I_{DS} with $\frac{V_R}{R_L}$ from before, we write:

$$\frac{V_{S+} - (V_{IN} - V_T)}{R_L} = \frac{K}{2}(V_{IN} - V_{S-} - V_T)^2$$

This equation can be solved with the quadratic equation to give two solutions. Picking the one that will keep the MOSFET on, (i.e. $V_{GS} \geq V_T$) we get $V_{IN} = 0.093V$.

The two conditions you need to think about are $v_{\mathrm{DS}} \geq v_{\mathrm{GS}} - V_{\mathrm{T}}$ and $v_{\mathrm{GS}} > V_{\mathrm{T}}$.
It is the second condition which will provide you with the minimum value of the input voltage.

posted 2 years ago by **Grove** (Community TA)

Grove (Community TA)
2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)

I will try and give you a sequence of steps without giving too much away.

The threshold condition for saturation gives you a relationship for V_{DS} in terms of V_{GS} and V_{T}.

Apply KVL to the output side, $V_{\mathrm{DS}} + V_{\mathrm{S-}} - V_{\mathrm{OUT}} = 0 \Rightarrow V_{\mathrm{OUT}} = V_{\mathrm{DS}} + V_{\mathrm{S-}}$.

Replace V_{DS} using the threshold saturation condition so you now have V_{OUT} in terms of $V_{\mathrm{S-}}$, V_{GS} and V_{T}.

Find V_{GS} in terms of V_{IN} and $V_{\mathrm{S-}}$ and use it to get V_{OUT} in terms of only V_{IN} and V_{T}.

Find I_{D} in terms of $V_{\mathrm{S+}}$, V_{OUT} and R_{L} and eliminate V_{OUT} from the expression to get I_{D} in terms of $V_{\mathrm{S+}}$, V_{OUT}, V_{T} and R_{L}.

Equate the expression to the $\dfrac{K}{2}(\ldots)^2$ formula having replaced V_{GS} so that you now have a quadratic in V_{IN}.

Grove (Community TA)
2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)

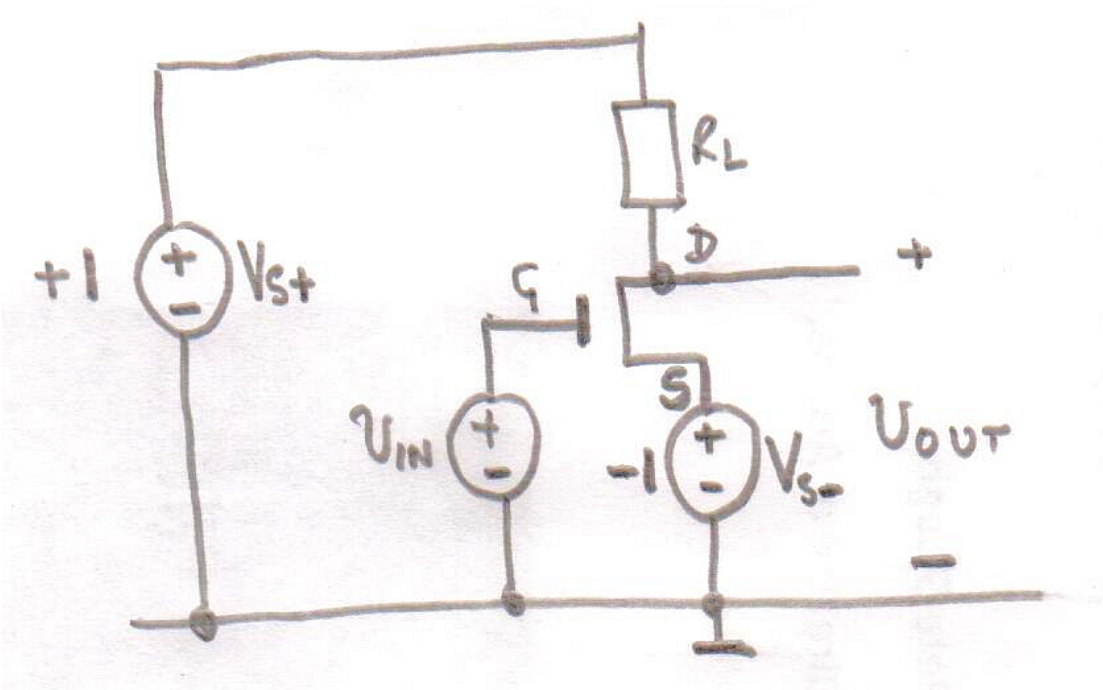
Oh!!

At times doubting my (and the course team's) solution I think that I have found your mistake!!

ID=(Vs+-VDS)/R

$I_{\mathrm{D}} \neq \dfrac{V_{\mathrm{S+}} - V_{\mathrm{DS}}}{R_{\mathrm{L}}}$ rather using KVL for the outer loop gives

$$-V_{\mathrm{S+}} + I_{\mathrm{D}} R_{\mathrm{L}} + V_{\mathrm{DS}} + V_{\mathrm{S-}} = 0 \Rightarrow I_{\mathrm{D}} = \frac{V_{\mathrm{S+}} - V_{\mathrm{S-}} - V_{\mathrm{DS}}}{R_{\mathrm{L}}}$$



```
vsp = 1; vsm = -1; k = 1*^-3; vt = 0.5;
(*VGS = VIN - VS- > VT for saturation*)
vsm + vt
(*KVL gives VOUT = VDS + VS-
    Substituting VDS = VGS - VT gives VOUT = VGS - VT + VS-
      Then VGS = VIN - VS- which gives VOUT = VIN - VT (cancelling out VS-)
        ID = (VS+-VOUT)/RL = K/2 (VIN-VS--VT)^2, then sub VIN=VOUT=0*)
Solve[vsp/rl == k/2 (vsm + vt)^2, rl]
Solve[(vsp - (vin - vt))/rl == k/2 (vin - vsm - vt)^2, vin] /. {rl → 8000}
```

Out[•]= -0.5

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{rl → 8000.}}

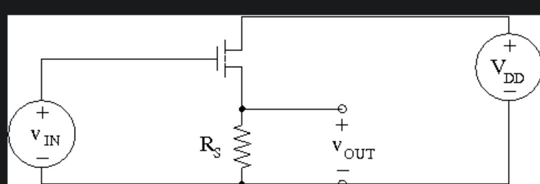
Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{vin → -1.34307}, {vin → 0.0930703}}

H1P2: Source Follower - Large Signal

7/7 points (graded)

One of the most valuable of the amplifier configurations is the source follower, shown in the figure below:



Here the source labeled V_{DD} is the power supply.

Although the source follower has no voltage gain (actually a small loss) it has power gain: it presents a very high resistance to the signal source so it takes no power from the source, but it can drive a low resistance load. Also, because of feedback, which we will learn about later, the source follower is close to linear. In this problem we will investigate the large-signal characteristics of a source follower.

As usual, the MOSFET is set up to operate in the saturated region.

Write an algebraic expression for i_{DS} in terms of K, v_{IN}, v_{OUT}, and V_T. Remember, algebraic expressions are case sensitive.

K/2*(vIN-vOUT-VT)^2 ✔ **Answer:** (K/2)*(vIN - vOUT - VT)^2

$$\frac{K}{2} \cdot (v_{\text{IN}} - v_{\text{OUT}} - V_T)^2$$

Write an algebraic expression for v_{OUT} in terms of i_{DS} and R_S.

iDS*RS ✔ **Answer:** iDS*RS

$i_{DS} \cdot R_S$

Solve the previous two results for i_{DS}. You will get a quadratic equation, and you will have to choose the correct root: make sure that $i_{DS} = 0$ when $v_{IN} = V_T$.

What is your solution for i_{DS}?

```
(1+K*RS*vIN-K*RS*VT-sqrt(1+2*K*RS*vIN-2*K*RS*VT))/(K*RS^2)
```
✔

Answer: (1+K*RS*(vIN-VT)-sqrt(1+2*K*RS*(vIN-VT)))/(K*RS^2)

$$\frac{1 + K \cdot R_S \cdot v_{\text{IN}} - K \cdot R_S \cdot V_T - \sqrt{1 + 2 \cdot K \cdot R_S \cdot v_{\text{IN}} - 2 \cdot K \cdot R_S \cdot V_T}}{K \cdot R_S^2}$$

So, now you have an expression for v_{OUT}. We won't ask you for it :-).

Let's crunch some numbers. Assume our power MOSFET has $K = 2.0A/V^2$ and $V_T = 2.0V$. Also, assume we are driving a $R_S = 16\Omega$ earphone load.

If we bias the input so that $V_{IN} = 6.2$V, what is the bias voltage V_{OUT} (in Volts) across the earphone?

3.71795 ✔ **Answer:** 3.718

We expect our signal to drive the input up and down by $1.0V$.

So, if the input goes up to $V_{IN} = 7.2$ V what will the output voltage v_{OUT} (in Volts) be?

4.66031 ✔ **Answer:** 4.66

And, if the input goes down to $V_{IN} = 5.2V$ what will the output voltage v_{OUT} (in Volts) be?

2.78295 ✔ **Answer:** 2.783

In order to allow this much swing on the output voltage, we must supply a sufficiently high supply voltage. What is the minimum value of V_{DD} (in Volts) that we must supply to keep the transistor in the region of saturated operation?

5.2 ✔ **Answer:** 5.2

Explanation:
(a) The gate to source voltage can be calculated as $v_{GS} = v_{IN} - v_{OUT}$ using KVL. Since the typical expression for the drain source current in saturation is $i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$, we can use the new equation of v_{GS} to get the answer i_{DS} in saturation is $\frac{K}{2}(v_{IN} - v_{OUT} - V_T)^2$.
(b) Notice that the current through the resistor is i_{DS} and v_{OUT} is the voltage drop across it. Therefore, the algebraic expression for v_{OUT} in terms of i_{DS} and R_S is simply $i_{DS} \cdot R_S$.
(c) Equating the current through the resistor, R_S, with the current through the MOSFET:

$$i_{DS} = \frac{v_{OUT}}{R_S} = \frac{K}{2}(v_{IN} - v_{OUT} - V_T)^2$$

$$\frac{v_{OUT}}{R_S} = \frac{K}{2} \cdot (v_{IN}^2 + v_{OUT}^2 + V_T^2 + 2 \cdot (v_{IN} \cdot (-v_{OUT}) + v_{IN} \cdot (-V_T) + (-v_{OUT}) \cdot (-V_T)))$$

$$\frac{v_{OUT}}{R_S} = \frac{K}{2} \cdot (v_{IN}^2 + v_{OUT}^2 + V_T^2 - 2 \cdot v_{IN} \cdot v_{OUT} - 2 \cdot v_{IN} \cdot V_T + 2 \cdot v_{OUT} \cdot V_T)$$

$$\frac{1}{R_S} \cdot v_{OUT} = \frac{K}{2} \cdot (v_{OUT}^2 - 2 \cdot (v_{IN} - V_T) \cdot v_{OUT} + v_{IN}^2 + V_T^2 - 2 \cdot v_{IN} \cdot V_T)$$

Collecting the terms:

$$0 = \frac{K}{2} \cdot {v_{OUT}}^2 + (-\frac{1}{R_S} - K \cdot (v_{IN} - V_T)) \cdot v_{OUT} + \frac{K}{2} \cdot (v_{IN}^2 - 2 \cdot v_{IN} \cdot V_T + {V_T}^2)$$

$$0 = \frac{K}{2} \cdot {v_{OUT}}^2 - (\frac{1}{R_S} + K \cdot (v_{IN} - V_T)) \cdot v_{OUT} + \frac{K}{2} \cdot (v_{IN} - V_T)^2$$

Applying the Quadratic Equation:

$$v_{OUT} = \frac{\frac{1}{R_S} + K\,(v_{IN} - V_T) \pm \sqrt{(-(\frac{1}{R_S} + K \cdot (v_{IN} - V_T)))^2 - 4 \cdot (\frac{K}{2}) \cdot (\frac{K}{2}) \cdot (v_{IN} - V_T)^2}}{K}$$

Substitute $i_{DS} = \frac{v_{OUT}}{R_S}$ and simplify. We use the negative root to enforce $i_{DS} = 0$ when $v_{IN} = V_T$

Substitute $i_{DS} = \frac{}{R_S}$ and simplify. We use the negative root to enforce $i_{DS} = 0$ when $v_{IN} = V_T$.

$$i_{DS} = \frac{\frac{1}{R_S} + K \cdot (v_{IN} - V_T) - \sqrt{\left(\frac{1}{R_S} + K \cdot (v_{IN} - V_T)\right)^2 - 4 \cdot \left(\frac{K}{2}\right)^2 \cdot (v_{IN} - V_T)^2}}{K \cdot R_S}$$

$$i_{DS} = \frac{1 + K \cdot R_S \cdot (v_{IN} - V_T) - R_S \cdot \sqrt{\left(-\left(\frac{1}{R_S} + K \cdot (v_{IN} - V_T)\right)\right)^2 - \left(2 \cdot \left(\frac{K}{2}\right)(v_{IN} - V_T)\right)^2}}{K \cdot R_S^2}$$

$$i_{DS} = \frac{1 + K \cdot R_S \cdot (v_{IN} - V_T) - \sqrt{R_S^2 \cdot \left(\frac{1}{R_S} + K \cdot (v_{IN} - V_T)\right)^2 - R_S^2 \cdot \left(2 \cdot \left(\frac{K}{2}\right)(v_{IN} - V_T)\right)^2}}{K \cdot R_S^2}$$

$$i_{DS} = \frac{1 + K \cdot R_S \cdot (v_{IN} - V_T) - \sqrt{(1 + K \cdot R_S \cdot (v_{IN} - V_T))^2 - (K \cdot R_S \cdot (v_{IN} - V_T))^2}}{K \cdot R_S^2}$$

$$i_{DS} = \frac{1 + K \cdot R_S \cdot (v_{IN} - V_T) - \sqrt{1 + 2 \cdot K \cdot R_S \cdot (v_{IN} - V_T) + (K \cdot R_S \cdot (v_{IN} - V_T))^2 - (K \cdot R_S \cdot (v_{IN} - V_T))^2}}{K \cdot R_S^2}$$

$$i_{DS} = \frac{1 + K \cdot R_S \cdot (v_{IN} - V_T) - \sqrt{1 + 2 \cdot K \cdot R_S \cdot (v_{IN} - V_T)}}{K \cdot R_S^2}$$

(d) The expression above (in part (c)) for i_{DS} when multiplied by R_S gives us an expression for v_{OUT}. Plugging in the given bias value for $v_{IN} = V_{IN}$ and known values of the other variables, we get: $v_{OUT} = V_{OUT}$ = 3.718
(e) The expression above (in part (c)) for i_{DS} when multiplied by R_S gives us an expression for v_{OUT}. Plugging in the given value for $v_{IN} = V_{IN}$ and known values of the other variables, we get: $v_{OUT} = V_{OUT}$ = 4.66
(f) The expression above (in part (c)) for i_{DS} when multiplied by R_S gives us an expression for v_{OUT}. Plugging in the given value for $v_{IN} = V_{IN}$ and known values of the other variables, we get: $v_{OUT} = V_{OUT}$ = 2.783
(g) The MOSFET is in saturation when $V_{DS} \geq V_{GS} - V_{OUT}$. From KVL we can get the expressions $V_{DS} = V_{DD} - V_{OUT}$ and $V_{GS} = V_{IN} - V_{OUT}$. Using these two equations in the condition for saturation, we get:
$V_{DD} - V_{OUT} \geq V_{IN} - V_{OUT} - V_T$, and therefore $V_{DD} \geq V_{IN} - V_T$. The maximum input voltage we expect to have is V_{IN}. Using this, we find that for the maximum input voltage V_{IN}: $V_{DD} \geq 7.2 - 2.0 = 5.2$

In[◦]:=
```
(*KVL gives us 2 eqns:
   V_OUT-V_DD+V_DS=0 and -V_IN+V_GS+V_OUT=0
    V_OUT=i_DS R_s and i_DS=K/2 (V_IN-V_OUT-V_T)^2*)
```

$\text{Solve}\left[i_{DS} == \frac{1}{2} K \left(-i_{DS} R_s + V_{IN} - V_T\right)^2, i_{DS}\right]$

```
(*Check each solution by subbing in V_IN=
 V_T. Second one seems right as we get i_DS=0.*)
```

$\frac{1 + K R_s V_{IN} - K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2}$ /. $\{V_{IN} \to V_T\}$

$-\frac{-1 - K R_s V_{IN} + K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2}$ /. $\{V_{IN} \to V_T\}$

```
(*Gotta find a way to avoid copy-pasta*)
```

$K = 2; V_T = 2; R_s = 16;$

$V_{IN} = 6.2;$

$i_{DS} = -\frac{-1 - K R_s V_{IN} + K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2};$

$V_{OUT} = i_{DS} * R_s$

$V_{IN} = 7.2;$

$i_{DS} = -\frac{-1 - K R_s V_{IN} + K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2};$

$V_{OUT} = i_{DS} * R_s$

$V_{IN} = 5.2;$

$i_{DS} = -\frac{-1 - K R_s V_{IN} + K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2};$

$V_{OUT} = i_{DS} * R_s$

Out[◦]= $\left\{\left\{i_{DS} \to \frac{1 + K R_s V_{IN} - K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2}\right\}, \left\{i_{DS} \to -\frac{-1 - K R_s V_{IN} + K R_s V_T + \sqrt{1 + 2 K R_s V_{IN} - 2 K R_s V_T}}{K R_s^2}\right\}\right\}$

Out[◦]= $\frac{2}{K R_s^2}$

Out[◦]= 0

Out[◦]= 3.71795

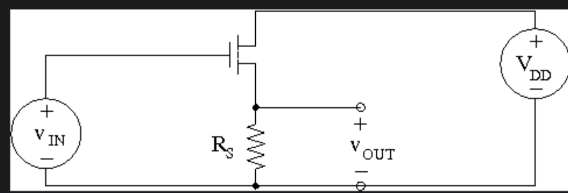
Out[◦]= 4.66031

Out[◦]= 2.78295

H1P3: Source Follower - Small Signal

3/3 points (graded)

Continuing with our analysis of the source follower, we now want to examine its small-signal behavior.



We have exactly the same situation as we had in H1P2 (but the numbers will be different.)

Derive an algebraic expression for the incremental output voltage v_{out} in terms of parameters K, V_T, and R_S, the input bias voltage V_{IN}, and the incremental input voltage v_{in}.

Hint: Remember that you computed the total output voltage v_{OUT} in H1P2. Also remember that $v_{out} = v_{in} \cdot \frac{\partial v_{OUT}}{\partial v_{IN}}|_{v_{IN}=V_{IN}}$.

So, in the space provided below, write your algebraic expression for v_{out}:

vin*(1-1/(sqrt((1+2*K*RS*(VIN-VT)))))   ✔ **Answer:** (1 - 1/sqrt(1 + 2*K*RS*(VIN-VT)))*vin

$$v_{\text{in}} \cdot \left(1 - \frac{1}{\sqrt{(1 + 2 \cdot K \cdot R_S \cdot (V_{\text{IN}} - V_T))}}\right)$$

Using the expression you just derived, let's crunch the numbers again. Remember, our power MOSFET has $K = 2.0$A/V^2 and $V_T = 2.0$. And we are driving a $R_S = 16\Omega$ earphone load. Assume here that our input bias voltage $V_{IN} = 6.2$ V.

If we have an incremental input of $v_i = 0.0035$V what is the incremental output v_o (in Volts)?

0.00328692   ✔ **Answer:** 0.00329

So, what is the incremental voltage gain v_o/v_i of this circuit?

0.939119   ✔ **Answer:** 0.939

Notice that although the voltage gain is less than one, we are driving a 16Ω earphone. The signal source is putting out (almost) no power, because the gate of the MOSFET takes (almost) no current. (Actually, because the gate has capacitance to the source and drain there is a small amount of current pulled, but we will learn about that later!) So this circuit has very large current gain.

One other problem is that usually we don't want our bias current to go through the earphone. We can use capacitors to solve that problem too, but later in the course.

Explanation:
(a) Consider the expression for V_{OUT} from H2P2(c):

$$V_{OUT} = I_{DS} \cdot R_S = \frac{1 + K \cdot R_S \cdot (V_{IN} - V_T) - \sqrt{1 + 2 \cdot K \cdot R_S \cdot (V_{IN} - V_T)}}{K \cdot R_S}$$

$$V_{OUT} = \frac{1}{R_S} + K \cdot (V_{IN} - V_T) - \frac{\sqrt{1 + 2 \cdot K \cdot R_S \cdot (V_{IN} - V_T)}}{K \cdot R_S}$$

We know that the small signal output voltage is: $v_{out} = v_{in} \cdot \frac{\partial V_{OUT}}{\partial V_{IN}}|_{v_{in}=V_{IN}}$. Taking the derivative:

$$\frac{\partial V_{OUT}}{\partial V_{IN}} = 1 - (2 \cdot K \cdot R_S \cdot (V_{IN} - V_T) + 1)^{-0.5} = 1 - \frac{1}{\sqrt{2 \cdot K \cdot R_S \cdot (V_{IN} - V_T) + 1}}$$

Multiplying this with V_{IN}, we get the small signal output voltage:

$$v_{out} = v_{in} \cdot \left(1 - \frac{1}{\quad}\right)$$

$$\sqrt{2 \cdot K \cdot R_S \cdot (V_{IN} - V_T) + 1}$$

(b) Using the solution from part (a), plug in the given values to get:

$$v_{out} = (0.0035V) \cdot \left(1 - \frac{1}{\sqrt{2 \cdot (2.0A/V^2) \cdot (16\Omega) \cdot ((6.2V) - (2.0V)) + 1}}\right) = 0.00329V$$

(c)

$$\frac{v_{out}}{v_{in}} = \frac{0.00329}{0.0035} = 0.939$$

```
(*Removing subscripts because it messed with differentiation.
  Mathematica simplification is inadequate so we work with what we got.*)
```

$$\text{vout[vIN\_] :=} \frac{\frac{1}{R} + K\,(\text{vIN} - \text{VT}) - \sqrt{\left(\frac{1}{R} + K\,(\text{vIN} - \text{VT})\right)^2 - 4\left(\frac{K}{2}\right)^2 (\text{vIN} - \text{VT})^2}}{K};$$

```
vin * vout'[VIN]
```

$$\text{Simplify}\left[\frac{\text{vin}\left(K - \frac{2K\left(\frac{1}{R} + K\,(\text{VIN} - \text{VT})\right) - 2K^2\,(\text{VIN} - \text{VT})}{2\sqrt{\left(\frac{1}{R} + K\,(\text{VIN} - \text{VT})\right)^2 - K^2\,(\text{VIN} - \text{VT})^2}}\right)}{K}\right]$$

```
K = 2; VT = 2; R = 16; VIN = 6.2; vin = 0.0035;
```

$$\text{vout} = \text{vin}\left(1 - \frac{1}{R\sqrt{\frac{1 + 2KR\,(\text{VIN} - \text{VT})}{R^2}}}\right)$$

```
vout / vin
```

Out[ ]= $$\frac{\text{vin}\left(K - \frac{2K\left(\frac{1}{R} + K\,(\text{VIN} - \text{VT})\right) - 2K^2\,(\text{VIN} - \text{VT})}{2\sqrt{\left(\frac{1}{R} + K\,(\text{VIN} - \text{VT})\right)^2 - K^2\,(\text{VIN} - \text{VT})^2}}\right)}{K}$$

Out[ ]= $$\text{vin}\left(1 - \frac{1}{R\sqrt{\frac{1 + 2KR\,(\text{VIN} - \text{VT})}{R^2}}}\right)$$

Out[ ]= 0.00328692

Out[ ]= 0.939119

Lab

Lab due Aug 9, 2022 15:35 +04 Completed

Lab 1

4/4 points (graded)

The goal of this lab is to analyze the performance of an inverting mosfet amplifier operating in its *linear region*, where the output waveform is an undistorted but amplified replica of the input waveform. Note that inversion of the signal introduced by the amplifier is not an issue in, for example, audio applications where the ear isn't sensitive to the 180 degree phase shift in the output signal.

You may find it useful to review Sections 7.5, 7.6 and Chapter 8 in the text.

To start, let's use a variant of the curve tracer we worked on in Lab 4 to plot the v_O vs. v_I curve for the amplifier so we can identify the range of input voltages that corresponding to the linear operating region of the amplifier.

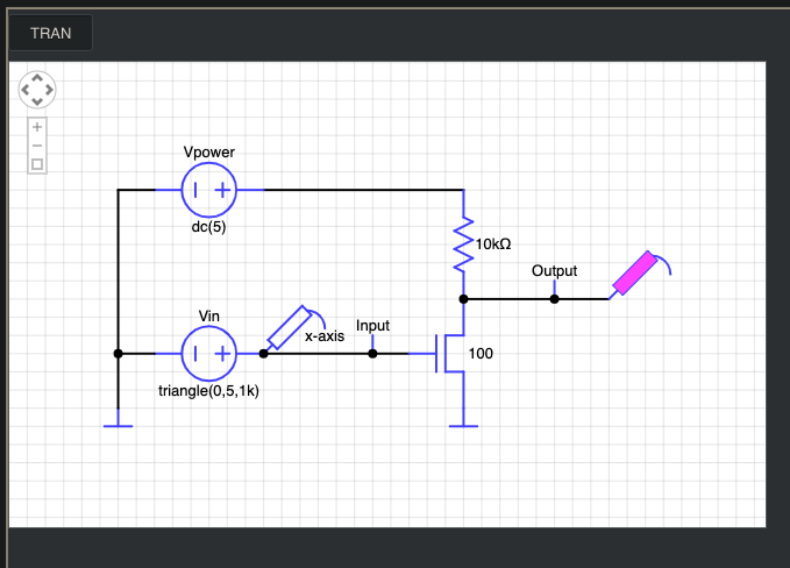


Figure 1. Circuit to determine v_O vs. v_I curve for inverting amplifier

The circuit shown in Figure 1 is using a 5V DC voltage source to power the amplifier and a triangle-wave voltage source to generate an input voltage that ramps from 0V to 5V over 500ms. Please click on TRAN in Figure 1 to generate the plot and then use the plot to determine the approximate range of input voltages during the amplifier will operate in its linear region.

Input voltage at lower end of linear operating range:

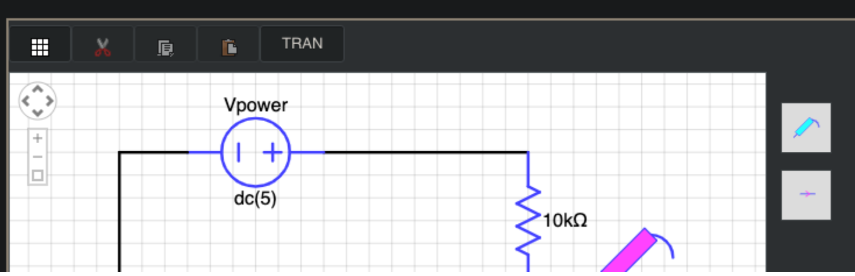
0.5 ✔ **Answer:** .6

Input voltage upper end of linear operating range:

1.1 ✔ **Answer:** 1.15

If we keep the input voltage within the range above, we expect the output signal will be a relatively undistorted replica of the input signal.

Next let's determine the gain of the amplifier when operating in its linear region. In the circuit shown in Figure 2 the input signal is a sum of a 1V DC bias voltage (to offet the signal into the linear operating range) and the test signal itself, in this case a 1kHZ sine wave with an amplitude of 0.1V.



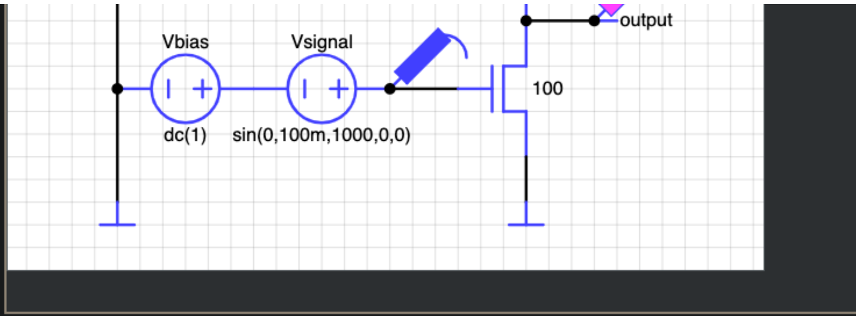


Figure 2. Circuit to measure amplifier gain

Click on TRAN in Figure 2 and measure the gain of the amplier, which is simply the ratio of the amplitude of the output signal to the amplitude of the input signal.

Measured gain:

9.8

✔ **Answer:** 9.75

Now change the amplitude of Vsignal in Figure 2 from 0.1V to 1V and rerun the TRAN analysis. You should see significant distortion in the output signal, in this case *clipping* or truncation of the max and min signal values. Experiment with amplitudes of Vbias and Vsignal to find the largest amplitude for Vsignal for which amplifier produces an unclipped output.

Largest input amplitude resulting in an undistorted output signal:

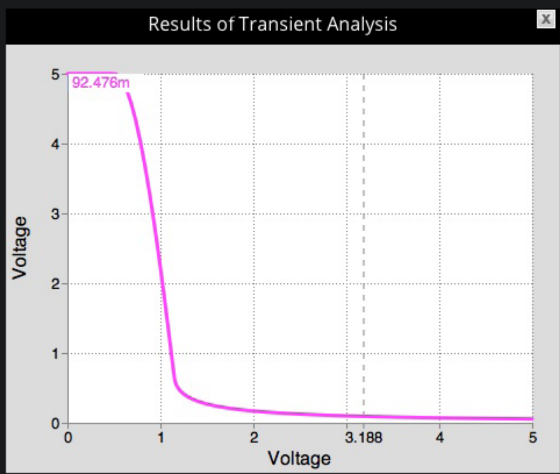
0.325

✔ **Answer:** 0.3

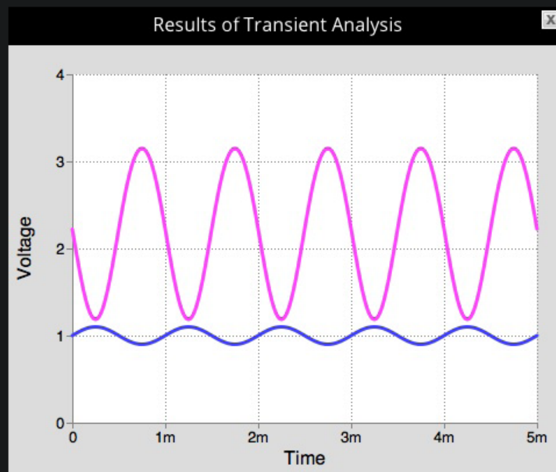
Think about how your final choices for the amplitudes of Vbias and Vsignal relate to the input voltage range you reported above for linear operation.

The interactive demonstration in the lecture sequence allows you to listen to how a similar amplifier sounds with a music signal as input.

Explanation:
Using the TRAN function on the circuit to determine the tranfer function, we end up with a graph that looks something like this:



From this we can determine the lower and upper bounds of the input voltages in the device's linear region. These voltages section off the diagonal, "linear" region of the curve. The lower boundary is at 700mV, and the upper one at 1.2V. Determining the gain is similarly straightforward. Running a TRAN simulation on the second circuit, we get a graph like this:



The gain is the ratio of amplitudes of the big curve to the small one. When we take this ratio, we find that the gain is about 10. To find the largest input amplitude to result in an undistorted output signal, we can experiment blindly with combinations of bias and input signal voltages until we get the right answer, but a little thinking will quicken this process considerably. The output voltage distorts, or "clips" when the input voltage brings the device outside of its linear region of operation. Thinking back to the first graph we saw, this means the input voltage can swing no lower than 700mV and no higher than 1.2V. To get the maximum output swing, and thus the maximum "gain," we should ideally start in the middle of this region (making the bias voltage 950mV). That way, we get maximum swing in both directions. The distance between the midpoint bias-voltage and the upper/lower bound voltage is 250mV, and so 250mV is the maximum amplitude we can achieve without distortion.

```
(*Beware, the explanation given,
my answer as well as the graders answer ALL don't match.
  Although I benefited from it, they were NOT doing the
 ±5% answer tolerance that they usually do.*)
```

In[∘]:= $\frac{3.14 - 1.18}{1.1 - 0.9}$

Out[∘]= 9.8

In[∘]:= `Mean[{1.15, 0.5}]`

Out[∘]= 0.825

In[∘]:= `1.15 - 0.825`

Out[∘]= 0.325

In[∘]:= `{0.3 - 0.05 * 0.3, 0.3 + 0.05 * 0.3}`

Out[∘]= {0.285, 0.315}