Week 13

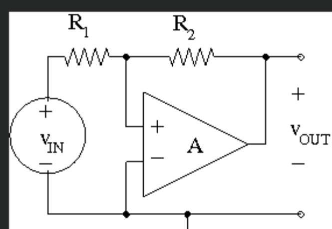
S25 - Op Amps Positive Feedback
S26 - Energy and Power
S27 - Energy and CMOS Design
S28 - Breaking the Abstraction Barrier

Lectures

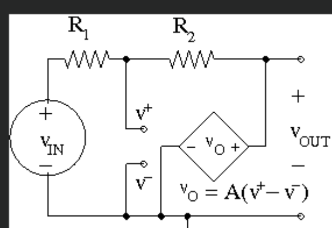
S25E1: Positive Feedback Gain

0 points possible (ungraded)

We are considering what happens when we hook up an op amp with positive feedback.



So let's analyze this very simple model:



Compute the circuit gain v_{OUT}/v_{IN} in terms of R_1, R_2, and A. Give an algebraic expression for that value in the space provided below:

A*R2/((1-A)*R1+R2) ✔ **Answer:** A*R2/(R1+R2-A*R1)

$$A \cdot \frac{R_2}{(1-A) \cdot R_1 + R_2}$$

Now let $A \to +\infty$. Give an algebraic expression for the circuit gain v_{OUT}/v_{IN}.

-R2/R1 ✔ **Answer:** -R2/R1

$$-\frac{R_2}{R_1}$$

Now let $A \to -\infty$. Give an algebraic expression for the circuit gain v_{OUT}/v_{IN}.

-R2/R1

✔ **Answer:** -R2/R1

$$-\frac{R_2}{R_1}$$

Hmmmmm... You should be worried...

Solution:

Part 1

Using KCL:

$$\frac{v_{IN} - V^+}{R_1} = \frac{V^+ - v_{OUT}}{R_2}$$

$$V^+ = \frac{v_{OUT}}{A} \rightarrow \frac{A v_{IN} - v_{out}}{A R_1} = \frac{(1-A)\, v_{OUT}}{A R_2}$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{A R_2}{(1-A)\, R_1 + R_2}$$

Part 2: Limits

$$\lim_{A \to \infty} \frac{A R_2}{(1-A)\, R_1 + R_2} \approx \frac{A R_2}{-A R_1} = -\frac{R_2}{R_1}$$

$$\lim_{A \to -\infty} \frac{A R_2}{(1-A)\, R_1 + R_2} \approx \frac{A R_2}{-A R_1} = -\frac{R_2}{R_1}$$

In[∘]:=
```
g[A_] := A r2 / ((1 - A) r1 + r2)

Limit[g[A], A → ∞]
Limit[g[A], A → -∞]
```

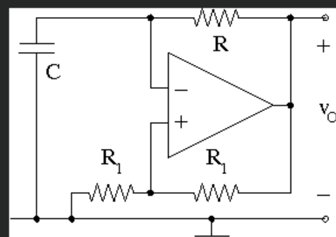
Out[∘]= $-\frac{r2}{r1}$

Out[∘]= $-\frac{r2}{r1}$

S25E2: Relaxation Oscillator Frequency

0 points possible (ungraded)

We have a relaxation oscillator circuit.



Suppose that we want to use it to make a 10kHz clock. That is, we want 10000.0 rising edges in every second.

Assume that $V_S = 5$V, $R = 1$kΩ and $R_1 = 1.5$kΩ. It remains to choose the capacitance C.

What is the value of C, in nanoFarads, required to make this circuit oscillate at 10kHz.

45.512 ✔ **Answer:** 45.511961331341865

Solution:

We want a 5 volt source to charge the capacitor from -2.5 V to 2.5 V in half the period, so $\frac{1}{20000}$ seconds. The voltage at the negative terminal of the op amp is: is:

$$v_{-}(t) = 7.5\left(1 - e^{-\frac{t}{RC}}\right) - 2.5$$

$$v_{-}\left(\frac{1}{20000}\right) = 2.5 = 7.5\left(1 - e^{-\frac{1}{20000RC}}\right) - 2.5$$

$$= C = \frac{1}{20000R \times \ln(3)} = 45.512nF$$

In[∘]:=
```
vs = 5; r = 1*^3; r1 - 1.5 * 63;
t = 1 / 20 000;
vm[t_] := -2.5 + 7.5 × (1 - Exp[-t / (r c)]);
Solve[vm[t] == 2.5, c]
```

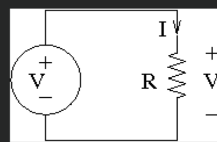
••• Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $\{\{c \to 4.5512 \times 10^{-8}\}\}$

S26E1: Power and Energy Review

0 points possible (ungraded)

Just a reminder of power and energy...



In this simple network, what is the power dissipated in the resistor, in terms of V and I?

V*I ✔ **Answer:** V*I

$V \cdot I$

What is the power dissipated in the resistor, in terms of V and R?

V^2/R ✔ **Answer:** V^2/R

$\frac{V^2}{R}$

In an interval T what is the energy dissipated in the resistor, in terms of V, I and T?

V*I*T ✔ **Answer:** V*I*T

$V \cdot I \cdot T$

Solution:

$$P = IV$$

$$V = IR \to I = \frac{V}{R} \to P = \frac{V}{R}V = \frac{V^2}{R}$$

$$E = \int_{t_0}^{t_0+T} P(t)\, dt = VIT$$

```
(*Minor thing:
  They use <> instead of [] to
   denote limit substitution for the definite integral*)
```

S26E2: Energy Sourced in T1

0 points possible (ungraded)

We have the energy supplied by the source as an integral

$$E = \int_0^{T_1} V_S i\,(t)\,dt$$

and we have the current that appears in the integrand

$$i\,(t) = \frac{V_S}{R_1} e^{\frac{-t}{R_1 C}}$$

In the space provided below write an algebraic expression for the energy E in terms of C, V_S, T_1, and R_1.

VS^2*C*(1-e^(-T1/(R1*C))) ✔ **Answer:** C*VS^2*(1 - e^(-T1/(R1*C)))

$V_S^2 \cdot C \cdot \left(1 - e^{-\frac{T_1}{R_1 \cdot C}}\right)$

For $T_1 \gg R_1 C$ what does E approach?

In the space provided below write an algebraic expression for the energy E, as $T_1 \to \infty$ in terms of C, V_S, T_1, and R_1.

VS^2*C ✔ **Answer:** C*VS^2

$V_S^2 \cdot C$

Notice that your answer here is independent of R_1. Hmmmm...

Solution:

$$E = \int_0^{T_1} V_S i\,(t)\,dt = \int_0^{T_1} \frac{V_S^2}{R_1} e^{\frac{-t}{R_1 C}}\,dt$$

$$E = \langle \frac{V_S^2}{R_1} \times (-R_1 C)\, e^{-\frac{-t}{R_1 C}} \rangle_0^{T_1} = V_S^2 C\,(1 - e^{-\frac{-T_1}{R_1 C}})$$

For large times:

$$\lim_{T_1 \to \infty} E = \lim_{T_1 \to \infty} V_S^2 C\,(1 - e^{-\frac{-T_1}{R_1 C}}) = V_S^2 C$$

This is what we expect from the energy provided by a battery in series with a resistor and capacitor. This is twice the energy stored on a capacitor, as the battery must provide both the energy on the capacitor and the energy the resistor dissipates.

In[ ]:=
```
Integrate[vs^2/r1 Exp[-t / (r1 c)], {t, 0, T1}]
e[T_] := c (1 - ⅇ^(-T/(c r1))) vs^2;
Limit[e[T], T → ∞]
```

Out[ ]= $c \left(1 - e^{-\frac{T1}{c\,r1}}\right) vs^2$

Out[ ]= $c\,vs^2$ if $vs^2 \in \mathbb{R}$ && $c\,r1 > 0$

S26E3: A Hot Processor

0 points possible (ungraded)

Suppose we are making an NMOS chip with 5 million gates, to run at a frequency of 3GHz. Our process makes MOSFETs with a gate-source capacitance of 1fF, and our pullups are $10\text{k}\Omega$. The $R_{ON} = 100\Omega \ll R_L$, so we can ignore it.

Suppose also that we chose to use a 5V power supply (a bad idea!).

What is the average static power, in milliWatts, that a gate in this process dissipates?

1e3/800 ✔ **Answer:** 1.25

What is the average dynamic power, in microWatts, that a gate in this process dissipates?

3e6/40000 ✔ **Answer:** 75.0

Now, we have 5 million gates, so what power does the whole chip dissipate? (In kiloWatts.)

6625e-3 ✔ **Answer:** 6.625

To put this into perspective, consider the Sun. The luminosity of the Sun (the total power output) is about $3.8600000000000003 \times 10^{26}$W. The radius of the Sun is about 695500.0km. Thus each square centimeter of the surface of the Sun is putting out 6.3500000000000005kW/cm^2.

But our chip is about a square centimeter; a difficult problem for cooling! We cannot make chips this way.

Solution:

Static Power

The key question here is whether to use R_L or R_{PU} in each calculation:

$$P_S = \frac{V_S^2}{2R_{PU}} = 1.25mW$$

Dynamic Power

This time, we will use both R_L and R_{on}. Thankfully the latter is negligible:

$$P_D = \frac{V_S^2 \times R_L^2 \times C_L}{(R_L + R_{on})^2 T} \approx \frac{V_S^2 C_L}{T} = V_S^2 C_L f = 75\mu W$$

Total power

$$P_{total} = n\left(P_D + P_S\right) = 5 \times 10^6 \left(1.25 \times 10^{-3} + 75 \times 10^{-6}\right) = 6.625kW$$

The bad news is is has the same power output as the Sun. The good news is that a sphere of your capacitors with radius $69500km$ is essentially a Sun.

In[]:=
```
n = 5*^6; f = 3*^9; c = 1*^-15; r = 10*^3; ron = 100;
vs = 5;
```

$$ps = \frac{vs^2}{2\,r}$$

$$pd = vs^2\,c\,f$$

$$n\,(ps + pd)$$

Out[]= $\frac{1}{800}$

Out[]= $\frac{3}{40\,000}$

Out[]= 6625

```
(*Interesting discussion, food for thought*)
```

Static vs. Dynamic power?

question posted 3 years ago by it1999

So Static power dissipation comes from current paths from the VS node to ground that only have resistors, no capacitors?

Then Dynamic power dissipation comes from current paths from the VS node to ground that also have the capacitor in the path?

Why is the static power so dominant over the dynamic power dissipated in the Inverter example? I understand numerically because of the VS^2 term, but why is this the case in general? Why is the power dissipated during the charging and discharging of the capacitor so much less significant?

This post is visible to everyone.

Grove (Community TA)
3 years ago - marked as answer 3 years ago by **MIT_Lover_UA** (Staff)

It is a "compounding" effect.

In the previous video the Professor showed that the dynamic power was equal to $CV_s^2 f$.

An alternative means of obtaining this formula is as follows.

The capacitor when fully charge is storing a charge Q with a potential difference of V_s across it.
The energy stored in the capacitor when fully charged is $\frac{1}{2}QV_s^2$ and the energy supplied by the voltage source is QV_s^2.
The difference between the energy supplied by the voltage source and the energy stored in the capacitor is due to the fact that R_1 has dissipated electrical energy, $\frac{1}{2}QV_s^2$, as heat.
Then when the capacitor is discharged the energy stored in the capacitor, $\frac{1}{2}QV_s^2$, is dissipated as heat.
Multiplying these energies by the frequency f gives the average power.

Let's look at the charge and discharge of the capacitor in a little more detail.
The time constant of the charging and discharging phase is CR.
A rule of thumb is that after four time constants a circuit has reached $e^{-4} \approx 2\,\%$ of the final values and after five time constants it is $e^{-5} \approx \frac{1}{2}\,\%$ of the final values.
The initial current during the charging and discharging phase is $\frac{V_s}{R}$ which is the same as the static current.
So interestingly, the initial electrical power supplied by the voltage source to charge the capacitor is the same as the static power.

However the static power stays at that value for all time whereas the initial dynamic power is transient.
The dynamic power drops as the the capacitor is charged (and discharged) and is supplied over a time $5CR$ which is much shorter than the time taken for the chrage/recharge cycle $\frac{1}{f}$.
These two effects acting in tandem results in the average dynamic power being much less than the static power.

Calc