Week 4

S7 - Incremental Analysis

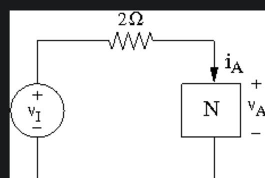
S8 - Dependent Sources and Amplifiers

Lectures

S7E1: Linearization

0 points possible (ungraded)

Consider the following circuit containing the nonlinear element N:



The i-v relation for N is given by $i_A = 10 \cdot \left(1 - e^{-v_A/5}\right)$, where i_A is in Amperes and v_A is in Volts.

Solve for the voltage v_A when $v_I = 5.0$V. Note that this requires that you solve the equation obtained using KVL iteratively for v_A.

(Hint: Use the exponential term to solve for v_A as a function of the assumed value of v_A, and then iterate. Taking logs on both sides may facilitate convergence.)

The voltage v_A is:

1.08822 ✔ **Answer:** 1.0882568359375

Find the incremental change in v_A for a 2% increase in v_I and use it to calculate the ratio $\frac{\Delta v_A}{\Delta v_I}$.

0.2376 ✔ **Answer:** 0.23193359375

Find the value for the incremental resistance of the nonlinear element N by linearizing the expression for i_A about the operating point when $v_I = 5.0$V.

0.621572 ✔ **Answer:** 0.6215767939645509

```
(*Food for thought, not sure if this works though*)
```

Grove (Community TA)
2 years ago

If you use this form of the equation for v_A

$$v_A = -a + b\,e^{-v_A/c}$$

where a, b and c are constants, there is a problem with convergence using a "trial and error" method..

Rearranging the equation to the form

$$v_A = c\,\ln\left(\frac{b}{v_A + a}\right)$$

will produce a rapid convergence.

In[∘]:=
```
r = 2; vi = 5;
i[v_] := 10 × (1 - Exp[-v / 5]);
(*Use KVL to get the f[x]==0 equation,
then use Newton's method to find the roots of f[x]*)
f[va_] := -vi + 2 i[va] + va;
n[x_] := x - f[x] / f'[x];
vasoln = NestList[n, 0.0, 100] // Last

vi2 = 1.02 * vi;(*2% increase*)
f2[va_] := -vi2 + 2 i[va] + va;
n2[x_] := x - f2[x] / f2'[x];
vasoln2 = NestList[n2, 0.0, 100] // Last;
r = (vasoln2 - vasoln) / (vi2 - vi)

(*Incremental resistance at 1st operation point*)
1 / i'[vasoln] // N
```

Out[∘]= 1.08822

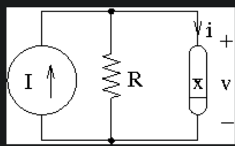
Out[∘]= 0.237529

Out[∘]= 0.621573

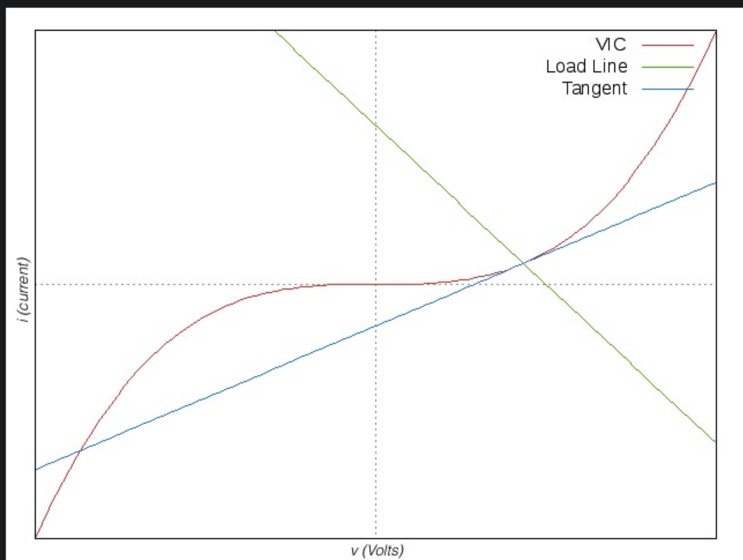
S7E2: Graphs

0 points possible (ungraded)

In the circuit shown, the strange nonlinear device is specified to have the characteristic $i = (1A/V^3) \cdot v^3$. We have seen this strange nonlinear device before, in problem S6E2. The current supplied by the current source is $I = 4.0$A, and the resistance of the parallel resistor is $R = 8.2\Omega$.



In the graph the horizontal axis is the voltage across the nonlinear device and the vertical axis is the current through the device. The red curve is the voltage-currrent characteristic of the device; the green straight line is the load line; and the blue line is the tangent to the voltage-currrent characteristic at the operating point.



Hint: use guess and check to solve for the operating point.

What is operating voltage (in Volts) of the device in this circuit?

1.5618 ✔ **Answer:** 1.5617947578430176

What is operating current (in Amperes) of the device in this circuit?

3.80954 ✔ **Answer:** 3.8095342488502437

What is incremental resistance (in Ohms) of the device, at the operating point, in this circuit?

0.136657 ✔ **Answer:** 0.136656666827831

```
r = 8.2;
(*Check S6E2: Load Line for more*)

(*Operating point*)
Solve[v == (4 - i) r && i == v^3, {v, i}, Reals]
(*Incremental resistance*)
i[v_] := v^3
1 / i'[1.5617951578401101`] // N
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

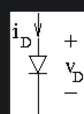
Out[•]= {{v → 1.5618, i → 3.80954}}

Out[•]= 0.136657

S7E3: Linearization

0 points possible (ungraded)

The v-i characteristic of a junction diode is well approximated by $i_D = I_0 \cdot \left(e^{v_D/V_T} - 1\right)$, with two parameters V_T and I_0.



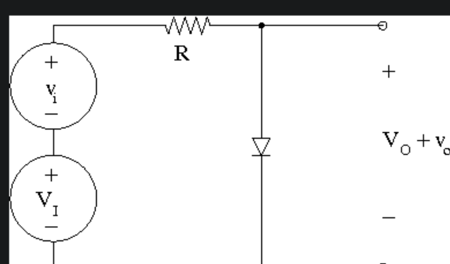
A diode has an incremental resistance that varies with the current through the diode.

In the space provided write an algebraic expression for the incremental resistance of a diode when biased with a voltage V_D. Your expression will include the parameters I_0 and V_T.

VT/(I0*(e^(VD/VT)))

Answer: 1/((I0/VT)*e^(VD/VT))

This variable incremental resistance characteristic makes a diode useful as part of a variable attenuator. For example, in the circuit below



the diode is the output leg of a voltage divider. The incremental resistance of the diode varies with the bias current, so the proportion of the incremental input voltage that appears as incremental output voltage can be varied by changing the bias voltage.

The parameter $V_T \approx 26\text{mV}$ at a temperature of 300K (about 27C). This is fixed by the physics of the semiconductor. For a typical small-signal silicon junction diode in a glass case (e.g. 1N914 or 1N4148) $I_0 \approx 8 \times 10^{-14}$A.

Assume that $R = 3.9\text{k}\Omega$.

For what value of the bias voltage V_I (in Volts) will the incremental resistance of the diode $v_d/i_d = 100.0\Omega$?

1.58 ✔ **Answer:** 1.5834499413534882

For what value of the bias voltage V_I (in Volts) will the incremental resistance of the diode $v_d/i_d = 1000.0\Omega$?

0.61 ✔ **Answer:** 0.6109827289356449

Explanation:
The incremental resistance can be found by taking the deriviative of the v-i characteristic:

$$r_d = \left(\frac{i_D}{v_D}|_{v_D=V_D}\right)^{-1}$$

$$\frac{i_D}{v_D} = I_0 \cdot \frac{1}{V_T} \cdot e^{v_D/V_T}$$

$$r_d = \left(\frac{I_0}{V_T} \cdot e^{V_D/V_T}\right)^{-1} = \frac{V_T}{I_0 \cdot e^{V_D/V_T}}$$

We know that the current through the resistor R and the current through the diode must be equal by KCL. Therefore the bias voltage V_I must be set by the following expression, according to KVL:

$$V_I = V_R + V_D = I_D \cdot R + V_D$$

Given that each value for incremental resistance $r_d = v_d/i_d$, we can find the voltage drop across the diode by rearranging the expression we found in the first part for incremental resistance:

$$r_d = \left(\frac{I_0}{V_T} \cdot e^{V_D/V_T}\right)^{-1} = \frac{V_T}{I_0 \cdot e^{V_D/V_T}}$$

$$e^{V_D/V_T} = \frac{V_T}{I_0 \cdot r_d}$$

$$V_D = V_T \cdot ln\left(\frac{V_T}{I_0 \cdot r_d}\right)$$

Then, we can substitute this value of V_D into the expression for I_D:

$$I_D = I_0 \cdot \left(e^{V_D/V_T} - 1\right)$$

For $r_d = 100.0\Omega$, we get:

$$V_D = (26mV) \cdot ln\left(\frac{(26mV)}{(8\times10^{-14}A)\cdot(100.0\Omega)}\right) = 569mV$$

$$I_D = (8 \times 10^{-14}A) \cdot \left(e^{(569mV)/(26mV)} - 1\right) = 0.00026A$$

$$V_I = I_D \cdot R + V_D = (0.00026A) \cdot (3.9k\Omega) + (0.569V) = 1.58V$$

For $r_d = 1000.0\Omega$, we get:

$$V_D = (26mV) \cdot ln\left(\frac{(26mV)}{(8\times10^{-14}A)\cdot(1000.0\Omega)}\right) = 510mV$$

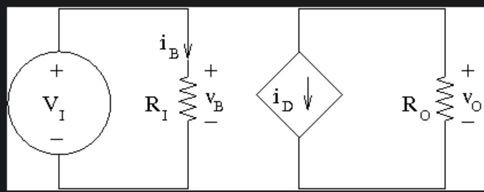
$$I_D = (8 \times 10^{-14}A) \cdot \left(e^{(510mV)/(26mV)} - 1\right) = 0.000026A$$

$$V_I = I_D \cdot R + V_D = (0.000026A) \cdot (3.9k\Omega) + (0.510V) = 0.61V$$

S8E0: Dependent Source

0 points possible (ungraded)

In the following diagram there is a dependent current source driving a resistor. The current in the dependent source may be a function of either v_B or i_B as indicated in the diagram.



For the different dependent current sources below, write the output voltage, v_O, as a function of the input voltage, V_I, and resistors, R_I and R_O.

If $i_D = K_1 \cdot v_B$ what is v_O?

-RO*K1*VI

✔ **Answer:** -RO*K1*VI

If $i_D = K_2 \cdot i_B$ what is v_O?

-(RO/RI)*K2*VI

✔ **Answer:** -(RO/RI)*K2*VI

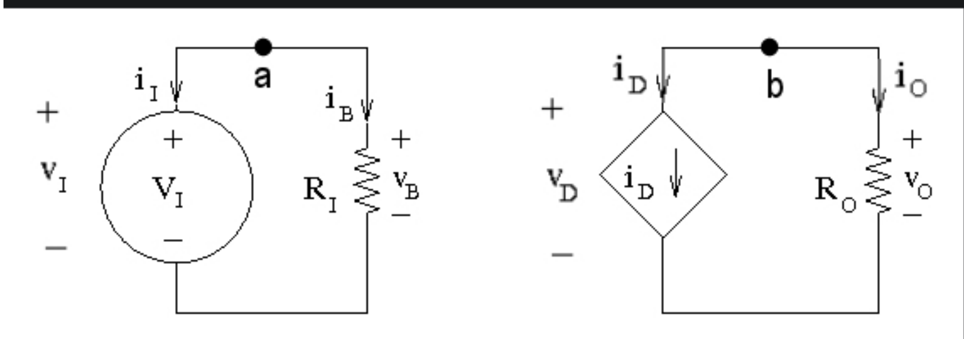
Submit

Show answer

Grove (Community TA)

2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)

I have fully labelled the two circuit diagrams using the sign convention we are using in this course.



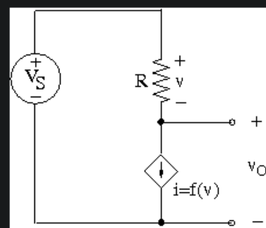
Apply KVL to the left hand loop and KCL at node b noting that $v_B = i_B R_B$ and $v_O = i_O R_O$.

(*Grove's comment should have vB=iB RI for the first eqn*)

S8E1: Dependent Current Source

0 points possible (ungraded)

The following figure shows a circuit with a nonlinear voltage-controlled current source:



Determine the voltage v_O (in Volts) across the dependent source given that $i = f(v) = \frac{K}{v^2}$. Assume that $R = 850.0\Omega$, $V_S = 5.0$V, and $K = 0.088A \cdot V^2$.

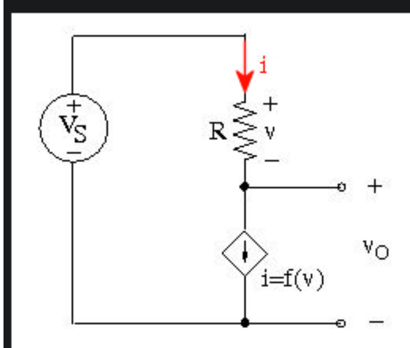
Output voltage v_O:

0.786589

✔ **Answer:** 0.78658860013969

Grove (Community TA)

3 years ago - marked as answer 3 years ago by **MIT_Lover_UA** (Staff)



Start with $v = iR$ and $i = \frac{K}{v^2}$ to find v and then use KVL clockwise $\Rightarrow -V_S + v + v_0 = 0$

hi,can you help me understand?

i didnt get why is v= iR only....isnt there any current flowing through R because of Vs?or is the current flowing due to Vs is already included in i?

posted 2 years ago by **Gln0212**

The current flowing around all the loop is the same, i.

posted 2 years ago by **Grove** (Community TA)

In[•]:=
```
vs = 5; k = 0.088; r = 850;
Solve[v == k/v^2 r, v, Reals]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{v → 4.21341}}

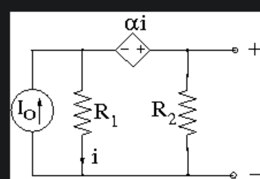
In[•]:=
```
Solve[-vs + v + vo == 0, vo] /. {v → 4.21341139986031`}
```

Out[•]= {{vo → 0.786589}}

S8E2: Dependent Voltage Source

0 points possible (ungraded)

The following circuit contains a linear current-controlled voltage source.



Find the Thevenin equivalent voltage and resistance of this circuit as seen from the indicated port. Assume that $I_O = 0.004$ A, $R_1 = 850.0\Omega$, $R_2 = 750.0\Omega$, and $\alpha = 4.0\Omega$.

Hint: Add an independent current source at the port and use superposition of independent sources. Do not suppress the dependent source!

Thevenin voltage V_{TH}:

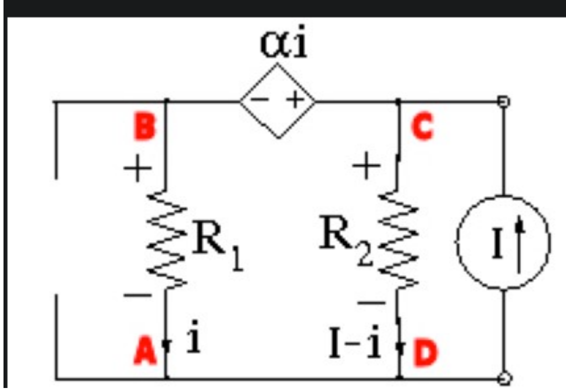
1.59726 ✔

Thevenin resistance R_{TH}:

399.314 ✔

Method 1

First to find the Thevenin resistance.
The independent current source has been removed and an open circuit has replaced it and a current source has been added to the output.



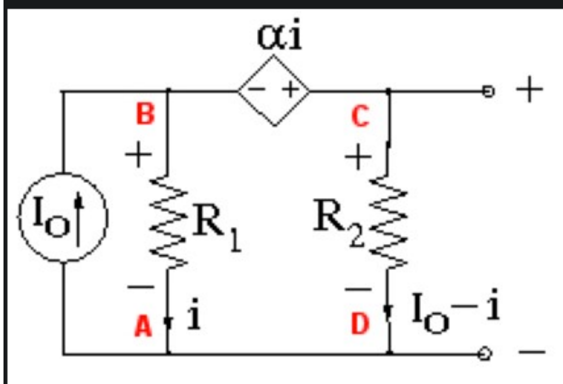
Use KVL on loop $ABCDA$ to find i in terms of I and then $R_{\mathrm{TH}} = \frac{V_{\mathrm{R}_2}}{I} = \frac{(I-i)\,R_2}{I}$

[*When you get the final formula for R_{TH} in terms of R_1, R_2 and α you may recognise it in another context especially if you change $R_1 + \alpha$ to R?*]

Once you have got R_{TH} then V_{TH} follows because the Thevenin resistance is equal to the Norton resistance.

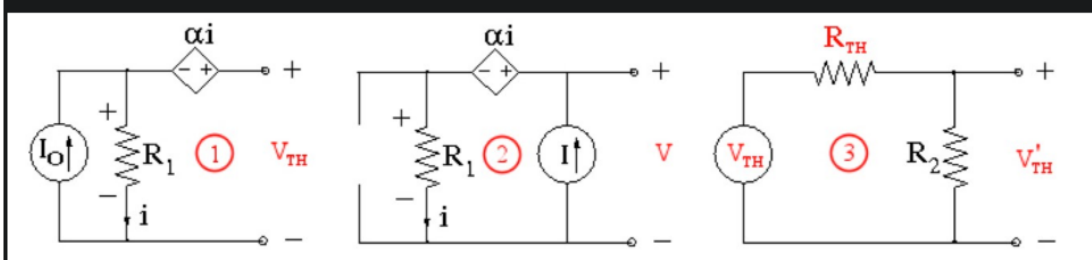
Method 2

Here you might use KVL on loop $ABCDA$ to find i in terms of I_0 and then find V_{TH} as the voltage across resistor R_2.



Method 3

This is a "two-step" way of solving the problem.



Ignore resistor R_2 and find the Thevenin voltage V_{TH} using circuit **1** and the Thevenin resistance $R_{\mathrm{TH}} = \frac{V}{I}$ using circuit **2**.

Now include resistor R_2 and find the Thevenin voltage V'_{TH} and Thevenin resistance of circuit **3**.

```
io = 0.004; r1 = 850; r2 = 750; α = 4;
(*Method 2*)
Solve[i r1 - (io - i) r2 + α i == 0, i];
vth = (io - i) r2 /. {i → 0.0018703241895261845`}
(*Method 1*)
Solve[i r1 - (Id - i) r2 + α i == 0, Id];
rth = ((Id - i) r2)/Id /. {Id → (802 i)/375} // N
```

Out[•]= 1.59726

Out[ ]= 399.314

Homework

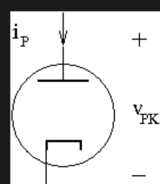
Homework due Jul 29, 2022 07:18 +04

H4P1: VACUUM DIODE

4/4 points (graded)

Although vacuum tubes are no longer commonly used in computer or consumer electronics, they still have a substantial niche in high-end audio, high-power radio transmitters, particle accelerators, and microwave ovens.

A vacuum diode, the simplest vacuum tube, is an interesting two-terminal device.



A vacuum diode's voltage-current characteristic is closely approximated by the *Childs-Langmuir Law*, with one parameter P called the perveance:

$$i_P = \begin{cases} P \cdot v_{PK}^{3/2} & \text{if } v_{PK} > 0 \\ 0 & \text{if } v_{PK} < 0 \end{cases}$$

In this problem, we will use $P \approx 2.0$mA/V$^{3/2}$ (see, for example, the 6AL5 twin diode).

What is the current I_P (in Amperes) for a bias voltage of $V_{PK} = 4.0$V?

0.016 ✔ **Answer:** 0.016

What is the incremental resistance (in Ohms) for the bias voltage of $V_{PK} = 4.0$V?

166.667 ✔ **Answer:** 166.67

What is the current I_P (in Amperes) for a bias voltage of $V_{PK} = 14.0$V?

0.104766 ✔ **Answer:** 0.1048

What is the incremental resistance (in Ohms) for the bias voltage of $V_{PK} = 14.0$V?

89.0871 ✔ **Answer:** 89.09

Explanation:
(a) The voltage across the vacuum tube is known to be $V_{PK} = 4.0V$, which implies that the device is operating in the $V_{PK} > 0$ region. The current going through the device can be calculated using Childs-Langmuir Law, as shown in the problem description. Substituting given values,

$$P = 2.0\frac{mA}{V^{3/2}} = 0.002\frac{A}{V^{3/2}}$$
$$V_{PK} = 4.0V$$

the current i_P can be evaluated as

$$i_P = P \cdot V_{PK}^{3/2} = (0.002)\left(4.0^{3/2}\right) = 0.016A$$

(b) The incremental resistance of any device is defined as the inverse of the rate of change of its current with respect to its voltage. Graphically, this is equal to the inverse of the slope of the IV curve, or mathematically $\frac{1}{R_{incr}} = \frac{di}{dv}$. In the case of the vacuum diode, we already know the relationship of current and voltage, so that by taking the derivative we can get the incremental resistance:

$$\begin{aligned} \frac{1}{R_{incr}} &= \left[\frac{di_p}{dV_{PK}}\right] \\ &= \left[\frac{d}{dV_{PK}}\left(P \cdot V_{PK}^{3/2}\right)\right] \\ &= (1.5)\, P \cdot \sqrt{V_{PK}} \\ &= (1.5)\,(0.002)\,\sqrt{4.0} \\ &= 0.006\,\frac{Amps}{Volt} \end{aligned}$$

Which means that $R_{incr} = 166.67\Omega$

(c) The same prodecure from part (a) can be done here. Therefore the current i_P evaluated at $V_{Pk} = 14.0V$

$$i_P = P \cdot V_{PK}^{3/2} = (0.002)\,(14.0)\,(\sqrt{14.0}) = 0.1048A$$

(d) Using the equation for incremental resistance discussed in part (b), we find that:

$$\begin{aligned} \frac{1}{R_{incr}} &= (1.5)\, P \cdot \sqrt{V_{PK}} \\ &= (1.5)\,(0.002)\,(\sqrt{14.0}) \\ &= 0.0112\,\frac{Amps}{Volt} \end{aligned}$$

which implies $R_{incr} = (0.0112)^{-1} = 89.09\Omega$

In[∘]:=
```
P = 2*^-3;
Vpk = 4;
Ip[Vpk_] := P * Vpk^(3/2) // N
Ip[Vpk]
1 / Ip'[Vpk]
Vpk = 14;
Ip[Vpk]
1 / Ip'[Vpk]
```

Out[∘]= 0.016

Out[∘]= 166.667

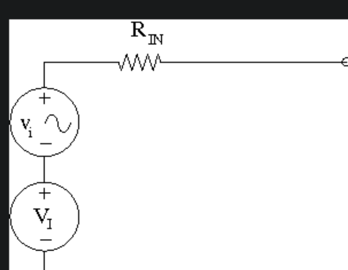
Out[∘]= 0.104766

Out[∘]= 89.0871

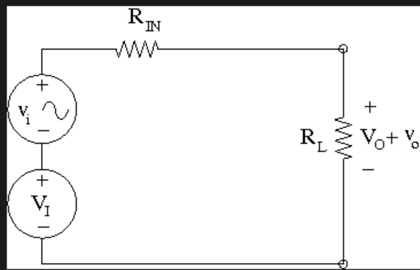
H4P2: ZENER REGULATOR

9/9 points (graded)

A non-ideal voltage source can be modeled by a series combination of an ideal DC voltage source, a resistance, and a small-signal voltage source. The small-signal source is included to represent the noise inherent in the source. This model is illustrated in the figure below:



In practice, connecting the non-ideal voltage source to a load may result in undesirable effects due to the noise voltage v_o appearing across the load.



This problem studies such effects and how a Zener diode may be used to ameliorate the problem. Assume that $V_I = 9.5$V, $R_{\text{IN}} = 1\ \text{k}\Omega$, and $v_i = 50.0$mV.

In the figure above, calculate the DC output voltage V_O, and the output noise voltage v_o for two values of the load resistance.

For $R_L = 2\ \text{k}\Omega$, the value of V_O (in Volts) is:

6.33333 ✔ **Answer:** 6.3333

and the value of v_o (in Volts, to at least 3 decimal places) is:

0.0333333 ✔ **Answer:** 0.033333333

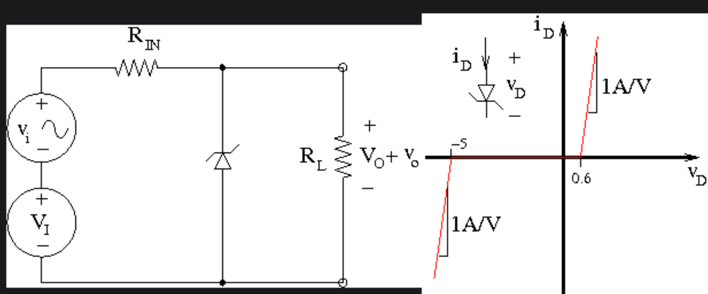
For $R_L = 4\ \text{k}\Omega$ the value of V_O (in Volts) is:

7.6 ✔ **Answer:** 7.6

and the value of v_o (in Volts, to at least 3 decimal places) is:

0.04 ✔ **Answer:** 0.04

Now, we can insert a Zener diode into the circuit, as shown below. The Zener diode is a nonlinear device and a piecewise-linear approximation to its i-v characteristic is shown graphically below.



Again, we calculate v_o (i.e. output noise) and V_O (i.e. DC output voltage) for R_L=2kΩ and R_L=4kΩ for this new circuit.

Hint: Your first job here is to determine which of the three regions of the piecewise-linear characteristic of the Zener is the one containing the operating point: we suggest you sketch a load line, but be very careful about the signs. Once you have determined where the operating point is, you can model the Zener in the circuit with a series combination of an independent voltage source and a resistor.

For $R_L = 2\ \text{k}\Omega$ the value of V_O (in Volts) is:

5.002 ✔ **Answer:** 5.002

And the value of v_o (in Volts, to at least 5 decimal places) is:

0.00004995 ✔ **Answer:** 4.9925e-05

For $R_L = 4\ \mathrm{k\Omega}$ the value of V_O (in Volts) is:

5.00325 ✔ **Answer:** 5.0032

And the value of v_o (in Volts, to at least 5 decimal places) is:

0.00004995 ✔ **Answer:** 4.9938e-05

What is the minimum value of R_L, in Ohms, that guarantees that the circuit will operate this way?

1110 ✔ **Answer:** 1111.1

Explanation:
This circuit is a voltage divider with a DC voltage source and a small-signal voltage source. We can apply superposition to find the DC and small-signal components separately. When $R_L = 2k\Omega$:
(a) Using the DC source first, we find:

$$V_o = \frac{V_1\,(R_L)}{R_L + R_{in}} = \frac{9.5\,(2k\Omega)}{3k\Omega} = 6.3333V$$

(b) Using the small-signal source:

$$v_o = \frac{V_i\,(R_L)}{R_L + R_{in}} = \frac{(0.05)\,(2k\Omega)}{3k\Omega} = 0.033333333V$$

When $R_L = 4k\Omega$, we use the same method as above in parts (a) and (b) to solve for the DC and small-signal output voltages:
(c)

$$V_o = \frac{V_1\,(R_L)}{R_L + R_{in}} = \frac{9.5\,(4k\Omega)}{5k\Omega} = 7.6V$$

(d)

$$v_o = \frac{V_i\,(R_L)}{R_L + R_{in}} = \frac{(0.05)\,(4k\Omega)}{5k\Omega} = 0.04V$$

The added Zener diode ameliorates the noise output. A complication, however, is that the Zener diode is a non-linear device. From the Zener I-V characteristic, we can see that it operates as a voltage source in series with a resistor for $V_D \le -5V$ and $V_D \ge 0.6V$. For $-5V \le V_D \le 0.6V$, the device acts as an open circuit. As we have three regions of operation, through the piece-wise linear method studied in week 4, we can solve this problem by determining in which region the Zener is operating with the given load resistances. Since $V_O + v_o$ is positive and therefore $V_D = -V_O - v_o$ is always negative, we can discard the region $V_D \ge 0$. Modeling the Zener as an open circuit will give us the schematic used in parts (a) - (d), for which $V_D < -5V$. This eliminates the $-5V \le V_D \le 0.6V$ range of operation. We can conclude therefore that the Zener operates in the $V_D \le -5V$ region. We model the Zener Diode as a $-5V$ DC Voltage source (with the same polarity as V_D) in series with a 1Ω resistor (to account for the $\frac{A}{V}$ slope of the I-V plot).
(e) Replacing the equivalent model in the circuit, we can solve for the output voltage using the node method. Again, we use superposition to find the DC and small signal components one at a time. For the DC component, for $R_L = 2k\Omega$, the node method gives:

$$\frac{V_I - V_O}{1000\Omega} + \frac{(5V) - V_O}{1\Omega} = \frac{V_O}{2000\Omega}$$

Which gives $V_O = 5.002V$
(f) For the small signal component, we turn off both independent sources so that we are left with a voltage divider again, with the output resistor being $R_L \parallel 1\Omega$ (where $R_L = 2k\Omega$):

$$v_o = \frac{v_i \cdot (R_L \parallel 1\Omega)}{(R_L \parallel 1\Omega) + 1k\Omega} = (9.985022 \times 10^{-4}) \cdot v_i = 4.9925e - 05V$$

(g) Using the equation from part (e) and replacing $R_L = 4k\Omega$:

$$\frac{V_I - V_O}{1000\Omega} + \frac{(5V) - V_O}{1\Omega} = \frac{V_O}{4000\Omega} \rightarrow V_O = 5.0032V$$

(h) Using the equation from part (f) and replacing $R_L = 4k\Omega$:

$$v_o = \frac{v_i \cdot (R_L \parallel 1\Omega)}{(R_L \parallel 1\Omega) + 1k\Omega} = (9.987514 \times 10^{-4}) \cdot v_i = 4.9938e - 05V$$

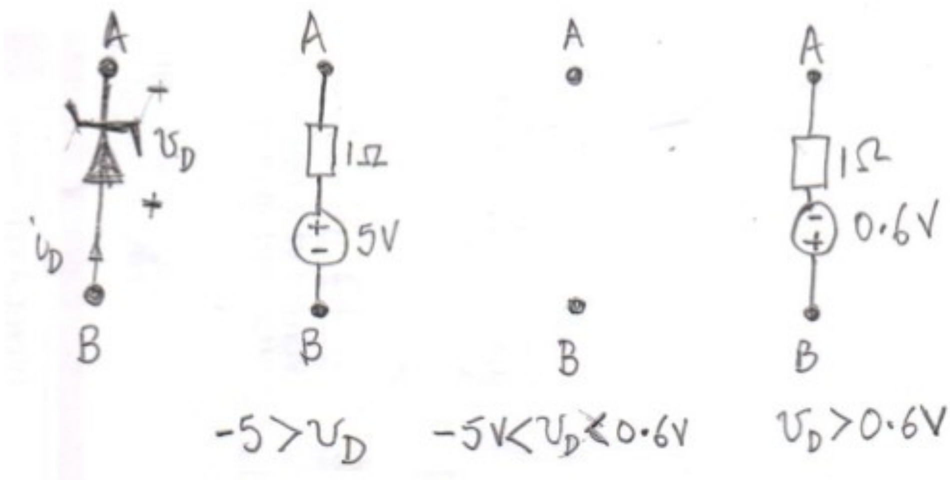
(i) We want to find the minimum value of R_L that guarantees that the Zener will operate in the $V_D \leq 5V$ region. Due to the continuity of the I-V curve of the Zener, the R_L should be the same whether we calculate it using the model of the Zener used in parts (e)-(h) (i.e. independent voltage source in series with resistance) or using the open circuit model. For simplicity we use the open circuit model, so that we have the circuit from part (a)-(d) again. Using that circuit, we set the output voltage $V_O = 5V$, which was set by the Zener diode. Using the circuit from parts (a) - (d), we solve for R_{Lmin}:

$$5V = V_i \cdot \frac{R_{Lmin}}{R_{Lmin} + 1k\Omega}$$

$$1k\Omega = R_L \cdot (\frac{9.5}{5} - 1)$$

$$R_{Lmin} = 1111.1\Omega$$

(*Zener diode behavior*)



In[◦]:=
```
VI = 9.5; Rin = 1*^3; vi = 50*^-3;
RL = 2*^3;
Vo = (RL*VI)/(RL + Rin) // N
vo = (RL*vi)/(RL + Rin) // N
RL = 4*^3;
Vo = (RL*VI)/(RL + Rin) // N
vo = (RL*vi)/(RL + Rin) // N
RL = 2*^3;
(*Choosing the node above the Zener diode*)
Solve[(x - VI)/Rin + (x - 5)/1 + x/RL == 0, x] // N
vo = (1*vi)/(1 + Rin) // N
RL = 4*^3;
Solve[(x - VI)/Rin + (x - 5)/1 + x/RL == 0, x] // N
vo = (1*vi)/(1 + Rin) // N
(*Check image for the minimum value of RL*)
```

Out[◦]= 6.33333

Out[◦]= 0.0333333

Out[◦]= 7.6

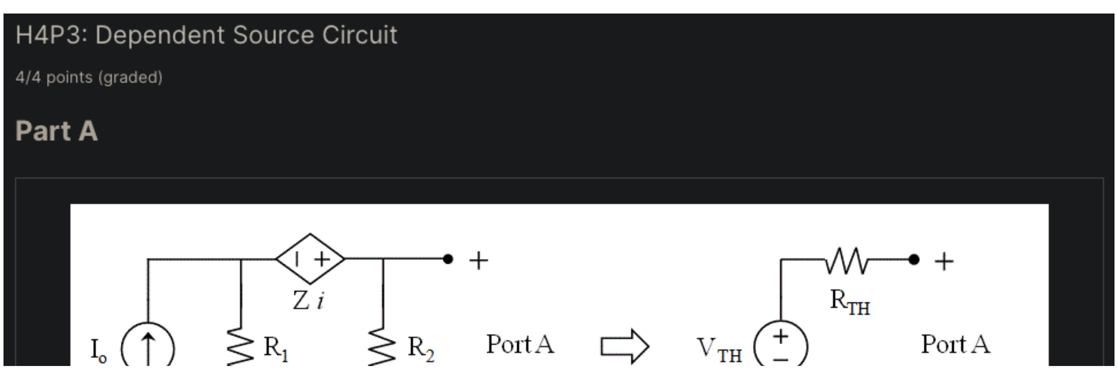
Out[◦]= 0.04

Out[◦]= {{x → 5.002}}

Out[◦]= 0.00004995

Out[◦]= {{x → 5.00325}}

Out[◦]= 0.00004995



The figure above shows a circuit with a linear current-controlled-voltage-source and its Thevenin equivalent model as seen from Port A. Given that $I_0 = 2A, Z = 2\Omega, R_1 = 2\Omega, R_2 = 4\Omega$, Determine the Thevenin voltage V_{TH} and the Thevenin resistance R_{TH}.

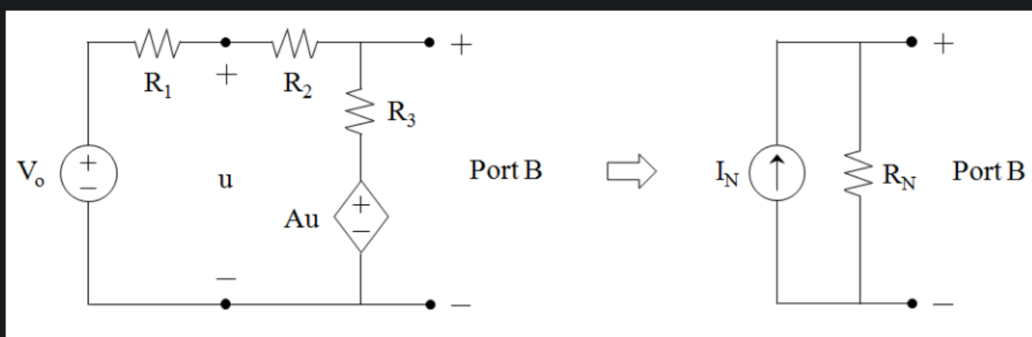
What is the Thevenin voltage across port A, in Volts?

4 ✔ **Answer:** 4

What is the Thevenin resistance as seen from port A in Ohms?

2 ✔ **Answer:** 2

Part B



The figure above shows a circuit with a linear voltage-controlled-voltage-source and its Norton equivalent model as seen from Port B. Given that $V_o = 5V, A = 2, R_1 = 1\Omega, R_2 = 3\Omega, R_3 = 5\Omega$, determine the Norton current I_N and Norton resistance R_N.

What is the Norton equivalent current in Amperes?

11/4 ✔ **Answer:** 2.75

What is the Norton resistance in Ohms?

30/11 ✔ **Answer:** 2.85

Explanation:
Part A
(i) First we are asked to compute the Thevenin voltage of the circuit. Applying the node method, we get two equations to solve for, with two unknowns:

$$\frac{e_1}{R_1} + \frac{V_{th}}{R_2} = I_o$$

$$V_{th} - e_1 = Z \cdot \frac{e_1}{R_1}$$

(Where e_1 is the potential at the node connecting the negative side of the dependent voltage source to the current source). From the second equation, we can get V_{th} in terms of e_1:

$$V_{th} = e_1 \cdot \left(1 + \frac{Z}{R_1}\right)$$

Using this partial result in the first equation, we get:

$$e_1 \cdot \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{Z}{R_1 R_2}\right] = I_o$$

$$e_1 = I_o \cdot \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{Z}{R_1 R_2}\right]^{-1}$$

Using this result in the expression for V_{th} found before, we get:

$$V_{th} = I_o \cdot \left(1 + \frac{Z}{R_1}\right) \cdot \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{Z}{R_1 R_2}\right]^{-1}$$

Substituting in the values $I_o = 2A$, $Z = 2\Omega$, $R_1 = 2\Omega$ and $R_2 = 4\Omega$, we get:

$$V_{th} = (2A) \cdot \left(1 + \frac{2\Omega}{2\Omega}\right) \cdot \left[\frac{1}{2\Omega} + \frac{1}{4\Omega} + \frac{2\Omega}{(2\Omega)\,(4\Omega)}\right]^{-1} = 4V$$

(ii) To find the Thevenin resistance, we can't just turn off the independent sources and leave the dependent ones, because dependent sources might be turned on by independent sources in the network, and that effect must be taken into account. The easiest way to calculate the Thevenin resistance is to connect a short circuit to the output terminals and calculate the current through it. R_{th} is then $\frac{V_{th}}{I_{sc}}$, where I_{sc} is the short-circuit current.

To solve for the short-circuit current, we apply the node method again, and get two equations:

$$I_o - \frac{e_1}{R_1} = I_{sc}$$

$$e_1 + e_1 \cdot \frac{Z}{R_1} = e_1 \cdot \left(1 + \frac{Z}{R_1}\right) = 0$$

The second equation necessitates $e_1 = 0V$, which transforms the first equation into:

$$I_o = I_{sc} = 2A$$

Using this result, we calculate the Thevenin resistance as:

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{4V}{2A} = 2\Omega$$

Part B

(i) To find the Norton current in this circuit, we connect a short circuit to the output terminals and compute the current through it. The Norton current is then that current. We can apply the node method to solve for $I_{sc} = I_N$:

$$I_{sc} = \frac{V_o}{R_1 + R_2} + \frac{Au}{R_3}$$

The value of "u" can be found using a voltage divider:

$$u = \frac{R_2}{R_1 + R_2} \cdot V_o$$

Using the second equation within the first one, we can compute the value of the short circuit current:

$$I_N = \frac{V_o}{R_1 + R_2} + \frac{A}{R_3} \cdot \frac{R_2}{R_1 + R_2} \cdot V_o$$

Using the values $V_o = 5V$, $A = 2$, $R_1 = 1\Omega$, $R_2 = 3\Omega$ and $R_3 = 5\Omega$:

$$I_N = \frac{5V}{1\Omega + 3\Omega} + \frac{2}{5\Omega} \cdot \frac{3\Omega}{1\Omega + 3\Omega} \cdot (5V) = 2.75A$$

(ii) To calculate the Norton resistance, we first calculate the Thevenin voltage by leaving the output terminal as an open circuit. The circuit can be solved by the node method, but it is also possible to use the loop method. With the loop method, we get one equation for an assumed current "i" with its positive sign leaving the positive terminal of the independent source V_o. The equation is:

$$V_o - R_1 \cdot i - R_2 \cdot i - R_3 \cdot i - Au = 0$$

The expression for "u" is found using KVL:

$$u = V_o - R_1 \cdot i$$

Therefore, using the given values, we can calculate the current to be:

$$i = \frac{V_o \cdot (1-A)}{R_1 \cdot (1-A) + R_2 + R_3} = -0.7143A$$

Note: the fact that the current is negative does not affect the procedure to calculate V_{th}. The Thevenin voltage is found through KVL:

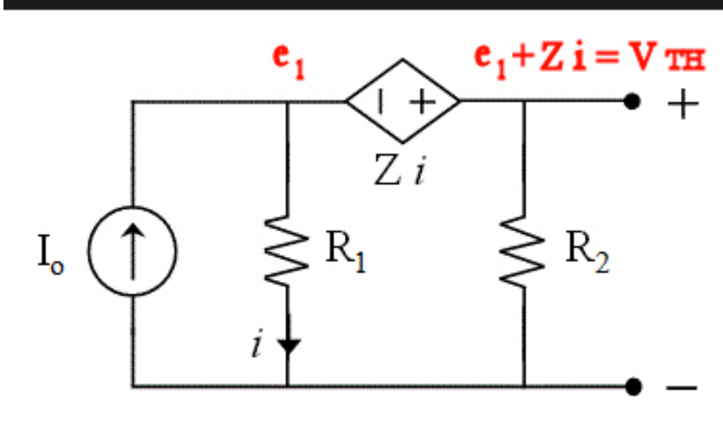
$$V_{th} = A \cdot u + R_3 \cdot i = A \cdot (V_o - R_1 \cdot i) + R_3 \cdot i$$

Substituting in the current value found earler into the equation above, we get:

$$V_{th} = (2) \cdot (5V - (1\Omega)(-0.7143A)) + (5\Omega) \cdot (-0.7143A) = 7.86V$$

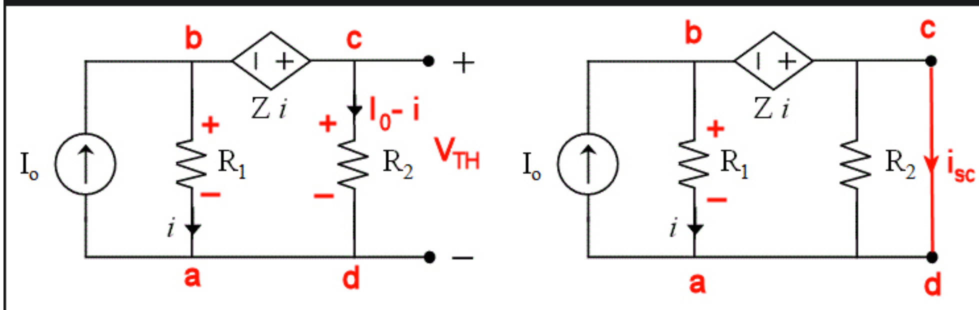
The Norton resistance R_N is calculated through Ohm's law:

$$R_N = \frac{V_{th}}{I_N} = \frac{7.86V}{2.75A} = 2.85\Omega$$



The potential difference across the voltage source is $Zi = V_{\mathrm{TH}} - e_1$.

Let me explain and then it will lead on to the Norton solution although you will realise there are many ways of solving such circuit problems.

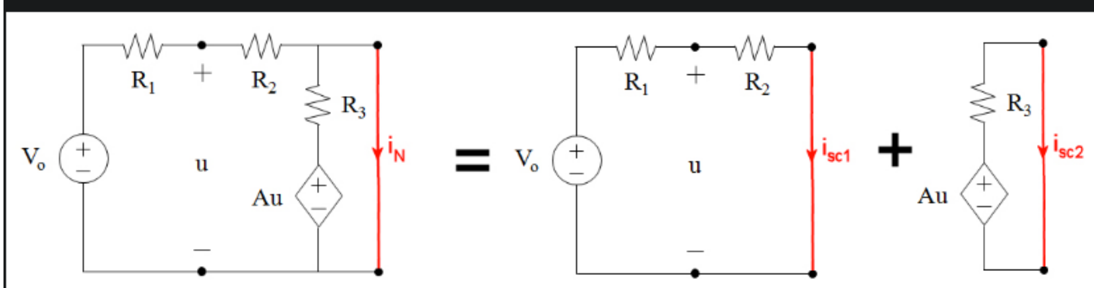


To find the Thevenin voltage, V_{TH}, one needs to find the open circuit voltage across nodes c and d as shown in the left hand diagram.
I have added some extra labels which might help with the analysis.
If current, i, can be found then knowing I_0 and R_2 the Thevenin voltage cab=n be found.
You can find i by applying KVL to loop $abcda$.

To find the Thevenin resistance R_{TH} short out the output nodes c and d and find the short circuit current i_{sc} noting that $R_{\mathrm{TH}} = \dfrac{V_{\mathrm{TH}}}{i_{\mathrm{sc}}}$.
Use of KVL for loop $abcda$ will give you a (possibly unexpected?) value of i.

Think of the Norton solution as the opposite of the Thevenin solution and again there are many ways of solving the problem.

This time start by shorting out the output nodes to find $I_{\mathrm{N}} = i_{\mathrm{sc1}} + i_{\mathrm{sc2}}$.



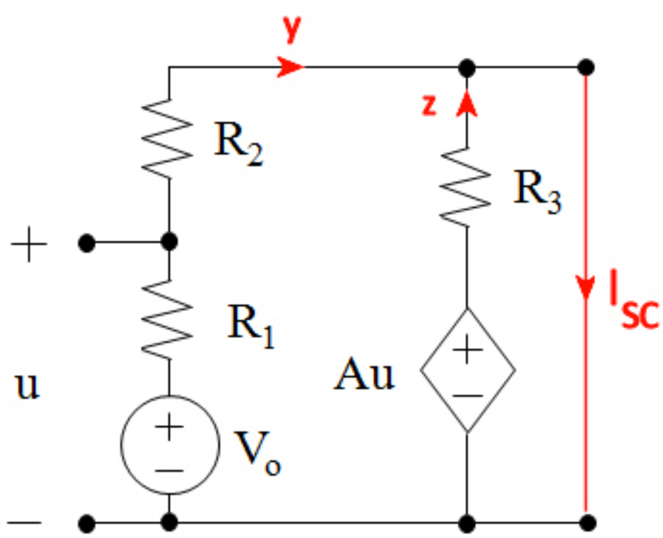
The middle diagram allows you to find i_{sc1} and also u whose value you can use to find i_{sc2} using KVL applied to the right hand diagram.

I leave you to figure out how to find the Norton resistance R_{N} - think open circuit voltage the equivalent of finding the short circuit current in the Thevenin method!

In[]:=
```
io = 2; z = 2; r1 = 2; r2 = 4;
(*KVL on loop abcda*)
Solve[-i r1 - z i + (io - i) r2 == 0, i];
(*Using the fig where node b is marked as e1*)
vth = i r1 + z i /. {i → 1}
(*Using the closed circuit isc would be the same as io*)
rth = vth/io /. {i → 1}
```

Out[]= 4

Out[]= 2




```
(*Can also use the diagram which clearly shows current superposition Isc =
 I_N = Isc1 + Isc2*)
vo = 5; a = 2; r1 = 1; r2 = 3; r3 = 5; u = vo;
y = vo/(r1 + r2); (*Same as Isc1*)
(*u would be measuring vo minus voltage drop over r1*)
u = vo - y r1;
z = a u/r3; (*Same as Isc2*)
in = y + z
(*Clearly Thevenin voltage is Au*)
vth = a u;
rn = vth / in
```

Out[]= $\frac{11}{4}$

Out[]= $\frac{30}{11}$

Lab

In[ ]:=
```
vds = 1; vgs = 3; v1 = 3;
Solve[4 x * 10 ^ -5 == k (2) ^ 2 / 2, k]
```

Out[ ]=
$$\left\{\left\{k \to \frac{x}{50\,000}\right\}\right\}$$