Week 12

S23 - The Operational Amplifier Abstraction

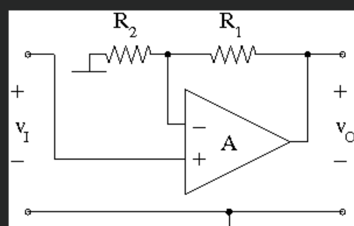
S24 - Operational Amplifier Circuits

Lectures

S23E1: Non-Inverting Amplifier

0 points possible (ungraded)

In the figure below we have a non-inverting amplifier. It is drawn in a way that we see in many professional circuit diagrams.



In our circuit $R_1 = 4000\Omega$ and $R_2 = 1000\Omega$.

If $A = 10$ what is the voltage gain v_O/v_I of this amplifier?

3.33333 **Answer:** 3.3333333333333335

If $A = 100$ what is the voltage gain v_O/v_I of this amplifier?

4.7619 **Answer:** 4.761904761904762

If $A = 100000$ what is the voltage gain v_O/v_I of this amplifier?

4.9975 **Answer:** 4.9997500124993755

Notice how the gain approaches $(R_1 + R_2)/R_2 = 5.0$ as $A \to \infty$.

Detailed Solution:

We can start by applying Kirchoff's Rules to the top branch and obtain:

$$v_2/R_2 + (v_2 - v_O)/R_1 = 0$$

$$v_2 = \frac{v_O}{R_1} \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

We can use our Amplifier Characteristic to write out an expression relating our voltages:

$$v_o = A(v_2 - v_I)$$

Combining the two equations and doing a little algebra, we get:

$$\frac{v_O}{v_I} = \frac{A}{1 + \frac{AR_2}{R_1 + R_2}}$$

R_1+R_2

Now, we can just plug in each A value to get an answer:

$$A=10 \rightarrow \frac{v_O}{v_I}=\frac{10}{1+\frac{10}{5}}=3.333$$

$$A=100 \rightarrow \frac{v_O}{v_I}=\frac{100}{1+\frac{100}{5}}=4.762$$

$$A=100000 \rightarrow \frac{v_O}{v_I}=\frac{100000}{1+\frac{100000}{5}} \approx \frac{10000}{\frac{100000}{5}}=5$$

It is important to note that large A values gives an new relation between our voltages:

$$\lim_{A \to \infty} v_2 = v_I$$

In[∘]:=
```
r1 = 4000; r2 = 1000;
g[A_] := A / (1 + (A r2)/(r1+r2));

g[10] // N

g[100] // N

g[10 000] // N

Limit[g[A], A → ∞] == (r1 + r2)/r2
```

Out[∘]= 3.33333

Out[∘]= 4.7619

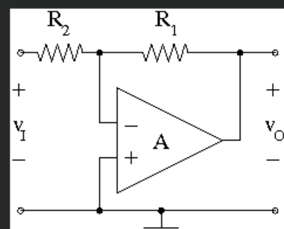
Out[∘]= 4.9975

Out[∘]= True

S23E2: Inverting Amplifier analysis

0 points possible (ungraded)

In the figure below we have an inverting amplifier. It is drawn in a way that we see in many professional circuit diagrams.



In our circuit $R_1 = 4000\Omega$ and $R_2 = 1000\Omega$.

If $A = 10$ what is the voltage gain v_O/v_I of this amplifier?

-2.66667 ✔ **Answer:** -2.6666666666666665

If $A = 100$ what is the voltage gain v_O/v_I of this amplifier?

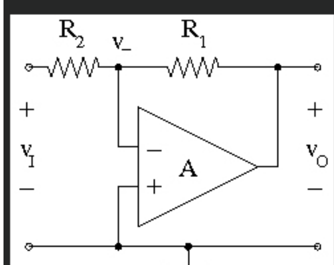
-3.80952 ✔ **Answer:** -3.8095238095238093

If $A = 100000$ what is the voltage gain v_O/v_I of this amplifier?

-3.998 ✔ **Answer:** -3.9998000099995

Notice how the gain approaches $-R_1/R_2 = -4.0$ as $A \to \infty$.

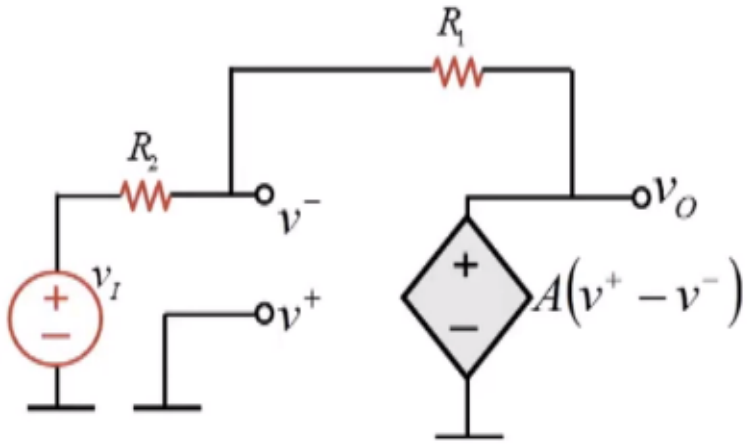
Grove (Community TA)
2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)



Because $i_- \approx 0$ you can say that the current through the two resistors is the same and so you can write an equation linking $v_I,\ v_-,\ v_O,\ R_1$ and R_2 via $i = \dfrac{\Delta v}{R}$.
You might find the algebra easier by first using conductance, $Y = \frac{1}{R}$ so $i = \Delta v\, Y$ and then changing back to resistance towards the end of the solution.

You can eliminate v_- from the equation because $A\,(v_+ - v_-) = v_O$ and $v_+ = 0$ leaving you with an equation from which you can find $\dfrac{v_O}{v_I}$ in terms of $A,\ R_1$ and R_2.

```
(* Next 3 cells are OPTIONAL
 Comment from the following lecture,
relevant to this exercise. Prof. redraws the circuit,
and uses superposition method. Will ignore this solution because it is a special
 case where we can set dependent source to zero, which is not usually allowed.*)
```



Clarification: Prof. Agarwal uses superposition here to analyze the subcircuit containing the two resistors R_1 and R_2 and the voltages v_I and v_O. In other words, he is taking v_O as a given, and carving out the subcircuit containing the two resistors and two voltage sources, in order to find the voltage v^- in terms of v_I and v_O. So the dependent voltage source does not figure in the computation and superposition works.

In general, if a dependent source is involved in the computation, then you cannot simply go and set it to 0.

You can also obtain the same expression for v^- by applying basic circuit methods such as KVL/KCL. Using KVL, you get:

$v^- = v_I + (v_O - v_I) \cdot \frac{R_2}{R_1+R_2} = v_I \cdot \frac{R_1}{R_1+R_2} + v_O \cdot \frac{R_2}{R_1+R_2}$

Yes, I read the comments below the video. Usually, Prof Agarwal lectures are excellent and very clear. I find the use of superposition in this lecture very confusing. How is a student to know when it is safe to use superposition as Prof Agarwal has done in this video? Section 15.3.2 in A & L textbook does not use superposition. In fact, it warns at the very end of this section about using superposition when dependent sources are involved. Unless this course can clearly define when it is safe to use superposition when dependent sources are involved, I suggest it would be better not to use superposition in videos like this one.

posted 3 years ago by Harry48

```
r1 = 4000; r2 = 1000;

(*
 Current through the two resistors is the same, so: (vI-v-)/R2 = (v--vo)/R1.
      Then from Grove's hint we get -Av- = vo.
    Combining the two we get the gain
   vo/vI (just do some algebra to get vI/vo, the inverted gain)
*)
g[A_] := (-r2/A ((1 + A)/r1 + 1/r2))^-1;

g[10] // N

g[100] // N

g[10 000] // N

Limit[g[A], A → ∞] == -r1/r2
```

Out[∘]= -2.66667

Out[∘]= -3.80952

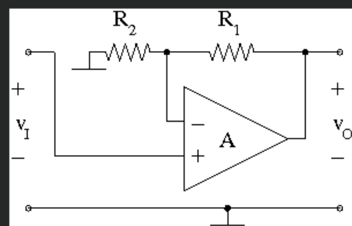
Out[∘]= -3.998

Out[∘]= True

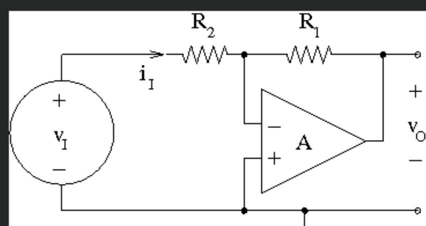
S23E3: L23AmplifierInputResistance

0 points possible (ungraded)

In the non-inverting amplifier the input resistance (the resistance looking into the input port) is infinite, because the operational amplifier does not take any current into its input terminals (in our model: this is only approximately true.)



However, the inverting amplifier has a finite input resistance.



In our circuit $R_1 = 4000\Omega$ and $R_2 = 1000\Omega$. Here we look at how the input resistance depends on the gain of the operational amplifier.

If $A = 10$ what is the input resistance v_I/i_I, in Ohms, of this amplifier?

1363.64 ✔ **Answer:** 1363.6363636363635

If $A = 100$ what is the input resistance v_I/i_I, in Ohms, of this amplifier?

1039.6 ✔ **Answer:** 1039.6039603960396

If $A = 100000$ what is the input resistance v_I/i_I, in Ohms, of this amplifier?

1000.04 ✔ **Answer:** 1000.039999600004

Notice how the resistance approaches $R_2 = 1000$ as $A \to \infty$.

Detailed Solution:

We can start again by applying Kirchoff's Rules to the node at the negative terminal and obtain:

$$i_I = v_I \times \frac{A+1}{R_2 \times A + R_2 + R_1}$$

Therefore, the resistance can be calculated as:

$$R_{in} = \frac{v_I}{i_I} = \frac{R_1 + (A+1)\,R_2}{A+1}$$

Now, we can just plug in each A value to obtain the following three conclusions: (note: To obtain the gain (V0/VI), please realize that the current going to two resistors is the same; and we have A(V+ - V-) = V0. Then you will find V0/VI is the function of A, R1, and R2. This part is left for your homework.):

$$A = 10 \rightarrow \frac{4 + 11 \times 1}{11} = 1363.6$$

$$A = 100 \rightarrow \frac{4 + 101 \times 1}{101} = 1039.6$$

$$A = 100000 \rightarrow \frac{4 + 100001 \times 1}{100001} \approx \frac{100001 \times 1}{100001} = 1000$$

In[∘]:=
```
r1 = 4000; r2 = 1000;

rin[A_] = (r1 + (A + 1) r2) / (A + 1);

rin[10] // N

rin[100] // N

rin[100 000] // N

Limit[rin[A], A → ∞] == r2
```

Out[∘]= 1363.64

Out[∘]= 1039.6

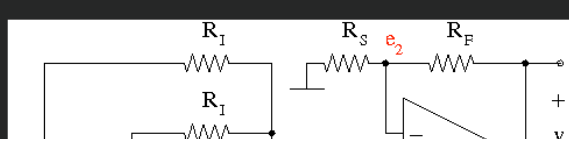
Out[∘]= 1000.04

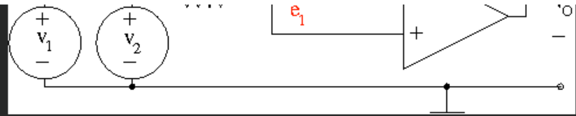
Out[∘]= True

S24E1: Summing Amplifier

0 points possible (ungraded)

Consider the following circuit:





Assume that the op amp is ideal, with infinite gain, infinite input resistance, and zero output resistance.

In the box provided below write an algebraic expression in voltages v_1 and v_2 for the node potential e_1.

(v1+v2)/2 ✔ **Answer:** (v1+v2)/2

$$\frac{v_1+v_2}{2}$$

In the box provided below write an algebraic expression in voltages v_1 and v_2 for the node potential e_2.

(v1+v2)/2 ✔ **Answer:** (v1+v2)/2

$$\frac{v_1+v_2}{2}$$

In the box provided below write an algebraic expression in voltages v_1 and v_2 for the output voltage v_O.

((RF+RS)/RS)*((v1+v2)/2) ✔ **Answer:** ((RF+RS)/RS)*((v1+v2)/2)

$$\left(\frac{R_F+R_S}{R_S}\right)\cdot\left(\frac{v_1+v_2}{2}\right)$$

If we want the output voltage to be $v_O = v_1 + v_2$ what should be the resistance R_F in terms of the other resistances? Write an algebraic expression for R_F in the box provided below.

RS ✔ **Answer:** RS

$$R_S$$

This circuit is called a "summing amplifier". As you see, we can control the scale of the output with the resistors R_F and R_S. We can also make a weighted sum by manipulating the resistances of the resistors labeled R_I. Do you see how to do that?

Detailed Solution:

Part 1

We can use the KCL on the node to obtain an expression:

$$\frac{e_1-v_1}{R_1}+\frac{e_1-v_2}{R_1}=0$$

$$e_1-v_1=v_2-e_1$$

$$e_1=\frac{v_1+v_2}{2}$$

Part 2

We know that at, for an ideal op amp, the positive and terminal nodes should have the same potential. Therefore,

$$e_2=e_1=\frac{v_1+v_2}{2}$$

Part 3

We can use the KCL on the op amp's negative terminal to obtain an expression:

$$\frac{e_2 - v_O}{R_F} + \frac{e_2}{R_S} = 0$$

$$\frac{v_O}{R_F} = e_2 \times (\frac{1}{R_F} + \frac{1}{R_S})$$

We already solved for the terminal nodes, so just plug that back in:

$$\frac{v_O}{R_F} = \frac{v_1 + v_2}{2} \times (\frac{1}{R_F} + \frac{1}{R_S})$$

$$v_O = \frac{(v_1 + v_2)(R_S + R_F)}{2R_S}$$

Part 4

We already have an expression for the output voltage, we can substitute the equations we already have to obtain an answer:

$$v_1 + v_2 = \frac{(v_1 + v_2)(R_S + R_F)}{2R_S}$$

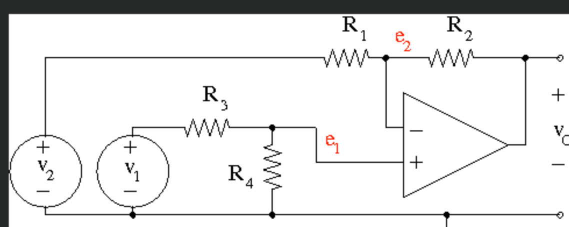
$$1 = \frac{(R_S + R_F)}{2R_S}$$

$$R_F = R_S$$

S24E2: Difference Amplifier

0 points possible (ungraded)

The following circuit is a generalization of the one you saw in lecture, in that the resistances are all distinct.



Assume that the op amp is ideal, with infinite gain, infinite input resistance, and zero output resistance.

In the box provided below write an algebraic expression in voltages v_1 and v_2 and the resistances for the node potential e_1.

(R4/(R3+R4))*v1

✔ **Answer:** v1*(R4/(R3+R4))

$$\left(\frac{R_4}{R_3 + R_4}\right) \cdot v_1$$

In the box provided below write an algebraic expression in voltages v_1 and v_2 and the resistances for the node potential e_2.

(R4/(R3+R4))*v1

✔ **Answer:** v1*(R4/(R3+R4))

$$\left(\frac{R_4}{R_3 + R_4}\right) \cdot v_1$$

In the box provided below write an algebraic expression in voltages v_1 and v_2 and the resistances for the output voltage v_O.

(R4/(R3+R4))*((R1+R2)/R1)*v1 - (R2/R1)*v2

✔ **Answer:** (R4/(R3+R4))*((R1+R2)/R1)*v1 - (R2/R1)*v2

$$\left(\frac{R_4}{R_3 + R_4}\right) \cdot \left(\frac{R_1 + R_2}{R_1}\right) \cdot v_1 - \left(\frac{R_2}{R_1}\right) \cdot v_2$$

Suppose we want a circuit that gives $v_O = 2v_1 - 3v_2$. And suppose we know that $R_1 = R_4 = 1\text{k}\Omega$.

Choose R_2 and R_3, in kΩ, to make this circuit perform this computation.

R_2 =

3 ✔ **Answer:** 3

R_3 =

1 ✔ **Answer:** 1

Detailed Solution:

Part 1

This potential is a voltage splitter between resistances 3 and 4:

$$e_1 = v_1 \times \frac{R_4}{R_4 + R_3}$$

Part 2

For an ideal op amp, the positive and negative terminals have the same potential:

$$e_2 = e_1 = v_1 \times \frac{R_4}{R_4 + R_3}$$

Part 3

We can use the KCL on the op amp's negative terminal to obtain an expression:

$$\frac{e_2 - v_2}{R_1} + \frac{e_2 - v_O}{R_2} = 0$$

$$\frac{v_O}{R_2} = e_2 \times \left(\frac{1}{R_1} + \frac{1}{R_1}\right) - \frac{v_2}{R_1}$$

We already solved for the terminal nodes, so just plug that back in:

$$\frac{v_O}{R_2} = v_1 \times \frac{R_4}{R_4 + R_3} \times \left(\frac{1}{R_2} + \frac{1}{R_1}\right) - \frac{v_2}{R_1}$$

$$v_O = v_1 \times \frac{R_4}{R_4 + R_3} \times \frac{R_2 + R_1}{R_1} - v_2 \times \frac{R_2}{R_1}$$

Part 4

Start substituting values into our equation for the output voltage:

$$v_O = 2v_1 - 3v_2 = v_1 \times \frac{1}{1 + R_3} \times \frac{R_2 + 1}{1} - v_2 \times \frac{R_2}{1}$$

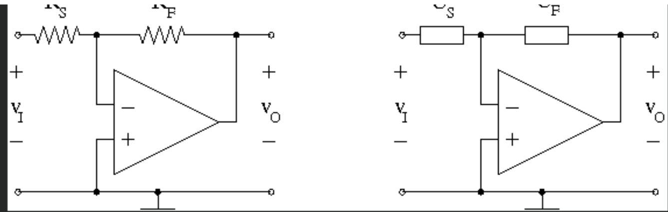
$$\frac{R_2}{1} = 3 \rightarrow R_2 = 3k\Omega$$

$$\frac{1}{1 + R_3} \times \frac{3 + 1}{1} = 2 \rightarrow R_3 = 1k\Omega$$

S24E3: Inverting Amplifier Generalized

0 points possible (ungraded)

The inverting amplifier is a very flexible concept.



We now know that for two resistors, the gain of this configuration approaches $\frac{v_O}{v_I} = -\frac{R_F}{R_S}$ as the op amp approaches the ideal.

We have also learned that this idea can be generalized to elements that store energy. For example, if we use a capacitor in the feedback path (U_F is a capacitor of capacitance C and U_S is a resistor of resistance R), then

$$v_O = -\frac{1}{RC} \int_{-\infty}^{t} v_I(t)\, dt$$

Suppose that we make U_S an inductor with inductance L and U_F a resistor with resistance R. Using the same reasoning we will also get an integrator of the form:

$$v_O = A \int_{-\infty}^{t} v_I(t)\, dt$$

In the space provided below give an algebraic expression for A in the formula above:

-R/L ✔ **Answer:** -R/L

$$-\frac{R}{L}$$

For even more fun, we could put a capacitor in for U_F and an inductor in for U_S, yielding

$$v_O = B \int_{-\infty}^{t} \int_{-\infty}^{t} v_I(t)\, dt^2$$

In the space provided below give an algebraic expression for B in the formula above:

-1/(L*C) ✔ **Answer:** -1/(L*C)

$$-\frac{1}{L \cdot C}$$

Detailed Solution:

Part 1

$$L\frac{di_L}{dt} = v_L \rightarrow i_L = \frac{1}{L}\int_{-\infty}^{t} v_L dt$$

$$i_L + i_R = 0 \rightarrow i_L = -i_S$$

$$i_L = -\frac{v_O}{R}$$

We know that the positive and negative terminals of an ideal op amp have the same potential:

$$v_L = v_I$$

$$-\frac{v_O}{R} = \frac{1}{L}\int_{-\infty}^{t} v_I\,dt$$

$$v_O = -\frac{R}{L}\int_{-\infty}^{t} v_I\,dt = A\int_{-\infty}^{t} v_I\,dt$$

A = -\frac{R}{L}

Part 2

As a quick shortcut, the op amp potentials have not changed and the equation for the inductor characteristics have not deviated:

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v_I\,dt$$

Now, derive some equations to describe the voltage and current on the capacitor:

$$i_C = \frac{dQ_c}{dt} = C\frac{dv_C}{dt}$$

We can use KCL to determine a relationship between these two currents:

$$i_L + i_C = 0 \rightarrow i_C = -i_L$$

$$C\frac{dv_C}{dt} = -\frac{1}{L}\int_{-\infty}^{t} v_I\,dt$$

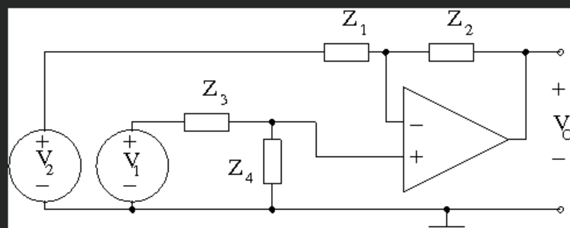
$$v_C = -\frac{1}{CL}\int_{-\infty}^{t}\int_{-\infty}^{t} v_I\,dt^2$$

$$B = -\frac{1}{LC}$$

S24E4: Generalization to impedances

0 points possible (ungraded)

A powerful generalization of the use of operational amplifiers is to complex amplitudes and impedances. For example, we can make the impedance generalization of the difference amplifier:



In the space provided below give an algebraic expression for the complex amplitude, V_O, of the output voltage in this circuit. Your expression will be in terms of the element impedances and V_1 and V_2, the complex amplitudes of the inputs.

((Z1 +Z2)/Z1)*(Z4/(Z3+Z4))*V1 - (Z2/Z1)*V2

✔ **Answer:** ((Z1 +Z2)/Z1)*(Z4/(Z3+Z4))*V1 - (Z2/Z1)*V2

$$\left(\frac{Z_1+Z_2}{Z_1}\right)\cdot\left(\frac{Z_4}{Z_3+Z_4}\right)\cdot V_1-\left(\frac{Z_2}{Z_1}\right)\cdot V_2$$

Detailed Solution:

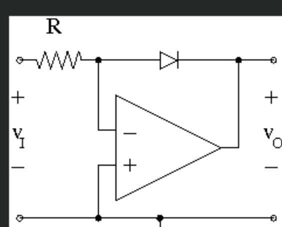
We solved for something very similar in exercise 2 of this same section. Because the only difference is that we replaced the resistances with the more general impedances, our derivation will be of a similar form (see exercise 2 for all the details, the only difference here is that $R_i \to Z_i$).:

$$v_O = v_1 \times \frac{Z_4}{Z_4+Z_3} \times \frac{Z_2+Z_1}{Z_1} - v_2 \times \frac{Z_2}{Z_1}$$

S24E5: Generalization to nonlinear elements

0 points possible (ungraded)

We can also generalize to nonlinear elements. For example, a junction diode has the characteristic $i_D = I_S\left(e^{v_D/V_T}-1\right)$.



In the space provided below give an algebraic expression for v_O in this circuit. Your expression will be in terms of V_T, I_S, R, and v_I.

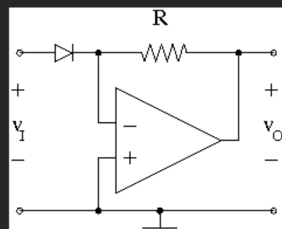
-VT*ln(vI/(R*IS) + 1)

✔ **Answer:** -VT*ln(vI/(R*IS) + 1)

$$-V_T\cdot\ln\left(\frac{v_I}{R\cdot I_S}+1\right)$$

Since I_S is always a very small number, for example $I_S = 10^{-14}$A is typical for a small junction diode, the "1" in the equation you just wrote can be ignored. So you should see why this circuit is called a "logarithmic amplifier".

Suppose, instead, we moved the diode into the other position



In the space provided below give an algebraic expression for v_O in this circuit. Your expression will be in terms of V_T, I_S, R, and v_I.

-R*IS*(e^(vI/VT) - 1)

✔ **Answer:** -R*IS*(e^(vI/VT) - 1)

$$-R \cdot I_S \cdot \left(e^{\frac{v_I}{V_T}} - 1\right)$$

This circuit is called an "exponential amplifier".

Detailed Solution:

Part 1

We can take a KCL at the negative terminal of the op amp v_-:

$$I_R = i_D$$

$$\frac{v_I}{R} = I_S\,(e^{v_D/V_T} - 1)\,, v_D = -v_O$$

$$\frac{v_I}{R} = I_S\,(e^{-v_O/V_T} - 1)$$

$$\frac{v_I}{I_S R} + 1 = e^{-v_O/V_T}$$

$$v_O = -V_T \ln\left(\frac{v_I}{I_S R} + 1\right)$$

Part 2

We can take a similar KCL on the same node. Note that the initial setup is the same, only that $v_D = v_I$ and $v_R = v_O$:

$$\frac{v_O}{R} = I_S\,(e^{v_I/V_T} - 1)$$

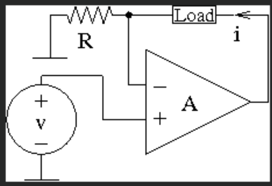
$$v_O = I_S R\,(e^{v_I/V_T} - 1)$$

Homework

H4P1: Current Source

3/3 points (graded)

We often need a precision current source in a circuit. An operational amplifier gives us one way to make a precisely controlled current flow through a load whose properties are not precisely known. The following handy circuit is often the answer.

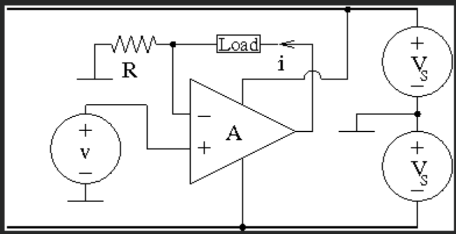


For example, suppose that we need to put precisely $i = 10$mA through a load that presents a resistance but the value of the resistance is not well controlled. Suppose also that we have a precision voltage source of $v = 5$V, but this voltage source is pretty wimpy: it cannot supply more than 0.1mA without smoke coming out of it. With the circuit shown above we can choose a resistor of resistance R that can accomplish this goal.

First, assuming that the op amp is ideal ($A = \infty$) give the value of the resistance R, in Ohms, that we need.

500 ✔ **Answer:** 500.0

Of course, our real op amp is imperfect, in many ways. For example, it is suspended between two power-supply rails with voltage $\pm V_S = 15$V, so the real circuit is more like this:



An op amp cannot put out a voltage that exceeds the power-supply rails. But assume our op amp can reach the rails.

Given the value of R that you deduced above, what is the maximum load resistance, in Ohms, that this current source can handle?

1000 ✔ **Answer:** 1000.0

Another problem is that the gain of the op amp is really finite.

Suppose that $A = 100$. Using the value of R that you computed above, and assuming a load resistance of 500Ω, what is the actual current, in Amperes, that will go through the load? Please give your answer to within 0.1% of the exact answer.

0.00980392 ✔ **Answer:** 0.00980392156862745

Explanation:
(a) Assuming the Op-Amp is ideal, we just need to perform KCL at the negative terminal:

$$i_{Load} = i_R$$

$$R = \frac{v}{i_{Load}} = \frac{5.0}{0.01} = 500.0\Omega$$

(b) In the worst case, the Op-Amp will have to drop $V_S - v$ volts across the load resistance, because the negative terminal is constrained to be at v volts and the Op-Amp's power rails are at V_S. Divide by the current we've constrained by v and R to get this worst-case load resistance:

$$R_{Load} = \frac{(15 - 5.0)}{0.01} = 1000.0\Omega$$

(c) Now that the Op-Amp has a finite gain, we have to remember the relation:

$$v_o = A\left(v^+ - v^-\right)$$

Solving for v_0:

$$v_o = \frac{Av}{1 + \frac{AR}{R_{Load}+R}}$$

So to find the actual current, we divide this output voltage by the total resistance the current goes through, $R_{Load} + R$:

$$i_{actual} = \frac{Av}{R_L + R\,(1+A)} = \frac{100 * 5.0}{500 + 500.0\,(1+100)} = 0.00980392156862745A$$

```
i = 10*^-3; v = 5;
(*In an ideal op amp (infinite gain) both terminals have the same voltage,
and current through both branches is zero.*)
r = v/i

(*Voltage across Rload is Vs-v*)
Vs = 15;
Rl = (Vs - v)/i

(*We use the non-inverting op amp relationship for finite gain - S23E1*)
A = 100; Rl = 500;
vo = (A v)/(1 + (A r)/(Rl+r));
vo/(Rl + r) // N
```

Out[ ]= 500

Out[ ]= 1000

Out[ ]= 0.00980392

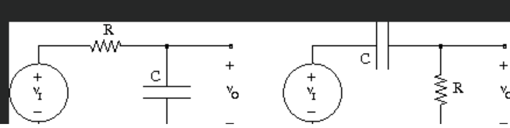
```
(*TODO: H4P2*)
```

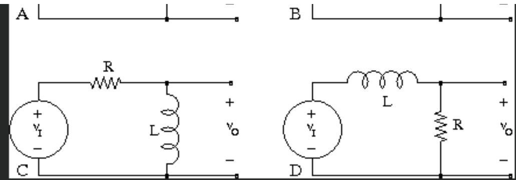
H4P3: Opamps and Filter Design

10/10 points (graded)

Op amps give us enormous flexibility of design of filters. For example, we need a filter if we want to measure or record some signals in the audio spectrum, but we are in a noisy environment where there is rumble at 1Hz and some ultrasonic noise at 200kHz that we would like to attenuate.

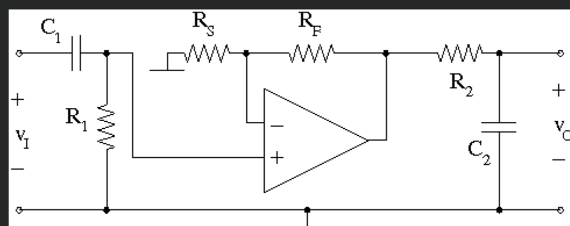
You have already seen a few fundamental modules that can be used to make filters. We have learned that circuit fragments A and D are low-pass filters and fragments B and C are high-pass filters.





An op amp can let us combine modules like these in such a way that the behavior of the compound is easily derived from the behaviors of the parts separately.

Let's design a filter that does not seriously attenuate audio frequencies, but discriminates against frequencies outside of that band. Here is one plan that we can use:



Our specification is that we want there to be two break frequencies for this filter: 10Hz and 20kHz. We also want midband frequencies, such as signals around 1kHz, to be amplified by about a factor of 2.

We have available a 1.5μF and a 220pF capacitor. We also have a variety of resistors between 100Ω and $100\text{k}\Omega$. However, our op amp really likes $R_S = 20\text{k}\Omega$.

Your job is to fill in the remaining details of this design.

What is the value of R_1, in Ohms?

10610.3 ✔ **Answer:** 10610.33

What is the value of R_2, in Ohms?

36171.6 ✔ **Answer:** 36171.58

What is the value of R_F, in Ohms?

20000 ✔ **Answer:** 20000.0

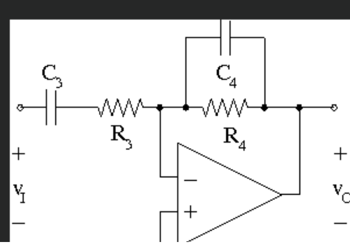
What is the value of C_1, in Farads? (Remember, you can use "u" for micro and "p" for pico.)

1.5e-6 ✔ **Answer:** 1.5e-06

What is the value of C_2, in Farads? (Remember, you can use "u" for micro and "p" for pico.)

220e-12 ✔ **Answer:** 2.2e-10

Alternatively, we could choose to implement this filter with the following circuit;



This time the frequency break point specifications are the same, but we don't have enough degrees of freedom to independently specify the midband gain.

We have the same parts available for this design as we had for the previous design.

Your job is to fill in the remaining details of this design.

What is the value of R_3, in Ohms?

10610.3 ✔ **Answer:** 10610.33

What is the value of R_4, in Ohms?

36171.6 ✔ **Answer:** 36171.58

What is the value of C_3, in Farads? (Remember, you can use "u" for micro and "p" for pico.)

1.5e-6 ✔ **Answer:** 1.5e-06

What is the value of C_4, in Farads? (Remember, you can use "u" for micro and "p" for pico.)

220e-12 ✔ **Answer:** 2.2e-10

What is the midband gain for this circuit?

-3.40467 ✔ **Answer:** -3.41

If you want to find out more about filters, a good place to look is here.

Explanation:
First, we realize that the network to the left of the Op-Amp is our high pass filter, and the network to the right our low pass one. Four our high pass filter, which determines the low-frequency cutoff, we want $C1$ to be the bigger capacitor (because $f_{low} = \frac{1}{2\pi RC}$). Then our only unknown is R_1, and solving:

$$C_1 = 1.5\mu F$$

$$R_1 = \frac{1}{2\pi f_{low} C_1} = 10610.33\Omega$$

Similarly, C_2 will be $220pF$ and we can solve for R_2:

$$C_2 = 220nF$$

$$R_2 = \frac{1}{2\pi f_{low} C_2} = 36171.58\Omega$$

And finally to find R_F, we note that the Op-Amp is a non-inverting amplifier, whose gain is given by:

$$K = (1 + \frac{R_F}{R_S})$$

$$R_F = (K - 1)\, R_S = 20000.0\Omega$$

For the second circuit, R_3 and C_3 again serve the purpose of the high pass filter, and R_4 and C_4 the purpose of the low pass filter. Therefore:

$$R_3 = R_1 = 10610.33\Omega$$

$$C_3 = C_1 = 1.5\mu F$$

$$R_4 = R_2 = 36171.58\Omega$$

$$C_4 = C_2 = 220pF$$

And this is how an Op-Amp in an inverting amplifier configuration, whose gain at the midband is given by:

$$G = -\frac{R_4}{R_3} = -3.41$$

Grove (Community TA)
3 years ago - marked as answer 3 years ago by **MIT_Lover_UA** (Staff)

All you need to do is identify which is the low pass filter and which is the high pass filter and use $f_{\text{break}} = \frac{1}{2\pi RC}$.

You have two capacitors and a range of resistors to choose from.

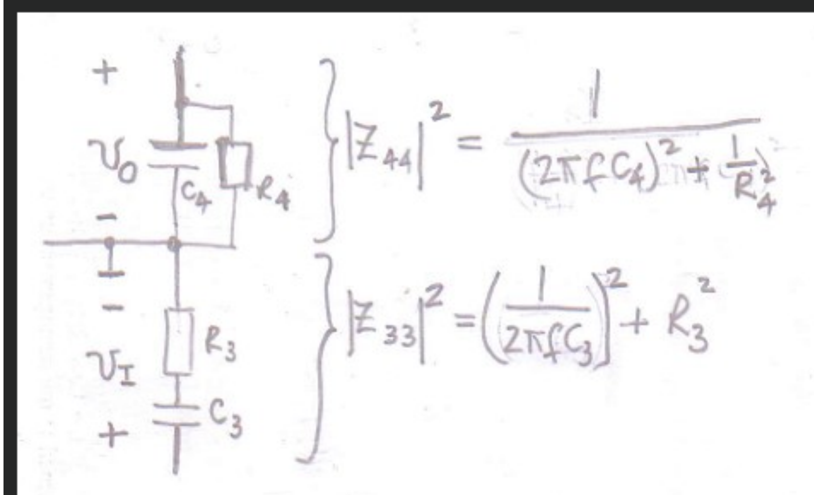
The op amp and the two resistors in the middle act as a buffer and amplifier.

You will also need to calculate (or make an "informed" guess) the value of $\left|\frac{v_{\text{OUT}}}{v_{\text{IN}}}\right|$ for each of the filters at $1\,\text{kHz}$ to do the final part.

(*Hint for the second circuit*)

Grove (Community TA)
3 years ago - endorsed 3 years ago by **MIT_Lover_UA** (Staff)

You can think of the circuit as follows with the gain depending on the ratio $\frac{Z_{44}}{Z_{33}}$.



At low frequencies $\frac{Z_{44}}{Z_{33}} \approx 2\pi f C_3 R_4 \propto f$ and at high frequencies $\frac{Z_{44}}{Z_{33}} \approx \frac{1}{2\pi f C_4 R_3} \propto \frac{1}{f}$

```
f1 = 10; f2 = 20*^3;
c1 = 1.5*^-6; c2 = 220*^-12;
rs = 20*^3;

(*Based on 1st hint, break frequency and capacitance
 are inversely prop. So we use a high cap for low break freq*)
```

$$\text{Solve}\left[f1 == \frac{1}{2\,\pi\,r1\,c1}, r1\right] \text{// N}$$

$$\text{Solve}\left[f2 == \frac{1}{2\,\pi\,r2\,c2}, r2\right] \text{// N}$$

```
f = 1*^3; r1 = 10610.32953945969`; r2 = 36171.57797543076`;
```

$$\text{Solve}\left[2 == \frac{rf + rs}{rs}, rf\right] \text{(*S23E1*)}$$

```
c3 = c1; c4 = c2;
```

$$\text{Solve}\left[f1 == \frac{1}{2\,\pi\,r3\,c3}, r3\right] \text{// N}$$

$$\text{Solve}\left[f2 == \frac{1}{2\,\pi\,r4\,c4}, r4\right] \text{// N}$$

```
(*Using 2nd hint*)
f = 1*^3; r3 = 10610.32953945969`; r4 = 36171.57797543076`;
```

$$z44magsq = \frac{1}{(2\,\pi\,f\,c4)^2 + 1 / r4^2};$$

$$z33magsq = \frac{1}{(2\,\pi\,f\,c3)^2} + r3^2;$$

$$g = -\sqrt{\frac{z44magsq}{z33magsq}} \text{ // N(*S23E2*)}$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{r1 → 10 610.3}}

Out[•]= {{r2 → 36 171.6}}

Out[•]= {{rf → 20 000}}

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{r3 → 10 610.3}}

Out[•]= {{r4 → 36 171.6}}

Out[•]= -3.40467

Lab

Lab 4

5/5 points (graded)

For this lab the design task is to build a resonant circuit *without using inductors*, a feat that's made possible by the use of Op Amps to create an active filter circuit. You'll find it useful to review Section 15.6 in the text.

One variant of an Op Amp active filter is shown in Figure 1.

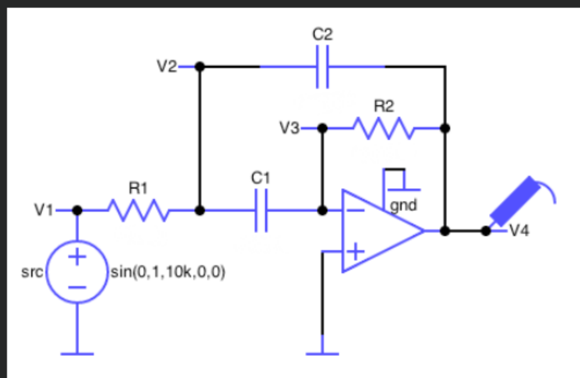


Figure 1. One variant of an Op Amp active filter

The goal is to create a filter with a resonant frequency $f_O = 10\ \mathrm{kHz} \pm 100\ \mathrm{Hz}$ and a $Q > 5$, i.e., a filter similar to that of the RLC circuit in Task 1 of Lab 11. With the appropriate choice of component values, the magnitude of the frequency response should be similar to the plot shown in Figure 2. The exact dB values aren't important, just the resonant frequency and Q of your circuit.

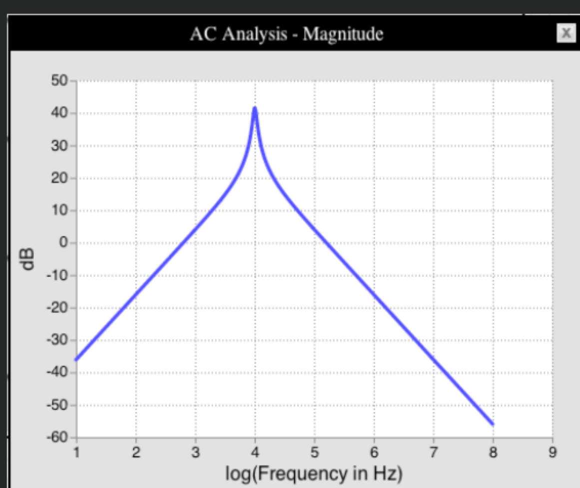


Figure 2. Desired frequency response of active filter

To start, use the Node Method and impedances to write KCL equations for the nodes labeled V2 and V3 in Figure 1. These can then be solved to give a system equation of the form

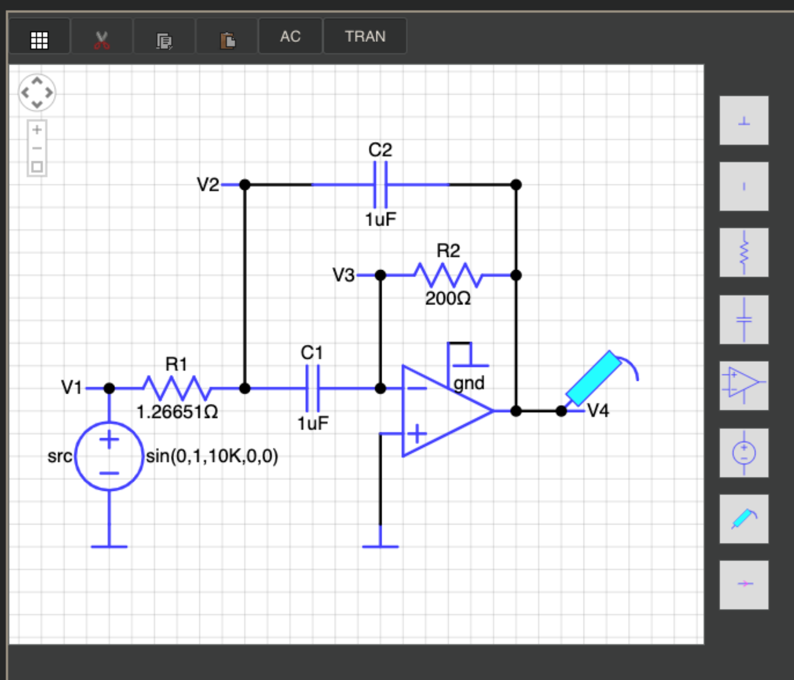
$$H(s) = \frac{V4}{V1} = \frac{numerator}{s^2 + 2\alpha s + \omega_0^2}$$

where the denominator is in the canonical form for second-order systems. If your math is correct, the expressions for α and ω_0 should be expressions involving R1, C1, R2 and C2. Recalling that $Q = \omega_0/2\alpha$, you should now be able to choose values for the resistors and capacitors.

If we actually wanted to build the design, the specified components would have to be available from parts suppliers, so please choose values from the listings of standard 1% resistance values and standard capacitance values.

Please use the sandbox below to enter the circuit and try out various component values. Run an AC analysis to verify the appropriate resonant frequency and Q.

Remember to connect the "gnd" pin of the Op Amp to the ground node in your circuit. So why does the Op Amp in the circuit sandbox have a "gnd" pin? The idealOp Amp is modeled as a dependent voltage source, as shown in Figure 15.6, which generates the output voltage v_o relative to a reference voltage shown as a ground connection in the figure. The "gnd" pin on the sandbox Op Amp is provided so that you can indicate which node in your circuit has been selected as the source of the reference voltage. (Note: For this lab the schematic is not checked, and you will receive a green check for any schematic. The lab checks your combination of resistor and capacitor values inputted in the fields below for correctness.)



When you've completed the design, please enter your final values for the components below. The on-line system will perform a calculation using the values to determine the resonant frequency and Q of your system. Note that the answers will either all be marked correct (ω_0 and Q are okay) or all be marked incorrect (a negative component value, or out of range values for ω_0 or Q).

R1 (in Ohms):

1.26651

C1 (in Farads):

1e-6

R2 (in Ohms):

R2 (in Ohms):

200 ✔

C2 (in Farads):

1e-6 ✔

Explanation:
To solve this problem we first find the transfer function $H(s)$ between input V_1 and output V_4. As recommended in the problem description, it is useful to find expressions for V_2 and V_3.
By noting that the non-inverting input of the op-amp is connected to ground, we can easily find $V_3 = 0$. Then, by using the node method we find the following equations:

$$\frac{V_2}{Z_{c1}} = -\frac{V_4}{R_2} \tag{1}$$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_4}{Z_{c2}} + \frac{V_2}{Z_{c1}} \tag{2}$$

From the first equation, we find:

$$V_2 = -\left(\frac{Z_{c1}}{R_2}\right) \cdot V_4 \tag{3}$$

Using this result in the second equation, we can now find an expression to relate V_1 and V_4:

$$\frac{V_1}{V_4} = -R_1 \cdot \left(\frac{1}{Z_{c2}} + \frac{1}{R_1} + \frac{1}{Z_{c1}}\right) \cdot \left(\frac{Z_{c1}}{R_2}\right) - \frac{R_1}{Z_{c2}} \tag{4}$$

Plugging in $Z_c = \frac{1}{sC}$ we get:

$$\frac{V_1}{V_4} = -R_1 \cdot \left(sC_2 + \frac{1}{R_1} + sC_1\right) \cdot \left(\frac{1}{sC_1 R_2}\right) - sC_2 R_1 \tag{5}$$

$$\frac{V_1}{V_4} = \frac{sC_2R_1 + 1 + sC_1R_1 + (sC_2R_1) \cdot (sC_1R_2)}{-sC_1R_2}$$

Re-arranging the terms of the expression above, we find:

$$\frac{V_1}{V_4} = \frac{s^2\left(C_1C_2R_1R_2\right) + s\left(R_1C_2 + R_1C_1\right) + 1}{-sR_2C_1} \tag{6}$$

Taking the inverse and arranging terms to obtain cannonial form of the transfer function, we get:

$$\frac{V_4}{V_1} = \frac{\frac{-sR_2C_1}{C_1C_2R_1R_2}}{s^2 + s\left(\frac{R_1\cdot(C_1+C_2)}{R_1R_2C_1C_2}\right) + \frac{1}{C_1C_2R_1R_2}} \tag{7}$$

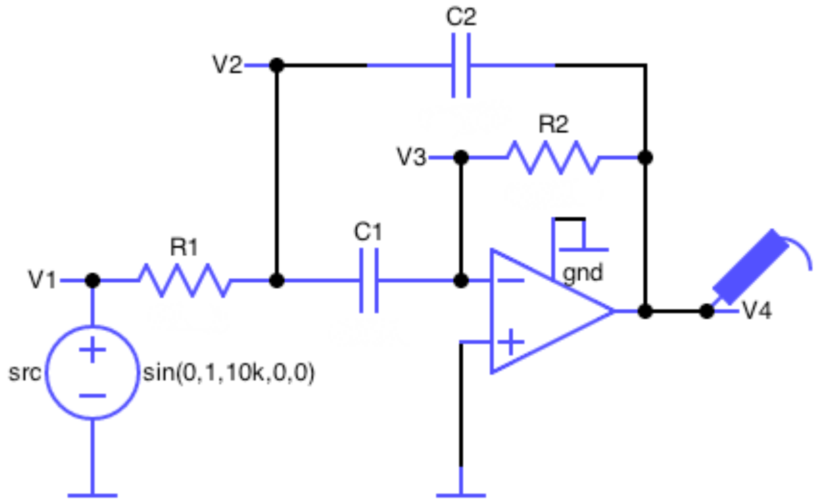
Therefore, we know α and ω_0 to be:

$$\alpha = \frac{1}{2}\left(\frac{C_1 + C_2}{R_2C_1C_2}\right) \tag{8}$$

$$\omega_0 = \sqrt{\frac{1}{C_1C_2R_1R_2}} \tag{9}$$

To find Q we then calculate:

$$Q = \frac{R_2C_1C_2}{(C_1 + C_2) \cdot \sqrt{R_1R_2C_1C_2}} \tag{10}$$



Thus, the corresponding damping factor in our Op Amp circuit is

$$\alpha = g_2 \frac{C_1 + C_2}{2 C_1 C_2} \tag{15.83}$$

and the undamped resonant frequency is

$$\omega_o = \sqrt{\frac{g_1 g_2}{C_1 C_2}}. \tag{15.84}$$

```
(*Op Amp active filter circuit
 Check out Pg 863, Section 15.6.3 of A&L*)
f = 10*^3;
ω = 2 π f;
Q = 2 π;(*Some value above 5*)

(*Fixing the capactance to 1uF begin with,
using some standard values from our "part bin". Fixing
 resistors first gives some ugly complex numbers*)
c1 = 1*^-6; c2 = 1*^-6;

(*Eqns 15.83, 15.84*)
```

$$\text{Solve}\left[\frac{\omega}{2\,Q} == g2 * \frac{c1 + c2}{2\,c1\,c2}\ \&\&\ \omega == \sqrt{\frac{g1\,g2}{c1\,c2}},\ \{g1,\ g2\}\right]\ //\ N$$

```
r1 = 1 / 0.7895683520871487`
r2 = 1 / 0.005`
```

Out[]= {{g1 → 0.789568, g2 → 0.005}}

Out[]= 1.26651

Out[]= 200.