Week 7

S13E1: Scaling Inductors

0 points possible (ungraded)

An inductor is a device that stores energy in the magnetic field. The characteristic parameter L of an inductor is called "inductance," which is measured in units of Henrys, after Joseph Henry, who discovered the phenomena of self inductance and of mutual inductance. See Henry for more historical information about this great scientist/engineer.

For a mechanical analogy, inductance is analogous to inertia. If voltage is analogous to force, and charge is analogous to displacement, then current is analogous to velocity and magnetic flux Φ is analogous to momentum. So the law for a linear inductor is analogous to Newton's law of motion:

$$\Phi = L \cdot i \iff p = m \cdot v$$

and

$$v = L \cdot \frac{di}{dt}$$

The inductance of an inductor is determined by its geometry and the magnetic properties of the materials it is made out of. For example, for a solenoidal inductor (with N turns of wire wrapped around a core of area A and length l we have:

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$

The constant of proportionality μ is called the "permeability" of the core on which the solenoid is wound. The permeability is a property of the material of the core.

Suppose we double the length, width, and depth of a solenoidal inductor, by what multiple does the inductance increase?

2 ✔ **Answer:** 2

Suppose we double the number of turns of wire on a solenoidal inductor, by what multiple does the inductance increase?

4 ✔ **Answer:** 4

The permeability of the vacuum is $\mu_0 = 4\pi \times 10^{-7}$. This is also called the magnetic constant.

Magnetic materials, such as manganese-zinc ferrite, have a higher permeability than the vacuum. The ratio of the permeability of a material to the permeability of the vacuum is called the "relative permeability" of the material. For example, the relative permeability of one kind of manganese-zinc ferrite is about 640. For more information see the Wikipedia page on permeability.

What is the approximate inductance (in Henrys) of a 100-turn solenoidal inductor wound on 1cm diameter cylindrical ferrite core 10cm long?

0.00631655 ✔ **Answer:** 0.006316546816697189

In[∘]:=
```
μ0 = 4*^-7 π; rp = 640; n = 100; r = 0.5*^-2; l = 10*^-2;
(*Relative permeability = permeability of a material / permeability of vacuum *)
```

$$L = \frac{\mu_0 * rp * n^2 * \pi\, r^2}{l}$$

Out[∘]= 0.00631655

S13E2: Inductors Store Energy

0 points possible (ungraded)

We have just learned the v-i characteristic for a linear inductor:

$$v = L\frac{di}{dt}$$

So the power entering the inductor (as in any two-terminal device) is:

$$P = vi = Li\frac{di}{dt}$$

If we integrate this between two times we get the energy change in the inductor over the interval:

$$\Delta E = \int_{t_0}^{t_1} P(t)\,dt = \int_{t_0}^{t_1} Li(t)\,\frac{di(t)}{dt}dt = \frac{1}{2}L(i(t_1))^2 - \frac{1}{2}L(i(t_0))^2$$

So the energy stored in an inductor, as a function of the current through it, is:

$$E = \frac{1}{2}L\cdot i^2$$

Inductors are used in buck-boost converters as part of the power supply of a mobile device such as a cell phone. For example, see the Texas Instruments TPS63000. In a typical application the inductor might have an inductance of 2.2μH and the peak current might be 1.0A.

How much energy, in Joules, is stored in the inductor at the time of the peak current?

1.1e-6 ✔ **Answer:** 1.1e-06

This energy, stored in the inductor, is dumped into a capacitor on a timescale of perhaps 0.25μs.

What is the peak power (in Watts) of this transaction?

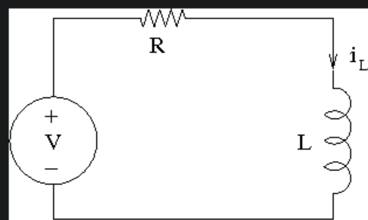
4.4 ✔ **Answer:** 4.4

```
In[]:= L = 2.2*^-6; i = 1;
       e = 1/2 L i^2
       t = 0.25*^-6;
       P = e/t
Out[]= 1.1 × 10^-6
Out[]= 4.4
```

S13E3: Thevenin Inductor Circuit

0 points possible (ungraded)

For the circuit shown below,



write an algebraic expression for the rate of change of the inductor current in terms of V, R, L, and i_L.

Rate of change of inductor current $\frac{di_L}{dt}$ =

(V - R*iL)/L ✔ **Answer:** (V-R*iL)/L

Let $R = 1.0\text{k}\Omega$ and $L = 10\mu\text{H}$. If at some time t we have $V(t) = 1.0\text{V}$ and $i_L(t) = 0.9\text{mA}$ what is the rate of change of inductor current? Express your answer in Amperes/millisecond.

10 ✔ **Answer:** 9.999999999999988

```
(*This is a voltage divider circuit. Total
  voltage V is split among resistor and inductor: V=V_R+V_L (KVL)
    Remember V_R=i_L R and V_L=L*di/dt*)
Solve[V == i_L R + L i', i']
V = 1; i = 0.9*^-3; R = 1*^3; L = 10*^-6;
(V - R i)/(L * 1*^3) (*Convert to ms*)
```

Out[•]= $\left\{\left\{i' \to \frac{V - R\, i_L}{L}\right\}\right\}$

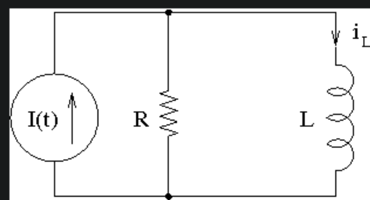
Out[•]= 10.

In[•]:= Quit[]

S13E4: First-Order Inductor Examples

0 points possible (ungraded)

We have a circuit with an inductor and a resistor driven by an independent current source with a current that is a function of time $I(t)$.



The resistance of the resistor is $R\Omega$ and the inductance of the inductor is LH.

Suppose the drive I rises in a step from 0A to 0.5A at the initial time $t=0$. Also suppose that the inductor's initial current $i_L(0)=0$A.

In the space provided below write an algebraic expression for the inductor current as a function of time, $i_L(t)$ for $t>0$.

0.5-0.5*e^(-t*R/L) ✔

$0.5-0.5\cdot e^{-t\cdot\frac{R}{L}}$

Now, suppose the drive falls in a step from 0.5A to 0A at the initial time $t=0$. Also suppose that the inductor's initial current $i_L(0)=0.5$A.

In the space provided below write an algebraic expression for the inductor current as a function of time, $i_L(t)$ for $t>0$.

0.5*e^(-t*R/L) ✔

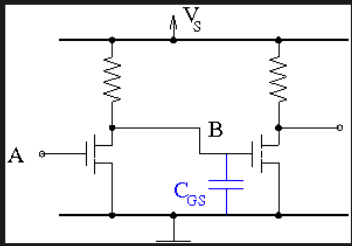
$0.5\cdot e^{-t\cdot\frac{R}{L}}$

(*We plug in the values for $i_L(t)=I_I+(I_0-I_I)e^{-t/\tau}$ where time constant $\tau=L/R$*)

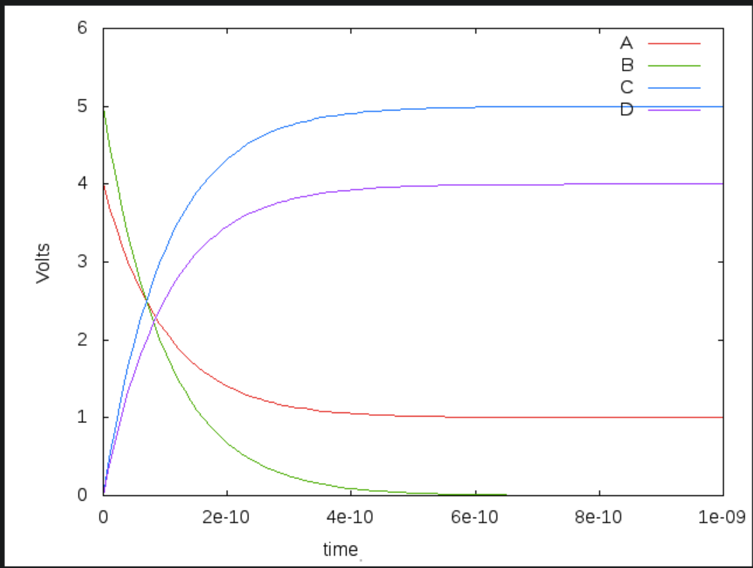
S14E1: Response to step down

0 points possible (ungraded)

In the circuit just shown:



When the voltage at node A falls in a step from $V_S = 5V$ to $0V$, the voltage at node B looks like one of the following traces.



Which trace is the correct one? Write the letter corresponding to the correct trace in the box provided below:

C

✔ **Answer:** C

Note: You need a very fast (and thus very expensive) oscilloscope to see traces like these. You need to use a very low capacitance probe and you must connect the oscilloscope to the circuit very carefully, with a very short ground.

Grove (Community TA)
2 years ago - endorsed 2 years ago by **MIT_Lover_UA** (Staff)

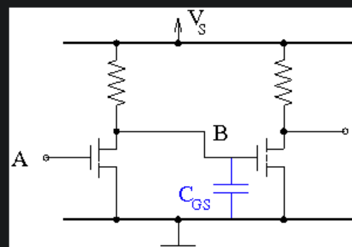
In this example the rise/fall time is of the order of 10^{-10} s so the CRO must have a fast response time.

$C_{\text{GS}} = 0.1\,\text{pF}$ so the probe capacitance must be smaller than this otherwise it will significantly change the rise/fall time.

A very short ground is to reduce ny stray/parasitic capacitance.

S14E2: Rise Time

0 points possible (ungraded)



Call the drain resistance drawn above R_L. Given that $R_L = 1\mathrm{k}\Omega$, $C_{GS} = 0.1$pF, and $V_S = 5.0$V. What is the time t_r, in nanoseconds, after the falling edge at node A, that it takes for the signal at node B to rise to $V_{OH} = 4.0$V?

0.160944 ✔ **Answer:** 0.16094379124341004

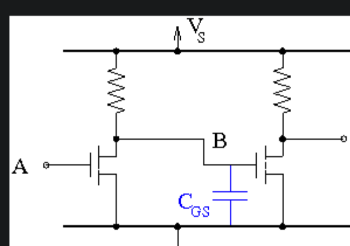
```
rl = 1*^3; cgs = 0.1*^-12; vs = 5; voh = 4;
(*Using the nice intuition developed in the lectures where:
  Exponential rise is (1-exp(-t))
   Exponential run is exp(-t)*)
Solve[voh == vs (1 - Exp[-t / (rl * cgs)])]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]= $\{\{t \to 1.60944 \times 10^{-10}\}\}$

S14E3: Fall Time Constant

0 points possible (ungraded)



Call the drain resistance drawn above R_L and the MOSFET on resistance R_{ON}. Given that the MOSFET is conducting, $R_{ON} = 10\Omega$, $R_L = 1\mathrm{k}\Omega$, and $C_{GS} = 0.1$pF:

What is the Thevenin equivalent resistance, in Ohms, as seen by the gate-source capacitance?

1000/101 ✔ **Answer:** 9.900990099009901

What is the time constant for this circuit, in nanoseconds?

0.000990099 ✔ **Answer:** 0.0009900990099009901

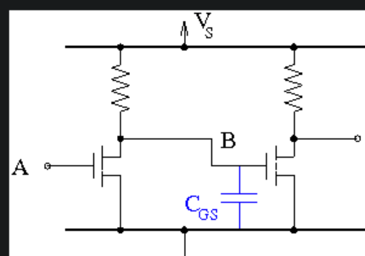
In[∘]:=
```
ron = 10; rl = 1*^3; cgs = 0.1*^-12;
(*If the MOSFET is conducting, in SR model we approximate it as a resistor R_ON*)
rth = 1/(1/rl + 1/ron)
τ = rth * cgs * 1*^9(*Convert to ns*)
```

Out[∘]= $\frac{1000}{101}$

Out[∘]= 0.000990099

S14E4: Fall Time

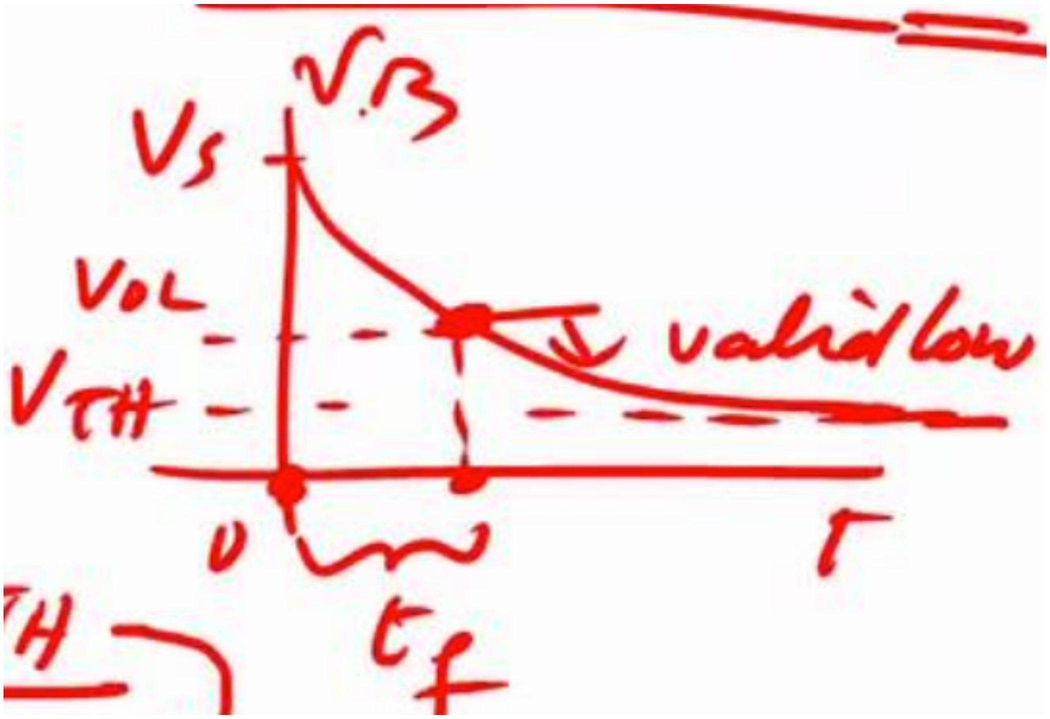
0 points possible (ungraded)



Call the drain resistance drawn above R_L and the MOSFET on resistance R_{ON}. Given that $V_S = 5.0V$, $R_{ON} = 10\Omega$, $R_L = 1\text{k}\Omega$, and $C_{GS} = 0.1$pF.

What is the time, in picoseconds, that it takes for the signal at node B to fall to the value of $V_{OL} = 1.0$V

1.63392 ✔ **Answer:** 1.6339206999548073



In[∘]:=
```
vs = 5; ron = 10; rl = 1*^3; cgs = 0.1*^-12;
vth = vs * ron/(rl + ron);
rth = 1/(1/rl + 1/ron);
vol = 1;
(*As derived in lecture.*)
τ = -rth cgs Log[(vol - vth)/(vs - vth)]
(*Equivalently here is the intuitive method*)
Solve[vol == vth + (vs - vth) Exp[-t / (rth cgs)], t]
```

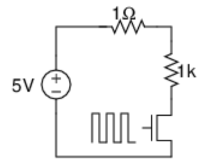
Out[∘]= 1.63392×10^{-12}

••• Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $\{\{t \to 1.63392 \times 10^{-12}\}\}$

Bypass Capacitor - Problem Statement

Given this circuit:



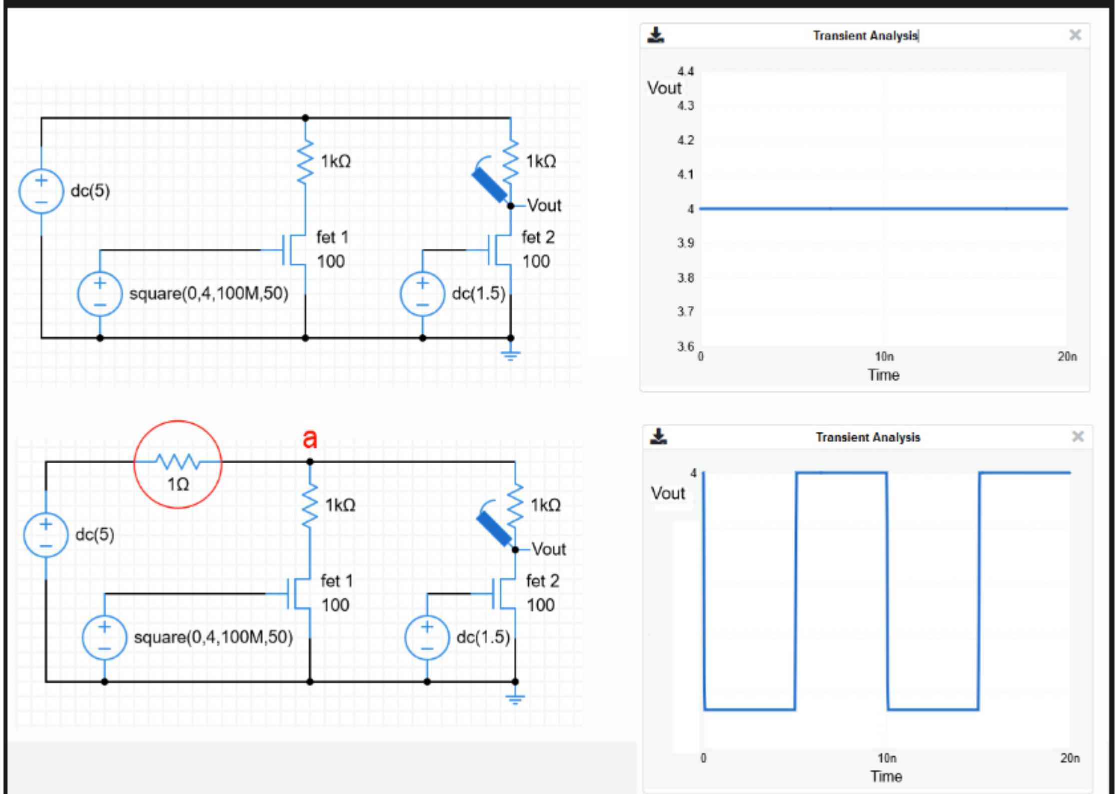
where the 1Ω resistor represents the impedance of the power line, and the 1k resistor and MOSFET form an inverter.

- If the inverter is driven with a 100MHz waveform, how much noise would you see on the power supply?
- What if we add a 1μF capacitor across the power rails?

You may assume a switch model for the MOSFET.

Grove (Community TA)
about a year ago - marked as answer about a year ago by **MIT_Lover_UA** (Staff)

This section is to show you how an unwanted signal (noise) might appear in a circuit and then how that noise might be reduced.



In the top circuit the output voltage of fet 2 (Vout) is constant at $4\ \mathrm{V}$ which is to be expected as there is a constant input voltage of $1.5\ \mathrm{V}$.
What is happening to fet 1 does not affect the output of fet 2.

In the bottom circuit the introduction of the $1\ \Omega$ resistor due to the resistance of the connecting wires results in the voltage at node a changing due to a varying voltage drop across the $1\ \Omega$ resistor because of varying current drawn by fet1 as it switches state .

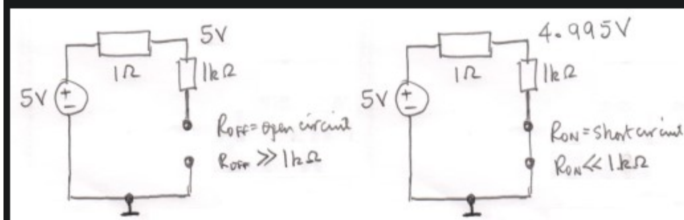
In turn the varying potential of node a produces changing and unwanted output from fet 2 which may cause the circuit to malfunction.

The introduction of a (reservoir) capacitor between node a and ground reduces the magnitude of the voltage fluctuation at node a.

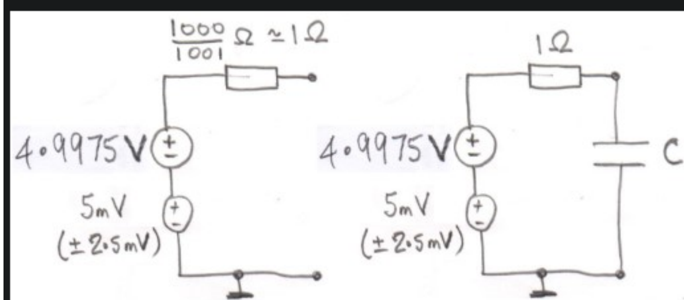
Grove (Community TA)
2 years ago - marked as answer 2 years ago by **MIT_Lover_UA** (Staff)

This is not an easy video to follow when you have so many new concepts to deal with.

I will go through the simplified analysis of the circuit in three steps with a little dialogue so that you can fill in the gaps, eg working out the Thevenin voltages and resistance values.
The MOSFET is assumed to act as an ideal switch as shown in the diagram below.

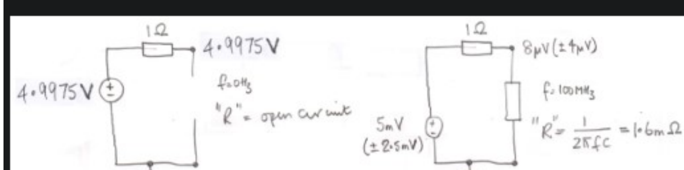


The net result is that the output voltage is between $4.995\,\mathrm{V}$ and $5.000\,\mathrm{V}$ which can be thought of as a steady (DC) voltage of $4.9975\,\mathrm{V}$ with a fluctuating voltage of $\pm 2.5\,\mathrm{mV}$, $5\,\mathrm{mV}$ peak to peak superimposed on it.



The analysis with the capacitor in circuit will split the Thevenin voltage source in two with a DC component of $4.9975\,\mathrm{V}$ and an AC component with frequency $100\,\mathrm{MHz}$ of $5\,\mathrm{mV}$ peak to peak and a Thevenin resistance of $1\,\Omega$ to a good approximation.

In this analysis the capacitor can be thought of as a circuit element whose impedance ("AC resistance") varies with frequency ($= \dfrac{1}{2\pi f C}$ where C is the capacitance and f the frequency).



With the capacitor added to the circuit the DC part of the voltage stays the same but the fluctuation, often called the ripple voltage, is reduced to $8\,\mu\mathrm{V}$ peak to peak ie $4.9975\,\mathrm{V} \pm 4\,\mu\mathrm{V}$.

Homework

Homework due Sep 5, 2022 20:16 +04

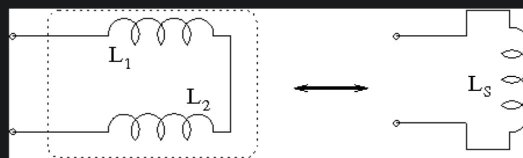
H3P1: Series and Parallel Inductors

4/4 points (graded)

Now we know that the voltage-current behavior of an inductor of inductance L is:

$$v_L = L\frac{di_L(t)}{dt}$$

It turns out that, just like with resistors, various combinations of inductors are terminal-equivalent to a single inductor with an inductance determined from the inductances of the parts of the combination. For each of the following circuits give the inductance of an equivalent inductor, as seen from the exposed terminals. First, let's look at two inductors in series:

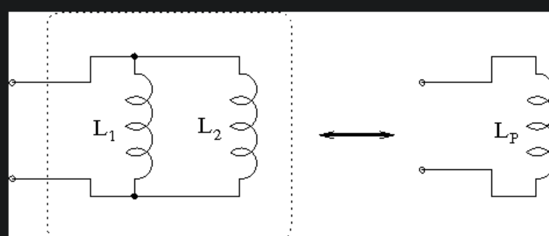


Think in terms of the geometry: how can two inductors in series be thought of as one?
In the space provided below give an algebraic expression for L_S in terms of L_1 and L_2 that makes these terminal equivalent.
L_S =

L1+L2 ✔ **Answer:** L1+L2

$L_1 + L_2$

Next, let's look at two inductors in parallel:

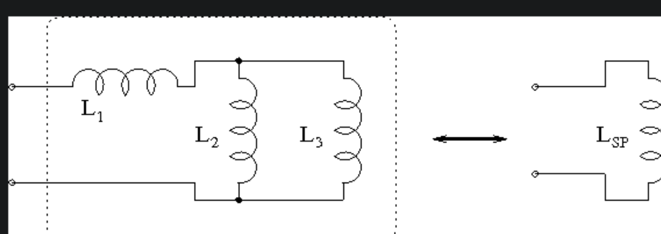


Think in terms of the geometry: how can two inductors in parallel be thought of as one?
In the space provided below give an algebraic expression for L_P in terms of L_1 and L_2 that makes these terminal equivalent.
L_P =

1/(1/L1+1/L2) ✔ **Answer:** (L1*L2)/(L1+L2)

$$\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}$$

Next, let's look at a combination

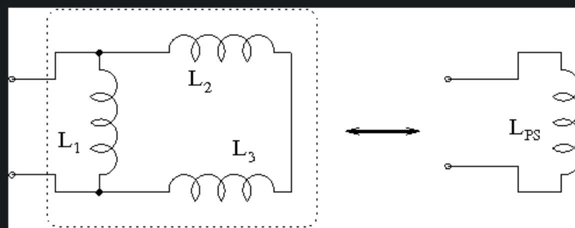


In the space provided below give an algebraic expression for L_{SP} in terms of L_1, L_2, and L_3 that makes these terminal equivalent.
L_{SP} =

L1+1/(1/L2+1/L3) ✔ **Answer:** L1+(L2*L3)/(L2+L3)

$$L_1 + \frac{1}{\frac{1}{L_2} + \frac{1}{L_3}}$$

Next, let's look at another combination



In the space provided below give an algebraic expression for L_{PS} in terms of L_1, L_2, and L_3 that makes these terminal equivalent.

L_{PS} =

1/(1/L1+1/(L2+L3))

✔ **Answer:** (L1*(L2+L3))/(L1+L2+L3)

$$\frac{1}{\frac{1}{L_1} + \frac{1}{L_2+L_3}}$$

Explanation:

In the first part of this problem, we wish to size L_S to be equivalent to the two series inductors L_1 and L_2. This means that for the same applied current $i\,(t)$, we expect both circuits to respond with the same terminal voltage:

$$v_1 + v_2 = v_S$$

Where v_1 and v_2 are the voltages across each series inductor and v_s is the voltage across our lumped inductor. v_1 and v_2 are summed because L_1 and L_2 are connected in series. Substituting the inductor characteristic equation yields:

$$L_1 \frac{d}{dt} i\,(t) + L_2 \frac{d}{dt} i\,(t) = L_S \frac{d}{dt} i\,(t)$$

Dividing by $\frac{di}{dt}$ gives the series combination for inductors:

$$L_S = L_1 + L_2 \quad (1)$$

In the second part of this problem, we wish to size L_P to be equivalent to the two parallel inductors L_1 and L_2. This means that for the same applied voltage $v_T\,(t)$, we expect both circuits to draw the same amount of current:

$$i_1 + i_2 = i_P$$

Substituting the inductor characteristic equation in its integral form, and noting that $v_1 = v_2 = v_P$ due to the parallel nature of the circuit:

$$\frac{1}{L_1}\int_{t_0}^{t_1} v_T \,\mathrm{d}t + \frac{1}{L_2}\int_{t_0}^{t_1} v_T \,\mathrm{d}t = \frac{1}{L_P}\int_{t_0}^{t_1} v_T \,\mathrm{d}t.$$

Dividing by $int_{t_0}^{t_1} v_T \,\mathrm{d}t$ gives the parallel combination rule for inductors:

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} \quad (2)$$

The final two parts of this problem can be easily solved by applying the series and parallel identities of the inductor to each circuit (shown in equations (1) and (2) respectively. With this, we can write:

$$\begin{aligned} L_{SP} &= L_1 + (L_2 \parallel L_3) \\ &= L_1 + \frac{L_2 \cdot L_3}{L_2 + L_3} \end{aligned}$$

and

$$L_{PS} = L_1 \parallel (L_2 + L_3)$$
$$= \frac{L_1 \cdot (L_2 + L_3)}{L_1 + L_2 + L_3}$$

This question represents a single time-constant circuit. One whose (voltage or current) response can be described by a first order linear differential equation. In order to satisfy the single time constant condition, the circuit must be capable of being reduced to one containing a single resistor and inductor initially in an unfluxed state.

$L\frac{di}{dt} + iR_s + iR_o = V$

The general solution for this equation:

$i\,(t) = \frac{V}{(R_s+R_o)} + K * e^{-\frac{(R_s+R_o)t}{L}}$

Now K can be determined using the initial condition $i\,(0^+) = 0$

$0 = \frac{V}{(R_s+R_o)} + K$, so then $K = -\frac{V}{(R_s+R_o)}$

And rearranging gives the instantaneous : $i\,(t) = \frac{V}{(R_s+R_o)} * (1 - e^{-\frac{(R_s+R_o)t}{L}})$

{Staff} correct me if I'm wrong about adding both series resistances $R_s + R_o$

Since we know the Time Constant for an Inductor is: $TC = \frac{L}{R}$

At time > 5 TC the total current will be $i_T = \frac{V}{R_s+R_o}$

So the fractional voltage of v_o will be $i_T * \frac{R_o}{R_s+R_o}$

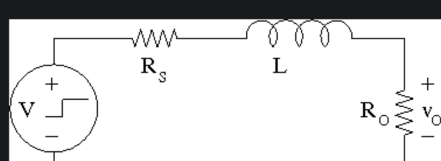
H3P3: The Curse of Lead Inductance

4/4 points (graded)

A wire is not a simple thing: it is not a node. A wire is a physical object that has properties. For example, in power transmission lines the resistance of the wire is very important. When we work at very high frequencies, the capacitance of nearby wires to each other and even the inductance of the wire and the mutual inductances between wires can be important. We call these unwanted interactions "parasitics". They break our lumped-circuit abstraction, so we need to know just how bad they are.

Let's look at the effect of the inductance of a wire lead on a component. A typical 1/4 Watt discrete resistor comes with 1.5 inch leads of AWG #20 wire, one for each terminal. We usually mount such a part using only a bit of that wire. Suppose we end up with a total of 2cm of wire in series with that resistor. AWG #20 wire has a diameter of 0.812mm. So 2cm of this wire has an inductance of about 15.44nH. (The estimate of inductance is computed from from a magic formula that is not part of this subject. You can learn about this in an electromagnetic theory subject.) We will ignore the resistance of the 2cm of wire...

Suppose we have a voltage source that turns on (it goes from 0 to V) at $t = 0$, and suppose it is connected to an R_O load through a resistor R_S and a wire with inductance L. Let's model the situation as follows:



Assume that the initial voltage across the load is zero. What is the final voltage across the load? Write an algebraic expression for this final value in terms of the parameters given:

(RO*V)/(RS+RO) ✔ **Answer:** (RO*V)/(RS+RO)

$$\frac{R_O \cdot V}{R_S + R_O}$$

Write an algebraic expression for the time constant of this circuit in terms of the parameters given:

L/(RS+RO) ✔ **Answer:** L/(RS+RO)

$$\frac{L}{R_S + R_O}$$

Now, let's get down to numbers. Let $V = 1.0$V, $R_S = 22.0\Omega$, $R_O = 50.0\Omega$, and $L = 15.44$nH. How much time, in nanoseconds, does it take for the output voltage to reach $v_O = 0.5$V?

0.27298 ✔ **Answer:** 0.27301000000000003

This is a pretty significant amount of time! For comparison, how long, in nanoseconds, does light take to go 2cm? Remember, the speed of light is 299792458m/s.

10000000/149896229 ✔ **Answer:** 0.06670999999999999

Explanation:
To calculate the final, steady-state DC value of the output voltage, we must first understand what happens to an inductor during DC steady state. We know that the voltage across an inductor is related to the rate of change of its current.

$$v_L = L \cdot \frac{d\,(i_L)}{dt}$$

In the DC steady-state, the system has reached equilibrium. The current is no longer changing, and the inductor has zero voltage drop, making it "invisible" to the rest of the circuit. Therefore in steady-state DC analysis, we can simply replace the inductor with a short circuit. The final output voltage is the voltage divider expression:

$$V_O = \frac{R_O}{R_O + R_S} \cdot V$$

To calculate the time constant of this circuit, we note that the arrangement is identical to the first part of H8P2. The time constant should be $\frac{L}{R}$, calculated using the Thevenin resistance at the terminals of the inductor:

$$\tau = \frac{L}{R_S + R_O}$$

We would like to calculate how long it takes for the output voltage to reach the 0.5 V. First we know that the equation for the current through the circuit is:

$$v_O = V_O\,(1 - e^{\frac{-t}{\tau}})$$

Where V_O and τ are defined according to the above equations. If we define the time when v_O reaches 0.5 as $t = t_0$, we can write:

$$0.5 = V_O\,(1 - e^{\frac{-t_0}{\tau}})$$

We can do some algebra and take the natural logarithm of both sides to cancel out the exponential, in order to arrive at an expression for t_0 in terms of τ:

$$1 - \frac{0.5}{V_O} = e^{\frac{-t_0}{\tau}}$$

$$\frac{-t_0}{\tau} = ln\left(1 - \frac{0.5}{V_O}\right)$$

$$t_0 = \tau \cdot ln\left(\frac{V_O}{V_O - 2.4}\right)$$

Substituting our values into the above equations, we find that V_O=0.6944, τ=2.1444e-10, and a time of t =0.27301000000000003 nanoseconds.
Comparing 0.27301000000000003 nanoseconds to the time it takes light to go 2cm:

$$time = \frac{distance}{speed} = \frac{2 \times 10^-2}{3 \times 10^8} \approx 0.06670999999999999ns$$

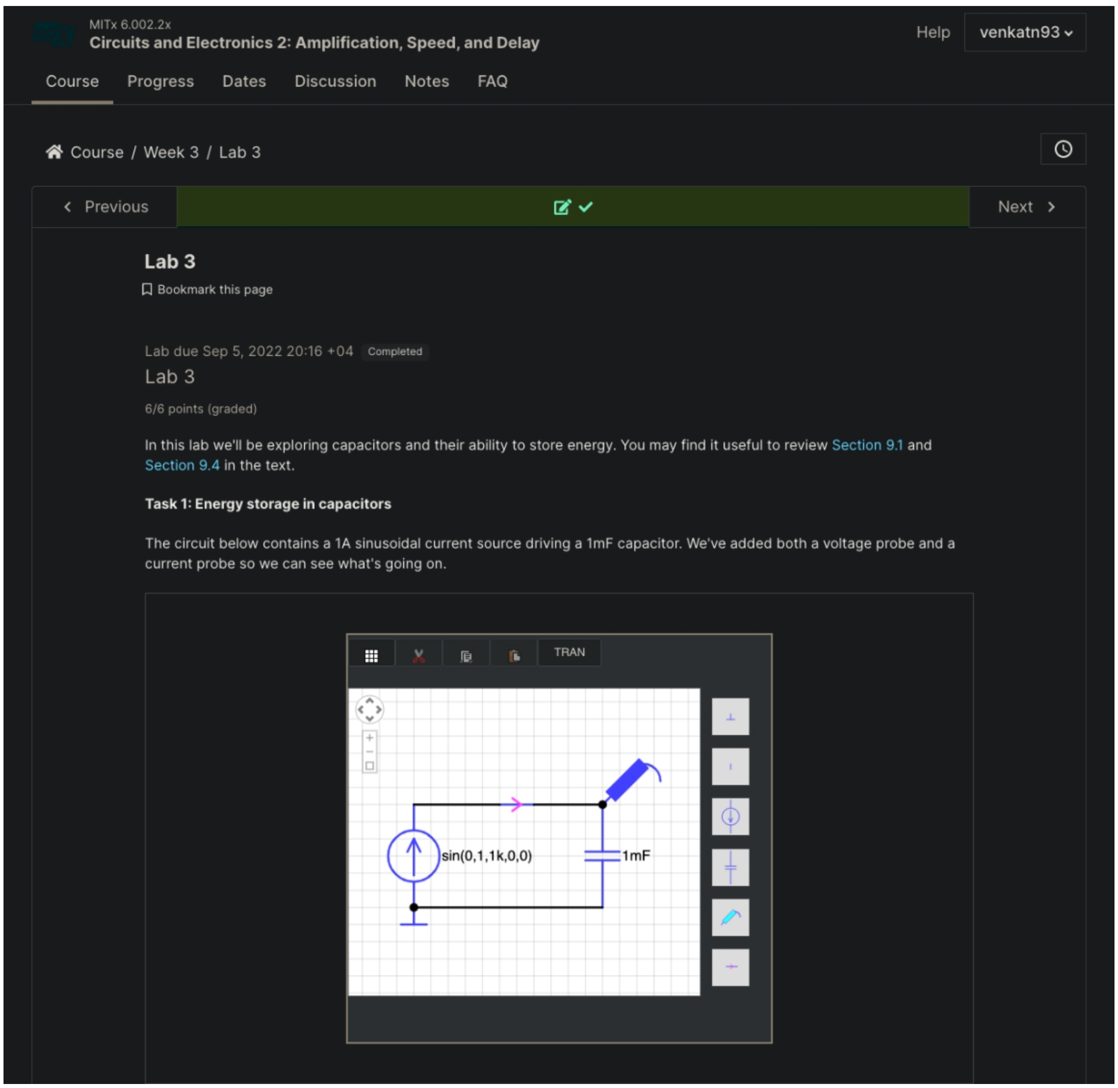
```
v = 1; rs = 22; ro = 50; l = 15.44*^-9;
vo = 0.5; τ = l / (rs + ro);
Solve[vo == (ro * v)/(rs + ro) (1 - Exp[-t / τ]), t]
c = 299 792 458; d = 2*^-2;
t = d/c * 1*^9 (*Converted to ns*)
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $\{\{t \to 2.7298 \times 10^{-10}\}\}$

Out[∘]= $\frac{10\,000\,000}{149\,896\,229}$

Lab



The current to/from the current source can only flow from/to one plate of the capacitor. The voltage across this capacitor at any point in time is given by Equation 9.12, shown below:

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

In this example, $C = .001$ and $i(t) = \sin(1000 * 2\pi * t)$. Please evaluate the integral using the known capacitance, current and initial conditions and enter the resulting formula for $v_C(t)$, the voltage across the capacitor, in terms of the capacitance C and time t. You can type "pi" to represent π in your formula. Assume that $i(t) = 0$ for $t \leq 0$ and simplify your equation accordingly.

Formula for v_C(t):

-0.159155*(-1 + cos(2000*pi*t)) ✔

Answer: (1/C)*(1/(1000*2*PI))*(-cos(1000*2*PI*t) + 1)

Please evaluate your formula for $t = 0.0005$, i.e., 0.5ms, half-way through the first cycle of current's sine wave and enter the result below. You can evaluate the result precisely, so please enter 6 digits of precision.

Computed value for v(0.0005) in volts:

0.31831 ✔ **Answer:** .318310

Now let's see what we measure from the simulation: Run a 2ms transient simulation on the circuit above, measure the voltage at 0.5ms and enter the result below.

Measured value for v(0.0005) in volts:

0.318128 ✔ **Answer:** .317585

There may be a small discrepancy between the measurement and the calculation since the simulator is estimating the current at only a discrete number of time points and therefore its estimate of the integral will be off a bit from the true answer (the measurement will converge to the computed value as we use more and more time points in the requested interval).

Finally, using Equation 9.18 and your measurement above, estimate the amount of electrical energy stored in the capacitor at time 0.5ms and enter the result below.

Estimated value for electrical energy stored at time 0.0005, in joules:

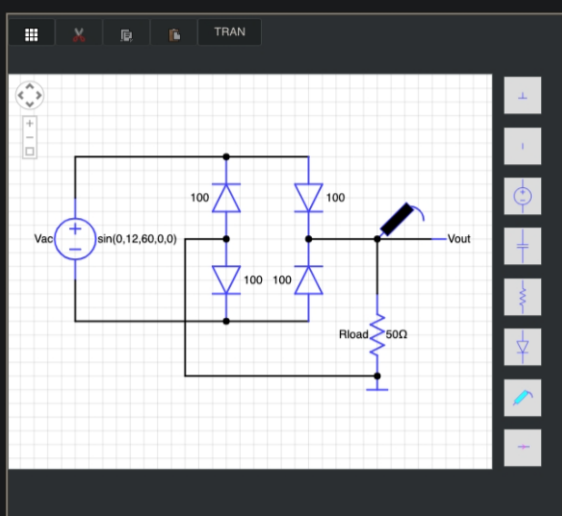
0.0000506606 ✔ **Answer:** 50.4e-6

The moral of the story: current flowing into a capacitor causes charge to accumulate on the capacitor's plates. The accumulated charge represents stored electrical energy that can be drawn upon later (see Task 2 below!).

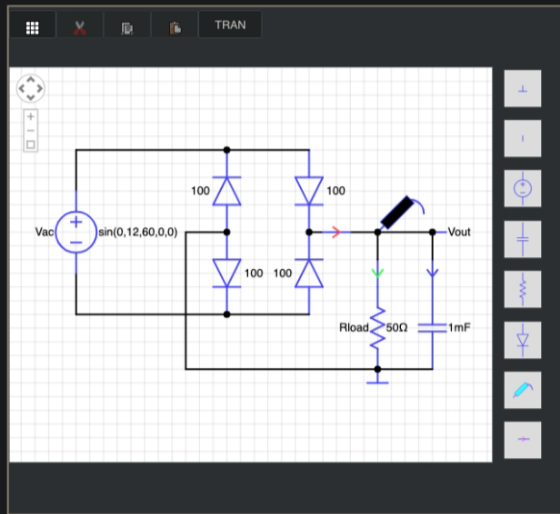
Task 2: Using energy storage in a circuit

Let's look at an example circuit where we'll use the energy storage capability of the capacitor to our advantage. The circuit shown below is the first draft of a power supply whose job it is convert a 12 VAC power source (such as might come from a transformer plugged into an electrical outlet) into a relatively stable DC voltage for driving a load.

The initial circuit, shown below, is a *bridge rectifier* built from four diodes wired so that the positive and negative transitions of the AC supply voltage are converted into a voltage that is always positive: basically the negative excursions in the AC waveform are converted into positive excursions. Run a 30ms transient analysis and examine the voltage waveform to see what's happening.



Sadly, the supply voltage delivered to the load is far from constant! Let's add a *reservoir capacitor* in parallel with the load along with some current probes to see what happens. Run a 30ms transient analysis on the circuit below and look at the plot.



First look at the black waveform, the voltage across the load. It's now relatively stable at around 10V, with a small 0.6V *ripple* (the positive and negative supply voltage variations about the average supply voltage). Looking the green curve, the current into the load resistor, we see that it's also relatively constant at about 200mA. Approximately how much power is being delivered to the load?

Power delivered to the load in watts:

2 ✔ **Answer:** 2

Now look at the red curve, the supply current flowing from the bridge recifitier into the resistor and capacitor. Notice that *the supply current is essentially zero* for 80% of the time.

But a relatively constant current is supplied to the load all the time, so where is it coming from the other 80% of the time? Compare the green curve (the load current) to the blue curve (the capacitor current). Notice they are *equal and opposite* during the 80% of the time the bridge recitifier current is essentially zero: the capacitor is supplying the load current when the bridge rectifier is not.

As a sanity check, let's estimate the power supplied by the bridge rectifier. The average supply voltage is about 10V. What is average current supplied by the bridge rectifier? (Hint: I thought the red current waveform looked like a right triangle during its non-zero portion, so using some measurements from the plot was able to compute the average current.)

Average current supplied by bridge rectifier, in amps:

0.2 ✔ **Answer:** .2

Multiply by 10V and you should get the same answer as the power delivered to the load -- so, yes, the bridge rectifier is delivering all the power required to drive the load, but it's doing it during only 20% of the cycle. 80% of the energy delivered by the bridge rectifier is stored in the capacitor and then fed back into the circuit as the load current when the bridge rectifier is not supplying current.

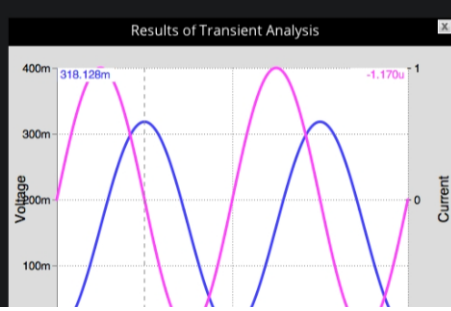
Explanation:
Task 1
(a) Evaluating the integral using the given expression for $i\,(t)$, we get:

$$v_C\,(t) = \frac{1}{2\pi 1000C} \cdot [1 - cos\,(2\pi 1000t)]$$

(b) Plugging in at $t = 5ms$, we get:

$$v\,(0.0005) = 0.318310V$$

(c) The Transient analysis is plotted below.



As we can see, $v = 0.318128V$

(d) Plugging in our answer for the previous part into formula 9.18:

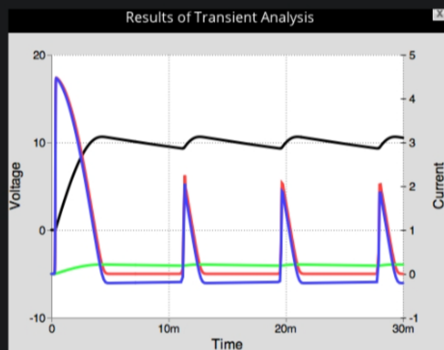
$$Energy = \frac{C \cdot v(0.0005)^2}{2} = 5.06 \cdot 10^{-5} Joules$$

Task 2

(a)

$$P_{avg} = i_{avg} \cdot v_{avg} = 2$$

(b) We see the plot below:



Ignore the big curve from the initial transience and ocus on the repeating spikes of current. Using the formula for the average value of a function:

$$f_{avg} = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx$$

And using the boundary values for a single period $(a = 11ms, b = 19ms)$, all we have to do is evaluate the definite integral, which is merely the area under the "right angle" formed by the red curve, about $0.0016A \cdot s$. This results in $i = 200mA$, as expected.

Submit You have used 5 of 25 attempts

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Answers are displayed within the problem

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Formula for vC(t)
I have realy problems to solve the first part. Firstly vc(t=0) = 0 because the current is finite. I did the integration and used tstart = 0 and tfinish = 1 ... 4

Formula for vC(t):
I cant figure out the formula for the voltage. To me it looks like a simple integral.. 1/C * integral of sine leaving -1/2*pi *cos(inside stays the same) b... 5

[Staff] Vc(t) formula
I read the chapters in the textbook, calculated 600 different options, went over the posts here and after spending almost all of the attempts I concl... 3

[Staff] Lab03
When I click to submit my Lab assignments, Its not sent neither do I get any notifications. I've tried severally for two days now 4

Vc(t) formula
I was not able to determine the formula for Vc(t) by doing the simple integration. After looking at the transient analysis I notice that the graph was s... 2

bridge rectifier
how to solve this problem -I have used (i^2)*r to find the power , i=210ma and r=50 but the answer was wrong. - also how to find the average curr... 2

What does "supply current is essentially zero for 80% of the time." mean?
The question says, the supply current is essentially zero for 80% of the time. but from the TRAN graph, it is a negative value, also the red and blue ... 5

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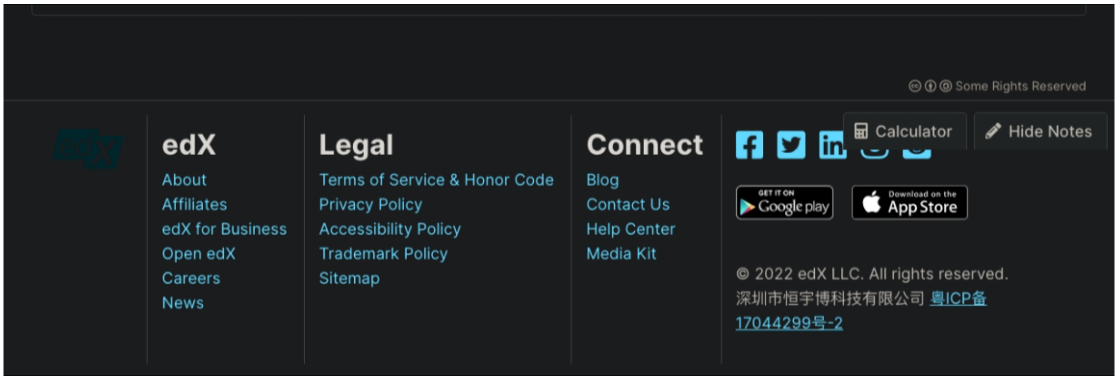

In[∘]:=
```
c = 0.001;
i[t_] := Sin[1000 * 2 π * t];
Integrate[1/c i[t], {t, 0, T}]
v[t_] := -0.15915494309189535` × (-1 + Cos[2000 π t]);
v[0.0005]
e = 1/2 c * (v[0.0005])^2
v = 10; i = 200*^-3;
p = v i
```

Out[∘]= $-0.159155 \times (-1 + \mathrm{Cos}[2000\,\pi\,T])$

Out[∘]= 0.31831

Out[∘]= 0.0000506606

Out[∘]= 2