GWEinstein:

OGRe: $\quad G_{tt} = G_{zz} \;\; = \;\; -\dfrac{2\left(\partial_u{}^2 L[u] + L[u]\,\partial_u \beta[u]^2\right)}{L[u]}$

$\qquad\qquad G_{tz} = G_{zt} \;\; = \;\; 2\left(\dfrac{\partial_u{}^2 L[u]}{L[u]} + \partial_u \beta[u]^2\right)$