The Case of the Vanishing Wavefunction

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Abstract

Spacetime singularities are often held to be pathologies which need to be resolved, and researchers working on the foundations of physics often pin their hopes on the elusive quantum theory of gravity to offer a way to resolve singularities. What is less agreed upon is what such a resolution would amount to: what criteria would a theory of quantum gravity have to fulfill to resolve spacetime singularities? In this paper, I critically analyze one criterion proposed within canonical quantum cosmology for the avoidance of the big bang singularity, to draw philosophical lessons about how to interpret such criteria. The criterion, when applied to symmetry-reduced cosmological models, claims to avoid the big bang by making the wavefunction of the universe vanish 'at' the singularity. I ask whether it works as intended, and even if it does, whether avoiding the singularity is the same as resolving the singularity.

keywords: singularity resolution; big bang; quantum gravity, quantum cosmology; minisuperspace models

The classical singularity theorems suggest that there is at least one cosmological singularity in our universe, characterized by the existence of an incomplete causal geodesic,[1] and this singularity is often taken to be the big bang singularity. What does it mean to 'resolve' this big bang? It is frequently said that we expect a fully developed quantum theory of gravity to resolve the spacetime singularities that plague general relativity (GR), the cosmological big bang singularity among them. But if that is correct, what exactly is it that we expect the theory of quantum gravity to do? There is a general notion of how we would like to not have to worry about the big bang singularity in the final quantum theory of gravity, and different approaches to quantum gravity attempt to make this worry precise to varying degrees, but do these efforts within the various models of quantum gravity accomplish what they aim to?[2] With this paper, I attempt to start addressing these questions, focusing on what has come to be known as the 'DeWitt criterion' for singularity avoidance.[3]

DeWitt (1967, 1129) first suggested the idea that the big bang singularity might be "conceivably alleviated" if the wavefunction of the universe vanishes when the scale factor of the universe is zero. This was more recently discussed as a criterion for singularity avoidance by Kiefer (2010; et al. 2019), who claims that application of this criterion "avoids" the big bang singularity in symmetry-reduced cosmological models. Does this criterion aid in the

[1]In the case of timelike geodesics, to be incomplete is to not be extendable for an infinite amount of time. (Similarly, incomplete null geodesics that those that cannot be extended for an infinite amount of the respective affine parameter.) Arguably, a definition of singularities in terms of curve incompleteness is sufficient for studying singular behavior (Curiel, 1999).

[2]Note that I do not question here whether it is reasonable to ever worry about the spacetime singularities in GR (Curiel (1999) addresses this question, for example) or whether the expectation that quantum gravity will resolve spacetime singularities is justified. I only ask the following question. *If* the singularities in GR are in need of resolution which may be expected to come from quantum gravity, what precisely is it that we are asking of quantum gravity by way of resolution?

[3]Singularity resolution is an extensively studied issue in quantum gravity, with distinct approaches to quantum gravity exploring resolution in a wide variety of cosmological models. A comparison of these approaches is out of the scope of this paper, which is deliberately focused on one criterion within (the relatively less popular approach of) canonical quantum gravity in order to highlight the philosophical issues that come up even in the simplest of cosmological models in the most straightforward of approaches to quantum gravity. Let me also clarify that throughout this paper, I do not mean 'canonical quantum cosmology' to include loop quantum cosmology (LQC), which shares some of the basic formalism with the former, but is significantly more established. (Calcagni, 2017) is a recent work that discusses resolution in 'mainstream' approaches like LQC and string cosmology, but also has an overview of the cosmologies of a few other approaches to quantum gravity.

alleviation/avoidance of the singularity as claimed? If so, does that allay the concerns about spacetime singularities captured by the broader singularity resolution program in quantum gravity? These are the particular questions I raise in this paper. I begin by describing the canonical formalism below, followed by a discussion of the criterion in detail. Sections 3 and 4 discuss two conceptual difficulties with the model, and in the last section, I suggest that the *avoidance* of the singularity, as the application of the DeWitt criterion is claimed to achieve, ought to be distinguished from the *resolution* that one hopes for from quantum gravity.

1 Canonical Quantum Cosmology

Quantum cosmology aims to provide a quantum mechanical description of the universe. One approach to quantum cosmology, the 'canonical approach', uses the canonical quantization procedure on the Hamiltonian formulation of GR, starting with the Wheeler-DeWitt equation (WDW). To simplify matters, quantum cosmologists study drastically reduced models of the universe, called 'minisuperspace models' (MSS models), which are models with all but few physical degrees of freedom truncated. We think of the configuration space of GR, or 'superspace' (Misner et al., 2017), as the space of all (spatial) 3-geometries (up to diffeomorphisms). These geometries evolve over time in superspace, and by restraining the number of degrees of freedom in this space to a select few (which we do by imposing symmetries), we are restraining the trajectories of these geometries to the minisuperspace sector of the full superspace. The DeWitt criterion for singularity avoidance I discuss in the next section is based on a two-dimensional MSS model, and I apply the canonical quantization procedure to a universe with only two degrees of freedom below.

In ordinary quantum mechanics[4] (QM), systems evolve in time according to the Schrödinger equation,

[4]By 'ordinary', I mean QM applied to subsystems of the entire universe.

$$i\hbar\dot{\psi}(x,t) = \hat{H}\psi(x,t), \tag{1}$$

where the wavefunction $\psi(x,t)$ represents the state of the system at time t and $\hat{H}$, the Hamiltonian operator, represents the energy of the system. The analog of the Schrödinger equation in quantum cosmology is WDW,

$$\hat{H}\Psi = 0. \tag{2}$$

WDW is also a differential equation like the Schrödinger equation, but unlike $\psi(x,t)$, Ψ is a functional of field configurations, and despite its being a functional, Ψ is often called the 'wavefunction of the universe'. WDW captures the argument that unlike wavefunctions in ordinary QM, Ψ cannot depend on time, as there is no external clock—there is nothing external to the universe—in quantum cosmology, and if Ψ has no time-dependence, then the total energy of the universe must stay zero forever.

Let's take a look at what WDW looks like in a particular model. I follow Kiefer's (2012, 262-66) quantization of a closed, Friedmann-Lemaître-Robertson-Walker (FLRW) universe, i.e. an isotropic, homogeneous, expanding or contracting universe. The MSS model under consideration has only two degrees of freedom, the scale factor of the universe, represented by '$a(t)$', which is a measure of the size of the universe, and a scalar, spatially constant matter field, represented by '$\phi(t)$'.[5]

The FLRW line element for the closed universe is given by

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega^2, \tag{3}$$

where $N(t)$ is the 'lapse function', and $\mathrm{d}\Omega^2 = \mathrm{d}\chi^2 + \sin^2\chi\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)$. This, when plugged into the Einstein-Hilbert action for quantum gravity, gives the minisuperspace action

[5] t here is the classical time parameter; it does not exist in the quantum theory.

$$S = \int \mathrm{d}t\, N \left(\frac{1}{2} G_{AB} \frac{\dot{q}^A \dot{q}^B}{N^2} - V(q) \right) \equiv \int \mathrm{d}t\, L(q, \dot{q}). \tag{4}$$

Here, $L(q, \dot{q})$ is the minisuperspace Lagrangian and $A, B \in \{a, \phi\}$ and $q^A := a$ and $q^B := \phi$. Denoting the cosmological constant by 'Λ',

$$V(q) \doteq V(a, \phi) = \frac{1}{2} \left(-a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2 \right).^{6} \tag{5}$$

One can also read off G_{AB}, the 'minisuperspace DeWitt metric' from the MSS gravitational and matter actions as

$$G_{AB} = \begin{pmatrix} -a & 0 \\ 0 & a^3 \end{pmatrix}. \tag{6}$$

Now, we calculate the canonical momenta:

$$p_N = \frac{\partial L}{\partial \dot{N}} \approx 0,^{7} \tag{7}$$

$$p_a = \frac{\partial L}{\partial \dot{a}} = -\frac{a\dot{a}}{N}, \text{ and} \tag{8}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{a^3 \dot{\phi}}{N}. \tag{9}$$

From these, we can get the Hamiltonian as follows.

[6] I have not provided the explicit forms of the gravitational and the matter counterparts of the MSS action here, but mass 'm' comes from the matter action, and 'Λ' from the gravitational part. See (Kiefer, 2012, 263-64) for details.

[7] '$\approx$' here is to remind us that this primary constraint is a 'Dirac weak equation', i.e. it cannot be used before working out the relevant Poission brackets.

$$
\begin{aligned}
H &= p_N \dot{N} + p_a \dot{a} + p_\phi \dot{\phi} - L \\
&= \frac{N}{2} \cdot \left(-\frac{p_a^2}{a} + \frac{p_\phi^2}{a^3} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2 \right) \\
&\equiv N. \left(\frac{1}{2} G^{AB} p_A p_B + V(q) \right),
\end{aligned}
\tag{10}
$$

where G^{AB} is the inverse of G_{AB}. Since the primary constraint, $p_N \approx 0$, has to hold at all times, $\{p_N, H\} \approx 0$, which leads us to $H \approx 0$, which we impose as a constraint on the wavefunction.

The final step of the canonical quantization procedure is to pick an operator ordering. Kiefer uses the Laplace-Beltrami ordering, by which $G^{AB} p_A p_B$ becomes $\hbar^2 \nabla^2_{\mathrm{LB}}$, which, when written out, is $-\frac{\hbar^2}{\sqrt{-G}} \partial_A \left(\sqrt{-G} G^{AB} \partial_B \right)$. Simplifying this, one gets

$$
\hbar^2 \nabla^2_{\mathrm{LB}} = \frac{\hbar^2}{a^2} \frac{\partial}{\partial a} \left(a \frac{\partial}{\partial a} \right) - \frac{\hbar^2}{a^3} \frac{\partial^2}{\partial \phi^2}. \tag{11}
$$

(∇^2_{LB} is the covariant version of the Laplacian.)

Now, Ψ in (2) reduces to a function of a and ϕ in this MSS model. It is effectively a functional since it ranges over all fields, but it is mathematically a function since the constant field is represented by one variable, ϕ. So our WDW takes the form,

$$
\hat{H} \psi(a, \phi) = 0. \tag{12}
$$

Plugging (11) and (5) into (10) gives us the MSS Hamiltonian operator, which, when plugged into WDW (12), gives us

$$
\frac{1}{2} \left(\frac{\hbar^2}{a^2} \frac{\partial}{\partial a} \left(a \frac{\partial}{\partial a} \right) - \frac{\hbar^2}{a^3} \frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2 \right) \psi(a, \phi) = 0. \tag{13}
$$

A special case is when $\Lambda = m = 0$. The solutions to WDW in that case, with the boundary condition that ψ goes to zero as a does, are given by

$$\psi_k(\alpha, \phi) = e^{-ik\phi} K_{ik/2} \frac{e^{2\alpha}}{2}, \tag{14}$$

where $\alpha \doteq \ln a$ and $K_{ik/2}$ is a Bessel function.

The same boundary condition appears as a singularity avoidance criterion in the model discussed in the following section, even though $\Lambda \neq 0$ and $m \neq 0$. The WDW in that model is similar to (13) for the closed FLRW universe, except that $V(q)$ takes a different form. We then look at the solutions to that WDW, and see what they can tell us, if anything, about singularity avoidance.

2 The DeWitt Criterion

As mentioned earlier, the vanishing wavefunction condition was introduced by DeWitt (1967) and recently revived by Kiefer (2010; et al. 2019).[8] That the classical singularity theorems do not say what the precise physical nature of a (cosmological) singularity is, beyond that the causal structure of spacetime as we know it breaks down 'there' leads DeWitt (1967) to think of a singularity as a "barrier" beyond which one cannot extend the solutions of the field equations of GR. He then asks if the barrier would also prevent the solutions of WDW from being extended. It is in this context that he introduces the condition, by claiming that if the wave functional Ψ vanishes at the barrier, then the quantum physicist would have "alleviated" their barrier, and there would be no problem in extending the solutions of WDW beyond the singularity (DeWitt, 1967, 1129).

Kiefer (2010) works out a concrete example in which one can see the criterion in action. He aims to show that the MSS model avoids big brake singularities, i.e., singularities where the expansion of the universe comes to an abrupt stop as a result of the pressure of the fluid

[8] DeWitt himself does not propose that the vanishing wavefunction condition ought to count as a *criterion* for singularity avoidance—what he does claim is that "*[p]rovided it does not turn out to be ultimately inconsistent*, [...] it makes the probability amplitude for catastrophic 3-geometries vanish" (DeWitt, 1967, 1129). The term 'DeWitt criterion' I borrow from Kiefer.

that models the matter field of the universe diverging,[9] if it satisfies the criterion. However, satisfying the criterion also makes the model avoid the big bang singularity. Consider a two-dimensional FLRW MSS with a scale factor a and scalar field ϕ. Near the singularity, ϕ is small, and hence WDW is approximately

$$\frac{\hbar^2}{2}\left(\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\phi^2}\right)\psi(\alpha,\phi)-\frac{\tilde{V}_0}{|\phi|}e^{6\alpha}\psi(\alpha,\phi)=0, \tag{15}$$

where $\kappa^2 = 8\pi G$, $\alpha = \ln a$ and $\tilde{V}_0 = \sqrt{(\rho p)/4}$ where ρ is the energy density and p the pressure of the Anti-Chaplygin gas.

The solutions of this WDW are

$$\psi(\alpha,\phi)=\sum_{k=1}^{\infty}A(k)k^{-3/2}\mathrm{K}_0\left(\frac{1}{\sqrt{6}}\frac{V_\alpha}{\hbar^2 k\kappa}\right)\times\left(2\frac{V_\alpha}{k}|\phi|\right)e^{-\frac{V_\alpha}{k|\phi|}}\mathrm{L}^1_{k-1}\left(2\frac{V_\alpha}{k}|\phi|\right), \tag{16}$$

where K_0 is a Bessel function, L^1_{k-1} are Laguerre polynomials, and $V_\alpha \equiv \tilde{V}_0 e^{6\alpha}$. As can be seen above, as $\alpha \to -\infty, \psi \to 0$, since the (linear) V_α quickly goes to zero as $\alpha \to -\infty$ (Kiefer, 2010, 216). This model thus satisfies the DeWitt criterion. But what does that show about the fate of the singularities in this model?

Heuristically, the argument for the DeWitt criterion being a criterion for singularity avoidance is that it makes the probability amplitude of singular 3-spaces zero at the quantum level. However, there is no consensus as to what the standard probability interpretation of QM amounts to in canonical quantum gravity, as we shall see below. Without a satisfactory probability interpretation, the heuristic argument fails. The only way to argue for the criterion is to say that from the mathematics, we can see that ψ goes to zero when a does, and this is what we mean by avoiding the singularity. This is exactly the move that Kiefer makes when he writes, "we shall interpret the vanishing of the wave function in the region of the classical singularity as singularity avoidance" (Kiefer, 2010, 215). But of course, this claim then begs the question.

[9] Typically, in this case, the matter content is represented by an 'Anti-Chaplygin gas' with the equation of state $p = A/\rho$, $A > 0$ where p the pressure of the gas, and ρ is its energy density.

Even if we accept that the vanishing wavefunction counts as a singularity avoidance criterion, we could ask if avoiding the two-dimensional MSS singularity says anything about whether the criterion would hold for more realistic, higher dimensional models. The original DeWitt criterion turns out to not work even when the MSS model has more than two dimensions. Kiefer et al. (2019) argue that because of the constraint nature of the Hamiltonian, the minisuperspace possesses a conformal structure, and since the DeWitt criterion is not conformally invariant, it can only be good for two-dimensional MSS models. Hence they (see also (Kiefer and Kwidzinski, 2019)) develop a generalized DeWitt criterion that holds for higher dimensional, anisotropic cosmologies. The generalized criterion states, "A singularity is said to be avoided if $\star|\Psi|^{\frac{2d}{d-2}} = |\Psi|^{\frac{2d}{d-2}}\mathrm{dvol} \to 0$ in the vicinity of the singularity". Here, d denotes the dimension of the MSS, $\star$ is the Hodge star, and dvol "contains" the square root of the (absolute value of the) determinant of the DeWitt metric (Kiefer et al., 2019, 4).

Two conceptual issues present themselves in the above discussion: (1) Even the generalized DeWitt criterion only works for finite-dimensional MSS models, and this is not enough reason to expect an infinite generalization of the criterion to be well-defined for 'full' universes with an infinite number of degrees of freedom. Indeed, as we shall see, MSS models are, at best, only approximations of the full universe, and unless the criterion is independent of the number of degrees of freedom of the universe, it is not clear if one can realistically consider it a criterion for singularity avoidance. (2) As mentioned above, the probability interpretation of the wavefunction is even less straightforward in canonical quantum gravity than it is in ordinary QM. And without a satisfactory interpretation, the heuristic argument for the DeWitt criterion has no bite, and it is unclear what the criterion would even mean physically. These issues form the contents of the following sections.

3 The Spherical Cow Issue

Since all of quantum cosmology is done using MSS models, and in particular, the DeWitt criterion—both in its original as well as generalized forms—is meant to hold only in MSS models, the first philosophical issue that one ought to address is how one would justify the use of these drastically reduced models. Even though this question has not received much attention in philosophy of physics, it is one that quantum cosmologists have acknowledged, and attempted to answer. I focus here on two papers written by Karel Kuchař and Michael Ryan.

In their 1986 paper, Kuchař and Ryan give a straightforward answer to whether "quantizing a system restricted classically to a sector of the available phase space" is the same as "quantizing the system on the full phase space and then following the statistics of a restricted class of canonical variables": it is not (Kuchař and Ryan, 1986, 453). What we want is a correct, but tractable model which we can use to make experimental predictions. Ideally, we would quantize gravity first and then reduce the number of degrees of freedom involved. However, there is no straightforward way to quantize the gravitational field without its being afflicted with untameable infinities, and the only way we have to get the model we want is to reduce the universe to a simplistic model, and then quantize it. But reducing the phase space and then quantizing does not give us the same results as models that are reduced versions of some 'full' quantum gravity. In other words, the quantization and the reduction do not commute. This is so because the MSS approximation violates Heisenberg's Uncertainty Principle (HUP) (Kuchař and Ryan, 1986, 453). The canonically conjugate variables in quantum field theory are the field and momentum operators. Assuming that fields can be represented by expansions of modes, what one does when one creates an MSS model is to artificially set the inhomogeneous field modes and their associated momenta to zero. However, this violates HUP, as this reduction would mean that on quantization, the inhomogeneous modes and their momenta commute, which, in fact, they do not. Any quantum model that violates HUP is not an accurate quantum model, and hence the result we get by quantizing an MSS model is definitively not the ‘real deal’.

One could ask if the MSS model is an *approximation* to the full quantum theory, and this is the question Kuchař and Ryan set out to answer. In their 1989 paper, they propose to study a hierarchy of exactly soluble models (which are MSS models), in which are embedded models of higher symmetries (“microsuperspaces”), to figure out what it would take for the microsuperspaces to count as approximations to the respective minisuperspaces. In the paper, they consider the embedding of the Taub model in the mixmaster universe (Bianchi IX). Bianchi IX models are similar to FLRW models in that they are homogeneous, but in addition to expanding and contracting, they can also change shape anisotropically. The Taub models are a special case of the Bianchi IX models. Kuchař and Ryan distinguish among three different senses of ‘approximation’ (Kuchař and Ryan, 1989, 3983-84):

1. When one ignores all but the variables that can be measured in the minisuperspace, the superspace wavefunction becomes a density operator that can be decomposed into a combination of pure minisuperspace states. Each term in the decomposition evolves according to an “autonomous” Schrödinger equation that is insensitive to the other terms. If the minisuperspace Schrödinger equation is similar to the “autonomous” Schrödinger equation barring small corrections, then the MSS model would count as an approximation.

2. Near a cosmological singularity, almost all the energy of the gravitational field flows into the homogeneous modes of the field, i.e. the modes that are not frozen out in MSS models. The MSS model would then count as an approximation to the full model at least near the singularity.

3. The “important parts” of the minisuperspace qualitatively behave in the same way as the corresponding parts in superspace, just like how a one-dimensional slice of a three-dimensional harmonic oscillator approximates the complete harmonic oscillator.

They conclude that the Taub model does not approximate the mixmaster universe in any of the three senses of ‘approximation’, and that perhaps the only reason to think that the former approximates the latter would be that the minisuperspace “stays near” the microsuperspace for some time (Kuchař and Ryan, 1989, 3994). It turns out that there is a finite time interval during which the solution to the microsuperspace WDW predicts the behaviour of the minisuperspace. However, the existence and the duration of this interval seems to be model-dependent. And hence Kuchař and Ryan further claim that there needs to be some criterion for when a model with more symmetry counts as an approximation to a less symmetric model, but as far as I can tell, no such criterion has been proposed to date.

It could be that by their very nature, MSS models cannot be approximations in general. Bojowald et al. (2012), for instance, write that “quantum cosmology is a truncation rather than an approximation, drastically cutting off unwanted degrees of freedom instead of providing a harmonious embedding of a simplified model within a fuller framework”. If there is no point in looking for a general approximation criterion that applies to all models, then it would turn out that the model-dependency Kuchař and Ryan discovered merely represents a coincidence. What is one to do then?

Given the infinite degrees of freedom one would otherwise have to deal with, it is, in general, reasonable to study these reduced models. In fact, Kiefer writes that MSS models can be treated as toy models to study a variety of issues that do not depend on the number of degrees of freedom (2012, 260). But the question we ought to ask is whether solving the singularity issue in a toy model is the same as solving the singularity issue. For the answer to this question to be in the affirmative, the singularity avoidance needs to be independent of the number of degrees of freedom involved in the model. Let’s take a look at the generalized DeWitt criterion again. It states that “a singularity is said to be avoided if $\star|\Psi|^{\frac{2d}{d-2}} = |\Psi|^{\frac{2d}{d-2}}\mathrm{dvol} \to 0$ in the vicinity of the singularity”, where d is the dimension of the MSS (Kiefer et al., 2019, 4). Obviously, the quantity $\star|\Psi|^{\frac{2d}{d-2}}$ is not independent of the number of dimensions; indeed it is not even well-defined for an infinite-dimensional model.

Where does that leave us? So far, we know that applying the DeWitt criterion makes Eq.

(13) well-defined when $a \to 0$. All we can say from this is that the evolution of an unrealistic toy model is not troubled by quantities 'blowing up'. Because the criterion does not allude to any physical characteristics of the singularity itself (other than that it is 'located around' $a \to 0$), there is no justification for expecting that application of the same criterion will accomplish anything by way of resolving the real big bang singularity in our universe. Note that this is not to say that MSS models are completely useless. Again, they could be very useful in studying issues that are independent of the number of degrees of freedom involved (Kiefer (2012, 260) mentions the examples of the problem of time, the role of observers, and the emergence of a classical world), but singularity resolution, or at least, singularity avoidance with the aid of the DeWitt criterion, does not appear to be one of them. It is worth noting at this point that singularity resolution in LQC has been shown to be robust; resolution as defined within the LQC framework does generalize to realistic models (see (Singh, 2015), for instance).

4 Unfreezing Time

Another issue with the DeWitt criterion that both DeWitt (1967) and Kiefer (2012) point out is the lack of a satisfactory probability interpretation of $\psi(a, \phi)$. In ordinary QM, if $\psi(x, t)$ is the state of a system, then $\int_x^{x+dx} |\psi(x,t)|^2 dx$ is the probability that the system will be found between x and $x + dx$ at any given time t. We expect the probability $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx$ to be conserved over time. However, such an expectation is problematic in quantum cosmology. The state of our system in this case is given by $\psi(a, \phi)$, which is independent of the classical time variable t. It would appear that the notion of a probability conserved over time makes no sense, because there is no time over which the probability could be conserved. So we ask: given $\psi(a, \phi)$ is independent of t, could the quantity $|\psi(a, \phi)|^2$ represent some kind of probability? If so, how? We seem to have several options:[10]

[10]This list is not exhaustive. See Kuchař (2011) for an extensive list of choices, and (potentially dispiriting) arguments for why none of them look promising.

(i) Define a physically meaningful, 'intrinsic' time measured by a quantity such as a or ϕ, and let $|\psi(a,\phi)|^2$ encode the probability that the universe has a field of strength ϕ (or respectively, a size a) when it has a size a (field strength ϕ).

(ii) Let $|\psi(a,\phi)|^2$ be related to the probability that an 'arbitrarily chosen' 3-geometry from superspace has a scale factor a and field strength ϕ.

(iii) Use a probability interpretation that is neither of the above.

(iv) Give up on extending the probability interpretation of ordinary QM to quantum cosmology.

The problem of the probability interpretation concerns all of quantum cosmology done using WDW. But it is particularly acute for singularity avoidance using the DeWitt criterion, because, as we have seen, the criterion says that the big bang singularity is avoided if, for $a \to 0$, $\psi \to 0$. When $\psi = 0$, $|\psi|^2 = 0$. And the heuristic argument for the criterion being a singularity avoidance criterion is that satisfying it makes the probability of singular 3-spaces zero. But without a proper notion of probability, i.e. without a story about how $|\psi|^2$ is a probability, this argument does not hold. So when we look at the ways people have suggested to work around the general problem below, we will also keep an eye on the specific problem concerning the DeWitt criterion.

DeWitt (1967) chooses (i). Here, I follow Vilenkin's (1986) description of DeWitt's argument. Suppose a is the intrinsic time. Then the probability density, $\rho(a)d\phi$ is taken to be the probability that the field is between ϕ and $\phi + d\phi$ when the scale factor is a. To impose normalizability, i.e. to ensure that $\int \rho(a)d\phi = 1$, $\rho(a)$ is identified with the conserved current in superspace, j. The argument for this identification goes as follows. Consider the continuity equation $\partial_\mu j^\mu = 0$, i.e. $\partial_a j^a + \partial_\phi j^\phi = 0$. The components of the current that would satisfy this equation (as can be shown using the WDW[11]) are

[11] Vilenkin uses a model in which there are interacting gravitational and scalar fields, and $p \sim 1$ is a factor-ordering parameter.

$$j^a = ia^p(\psi^* \partial_a \psi - \psi \partial_a \psi^*), \tag{17}$$

$$j^\phi = -ia^{p-2}(\psi^* \partial_\phi \psi - \psi \partial_\phi \psi^*). \tag{18}$$

Now, if we integrate the continuity equation over all ϕ, assuming ϕ vanishes at the end points, we get that $\partial_a \int j^a d\phi = 0$. This is consistent with $\int j^a d\phi$ being 1, and so identifying the probability density with the conserved current component would satisfy normalizability (Vilenkin, 1986, 3565). In terms of the DeWitt criterion, as can be seen from (17) and (18), if a vanishes, so would j, and hence, ψ.

The problem with this approach that DeWitt (1967) points out (which, as he notes, also affects the interpretability of the condition for singularity avoidance) is that j^a is not necessarily positive-definite, and hence we could, in theory, end up with negative probability densities. He attempts to solve the problem in the special case of a Friedmann universe. Vilenkin also takes up the problem, and argues that we should not require that our definition of probability be necessarily positive-definite for an arbitrary wavefunction. This is so, he argues, because ψ, the wavefunction of the universe, is the unique result of imposing a set of boundary conditions on the WDW. It is possible for this wavefunction to have only positive- or negative-frequency components with respect to some time variable, and it is not necessary for the same variable to be the global time variable across superspace. Hence we could always find a set of "good" time variables in overlapping regions of superspace such that there would be no negative probabilities. He also formulates the conditions for what makes a time variable good: it should be semiclassical, and monotonic (Vilenkin, 1986, 3567). The requirement of semiclassicality, however, turns out to be more trouble than it is worth. As Kuchař (2011) notes, this interpretation calls for a split of the gravitational variables into Vilenkin's 'good' time variables that need to become classical, and the variables that represent the dynamics of the system and remain quantum. And it is far from obvious that such a split is possible and can be meaningful. For detailed arguments for why this is the case, I direct the reader to (Kuchař, 2011).

Hawking and Page (1986) cite the problem of the current not being positive-definite to reject (i) in favour of (ii). They define $|\psi|^2$ as being proportional to the probability of finding a 3-geometry in superspace with scale factor a and field configuration ϕ. The major objection Kuchař (2011) raises against this choice is that there is no dynamics in this interpretation. It can give you a probability for a particular universe, but it cannot answer the question of what the probabilities of particular evolutions of the same universe are. This might not be a problem when we isolate the issue of singularity resolution. If, by $|\psi(a=0,\phi)|^2 = 0$, all we mean is that the probability of there being a universe with $a = 0$ is zero, then one could argue that the big bang singularity is avoided. But of course, this interpretation would be of no help in predicting the evolution of a non-singular universe so chosen by the imposition of the DeWitt criterion, and so does not solve the probability interpretation problem in the broader quantum cosmology program.

Surveying all the possibilities under (iii) is out of the scope of this paper, but, again, Kuchař (2011) studies many of these, and deems them all ultimately unsuccessful. But even if one can find a satisfactory probability interpretion, it is unclear that the heuristic argument for singularity avoidance using the DeWitt criterion necessarily holds. As Blyth and Isham (1975, 774) note, a has a continuous spectrum, and so $\psi(0,\phi)$ being 0 does not necessarily signal the "absence" of the singularity, since it is not clear where precisely the line is between singular and non-singular behavior as a approaches 0. The singularity is generally just assumed to be localized to the point $a = 0$, and this assumption needs to be justified before one can claim that the wavefunction vanishing at the point $a = 0$ avoids the singularity.

For (iv) to be the right choice, one would have to provide an argument for why the universe is a special system. That there are no observers external to the universe could be such an argument. But then, again, the heuristic argument for the DeWitt criterion being a sufficient criterion for singularity avoidance would not hold. However, this may not be a big problem, if all we are looking for is a criterion that helps to only formally avoid the singularity. The question of whether that is all one wants from singularity resolution is taken up in the next section.

5 Groundwork for a Distinction between Avoidance and Resolution

Historically, several terms have been used to describe the process of doing away with singularities, from DeWitt's talk of "alleviating" singularities (DeWitt, 1967) and Kiefer's singularity "avoidance" (Kiefer, 2010) to Wüthrich's talk of singularities being "spirited away" (Wüthrich, 2006). Such a wide variety of terms being used is itself indicative of a lack of precision in thinking about singularity resolution.

Satisfying the DeWitt criterion lets the relevant MSS models *avoid* the big bang singularity. The criterion provides no explanation as to what happens 'at' the singularity, or shed any light on its nature. Avoidance criteria thus help models avoid singularities in the same way one would avoid a pothole: you acknowledge there is a pothole and that it is dangerous to step into it, so you sidestep it. This notion of avoidance can be further split into two senses. The way Kiefer sets the criterion up, one assumes that there is a satisfactory probability interpretation, and then claims that satisfying the criterion results in zero chance of the universe actually stepping into a pothole. But there is another sense in which satisfying the criterion could be said to avoid the singularity, a formal sense in which (13) is not well-defined unless the model satisfies the criterion, i.e. ψ goes to zero as a does. For singularity avoidance in this sense, we do not need a probability interpretation to say that the vanishing wavefunction makes the quantum theory safe from singularities.

One could argue that a true *resolution* of the singularity calls for a physically meaningful story about what the quantum theory of gravity did to the classical singularity other than merely make the universe go around it. In other words, a quantum gravitational account of singularity resolution would show that the singularity exists only because we try to derive classical conclusions about quantum phenomena, that there is a quantum gravitational explanation of what really goes on 'at' the singularity. A historical example of such a singularity resolution would be how the 'singularity' caused by the electron spiraling into the nucleus in the classical model of the atom was resolved by the quantum theory. What made it count as *resolution*, I would argue, was that the new theory gave an explanation, in a language unfamiliar within the framework of the old theory, for what the electron really does that makes the classical prediction wrong. If all the quantum theory had done was to make the evolution of the electron formally free from infinities, or predicted that despite there being a possibility of it happening, the probability of the electron actually spiraling into the nucleus is zero, then the quantum theory would only have *avoided* the singularity. Having thus made the distinction between singularity *avoidance* and *resolution*, I would claim that the latter is more desirable, and indeed, what one looks for in a theory of quantum gravity, when it comes to cosmological singularity resolution. Various approaches to quantum gravity do aim to provide, beyond a description of formal avoidance, an explanation for what replaces the classical singularity in the quantum theory. There is a lot to be said about the extent to which these approaches are successful, and this is a matter I defer to another time.

My point in this paper has not been that studying the DeWitt criterion is futile unless it is a singularity resolution criterion as opposed to merely being a singularity avoidance criterion. Kiefer (2010) writes that the DeWitt criterion is a sufficient, but not necessary criterion for singularity avoidance, and all I am doing here is to question this claim. Given the MSS and probability interpretation problems, I am inclined to rule that the DeWitt criterion cannot even count as being sufficient, unless what all one is looking to do is to formally avoid the singularity at $a = 0$ in a toy model. That said, the investigation of the DeWitt criterion has highlighted some potential conceptual pitfalls we ought to be wary of in our search for a quantum gravitational singularity resolution. So even if the case of the vanishing wavefunction is now closed, it paves the way for attempts to deal with spacetime singularities that amount to more than mere formal avoidance.

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