Big Bang Resolution in Quantum Gravity*

by

NIRANJANA WARRIER
B.A., Brandeis University, 2017
M.A., University of Illinois at Chicago, 2018
M.S., University of Illinois at Chicago, 2023

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Defense Committee:
Rachel Goodman
Aidan Gray
Jon Jarrett
Nick Huggett, Chair and "Advisor"
Christian Wüthrich, University of Geneva

** My biggest source of shame and regret is to ever have been associated with the program I was in while I wrote this. No part of it was enhanced because of the association; in fact, most of it was severely impoverished by my unfortunate choice of affiliation. That this work has insights I chanced upon in spite of the circumstances is the sole reason I still admit to writing it. Distinguishing between the parts that are original and those that are demonstrably inaccurate is left as an exercise to the reader.*

To Acchan and Amma—

if you had given me better genes,

I would never have become a philosopher

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF ABBREVIATIONS

EFEs	Einstein field equations
FLRW	Friedmann-Lemaître-Robertson-Walker
GR	general relativity
HUP	Heisenberg's uncertainty principle
LQC	loop quantum cosmology
LQG	loop quantum gravity
MSS	minisuperspace
QG	quantum gravity
QM	quantum mechanics
QRLG	quantum reduced loop gravity
SFD	scale factor duality
WDW	Wheeler-DeWitt equation

SUMMARY

Ask any theoretical physicist to name one unsolved mystery in physics, and the odds are that they will choose to talk about the hunt for the correct theory of quantum gravity (QG). Early twentieth century physics had two crowning jewels, the general theory of relativity (GR), and quantum mechanics (QM). Despite its experimental success, GR is plagued by what are called "spacetime singularities", regions of spacetime about which the equations of GR fail to make any predictions. This suggests that spacetime itself breaks down at a singularity, and hence a theory more fundamental than GR might be needed to make sense of singularities. Since singularities are typically characterized by the presence of immensely dense matter of the kind that GR deals with, and minute scales of length that fall under the purview of QM, many scholars believe that if we could combine the two theories, the resulting theory would no longer be afflicted with singularities. As of this writing, the search for such a theory, or QG, is over half a century old. While there exist dozens of incomplete candidate theories, no part of any has been experimentally confirmed. And yet, many of these speculative theories abound with proposals to resolve the singularities of GR, and the realm of these proposals that speaks especially to the resolution of the big bang singularity is what this dissertation explores.

In particular, I claim that a quantum gravitational *resolution* of the big bang ought to involve a story in a language unfamiliar to GR about what it is that GR gets wrong, that only a quantum theory of gravity can explain. Such *resolution*, I distinguish from big bang *avoidance*, which is what one gets by formally avoiding the problems identified with the big bang within GR, like the involvement of infinities in the mathematics, or abrupt ends to the "evolution" of the universe. Then I argue that even in the simple cosmological models that can be built using the techniques of canonical quantum gravity, loop quantum gravity and string theory, despite their claims of big bang *resolution*, only big bang *avoidance* can be said to occur, and in some cases, not even that. Furthermore, the overarching goal of my work has been to inform QG research, which itself is an actively developing area of study in physics. The hope is that given big bang resolution is expected of QG, specifying what resolution is not, might count as a step towards more clarity about what it is that we ought to expect of QG.

CHAPTER 1

A PHILOSOPHER LOOKS AT QUANTUM COSMOLOGY

This project aims to study an area of physics hitherto largely left unexplored by philosophers of physics, that of quantum cosmology. Quantum cosmology is what one gets when one applies quantum mechanics (QM) to the only true "closed" system: the entire universe. This in itself causes a multitude of conceptual problems, as in ordinary QM[1] the classical observer observing a system plays a role in the evolution of the system over time, but when the system comprises of everything, the observer him/herself has to also be a part of the system. Quantum cosmology has to also concern itself with the gravitational force, which is negligible in the realm of ordinary QM, but is very strong in a cosmological context. This in turn means that quantum cosmology is intricately related to quantum gravity, the attempt to unify the two major twentieth century discoveries in physics, QM and general relativity (GR).[2] Indeed, quantum cosmology can be thought of as a potential observational counterpart to quantum gravity in that arguably, the predictions made by the correct theory of quantum gravity will have cosmologically testable consequences. And this is the stance I take throughout the project, that hopes of experimental confirmation of quantum gravity are closely tied to progress in quantum cosmology, and hence a study of the latter might shed some light on what to look for when we look for a theory of quantum gravity.

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[1] By "ordinary", I mean the kind of QM that takes subsystems of the universe as systems.

[2] While I consider "quantum cosmology" to be a very broad umbrella term that encompasses all serious enquiries into the quantum nature of cosmological phenomena, sometimes the term is reserved solely for the cosmological applications of certain approaches to quantum gravity, like canonical quantum gravity, or loop quantum gravity. Where I want to emphasize that I am talking about all approaches to quantum gravity, I use the term "quantum gravitational cosmology" in this dissertation.

Quantum cosmology as a field came into being in the late 60s, with Misner (1969b) coining the term "quantum cosmology" to denote the quantization of a homogeneous universe. The initial attempts to do quantum cosmology aimed to study quantum gravity in tractable systems, but interest seems to have had petered out due to conceptual problems, many of which I take up in this project. The second wave of interest in quantum cosmology surfaced when Hartle and Hawking (1983) proposed an initial condition that has to be satisfied by the wavefunction of the universe. But this proposal too is plagued by many conceptual issues, as we shall see. Quantum cosmologists today focus on particular problems within the field and ways to solve the problems using different approaches to quantum gravity. I take up one such problem in this project, that of singularity resolution.

Since its inception in the early twentieth century, GR has been successfully confirmed several times. However, GR also predicts the existence of singularities, "where" the theory breaks down. Singularities are broadly classified into two, cosmological singularities and black holes. The former, unlike the latter, are not covered by boundaries called "event horizons", and hence are also called "naked" singularities. I focus on cosmological singularities in this project, and in particular, on the "initial" cosmological singularity. There are reasons—the most important of which is provided by the singularity theorems—to think that our universe has at least one cosmological singularity, what is often called the big bang singularity.[3] Since GR breaks down at the big bang singularity, physicists turn to quantum gravity to resolve the problem. One of the most hackneyed statements in the field is that once we have the correct theory of quantum gravity, we will no longer have to

[3]It is also sometimes called the "initial" singularity because in the big bang model of the universe, the universe started with, well, a big bang. I, too, use the term "initial singularity" sometimes to refer to the big bang singularity in this dissertation, but solely so that I have a variety of terms to refer to the same entity, not because I endorse the identification of the big bang singularity with cosmogenesis. I discuss problems with this identification in the final chapter. One might also argue that even in models in which the big bang is not *the* beginning, it is often the beginning of the present epoch, and so the use of "initial" is justified.

worry about the big bang. Indeed, the hope is so palpable in the community that it often feels as if we are all the princess trapped in a tower, waiting for the quantum gravitational prince to come rescue us.[4] Of course, this hope is justified. But what I suggest in this project is that we need not idly wait for the prince, that our attempts to resolve singularities might teach us valuable lessons about the underlying theory of quantum gravity.[5]

This introduction is largely a discussion of what a singularity is. Many of the terms used in the singularity resolution literature, like "singularity" and "resolution" to begin with, are vaguely defined, if at all, which often distorts our understanding of what it is that they refer to. The present attempt will not be successful in rigourously defining all of these terms. Rather, it is to be looked at as a call for clearer use of terminology. What follows will, at the very least, flag the misconceptions that stem from what I will argue are improper uses of terminology, which themselves arise because of a lack of comprehension of what it is that the terms attempt to describe. Put another way, I claim that the absence of a thorough understanding of the relevant concepts is both reflected in, and made worse by, the terminology often chosen to refer to those concepts, and that more clarity in how we use words to refer to quantum cosmological ideas will lead to a better understanding of what it is that we do when we try to resolve singularities in quantum cosmology.

Each of the following chapters argues for the main thesis of this dissertation, that none of the extant approaches to quantum gravity, qua replacements for GR, *resolves* the big bang singularity. As we shall see, some of the reasons why this is the case depend on the approach, and have to do with the technicalities of the specified approach, but the fundamental reason is the same across the

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[4]See any text ever written on quantum gravity, all the way from popular books such as Smolin (2000) to textbooks like Calcagni (2017).

[5]Note that I do not question here whether it is reasonable to ever worry about the spacetime singularities in GR. (Curiel (1999) addresses this question, for example.) This is a question worth asking; it simply is not one I address in this project.

board—as proposed quantum replacements for GR, none of these satisfactorily explain what it is that GR gets wrong about a reality that is not classical.

1.1 What is a Spacetime Singularity?

We know that GR breaks down at spacetime singularities, but beyond that, trying to make our intuitions of a singularity precise has proven to be difficult.[6] Wald (2010), for instance, begins his discussion on singularities by introducing the idea of a singularity being "a 'place' where the spacetime curvature 'blows up' or other 'pathological behavior' of the metric takes place" (212). He then promptly goes on to show why the terms in quotes are problematic, and hence the definition is not good enough. Curiel (1999) argues that there is no single, universal definition of singularities, and that we do not need such a definition. According to him, each problem presents its own specific definition, and a general definition in terms of 'curve incompleteness' is good enough for most problems. Even so, the stance I shall take throughout this dissertation is that the answer to the question, "What is a singularity?" is not "Geodesic incompleteness." The latter indicates the existence of the former, but does not define it. Faced with the lack of a better alternative, to answer the question of what a singularity is in GR, we take the route physicists take: admit that we do not have a satisfactory, universal definition, assume that singularities have something to do with geodesic incompleteness and then discuss the singularity theorems. Let me emphasize here that all of this only applies to defining a spacetime singularity in GR. The discussion of whether the putative definition is of any relevance to singularity resolution in quantum gravity will unfold as we progress through the more substantial chapters of the dissertation.

An incomplete geodesic is one which cannot be extended for an infinite amount of time. An observer travelling along it will encounter a singularity after a finite period of time. The singularity

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[6] "Singularity" stands for "spacetime singularity" throughout this dissertation.

theorems prove that given a set of reasonable assumptions, our universe has at least one incomplete geodesic. There are various singularity theorems, but here I sketch out the most general one proved by Hawking, Penrose, and Bondi (1970). The theorem states that if a spacetime (M, g_{ab})—where M is a manifold and g_{ab} is a metric—satisfies the following conditions, then it must have at least one incomplete timelike or null geodesic.

1. The Einstein field equations (EFEs) hold, with the cosmological constant being zero or negative.
2. The energy condition, $R_{ab}v^a v^b \geq 0$ holds for all timelike and null v^a.
3. There exists no closed timelike curve.
4. Each timelike and null geodesic has at least one point at which $R_{abcd}\xi^a\xi^b \neq 0$.
5. At least one of the following three holds:
   (a) (M, g_{ab}) is a closed universe.
   (b) M contains a trapped surface.
   (c) There exists a point p in M for which the convergence of all the null geodesics through p changes sign somewhere to the past of p.

Condition (1) requires the cosmological constant to not be positive, which would make it seem like the theorem cannot be applied to our universe, but the argument Hawking, Penrose, and Bondi (1970, pg. 531) make is that in both a gravitational collapse and a big bang situation, we expect the curvature of the spacetime to be large, and the larger the curvature, the smaller the significance of the value of the cosmological constant. The second condition physically translates to the claim that gravitation is always attractive, while the third ensures that the spacetime is "causally well-behaved". Condition (4), dubbed the "generic condition", requires that every timelike and null geodesic enter at least one region of spacetime where the curvature is "not specially alined [sic] with the geodesic"—this is expected to hold in any generic, physically realistic model (Hawking, Penrose, and Bondi, 1970, pg. 540). Condition (5) captures the generality of the theorem: it applies to a spatially closed universe or a collapsing star (which is generically associated with the existence of a "trapped surface") or "if the apparent solid angle subtended by an object of a given intrinsic size reaches some minimum when the object is at a certain distance from us" (Hawking, Penrose, and Bondi, 1970, pg. 532). The point $\mathfrak{p}$ in (5c) is assumed to be on earth at the present time, and Hawking and Penrose show that there is enough matter on the past lightcone of $\mathfrak{p}$ for the condition to hold for our universe.

These assumptions are reasonable to make of our universe, but not incontrovertible facts as such. It is easy to see that it is logically, and indeed, physically, possible for them to be violated. Gödel universes, for instance, have closed timelike curves. At best, then, what we get from the theorems is some reason to assume that classically, there is a cosmological singularity in our universe. Famously, no version of the theorem tells us anything about the nature of a singularity; in particular, none of them mentions spacetime curvature blowing up. In other words, the theorems indicate geodesic incompleteness, but say nothing about why there is geodesic incompleteness. Nor should they. The theorems are, after all, classical theorems, and if singularity resolution is expected from a quantum theory, then it is expected that there are limits to what any classical treatment can tell us about the nature of singularities. Attempts to provide explanations for how the quantum theory circumvents the problem of the singularities are made within the quantum theory of the universe, i.e. quantum cosmology.

For now, I propose that we heuristically think of singularities as indication of GR breaking down, of GR not being fundamental: no more, no less. They are simply symptoms of there being a tension between classical GR and whatever theory of gravity is more fundamental. (Let's call this theory "quantum gravity", while noting that it is still logically possible for this theory to not really be a quantum theory.) This seems to me to be the most general way to describe a singularity.

Given this proposal, I am in accord with Earman (1995), who suggests that there is a tension between the noun and the adjective forms of "singularity". It makes a lot more sense to talk about singular spacetimes than of singularities. The argument Earman gives for this is that the adjective form avoids the (erroneous) notion of the singularity being *in* spacetime. Favoring the use of the adjective will help us to not think of the big bang and whatever the event horizons of black holes cover as objects (being extended in space and such), which they simply are not.[7] But there is a further reason why I suggest eschewing the noun form. The way I describe a singularity, there is no information about "its" nature. The hope is that whatever theory is more fundamental than GR will explain what caused GR to break down. As such, within the languauge of this fundamental theory, the singularity is not a singularity. When a singularity is resolved, what an observer would see in "its" "place" is a region GR just isn't enough to describe. All this said, I do still use the noun "singularity" where the noun form is called for, simply because it sounds better than "singularness".

1.2 ***Resolving* the Big Bang**

If the big bang is in need of resolution which may be expected to come from quantum gravity, what precisely is it that we are asking of quantum gravity by way of resolution? Surely, it is some kind of an explanation of the classical/quantum divide? Is that what various approaches to quantum cosmology mean by "resolution"? This is the question we shall explore in this dissertation. We do so by cataloging and critically analyzing a variety of proposals put forth in the context of a handful of different approaches to quantum gravity that claim to resolve the big bang. Singularity

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[7] Black holes are usually thought of as cosmological "objects", but a black hole is generally the package consisting of a region of singular spacetime and an event horizon covering that region; the object description applies more to the event horizon itself, I'd argue, rather than the entire black hole.

resolution has not been completely ignored by philosophers before me. For instance, Smith and Weingard (1990) explore how the big bang singularity is "smeared" in a certain cosmological model they study, and Wüthrich (2006) discusses the fate of the big bang in loop quantum gravity. Absent is a comparison of such proposals across various models to establish what *resolution* entails, and such a comparison is what this project aims to accomplish.

Chapter 2 looks at arguably the simplest quantum cosmological model, one that falls under the approach to quantum gravity called canonical quantum gravity. In Chapter 3, we look at a model within the pre-big bang program of string cosmology. Chapter 4 studies loop quantum cosmology. And in Chapter 5, we look at a few more proposals for quantum singularity resolution. My claim throughout will be that singularities are singularities because we try to derive classical conclusions from quantum "events", that the singularity is in the incompatibility between the classical and the quantum theories, not in the infinite curvature or abrupt end to evolution, as is often assumed without argument. The thing to do with singularities then is not to be alarmed, or try to *remove* them or *alleviate* them or use a number of other verbs that have been used to say what we ought to do with them. What we want is for singularities to be *explained*. Providing such an explanation for where the classical and the quantum disagree is what I refer to as *resolution*. It will turn out that the canonical quantum cosmology model in Chapter 2 only *avoids* and does not *resolve* the big bang, and for more complicated reasons, the same may be said of the string and loop models in Chapters 3 and 4. In Chapter 5, I show that prescribing the initial conditions of the universe or showing that a given model violates some of the assumptions of the classical singularity theorems do not do a whole lot for *resolving* the big bang either. The final chapter also addresses the issue: what, if anything, does the answer to the question of *what resolution is* teach us about the nature of quantum gravity?

1.3 The Rules of the Game

The following are assumed (not necessarily independently of each other) in this dissertation.

- The big bang model is the correct classical cosmological model.
- GR is the classical approximation to a more fundamental theory.
- This theory is a quantum theory of gravity.
- Spacetime singularities ought to be resolved.
- This resolution is expected to come from the quantum theory of gravity.
- When we study the physics of our universe, it is kosher to pretend that there exists, classically, a “cosmic time”.[8]

Of course, all of these could be challenged. Take the one about cosmic time, for instance. First of all, it is a direct consequence of the first assumption about the big bang theory being correct, since the theory assumes the existence of cosmic time. Cosmic time is the time measured by a clock that is at rest with respect to the expanding universe; it can be thought of as representing an “objective” flow of time in the background of a relativistic universe. There is only the small problem that it very likely does not exist. The argument against its existence that I am most familiar with is from (Gödel, 1949) via (Yourgrau, 1999). Gödel’s argument is a modal argument that essentially goes as follows: if time in the “intuitive” sense exists, then necessarily, there is objective universal becoming, which is what cosmic time would measure. Provably, there can be no cosmic time in Gödel universes. Both Gödel universes and our universe are solutions to the EFEs, and they only differ in the particular ways in which, to quote Gödel, “matter and its motion

[8]We are also assuming that cosmology aims to emcompass all the degrees of freedom of the universe, not just the “large” ones.

are arranged" in each of them. It is not philosophically satisfactory to argue that the objective lapse of time depends solely on the arrangement of matter and its motion, and thus, it can "hardly be considered as satisfactory" to maintain that cosmic time exists in our universe (Gödel, 1949).[9] What is worth remembering for the purposes of this dissertation is that as a consequence of the standard big bang model assuming the existence of a cosmic time, all the classical models we study here include a parameter $\mathtt{t}$ to represent it.

A few general cautionary remarks are in order. I come to this project with zero preference for any one candidate theory of quantum gravity. I neither think of any candidate as "leading the race" to be eventually shown to be the "correct" theory of quantum gravity, nor do I find anything else of particular philosophical interest in any single candidate theory. As someone who is deeply troubled (and, let's face it, to a great degree, also amused) by how the sociology of quantum gravity research has panned out in the past half century, with proponents of individual candidate theories forming separate cults, cults which seem to also have recruited philosophers of physics, I have made every effort to present the various models in this dissertation in a way that reflects my impartiality. Where I discuss matters about which I am biased, however, I have not attemped to hide my biases. There are places where my exasperation with the proponents of specific models becomes apparent, for example. The reader is reminded that in these cases, my attitude is prompted by my being forced to read (or listen to in person, in some particularly unfortunate cases) overweening accounts of these models than by any prejudice I have against the models or approaches themselves. A related warning pertains to my style of writing, which, I have realized, is not at all in accord with the kind of anxious writing one finds in at least some of present-day philosophy of physics, in which the authors go to great lengths to not antagonize

[9] Many thanks to Palle Yourgrau for helping me understand this argument.

especially the physicists whose physics they analyze, but also other philosophers. Anticipating a reasonable number of substantial philosophical responses is one thing, desperately aiming to please is quite another, and I believe the job description of a philosopher only includes the former. And so, for instance, when I call the cliques of quantum gravity researchers “cults” (or “cliques”, for that matter), I do not feel the need to mollify my targets by adding a footnote about how not all of them belong to cults or with citations of works that have attempted to cut across the boundaries etc.

In a fashion also unfamiliar to most of the literature in this field, there are allusions to literary works and cultural references peppered throughout this otherwise “standard” work in the philosophy of physics, and it may very well be that save for a miniscule number of readers, they will not mean anything. Indeed, they might come across as a nuisance to most of the readers.[10] All I can say to the majority is that I do not think about the kind of philosophy I have taken up here in isolation from all the other things that I think about, and I cannot resist writing about at least some of the external connections I have made with some of the thoughts that feature in this work. To the reader who finds my writing irritating for this and/or other reasons, I can still assure you that the physics in this work is accurate to the best of my knowledge, and the philosophy, earnest.

[10] “Lord, what would they say/ Did their Catullus walk that way?” I wonder, as did W. B. Yeats in “The Scholars”.

CHAPTER 2

THE CASE OF THE VANISHING WAVEFUNCTION[11]

DeWitt (1967) first suggested the idea that the big bang singularity might be "conceivably alleviated" if the wavefunction of the universe vanishes when the scale factor of the universe is zero. This was more recently discussed as a criterion for singularity avoidance by Kiefer (2010; et al. 2019), who claims that satisfaction of this criterion "avoids" the big bang singularity in symmetry-reduced cosmological models. Does a model's satisfying this criterion amount to the alleviation/avoidance of the singularity as claimed? If so, does that allay the concerns about spacetime singularities captured by the broader singularity resolution program in quantum gravity? These are the particular questions I raise in this chapter. I begin by describing the canonical formalism within which the criterion is introduced below, followed by a discussion of the criterion itself in detail. Sections 2.3 and 2.4 discuss two conceptual difficulties with the model, and in the last section, I suggest that the *avoidance* of the singularity, as the application of the vanishing wavefunction criterion is claimed to achieve, ought to be distinguished from the *resolution* that one hopes for from quantum gravity.

2.1 Canonical Quantum Cosmology

As we saw in the introduction, quantum cosmology aims to provide a quantum mechanical description of the universe. There are two equivalent approaches, corresponding to the two waves of interest in quantum cosmology, which aim to achieve such a description. The first one, the "canonical approach", uses the canonical quantization procedure on the Hamiltonian formulation of GR, starting with the Wheeler-DeWitt equation (WDW). The "path integral" approach frames

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[11] A version of this chapter was published as (Warrier, 2022).

the wavefunction of the universe as a path integral and then evaluates the integral in specific models. In either case, to simplify matters, quantum cosmologists study drastically reduced models of the universe, called "minisuperspace models" (MSS models), which are models with all but few physical degrees of freedom truncated. We think of the configuration space of GR (Misner et al., 2017), as the space of all (spatial) 3-geometries (up to diffeomorphisms). These geometries evolve over time in superspace, and by restraining the number of degrees of freedom in this space to a select few (which we do by imposing symmetries), we are restraining the trajectories of these geometries to the minisuperspace sector of the full superspace. The vanishing wavefunction criterion for singularity avoidance I discuss in the next section is based on a two-dimensional MSS model, and so I apply the canonical quantization procedure to a universe with only two degrees of freedom below.

In ordinary QM, systems evolve in time according to the Schrödinger equation,

$$i\hbar\dot{\psi}(x,t) = \hat{H}\psi(x,t), \tag{2.1}$$

where the wavefunction $\psi(x,t)$ represents the state of the system at time t and $\hat{H}$, the Hamiltonian operator, represents the energy of the system. The analogue of the Schrödinger equation in quantum cosmology is called the Wheeler-DeWitt equation,

$$\hat{H}\Psi = 0. \tag{2.2}$$

WDW is also a differential equation like the Schrödinger equation, but unlike $\psi(x,t)$, Ψ is a functional of field configurations. Despite its being a functional, Ψ is often called the "wavefunction of the universe". WDW captures the argument that unlike wavefunctions in ordinary QM, Ψ cannot depend on time, as there is no external clock—there is nothing external to the universe—in quantum cosmology, and if Ψ has no time-dependence, then the total energy of the universe must stay zero "forever". Historically, this has also been a reason to doubt the validity of WDW, as it seems to paint the picture of a universe frozen in time.

Let's take a look at what WDW looks like in a particular model. I follow Kiefer's (2012, pg. 262-6) quantization of a closed, Friedmann-Lemaître-Robertson-Walker (FLRW)—an isotropic, homogeneous, expanding or contracting—universe. The MSS model under consideration has only two degrees of freedom, the scale factor of the universe, represented by '$a(t)$', which is a measure of the size of the universe, and a scalar, spatially constant matter field, represented by '$\phi(t)$'.[12]

The FLRW line element for the closed universe is given by

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega^2, \tag{2.3}$$

where $N(t)$ is the "lapse function", and $d\Omega^2 = d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$.[13] This, when plugged into the Einstein-Hilbert action for quantum gravity, gives the minisuperspace action

$$S = \int dt N(t)\left(\frac{1}{2}G_{AB}\frac{\dot{q}^A\dot{q}^B}{N(t)^2} - V(q)\right) \equiv \int dt L(q, \dot{q}). \tag{2.4}$$

Here, $L(q, \dot{q})$ is the minisuperspace Lagrangian and $q^A := a$ and $q^B := \phi$. Denoting the cosmological constant by 'Λ',

$$V(q) \doteq V(a, \phi) = \frac{1}{2}\left(-a + \frac{\Lambda a^3}{3} + m^2a^3\phi^2\right).^{14} \tag{2.5}$$

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[12] t here is the classical cosmic time parameter; it does not exist in the quantum theory.

[13] The standard line element in spherical coordinates is $ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$; $d\Omega^2$ is the line element on the surface of a 3-sphere with $r = 1$, representing a closed universe.

[14] I have not provided the explicit forms of the gravitational and the matter counterparts of the MSS action here, but mass 'm' comes from the matter action, and 'Λ' from the gravitational part. See (Kiefer, 2012, pg. 263-4) for details.

One can also read off G_{AB}, the "minisuperspace DeWitt metric" from the MSS gravitational and matter actions as

$$G_{AB} = \begin{pmatrix} -a & 0 \\ 0 & a^3 \end{pmatrix}. \tag{2.6}$$

Now, we calculate the canonical momenta:

$$p_N = \frac{\partial L}{\partial \dot{N}} \approx 0, \tag{2.7}$$

$$p_a = \frac{\partial L}{\partial \dot{a}} = -\frac{a\dot{a}}{N}, \tag{2.8}$$

and

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{a^3 \dot{\phi}}{N}. \tag{2.9}$$

From these, we can get the Hamiltonian as follows.

$$\begin{aligned} H &= p_N \dot{N} + p_a \dot{a} + p_\phi \dot{\phi} - L \\ &= \frac{N}{2} . \left(-\frac{p_a^2}{a} + \frac{p_\phi^2}{a^3} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2 \right) \\ &\equiv N . \left(\frac{1}{2} G^{AB} p_A p_B + V(q) \right), \end{aligned} \tag{2.10}$$

where G^{AB} is the inverse of G_{AB}. Since the primary constraint, $p_N \approx 0$, has to hold at all times, $\{p_N, H\} \approx 0$, which leads us to $H \approx 0$, which we impose as a constraint on the wavefunction.

The final step of the canonical quantization procedure is to pick an operator ordering. Kiefer uses the Laplace-Beltrami ordering, by which $G^{AB} p_A p_B$ becomes $\hbar^2 \nabla^2_{\mathrm{LB}}$, which, when written out, is $-\frac{\hbar^2}{\sqrt{-G}} \partial_A \left(\sqrt{-G} G^{AB} \partial_B \right)$. Simplifying this, one gets

$$\hbar^2\nabla^2_{\mathrm{LB}} = \frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2}. \tag{2.11}$$

(∇^2_{LB} is the covariant version of the Laplacian.)

Now, Ψ in (2.2) reduces to a function of a and ϕ in this MSS model. It is effectively a functional since it ranges over all fields; however, it is mathematically a function since the constant field is represented by one variable, ϕ. So our WDW takes the following form:

$$\hat{H}\psi(a,\phi) = 0. \tag{2.12}$$

Plugging (2.11) and (2.5) into (2.10) gives us the MSS Hamiltonian operator, which, when plugged into the WDW (2.12), gives us

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0. \tag{2.13}$$

A special case is when $\Lambda = m = 0$. The solutions to the WDW in that case with the boundary condition that ψ goes to zero as a does are given by

$$\psi_k(\alpha,\phi) = e^{-ik\phi}K_{ik/2}\frac{e^{2\alpha}}{2}, \tag{2.14}$$

where $\alpha \doteq \ln a$ and $K_{ik/2}$ is a Bessel function.

The same boundary condition appears as a singularity avoidance criterion in the following section, even though $\Lambda \neq 0$ and $m \neq 0$. The WDW in that model is similar to (2.13) for the closed FLRW universe, except that $V(q)$ takes a different form. We then look at the solutions to that WDW, and see what they can tell us, if anything, about singularity avoidance.

2.2 The Vanishing Wavefunction Criterion

The vanishing wavefunction condition was introduced by DeWitt (1967) and recently revived by Kiefer (2010; et al. 2019).[15] That the classical singularity theorems do not say what the precise physical nature of a (cosmological) singularity is, beyond that the causal structure of spacetime as we know it breaks down "there" leads DeWitt (1967) to think of a singularity as a "barrier" beyond which one cannot extend the solutions of the field equations of GR. He then asks if the barrier would also prevent the solutions of WDW from being extended. It is in this context that he introduces the criterion, by claiming that if the wave functional Ψ vanishes at the barrier, then the quantum physicist would have "alleviated" his/her barrier, and there would be no problem in extending the solutions of WDW beyond the singularity (DeWitt, 1967, pg. 1129).

Kiefer (2010) works out a concrete example in which one can see the criterion in action. He aims to show that the MSS model avoids big brake singularities, i.e., singularities where the expansion of the universe comes to an abrupt stop as a result of the pressure of the fluid that models the matter field of the universe diverging, if it satisfies the criterion.[16] However, satisfying the criterion also makes the model avoid the big bang singularity. Consider a two-dimensional FLRW MSS with a scale factor a and scalar field ϕ. Near the singularity, ϕ is small, and hence the WDW is approximately

$$\frac{\hbar^2}{2}\left(\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\phi^2}\right)\psi(\alpha,\phi)-\frac{\tilde{V}_0}{|\phi|}e^{6\alpha}\psi(\alpha,\phi)=0, \tag{2.15}$$

[15] DeWitt himself does not propose that the vanishing wavefunction condition ought to count as a *criterion* for singularity avoidance—what he does claim is that "*[p]rovided it does not turn out to be ultimately inconsistent*, [...] it makes the probability amplitude for catastrophic 3-geometries vanish" (1967, pg. 1129).

[16] Typically, in this case, the matter content is represented by an "Anti-Chaplygin gas" with the equation of state $p = A/\rho$, $A > 0$ where p the pressure of the gas, and ρ is its energy density.

where $\kappa^2 = 8\pi G$, $\alpha = \ln a$ and $\tilde{V}_0 = \sqrt{(\rho p)/4}$ where ρ is the energy density and p the pressure of the Anti-Chaplygin gas.

The solutions of this WDW are

$$\psi(\alpha, \phi) = \sum_{k=1}^{\infty} A(k) k^{-3/2} K_0 \left(\frac{1}{\sqrt{6}} \frac{V_\alpha}{\hbar^2 k \kappa} \right) \times \left(2 \frac{V_\alpha}{k} |\phi| \right) e^{-\frac{V_\alpha}{k|\phi|}} L^1_{k-1} \left(2 \frac{V_\alpha}{k} |\phi| \right), \tag{2.16}$$

where K_0 is a Bessel function, L^1_{k-1} are Laguerre polynomials, and $V_\alpha \equiv \tilde{V}_0 e^{6\alpha}$. As can be seen above, as $\alpha \to -\infty, \psi \to 0$, since the (linear) V_α quickly goes to zero as $\alpha \to -\infty$ (Kiefer, 2010, pg. 216). This model thus satisfies the vanishing wavefunction criterion. But what does that show about the fate of the singularities in this model?

Heuristically, the argument for the vanishing wavefunction criterion being a criterion for singularity avoidance is that it makes the probability amplitude of singular three spaces zero at the quantum level. However, there is no consensus as to what the standard probability interpretation of QM amounts to in canonical quantum gravity, as we shall see below. Without a satisfactory probability interpretation, the heuristic argument fails. The only way to argue for the criterion is to say that from the mathematics, we can see that ψ goes to zero when a does, and this is what we mean by avoiding the singularity. This is exactly the move that Kiefer makes when he writes, "we shall interpret the vanishing of the wave function in the region of the classical singularity as singularity avoidance" (Kiefer, 2010, pg. 215). But of course, this claim then begs the question.

Even if we accept, for whatever reason, or none, that the wavefunction vanishing counts as singularity avoidance, we could ask if avoiding the two-dimensional MSS singularity says anything about whether the criterion would hold for more realistic, higher-dimensional models. The original vanishing wavefunction criterion turns out to not work even when the MSS model has more than two dimensions. Kiefer et al. (2019) argue that because of the constraint nature of the Hamiltonian, minisuperspace possesses a conformal structure, and since the vanishing wavefunction criterion is not conformally invariant, it can only be good for two-dimensional MSS models. Hence they (see also (Kiefer and Kwidzinski, 2019)) develop a generalized vanishing wavefunction criterion that holds for higher dimensional, anisotropic cosmologies. The generalized criterion states, "A singularity is said to be avoided if $\star|\Psi|^{\frac{2d}{d-2}} = |\Psi|^{\frac{2d}{d-2}}\text{dvol} \to 0$ in the vicinity of the singularity". Here, d denotes the dimension of the MSS, $\star$ is the Hodge star, and dvol "contains" the square root of the (absolute value of the) determinant of the DeWitt metric (Kiefer et al., 2019, pg. 4).

Two conceptual issues present themselves in the above discussion: (1) Even the general vanishing wavefunction criterion only works for finite-dimensional MSS models, and this is not enough reason to expect an infinite generalization of the criterion to be well-defined for "full" universes with an infinite number of degrees of freedom. Indeed, as we shall see, MSS models are, at best, only approximations of the full universe, and unless the criterion is independent of the number of degrees of freedom of the universe or the approximate version of the criterion is identical to the full version, it is not clear if one can realistically consider it a criterion for singularity avoidance. (2) As mentioned above, the probability interpretation of the wavefunction is even less straightforward in canonical quantum gravity than it is in ordinary QM. And without a satisfactory interpretation, the heuristic argument for the vanishing wavefunction criterion has no bite, and it is unclear what the criterion would even mean physically. These issues form the contents of the following sections.

2.3 The Spherical Cow Issue

Since all of quantum cosmology is done using MSS models, and in particular, the vanishing wavefunction criterion, both in its original as well as generalized forms, is meant to hold only in MSS models, the first philosophical issue that one ought to address is how one would justify the use of these drastically reduced models. Even though this question has not received much attention in philosophy of physics, it is one that quantum cosmologists have acknowledged, and attempted to answer. I focus here on two papers written by Karel Kuchař and Michael Ryan.

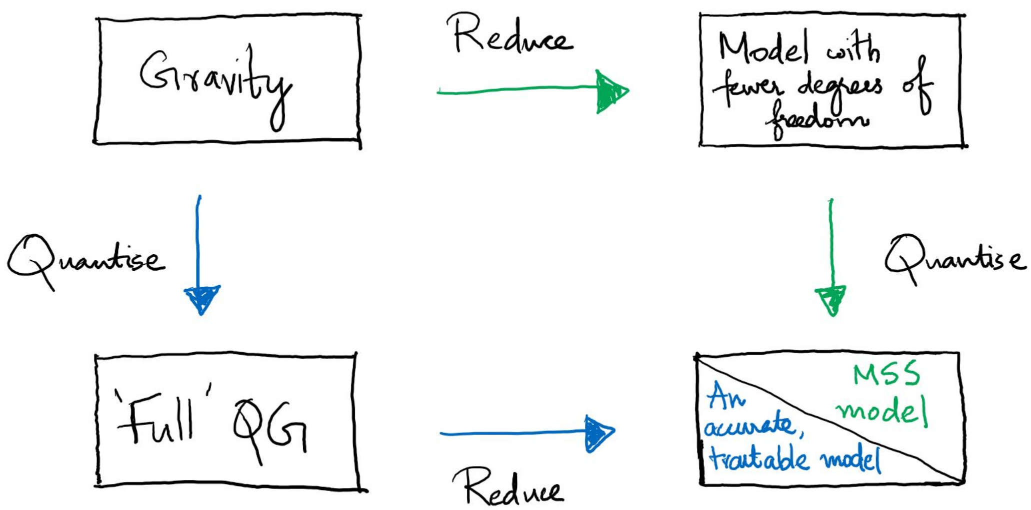

Figure 1: Creating a tractable quantum gravititational model

In their 1986 paper, Kuchař and Ryan give a straightforward answer to whether "quantizing a system restricted classically to a sector of the available phase space" is the same as "quantizing the system on the full phase space and then following the statistics of a restricted class of canonical variables": it is not (Kuchař and Ryan, 1986, pg. 453). The problem is visualized in Figure Figure 1. What we want is a correct, but tractable model which we can use to make experimental predictions. Ideally, we would follow the blue path, quantizing gravity first and then reducing the number of degrees of freedom involved. However, there is no straightforward way to quantize the gravitational field without its being plagued by insurmountable technical challenges, and the only way we have to get the model we want is to follow the green path, i.e. to reduce the universe to a simplistic model, and then quantize it. But reducing the phase space and then quantizing does not give us the same results as models that are reduced versions of some "full" quantum gravity. In other words, the quantization and the reduction do not commute. This is so because the MSS approximation violates Heisenberg's uncertainty principle (HUP) (Kuchař and Ryan, 1986, pg. 453). The canonically conjugate variables in quantum field theory are the field and momentum operators. Assuming that fields can be represented by expansions of modes, what one does when one creates an MSS model is to artificially set the (spatially) inhomogeneous field modes and their associated momenta to zero. However, this violates HUP, as this reduction would mean that on quantization, the inhomogeneous modes and their momenta (technically, the corresponding operators) commute, which in fact, they do not. Any quantum model that violates HUP is not an accurate quantum model, and hence the result we get by quantising an MSS model is definitively not the “real deal” (which is what we would expect to get if we follow the blue path).

One could ask if the MSS model is an *approximation* to the full quantum theory, and this is the question Kuchař and Ryan set out to answer. In their 1989 paper, they propose to study a hierarchy of exactly soluble models (which are themselves MSS models), in which are embedded models of higher symmetries (“microsuperspaces”), to figure out what it would take for the microsuperspaces to count as approximations to the respective minisuperspaces. In the paper, they consider the embedding of the Taub model in the mixmaster universe (Bianchi IX). Bianchi IX models are similar to the FLRW models in that they are homogeneous, but in addition to expanding and contracting, they can also change shape anisotropically. The Taub models are a special case of the Bianchi IX models. Kuchař and Ryan (1989, pg. 3983-84) distinguish among three different senses of “approximation”:

1. When one ignores all but the variables that can be measured in the minisuperspace, the superspace wavefunction becomes a density operator that can be decomposed into an incoherent combination of pure minisuperspace states. Each term in the decomposition evolves according to an “autonomous” Schrödinger equation that is insensitive to the other terms. If the minisuperspace Schrödinger equation is similar to the “autonomous” Schrödinger equation barring small corrections, then the MSS model would count as an approximation.

2. Near a cosmological singularity, almost all the energy of the gravitational field flows into the homogeneous modes of the field, i.e. the modes that are not frozen out in MSS models. (This sense of approximation is a conjecture originally made by Misner (1969a).) The MSS model would then count as an approximation to the full model at least near the singularity.

3. The "important parts" of the minisuperspace qualitatively behave in the same way as the corresponding parts in the superspace, just like how a one-dimensional slice of a three-dimensional harmonic oscillator approximates the complete harmonic oscillator.

They conclude that the Taub model does not approximate the mixmaster universe in any of the three senses of "approximation", and that perhaps the only reason to think that the former approximates the latter would be that the minisuperspace "stays near" the microsuperspace for some time (Kuchař and Ryan, 1989, pg. 3994). It turns out that there is a finite time interval during which the solution to the microsuperspace WDW predicts the behaviour of the minisuperspace. However, the existence and the duration of this interval seems to be model-dependent. And hence Kuchař and Ryan further claim that there needs to be some criterion for when a model with more symmetry counts as an approximation to a less symmetric model, but as far as I can tell, no such criterion has been proposed to date.

It could be that by their very nature, MSS models cannot be approximations in general. Bojowald et al. (2012), for instance, write that "quantum cosmology is a truncation rather than an approximation, drastically cutting off unwanted degrees of freedom instead of providing a harmonious embedding of a simplified model within a fuller framework". Given the infinite degrees of freedom one would otherwise have to deal with, it is reasonable to study these reduced models. In fact, Kiefer (2012, pg. 260) writes that MSS models can be treated as toy models to study a variety of issues that do not depend on the number of degrees of freedom. But the question we ought to ask is whether solving the singularity issue in a toy model is the same as solving the singularity issue. For the answer to this question to be in the affirmative, the singularity avoidance needs to be independent of the number of degrees of freedom involved in the model. Let's take a look at the generalized vanishing wavefunction criterion again. It states that "a singularity is said to be avoided if $\star|\Psi|^{\frac{2d}{d-2}} = |\Psi|^{\frac{2d}{d-2}}\mathrm{dvol} \to 0$ in the vicinity of the singularity", where d is the dimension of the MSS, $\star$ is the Hodge star, and dvol "contains" the square root of the (absolute value of the) determinant of the DeWitt metric (Kiefer et al., 2019, pg. 4). Obviously, the quantity $\star|\Psi|^{\frac{2d}{d-2}}$ is not independent of the number of dimensions; in fact it is not even well-defined for an infinite-dimensional model.

Where does that leave us? So far, we know that applying the vanishing wavefunction criterion makes (2.13) well-defined when $a \to 0$. All we can say from this is that the evolution of a drastically unrealistic toy model is not troubled by quantities "blowing up". Because the criterion does not allude to any physical characteristics of the singularity itself (other than that it is "located around" $a \to 0$), there is no justification for expecting that application of the same criterion will accomplish anything by way of resolving the real big bang singularity in our universe. Note that this is not to say that MSS models are completely useless. Again, they could be very useful in studying issues that are independent of the number of degrees of freedom involved (Kiefer (2012, pg. 260) mentions the examples of the problem of time, the role of observers, and the emergence of a classical world), but singularity resolution, or at least, singularity avoidance with the aid of the vanishing wavefunction criterion, does not appear to be one such issue.

2.4 Unfreezing Time

Another issue with the vanishing wavefunction criterion that both DeWitt (1967) and Kiefer (2012) point out is the lack of a satisfactory probability interpretation of $\psi(a, \phi)$. In ordinary QM, if $\psi(x, t)$ is the state of a system, then $\int_x^{x+dx} \psi(x, t)^2 dx$ is the probability that the system will be found between x and $x + dx$ at any given time t. We expect the probability $\int_{-\infty}^{\infty} \psi(x, t)^2 dx$ to be conserved over time. However, such an expectation is problematic in quantum cosmology. The state of our system in this case is given by $\psi(a, \phi)$, which is independent of the classical time variable t. It seems as if the notion of a probability conserved over time makes no sense, because there is no time over which the probability could be conserved. So we ask: given $\psi(a, \phi)$ is independent of t, could the quantity $\psi(a, \phi)^2$ be related to some kind of probability? If so, how? We seem to have several options:[17]

(i) Define a physically meaningful, "intrinsic" time measured by a quantity such as a or ϕ, and let $\psi(a, \phi)^2$ encode the probability that the universe has a field of strength ϕ (or respectively, a size a) when it has a size a (field strength ϕ).

(ii) Let $\psi(a, \phi)^2$ be related to the probability that an "arbitrarily chosen" 3-geometry from the superspace has a scale factor a and field strength ϕ.

(iii) Use a probability interpretation that is neither of the above.

(iv) Give up on extending the probability interpretation of ordinary QM to quantum cosmology.

The problem of the probability interpretation concerns all of quantum cosmology done using WDW. But it is particularly acute for singularity avoidance using the vanishing wavefunction criterion, because, as we have seen, the criterion says that the big bang singularity is avoided if, for $a \to 0$, $\psi \to 0$. When $\psi = 0$, $\psi^2 = 0$. And the heuristic argument for the criterion is that satisfying it makes the probability of singular 3-spaces zero. But without a proper notion of probability, i.e. without a story about how ψ^2 is a probability, this argument does not hold. So when we look at

[17] This list is not exhaustive. I have not mentioned interpretations of QM, but it is worth noting that the Copenhagen interpretation (or similar collapse interpretations) would not work with canonical quantum cosmology. This is because collapses require external observers to make measurements, and in quantum cosmology, there are no observers external to the universe. The alternative interpretations offer their own interpretations of probability. See Kuchař (2011) for a more extensive list of (ultimately unsuccessful) attempts to unfreeze time.

the ways people have suggested to work around the general problem below, we will also keep an eye on the specific problem concerning the vanishing wavefunction criterion.

DeWitt (1967) chooses (i). Here, I follow Vilenkin's (1986) description of DeWitt's argument. Suppose a is the intrinsic time. Then the probability density, $\rho(a)d\phi$ is taken to be the probability that the field is between ϕ and $\phi + d\phi$ when the scale factor is a. To impose normalizability, i.e. to ensure that $\int \rho(a)d\phi = 1$, $\rho(a)$ is identified with the conserved current in the superspace, j. The argument for this identification goes as follows: Consider the continuity equation $\partial_\mu j^\mu = 0$, i.e. $\partial_a j^a + \partial_\phi j^\phi = 0$. The components of the current that would satisfy this equation (as can be shown using the WDW[18]) are

$$j^a = ia^p(\psi^* \partial_a \psi - \psi \partial_a \psi^*) \tag{2.17}$$

$$j^\phi = -ia^{p-2}(\psi^* \partial_\phi \psi - \psi \partial_\phi \psi^*) \tag{2.18}$$

Now, if we integrate the continuity equation over all ϕ, assuming ϕ vanishes at the end points, we get that $\partial_a \int j^a d\phi = 0$. This is consistent with $\int j^a d\phi$ being 1, and so identifying the probability density with the conserved current component would satisfy normalizability (Vilenkin, 1986, pg. 3565). In terms of the vanishing wavefunction criterion, as can be seen from 2.17 and 2.18, if a vanishes, so would j, and hence, ψ.

The problem with this approach that DeWitt (1967) points out (which, as he notes, also affects the interpretability of the criterion for singularity avoidance) is that j^a is not necessarily positive-definite, and hence we could, in theory, end up with negative probability densities. He

[18] Vilenkin uses a model in which there are interacting gravitational and scalar fields, and $p \sim 1$ is a factor-ordering parameter.

attempts to solve the problem in the special case of a Friedmann universe. Vilenkin also takes up the problem, and argues that we should not require that our definition of probability be necessarily positive-definite for an arbitrary wavefunction. This is so, he argues, because ψ, the wavefunction of the universe, is the unique result of imposing a set of boundary conditions on the WDW. It is possible for this wavefunction to have only positive- or negative-frequency components with respect to some time variable, and it is not necessary for the same variable to be the time variable across superspace. Hence we could always find a set of "good" time variables in overlapping regions of superspace such that there would be no negative probabilities. He also formulates the conditions for what makes a time variable good: it should be semiclassical, and monotonic (Vilenkin, 1986, pg. 3567).

The obvious problem here is that the semiclassical treatment is not good enough for the sub-Planckian regime with which we are concerned in our study of singularity resolution. But even if one were only looking at the non-singular parts of superspace, this way of interpreting probabilities appears to be more trouble than it's worth. Kuchař (2011) writes, "the semiclassical interpretation does not solve the standard problems of time. It merely obscures them by the approximation procedure and, on the way, creates more problems."

Hawking and Page (1986) cite the problem of the current not being positive-definite to reject (i) in favour of (ii). They define ψ^2 as being proportional to the probability of finding a 3-geometry in the superspace with scale factor a and field configuration ϕ. One objection Kuchař (2011) raises against this choice is that there is no dynamics in this interpretation. It can give you a probability for a particular universe, but it cannot answer what the probabilities of particular evolutions of the same universe are. This, however, might not be a problem if we are only concerned about singularity avoidance. If, by $\psi(a = 0, \phi)^2 = 0$ we mean that the probability of there being a universe with $a = 0$ is zero, then one could argue that the big bang singularity is avoided.

Surveying all the possibilities under (iii) that have so far been explored by physicists is out of the scope of this dissertation, but Kuchař (2011) studies many of these, and deems them all ultimately unsuccessful. But even if one could find a satisfactory probability interpretion, it is unclear that the heuristic argument for singularity avoidance using the vanishing wavefunction criterion necessarily holds. As Blyth and Isham (1975, pg. 774) note, α has a continuous spectrum, and so $\psi(0, \phi)$ being 0 does not necessarily signal the "absence" of the singularity, since it is not clear where precisely the line is between singular and non-singular behavior as α approaches 0. The singularity is generally just assumed to be localized to the point $\alpha = 0$, and this assumption needs to be justified before one can claim that the wavefunction vanishing at the point $\alpha = 0$ avoids the singularity.

For (iv) to be the right choice, one would have to provide an argument for why the universe is a special system. That there are no external observers in quantum cosmology could be such an argument. But then, again, the heuristic argument for the vanishing wavefunction criterion being a sufficient criterion for singularity avoidance would not hold. However, this may not be a big problem, if all we are looking for is a criterion that helps to only formally avoid the singularity. The question of whether that is all one wants from singularity resolution is taken up in the next section.

2.5 Is Avoidance the Same as Resolution?

Historically, several terms have been used to describe the process of doing away with singularities, from DeWitt (1967)'s talk of "alleviating" singularities and Kiefer (2010)'s singularity "avoidance" to Wüthrich (2006)'s talk of singularities being "spirited away". Such a wide variety of terms being used is itself indicative of a lack of clarity in thinking about singularity resolution.

Satisfying the vanishing wavefunction criterion lets the relevant MSS models *avoid* the big bang singularity. The criterion provides no explanation as to what happens at the singularity, or shed any light on its nature. Avoidance criteria thus help the models avoid singularities in the same way one would avoid a pothole: you acknowledge there is a pothole and that it is dangerous to step into it, so you sidestep it. This notion of avoidance can be further split into two senses. The way Kiefer sets the criterion up, one assumes that there is a satisfactory probability interpretation, and then claims that satisfying the criterion results in zero chance of the universe actually stepping into a pothole. But there is another sense in which satisfying the criterion could be said to avoid the singularity, a formal sense in which (2.13) is not well-defined unless the model satisfies the criterion, i.e. ψ goes to zero as α does. For singularity avoidance in this sense, we do not need a probability interpretation to say that the vanishing wavefunction makes the quantum theory safe from singularities. Of course, then we would not know how to interpret ψ as a physical state of the universe.

I would argue that a true *resolution* of the singularity calls for a physically meaningful story about what the quantum theory of gravity did to the classical singularity other than merely make the universe go around it. In other words, a quantum gravitational account of singularity resolution would show that the singularity exists only because we try to derive classical conclusions about quantum phenomena, that there is a quantum gravitational explanation of what really goes on "at" the singularity. A historical example of such a resolution would be how the "singularity" caused by the electron spiraling into the nucleus in the classical model of the atom was resolved by the quantum theory. What made it count as resolution, I would argue, was that the new theory gave an explanation, in a language unfamiliar within the framework of the old theory, for what the electron really does that makes the classical prediction wrong. If all the quantum theory had done was to make the evolution of the electron formally free from infinities, or predicted, without providing a reason why, that despite there being a possibility of it happening, the probability of the electron actually spiraling into the nucleus is zero, then the quantum theory would only have avoided the singularity. To clarify, in the case of the electron, there never was a spacetime singularity. All there was was the classical prediction that the electron would collapse into the nucleus. We already knew that the atom was stable, and hence, that the classical prediction must be wrong. We have no such certainties to explain about the big bang using quantum gravity. That the geodesic of the observer ought not end abruptly "at" the big bang is not a fact of our world, as far as we know. The electron analogy is thus far from being perfect for discussing spacetime singularity resolution, but it is useful to a small extent to talk about what happens when we expect a quantum theory to replace a classical theory. The explanation of the evolution of the electron in terms of orbitals etc., which the classical theory could never have foreseen, is what makes the quantum theory of the electron *resolve* the puzzle of the collapsing electron.

For quantum gravitational *resolution* of the big bang then, given that a similar language disparity between the classical and the quantum is at play, what one will want to look for is a quantum explanation for what it is that really "happens" "at" the big bang that is simply beyond the reaches of GR to describe. Any attempt that falls short of such an explanation, but still formally makes sure nothing goes awry "at" the singularity, would only count as an attempt to *avoid* the singularity. These could be attempts to show that the universe "evolves" to classically forbidden regions, or that there are no infinities in the formalism, but they could also be other kinds of formal patchery; I use *avoidance* to refer to all such attempts, even though these two specific kinds are what one usually encounters in quantum cosmology.[19] More elaborate approaches to quantum gravity do claim to provide, beyond a description of formal avoidance, an explanation for what replaces the classical singularity in the quantum theory. There is a lot to be said about the extent to which these approaches are successful, and this is what the following chapters take up.

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[19]In this sense, there is also *avoidance* in the electron's story. In the quantum theory, since the electron does not collapse, there is neither an abrupt end to its evolution, nor blowing up of any physical quantity as a result of the collapse. But if that were all the quantum theory told us, without a positive descripton of what the electron *does* do, there would have been no *resolution*.

My point in this chapter has not been that the vanishing wavefunction criterion is useless unless it is a resolution criterion as opposed to merely an avoidance criterion. Kiefer (2010) writes that the vanishing wavefunction criterion is a sufficient, but not necessary criterion for singularity avoidance, and all I am doing here is to question this claim. Given the MSS and probability interpretation problems, I am inclined to rule that the vanishing wavefunction criterion cannot even count as being sufficient, unless what all one is looking to do is to formally avoid the singularity at $a = 0$ in a toy model. That said, the investigation of the vanishing wavefunction criterion has highlighted some potential conceptual pitfalls we ought to be wary of in our search for a quantum gravitational singularity resolution. So even if the case of the vanishing wavefunction is now closed, it paves the way for attempts to deal with spacetime singularities that amount to more than mere formal avoidance.

CHAPTER 3

SEEKING COSMIC GRACE: BIG BANG RESOLUTION IN STRING COSMOLOGY

If one could show that our universe's present evolution is a result of a transition through the big bang singularity from a "pre-big bang" phase of evolution, would one have resolved the singularity? This is the question this chapter aims to address, using the string theoretic framework of the "pre-big bang cosmology" program. The answer to this question, it turns out, depends on whether there is a solution to what has been called the "graceful exit problem". The graceful exit problem—the problem by which the universe cannot stop inflating, or rather, cannot gracefully exit its inflationary phase—first plagued the original inflation model of the universe (Guth, 1981). In the pre-big bang program too, there is no solution for how the universe is supposed to end its inflationary phase and transition through the big bang. However, the way the program is presented often suggests that its proponents think the solution to the graceful exit problem is the last piece of the puzzle before we can have a string theoretic resolution of the big bang singularity. Whether that is the case is what we shall investigate in this chapter.

In the pre-big bang model, first proposed by Gasperini and Veneziano (1993), the classical description of the evolution of the background spacetime is that of an accelerating (inflationary) universe with a growing curvature "preceding" (with respect to the cosmic time, $\mathtt{t}$) the current phase of the universe, which is distinguished by its decelarating expansion with a decreasing curvature. These "pre-" and "post-" big bang phases[20] are assumed to be related to one another

[20] I use "phase", "branch" and "epoch" interchangeably to refer to these separate solutions on the half line in $\mathtt{t}$; none of these terms are to be confused to have other meanings that they are sometimes ascribed.

via a particularly stringy property of the effective string action, the “scale factor duality” (SFD). The classical description, however, does not apply to the phase around the big bang when the background has to exit the inflationary phase. The issue of explaining how this exit, or equivalently, the transition (again, with respect to t) of the universe from the pre- to the post-big bang phase, happens, has come to be known as the graceful exit problem. Whether the big bang singularity is resolved in string cosmology, thus, depends on whether there is a satisfactory solution to the graceful exit problem.

Regrettably, several formal no-go theorems show that there could be no classical solution to the graceful exit problem (see, for eg., (Kaloper et al., 1995)). But the proponents of the pre-big bang model are hopeful that quantum theory will come to the rescue. In particular, in quantum string cosmology, which is what one gets when one applies the same procedure as in canonical quantum gravity to the effective string action, it is claimed that the probability of the universe transitioning from one to the other phase is not zero. Quantum string cosmology was first discussed by Bento and Bertolami (1995), and further investigated by Gasperini et al. (1996). I draw most of the formal content for this chapter from (Gasperini, 2007).

The next section introduces the classical set up of the pre-big bang scenario that argues for the existence of separate classical branches of universal evolution separated by the big bang, and the section that follows discusses the quantum picture in which transition between these two branches is shown to be not impossible (along with a brief aside on cyclicity). The rest of the chapter asks if the big bang can be said to be resolved in this picture, and in doing so, raises a few different conceptual issues with the pre-big bang program. In particular, the two conceptual concerns from the previous chapter make a brief comeback in Section 3.4, Section 3.5 discusses whether the principle of self-duality, which is central to the program, is an ad hoc principle, Section 3.6 takes up the issue of interpreting the pre-big bang evolution of the universe through the big bang as tunneling, and the final section raises the most significant objection against big bang resolution in pre-big bang cosmology, that the prime ingredient of the principle of self-duality, scale factor duality, cannot be used to show that the universe evolves between branches pre- and post-big bang.

3.1 The Classical Picture

Consider a flat, homogeneous, anisotropic, $(d+1)$-dimensional, vacuum universe with no antisymmetric tensor potential or dilaton potential. Let the scale factor of the universe be denoted by a (with the a_i's denoting the different scale factors along the i directions of an anisotropic universe), the Hubble parameter by H, and the dilaton field by ϕ. In terms of the "shifted dilaton" variable $\bar{\phi}$,

$$\bar{\phi} = \phi - \ln(\Pi_i a_i) = \phi - \sum_i \ln a_i, \tag{3.1}$$

the equations of motion derived from the effective string action,

$$S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left[R + (\nabla\phi)^2 + V(\phi, g) \right], \tag{3.2}$$

are

$$\begin{aligned} \dot{\bar{\phi}}^2 - \sum_i H_i^2 &= 0 \\ \dot{H}_i - H_i \dot{\bar{\phi}} &= 0 \\ 2\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 - \sum_i H_i^2 &= 0. \end{aligned} \tag{3.3}$$

These equations are invariant under the time-reversal transformation, $t \to -t$, which holds for the EFEs as well. But the stringiness introduces another invariance, under what are called "scale factor duality" (SFD) transformations: $a_i \to \widetilde{a}_i = a_i^{-1}$ (at fixed t) and $\bar{\phi} \to \bar{\phi}$, for which

$$H \rightarrow \widetilde{H} = a\left(\frac{da^{-1}}{dt}\right) = -H. \tag{3.4}$$

By inverting all the scale factors, one gets (Gasperini, 2007, pg. 137)

$$\{a_i, \phi\} \rightarrow \left\{a_i^{-1}, \phi - 2\sum_{i=1}^{d} \ln a_i\right\}. \tag{3.5}$$

With both the symmetries, that of time reversal, and of the scale factor duality, for any given solution $a_i(t)$ of the equations of motion derived from the string action, one could consider three additional (what Gasperini claims to be "in principle, physically distinct" (Gasperini, 2007, pg. 137)) phases:

$$a_i(t), \quad a_i(-t), \quad a_i^{-1}(t), \quad a_i^{-1}(-t). \tag{3.6}$$

These solutions are all defined on the half-line in t, with the big bang conventionally chosen to be "at" $t = 0$. The crux of the argument in favor of the pre-big bang model lies in showing that one of the solutions in $-t$ is related to one of the solutions in t in such a way that one can make a coherent, complete story about how the universe evolves from a pre-big bang phase to a post-big bang phase.

The proponents of the model craft this story by introducing the principle of "self-dual" cosmological evolution that is characterized by a solution satisfying $a(t) = a^{-1}(-t)$ (Gasperini, 2007, pg. 139).[21] The full picture of cosmic evolution then has a universe that begins in a low-curvature state (at the lowest t) which then accelerates while its curvature increases until it reaches a maximum, finite scale possibly dictated by the string length; this is when the big bang happens. After the big bang,

[21] i.e. "self-duality"=SFD+time reversal

the curvature decreases, and in this phase, the universe decelerates and expands to match the current observational data. To reiterate, this picture follows from the assumption of the principle of self-duality, i.e. "assuming that the past evolution of our [u]niverse should represent the dual 'complement' of the present one" (Gasperini, 2007, pg. 146-7). The study of the classical model thus establishes the existence of these separate branches. The possibility of actual transition between the branches is illustrated by the quantum theory, as seen in the next section.[22]

3.2 Quantum Gravity and Grace

To quantize this situation, in keeping with the theme of working with simple, not necessarily realistic models, we start again with the gravi-dilaton effective action (i.e, we ignore higher-order corrections of both the curvature and the coupling)

$$S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x\sqrt{|g|}e^{-\phi}\left[R + (\nabla\phi)^2 + V(\phi, g)\right], \tag{3.2}$$

where λ_s is the string length, R the scalar curvature, g the metric, and ϕ denotes the dilaton field (Gasperini, 2007, Appendix 6A). The (isotropic) minisuperspace is parametrised by $\beta = \sqrt{d}\ln a$, where a is the scale factor of the universe as before ($0 \leq a \leq \infty$, i.e. $-\infty \leq \beta \leq \infty$), and $\bar{\phi} = \phi - \sqrt{d}\beta - \ln\int d^d x \lambda_s^{-d}$. For a constant potential $V(\beta, \bar{\phi}) = \Lambda$, we get the Wheeler-DeWitt equation (WDW)

$$\left(\partial_{\bar{\phi}}^2 - \partial_\beta^2 + \lambda_s^2 \Lambda e^{-2\bar{\phi}}\right)\Psi(\beta, \bar{\phi}) = 0. \tag{3.7}$$

The general solution to this equation is a linear combination of Bessel functions. The particular solution to this equation, up to a normalization factor N_k, is

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[22] Discussions of more realistic cosmologies than the simplest model that I've chosen here can be found in (Gasperini, 2007, Appendix 4B).

$$\Psi_k(\beta, \bar{\phi}) = N_k J_{-ik}\left(\lambda_s\sqrt{\Lambda}e^{-\bar{\phi}}\right)e^{-ik\beta}, \tag{3.8}$$

where the Js are Bessel functions, and we have assumed that the initial geometric configuration of the universe is flat and perturbative, i.e. we choose the initial wavefunction to be coming from $\bar{\phi} \to -\infty, \beta \to -\infty$, so that only right-moving waves approach the singularity—this is the Vilenkin tunneling boundary condition; see Section 3.6. The solution, expanded in the perturbative limit $\bar{\phi} \to -\infty$, is a superposition of the initial incoming state, and the component that gets reflected to the post-big bang phase ($k \doteq \lambda_s\dot{\beta}e^{-\bar{\phi}}$; $z \doteq \lambda_s\sqrt{\Lambda}e^{-\bar{\phi}}$):

$$\begin{aligned} \lim_{\bar{\phi}\to-\infty} \Psi_k(\beta, \bar{\phi}) &= \frac{N_k e^{-ik\beta}}{(2\pi z)^{1/2}}\left[e^{-i(z-\pi/4)}e^{k\pi/2} + e^{i(z-\pi/4)}e^{-k\pi/2}\right] \\ &\equiv \Psi_k^- + \psi_k^+. \end{aligned} \tag{3.9}$$

The transition probability is given by the reflection coefficient:

$$R_k = \frac{|\psi_k^+|^2}{|\Psi_k^-|^2} = e^{-2\pi k} \neq 0. \tag{3.10}$$

Thus, even when self-duality cannot explain the cosmic evolution through the singularity ("of the curvature invariants and of the effective string coupling" (Gasperini, 2007, pg. 243)) at $t = 0$ classically, quantum mechanically, such an evolution does not look impossible or unexplainable. That such a transition between phases appears possible in the quantum theory is what leads string theorists to hope that the solution to the graceful exit problem lies in the quantum theory (Gasperini et al., 1996). However, the possibility of continued evolution through the singularity would at most signal an avoidance of the singularity. Indeed, it does not quite solve the graceful exit problem. All the existence of the nonzero transition probability tells us is that there is hope yet for graceful exit not being prohibited in the quantum theory, as it is in the classical theory. To solve the problem, there would have to be an explanation for *how* the graceful exit of the universe can be interpreted as the reflection of the wavefunction, and as far as I know, no such explanation exists.

An obvious point that is still worth mentioning here is that the quantization here is formally very similar to the canonical quantization presented in the previous chapter. The major difference between the two is that there are no transition probabilities involved in the canonical model, because in it, at the classical level, there is no justification for there being multiple branches, unlike in the string model, which can argue for the existence of these branches because of the scale factor duality. Yet another way in which stringy models differ from canonical quantum cosmology has to do with whether the models can be cyclic. This is discussed in the next section.

3.3 The Cyclic Universe

String cosmology, of course, is not limited to the pre-big bang model. One alternative to the pre-big bang program that seems to have gained a lot more traction is "ekpyrosis". First introduced in Khoury et al. (2001), the ekpyrotic model involves colliding branes that classically appear as bangs, crunches etc. The extended version of the model paints the picture of a cyclic universe in which the big bang is but one repeatable transition within multiple cycles of the universe, and hence need not be conflated with "the beginning of the universe", as one often does, in say, the standard models studied in canonical quantum gravity. This cyclicity is marketed as one of the most attractive features of ekpyrosis by its proponents.

I find my life to be too short to wade through the overbearing presentation of the ekpyrotic program that one finds far too often in the literature, to figure out if there is anything of merit in ekpyrosis that speaks to singularity resolution (and indeed, in general). But cyclicity simpliciter has philosophical import in that it potentially helps separate the question about the beginning of existence from the mysteries surrounding the big bang singularity, and hence is a feature worth exploring when thinking about singularity resolution. The pre-big bang program could also, in theory, be extended to include a cyclic version.[23] One obvious problem in accomplishing this is that ϕ in the pre-big bang program increases monotonically from the beginning of the universe (unlike in ekpyrosis, where it decreases before the branes collide, and increases after), and to reproduce the conditions that lead to the big bang, ϕ needs to be brought back down in the $t > 0$ branch. Gasperini (2007, pg. 532-33) discusses two ways in which this may be possible: by adding a "duality-breaking" effective potential, or by considering a higher-dimensional Kasner model in which some of the dimensions contract while the others expand in the $t > 0$ branch. The details of how this works are not relevant here. But when we think about what we ought to expect from a theory that resolves the big bang singularity, it does make a difference whether we also expect that theory to necessarily have a story about the beginning of the universe solely because of its claim to resolve the initial singularity. And what string cosmology, in general, seems to suggest is that it might be possible to set aside weightier questions about the beginning of existence in the pursuit of a resolution to the big bang. We shall come back to cosmogenesis in the last chapter.

In the meantime, with the formal setup at hand, we return to our original question: is the big bang singularity resolved in pre-big bang cosmology? Well, no. Again, the graceful exit problem remains unsolved, and until there is a description for *how* the pre- to post- branch transition occurs, there is no definite sense in which the singularity may be said to be resolved. Of course, pre-big bang cosmology does not yet claim to have resolved the big bang; all it has done so far is to hint that we need not consider the big bang singularity to be an abrupt beginning of the universe. But does it even do that? In other words, are string theorists justified in thinking that self-dual

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[23] The pre-big bang model as discussed until now does not conflate the big bang and *the* beginning of the universe. The latter in this model corresponds to the state of the universe at the lowest t. I only bring up cyclicity here because discussions of string cosmology today are dominated by ekpyrosis, and its proponents claim cyclicity to be among its biggest selling points. That the pre-big bang program need not be abandoned even if we expect the quantum theory of gravity that resolves the big bang to be cyclic is what I want to emphasize here.

universes take us further towards singularity resolution, that it is just a matter of figuring out what happens where the duals “meet”? Addressing this question brings up the conceptual holes in the proposal that are not easily explained. In the following sections, we shall see that some of these conceptual issues are the same as those in the canonical quantum gravitational model discussed in the previous chapter, while some of them are specific to string theory, and that all of them warrant deeper reflection.

3.4 Same Old, Same Old

Two conceptual concerns were isolated in the previous chapter on canonical quantum cosmology, one that had to do with the limitations of the minisuperspace approximation, and one that involved problems with the lack of a fixed time parameter in the quantum theory. It should not come as too much of a surprise that both of these problems remain in the pre-big bang framework. In fact, the problem with approximations is now two-fold. The effective string action ignores higher-order terms, and hence the validity of perturbation theory can be questioned, i.e. one can ask whether even if the model discussed may be claimed to go some distance in resolving the singularity, the same can be said of the “full” theory. In addition, the minisuperspace problem is exactly the same as before: the reduction and the quantization do not commute, and there is no reason to expect that what we see in these models resembles what really happens in the “full” (in a different way this time) theory. It is worth noting, however, that claims of singularity resolution do not obviously depend on the number of dimensions or perturbation terms involved here, as was the case with the generalized vanishing wavefunction criterion, and so it very well might be that even though the general problems with idealization remain, they do not affect singularity resolution.

On the other hand, $|\psi|^2$ is assumed to be representing a probability here as well, in reading the reflection coefficient as providing the transition probability, for instance. But what it is a probability of, is not explained, and hence, the probability issue, as far as I can tell, remains an unresolved problem in pre-big bang cosmology. Since these issues were discussed at length in the previous chapter, I do not go into them again here.

3.5 Deus Ex Machina?

One charge against the pre-big bang model is that the principle of self-duality is ad hoc (see, for eg., Brustein and Veneziano (1994)). Literally, a proposition is "ad hoc" if it is introduced specifically for a purpose, and in science, hypotheses get marked as being ad hoc when they are proposed solely for solving particular problems, typically problems with there not being explanations for certain experimental data, within a theory, unmotivated by the machinery of the rest of the theory. Such an appearance, seemingly out of thin air, lends ad hoc propositions an air of inauthenticity.

Investigating whether it is fair to accuse the principle of self-duality of ad hocness becomes tricky, because most standard attempts of defining ad hocness allude to how ad hoc assumptions help explain away experimentally discovered discrepancies that cannot otherwise be explained by the theory in question. Hempel (1966, pg. 219), for instance, suggests the following as "guidance" (as opposed to a definitive definition) for when an assumption may be considered to be ad hoc: if it is introduced "for the sole purpose of saving a hypothesis seriously threatened by adverse evidence". String theory famously has very little to do with experiments. Self-duality, in the pre-big bang program, is not being proposed to explain an "embarrassing" piece of experimental evidence. Rather, it is invoked to complete a theoretical story, and hence, such a definition of ad hocness in terms of experimental evidence is unlikely to be helpful to us at this stage.

Further, going back to the literal meaning of ad hocness, it is obvious that the principle of self-duality is ad hoc. However, does that warrant the sort of panic that an accusation of ad hocness typically fosters? Here, I am inclined to say that self-duality is not to be dismissed solely because of its ad hocness. Ad hocness as studied in philosophy of science limits itself to more-or-less complete theories, and thus it may be hasty to accuse principles that come into play in various quantum gravitational models of being ad hoc. Who's to say what appears ad hoc now will not be justified as the theory develops? There are no criteria to evaluate ad hocness within a developing theory, and hence it is too early to deem any part of quantum gravity to be ad hoc. A much graver charge against self-duality's being of any use in resolving the big bang singularity is discussed in Section 3.7 below.

3.6 The Real Problem with Appearing Out of Thin Air

Interestingly, the boundary condition used to find the particular solution to the WDW is the tunnelling boundary condition, which dictates that only outgoing waves from $\bar{\phi} \to -\infty, \beta \to -\infty$, traveling along the direction of increasing $\bar{\phi}$, approach the singularity at $\bar{\phi} \to \infty$ (Vilenkin, 1988; Gasperini, 2007). This ensures, in the tunneling picture, that at singular boundaries of superspace, there are only modes that carry flux out of superspace, i.e that singularities "occur" in the final state of the universe, but not in its initial state. The formal similarity between (3.10) and the reflection coefficient in the original tunneling picture leads Gasperini to claim that the pre-big bang quantum reflection can also be interpreted as a tunneling process (Gasperini, 2007). Tunneling à la Vilenkin is studied in detail in Chapter 5, but to summarize here, we have a universe that nucleates when $t > 0$, by tunneling through a potential barrier at $a = H^{-1}$, where $H = (8\pi G\rho)^{1/2}$. This tunneling happens from "nothing" (Vilenkin, 1988) in the sense that beyond the barrier, there is no space or time or matter. All there is, is a vague "existence" of the laws of physics that make the tunneling possible. Singularity resolution in the Vilenkin tunneling picture is purported to work by way of analogy with tunneling in ordinary QM (see Chapter 5).

In string cosmology then, we have a second-hand analogy. (3.10) may be formally similar to the reflection coefficient equation in standard tunneling, but it itself borrows its interpretation from yet another framework that is formally analogous to the second. *If* the string tunneling is to resolve the singularity, these three situations—that of ordinary QM tunneling, Vilenkin tunneling, and string tunneling—would have to be related by analogy in a way that, as Plato said in the *Timaeus* (albeit in a different context), "[the middle one] is such that, as the first is to it, so is it to the last, and conversely as the last is to the middle, so is the middle to the first". We shall see, in Chapter 5, that the analogy between ordinary QM and Vilenkin tunneling is not enough for the latter to claim to have resolved the big bang singularity. As such, if the string tunneling borrows its interpretation from Vilenkin tunneling, then it cannot resolve the singularity either, for exactly the same reasons. But since tunneling is only offered as an alternative interpretation to what happens in pre-big bang cosmology, let's go back to the original interpretation to see if it helps make the case for resolution.

3.7 Interpreting Scale Factor Duality

The answer to what, if any, kind of singularity resolution happens in the pre-big bang picture boils down to how we make sense of the dual universes that are separated by the singularity, and linked by SFD and time reversal. The claim is that both duals represent the same universe; it's just that one of the duals describes its evolution *pre* big bang, and the other, after the singularity.

That this is necessarily the case is far from obvious. The arguments for interpreting self-duality so generally start by mentioning target space duality ("T-duality"), to then consider SFD as its "extension" to time-dependent gravi-dilaton backgrounds (see, for eg., (Gasperini, 2007, pg. 136)). Let's thus first figure out how we ought to make sense of T-duality. T-duality is a symmetry of string theory, by which a closed string that lives in a space with radius R is physically equivalent to one that lives in a space with radius λ_s^2/R, where λ_s is the "characteristic string length". The philosophy of T-duality that I take for granted here is from (Huggett, 2017), the relevant lessons of which are that dual theories offer different descriptions of the same physics, and that what the duals agree on, i.e. what physical features are "determinate" between the duals, are the only physical facts to which they are committed.

Putting aside time-reversal for a second, let's see what happens if we extend this intepretation to two branches of a universe connected by SFD. If we are to think of the duals in this case exactly as the T-duals, then we would have that (a) the same physical situation is being described by both, and that (b) what is indeterminate between the duals is not a physical fact of the universe, only what is determinate is. Why (a) is problematic for the pre-big bang program is at once apparent: at the heart of the pre-big bang picture is the story about how the two branches are physically inequivalent and separated by the singularity, but according to this intepretation both the duals describe the same spacetime region! The duals could very well be separated by a singularity, but evolution through the singularity would not be to a different region of spacetime, but rather to a different description of the very same region. If we added time-reversal to the mix to look at the full pre-big bang picture, then it would seem that all we would have would be two different descriptions of our universe, one that accords with our standard perception of the flow of time, and the other in which we would live the life of Benjamin Button (except for the original Benjamin Button, I guess, who probably would have a perfectly normal life in that universe). In other words, it wouldn't be just that pre-big bang cosmology does not resolve the initial singularity, it could be argued that it does not even count as having attempted to resolve the singularity.

What if intepreting SFD is not quite as simple as just thinking of the SFD-duals as cosmic versions of T-duals? In the paper that eventually formed the basis for the pre-big bang program, Veneziano (1991) argues for considering SFD less as a symmetry and more as a group that transforms physically inequivalent solutions into each other. His argument relies on the following example:[24]

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[24]Note that this paper predates the pre-big bang program, and hence there is no mention of the time-reversal part of self-duality in it.

> "[C]onsider the case of a single circle of fixed radius R and a time-dependent scale factor $a(t)$. A naive extension of [T]-duality would connect this situation to the one in which $aR/\lambda_s \to (aR/\lambda_s)^{-1}$. However, these [T]-duality-related situations both describe a contracting or expanding circle according to whether aR/λ_s approaches the fixed point value 1 or moves away from it. Instead, by fixing $a(0) = a^{-1}(0) = 1$ and by letting the two evolve according to
>
> $$a_i(t) \to a_i{}^{-1}(t); \phi \to \phi - 2\ln(a_i),$$
>
> we are connecting, through SFD, a physically expanding $(aR/\lambda_s \to \infty)$ to a physically contracting $(aR/\lambda_s \to 1)$ [u]niverse." (Veneziano, 1991, pg. 5)

The idea seems to be that in the "naive" extension, from (b) above, we can say that whether the circle expands or contracts is "determinate" because the both duals agree on this. However, when we add an initial value for a and a^{-1} and impose SFD, the "duals" stop agreeing on whether the circle is expanding or contracting. In this case, when one dual describes an expanding circle, the other necessarily describes a contracting one (with an emphasis on the expansion/contraction being "physical", which I take it is stressed because of the involvement of ϕ, which is unique to SFD). These considerations lead Veneziano (1991) to conclude that "the correct interpretation of SFD is not that of a true symmetry, but rather of a group acting on the vacuum manifold and transforming solution of the field equations into other (generally inequivalent) solutions".

But this is mathematically incorrect. If the quantity aR/λ_s going towards or away from 1 is what determines whether the circle is contracting or expanding, then the quantity does go away from 1 in one T-dual when it goes towards 1 in the other dual, i.e. the situation is not at all different from the SFD case. This is, of course, assuming that the scale factor in one dual would be

a, while that in the other would be λ^2/aR^2. On the other hand, if we are only focusing on aR/λ_s for both duals, where a is the scale factor of only one of the duals, then I don't see why the same does not apply to the SFD case as well. In short, as an argument for SFD betokening transition of the universe between two physically different branches, this one is not convincing at all.

Of course, one could take the position that SFD *is* a duality, but that that does not mean the duals should agree on every detail, that the mirroring only happens at a coarse-grained level, that time reversal in addition to SFD would not lead to Benjamin Button-duals. But then, again, no big bang resolution comes from the universe transitioning from one description to another description, however dissimilar the branches being described are, and there is no case for singularity resolution via pre- to post-big bang evolution. Indeed, there would not even be a case for why the pre-big bang branch is, indeed, *pre-* big bang. If all that was happening at the big bang was a transition between two descriptions, then what is claimed to be the pre-big bang dual could just as well also be physically describing post-big bang evolution, only with t running the other way.

In short, the pre-big bang program aims to show that the universe's evolution does not abruptly start at the singularity, but can be shown to be traceable backwards in time to another branch of evolution. However, as I have been arguing, mere demonstration of continued evolution does not a singularity resolution make. And to make matters worse, because of the issues with interpreting SFD in particular, the claim that there even is such a demonstration to be found in pre-big bang cosmology appears to be dubious.

CHAPTER 4

TREATING THE SYMPTOMS: BIG BANG RESOLUTION IN LOOP QUANTUM COSMOLOGY

Among the extant approaches to quantum gravitational cosmology, arguably, the most extensive claims about big bang resolution are found in loop quantum cosmology (LQC), the field that studies the application of the techniques of loop quantum gravity (LQG) to MSS models. Mirroring the two approaches to LQG, the "canonical" and the "covariant", there are, broadly, two approaches to LQC as well. However, both because canonical LQC is more or less an extension of the canonical formalism introduced in Chapter 2, and because covariant LQC, or spinfoam cosmology as it is more popularly known, is not yet developed enough to make significant claims about singularity resolution, in this chapter, I focus on the canonical approach.[25] As before, I start by describing the formalism in the first section, and then analyze the singularity resolution claims in the following section. Section 4.2 introduces the claims of singularity resolution in LQC, Section 4.3 consists of a brief discussion of the irrational fear of infinities. Section 4.4 discusses the framework of quantum reduced loop quantum gravity, which promises to solve the MSS problem, and finally, Section 4.5 argues against resolution in LQC.

[25]This, of course, is not to say that spinfoam cosmology is to date completely silent on the topic of singularity resolution. Rovelli and Vidotto (2014, pg. 225) write, for instance, that there is an "indirect element of evidence" of big bang resolution in spinfoam cosmology because it predicts that there is a maximal acceleration for observers and hence a maximal energy density for our universe. But as I have been saying, and will say again in this chapter as well as the next, the finitude alone, if achieved at all, makes for little more than the avoidance of the mathematical singularity, which is often taken to be coreferential with the spacetime singularity without argument, which is incorrect. In the terminology introduced in this chapter, the absence of infinite quantities, at best, amounts to avoiding the "kinematic" aspect of the singularity. The actual resolution of the singularity would have to, if at all, be present in the as yet undeveloped parts of the theory.

4.1 Canonical LQC

The scale factor a and momentum p, which were the canonical configuration and momentum variables, respectively, in canonical quantum gravity, no longer retain their statuses in LQC. To create the canonical variables in LQC (also called Ashtekar variables), we start with mathematical objects called connections and triads. In very broad terms, triads are sets of three spacelike vector fields, $e^a_i(x)$ at spatial points denoted by x, where both a and i range from 1 to 3, with a being the spatial index and i being the index that labels the vectors. The variant of the triad that is relevant to LQC is the densitized triad, $E^a_i(x) := \sqrt{h(x)}e^a_i(x)$, for a given metric $h(x)$. The connection is an object that is introduced in the definition of the covariant derivative of a vector V^i (i.e. it shows up when one thinks about parallel transporting vectors along curves): $\nabla_\mu V^\nu := \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda}A^\lambda$, where Γ is the connection. With these in mind, let's take a look at an FLRW model with a scalar field. (I follow Kiefer's (2012) presentation here.) As with the canonical quantum cosmology model in Chapter 2, here too, we work with a symmetry-reduced model. As the new canonical variables, we consider the functions $\tilde{p}$ and $\tilde{c}$ such that the densitized triad, $E^a_i = \tilde{p}\delta^a_i$ and the connection, $A^i_a = \tilde{c}\delta^i_a$. Assuming a universe with a finite volume V_0 and either positive or null curvature ($k = 1$ or $k = 0$), in terms of our old canonical variables,

$$|\tilde{p}| = a^2, \tag{4.1}$$

and

$$\tilde{c} = k + \beta\dot{a}. \tag{4.2}$$

Here, β is the Barbaro-Immirzi parameter, a non-zero constant that can take any (even a complex) value. Now, we calculate the Poisson bracket. We saw in the canonical quantum cosmology case of the same model that $p_a = \frac{-3V_0}{4\pi G}\frac{a\dot{a}}{N}$, and hence we have

$$\dot{a} = \frac{-4\pi G}{3V_0}\frac{Np_a}{a}. \tag{4.3}$$

From this, we can calculate

$$\{\tilde{c}, \tilde{p}\} = \frac{8\pi G\beta}{3V_0}. \tag{4.4}$$

Rescaling $\tilde{p} := V_0^{2/3}p, \tilde{c} := V_0^{-1/3}c$, we get

$$\{c, p\} = \frac{8\pi G\beta}{3}. \tag{4.5}$$

It is interesting to note that in this framework, the big bang is at $p = 0$, not at the boundary of (mini)superspace. Another difference between this and the canonical quantum cosmological model is that the "evolution" equation that comes from the Hamiltonian constraint in the quantum theory here will turn out to be a difference equation as opposed to a differential equation. But first: the quantization.

Now, the connection itself cannot be an operator, rather, it is the holonomy of the connection that ends up being the observable, along with the triad flux. Again in very broad terms, the holonomy captures the idea of a quantity being parallel-propagated along a closed curve in a way that is "as parallel to itself as possible" (Gambini and Pullin, 2011, pg. 67). Following Gambini and Pullin (2011), let's assume that homogeneous space can be thought of as one elementary cell (with a side of length μ) repeated over and over again. Then, the holonomy of the connection A_a^i along a loop (hence the "loop" in "loop quantum cosmology") going around the elementary cell is given by exp($i\mu c$). The state space of the quantum theory then is generated using the holonomies as creation operators:

$$\psi(c) = \Sigma_\mu \psi_\mu \exp(i\mu c/2). \tag{4.6}$$

The space of these functions is the Bohr compactification of the real line, or $\mathbb{R}_{\text{Bohr}}$. (These functions are also known as "almost-periodic functions".) The Hilbert space chosen is the space of all square integrable functions on this compactification, and the basis states are chosen to be $\langle c|\mu\rangle = \exp(i\mu c/2)$. Since p is conjugate to c, it is represented by a derivative operator,

$$\hat{p} = \frac{-i8\pi\beta l_P{}^2}{3}\frac{d}{dc}, \tag{4.7}$$

where $l_P = \sqrt{G}$. p has a discrete spectrum.

The Hamiltonian in terms of the old variables was

$$H = \frac{N}{2}\left(-\frac{2\pi G}{3}\frac{p_a^2}{V_0 a} + \frac{p_\phi^2}{a^3} - \frac{3}{8\pi G}V_0 k a + \frac{V_0 \Lambda a^3}{8\pi G} + m^2 a^3 \phi^2\right). \tag{2.10}$$

In terms of the new variables, the Hamiltonian constraint thus is,

$$H = -\frac{3}{8\pi G}\left(\frac{(c-k)^2}{\beta^2} + k^2\right)\sqrt{|p|} + H_{\text{matter}} \approx 0, \tag{4.8}$$

where, in the case of the scalar field,

$$H_{\text{matter}} = \frac{1}{2}\left(|p|^{-3/2}p_\phi^2 + |p|^{3/2}\mathcal{V}(\phi)\right), \tag{4.9}$$

where, $\mathcal{V}(\phi) := \Lambda/3 + m^2\phi^2$.

Expanding the general solution to $H|\psi\rangle \approx 0$ in terms of volume eigenstates,

$$|\psi\rangle = \sum_{\mu} \psi_\mu |\mu\rangle,$$

one gets an awkward sum which can be rearranged to get the following difference equation:

$$\begin{aligned}&(V_{\mu+5\delta} - V_{\mu+3\delta})\,\psi_{\mu+4\delta}(\phi) - 2\,(V_{\mu+\delta} - V_{\mu-\delta})\,\psi_\mu(\phi) \\ &+ (V_{\mu-3\delta} - V_{\mu-5\delta})\,\psi_{\mu-4\delta}(\phi) = -\frac{128\pi^2 G\beta^2\delta^3 l_P^2}{3}\hat{H}_{\text{matter}}(\phi)\psi_\mu(\phi),\end{aligned} \tag{4.10}$$

where $V_\mu := |p_\mu|^{3/2}$. For large a, this equation is the same as the Wheeler-DeWitt equation. There being this difference equation at the fundamental level forms the basis for claims about singularity resolution in LQC, since it represents how there are two regions in the formalism, one for each sign of p_μ, which are separated at $\mu = 0$. It is claimed that ψ_μ can evolve through $\mu = 0$ from one region to the other, and hence, "the classical singularity is avoided in the quantum theory" (Kiefer, 2012, pg. 296). Yet another feature of the formalism that is claimed to indicate singularity resolution is the fact that there appears to be a quantum "bounce" caused a new repulsive potential in the effective Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right), \tag{4.11}$$

where $\rho_c = \sqrt{3}/(32\pi^2\beta^2 G^2\hbar) \approx 0.41\rho_{\text{Planck}}$, and the last term with the critical density ρ_c is responsible for the bounce. In the quantum theory, the operator corresponding to the density can be shown to have an upper bound that is identical to ρ_c.

4.2 Dreams of Singularity Resolution

There are two senses in which the big bang singularity is claimed to be "resolved" in LQC. In much of LQC-related discussions of singularity resolution, one finds a distinction between "dynamical" and "kinematic" singularities. In broad terms, dynamical spacetime singularities are those that prevent the extrapolation of the evolution of the universe through and beyond them (i.e. in DeWitt's language from two chapters ago, they pose a "barrier" beyond which the evolution cannot be continued), and kinematic spacetime singularities are those "at" which physical quantities like the curvature diverge. When I am not mentioning or using terminology from elsewhere, I shall not follow this practice of classifying singularities. Instead, I shall speak in terms of dynamical and kinematic *aspects* of singularities. As far as GR is concerned, the big bang singularity has both these aspects. The proponents of LQC then proceed to show how the quantization involved in LQC takes care of both these aspects of the big bang, which, they claim, amounts to big bang resolution.

Following Wüthrich (2006, Chs. 6, 8), let us define a few concepts involved in evaluating these claims. Let an *expectation value singularity* be the quantum version of the classical kinematic singularity, wherein the the operators representing observables do not have spectra that are bounded from above, and let a *dynamical quantum singularity* be the one possessed by a system whose state at any given instant in time does not uniquely determine the dynamical evolution of the system for all other times. Further, let *evolution** be the kind of evolution described by a difference equation that must be solved recursively rather than that described by a standard, differential equation. Then, the evolution* of a system governed by a difference equation is *deterministic* if and only if the states of the system over a finite period of time, along with the difference equation, fix the state of the system at all times.[26] The claims for the big bang being "resolved" in LQC are then twofold: (1) the spectrum of the inverse scale operator $\widehat{p^{-1}}$ is bounded from above, suggesting the curvature is finite in LQC (the curvature goes as a^{-2}), which implies that there is no expectation

[26]To clarify, the terms in italics here are the ones used by Wüthrich (2006), and defined in exactly the same ways as he does, with the small change that what he calls "Laplacean determinism", I simply call "determinism".

value singularity, and (2) LQC models are deterministic in the way determinism was defined above, suggesting that there is no dynamical quantum singularity either.

Of course, things don't work out quite that neatly for LQC, as Wüthrich (2006) points out. For one, $\widehat{\mathsf{p}^{-1}}$ does not represent an observable, and hence its spectrum being bounded has no effect on the kinematic aspect of the singularity whatsoever. This is so, because, $\hat{\mathsf{p}}$, we saw earlier, has a discrete spectrum, which implies that $\widehat{\mathsf{p}^{-1}}$ is not densely defined around zero, which, of course, means that there is no way to directly create the operator $\widehat{\mathsf{p}^{-1}}$. Wüthrich (2006) discusses a "principled way" of attempting to create such an operator, and suggests that "the jury is still [as of 2006] out" on whether such a construction is correct. For another, LQC models turn out to not be deterministic, since the evolution* "at" the big bang is not determined by the difference equation. All these models satisfy is a weakened form of determinism, or *quasi-determinism*, which is what obtains when the states of the system under consideration, along with the difference equation that governs its evolution*, fix the state of the system for all times except for a subset of points of Lebesgue-measure zero, which, unsurprisingly, represents the big bang itself. Singularity "resolution" in LQC, thus, is at best achieved with substantial qualifications. Wüthrich (2006) summarizes the situation thus:

> In isotropic and homogeneous models of loop quantum cosmology the initial singularity of the classical model seems to disappear in two different senses: first the curvature does not increase without bound for arbitrarily small scale factors; and second, there exists a principled—though perhaps not correct—way of extending the models through the initial singularity into a mirror world, thereby circumventing, to some extent at least, the classical singularity (134).

Evidently, Wüthrich (2006) discusses the philosophical issues associated with the technicalities of the LQC formalism in much detail, especially those that are relevant to singularity resolution. Hence, in a slight deviation from the structure of the previous two chapters, I do not include any analyses of the technical aspects of LQC in this chapter. What I wish to draw attention to in the rest of the chapter, instead, is the fact that even if all such philosophical issues are resolved, the conceptual foundations for the technicalities are still on shaky grounds, and so LQC, qua a quantum gravitational replacement for general relativistic cosmology, does not resolve the big bang singularity.

4.3 Cantor's Paradise Lost

Before we discuss the extent to which the big bang can be said to be resolved in LQC, I would like challenge an assumption about the kinematic aspect of singularities that I have never seen challenged. It is universally accepted, for reasons I genuinely do not understand, that the kinematic "problem" with singularities is that physical observables tend to be infinite, that any resolution should thus make sure that "nothing bad happens" in this way. Why are infinite quantities "bad"?

That we have only ever encountered finite quantities is hardly a reason to be completely dismissive of the possibility of the existence of infinite quantities, which, incidentally, is an argument that holds regardless of whether one is pondering on the existence of physical infinity or black swans. Is the worry that an infinite quantity might not be measurable by finite experimental devices? Such a worry is justified to some extent, because if we were to classify infinities into potential and actual in the tradition of Aristotle, and the quantity in question is a potential infinity, then there can be no empirical witness to the potential infinity. But what is the argument for a finite device not being able to measure actual infinity? And even if it were the case that finite beings and their devices cannot grasp infinity, how does that show that the infinity itself does not exist, and that hence the resolution of any problem involving infinities involves forcing all the relevant quantities to be finite?

If the worry is that the kind of mathematics we have been using is not well-defined for infinite quantities, that is fair enough. However, I fail to see what the argument is for assuming, based on this worry, that what is problematic is the non-finitude of the quantities involved, instead of wondering whether it is the mathematics itself that breaks down. Especially given that there exists a mathematical theory of transfinite numbers, I do not see why transfinite algebra is never considered as a possibility to describe a reality that is not obviously finite.

There is nothing in this discussion specific to LQC. It just so happens that it is easier to discuss the infinity issue with the distinction of the kinematic and dynamical aspects of a singularity at hand, and this distinction is clearly spelled out in the LQC literature. All extant treatments of spacetime singularity resolution are guilty of trying to get rid of the infinity without an argument for why it needs to be avoided. I would merely like to flag here that it is not at all clear to me that the avoidance of the kinematic aspect of a singularity necessarily involves forcing all quantities in quantum gravity to be finite. Quantities "blowing up" in GR could be used to indicate the presence of a singularity, but I do not see why that necessarily leads to requiring the complete absence of infinite quantities in quantum gravity.

4.4 A Solution to the Spherical Cow Issue?

We have been looking at an MSS model in this chapter, just like in the previous chapters. The issue of minisuperspace idealization from before thus equally affects the LQC model that we have been considering. However, there are developments in LQG that claim to finally have solved the problem. In particular, "quantum reduced loop gravity" (QRLG) aims to "bridge the gap" between LQC and the full theory, by first quantizing and then reducing, instead of the other way around as we have been doing thus far, to get tractable models (Alesci et al., 2018). Because the mathematics of QRLG is quite involved for something with no payoff for our discussion about singularity resolution as we shall see in a second, I do not attempt to repeat the formalism here. All I shall say here is that the procedure to get a tractable model involves starting at the quantum level, with the "kinematical" Hilbert space of LQG, on which we "implement the gauge fixing conditions restricting to a diagonal metric tensor and to diagonal triads" and then reducing, "on a dynamical level, by considering only that part of the scalar constraint which generates the evolution of the homogeneous part of the metric" (Alesci and Cianfrani, 2016).

QRLG (specifically, "the isotropic sector" of QRLG) replaces the LQC bounce to tell the story of an "emergent-bouncing" universe. Alesci et al. (2018) report, "the [u]niverse emerges from the infinite past with a finite Planckian volume and [it remains stationary until it] eventually undergoes a transient phase during which expanding and contracting phases succeed until the geometric energy density gets enough diluted [...] to leave the [u]niverse expanding forever according to the classical dynamics". The last of these bounces in this picture is supposed to match LQC's bounce, and the dynamics thereafter matches LQC's as well.

Since we start with the "full" Hilbert space in this picture, there is no violation of the uncertainty principle anymore, and so the argument from Chapter 2 does not hold against these tractable models. However, I am not sure the broader issue of idealizations has been fully solved. In a lecture, Aurélien Barrau claims that gauge-fixing is the "price we pay" to be able to quantize first and then reduce.[27] I take it that what he means by this is that by restricting our attention to only a diagonal metric, for instance, we are willfully ignoring known metrics, like the Kerr metric, that are not diagonal. So the cow may not be spherical anymore, but still does not quite have the shape of an actual cow.

More importantly, changing the order of the reduction and the quantization, in this case, does not do much by way of making a case for a loopy singularity resolution. We do not need the full details of a QRLG model to see that it "resolves" both the kinematic and dynamical aspects of singularities in the same way as LQC. ((Alesci et al., 2017) has a comparison of the formal

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[27] "Master-Level Lecture on Loop Quantum Cosmology" `https://www.youtube.com/watch?v=-ubu07Rk9j4`

differences between the two kinds of "resolution", but conceptually, they work the same way.) I shall argue below that such "resolutions" still do not count as quantum gravitational resolutions of singularities.

4.5 Defining Singularities

As we saw, the claims for singularity resolution in LQC assume that singularities have precise mathematical definitions which let them be classified into two, dynamical and kinematic singularities, and "resolution" proceeds by showing that the formalism of LQC (contrasted with attempts that do not utilize loop quantization) does away with both kinds (with qualifications, as Wüthrich (2006) notes). I'd like to question here the assumption that there exist precise mathematical definitions for spacetime singularities. Spacetime singularities are not mathematical artifacts. Nor are they physical artifacts with precise mathematical definitions in quantum gravity. The classical singularity theorems do consider geodesic incompleteness as an essential characteristic of all singularities, and then prove that given reasonable (but not incontrovertible) assumptions, our universe has at least one incomplete geodesic. This leads physicists to make the claim, as, for instance, Kiefer (2012, pg. 294) does, that "[i]n classical GR, there exist *mathematically precise definitions* [emphasis mine] of singularities, as well as rigorous theorems for their occurence". Curiel (1999) discusses the issue of defining singularities and concludes that "for most purposes," a definition in terms of a more general sort of curve incompleteness than geodesic incompleteness ("b-incompleteness") suffices.[28] I concur that within GR, the mathematical prescription for what a singularity is works perfectly fine as a functional definition of a singularity.

However, quantum gravity isn't GR. Indeed, the hope is that it will replace GR as a more fundamental theory of our reality. And hence, a quantum gravitational resolution of singularities

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[28] A 'b-complete' spacetime is one in which every inextendible curve has an unbounded generalized affine parameter.

cannot take the GR definitions of singularities as definitions at all—in the world of quantum gravity, they are *nonsensical* (in the Wittgensteinian sense)! Unfortunately, all claims of singularity resolution in LQC, as we saw, stem from the fact that it can be shown that LQC models do not have the problems identified in the various defining features of a singularity as defined in the language of GR. And this is reflected in almost all discussions of singularity resolution in LQC, wherein singularities are defined at the outset in GR terms, and then it is shown that the models in question avoid the problems that the definition expresses.

Of course, if one were to define a singularity in terms of geodesic incompleteness and curvature pathology (which is standard, but not necessarily correct - see (Curiel, 1999)), the way to "demonstrate" singularity resolution is fairly straightforward: show how the new theory makes all the relevant geodesics complete and the observables finite, and there is no singularity anymore. This is exactly what the myriad attempts in LQC to resolve the singularity amount to, and thus, they all do little more than "resolve" the particular problems the definition lists. As an example of such work, consider (Singh, 2009), which discusses a "generic" resolution of singularities in LQC. Singh defines singularities as "the boundaries of spacetime which can be reached by an observer in a finite proper time where the spacetime curvature and tidal forces become infinite", and then goes on to show how different types of singularities are "resolved" in LQC as a result of taming infinities or extending geodesics.

But geodesic incompleteness and infinite observables do not answer the question of *what* a spacetime singularity is in quantum gravity. Attempting to make these particular problems go away in the hope of resolving singularities as such is very much like treating the symptoms hoping the disease would go away. While defining the quantum versions of the classical dynamical and kinematic singularities, as we did earlier following (Wüthrich, 2006), is an integral part of arguing for the kind of "resolution" the proponents of LQC claim their theory accomplishes, those definitions cannot answer the question of *what* a singularity is, since if we expect quantum gravity to resolve the singularity, and that we do has been the assumption throughout this dissertation, then once it is resolved, there can be no singularity in the quantum case to define! So, whereas Wüthrich urges, on technical grounds, that claims of singularity "resolution" in LQC ought to be significantly qualified, what I'm suggesting is that "resolution" in LQC is directed towards the wrong targets. Infinite curvature does not define a singularity in quantum gravity, it merely signals one in GR.

And what needs to not come to an abrupt end as one traces cosmic time back to the big bang is *not* the evolution of the universe, but the description of the universe. These things are not the same in a context where there is no external time with respect to which talk of evolution is possible, which is the case in quantum cosmology. This point is reflected, to a certain extent, in the introduction of evolution*, but to my mind, the distinction between evolution and evolution* is one without much difference. In defining the determinism of evolution*, a clock in the background is assumed, a clock which has no business being there in quantum cosmology, suggesting that evolution* itself is not timeless. Now, Wüthrich (2006, pg. 132) makes the point that (4.10) is not an evolution equation, that it ought to be interpreted "such that physical states do not undergo a dynamical evolution through the values of μ to be summed over, but rather as a *superposition* of kinematical states corresponding to spatial universes of different sizes, where the manner [in which] these kinematic states superpose is constrained by [(4.10)]". This, I completely agree with. However, he goes on to say that one ought to be wary of interpreting (4.10) as describing "evolution" with respect to the partial observable $\widehat{p^{-1}}$ because such an "evolution" is not deterministic. The issue, it seems to me, cuts deeper than the evolution not being deterministic; the very definition of determinism in this scenario is faulty if it depends on clocks it has no access to!

Thus, despite the significant progress made in LQC regarding singularity "resolution" in terms of investigating the behaviors of a variety of models, there are still no grounds for claiming any actual resolution of singularities; the singularity is still only being avoided. Note that it follows from this discussion of evolution* that if there were to be any notion at all about how the difference equation describes the state of the universe with absolutely no reference to time (whether in terms of time as we know and love it, or some "fiducial" time), i.e., if evolution* can be defined with no recourse to anything temporal, then, there just might be a case for a loopy resolution of the big bang.[29]

In summary, much has been done in LQC and the philosophy of LQC to attempt to precisely define concepts related to singularity resolution in a way that makes it possible to argue for the the big bang "singularity" being "resolved" in LQC, given these terms are defined in the specific ways laid out by the proponents of the theory. Unfortunately, these definitions, I have argued, hold on to a lot of general relativistic baggage, and thus, do not, cannot, make sense in quantum gravity. And thus, despite the volume of work done in LQC on singularity resolution, it still does not fare much better than the other attempts we have seen in this dissertation.

A brief comparison might be in order here, between the discussion of string cosmology from the previous chapter, and LQC. Could one not make the same argument against the former as the latter, in terms of general relativistic definitions not being sufficient in quantum gravity? Absolutely. Essentially, all quantum gravitational attempts to "resolve" the singularity to date (at least the ones that I'm aware of) only aim to avoid the kinematic and dynamical aspects of the classical singularity. Why did I not raise this argument in the previous chapter then? Because as we saw, the biggest case against pre-big bang cosmology is that SFD does not help in actually letting the universe evolve through the singularity to a different branch, so even if the goal there is the

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[29]It is worth noting that Wüthrich (2006) seems to agree with my general sentiment that we ought to not be talking about evolution in timeless quantum cosmology. He writes about how the evolution rhetoric within LQC is beginning to be toned down at the time of writing. As far as I can tell, however, the emphasis on evolution is still not sufficiently absent in discussions of big bang resolution in almost all approaches to quantum gravitational cosmology, including LQC.

avoidance of the dynamical aspect of the singularity, that goal is not achieved. In a sense then, the model in the previous chapter is one step behind the one in this chapter in *not* resolving the singularity.

CHAPTER 5

BIG BANG RESOLUTION IN QUANTUM GRAVITY

को अद्धा वेद क इह प्र वोचत्कुत आजाता कुत इयं विसृष्टिः।
अर्वाग्देवा अस्य विसर्जनेनाथा को वेद यत आबभूव॥
इयं विसृष्टिर्यत आबभूव यदि वा दधे यदि वा न।
यो अस्याध्यक्षः परमे व्योमन्त्सो अङ्ग वेद यदि वा न वेद॥

Ṛg Veda, 10.129[30]

The previous chapters set up the technical aspects of a few simple cosmological models within different "mainstream" approaches to quantum gravity, and analyzed claims of big bang resolution in those models. A thorough appraisal of all extant approaches to quantum gravity beyond what has already been discussed and their success with singularity resolution is out of the scope of this dissertation. Indeed, many of these approaches are still in their infancy, and do not yet have a whole lot to say about singularity resolution. This final chapter begins with discussions of claims of quantum "resolution" of the classical singularity that are not necessarily confined to any one approach to quantum gravity. These constitute the first two sections of this chapter. Various issues that are not specific to any one approach, but are often discussed when big bang resolution is brought up, warrant exploration, which constitutes Section 5.3. In the penultimate section, I ask what, if anything, our investigation of quantum gravitational big bang resolution teaches us about what we should look for in a theory of quantum gravity. I end this chapter, and thus, the dissertation, by discussing the distinction between singularity avoidance and resolution.

[30]Who really knows, who here can say, whence this creation? / The gods came after; who knows from what it originated? / That which gave cause to this creation, whether It put all this in place, or It did not, / That which is its overseer in the supreme heavens, must know - or perhaps It knows not. (my translation)

In Chapter 2, we saw that canonical quantum gravity at best has claims of avoiding the big bang, but even those claims need to be qualified to say that in simple, unrealistic models of canonical quantum gravity with a miraculous probability interpretation that has so far eluded us, the big bang is avoided. Chapter 3 saw the case against resolution in pre-big bang string cosmology, where the most severe charge against the program was that all there is, is continued evolution through the big bang, which merely counts as singularity avoidance, but even that continued evolution can be contested by arguing, as I have, that the scale factor duality cannot be interpreted as arranging for the physical transition of the universe between branches. And in Chapter 4, I argued against anything more than mere avoidance of the big bang in loop cosmology, while drawing attention to Wüthrich (2006)'s work that suggests that even those claims of avoidance need to be significantly qualified. I continue with the *via negativa* approach in this chapter as well.

5.1 "Resolution" by Initial Conditions

As a species, we have made a few attempts at thinking about what our universe must have been like at the very beginning over the years. The ancient Hindus thought that the initial state was represented by Hiranyagarbha, the golden embryo from which the phenomenal world came into existence, the ancient Greeks had the Orphic Egg, and curiously, just about everyone else in between had their own versions of egg-based cosmogony (among other things). Thousands of years hence, we are still in the business of attempting to describe the initial state of our universe. This section analyzes two such quantum cosmological descriptions, cosmological tunneling, and the no-boundary proposal. Since the big bang singularity is often considered to be the "initial" singularity, one could ask if specifying the initial state of the universe considered as a single quantum system would achieve anything by way of singularity resolution. This is the line of thought pursued in this section. I briefly describe both proposals in their own sections below, highlighting only the technicalities that will be relevant to discussing singularity resolution, and then raise the question of whether the big bang may be said to be resolved in either of these frameworks.

5.1.1 Being from Nothingness

Vilenkin introduces his tunneling proposal as a scenario in which "the universe is created from literally *nothing*" (1982, pg. 26). For a closed FLRW metric, the solution to the Friedmann equation gives the scale factor,

$$a(t) = H^{-1}\cosh(Ht), \tag{5.1}$$

where the Hubble parameter, $H = \dot{a}/a = (8\pi G\rho/3)^{1/2}$, and t denotes cosmic time. It describes a universe that contracts when $t < 0$, reaches a minimum, but non-zero size at $t = 0$, and expands when $t > 0$. *By analogy* (emphasized because this will be important later) with the familiar case of the particle tunneling through a potental barrier in ordinary QM (in this case, a barrier at $a = H^{-1}$), Vilenkin suggests that "the birth of the universe might be a quantum tunneling effect" by which the universe appeared out of nowhere with a finite size (1982, pg. 26).[31] He calculates the tunneling probability to be proportional to $\exp(-3/8G^2\rho)$ (1982; 1984). The similarities between this model and the pre-big bang model were discussed in Chapter 3.

As advantages of this cosmological picture, Vilenkin writes that this model "does not have a singularity at the big bang" and that "[t]he structure and evolution of the universe(s) are totally determined by the laws of physics" (1982, pg. 27-8). We will return to the claim about the singularity in Section 5.1.3, but a note here about the "nothingness" from which the universe would have to nucleate in this model. In another paper, Vilenkin clarifies that by "nothing", he

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[31] "Birth" here refers to the "beginning" of the $t > 0$ epoch, at $t = 0$, which also coincides with the big bang.

means a state with no classical spacetime, and that "the initial state of the [u]niverse is determined by the laws of physics" (1984). For the initial state to be determined by the laws of physics, the nothingness that is devoid of classical spacetime must still have some notion of the laws of physics. Vilenkin writes that "[t]he concept of the universe being created from nothing is a crazy one" (1982, pg. 26), but the craziness, to me, seems to lie not so much in the creation part itself as in the idea of the "nothing" that raises many more questions than the tunneling. How do the laws of physics exist in this "nothing"? And if spacetime itself emerges only after the tunneling, then are we to assume that these laws dictate this emergence as well? If all we can say about the "nothing" is that it is a unique state of the universe from which every successive state has come to be, which already contained the seeds for the structure and evolution of the universe, then how far really have we come from the cosmic egg?

5.1.2 A Boundary-less Universe

In a conceptually not far-off alternative proposal for the initial state of this universe, Hartle and Hawking (1983) prescribe that the initial conditions for the universe must be that it has no boundaries, that the ground state of the universe gives the amplitude for it to, again, appear from "nothing". One of the differences between this no-boundary proposal and tunneling is that the latter is a prescription for the intitial state only inasmuch as the description of the initial state follows from the proposed interpretation of how the universe may be said to tunnel into existence—it is the interpretation itself that is novel in the tunneling program. Hartle and Hawking (1983, pg. 2961), on the other hand, go the other way around to propose what the initial state must be, and then attempt to show that it can indeed be considered as the ground state in that it is a "state of minimum excitation corresponding to the classical notion of a geometry of high symmetry".

To that end, they propose that the ground state wavefunction of the universe be expressed as a functional integral,

$$\Psi_0[h_{ij}, \phi] = N \int_c g\phi e^{-I[g,\phi]}, \tag{5.2}$$

where the class of geometries that are summed over is that of compact four-geometries on the boundaries of which the induced metric is h_{ij}, and that of field configurations is that of regular fields that match ϕ on the boundaries of the four-geometries. (I is the Euclidean action.) What this amounts to is that in the Euclidean regime, the universe has no boundaries, and this functional integral is interpreted as "giving the amplitude for the initial three-geometry to arise from a zero three-geometry, i.e., a single point" (Hartle and Hawking, 1983, pg. 2961). They continue, "[i]n other words, the ground state is the amplitude for the [u]niverse to appear from nothing" (Hartle and Hawking, 1983, pg. 2961).

The demonstration of how this state may be considered the ground state happens within a minisuperspace model, the details of which are not really relevant here. I will simply say here that similar conceptual questions as in the canonical quantum cosmological model in Chapter 2 are to be anticipated here as well. Even if one ignores the spherical cow issue, the question about interpreting probabilities shows up in a major way again in that it is not at all obvious that (5.2) does indeed give the probability amplitude for the universe to appear out of nothing. Hartle and Hawking (1983, pg. 2963) say about the wavefunction that "[i]f the [u]niverse is in a quantum state specified by a [wavefunction] Ψ then that [wavefunction] describes the correlations between observables to be expected in that state [...] This is the only interpretive structure we shall propose or need". This is all they propose by way of interpreting the wavefunction, but precisely because of all the reasons listed in Chapter 2, they will need a satisfactory answer to the question of how to interpret the quantum cosmological wavefunction as giving an amplitude to make the case for their proposed ground state wavefunction actually giving such an amplitude for cosmogenesis from "nothing". But the goal of this project is not to critique the no-boundary proposal; we are only concerned about whether it can lay claim to have resolved the big bang, and that is the issue the next section addresses. We shall see that the proponents of the no-boundary proposal too, like Vilenkin, discuss singularity resolution *by analogy* with ordinary QM, and I will claim that the analogy does not hold.

5.1.3 The Fate of the Big Bang

Both the proposals above claim to have allayed worries about the initial singularity. Vilenkin writes that his model "does not have a singularity at the big bang" (1982, pg. 27), and Hartle and Hawking write that the situation is "improved" in their model, by analogy to ordinary QM in which particles can be shown to evolve through classically forbidden regions of spacetime. While Vilenkin uses the case of the quantum particle tunneling through a classical energy barrier, Hartle and Hawking are thinking about the electron spiraling into the nucleus. We have to ask, can ordinary quantum mechanical tunneling (the case of the electron was discussed in Chapter 2) be counted as singularity resolution?

There is nothing resolutionary, as far as I know, about a particle tunneling through a classical barrier. The barrier is not a singularity, and is never spoken of as being "resolved" because of the particle's being able to tunnel through it. It is true that the tunneling re-establishes predictability in that quantum mechanically, one can still make predictions of events in the classically forbidden region. And hence, thinking in terms of the terminology from Chapter 4, there is a dynamical story about how the barrier does not stop the quantum particle's evolution. But singularity resolution is more than mere re-establishment of predictability, i.e. to repeat the claim from the previous chapter, there is more to quantum gravitational spacetime singularity resolution than mere dynamical avoidance. Formally, one could interpret both kinds of initial states as states of the universe's immediately "after" having appeared out of nowhere, much in the same way the ordinary quantum particle finds itself traversing the classically forbidden region to appear "on the other side". But in the quantum mechanics of the closed universe, the demonstration of continued evolution is an "improvement" only insofar as it no longer has the abruptness of the classical theory. The analogy with the case of the tunneling particle thus does not help make the case for singularity resolution via tunneling.

Considering neither of these proposals aims to resolve the "initial" singularity, the fact that they avoid the dynamical aspect (and indeed, the kinematic aspect as well, although it is not emphasized as much) is fine. But I would like to ask here whether providing a description of the initial state would ever work as a strategy for singularity resolution. Let's take a step back. Why do I rule that there is no singularity resolution "by initial conditions" in these two models? Because (a) there is no necessity that resolving the singularity must have anything to do with cosmogenesis (more on this later), and so specifying the initial state does not automatically resolve the singularity, even if it gives us information about the provenance of the universe; (b) these models have issues specific to them outlined above, like the lack of a satisfactory probability interpretation, or lack of a precise understanding of "nothingness"; (c) the specified ground state tells us nothing about why GR could not describe it. Given these, it seems to me that it is still logically possible for a specification of the initial state to at least help in singularity resolution, but only if the big bang is the beginning of the universe and the state so proposed is backed by a theory with concepts that are obviously indescribable in the language of GR.

5.2 Violating the Assumptions of the Classical Theorems

The classical singularity theorems, reviewed in the introduction, form the main supporting evidence for the argument for the existence of spacetime singularities. As such, if a theory were able to violate one or more assumptions of the theorems, would the singularities be resolved? There seem to be two types of ways in which the assumptions of the theorems are used in the context of singularity resolution. The first is a "direct attack" on the theorems, specifically aimed with the intention of resolving singularities. The direct attacks, when presented within a theory of quantum gravity, represent a "principle" of quantum gravity (more on principles of quantum gravity in a bit) which seems to state something along the lines of *the correct theory of quantum gravity will violate one or more of the assumptions of the classical singularity theorems.* Presumably, the hope is that specifically looking, in this manner, for ways to refute the assumptions of the theorems will inevitably lead to singularity resolution, and hence, the theory constructed with this being one of the principles will be a theory that resolves the singularities of GR. An example of such work is (Kuipers and Calmet, 2020), which shows that effective quantum fields can "resolve" singularities by violating the energy conditions of the classical theorems.

Another way in which the classical theorems are used is in what I shall call "indirect attacks" at their assumptions. Kiefer (2010), for instance, uses this method, in such a way that his models might violate the energy condition, but the claim for singularity "resolution" (full disclosure: Kiefer uses "avoidance", as the reader might remember from Chapter 2) does not rest solely on the violation of the assumptions of the classical theorems. The violation is more so a supporting piece of evidence for the argument for singularity avoidance than the only justification for the argument, or indeed, the definition of avoidance.

Obviously, in neither of these cases is the violation of the classical assumptions a sufficient reason for claiming that the singularities have been resolved. No theory can *resolve* singularities solely by demonstrating that the assumptions of the classical theorems fail to hold in it. Simply having a quantum gravitational account that violates, say, the energy conditions, and then stating that what GR gets wrong about reality is that it assumes these energy conditions is not the full explanation one ought to expect from a quantum gravitational resolution of the singularity. There would have to also be an account *why* it is that GR makes the wrong assumption.

Is it necessary for quantum gravitational singularity resolution that the assumptions be violated at the quantum level? It seems to me that it is impossible to speak in absolute terms about what the status of the classical theorems will be in a theory that resolves the singularities. It is logically, and indeed, physically, possible that the assumptions of the theorems do not even hold classically. As has been mentioned at several points in this dissertation, the theorems make reasonable, but not incontrovertible assumptions about our universe. It would not be a big surprise then, if it turned out that the theory that did resolve the singularities did not really violate any of the assumptions of the classical theorems. That said, the theorems form the most concrete reason for us to think there exist spacetime singularities to begin with, and hence, it is likely that the theory that resolves them will violate the assumptions of the theorems. Of course, one cannot build this theory solely out of the principle that it violates the assumptions, and hence, attacks on the assumptions, direct or indirect, are going to have to be supported by accounts about what it is that GR gets wrong in making these assumptions.

5.3 More Things to Consider

While this dissertation makes no claim of being a comprehensive review of big bang resolution, there are a few strands of thought, some of which came up in discussions within the previous chapters, which are often voiced in relation to big bang resolution, and cannot be left unexamined. These thoughts form the content of this section.

5.3.1 Are Singularity Theorems Incompleteness Theorems?

Sometimes, the classical singularity theorems are mentioned in the same breath as Gödel's incompleteness theorems, often to suggest that just as the latter show arithmetic is incomplete, the former show GR is "incomplete".[32] This is a blatantly false analogy. The singularity theorems

[32] See, for instance, (Kiefer, 2023).

state that given a set of reasonable assumptions about our universe, there have to be parts of it that cannot be described by GR.[33] The first incompleteness theorem states that every axiomatic system of arithmetic is such that it cannot, using only its own tools, prove all the true statements about arithmetic. These sound very similar, since they both talk about the limitations of a given formal theory. But there is an important sense in which Gödel's incompleteness does not apply to GR.

For Gödel, incompleteness definitively shows that for any formal system we choose, there are things about mathematics we can know for certain, but cannot be proved by the system. We do not *know* that cosmological singularities exist, it just would be likely that they do, if GR is the correct classical theory of gravity, the assumptions of the theorems hold, and if curve incompleteness can at least broadly be taken to signal singularity. And therein lies the biggest difference between the two: because we have no "intuitive" knowledge of there being a singularity, we can, quite reasonably, hope that a theory less limited in scope than GR might fix the singularity problem, i.e. a "deeper" theory could make the singularity go away without affecting any part of our "intuitions" about the world in which we live. There is no such hope for arithmetic, where the unprovability of true statements is not a matter of the "fundamentality" of the theory—Gödel showed that it was a matter of the limitations of formal methods themselves. GR, as a theory, is not incomplete. At worst, it is simply not sufficient in describing all of reality. Thus, the classical singularity theorems are, in no shape or form, incompleteness theorems, and there still is much hope in the theory that will eventually replace GR, be it something that falls under the umbrella of quantum gravity, or something else entirely, being able to resolve the singularities of GR.

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[33] Let me remind the reader here of the discussion in Section 1.1 about thinking about singularities as nothing more nor less than indications of GR breaking down.

5.3.2 On Origin Stories

Cosmogonical stories abound in the history of human thought. A relatively recent number in this series that has gained traction is what has come to be called the "big bang theory", according to which, in the beginning was a bang, and the bang was big.[34] In even more boring formulations of this theory, as we trace the history of our universe back in cosmic time, we get hotter and denser states as we approach the initial singularity, but there is not necessarily a specification of an initial state involving bangs. At any rate, in Chapter 3, we saw that string cosmology models do not take the big bang to be the beginning of our universe. Indeed, the pre-big bang model can have cyclic versions, and string cosmology research today (especially on ekpyrosis) seems to emphasize the cyclicity as an attractive quality that is featured in the string models. From the discussion of LQC in Chapter 4, especially about the "resolution" of the dynamical aspect of singularities, it is evident that LQC models cannot consider the big bang to be the beginning either. Other approaches to quantum cosmology do not seem to insist on our universe "evolving through" the big bang or going through cycles, but none of those are as developed as the approaches studied in this dissertation, so it's still too early to tell exactly what the big picture looks like in those theories. But simply because both the present "frontrunners" of quantum gravity insist on at least avoiding the dynamical aspect of the singularity by having the evolution of the universe continue through the big bang, it is worth asking ourselves, is it necessary to have cosmogonical answers to have resolved the big bang in quantum gravity?

I am inclined to speculate that resolving the big bang quantum gravitationally will not tell us anything definitive about the origins of the universe. Providing an origin story was never one of the

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[34] In his special *What It Is*, comedian Dylan Moran laments, "Look at the scientific explanation for the origin of life as we know it [...] It doesn't sound very scientific. There was a BIG BANG, and then we all came from monkeys [...] I need more than that!" (Also: "It's such a boring theory anyway. It's much more interesting if you reverse the order.")

aims of quantum gravity. Moreover, the big bang "was" (very likely) not the origin of the universe, it is just a patch of reality that GR cannot describe. Hence, singularity resolution is a goal for whatever theory replaces GR. This theory is not necessarily cyclic, but it will (almost) necessarily leave cosmogonical questions open.

5.3.3 Causes and Effects

It's not uncommon to find the big bang portrayed as an effect with an unknown cause, and the ignorance of this cause being identified with the singularity problem. For instance, Ryan and Shepley (1975) write in the introduction to their textbook on cosmology, "It is that beginning singularity that is disturbing, for it is an effect without a cause". Given how I have been arguing for singularity resolution to be the explanation of the disagreement between the classical and the quantum, one might wonder whether resolution comes from finding a quantum cause for a classical effect.

Granted that it is tempting to think of the big bang singularity as a physical effect, especially when one takes it to be a literal big bang, maybe even one that kick started our universe. However, the temptation is only a result of our conditioning ourselves to think about the physical universe in solely classical terms. It is dangerous to talk about causes and effects in the context of quantum cosmological singularity resolution, because, as we have seen multiple times in this dissertation, what we are dealing with in quantum cosmology is a timeless universe. Causal relationships, on the other hand, are necessarily temporal. If time itself only emerges at the big bang—and that it is so seems to be the consensus as of this writing (see the next section)—then we cannot talk of the "cause" that resulted in the "effect" that created time!

Whatever the quantum gravitational explanation for the classical singularity turns out to be, then, will not be a quantum gravitational "cause". What the relationship between the quantum description and the classical singularity is, is for us to find out when we resolve the singularity.

5.3.4 Spacetime Emergence

Much has been, and continues being, said about spacetime emergence, particularly spacetime emergence "at" the big bang, in the philosophy of quantum gravity literature (see, for instance, (Huggett and Wüthrich, 2018)). Evaluating such work is not a part of the agenda of this dissertation. But I would like to point out that nothing that has been said in this dissertation conflicts with the possibility of spacetime emerging "at" the big bang. The argument for spacetime emergence, however, can be, and perhaps ought to be, separated from that for singularity resolution. It may very well be that it is a principle of quantum gravity that spacetime emerges "at" the big bang, and hence the correct theory of quantum gravity will necessarily have a description of how this emergence comes to be. But showing that spacetime emerges "at" the big bang from non-spatiotemporal entities of any given theory of quantum gravity is the same as showing the big bang is resolved in that particular theory if and only if the sole area of disagreement between the theory in question and GR is that the theory, unlike GR, is not spatiotemporal.

The burden is on individual candidate theories of quantum gravity to show that the right side of this biconditional holds within these theories, if they are to claim that demonstration of emergence of spacetime is identical to the resolution of the singularity. It may very well be that for someone who is partial to an approach to quantum gravity that quantizes spacetime, it is obvious that demonstration of spacetime emergence is identical to the demonstration of singularity resolution, but such an identification is not necessary, and especially not necessary if what we ought to quantize are not the classical spacetime degrees of freedom. In summary, then, spacetime emergence has little to no bearing on everything that has been said in this dissertation.

5.4 What Have We Learned About Quantum Gravity?

A while ago, addressing an audience consisting of philosophers including yours truly, physicist Lee Smolin presented an overview of a wide range of approaches to quantum gravity and asked the philosophers to help the physicists "get out of this mess that we're in [i.e. with so many candidate theories, and not getting much close to choosing one of them as the 'correct' theory of quantum gravity]" (or something to that effect). In a sense, this project was created in response to this plea for help. I figured, if we could clarify what it is precisely that we expect quantum gravity to do when we expect it to resolve the big bang, then we might be able to winnow down our options based on which of the extant candidate theories accomplish resolution in the right sense. However, what I have found out is that the extant theories either do not have anything to say about resolution yet, or, if they do, they take "resolution" to mean surface-level, formal repair, which, I have been stressing throughout this dissertation, is not what a theory that is meant to replace GR ought to accomplish by way of resolution. My apologies to the physicists for not being of more help.

Put another way, consider the statement,

$\mathcal{P}$: *The correct theory of quantum gravity will necessarily resolve the initial cosmological singularity.*

Could $\mathcal{P}$ be a principle of quantum gravity?[35] By this, I mean, despite the fact that the present state of affairs does not let us narrow down the candidate theories of quantum gravity, could we hope that we could choose from among our options once they attempt to achieve resolution in the right sense? I have been claiming that resolution will be in the explanation of the division between the classical and the quantum. Once this explanation exists, there would not need to be a separate argument for resolution, since the explanation *is* the resolution. As such, $\mathcal{P}$ is not going to help us choose from among potential candidates. One looking for the "correct" theory of quantum gravity should, thus, do exactly the opposite of what this dissertation has done, s/he should not focus on singularity resolution. More precisely, s/he should avoid targeting singularity resolution in the

[35] Crowther (2018) differentiates the roles that principles could take in theory-development and assessment into several, not necessarily sharpily distinct categories. What I mean by "principle" is more or less what Crowther means by "fallible constraints", but they are necessary, I think.

hope that resolved singularities will lead to quantum gravity. Wherever your quantum theory of gravity is, there you will find your singularity resolution also, but not the other way around. Like the old joke about a drunkard looking for his keys far away from where he lost them because that was where the light was, all the proposed ways to resolve the big bang so far seem to use the tools that they do have access to, to make formal accomplishments that they deem to be resolution, instead of focusing on developing the as yet undeveloped parts of their respective theories where the actual resolution might be found.

5.5 The Last Word

What I have attempted to show in this dissertation is that the extant approaches to quantum gravity and their respective applications to cosmology, insofar as they say anything at all about big bang resolution, do not say quite the right thing about it expected of them as potential replacements for GR. Along the way, several philosophical issues with the formal aspects of the various models came to light. But the most useful takeaway from this entire exercise, it seems to me, would be the distinction between singularity resolution and avoidance, and I would like to end by recapitulating some of what has been said in this regard, but also by exploring if the two are logically related.

Resolution, for the last time, when expected of a quantum theory of gravity meant to replace GR, constitutes an explanation of what exactly it is about reality that is beyond the powers of the classical theory to describe, that the quantum theory is nevertheless capable of describing. It is impossible to provide a historical example of singularity resolution because no spacetime singularity has ever been resolved. All I can say is, resolution will not be describable in the language of GR, be it in terms of avoiding infinite curvature or in terms of continuance of classical evolution.

Avoidance implies some kind of kinematic patching up in some cases, or perhaps a dynamical repair in other cases, or both, or neither and instead, some other way of avoiding to resolve the singularity. There is no explicit list of all the attempts that count as avoidance. Everything that is not resolution, that formally gets rid of infinities or abruptness or other things about singularities that people might (rationally or not) worry about, counts as avoidance.

As such, it is not necessary at all that resolution implies avoidance. A resolved "singularity" in quantum gravity, such as it is, could involve infinite quantities, for instance. On the other hand, it is also not impossible to imagine that some kind of avoidance would follow from resolution, and hence, as far as I can tell, there is no obvious logical relationship between resolution and avoidance. The theory poised to replace GR might thus do well to avoid being led down rabbit holes in search of irrelevant formal band-aids. Such is my conviction.

CITED LITERATURE

Emanuele Alesci and Francesco Cianfrani. Quantum Reduced Loop Gravity and the foundation of Loop Quantum Cosmology. *International Journal of Modern Physics D*, 25(08):1642005, July 2016. ISSN 0218-2718, 1793-6594. doi: 10.1142/S0218271816420050.

Emanuele Alesci, Gioele Botta, Francesco Cianfrani, and Stefano Liberati. Cosmological singularity resolution from quantum gravity: The emergent-bouncing universe. *Physical Review D*, 96(4): 046008, August 2017. ISSN 2470-0010, 2470-0029. doi: 10.1103/PhysRevD.96.046008.

Emanuele Alesci, Aurélien Barrau, Gioele Botta, Killian Martineau, and Gabriele Stagno. Phenomenology of Quantum Reduced Loop Gravity in the isotropic cosmological sector. *Physical Review D*, 98(10):106022, November 2018. ISSN 2470-0010, 2470-0029. doi: 10.1103/PhysRevD.98.106022.

Maria da Conceicao Bento and Orfeu Bertolami. Scale Factor Duality: A Quantum Cosmological Approach. *Classical and Quantum Gravity*, 12(8):1919–1925, August 1995. ISSN 0264-9381, 1361-6382. doi: 10.1088/0264-9381/12/8/009.

William Blyth and Chris Isham. Quantization of a Friedmann universe filled with a scalar field. *Physical Review D*, 11(4):768–778, February 1975. ISSN 0556-2821. doi: 10.1103/PhysRevD.11.768.

Martin Bojowald, Claus Kiefer, and Paulo Vargas Moniz. Quantum Cosmology for the XXIst Century: A Debate. In *The Twelfth Marcel Grossmann Meeting*, pages 589–608. World Scientific, 2012. ISBN 978-981-4374-51-4. doi: 10.1142/9789814374552_0030.

Ram Brustein and Gabriele Veneziano. The Graceful Exit Problem in String Cosmology. *Physics Letters B*, 329(4):429–434, June 1994. ISSN 03702693. doi: 10.1016/0370-2693(94)91086-3.

Gianluca Calcagni. *Classical and Quantum Cosmology*. Graduate Texts in Physics. Springer International Publishing, Cham, 2017. ISBN 978-3-319-41125-5 978-3-319-41127-9. doi: 10.1007/978-3-319-41127-9.

Karen Crowther. Defining a crisis: The roles of principles in the search for a theory of quantum gravity. *Synthese*, October 2018. ISSN 0039-7857, 1573-0964. doi: 10.1007/s11229-018-01970-4.

Erik Curiel. The Analysis of Singular Spacetimes. *Philosophy of Science*, 66:S119–S145, 1999. ISSN 0031-8248, 1539-767X. doi: 10.1086/392720.

Bryce DeWitt. Quantum Theory of Gravity. I. The Canonical Theory. *Physical Review*, 160(5): 1113–1148, 1967. ISSN 0031-899X. doi: 10.1103/PhysRev.160.1113.

John Earman. *Bangs, Crunches, Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*. Oxford University Press, Oxford, New York, November 1995. ISBN 978-0-19-509591-3.

Rodolfo Gambini and Jorge Pullin. *A First Course in Loop Quantum Gravity*. Oxford University Press, Oxford, New York, November 2011. ISBN 978-0-19-959075-9.

Maurizio Gasperini. *Elements of String Cosmology*. Cambridge University Press, July 2007. ISBN 978-1-139-46631-8.

Maurizio Gasperini and Gabriele Veneziano. Pre-Big-Bang in String Cosmology. *Astroparticle Physics*, 1(3):317–339, July 1993. ISSN 09276505. doi: 10.1016/0927-6505(93)90017-8.

Maurizio Gasperini, Jnanadeva Maharana, and Gabriele Veneziano. Graceful exit in quantum string cosmology. *Nuclear Physics B*, 472(1-2):349–360, July 1996. ISSN 05503213. doi: 10.1016/0550-3213(96)00201-5.

Kurt Gödel. A remark about the relationship between relativity theory and idealistic philosophy. In Paul Arthur Schilpp, editor, *Albert Einstein: Philosopher-Scientist*. Harper & Row, 1949.

Alan Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23(2):347–356, January 1981. doi: 10.1103/PhysRevD.23.347.

James Hartle and Stephen Hawking. Wave function of the Universe. *Physical Review D*, 28(12): 2960–2975, December 1983. ISSN 0556-2821. doi: 10.1103/PhysRevD.28.2960.

Stephen Hawking and Don Page. Operator ordering and the flatness of the universe. *Nuclear Physics B*, 264:185–196, 1986. ISSN 0550-3213. doi: 10.1016/0550-3213(86)90478-5.

Stephen William Hawking, Roger Penrose, and Hermann Bondi. The singularities of gravitational collapse and cosmology. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 314(1519):529–548, January 1970. doi: 10.1098/rspa.1970.0021.

Carl Hempel. *Philosophy of Natural Science*. Prentice-Hall, 1966. ISBN 978-0-13-663823-0.

Nick Huggett. Target Space $\neq$ Space. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 59:81–88, August 2017. ISSN 13552198. doi: 10.1016/j.shpsb.2015.08.007.

Nick Huggett and Christian Wüthrich. The (A)temporal Emergence of Spacetime. *Philosophy of Science*, 85(5):1190–1203, December 2018. ISSN 0031-8248. doi: 10.1086/699723.

Nemanja Kaloper, Richard Madden, and Keith Olive. Towards a singularity-free inflationary universe? *Nuclear Physics B*, 452(3):677–702, October 1995. ISSN 0550-3213. doi: 10.1016/0550-3213(95)00398-C.

Justin Khoury, Burt Ovrut, Paul Steinhardt, and Neil Turok. The Ekpyrotic Universe: Colliding Branes and the Origin of the Hot Big Bang. *Physical Review D*, 64(12):123522, November 2001. ISSN 0556-2821, 1089-4918. doi: 10.1103/PhysRevD.64.123522.

Claus Kiefer. Can singularities be avoided in quantum cosmology? *Annalen der Physik*, 19(3-5): 211–218, 2010. ISSN 1521-3889. doi: 10.1002/andp.201010417.

Claus Kiefer. *Quantum Gravity.* Oxford University Press, third edition, 2012.

Claus Kiefer. What does it mean for our world-view if we assume with gödel the non-existence of time? In Oliver Passon, Christoph Benzmüller, and Brigitte Falkenburg, editors, *On Gödel and the Nonexistence of Time – Gödel und die Nichtexistenz der Zeit: Kurt Gödel Essay Competition 2021 – Kurt-Gödel-Preis 2021*. Springer Spektrum, 2023.

Claus Kiefer and Nick Kwidzinski. Singularity avoidance in anisotropic quantum cosmology. *Journal of Physics: Conference Series*, 1275:012011, 2019. ISSN 1742-6588, 1742-6596. doi: 10.1088/1742-6596/1275/1/012011.

Claus Kiefer, Nick Kwidzinski, and Dennis Piontek. Singularity avoidance in Bianchi I quantum cosmology. *The European Physical Journal C*, 79(8):686, 2019. ISSN 1434-6044, 1434-6052. doi: 10.1140/epjc/s10052-019-7193-6.

Karel Kuchař. Time and Interpretations of Quantum Gravity. *International Journal of Modern Physics D*, 20(supp01):3–86, 2011. ISSN 0218-2718, 1793-6594. doi: 10.1142/S0218271811019347.

Karel Kuchař and Michael Ryan. Can mini-superspace quantization be justified? In Humitaka Sato and Takashi Nakamura, editors, *Gravitational Collapse and Relativity: Proceedings of Yamada Conference XIV*, pages 451–465. World Scientific, 1986.

Karel Kuchař and Michael Ryan. Is minisuperspace quantization valid?: Taub in mixmaster. *Phys.Rev.*, D40:3982–3996, 1989. doi: 10.1103/PhysRevD.40.3982.

Folkert Kuipers and Xavier Calmet. Singularity theorems in the effective field theory for quantum gravity at second order in curvature. (arXiv:1911.05571v1 [gr-qc]). *arXiv General Relativity and Quantum Cosmology*, 2020. doi: arXiv:1911.05571v1.

Charles Misner. Absolute Zero of Time. *Physical Review*, 186(5):1328–1333, 1969a. doi: 10.1103/PhysRev.186.1328.

Charles Misner. Quantum Cosmology. I. *Physical Review*, 186(5):1319–1327, October 1969b. doi: 10.1103/PhysRev.186.1319.

Charles Misner, Kip Thorne, and John Wheeler. *Gravitation.* Princeton University Press, 2017. ISBN 978-1-4008-8909-9.

Carlo Rovelli and Francesca Vidotto. *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory.* Cambridge University Press, Cambridge, 2014. ISBN 978-1-107-06962-6. doi: 10.1017/CBO9781107706910.

Michael Ryan and Lawrence Shepley. *Homogeneous Relativistic Cosmologies*. Princeton Series in Physics. Princeton University Press, Princeton, N.J, 1975. ISBN 978-0-691-08146-5.

Parampreet Singh. Are loop quantum cosmos never singular? *Classical and Quantum Gravity*, 26 (12):125005, June 2009. ISSN 0264-9381, 1361-6382. doi: 10.1088/0264-9381/26/12/125005.

Gerrit Smith and Robert Weingard. Quantum Cosmology and the Beginning of the Universe. *Philosophy of Science*, 57(4):663–667, 1990. doi: 10.1086/289585.

Lee Smolin. *Three Roads to Quantum Gravity*. Basic Books, 2000. ISBN 978-0-465-07835-6.

Gabriele Veneziano. Scale factor duality for classical and quantum strings. *Physics Letters B*, 265 (3-4):287–294, August 1991. ISSN 0370-2693. doi: 10.1016/0370-2693(91)90055-U.

Alexander Vilenkin. Creation of Universes from Nothing. *New Scientist*, 192(2578):51, 1982. ISSN 02624079. doi: 10.1016/S0262-4079(06)61121-4.

Alexander Vilenkin. Quantum creation of universes. *Physical Review D*, 30(2):509–511, July 1984. ISSN 0556-2821. doi: 10.1103/PhysRevD.30.509.

Alexander Vilenkin. Boundary conditions in quantum cosmology. *Physical Review D*, 33(12): 3560–3569, 1986. ISSN 0556-2821. doi: 10.1103/PhysRevD.33.3560.

Alexander Vilenkin. Quantum cosmology and the initial state of the Universe. *Physical Review D*, 37(4):888–897, February 1988. ISSN 0556-2821. doi: 10.1103/PhysRevD.37.888.

Robert Wald. *General Relativity*. University of Chicago Press, May 2010. ISBN 978-0-226-87037-3.

Niranjana Warrier. The case of the vanishing wavefunction. *Studies in History and Philosophy of Science*, 96:135–140, December 2022. ISSN 0039-3681. doi: 10.1016/j.shpsa.2022.09.015.

Christian Wüthrich. *Approaching the Planck Scale from a Generally Relativistic Point of View: A Philosophical Appraisal of Loop Quantum Gravity*. University of Pittsburgh ETD, University of Pittsburgh, 2006.

Palle Yourgrau. *Gödel Meets Einstein: Time Travel in the Gödel Universe*. Open Court, 1999.

APPENDIX

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VITA

NAME	Niranjana Warrier
EDUCATION	Ph.D, Philosophy, University of Illinois at Chicago, Chicago, IL, 2023 M.S., Physics, University of Illinois at Chicago, Chicago, IL, 2023 M.A., Philosophy, University of Illinois at Chicago, Chicago, IL, 2018 B.A., Philosophy, Physics, Brandeis University, Waltham, MA, 2017
PUBLICATION	Niranjana Warrier. The case of the vanishing wavefunction. *Studies in History and Philosophy of Science* 96: 135-40, 2022.
PRESENTATIONS	"Singularity Resolution: The Case of the Vanishing Wavefunction," Biennial Meeting of the Philosophy of Science Association, Baltimore, 2021. "A Philosopher Looks at Quantum Cosmology," UIC Philosophy Work in Progress, Chicago, 2019.
TEACHING	Primary Instructor, *Introductory Logic*, Spring 2022 Primary Instructor, *Introductory Logic*, Fall 2021 Teaching Assistant to Prof. David Hilbert, *Introduction to the Philosophy of Science*, Spring 2020 Teaching Assistant to Prof. Nick Huggett, *Introduction to the Philosophy of Science*, Fall 2019 Teaching Assistant to Prof. Georgette Sinkler, *Introduction to Philosophy*, Spring 2019 Teaching Assistant to Prof. Aidan Gray, *Introductory Logic*, Fall 2018
RECOGNITION	Dean's Scholar Fellowship, University of Illinois at Chicago, 2022-23 Provost's Graduate Internship Award, University of Illinois at Chicago, 2020 University Fellowship, University of Illinois at Chicago, 2017-21 Doris Brewer Cohen Award in the Humanities for the best senior thesis, Brandeis University, 2017 Philosophy Prize, Brandeis University, 2017 South Asian Scholarship, Brandeis University, 2015-17 Alumni and Friends Scholarship, Brandeis University, 2014-15