A Modified Otsu Image Segment Method Based on the Rayleigh Distribution

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***Abstract*—Image segmentation by thresholding is a usual way in im- age processing and analysis. With some measures of differ- ence between images, some new methods for image thresh- old selection are put forward based on the principle that the difference between two parts from an good thresholding segmentation should be big and the differences between original image and two parts are both big.The OTSU algorithm(Maximization of interclass variance)is one of the superior threshold selection methods.Otsu's method of im- age segmentation selects an optimum threshold by maxi- mizing the between-class variance in a gray image. Un- der studying the principle of Otsu method, we found it still deals directly with the gray-level histogram by parametric techniques, and the histogram is approximated in the least square sense by a sum of Gaussian distributions. However, the low-bandwidth Gaussian randomized procedure will be a more excellent model because of the low-bandwidth fre- quency response of the image transmission and acquisition system. In this case, the object and the background in im- age obey Rayleigh distributions, an improved threshold im- age segmentation algorithm based on the Otsu method is developed.The new improved algorithm takes into account that the object and the background in image obey Rayleigh distributions, and the maximum between-cluster variance is modified based on the model.From the experiment, the results show that the new improved algorithm has these ad- vantages such as high segmentation precision and fast com- putation speed.**

I. INTRODUCTION

Image segment is a basic component of many computer vision systems. Automatic thresholding is an important technique in image segmentation and machine vision applications. The basic idea of automatic thresholding is to automatically select an optimal gray-level threshold value for separating objects of interest in an image from the background based on their gray-level distribution. While humans can easily differentiable an object from complex background, and image thresholding is a difficult task for separate them. The gray-level histogram of an image is usu- ally considered as efficient tools for development of image thresholding algorithms. The main objective is to deter- mine a threshold for bi-level thresholding or several thresh- olds for multi-level thresholding for image segmentations. Several algorithms of multi-level thresholding have been proposed in literature that included the works of Kapur et al [1], Otsu [2] and fast Ostu's implementation [3]. Among the tremendous amount of image thresholding tech- niques, entropy-based approaches have drawn many atten- tions. Yin

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[4] proposed a new method that adopts the parti- cle swarm optimization to select the thresholds based on the minimum cross-entropy. Madhubanti et al. Usesthe hybrid cooperative-comprehensive learning PSO algorithm based on maximum entropy criterion [5].

As a classical image segmentation method, Otsu adaptive threshold algorithm has been applied widely in image processing.Otsu's method [2] had been proposed in 1979, and it is deduced by least square(LS) method based on gray histogram. As we all known, this method not only has the best threshold value in the statistical sense, but also is the most stable method in the image threshold segmentation at present. In fact,Otsu's method still deals directly with the gray-level histogram by parametric techniques, and the histogram is approximated in the least square sense by a sum of Gaussian distributions. Moreover, in many cases, the Gaussian distributions turn out to be a meager approximation of the real modes.The low-bandwidth Gaussian randomized procedure will be a more excellent model because of the frequency response of the image transmission and acquisition system. So the Rayleigh distributions of histogram can be adopted.In this paper, we proposed a modified Otsu image segment method based on the Rayleigh model.

II. THE OTSU'S METHOD

Let the pixels of a given picture be represented in L gray levels $[1; 2; \cdots; L]$ .The number of pixels at level i is denoted by n_i and the total number of pixels by $N = n_1 + n_2 + \cdots; +n_L$.In order to simplify the discussion, probability distribution:

$$p_i = \frac{n_i}{N}, p_i \geq 0, \sum_{i=1}^{L} p_i = 1$$

,Now suppose that we dichotomize the pixels into two classes C_0 and C_1 (background and objects, or vice versa)by a threshold at level k ,C_0 denotes pix- els with levels$[1; 2; \cdots; k]$,and C_1 denotes pixels with levels$[k + 1; \cdots ; L]$.Then the probabilities of class occur- rence and the class mean levels, respectively, are given by:

$$\omega 0 = p_r(C_0) \sum_{i=1}^{L} p_i = \omega(k)$$

$$\omega 1 = pr(C_1) \sum_{i=k+1}^{L} pi = 1 - \omega(k)$$

, and

$$\mu_0 = \sum_{i=1}^{k} iPr(i|C_0) = \sum_{i=1}^{k} i\frac{p_i}{\omega_0} = \frac{\mu(k)}{\omega(k)}$$

$$\mu_1 = \sum_{i=k+1}^{L} iPr(i|C_1) = \sum_{i=k+1}^{L} i\frac{p_i}{\omega_1} = \frac{\mu_T - \mu(k)}{1 - \omega(k)}$$

where

$$\omega(k) = \sum_{i=1}^{k} p_i$$

and

$$\mu(k) = \sum_{i=1}^{k} ip_i$$

are the zeroth- and the first-order cumulative moments of the histogram up to the kth level,respectively, and

$$\mu_T = \mu(L) \sum_{i=1}^{L} ip_i$$

is the total mean level of the original picture. We can easily verify the following relation for any choice of k:

$$\mu 0 \omega 0 + \mu 1 \omega 1 = \mu_T, \omega 0 + \omega 1 = 1$$

The class variances are given by:

$$\sigma_0^2 = \sum_{i=1}^{k} (i - \mu_0)^2 Pr(i|C_0) = \sum_{i=1}^{k} (i - \mu_0)^2 \frac{p_i}{\omega_0}$$

$$\sigma_1^2 = \sum_{i=k+1}^{L} (i - \mu_1)^2 Pr(i|C_1) = \sum_{i=k+1}^{L} (i - \mu_1)^2 \frac{p_i}{\omega_1}$$

These require second-order cumulative moments (statistics). In order to evaluate the "goodness" of the threshold (at level k), we shall introduce the following discriminant criterion measures (or measures of class separability) used in the discriminant analysis:

$$\eta = \frac{\sigma_B^2}{\sigma_T^2}$$

where

$$\sigma_W^2 = \omega 0 \sigma_0^2 + \omega 1 \sigma_1^2$$

$$\sigma_B^2 = \omega_0(\mu_0 - \mu_T)^2 + \omega_1(\mu_1 - \mu_T)^2 = \omega_0\omega_1(\mu_1 - \mu_0)^2$$

and

$$\sigma_T^2 = \sum_{i=1}^{L} (i - \mu_T)^2 p_i$$

are the within-class variance,the between-class variance, and the total variance of levels, respectively. Then our prob- lem is reduced to an optimization problem to search for a threshold k that maximizes one of the object functions (the criterion measures). This standpoint is motivated by a conjecture that well-thresholded classes would be separated in gray levels, and conversely, a threshold giving the best separation of classes in gray levels would be the best threshold. Because the following basic relation always holds:

$$\sigma_W^2 + \sigma_B^2 = \sigma_T^2$$

It is noticed tha σ_W^2 and σ_B^2 are functions of threshold level *k*,but σ_T^2 is independent of *k*.It is also noted that σ_W^2 is based on the second-order statistics (class variances), while σ_W^2 is based on the first-order statistics (class means). Therefore, η is the simplest measure with respect to *k*.Thus we adopt η as the criterion measure to evaluate the "goodness" (or sep- arability) of the threshold at level *k*.

The optimal threshold *k**that maximizes η ,or equivalently maximizes σ_B^2,is selected in the following sequential search by using:

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_T^2}$$

$$\sigma_B^2(k) = \frac{[\mu_T\omega(k) - \mu(k)]^2}{\omega(k)[1 - \omega(k)]}$$

and the optimal threshold *k**is

$$\sigma_B^2(k^*) = max\sigma_B^2(k), 1 \le k < L$$

III. A Modified Otsu Image Segment Method Based On The Rayleigh Model

A. Image Distribution

The Gaussian probability distribution is perhaps the most used distribution in all of science.Sometimes it is called normal distribution and described as:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

η =mean of distribution (also at the same place as mode and median), σ^2 =variance of distribution,*x* is a continuous variable $(-\infty \le x \le \infty)$.

In many cases, the Gaussian distributions turn out to be a meager approximation of the real modes.The images could been described as low-bandwidth Gaussian procedure through the transmission system.So images could been considered as the Rayleigh distributions.

We obtain the well known Rayleigh model for the ampli- tude distribution:

$$f(x) = \frac{x}{\lambda^2} e^{-\frac{x^2}{2\lambda^2}}$$

$(\frac{\pi\sigma}{2})^{\frac{1}{2}}$ =mean of distribution (also at *x* is a continuous vari-able $(0 < x)$.

An image, as a Gaussian procedure, supplies into a low-bandwidth linearly system, then the output will obey Rayleigh distributions, and the parameters of these two model satisfy the following equations:

$$\lambda^2 = \frac{\mu^2 + \sigma^2}{2}$$

where η and σ are mean and variance of Gaussian proce-dure, λ is parameter of Rayleigh model.

B. Improvement of Otsu Algorithm

In order to evaluate the "goodness" of the threshold, I propose the following discriminant criterion measures (or measures of class separability) used

$$\lambda_B^2 = \omega_0(\lambda_0 - \lambda_T)^2 + \omega_1(\lambda_1 - \lambda_T)^2 = \omega_0\omega_1(\lambda_1 - \lambda_0)^2$$

, where

$$\lambda_T^2 = \frac{\mu_T^2 + \sigma_T^2}{2}$$

, and

$$\lambda_0^2 = \frac{\mu_0^2 + \sigma_0^2}{2}$$

,

$$\lambda_1^2 = \frac{\mu_1^2 + \sigma_1^2}{2}$$

and the optimal threshold k*is

$$\lambda_B^2(k^*) = max\lambda_B^2(k), 1 \le k < L$$

IV. Experiments and Results

Several examples of experimental results are shown in Figs. 1-3. Throughout these figures, Figs. 1(First row) are original gray-level pictures;Figs. 2(Second row) are segment results using Otsu;Figs. 3(Third row) are segment results using Improvement of Otsu.The numbers of gray levels of the original pictures are all 256 in Figs. 1-3. The ob-

Figure 1. The original images of cameraman,lena and 1573

Figure 2. The segment results using Otsu

Figure 3. The segment results using Improve-ment of Otsu

ject function $\lambda_B^2(k)$ is always smooth and unimodal, as can be seen in the experimental results in Figs.3.It may attest to an advantage of the suggested criterion and may also imply the stability of the method. All the parameters in computa- tion are shown in the Table.1 and Table.2

TABLE I. THE RESULTS USING OTSU ALGORITHM

	$\sigma0$	$\sigma1$	σB	K*
cameraman	3.8862e+003	2.1916e+004	0.2751	88
Lenan	2.2804e+003	2.1122e+004	0.2875	101
1573	1.3898e+003	1.7240e+004	61.9323	149

TABLE II. THE RESULTS USING IMPROVED ALGORITHM

	$\lambda0$	$\lambda1$	λB	K*
cameraman	8.9906e+003	4.2963e+004	0.1930	77
Lenan	6.6249e+003	3.9843e+004	0.2132	95
1573	7.1744e+003	3.7420e+004	46.0170	176

V. Conclusions

A method to select a threshold automatically from a gray level histogram has been derived from the view- point of discriminant analysis. A image will be a low- bandwidth Gaussian randomized procedure, because of the low-bandwidth frequency response of the image transmis- sion and acquisition system, and the object and the back- ground in image obey Rayleigh distributions.In this paper, we proposed a modified Otsu image segment method based on the Rayleigh model. This directly deals with the prob- lem of evaluating the goodness of thresholds. An optimal threshold is selected by the discriminant criterion; namely, by maximizing the discriminant measure λB . The proposed method is characterized by its nonparametric and unsupervised nature of threshold selection.The range of its applications is not restricted only to the thresholding of the gray-level picture, such as specifically described in the foregoing, but it may also cover other cases of unsupervised classification in which a histogram of some characteristic (or feature) discriminative for classifying the objects is available.So this method may be used as a simple and standard one for automatic threshold selection that can be applied to various practical problems.

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