CS 763: Assignment 3

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March 14, 2016

Problem 1

$$I = L\rho \frac{pl_x + ql_y + 1}{\sqrt{l_x^2 + l_y^2 + 1}\sqrt{p^2 + q^2 + 1}} \tag{1}$$

As the geometry is known, we know p and q at each point. We can eliminate the L and ρ terms which are unknown by dividing the equations of intensities of 2 points.

$$\frac{I_i}{I_j} = \frac{p_i l_x + q_i l_y + 1}{p_j l_x + q_j l_y + 1} \frac{\sqrt{p_j^2 + q_j^2 + 1}}{\sqrt{p_i^2 + q_i^2 + 1}} \tag{2}$$

$$\therefore \left(I_i p_j \sqrt{p_i^2 + q_i^2 + 1} - I_j p_i \sqrt{p_j^2 + q_j^2 + 1}\right) l_x + \left(I_i q_j \sqrt{p_i^2 + q_i^2 + 1} - I_j q_i \sqrt{p_j^2 + q_j^2 + 1}\right) = I_j \sqrt{p_j^2 + q_j^2 + 1} - I_i \sqrt{p_i^2 + q_i^2 + 1} \tag{3}$$

Taking N such pairs of points in the image, we have a system of equations of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} \tag{4}$$

which can be solved using pseudoinverse:

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}b \tag{5}$$

Problem 2

$$R(p,q) = \frac{1 + pp_s + qq_s}{\sqrt{1 + p_s^2 + q_s^2}\sqrt{1 + p^2 + q^2}} \tag{6}$$

$$= \mathbf{l}^T\mathbf{N} \tag{7}$$

where

$$\mathbf{l} = \frac{1}{\sqrt{p_s^2 + q_s^2 + 1}} \begin{pmatrix} -p_s \\ -q_s \\ 1 \end{pmatrix} \text{ and } \mathbf{N} = \frac{1}{\sqrt{p^2 + q^2 + 1}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix} \tag{8}$$

As $R(p,q)$ is a dot product defined as above, it is maximized when surface normal is parallel to the light source direction, i.e. when $p = p_s$ and $q = q_s$ and it is 0 when $p = -p_s$ and $q = -q_s$.