A Partial Identification Approach to Identifying the Determinants of Human Capital Accumulation: An Application to Teachers

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Abstract

Teacher quality exhibits substantial growth over teachers' careers, but why it improves is not well understood. I use a human capital production function nesting On-the-Job-Training (OJT) and Learning-by-Doing (LBD) and experimental variation from Glewwe et al. (2010), a teacher incentive pay experiment in Kenya, to discern the presence and relative importance of these forces. The identified set for the OJT and LBD components has a closed-form solution, which depends on experimentally estimated average treatment effects. I find that the LBD component is indeed present in the human capital production function, and also estimate an informative upper bound on the OJT component.

Keywords: partial identification, human capital, teacher quality, on-the-job training, learning-by-doing

JEL codes: I2, I28, J2, J24, J45, C1

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1 Introduction

Teacher quality, typically measured by a teacher's value added to student achievement, is an important determinant of student achievement and economic growth (Rivkin et al., 2005; Hanushek, 2011). While the research and policy focus has typically been on cross-sectional variation in teacher quality, researchers have also documented substantial growth in quality over teachers' careers (Harris and Sass, 2011; Wiswall, 2013; Papay and Kraft, 2015). For example, using data from North Carolina, Wiswall (2013) finds that the average growth in teacher quality over 35 years of experience is equal to almost one standard deviation of the cross-sectional teacher quality among novice teachers.

While the growth in quality over teachers' careers is clearly important, little is known about why it occurs. Economists have developed two main theories to explain how workers accumulate human capital, which is then used to produce output: *On-the-Job Training* (OJT) and *Learning-by-Doing* (LBD). In the "pure OJT" model, workers (in the current application, teachers) allocate work time away from production (e.g., classroom teaching) to invest in their human capital (e.g., teachers' professional development) (Becker, 1964; Ben-Porath, 1967). This model implies a tradeoff between current and future production because of the multitask problem (Hölmstrom and Milgrom, 1991), which emerges because OJT investments are unobserved and, therefore, noncontractible. This tradeoff is not present under the "pure LBD" model, wherein workers accumulate human capital via the act of production (Rosen, 1972; Weiss, 1972; Blinder and Weiss, 1976).[1]

The importance of these potential determinants of human capital has implications for the design of effective education policy, and, more generally, optimal employee compensation in the presence of market failures. For example, if OJT was the main driver of teacher human capital accumulation then an incentive pay scheme that increased current achievement could reduce long-run teacher quality, by diverting teachers from making human capital investments. This would not be the case if instead LBD were the dominant force. More generally, the importance of OJT vs. LBD forces has also been shown to have implications for the design of tax policy (Heckman et al., 2002; Blandin, 2018) and even the steady state of an economy (Hansen and İmrohoroğlu, 2009).

Unfortunately, we do not know about the importance of OJT versus LBD forces among any type of worker, let alone teachers. Killingsworth (1982) shows there are no clear general testable implications of the OJT versus LBD theories of human capital accumulation when using standard observational data on workers. Additionally, as Heckman et al. (2002) discuss, identification in such contexts is further hampered by equilibrium re-adjustments that occur in the private sector.[2] This

[1] For an example of an OJT-intensive job, consider being a student in school. For an example of an LBD-intensive job, consider "chicken sexing", where workers gain an ability to determine the sex of baby chickens through their experience sorting chicks (McWilliams, 2018).

[2] Belley (2017) uses "excess variation" in wage growth to test for the presence of a non-LBD determinant in human capital production, rejecting the specifications lacking non-LBD determinant for women; he is careful not to ascribe this variation to the presence of OJT. Researchers using the OJT framework (see, e.g., Haley, 1976; Heckman, 1976) typically circumvent the fact that OJT investments are unobserved by solving for the optimal investment path in a worker's dynamic program, which yields estimates of the importance of OJT in the technology of human capital

has left researchers and policymakers lacking even basic, qualitative information about the presence of OJT versus LBD forces for workers of any type. This helps explain why prior empirical research has adopted "pure" specifications, only allowing for either OJT or LBD as the force generating human capital.[3]

To fill this gap, this paper develops a framework that yields new findings about the roles played by OJT and LBD forces in the technology governing teacher human capital accumulation. In light of the prominent identification difficulties, I adopt a partial identification approach. The identified set contains all of the values of the OJT and LBD components of teacher human capital accumulation consistent with the data, which can then be projected onto the marginal identified set for either component. I estimate the identified set using data from an experimental intervention, Glewwe et al. (2010), which studies a teacher performance pay scheme enacted across many sites across Kenya. Labor prices are often fixed in the education sector because teachers are typically paid according to public salary schedules (Podgursky and Springer, 2011) and worker-specific output is measured, even if noisily so, making teacher quality trajectories a particularly attractive context for discriminating between OJT and LBD.

The starting point is workhorse models that have become ubiquitous in empirical research over the last five decades. The value-added (VA) model (Hanushek, 1971; Murnane, 1975), which measures the contribution of different educational inputs to the production of student achievement, is used in the overwhelming mass of research on teacher quality (for more recent examples, see Kane et al., 2013; Chetty et al., 2014).[4] The literature studying human capital accumulation has from its inception typically used log-linear specifications when considering either OJT (Brown, 1976; Haley, 1976; Heckman, 1976) or LBD (Blinder and Weiss, 1976), and these specifications continue to dominate in more recent research (see, e.g., Heckman et al., 1998; Fan et al., 2015; Blandin, 2018).[5]

I extend the modal specifications in the literature to nest both OJT and LBD forces. The resulting specification leverages experimentally estimated average treatment effects (ATEs) to report key information about average tendencies and guarantees that identification is not driven by assumed nonlinearities in functional forms, while not being very restrictive compared to specifications typically used in the literature. As is typically the case in the literature, I model investment and production inputs as unobserved.[6] This closely matches the teacher quality application, where vari-

accumulation. See Kuruscu (2006) for an example using a different approach.

[3]This is also true of recent research studying heterogeneity in the returns to experience for teachers (Kraft and Papay, 2014) or other types of workers (see, e.g., Shaw and Lazear, 2008; Haggag et al., 2017). Here, too, the distinction between OJT versus LBD has implications for policy, as heterogeneous impacts of interventions would be exacerbated or attenuated, depending on how human capital was generated.

[4]See McCaffrey et al. (2003) and Hanushek and Rivkin (2012) for detailed discussions.

[5]As discussed by Willis (1985), these specifications for human capital production had already had a long history of use in labor economics as of three decades ago. These specifications are also consistent with those in the statistical literature spawned by Abowd et al. (1999). See Shaw (1989) for an empirical model of LBD using a different specification and see Fu et al. (2021) for an empirical model of OJT using a different specification.

[6]Kuruscu (2006), who considers general worker contexts, argues that it is more appropriate to treat on-the-job investments as unobserved because (at least) some training may not be observed by researchers. While this

ables that might seem like natural measures of training, such as formal professional development, or additional certification or education (e.g., Master's degrees), do not help predict teacher quality (Hanushek, 2003; Hanushek and Rivkin, 2006; Harris and Sass, 2011; Podgursky and Springer, 2011; Jackson et al., 2014), and the general lack of availability of data on how teachers allocate their work time (Hanushek and Rivkin, 2012).[7] This makes it most natural to treat the relevant (i.e., productive) teacher inputs as unobserved.

Given the literature's prevailing treatment of post-schooling OJT investments as unobserved and the associated lack of concrete examples of such investments, it might be useful to elaborate on the distinction between OJT and LBD activities in the current context. At their core, OJT investments capture any activity that reduces output today to increase output in the future. This tradeoff between current and future output is not present under LBD. I can distinguish between these theories without data on production or investment activities because *the same dynamic tradeoff generating implications testable within my framework underlies why researchers and policymakers care about separately identifying the OJT and LBD forces in the first place.*[8]

Identification is possible in the current paper because Glewwe et al. (2010) includes a follow-up measurement of the effects of the program. The intuition can be outlined in the following example: suppose that achievement increased in the treatment group while the incentive scheme was in place (as was the case in Glewwe et al. 2010) and that the random assignment of the intervention was balanced (as was also the case). Then the increase in achievement during the intervention must have come from an increase in treatment-group teachers' human capital allocated to production. Further, a positive treatment effect on achievement after the intervention ended, beyond that accounted for by the persistence of the during-intervention increase in achievement, would imply that teacher quality increased, pointing to the presence of the LBD component. On the other hand, a post-intervention treatment effect lower than that which could be explained by the persistence of the prior achievement effect would imply that teacher quality decreased, pointing to the presence of the OJT component. Without the follow-up data the identified set would be uninformative, meaning we would remain in the typical case in which we could not separate OJT and LBD forces.[9]

I find that the 95% confidence set for the identified set for each parameter is informative. Specifically, the lower bound on the LBD component is greater than zero and the upper bound on

complicates identification in the typical setting, the current paper's approach allows us to learn about the importance of OJT and LBD forces despite this fundamental data limitation.

[7]In his recent review of human capital in education, Burgess (2016) writes that "[w]e don't really know what effective teachers do that makes them effective," (p. 72).

[8]While not necessary for the analysis or results, it may be useful to fix ideas with some examples of production and investment for teachers. Examples of production activities include classroom teaching and providing feedback to students; examples of investment activities include refining one's teaching practice or developing new pedagogical tools. Again, while they might help provide a more concrete picture, one should not take these specific examples as the only activities corresponding to production or investment. Indeed, the analysis could principle inform us about whether such a tradeoff even exists: If there were no scope for making OJT investments, say, because teachers were inundated with classroom teaching duties, this would show up in the estimates because I allow there to be no role played by this force.

[9]As the model makes clear, these follow-up data are not sufficient because the bounds also depend on mean changes in leisure, which are also measured.

the OJT component is lower than its uninformative level. I reject the "pure OJT" specification (in which LBD plays no role) at the 5% significance level, but I cannot reject the "pure LBD" specification, as the confidence set for the OJT component contains zero. When further imposing returns-to-scale type assumptions (i.e., restrictions on the sum of the OJT and LBD components in human capital production), bounds are tighter. Under the strongest assumption, constant returns to scale, the 95% confidence set for the LBD component lies strictly above that for the OJT component, meaning in this case we can also say with appreciable certainty that the LBD component is larger than the OJT component.

This paper's framework generates several contributions. First, it works with a human capital production function that nests both the OJT and LBD mechanisms, as opposed to the "pure" specifications used in prior empirical work. This allows for standard hypothesis testing about important qualitative features, in particular, the presence of either force in the human capital production function. Second, it uses a partial identification approach that features a closed-form solution for the identified set: it is a locus characterized by a line. This characterization sharply contrasts with the literature's lack of a proof showing how to separately identify OJT and LBD forces, yet obtains quite naturally here. Moreover, the primary input to the identified set is an experimentally estimated ATE, commonly viewed as the "gold standard" in causal inference. Thus, it is also easy to estimate the identified set and to do inference on it, which is rare in the partial identification literature (Imbens and Manski, 2004; Stoye, 2009; Tamer, 2010).

Finally, in contrast to the aforementioned work estimating "pure" LBD or OJT models, the approach does not have to make assumptions regarding the optimality of input choices or outcomes observed in the data. Researchers with different goals, say, of examining heterogeneous impacts or understanding how teacher quality would change under counterfactual incentive pay schemes, would have to make stronger statistical assumptions or impose behavioral assumptions. These would naturally represent fruitful and complementary avenues for future research. That being said, there is a vast body of research focused on estimating production functions. As discussed by Griliches and Mairesse (1995), estimates of production functions can serve as a good starting point to answer many important questions. In addition to their intrinsic scientific merit, such estimates help inform researchers and policymakers about important questions in myriad domains, for example, the effects of inputs to the production of children's' cognitive achievement (see, e.g., Todd and Wolpin, 2003; Cunha et al., 2010) to the effects of deregulation on aggregate firm productivity (Olley and Pakes, 1996) to how health care providers trade off quality and quantity (Grieco and McDevitt, 2017).

This paper seeks to understand the evolution of teacher quality from the perspective of the dominant theories of human capital accumulation. Somewhat separately from the aforementioned human capital literature, a small but substantively important set of papers examines other channels underlying teachers' improvement as they gain more experience. Ost (2014), which measures the returns to teachers' general and grade-specific experience, finds both to be important determinants of teacher quality growth. Cook and Mansfield (2016) extends this work to also allow for general and context-specific permanent components to teacher quality. The current paper nicely complements these papers by explicitly viewing teacher quality through the lens of the main conceptual frameworks for human capital, and by identifying and separating the OJT and LBD channels of human capital accumulation, which are not necessary to distinguish given these other papers' goals.[10] This paper also complements the extensive literature studying teacher quality more generally, recently discussed in Hanushek and Rivkin (2006); Jackson et al. (2014); Strøm and Falch (2020), and the sub-literature on teacher incentives (see, e.g., Hanushek and Raymond, 2005; Muralidharan and Sundararaman, 2011; Imberman and Lovenheim, 2015; Petronijevic, 2016).[11]

2 Data and Variables

Study Design Glewwe et al. (2010) implemented a teacher incentive pay scheme in 100 primary schools in Kenya, which provided bonuses (in the form of in-kind prizes, e.g., wall clocks or bells) to teachers and headmasters at schools where students did well on standardized exams administered as part of the standard curriculum in Kenya. The scheme applied to students in grades 4-8, and bonuses were based on school-level averages to discourage competition between teachers. The treatment group (comprising 50 schools) was exposed to the incentive scheme in 1998 and 1999, which I refer to as the "active-treatment" years. In addition to being observed then, outcomes for both the treatment and control groups were also observed for both a pre-treatment year (1997), and a post-active-treatment year (2000). There were 7,492 students in the treatment schools and 8,226 students in the control schools in the pre-treatment year.[12] Next, I provide an overview of the variables used in the current analysis; please see Glewwe et al. (2010) for additional details about the data, and Section 4 for a more detailed discussion of how I map the data to the model (which necessarily follows the model, presented in Section 3).

Variables I measure output using the incentivized standardized achievement test presented in the main results of Glewwe et al. (2010), which is an average of seven standardized subject-specific achievement tests, where standardization was performed with respect to the means and standard deviations of the control group. It is important to note that the experiment was balanced. In particular, mean test scores in the pre-active-treatment year were balanced, and teacher exit and entry also did not significantly differ between the control and treatment groups. The data also include information about whether teachers were present during random, unannounced, site visits

[10] These papers may be viewed as part of the substantial empirical literature studying task-specific human capital for more general worker contexts. For recent examples, see Poletaev and Robinson (2008); Sanders (2010); Yamaguchi (2012); Robinson (2018). Sanders and Taber (2012) contains a detailed discussion of this and other extensions of the one-dimensional human capital model.

[11] Although less tightly related, this paper also relates to the literature studying teacher labor markets (see, e.g., Dolton and Klaauw, 1999; Stinebrickner, 2001; Behrman et al., 2016; Tincani, 2021; Biasi, 2021; Bobba et al., 2021; Biasi et al., 2021).

[12] As in Glewwe et al. (2010), I exclude one of the districts (Teso), which did not offer the achievement test in the first year.

to schools. Averaging over all teachers in either the treatment or control group provides a good measure of the probability a teacher was present at the school for that group. Under uniformity, the probability that a teacher was present during site visits would be affine transformations of hours teachers were at work, a common measure of labor supply.

3 Model

The model provides the foundation for the partial identification of the components governing human capital accumulation. It relates output (i.e., student achievement) and teacher human capital to production and investment inputs. The specifications for the production of human capital and output enable a closed-form, constructive, identification proof and calculation of bounds using ATE estimates, meaning the analysis is informative about average behavior.[13] They are also not very restrictive either in terms of implications for behavior,[14] or compared with specifications commonly used in the literature.[15]

3.1 Environment

Let the periods be indexed by $t = 0, 1, 2$, where $t = 0$ is the pre-treatment period, $t = 1$ is the "active-treatment" period, in which teachers in the treatment group are offered output-based incentives, and $t = 2$ is the post-active-treatment period, where teachers in either treatment or control group no longer are offered the incentives.

The human capital of teacher i in period t, θ_{it}, is produced according to

$$\theta_{it} = \delta_\theta \theta_{it-1} + \delta_I I_{it-1} + \delta_h h_{it-1}, \tag{1}$$

where θ_{it-1} is teacher i's human capital last period, I_{it-1} is teacher i's investment in human capital last period, and h_{it-1} is teacher i's input to production (e.g., actively teaching, engaging in teaching preparation, providing feedback to students, etc.) last period. Output produced by teacher i teaching student j in period t, y_{ijt}, which is measured by performance on a standardized achievement test, follows a standard value-added specification (Hanushek, 1971; Murnane, 1975;

[13] While averages are not the only conceivable objects of policy interest, they certainly have garnered a vast amount of interest by researchers and policymakers, as they may average out unobserved heterogeneity and also may be useful for certain normative considerations (e.g., maximizing output).

[14] The human capital production function does not assume that growth in human capital is constant over time. Indeed, as the model does not assume optimality of teacher behavior, it allows for, e.g., decreasing OJT investments over the course of a teacher's career, which would result in the concave value added profiles documented in the literature.

[15] The model developed in this section could be viewed as a log-linear approximation to a specification considering the behavior of a representative teacher and student in each of the control and treatment groups (see Appendix A), and is also consistent with (specifically, nested by) the commonly used translog specification used in the burgeoning literature using dynamic factor models to understand skill growth (see, e.g., Agostinelli and Wiswall, 2023; Del Bono et al., 2022; Freyberger, 2021). While the link developed in the appendix might provide useful context, it is in no way necessary to this paper's analysis.

Hanushek, 1979):[16]

$$y_{ijt} = \theta_{it} + \beta_h h_{it} + \beta_y y_{jt-1} + \epsilon_{ijt}, \tag{2}$$

where y_{jt-1} is student j's prior test achievement (meaning β_y measures the persistence of student knowledge), $\theta_{it}+\beta_h h_{it}$ is teacher i's value added to student j's achievement in period t (i.e., *quality*), and ϵ_{ijt} is an ex post IID productivity shock.[17]

It is not uncommon to fix leisure when operating within the "pure OJT" framework (see, e.g., Ben-Porath, 1967; Heckman et al., 1998; Kuruscu, 2006; Huggett et al., 2011), but doing so would hamstring the LBD force, so given this paper's goals it is important to allow for leisure responses. The teacher's input (or resource) constraint links production, h_{it}, investment, I_{it}, and leisure, l_{it}:

$$h_{it} + I_{it} + l_{it} = M. \tag{3}$$

The variables h, I, and l could be viewed as shares of a teacher's total time, or of a teacher's overall focus or potential effort respectively allocated to production, investment, and leisure. Under the latter interpretations, a teacher has a fixed "budget" of focus/potential effort, which can be allocated between h, I, and l; naturally, a teacher must be working to engage in production or investment. While, for consistency with the human capital literature (see, e.g., Ben-Porath, 1967; Heckman, 1976; Killingsworth, 1982), I typically use the term "time", one should keep these alternative interpretations in mind. I refer to $h_{it} + I_{it}$ as *total work time* or *total inputs*; eq. (3) then captures the notion that total work time should not increase if leisure has increased. As is standard, neither h_{it} nor I_{it} are observed, and, further, they have no inherent scale. However, we can identify total work time because the data contain measures of its complement, leisure time, which means the scale of $h_{it} + I_{it}$ is fixed by the scale of the measure of l_{it}.[18]

I assume that $\delta_k \in D_k$, where $D_k = [0, \overline{\delta}]$, for $k = \theta, I, h$, and that $\beta_k \in B_k$, where $B_k = [0, \overline{\beta}]$, for $k = h, y$. The lower bound of zero for each parameter captures the natural assumption that inputs cannot have negative effects. The upper bound for each parameter (i.e., $\overline{\delta}$ or $\overline{\beta}$) is taken to be large; I discuss below how the specific values of $\overline{\delta}$ and $\overline{\beta}$ do not affect this paper's main findings.

I define the "pure OJT" specification for the human capital production function as $\delta_I > 0$ and $\delta_h = 0$. Analogously, in the "pure LBD" specification, we have $\delta_h > 0$ and $\delta_I = 0$. Some scenarios representing different possibilities for true combinations of (δ_h, δ_I) are illustrated in Figure 1 (bounds for the *identified set* for (δ_h, δ_I), which is denoted $d_h \times d_I$, are derived in Section 3.2). For

[16] Recent research supports the view that controlling for prior achievement, as is done in a value-added model, does a reasonably good job of controlling for unobserved prior inputs (see, e.g., Kinsler, 2012; Chetty et al., 2014).

[17] Note that the scale of θ is determined by y. This implicit normalization, in eq. (2), has no bearing at all on the returns to teacher human capital, which are governed by δ_θ. I show below that this parameter is unidentified in this paper's framework, and is therefore immaterial to any of this paper's results (see Agostinelli and Wiswall, 2023, for a discussion of some of the pitfalls to avoid when (point-) estimating the returns to scale for skills). Also, I abstract from student characteristics here; as will become clear, what matters for this paper's approach is obtaining a consistent estimate of β_y.

[18] The value of the endowment, M (on the right-hand side of eq. (3)), is immaterial as it differences out in the analysis below.

example, the point labeled "pure OJT", on the horizontal axis, features a positive OJT component, with no LBD component, in contrast to the point labeled "pure LBD", on the vertical axis. The interior point, labeled "OJT and LBD both present", represents the possibility that teachers accumulate human capital via both OJT and LBD components; in contrast, the "pure" versions of the OJT and LBD human capital production functions are mutually exclusive.

3.2 Derivation of Bounds

This section develops bounds for (δ_h, δ_I) that only depend on period-specific estimates of the average treatment effects of the intervention on student achievement and on labor supply, and an estimate of the persistence of student knowledge (which could be estimated using the same dataset, or obtained from another source). The bounds are sharp, meaning they contain only the values of (δ_h, δ_I) that cannot be rejected given the data. For example, the sharp bound on δ_I would increase the lower bound for δ_I as much as possible while still being consistent with the data; if the lower bound were greater than zero one would reject the pure LBD specification.

Define the mean difference between the treatment and control groups for variable z in period t as $\Delta z_t := \overline{z_t^T} - \overline{z_t^C}$, where $\overline{z_t^T}$ and $\overline{z_t^C}$ respectively denote the treatment and control group means of z in t.

First, note that in any period the constant time endowment implies that

$$\Delta h_t + \Delta I_t + \Delta l_t = 0. \tag{4}$$

For the pre-treatment period, $t = 0$, the mean difference in achievement between the treatment and control groups is

$$\Delta y_0 = \underbrace{\Delta\theta_0}_{=0} + \beta_h \underbrace{\Delta h_0}_{=0} + \beta_y \underbrace{\Delta y_{-1}}_{=0} + \underbrace{\Delta\epsilon_0}_{=0} = 0, \tag{5}$$

i.e., a balanced experimental design implies there will be no average difference in the pre-treatment average scores. Consistent with Glewwe et al. (2010), I assume the experiment was implemented with high fidelity, and consequently omit $\Delta\epsilon_t$ hereafter.

For the active-treatment period, $t = 1$, we have

$$\Delta y_1 = \underbrace{\Delta\theta_1}_{=0} + \beta_h \Delta h_1 + \beta_y \underbrace{\Delta y_0}_{=0} = \beta_h \Delta h_1. \tag{6}$$

Equation (6) shows that the difference between treatment and control achievement in the active-treatment period can only come from the change in mean production time Δh_1, as the fact that θ_{it} depends on lagged inputs and balance between the treatment and control groups implies that $\Delta\theta_0 = 0$, while, as shown in eq. (5), balance between the treatment and control groups implies that $\Delta y_0 = 0$.

The mean difference in achievement between the treatment and control groups for the post-active-treatment period, $t = 2$, is

$$\begin{aligned}\Delta y_2 &= \Delta\theta_2 + \beta_h \Delta h_2 + \beta_y \Delta y_1 \\ &= \delta_I \Delta I_1 + \delta_h \Delta h_1 + \beta_h \Delta h_2 + \beta_y \Delta y_1 \\ &= \delta_I \underbrace{[-\Delta l_1 - \Delta h_1]}_{=\Delta I_1, \text{ from eq. (4)}} + [\delta_h + \beta_h \beta_y]\Delta h_1 + \beta_h \Delta h_2 \\ &= -\delta_I \Delta l_1 + \left[\frac{\delta_h - \delta_I + \beta_h \beta_y}{\beta_h}\right] \underbrace{\Delta y_1}_{\beta_h \Delta h_1, \text{ from eq. (6)}} + \beta_h \Delta h_2, \end{aligned} \tag{7}$$

which uses $\Delta\theta_1 = 0$ to go from the first to the second line. Equation (7) shows that, in general, only a locus of (δ_h, δ_I) will be identified, and, further, that many other variables appear in the same equation: the achievement production function parameters (β_h, β_y) and, even after using eq. (6) to eliminate Δh_1, the quantities $(\Delta y_1, \Delta y_2, \Delta l_1, \Delta h_2)$.[19] The upper bound on the parameter spaces for δ_h and δ_I, $\overline{\delta}$, is also unknown. I estimate $(\beta_y, \Delta y_1, \Delta y_2, \Delta l_1)$ in Section 4.1. However, Δh_2 is unobserved and it cannot be eliminated, as was Δh_1 via eq. (6).

I now discuss how I arrive at values for $(\beta_h, \overline{\delta}, \Delta h_2)$. I show below that Δy_1 is significantly greater than zero (i.e., there was a positive effect of the intervention during the active-treatment period). In light of eq. (6) and the fact that $\beta_h \geq 0$, it is then reasonable to treat β_h as strictly positive, as the positive effect on achievement during the active-treatment period could only be rationalized by an increase in time allocated to production (i.e., $\Delta h_1 \geq 0$). Because the scale of β_h is not identified separately from $\overline{\delta}$, I fix $\beta_h = 1$ and $\overline{\delta} = 1$ hereafter.[20] It is important to note that the specific values of β_h and $\overline{\delta}$ do not affect the main findings, such as whether bounds are informative or whether I can reject either the "pure OJT" or "pure LBD" specifications.[21]

Finally, I assume $\Delta h_2 = 0$, i.e., that production after the active-treatment period is the same in the control and treatment groups. While any potential income effects were dominated by substitution effects during active treatment (as discussed above, the data are consistent with $\Delta h_1 > 0$), the substitution effects would no longer be present afterwards. The structure of the incentive scheme, however, suggests that income effects, if present, would not be very large: the rewards were non-fungible prizes, which 24 of the 50 treatment schools received, and the prize values ranged from less than 2% to 4% of the typical teacher's annual salary (Glewwe et al., 2010, p. 208). While this does not preclude there being an average difference in production after active treatment, it also seems

[19] Note that δ_θ does not appear in eq. (7). Intuitively, this parameter measures how differences in teacher human capital emanating from differences in inputs from two periods ago affect production today; the balanced experimental design means these differences are all zero, causing δ_θ to drop out.

[20] If the researcher viewed the model as a (log-linearized) approximation to a nonlinear model (see Appendix A for discussion), it would be natural to have $\beta_h = 1$ (i.e., the same as the scale of θ, which is consistent with the interpretation that a teacher's output equals her share of human capital allocated to production), and also to not allow δ_h or δ_I to exceed 1 (i.e., $\overline{\delta} = 1$).

[21] There are possible parameter values where the results would be affected by the choice of β_h relative to $\overline{\delta}$, but the estimated parameters are far from this region.

likely that Δh_2 would be small, if it were indeed different from zero. It is important to note that the assumption that $\Delta h_2 = 0$ does not rule out the intervention increasing a teacher's productivity by way of, e.g., preparing materials that could also be used in subsequent years. This increased productivity would be captured by a higher level of θ_2, which could be generated by either OJT or LBD forces.

Assumption 1 summarizes the parameter values discussed thus far. Assumption 1(i) is maintained hereafter. Assumption 1(ii) corresponds to making no assumption on returns to scale for the human capital production function. Section 3.4 explores how stronger assumptions about the returns to scale would tighten the identified set.

Assumption 1 (Baseline, no returns to scale assumption)**.**

(i) $\beta_h = 1$, $\Delta h_2 = 0$

(ii) $(\delta_h, \delta_I) \in [0, 1] \times [0, 1]$.

Using the values obtained thus far, eq. (7) becomes

$$\Delta y_2 = [\delta_h - \delta_I + \beta_y]\, \Delta y_1 - \delta_I \Delta l_1,$$

which we can rearrange to get the following expression for the LBD component δ_h as a function of the OJT component δ_I and remaining parameters:

$$\delta_h = \left[\frac{\Delta y_2}{\Delta y_1} - \beta_y\right] + \delta_I \left[1 + \frac{\Delta l_1}{\Delta y_1}\right], \tag{8}$$

i.e., eq. (8) characterizes the identified set, $d_h \times d_I$. We can find the identified set for either parameter by projecting the locus characterized by eq. (8) onto the relevant axis, e.g., d_h can be obtained by projecting $d_h \times d_I$ onto the δ_h-axis. It will be convenient to rewrite eq. (8) as

$$\delta_h = \pi_{\text{icept}} + \pi_{\text{slope}} \delta_I, \tag{9}$$

where the intercept of the locus of permissible combinations of (δ_h, δ_I) is $\pi_{\text{icept}} := \frac{\Delta y_2}{\Delta y_1} - \beta_y$ and the slope of the locus is $\pi_{\text{slope}} := 1 + \frac{\Delta l_1}{\Delta y_1}$.

Note that, even with the parameter values obtained so far, the relationship between (δ_h, δ_I) still depends on $(\beta_y, \Delta y_1, \Delta y_2, \Delta l_1)$. Section 4.1 discusses estimation of these parameters. Briefly, all but β_y are estimated using the experimental variation. Standard methods (treatment-year fixed effects in the production function, coupled with the assumption that the experiment was balanced) yield consistent estimates of β_y when using just control-group data.[22]

[22]This is because, for teacher-student pair ij in group g (i.e., treatment or control), we have $\mathrm{E}_{ij \in g}[y_{ijt}] = \mathrm{E}_{ij \in g}[\theta_{ijt} + \beta_h h_{it}] + \beta_y \mathrm{E}_{ij \in g}[y_{ijt-1}] + \underbrace{\mathrm{E}_{ij \in g}[\epsilon_{ijt+1}]}_{=0}$ and $\mathrm{E}_{ij \in g}[\theta_{ijt} + \beta_h h_{it}]$ can be measured by the coefficient on a group-period indicator variable.

3.3 Comparative Statics

We can understand the restrictions afforded by eq. (8) by considering some cases, where for the sake of illustration assume $\Delta y_1 > 0$. I start by maintaining Assumption 1 and also consider a very simple special case, to highlight the difficulty in separating OJT and LBD forces. Specifically, suppose that $\Delta l_1 = 0$ and suppose that $\frac{\Delta y_2}{\Delta y_1} - \beta_y = \pi_{\text{icept}} = 0$; this corresponds to case (a) in Table 1. Intuitively, the increase in test score coming via the positive effect of Δh_1 on Δy_1 entirely accounts for Δy_2. The first line of eq. (7) then implies that $\Delta\theta_2 = 0$, i.e., teacher human capital post-active-treatment is on average the same in the treatment and control groups. Further, $\Delta l_1 = 0$ (i.e., $\pi_{\text{slope}} = 1$) implies that $\Delta I_1 = -\Delta h_1$, meaning the only way to satisfy eq. (8) is for $\delta_I = \delta_h$. That is, any increase in δ_I can satisfy the condition by a concomitant increase in δ_h. Without further information on either of these parameters, we cannot shrink the identified set. This scenario corresponds to the dashed, 45-degree, line in the left panel of Figure 2. Projecting the identified set onto each axis, we can see that the marginal identified set for either δ_I or δ_h (depicted by the dashed lines just outside that parameter's axis) has not shrunk at all, because any feasible value of, e.g., δ_I, can be rationalized by the same value for the coefficient on the other input to teacher human capital (in this example, δ_h). That is, the bounds in this case would be uninformative.

Next consider case (b), which differs from case (a) in that $\frac{\Delta y_2}{\Delta y_1} - \beta_y < 0$, i.e., teacher human capital is lower in the treatment group, post active treatment ($\Delta\theta_2 < 0$). Because we still have $\Delta I_1 = -\Delta h_1$, the fact that $\Delta\theta_2 < 0$ means that we can rule out very low values of δ_I—the OJT component—and very high values of δ_h—the LBD component. Intuitively, if the net effect of increasing time spent on production on teacher human capital is negative, knowing that $\Delta I_1 = -\Delta h_1$ implies that the OJT parameter must be larger than the LBD one. At the same time, the slope of eq. (8) is unaffected because the one-to-one tradeoff between different time uses in the budget constraint (due to there being no average change in leisure time) implies a one-to-one tradeoff between δ_I and δ_h. This case is depicted by the dotted lines in the left panel of Figure 2.

In cases (a) and (b), there was no average difference in leisure time between the treatment and control groups during the active-treatment period (i.e., $\Delta l_1 = 0$, or $\pi_{\text{slope}} = 1$). Consider now case (c), where we start from case (b) but now assume that $\Delta l_1 < 0$ (here, $\pi_{\text{slope}} < 1$). Here, we know that $\Delta I_1 > -\Delta h_1$, i.e., the absolute difference in investment is smaller than the absolute difference in production time. As shown in the dash-dotted lines in the left panel, this rotates the locus eq. (8) downward from the intercept (which was already negative, as the starting point was case (b)), increasing the lower bound on δ_I and decreasing the upper bound on δ_h. Intuitively, all else equal, a smaller change in investment must be coupled with a relatively bigger technological effect of investment (δ_I) to rationalize the same data.

The cases discussed above are not exhaustive. For example, the signs of $\frac{\Delta y_2}{\Delta y_1} - \beta_y$ or Δl_1 could be opposite to those considered in cases (b) or (c), in which case the sharp bounds would be different. An example of this is case (d), which is depicted in the right panel of Figure 2, which illustrates how strengthening returns-to-scale-type assumptions yields tighter sharp bounds for the identified set. The next section explores this.

3.4 Bounds with Increasing Assumption Strength

A researcher might further find it natural to restrict the returns to scale in the human capital production function, by assuming they are nonincreasing. This is Assumption 2 below. Even stronger, the researcher might believe it reasonable to assume constant returns to scale (Assumption 3). This exploration of how assumptions about $\delta_h + \delta_I$ affect the identified sets is in the spirit of the "worst-case" approach of Horowitz and Manski (2000), which examines the sensitivity of findings to stronger sets of assumptions, some of which are made in the literature. It does not constitute an endorsement of making these stronger assumptions.

Assumption 2 (Nonincreasing returns to scale (NIRS))**.** *Assumption 1(i) and* $\delta_h + \delta_I \leq 1$*.*

Assumption 3 (Constant returns to scale (CRS))**.** *Assumption 1(i) and* $\delta_h + \delta_I = 1$*.*

The right panel of Figure 2 shows the additional information embedded in assumptions about the returns to scale for (δ_h, δ_I), starting with Assumption 1 (case (d)); these cases are summarized in the lower part of Table 1. Case (d2) further imposes the restriction that $\delta_h + \delta_I \leq 1$ (Assumption 2), which means permissible combinations of (δ_h, δ_I) lie in the south/west right triangle.[23] By looking at the dotted lines in the right panel of Figure 2, corresponding to this case, we can see that the marginal identified sets are tighter than those in case (d). Intuitively, the tighter upper bound for δ_h, combined with the nonincreasing returns to scale assumed in case (d2), yields a tighter upper bound for δ_I. Case (d3) further imposes the restriction that $\delta_h + \delta_I = 1$ (Assumption 3), which affords point identification (interior solid point in the right panel of Figure 2).

3.5 Expressions for Marginal Identified Sets

This section ends by characterizing the marginal identified sets for δ_h and δ_I, which are used in estimating the parameters' bounds and confidence sets. Under Assumption 1, i.e., when making no additional assumptions about returns to scale, we can derive the identified set for δ_h, d_h, by varying δ_I over its domain, resulting in

$$\delta_h \in [\max\{\pi_{\text{icept}}, 0\}, \min\{\pi_{\text{icept}} + \pi_{\text{slope}}, 1\}]. \tag{10}$$

Solving eq. (9) instead for δ_I, we can analogously obtain d_I by varying δ_h over its domain:

$$\delta_I = \frac{\delta_h - \pi_{\text{icept}}}{\pi_{\text{slope}}} \Rightarrow \delta_I \in \left[\max\left\{\frac{-\pi_{\text{icept}}}{\pi_{\text{slope}}}, 0\right\}, \min\left\{\frac{1 - \pi_{\text{icept}}}{\pi_{\text{slope}}}, 1\right\}\right]. \tag{11}$$

[23] Note that this restriction in itself would not yield any information; without data (e.g., information about π_{icept}), the bounds on either parameter, obtained from projecting the south/west right triangle onto either axis, would be uninformative.

Table 1: Bounds: Comparative Statics and Effects of Returns-to-Scale Assumptions

Case	$(\delta_h, \delta_I) \in$	Δl_1	$\frac{\Delta y_2}{\Delta y_1} - \beta_y$	Notes on bounds
(a)	$\delta_h \leq 1,\ \delta_I \leq 1$	0	0	Uninformative
(b)	$\delta_h \leq 1,\ \delta_I \leq 1$	0	< 0	Informative
(c)	$\delta_h \leq 1,\ \delta_I \leq 1$	< 0	< 0	Tighter than (b)
(d)	$\delta_h \leq 1,\ \delta_I \leq 1$	0	> 0	Informative
(d2)	$\delta_h + \delta_I \leq 1$	0	> 0	Tighter than (d)
(d3)	$\delta_h + \delta_I = 1$	0	> 0	Point identification

Note: $\Delta y_1 > 0$. Cases (a)-(c) correspond to comparative statics, and cases (d)-(d3) correspond to different returns-to-scale assumptions.

Invoking non-increasing returns to scale (Assumption 2) does not affect the lower bound of d_h, but does tighten its upper bound:

$$\begin{aligned}
&\delta_h = \pi_{\text{icept}} + \pi_{\text{slope}}\delta_I \Rightarrow \delta_h \leq \pi_{\text{icept}} + \pi_{\text{slope}}[1-\delta_h] \Rightarrow \\
&\delta_h \in \left[\max\left\{\pi_{\text{icept}}, 0\right\}, \min\left\{\frac{\pi_{\text{icept}} + \pi_{\text{slope}}}{1+\pi_{\text{slope}}}, 1\right\}\right]. \qquad (12)
\end{aligned}$$

We can analogously tighten the upper bound on d_I by applying Assumption 2:

$$\begin{aligned}
&\delta_h = \pi_{\text{icept}} + \pi_{\text{slope}}\delta_I \Rightarrow [1-\delta_I] \geq \pi_{\text{icept}} + \pi_{\text{slope}}\delta_I \Rightarrow [1+\pi_{\text{slope}}]\delta_I \leq 1 - \pi_{\text{icept}} \\
&\delta_I \in \left[\max\left\{\frac{-\pi_{\text{icept}}}{\pi_{\text{slope}}}, 0\right\}, \min\left\{\frac{1-\pi_{\text{icept}}}{1+\pi_{\text{slope}}}, 1\right\}\right]. \qquad (13)
\end{aligned}$$

Finally, imposing Assumption 3, we achieve point identification for both parameters:

$$\delta_h = \pi_{\text{icept}} + \pi_{\text{slope}}[1-\delta_h] \Rightarrow \delta_h = \frac{\pi_{\text{icept}} + \pi_{\text{slope}}}{1+\pi_{\text{slope}}} \qquad (14)$$

$$[1-\delta_I] = \pi_{\text{icept}} + \pi_{\text{slope}}\delta_I \Rightarrow \delta_I = \frac{1-\pi_{\text{icept}}}{1+\pi_{\text{slope}}}. \qquad (15)$$

4 Empirical Results

4.1 Estimation of Parameters Determining Identified Set for (δ_h, δ_I)

From eq. (8), the identified set for (δ_h, δ_I) depends on $(\Delta y_1, \Delta y_2, \beta_y, \Delta l_1)$. This section discusses how these parameters are estimated, and Section 4.2 shows how the resulting marginal identified sets for (δ_h, δ_I) are estimated.

I pool the two active-treatment years to map the empirical application, which spans four years

Table 2: Estimates of ATEs on Achievement

	Dependent variable:
	Test score y_{ijt}
Active-treatment ($t = 1$)	0.001
	(0.019)
Post-active-treatment ($t = 2$)	-0.001
	(0.017)
Active $*$ treated (Δy_1)	0.089^{***}
	(0.013)
Post $*$ treated (Δy_2)	0.098^{***}
	(0.025)
Constant	0.007
	(0.008)
Observations	26,537

Note: *p<0.1; **p<0.05; ***p<0.01. The t corresponds to a pooled period.

of data, to the three-period structure of the model. The pre-treatment year (1997) corresponds to model period $t = 0$. Both active-treatment years (1998 and 1999) are pooled into one period, corresponding to $t = 1$ in the model, and the post-active-treatment year (2000) corresponds to $t = 2$. While the estimates presented here are based on the pooled data, I have examined their sensitivity to using unpooled data when feasible (i.e., for β_y, which can be estimated without the experimental variation), and the results were essentially unaffected.

Table 2 presents the achievement ATE results for the active-treatment period (Active $*$ treated) and the post-active treatment period (Post $*$ treated). I also include the treatment year as a regressor (Active-treatment, Post-active-treatment) to control for secular trends. The results for the treatment group in the active-treatment period indicate that the average treatment effect pooled over both active-treatment years, Δy_1, is positive and significantly different than zero (0.089). The next row indicates that student achievement in the post-active-treatment year, Δy_2, remained significantly higher (0.098) in the treatment group. Through the lens of the model, this positive ATE could be due to persistence of higher achievement from the active-treatment period and (potentially) higher teacher human capital for the treatment group in the post-active-treatment period. Establishing the relative importance of these effects is the goal of the next section.

Table 3 presents an estimate of the persistence component β_y using control group data, where the lagged score is that from the previous pooled period. I estimate the persistence component to be 0.544, and statistically greater than zero. The estimate of persistence is not driven by

Table 3: Estimate of the Persistence Component

	Dependent variable:
	Test score y_{ijt}
Lagged score $y_{j,t-1}$ (β_y)	0.544^{***}
	(0.010)
Constant	-0.047^{***}
	(0.008)
Observations	5,007

Note: *p<0.1; **p<0.05; ***p<0.01. These estimates are obtained from a regression of current test score on the lagged test score, run on the pooled data for pooled periods 1 and 2, for the control group. Specifically, the (period) t subindex on the test score y_{ijt} refers to a pooled period, meaning student j's lagged test score $y_{j,t-1}$ is the one from the previous pooled period.

the use of pooled data: when instead using unpooled data the estimate is 0.618. Even more to the point, this estimate is in the range of those in Andrabi et al. (2011), which estimated the persistence of a variety of cognitive skills using a dynamic panel data model applied to Pakistani schoolchildren. Allowing for both measurement error and unobserved student heterogeneity (in contrast with the specifications researchers have typically used to estimate achievement production functions), they estimate the (annual) persistence of cognitive skills to range from 0.2 to 0.55, across a variety of subjects. Indeed, they argue that the upper part of their range is possibly too high. This matters because a smaller value of β_y will further tighten the estimated bounds, in light of the achievement ATE estimates, which already yield a positive, significant, estimate of π_{icept}. Intuitively, the post-active treatment achievement ATE is higher than can be explained by the active-treatment achievement ATE persisting into post-active treatment, pointing to the presence of an LBD component; lowering the value of β_y would only make this effect more prominent.

I now discuss how I obtain a value for the effect of the intervention on leisure during the active-treatment period, Δl_1. The literature estimating OJT models of human capital accumulation that also models labor supply typically treats labor supply as observed (see, e.g., Brown, 1976; Heckman, 1976; Blandin, 2018; Fu et al., 2021). I follow this literature and use measures of total work time, which are sufficient for the current paper because the change in total work time ($\Delta I_t + \Delta h_t$) is the complement of the change in leisure (Δl_t). The Glewwe et al. (2010) data contain two measures of teachers' total work time in a period: the fraction of teachers in attendance at the school during random, unannounced, site visits by the research team that period and the fraction of teachers present in their classroom during the site visits.[24] While either could serve as a measure of total work time, it seems more appropriate to use the share of teachers present at the school, as teachers could make investments outside the classroom. Table 4 shows the treatment effect on the share

[24] I use school-period-level averages because teacher-level data on either measure were not available.

Table 4: Estimate of ATE on Teacher Attendance

	Dependent variable:
	Present at school in period t
Active-treatment ($t = 1$)	0.012
	(0.025)
Post-active-treatment ($t = 2$)	0.052**
	(0.025)
Active $*$ treated ($-\Delta l_1$)	-0.017
	(0.029)
Post $*$ treated	-0.011
	(0.041)
Constant	0.833***
	(0.014)
Observations	202

Note: *p<0.1; **p<0.05; ***p<0.01. Dependent variable is school-level average share of site visits during which teachers were in attendance. The t corresponds to a pooled period.

of teachers in attendance, -0.017, is not significantly different from zero.[25] This means the point estimate for the effect on leisure, which is the negative of the effect on total work time, is positive, at 0.017 (and, naturally, also not significantly different from zero). It is important to note that, even though the estimated effect on leisure is not significantly different from zero, I use the estimate of Δl_1 (and the associated uncertainty) when estimating bounds and conducting inference.

4.2 Estimates of Bounds

The parameters characterizing the identified set for (δ_h, δ_I), π_{icept} and π_{slope}, can be estimated using $\widehat{\pi}_{\text{icept}} := \widehat{\frac{\Delta y_2}{\Delta y_1} - \beta_y}$ and $\widehat{\pi}_{\text{slope}} := \widehat{1 + \frac{\Delta l_1}{\Delta y_1}}$, which are computed using plug-in estimators. To simulate the joint distribution of $(\widehat{\pi}_{\text{icept}}, \widehat{\pi}_{\text{slope}})$, taking into account the variability of the inputs to the plug-in estimators, I bootstrap the joint distribution of $(\widehat{\Delta y}_1, \widehat{\Delta y}_2, \widehat{\beta}_y, \widehat{\Delta l}_1)$, where in each bootstrap replication, the individual elements are estimated as described just above.[26]

I provide an overview here of how I estimate the bounds and confidence sets for the parameters; see Algorithm 1 in Appendix B for more detail. I first construct the identified set, $\hat{d}_h \times \hat{d}_I$, and then project this set onto each marginal dimension, and then construct confidence sets to contain

[25] I also include the treatment year as a regressor here to control for secular trends. This finding is consistent with that of Glewwe et al. (2010), who find no evidence that teachers in the treatment group on average altered their school attendance or classroom presence (see Table 5, Panels A and B, columns (2) and (3), of that paper).

[26] I bootstrap using 100,000 replications of the data, stratified by treatment status.

each parameter (not the identified set for each parameter) at the pre-specified significance level (I use a confidence level of 95%). Estimated bounds are the average across the identified sets. By projecting the identified set onto the marginal dimensions, I avoid the problem of overly conservative confidence sets described by Kaido et al. (2019). That being said, the estimated confidence sets are virtually identical when containing parameter or identified set with particular probability, because each confidence set contains at least one end point of the parameter space.

Results Figure 3 illustrates the estimated bounds and 95% confidence sets for δ_h and δ_I under the different assumptions about $\delta_h + \delta_I$. The table below presents the corresponding estimates and also reports bound widths. Starting with the LBD parameter in panel (a), we can see that when we do not impose a returns-to-scale-type assumption ("None"), the estimated upper and lower bounds (thick, black, line) for δ_h are informative, and the lower bound is greater than zero at the 95% confidence level (thin, grey, line), leading us to reject the pure OJT specification (in which $\delta_h = 0$). Similarly, in panel (b) we can see that the estimated upper and lower bounds for δ_I are informative, and that the 95% confidence set for δ_I does not contain the upper bound of 1 when no assumption about returns to scale is made ("none").

At this point it would be natural to ask how reasonable it would be to surmise that LBD could underlie the observed patterns, in light of the fact that LBD operates via changes in production time during active treatment h_1, coupled with the previous finding that there was not a significant change in leisure during the active-treatment period. This is fine for two reasons. First, while the identified set for δ_I does include zero, it also includes positive values; such parameter values would naturally yield positive controls levels of OJT investments that could decrease in the active-treatment period. Second, by using the estimates in Table 4 we can see that the 95% confidence interval for the change in total work time ($-\Delta l_1$) is $[-0.074, 0.04]$, which includes increases in total work time of 9% over to the pre-intervention mean of 0.833.

Imposing non-increasing returns to scale ("NIRS" on the horizontal axis in each panel) tightens the estimated upper bound for δ_h, although it does not affect the 95% confidence set, because the estimated lower bound, which governs the lower bound of the confidence set, is unchanged. However, imposing NIRS does tighten the estimated upper bound and upper bound on the confidence sets for δ_I. Intuitively, high values of both δ_h and δ_I are no longer mutually feasible under NIRS, lowering the upper bounds for both parameters. Consequently, we can see in the accompanying table that the width of the estimated bounds falls by about one half for both parameters when imposing NIRS (e.g., from 0.433 to 0.233 for δ_h), and the same is true of the width of the 95% confidence set for δ_I. Finally, imposing constant returns to scale ("CRS" on the horizontal axis in each panel) yields point identification for both parameters, corresponding to the smallest width confidence sets. The estimated bounds are of course zero width in this case.

Overall, under all the assumptions about returns to scale, the achievement ATE in the post-active-treatment period is larger than would be accounted for by the positive ATE in the active-treatment period (caused by an increase in h_1, an input to contemporaneous student achievement) and persistence of this increased student achievement. That is, the OJT component, operating through $\Delta I_1 \leq 0$, is dominated by the LBD component, operating through $\Delta h_1 > 0$. We can reject the "pure OJT" specification in which $\delta_h = 0$ across all returns-to-scale assumptions at the 95% confidence level. However, we cannot reject that $\delta_I = 0$ in any of the returns-to-scale assumptions at the 95% confidence level; this means we cannot reject the "pure LBD" specification. Further, if one were willing to assume CRS, then one could infer that δ_h was greater than δ_I, as the 95% confidence sets under CRS do not overlap; the same cannot be said under either of the weaker assumptions.

Discussion The finding that LBD, at least in part, explains growth in teacher quality means that, on average, teachers improve by teaching their students. Of course, teachers might also improve by making OJT investments, as the confidence set for the OJT component includes strictly positive values. While this might seem quite intuitive, economists to date had not been able to identify the force within human capital theory behind why teacher quality should increase with experience.

Coming back to the specific context underlying this paper's estimates, Glewwe et al. (2010) assessed that the intervention may have increased "teaching to the test", not more general student knowledge. They surmised this because, in contrast to the results for the incentivized exam (used in the current paper), student achievement for a non-incentivized exam that covered similar material did not significantly increase in the treatment group during active treatment. That being said, it is important to note that the associated change in teacher human capital was not limited to a one-time change contained only to the active treatment period, as it did show up in the post-active-treatment period, meaning that the current paper does identify a force generating a form of teacher human capital. However, just as the above assessment of Glewwe et al. (2010) warrants a modicum of caution when interpreting the findings of that paper, it does so for those of this paper too.

While I use the incentivized test to most clearly demonstrate the current methodology for separately identifying OJT and LBD, we can also use the developed framework to help think about the importance of LBD versus OJT in generating (desirable) teacher human capital, even in light of the above caveat. Somewhat loosely, imagine there were two forms of OJT investment, one for "incentivized teacher human capital", and another for more general (or unincentivized) teacher human capital, and analogously that a teacher can allocate her time to production of either form of human capital in her students (which could manifest in different degrees in student achievement). Through the lens of the model it still must be the case that $\Delta h_1 > 0$, i.e., teacher human capital allocated to production increased in the treatment group during the active-treatment period. While all we know is that this increase in production time may have been directed to incentivized (and not more general) student knowledge, my findings suggest the OJT investment likely decreased for either type of human capital. Then, the fact that the post-active treatment effect was still greater than that which would be explained by the persistence of the active-treatment achievement effect is still consistent with OJT not playing the paramount role, insofar as achievement depended on both test-specific and general inputs.

5 Conclusion

I develop a framework nesting the OJT and LBD forces of human capital accumulation, and derive theoretical bounds for OJT and LBD components. The developed bounds are sharp, and yield novel information about the presence and relative importance of the forces generating human capital. The derived bounds only require information about ATEs and the persistence of student achievement. The estimated bounds are informative, and under even the weakest assumptions about the returns to scale allow one to reject the "pure OJT" model. That is, the data are consistent with the presence of an LBD component to teacher human capital accumulation. This suggests the dynamic multitasking problem inherent to the "pure OJT" model is at least tempered by the presence of an LBD component to human capital accumulation.

Overall, this paper constitutes an important step towards designing effective educational policy that targets teachers and also shows how a partial identification approach can exploit existing data, designed for another purpose, to answer an important policy relevant question. This paper's framework could also be applied to other contexts, in education and otherwise. It is common to collect follow-up measures to gauge the longer-run effects of educational interventions. The framework developed in this paper provides a way to interpret such follow-up data on interventions targeting teachers, through the lens of classic conceptual frameworks for human capital: longer-run effects stem from the persistence of student knowledge and changes in teacher human capital. Future teacher incentive pay experiments that collected follow-up data would be able to apply this paper's methodology to identify the forces underlying teacher human capital growth. The strategy developed in this paper could also be adapted to other applications in which there were outcome-based incentives and a followup measure of output, to quantify the importance of different channels underlying human capital development.

In light of the well known identification difficulties, it may be surprising that we can learn something new about human capital accumulation, even under the transparent and relatively simple approach taken here. A very promising, complementary, tack would be the structural econometric approach, which would require different (some stronger) assumptions but could then also answer other important questions about the importance of OJT and LBD forces in teachers' human capital accumulation. Such an approach would also be well-suited to rationalize the observed patterns in the data, and would yield other benefits, such as allowing for heterogeneity in teacher human capital accumulation trajectories and, thus, heterogeneity in the growth of teacher quality. It would also permit simulation of behavior and outcomes under counterfactual incentive schemes. This is left for future research.

APPENDIX

A Relationship to a Log-Linear Specification

This section illustrates one way in which the linear technologies (1)-(2) relate to nonlinear specifications. The illustration considers a representative teacher, teaching a representative student, in each of the control and treatment groups; therefore I suppress the teacher and student subscripts in this section. I maintain the assumption of balance of the experimental design.

Consider the following production function for teacher human capital, denoted here as κ_t:

$$\kappa_t = \kappa_{t-1}^{\gamma_\kappa}[\kappa_{t-1}\iota_{t-1}]^{\gamma_\iota}[\kappa_{t-1}\zeta_{t-1}]^{\gamma_\zeta}, \tag{16}$$

where κ_{t-1} is the teacher's human capital last period (which may depreciate), ι_{t-1} is the share of the teacher's human capital last period spent on OJT investment, and ζ_{t-1} is the share of the teacher's human capital last period spent on production. The parameters of interest, respectively representing the OJT and LBD components of human capital accumulation in (16), are $(\gamma_\iota, \gamma_\zeta) \in [0,1]^2$.[27]

Using $\tilde{}$ to denote the log of a variable, we can write the log-linearized version of the human capital production function, eq. (16):

$$\tilde{\kappa}_t = [\gamma_\kappa + \gamma_\iota + \gamma_\zeta]\tilde{\kappa}_{t-1} + \gamma_\iota\tilde{\iota}_{t-1} + \gamma_\zeta\tilde{\zeta}_{t-1}. \tag{17}$$

Let w_t measure the cognitive skill of the student in period t, which is produced according to[28]

$$w_t = \kappa_t^{\lambda_\kappa}\zeta_t^{\lambda_\zeta}w_{t-1}^{\lambda_w}, \tag{18}$$

which, in logs, is

$$\tilde{w}_t = \lambda_\kappa\tilde{\kappa}_t + \lambda_\zeta\tilde{\zeta}_t + \lambda_w\tilde{w}_{t-1}. \tag{19}$$

This equation is a log-linearized value added specification for cognitive achievement, where the value added to log achievement is $\lambda_\kappa\tilde{\kappa}_t + \lambda_\zeta\tilde{\zeta}_t$.

As before, the bounds on the parameters of interest will depend on the ATEs for achievement and leisure. With a representative teacher and student, we have $\Delta\tilde{z}_t = \tilde{z}_t^T - \tilde{z}_t^C$ for $z = \kappa, \iota, \zeta, w$.

For the pre-treatment period, $t = 0$, the mean difference in achievement between the treatment

[27] Note that the unit interval is conservative, as it allows for parameter values that could yield explosive growth and would thus likely be ruled out a priori if estimating this model. As is the case with any model, this specification is meant to serve as an approximation to reality. If taken literally (and augmented with additional assumptions to rationalize input choices) then a pure LBD specification would likely yield an optimal OJT investment of $\iota_{t-1} = 0$, which would imply $\kappa_t = 0$. Even here, however, even small positive values of γ_ι would avoid this problem.

[28] I have not included an error because there is a representative student in each group.

and control groups is

$$\Delta\tilde{w}_0 = \lambda_\kappa \underbrace{\Delta\tilde{\kappa}_0}_{=0} + \lambda_\zeta \underbrace{\Delta\tilde{\zeta}_0}_{=0} + \lambda_w \underbrace{\Delta\tilde{w}_{-1}}_{=0} = 0, \tag{20}$$

i.e., a balanced experimental design implies there will be no average difference in the pre-treatment average scores, as was also the case in Section 3.2.

For the active-treatment period, $t = 1$, we have

$$\Delta\tilde{w}_1 = \lambda_\kappa \underbrace{\Delta\tilde{\kappa}_1}_{=0} + \lambda_\zeta \Delta\tilde{\zeta}_1 + \lambda_w \underbrace{\Delta\tilde{w}_0}_{=0} = \lambda_\zeta \Delta\tilde{\zeta}_1. \tag{21}$$

Similar to eq. (6), eq. (21) shows that the difference between treatment and control achievement in the active-treatment period can only come from the change in mean working time, $\Delta\tilde{\zeta}_1$.

The mean difference in achievement between the treatment and control groups for the post-active-treatment period, $t = 2$, is

$$\begin{aligned} \Delta\tilde{w}_2 &= \lambda_\kappa \Delta\tilde{\kappa}_2 + \lambda_\zeta \Delta\tilde{\zeta}_2 + \lambda_w \Delta\tilde{w}_1 \\ &= \lambda_\kappa \gamma_\iota \Delta\tilde{\iota}_1 + \lambda_\kappa \gamma_\zeta \Delta\tilde{\zeta}_1 + \lambda_\zeta \Delta\tilde{\zeta}_2 + \lambda_w \Delta\tilde{w}_1, \end{aligned} \tag{22}$$

which uses $\Delta\theta_1 = 0$ to go from the first to the second line. Assuming for simplicity that the intervention had no effect on leisure, we have

$$\Delta\zeta_t + \Delta\iota_t = 0, \tag{23}$$

i.e., the effect on OJT investment is opposite that on production shares. Substituting using eqs. (21) and (23) and maintaining the assumption that post-active treatment production shares will not be different between the control and treatment groups (i.e., $\Delta\tilde{\zeta}_2 = 0$), eq. (22) becomes

$$\Delta\tilde{w}_2 = \left[\frac{\lambda_\kappa \gamma_\zeta - \lambda_\kappa \gamma_\iota + \lambda_\zeta \lambda_w}{\lambda_\zeta}\right] \underbrace{\Delta\tilde{w}_1.}_{\lambda_\zeta \Delta\tilde{\zeta}_1,\text{ from eq. (21)}} \tag{24}$$

The last step is to obtain values for the relevant quantities in (24), $\lambda_\kappa, \lambda_\zeta, \lambda_w, \Delta\tilde{w}_1, \Delta\tilde{w}_2$. Analogous to Section 4, I set $\lambda_\kappa = 1$ and $\lambda_\zeta = 1$; both of these are effectively normalizations,[29] and these parameters having the same value is consistent with a teacher's value added being the share of her human capital allocated to production. Next consider the remaining parameters, $\lambda_w, \Delta\tilde{w}_1, \Delta\tilde{w}_2$. It is well known that test scores measuring, e.g., cognitive skill, have no inherent scale, meaning any

[29] λ_κ fixes the scale of pre-treatment teacher human capital, which is unobserved and yields no direct testable implications; this normalization was also made in the model in the main text. Setting $\lambda_\zeta = 1$ may not strictly be a pure normalization, as a null ATE on achievement in the active-treatment period could stem from $\lambda_\zeta = 0$ and/or $\Delta\tilde{\zeta}_1 = 0$. However, both of these parameters must be nonzero to match the positive ATE in the data; given this, then, setting $\lambda_\zeta = 1$ is innocuous.

monotonic (increasing) transformations (e.g., logarithms) are also valid measures (see, e.g., Cunha and Heckman, 2008). Given the earlier argument that qualitative differences in inputs drive the analysis, and the known sensitivity of achievement tests to monotonic transformations (Bond and Lang, 2013), it is reasonable to use the estimator of β_y as an approximation for λ_w. In a similar way, we can use Δy_t, which corresponds to the estimated difference in value added between the treatment and control groups, to measure $\Delta\tilde{w}_t$, the difference in achievement for the students respectively representing the treatment and control groups. Putting all of this together, the bounds obtained for (δ_I, δ_h) would also apply to $(\gamma_\iota, \gamma_\zeta)$.

B Estimation of Confidence Sets

This appendix describes the algorithm used to estimate the confidence sets for δ_h and δ_I.

Algorithm 1 Bootstrap estimation of confidence sets

for $s = 1 \ldots nSamp$ **do**
    Sample observations from treatment and control groups (stratified by treatment group)
    Estimate $(\hat{\beta}^s_y, \hat{\Delta}l^s_1, \hat{\Delta}y^s_1, \hat{\Delta}y^s_2)$
    Use simulated values to create locus defined by eq. (8)
    Project locus onto marginals, obtaining random intervals $\hat{d}^s_h, \hat{d}^s_I$
end for
for $k = h, I$ **do**
    for $d^{try}_k \subseteq D_k$ **do**
        retain d^{try}_k iff $\frac{1}{nSamp}\sum_{s=1}^{nSamp} \mathbf{1}\{\hat{d}^s_k \subseteq d^{try}_k\} \geq 1-\alpha$
    end for
end for
$(1-\alpha)\%$ confidence set for δ_k is $\arg\min_{d^{try}_k \text{ retained}}\{\max\{d^{try}_k\} - \min\{d^{try}_k\}\}$

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Figure 1: Examples of OJT and LBD Specifications

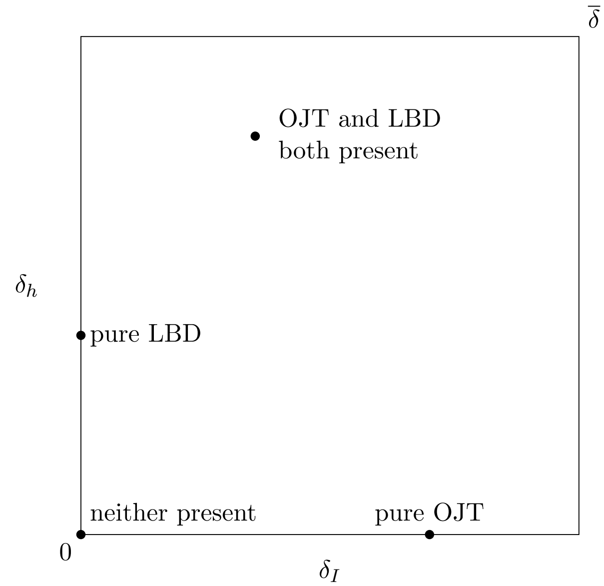
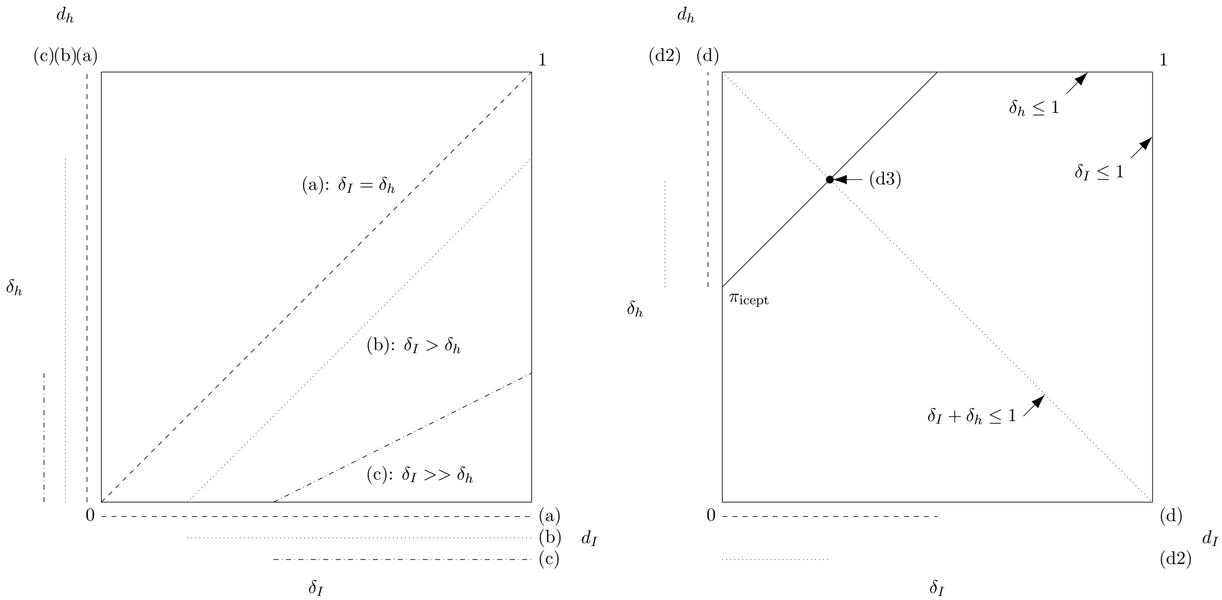


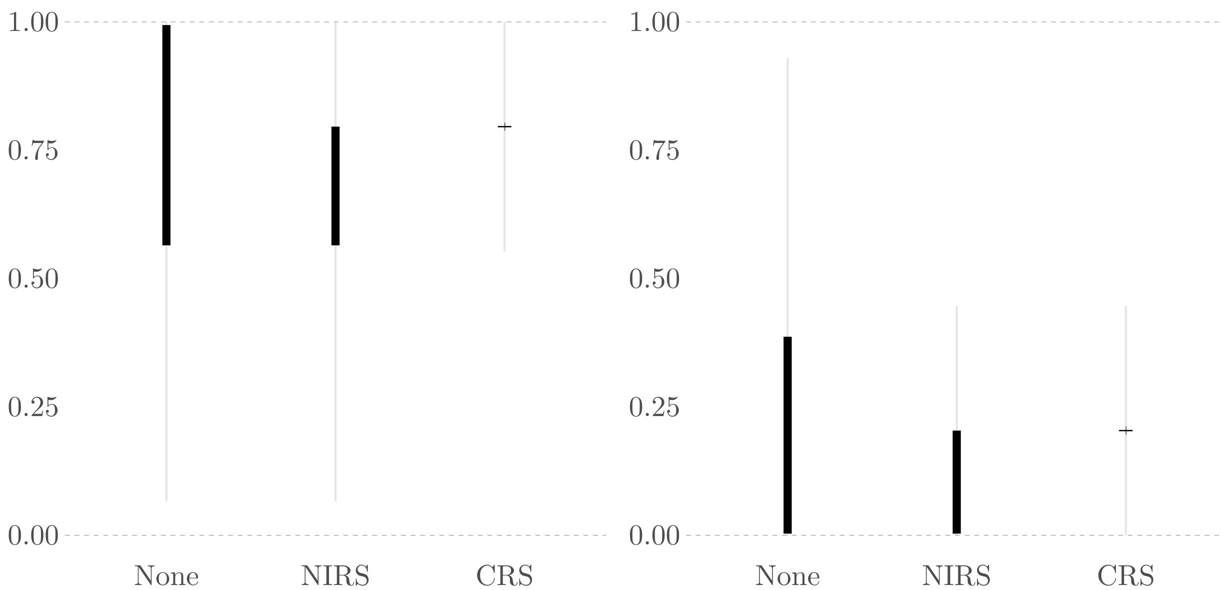
Figure 2: Illustration of Example Identified Set Cases



Note: Marginal identified sets for parameters are indicated by the lines outside their respective axes. Cases depicted in the figure are summarized in Table 1. The left panel corresponds to comparative statics, cases (a)-(c). The right panel corresponds to different returns-to-scale assumptions, cases (d)-(d3).

Figure 3: Estimated Bounds and Confidence Sets

(a) δ_h: LBD　　　　　(b) δ_I: OJT



Notes: The left panel depicts estimated bounds (▬) and 95% confidence sets (—) for δ_h, under Assumptions 1, 2, and 3 (denoted on the bottom axis via "None", "NIRS", and "CRS", respectively). The right panel depicts the analogous results for δ_I.

	Assumption	Estimated bounds			95% confidence set		
Parameter	about $\delta_h + \delta_I$	Min.	Max.	Width	Min.	Max.	Width
δ_h	1: None	0.565	0.998	0.433	0.067	1	0.933
	2: NIRS	0.565	0.798	0.233	0.067	1	0.933
	3: CRS	0.798	0.798	0	0.562	1	0.438
δ_I	1: None	0.003	0.378	0.375	0	0.853	0.853
	2: NIRS	0.003	0.202	0.199	0	0.438	0.438
	3: CRS	0.202	0.202	0	0	0.438	0.438

Notes: Assumption 1 corresponds to no assumption about the returns to scale for $\delta_h + \delta_I$, and Assumptions 2 and 3 respectively correspond to non-increasing and constant returns to scale.