Agenda

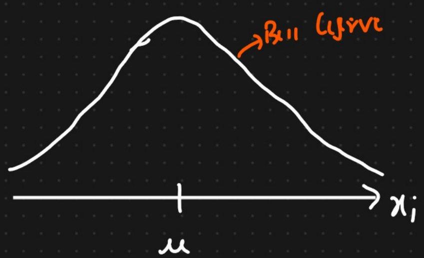
1. Central Limit Theorem ✓
2. Z-score And z stats ✓
3. Z-test, t-test } Solve problems   Assignment

} Infuential stats (inferential)
Hypothesis Testing.

① Central Limit Theorem

① $X \approx N(\mu, \sigma)$

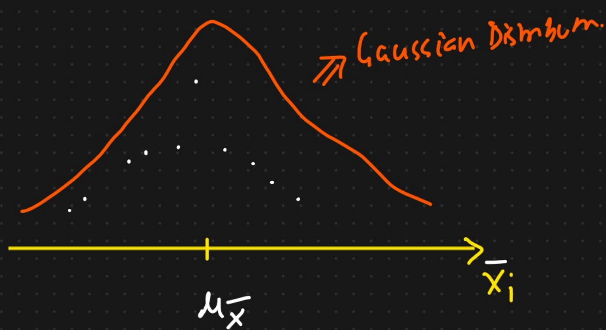
Suppose we have random variable x that belongs to a normal/gaussian distribution with some mean mu and standard deviation sigma



n = sample size => any value (1, 2, 3, 4, 5)

$$S_1 = \{x_1\ x_2\ x_3 \ldots x_n\} = \bar{x}_1$$
$$S_2 = \{x_2, x_3, x_4, x_6 \ldots x_n\} = \bar{x}_2$$
$$S_3 = \{\ \} = \bar{x}_3$$
$$S_4 = \{\ \} = \bar{x}_4$$
$$S_5 = \{\ \} = \ldots$$
$$\vdots$$
$$S_m = \{\ \}\ \bar{x}_m$$

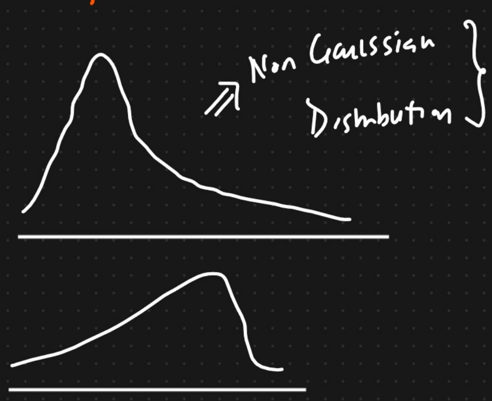
CLT



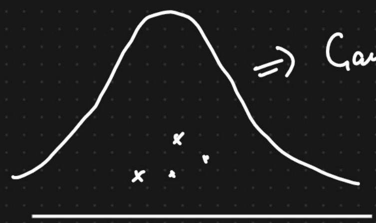
n = sample size

$\boxed{n \geq 30}$

② $X \not\approx N(\mu, \sigma)$ does not belong to gaussian dist.



$$S_1 = \{\ \} = \bar{x}_1$$
$$S_2 \quad = \bar{x}_2$$
$$S_3 \quad = \bar{x}_3$$
$$S_4 \quad \vdots$$
$$\bar{x}_m$$

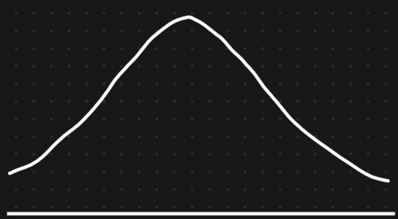


⇒ Gaussian/ Normal Distribution

① Central Limit Theorem

The central limit theorem relies on the concept of a sampling distribution, which is the probability distribution of a statistic for a large number of samples taken from a population.

The central limit theorem says that the sampling distribution of the mean will always be normally distributed, as long as the sample size is large enough. Regardless of whether the population has a normal, Poisson, binomial, or any other distribution, the sampling distribution of the mean will be normal.



$$X \sim N(\mu, \sigma)$$

σ = population std

μ = population mean

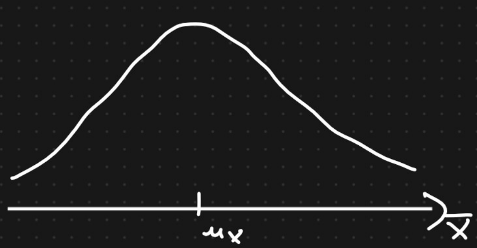
X' will have approximately the same mean as population mean and standard deviation as written below called as standard error

Sampling Distribution of the mean

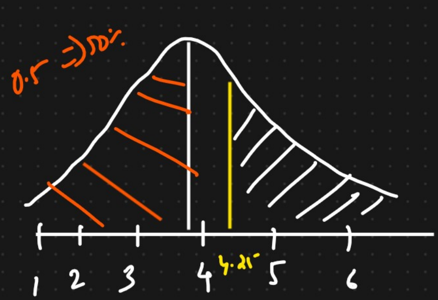
$$\bar{X} \sim N\left(\mu, \left\{\frac{\sigma}{\sqrt{n}}\right\}\right)$$

→ Standard error.

n = Sample size



① $X \sim N(4, 1)$



we have already read about Z-score, so I am not going to give the definition again

$x_i = 4.25$

$$Z\text{-score} = \frac{4.25 - 4}{\sigma} = 0.25$$

here basically, we need to find the area of white portion or can also said as

④ What percentage of score falls above 4.25?

Using Z score = 0.25 and Z-table, we can easily find the area of red portion, and then by subtracting it from 1, we can find the area of yellow portion

$1 - 0.59871 = 0.4013 = 40.13\%$

⑤ What percentage of score lies between 3.5 to 4.5?

$$Z\text{-score} = \frac{4.5-4}{1} = +0.5 = 0.69146$$

$$Z\text{-score} = \frac{3.5-4}{1} = -0.5 = 0.30854$$

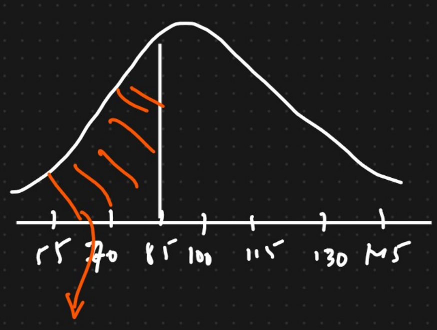
$$= 0.3829$$

$$= 38.29\%$$

⑥ In India the average IQ is 100, with a standard deviation of 15. What is the percentage of the population would you expect to have an IQ lower than 85?

Ans). $\mu = 100$ $\sigma = 15$

$$Z\text{-score} = \frac{85-100}{15} = -1$$



$0.15866 = 15.86\%$

(A) $IQ \geq 85$

$1 - 0.1586 = 84.13\%$

$\boxed{75 \geq IQ \leq 100}$ ⇒ Internal Assignment

① Hypothesis Testing And Statistical Analysis

① Z test
② t test
③ CHI SQUARE
④ ANNOVA

① Z-test

① 1) The average heights of all residents in a city is 168cm, with a $\sigma = 3.9$. A doctor believes the mean to be different. He measured the height of 36 individuals and found the average height to be 169.5 cm.

(a) State null and Alternate Hypothesis
(b) At a 95% confidence level, is there enough evidence to reject the null hypothesis.

Ans) $\mu = 168cm$ $\quad \sigma = 3.9$ $\quad n = 36$ $\quad \bar{x} = 169.5$ $\quad C.I = 0.95$ (confidence interval) $\quad \alpha = 1 - C.I = 0.05\%$

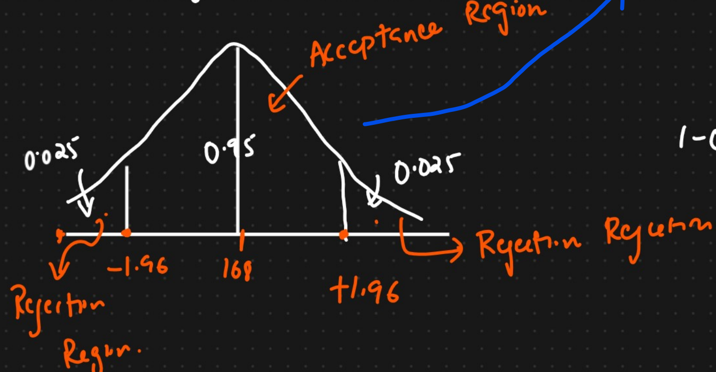
① Null Hypothesis H_0 : $\mu = 168cm$

Alternate Hypothesis H_1 : $\mu \neq 168cm$

z-test and p-test sirf normal dist. mei hi lgta hai

② Decision Boundary based C.I.

ab hme pata hai rejection area ka area 0.25 hai on one side, toh with the help of z-table, we can find ki koi particular point kitna standard deviation dur hai mean se, finally we get -1.96 to +1.96 as our decision boundary



2 Tailed Test

$1 - 0.025 = 0.9750$

isliye kyuki rejection of null hypotheis can lie on any side of the rejection region i.e. kyuki uska alt. hypothesis yehi hai ki mean != 168. Agr uska alt. hypothesis hota ki mean>168 toh ek hi rejection region hota(right one), then it will be called as 1 tailed test

If Z test is less than −1.96 or greater than +1.96, Reject the Null Hypothesis

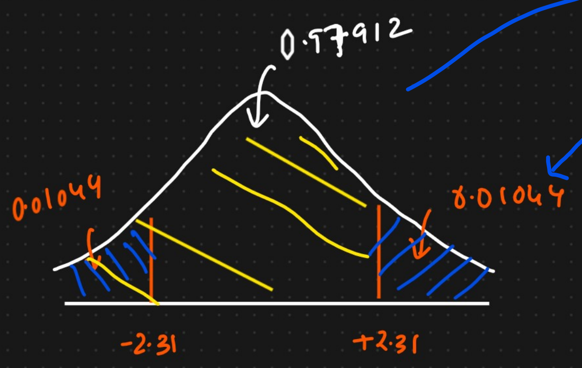
④ $Z\_test = \dfrac{\bar{x} - \mu}{\{\sigma/\sqrt{n}\}} = \dfrac{169.5 - 168}{3.9/\sqrt{36}} = \boxed{2.31}$

$\{\sigma/\sqrt{n}\} \Downarrow \{CLT\}$

Conclusion

2.31 > 1.96 Reject the Null Hypothesis. Z-test ka use krk hi P-score nikaalte hai

$1 - 0.98956 = 0.01044$



C.I = 99% Assignment

① p value = 0.01044 + 0.01044

= 0.02088

P < 0.05 ⇒ Reject the Null Hypothesis.

② A factory manufactures bulbs with a average warranty of 5 years with standard deviation of 0.50. A worker believes that the bulb will malfunction in less than 5 years. He tests a sample of 40 bulbs and find the average time to be 4.8 years.

(a) State null and alternate hypothesis

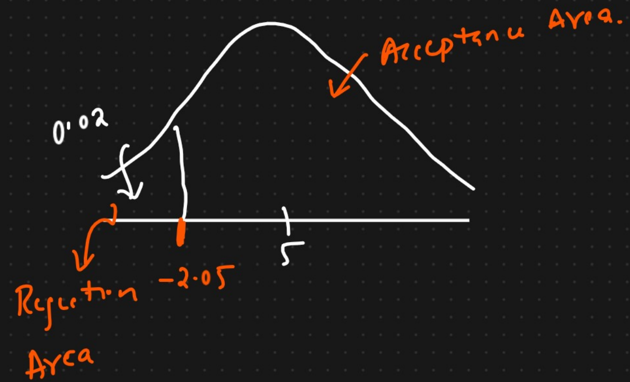
(b) At a 2% significance level, is there enough evidence to support the idea that the warranty should be revised?

Ans) $\mu = 5$ $\sigma = 0.50$ $n = 40$ $\bar{x} = 4.8$ years. C.I = 0.98 $\alpha = 0.02$

① H_0 $\mu = 5$

H_1 $\mu < 5$ {1 Tail Test}.

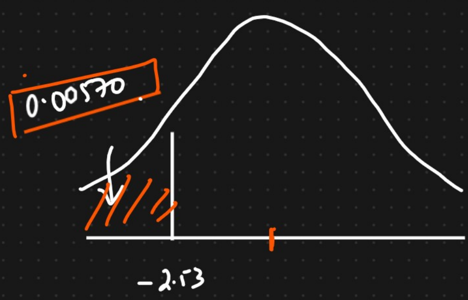
② Decision Boundary



Z test < -2.05 Reject the Null hypothesis.

③ $$Z_{test} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{4.8 - 5}{0.50 / \sqrt{40}} = \boxed{-2.53}$$

-2.53 < -2.05 ⇒ True ⇒ Reject the Null hypothesis.



Pvalue

$\Downarrow$ α

$\boxed{0.00570 < 0.02}$ 95%

$\Downarrow$

Reject the Null Hypothesis.

② T Test

① In the population the average IQ is 100. A team of researchers want to test a new medication to see if it has either a positive or negative effect on intelligence, or no effect at all. A sample of 30 participants who have taken the medication has a mean of 140 with a standard deviation of 20. Did the medication affect intelligence? C.I = 95% $\alpha = 0.05$

given

$n = 30$ $\mu = 100$ $\bar{x} = 140$ $s = 20$ C.I = 95

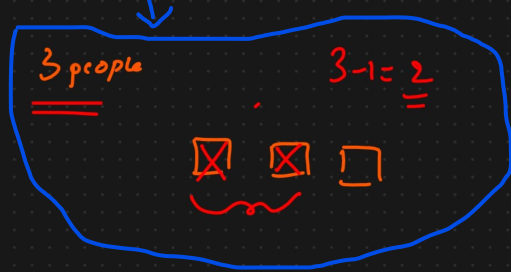
$\alpha = 0.05$

Ans) ① $H_0 = \mu = 100$

$H_1 = \mu \neq 100$ 2-tail test

② $\alpha = 0.05$

maanle hmaare paas 3 chair hai, first person has 3 choice, second person has 2 choices but the third person has no choice. So for 3 person dof = 2



Degree of freedom

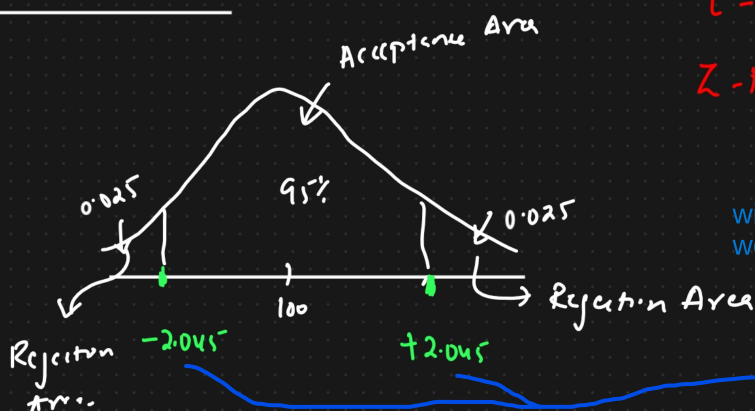
$dof = n-1 = 30-1 = 29$

③ **Decision Rule**

for t-test, we use t-table

t-test ⇒ sample std.

Z-test ⇒ population std



with the help of dof, tail area = 0.05 and t-table, we can find this value

If t test is less than -2.045 and greater than 2.045, Reject the Null Hypothesis.

④ **T Test statistics**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{140 - 100}{20/\sqrt{30}} = \frac{40}{3.65} = 10.96$$

$t > 2.045$ {Reject the Null hypothesis}.

Conclusion : Medication has a +ve effect on intelligence.

BANK A

A

B

95% 99%

Hypothesis Testing

2000

200 - 300 =>