Convolutional neural networks

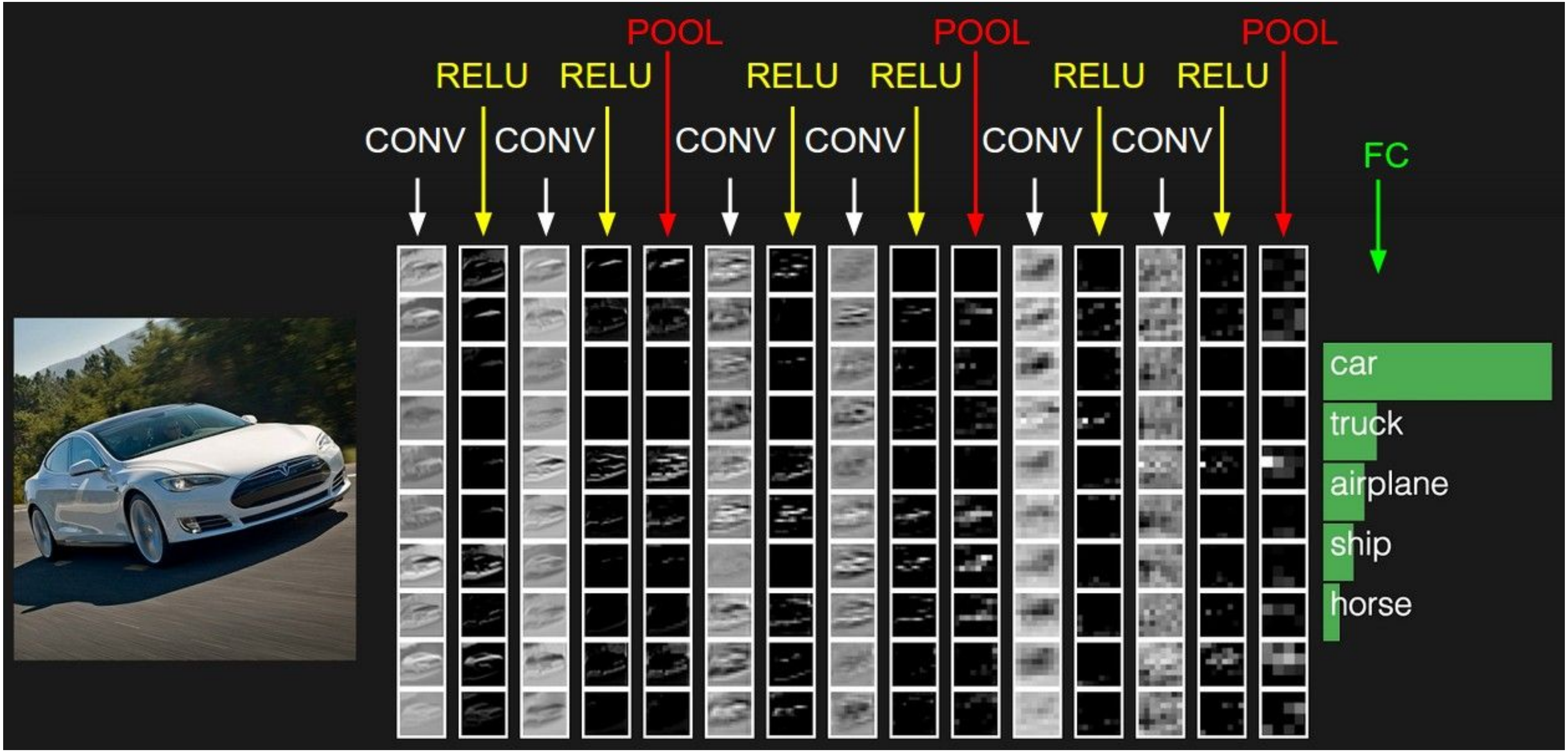


Image Source: cs231n, Stanford University

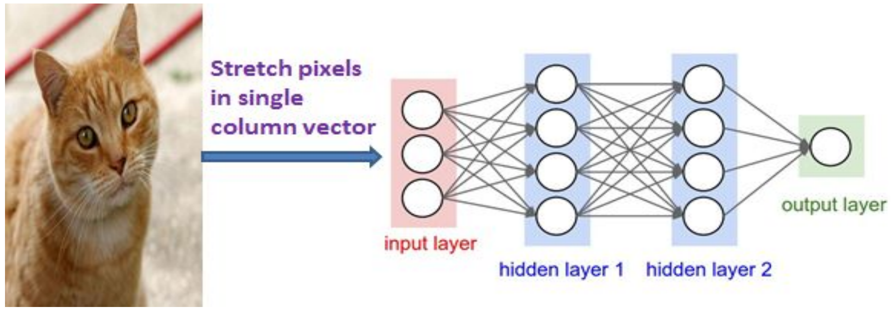
Outline

- Building blocks
  - Convolutional layers and backprop rules
  - Pooling layers and nonlinearities

This Class

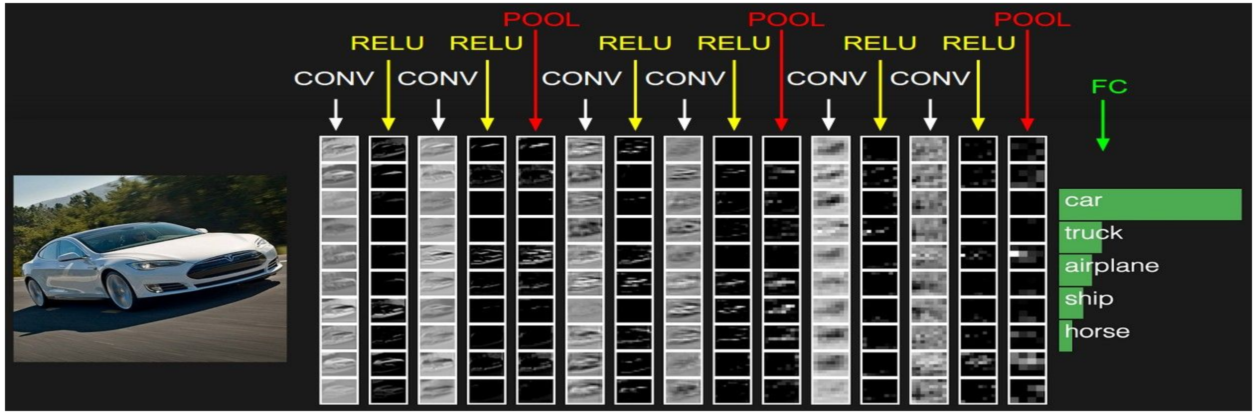
Neural Network and Image

- Dimensionality
- Local relationship



Convolutional Neural Network (CNN)

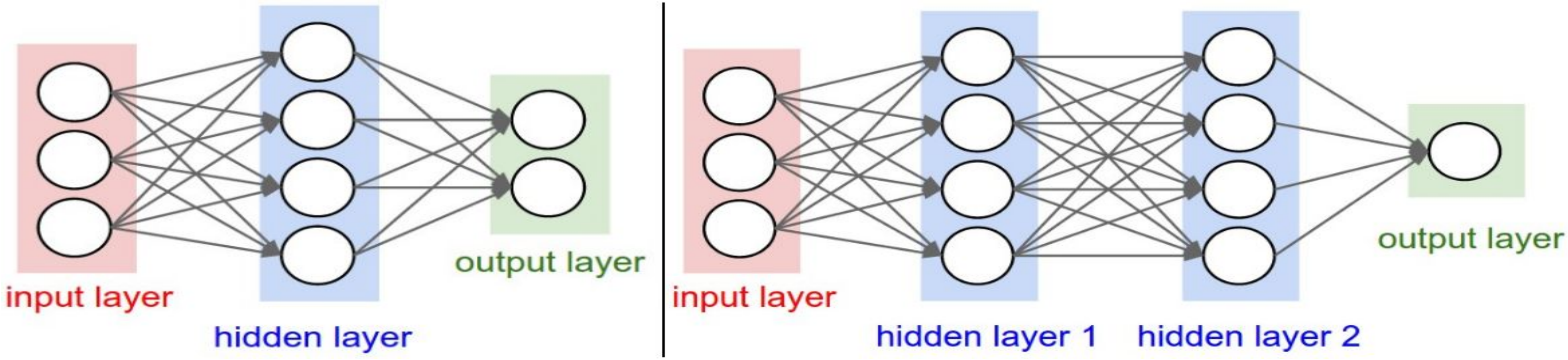
- Convolution Layer
- Non-linearity Layer
- Pooling Layer
- Fully Connected Layer
- Classification Layer



ImageNet Challenge

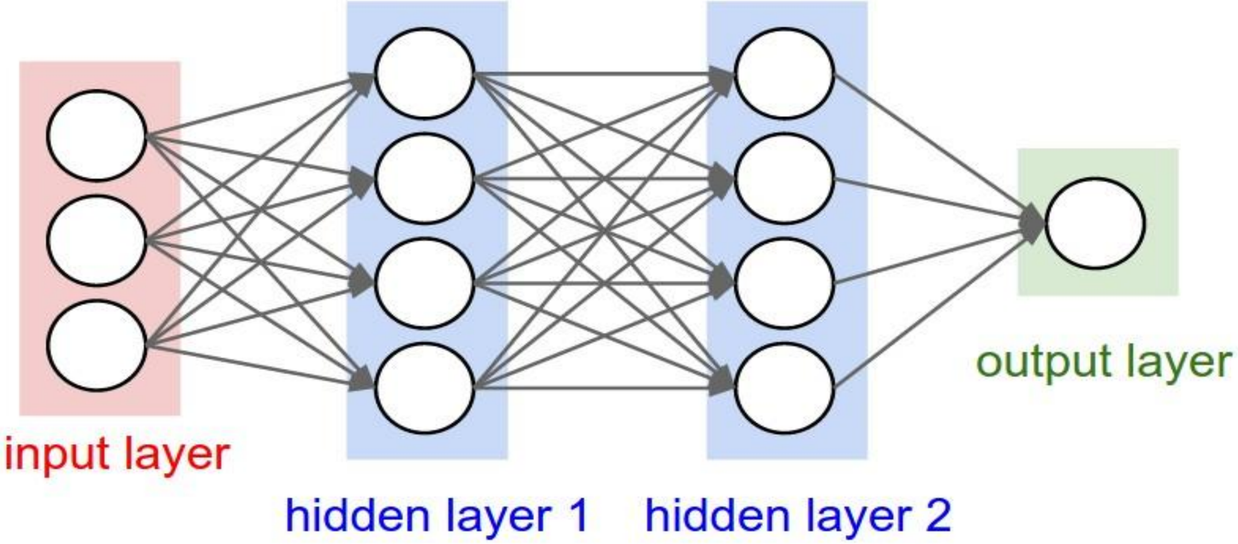
- Progress
- Human Level Performance

Neural Networks



Source: http://cs231n.github.io

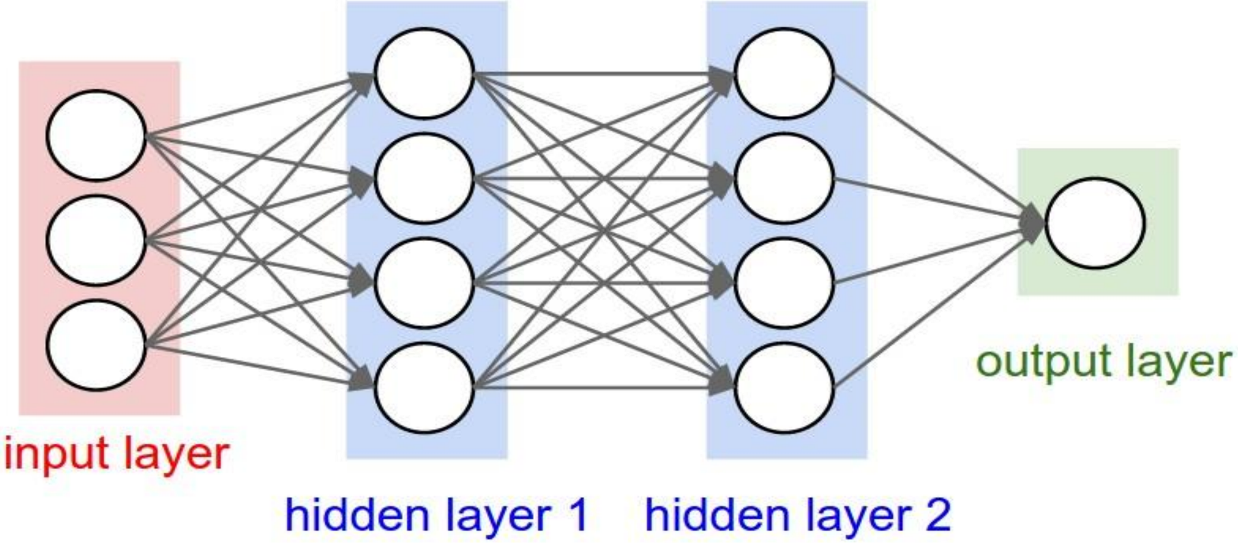
Multi-layer Neural Network & Image



How to apply NN over Image?

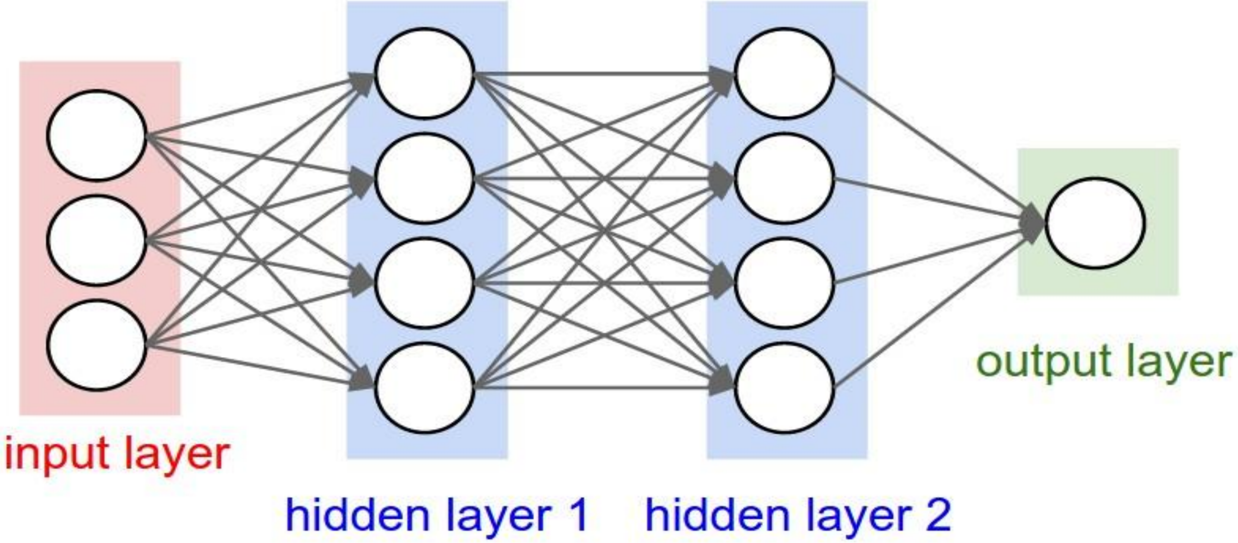
Multi-layer Neural Network & Image

Stretch pixels in single column vector



Multi-layer Neural Network & Image

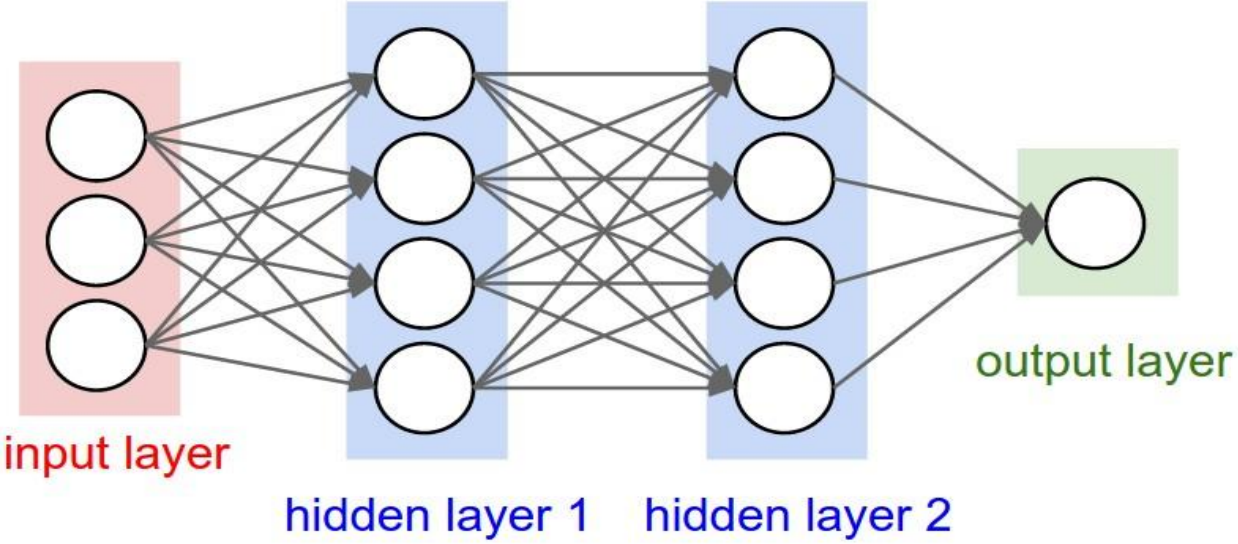
Stretch pixels in single column vector



Problems ?

Multi-layer Neural Network & Image

Stretch pixels in single column vector



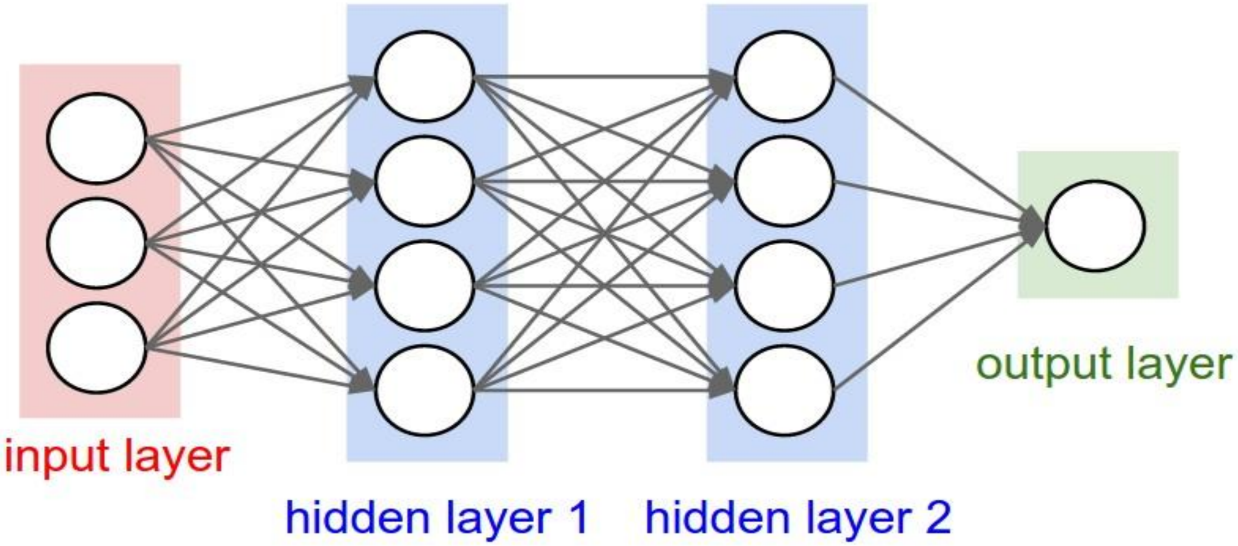
Problems:

High dimensionality

Local relationship

Multi-layer Neural Network & Image

Stretch pixels in single column vector



Problems:

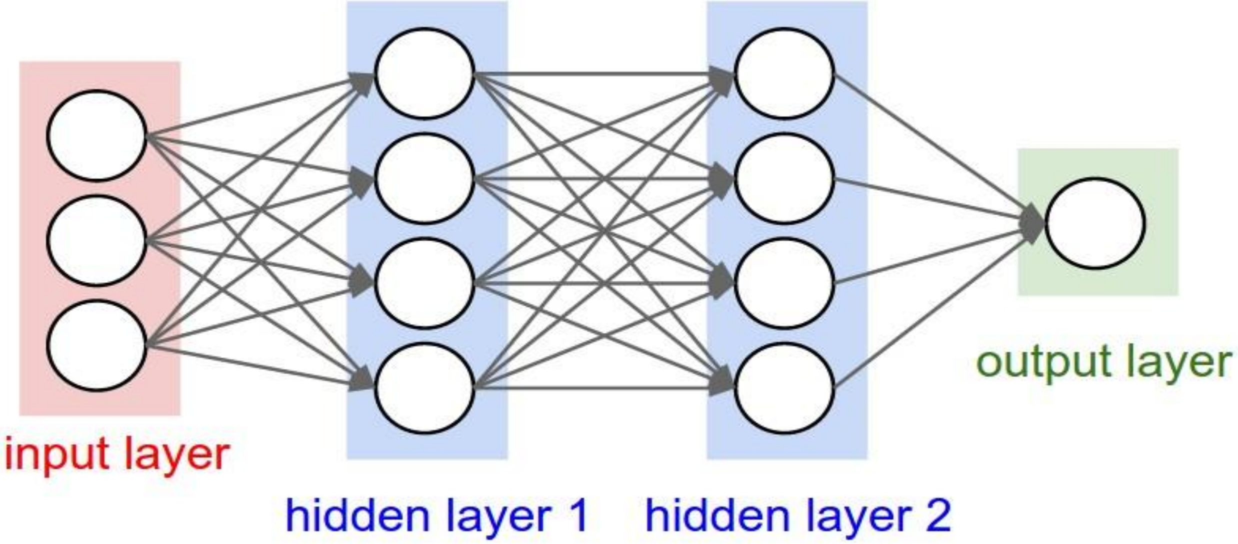
High dimensionality

Local relationship

Solution ?

Multi-layer Neural Network & Image

Stretch pixels in single column vector



Problems:

High dimensionality

Local relationship

Solution:

Convolutional Neural Network

Convolutional Neural Networks

Also known as

CNN,

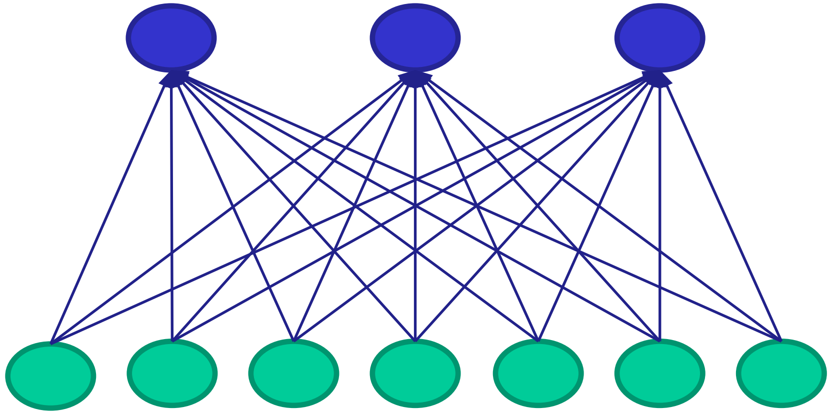
ConvNet,

DCN

CNN = a multi-layer neural network with

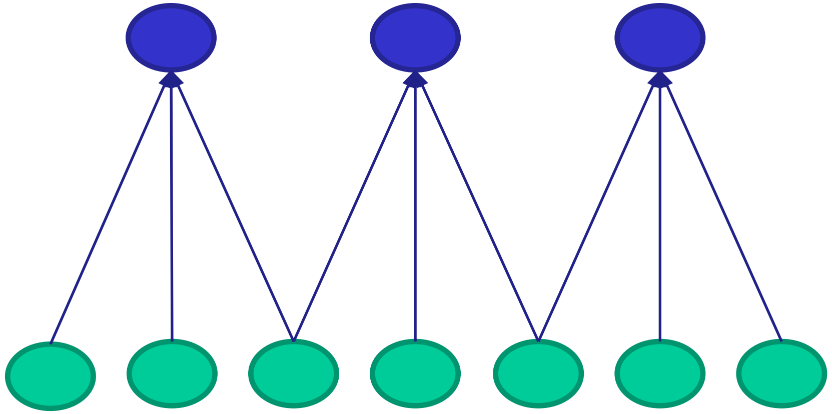
1. Local connectivity
2. Weight sharing

CNN: Local Connectivity



Hidden layer

Input layer



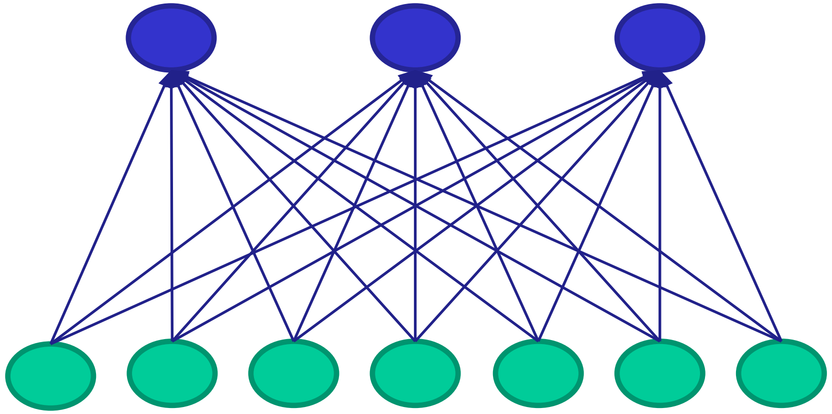
Global connectivity

Local connectivity

input units (neurons): 7

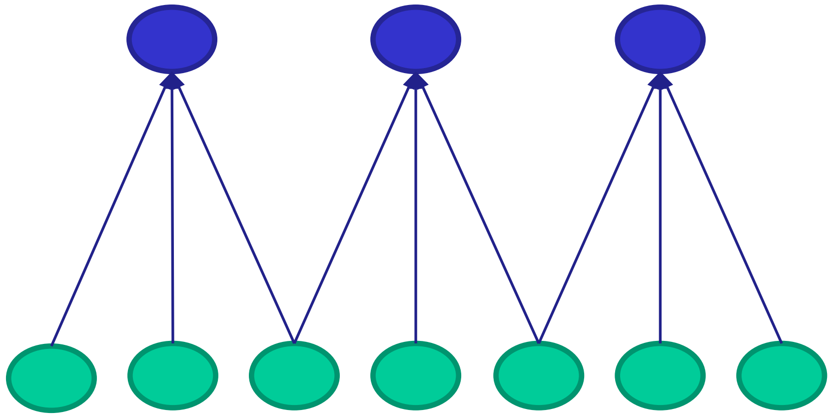
hidden units: 3

CNN: Local Connectivity



Hidden layer

Input layer



Global connectivity

Local connectivity

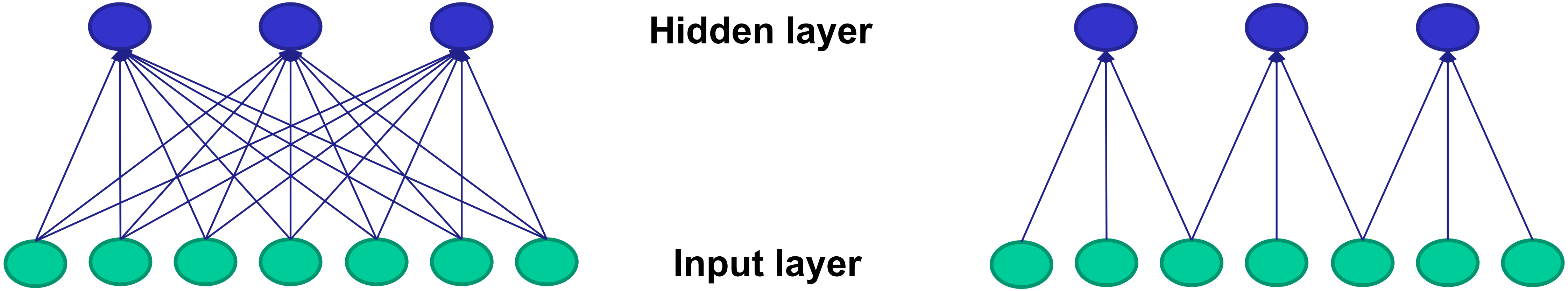
input units (neurons): 7

hidden units: 3

Number of parameters

- Global connectivity: ?
- Local connectivity: ?

CNN: Local Connectivity



Global connectivity

Local connectivity

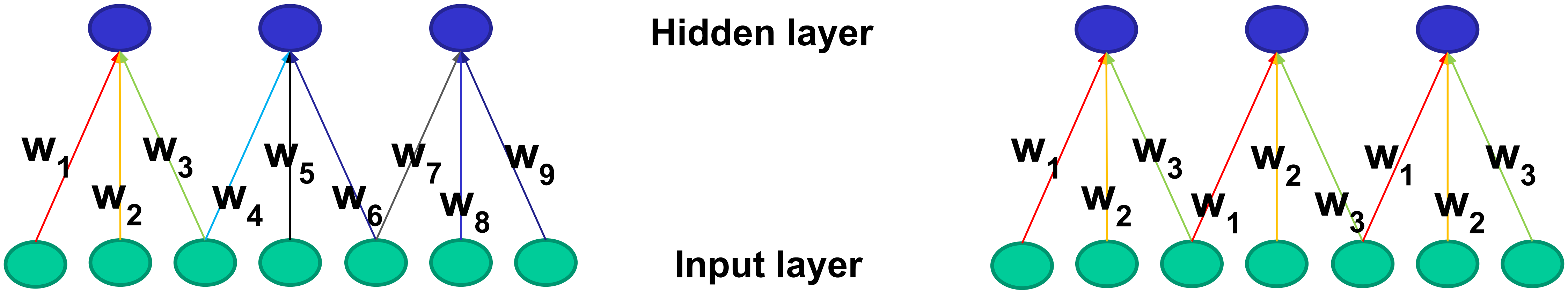
input units (neurons): 7

hidden units: 3

Number of parameters

- Global connectivity: **3 x 7 = 21**
- Local connectivity: **3 x 3 = 9**

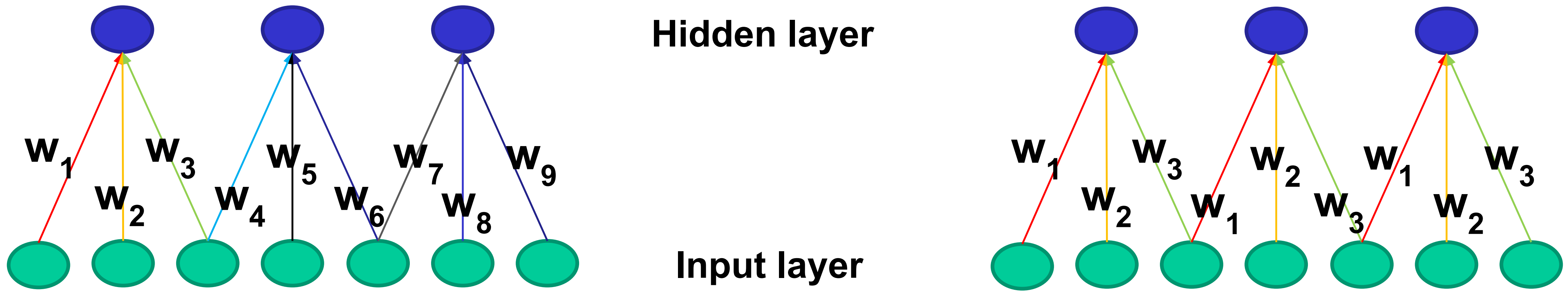
CNN: Weight Sharing



Without weight sharing **With weight sharing**

- **# input units (neurons): 7**
- **# hidden units: 3**

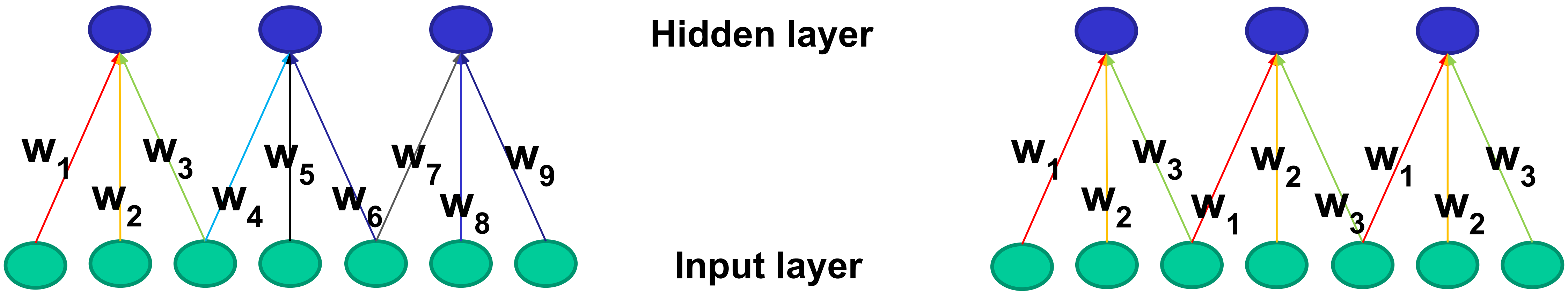
CNN: Weight Sharing



Without weight sharing **With weight sharing**

- **# input units (neurons): 7**
- **# hidden units: 3**
- **Number of parameters**
  - **Without weight sharing: ?**
  - **With weight sharing : ?**

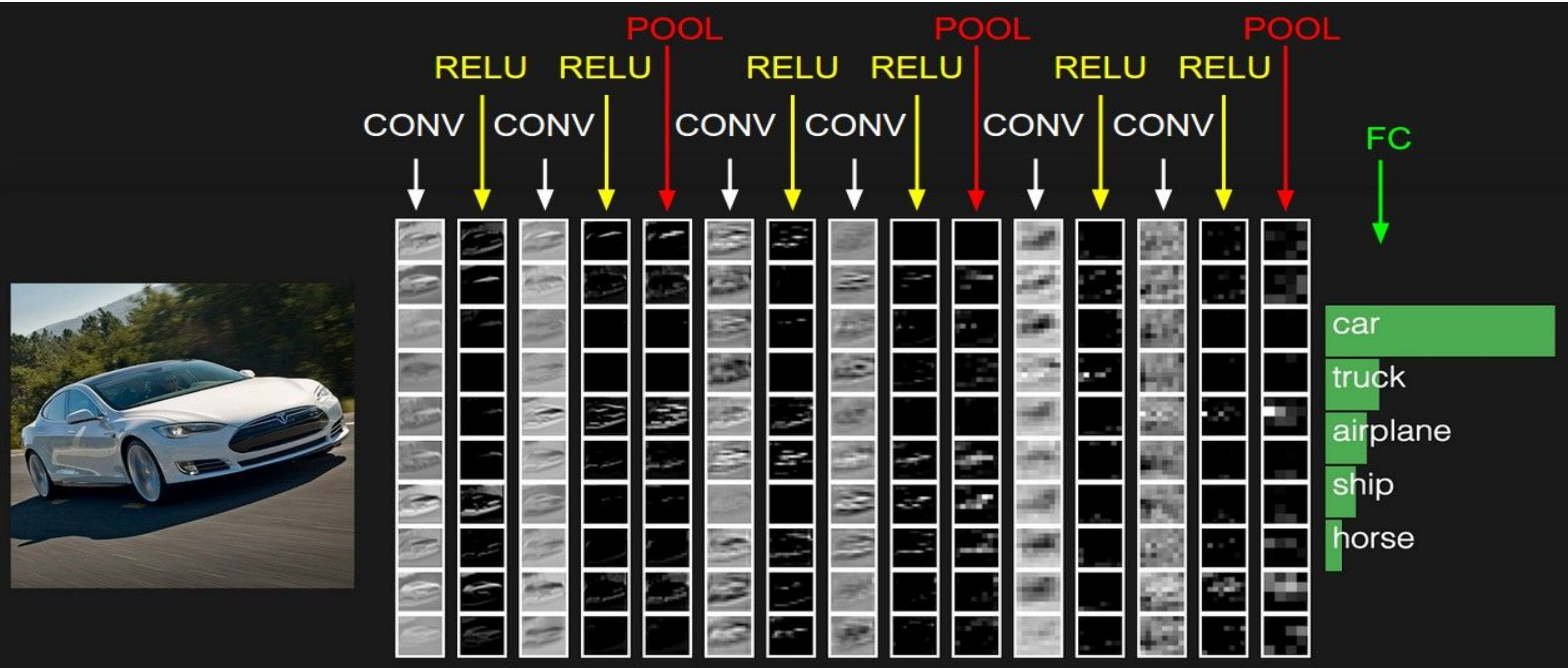
CNN: Weight Sharing



Without weight sharing **With weight sharing**

- **# input units (neurons): 7**
- **# hidden units: 3**
- **Number of parameters**
  - **Without weight sharing: 3 x 3 = 9**
  - **With weight sharing : 3 x 1 = 3**

Convolutional Neural Networks



Source: cs231n, Stanford University

Layers used to build ConvNets

Input Layer (Input image)

Convolutional Layer

Non-linearity Layer (such as Sigmoid, Tanh, ReLU, PReLU, ELU, Swish, etc.)

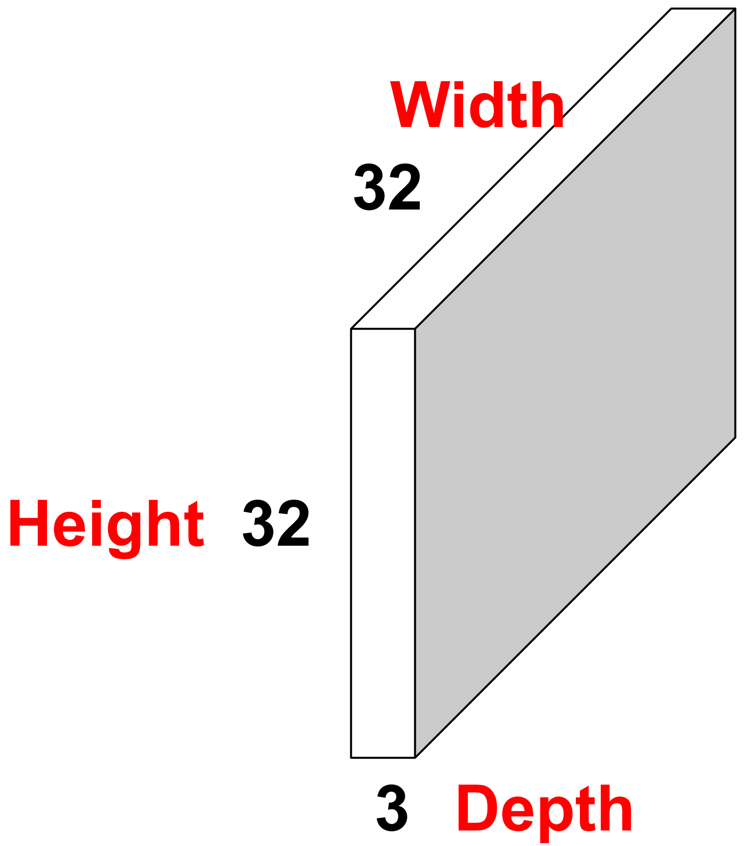
Pooling Layer (such as Max Pooling, Average Pooling, etc.)

Fully-Connected Layer

Classification Layer (Softmax, etc.)

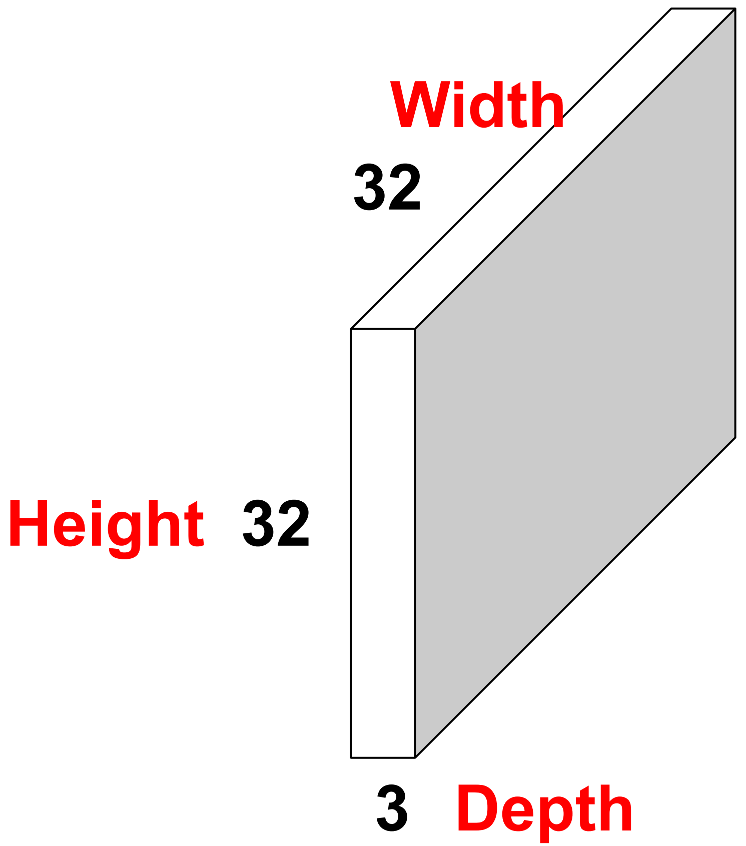
Convolutional Layer

32×32×3 Image -> preserve spatial structure

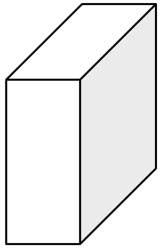


Convolutional Layer

32×32×3 Image



5×5×3 Filter



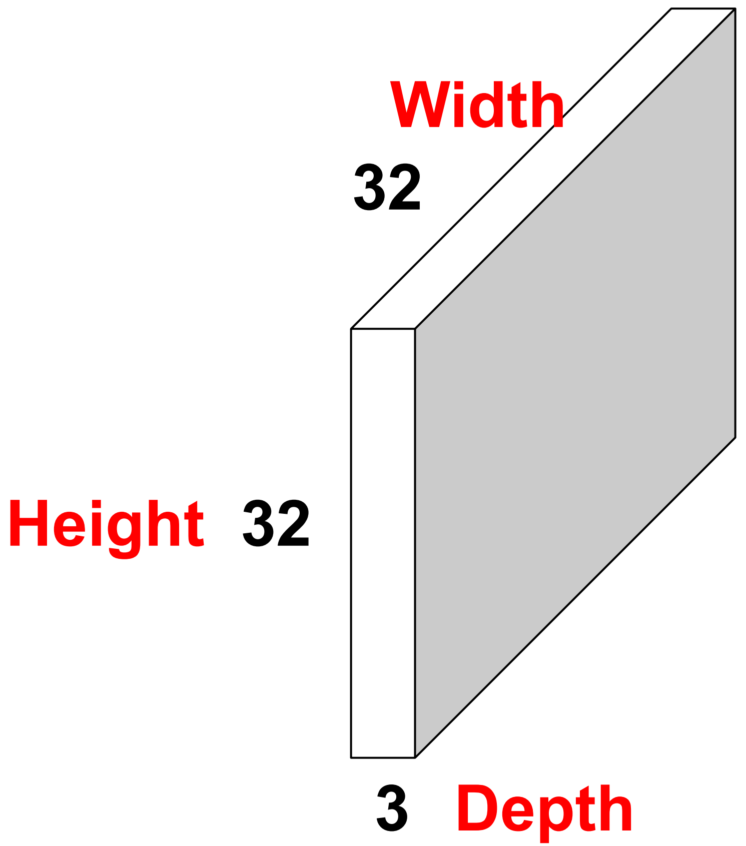
Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Convolutional Layer

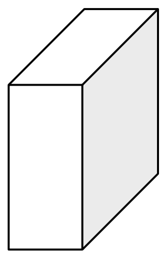
Handling multiple input channels

Filters always extend the full depth of the input volume

32×32×3 Image

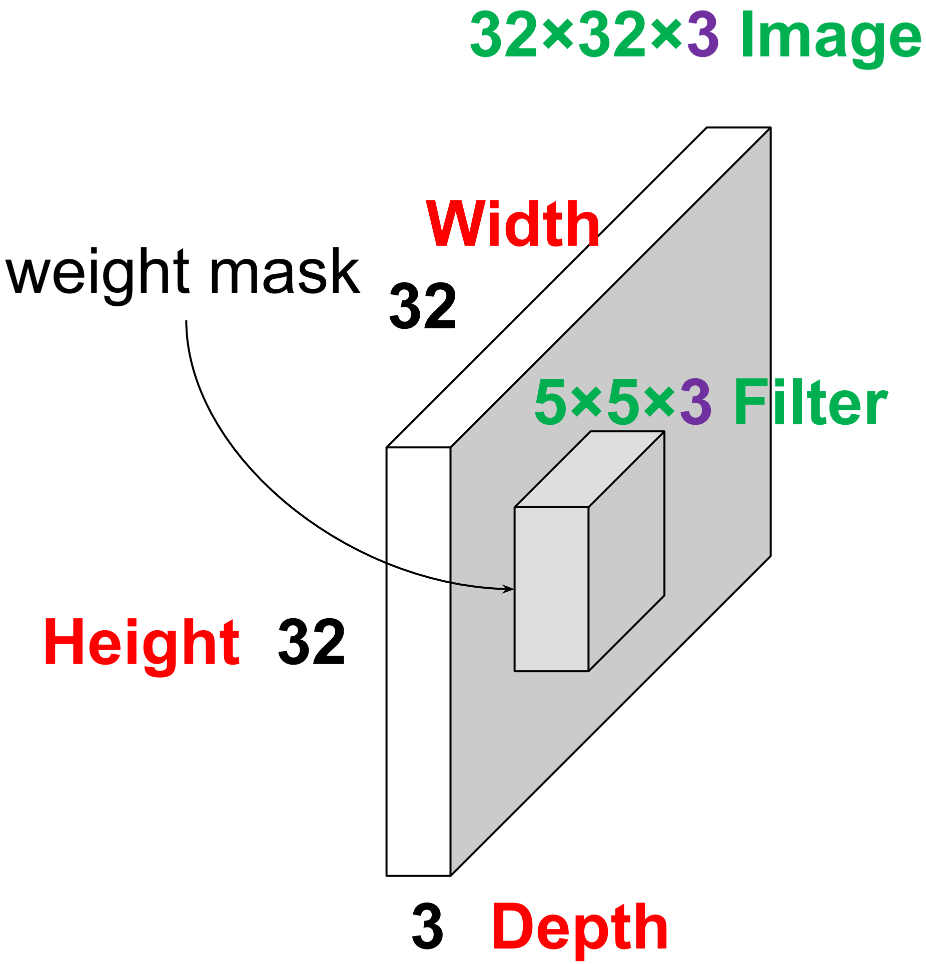


5×5×3 Filter

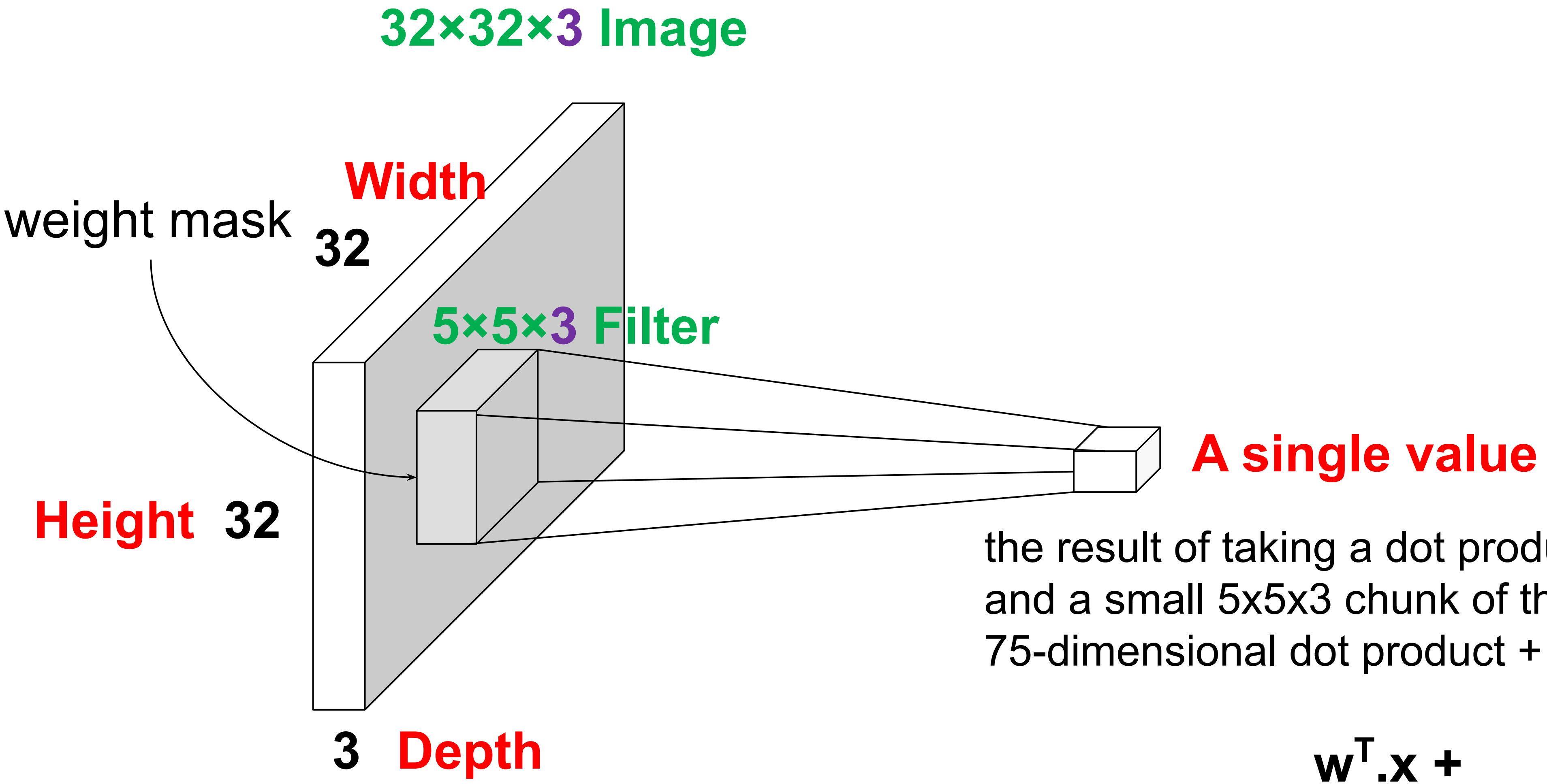


Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Convolutional Layer



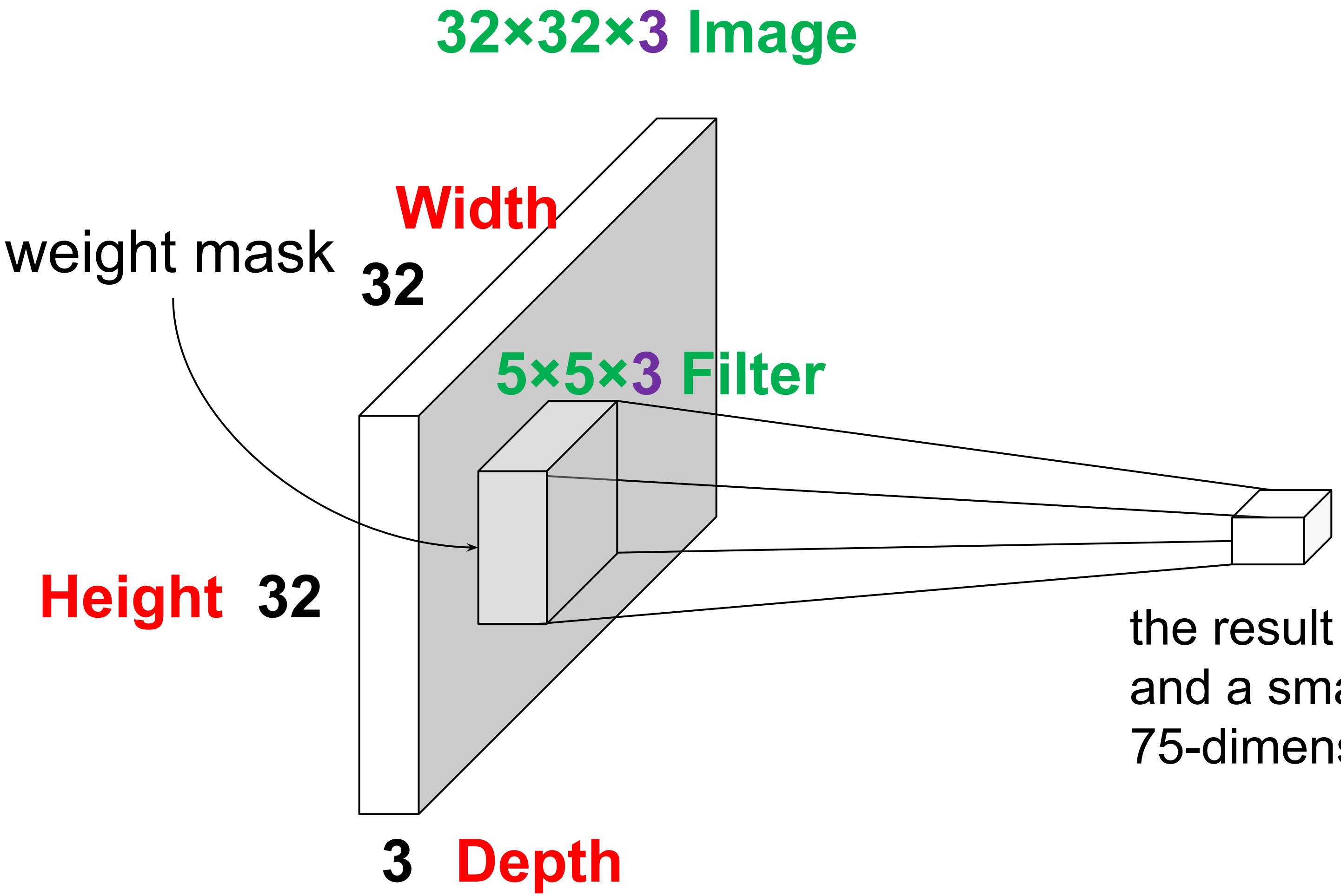
Convolutional Layer



the result of taking a dot product between the filter and a small 5x5x3 chunk of the image (i.e. 5*5*3 = 75-dimensional dot product + bias)

$\mathbf{w}^T.\mathbf{x} + \mathbf{b}$

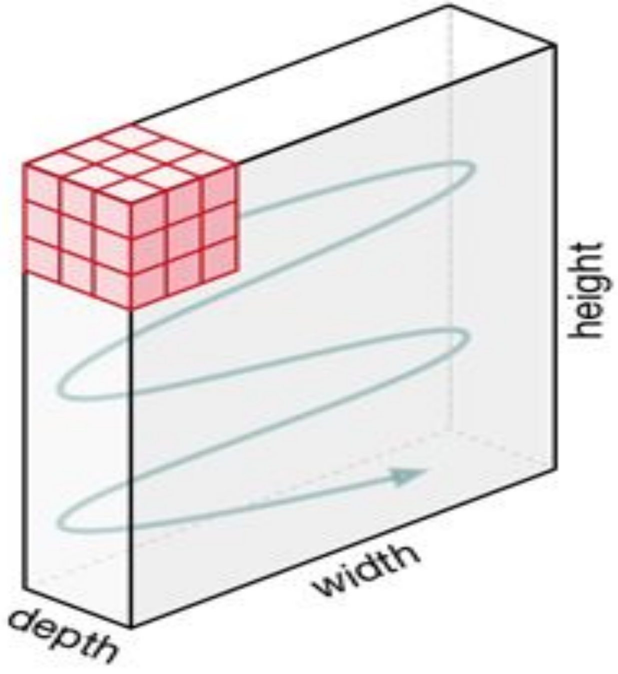
Convolutional Layer



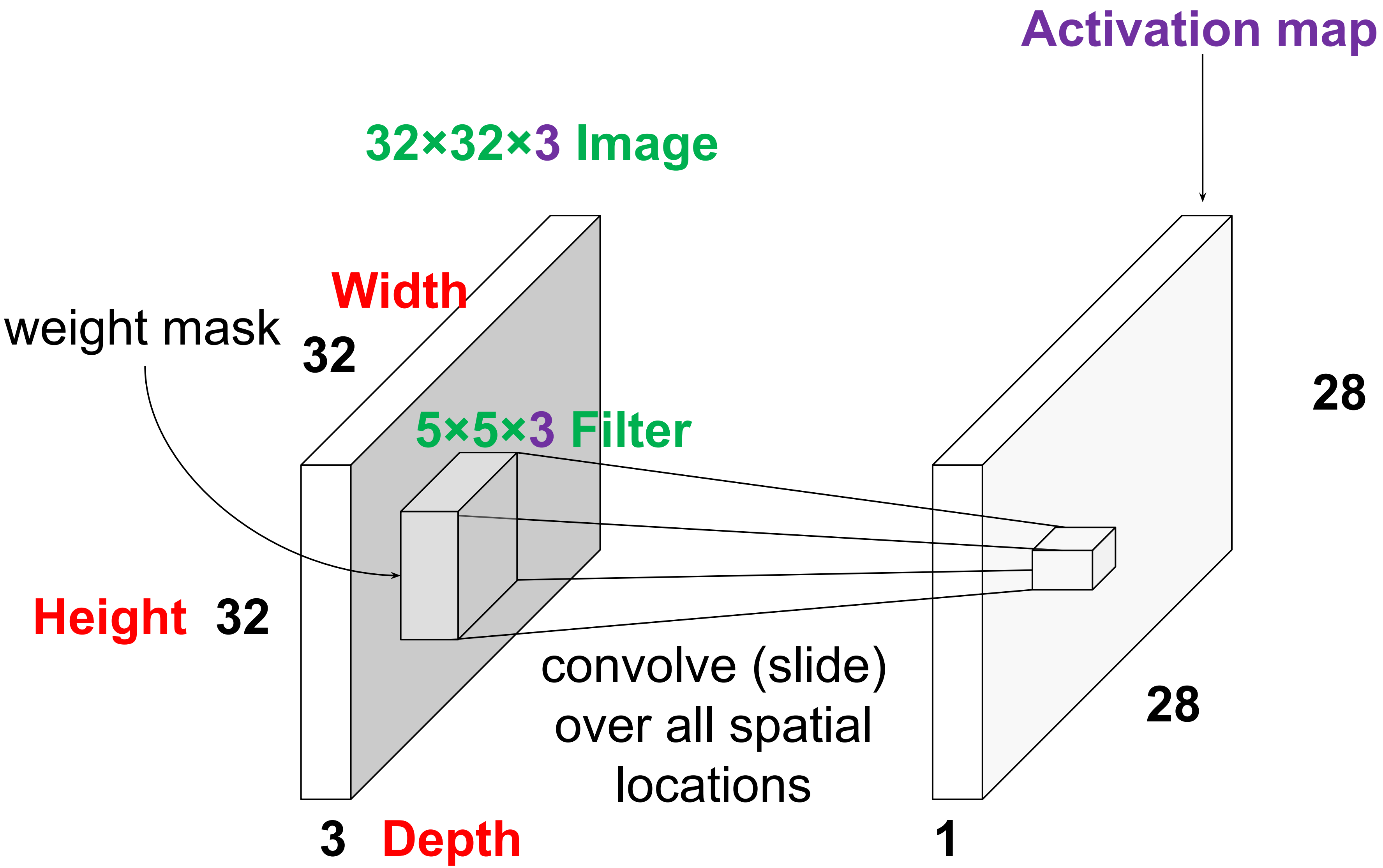
A single value

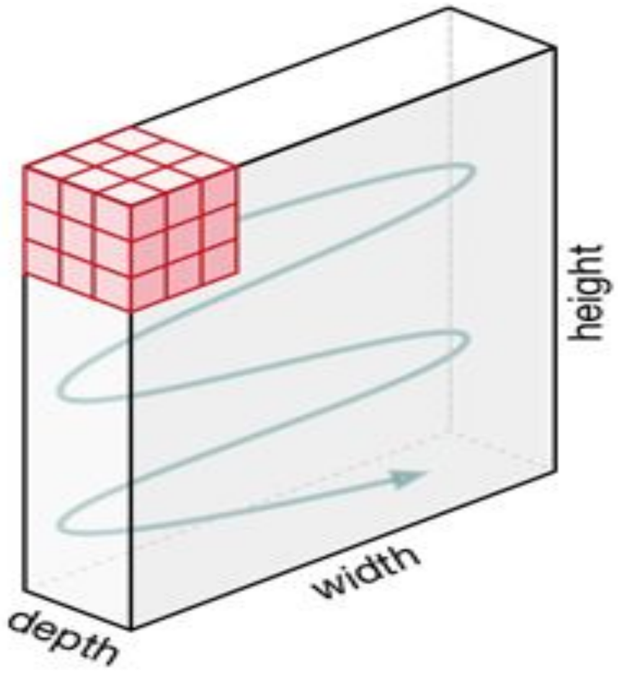
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image (i.e. 5*5*3 = 75-dimensional dot product + bias)

$w^T.x + b$



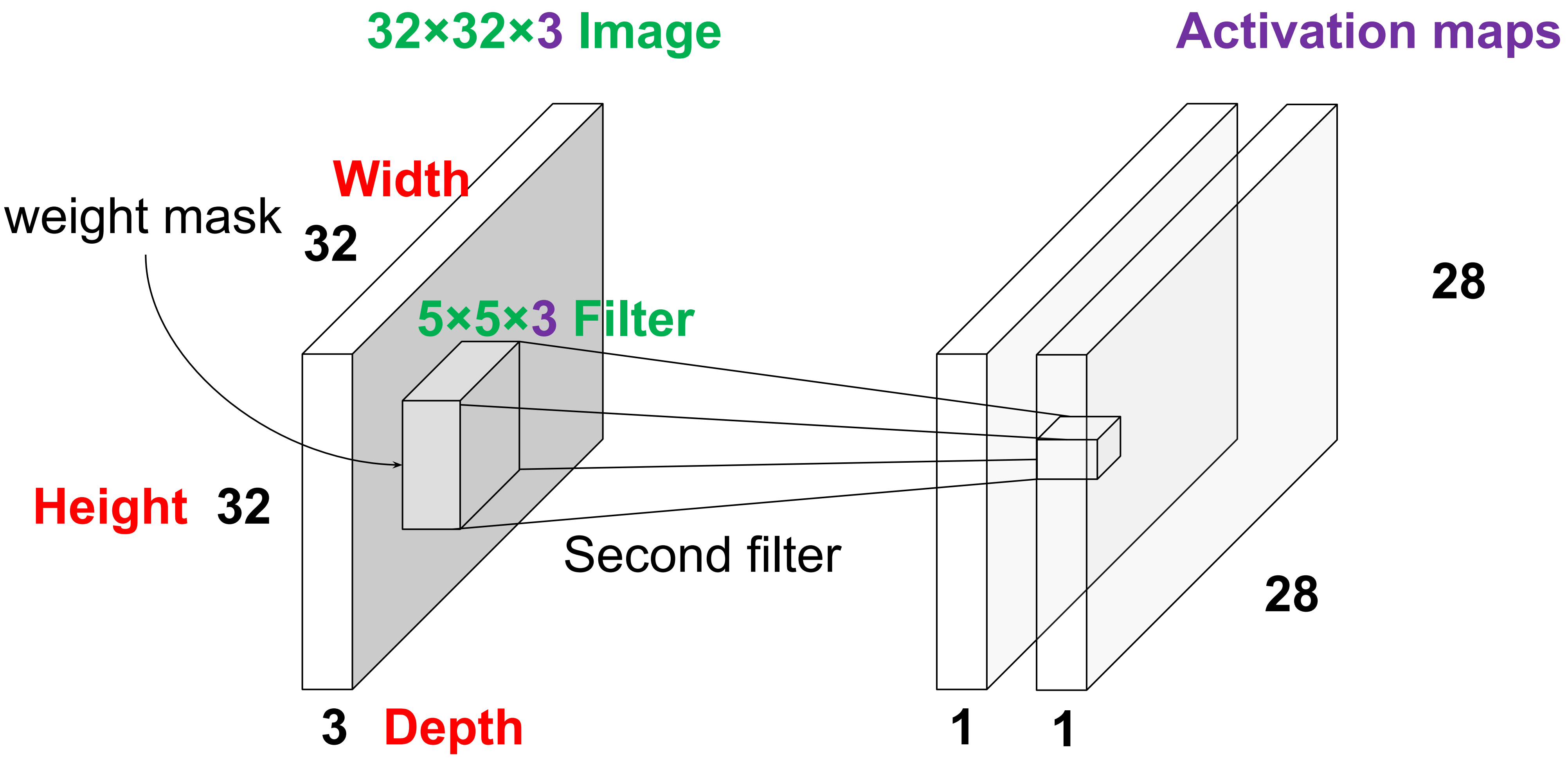
Convolutional Layer



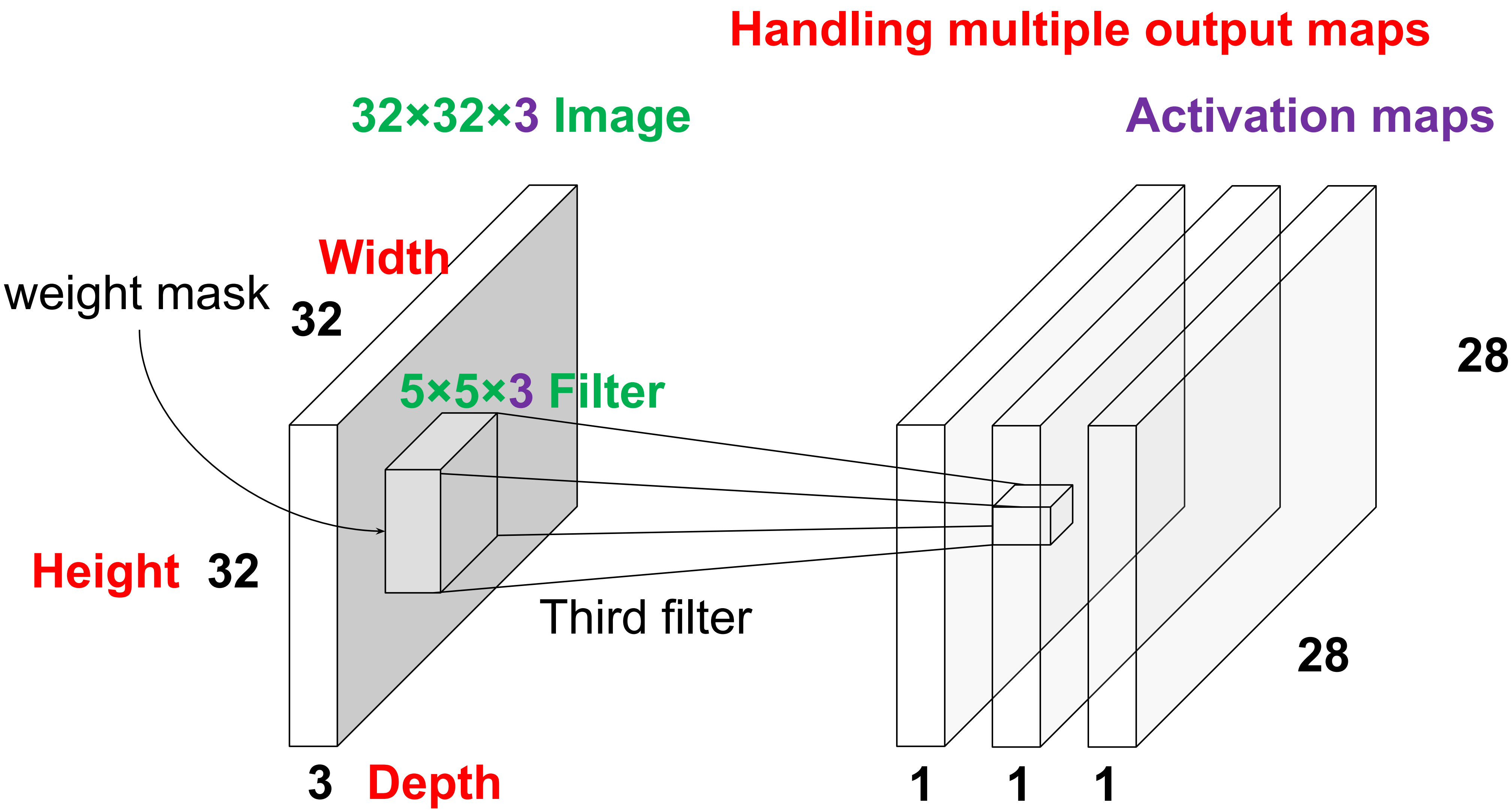


Convolutional Layer

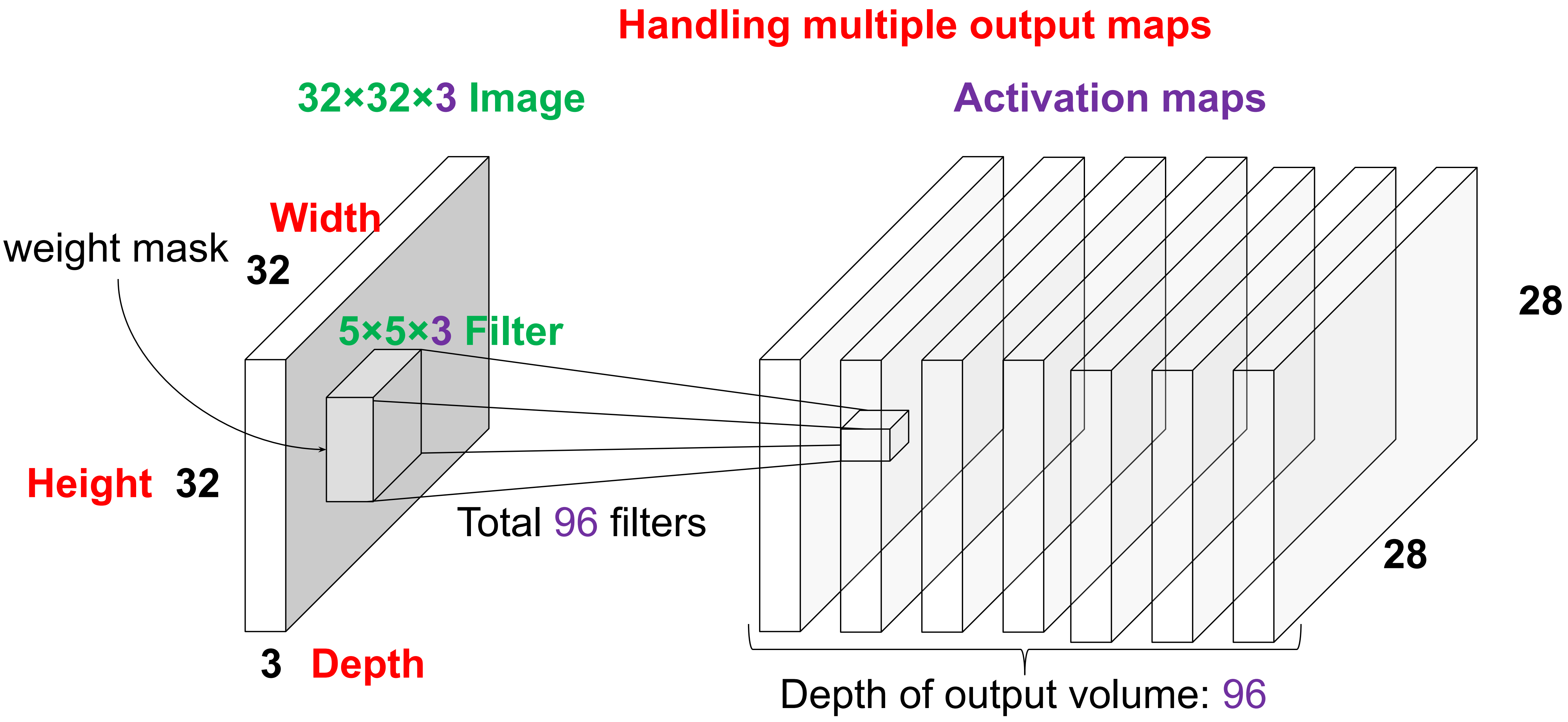
Handling multiple output maps



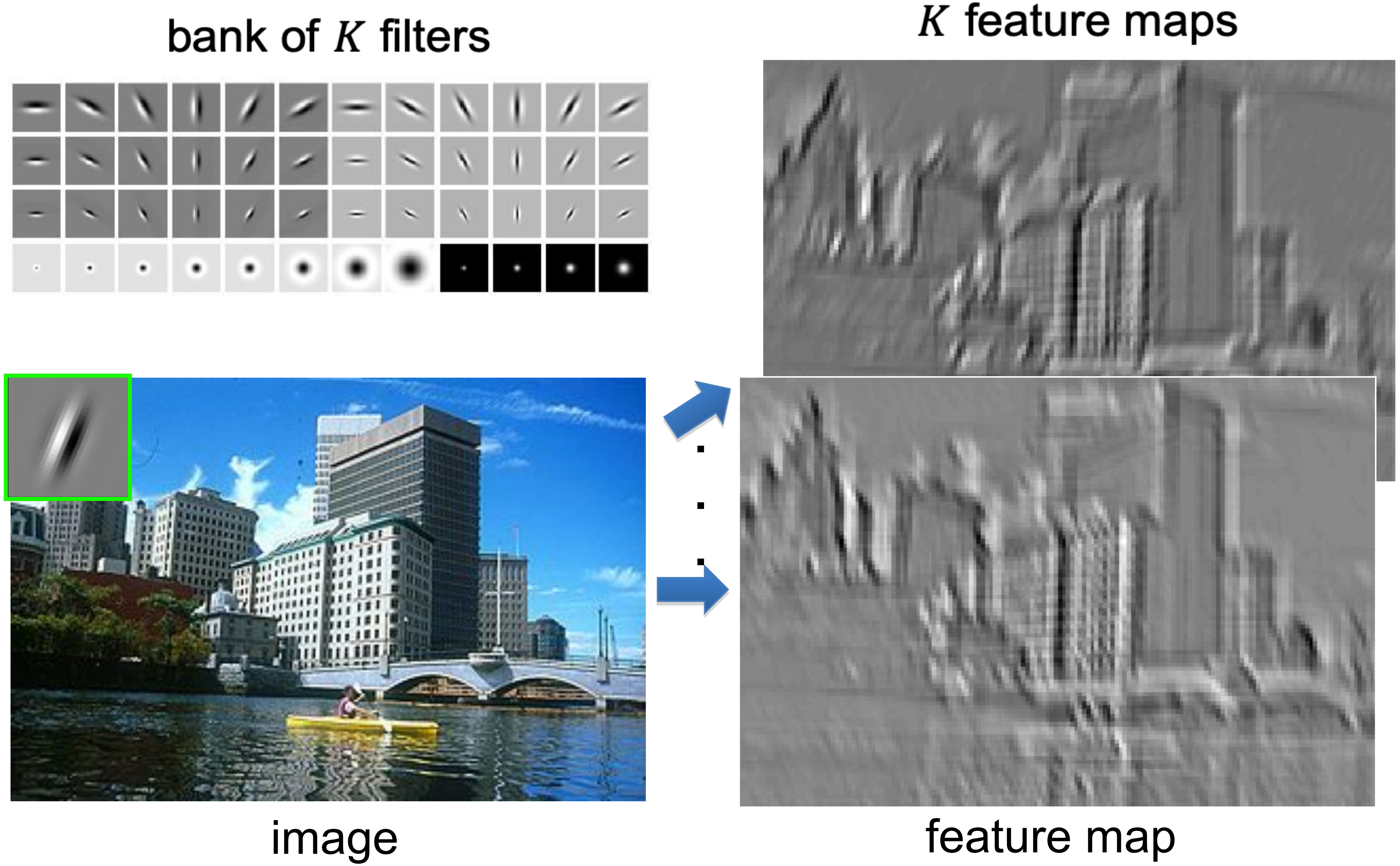
Convolutional Layer



Convolutional Layer



Convolution and traditional feature extraction



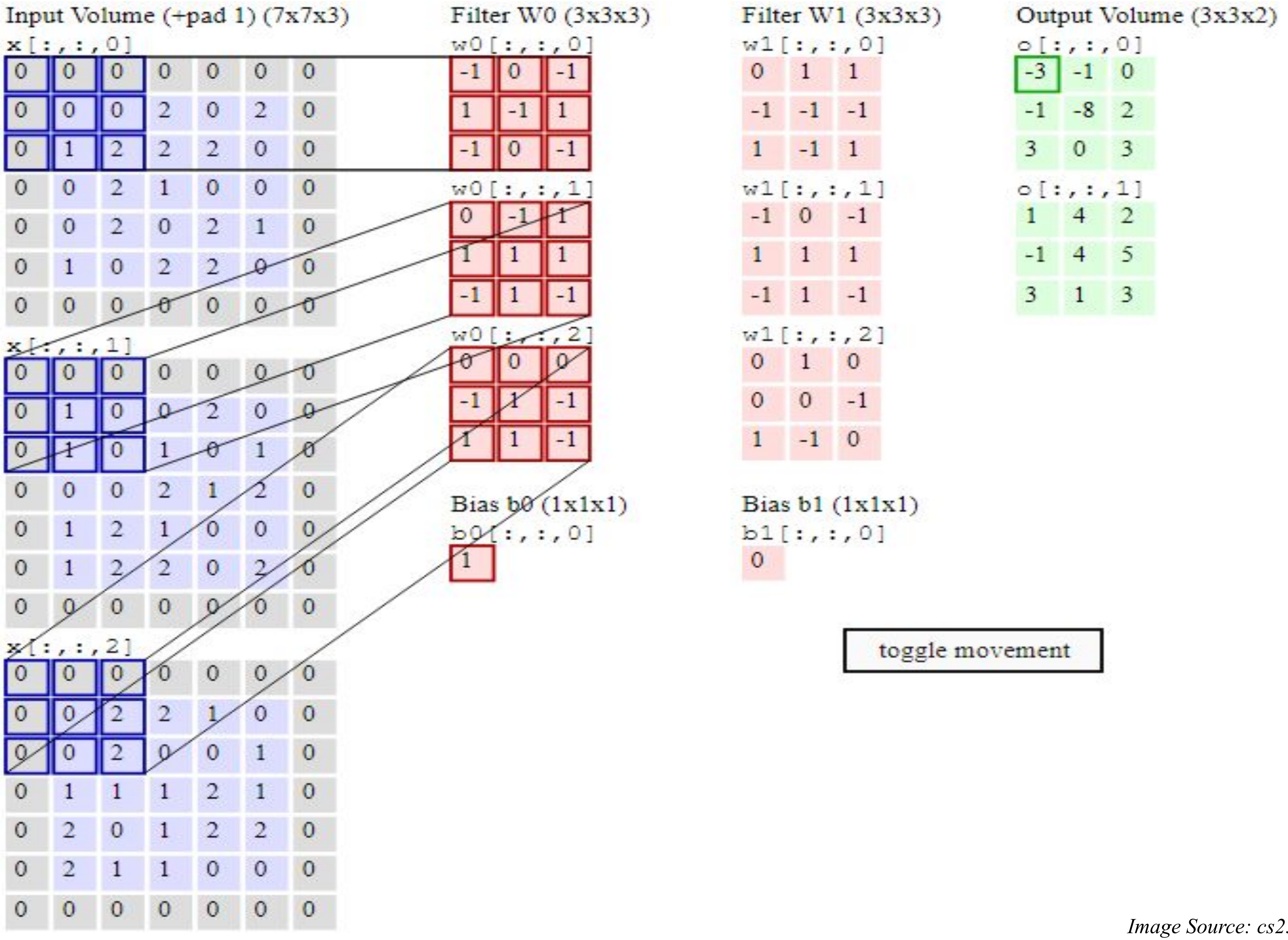


Image Source: cs231n, Oxford University

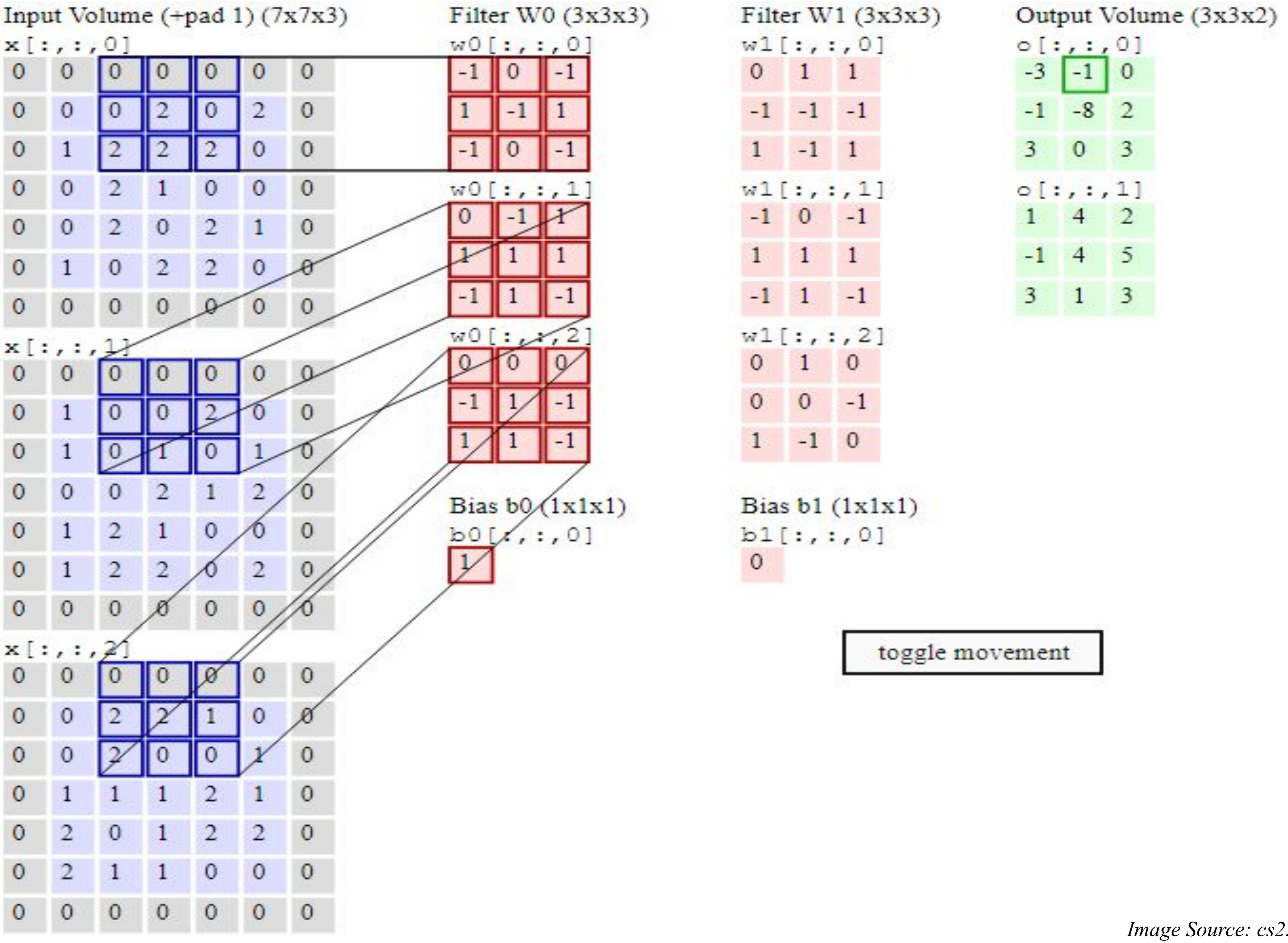


Image Source: cs231n, Oxford University

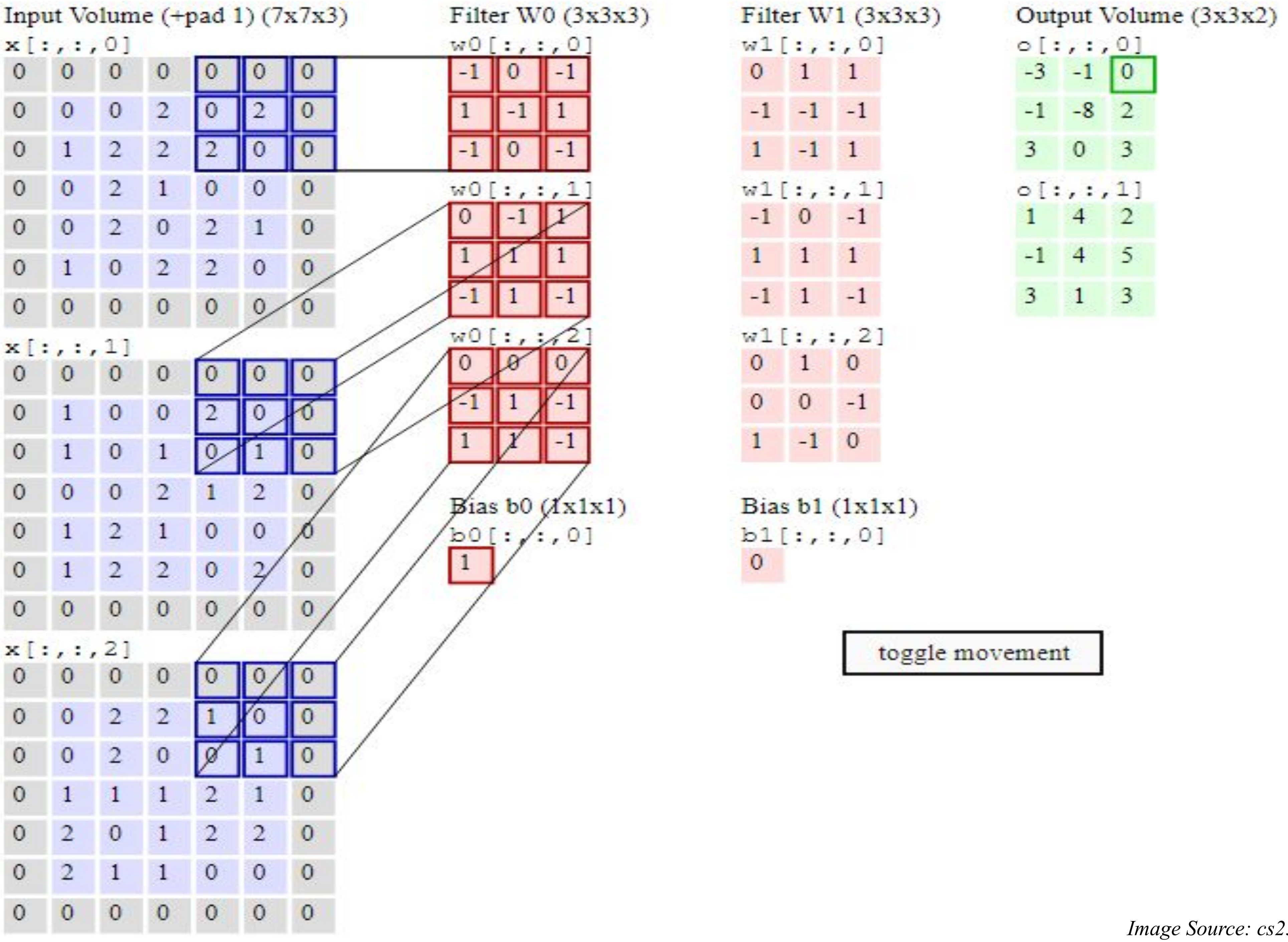
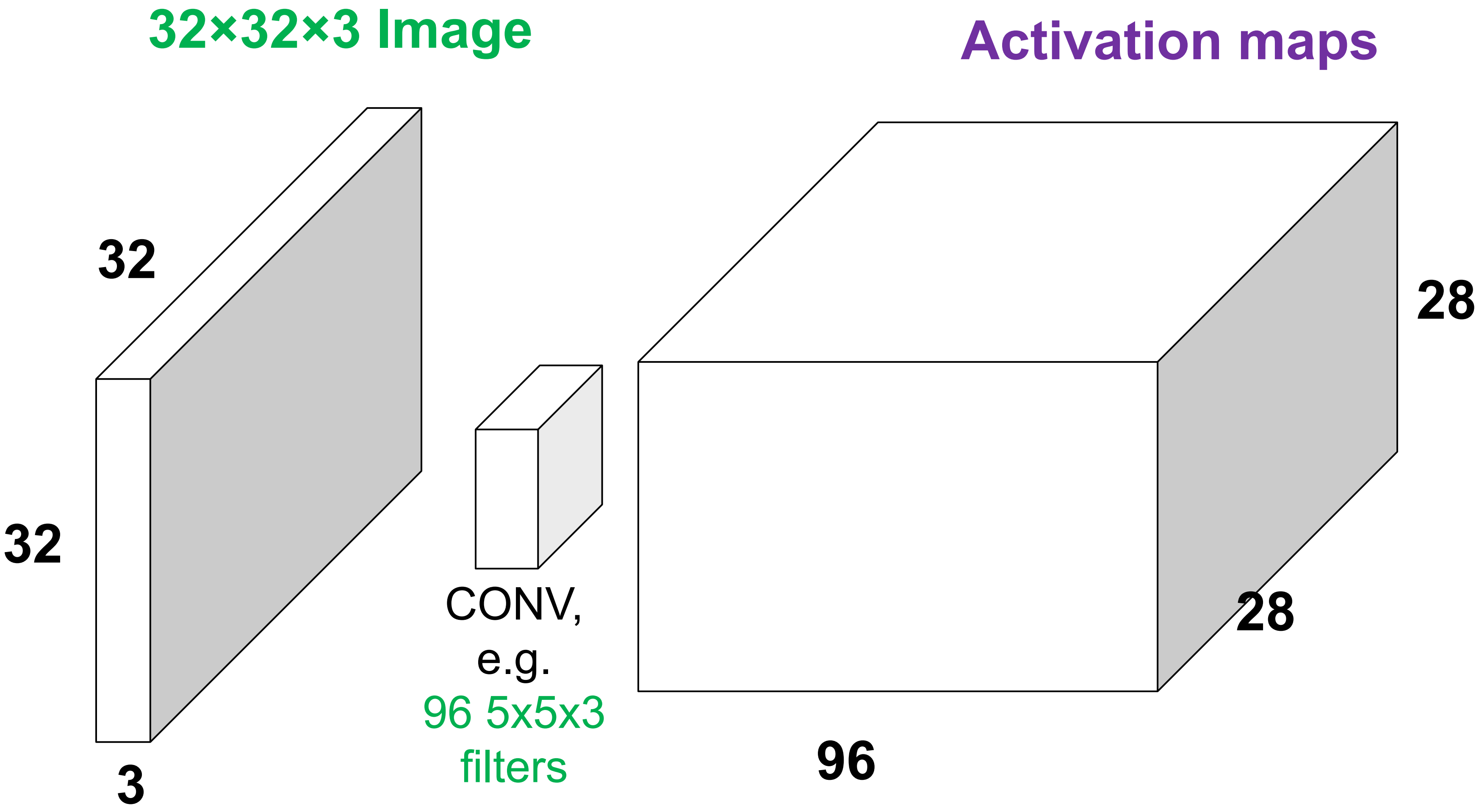


Image Source: cs231n, Oxford University

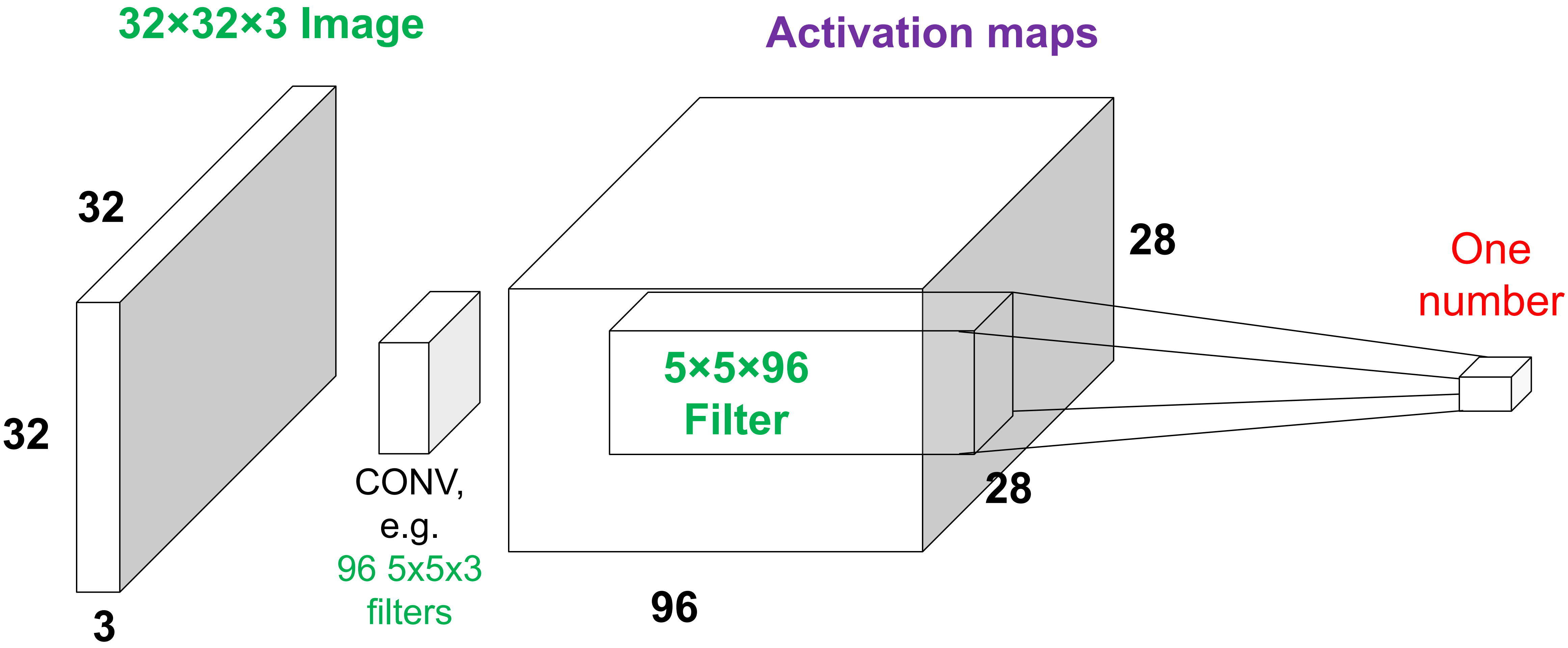
Convolutional Layer

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



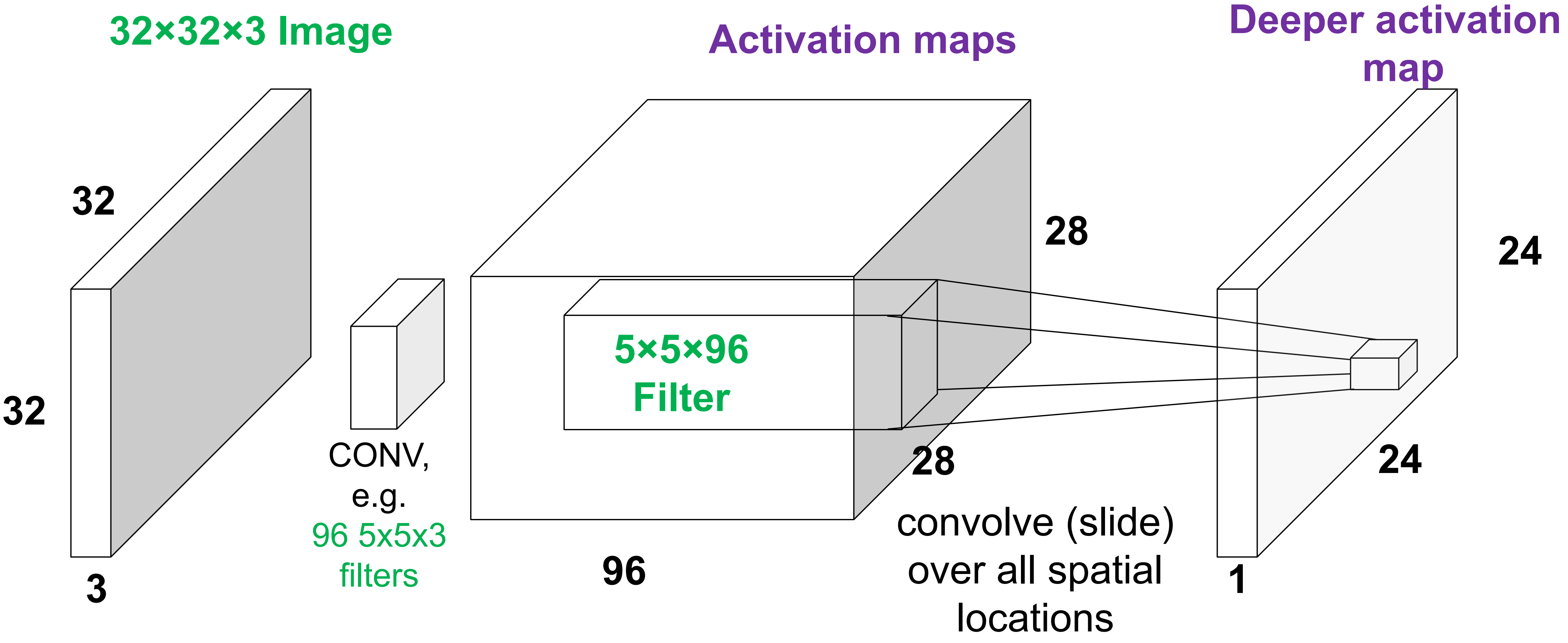
Convolutional Layer

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



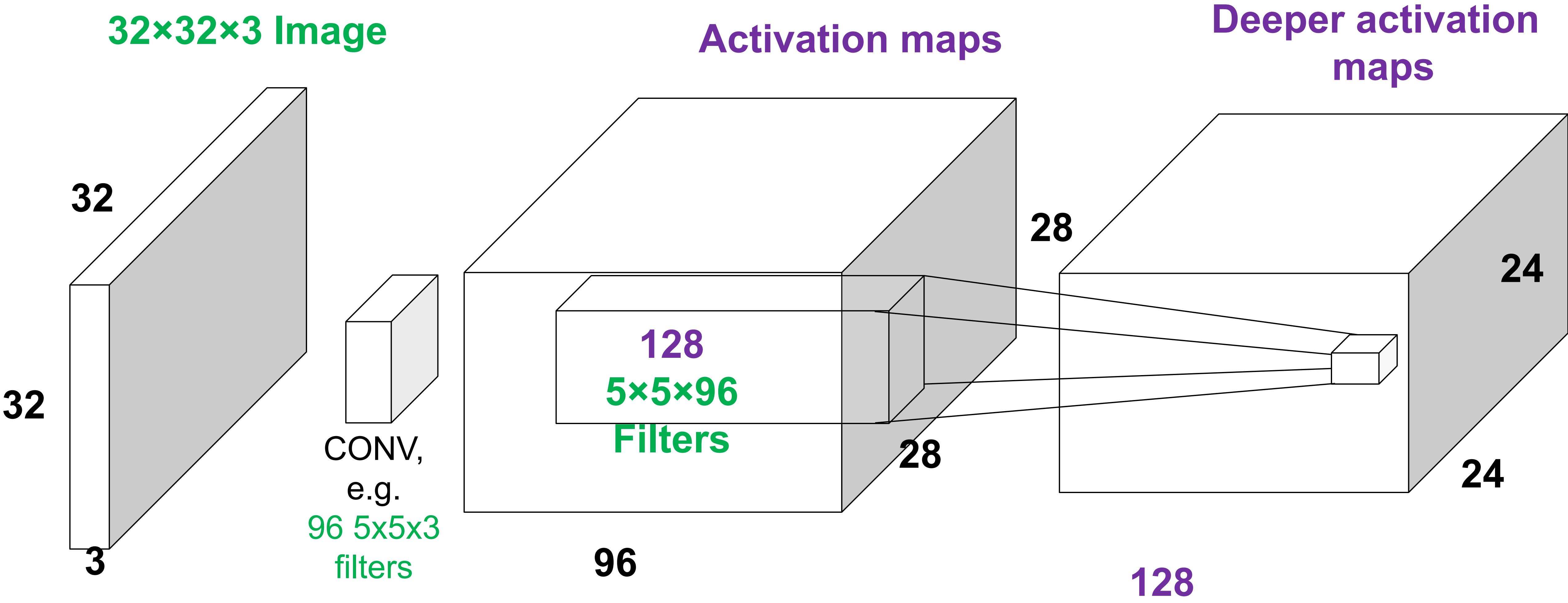
Convolutional Layer

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



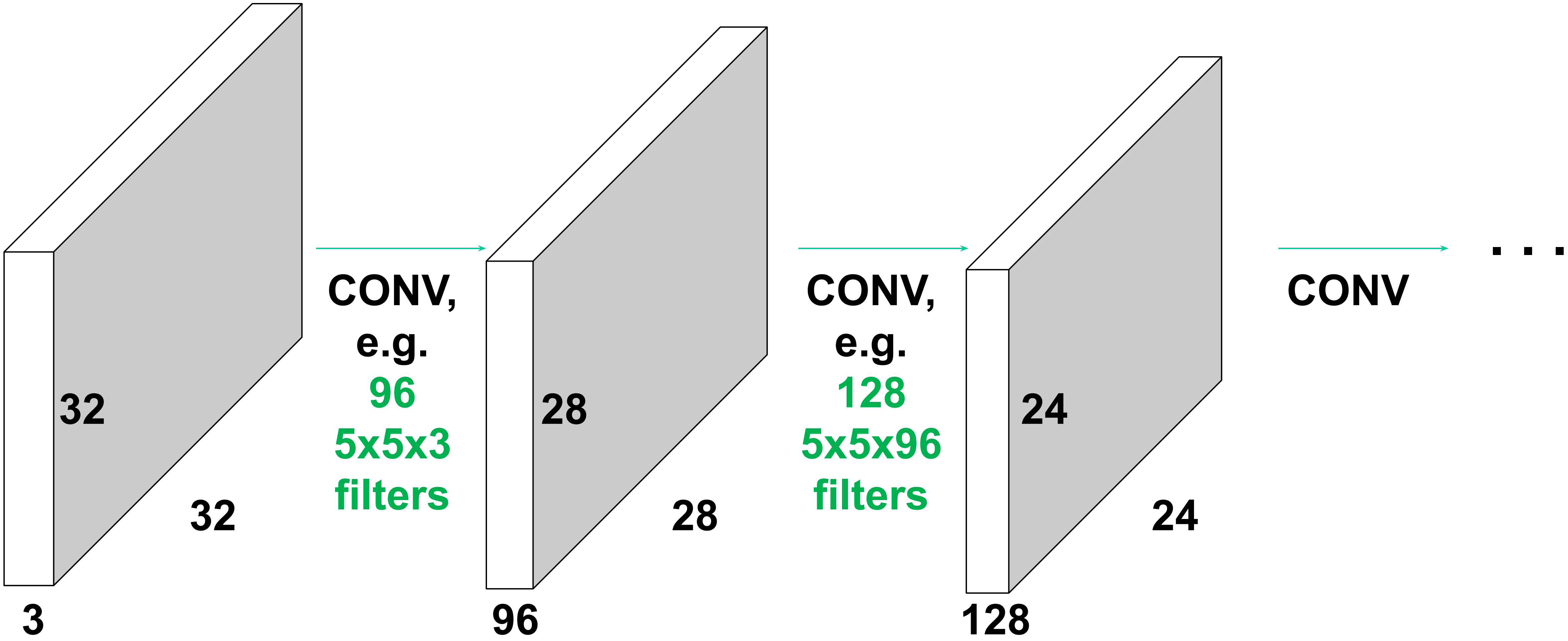
Convolutional Layer

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



Multilayer Convolution

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



Any Convolution Layer

Local connectivity

Weight sharing

Handling multiple input channels

Handling multiple output maps

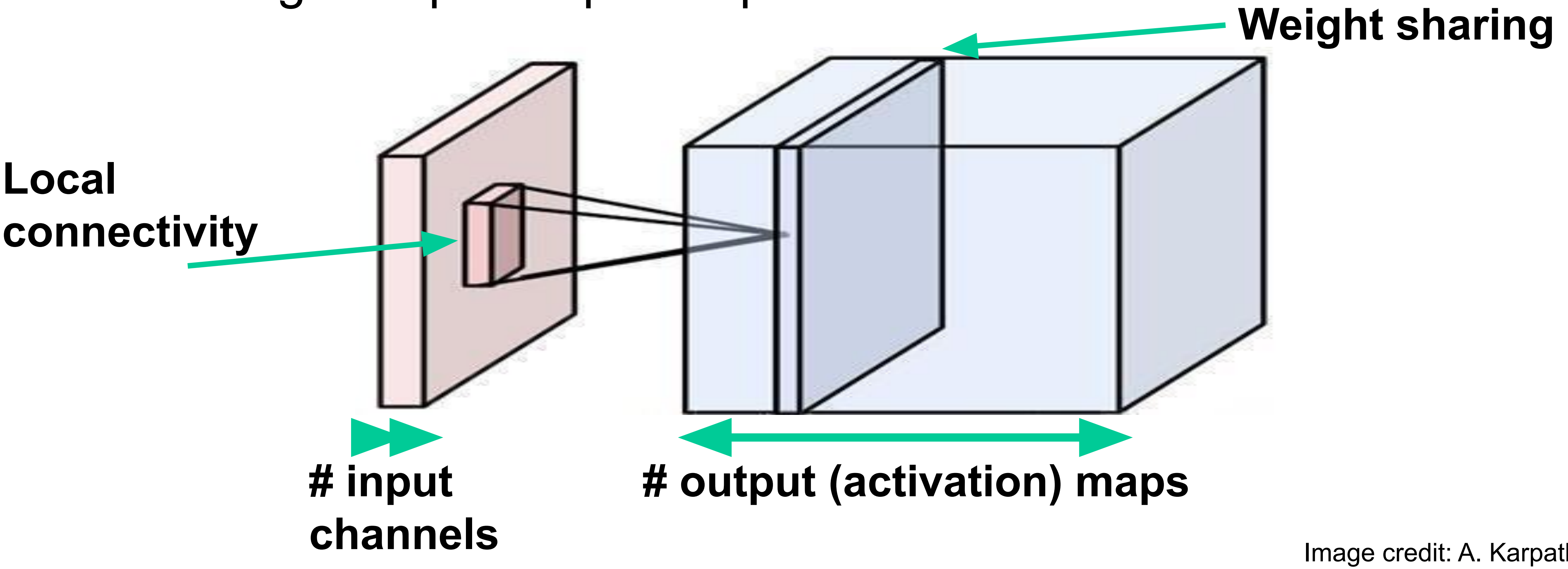
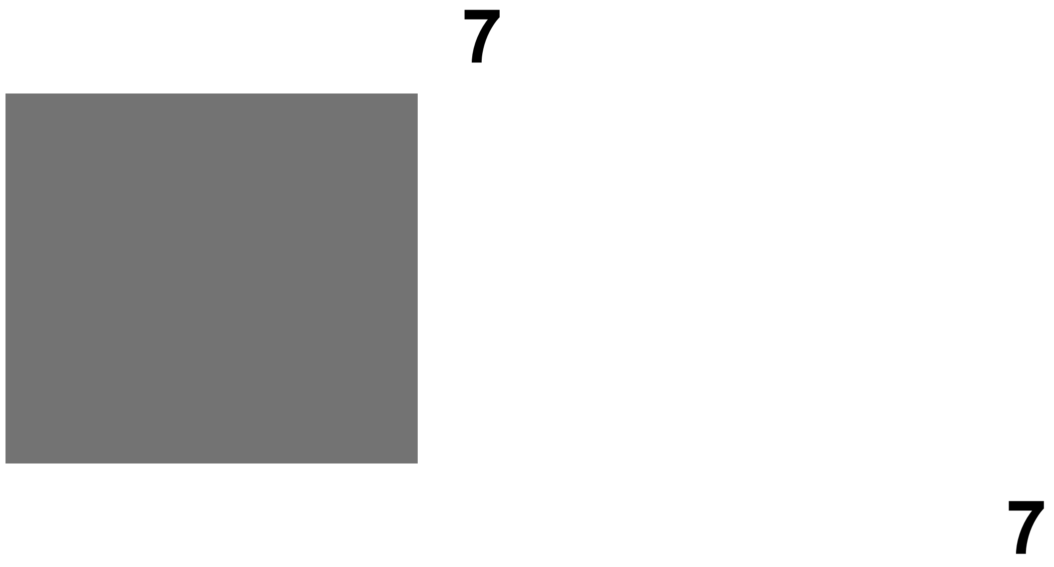


Image credit: A. Karpathy

A closer look at spatial dimensions



7×7 input (spatially)
assume 3×3 filter

A closer look at spatial dimensions

7

7

7×7 input (spatially)
assume 3×3 filter

A closer look at spatial dimensions

7

7

7×7 input (spatially)
assume 3×3 filter

A closer look at spatial dimensions

7

7

7×7 input (spatially)
assume 3×3 filter

A closer look at spatial dimensions

7

7

7×7 input (spatially)
assume 3×3 filter

5×5 output

A closer look at spatial dimensions

7

7×7 input (spatially)
assume 3×3 filter
applied with stride 2

7

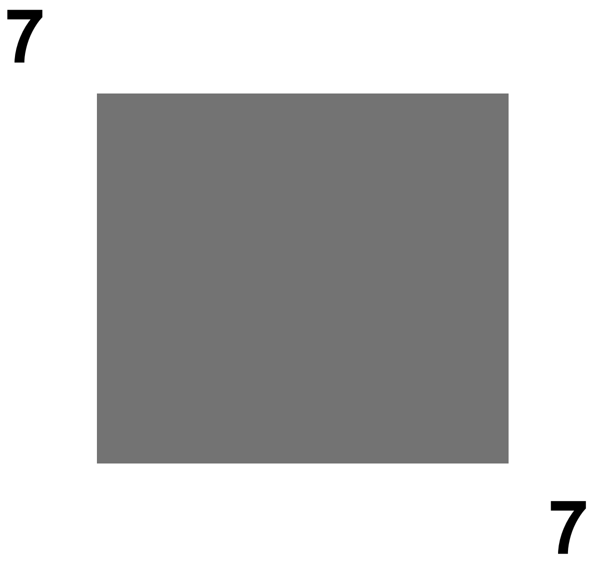
A closer look at spatial dimensions

7

7

7×7 input (spatially)
assume 3×3 filter
applied with stride 2

A closer look at spatial dimensions



7×7 input (spatially)
assume 3×3 filter
applied with stride 2

3×3 output

A closer look at spatial dimensions

7×7 input (spatially)
assume 3×3 filter
applied with stride 3

A closer look at spatial dimensions

7

7×7 input (spatially)
assume 3×3 filter
applied with stride 3

7

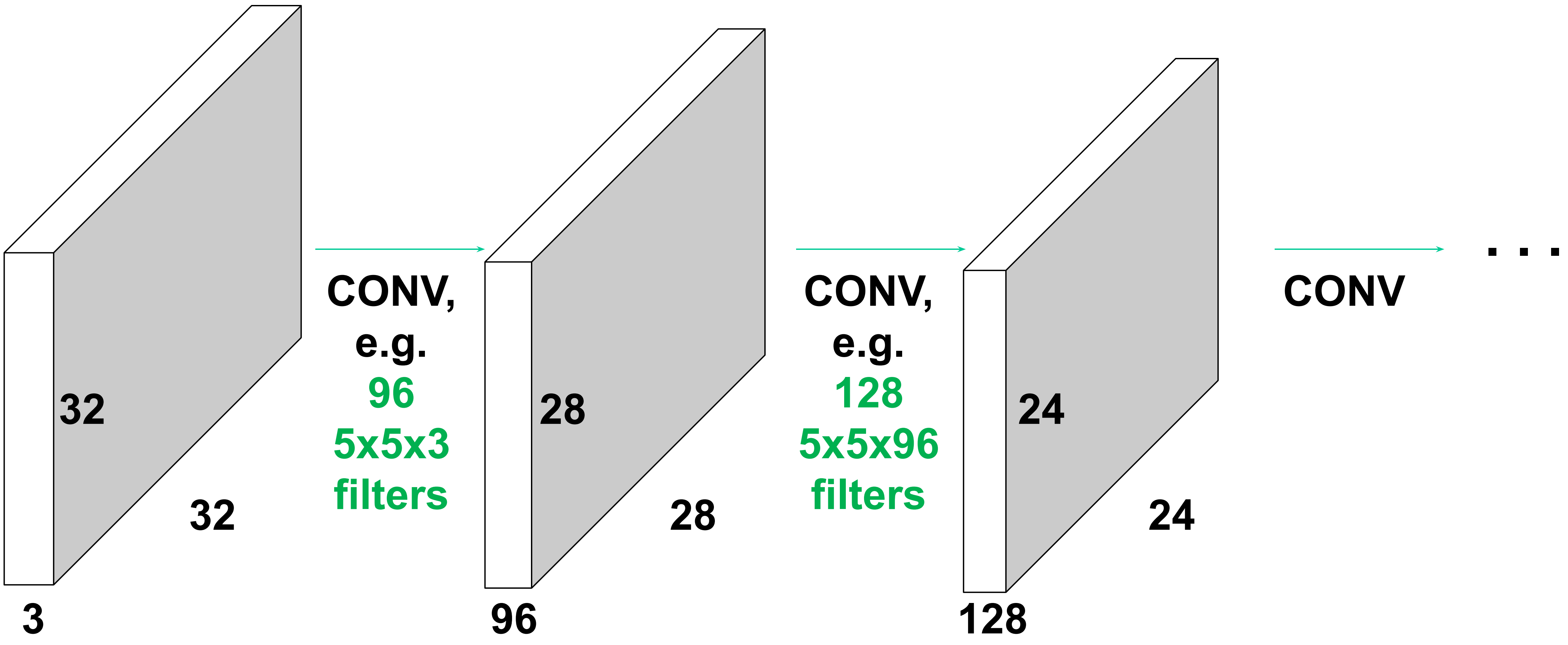
doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.

A closer look at spatial dimensions

Output size
(N - F) / stride + 1

e.g. N = 7, F = 3
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33

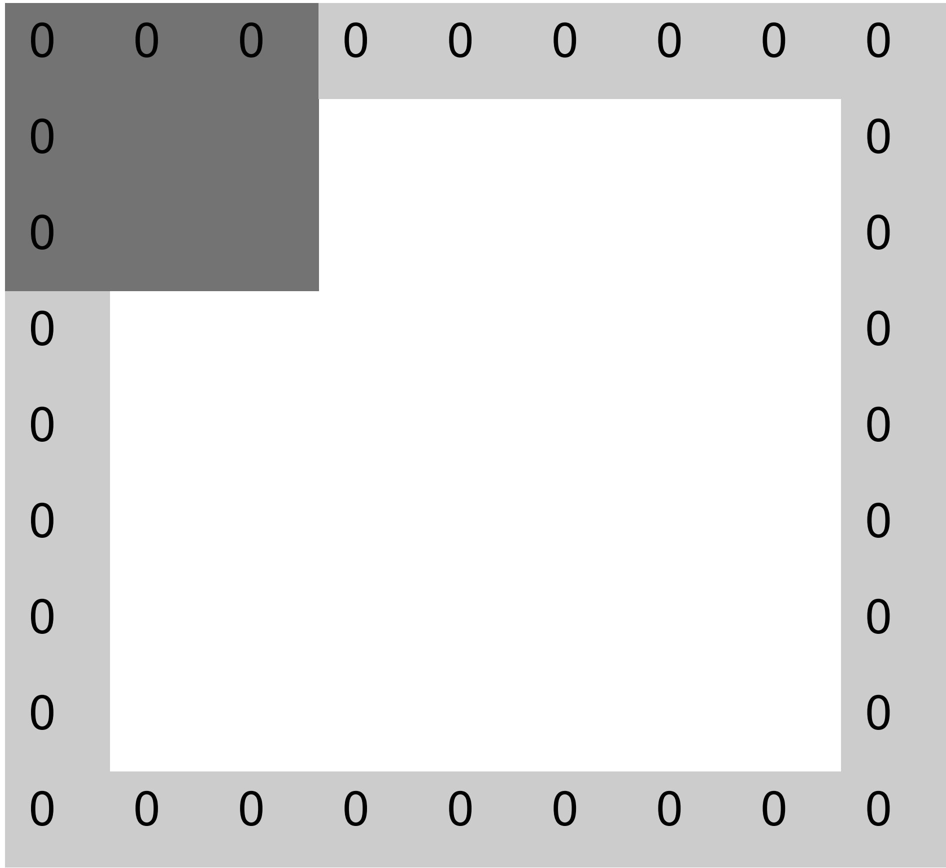
A closer look at spatial dimensions



E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially!
(32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn’t work well.

Source: cs231n, Stanford University

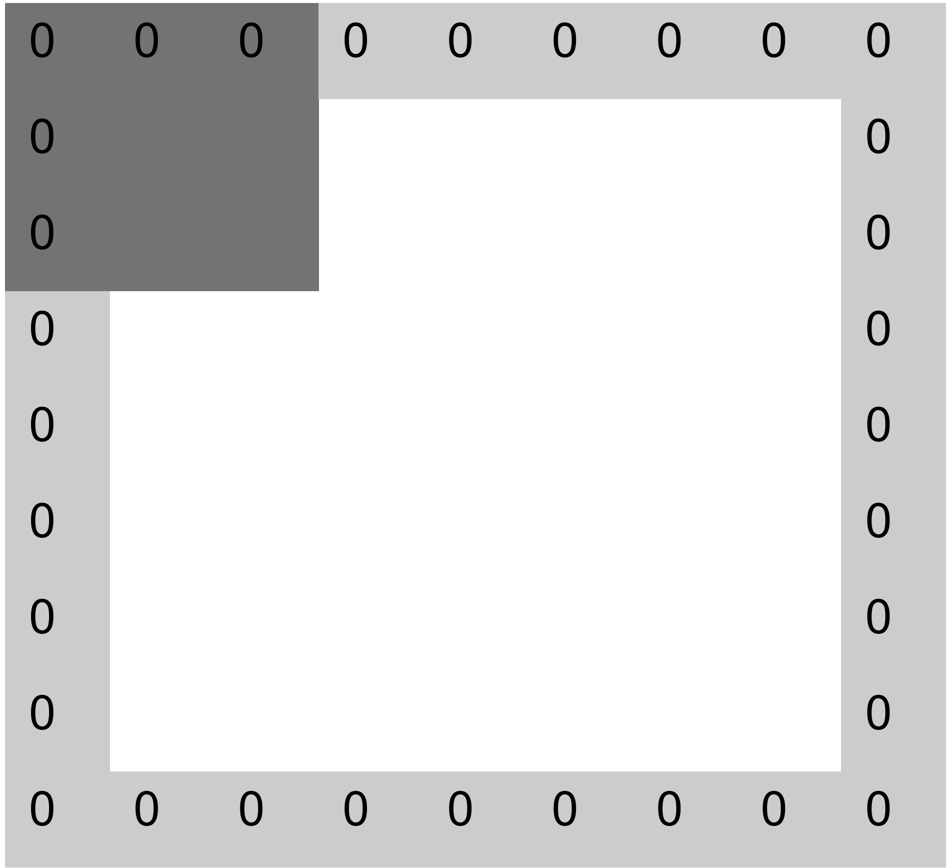
In practice: common to zero pad



e.g. input 7×7 (spatially)
3×3 filter, applied with stride 1
pad with 1 pixel border

What is the output dimension?

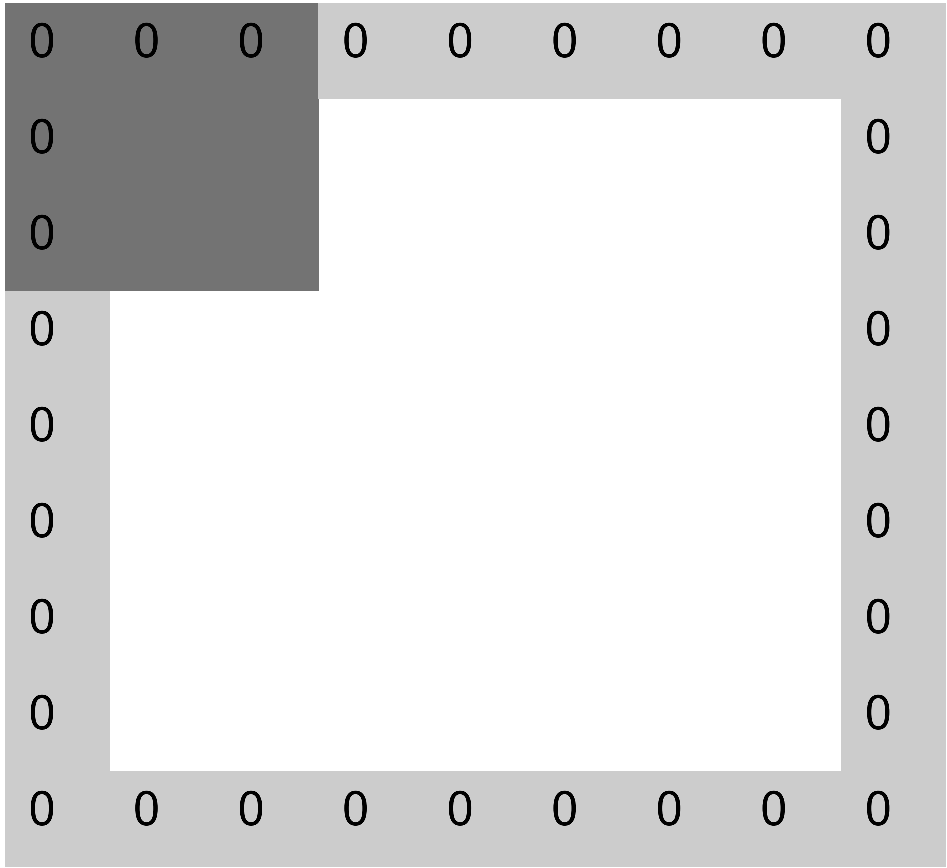
In practice: common to zero pad



e.g. input 7×7 (spatially)
3×3 filter, applied with stride 1
pad with 1 pixel border

7×7 Output

In practice: common to zero pad



e.g. input 7×7 (spatially)
3×3 filter, applied with stride 1
pad with 1 pixel border

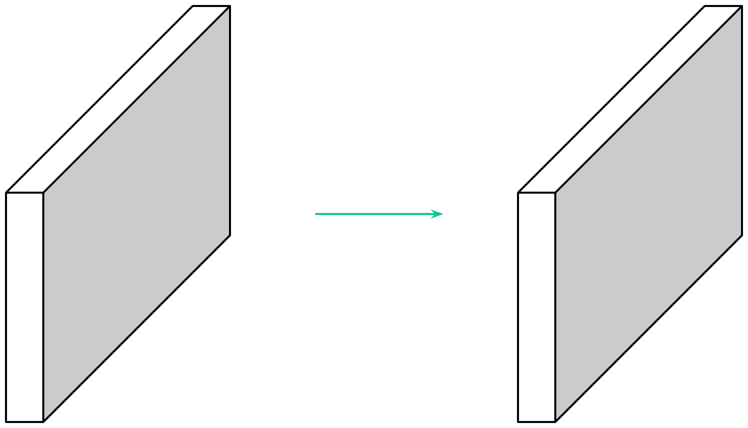
7×7 Output

in general, common to see CONV layers with stride 1, filters of size F×F, and zero-padding with
(F-1)/2. (will preserve size spatially)
e.g.
F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Example

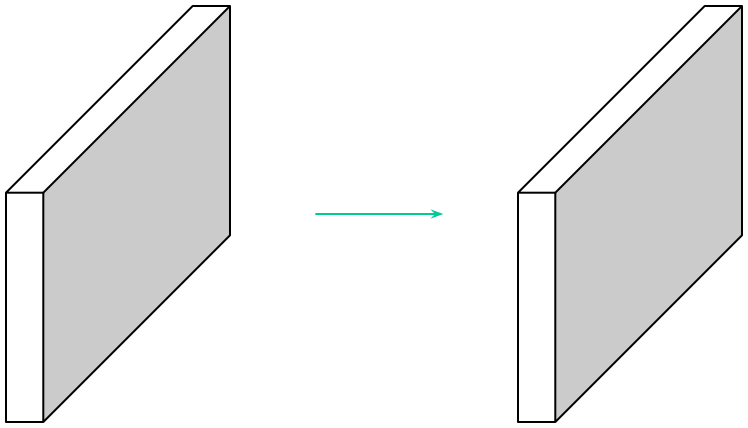
Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2

Output volume size: ?



Example

Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2

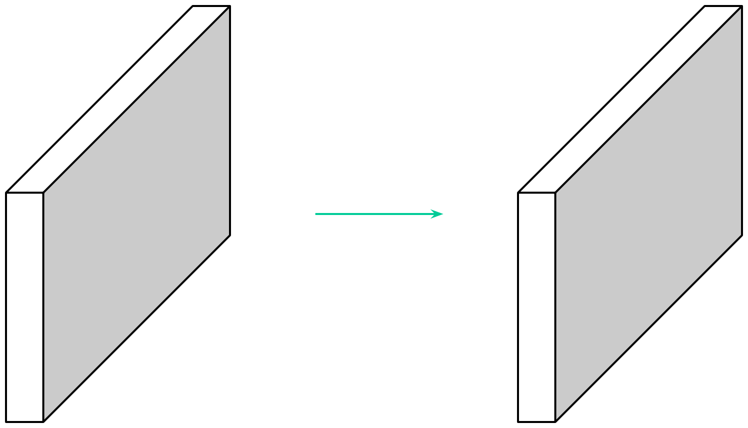


Output volume size:
(32+2*2-5)/1+1 = 32 spatially,
so
32x32x10

Example

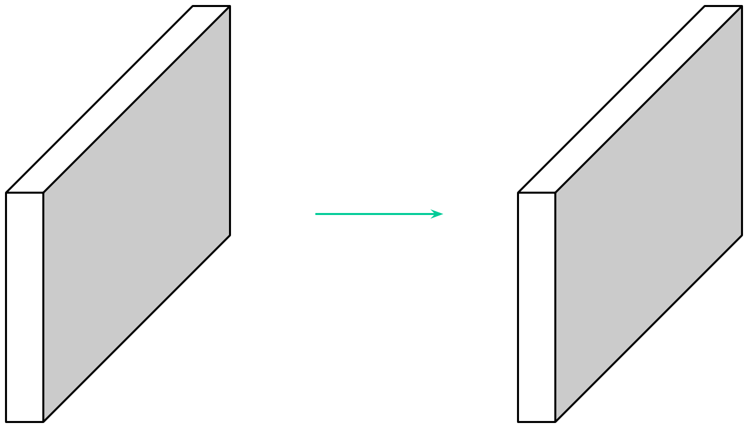
Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?



Example

Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

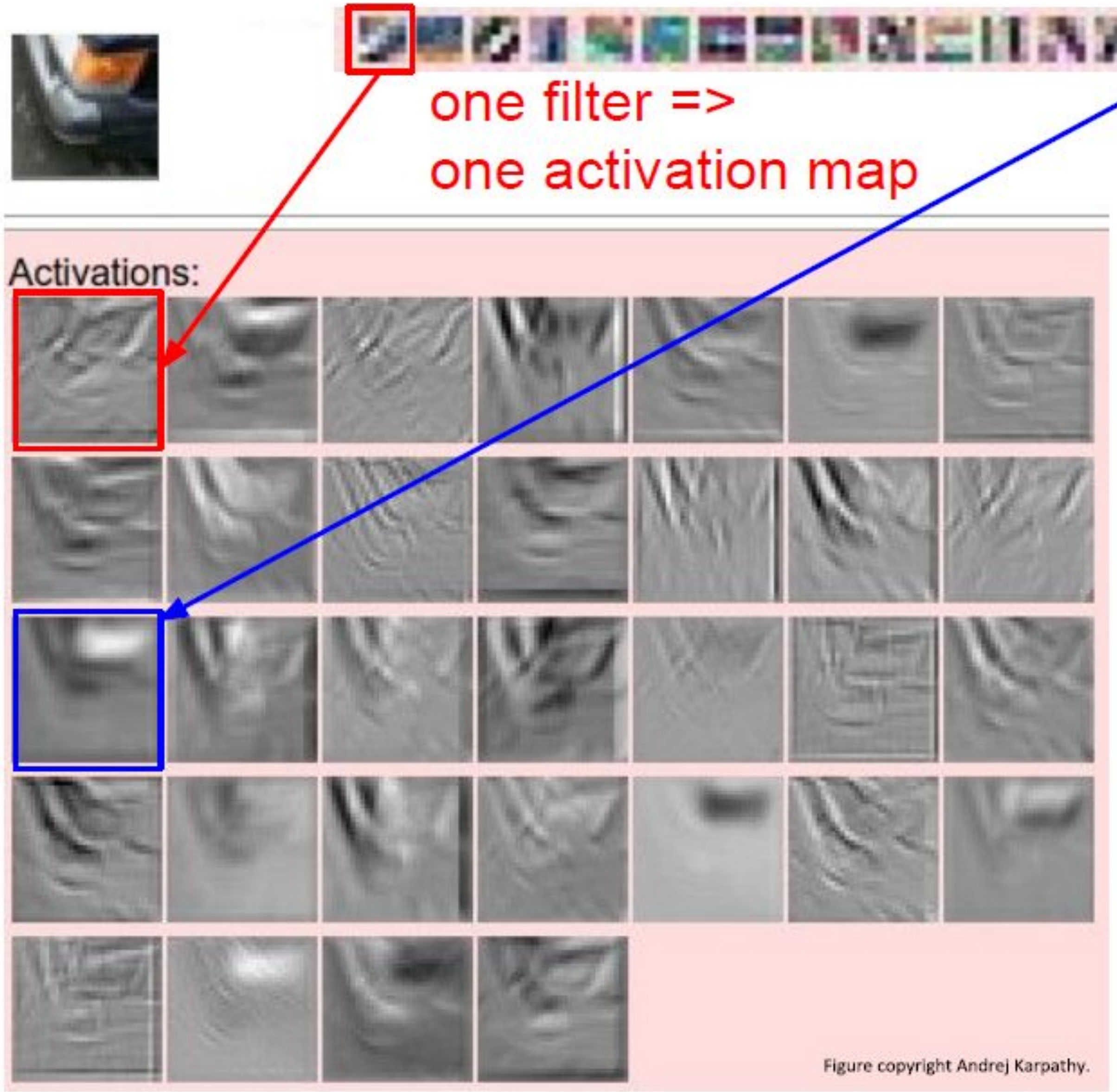
each filter has
5*5*3 + 1 = 76 params (+1 for bias)
=> 76*10 = 760

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.

Source: cs231n, Stanford University

Convolution as feature extraction



example 5x5 filters
(32 total)

We call the layer convolutional because it is related to convolution of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

elementwise multiplication and sum of a filter and the signal (image)

Source: cs231n, Stanford University

Efficient implementation of convolutions

- Reshape all image neighborhoods into columns (im2col operation), do matrix-vector multiplication

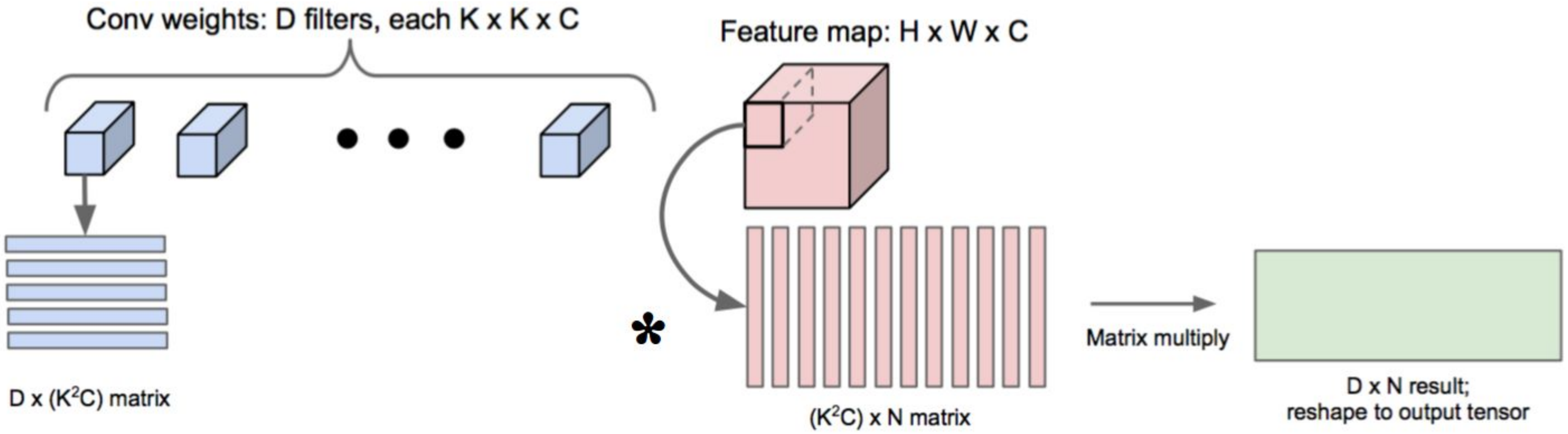
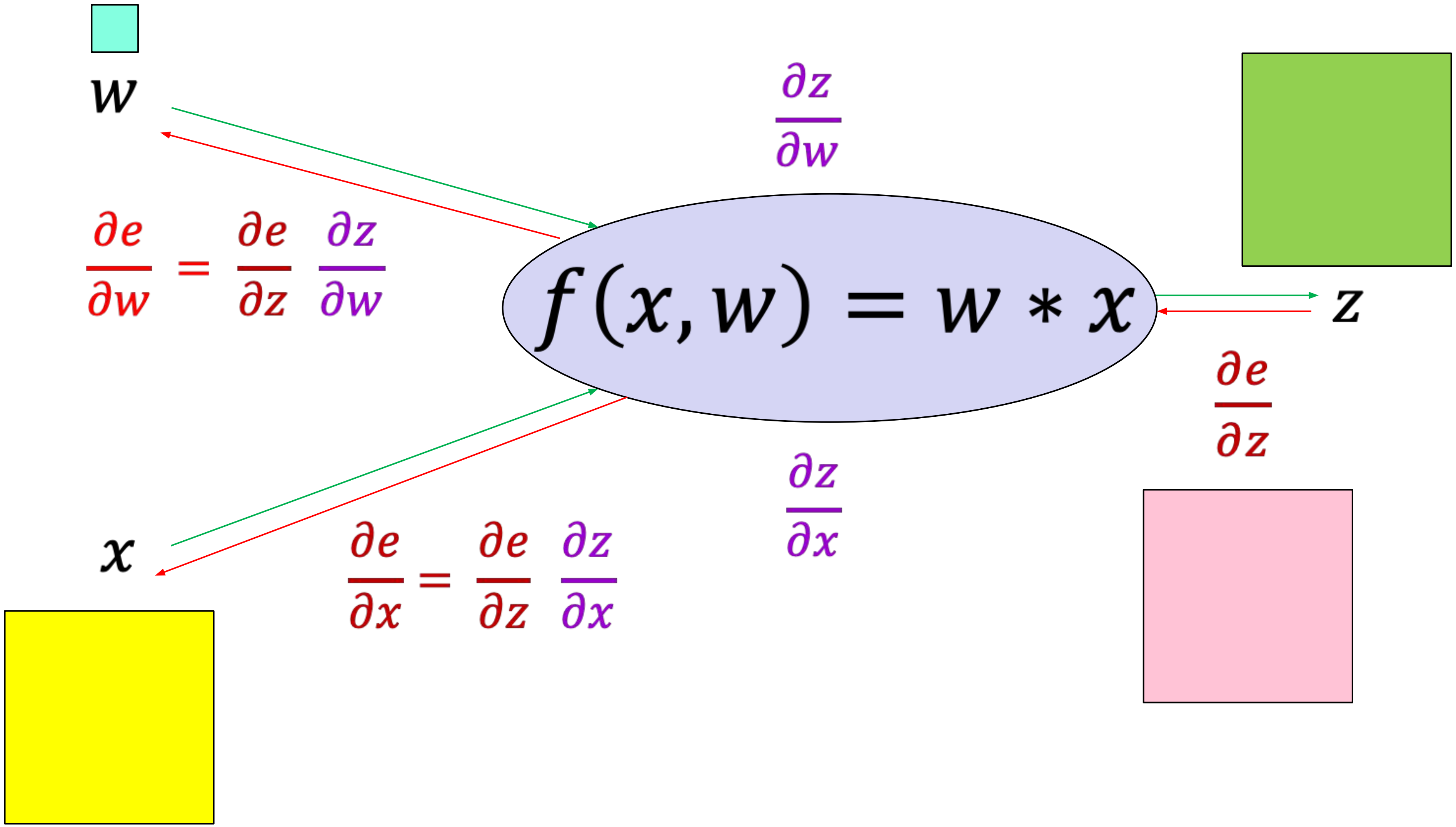


Image source

Backpropagation for convolutional layer



Backpropagation for convolutional layer

$$\frac{\partial e}{\partial w_{ij}}$$

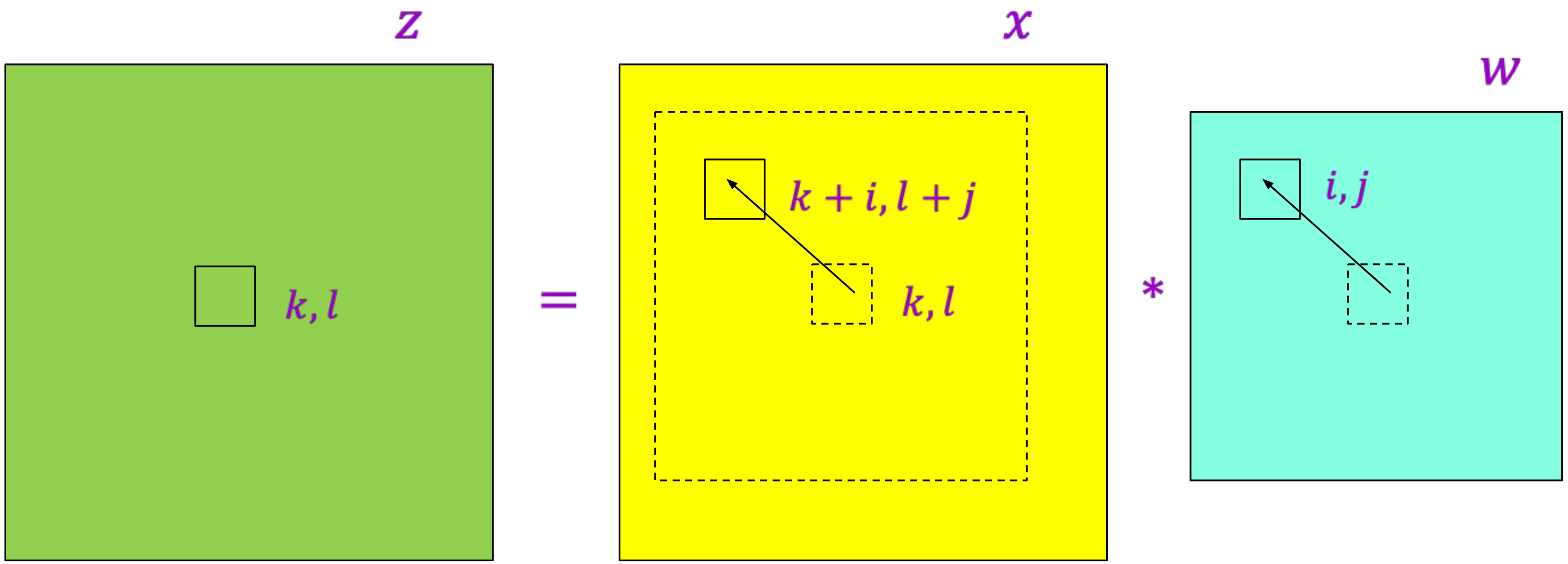
Backpropagation for convolutional layer

$$\frac{\partial e}{\partial w_{ij}} = \frac{\partial e}{\partial z} \frac{\partial z}{\partial w_{ij}} = \sum_{k,l} \frac{\partial e}{\partial z_{kl}} \frac{\partial z_{kl}}{\partial w_{ij}}$$

$$z_{kl} = \sum_{i,j=-f}^{f} w_{ij} x_{k+i,\, l+j}$$

For simplicity, assume filter indices go from $-f$ to f

$$\frac{\partial z_{kl}}{\partial w_{ij}} = x_{k+i,\, l+j}$$



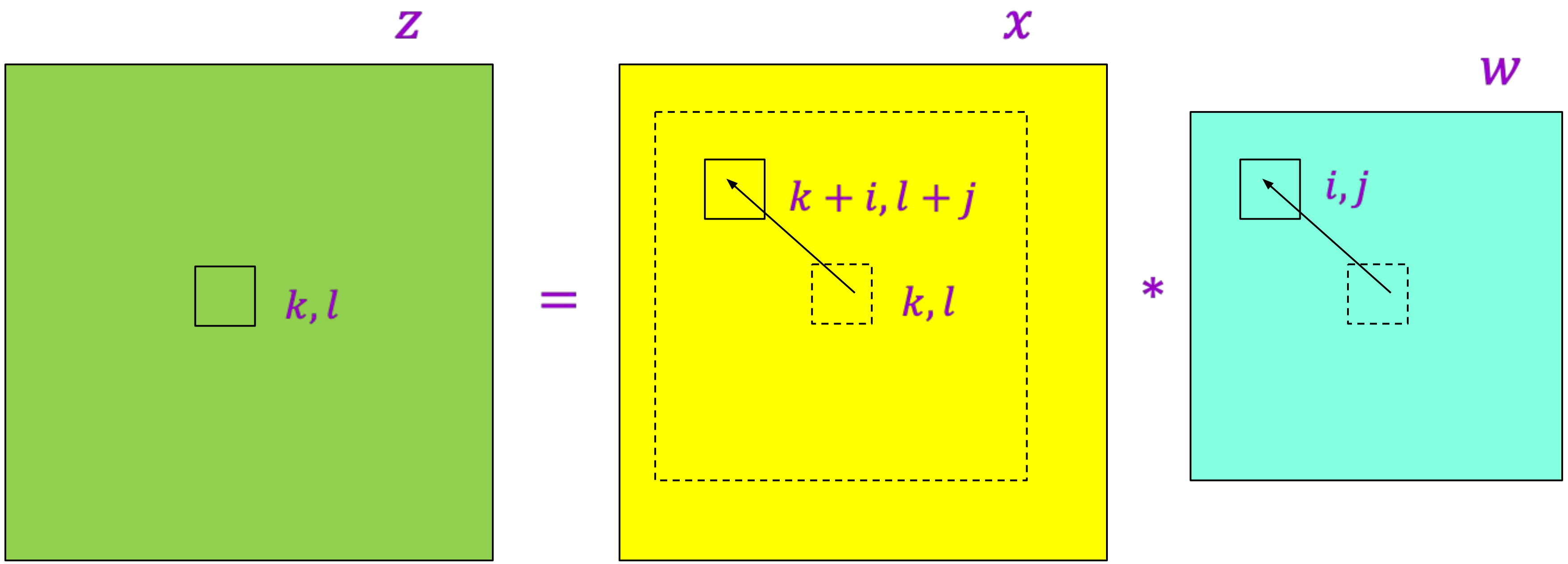
Backpropagation for convolutional layer

$$\frac{\partial e}{\partial w_{ij}} = \frac{\partial e}{\partial z}\frac{\partial z}{\partial w_{ij}} = \sum_{k,l}\frac{\partial e}{\partial z_{kl}}\frac{\partial z_{kl}}{\partial w_{ij}} = \boxed{\sum_{k,l}\frac{\partial e}{\partial z_{kl}} x_{k+i,\,l+j}}$$

$$z_{kl} = \sum_{i,j=-f}^{f} w_{ij} x_{k+i,\,l+j}$$

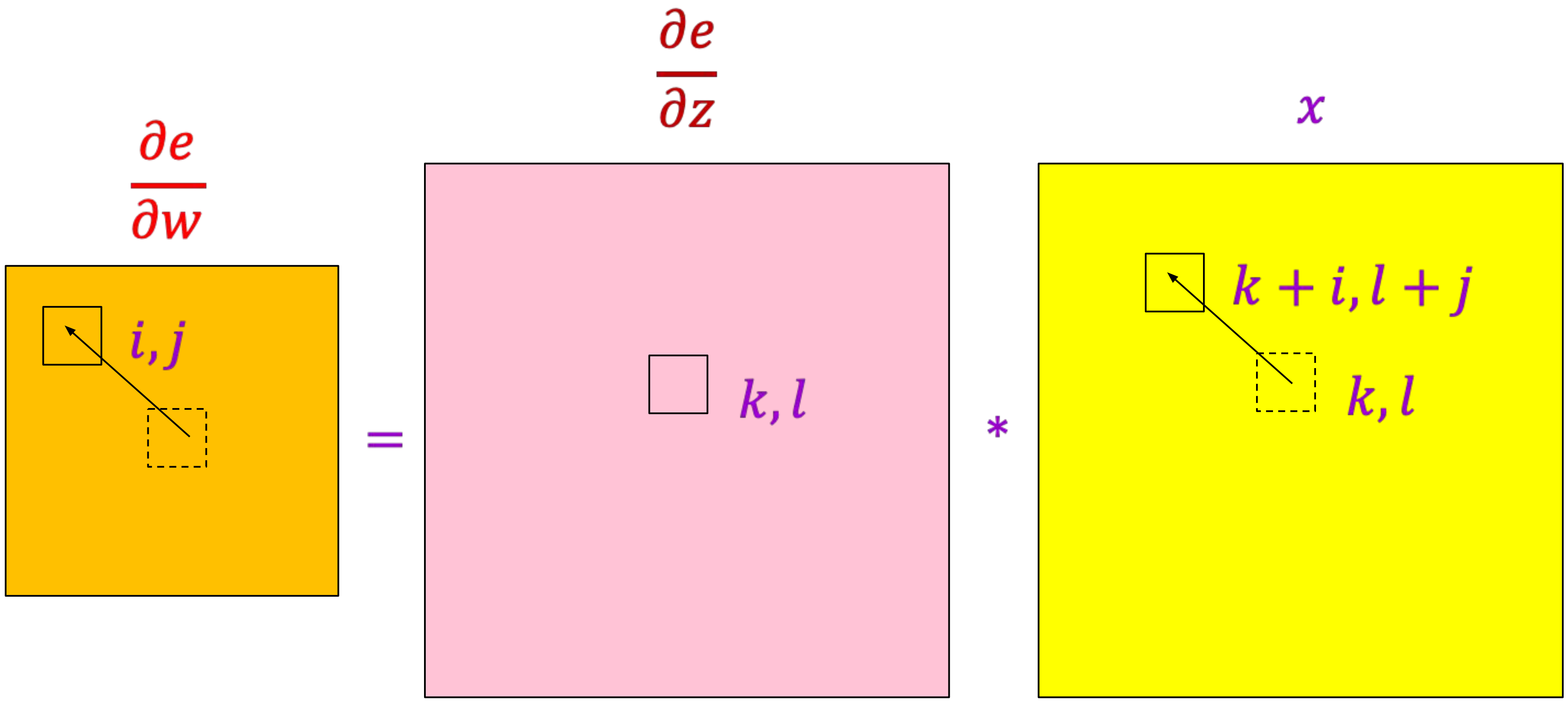
For simplicity, assume filter indices go from $-f$ to f

$$\frac{\partial z_{kl}}{\partial w_{ij}} = x_{k+i,\,l+j}$$



Backpropagation for convolutional layer

$$\boxed{\frac{\partial e}{\partial w_{ij}} = \sum_{k,l} \frac{\partial e}{\partial z_{kl}} x_{k+i,\, l+j}}$$

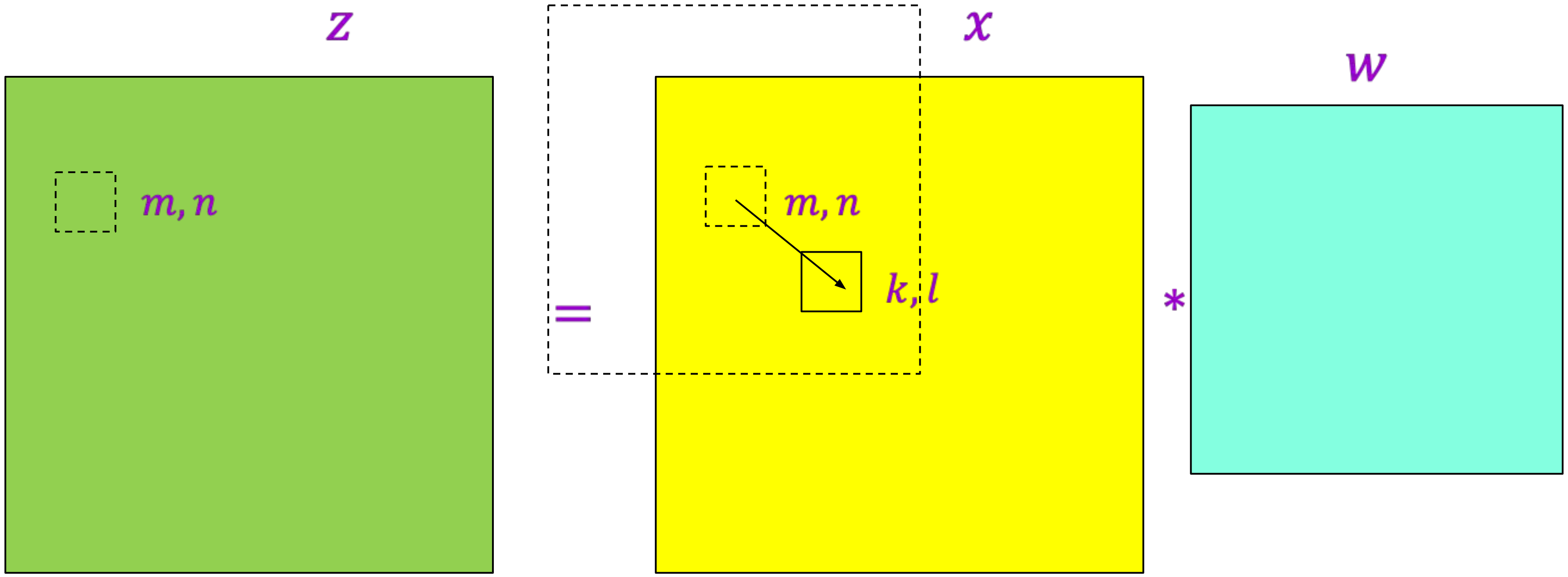


Backpropagation for convolutional layer

$$\frac{\partial e}{\partial x_{kl}}$$

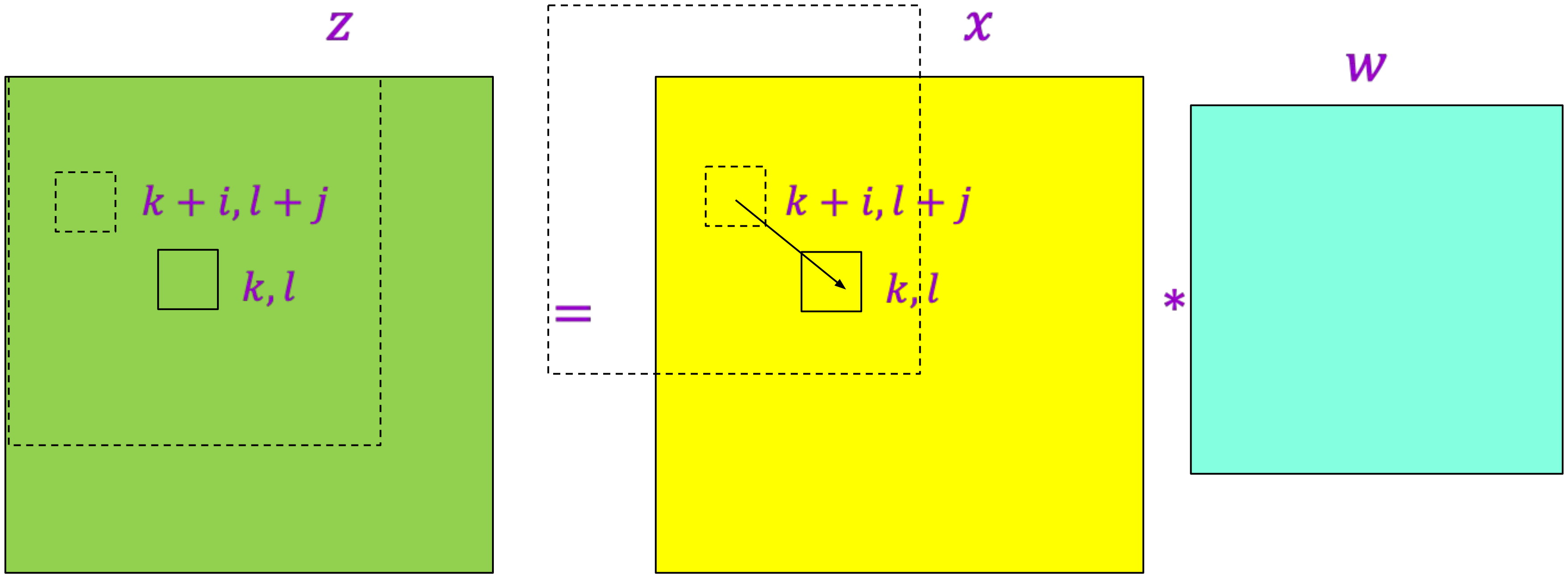
Backpropagation for convolutional layer

$$\frac{\partial e}{\partial x_{kl}} = \frac{\partial e}{\partial z} \frac{\partial z}{x_{kl}} = \sum_{m,n} \frac{\partial e}{\partial z_{mn}} \frac{\partial z_{mn}}{\partial x_{kl}}$$



Backpropagation for convolutional layer

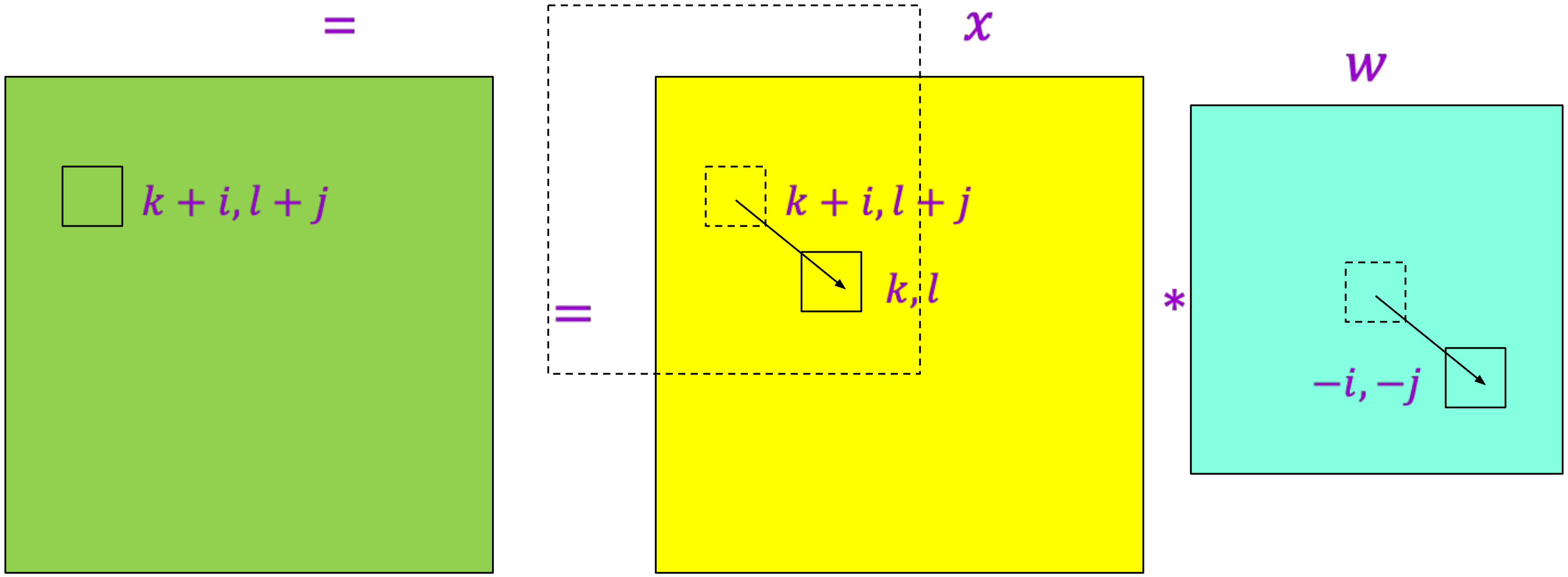
$$\frac{\partial e}{\partial x_{kl}} = \frac{\partial e}{\partial z}\,\frac{\partial z}{x_{kl}} = \sum_{m,n} \frac{\partial e}{\partial z_{mn}}\frac{\partial z_{mn}}{\partial x_{kl}} = \sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}}\frac{\partial z_{k+i,l+j}}{\partial x_{kl}}$$



Backpropagation for convolutional layer

$$\frac{\partial e}{\partial x_{kl}} = \sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}} \frac{\partial z_{k+i,l+j}}{\partial x_{kl}}$$

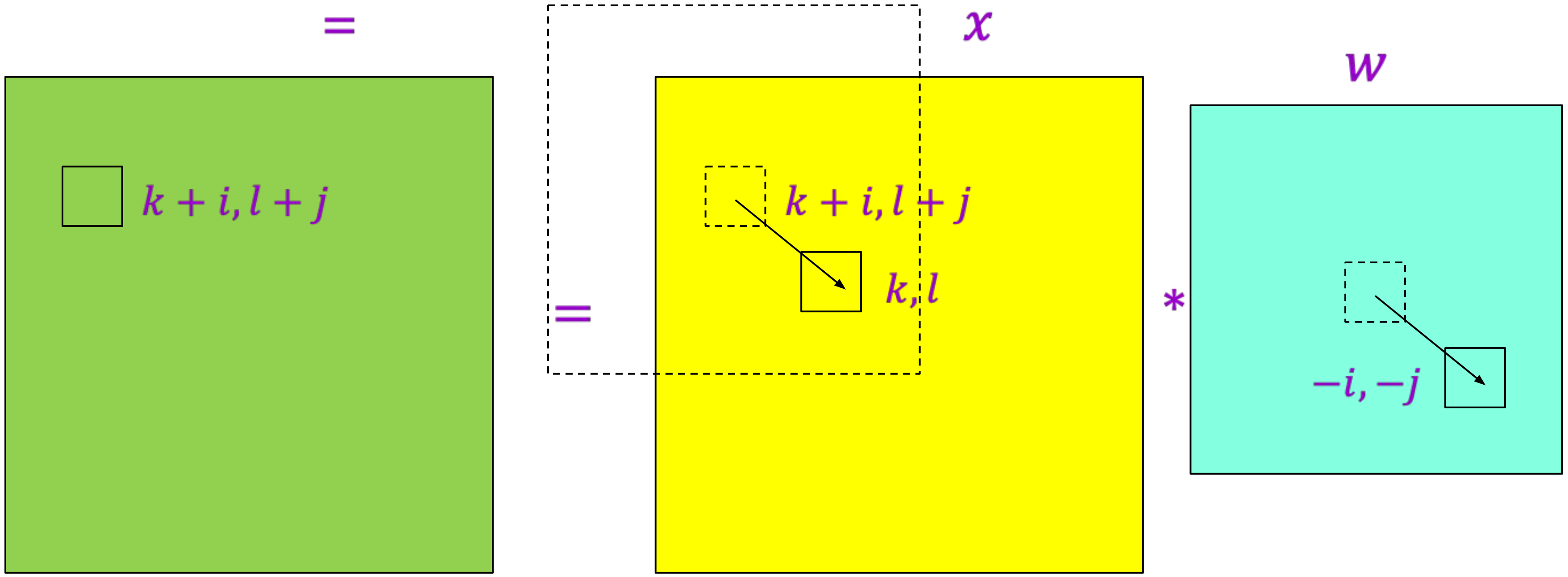
$$\frac{\partial z_{k+i,l+j}}{\partial x_{kl}} = w_{-i,-j}$$



Backpropagation for convolutional layer

$$\frac{\partial e}{\partial x_{kl}} = \sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}} \frac{\partial z_{k+i,l+j}}{\partial x_{kl}} = \boxed{\sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}} w_{-i,-j}}$$

$$\frac{\partial z_{k+i,l+j}}{\partial x_{kl}} = w_{-i,-j}$$

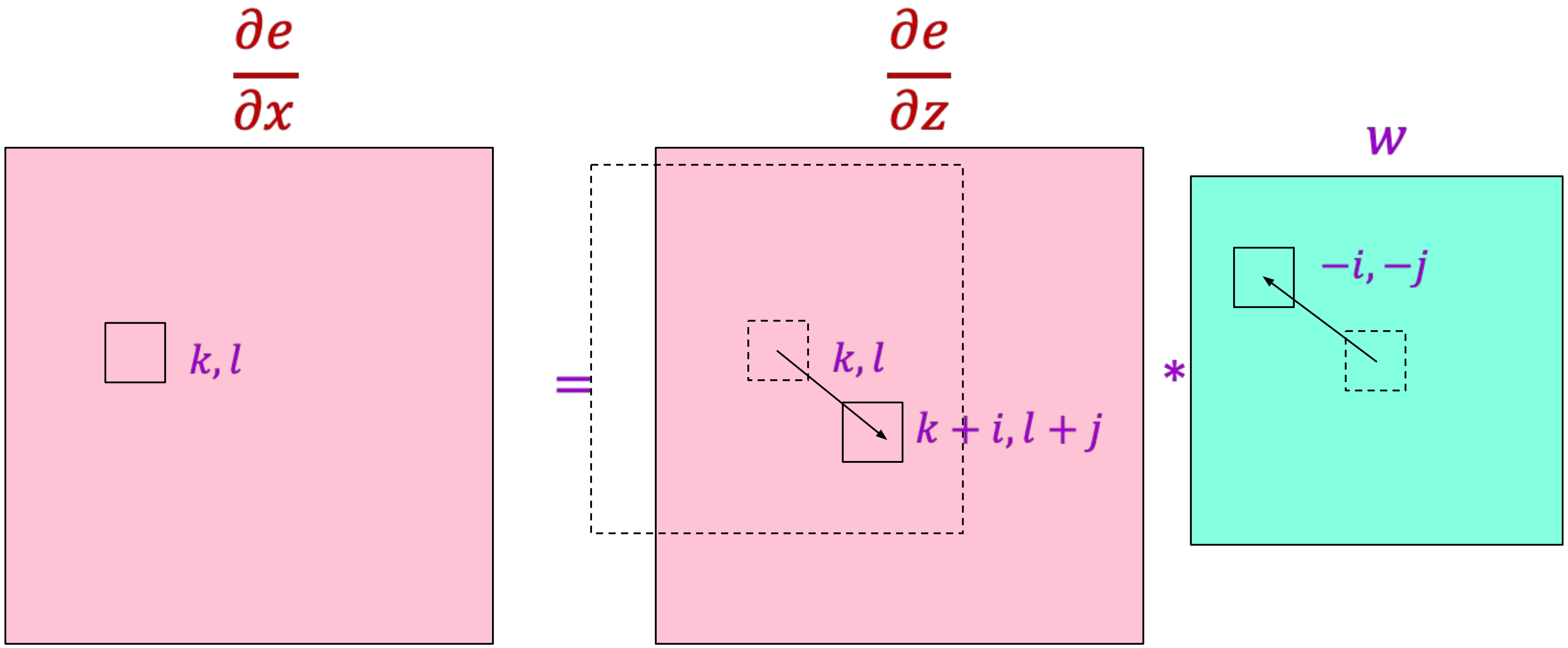


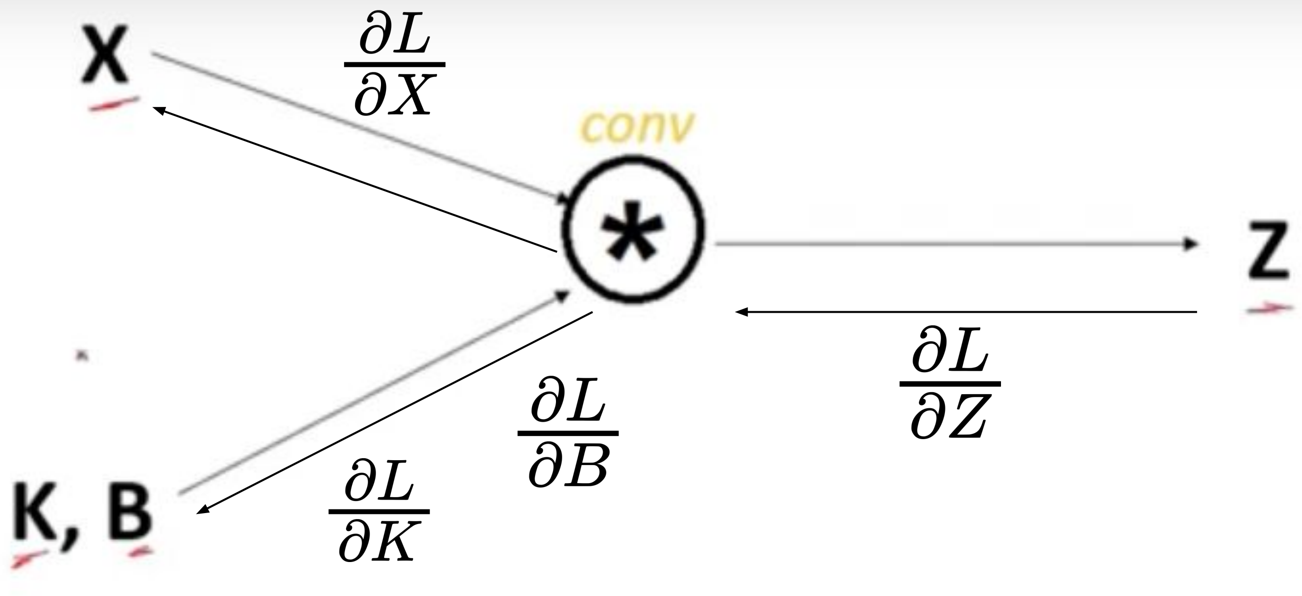
Backpropagation for convolutional layer

$$\boxed{\frac{\partial e}{\partial x_{kl}} = \sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}} w_{-i,-j}}$$

Backpropagation for convolutional layer

$$\boxed{\frac{\partial e}{\partial x_{kl}} = \sum_{i,j} \frac{\partial e}{\partial z_{k+i,l+j}} w_{-i,-j}}$$





$$W = W - \eta \frac{\partial L}{\partial W}$$

$$K = K - \eta \frac{\partial L}{\partial K}$$

$$B = B - \eta \frac{\partial L}{\partial B}$$

$$\frac{\partial L}{\partial K} = \left(\frac{\partial L}{\partial Z}\right) \cdot \left(\frac{\partial Z}{\partial K}\right)$$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \circledast \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} + B = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z_{11} = X_{11}K_{11} + X_{12}K_{12} + X_{21}K_{21} + X_{22}K_{22} + B$$

$$Z_{12} = X_{12}K_{11} + X_{13}K_{12} + X_{22}K_{21} + X_{23}K_{22} + B$$

$$Z_{21} = X_{21}K_{11} + X_{22}K_{12} + X_{31}K_{21} + X_{32}K_{22} + B$$

$$Z_{22} = X_{22}K_{11} + X_{23}K_{12} + X_{32}K_{21} + X_{33}K_{22} + B$$

$$\frac{\partial L}{\partial K_{mn}} = \sum \frac{\partial L}{\partial Z_{ij}} \cdot \frac{\partial Z_{ij}}{\partial K_{mn}}$$

$$\frac{\partial L}{\partial K} = \begin{bmatrix} \frac{\partial L}{\partial K_{11}} & \frac{\partial L}{\partial K_{12}} \\ \frac{\partial L}{\partial K_{21}} & \frac{\partial L}{\partial K_{22}} \end{bmatrix}$$

$$Z_{11} = X_{11}K_{11} + X_{12}K_{12} + X_{21}K_{21} + X_{22}K_{22} + B$$

$$Z_{12} = X_{12}K_{11} + X_{13}K_{12} + X_{22}K_{21} + X_{23}K_{22} + B$$

$$Z_{21} = X_{21}K_{11} + X_{22}K_{12} + X_{31}K_{21} + X_{32}K_{22} + B$$

$$Z_{22} = X_{22}K_{11} + X_{23}K_{12} + X_{32}K_{21} + X_{33}K_{22} + B$$

$$\frac{\partial L}{\partial K_{11}} = \left(\frac{\partial L}{\partial Z_{11}} \ast \frac{\partial Z_{11}}{\partial K_{11}}\right) + \left(\frac{\partial L}{\partial Z_{12}} \ast \frac{\partial Z_{12}}{\partial K_{11}}\right) + \left(\frac{\partial L}{\partial Z_{21}} \ast \frac{\partial Z_{21}}{\partial K_{11}}\right) + \left(\frac{\partial L}{\partial Z_{22}} \ast \frac{\partial Z_{22}}{\partial K_{11}}\right)$$

$$\frac{\partial L}{\partial K_{12}} = \left(\frac{\partial L}{\partial Z_{11}} \ast \frac{\partial Z_{11}}{\partial K_{12}}\right) + \left(\frac{\partial L}{\partial Z_{12}} \ast \frac{\partial Z_{12}}{\partial K_{12}}\right) + \left(\frac{\partial L}{\partial Z_{21}} \ast \frac{\partial Z_{21}}{\partial K_{12}}\right) + \left(\frac{\partial L}{\partial Z_{22}} \ast \frac{\partial Z_{22}}{\partial K_{12}}\right)$$

$$\frac{\partial L}{\partial K_{21}} = \frac{\partial L}{\partial Z_{11}} \ast \frac{\partial Z_{11}}{\partial K_{21}} + \frac{\partial L}{\partial Z_{12}} \ast \frac{\partial Z_{12}}{\partial K_{21}} + \frac{\partial L}{\partial Z_{21}} \ast \frac{\partial Z_{21}}{\partial K_{21}} + \frac{\partial L}{\partial Z_{22}} \ast \frac{\partial Z_{22}}{\partial K_{21}}$$

$$\frac{\partial L}{\partial K_{22}} = \frac{\partial L}{\partial Z_{11}} \ast \frac{\partial Z_{11}}{\partial K_{22}} + \frac{\partial L}{\partial Z_{12}} \ast \frac{\partial Z_{12}}{\partial K_{22}} + \frac{\partial L}{\partial Z_{21}} \ast \frac{\partial Z_{21}}{\partial K_{22}} + \frac{\partial L}{\partial Z_{22}} \ast \frac{\partial Z_{22}}{\partial K_{22}}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} * X_{11} + \frac{\partial L}{\partial Z_{12}} * X_{12} + \frac{\partial L}{\partial Z_{21}} * X_{21} + \frac{\partial L}{\partial Z_{22}} * X_{22}$$

$$\frac{\partial L}{\partial K_{12}} = \frac{\partial L}{\partial Z_{11}} * X_{12} + \frac{\partial L}{\partial Z_{12}} * X_{13} + \frac{\partial L}{\partial Z_{21}} * X_{22} + \frac{\partial L}{\partial Z_{22}} * X_{23}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} * X_{21} + \frac{\partial L}{\partial Z_{12}} * X_{22} + \frac{\partial L}{\partial Z_{21}} * X_{31} + \frac{\partial L}{\partial Z_{22}} * X_{32}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} * X_{22} + \frac{\partial L}{\partial Z_{12}} * X_{23} + \frac{\partial L}{\partial Z_{21}} * X_{32} + \frac{\partial L}{\partial Z_{22}} * X_{33}$$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \circledast \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial K_{11}} & \frac{\partial L}{\partial K_{12}} \\ \frac{\partial L}{\partial K_{21}} & \frac{\partial L}{\partial K_{22}} \end{bmatrix} = \frac{\partial L}{\partial K}$$

$$\frac{\partial L}{\partial K} = \mathrm{conv}\left(X, \frac{\partial L}{\partial Z}\right)$$

$$\frac{\partial L}{\partial B} = \frac{\partial L}{\partial Z_{ij}} \times \frac{\partial Z_{ij}}{\partial B}$$

$$\frac{\partial}{\partial B} Z_{11} = X_{11}K_{11} + X_{12}K_{12} + X_{21}K_{21} + X_{22}K_{22} + B \leftarrow 1$$

$$Z_{12} = X_{12}K_{11} + X_{13}K_{12} + X_{22}K_{21} + X_{23}K_{22} + B$$

$$Z_{21} = X_{21}K_{11} + X_{22}K_{12} + X_{31}K_{21} + X_{32}K_{22} + B$$

$$Z_{22} = X_{22}K_{11} + X_{23}K_{12} + X_{32}K_{21} + X_{33}K_{22} + B$$

$$\frac{\partial L}{\partial B} = \sum \frac{\partial L}{\partial Z_{ij}} * \frac{\partial Z_{ij}}{\partial B}$$

$$= \frac{\partial L}{\partial Z_{11}} * \left(\frac{\partial Z_{11}}{\partial B}\right)^{1} + \frac{\partial L}{\partial Z_{12}} * \left(\frac{\partial Z_{12}}{\partial B}\right)^{1} + \frac{\partial L}{\partial Z_{21}} * \frac{\partial Z_{21}}{\partial B}^{1} + \frac{\partial L}{\partial Z_{22}} * \frac{\partial Z_{22}}{\partial B}^{1}$$

$$\frac{\partial L}{\partial B} = \frac{\partial L}{\partial Z_{11}} + \frac{\partial L}{\partial Z_{12}} + \frac{\partial L}{\partial Z_{21}} + \frac{\partial L}{\partial Z_{22}}$$

$$\frac{\partial L}{\partial B} = sum\left(\frac{\partial L}{\partial Z}\right)$$

$$\frac{\partial L}{\partial x_{mn}} = \sum_{i} \frac{\partial L}{\partial z_{ij}} * \frac{\partial z_{ij}}{\partial x_{mn}}$$

$$Z_{11} = X_{11}K_{11} + X_{12}K_{12} + X_{21}K_{21} + X_{22}K_{22} + B$$

$$Z_{12} = X_{12}K_{11} + X_{13}K_{12} + X_{22}K_{21} + X_{23}K_{22} + B$$

$$Z_{21} = X_{21}K_{11} + X_{22}K_{12} + X_{31}K_{21} + X_{32}K_{22} + B$$

$$Z_{22} = X_{22}K_{11} + X_{23}K_{12} + X_{32}K_{21} + X_{33}K_{22} + B$$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \circledast \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} + B = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\frac{\partial L}{\partial X_{mn}} = \sum \frac{\partial L}{\partial Z_{ij}} * \frac{\partial Z_{ij}}{\partial X_{mn}}$$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial Z_{11}} * \left(\frac{\partial Z_{11}}{\partial X_{11}}\right) = \frac{\partial L}{\partial Z_{11}} * K_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \left(\frac{\partial L}{\partial Z_{11}} * \frac{\partial Z_{11}}{\partial X_{12}}\right) + \left(\frac{\partial L}{\partial Z_{12}} * \frac{\partial Z_{12}}{\partial X_{12}}\right) = \frac{\partial L}{\partial Z_{12}} * K_{12} + \frac{\partial L}{\partial Z_{12}} * K_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial Z_{12}} * \frac{\partial Z_{12}}{\partial X_{13}} = \frac{\partial L}{\partial Z_{12}} * K_{12}$$

o

o

o

$$\frac{\partial L}{\partial X_{22}} = \left(\frac{\partial L}{\partial Z_{11}} * K_{22}\right) + \left(\frac{\partial L}{\partial Z_{12}} * K_{21}\right) + \left(\frac{\partial L}{\partial Z_{21}} * K_{12}\right) + \left(\frac{\partial L}{\partial Z_{22}} * K_{11}\right)$$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial Z_{11}} * K_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial Z_{11}} * K_{12} + \frac{\partial L}{\partial Z_{12}} * K_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial Z_{12}} * K_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial Z_{11}} * K_{21} + \frac{\partial L}{\partial Z_{21}} * K_{11}$$

$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial Z_{11}} * K_{22} + \frac{\partial L}{\partial Z_{12}} * K_{21} + \frac{\partial L}{\partial Z_{21}} * K_{12} + \frac{\partial L}{\partial Z_{22}} * K_{11}$$

$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial Z_{12}} * K_{22} + \frac{\partial L}{\partial Z_{22}} * K_{12}$$

$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial Z_{21}} * K_{21}$$

$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial Z_{21}} * K_{22} + \frac{\partial L}{\partial Z_{22}} * K_{21}$$

$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial Z_{22}} * K_{22}$$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial Z_{11}} * K_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial Z_{11}} * K_{12} + \frac{\partial L}{\partial Z_{12}} * K_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial Z_{12}} * K_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial Z_{11}} * K_{21} + \frac{\partial L}{\partial Z_{21}} * K_{11}$$

$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial Z_{11}} * K_{22} + \frac{\partial L}{\partial Z_{12}} * K_{21} + \frac{\partial L}{\partial Z_{21}} * K_{12} + \frac{\partial L}{\partial Z_{22}} * K_{11}$$

$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial Z_{12}} * K_{22} + \frac{\partial L}{\partial Z_{22}} * K_{12}$$

$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial Z_{21}} * K_{21}$$

$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial Z_{21}} * K_{22} + \frac{\partial L}{\partial Z_{22}} * K_{21}$$

$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial Z_{22}} * K_{22}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \xrightarrow{180^\circ} \begin{bmatrix} K_{22} & K_{21} \\ K_{12} & K_{11} \end{bmatrix}$$

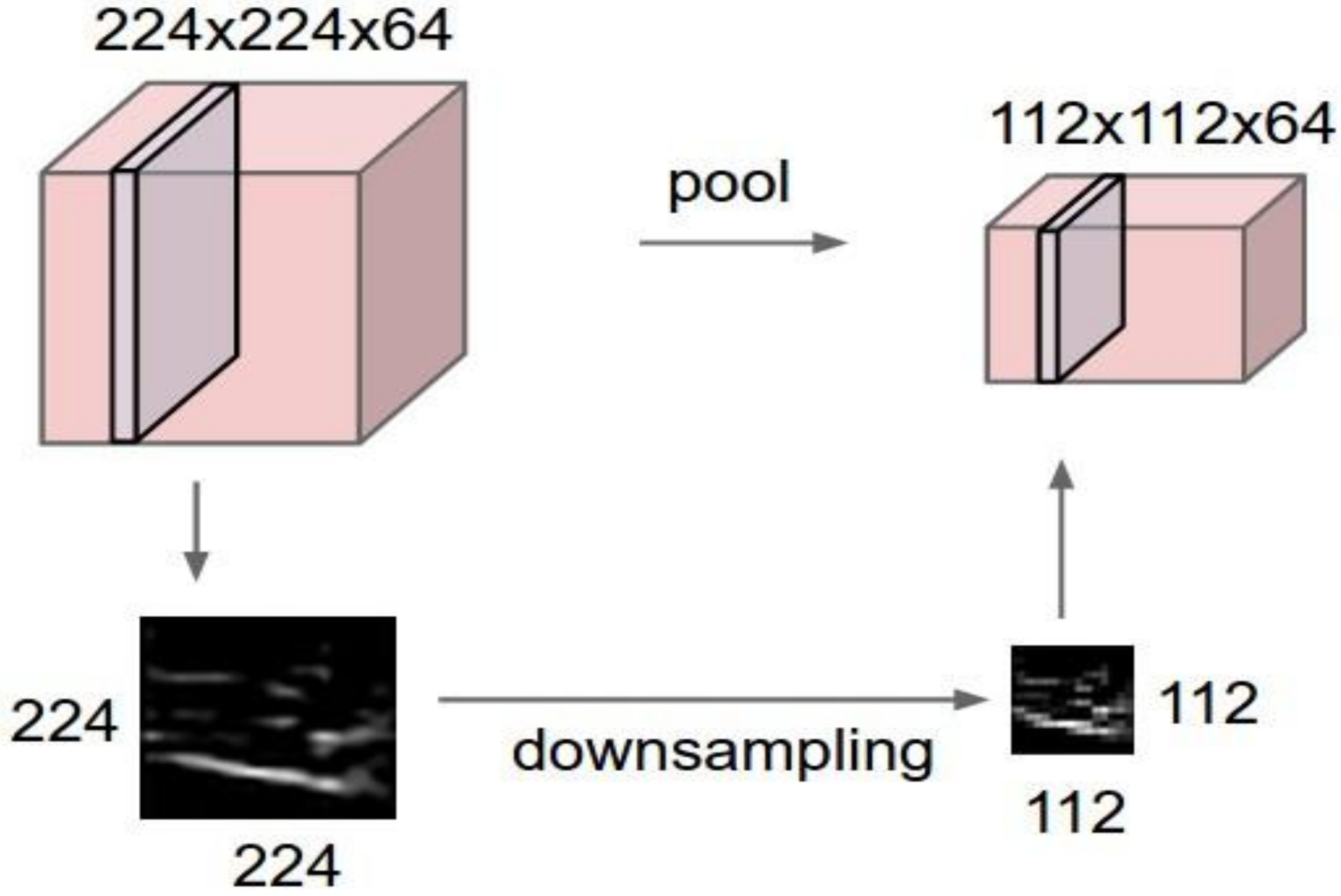
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} & 0 \\ 0 & \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial L}{\partial X} = \text{conv}\left(padded\left(\frac{\partial L}{\partial Z}\right), 180^\circ \text{ rotated filter K}\right)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} & 0 \\ 0 & \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circledast \begin{bmatrix} K_{22} & K_{21} \\ K_{12} & K_{11} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial X_{11}} & \frac{\partial L}{\partial X_{12}} & \frac{\partial L}{\partial X_{13}} \\ \frac{\partial L}{\partial X_{21}} & \frac{\partial L}{\partial X_{22}} & \frac{\partial L}{\partial X_{23}} \\ \frac{\partial L}{\partial X_{31}} & \frac{\partial L}{\partial X_{32}} & \frac{\partial L}{\partial X_{33}} \end{bmatrix}$$

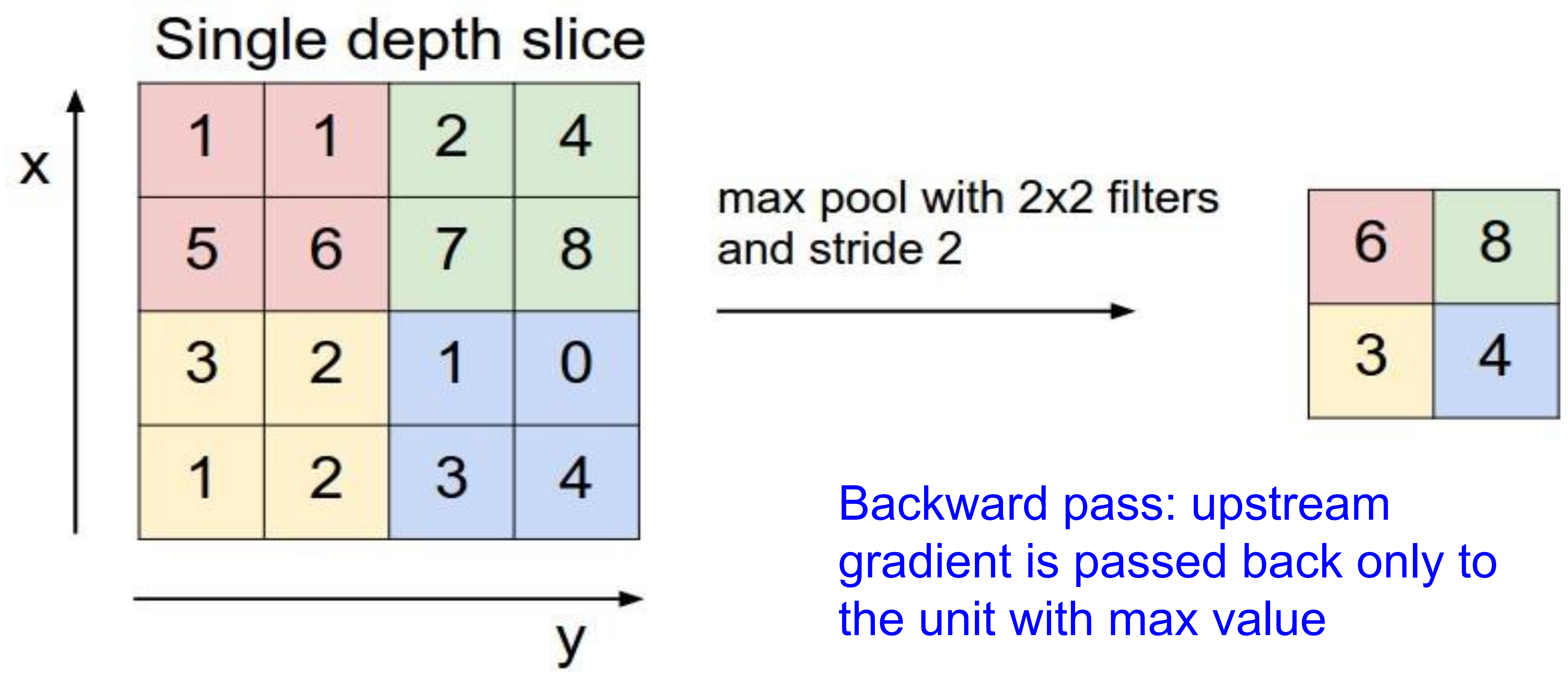
Pooling Layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



Source: cs231n, Stanford University

Max Pooling



Backward pass: upstream gradient is passed back only to the unit with max value

Source: cs231n, Stanford University

Pooling Layer

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - their spatial extent F,
  - the stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- Note that it is not common to use zero-padding for Pooling layers

Source: cs231n, Stanford University

Fully Connected Layer

- **Connect every neuron in one layer to every neuron in another layer**
- **Same as the traditional multi-layer perceptron neural network**

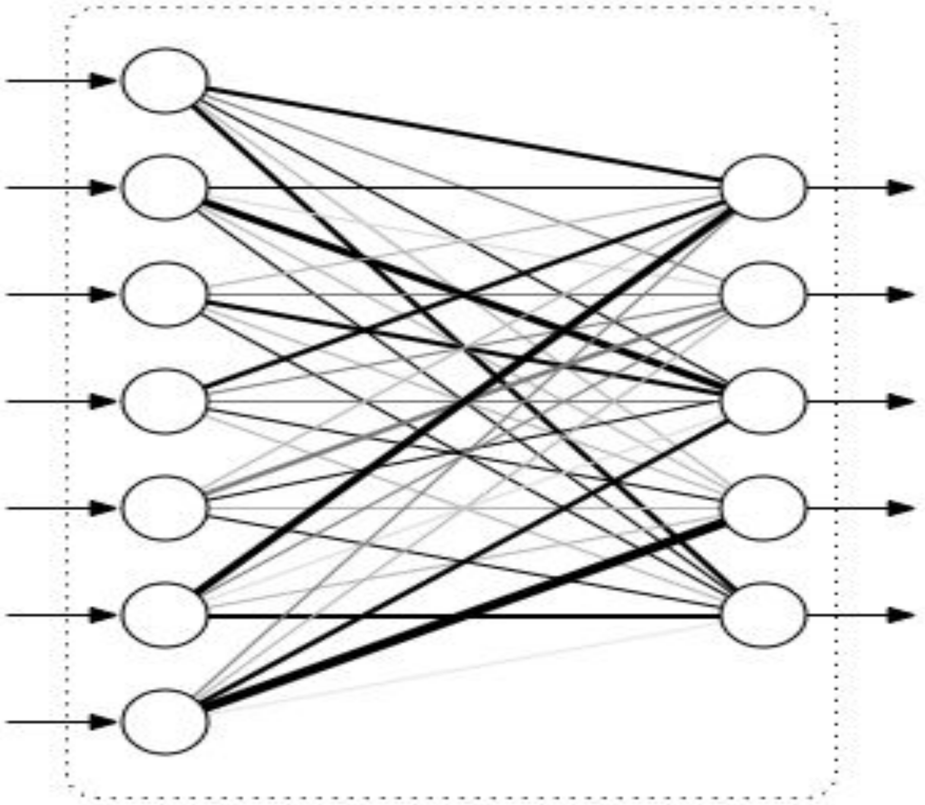


Image Source: machinethink.net

Fully Connected Layer

- **Connect every neuron in one layer to every neuron in another layer**
- **Same as the traditional multi-layer perceptron neural network**

No. of Neurons (Last FC)
= No. of classes

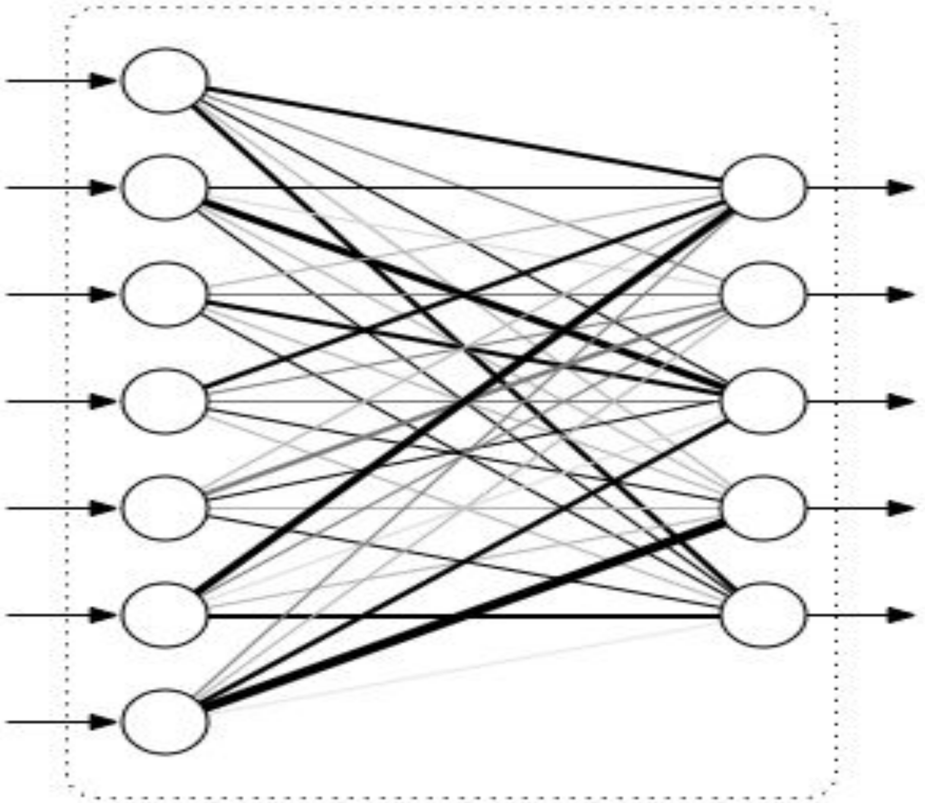
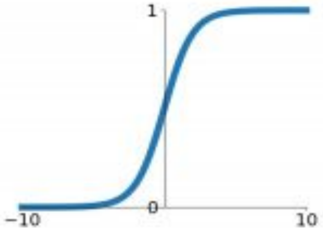


Image Source: machinethink.net

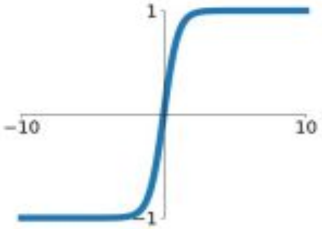
Non-linearity Layer

Activation Functions

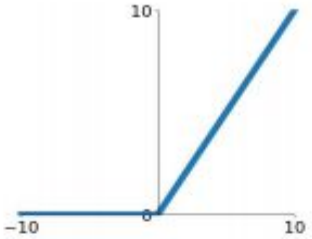
Sigmoid
$\sigma(x) = \frac{1}{1+e^{-x}}$



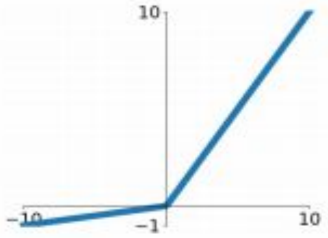
tanh
$\tanh(x)$



ReLU
$\max(0, x)$

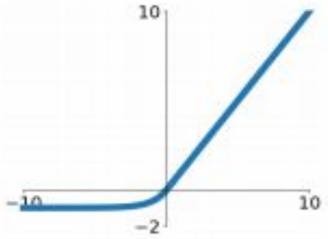


Leaky ReLU
$\max(0.1x, x)$



Maxout
$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Source: cs231n, Stanford University

Classification/Loss Layer

SVM Classifier

- SVM Loss/Hinge Loss/Max-margin Loss

Softmax Classifier

- Softmax Loss/Cross-entropy Loss

A typical CNN structure

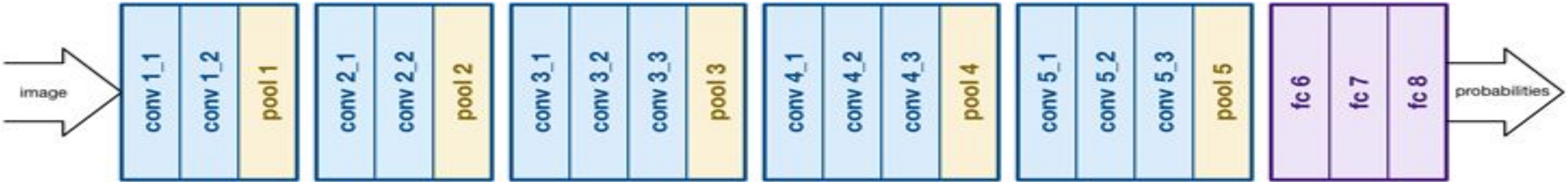
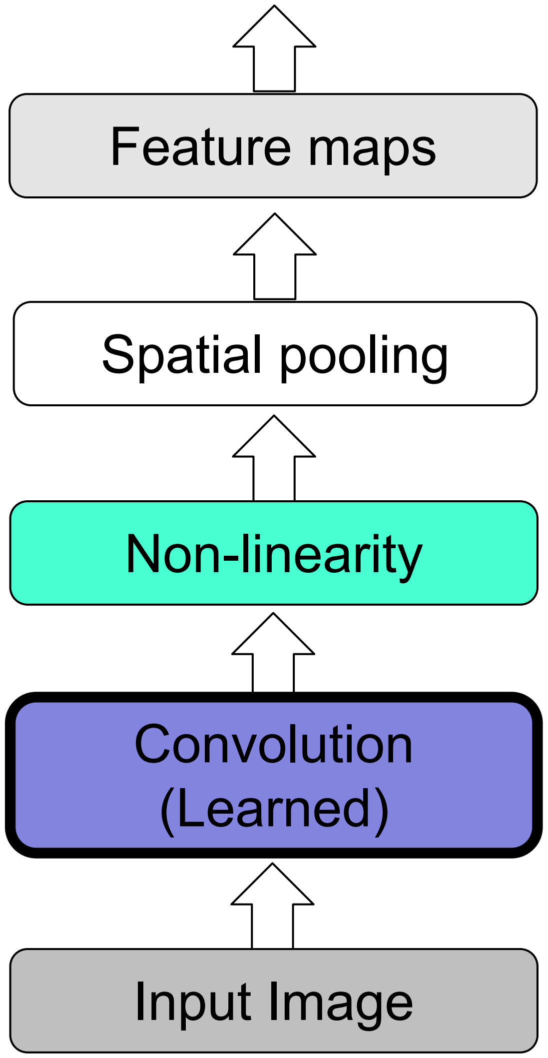
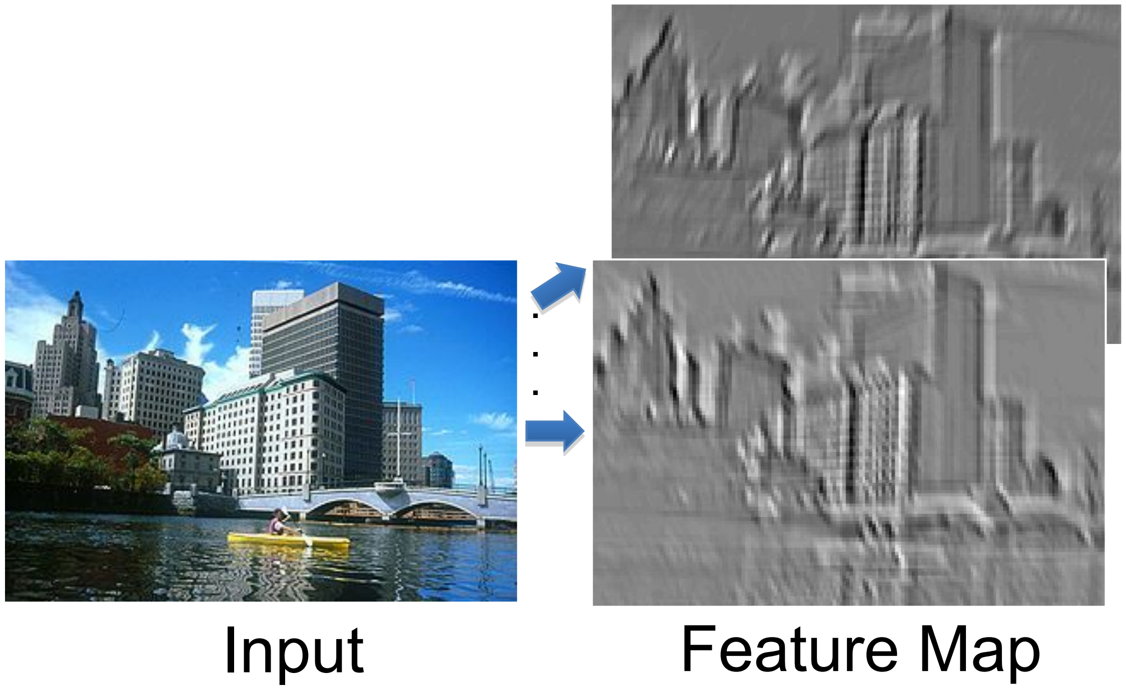


Image Source: machinethink.net

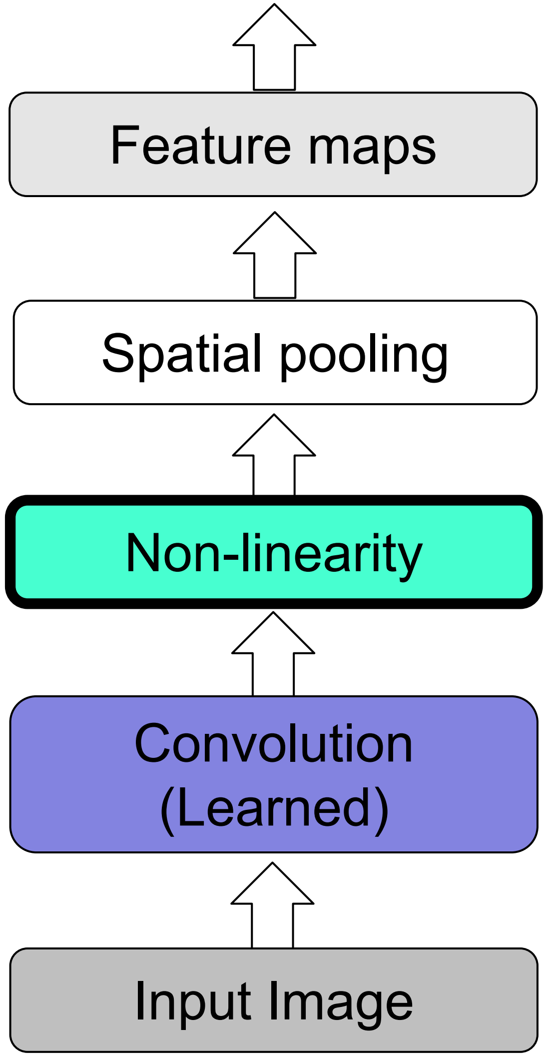
Summary: CNN pipeline





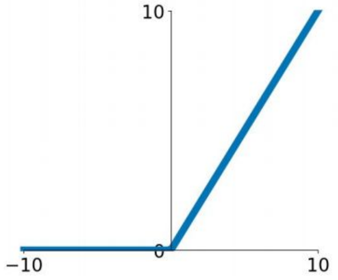
Source: R. Fergus, Y. LeCun

Summary: CNN pipeline



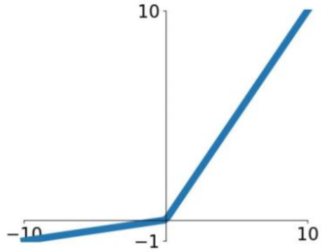
ReLU

$\max(0, x)$



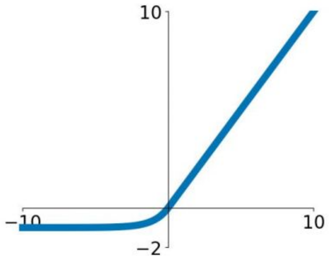
Leaky ReLU

$\max(0.1x, x)$



ELU

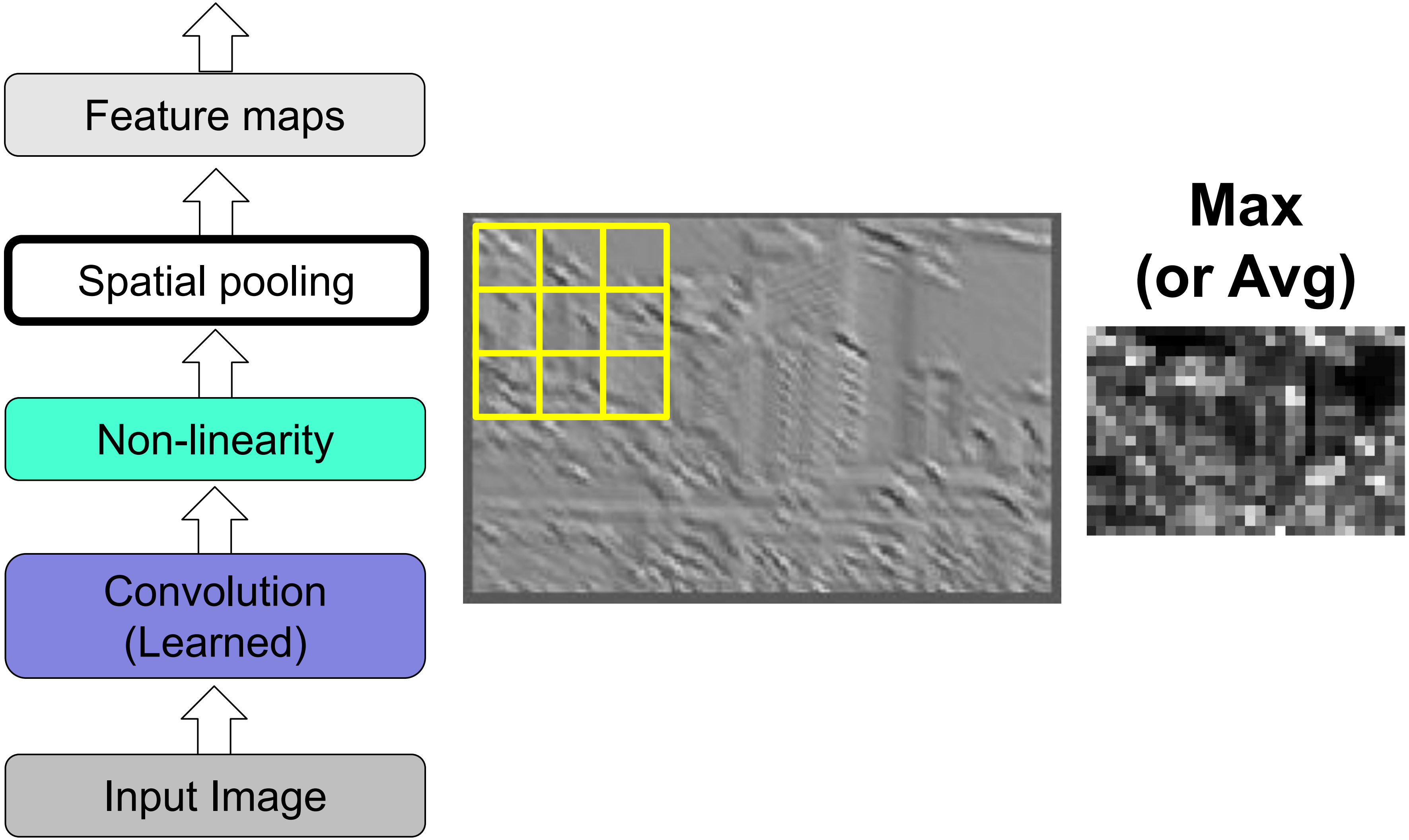
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Source: R. Fergus, Y. LeCun

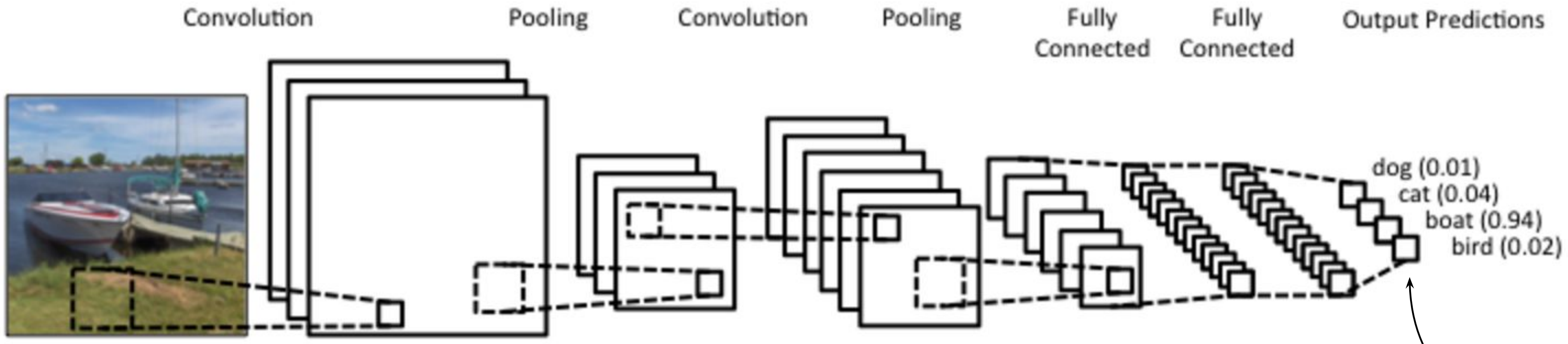
Source: Stanford 231n

Summary: CNN pipeline



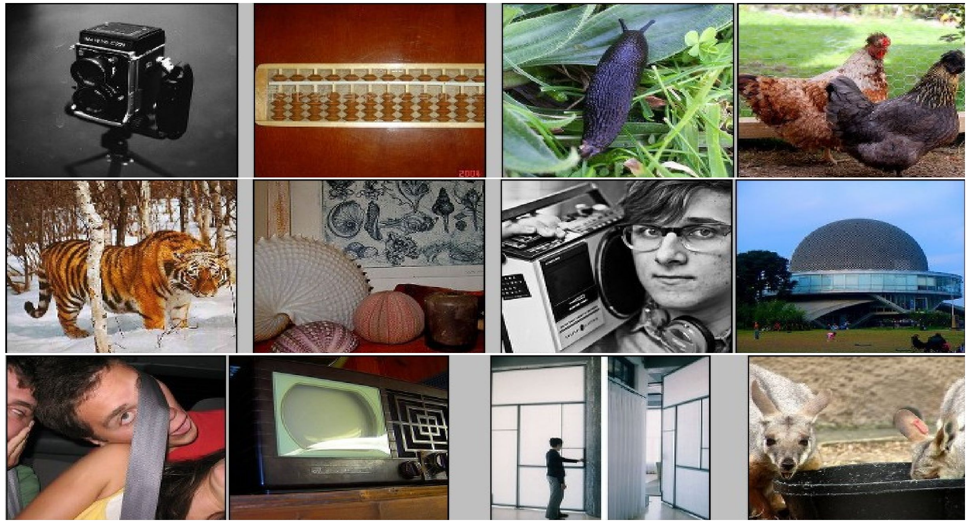
Source: R. Fergus, Y. LeCun

Summary: CNN pipeline



Softmax layer:

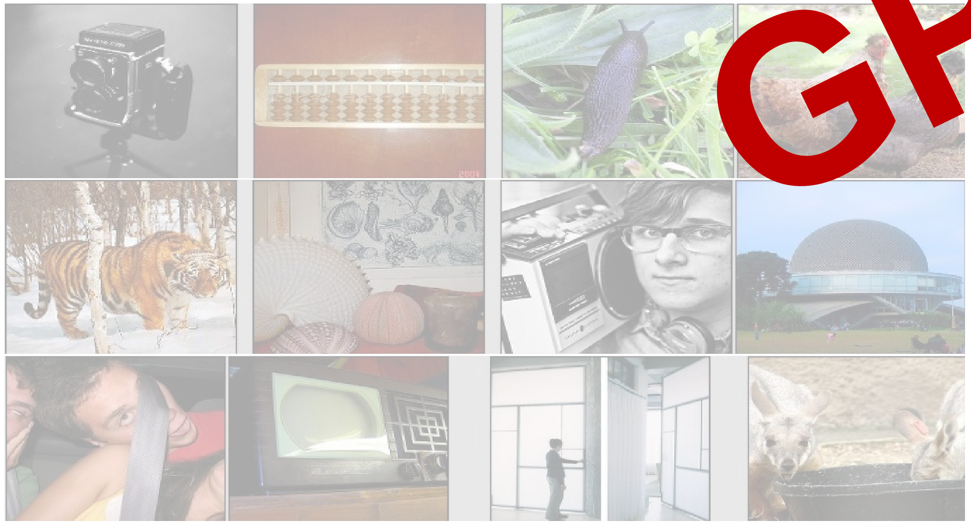
ImageNet Challenge



- ~14 million labeled images, 20k classes
- Images gathered from Internet
- Human labels via Amazon MTurk
- Challenge: 1.2 million training images, 1000 classes

www.image-net.org/challenges/LSVRC/

ImageNet Challenge



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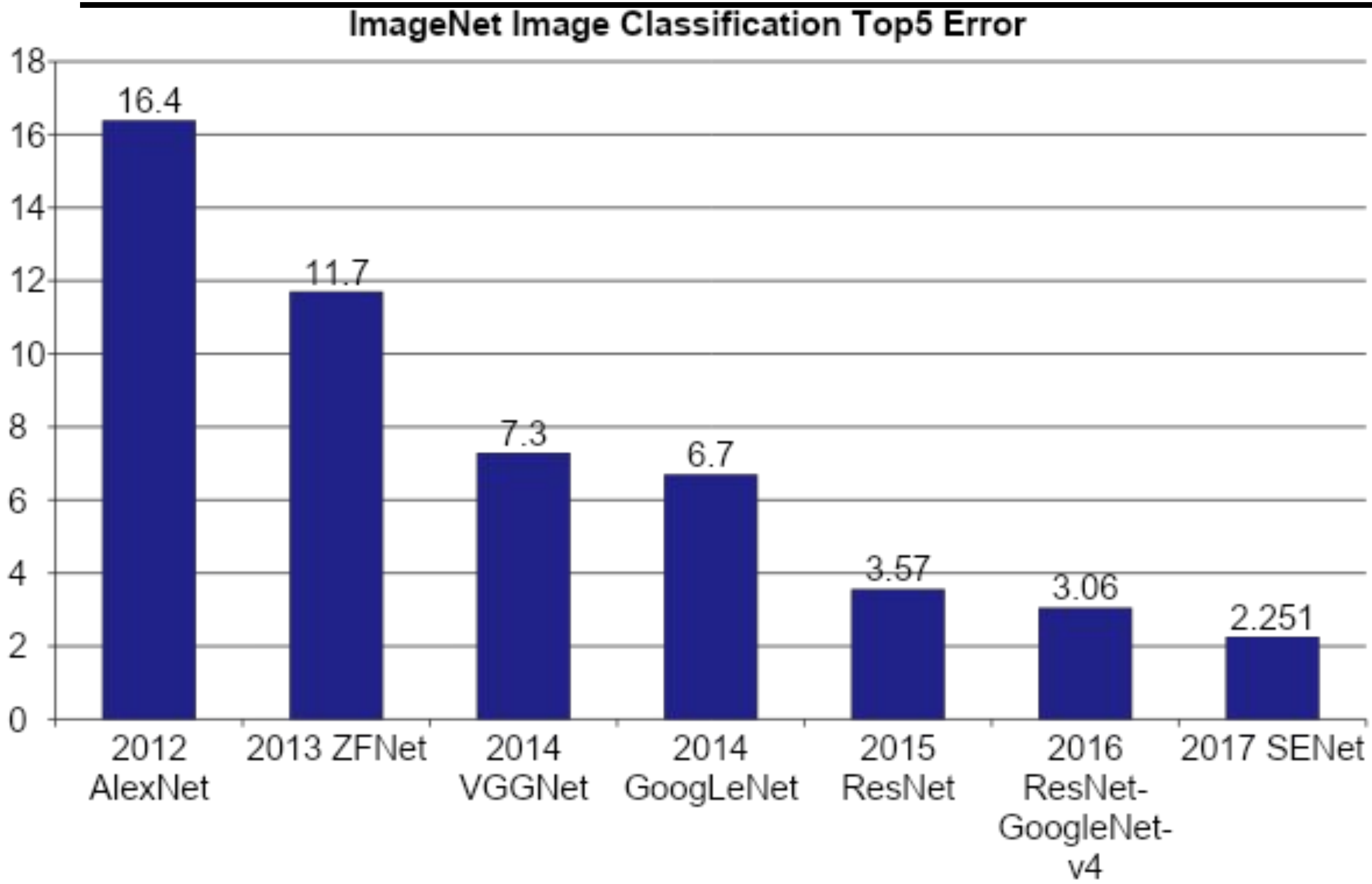
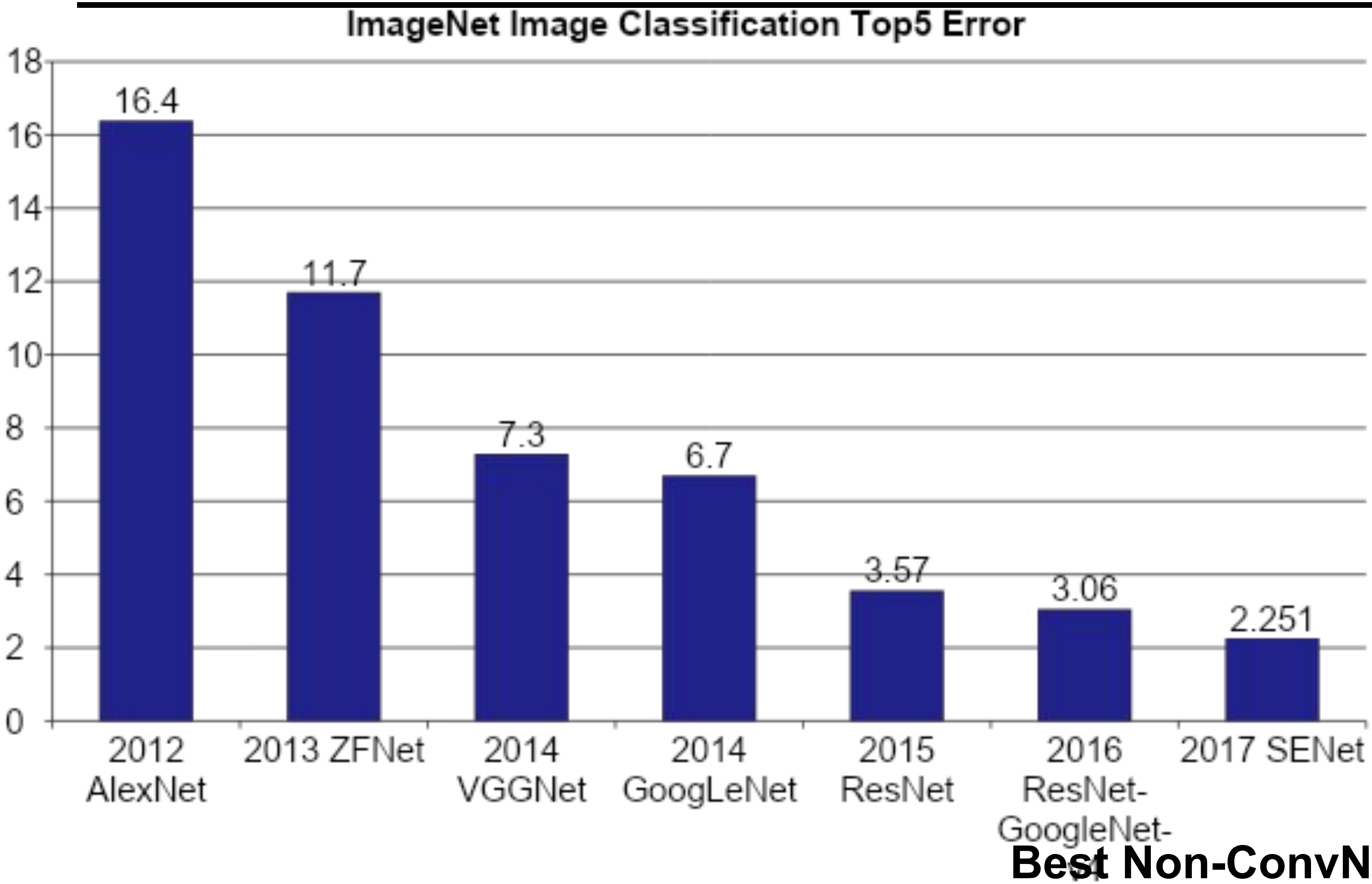
GPUs + Data

www.image-net.org/challenges/LSVRC/

Progress on ImageNet Challenge



Progress on ImageNet Challenge



Best Non-ConvNet in 2012: 26.2%

Things to remember

Neural network and Image

- Neuroscience, Perceptron, Problems due to High Dimensionality and Local Relationship

Convolutional neural network (CNN)

- Convolution Layer,
- Nonlinearity Layer,
- Pooling Layer,
- Fully Connected Layer,
- Loss/Classification Layer

Progress on ImageNet challenge

- Latest SENet, Winner 2017

Code

Convolutional_convLncode

Acknowledgement

Thanks to the following courses and corresponding researchers for making their teaching/research material online

- Deep Learning, Stanford University
- Introduction to Deep Learning, University of Illinois at Urbana-Champaign
- Introduction to Deep Learning, Carnegie Mellon University
- Convolutional Neural Networks for Visual Recognition, Stanford University
- Natural Language Processing with Deep Learning, Stanford University
- And Many More ......

Next Classes

Training Aspects of CNN

Activation Functions

Dataset Preparation

Data Preprocessing

Weight Initialization

Optimization Methods

Learning Rate

Transfer Learning

Generalization