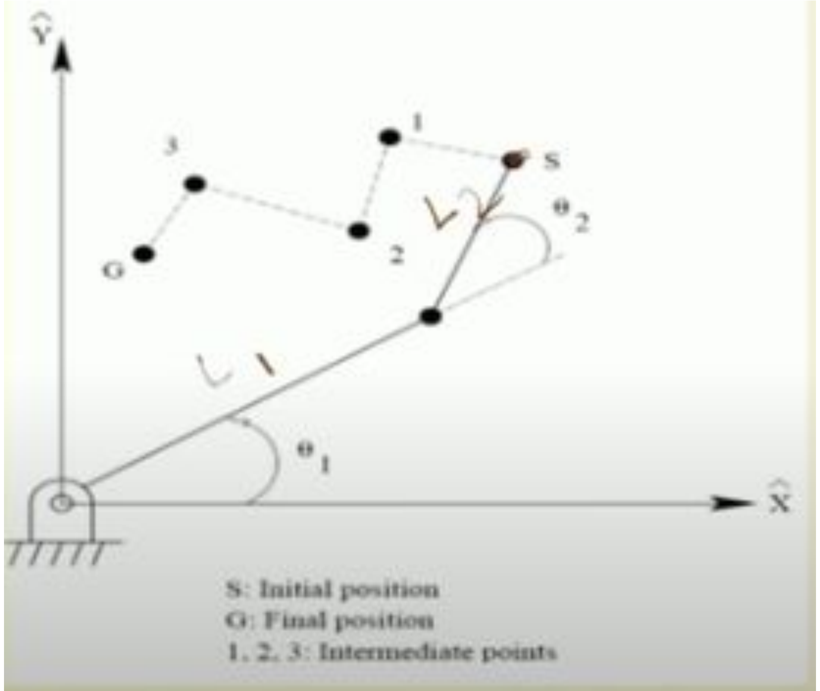
Trajectory Planning

Lecture-15

Aim: to determine time history of position, velocity and acceleration of end-effector of a manipulator, while moving from an initial position to a final position through some intermediate /via points



Points	**Cartesian scheme**	**Joint-space scheme**
S	$(X_{s,}Y_s)$	$\left(\Theta_{1,}^{s}\Theta_{2,}^{s}\right)$
1	$(X_{1,}Y_1)$	$\left(\Theta_{1,}^{1}\Theta_{2,}^{1}\right)$
2	$(X_{2,}Y_2)$	$\left(\Theta_{1,}^{2}\Theta_{2,}^{2}\right)$
3	$(X_{3,}Y_3)$	$\left(\Theta_{1,}^{3}\Theta_{2,}^{3}\right)$
G	$(X_{G,}Y_G)$	$\left(\Theta_{1,}^{G}\Theta_{2,}^{G}\right)$

Trajectory Planning

Cartesian scheme

- Computationally expensive, as inverse kinematics problem has to be solved, on-line

- Joint-Space scheme

Joint-space scheme

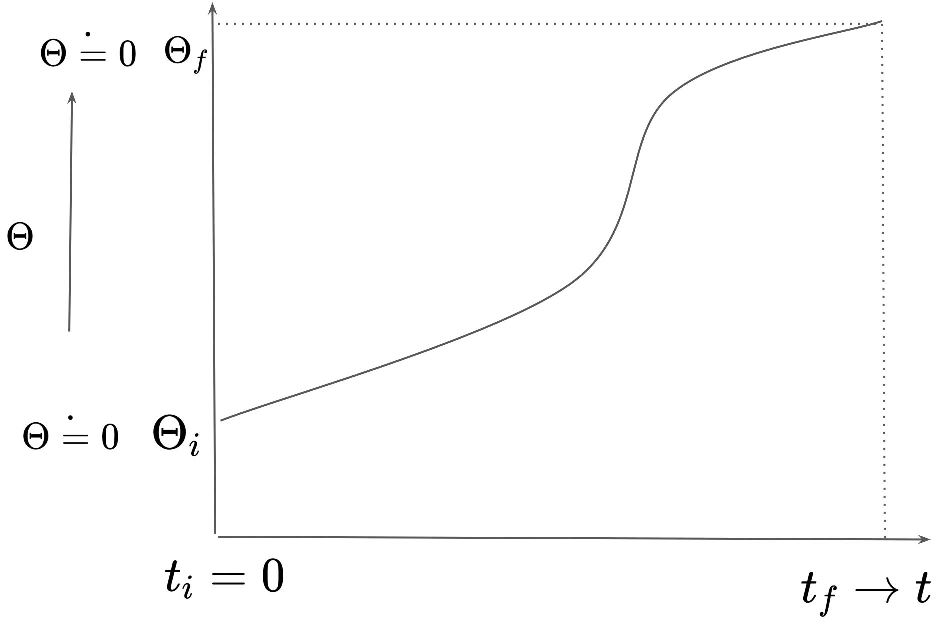
- To fit a smooth (continuous) curve through ($\Theta_1^s, \Theta_1^1, \Theta_1^2, \Theta_1^3, \Theta_1^G$)
- First and second order derivatives must be continuous.

Various Trajectory Functions

- Cube polynomial
- Fifth-order polynomial
- Linear trajectory function

Polynomial Trajectory function

Case-1: Initial and final values of joint angle are known, and angular velocities at the beginning and end of the cycle are kept equal to zero.



$$\text{At}\, t = t_i = 0;\ \Theta = \Theta_i, \dot{\Theta} = 0$$

$$\text{At}\, t = t_f;\ \Theta = \Theta_f, \dot{\Theta} = 0$$

Let us consider cubic polynomial

$$\Theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$

Angular displacement

where $C_0, C_1, C_2,\ C_3$ are the coefficients

Differentiate $\Theta(t)$ with respect to time to get angular velocity

$$\dot{\Theta} = C_1 + 2C_2 t + 3C_3 t^2$$

Angular velocity

Apply the initial conditions to angular displacement and velocity equations. We get,

$$C_0 = \Theta_i - - - - - - 1$$

$$C_1 = 0 - - - - - - 2$$

$$C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 = \Theta_f - - - -3$$

$$C_1 + 2C_2 t_f + 3C_3 t_f^2 = 0 - - - -4$$

Solving above equations, we get

$$\Theta(t) = \Theta_i + \frac{3(\Theta_f - \Theta_i)}{t_f^2} t^2 - \frac{2(\Theta_f - \Theta_i)}{t_f^3} t^3$$

Case-2

Initial and final values of joint angle are known and angular velocities at the beginning and end of the cycle are assumed to have non zero values.

$$At \quad t = t_i = 0; \Theta = \Theta_i, \ \dot{\Theta} = \dot{\Theta}_i$$

$$At \quad t = t_f; \Theta = \Theta_f, \ \dot{\Theta} = \dot{\Theta}_f$$

Let us consider a third order polynomial of the form:

$$\Theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$

where $C_0, C_1, C_2, \ C_3$ are the coefficients

Differentiate $\Theta(t)$ with respect to time to get angular velocity

$$\dot{\Theta}(t) = C_1 + 2C_2t + 3C_3t^2$$

Apply the initial conditions to angular displacement and velocity equations. We get,

$$C_0 = \Theta_i - - - 1$$

$$C_1 = \Theta_i - \dot{-} - -2$$

$$C_0 + C_1t_f + C_2t_f^2 + C_3t_f^3 = \Theta_f - - - -3$$

$$C_1 + 2C_2t_f + 3C_3t_f^2 = \dot{\Theta}_f - - - -4$$

Solving above equations, we get

$$C_0 = \Theta_i$$

$$C_1 = \dot{\Theta}_i$$

$$C_2 = \frac{3(\Theta_f - \Theta_i)}{t_f^2} - \frac{2}{t_f}\dot{\Theta}_i - \frac{1}{t_f}\dot{\Theta}_f$$

$$C_3 = \frac{2(\Theta_f - \Theta_i)}{t_f^3} + \frac{1}{t_f^2}\left(\dot{\Theta}_f + \dot{\Theta}_i\right)$$

Case-3

Initial and final values of joint angle are known and angular velocities and accelerations at the beginning and end of the cycle are assumed to have non zero values.

$$At \quad t = t_i = 0;\ \Theta = \Theta_i,\ \dot{\Theta} = \dot{\Theta}_i, \ddot{\Theta} = \ddot{\Theta}_i$$

$$At \quad t = t_f;\ \Theta = \Theta_f,\ \dot{\Theta} = \dot{\Theta}_f, \ddot{\Theta} = \ddot{\Theta}_f$$

Let us consider a fifth-order polynomial as follows:

$$\Theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5$$

Differentiate $\Theta(t)$ with respect to time once to get angular velocity and twice to get angular acceleration

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$

Apply the initial conditions to angular displacement, velocity and acceleration equations. We get

$$c_0 = \theta_i$$

$$c_1 = \dot{\theta}_i$$

$$c_2 = \frac{1}{2}\ddot{\theta}_i$$

$$c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3 + c_4 t_f^4 + c_5 t_f^5 = \theta_f$$

$$c_1 + 2c_2 t_f + 3c_3 t_f^2 + 4c_4 t_f^3 + 5c_5 t_f^4 = \dot{\theta}_f$$

$$2c_2 + 6c_3 t_f + 12c_4 t_f^2 + 20c_5 t_f^3 = \ddot{\theta}_f$$

Trajectory Planning

Lecture-16

A Numerical example

- A single-link robot with a revolute joint is motionless at $\Theta = 20^o$ It is desired to move the joint in a smooth manner to $\Theta = 80^o$ in 4.0 seconds. Find a suitable cubic polynomial to generate this motion and bring the manipulator to rest at the goal.

Solution:

$$\Theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$

Conditions:

At time $$t = t_i = 0,\ \Theta = \Theta_i = 20^o, \dot{\Theta} = 0;$$

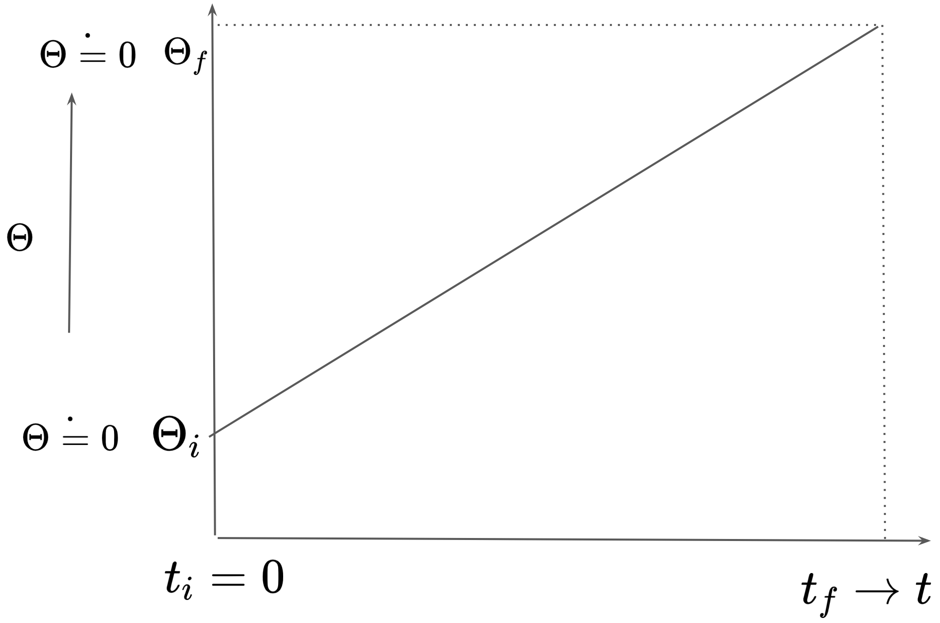
At time $$t = t_f = 4.0s,\ \Theta = \Theta_f = 80^o, \dot{\Theta} = 0;$$

$$\Theta(t) = \Theta_i + \frac{3(\Theta_f - \Theta_i)}{t_f^2}t^2 - \frac{2(\Theta_f - \Theta_i)}{t_f^3}t^3$$

$$\Theta(t) = 20 + \frac{3(80 - 20)}{(4.0)^2}t^2 - \frac{2(80 - 20)}{(4.0)^3}t^3$$

$$= 20 + 11.25^2 - 1.875t^3$$

Linear Trajectory function

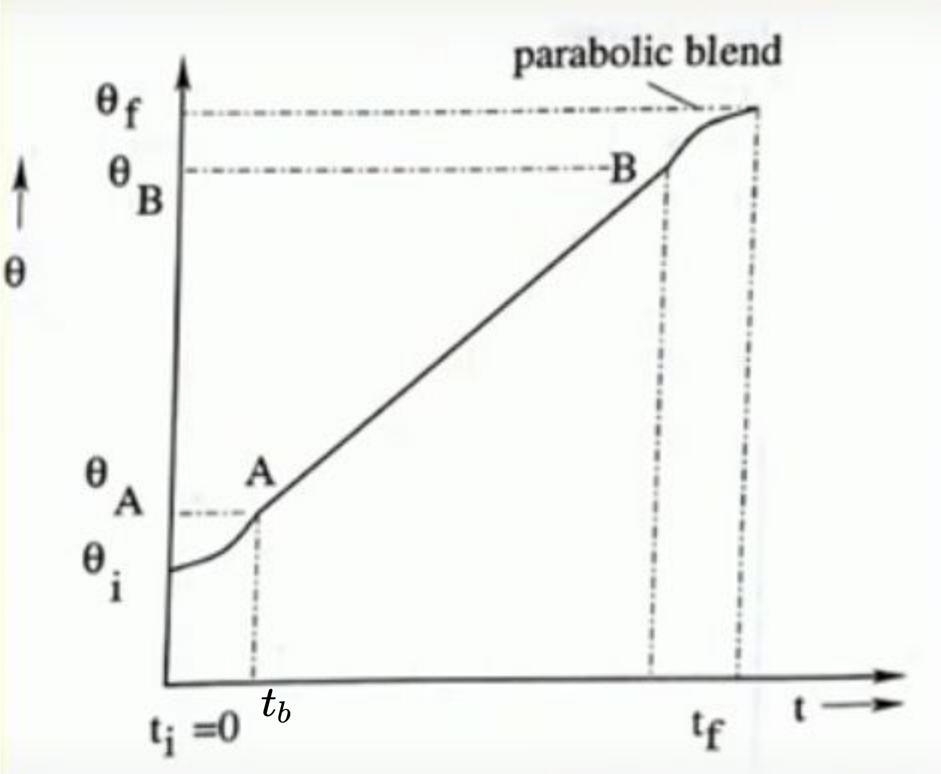


$$\text{At } t = t_i = 0;\ \Theta = \Theta_i, \dot{\Theta} = 0$$

$$\text{At } t = t_f;\ \Theta = \Theta_f, \dot{\Theta} = 0$$

Pure linear Trajectory function

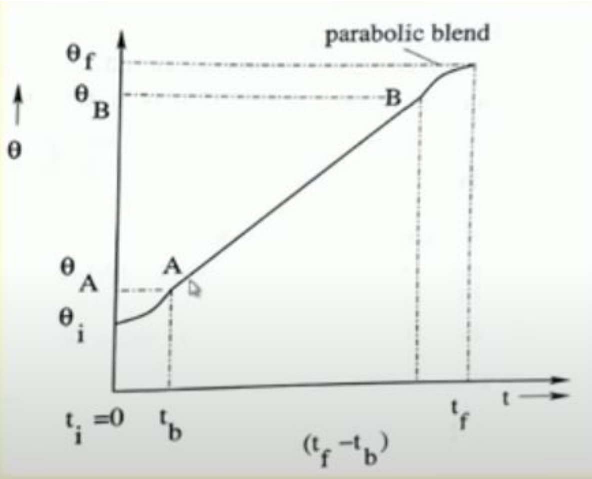
Note:Infinite acceleration and declaration



Modified linear trajectory function with parabolic blends

A numerical example-1

A linear trajectory function with parabolic blends at its two ends is to be obtained to satisfy the following conditions given below.



At time $t = t_i = 0,\ \Theta = \Theta_i = 20^o, \dot{\Theta} = 0;$

At time $t = t_f = 12.0s,\ \Theta = \Theta_f = 74^o, \dot{\Theta} = 0;$

Total cycle time $t_c = t_f - t_i = 12.0s$

Time duration at each of the blend portion $t_b = 3.0s$

Magnitude of acceleration/deceleration $\ddot{\Theta} = 2.0 \deg ree/s^2$

Determine angular displacement and velocity at two junctions of parabolic blends with the straight portion of trajectory function.

$$s = ut + \frac{1}{2} f t^2$$

Solution:

At point A

Angular displacement

$$\Theta_A = \Theta_i + \frac{1}{2} \times \ddot{\Theta} \times t_b^2 = 20.0 + \frac{1}{2} \times (2.0) \times (3.0)^2 = 29.0^o$$

Angular velocity

$$\dot{\Theta}_A = \dot{\Theta}_i + \ddot{\Theta} \times t_b = 0.0 + 2.0 \times 3.0$$

$$= 6.0\,\mathrm{degree}/s$$

$$V = U + ft$$

At point B: From the symmetry of the trajectory function,

$$\Theta_f - \Theta_B = \Theta_A - \Theta_i$$

$$74.0 - \Theta_B = 29.0 - 20.0$$

$$\Theta_B = 65.0^o$$

Angular velocity in the linear portion of trajectory function

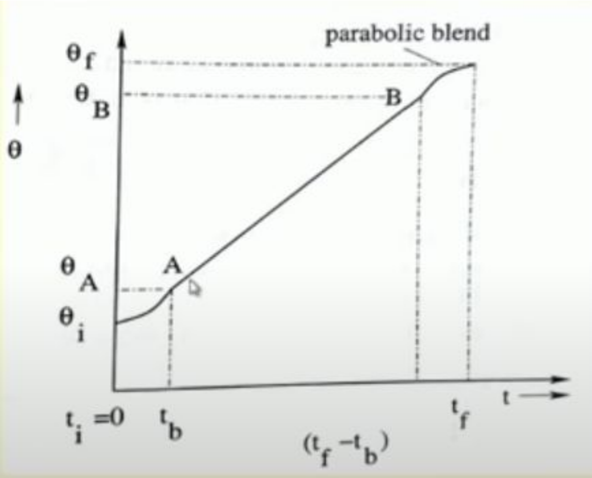
$$= \frac{\Theta_B - \Theta_A}{t_f - 2t_b}$$

$$= \frac{65.0 - 29.0}{12.0 - 2 \times 3.0} = \frac{36.0}{6.0} = 6.0\,\mathrm{degree/s}$$

To maintain continuity of the trajectory function at point B, $\dot{\Theta}_B$ should be equal to the velocity of the linear portion, that is, 6.0 degree per second.

A numerical example-2

A linear trajectory function with parabolic blends at its two ends is to be obtained to satisfy the following conditions given below.



At time $t = t_i = 0,\ \Theta = \Theta_i = 10^o, \dot{\Theta} = 0;$

At time $t = t_f = 9.0s,\ \Theta = \Theta_f = 64^o, \dot{\Theta} = 0;$

Total cycle time $t_c = t_f - t_i = 9.0s$

Time duration at each of the blend portion $t_b = 2.0s$

Magnitude of acceleration/deceleration $\ddot{\Theta} = 2.0 \deg ree/s^2$

Determine angular displacement and velocity at two junctions of parabolic blends with the straight portion of trajectory function.

Singularity Checking

Lecture-17

Singularity Checking

Singularity checking condition is a condition during which manipulator will loss either one or more degrees of freedom

Singularity Checking Through Jacobian

Multi-dimensional form of derivatives

$$f = f_1(x_{1,\ldots,}x_n)$$

$$\frac{\partial f_1}{\partial x_1}, \frac{\partial f_1}{\partial 2}, \ldots, \frac{\partial f_1}{\partial n}$$

This could be used to check singularity

Singularity Checking through Jacobian

Let us consider six functions and each of which is a function of six independent variables.

$$y_1 = f_1(x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,})$$

$$y_2 = f_2(x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,})$$

.

.

$$y_6 = f_6(x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,})$$

In vector notation: $Y = F(X)$

Now

$$\delta y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \frac{\partial f_1}{\partial x_2}\delta x_2 + \ldots + \frac{\partial f_1}{\partial x_6}\delta x_6$$

$$\delta y_2 = \frac{\partial f_2}{\partial x_1}\delta x_1 + \frac{\partial f_2}{\partial x_2}\delta x_2 + \ldots + \frac{\partial f_2}{\partial x_6}\delta x_6$$

•

•

$$\delta y_6 = \frac{\partial f_6}{\partial x_1}\delta x_1 + \frac{\partial f_6}{\partial x_2}\delta x_2 + \ldots + \frac{\partial f_6}{\partial x_6}\delta x_6$$

In vector notation: $\delta Y = J(X)\delta X$

Where $J(X)$ is jacobian.

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_6} \\ . & . & . & . \\ . & . & . & . \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix}$$

6X6

Now

$$\underbrace{\lim}_{\delta t \to 0} \frac{\delta Y}{\delta t} = \underbrace{\lim}_{\delta t \to 0} J(X) \frac{\delta X}{\delta t}$$

$$\frac{dY}{dt} = J(X) \frac{dX}{dt}$$

$$\dot{Y} = J(X)\dot{X}$$

In Robotics,

$$V = J(\Theta)\dot{\Theta}$$

Cartesian Velocity

Jacobian

Joint Velocity

Now

$$\underbrace{\lim}_{\delta t \to 0} \frac{\delta Y}{\delta t} = \underbrace{\lim}_{\delta t \to 0} J(X) \frac{\delta X}{\delta t}$$

$$\dot{Y} = J(X)\dot{X}$$

In Robotics,

$$V = J(\Theta)\dot{\Theta}$$

$$J^{-1}(\Theta)V = J^{-1}(\Theta)J(\Theta)\dot{\Theta}$$

$$J^{-1}(\Theta)V = \dot{\Theta}$$

Two DoF Serial Manipulator

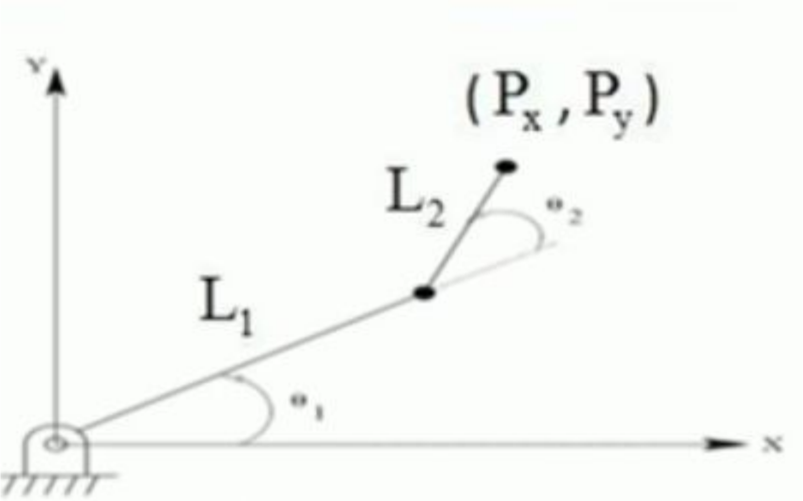
$$J^{-1}(\Theta) = \frac{\text{adj J}(\Theta)}{|J(\Theta)|}$$

Has to be non zero

$$|J(\Theta)| = 0$$

To check singularity of a manipulator

Two DoF serial manipulator



$$P_x = L_1 \cos\Theta_1 + L_2 \cos(\Theta_1 + \Theta_2)$$

$$\frac{\partial P_x}{\partial \Theta_1} = -L_1 \sin\Theta_1 - L_2 \sin(\Theta_1 + \Theta_2)$$

$$\frac{\partial P_x}{\partial \Theta_2} = -L_2 \sin(\Theta_1 + \Theta_2)$$

$$P_y = L_1 \sin\Theta_1 + L_2 \sin(\Theta_1 + \Theta_2)$$

$$\frac{\partial P_y}{\partial \Theta_1} = L_1 \cos\Theta_1 + L_2 \cos(\Theta_1 + \Theta_2)$$

$$\frac{\partial P_x}{\partial \Theta_2} = L_2 \cos(\Theta_1 + \Theta_2)$$

$$J(\Theta_1, \Theta_2) = \begin{bmatrix} \frac{\partial P_x}{\partial \Theta_1} & \frac{\partial P_x}{\partial \Theta_2} \\ \frac{\partial P_y}{\partial \Theta_1} & \frac{\partial P_y}{\partial \Theta_2} \end{bmatrix}$$

Jacobian

$$J(\Theta) = \begin{bmatrix} -L_1S_1 - L_2S_{12} & -L_2S_{12} \\ L_1C_1 + L_2C_{12} & L_2C_{12} \end{bmatrix}$$

Now $\dot{\Theta} = J^{-1}(\Theta)V$

$J^{-1}(\Theta)$ Should exist, that is, $|J(\Theta)| \neq 0$

$$|J(\Theta)| = -L_1L_2S_1C_{12} - L_2^2S_{12}C_{12} + L_1L_2S_{12}C_1 + L_2^2S_{12}C_{12}$$

$$= L_1L_2\sin(\Theta_1 + \Theta_2 - \Theta_1)$$

$$= L_1L_2\sin(\Theta_2)$$

For singularity Checking

$$|J(\Theta)| = 0$$

$$\implies L_1 L_2 S_2 = 0$$

Now

$$L_1 \neq 0,\ L_2 \neq 0$$

$$S_2 = 0$$

So,

$$\sin \Theta_2 = 0$$

$$\Theta_2 = 0^o \; or \; 180^o$$

When

$$\Theta_2 = 0^o; \longrightarrow \text{Fully- Stretched}$$

$$\Theta_2 = 180^o; \longrightarrow \text{Folded-back situation}$$