Labor Markets during Pandemics

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Abstract

The COVID-19 pandemic has killed millions across the globe and government responses have led to tens of millions of jobs lost. This paper combines the SIR epidemic model with a frictional labor market to examine the interaction between infection, wages and unemployment. Wages fall during the early phases of the pandemic, and then rise as the pandemic progresses. The unemployment rate increases among those not yet infected, decreases among those recovered and increases overall.

The labor market is not efficient during the pandemic. Optimal policies show that it is optimal to shut down businesses and impose a quarantine before the pandemic peaks. A quarantine itself is not enough, however, and must be complemented by additional policies. The policies are not unique and include a Pigouvian "infection tax" on those infected, a tax on susceptible individuals, higher unemployment benefits and a tax on vacancy creation. The optimal policies can reduce the fraction of people infected by about 22 percentage points.

1 Introduction

As the COVID-19 pandemic spreads through the world, it became clear that the nature and scale of the pandemic crisis is unprecedented in modern economic history. By the end of May 2021, more than 175 million cases have been confirmed throughout the world and more than 3.7 million people have died. The impact on the economy has been fast and deep: the U.S. unemployment rate soared to 14.7 percent in April 2020, up from 4.4 percent about a month earlier. While the pandemic affects almost every aspect of economic activity, labor markets are especially exposed. Labor market activities involve

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social interactions at the workplace, commuting and other related activities, directly contributing to the spread of the infection. At the same time, the pandemic can also reduce the willingness to participate in the labor market if people are worried about getting infected, disrupt the existing labor market mechanisms and change the dynamics of wages and unemployment. These issues, then, call for an understanding of optimal labor market policy responses.

To study the effects a pandemic can have in an economy, a standard Mortensen-Pissarides (Pissarides (1985) Mortensen and Pissarides (1994)) model of a frictional labor market is combined with a canonical epidemiological, SIR, model (Kermack and McKendrick (1927)). There are two key ingredients in the model. First, as is standard in the SIR models, the probability of getting infected depends on the number of infected people in the economy. Second, the probability of transmitting an infection is higher if one participates in the labor market activities. Individuals take into account that they have a higher chance of getting infected as employed when making their decisions, and this leads to novel implications about the dynamics of wages and unemployment during the pandemic. On the other hand, individuals do not take into account that they may increase the infection rate of others, and this creates, temporarily, a negative externality on others.

We start by characterizing the equilibrium in the labor market during a pandemic in the absence of any government intervention. We find that the labor market during the pandemic temporarily separates into two: a labor market for the people that have already recovered, and a labor market for those that have not yet been infected and are susceptible to the infection. Each of the two markets experiences a different sequence of wages and unemployment rates. The effect on wages for susceptible workers is non-monotonic, with wages first falling and then rising as the pandemic progresses. On one hand, workers are wary of becoming infected while employed, leading to increased reservation wages. Moreover, as more people become infected this effect gets stronger. On the other hand, the value of a match declines and reaches a minimum when the probability of becoming infected is highest. At the beginning of the pandemic wages fall as the job value effect dominates. As the pandemic progresses, the reservation wage effect dominates and wages rise.

The overall unemployment rate rises and labor market tightness (vacancies to unemployment ratio) decreases as the pandemic takes hold. In the absence of a government intervention, the unemployment rate at the peak of the pandemic is around 9% in our baseline calibration. This result is driven by the labor market for the susceptible workers who are less willing to participate in the labor market, and are also more costly for the potential employers because of a shorter expected tenure and higher cost. The unemployment rate for recovered workers, on the other hand, falls. This is because of a selection effect: employed people are more likely to get infected and so enter the pool of recovered people at a higher rate than the unemployed. We also investigate a variant where wages are rigid and do not move during the pandemic. If wages are rigid unemployment increases less during the pandemic, increasing the severity of the externality of the infection propagation. Flexible wages thus dominate rigid wages not only in terms of labor market efficiency, but also in terms of its effect on the pandemic spread.

The infection externality creates a wedge between the private and the social value of a labor market match. We allow the social planner is able to move people out of employment, essentially shutting down businesses and quarantining workers. Quarantine has an important role in our scenario, because, in its absence, the social value of the match may become negative when the pandemic peaks, despite the fact that the private value is still positive and firms would like to create more vacancies. It is then optimal for the planner to impose a quarantine and destroy labor market matches up to a point where the social value of a match is zero. In our baseline calibration the social planner does so only once, at the onset of the pandemic, and reallocates about 30% of the employed people out of employment. This increases the unemployment rate to 40% and saves a substantial number of lives: the fraction of people infected or dead is reduced by around 22 percent under quarantine, saving approximately 714,700 lives. Quarantine itself is, however, not enough to implement the efficient allocation. Even when the value of a match is positive, vacancies are created at a rate that is too high. Additional tools are still needed to restore efficiency, even if their role is smaller under quarantine than if the planner is not allowed to quarantine people, a case which we also consider.

What additional tools are needed for efficiency? We find that there is no single answer to this question. A vast array of policies can restore efficiency and all those policies are therefore, within the context of the model, equivalent. Conceptually the most straightforward policy is a Pigouvian "correction" tax: a tax imposed on those that create the external cost, that is, the infected individuals. The infection tax is astronomical at the beginning of the infection, around 100-650 times weekly wages depending on the scenario. This is not surprising, since the cost of spreading the pandemic at the beginning is the largest: the first person infected, for example, makes the difference between the whole pandemic and no pandemic at all. The infection tax does not converge to zero even in the long run, because it is proportional to the fraction of susceptible individuals, and this fraction stays positive.

The infection tax can perhaps be seen as unjust for reasons that are beyond the scope of our model. Fortunately, other policies that are equivalent to the infection tax evade those problems. Their common feature is that they shift the expected tax burden to an earlier susceptible stage. The most obvious policy is a tax on susceptibles, differentiated according to the employment status, and being larger for the employed workers, who are more likely to be infected. There are two ways to reinterpret the tax on susceptibles. The first one is that, by being higher for the employed than for the unemployed, it discourages workers from searching for jobs. We show that this can be achieved directly by a simple temporary increase in unemployment benefits during the pandemic. The optimal increase in unemployment benefits is substantial: they increase from a steady-state 71% of wages to 95% under quarantine and to 150% without quarantine. They peak with the pandemic and then slowly revert to the steady state value. Yet another way to reinterpret the tax on susceptibles is that it aims to reduce the equilibrium vacancy creation. We again show that this can be achieved more directly by imposing a tax on vacancy creation that increases during the pandemic and then converges back to zero.

In addition to our baseline calibration which, in our view, corresponds to the perception of the COVID-19 pandemic in the spring of 2020, we also consider an alternative calibration that takes into account the realized path of the pandemic up to April 2021 and is calibrated to replicate the time series of infected individuals. The exercise is useful in assessing the U.S. government policies and comparing them to the optimal ones. Our main findings are two. First, while the U.S. unemployment rate peaked rapidly in April 2020 to 14.7 percent, the optimal unemployment rate exhibits only a gradual increase to $7-12.3$ percent, reaching its peak only in April 2021. While the initial phase of the pandemic was characterized by a rapid spread of the infection, the level of the infection was too low to justify such a substantial disruption of the labor market. Second, the 600 USD temporary increase in unemployment benefits under the CARES Act is too large relative to the optimal value of 200-270 USD per week; the second temporary increase under the Consolidated Appropriations Act in December 2020 is, on the other hand, much closer to the optimal level.

2 Related Literature on Pandemics

The progression of an epidemic has been typically analyzed by means of epidemiology models, typically a variant of a SIR model (Kermack and McKendrick (1927)), that characterizes the dynamics of people that are susceptible to an infection, infected, and recovered. The epidemiology models are useful for tracking out pandemics for a given set of transition probabilities, but are, by themselves, not useful for studying the mutual interaction between epidemics and the economy. They are also not a useful framework for studying optimal policy responses because of their reduced form nature (Lucas (1976)).

This project joins a recent pool of literature that tries to understand and quantify the forces present in the SIR model (or the SEIR model, which adds an exposed stage), and to embed them within an economic framework. Atkeson (2021) and Fernández-Villaverde and Jones (2020) solve for various scenarios of the COVID-19 pandemic within a canonical SIR model. Alvarez et al. (2021) and Piguillem and Shi (2020) study optimal lockdowns, while Berger et al. (2021) study optimal testing and quarantines, within essentially the same framework of a standard SIR model. However, none of those papers explicitly models labor markets during the pandemic. Labor market is explicitly studied by Jackson and Ortego-Marti (2021), who extend our framework by incorporating human capital losses from unemployment and show that the pandemic then leads to a sizeable reduction of total factor productivity.

Eichenbaum et al. (2021) merge the SIR model with a dynamic representative agent framework study optimal policy responses to a pandemic. Like in our project, one of the central features of their model is an externality generated by individual behaviour. In their case it is both higher labor supply and higher consumption that increase the probability of spreading the infection. Brotherhood et al. (2020) focus on the role of age heteorogeneity in a behavioral SIR model, and study age-specific policies. Their model also incorporates uncertainty about one's health status. Glover et al. (2020) show how the epidemics can bring about intergenerational conflicts and new trade-offs for the government. In contrast to those papers, the focus of this paper is on the behaviour of labor markets and its role in spreading (or slowing down) the pandemic, and on labor market policies.

Literature that uses a search and matching framework to provide insights into a pandemic is limited. Garibaldi et al. (2020) explicitly model social contacts to show how forward looking behavior can affect transitions and the consequent spread of an epidemic, and characterize the externalities arising from individual choice of social contacts. Gregory et al. (2020) use directed search to show conditions under which a drop in productivity, for example due to a lockdown, leads to a protracted ("L-shaped") recession rather than a short-lived ("V-shaped") recession. Unlike us, they do not model the epidemic itself.

Guerrieri et al. (2020) show how a negative supply shock can create negative demand shocks of even larger magnitudes if there is a sufficient heterogeneity across sectors of the economy, just like a heterogeneity in the labor market status is a key aspect of our model. However, their mechanisms are quite different from ours, and they do not explicitly model the propagation of epidemics itself. Naturally, some of the policy implications are different as well: while in our model it is efficient to move people out of employment to reduce the externality, in their model it is optimal to subsidize businesses to reduce exits and stimulate demand. It is, of course, natural, that both mechanisms are in place at some point during the pandemic, perhaps with different intensities at different stages.

3 An Accounting Framework

We first describe the flows in the economy, both across health status and across employment status. We will later develop a model that endogenizes those flows, and the associated transition probabilities. The initial population is normalized to one. The population of living people at time t is denoted by N_t and the population that has died by the end of period t is denoted by D_t. Since people can be either dead or alive, the sum of both is equal to one:

$$N_t + D_t = 1.$$

The living population is categorized in two dimensions. Along the labor market dimension, people can be either employed (E) or unemployed (U). The other one is health status: people can be susceptible to infection but not yet infected (S), infected (I), or recovered (R). Recovered people are no longer susceptible to repeated infection. The total number of unemployed in period t, U_t, is the sum of unemployment across the health categories, and E_t is the sum of the employment across the health categories:

$$U_t = US_t + UI_t + UR_t$$
$$E_t = ES_t + EI_t + ER_t,$$

where US_t denotes the stock of susceptible unemployed in the beginning of period t, UI_t denotes the stock of infected unemployed in the beginning of period t, and UR_t denotes the stock of recovered unemployed in the beginning of period t. Similarly, ES_t, EI_t and ER_t are employed susceptible, employed infected, and employed recovered. Aggregating across health status, the total number of susceptible, infected and recovered are given as:

$$S_t = US_t + ES_t$$
$$I_t = UI_t + EI_t$$
$$R_t = UR_t + ER_t.$$

The total living population consists of only unemployed and employed or, alternatively, of only the susceptible, infected and recovered, and so the following equalities hold:

$$N_t = U_t + E_t = S_t + U_t + R_t.$$

Health transition probabilities. The rate at which susceptible people get infected depends on their labor market status. Employed individuals have probability π_t^{EI} of getting infected in the course of their employment, while unemployed individuals have a probability π_t^{UI} of getting infected by interacting with other people. These probabilities reflect the number of social interactions. The underlying assumption is that the unemployed interact less with other people because they do not go to work, and their probability of getting infected is lower, therefore, $\pi_t^{UI} < \pi_t^{EI}$.

Once people are infected they recover with probability π_R. With probability π_D, they die from the infection. With the remaining probability $1 - \pi_R - \pi_D$ they continue being infected next period. Both probabilities are assumed to be independent of employment status.

Employment transition probabilities. The dynamics of the labor market are driven by the job finding and job separation probabilities. The probability of finding a job depends on health status. Susceptible people find a job with probability p_t^S while recovered people find a job with probability p_t^R, and infected people cannot look for a job at all. If they get infected while being unemployed, they will stay unemployed until they recover or die. If a susceptible unemployed finds a job and gets infected at the same time, the job remains unfilled and the searcher remains unemployed.

On the other hand, the probability with which the employer and employee separate, λ, is independent of the health status of the employee. That is, employees cannot be fired just because they get infected. If a susceptible employed gets infected, they cannot be fired and remains employed, but is placed on temporary sick leave and is unproductive.

There are six types of living people in the economy, given by the combination of the three health states of living people and the two employment states. The transitional probabilities above determine the laws of motion for each category. For employed people, the inflows and outflows are as follows:

$$ES_{t+1} = (1-\lambda)(1-\pi_t^{EI})ES_t + p_t^S(1-\pi_t^{UI})US_t \tag{1a}$$

$$EI_{t+1} = (1-\lambda)(1-\pi_R-\pi_D)EI_t + (1-\lambda)\pi_t^{EI}ES_t \tag{1b}$$

$$ER_{t+1} = (1-\lambda)ER_t + (1-\lambda)\pi_R EI_t + p_t^R UR_t. \tag{1c}$$

The number of employed susceptible next period is reduced because a fraction $(1-\lambda)\pi_t^{EI}$ is not separated from the employer but gets infected, a fraction $\lambda(1-\pi_t^{EI})$ does not get infected but gets separated, and a fraction $\lambda\pi_t^{EI}$ gets both separated and infected. Thus, only a fraction $(1-\lambda)(1-\pi_t^{EI})$ remains. On the other hand, a fraction $p^S(1-\pi_t^{UI})$ of unemployed susceptible find a job and do not get infected, becoming employed and susceptible. The number of employed infected is reduced because a fraction $(1-\lambda)\pi_R$ recovers while keeping the job, a fraction $\lambda(1-\pi_R-\pi_D)$ loses their job and continues being infected (i.e. does not die or recover), $\lambda\pi_R$ gets separated and recover, and a fraction π_D die. On the other hand, the stock is increased by a fraction $(1-\lambda)\pi_t^{EI}$ of susceptible employees who become infected. Finally, the number of employed recovered decreases because a fraction λ lose their job, but a fraction $(1-\lambda)\pi_R$ of infected employees recover and stay on the job,and a fraction p_t^R of the unemployed and recovered find a job.

The dynamics of the unemployed categories is determined by the following flows:

$$US_{t+1} = (1-p_t^S)(1-\pi_t^{UI})US_t + \lambda(1-\pi_t^{EI})ES_t \tag{2a}$$

$$UI_{t+1} = (1-\pi_R-\pi_D)UI_t + \pi_t^{UI}US_t + \lambda\pi_t^{EI}ES_t + \lambda(1-\pi_R-\pi_D)EI_t \tag{2b}$$

$$UR_{t+1} = (1-p_t^R)UR_t + \pi_R UI_t + \lambda\pi_R EI_t + \lambda ER_t \tag{2c}$$

The pool of unemployed susceptible decreases because a fraction $(1-p_t^S)\pi_t^{UI}$ do not find a job but become infected, a fraction $p_t^S\pi_t^{UI}$ find a job and become infected, and a fraction $p_t^S(1-\pi_t^{UI})$ find a job but do not get infected. A fraction $\lambda(1-\pi_t^{EI})$ of employed susceptible workers get separated without catching the virus and adds to pool. The unemployed infected are not looking for a job and so the pool gets reduced only by recovering (fraction π_R) or by dying (fraction π_D). Increasing the pool of unemployed infected are susceptible unemployed who get infected (regardless of their job market outcome) with probability π^{UI} and susceptible employed who get infected and also happen to get fired with probability $\lambda\pi_t^{EI}$. The infected employed can also become infected unemployed by losing their job while avoiding dying or recovering with probability $\lambda(1-\pi_R-\pi_D)$. Finally, the stock of unemployed recovered is reduced because a fraction p_t^R find a job, and increases because a fraction π_R of unemployed infected recover, a fraction $\lambda\pi_R$ of employed infected are fired but recover at the same time, and a fraction λ of employed recovered lose their job.

It follows that the number of dead people keeps growing, with the increments being a fraction π_D of the infected people:

$$D_{t+1} = D_t + \pi_D(EI_t + UI_t).$$

Aggregating across health status gives the law of motion for the total number of employed and unemployed:

$$\begin{aligned}
E_{t+1} &= (1-\lambda)\left(E_t - \pi_D EI_t\right) + p_t^S US_t + p_t^R UR_t \\
U_{t+1} &= (1-p_t^S)US_t + (1-p_t^R)UR_t + (1-\pi_D)UI_t + \lambda(E_t - \pi_D EI_t)
\end{aligned}$$

The evolution of aggregate employment and unemployment cannot be expressed in terms of the labor market aggregates themselves for three reasons. First, the number of people who leave the labor market by kicking the bucket depends only on the number of infected people. Second, infected people cannot look for a job, and so the rate at which people transition to the employment status depends on the number of infected unemployed. Third, job finding probabilities also depend on whether people are susceptible or recovered.

Aggregating across the employment status yields

$$
\begin{aligned}
S_{t+1} &= (1 - \pi_t^{UI})S_t - (\pi_t^{EI} - \pi_t^{UI})ES_t \\
I_{t+1} &= (1 - \pi_R - \pi_D)I_t + \pi_t^{UI}S_t + (\pi_t^{EI} - \pi_t^{UI})ES_t \\
R_{t+1} &= R_t + \pi_R I_t.
\end{aligned}
$$

The aggregation is again only partial, this time because the unemployed and employed have different probabilities of being infected. Only when $\pi_t^{UI} = \pi_t^{EI}$ can the law of motion be expressed entirely in terms of health status aggregates S, I and R.

4 The Model

The goal now is to provide a theory of how some of the transition probabilities in the above accounting framework are determined in equilibrium. In particular, to characterize the job hiring probabilities p_t^S and p_t^R, and their dynamics during the transition and also endogenizing the infection probabilities π_t^{EI} and π_t^{UI}. The theory will then be used to characterize the dynamics of wages over the transition, welfare consequences of the pandemic, and optimal policies.

4.1 Labor Market

The labor market is frictional. Assuming virus testing, the labor market for the susceptible people can be separated from the labor market for the recovered people. It is beneficial for the recovered people to signal that they have already recovered because they no longer face any potential disruptions to their productivity. The key assumption is that susceptible people cannot pretend to have recovered. It is reasonable to assume that it would be in the overall public interest to separate the two groups. The assumption also simplifies the model substantially, because potential employers do not have to solve a screening problem that would otherwise arise.

Firms create vacancies in each of the two labor markets, VS_t and VR_t. The number of new jobs created among the susceptible and recovered, HS_t and HR_t, depends on the total number of unemployed and the number of vacancies in each labor market, and is given by a matching function m:

$$HS_t = m(US_t, VS_t)$$
$$HR_t = m(UR_t, VR_t).$$

The function m is common in both labor markets with constant returns to scale, differentiable, increasing and concave in both arguments.

The labor market tightness will differ across both labor markets. Labor market tightness in the susceptible labor market is given by $\theta_t^S = VS_t/US_t$, and in the recovered labor market by $\theta_t^R = VR_t/UR_t$. The job finding probabilities in both markets depend only on their respective labor market tightness:

$$p_t^S = \frac{HS_t}{US_t} = p(\theta_t^S) \tag{3a}$$
$$p_t^R = \frac{HR_t}{UR_t} = p(\theta_t^R), \tag{3b}$$

where $p(\theta) = m(1, \theta)$. The probabilities of filling a vacancy in each market are $q_t^S = p(\theta_t^S)/\theta_t^S$ and $q_t^R = p(\theta_t^R)/\theta_t^R$. Labor market tightness, and the associated job hiring probabilities, will only differ during the outbreak of the epidemic. Before and after the epidemic, they will be the same because the matching function is the same.

4.2 Infection Propagation

The probability that a worker gets infected is endogenous and depends on interactions with other people. Assume that if there are more infected workers around, the probability of getting infected increases.[1] The probability that employed and unemployed workers get infected is proportional to the number of currently infected people:

$$\pi_t^{EI} = \frac{EI_t}{ES_t} = s^E I_t \tag{4a}$$
$$\pi_t^{UI} = \frac{UI_t}{US_t} = s^U I_t. \tag{4b}$$

[1] It is reasonable to conjecture that, once a worker is revealed to be infected, they are quarantined and no longer interact with other workers. The incubation period, during which a worker interacts with other workers, is not modeled explicitly for simplicity.

The underlying assumption is that both employed and unemployed interact with all infected people, although at different rates, given by constants s^E and s^U.[2]

4.3 Workers

A job is formed by matching a firm with a worker. A match between susceptible or recovered workers and a firm produces y units of output per period, where y is constant and common across all matches. Once a match is formed, the worker and the firm bargain over the wage, described in detail below. All unemployed workers receive non-monetary benefits b per period, regardless of their health status. Infected workers in addition incur disease cost c that represents pain and additional non-monetary costs associated with the infection.

The value functions in the recovered, infected and susceptible stage of employed workers are denoted K_t^{ER}, K_t^{EI} and K_t^{ES}. The corresponding value functions for the the unemployed workers are K_t^{UR}, K_t^{UI} and K_t^{US}.

Recovered workers. We start by describing the recovered workers. Since the recovered workers cannot revert back to the infected or susceptible stage, and are immune to the infection, they face a standard labor search problem. The value function of the recovered employed is

$$K_t^{ER} = w_t^R + \beta \left[\lambda K_{t+1}^{UR} + (1-\lambda) K_{t+1}^{ER} \right], \tag{5}$$

while the value of being recovered unemployed is

$$K_t^{UR} = b + \beta \left[p_{t+1}^R K_{t+1}^{ER} + (1 - p_{t+1}^R) K_{t+1}^{UR} \right]. \tag{6}$$

Infected workers. Employed infected workers are unproductive until they recover. As mentioned previously, the worker cannot be fired just because of being infected. Separations of infected workers happen only for other reasons, with probability λ, or because the worker dies of the infection, with probability π_D. During sickness, a worker collects a fraction of the wage, that is negotiated with the employer. If the worker recovers, productivity is again y. Infected employees also incur a utility cost c. Their value function

[2] Similar results are obtained where the employed people interact more with other employed people, and their infection probability depends on EI instead of I.

satisfies

$$K_t^{EI} = w_t^I - c + \beta\lambda \left[\pi_R K_{t+1}^{UR} + (1 - \pi_R - \pi_D) K_{t+1}^{UI}\right] \\ + \beta(1-\lambda) \left[\pi_R K_{t+1}^{ER} + (1 - \pi_R - \pi_D) K_{t+1}^{EI}\right] + \pi_D D, \tag{7}$$

where D is the value of death. The infected unemployed incur disease cost c but also face different transition probabilities. In particular they can recover or die, but cannot become employed. Their value function is:

$$K_t^{UI} = b - c + \beta \left[\pi_R K_{t+1}^{UR} + \pi_D D + (1 - \pi_R - \pi_D) K_{t+1}^{UI}\right]. \tag{8}$$

Susceptible workers. Susceptible workers are as productive as recovered workers, but face the possibility of becoming infected. The value function of susceptible employees solves

$$K_t^{ES} = w_t^S + \beta \left\{\lambda \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{US} + \pi_{t+1}^{EI} K_{t+1}^{UI}\right] + (1-\lambda) \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{ES} + \pi_{t+1}^{EI} K_{t+1}^{EI}\right]\right\}. \tag{9}$$

The unemployed susceptible search for a new job during the period. At the end of the period, it is revealed whether the worker has found a job and was or was not infected during the period. A worker who has found a match but is infected at the same time is prohibited from signing a contract and the match is not formed. The wage, determined below, is then formed only for workers that are susceptible but will be able to work during the first period. Their value function is

$$K_t^{US} = b + \beta \left[p_{t+1}^S (1 - \pi_{t+1}^{UI}) K_{t+1}^{ES} + \pi_{t+1}^{UI} K_{t+1}^{UI} + (1 - p_{t+1}^S)(1 - \pi_{t+1}^{UI}) K_{t+1}^{US}\right]. \tag{10}$$

4.4 Firms

Firms can post vacancies in the labor market for susceptible workers or in the labor market for recovered workers. Firms in the susceptible market take into account that a susceptible worker may find a job and get infected, either immediately, in which case a match is not formed, or later, after employment. To characterize the firm's problem, consider the value of a filled job conditional on health status of the worker, given by J_t^R,

J_t^I and J_t^S. The value of having a recovered worker is

$$J_t^R = y - w_t^R + \beta \left[\lambda V_{t+1} + (1-\lambda) J_{t+1}^R \right], \tag{11}$$

where V_{t+1}^R is the value of a vacancy in period $t+1$. The value of having an infected worker is

$$J_t^I = -w_t^I + \beta \left\{ [\pi_D + \lambda(1-\pi_D)] V_{t+1} + (1-\lambda)\pi_R J_{t+1}^R + (1-\lambda)(1-\pi_R-\pi_D) J_{t+1}^I \right\}, \tag{12}$$

where the current period payoff is negative, because the firm has to pay the negotiated wage to the unproductive worker. Finally, the value of having a susceptible worker is

$$J_t^S = y - w_t^S + \beta \left\{ \lambda V_{t+1} + (1-\lambda) \left[\pi_{t+1}^{EI} J_{t+1}^I + (1-\pi_{t+1}^{EI}) J_{t+1}^S \right] \right\}, \tag{13}$$

Free entry. Posting a vacancy has a cost k irrespective of the market and tirms are free to enter any of the two markets anytime. This means that the value of a vacancy V_t equals zero. That is, in equilibrium, the cost of creating a vacancy equals the expected benefits from doing so,

$$\beta q_{t+1}^R J_{t+1}^R = k \tag{14}$$

$$\beta q_{t+1}^S (1-\pi_{t+1}^{UI}) J_{t+1}^S = k, \tag{15}$$

4.5 Wage determination

The wages in both markets are determined as a result of Nash bargaining. We assume that there are no long-term contracts and the wage is renegotiated continuously at the beginning of the period, including periods when the worker is infected and is on sick leave. Both parties know the health status of the worker. Note that we also solve the model for rigid wages in Section 7.2 .

The wage in the three stages is a solution to

$$w_t^S = \arg\max \left(J_t^S\right)^{1-\phi} \left(K_t^{ES} - K_t^{US}\right)^{\phi} \tag{16}$$

$$w_t^I = \arg\max \left(J_t^I\right)^{1-\phi} \left(K_t^{EI} - K_t^{UI}\right)^{\phi} \tag{17}$$

$$w_t^R = \arg\max \left(J_t^R\right)^{1-\phi} \left(K_t^{ER} - K_t^{UR}\right)^{\phi}. \tag{18}$$

where $\phi \in [0,1]$ is the bargaining weight of the worker. The bargaining weight is identical in all markets, and the bargaining problem already incorporates the result that the value of vacancies, the outside option for the firm, is equal to zero. Since the value functions K^{UI}, K^{US} and K^{UR} are all independent of the bargained wage and there is a fixed surplus to be divided, the first-roder conditions imply a standard result that the ratio of the worker's surplus and the firm surplus is equal to $\phi/(1-\phi)$ in all stages.

4.6 Equilibrium

The initial conditions are given by (US_0, ES_0, UI_0, EI_0), where the values of UI_0 and EI_0 can be thought of as being small, and no recovered people, $UR_0 = ER_0 = 0$. The equilibrium is given by type allocations $\{UR_t, ER_t, US_t, ES_t, UI_t, EI_t, D_t\}$, worker's value functions $\{K_t^{ER}, K_t^{UR}, K_t^{ES}, K_t^{US}, K_t^{EI}, K_t^{UI}\}$, firm value functions $\{J_t^R, J_t^I, J_t^S\}$, wages $\{w_t^R, w_t^S\}$, labor market tightness $\{\theta_t^R, \theta_t^S\}$ such that i) worker value functions satisfy (5)-(10), ii) firm value functions satisfy (11)-(13), iii) free entry conditions (14) and (15) hold, iv) wages solve (16)- (18), v) aggregates evolve according to (1) and (2), and vi) job finding and infection probabilities are given by (3) and (4).

4.7 Solving the Model

Since individuals always proceed from being susceptible to infected to recovered, the model can be solved recursively, starting from the recovered stage, then proceeding to the infected stage, and finally to the susceptible stage.

Recovered stage. The recovered stage has a time invariant solution. Market tightness, the wage, and all value functions are independent of time and identical to their value before the pandemic started. The dynamics of the recovered stage is reflected only in the unemployment rate. The solution is a textbook one and is provided mainly to be compared to the solution in other stages. Define the total surplus of the match in the recovered stage as $F^R = J^R + K^{ER} - K^{UR}$. The value of the match is independent of the wage. Then $K^{ER} - K^{UR} = \phi F^R$ and $J^R = (1-\phi)F^R$. Adding together the value functions in the recovered stage, the value of the surplus is,

$$F^R = \frac{y-b}{1-\beta[1-\lambda-\phi p(\theta^R)]},$$

where the relationship between the probability of finding a job and the market tightness in the recovered stage is made explicit. Combining with the free entry condition (14) yields the textbook equation in market tightness θ^R:

$$\frac{y-b}{k} = \frac{1/\beta - 1 + \lambda + \phi p(\theta^R)}{(1-\phi)q(\theta^R)}.$$

To characterize the equilibrium wage, define the reservation wage as a wage that, if paid in the current period (not permanently), makes the worker indifferent between accepting and not accepting the job offer. It is defined by $K^{ER}(\overline{w}^R) = K^{UR}$, therefore,

$$\overline{w}^R = b + \beta p^R \phi F^R - \beta(1-\lambda)\phi F^R. \tag{19}$$

The reservation wage increases above the unemployment benefits because there is an option, with probability p^R, of waiting until next period, negotiating, and getting a fraction of the surplus ϕF^R tomorrow. On the other hand, the reservation wage decreases because there is an option of continuing on the job tomorrow, with probability $1-\lambda$.[3] The equilibrium wage w^R is then a sum of the reservation wage plus a fraction ϕ of the surplus of the match:

$$w^R = \overline{w}^R + \phi F^R. \tag{20}$$

The worker thus gets the reservation wage plus a share of the match, given by the bargaining power. Replacing the reservation wage and the value of the surplus yields the equilibrium wage as a direct function of market tightness,

$$w^R = (1-\phi)b + \phi(y + k\theta^R)$$

[3] While the expression can be further simplified, it is kept in the current form to facilitate a comparison to the reservation wages in other stages.

As usual, higher market tightness means that workers are more likely to find a job and can command a higher wage.

Infected stage Once the solution to the problem in the recovered stage is known, the infected stage problem can be solved. Since the probabilities of recovery and death are constant over time, by assumption, the infected stage has a time invariant solution as well. The value of a match in the infected stage is $F^I = J^I + K^{EI} - K^{UI}$. The bargaining protocol again implies that it is divided according to the bargaining power. Since the problem is time invariant, F^I_t is also constant over time and is a solution to

$$F^I = \frac{-b + \beta(1-\lambda)\pi_R F^R}{1 - \beta(1-\lambda)(1-\pi_R-\pi_D)}.$$

The expression is different from F^R in one important aspect. The current surplus is $-b$ rather than $y - b$ because nothing is produced in the infected stage. Instead, the value of the match is given purely by its future surplus, once the worker recovers and becomes productive again. This is represented by the second term $\beta(1-\lambda)\pi_R F^R$, where the probability $(1-\lambda)\pi_R$ denotes the probability that the worker recovers and the match continues. The reservation wage in the infected stage again leaves the worker indifferent between employment and unemployment and is given by the following expression:

$$\overline{w}^I = b - \beta\pi_R(1-\lambda)\left(K^{ER} - K^{UR}\right).$$

The reservation wage is lower than the unemployment benefits because infected people are not able to look for a job directly and so must enter the recovered stage as unemployed. If they could, they would then be willing to take a wage cut, relative to benefits, in order to obtain a job now and enter the recovered stage as employed. The size of the cut, $\beta\pi_R(1-\lambda)\left(K^{ER} - K^{UR}\right)$, depends on the gain from being employed in the recovered stage, $K^{ER} - K^{UR}$, and the probability that the worker transits employed to the recovered stage, $\pi_R(1-\lambda)$.

Analogously to (20), the equilibrium wage w^I is a weighted average of the reservation wage $\overline{w}^I$, and the surplus of the match. The surplus of the match is now different, however. It is given by $\phi\beta(1-\lambda)\pi_R J^R$ and so the wage is

$$w^I = \phi\beta(1-\lambda)\pi_R J^R + (1-\phi)\overline{w}^I. \tag{21}$$

Combining both expressions, the equilibrium wage in the infected stage reduces to

$$w^I = (1 - \phi)b.$$

The future surplus $\beta(1 - \lambda)\pi_R J^R$ in (21) drives the wage of the infected above the reservation wage. On the other hand, the reservation wage is lower than unemployment benefits. In equilibrium, the second effect dominates and the wage is a fraction of unemployment benefits. The equilibrium wage in turn yields the equilibrium value of the firm J^I,

$$J^I = \frac{-(1 - \phi)b + \beta(1 - \lambda)\pi_R J^R}{1 - \beta(1 - \lambda)(1 - \pi_R - \pi_D)},$$

the value of an unemployed infected,

$$K^{UI} = \frac{b - c + \beta\pi_R K^{UR}}{1 - \beta(1 - \pi_R - \pi_D)},$$

and, finally, the value of infected employees:

$$K^{EI} = \frac{(1 - \phi)b - c + \beta\pi_R \left[\lambda K^{UR} + (1 - \lambda)K^{ER}\right] + \beta\lambda(1 - \pi_R - \pi_D)K^{UI}}{1 - \beta(1 - \lambda)(1 - \pi_R - \pi_D)}.$$

Susceptible stage. Unlike the last two stages, the problem in the susceptible stage does not have a time invariant solution because the probabilities of being infected are not constant over the pandemic. Again, define the value of a match in the susceptible stage by $F_t^S = J_t^S + K_t^{ES} - K_t^{US}$ and divide it between both parties according to their bargaining power, $K_t^{ES} - K_t^{US} = \phi F_t^S$ and $J_t^S = (1 - \phi)F_t^S$.

To characterize the law of motion for the value of the output of the match, note that the current value F_t^S depends not only on future match values F_{t+1}^S and F^I, but also on the utility loss that a susceptible person gets from getting infected

$$\Delta_{t+1}^S = K^{UI} - K_{t+1}^{US}.$$

By rearranging the value functions for the firm and the workers, the law of motion is given by

$$F_t^S = y - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) F_{t+1}^S\right] - \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) F_{t+1}^S \\ + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S \tag{22}$$

The law of motion for the loss from getting infected, Δ_t^S, is

$$\Delta_t^S = (1-\beta)K^{UI} - b - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S F_{t+1}^S - \Delta_{t+1}^S\right). \tag{23}$$

The loss from getting infected consists of several parts. First, there is the difference between the period utilities $(1-\beta)K^{UI} - b$, which is constant over time. Second, the loss in utility comes from the fact that an individual cannot get a job while being infected, while a susceptible worker gets a job with a probability $(1-\pi_{t+1}^{UI})p_{t+1}^S$, in which case the worker gets a share ϕ of the surplus. Finally, the worker might get infected tomorrow, in which case there is the additional loss Δ_{t+1}^S.

In addition to (22) and (23), the free entry condition (15) must hold as well, providing a third set of equations. Those equations are to be solved for equilibrium sequences $\{F_t^S, \Delta_t^S, \theta_t^S\}$. The system can be solved as follows. Using the above system of equations, given F_{t+1}^S, Δ_{t+1}^S and θ_{t+1}^S, solve for F_t^S and Δ_t^S. Then use the free entry condition to obtain θ_t^S. Iterate until the beginning. The system is started by assuming it is in steady state after some sufficiently distant date T.

The sequence of wages, $\{w_t^S\}$, can now be characterized. The reservation wages $\{\overline{w}_t^S\}$ are such that a currently susceptible worker is indifferent between working and not working, $K_t^{US} = K_t^{ES}(\overline{w}_t^S)$. Rearranging yields

$$\overline{w}_t^S = b + \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) F_{t+1}^S - \beta(1-\lambda)\phi\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) F_{t+1}^S\right] \\ - \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S. \tag{24}$$

The reservation wage is a product of several forces. The second and third term on the right-hand side are analogous to the second term on the right-hand side of the reservation wage in the recovered stage (19). An option of waiting, not getting infected and negotiating tomorrow increases the reservation wage, while an option of continuing on the job tomorrow decreases the reservation wage. A reasonable guess is that a person is more likely to be an employed susceptible tomorrow if they take the job today,

$(1-\lambda)(1-\pi^{EI}_{t+1}) > p^S_{t+1}(1-\pi^{UI}_{t+1})$: while the infection probability is somewhat higher on the job, the probability of losing the job λ is likely to be much smaller than the probability of finding a job p^S_{t+1}. Under those conditions, a higher value of a susceptible match F^S_{t+1} makes the workers more willing to accept the job and reduces the reservation wage. The last term in the reservation wage equation (24) is novel. It decreases the reservation wage, because accepting a job increases the probability of getting infected.[4] For example, if both infection probabilities get proportionally higher, the reservation wage will decrease. The equilibrium wage is then again equal to the reservation wage plus a fraction ϕ if the match:

$$w^S_t = \overline{w}^S_t + \phi F^S_t. \tag{25}$$

The equation (25) shows that the equilibrium wage is a product of two forces that will be under detailed scrutiny later, the reservation wage and the value of the match.

5 Efficient Allocations and Optimal Quarantine

The equilibrium of the model is not efficient, even when the Hosios condition holds, because of the externality coming from the propagation of the infection. We consider two versions of the planning problem. The first, more authoritative version, allows the planner to terminate job matches and move people from employment to unemployment in excess of what is dictated by the separation rate. The possibility of forcibly moving people out of employment and destroying labor market matches is what we call a quarantine. Correspondingly, we call the outcomes of this problem as *efficient allocations with quarantine*. The second version takes the separation rate λ as given and does not allow the planner to forcibly move individuals from employment to unemployment. The outcomes of the second version of the planning problem are called *efficient allocations without quarantine*. We note that a quarantine may or may not be efficient during a pandemic, depending on whether the social value of a match becomes negative or not. Absent a pandemic, however, both problems always coincide, because the equilibrium would be efficient and the planner would not want to voluntarily destroy any matches.

In the planning problem with quarantine, the social planner chooses the labor mar-

[4] The reservation wage increases because $\pi^{EI}_{t+1} - \pi^{UI}_{t+1} > 0$ and the Δ^S_{t+1} denotes the loss from getting infected, and so is negative.

ket flows $\{US_t, ES_t, UI_t, EI_t, UR_t, ER_t, D_t\}$, the number of vacancies $\{VS_t, VR_t\}$, and the number of quarantined people $\{Q_t\}$ to maximize the present value of resources,

$$\sum_{t=0}^{\infty} \beta^t \left[(ES_t + ER_t)y + (US_t + UI_t + UR_t)b + (UI_t + EI_t)(-c + \beta\pi_D D) - (VS_t + VR_t)k \right],$$

subject to the laws of motion (1b), (1c), (2b), (2c), and

$$ES_{t+1} = (1-\lambda)(1-\pi^{EI})ES_t + p^S(1-\pi^{UI})US_t - Q_t \tag{1a'}$$

$$US_{t+1} = (1-p^S)(1-\pi^{UI})US_t + \lambda(1-\pi^{EI})ES_t + Q_t, \tag{2a'}$$

where the probabilities of being hired are given by (3) and the infection probabilities are given by (4a) and (4b), and subject to the nonnegativity constraint $Q_t \geq 0$. Note that (1a′) and (2a′) modify their original counterparts (1a) and (2a) to take into account quarantined people.[5] In the planning problem without quarantine, the planner simply sets the number of quarantined people $\{Q_t\}$ to zero. Additional details of the solution to both planning problems are in Appendix A.[6]

Note that in both planning problems, the objective function is somewhat restrictive in that it assumes that the private and social value of one's life coincide. It is, of course, reasonable to argue that a loss of life has higher social cost, not internalized by individuals, as in, for example, Alvarez et al. (2021). The policies that solve our planning problem can then be seen as a lower bound on such policies when a life has larger cost.[7]

The efficient allocations can be characterized by the employment and unemployment flows, and labor market tightness. The recovered stage is efficient as long as the Hosios condition holds. The infected and susceptible stage are not efficient. Denoting μ_t^{UR} and μ_t^{ER} the social values of unemployment and employment in the recovered stage, it can be shown that they are equal to their equilibrium counterparts K^{UR} and $K^{ER} + J^R$, the sum of the firm value and the worker's value. Let μ_t^{UI} be the social value of unemployment in period t and μ_t^{EI} be the social value of employment in period t in the infected stage. The solution to the social planner's problem yields the following first-order conditions

[5]The social planner is not allowed to move people in the infected or recovered stage. This assumption is not restrictive, as the social planner has nothing to gain by doing so.

[6]There is one additional constraint: ES_{t+1} must be nonnegative and assume that this constraint does not bind. This will be the case in our quantitative simulations as well.

[7]See Hall et al. (2020) for calculations regarding the trade-off between lost consumption and lives.

in the infected stage:

$$\mu_t^{UI} = b - c + \beta\left[(1-\pi_R-\pi_D)\mu_{t+1}^{UI} + \pi_R\mu^{UR}\right] + \beta\pi_D D - \psi_t \tag{26}$$

$$\mu_t^{EI} = -c + \beta\lambda\left[(1-\pi_R-\pi_D)\mu_{t+1}^{UI} + \pi_R\mu_{t+1}^{UR}\right] + \beta(1-\lambda)\left[(1-\pi_R-\pi_D)\mu_{t+1}^{EI} + \pi_R\mu_{t+1}^{ER}\right] + \beta\pi_D D - \psi_t, \tag{27}$$

where ψ_t is the externality from the infected individuals, characterized below. The equilibrium counterparts of μ_t^{UI} and μ_t^{EI} are the private value of unemployment K_t^{UI} and private value of employment $K_t^{EI} + J_t^I$.[8] The externality ψ_t is given by

$$\psi_{t-1} = \beta S_t\Big[s^U p_t^S u_t^S(\mu_t^{ES}-\mu_t^{US}) + (1-\lambda)s^E(1-u_t^S)(\mu_t^{ES}-\mu_t^{EI}) + [s^U u_t^S + \lambda s^E(1-u_t^S)](\mu_t^{US}-\mu_t^{UI})\Big], \tag{28}$$

where μ_t^{ES} and μ_t^{US} are the social values of employment and unemployment in the susceptible stage, and u_t^S is the unemployment rate among the susceptible individuals. The externality is proportional to the number of suspected people, since they represent the pool of individuals that may be, directly or indirectly, affected by one's labor market behavior. The externality will therefore tend to be larger at the beginning of the epidemic when the number of susceptible individuals is the largest. All costs accrue only tomorrow and so are discounted.

The externality ψ_t further consists of three terms. They correspond to three types of external effects that an infected individual imposes on others: it reduces the effective job finding rate, generates a loss of value from employment, and generates a loss of value from unemployment. The first term $s^U p_t^S u_t^S(\mu_t^{ES}-\mu_t^{US})$ represents the reduction of the effective job finding rate. The effective job finding rate in the model is $p_t^S(1-\pi_t^{UI}) = p_t^S(1-s^U I_t)$, because an unemployed susceptible who becomes infected cannot accept a job, even if a match is found. A marginal increase of infected individuals decreases the job acceptance probability by $s^U p_t^S$. This value is multiplied by the fraction of unemployed people u_t^S and the corresponding loss of value $\mu_t^{ES} - \mu_t^{US}$. The second term $(1-\lambda)s^E(1-u_t^S)(\mu_t^{ES}-\mu_t^{EI})$ represents an increase in the probability that employed people do not lose job but get infected. Those people lose value $\mu_t^{ES} - \mu_t^{EI}$, and there

[8] Comparing conditions (26) and (27) to (8) and the sum of (7) and (12) shows that the difference is exactly the value of the externality ψ_t.

is $(1-\lambda)(1-u_t^S)$ of them. Finally, the third term $[s^U u_t^S + \lambda s^E(1-u_t^S)](\mu_t^{US} - \mu_t^{UI})$ represents the marginal increase in the loss for unemployed people. They are unemployed tomorrow either because they did not find a job, or because they lost a job, and each of them loses value $\mu_t^{US} - \mu_t^{UI}$.

While it can only be resolved quantitatively which of these terms are the most important, an educated guess can be made based on the number of people affected. The number of people affected by the reduction of the effective job finding rate is $p_t^S u_t^S$, and both of the terms are relatively small. The loss of value from unemployment likewise affects a relatively few people, because both u_t^S and $\lambda(1-u_t^S)$ are small. Thus, the most important term appears to be the middle term, the loss of value from employment. We note here that this preliminary conclusion is consistent with our quantitative findings in Section 7.4.

Finally, we note that the externality does not converge to zero over time, even if the fraction of infected converges to zero. The size of the externality is proportional to the number of suspected individuals, and that will typically remain strictly positive in the long run.

6 Policy Implications

The goal of the optimal tax policy is to align the private and social value of a match. A given tax policy *implements* the efficient allocation if the equilibrium value of the match F_t^S and F^R equals the optimal value of the match $\mu_t^{ES} - \mu_t^{US}$ and μ^R in all periods, and the equilibrium market tightness in the susceptible and recovered stage equals the optimal market tightness in the susceptible and recovered stage in all periods.[9] Since both the value of the match and the market tightness in the recovered stage is already equal to the optimal values in the absence of any government intervention, we restrict attention to tax policies in the infected and susceptible stage.

In order to implement the efficient allocation, the tax policy must reduce both the value of the match $K_t^{EI} + J_t^I$ and the value of unemployment in the infected stage K_t^{UI} by the size of the externality ψ_t. One way is a tax on those who create the externality in the model, the infected individuals. This implementation has the advantage of being most straightforward and conceptually the easiest: the externality ψ_t, after all, directly

[9]It is not required that the value of the match in the infected stage is also equal, because no decisions are made in the infected stage.

decreases the values of the infected stage in equations (26) and (27). However, we show that it is not the only way of how to implement the efficient allocation. The general characterization of the set of policies that implement the optimum is shown in Appendix B. Below we describe alternative implementations that have intuitive appeal: taxing susceptible workers appropriately, increasing unemployment benefits, and taxing the firms who create vacancies.

Implementation #1: Infection Tax. Consider a Pigouvian tax on the infected people, an *infection tax* at the amount equal to the net social harm ψ_t:

$$\tau_t^I = \psi_t. \tag{29}$$

The infection tax is paid by the unemployed workers and either the employed workers or employers. The model does not determine whether the employed workers or employers pay the tax; if the tax burden is shifted from one party to the other, the equilibrium wage adjusts appropriately. Assume for simplicity that it is the employed worker who pays the infection tax. The value functions for the workers in the infected stage change from (8) and (7) to

$$K_t^{UI} = b - c - \tau_t^I + \beta\left[\pi_R K^{UR} + (1-\pi_R-\pi_D)K_{t+1}^{UI}\right] \tag{30}$$

$$\begin{aligned} K_t^{EI} &= w - c - \tau_t^I + \beta\lambda\left[\pi_R K^{UR} + (1-\pi_R-\pi_D)K_{t+1}^{UI}\right] \\ &\quad + \beta(1-\lambda)\left[\pi_R K^{ER} + (1-\pi_R-\pi_D)K_{t+1}^{EI}\right]. \end{aligned} \tag{31}$$

To see that the infection tax $\{\tau_t^I\}$ *implements*, note that $K^{UR} = \mu^{UR}$ and $K^{ER} = \mu^{ER}$ because the recovered stage is efficient. By comparing (30) and (30) with their counterparts in the optimum and by using (29), such that $K_t^{UI} = \mu_t^{UI}$ and $K_t^{EI} + J_t^I = \mu_t^{EI}$ solves the dynamic program. It follows that the susceptible stage is efficient as well.

The properties of the infection tax follow from the properties of the externality ψ_t. First, the tax varies with time, but is the same for infected employed and infected unemployed. This is a consequence of the fact that the externality is the same regardless of the worker's labor market status. Second, the infection tax does not converge to zero over time and is in the steady state proportional to the fraction of the population that remains susceptible. This means that the long run infection tax is *complementary* to quarantine. Since quarantine reduces the number of infected people, the fraction of susceptibles remains higher in the steady state, and the infection tax must be higher.

Implementation #2: Taxing Susceptibles. The tax on the infected may have its disadvantages, not captured in our simple model. It could be considered unfair since it is paid by those who are sick, or people may not take it into account properly when making decisions in the susceptible stage. We now show that the tax can be "shifted" to the susceptible stage, where all the relevant decisions are made.

Shifting the tax burden to the susceptible stage is done in two steps. First, consider replacing the infection tax τ_t^I, which is paid every period as long as the worker remains sick, by its expected present value, which is paid only once at the beginning of the infection. Since the workers are risk neutral, they are indifferent between both options. The expected present value of the infection tax is, for both employed and unemployed,

$$\overline{\omega}_t = \sum_{j=0}^{\infty} \beta^j (1 - \pi_R - \pi_D)^j \tau_{t+j}^I.$$

In step two, consider shifting the present value of the infection tax from the beginning of the infection to the susceptible stage. The expected value of the taxes is

$$\tau_t^{US} = \beta \pi_{t+1}^{UI} \overline{\omega}_{t+1} \tag{32}$$

$$\tau_t^{ES} = \beta \pi_{t+1}^{EI} \overline{\omega}_{t+1}. \tag{33}$$

Appendix B shows that if the infection tax $\{\tau_t^I\}$ implements the efficient allocation, then the tax on susceptibles $\{\tau_t^{US}, \tau_t^{ES}\}$ implements the efficient allocation as well.

The tax on susceptibles is the present value of the infection tax multiplied by the probability of getting infected, furter discounted to the current period. It now differs for the employed and unemployed workers. Note that, by construction, $\tau_t^{ES} > \tau_t^{US}$. The tax on susceptible workers thus discourages employment by increasing the utility from unemployment relative to the utility from employment. We will later show that, quantitatively, the size of externality, and the infection tax, is the largest at the beginning of the pandemic, when the probability of getting infected is the smallest. The tax on susceptibles is thus a product of two forces that move in the opposite direction.

Implementation #3: Unemployment Insurance Benefits. The fact that the tax on susceptible employed workers exceeds the tax on susceptible unemployed workers means that the return from working is temporarily reduced. This looks a lot like a temporary increase in unemployment benefits. In fact, one can implement the efficient allocation solely through appropriately chosen increase in unemployment insurance benefits that is essentially, equal to the difference between the tax on unemployed susceptibles, and a tax on employed susceptibles. The details of the calculations are in Appendix B. In particular, we show that the increase in unemployment benefits is proportional to the difference between the infection probabilities $\pi^{EI}_{t+1} - \pi^{UI}_{t+1}$.

Implementation #4: Tax on Vacancy Creation. The tax on employed susceptibles τ^{ES}_t is paid while the match lasts. There is another equivalent representation of the optimal tax by shifting the tax burden forward to the vacancy creation stage to be paid by any firm that posts a vacancy. Consider again a two-stage transformation. Let $\bar{\tau}^{ES}_t$ be the expected present value of the tax at time t.[10] Assume that it is the firm that pays the tax on susceptibles, rather than the employer. The firm is indifferent between paying the expected present value of the tax $\bar{\tau}^{ES}_t$ once the match has been formed and paying the tax on a susceptible match τ^{ES}_t as long as the match lasts and the worker is susceptible. One can further shift the tax τ^{VS}_t to the time of a vacancy creation, as an additional cost of creating a vacancy. At the time the vacancy is created, the expected value of the tax on a susceptible match, denoted by τ^{VS}_t, is

$$\tau^{VS}_t = \beta q^S_t (1 - \pi^{UI}_t) \bar{\tau}^{ES}_t.$$

The equilibrium condition for the vacancy creation is now

$$(1-\phi) F^S_t = \frac{k + \tau^{VS}_t}{\beta (1 - \pi^{UI}_t) q(\theta^S_t)}.$$

One can again show that if the tax on susceptibles $\{\tau^{US}_t, \tau^{ES}_t\}$ implements the efficient allocation, then the tax on susceptible workers combined with the tax on vacancy creation $\{\tau^{US}_t, \tau^{VS}_t\}$ implements the efficient allocation as well.

Since the value of matches is always lower in the efficient allocations (as implied, for example, by figure 5a), the tax on vacancy creation is always positive, and the largest throughout the peak of the epidemic. The policy recommendation contrasts here with Guerrieri et al. (2020), who advocate policies that reduce the number of business shut-

[10] The expected present value does not have a closed form solution. Its computation is relatively straightforward, however, and is not shown.

downs and exits. In their case, the externality from keeping businesses afloat is positive, because it increases aggregate demand. In our model, the externality is negative, because it increases the spread of the pandemic.

7 Baseline Calibration

We now calibrate the model to a scenario that roughly corresponds to the perception of the COVID-19 pandemic in the Spring of 2020, and is consistent with a dynamic of a standard SIR model. We will later recalibrate some aspects of the model to take into account the actual realization of the COVID-19 pandemic.

A period is one week. An annual interest rate is 2 percent and so the discount factor is set to $\beta = 0.98^{1/52}$. We set output $y = 1.014$ which normalizes the pre-pandemic equilibrium wage to one. Following Hall and Milgrom (2008), we set the value of being unemployed to 0.71. The monthly separation rate is set to 1.4 percent and we target a monthly job finding probability of 0.277, as reported by Fujita and Moscarini (2017). The monthly values translate to a weekly separation probability $\lambda = 0.0033$ and a weekly job finding probability of 0.072. The matching function is Cobb-Douglas:

$$m(U, V) = aU^{\alpha}V^{1-\alpha}. \tag{34}$$

The elasticity of the matching function α is set to 0.5. The bargaining power of the workers is 0.5 as well, ensuring that the Hosios condition holds and the only source of inefficiency comes from the transmission of the epidemic. The steady state labor market tightness is normalized to one. This produces the value of a equal to the job finding probability and the value of a vacancy cost $k = 0.276$.

Based on the CDC data on cases and deaths by age[11] we compute the probability of dying from COVID-19 in the 18-64 age category, our target demographics, to be 0.00462. Following Eichenbaum et al. (2021), we assume that it takes 18 days on average to either recover or die from the infection. This gives weekly values $\pi_D = 7 \times 0.00462/18 = 0.0018$, and the probability of recovery to $\pi_R = 7 \times (1 - 0.00462)/18 = 0.387$. The ratio s^U/s^E determines the probability that the unemployed get infected relative to the employed. The ratio reflects the relative frequency of social interactions of people who are employed and people who are not, and the empirical evidence for that is relatively

[11] https://covid.cdc.gov/covid-data-tracker/#demographics

scarce. According to a Gallup panel survey on social distancing during the COVID-19 pandemic, workers generate 13.9 social contacts per day and nonworkers generate 4 contacts per day.[12] As a result, $s^U/s^E = 4/13.9 = 0.288$. In our benchmark calibration we set s^U and s^E to be constant over the course of the pandemic and obtain the values of s^E and s^U by targeting a steady state value of infected and recovered people to be two thirds. This yields $s^E = 0.6783$ and $s^U = 0.1953$. Table 1 shows the calibrated parameters.

The pre-pandemic calibration yields the expected present value of a worker's earnings to be 2532 times weekly wage, normalized in the model to one. Since the average weekly wage is about 1000 dollars, this value corresponds to 2.532 million USD. We target the value of life to be 5 million USD, which yields $D = -2458$, and the expected value of death for the infected to be $pi_D * D = -4.42$. Finally, since the value of death cannot be identified separately from c, we normalize $c = 0.0$.

Table 1: Benchmark Parameters

parameter	value	target
β	0.9996	2% annual interest rate
y	1.0139	wage normalized to one
b	0.71	Hall and Milgrom (2008)
c	0	normalization
D	-2458	value of death 5 million USD
λ	0.0077	monthly job separation probability 1.4 percent
α	0.5	
k	0.2761	monthly job finding probability 27.7 percent
a	0.0719	normalization
ϕ	0.5	Hosios condition
π_D	0.0018	CDC data
π_R	0.3871	CDC data
s^U	0.1968	relative probability of infection
s^E	0.6835	2/3 of the population eventually infected

We initialize the pandemic by assuming that 0.1 percent of the population gets infected for exogenous reasons and assume that the proportion of infected workers is the same among employed and unemployed workers.

[12] https://news.gallup.com/opinion/gallup/308444/americans-social-contacts-during-covid-pandemic.aspx

Reproduction number. The basic reproduction number R_0 assumes that everyone in the population is susceptible and computes the number of individuals that are expected to be infected by one infected person. In the model, the basic reproduction number at the beginning of the infection is

$$R_0 = \frac{s^{EI}(1-u) + s^{UI}u}{\pi_r + \pi_D},$$

where u is the initial unemployment rate. This yields $R_0 = 1.70$. This number is lower than some commonly reported values, for example Ferguson et al. (2020) use the value of 2.4 as their baseline value. But that is an appropriate choice, because there are additional mechanisms that reduce the transmission of the virus and that are not considered in the paper (for example, wearing masks or spending more time at home). Our basic reproduction number takes those unmodeled aspects already into account.

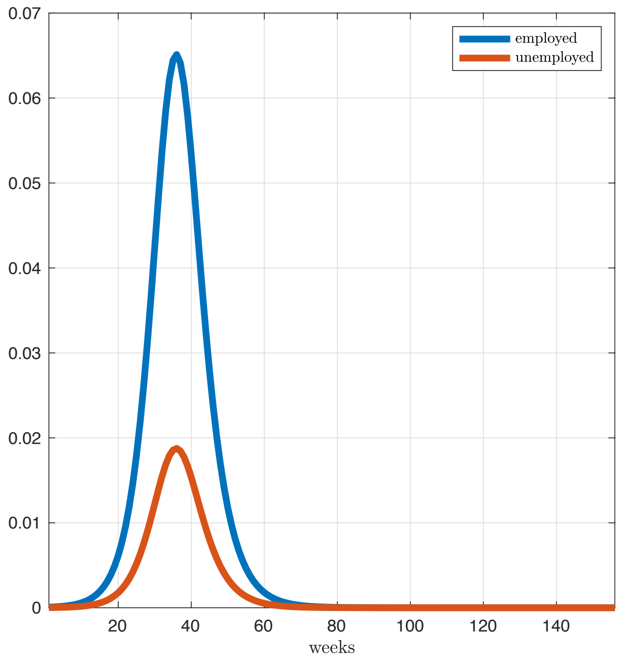
The effective reproduction number R_t takes into account that a fraction of the population may be immune to the infection and computes the number of individuals that are expected to be infected by one infected person, if less than the whole population is susceptible. The effective reproduction number is

$$R_t = S_t \frac{s^{EI}(1-u_t^S) + s^{UI}u_t^S}{\pi_r + \pi_D}.$$

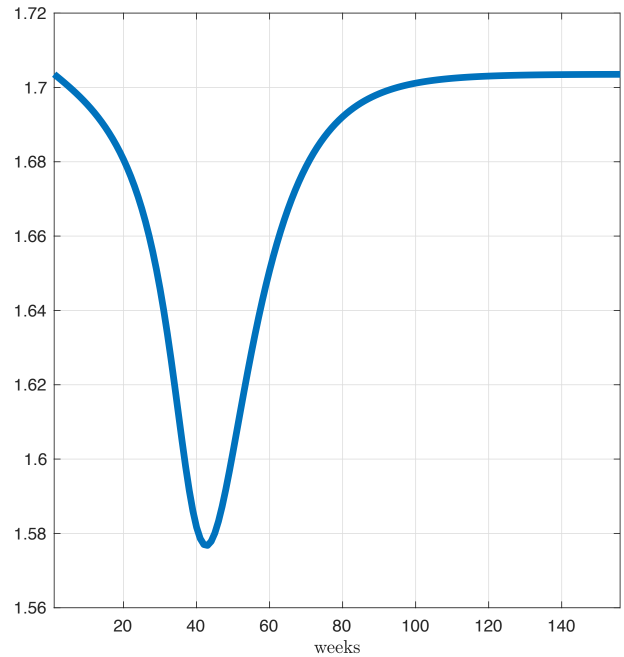
In the long run, the unemployment rate u_t^S converges to the initial unemployment rate u and so the effective reproduction number becomes simply the basic reproduction number multiplied by the fraction of susceptible individuals. Along the transition, the ratio of the effective reproduction number to the fraction of susceptibles varies and the dynamics will be examined below.

7.1 Model Predictions

Figure 1 shows how the infection spreads through the population. Panel 1a shows the equilibrium probability of getting infected over the course of the first one hundred weeks. The infection peaks after 36 weeks, when about 6.5 percent of employed workers and 1.9 percent of unemployed workers become infected. After approximately 70 weeks the probabilities of getting infected drop close to zero and the fraction of people who are either dead or infected stabilizes at two thirds of the population. By calibration, the



(a) Probability of getting infected



(b) Ratio of the effective reproduction number and the number of susceptibles, R_t/S_t.

Figure 1: Infection propagation.

pandemic costs lives of 0.67 percent of the population.

Figure 1b shows the ratio of the effective reproduction number R_t to the population of the susceptible individuals S_t. The ratio R_t/S_t starts, and converges to, the value of $R_0 = 1.70$. Over the course of the pandemic, this ratio declines, but the decline is rather small: the ratio drops only to 1.58 when the pandemic peaks. Labor markets thus, by themselves, do not mitigate the spread of the pandemic substantially.

Two factors are critical for the dynamics of the labor market: the value of the match and labor market tightness. Both are shown in Figure 2. The left panel 2a shows the match value for all three categories of workers. The value of the match is obviously the highest once the workers recover, because there are no additional disruptions to productivity. The value of the match in the infected stage is lower, because the workers are temporarily unproductive. In both cases, the values are constant over time. The value of the match in the susceptible stage on the other hand exhibits substantial changes over the course of the pandemic. It clearly co-moves negatively with the probability of getting infected, its main determining factor. It has a U-shaped profile with its lowest value during the peak of the pandemic. Interestingly, the match in the susceptible stage is lower than the match in the infected stage between weeks 4 and 45. The reason is that the value of the match in the susceptible stage takes into account that workers have a higher probability of getting infected, but this factor does not appear in the infected stage. When the probability of that happening is large, as it is in the peak weeks, the value of the match decreases substantially.

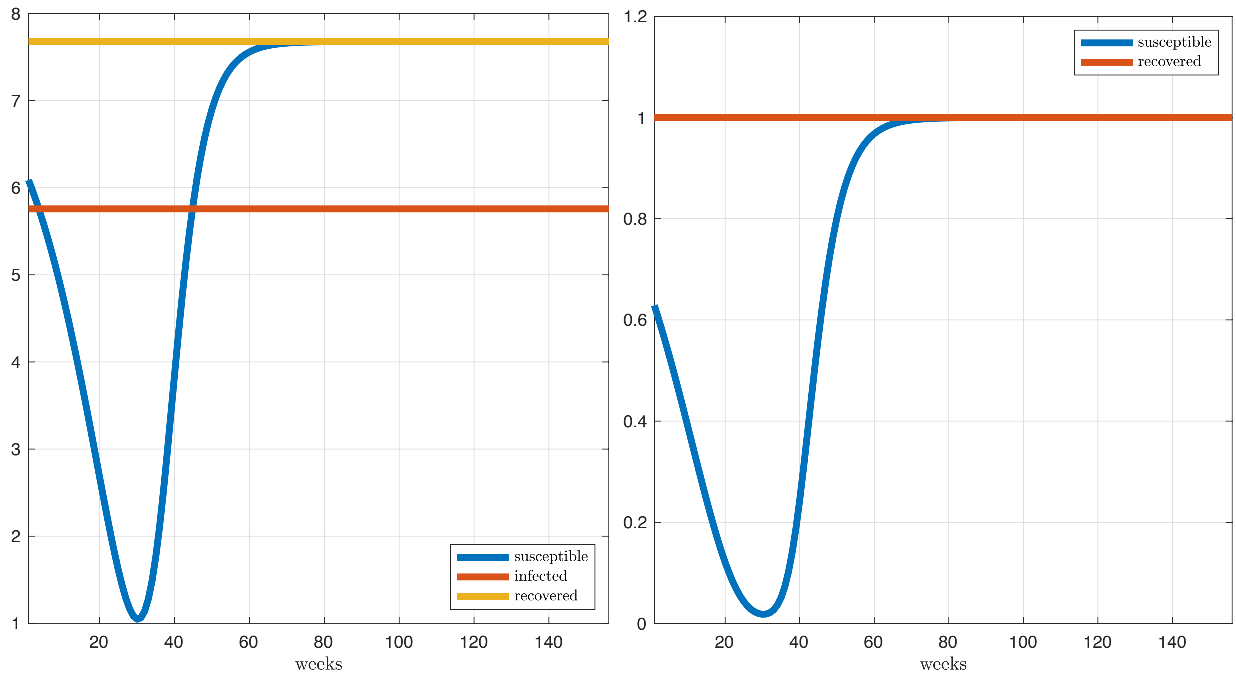
Labor market tightness is shown in the right panel 2b. Labor market tightness drops substantially in the susceptible market: at its minimum, in week 30, it drops to less than 20 percent of its steady state value of one. To understand why, it is useful to rewrite and inspect the free entry condition in the susceptible stage (15):

$$\frac{k}{q(\theta_t^S)} = \beta(1-\phi)(1-\pi_t^{UI})F_t^S.$$

The left-hand side is increasing in θ_t^S. Market tightness will then be lower if the value of the match is lower because it does not pay off to post vacancies. Market tightness will also be lower if the probability of getting infected is higher because this increases the chance that the match will be unproductive. Figures 1a and 2a show that both factors reinforce each other because the value of a match is the lowest when the probability of getting infected is the highest. As a result, labor market tightness co-moves strongly negatively with the probabilities of getting infected. In the long run, as the probability of getting infected converges to zero, labor market tightness converges to its steady state level of one.

Changes in the match value and labor market tightness have direct consequences for the equilibrium wage rate, shown in Figure 3a. Immediately after the pandemic starts, the wage in the market for susceptible workers drops below its steady state value and reaches a local trough after 16 weeks. As the probability of getting infected rapidly increases, the trend reverses, and the wage rate increases above its long-run value. After the epidemic peaks, the wage rate in the susceptible market starts declining again. At its peak, the wage rate is 31 percent above the recovered wage, while at its minimum it is about 10 percent below. The model thus predicts, first, a relatively strong segmentation in the labor market and, second, substantial volatility of wages in the economy.

What explains the reversal in the wage rate? The wage equation (25) reveals that the equilibrium wage is a product of two factors: the reservation wage, and the surplus of the match. Figure 2a has shown that the match surplus is U-shaped. That tends to decrease the equilibrium wage rate. The reservation wage, on the other hand, moves



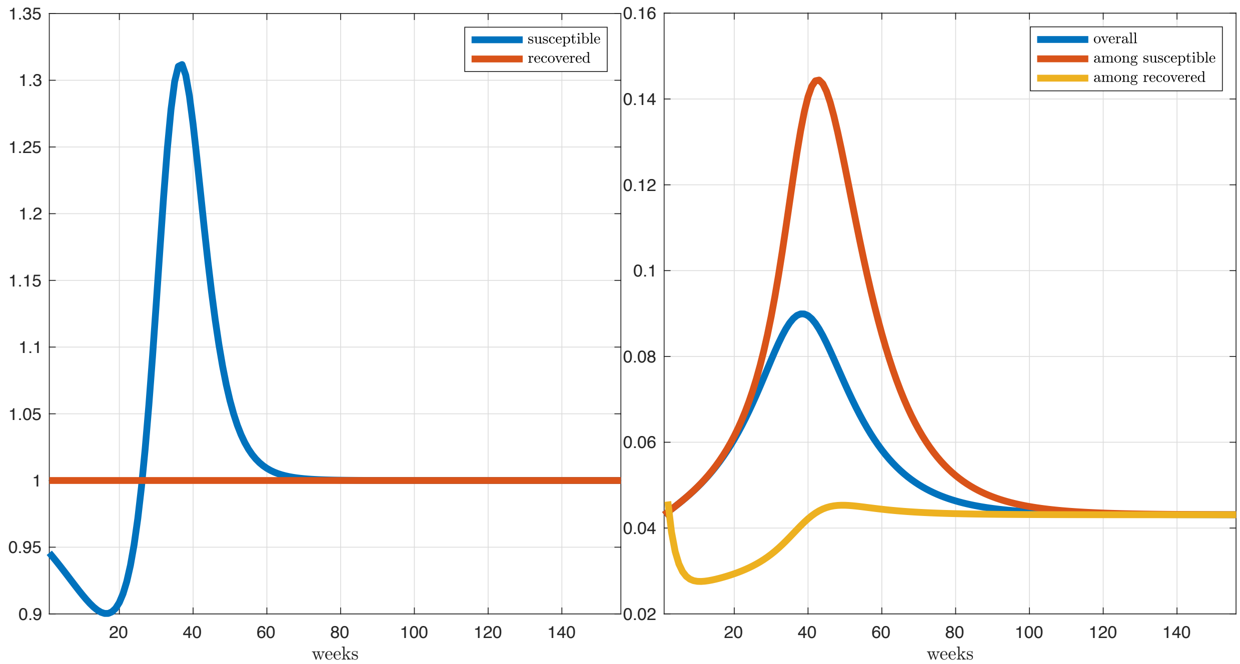
(a) Match value in the susceptible, infected and recovered stage, F_t^S, F^I and F^R.

(b) Labor market tightness in the susceptible and recovered market θ_t^S and θ^R.

Figure 2: Labor market tightness and match value.

in the opposite direction. The expression for the reservation wage (24) makes it clear why. It is both because the value of the future match is low, and so the expected future gains are relatively low, and because there is a higher chance of getting infected when employed, and that chance is particularly high in those periods. So both forces move the equilibrium wage in the opposite direction. But the reservation wage is more forward-looking than the match value and so starts increasing earlier than the match value starts decreasing and then starts decreasing earlier, producing the pattern in Figure 3a.

Finally, consider the unemployment rates in Figure 3. As the wage among susceptible workers starts rising, fewer vacancies are created, market tightness starts dropping and the unemployment rate rises. The unemployment rate among susceptibles peaks in week 43, 13 weeks after the pandemic peaks, when it reaches more than 14 percentage points, more than triple of its steady state level. On the other hand, the unemployment rate among the recovered starts decreasing right after the pandemic begins and stays lower for almost all periods. This is due to a selection effect. Since employed people are more likely to get infected, they enter the pool of recovered people at a higher rate than the unemployed and drive down the unemployment rate. Overall, the unemployment rate



(a) Wage rate in the susceptible and recovered market

(b) Unemployment rate in the susceptible and recovered stage, and overall unemployment rate.

Figure 3: Wages and unemployment rate

rises to 9 percentage points shortly after the pandemic peaks.

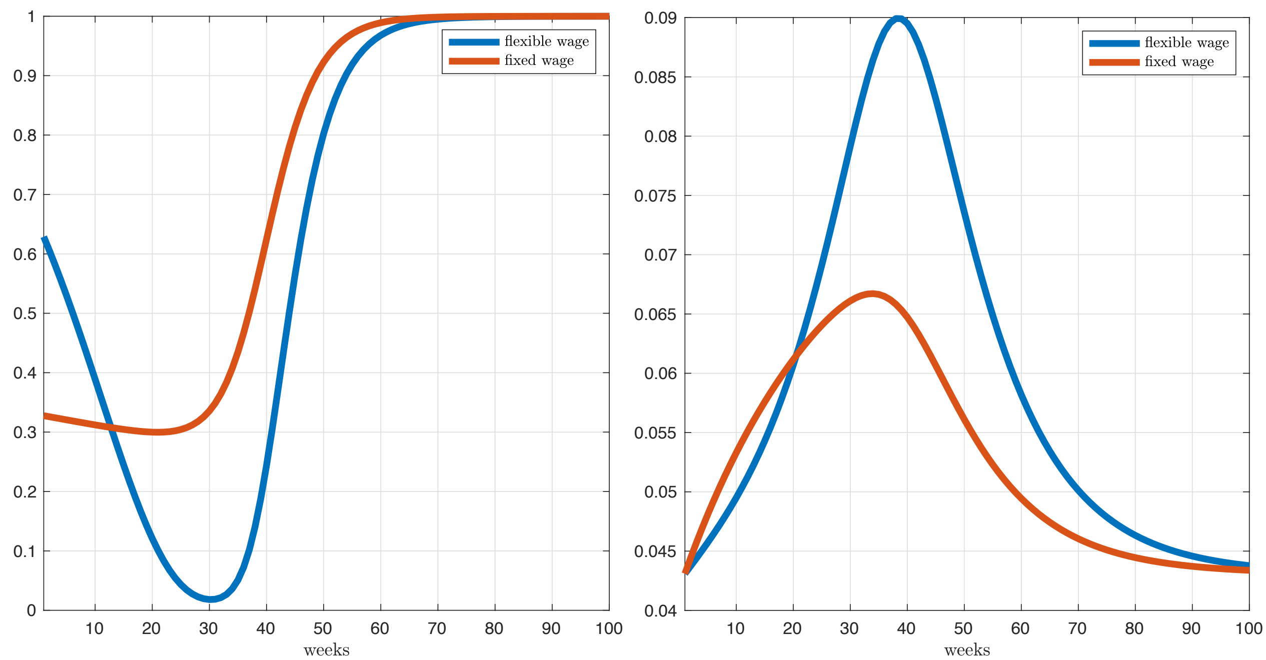
7.2 Rigid Wages

It is reasonable to ask to what extent is the behavior of labor markets driven by our assumption that wages are continuously renegotiable. To that end, consider alternative labor market arrangements, where wages, instead of being renegotiated continuously every period, are fully rigid and stay the same throughout the epidemic, at their initial steady state value w_0. The value functions of the firm in the susceptible stage are now[13]

$$J_t^S = y - w_0 + \beta(1-\lambda)\left[\pi_{t+1}^{EI} J^I(w_0) + (1-\pi_{t+1}^{EI}) J_{t+1}^S\right]$$

$$J^I = \frac{-w_0 + \beta(1-\lambda)\pi_R J^R}{1-\beta(1-\lambda)(1-\pi_R-\pi_D)}.$$

[13]For brevity, only the dynamics of the firm's problem in the infected and susceptible stage and the determination of the labor market tightness in the susceptible stage are characterized.



(a) Labor market tightness in the susceptible stage

(b) Overall unemployment rate.

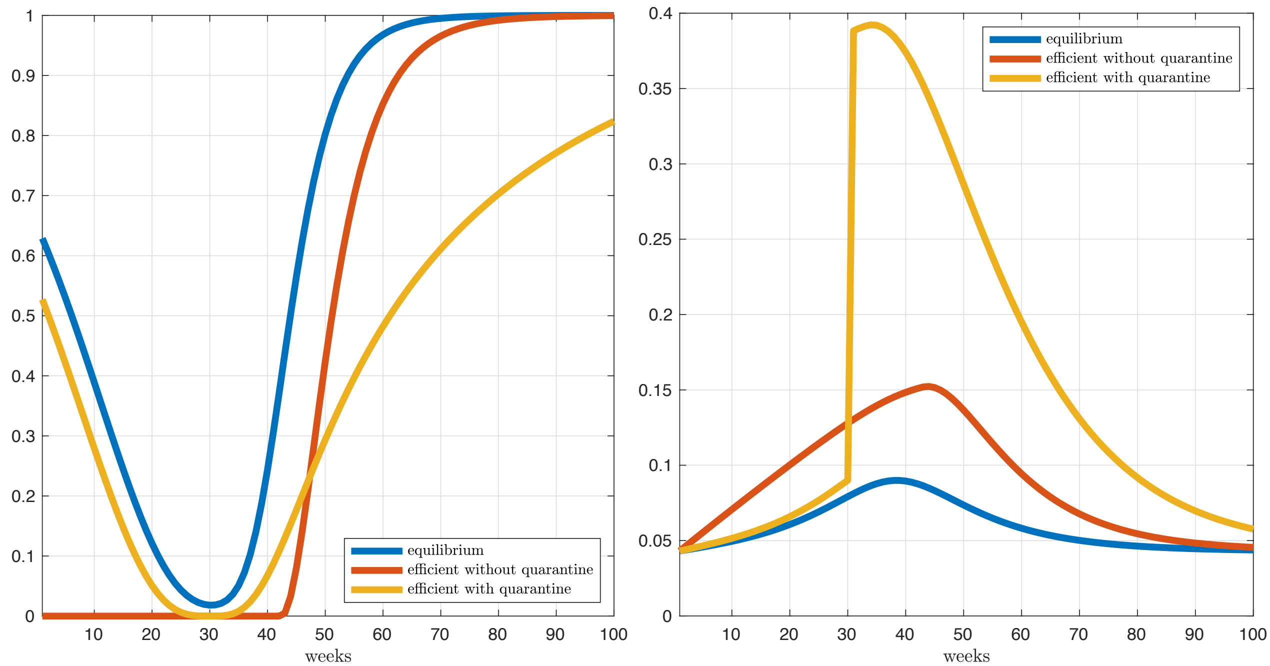
Figure 4: Labor market: flexible vs rigid wages.

Since the economy returns to the pre-epidemic steady state, the value of the firm in the recovered stage happens to be the same as in the model with flexible wages. Labor market tightness solves

$$J_t^S = \frac{k}{\beta(1-\pi_{t+1}^{UI})q(\theta^S)}.$$

If the wage w_0 is higher than the flexible wage, labor market tightness decreases relative to its value under flexible wages and vice versa. Figure 3a has shown that flexible wages first drop below the steady state value, and then, before the epidemic peaks, rise above. Labor market tightness under rigid wages must therefore be initially lower. This is exactly what happens, as figure 4a shows. As a result, the unemployment rate increases more initially. However, as the flexible wage rises above its long term value, the situation is reversed. Labor market tightness is higher under fixed wage as firms are more willing to open up vacancies, and the unemployment rate increases relatively less. Overall, the second phase is dominant, and the overall unemployment rate peaks at 6.5 percent, instead of 9 percent under flexible wages (figure 4b).

Lower unemployment under rigid wages means that the infection spreads at a faster



(a) Labor market tightness, susceptible stage.

(b) Overall unemployment rate.

Figure 5: Labor market: equilibrium vs. efficient allocation

rate during the critical phases of the pandemic. Overall, the fraction of recovered and dead increases, by 1.9 percentage point, to 68.6 percent. Coupled with the fact that rigid wages create an inferior labor market allocation, overall welfare slightly decreases.

7.3 Efficient Allocations

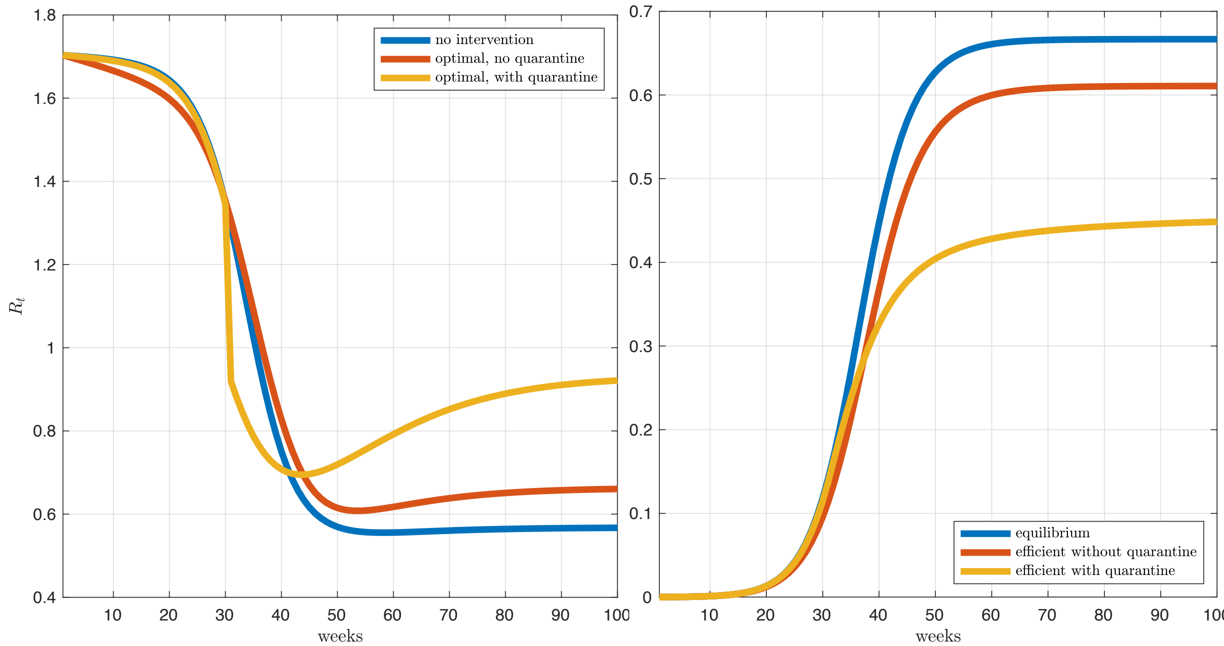
Next, we investigate the efficient allocations during the pandemic, with and without quarantines, and how they differ from the equilibrium allocations in the absence of any government intervention. The red line in Figure 5a shows that, if the planner is not allowed to use quarantines, the market tightness drops to zero in the first 42 weeks, during the rise and peak of the pandemic. This is because the social value of the match drops below zero: when the pandemic is on the rise, the negative externality from working is the strongest. In contrast, as evidenced from a strictly positive labor market tightness (as well as from Figure 2a directly), the private value of the match in the equilibrium allocation is strictly positive and new jobs are still being created.

When the social value of the match becomes negative, the planner wants to destroy matches. This is exactly what a quarantine does. The quarantine happens in week

30. The number of quarantined people is such that labor market tightness becomes exactly zero, as the yellow line in Figure 5a shows; this amounts to 29.5 percent of the population. It is worth noting that it is optimal to move people from employment to unemployment only once, before the pandemic peaks, even though the planner has the option to do it repeatedly. This is because the rate at which people get back to employment is low when the pandemic peaks and so there is no need to continue removing people from employment later. The job finding rate only accelerates after the pandemic peaks, but then the externalities are too small to warrant another quarantine. Note also that labor market tightness is higher before the quarantine relative to the efficient allocation without quarantine. It is less costly to create matches because they can be destroyed during quarantine. In contrast, if quarantine is not feasible, the matches will persist, will contribute to the epidemics, and their social value is lower.

Figure 5b shows the efficient unemployment rate. In both planning problems, the overall unemployment rate is significantly above the equilibrium one. While the equilibrium unemployment rate is 9 percent, in the optimum it almost doubles to more than 15 percent even without the use of a quarantine. If a quarantine is used, then it naturally rises even more due to the quarantined workers and, as the results show, substantially more: 29.5 percent of all employees are optimally quarantined in period 30, and the unemployment rate rises to 39 percent. After that, the quarantined people, now mixed with the people who lost their jobs in a regular way, slowly get back to the pool of employed people. Although the unemployment rate without quarantine is only two fifths of what it is under quarantine, it starts rising almost immediately after the epidemic peaks. This is because, in the absence of a quarantine, the decrease in employment needs time to build up. Under quarantine, the unemployment rate is relatively close to the laissez-faire unemployment rate before the quarantine is imposed, because matches can be terminated immediately.

What are the extra benefits gained in both efficient allocations? The progression of the pandemic throughout the population is shown in Figure 6. How the efficient allocation reduces the effective reproduction number depends very much on whether a quarantine is used. If there is no quarantine, R_t is reduced mainly before the pandemic peaks. The reduction is not substantial, but it helps to "flatten the epidemic curve". As a result, the fraction of the recovered and dead ultimately decreases by 5.6 percent to 61.1 percent. Since the death rate is one percent, it saves 0.056 percent of the population, which is about 183,100 lives, given a US population of 327 million.



(a) Effective reproduction number R_t.

(b) Fraction of recovered and dead.

Figure 6: Pandemic: equilibrium vs. efficient allocation

If a quarantine is used, the effective reproduction number behaves differently. Up until the quarantine, it is almost identical to the effective reproduction number in the absence of any intervention. The optimal quarantine then instantaneously reduces R_t from 1.35 to 0.9, and it continues falling to 0.7. The effective reproduction number then again increases to about 0.9, as the quarantined people are being rehired on the labor market. Overall, the fraction of the recovered and dead decreases by 21.9 percentage points, to 44.8 percent. This translates into savings of 714,700 lives. Note that the steady state effective reproduction number is higher in the efficient allocation than in the equilibrium without an intervention because more people remain susceptible.

Output loss vs loss of lives. The loss of lives, as well as the loss of output under various scenarios is shown in Table 2. Letting the epidemic progress without government intervention results in a 5.9 percent loss of output during the first year of the pandemic and the calibrated value of two thirds of one percent loss of the population. The output loss under rigid wages is somewhat smaller, and the loss of lives is higher by 1.9 percent. The efficient outcome, on the other hand, dictates a larger loss of output: 9.9 percent without a quarantine, and 16.6 percent with quarantine, with proportionally

Table 2: Loss of Output and Lives

	Loss of output annual	Loss of output peak	Loss of lives
Equilibrium, flexible wages	5.9 %	14.4 %	0.667 %
Equilibrium, rigid wages	5.1 %	12.6 %	0.686 %
Efficient, no quarantine	9.9 %	18.6 %	0.611 %
Efficient with quarantine	16.6 %	43.0 %	0.448 %

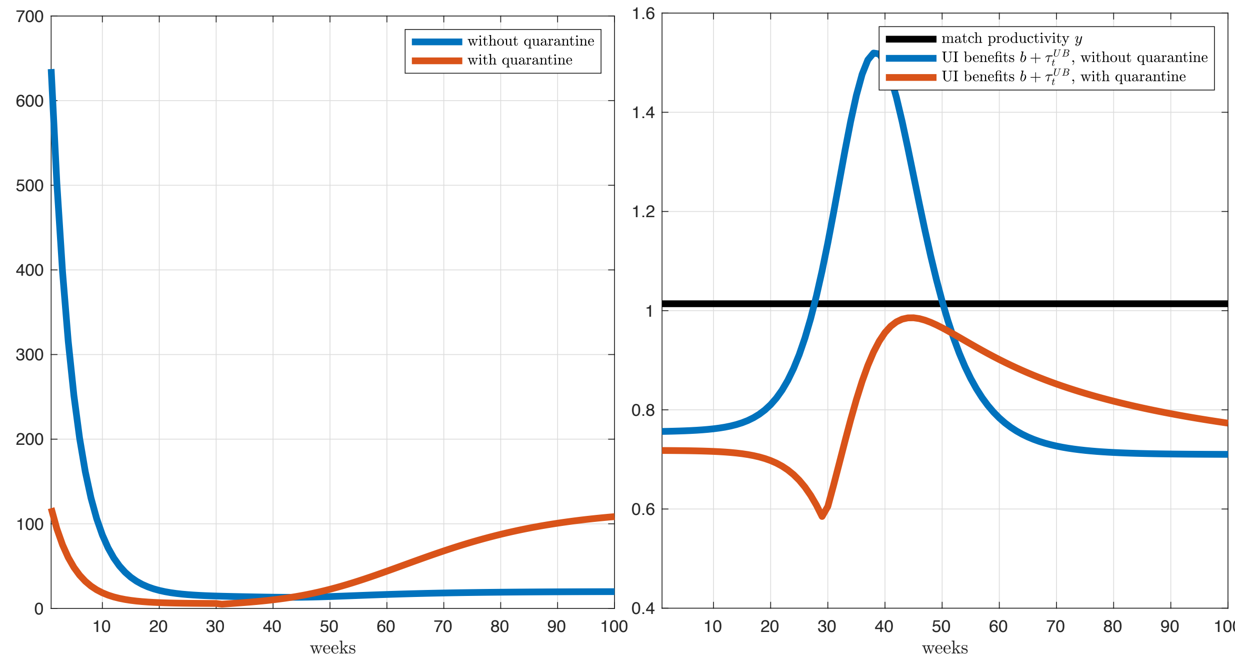
Loss of output and loss of lives in the four scenarios. Annual loss of output is the fraction of output lost during the first year. Peak loss of output is the largest weekly loss of output. One tenth of one percent loss of lives represents approximately 327 000 lives.

larger savings of lives, as discussed before. The recession produced under the quarantine is thus almost three times as large as it would be in the absence of the government intervention. The loss of output during the worst week of the pandemic is much larger: 14.4 percent under flexible wages, and 43.0 percent under quarantine.

7.4 Optimal Policies

We now characterize the optimal policies that implement the efficient allocation. For the sake of brevity, we focus on the infection tax and unemployment benefits only. Figure 7a shows the optimal infection tax, with and without quarantine. The infection tax is substantial. It is U-shaped, and the highest at the beginning of the epidemic, when it is equal to more than 600 weekly wages with quarantine, and to more than 100 weekly wages without quarantine. With weekly wages around 1000 dollars, the infection tax is around 600 and 100 thousand dollars in the two cases. Those initial magnitudes are astronomical. To understand them, note that the negative externality is the largest in the beginning, since the number of people directly of indirectly infected is the highest: the first person infected in the country is, in the absence of any additional contacts with the outside world, responsible for the whole pandemic.

The infection tax consists of three components, corresponding to the three terms in the externality: a decrease in the job finding rate, an infection of the employed people, and infection of the unemployed people. The first component, a decrease in the job finding rate, is insignificant in both cases. By far the largest component is the second one, a correction for infecting the employed people. This is because the employed people



(a) Infection tax, multiples of weekly wages. (b) Unemployement insurance benefits.

Figure 7: Tax on susceptibles and unemployment benefits

are the most significant group. While the loss from infecting one employed person, $\mu_t^{ES} - \mu_t^{EI}$, is comparable in magnitude to the loss from infecting one unemployed person, $\mu_t^{US} - \mu_t^{UI}$, there are more employed workers in the economy than unemployed ones. The loss from infecting the unemployed people only becomes significant during the peak of the pandemic, when it constitutes up to 10 percent (without quarantine), or up to 30 percent (with quarantine) of the overall externality.

The optimal unemployment benefits are shown in Figure 7b. Recall that the unemployment benefits are set to 71 percent of productivity before the pandemic. During the pandemic, they rise substantially. Under quarantine, they rise to 95 percent of productivity at the peak. If quarantine is not allowed, they rise even higher than productivity, in weeks 28-50. Since it is efficient to have zero labor market tightness in this case, the UI benefits are therefore set at such level so as to prevent any hiring in the labor market in those weeks. After the pandemic the efficient unemployment benefits return to its pre-pandemic level, but the convergence is slower with quarantine.

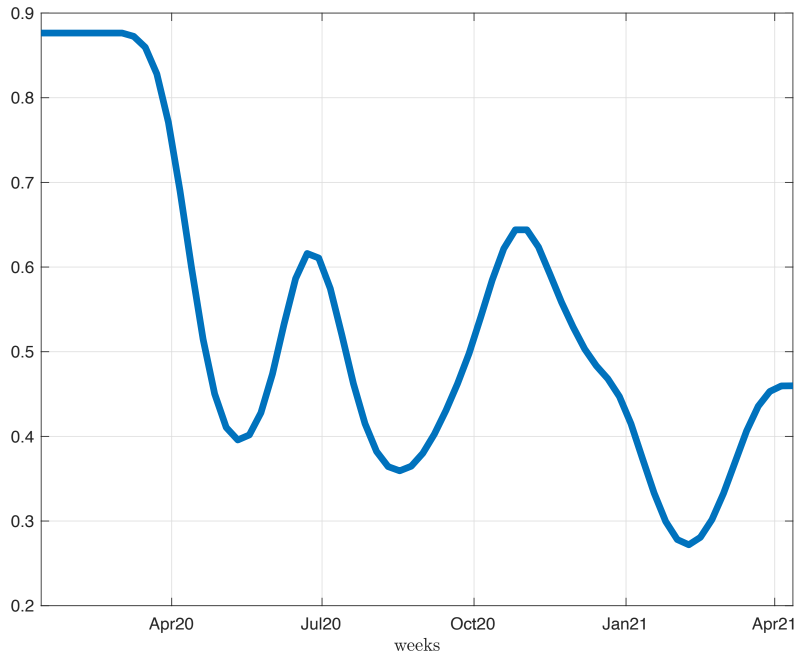


Figure 8: Infection propagation parameter s^E, January 2020 - April 2021.

8 2020-2021 Pandemic

We now calibrate the model specifically to the 2020-2021 COVID-19 pandemic. To do so, we make the infection propagation parameters s^E and s^U time varying, but keep their ratio constant at 0.288, as in the benchmark calibration. We choose their time series so as to match the fraction of susceptible individuals in the U.S. population and smooth the resulting time series. We assume that s^E stays constant after April 2021.[14] Since the the 2020 CARES Act temporarily increased the unemployment benefits from March 30, 2020 to August 10, 2020 by an additional $600 USD per week and the Consolidated Appropriations Act similarly increased the unemployment benefits from December 27, 2020 t0 March 14, 2021 by $300 USD, we correspondingly increase the model unemployment benefits from 0.71 by 0.6 in the weeks corresponding to the first period and by 0.3 in the weeks corresponding to the second period.[15] The remaining parameters of the model are identical to the benchmark calibration.

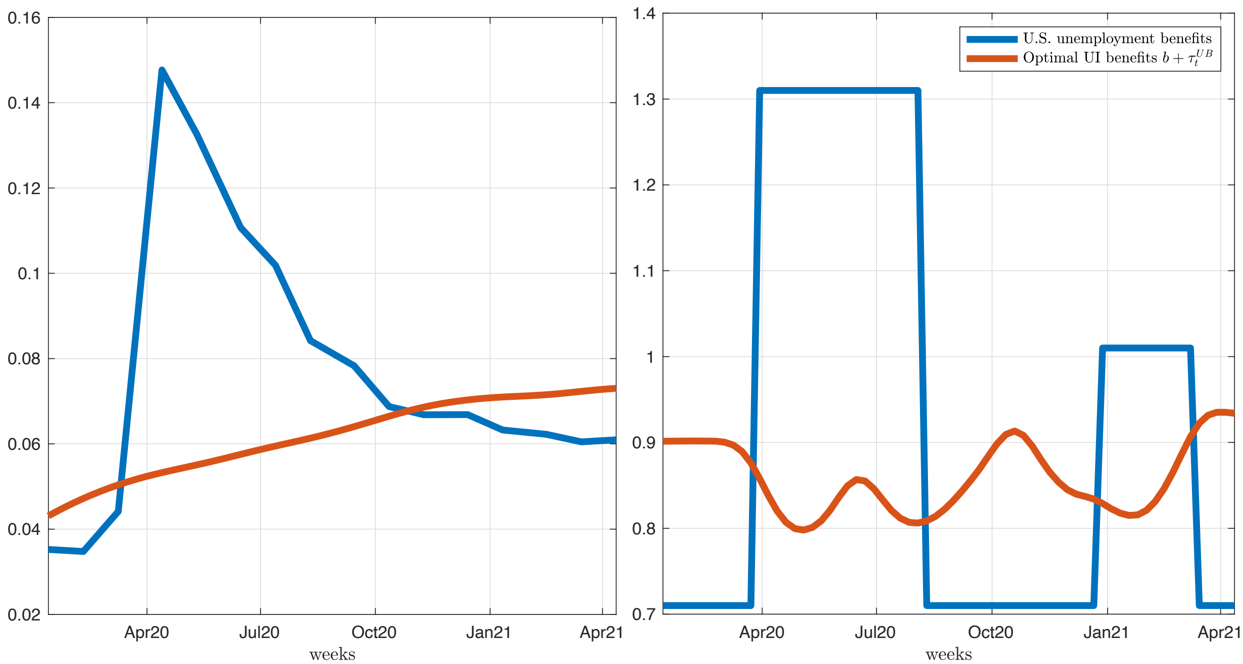
[14] We compute the fraction of susceptible individuals in the U.S. population as one minus the cumulative number of cases divided by U.S. population in 2020. The cumulative number of cases is obtained from `ourworldindata.com`.

[15] The fact that the replacement rates are temporarily substantially aboe 100 percent is consistent with the estimates in Ganong et al. (2020) and Petrosky-Nadeau and Valletta (2021).

Figure 8 shows the resulting infection propagation parameter s^E. The parameter s_E clearly mirrors the initial faster spread of the pandemic in February-March 2020, when its value is is the highest at 0.876, and the subsequent waves in the summer and fall of 2020. Overall, the parameter shows a downward trend, with the terminal value of 0.456. The mean value from January 2020-April 2021 is 0.536, somewhat lower than in our baseline calibration. The calibrated model produces an effective reproduction number around 2.2 in February-April 2020, which is roughly consistent with its early estimates. The effective reproduction number subsequently declines to around 0.7-1.6, with lower resp. higher values corresponding to the troughs and peaks of s^E .

For the sake of brevity, we concentrate on a comparison between the optimal unemployment rate and unemployment benefits, and their empirical counterparts. Figure 9a compares the U.S. unemployment rate and the fraction of susceptibles, and their optimal counterparts. The optimal unemployment rate is different from the U.S. one in two substantial aspects. First, the spike in the unemployment rate in April 2020, is clearly suboptimal. As shown in Figure 8, the model takes into account that the infection was spreading faster in the spring of 2020. That, however, is not enough to warrant such a substantial increase in the unemployment rate. Quarantine is now not optimal at any point, because the level of the number of infected was too small. Second, contrary to the U.S. unemployment rate, the optimal one is increasing throughout 2020, and reaches its maximum of somewhat more than 7 percent only in April 2021, and monotonically falls afterwards.

Figure 9b compares the underlying unemployment benefits as a fraction of earnings. It should be noted that, in the absence of the pandemic, the unemployment benefits are at their long term calibrated value of 0.71, and so any increase above that level is solely due to the pandemic. The contrast between the U.S. policy and the optimal one is again substantial. The level of optimal unemployment benefits is nowhere near the U.S. level, and fluctuates around 0.9. That corresponds to additional unemployment benefits of around $200 USD per week, a third of what was provided under the CARES Act. The timing of changes differs as well: the optimal unemployment benefits are decreasing when the CARES Act came into effect, and start increasing at a time when it expired. Overall, the optimal policy response exhibits substantially lower volatility and is positively correlated with the pandemic waves, in contrast to the U.S. policy response. We compute that, over the course of the whole pandemic, the optimal policy response reduces the number of infected or dead by 1.2 percent, from 19.8 percent to 18.6 percent,



(a) Unemployment rate (b) Unemployment benefits

Figure 9: Infection propagation.

and mitigates the drop in output by 1.5 percent.

To check the robustness of our results, we also considered doubling the value of death to \$10 million USD. That indeed increases the optimal unemployment rate to its peak value of 12.3 percent, also reached in the spring of 2021. However, the pattern is very similar to the one in Figure 9a: it gradually increases throughout 2020, rather than being hump shaped as in the data. In addition, the unemployment benefits increase only modestly relative to the baseline calibration, by about \$70 USD per week, and keep the pattern of Figure 9b. Higher unemployment rates are thus mostly due to individual responses, rather than due to increased policy interventions. It should, however, be noted, that the second increase under the Consolidated Appropriations Act is now approximately optimal in terms of its level.

9 Conclusions

A standard search and matching theory is integrated with the SIR model to study the impact that a COVID-19 pandemic has on the behavior of labor markets and on the optimal policies during the pandemic. The labor market separates the recovered and susceptible individuals. The epidemic has two opposing effects on wage formation of the people who are susceptible to the infection: the reservation wage increases because labor market activities make one more likely to get infected, but the value of the match decreases because job tenure is shorter and potentially less productive. The second effect dominates before the pandemic peaks, while the first one dominates during and after the peak so that the wage first decreases and then increases. The unemployment rate increases substantially among the susceptible, decreases among the recovered, and increases overall by about 3 percentage points.

Since the equilibrium is not efficient, due to the infection externality, it is important to study the efficient allocations: One where the government is not allowed to destroy matches and one where the government can quarantine people and destroy matches. In both cases the government wants to tax vacancy creation, especially around the peak of the epidemic, and slow down the spread of the virus. Unlike the private value, the social value of the match becomes negative before the epidemic peaks and the government optimally quarantines about a quarter of the employed individuals. This has a substantial output cost, almost 40 percent of the output during the weeks when the epidemic peaks, but saves more than 500,000 lives.

We also consider the actual path of the pandemic as it transpired and how the various policies affected the transmission. In particular, the model can closely replicate the "waves" that occurred throughout 2020. The optimal policy, however, looks much different than the actual US policy. The optimal policy saves lives and mitigates the health effects of the pandemic.

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Appendix A: The Efficient Allocations

Assuming the matching function takes the form assumed in the calibration, the problem is to choose the flows $\{US_t, ES_t, UI_t, EI_t, UR_t, ER_t, D_t\}$ and $\{VS_t, VR_t\}$ maximize

$$\sum_{t=0}^{\infty} \beta^t \left[(ES_t + ER_t)y + (US_t + UI_t + UR_t)b + (UI_t + EI_t)(-c + \beta\pi_D D) - (VS_t + VR_t)k\right]$$

subject to the constraints

$$
\begin{aligned}
US_{t+1} &= [1 - s^U(EI_t + UI_t)]\,[US_t - m(US_t, VS_t)] + \lambda[1 - s^E(EI_t + UI_t)]ES_t + Q_t \\
ES_{t+1} &= (1-\lambda)ES_t - (1-\lambda)s^E(EI_t + UI_t)ES_t + [1 - s^U(EI_t + UI_t)]m(US_t, VS_t) - Q_t \\
UI_{t+1} &= (1 - \pi_R - \pi_D)UI_t + s^U(EI_t + UI_t)US_t + \lambda s^E(EI_t + UI_t)ES_t + \lambda(1 - \pi_R - \pi_D)EI_t \\
EI_{t+1} &= (1-\lambda)(1 - \pi_R - \pi_D)EI_t + (1-\lambda)s^E(EI_t + UI_t)ES_t \\
UR_{t+1} &= UR_t - m(UR_t, VR_t) + \pi_R UI_t + \lambda\pi_R EI_t + \lambda ER_t \\
ER_{t+1} &= (1-\lambda)ER_t + (1-\lambda)\pi_R EI_t + m(UR_t, VR_t),
\end{aligned}
$$

and a nonnegativity constraint $Q_t \geq 0$. Let $\mu_t\beta^{ER}_{t+1}$, $\mu_t\beta^{UR}_{t+1}$, $\mu_t\beta^{ES}_{t+1}$, $\mu_t\beta^{EI}_{t+1}$, $\mu_t\beta^{US}_{t+1}$, $\mu_t\beta^{UI}_{t+1}$ be the Lagrange multipliers on the corresponding constraints. Recursively, first solving the recovered stage and then jointly the susceptible and infected stage.

Recovered stage. The first-order conditions for the recovered stage are

$$
\begin{aligned}
k &= \beta\left(\mu^{ER}_t - \mu^{UR}_t\right) m_V(1, \theta^R_t) \\
\mu^{UR}_t &= b + \beta\left[\mu^{ER}_{t+1} m_U(1/\theta^R_{t+1}, 1) + \mu^{UR}_{t+1}\left[1 - m_U(1/\theta^R_{t+1}, 1)\right]\right] \\
\mu^{ER}_t &= y + \beta\left[\mu^{ER}_{t+1}(1-\lambda) + \mu^{UR}_{t+1}\lambda\right]
\end{aligned}
$$

The recovered stage is again independent of time and the values of θ^R and the multipliers μ^{ER} and μ^{UR} are independent of time.

Infected stage. The first-order conditions in the employment and unemployment flows for the infected stage are:

$$
\begin{aligned}
\mu^{UI}_t = b - c + \beta\pi_D D + \beta\Big[&(1 - \pi_R - \pi_D)\mu^{UI}_{t+1} + \pi_R\mu^{UR} \\
&- (\mu^{ES}_{t+1} - \mu^{US}_{t+1})s^U m(1, \theta^S_{t+1})US_{t+1} - (\mu^{ES}_{t+1} - \mu^{EI}_{t+1})(1-\lambda)s^E ES_{t+1} \\
&- (\mu^{US}_{t+1} - \mu^{UI}_{t+1})(s^U US_{t+1} + \lambda s^E ES_{t+1})\Big] \qquad (35) \\
\mu^{EI}_t = -c + \beta\pi_D D + \beta\Big[&\lambda\left((1 - \pi_R - \pi_D)\mu^{UI}_{t+1} + \pi_R\mu^{UR}_{t+1}\right) + (1-\lambda)\left((1 - \pi_R - \pi_D)\mu^{EI}_{t+1} + \pi_R\mu^{ER}_{t+1}\right) \\
&- (\mu^{ES}_{t+1} - \mu^{US}_{t+1})s^U m(1, \theta^S_{t+1})US_{t+1} - (\mu^{ES}_{t+1} - \mu^{EI}_{t+1})(1-\lambda)s^E ES_{t+1} \\
&- (\mu^{US}_{t+1} - \mu^{UI}_{t+1})(s^U US_{t+1} + \lambda s^E ES_{t+1})\Big] \qquad (36)
\end{aligned}
$$

The second and third lines of the conditions represent new terms that reflect the externality of the problem. Unlike the equilibrium value functions in the infected stage, the values μ_t^{UI} and μ_t^{EI} depend on time, because the size of the externality is time varying. However, it is easy to show, that its difference, $\mu_t^{EI} - \mu_t^{UI}$, is independent of time. This is so because the probability of getting infected depends only on the total number of infected people and so the externality from one additional employed infected person and one additional unemployed infected person is the same. Hence, the difference $\mu_t^{EI} - \mu_t^{UI}$ is independent of the externality, and time independent.

Susceptible stage. The optimal allocations in the susceptible stage solve

$$\mu_t^{ES} = y + \beta \left[(1-\lambda) \left((1-\pi_{t+1}^{EI})\mu_{t+1}^{ES} + \pi_{t+1}^{EI}\mu_{t+1}^{EI} \right) + \lambda \left((1-\pi_{t+1}^{EI})\mu_{t+1}^{US} + \pi_{t+1}^{EI}\mu_{t+1}^{UI} \right) \right]$$
$$\mu_t^{US} = b + \beta \Big[(1-\pi_{t+1}^{UI}) m_U(1/\theta_{t+1}^S, 1)\mu_{t+1}^{ES} + (1-\pi_{t+1}^{UI}) \left(1 - m_U(1/\theta_{t+1}^S, 1)\right) \mu_{t+1}^{US} + \pi_t^{UI}\mu_{t+1}^{UI} \Big],$$

where $\pi_t^{EI} = s^E I_t$, $\pi_t^{UI} = s^U I_t$. The first-order conditions are analogous to the corresponding equilibrium conditions. When the matching function is given by (34), the expected difference is that the bargaining weight ϕ is now replaced by the elasticity of the matching function α.

The first-order condition in the susceptible vacancies V_t^S is

$$k = \beta \left(\mu_t^{ES} - \mu_t^{US} \right) (1 - s^U I_t) m_V(1, \theta_t^S).$$

In the planning problem without quarantine, Q_t is set equal to zero. The first-order conditions are solved recursively, together with the laws of motion (1) and (2). In the planning problem with quarantine, Q_t is chosen by the planner, but must be nonnegative. This yields the complementarity-slackness condition in Q_t:

$$\mu_t^{ES} - \mu_t^{US} \geq 0, \quad Q_t \geq 0, \quad (\mu_t^{ES} - \mu_t^{US})Q_t = 0.$$

Efficiency of equilibrium. There is a close correspondence between the Lagrange multipliers in the planning problem and the value functions of the workers, and of the firm. The Lagrange multipliers μ_t^{ES}, μ_t^{EI} and μ_t^{ER} represent the social values of employment in the three stages, and correspond to the total value of the match for the firm and worker together, $J_t^{ES} + K_t^{ES}$, $J_t^{EI} + K_t^{EI}$ and $J_t^{ER} + K_t^{ER}$. The Lagrange multipliers μ_t^{US}, μ_t^{UI}

and μ_t^{UR} represent the social value of unemployment and have their counterparts in the private value of unemployment K_t^{US}, K_t^{UI} and K_t^{UR}.

The optimal value of θ^R in the recovered stage satisfies

$$k = \beta \frac{y - b}{1 - \beta\left[1 - \lambda - m_U(1/\theta^R, 1)\right]} m_V(1, \theta^R).$$

This condition is identical to the equilibrium condition for θ^R if the matching function takes the form in (34) and $\phi = \alpha$, which is the Hosios condition.

Appendix B: Equilibrium with Taxes

The appendix generalizes the equilibrium conditions to allow for the following types of taxes: the infection tax τ_t^{UI} and τ_t^{EI}, paid every period during the infection but possibly different for employed and unemployed, and τ_t^{US} and τ_t^{ES}, paid by the susceptible workers, again possibly different for employed and unemployed. There are no taxes in the recovered stage, and no taxes on vacancy creation. The appendix further characterizes conditions under which various tax systems are equivalent.

Value functions. The value of the match in the infected stage and the value of the match and of the utility loss from getting infected in the susceptible stage is

$$F_t^I = \tau_t^{UI} - \tau_t^{EI} - b + \beta(1-\lambda)\left[\pi_R F^R + (1 - \pi_R - \pi_D) F_{t+1}^I\right] \tag{37}$$

$$\begin{aligned} F_t^S = {} & y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI} F_{t+1}^I + (1 - \pi_{t+1}^{EI}) F_{t+1}^S\right) - \beta\phi p_{t+1}^S (1 - \pi_{t+1}^{UI}) F_{t+1}^S \\ & + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S \end{aligned} \tag{38}$$

$$\Delta_t^S = K_t^{UI} - \beta K_{t+1}^{UI} - b + \tau_t^{US} - \beta(1 - \pi_{t+1}^{UI})\left(\phi p_{t+1}^S F_{t+1}^S - \Delta_{t+1}^S\right). \tag{39}$$

For completeness, the equilibrium wages are also reported. The equilibrium wage is $w_t^I = (1-\phi)\left(b - \tau_t^{UI} + \tau_t^{EI}\right)$. The wage in the susceptible stage is a solution to

$$w_t^S = y + (1-\phi)\left[\beta(1-\lambda)\left(\pi_{t+1}^{EI} F_{t+1}^I + (1 - \pi_{t+1}^{EI}) F_{t+1}^S\right) - F_t^S\right].$$

The value functions for the employed, unemployed and the firm are not reported here to save space, but they are straightforward generalizations of their laissez-faire counterparts.

Equivalence classes of tax systems. Denote the match value and the unemployment value in the infected stage as a function of a sequence of infection taxes by $F_t^I(\{\tau_t^{UI}, \tau_t^{EI}\})$, and $K_t^{UI}(\{\tau_t^{UI}, \tau_t^{EI}\})$. Then it is easy to show that $F_t^I\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) = F_t^I(0,0) + \eta_t$ and $K_t^{UI}\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) = K_t^{UI}(0,0) - \omega_t$, where

$$\omega_t = \sum_{j=0}^{\infty} \beta^j (1 - \pi_R - \pi_D)^j \tau_{t+j}^{UI} \tag{40}$$

$$\eta_t = \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j (1 - \pi_R - \pi_D)^j (\tau_{t+j}^{UI} - \tau_{t+j}^{EI}) \tag{41}$$

are the present values of the tax liabilities in the unemployed state and in the match overall. That is, paying the infection tax every period and paying its present values, appropriately discounted, yields the same values in the infected stage.

Two tax systems $\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}$ and $\{\tilde{\tau}_t^{US}, \tilde{\tau}_t^{ES}, \tilde{\tau}_t^{UI}, \tilde{\tau}_t^{EI}\}$ are *equivalent* if they generate, for given values of market tightness and infection probabilities $\{\theta_t^S, \pi_t^{UI}, \pi_t^{EI}\}$, the same sequence of values of the match in the susceptible stage $\{F_t^S\}$. To further simplify the exposition, set the optimal infection tax $\{0, 0, \overline{\tau}_t^I, \overline{\tau}_t^I\}$ as out baseline tax system, and relate other tax systems to it, giving the following result:

Proposition 1. *A tax system $\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}$ is equivalent to the baseline tax system $\{0, 0, \overline{\tau}_t^I, \overline{\tau}_t^I\}$ if and only if it satisfies*

$$\tau_t^{US} - \tau_t^{ES} = \beta\left[\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\zeta_{t+1} - (1-\lambda)\eta_{t+1}\right], \tag{42}$$

where the values of $\{\zeta_t\}$ solve

$$\zeta_t = -\tau_t^{US} + \omega_t - \overline{\omega}_t + \beta\left(\overline{\omega}_{t+1} - \omega_{t+1}\right) + \beta(1 - \pi_{t+1}^{UI})\zeta_{t+1}, \tag{43}$$

η_t^I and ω_t are given by (40) and (41) for the alternative infection tax, and $\overline{\omega}_{t+1}$ is given by (40) for the baseline infection tax.

Proof. Let $F_t^I\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right)$ be the value of the match in the infected stage, as a function of the taxes in the infected stage, and $\Delta_t^S\left(\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}\right)$ be the value of the loss from being infected, as a function of the whole tax system. It follows from (38) that the values of the match in the susceptible stage $\{F_t^S\}$ generated by the baseline tax system satisfy the Bellman equation

$$F_t^S = y - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI} F_{t+1}^I\left(\{\overline{\tau}_{t+1}^I, \overline{\tau}_{t+1}^I\}\right) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right)$$
$$- \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{0, 0, \overline{\tau}_{t+1}^I, \overline{\tau}_{t+1}^I\}\right).$$

Take now any tax system $\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}$. If it is to generate the same values of the match $\{F_t^S\}$, it must satisfy

$$F_t^S = y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI} F_{t+1}^I\left(\{\tau_{t+1}^{UI}, \tau_{t+1}^{EI}\}\right) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right)$$
$$- \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{\tau_{t+1}^{US}, \tau_{t+1}^{ES}, \tau_{t+1}^{UI}, \tau_{t+1}^{EI}\}\right).$$

Using the result that $F_t^I\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) = F_t^I(0,0) + \eta_t$ and cancelling terms yields

$$\tau_t^{US} - \tau_t^{ES} = \beta\left[\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\zeta_{t+1} - (1-\lambda)\eta_{t+1}\right], \tag{44}$$

where $\zeta_t = \Delta_t^S\left(\{0, 0, \overline{\tau}_t^I, \overline{\tau}_t^I\}\right) - \Delta_t^S\left(\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}\right)$ is the difference in the utility loss. Both utility losses satisfy (39). Rearranging yields

$$\zeta_t = -\tau_t^{US} + \omega_t - \overline{\omega}_t + \beta\left(\overline{\omega}_{t+1} - \omega_{t+1}\right) + \beta(1-\pi_{t+1}^{UI})\zeta_{t+1}, \tag{45}$$

where $\overline{\omega}_t$ is the present value of thee tax liabilities in the unemployed state under the baseline tax system. □

The following cases are in the equivalence class with the optimal infection tax.

1. **Tax on susceptibles.** Consider a tax system $\{\tau_t^{US}, \tau_t^{ES}, 0, 0\}$, where $\tau_t^{US} = \beta\pi_{t+1}^{UI}\overline{\omega}_{t+1}$ and $\tau_t^{ES} = \beta\pi_{t+1}^{EI}\overline{\omega}_{t+1}$. This tax system shifts all the tax burden to the susceptible stage. It is easy to verify that it satisfies (44) and (47), with $\zeta_t = -\overline{\omega}_t$.

2. **Unemployment insurance benefits.** Consider a tax system $\{\tau_t^{US}, 0, 0, 0\}$ that sets the tax on the employed to be zero in both stages, and the tax on unemployed to be zero in the infected stage. The values of $\{\tau_t^{US}, \zeta_t\}$ are a solution to

$$\tau_t^{US} = \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\zeta_{t+1}, \tag{46}$$
$$\zeta_t = -\tau_t^{US} - \overline{\omega}_t + \beta\overline{\omega}_{t+1} + \beta(1-\pi_{t+1}^{UI})\zeta_{t+1}, \tag{47}$$

The values of $b + \tau_t^{UB} := b - \tau_t^{US}$ are reported in Figure 7b.