

Binomial & Black Scholes Option Pricing Model

Q.1

The three stocks which we have taken from the market are:

1. Apple
2. Microsoft
3. Amazon

As we are looking for the options whose expiry date is within 2-3 months from present, we took the last two months' closing price data of the stocks and calculated their volatility.

The volatility of the stocks are:

	VARIANCE	STANDARD DEVIATION
AAPL	0.000335129	0.018306517
MSFT	0.000432359	0.020793244
AMZN	0.000593705	0.02436607

To calculate annual volatility, we consider 252 trading days in a year. Therefore the annual volatility of the stocks are :

	Volatility (σ) (Daily)	Volatility (σ) (Annual)
AAPL	1.83%	4.48%
MSFT	2.08%	5.09%

AMZN 2.44%

5.97%

Q.2 & Q.3 Binomial model to price the corresponding European call option with different strike price and expiry date and using Black Scholes equation value of European Call options

We have compared the actual price of the options with the price using the binomial 2 step, 10 step, 100 step model and also with black scholes model. Below tables also show the comparison between the price by binomial model and the price by black scholes model. We have done this project using Excel, so for multiple step binomial, we have used Excel VBA. Comparison for all the three stocks one by one are as follow:

1. APPLE INC. (AAPL)

Option Type	Actual Option Price	Time to Maturity (in years)	Strike Price	Option Price using 2 step Binomial	Option Price using 10 step Binomial	Option Price using 100 step Binomial	Option Price using Black Scholes
				model	model	Model	Model
CALL	\$ 142.10	0.234	\$ 28.75	\$ 150.544	\$ 150.544	\$ 150.544	\$ 150.544
CALL	\$ 44.30	0.555	\$ 130.00	\$ 43.64	\$ 43.64	\$ 43.65	\$ 43.651
CALL	\$ 111.95	1.230	\$ 60.00	\$ 113.66	\$ 113.66	\$ 113.668	\$ 113.669
CALL	\$ 45.45	0.0714	\$ 125	\$ 43.5	\$ 43.5	\$ 43.535	\$ 45.535
CALL	\$ 65.55	0.119	\$ 105	\$ 65.7	\$ 65.7	\$ 65.712	\$ 65.713
PUT	\$ 39.45	1.230	\$ 200	\$ 17.98	\$ 17.980	\$ 18.04	\$ 18.055
PUT	\$ 17.90	0.234	\$ 185	\$ 12.758	\$ 12.758	\$ 12.757	\$ 12.757

2. Microsoft Corporation (MSFT)

Option Type	Actual Option Price	Time to Maturity (in years)	Strike Price	Option Price using 2 step Binomial model	Option Price using 10 step Binomial model	Option Price using 100 step Binomial Model	Option Price using Black Scholes Model
CALL	\$ 149.45	0.333	\$ 150	\$ 151.6	\$151.63	\$ 151.636	\$ 151.639
CALL	\$ 137.85	0.674	\$ 180	\$ 124.7	\$ 124.7	\$ 124.75	\$ 124.760
CALL	\$ 159	0.139	\$ 140	\$ 175.9	\$ 175.9	\$ 175.938	\$ 175.939
CALL	\$ 194	0.916	\$ 105	\$ 196.6	\$ 196.6	\$ 196.670	\$ 196.673
CALL	\$ 126.5	1.11	\$ 185	\$ 122.3	\$ 122.3	\$ 122.3065	\$ 122.307
PUT	\$ 16.2	0.198	\$ 305	\$ 5.71	\$ 5.88	\$ 5.92	\$ 5.92
PUT	\$ 46.6	0.488	\$ 335	\$ 13.06	\$ 13.62	\$ 13.76	\$ 13.77

3. Amazon.com, Inc. (AMZN)

Option Type	Actual Option Price	Time to Maturity (in years)	Strike Price	Option Price using 2 step Binomial model	Option Price using 10 step Binomial model	Option Price using 100 step Binomial Model	Option Price using Black Scholes Model
CALL	\$ 1694.8	0.095	\$ 1400	\$ 1879.6	\$ 1879.6	\$ 1879.63	\$ 1879.641
CALL	\$ 1761.05	0.214	\$ 1340	\$ 1955.3	\$ 1955.3	\$ 1955.362	\$ 1955.381
CALL	\$ 960.20	0.643	\$ 2200	\$ 805.84	\$ 805.84	\$ 805.846	\$ 805.9399
CALL	\$ 1040.9	0.933	\$ 2150	\$ 1334.1	\$ 1334.1	\$ 1334.19	\$ 1334.247
CALL	\$ 1603.85	0.552	\$ 1580	\$ 1691.2	\$ 1691.2	\$ 1691.28	\$ 1691.25

Annual risk free rate which we have considered is 5%.

Formulas related to Black & Scholes Model:

$$Call = SN(d_1) - Ke^{-r \cdot T} N(d_2)$$

$$Put = Ke^{-r \cdot T} N(-d_2) - SN(-d_1)$$

Where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2) \cdot T}{\sigma\sqrt{T}}; d_2 = \frac{\ln(S/K) + (r - \sigma^2/2) \cdot T}{\sigma\sqrt{T}}$$

Q4. Black Scholes formula is limiting case of Binomial model as n goes to infinity its approaches the black scholes call option value

As we increase the number of steps in the Binomial Model, the binomial model tends to the Black Scholes model. In the above tables if we compare the results of Binomial Model with 100 steps and Black Scholes Model, they are approximately equal. Which verifies that the Black Scholes formula is the limiting case of the Binomial Model.

In fact the Black-Scholes model for European options is really a special case of the binomial model where the number of binomial steps is infinite. In other words, the binomial model provides discrete approximations to the continuous process underlying the Black-Scholes model.

Following are the values of Option price using 100 step Binomial Model and Option price using Black Scholes formula. These values are very close which verifies that Black Scholes formula is the limiting case of the Binomial Model.

For APPLE

Option Price using 100 step Binomial Model	Option Price using Black Scholes Model
\$ 150.544	\$ 150.544
\$ 43.65	\$ 43.651
\$ 113.668	\$ 113.669
\$ 43.535	\$ 45.535
\$ 65.712	\$ 65.713
\$ 18.04	\$ 18.055
\$ 12.757	\$ 12.757

For Microsoft

Option Price using 100 step Binomial Model	Option Price using Black Scholes Model
\$ 151.636	\$ 151.639
\$ 124.75	\$ 124.760
\$ 175.938	\$ 175.939
\$ 196.670	\$ 196.673
\$ 122.3065	\$ 122.307
\$ 5.92	\$ 5.92
\$ 13.76	\$ 13.77

For Amazon

Option Price using 100 step Binomial Model	Option Price using Black Scholes Model
\$ 1879.63	\$ 1879.641
\$ 1955.362	\$ 1955.381
\$ 805.846	\$ 805.9399
\$1334.19	\$ 1334.247
\$ 1691.28	\$1691.25

Q.5 Value of Delta for every week of three stocks

In this question we have applied implied volatility concept on one stock i.e. Amazon, We have selected the call option from the market and calculated the value of Delta and implied volatility for each week.

The Call Option which we have chosen for AMAZON from market is as following :

Previous Close	0.00
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Open	273.61
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Bid	239.60
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Ask	257.25
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Strike	2,910.00
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Expire Date	2022-05-06
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Day's Range	273.61 273.61
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Contract Range	N/A
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Volume	1
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Open Interest	2
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The time $t = 0$ is 08/04/2022.

As there are 4 week and 5 trading days in each week between 8 april 2022 to 6 may 2022. We took the value of initial volatility at $t = 0$ from the previous data and then minimizing the difference between actual option price and Black Scholes Options price by changing the value of volatility, which is basically implied volatility.

- At $t = 0$

$$\Delta(0) = 0.724$$

$$\text{Implied Volatility} = 42.23 \%$$

- At $t = 1$ week = 5 days

$$\Delta(1) = 0.7188$$

$$\text{Implied Volatility} = 49.54 \%$$

- At $t = 2$ week = 10 days

$$\Delta(2) = 0.7138$$

$$\text{Implied Volatility} = 61.60 \%$$

- At $t = 3$ week = 15 Days

$$\Delta(3) = 0.7088$$

$$\text{Implied Volatility} = 88.41 \%$$

Q.6 Justification for why prices have different value for market and value obtained by using formula

Option pricing theory uses variables (stock price, exercise price, volatility, interest rate, time to expiration) to theoretically value an option. Essentially, it provides an estimation of an option's fair value which traders incorporate into their strategies to maximize profits. The models which we have used to value the options are Binomial and Black Scholes. These theories have wide margins for error due to deriving their values from other assets, usually the price of a company's common stock.

The two main factors we consider because of which the option price is different for the market are volatility and risk free rate of interest. The volatility is subjective and varies person to person, it depends on the number of past days we are considering to calculate the volatility of the stock. The best practice is to take the past data of the same time to calculate the volatility which is remaining in the maturity of the option.

Another reason which we can incorporate is the interest rate. The risk free rate of interest which we are considering in both the models depends on so many factors and can not be exactly known. We generally take a risk free rate by making some assumption or using government bond which is not exactly same as the rate which is being used in the market.

