Sorting

Introduction

- Sorting refers to arranging a set of data in some logical order

- For ex. A telephone directory can be considered as a list where each record has three fields - name, address and phone number.

- Being unique, phone number can work as a key to locate any record in the list.

Sorting refers to the operation of arranging data in some give order such as increasing or decreasing order with numerical data or alphabetically with character data.

Given record r1,r2,…rn with key values k1,k2….kn. Such that k1≤k2≤…kn
The complexity of sorting algorithm can be measured in terms of:

i) No of algorithm steps to sort the records.
ii) No of comparison between key(s).

Introduction

- Sorting is among the most basic problems in algorithm design.

- We are given a sequence of items, each associated with a given key value. And the problem is to rearrange the items so that they are in an increasing(or decreasing) order by key.

- The methods of sorting can be divided into two categories:
  - **Internal** Sorting
  - **External** Sorting

- **Internal Sorting**
  - ✓If all the data that is to be sorted can be adjusted at a time in main memory, then internal sorting methods are used

- **External Sorting**
  - ✓When the data to be sorted can't be accommodated in the memory at the same time and some has to be kept in auxiliary memory, then external sorting methods are used.

❖NOTE: We will only consider **internal sorting**

Efficiency of Sorting Algorithm

- The complexity of a sorting algorithm measures the running time of a function in which **n** number of items are to be sorted.

- The choice of sorting method depends on efficiency considerations for different problems.

- Tree most important of these considerations are:
  - The length of time spent by programmer in coding a particular sorting program
  - Amount of machine time necessary for running the program
  - The amount of memory necessary for running the program

Types of Sorting

Internal Sorting

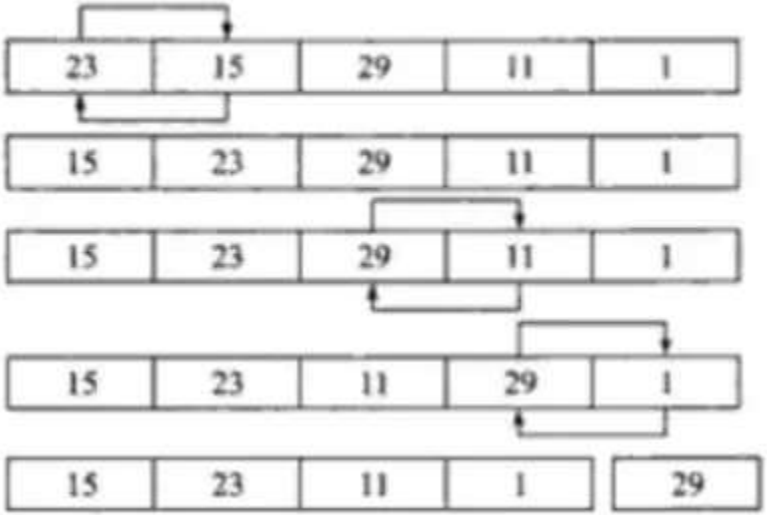
- Exchange Sort
- Quick Sort
- Bubble Sort
- Heap Sort
- Insertion Sort
- Selection Sort
- Shell Sort

External Sorting

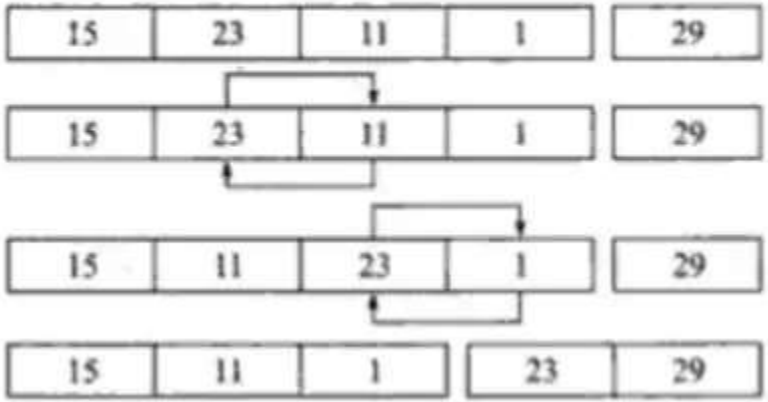
- Merge Sort
- Radix Sort
- Polyphase Sort

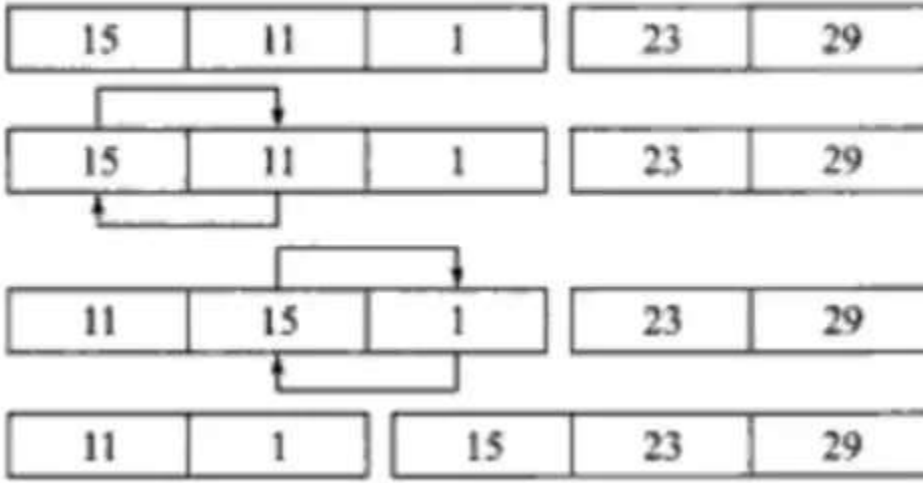
BUBBLE SORT

- In bubble sort, each element is compared with its adjacent element.
- We begin with the 0^{th} element and compare it with the 1^{st} element.
- If it is found to be greater than the 1^{st} element, then they are interchanged.
- In this way all the elements are compared (excluding last) with their next element and are interchanged if required
- On completing the first iteration, largest element gets placed at the last position. Similarly in second iteration second largest element gets placed at the second last position and so on.

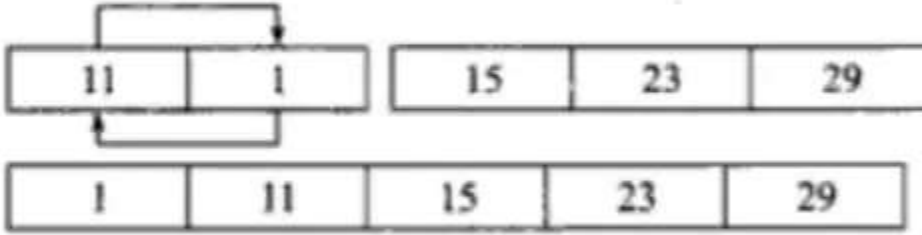


First iteration





Third iteration



Fourth iteration

Fig. 11.1 *Bubble Sort*

Algorithm

BUBBLED(Data, N)
Here Data is in array with N elements. This algorithm sorts the elements in Data.
Step1: Repeat step 2 and step 3 for K=1 to N-1
Step2: Set Ptr:=1
Step3: Repeat while Ptr<=N-K
    a. If Data[Ptr]>Data[Ptr+1], Interchange Data[Ptr] andData[Ptr+1]
    b. Set Ptr:=Ptr+1
Step4: Exit

TIME COMPLEXITY

- The time complexity for bubble sort is calculated in terms of the number of comparisons f(n) (or of number of loops)

- Here two loops(outer loop and inner loop) iterates(or repeated) the comparison.

- The inner loop is iterated one less than the number of elements in the list (i.e., n-1 times) and is reiterated upon every iteration of the outer loop

$$f(n) = (n-1) + (n-2) + \ldots\ldots + 2 + 1$$
$$= n(n-1) = O(n2).$$

TIME COMPLEXITY

- Best Case
  - sorting a sorted array by bubble sort algorithm
  - In best case outer loop will terminate after one iteration, i.e it involves performing one pass which requires n-1 comparison

$$f(n) = O(n^2)$$

- Worst Case
  - Suppose an array [5,4,3,2,1], we need to move first element to end of an array
  - n-1 times the swapping procedure is to be called

$$f(n) = O(n^2)$$

- Average Case
  - Difficult to analyse than the other cases
  - Random inputs, so in general

$$f(n) = O(n^2)$$

- <u>Space Complexity</u>
  - O(n)

Exchange Sort

The exchange sort is almost similar as the bubble sort. In fact some refer to the exchange sort as just a different bubble sort.

The exchange sort compares each element of an array and swap those elements that are not in their proper position, just like a bubble sort does. The only difference between the two sorting algorithms is the manner in which they compare the elements.

Exchange Sort

```
//Algorithm
        for(i = 0; i < (length -1); i++)
        {
                for (j=(i + 1); j < length; j++)
                {
                        if (array[i] < array[j])
                        {
                                temp = array[i];
                                array[i] =
array[j];
                                array[j] = temp;
                        }
                }
        }
```

SELECTION SORT

- Find the least(or greatest) value in the array, swap it into the leftmost(or rightmost) component, and then forget the leftmost component, Do this repeatedly.

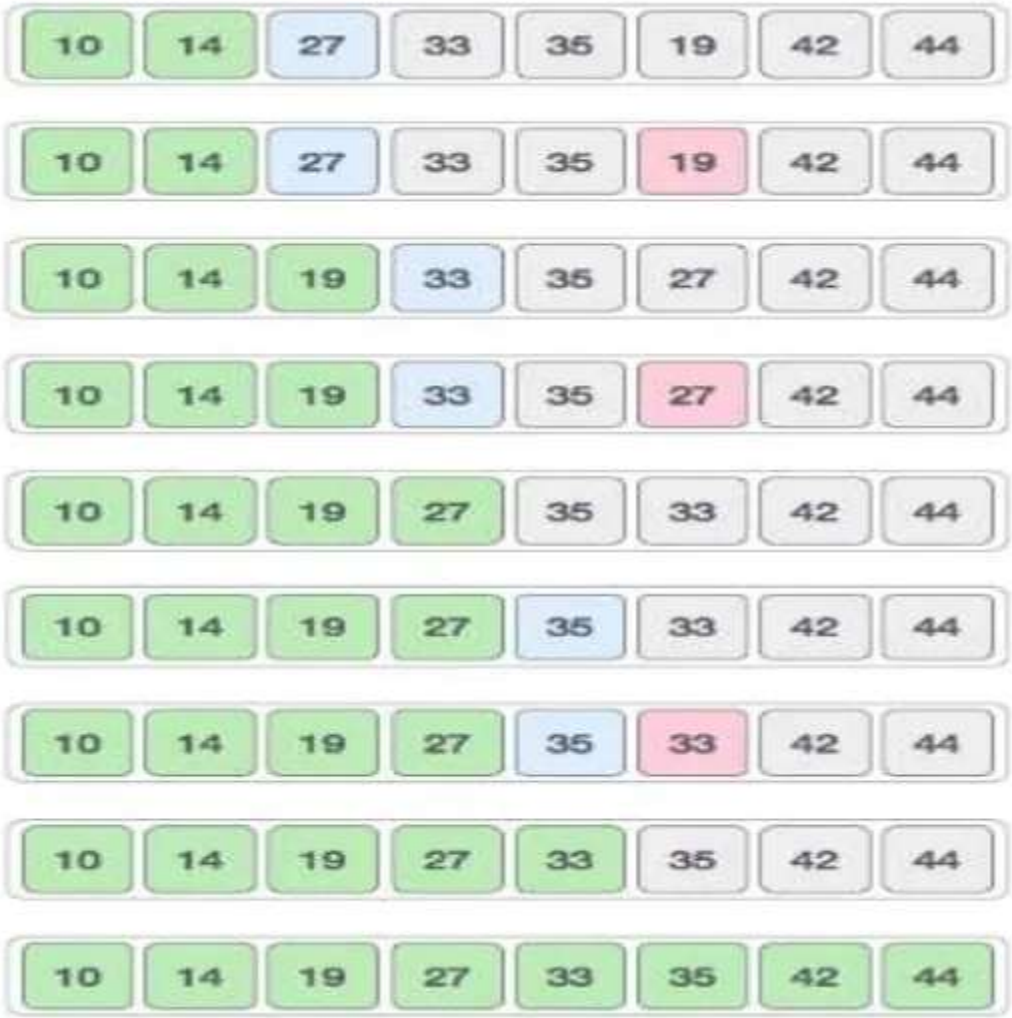
- Let a[n] be a linear array of n elements. The selection sort works as follows:

- Pass 1: Find the location **loc** of the smallest element in the list of n elements a[0], a[1], a[2], a[3], ……,a[n-1] and then interchange a[loc] and a[0].

- Pass 2: Find the location loc of the smallest element int the sub-list of n-1 elements a[1], a[2], a[3], ……,a[n-1] and then interchange a[loc] and a[1] such that a[0], a[1] are sorted.

- Then we will get the sorted list

  a[0]<=a[2]<=a[3]……<=a[n-1]


10 14 27 33 35 19 42 44
10 14 27 33 35 19 42 44
10 14 19 33 35 27 42 44
10 14 19 33 35 27 42 44
10 14 19 27 35 33 42 44
10 14 19 27 35 33 42 44
10 14 19 27 35 33 42 44
10 14 19 27 33 35 42 44
10 14 19 27 33 35 42 44

Algorithm:

```
SelectionSort(A)
{
        for( i = 0;i < n ;i++)
        {
                least=A[i];
                p=i;
                for ( j = i + 1;j < n ;j++)
                {
                        if (A[j] < A[i])
                        least= A[j]; p=j;
                }
        }
        swap(A[i],A[p]);
}
```

Time Complexity

- Inner loop executes (n-1) times when i=0, (n-2) times when i=1 and so on:
- Time complexity = (n-1) + (n-2) + (n-3) + ... +2+1

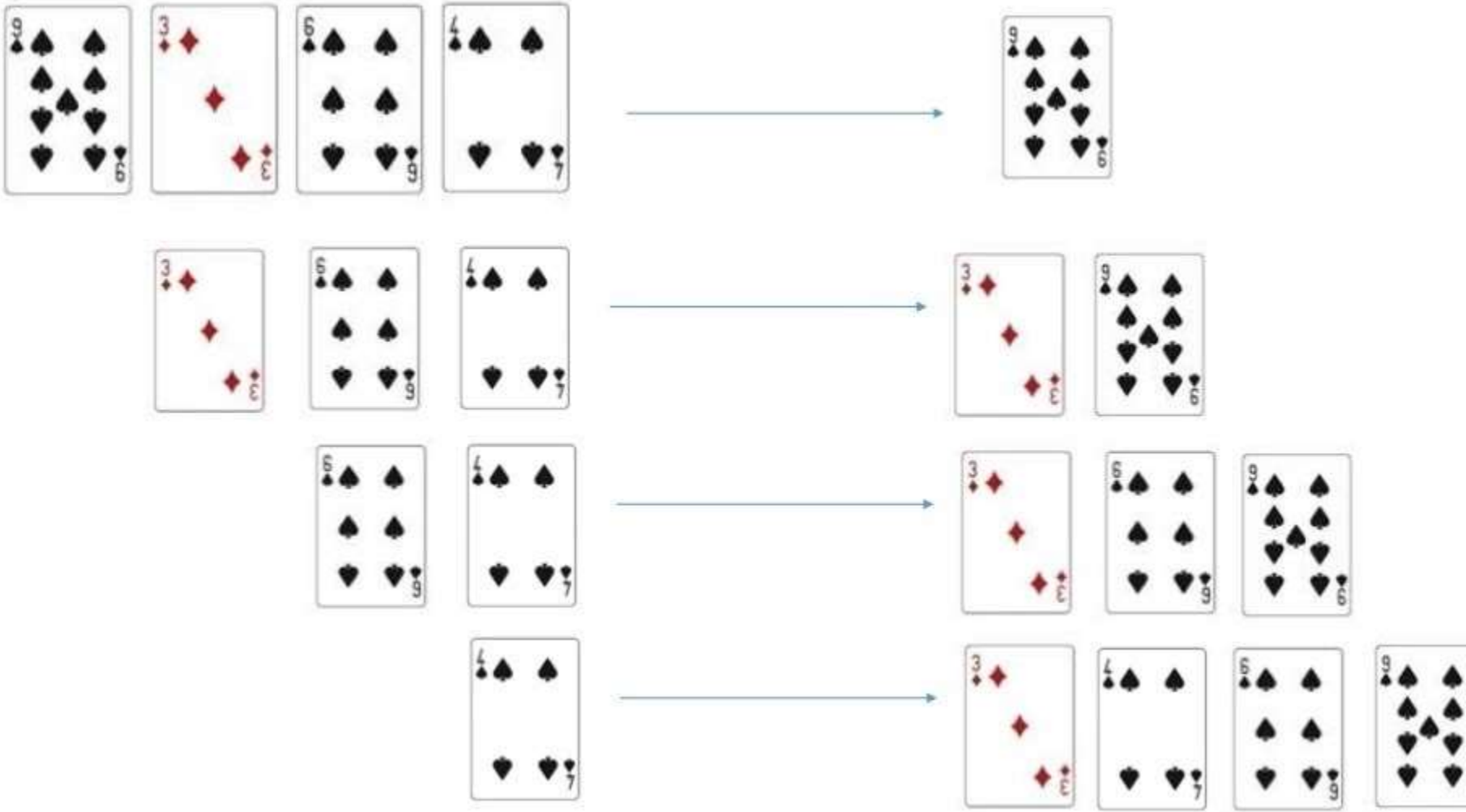
  $= O(n^2)$

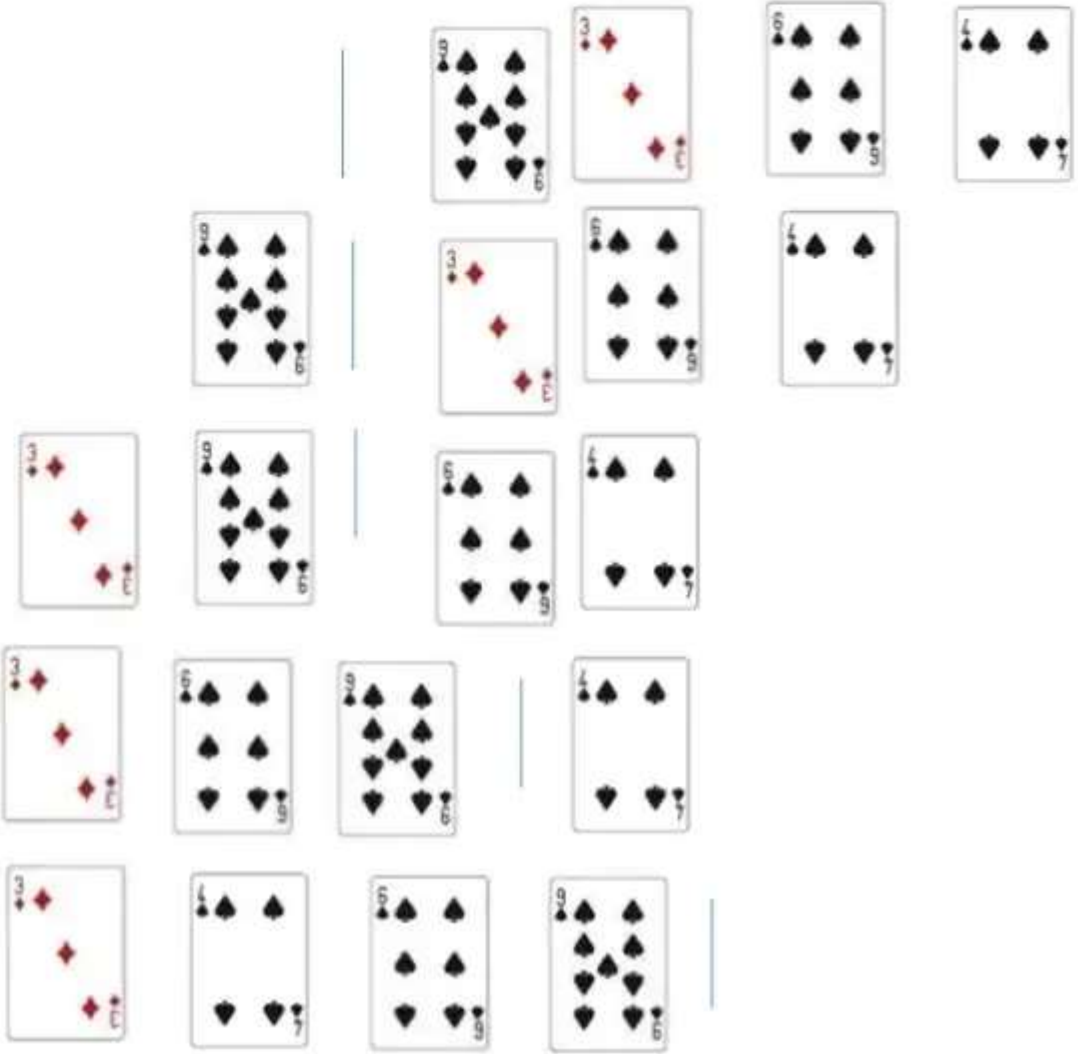
Space Complexity

- Since no extra space beside n variables is needed for sorting so
- O(n)

Insertion Sort

- Like sorting a hand of playing cards start with an empty hand and the cards facing down the table.
- Pick one card at a time from the table, and insert it into the correct position in the left hand.
- Compare it with each of the cards already in the hand, from right to left
- The cards held in the left hand are sorted.





Insertion Sort

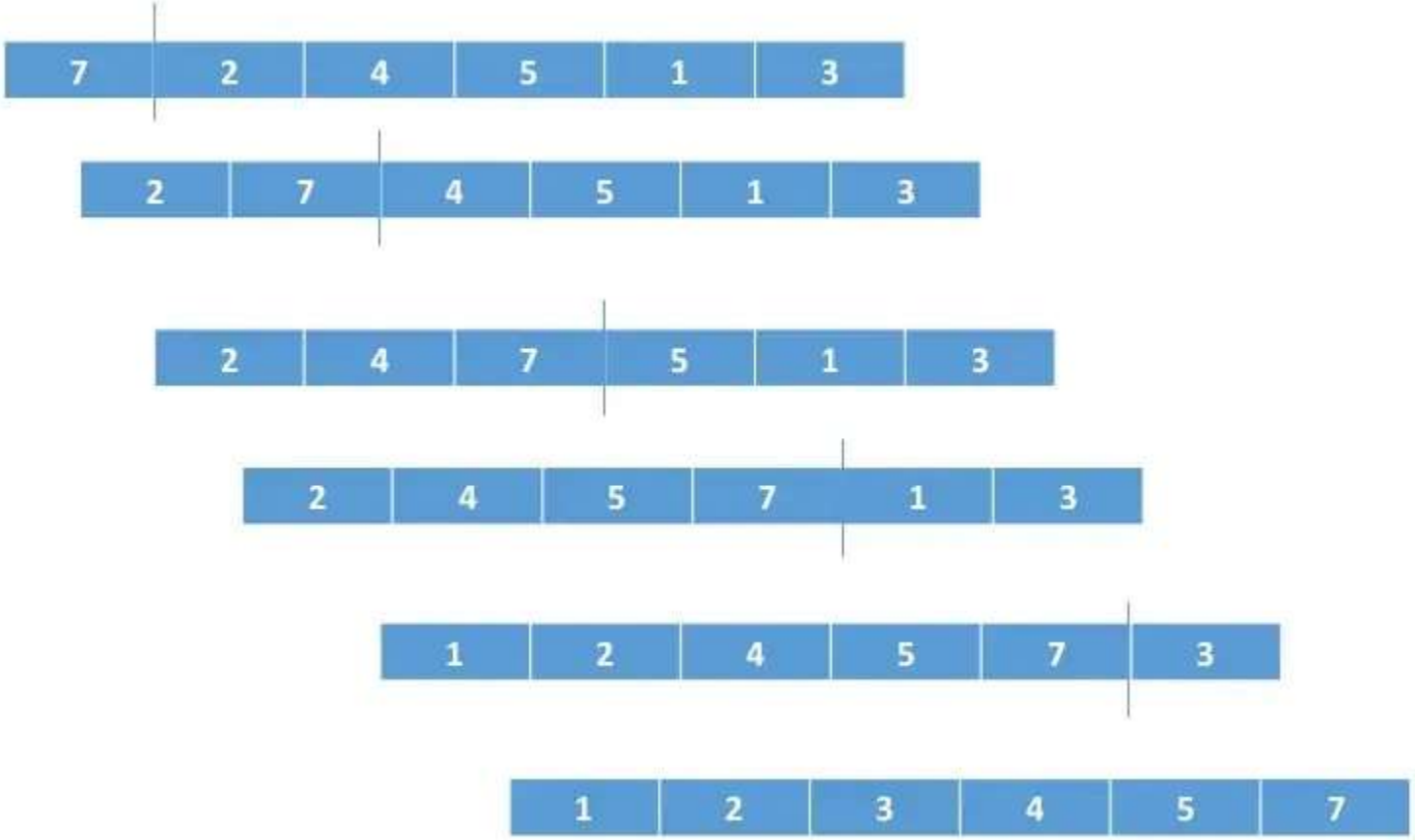
- Suppose an array a[n] with n elements. The insertion sort works as follows:

Pass 1: a[0] by itself is trivially sorted.

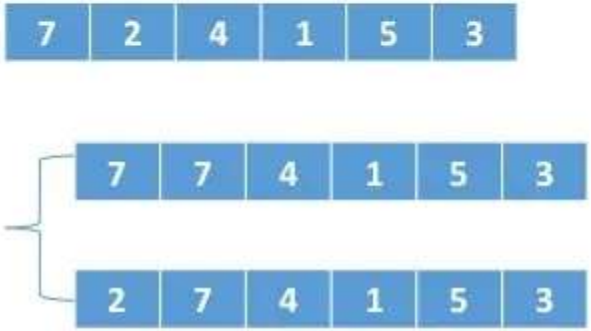
Pass 2: a[1] is inserted either before or after a[0] so that a[0], a[1] is sorted.

Pass 3: a[2] is inserted into its proper place in a[0],a[1] that is before a[0], between a[0] and a[1], or after a[1] so that a[0],a[1],a[2] is sorted.

pass N: a[n-1] is inserted into its proper place in a[0],a[1],a[2],.....,a[n-2] so that a[0],a[1],a[2],.....,a[n-1] is sorted with n elements.


7 2 4 5 1 3
2 7 4 5 1 3
2 4 7 5 1 3
2 4 5 7 1 3
1 2 4 5 7 3
1 2 3 4 5 7

Algorithm



i	value	hole
1	2	1
1	2	0

```
InsertionSort(){
        for (i=1;i<n;i++){
                value=C[ i ];
                hole= i ;
                while(hole>0 && C[hole-1]>value){
                        C[hole]=C[hole-1];
                        hole=hole-1;
                }
                C[hole]=value;
        }
}
```

Time Complexity

- Best Case:
  - If the array is all but sorted then
  - Inner Loop wont execute so only some constant time the statements will run
  - So Time complexity= O(n)

- Worst Case:
  - Array element in reverse sorted order
  - Time complexity=$O(n^2)$

- Space Complexity
  - Since no extra space beside n variables is needed for sorting so
  - Space Complexity = O(n)

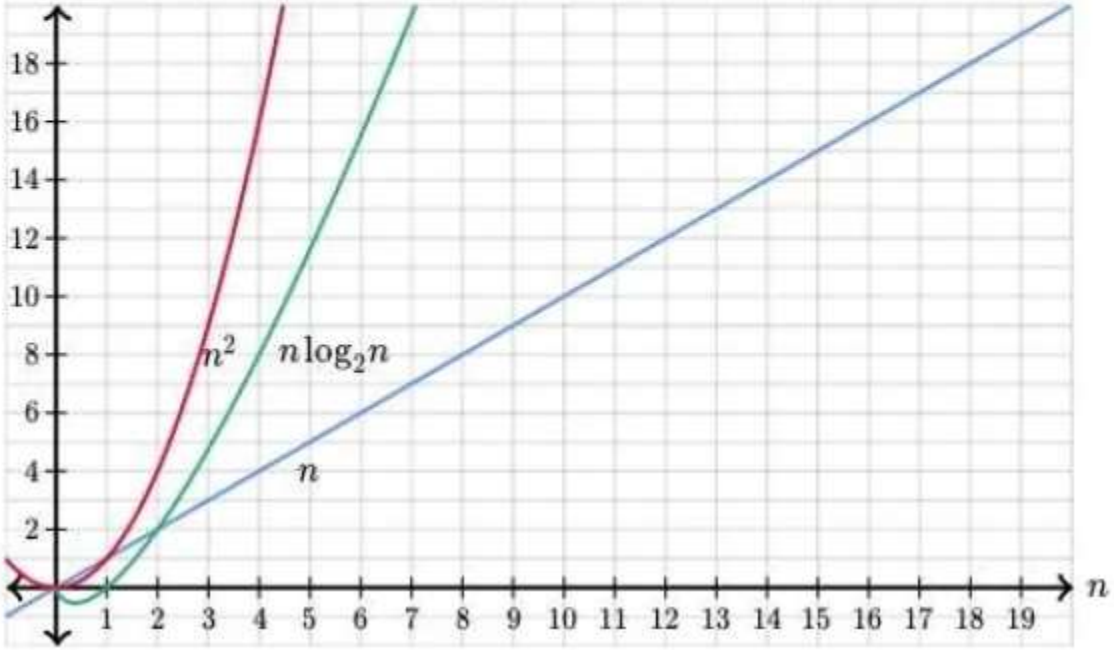
Divide and conquer algorithms

- The sorting algorithms we've seen so far have worst-case running times of $O(n^2)$

- When the size of the input array is large, these algorithms can take a long time to run.

- Now we will discuss two sorting algorithms whose running times are better
  - Merge Sort
  - Quick Sort

Algorithm	Worst-case running time	Best-case running time	Average-case running time
Selection sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Insertion sort	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n^2)$
Merge sort	$\Theta(n \lg n)$	$\Theta(n \lg n)$	$\Theta(n \lg n)$
Quicksort	$\Theta(n^2)$	$\Theta(n \lg n)$	$\Theta(n \lg n)$

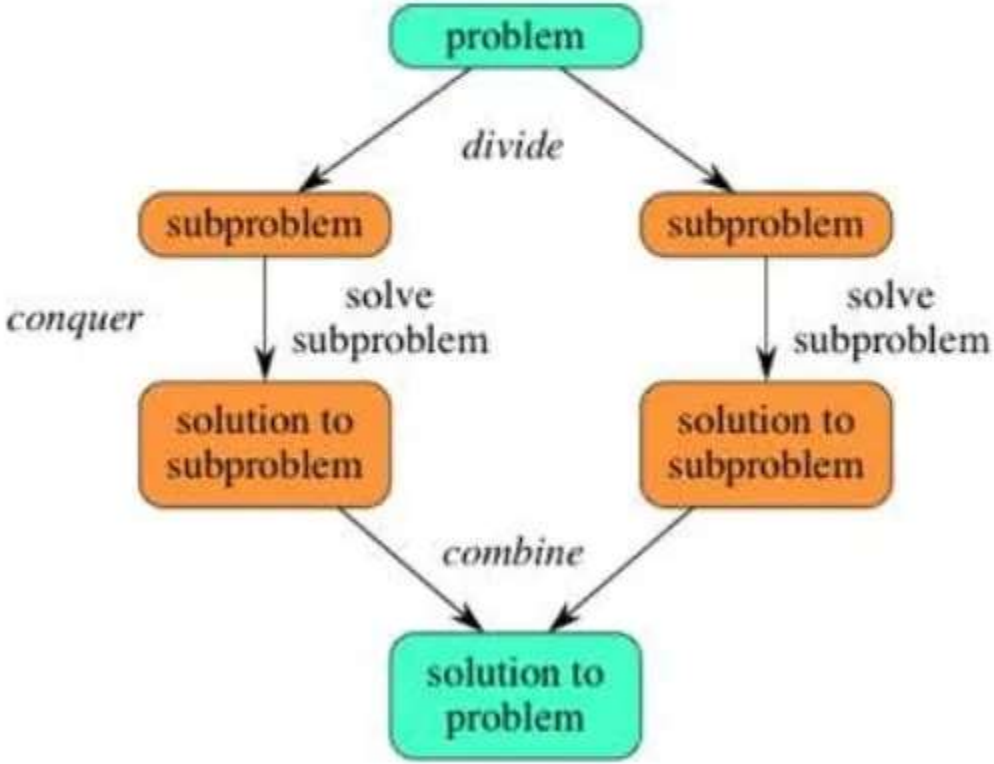


Divide-and-conquer

- **Divide-and-conquer**, breaks a problem into sub problems that are similar to the original problem, recursively solves the sub problems, and finally combines the solutions to the sub problems to solve the original problem.

- Think of a divide-and-conquer algorithm as having three parts:
  - **Divide** the problem into a number of subproblems that are smaller instances of the same problem.
  - **Conquer** the subproblems by solving them recursively. If they are small enough, solve the subproblems as base cases.
  - **Combine** the solutions to the subproblems into the solution for the original problem.

Divide-and-conquer



Merge Sort

- Merge sort is a sorting technique based on divide and conquer technique.

- Merge sort first divides the array into equal halves and then combines them in a sorted manner.

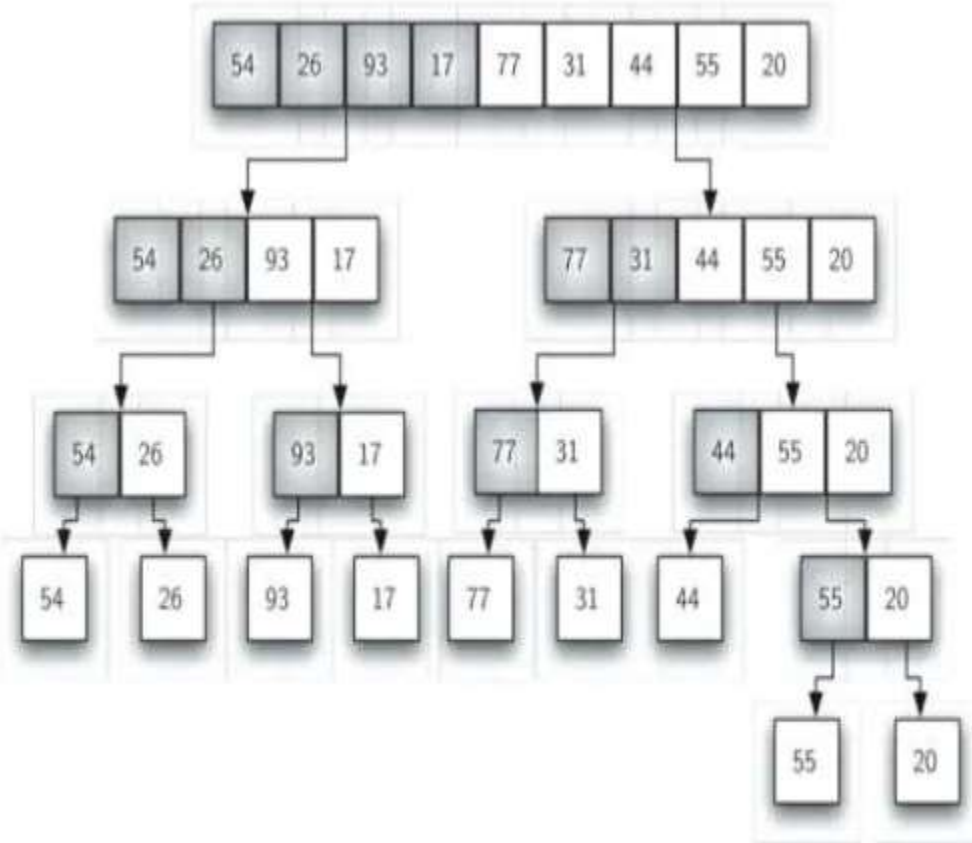
- With worst-case time complexity being **O(n log n)**, it is one of the most respected algorithms.

Merge Sort

- Because we're using divide-and-conquer to sort, we need to decide what our sub problems are going to be.

- Full Problem: Sort an entire Array
- Sub Problem: Sort a sub array

- Lets assume **array[p..r]** denotes this subarray of array.
- For an array of n elements, we say the original problem is to sort **array[0..n-1]**


54 26 93 17 77 31 44 55 20
54 26 93 17
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54 26
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77 31
44 55 20
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26
93
17
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55 20
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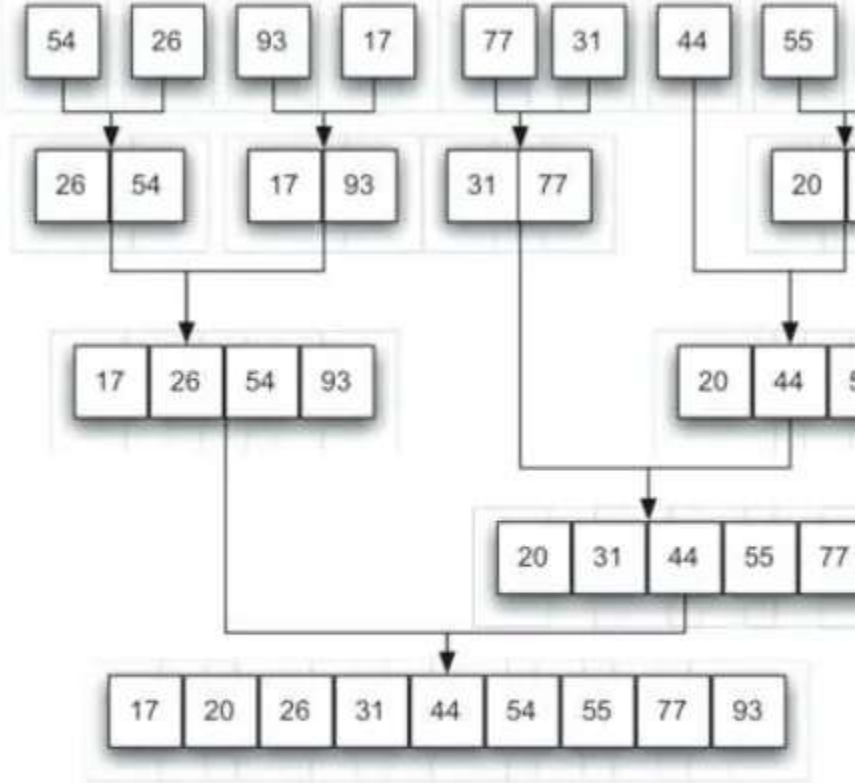
54 26 93 17 77 31 44 55
26 54
17 93
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17 26 54 93
20 44
20 31 44 55 77
17 20 26 31 44 54 55 77 93

Merge Sort

- Here's how merge sort uses divide and conquer
  1. ***Divide*** by finding the number **q** of the position midway between **p** and **r**. Do this step the same way we found the midpoint in binary search: add p and r, divide by 2, and round down.

  2. ***Conquer*** by recursively sorting the subarrays in each of the two sub problems created by the divide step. That is, recursively sort the subarray **array[p..q]** and recursively sort the subarray **array[q+1..r].**

  3. ***Combine*** by merging the two sorted subarrays back into the single sorted subarray **array[p..r].**

Merge Sort

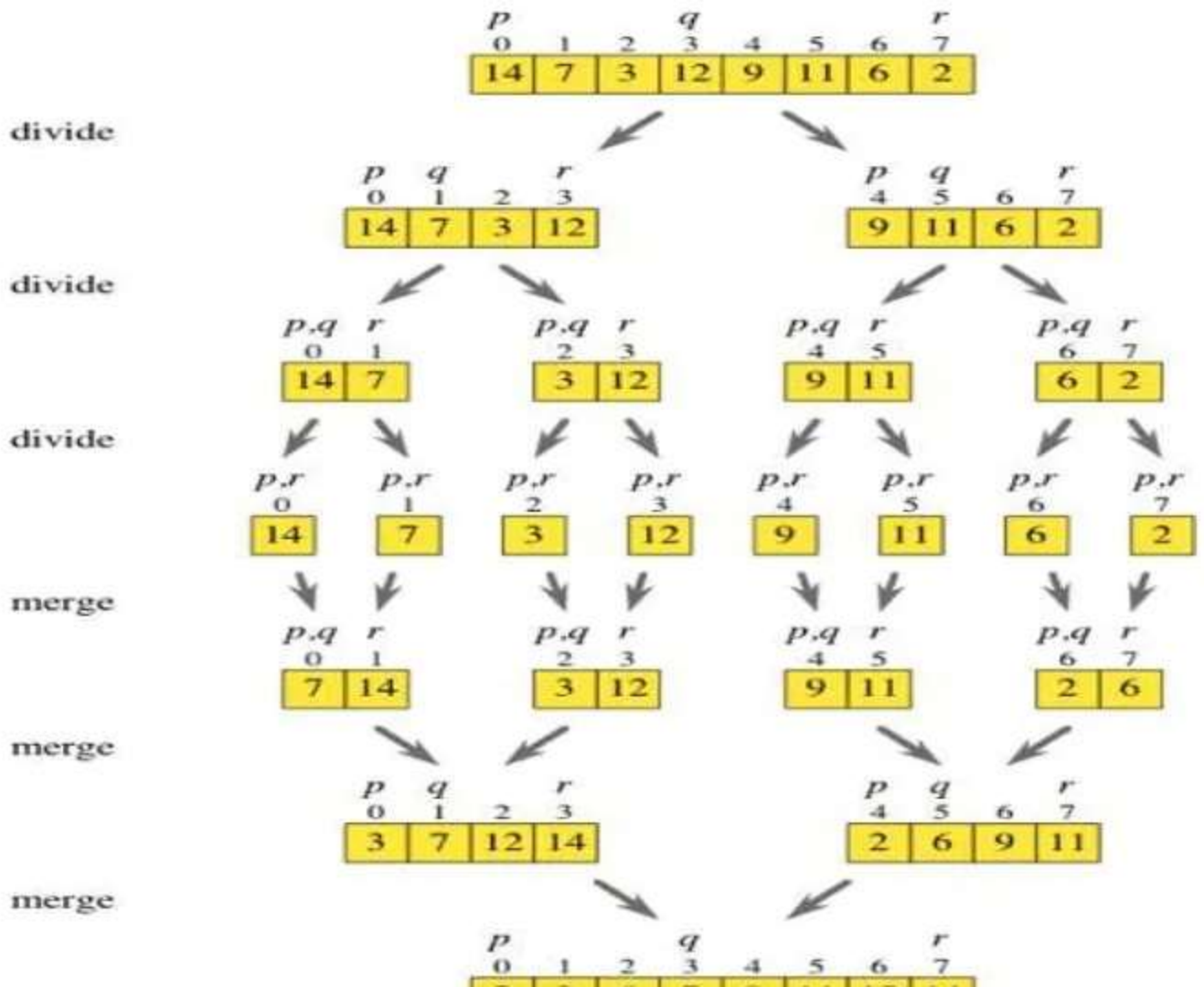
- Let's start with array holding [14,7,3,12,9,11,6,2]
- We can say that **array[0..7]** where p=0 and r=7
- In the **divide** step we compute **q=3**
- The **conquer** step has us sort the two subarrays
  - array[0..3] = [**14,7,3,12**]
  - array[4..7]= [**9,11,6,2**]
  - When we comeback from the conquer step, each of the two subarrays is sorted i.e.
  - array[0..3] = [3,7,12,14]
  - array[4..7]= [2,6,9,11]
- Finally, the **combine** step merges the two sorted subarrays in first half and the second half, producing the final sorted array **[2,3, 6,7,9, 11, 12,14]**

How did the subarray array[0..3] become sorted?

- It has more than two element so it's not a base case.
- So with p=0 and r=3, compute q=1, recursively sort array[0..1] and array[2..3], resulting in array[0..3] containing [7,14,3,12] and merge the fist half with the second half, producing [3,7,12,14]

How did the subarray array[0..1] become sorted?

- With p=0 and r=1, compute q=0, recursively sort array[0..0] ([14]) and array[1..1] ([7]), resulting in array[0..1] still containing [14, 7], and merge the first half with the second half, producing [7, 14].


p q r
0 1 2 3 4 5 6 7
14 7 3 12 9 11 6 2
divide
p q r
0 1 2 3
14 7 3 12
p q r
4 5 6 7
9 11 6 2
divide
p,q r
0 1
14 7
p,q r
2 3
3 12
p,q r
4 5
9 11
p,q r
6 7
6 2
divide
p,r
0
14
p,r
1
7
p,r
2
3
p,r
3
12
p,r
4
9
p,r
5
11
p,r
6
6
p,r
7
2
merge
p,q r
0 1
7 14
p,q r
2 3
3 12
p,q r
4 5
9 11
p,q r
6 7
2 6
merge
p q r
0 1 2 3
3 7 12 14
p q r
4 5 6 7
2 6 9 11
merge
p q r
0 1 2 3 4 5 6 7

Analysis of merge Sort

- We can view merge sort as creating a tree of calls, where each level of recursion is a level in the tree.

- Since number of elements is divided in half each time, the tree is balanced binary tree.

- The height of such a tree tend to be **log n**

Analysis of merge Sort

- Divide and conquer
- Recursive
- Stable
- Not In-place
- 0(n) space complexity
- 0(nlogn) time complexity

Recursive function calls and finally merge	Merge function merges the whole block of array

void MergeSort(int *A,int n) {
int mid,i, *L, *R;
if(n < 2) return;
mid = n/2;
L = (int*)malloc(mid*sizeof(int));
R = (int*)malloc((n- mid)*sizeof(int));
 	
for(i = 0;i<mid;i++)
L[i] = A[i]; // creating left subarray
for(i = mid;i<n;i++)
R[i-mid] = A[i]; // creating right subarray
 	
MergeSort(L,mid);
MergeSort(R,n-mid);
Merge(A,L,mid,R,n-mid);
free(L);
free(R);
}
	void Merge(int *A,int *L,int leftCount,int *R,int rightCount) {
	int i,j,k;
	i = 0; j = 0; k =0;
	while(i<leftCount && j< rightCount) {
	if(L[i] < R[j]) A[k++] = L[i++];
	else A[k++] = R[j++];
	}
	while(i < leftCount)
	A[k++] = L[i++];
	while(j < rightCount)
	A[k++] = R[j++];
	}

Analysis of merge Sort

- The time to merge sort n numbers is equal to the time to do two recursive merge sorts of size n/2 plus the time to merge, which is linear.
- We can express the number of operations involved using the following recurrence relations

$$T(1)=1$$

$$T(n) =2T(n/2)+n$$

- Going further down using the same logic

$$T(n/2)=2T(n/2)+n/2$$

- Continuing in this manner, we can write

$$T(n)=nT(1)+n\log n$$

$$=n+n\log n$$

$$T(n)=0(n\log n)$$

Analysis of merge Sort

- Although merge sort's running time is very attractive it is not preferred for sorting data in main memory.

- Main problem arises when the it uses linear extra memory as we need to copy the original array into two arrays of half the size and the additional work spent on copying to the temporary array and back

- This might considerably slow down the time to sort

- However merge sort can work very well with linked list as it doesn't require additional space. Since we only need to change pointer links rather than shifting the element.

Quick Sort

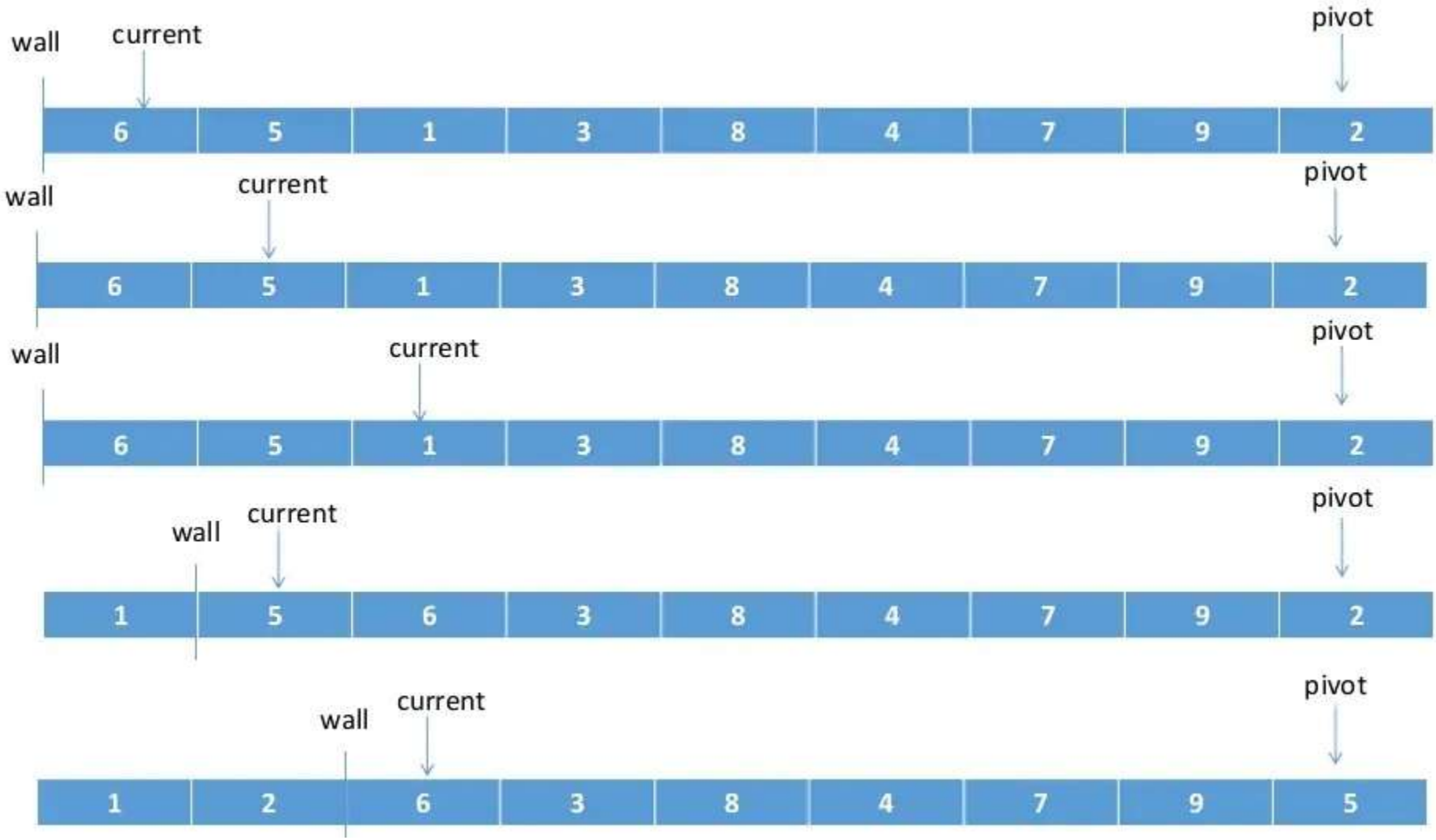
- Quick sort is one of the most popular sorting techniques.
- As the name suggests the quick sort is the fastest known sorting algorithm in practice.
- It has the best average time performance.
- It works by partitioning the array to be sorted and each partition in turn sorted recursively. Hence also called **partition exchange sort.**

Quick Sort

- In partition one of the array elements is choses as a pivot element

- Choose an element pivot=a[n-1]. Suppose that elements of an array a are partitioned so that pivot is placed into position I and the following condition hold:
  - Each elements in position 0 through i-1 is less than or equal to pivot
  - Each of the elements in position i+1 through n-1 is greater than or equal to key

- The pivot remains at the i^{th} position when the array is completely sorted. Continuously repeating this process will eventually sort an array.


wall
current
pivot
6 5 1 3 8 4 7 9 2
wall
current
pivot
6 5 1 3 8 4 7 9 2
wall
current
pivot
6 5 1 3 8 4 7 9 2
wall
current
pivot
1 5 6 3 8 4 7 9 2
wall
current
pivot
1 2 6 3 8 4 7 9 5

Algorithm

- Choosing a pivot
  - To partition the list we first choose a pivot element

- Partitioning
  - Then we partition the elements so that all those with values less than pivot are placed on the left side and the higher vale on the right
  - Check if the current element is less than the pivot.
    - If lesser replace it with the current element and move the wall up one position
    - else move the pivot element to current element and vice versa
- Recur
  - Repeat the same partitioning step unless all elements are sorted

```
function partitionFunc(left, right, pivot)
   leftPointer = left
   rightPointer = right - 1

   while True do
      while A[++leftPointer] < pivot do
         //do-nothing
      end while

      while rightPointer > 0 && A[--rightPointer] > pivot do
         //do-nothing
      end while

      if leftPointer >= rightPointer
         break
      else
         swap leftPointer,rightPointer
      end if

   end while

   swap leftPointer,right
   return leftPointer

end function
```

```
procedure quickSort(left, right)

   if right-left <= 0
      return
   else
      pivot = A[right]
      partition = partitionFunc(left, right,
pivot)
      quickSort(left,partition-1)
      quickSort(partition+1,right)
   end if

end procedure
```

Analysis of Quick Sort

- Best case
  - The best case analysis assumes that the pivot is always in the middle
  - To simplify the math, we assume that the two sublists are each exactly half the size of the original T(N)=T(N/2)+T(N/2)….+1 leads to T(N)=**O(nlogn)**

- Average case
  - T(N)=**O(nlogn)**

- Worst case
  - When we pick minimum or maximum as pivot then we have to go through each and every element so
  - T(N) = **$O(n^2)$**

Heap

- *A heap is defined as an almost complete binary tree o nodes n such that value of each node is less than or equal to the value of the father OR greater than or equal to the value of the father.*

  - ***Descending heap*** is an almost complete binary tree in which the value of each node is smaller or equal to the value of its father

  - ***Ascending heap*** is an almost complete binary tree in which the value of each node is greater or equal to the value of its father

Heap

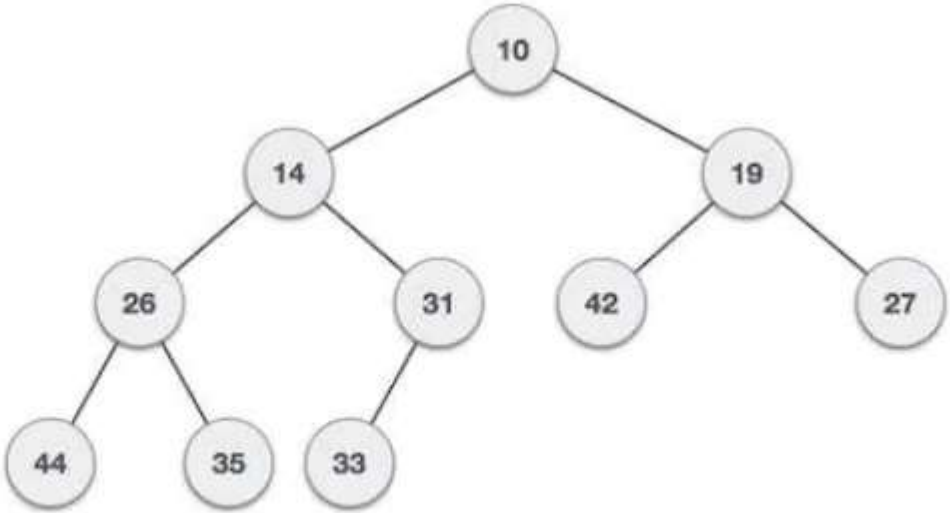
- Heap is a special case of balanced binary tree data structure where root-node key is compared with its children and arranged accordingly.

- If α has child node β then −
  - Key(α) >= key(β)

- As the value of the parent is greater than that of child, this property generates MAX heap. Based on this criteria a heap can be of two types
  - Min - Heap
  - Max - Heap

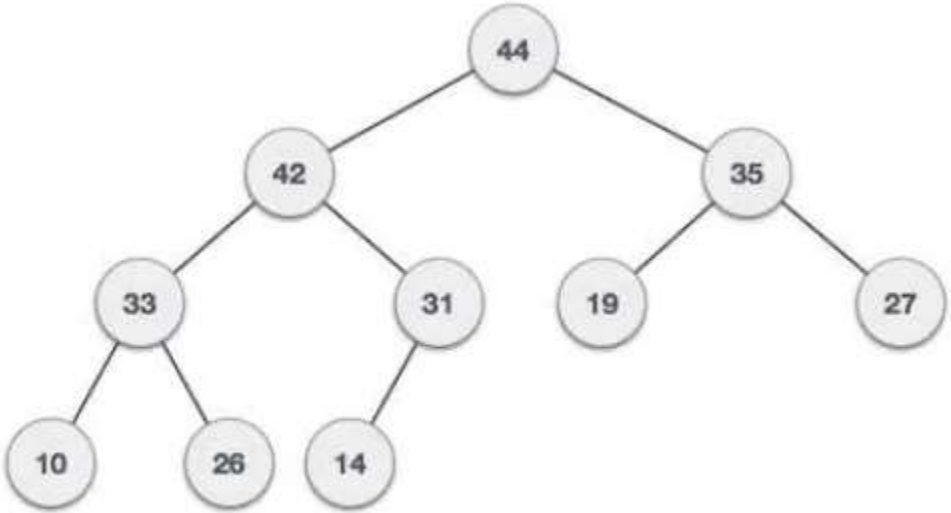
Heap

- Min-Heap
  - Where the value of the root node is less than or equal to either of its children
  - For input **35 33 42 10 14 19 27 44 26 31**



Heap

- **Max-Heap** –
  - where the value of root node is greater than or equal to either of its children.
  - For input **35 33 42 10 14 19 27 44 26 31**



Max Heap Construction Algorithm

Step 1 – Create a new node at the end of heap.

Step 2 – Assign new value to the node.

Step 3 – Compare the value of this child node with its parent.

Step 4 – If value of parent is less than child, then swap them.

Step 5 – Repeat step 3 & 4 until Heap property holds.

Input 35 33 42 10 14 19 27 44 26 31

Max Heap Deletion Algorithm

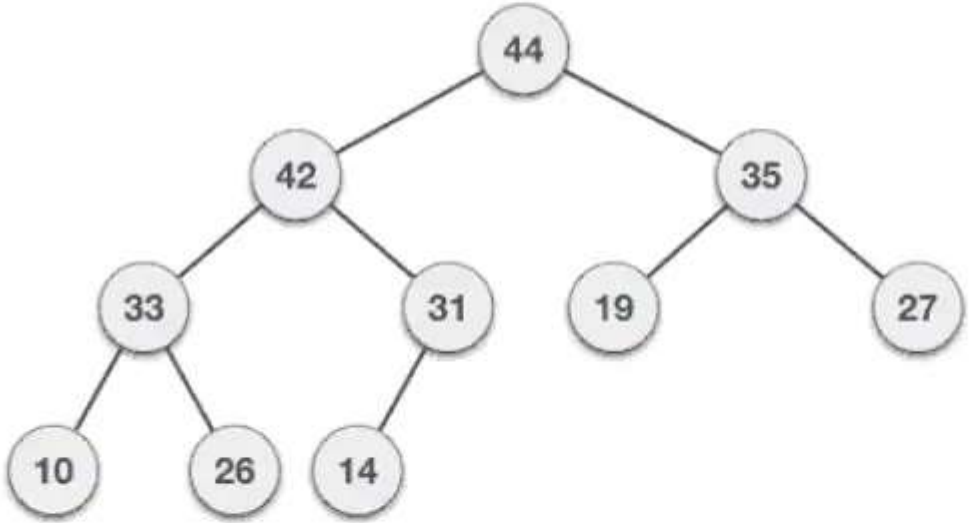
Step 1 – Remove root node.

Step 2 – Move the last element of last level to root.

Step 3 – Compare the value of this child node with its parent.

Step 4 – If value of parent is less than child, then swap them.

Step 5 – Repeat step 3 & 4 until Heap property holds.


44
42
35
33
31
19
27
10
26
14

Analysis of Heap Sort

- The worse case complexity of the heap sort is O(nlogn), therefore, Heap sort is superior to quick sort in the worst case

- Heap sort is not very efficient for small array because of the overhead of initial heap creation and computation.

- The space requirement for the hap sort is only one additional record to hold the temporary value.

Radix Sort

- The idea behind radix sort is slightly more complex than that of bucket sort.

- Algorithm:
  - Take the least significant digit of each element in the unsorted array
  - Perform a stable sort based on that key
  - Repeat the process sequentially with each more significant digit

Radix Sort

- For example suppose we want to sort the list
  849,770,67,347,201,618,66,495,13,45

- Sorting by least significant digit(i.e. ones digit)
  770,201,13,495,45,66,67,347,618,849

- Stability is maintained. Now sort by more significant tens digit,
  210,13,618,45,347,849,66,67,770,495

- We now consider the hundreds digit to be zero
  12,45,66,67,201,347,495,618,770,849

Shell Sort

Shell sort is a highly efficient sorting algorithm and is based on insertion sort algorithm. This algorithm avoids large shifts as in case of insertion sort, if the smaller value is to the far right and has to be moved to the far left.
This algorithm uses insertion sort on a widely spread elements, first to sort them and then sorts the less widely spaced elements. This spacing is termed as **interval**. This interval is calculated based on Knuth's formula as −
h = h * 3 + 1
where −
h is interval with initial value 1

Algorithm
Step 1 − Initialize the value of h
Step 2 − Divide the list into smaller sub-list of equal interval h
Step 3 − Sort these sub-lists using insertion sort
Step 3 − Repeat until complete list is sorted

Eg
13 17 19 10 15 11 16 18 14 12 10 16 11 15 17 13 14 19 18 12
Writing in two dimension[i,e doing partition]
13 17 19 10 15 11 16
18 14 12 10 16 11 15
17 13 14 19 18 12
Sorting within the list
13 13 12 10 15 11 15
17 14 14 10 16 11 16
18 17 19 19 18 12

13 13 12 10 15 11 15 17 14 14 10 16 11 16 18 17 19 19 18 12
Now dividing in three list
13 13 12
10 15 11
15 17 14
14 10 16
11 16 18
17 19 19
18 12

10 10 11
11 12 12
13 13 14
14 15 16
15 16 18
17 17 19
18   19

Final sorted list
10 10 11 11 12 12 13 13 14 14 15 15 16 16 17 17 18 18 19 19