# CV202, HW2

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## Problem 1

The condition is that should be an invertible matrix. Then .

## Problem 2

For to be a linear map it must satisfy*:*

In our case, considering :

If we want to be linear, we must have:

This is only true if , which is not the case, therefore is not a linear map.

## Problem 3

Let’s observe our :

For to be invertible, we need to find such that :

Choosing :

As we can see, this is the only possible inverse for .

But its existence assumes two major conditions! must be invertible (not only right/left invertible), which also means it must be a square matrix, meaning .

So in conclusion can be invertible only on the conditions that and is invertible.

## Problem 4

Define , and .

Therefore is affine .

## Problem 5

We have invertible affine maps.

This means:

Such that are invertible (we’ve shown that in problem 3).

Now by definition is an affine map (as we’ve shown before), of the following structure:

And knowing that exist, we can see that is invertible (by ), therefore we can conclude from what we’ve proven in problem 3 that is invertible affine map.

## Problem 6

We will show that G1, G2 and G3 definitions are satisfied:

*G1:*

*G2:*

*G3: define*

## Problem 7

Define:

*We found 2 matrices that belongs to GL(2) so that the Abelian definition is not satisfied, therefore GL(n) is not Abelian.*

## Problem 8

We will show that G1, G2 and G3 definitions are satisfied:

*G1:*

*G2:*

*G3:*

## Problem 9

Lets take a look at G2 definition:

## Problem 10

## Problem 11

We will show that G1, G2 and G3 definitions are satisfied:

*G1:* so

*G2:*

*G3:*

## Problem 12

## Problem 13

We can see that the matrix satisfies the following:

* for all
* for all

Therefore the matrix is the zero element.

## Problem 14

A linear subspace of a linear space must contain the 0 element.

We know that if A belongs to a matrix group there exists

However the is clearly non-invertible, therefore can’t be in any matrix group.

Therefore any matrix group cannot be a linear subspace of .

## Problem 15

We will show that G1, G2 and G3 definitions are satisfied:

*G1:*

*G2:*

We know as are in the affine group.

So

*G3:*

Define:

## Problem 16

We will show that G1, G2 and G3 definitions are satisfied:

*G1:*

*G2:*

We know as are in our group.

So

So is in our group.

*G3:* be a matrix in our group.

Define:

We know , and which means also.

So we can conclude:

## Problem 17

Lets look at G2:

But which means is not in our group, therefore the group is not a matrix group.

## Problem 18

The answer is no. There is no continuous curve , from the unit interval to the affine group, such that and .

We will show that by observing any possible continuous curve , , such that:

We know that:

is a continuous curve, so for any :

And is continuous!

Now, as we know from the Leibniz formula, a determinant of a matrix is a polynomial expression of the matrix entries. Every entry in our is continuous over , therefore we can say is a continuous curve as well!

Now we know it begins in , and ends at , therefore exists such that !

So we can see that we do not have the case , showing no continuous curve exists between the two affine maps.

## Problem 19

### Part (i):

We need to find the least square estimator for .

We want to find such that for all : .

So our problem is described by:

Now let’s observe :

Now let:

**,**

And we get:

Now:

And as we’ve learned, the minimizer satisfies:

### Part (ii):

For this part we will note that we are looking for (let be our probability density function):

We know the probability density function of , so:

Where:

### Part (iii):

The maximum likelihood estimator for will be achieved by getting the maximum of the **log likelihood function** (as it is a monotonic non-decreasing function):

To get the maximum , we will look for a gradient of .

We will start with :

And now for :

## Problem 20

We need to find:

Let’s observe:

And this is of LS form, so we know our solution is:

## Problem 21

So we need to find:

But means:

for some

So we can rewrite our problem:

As when we find the right , we can get our .

Now, as we know is a monotonic non-decreasing function, so we can rewrite again:

And now we can it’s a LS problem. For :

So now we can use our known solution:

## Problem 22

Definition:

## Problem 23

First of all, we have seen in definition 9 that for all .

So all we need to show is that for all .

**Intuition:** if at (0,0) the value of is 1 and otherwise its 0, it means that it preserve only the pixel that the convolution is affecting, without any influence by other pixels (they are multiplied by 0).

Therefore

## Problem 24

First of all let’s be clear about our “column vector” versions of and :

If we want to do filter () on the definition of it is:

But we know , so:

Now let’s re-purpose our indexes, marking . Recall we are using the zero-boundary assumption.

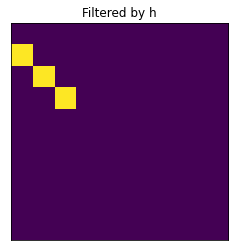
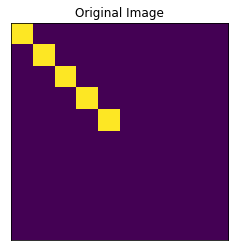
Now let’s be clear about our goal. We need to define for all such that:

and are both column vectors, so according to our goal, lets see what each value (which is a row) of should be:

Now using this with our , we can safely define :

## Problem 25

The effect of convolving an image with this given , is moving the entire image 2 pixels to the left, and 1 pixel upwards. This is a very small change, barely noticeable in sizeable images, so we can see this with this very small image example:



## Problem 26

### Part (i)

### Part (ii)

From , we get by definition of the inverse matrix. Therefore directly we can see that:

Proving that is an orthogonal square matrix.

### Part (iii)

For any , directly from part (ii), by definition of matrix multiplication we get (as is a column of ):

### Part (iv)

For any , by definition:

### Part (v)

Similarly to part (iii), for any :

This means the angle between and is .

### Part (vi)

We will prove this by definition. We mark this group of all orthogonal matrices .

This is clear because , meaning it’s orthogonal so in .

1. We need to show that if , so is .

Let’s observe, remembering both are orthogonal matrices:

is an orthogonal matrix so in .

1. We need to show that if , exists and it’s in .

is orthogonal so meaning exists and .

We’ve proven before that is also orthogonal, meaning it’s in , so is in as we wanted.

And that shows by definition that our group is a matrix group.

## Problem 27

is a separable filter. Is it invertible? Well a matrix is invertible if and only if it has a full rank. So we need to see whether .

is separable so:

Such that is a diagonal matrix with values , and **only on of them** is a non-zero!

This directly means !

And from fact 3 we know:

So we got (it’s actually 1 otherwise it’s a matrix). So unless which is a very trivial case, is not invertible.

## Problem 28

Bilateral filtering is not a linear operation, because as we can see in equation (28), the filter operation is dependent on the in a non-linear way.

Any change to the input image will not result in a proportional change in the output.  
First of all is not necessarily linear, and even if it is: It’s input is in absolute value which is not linear.  
In addition, the component which has in it, is not linear in any way.

## Problem 29

First derivative:

Second derivative:

## Problem 30

First derivatives:

Second derivatives:

Now finally we can see: