CV2020-HW5

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Problem 1

The key difference between Python 2.x and Python 3.x that will cause an issue with Exercise 7 is the different ways they handle **division**.

In Python 2.x we have $1/2 = 0$ in contrast to Python 3.x which calculates properly $1/2 = 0.5$. This causes our F and G functions to not work properly when $Temp = 2$. In Python 2.x they get $1/Temp = 1/2 = 0$, which causes them to be equal 1 for any row. This creates the **uniform distribution**, which is the "completely random" output the Python 2.x gets.

Problem 2

Firstly we need to note the meaning of these empirical expectations. They represent the *expected value of the multiplication of two sites of the lattice*. Because the sites are of the values $\{-1, +1\}$, the multiplication represents whether the two sites have the same value or not: it's $+1$ when they are the same, and -1 otherwise. So the closer the expected value is to 1, the more we should expect the two sites to have the same value.

Now to the results we got:

$$\begin{aligned}
Temp = 1: &\quad \hat{E}_{Temp}(X_{(1,1)}X_{(2,2)}) \approx 0.9542, \quad \hat{E}_{Temp}(X_{(1,1)}X_{(8,8)}) \approx 0.903 \\
Temp = 1.5: &\quad \hat{E}_{Temp}(X_{(1,1)}X_{(2,2)}) \approx 0.7672, \quad \hat{E}_{Temp}(X_{(1,1)}X_{(8,8)}) \approx 0.5414 \\
Temp = 2: &\quad \hat{E}_{Temp}(X_{(1,1)}X_{(2,2)}) \approx 0.5194, \quad \hat{E}_{Temp}(X_{(1,1)}X_{(8,8)}) \approx 0.1084
\end{aligned}$$

We can explain these results from two different perspectives.

Looking from the **temperature** perspective, we can see that for a bigger $Temp$, we expect more differences in values, for any site pairs (The reason for that we explained and understood in Problem 4 of HW4), causing the lower expected value we see here (for each pair on its own).

Regarding the **spatial distance** perspective (which is the difference between the two pairs), we see that the $X_{(1,1)}$ is less likely to have equal value with $X_{(8,8)}$ than $X_{(2,2)}$. This is because the $(8,8)$ site is "further away": the case that $(1,1)$ and $(8,8)$ have the same value, requires equal value on the many "routes" of neighbors between them (or the more unlikely case of "switching" values twice). This case is much less likely than the $(1,1)$ and $(2,2)$ case, especially for bigger temperatures, which explains the lower expected value of the $X_{(1,1)}X_{(8,8)}$.