Simpson’s paradox

Data Science in a Box

datasciencebox.org

Case study: Berkeley admission data

Berkeley admission data

- Study carried out by the Graduate Division of the University of California, Berkeley in the early 70's to evaluate whether there was a gender bias in graduate admissions.
- The data come from six departments. For confidentiality we'll call them A-F.
- We have information on whether the applicant was male or female and whether they were admitted or rejected.
- First, we will evaluate whether the percentage of males admitted is indeed higher than females, overall. Next, we will calculate the same percentage for each department.

Data

```
## # A tibble: 4,526 × 3
##    admit    gender dept 
##    <fct>    <fct>  <ord>
##  1 Admitted Male   A    
##  2 Admitted Male   A    
##  3 Admitted Male   A    
##  4 Admitted Male   A    
##  5 Admitted Male   A    
##  6 Admitted Male   A    
##  7 Admitted Male   A    
##  8 Admitted Male   A    
##  9 Admitted Male   A    
## 10 Admitted Male   A    
## 11 Admitted Male   A    
## 12 Admitted Male   A    
## 13 Admitted Male   A    
## 14 Admitted Male   A    
## 15 Admitted Male   A    
## # … with 4,511 more rows
```

```
## # A tibble: 2 × 2
##   gender     n
##   <fct>  <int>
## 1 Female  1835
## 2 Male    2691
```

```
## # A tibble: 6 × 2
##   dept      n
##   <ord> <int>
## 1 A       933
## 2 B       585
## 3 C       918
## 4 D       792
## 5 E       584
## 6 F       714
```

```
## # A tibble: 2 × 2
##   admit        n
##   <fct>    <int>
## 1 Rejected  2771
## 2 Admitted  1755
```

> What can you say about the overall gender distribution? Hint: Calculate the following probabilities: $P(Admit|Male)$ and $P(Admit|Female)$.

```r
ucbadmit %>%
  count(gender, admit)
```

```
## # A tibble: 4 × 3
##   gender admit        n
##   <fct>  <fct>    <int>
## 1 Female Rejected  1278
## 2 Female Admitted   557
## 3 Male   Rejected  1493
## 4 Male   Admitted  1198
```

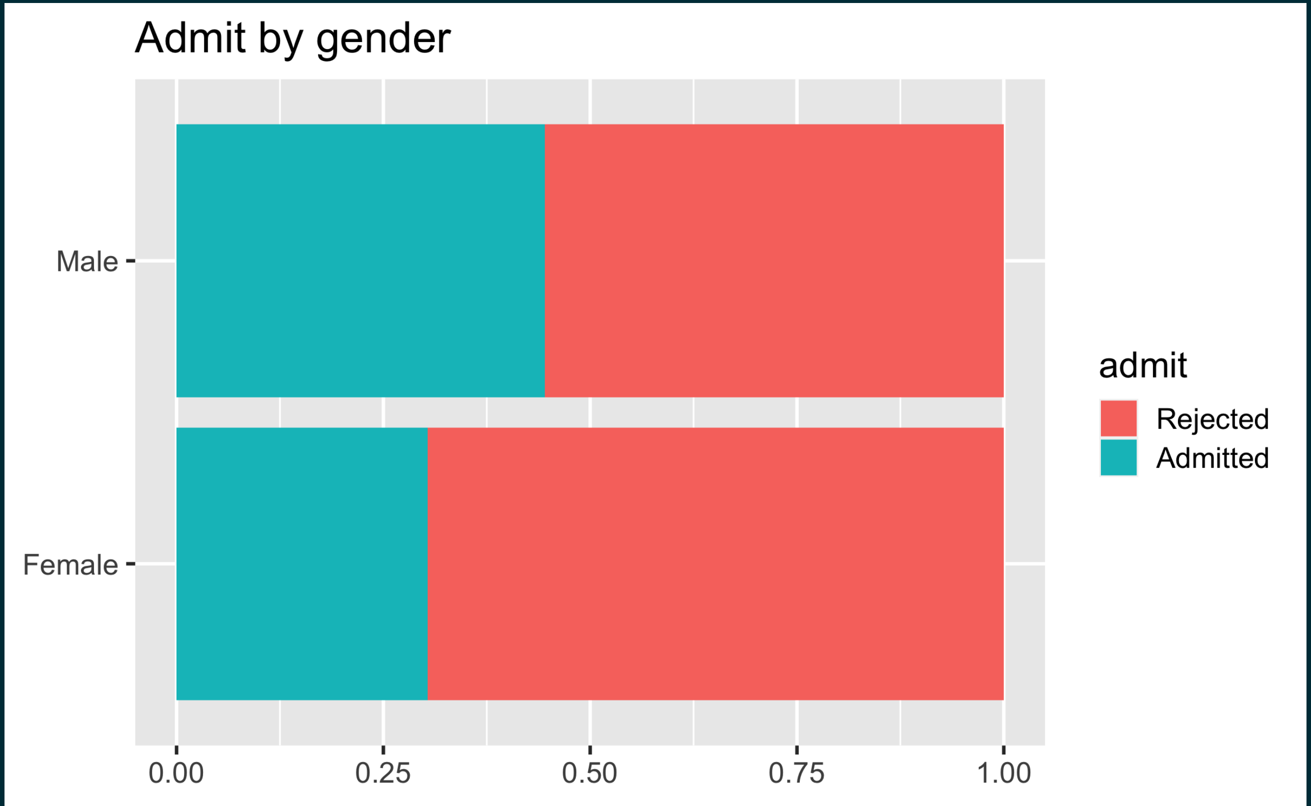
```r
ucbadmit %>%
  count(gender, admit) %>%
  group_by(gender) %>%
  mutate(prop_admit = n / sum(n))
```

```
## # A tibble: 4 × 4
## # Groups:   gender [2]
##   gender admit        n prop_admit
##   <fct>  <fct>    <int>      <dbl>
## 1 Female Rejected  1278      0.696
## 2 Female Admitted   557      0.304
## 3 Male   Rejected  1493      0.555
## 4 Male   Admitted  1198      0.445
```

- $P(Admit | Female) = 0.304$
- $P(Admit | Male) = 0.445$

Overall gender distribution

Plot Code



Overall gender distribution

Plot Code

```r
ggplot(ucbadmit, aes(y = gender, fill = admit)) +
  geom_bar(position = "fill") +
  labs(title = "Admit by gender",
       y = NULL, x = NULL)
```

What can you say about the gender distribution by department ?

```
ucbadmit %>%
  count(dept, gender, admit)
```

```
## # A tibble: 24 × 4
##   dept  gender admit        n
##   <ord> <fct>  <fct>    <int>
## 1 A     Female Rejected    19
## 2 A     Female Admitted    89
## 3 A     Male   Rejected   313
## 4 A     Male   Admitted   512
## 5 B     Female Rejected     8
## 6 B     Female Admitted    17
## # … with 18 more rows
```

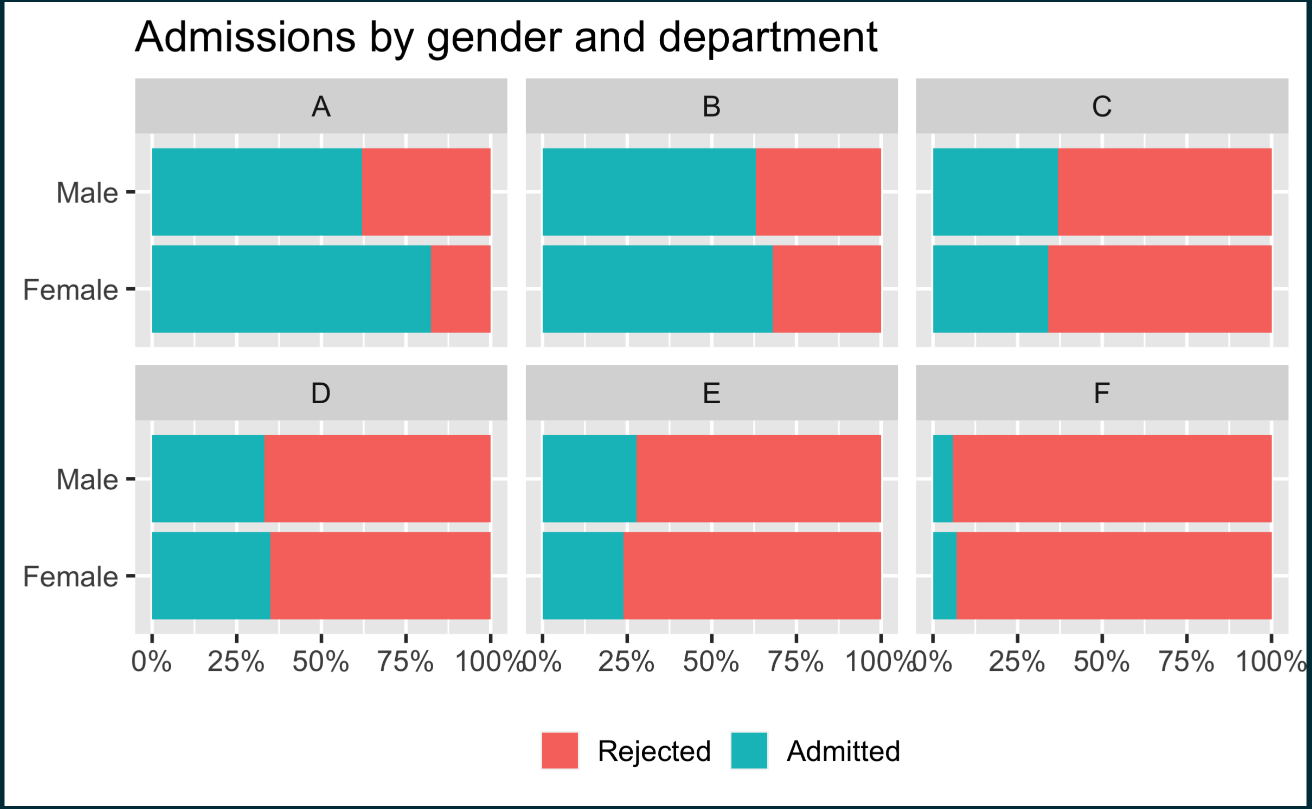
Let's try again... What can you say about the gender distribution by department?

```r
ucbadmit %>%
  count(dept, gender, admit) %>%
  pivot_wider(names_from = dept, values_from = n)
```

```
## # A tibble: 4 × 8
##   gender admit        A     B     C     D     E     F
##   <fct>  <fct>    <int> <int> <int> <int> <int> <int>
## 1 Female Rejected    19     8   391   244   299   317
## 2 Female Admitted    89    17   202   131    94    24
## 3 Male   Rejected   313   207   205   279   138   351
## 4 Male   Admitted   512   353   120   138    53    22
```

Gender distribution, by department

Plot Code

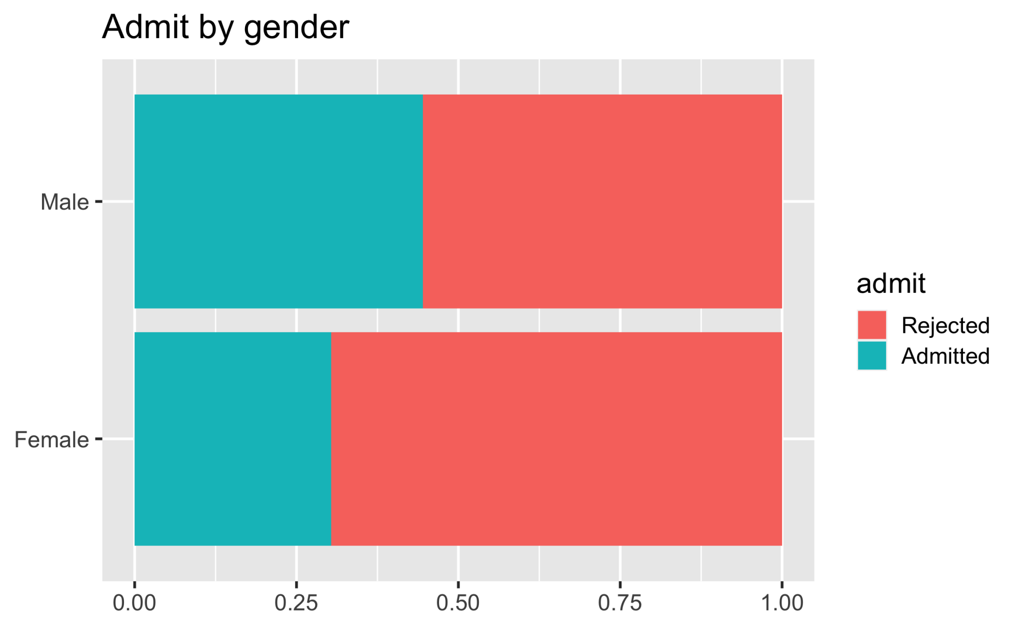


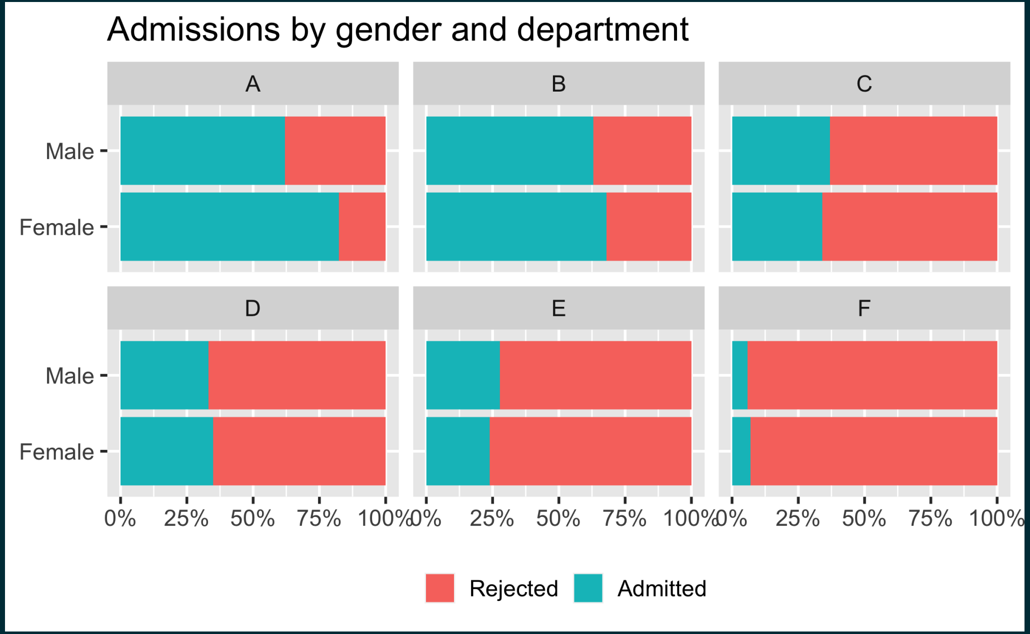
Gender distribution, by department

Plot Code

```r
ggplot(ucbadmit, aes(y = gender, fill = admit)) +
  geom_bar(position = "fill") +
  facet_wrap(. ~ dept) +
  scale_x_continuous(labels = label_percent()) +
  labs(title = "Admissions by gender and department",
       x = NULL, y = NULL, fill = NULL) +
  theme(legend.position = "bottom")
```

Case for gender discrimination?





Closer look at departments

Output Code

```
## # A tibble: 12 × 5
## # Groups:   dept, gender [12]
##    dept  gender n_admitted n_applied prop_admit
##    <ord> <fct>       <int>     <int>      <dbl>
##  1 A     Female         89       108     0.824
##  2 A     Male          512       825     0.621
##  3 B     Female         17        25     0.68
##  4 B     Male          353       560     0.630
##  5 C     Female        202       593     0.341
##  6 C     Male          120       325     0.369
##  7 D     Female        131       375     0.349
##  8 D     Male          138       417     0.331
##  9 E     Female         94       393     0.239
## 10 E     Male           53       191     0.277
## 11 F     Female         24       341     0.0704
## 12 F     Male           22       373     0.0590
```

Closer look at departments

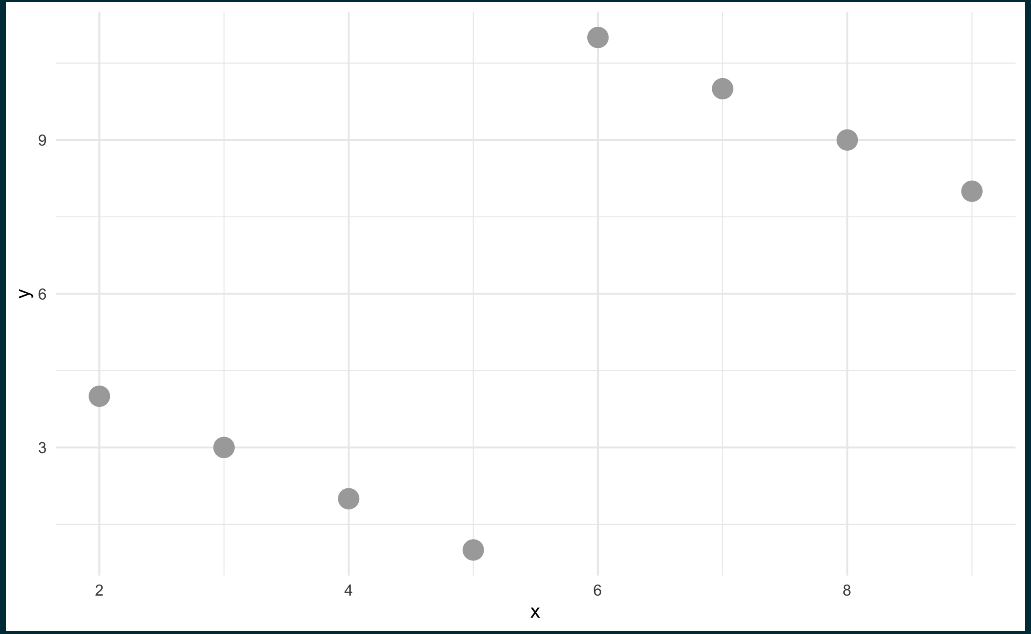
Output Code

```
ucbadmit %>%
  count(dept, gender, admit) %>%
  group_by(dept, gender) %>%
  mutate(
    n_applied  = sum(n),
    prop_admit = n / n_applied
    ) %>%
  filter(admit == "Admitted") %>%
  rename(n_admitted = n) %>%
  select(-admit) %>%
  print(n = 12)
```

Simpson's paradox

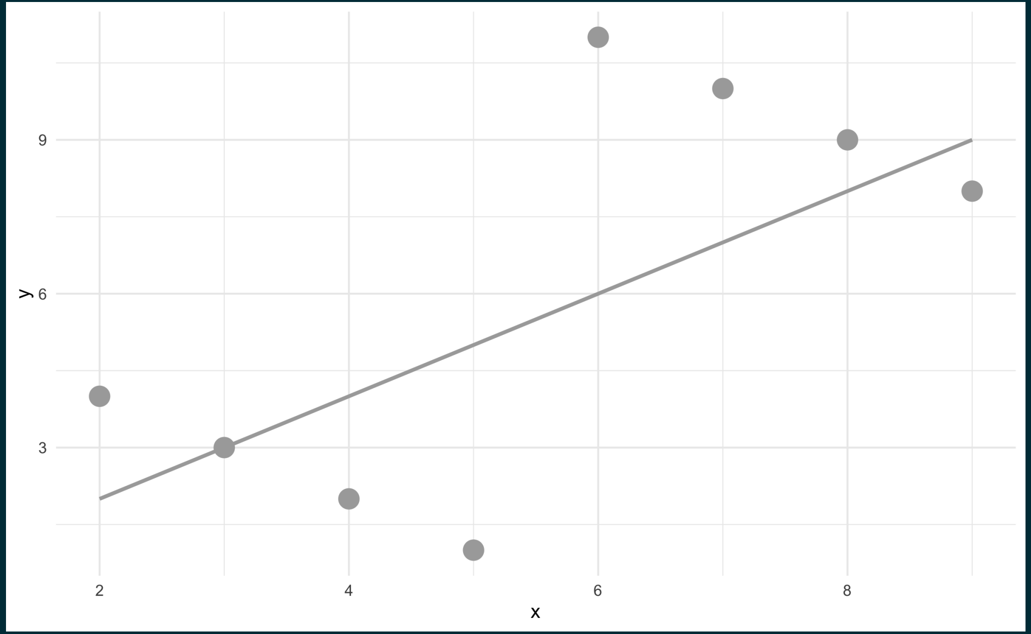
Relationship between two variables

```
## # A tibble: 8 × 3
##         x     y z    
##     <dbl> <dbl> <chr>
## 1       2     4 A    
## 2       3     3 A    
## 3       4     2 A    
## 4       5     1 A    
## 5       6    11 B    
## 6       7    10 B    
## # … with 2 more rows
```



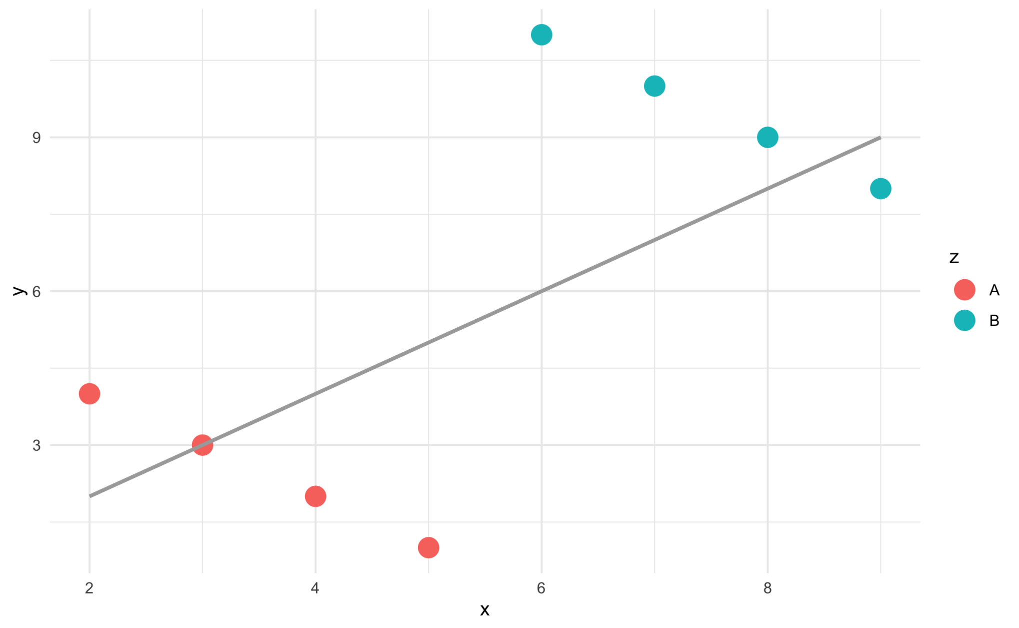
Relationship between two variables

```
## # A tibble: 8 × 3
##         x     y z    
##     <dbl> <dbl> <chr>
## 1       2     4 A    
## 2       3     3 A    
## 3       4     2 A    
## 4       5     1 A    
## 5       6    11 B    
## 6       7    10 B    
## # … with 2 more rows
```



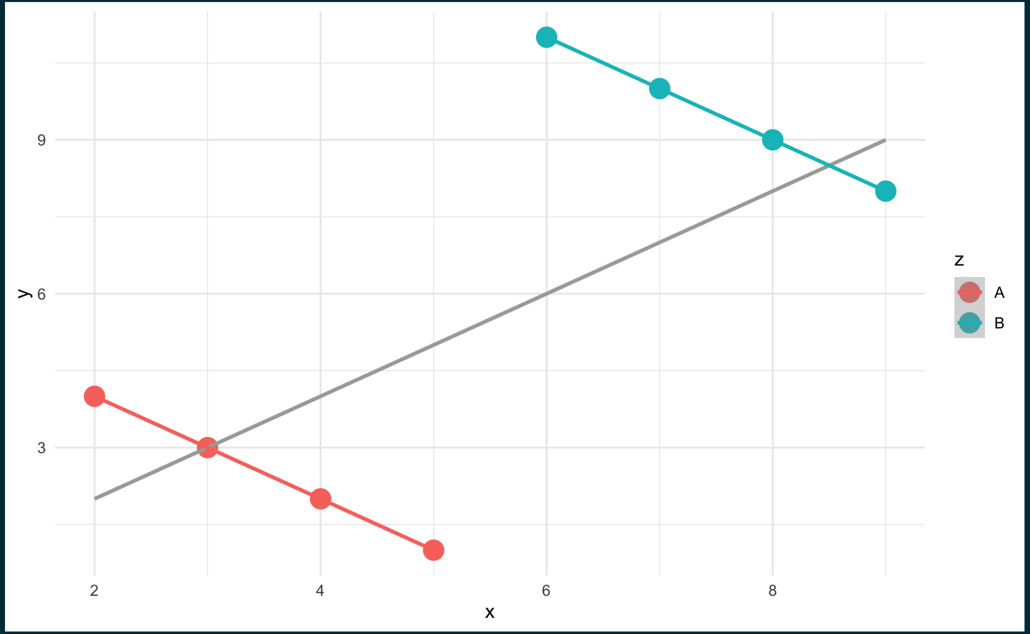
Considering a third variable

```
## # A tibble: 8 × 3
##       x     y z    
##   <dbl> <dbl> <chr>
## 1     2     4 A    
## 2     3     3 A    
## 3     4     2 A    
## 4     5     1 A    
## 5     6    11 B    
## 6     7    10 B    
## # … with 2 more rows
```



Relationship between three variables

```
## # A tibble: 8 × 3
##       x     y z    
##   <dbl> <dbl> <chr>
## 1     2     4 A    
## 2     3     3 A    
## 3     4     2 A    
## 4     5     1 A    
## 5     6    11 B    
## 6     7    10 B    
## # … with 2 more rows
```



Simpson's paradox

- Not considering an important variable when studying a relationship can result in **Simpson's paradox**
- Simpson's paradox illustrates the effect that omission of an explanatory variable can have on the measure of association between another explanatory variable and a response variable
- The inclusion of a third variable in the analysis can change the apparent relationship between the other two variables

Aside: `group_by()` and `count()`

What does group_by() do?

`group_by()` takes an existing data frame and converts it into a grouped data frame where subsequent operations are performed "once per group"

```
ucbadmit
```

```
## # A tibble: 4,526 × 3
##   admit    gender dept 
##   <fct>    <fct>  <ord>
## 1 Admitted Male   A    
## 2 Admitted Male   A    
## 3 Admitted Male   A    
## 4 Admitted Male   A    
## 5 Admitted Male   A    
## 6 Admitted Male   A    
## # … with 4,520 more rows
```

```
ucbadmit %>%
  group_by(gender)
```

```
## # A tibble: 4,526 × 3
## # Groups:   gender [2]
##   admit    gender dept 
##   <fct>    <fct>  <ord>
## 1 Admitted Male   A    
## 2 Admitted Male   A    
## 3 Admitted Male   A    
## 4 Admitted Male   A    
## 5 Admitted Male   A    
## 6 Admitted Male   A    
## # … with 4,520 more rows
```

What does group_by() not do?

`group_by()` does not sort the data, `arrange()` does

```r
ucbadmit %>%
  group_by(gender)
```

```
## # A tibble: 4,526 × 3
## # Groups:   gender [2]
##   admit    gender dept 
##   <fct>    <fct>  <ord>
## 1 Admitted Male   A    
## 2 Admitted Male   A    
## 3 Admitted Male   A    
## 4 Admitted Male   A    
## 5 Admitted Male   A    
## 6 Admitted Male   A    
## # … with 4,520 more rows
```

```r
ucbadmit %>%
  arrange(gender)
```

```
## # A tibble: 4,526 × 3
##   admit    gender dept 
##   <fct>    <fct>  <ord>
## 1 Admitted Female A    
## 2 Admitted Female A    
## 3 Admitted Female A    
## 4 Admitted Female A    
## 5 Admitted Female A    
## 6 Admitted Female A    
## # … with 4,520 more rows
```

What does group_by() not do?

`group_by()` does not create frequency tables, `count()` does

```
ucbadmit %>%
  group_by(gender)
```

```
## # A tibble: 4,526 × 3
## # Groups:   gender [2]
##   admit    gender dept
##   <fct>    <fct>  <ord>
## 1 Admitted Male   A
## 2 Admitted Male   A
## 3 Admitted Male   A
## 4 Admitted Male   A
## 5 Admitted Male   A
## 6 Admitted Male   A
## # … with 4,520 more rows
```

```
ucbadmit %>%
  count(gender)
```

```
## # A tibble: 2 × 2
##   gender     n
##   <fct>  <int>
## 1 Female  1835
## 2 Male    2691
```

Undo grouping with ungroup()

```
ucbadmit %>%
  count(gender, admit) %>%
  group_by(gender) %>%
  mutate(prop_admit = n / sum(n)) %>%
  select(gender, prop_admit)
```

```
## # A tibble: 4 × 2
## # Groups:   gender [2]
##   gender prop_admit
##   <fct>       <dbl>
## 1 Female      0.696
## 2 Female      0.304
## 3 Male        0.555
## 4 Male        0.445
```

```
ucbadmit %>%
  count(gender, admit) %>%
  group_by(gender) %>%
  mutate(prop_admit = n / sum(n)) %>%
  select(gender, prop_admit) %>%
  ungroup()
```

```
## # A tibble: 4 × 2
##   gender prop_admit
##   <fct>       <dbl>
## 1 Female      0.696
## 2 Female      0.304
## 3 Male        0.555
## 4 Male        0.445
```

count() is a short-hand

`count()` is a short-hand for `group_by()` and then `summarise()` to count the number of observations in each group

```
ucbadmit %>%
  group_by(gender) %>%
  summarise(n = n())
```

```
## # A tibble: 2 × 2
##   gender     n
##   <fct>  <int>
## 1 Female  1835
## 2 Male    2691
```

```
ucbadmit %>%
  count(gender)
```

```
## # A tibble: 2 × 2
##   gender     n
##   <fct>  <int>
## 1 Female  1835
## 2 Male    2691
```

count can take multiple arguments

```
ucbadmit %>%
  group_by(gender, admit) %>%
  summarise(n = n())
```

```
## # A tibble: 4 × 3
## # Groups:   gender [2]
##   gender admit        n
##   <fct>  <fct>    <int>
## 1 Female Rejected  1278
## 2 Female Admitted   557
## 3 Male   Rejected  1493
## 4 Male   Admitted  1198
```

```
ucbadmit %>%
  count(gender, admit)
```

```
## # A tibble: 4 × 3
##   gender admit        n
##   <fct>  <fct>    <int>
## 1 Female Rejected  1278
## 2 Female Admitted   557
## 3 Male   Rejected  1493
## 4 Male   Admitted  1198
```

summarise() after group_by()

- count() ungroups after itself
- summarise() peels off one layer of grouping by default, or you can specify a different behaviour

```
ucbadmit %>%
  group_by(gender, admit) %>%
  summarise(n = n())
```

```
## # A tibble: 4 × 3
## # Groups:   gender [2]
##   gender admit        n
##   <fct>  <fct>    <int>
## 1 Female Rejected  1278
## 2 Female Admitted   557
## 3 Male   Rejected  1493
## 4 Male   Admitted  1198
```