Prediction and overfitting

Data Science in a Box datasciencebox.org



Prediction

Goal: Building a spam filter

- Data: Set of emails and we know if each email is spam/not and other features
- Use logistic regression to predict the probability that an incoming email is spam
- Use model selection to pick the model with the best predictive performance
- Building a model to predict the probability that an email is spam is only half of the battle!
 We also need a decision rule about which emails get flagged as spam (e.g. what probability should we use as out cutoff?)
- A simple approach: choose a single threshold probability and any email that exceeds that probability is flagged as spam

A multiple regression approach

Output Code

```
## # A tibble: 22 × 5
##
      term
                   estimate std.error statistic p.value
##
      <chr>
                                 <dbl>
                       < ldb>
                                           <1db>>
                                                     <dh1>
                   -9.09e+1
                                        -0.00928 9.93e- 1
    1 (Intercept)
                               9.80e+3
   2 to_multiple1 -2.68e+0
                               3.27e-1
                                        -8.21
                                                 2.25e-16
   3 from1
                   -2.19e+1
                                        -0.00224 9.98e- 1
##
                               9.80e+3
##
   4 cc
                    1.88e-2
                               2.20e-2
                                         0.855
                                                 3.93e- 1
   5 sent email1 -2.07e+1
                               3.87e+2
                                        -0.0536
                                                 9.57e- 1
##
   6 time
                    8.48e-8
                               2.85e-8
                                         2.98
                                                 2.92e- 3
   7 image
                   -1.78e+0
                               5.95e-1
                                        -3.00
                                                 2.73e- 3
##
                    7.35e-1
                                                 3.61e- 7
   8 attach
                               1.44e - 1
                                         5.09
   9 dollar
                   -6.85e-2
                               2.64e-2
                                        -2.59
                                                 9.64e- 3
                    2.07e+0
                               3.65e-1
                                         5.67
                                                 1.41e- 8
## 10 winnerves
## 11 inherit
                    3.15e-1
                               1.56e-1
                                         2.02
                                                 4.32e- 2
## 12 viagra
                    2.84e+0
                               2.22e+3
                                         0.00128 9.99e- 1
                               2.97e-1
## 13 password
                   -8.54e-1
                                        -2.88
                                                 4.03e- 3
## 14 num char
                    5.06e-2
                               2.38e-2
                                         2.13
                                                 3.35e- 2
## 15 line breaks -5.49e-3
                               1.35e-3
                                        -4.06
                                                 4.91e-5
## 16 format1
                   -6.14e-1
                               1.49e-1
                                        -4.14
                                                 3.53e- 5
## 17 re subj1
                   -1.64e+0
                               3.86e-1
                                                 2.16e-5
                                        -4.25
                    1.42e-1
                               2.43e-1
                                         0.585
                                                 5.58e- 1
## 18 exclaim subj
## 19 urgent subj1
                    3.88e+0
                               1.32e+0
                                         2.95
                                                 3.18e- 3
## 20 exclaim mess
                   1.08e-2
                               1.81e-3
                                         5.98
                                                 2.23e-9
## 21 numbersmall -1.19e+0
                               1.54e-1
                                        -7.74
                                                 9.62e-15
## 22 numberbig
                   -2.95e-1
                               2.20e-1 -1.34
                                                 1.79e- 1
```

A multiple regression approach

Output

Code

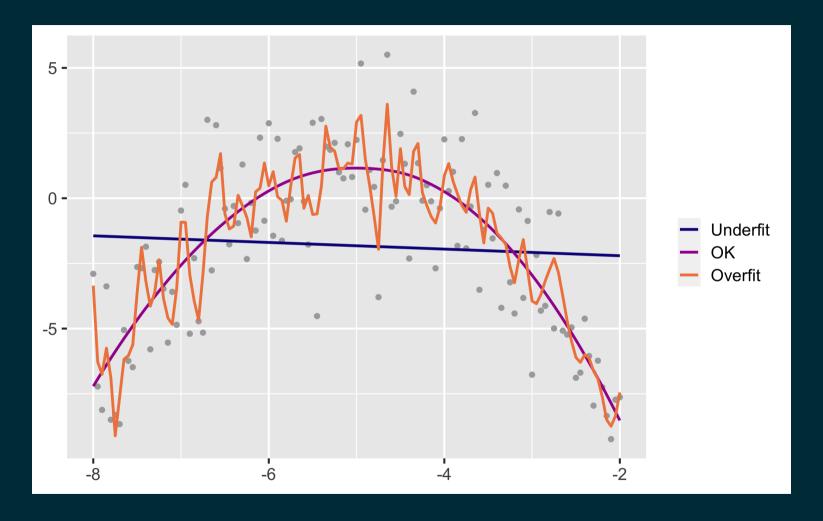
```
logistic_reg() %>%
  set_engine("glm") %>%
  fit(spam ~ ., data = email, family = "binomial") %>%
  tidy() %>%
  print(n = 22)
```

Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

Prediction

- The mechanics of prediction is **easy**:
 - Plug in values of predictors to the model equation
 - Calculate the predicted value of the response variable, \hat{y}
- Getting it right is **hard**!
 - There is no guarantee the model estimates you have are correct
 - Or that your model will perform as well with new data as it did with your sample data

Underfitting and overfitting



Spending our data

- Several steps to create a useful model: parameter estimation, model selection, performance assessment, etc.
- Doing all of this on the entire data we have available can lead to overfitting
- Allocate specific subsets of data for different tasks, as opposed to allocating the largest possible amount to the model parameter estimation only (which is what we've done so far)

Splitting data

Splitting data

Training set:

- Sandbox for model building
- Spend most of your time using the training set to develop the model
- Majority of the data (usually 80%)

Testing set:

- Held in reserve to determine efficacy of one or two chosen models
- Critical to look at it once, otherwise it becomes part of the modeling process
- Remainder of the data (usually 20%)

Performing the split

```
# Fix random numbers by setting the seed
# Enables analysis to be reproducible when random numbers are used
set.seed(1116)

# Put 80% of the data into the training set
email_split <- initial_split(email, prop = 0.80)

# Create data frames for the two sets:
train_data <- training(email_split)
test_data <- testing(email_split)</pre>
```

Peek at the split

glimpse(train_data)

```
## Rows: 3,136
## Columns: 21
## $ spam
               <fct> 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, ...
## $ to multiple
               <fct> 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, ...
## $ from
               <int> 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 35,...
## $ cc
               <fct> 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ sent email
## $ time
               <dttm> 2012-01-26 00:46:55, 2012-01-03 07:28:28,...
## $ image
               <dbl> 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ attach
               <dbl> 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ dollar
               <dbl> 10, 0, 0, 0, 0, 0, 13, 0, 0, 0, 2, 0, 0, 0...
## $ winner
               <fct> no, no, no, no, no, no, yes, no, no, n...
## $ inherit
               <dbl> 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ viagra
               ## $ password
## $ num char
               <dbl> 23.308, 1.162, 4.732, 42.238, 1.228, 25.59...
## $ line breaks
               <int> 477, 2, 127, 712, 30, 674, 367, 226, 98, 6...
## $ format
               <fct> 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, ...
## $ re subj
               <fct> 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, ...
## $ exclaim subj <dbl> 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, ...
               ## $ urgent subj
## $ exclaim mess <dbl> 12, 0, 2, 2, 31, 2, 0, 0, 1, 0, 1, 2, 0...
## $ number
               <fct> small, none, big, big, small, small...
```

glimpse(test_data)

```
## Rows: 785
## Columns: 21
## $ spam
              ## $ to multiple
              <fct> 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, ...
## $ from
              ## $ cc
              <int> 0, 1, 0, 1, 4, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ sent email
              <fct> 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
## $ time
              <dttm> 2012-01-01 19:55:06, 2012-01-01 21:38:32,...
## $ image
              <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, ...
## $ attach
## $ dollar
              <dbl> 0, 0, 5, 0, 0, 0, 0, 5, 4, 0, 0, 0, 21, 0,...
## $ winner
              <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, ...
## $ inherit
## $ viagra
              ## $ password
              <dbl> 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, ...
## $ num char
              <dbl> 4.837, 15.075, 18.037, 45.842, 11.438, 1.4...
## $ line breaks
              <int> 193, 354, 345, 881, 125, 24, 296, 13, 192,...
## $ format
              <fct> 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, ...
## $ re subj
              <fct> 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, ...
## $ exclaim subj
              <dbl> 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
              ## $ urgent subj
## $ exclaim mess <dbl> 1. 10. 20. 5. 2. 0. 0. 0. 6. 0. 0. 1. 3. 0...
              <fct> big, small, small, big, small, none, small...
## $ number
```

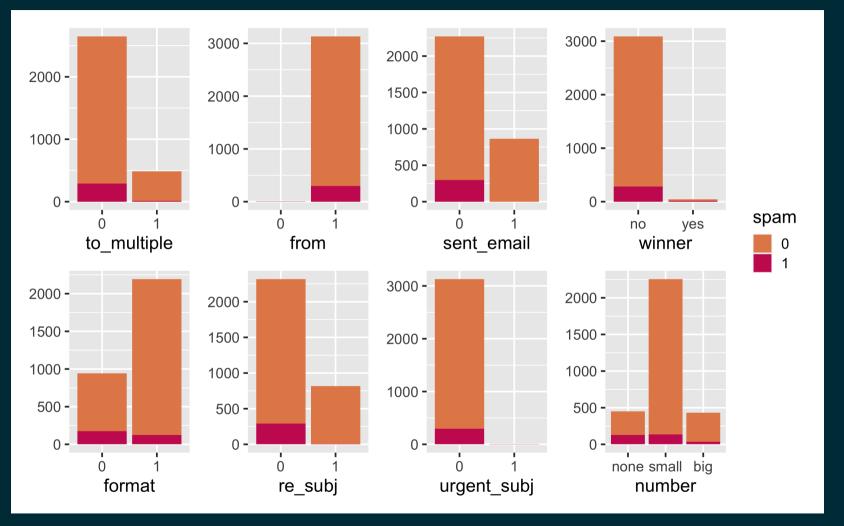
Modeling workflow

Fit a model to the training dataset

```
email_fit <- logistic_reg() %>%
  set_engine("glm") %>%
  fit(spam ~ ., data = train_data, family = "binomial")
```

Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

Categorical predictors



from and sent_email

 from: Whether the message was listed as from anyone (this is usually set by default for regular outgoing email)

```
train_data %>%
  count(spam, from)
```

```
## # A tibble: 3 × 3
## spam from n
## <fct> <fct> <int>
## 1 0 1 2837
## 2 1 0 3
## 3 1 1 296
```

 sent_email: Indicator for whether the sender had been sent an email in the last 30 days

```
train_data %>%
  count(spam, sent_email)
```

```
## # A tibble: 3 × 3
## spam sent_email n
## <fct> <fct> <int>
## 1 0 0 1972
## 2 0 1 865
## 3 1 0 299
```

Numerical predictors

##											
	able type										
## skim	_variable	spam	n_missing complet	e_rate	mean		þ	p25	p50	p75	p100
## 1 cc		0	0	1	0.393	2.62	0	0	0	0	68
## 2 cc		1	0	1	0.388	3.25	0	0	0	0	50
## 3 imag	e	0	0	1	0.0536	0.503	0	0	0	0	20
## 4 imag	e	1	0	1	0.00334	0.0578	0	0	0	0	1
## 5 atta	ch	0	0	1	0.124	0.775	0	0	0	0	21
## 6 atta	ch	1	0	1	0.227	0.620	0	0	0	0	2
## 7 doll	ar	0	0	1	1.56	5.33	0	0	0	0	64
## 8 doll	ar	1	0	1	0.779	3.01	0	0	0	0	36
## 9 inhe	rit	0	0	1	0.0352	0.216	0	0	0	0	6
## 10 inhe	rit	1	0	1	0.0702	0.554	0	0	0	0	9
## 11 viag		0	0	1	0	0	0	0	0	0	0
## 12 viag	ra	1	0	1	0.0268	0.463	0	0	0	0	8
## 13 pass	word	0	0	1	0.112	0.938	0	0	0	0	22
## 14 pass	word	1	0	1	0.0201	0.182	0	0	0	0	2
## 15 num_	char	0	0	1	11.4	14.9	0.00	3 1 . 97	6.83	15.7	190.
## 16 num_		1	0	1	5.63	15.7	0.00	0.468	0.999	3.55	
## 17 line	_breaks	0	0	1	247.	326.	2	42	138	318	4022
## 18 line	_breaks	1	0	1	108.	321.	1	14	23	66.5	3729
## 19 excl	aim_subj	0	0	1	0.0783	0.269	0	0	0	0	1
## 20 excl	aim_subj	1	0	1	0.0769	0.267	0	0	0	0	1
## 21 excl	aim_mess	0	0	1	6.68	50.2	0	0	1	5	1236
## 22 excl	aim_mess	1	0	1	8.75	88.4	0	0	0	1	1209

Fit a model to the training dataset

```
email_fit <- logistic_reg() %>%
  set_engine("glm") %>%
  fit(spam ~ . - from - sent_email - viagra, data = train_data, family = "binomial")
```

Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

```
email_fit
```

```
## parsnip model object
##
## Call: stats::qlm(formula = spam \sim . - from - sent email - viagra, family = stats::binomial,
       data = data
##
## Coefficients:
   (Intercept) to multiple1
                                                    time
                                                                                             dollar
                                                                               attach
                                                                  image
                                        CC
    -9.867e+01
                  -2.505e+00
                                 1.944e-02
                                                7.396e-08
                                                            -2.854e+00
                                                                            5.070e-01
                                                                                         -6.440e-02
##
     winneryes
                                                                                           re subj1
                      inherit
                                  password
                                               num char
                                                            line breaks
                                                                              format1
##
                   4.499e-01
                                -7.065e-01
                                                5.870e-02
                                                            -5.420e-03
                                                                           -9.017e-01
                                                                                         -2.995e+00
     2.170e+00
## exclaim subj urgent subj1
                              exclaim mess
                                              numbersmall
                                                          numberbig
                    3.572e+00
                                 1.009e-02
                                               -8.518e-01
                                                            -1.329e-01
##
     1.002e-01
## Degrees of Freedom: 3135 Total (i.e. Null); 3117 Residual
## Null Deviance:
## Residual Deviance: 1447
                              AIC: 1485
```



Predict outcome on the testing dataset

```
predict(email_fit, test_data)
```

Predict probabilities on the testing dataset

```
email_pred <- predict(email_fit, test_data, type = "prob") %>%
  bind_cols(test_data %>% select(spam, time))
email_pred
```

```
## # A tibble: 785 \times 4
     .pred_0 .pred_1 spam
##
                          time
      <dbl> <dbl> <fct> <dttm>
##
    0.993 0.00709 0
## 1
                           2012-01-01 19:55:06
## 2
     0.998 0.00181 0
                           2012-01-01 21:38:32
## 3 0.981 0.0191 0
                           2012-01-02 07:42:16
## 4 0.999 0.00124 0
                           2012-01-02 17:12:51
## 5 0.988 0.0121 0
                           2012-01-02 18:45:36
## 6 0.830 0.170
                           2012-01-02 23:55:03
## # ... with 779 more rows
```

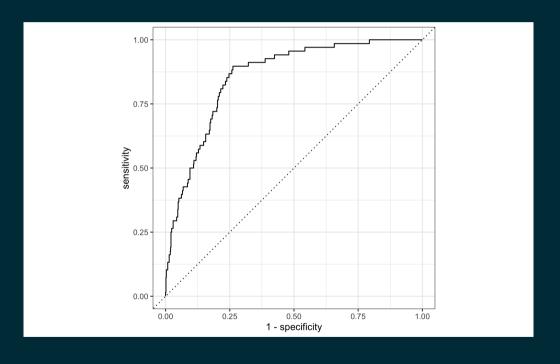
A closer look at predictions

```
email pred %>%
   arrange(desc(.pred 1)) %>%
   print(n = 10)
    A tibble: 785 \times 4
##
      .pred 0 .pred 1 spam
                             time
##
        <dbl>
                <dbl> <fct> <dttm>
                0.903 1
##
   1 0.0972
                             2012-02-13 14:15:00
##
       0.167
                0.833 0
                             2012-01-27 22:05:06
       0.267
                0.733 1
                             2012-03-17 12:13:27
##
       0.317
                0.683 1
                             2012-03-21 14:33:12
                0.626 1
##
    6 0.374
                             2012-02-08 10:00:05
   8 0.403
                0.597 1
                             2012-01-07 18:11:49
##
##
       0.462
                0.538 1
                             2012-03-06 13:46:20
  10
       0.463
                0.537 0
                             2012-02-18 00:54:16
##
## # ... with 775 more rows
```

Evaluate the performance

Receiver operating characteristic (ROC) curve⁺ which plot true positive rate vs. false positive rate (1 - specificity)

```
email_pred %>%
  roc_curve(
    truth = spam,
    .pred_1,
    event_level = "second"
) %>%
  autoplot()
```



⁺Originally developed for operators of military radar receivers, hence the name.

Evaluate the performance

Find the area under the curve:

```
email_pred %>%
  roc_auc(
    truth = spam,
    .pred_1,
    event_level = "second"
)
```

