Dual Simplex Method.

1) Convert the objective function and the constraints into the standard form. Either to Min^n / Max^n. in such a ~~way~~ way that the Soln col^m/row values should be -ve if possible to choose the Pivot row. such as (most -ve value)
2) then Select the Pivot col^n by using Z-row and Pivot row by finding ratio & its value choose (Min^m +ve value)
3) Then find the Pivot row as usual simplex method. with the all other rows including Z-row.
4) Continue until all Z-row values ≤ 0 and Soln col/n values be +ve.

1) Max. $Z = -2x_1 - 3x_2$

STC; $x_1 + x_2 \geq 2$

$2x_1 + x_2 \leq 10$

$x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$

Min $Z = 2 * 2 = 4$

$\therefore$ Max $Z = -4$

$Z = -2 * 2 = -4$

Min $Z = 2x_1 + 3x_2$

$\therefore Z - 2x_1 - 3x_2 + 0S_1 + 0S_2 + 0S_3$

STC; $x_1 + x_2 - S_1 = 2 \Rightarrow -x_1 - x_2 + S_1 = -2$

$2x_1 + x_2 + S_2 = 10$

$x_1 + x_2 + S_3 = 8$

Basic	x_1	x_2	S_1	S_2	S_3	Soln
Z	-2	-3	0	0	0	0
S_1	-1	-1	1	0	0	-2
S_2	2	1	0	1	0	10
S_3	1	1	0	0	1	8
Ratio	2	3	—	—	—	

Pivot col^n = row with the ratio col^n

$0 - (-2) * -2 = -4$

Basic	x_1	x_2	S_1	S_2	S_3	Soln
Z	2	-3	0	0	0	-4
x_1	1	1	-1	0	0	2
S_2	0	-1	2	1	0	6
S_3	6	0	0	1	0	6

(-4)

Hence it is not

O.R

(5)

2) Min $Z = 2x_1 + x_2 + 3x_3$

S.T.C; $x_1 - 2x_2 + x_3 \geq 4$

$2x_1 + x_2 + x_3 \leq 8$

$x_1 - x_3 \geq 0$

$x_1, x_2, x_3 \geq 0$

Modified Z

$Z - 2x_1 - x_2 - 3x_3 + 0S_1 + 0S_2 + 0S_3 = 0$

S.T.C;

$-x_1 + 2x_2 - x_3 + S_1 = -4$

$2x_1 + x_2 + x_3 + S_2 = 8$

$-x_1 + x_3 + S_3 = 0$

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol^n
Z	-2	-1	-3	0	0	0	0
S_1	-1	2	-1	1	0	0	-4
S_2	2	1	1	0	1	0	8
S_3	-1	0	1	0	0	1	0
Ratio	2	-1/2	3	—	—	—	

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol^n
Z	0	-5	-1	-2	0	0	8
x_1	1	-2	1	-1	0	0	4
S_2	0	5	-1	2	1	0	0
S_3	0	-2	2	-1	0	1	4

$Z - 2x_1 - x_2 - 3x_3 = 0$

Min $Z = 2 * 4$

$= 8$

S.T.C;

$x_1 - 2x_2 + x_3 - S_1 = 4$

$2x_1 + x_2 + x_3 + S_2 = 8$

$x_1 - x_3 - S_3 = 0$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

/Dual

3) Minimize:

$Z = 3x_1 + 2x_2$

STC; $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + x_2 \leq 3,\ x_1, x_2 \geq 0$

$\Rightarrow$

$Z - 3x_1 - 2x_2 = 0$

STC $-3x_1 - x_2 + S_1 = -3$

$-4x_1 - 3x_2 + S_2 = -6$

$x_1 + x_2 + S_3 = 3$

Basic		x_1	x_2	S_1 x_3	S_2 x_4	S_3 x_5	Soln.
	Z	−3	−2	0	0	0	0
x_3	S_1	−3	−1	1	0	0	−3
x_4	S_2	−4	−3	0	1	0	−6
x_5	S_3	1	1	0	0	1	3

Variables	x_1	x_2	S_1 x_3	S_2 x_4	S_3 x_5
Z row	−3	−2	0	0	0
x_4/S_2 row	−4	−3	0	1	0
Ratio	①/② 1	3/4	2/3	—	—	—

Basic		x_1	x_2	x_3	x_4	x_5	Soln
	Z	−1/3	0	0	−2/3	0	[illegible] 4
S_1	x_3	−5/3	0	1	−1/3	0	−1
S_2	x_2	4/3	1	0	−1/3	0	2
S_3	x_5	−1/3	0	0	1/3	1	1
Ratio		1/5	—	—	2	—	

Basic	x_1	x_2	x_3	x_4	x_5	Sol^n
Z	0	0	$-1/5$	$-3/5$	0	$21/5$
x_1	1	0	$-3/5$	$1/5$	0	$3/5$
x_2	0	1	$4/5$	$3/5$	0	$6/5$
x_3	0	0	$-1/5$	$2/5$	1	$6/5$

$\therefore Z = 3 \times 3/5 + 2 \times 6/5$

$= 9/5 + 12/5 = 21/5.$

4) $Z_{max} = -3x_1 - 2x_2$

S.T.C; $x_1 + x_2 \geq 1$

$x_1 + x_2 \geq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \geq 3$

$x_1, x_2 \geq 0$:

$\max Z = -3x_1 - 2x_2$

S.T.C.

$-x_1 - x_2 \leq -1$

$-x_1 - x_2 \leq -7$

$-x_1 - 2x_2 \leq -10$

$-x_2 \leq -3$

Convert inequalities to equalities.

$-x_1 - x_2 + S_1 = -1$

$-x_1 - x_2 + S_2 = -7$

$-x_1 - 2x_2 + S_3 = -10$

$-x_2 + S_4 = -3$

Modified objective function,

$\max Z = -3x_1 - 2x_2$

$\min Z = 3x_1 + 2x_2$

$\min Z = -3x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

P.C

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Soln
Z-row	-3	-2	0	0	0	0	0
S_1	-1	-1	1	0	0	0	-1
S_2	-1	-1	0	1	0	0	-7
S_3	-1	-2	0	0	1	0	-10 ⇐ PR
S_4	0	-1	0	0	0	1	-3

All the values in Z-row is ≤ 0 but all soln are also -ve. Hence it is not feasible.

	x_1	x_2	S_1	S_2	S_3	S_4	
Z-row	-3	-2	0	0	0	0	
S_3-row	-1	-2	0	0	1	0	
Ratio ①/②	3	1	0	0	0	0	$0-(-2)\times\frac{1}{2}$
		⇑					$= 10$

P.C

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Soln
Z-row	-2	0	0	0	-1	0	+10
S_1	-1/2	0	1	0	-1/2	0	4
S_2	-1/2	0	0	1	-1/2	0	-2 ⇐ PR
x_2	1/2	1	0	0	-1/2	0	5
S_4	1/2	0	0	0	-1/2	1	2

In Z-row all the values are ≤ 0

But in Soln col'n in the 2nd row i.e., S_2 has -ve value. Hence it is not feasible. Hence Proceed...

	x_1	x_2	S_1	S_2	S_3	S_4	
Z-row	−2	0	0	0	−1	0	—①
Pivot-row (S_2-row)	−1/2	0	0	1	−1/2	0	—②
Ratio ①/②	4	0	0	0	2 ⇑	0	

Basic variable	x_1	x_2	S_1	S_2	S_3	S_4	Soln
Z-row	−1	0	0	−2	0	0	+14
S_1	0	0	1	−1	0	0	6
S_3	1	0	0	−2	1	0	4 ← PR
x_2	1	1	0	−1	0	0	7
S_4	1	0	0	−1/2	0	1	4