NLTS Hamiltonians from good quantum codes

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* prev. Berkeley

Understanding classical proofs

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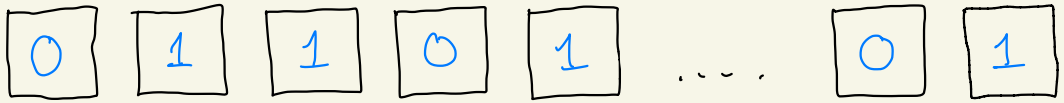

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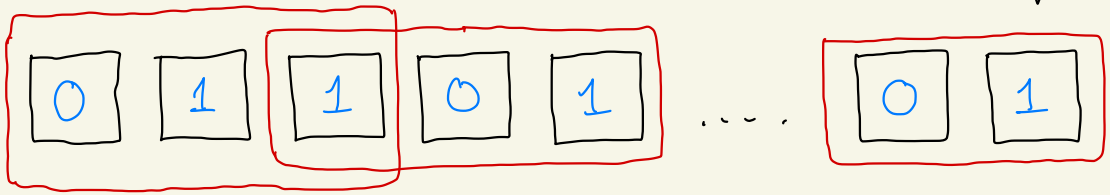

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local check $C_i = x_1 \oplus x_2 \oplus x_3 = 0$.

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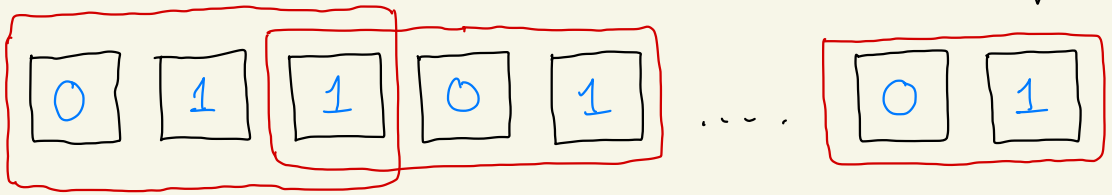

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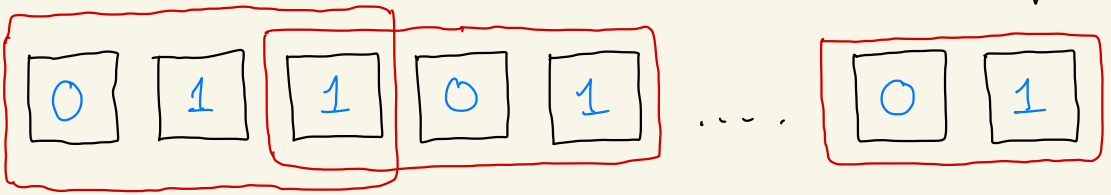

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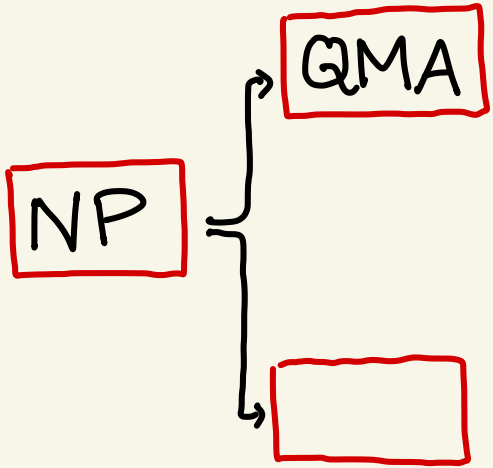
Decide if

① $\exists\, x,\ C(x) = 0$.

② $\forall\, x,\ C(x) \geq 1$.

Two extensions of the notion of proofs



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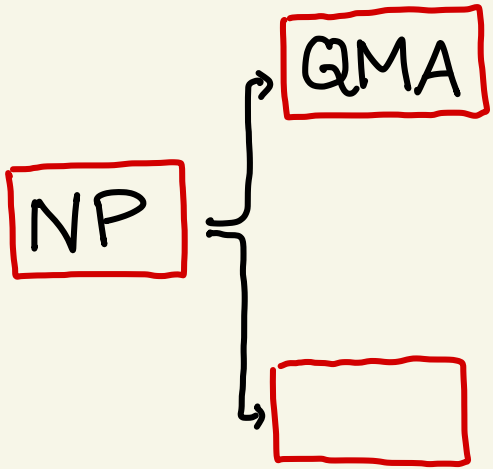

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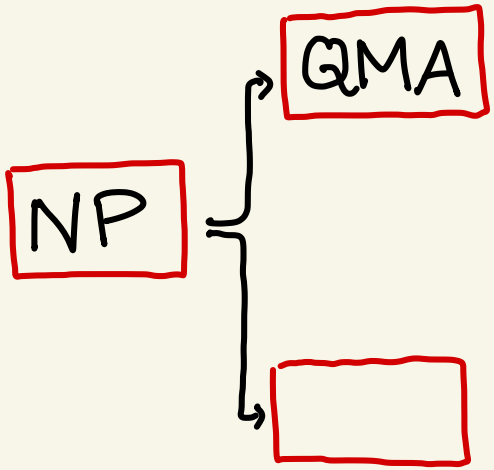

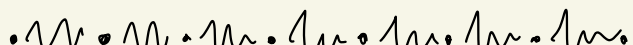

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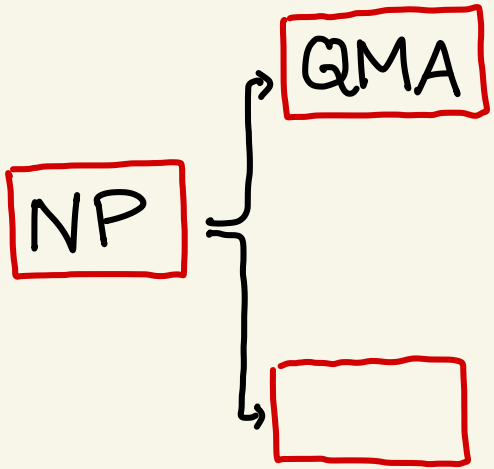

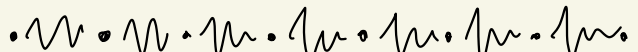

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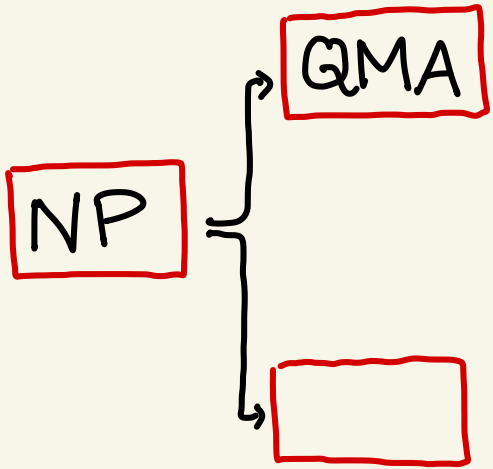

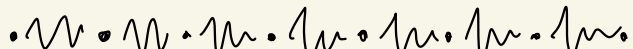

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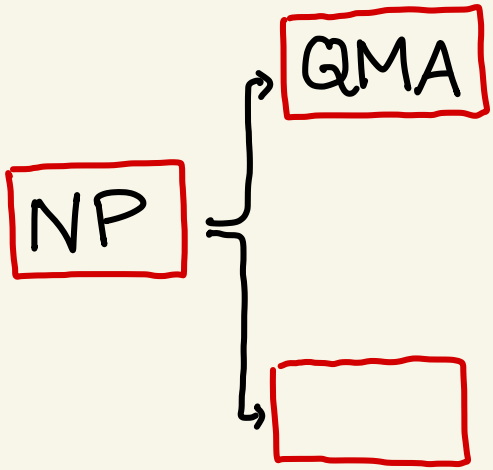

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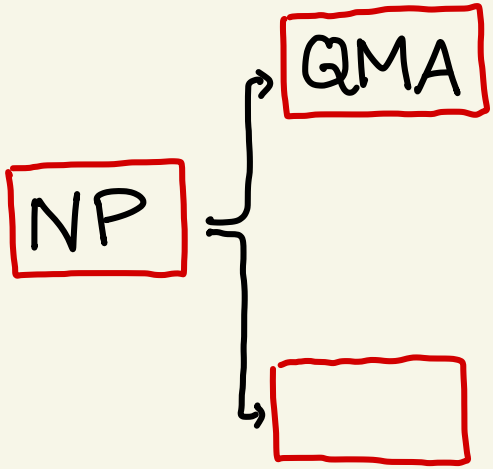

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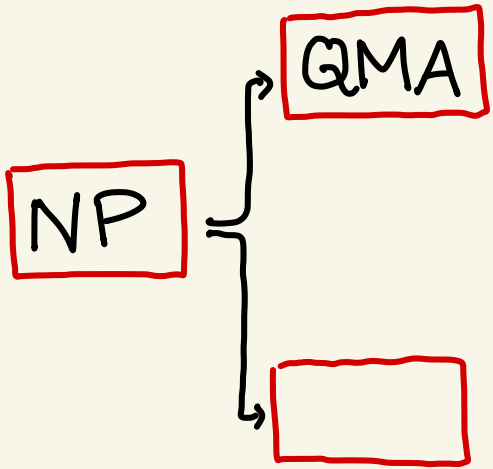

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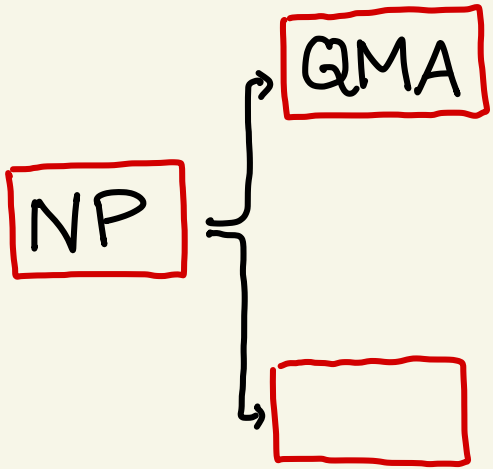

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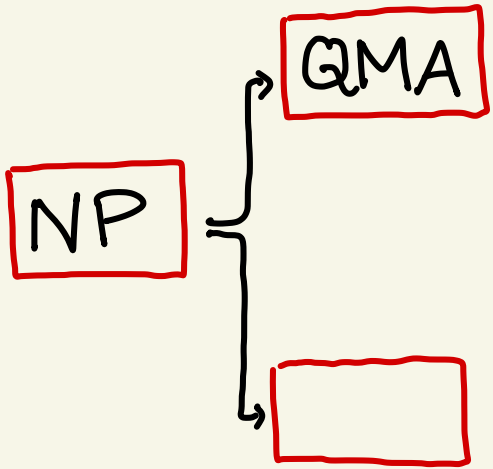

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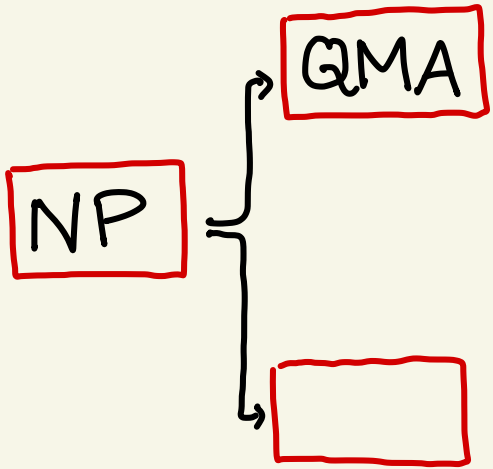

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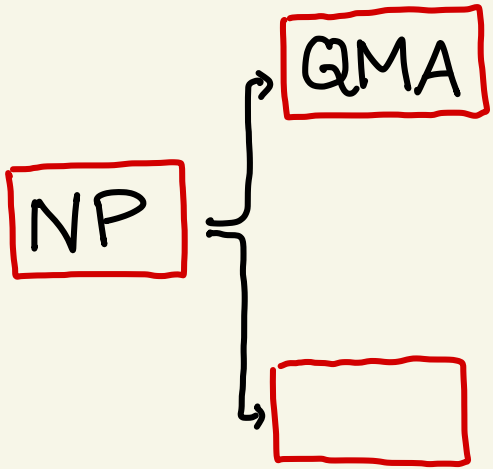

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Therefore, not all groundstates of local Hamiltonians can be classically described (in an efficiently verifiable manner)

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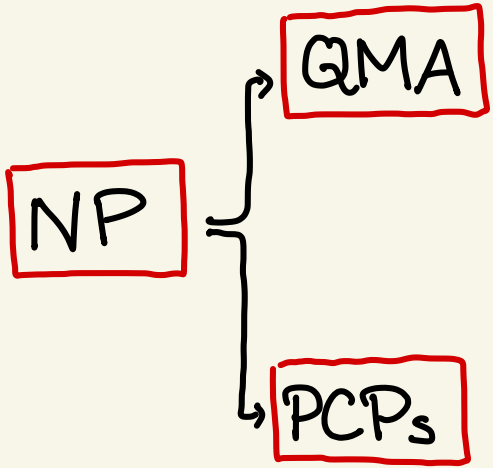

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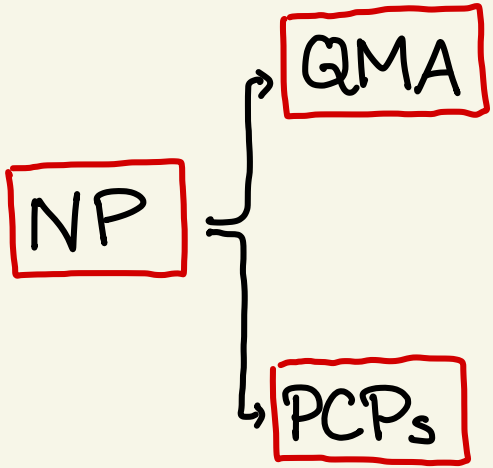

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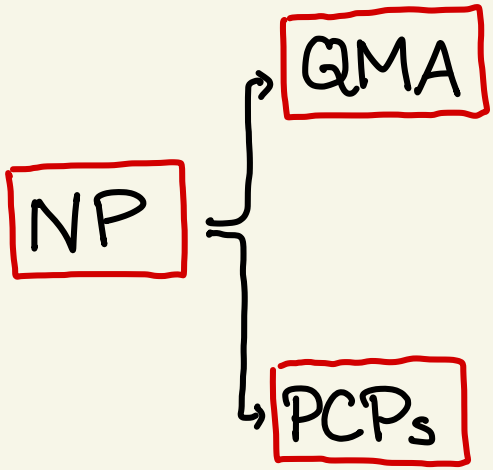

PCP theorem Every NP problem (i.e. every pf.) can be converted into a form s.t. only $O(1)$ bits need to be read to be 99% confident in validity.

Arora-Safra et al. 98, Dinur

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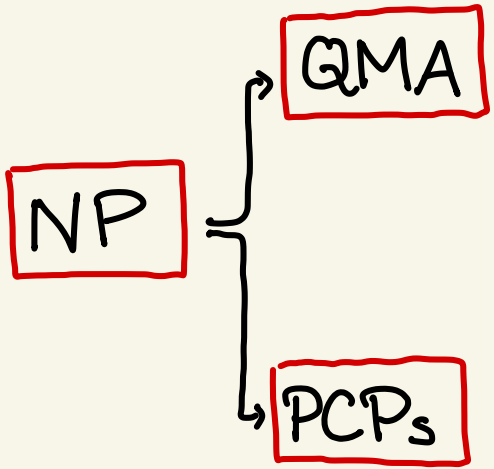

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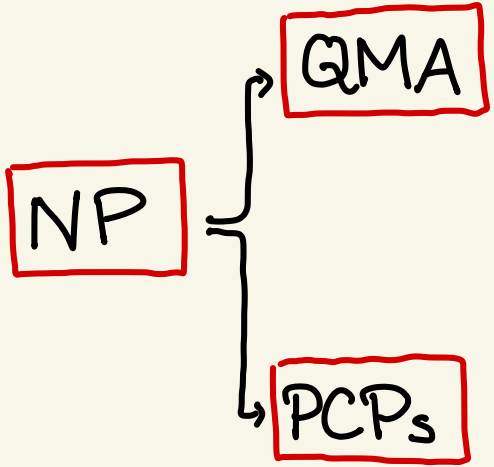

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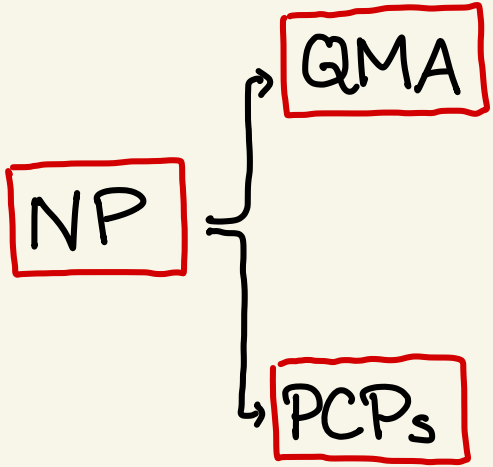

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Any x s.t. $C(x) < \frac{m}{4}$ can be prob. verified with $O(1)$ queries.

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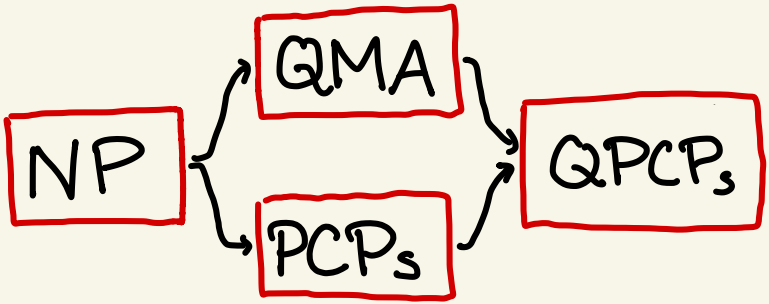

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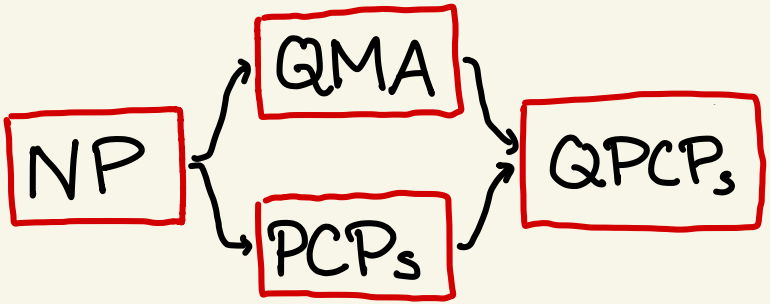

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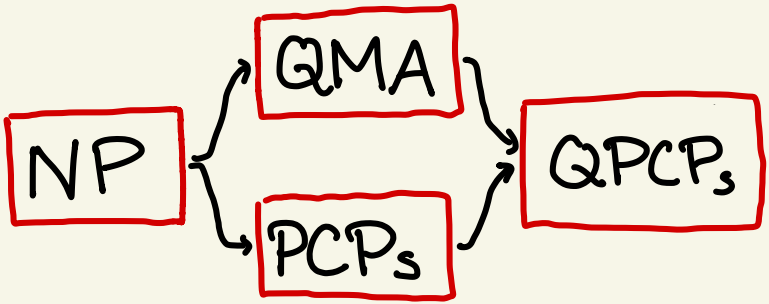

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Conj. For $\varepsilon > 0$, it's QMA-hard to decide

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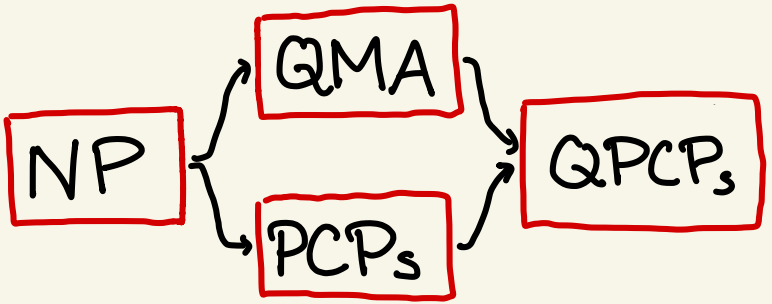

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Similar to PCP theorem, every state of energy $\leq \frac{\varepsilon}{2} m$ is a valid pf. for a QPCP local Hamiltonians.

Set of pfs is much larger!

An important consequence of QPCPs

(A) (if NP ≠ QMA) quantum pfs. cannot be classically described (in any efficiently checkable manner)

(B) low. energy states of QPCP local Hamiltonians are also valid pfs (since they are noisy pfs.)

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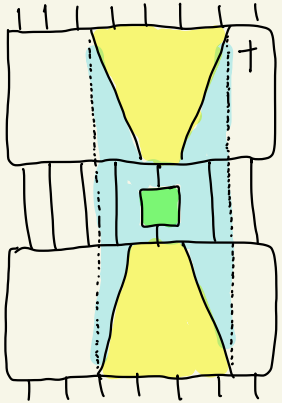
$\exists\, \varepsilon > 0$, and Hamiltonian family $\mathbf{H}$ s.t. every state ψ of energy $\leq \varepsilon n$, the minimum depth circuit to generate ψ is $\Omega(\log n)$.

Proof sketch of the NLTS theorem

(1) Trivial states $\Rightarrow$ Local Hamiltonians

$\Rightarrow$ Circuit depth lower bounds



Lightcones for low depth circuits

Proof sketch of the NLTS theorem

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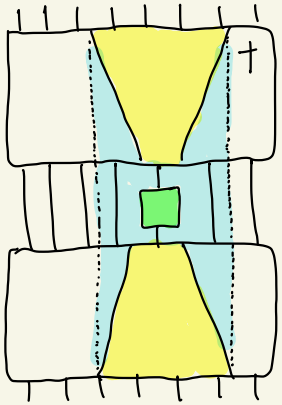
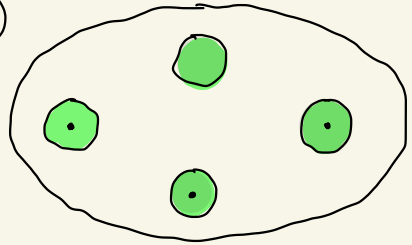


Lightcones for low depth circuits

Error Correction Codes (ECC)

(2)



↙ low energy subspace of expanding codes.

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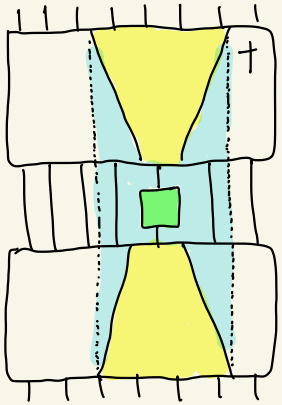

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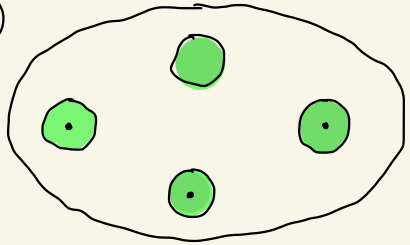
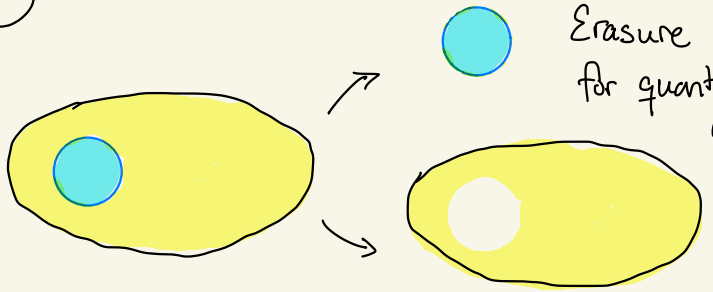


③



Erasure errors for quantum codes

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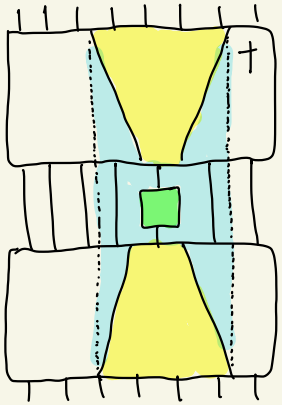

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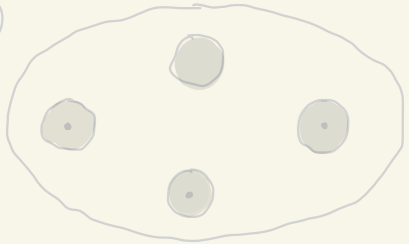

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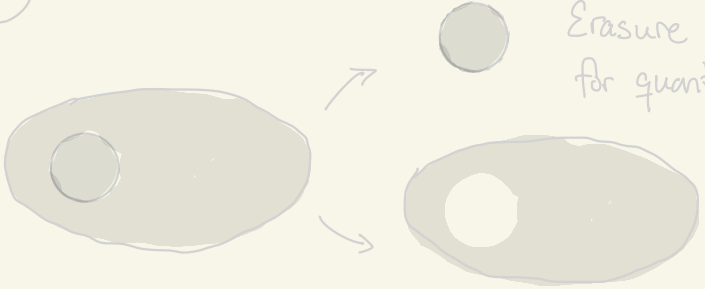


Erasure errors for quantum codes

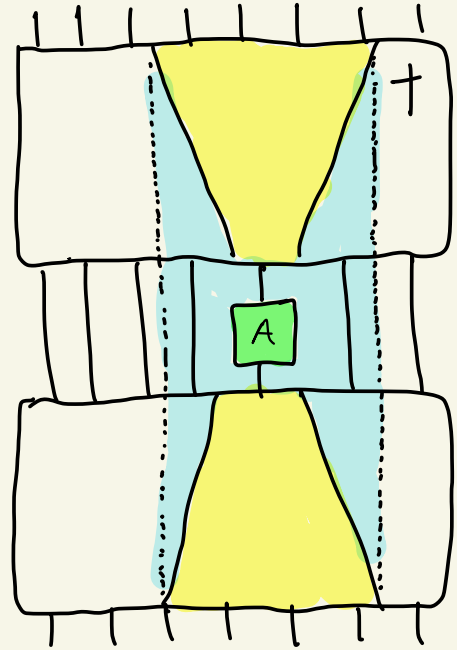
Lightcones and quantum circuits

Lightcones and quantum circuits

Low-depth states are classical witnesses for energy

Lightcones and quantum circuits

If A is a local operator and $\mathcal{U}$ is a q. circuit of depth t, then $\mathcal{U}^\dagger A \mathcal{U}$ is a $\leq 2^t \cdot |A|$ local operator.



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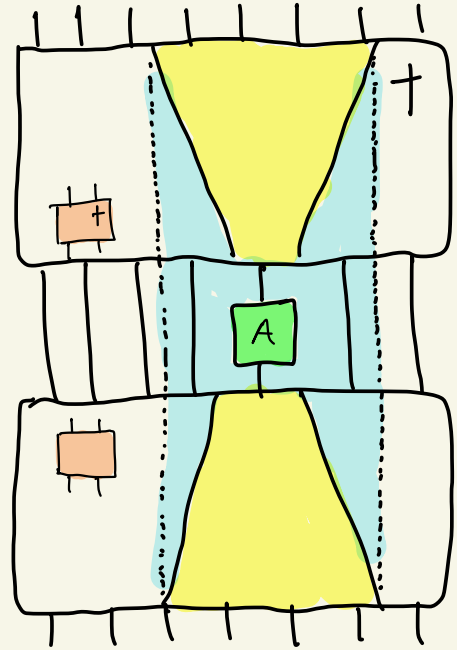

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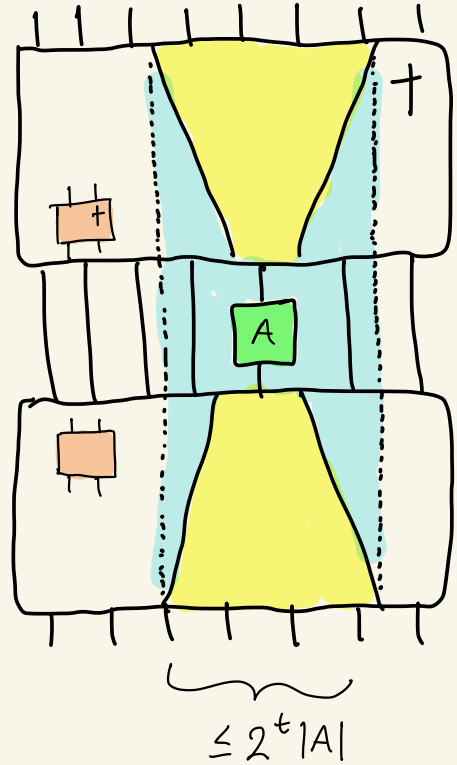

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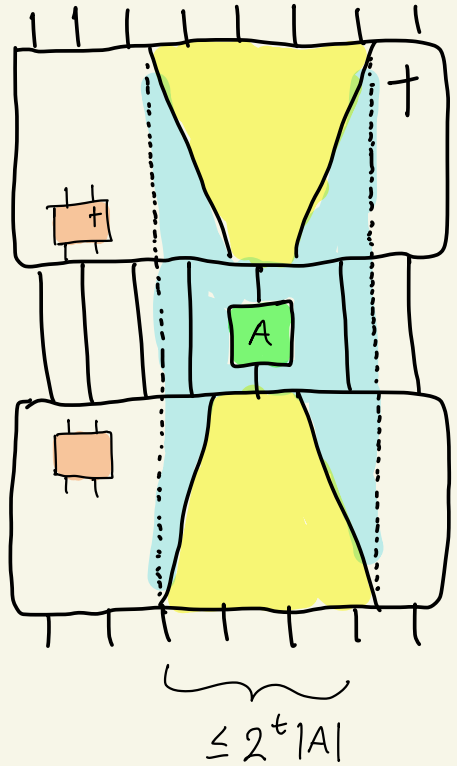

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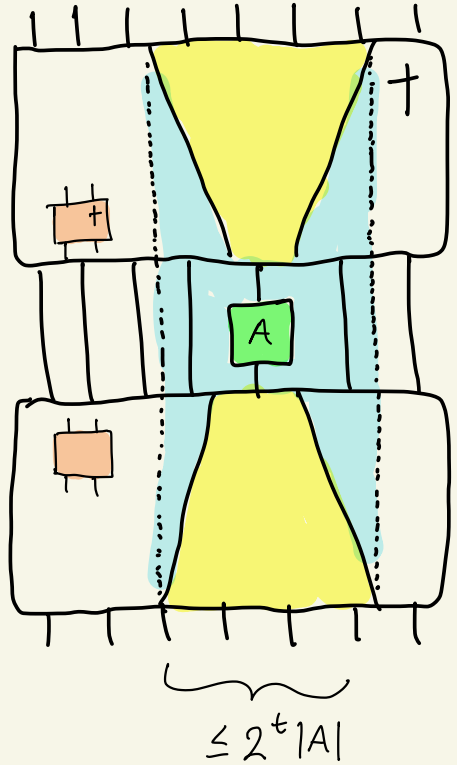

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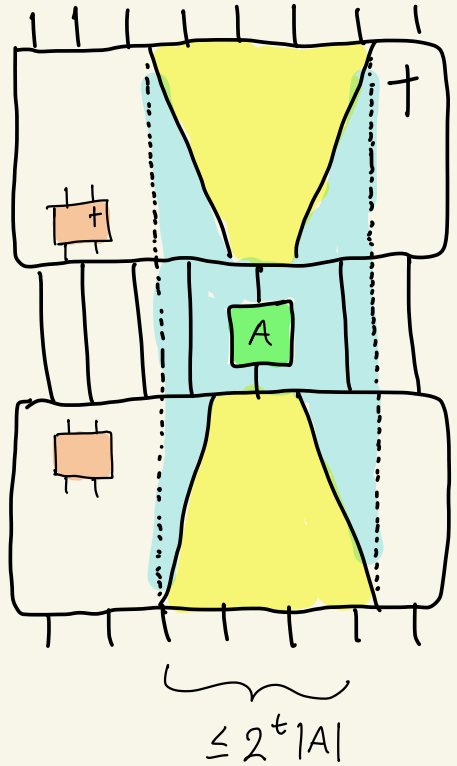

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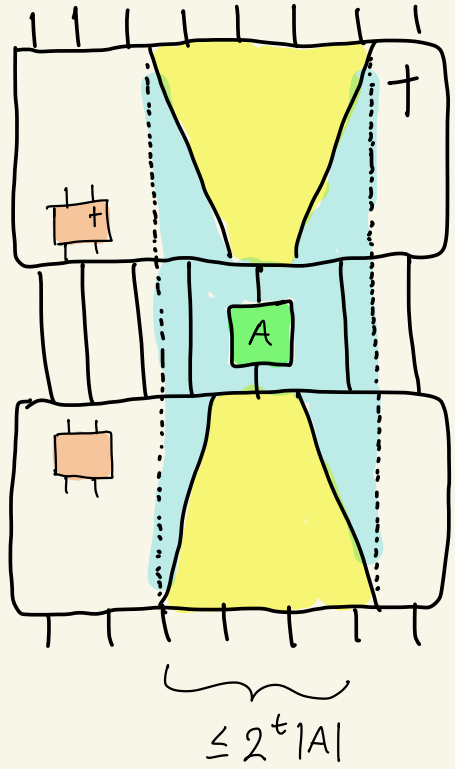

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And H_U is a 2^t-local Hamiltonian.

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Two states $|\psi\rangle$ and $|\psi'\rangle$ are d- locally indistinguishable if for every region S of size $\leq d$, $\psi_{-S} = \psi'_{-S}$.

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Any strict reduced density matrix equals

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But groundstate $|\psi\rangle$ is unique! ⇒ $|\psi\rangle = |\psi'\rangle$, a contradiction!

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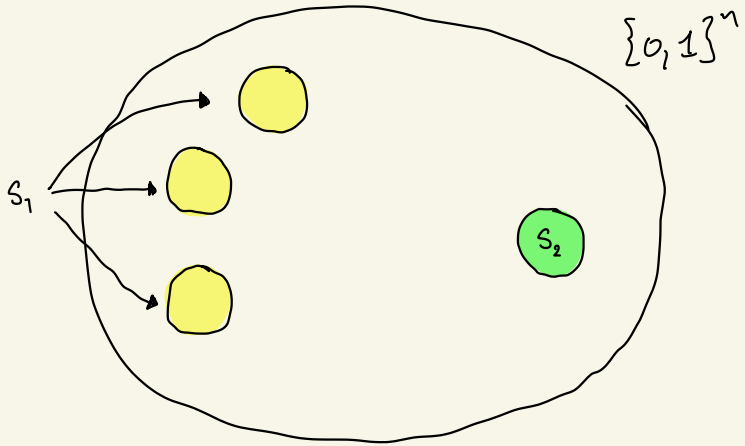

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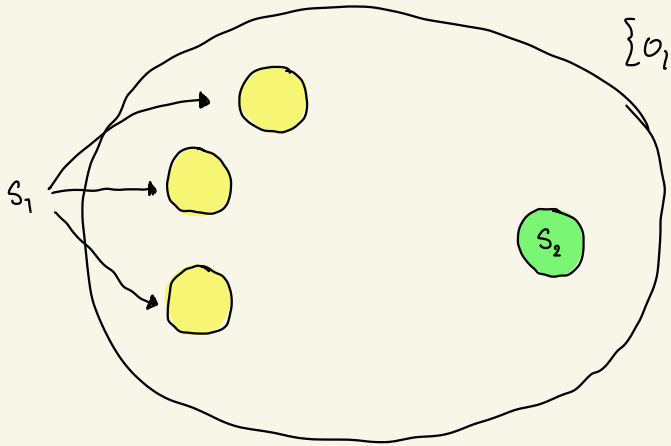

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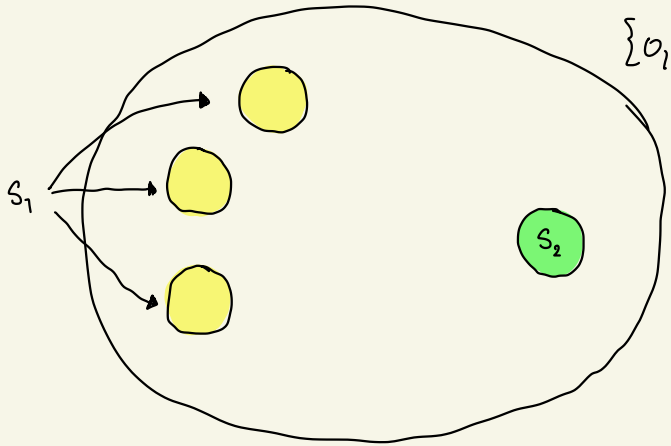

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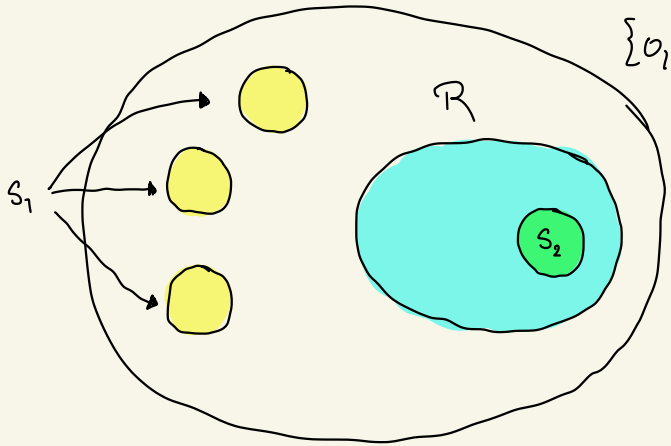

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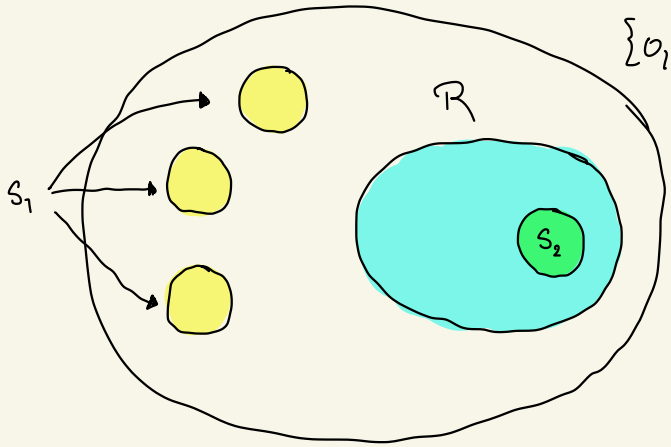

Pf sketch. Let $|\psi\rangle$ generate p. Then $\exists$ region R s.t. $|\psi'\rangle$ = "flip sign of $|\psi\rangle$ on R" and $|\psi\rangle$ and $|\psi'\rangle$ are approx. locally indistinguishable.

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When $\text{dist}(S_1,S_2) \geq \omega(\sqrt{n})$ and $\mu = \Omega(1)$,

we call such distributions well spread. To prove NLTS, we need to show $\exists$ a local Hamiltonians whose entire low-energy subspace induces well-spread distributions.

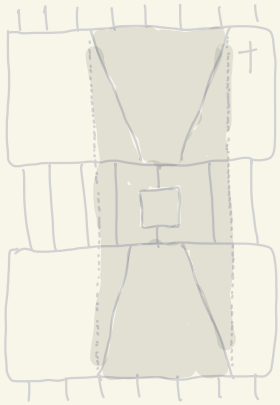
Expanding codes & Tanner codes

A linear code $\subseteq \{0,1\}^n$ can be expressed as $\ker H$ for $H \in \mathbb{F}_2^{m \times n}$

Proof sketch of the NLTS theorem

① Trivial states ⇒ Local Hamiltonians

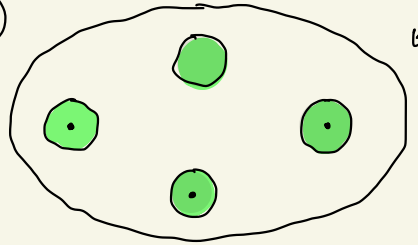
⇒ Circuit depth lower bounds



Lightcones for low depth circuits

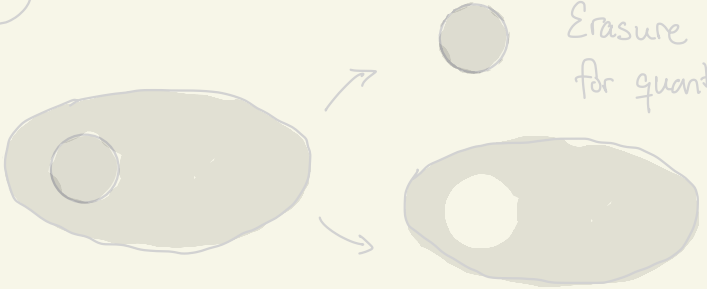
Error Correction Codes (ECC)

②



low energy subspace of expanding codes.

③



Erasure errors for quantum codes

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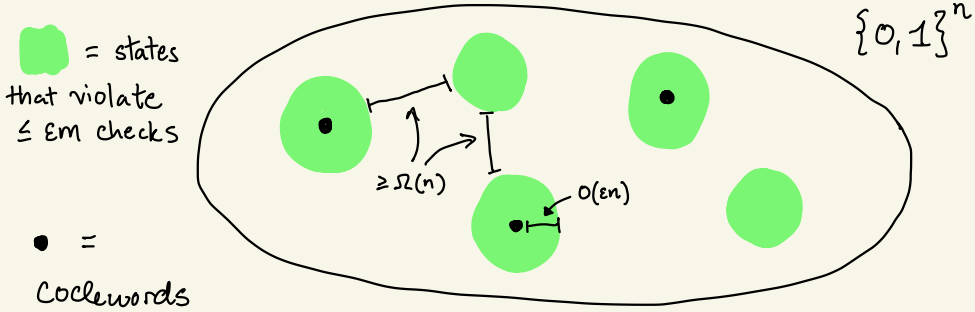

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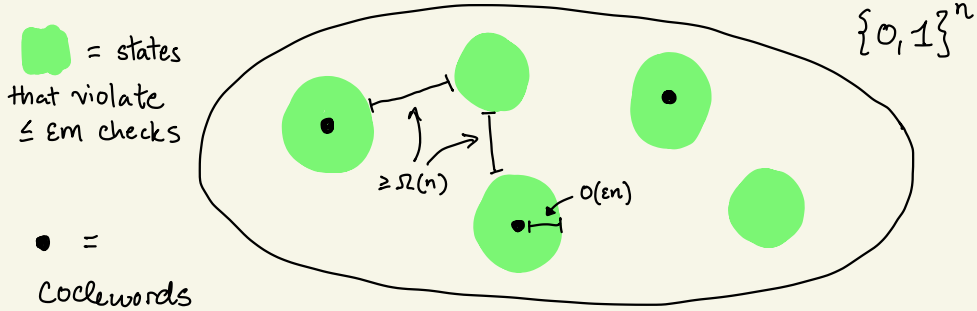

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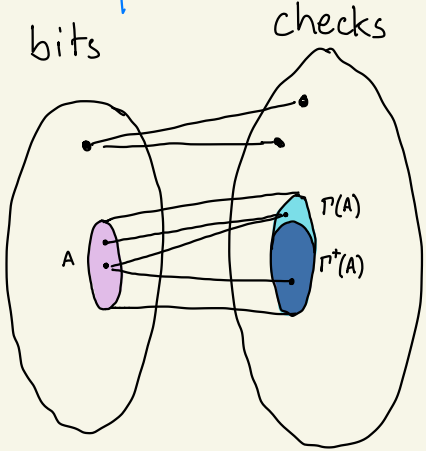

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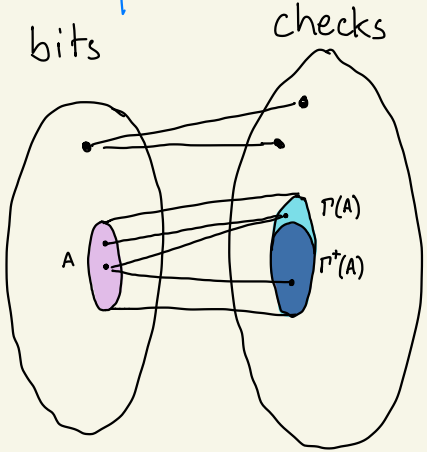

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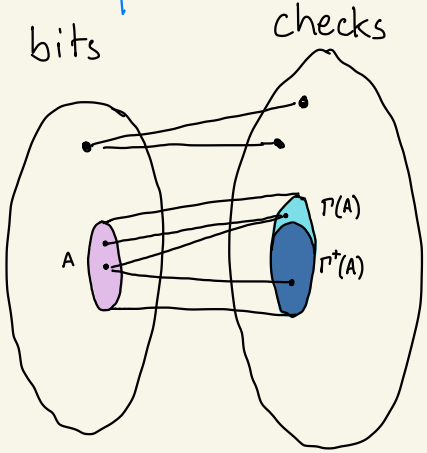

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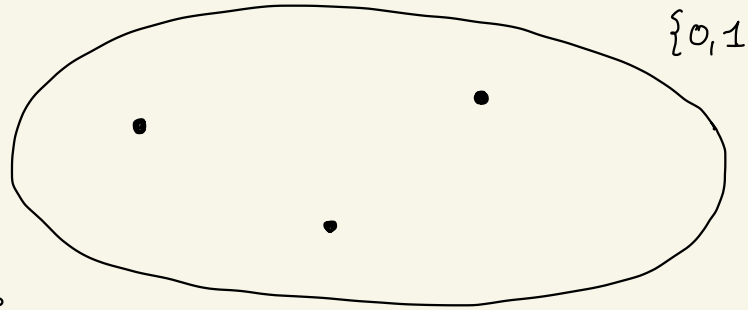

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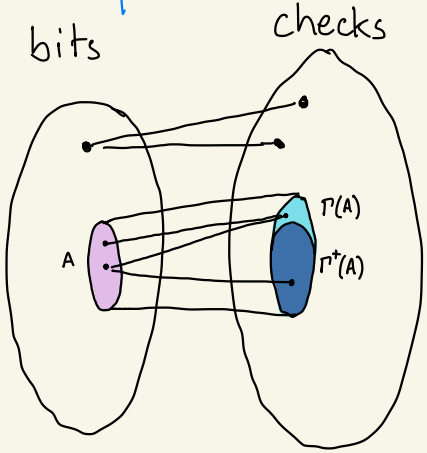

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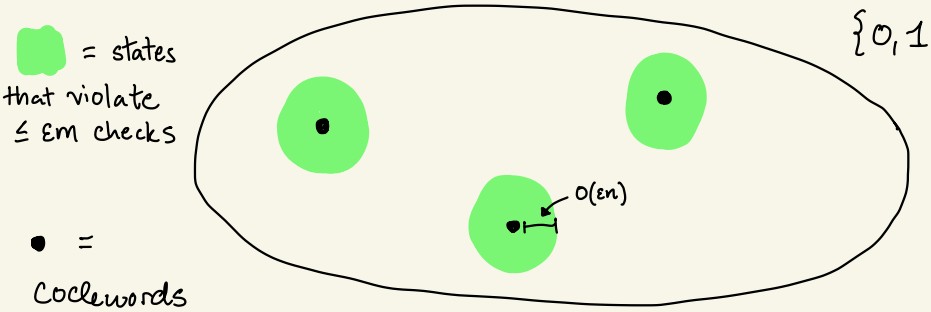

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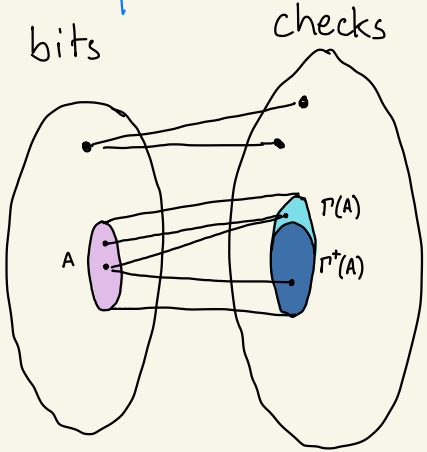

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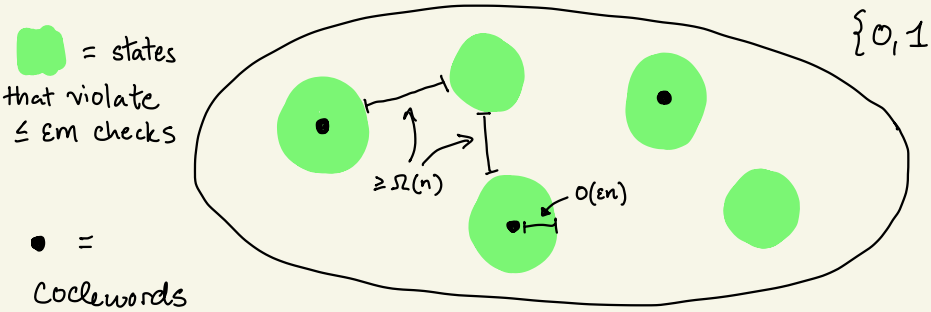

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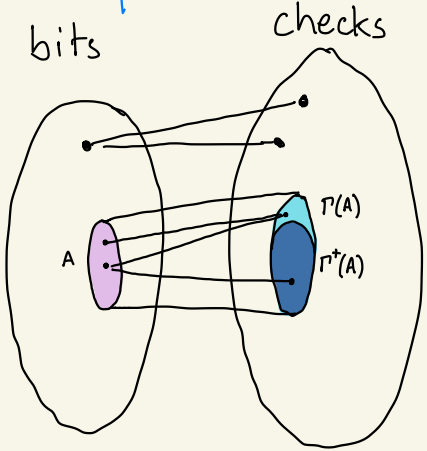

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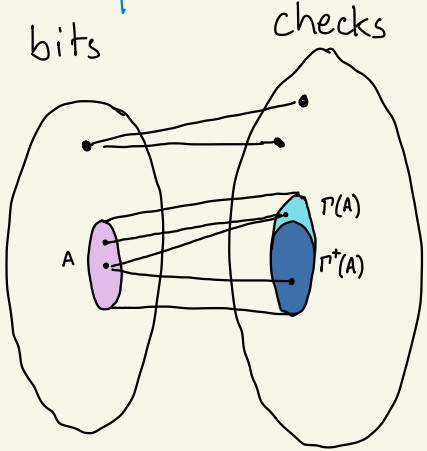

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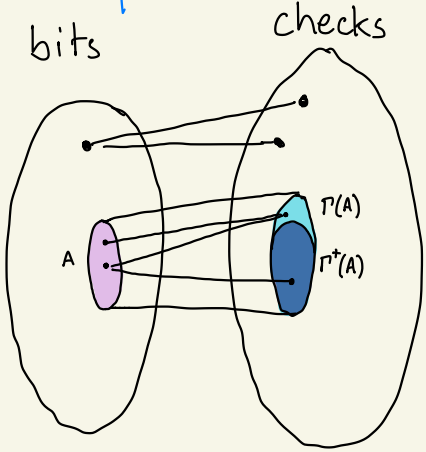

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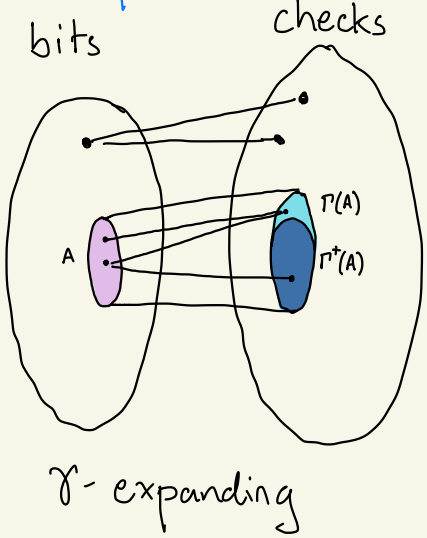

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Pf sketch: $A = \text{supp}(y)$. $\Gamma^+(A)$ = unique neighbors of $|A|$.

$|\Gamma^+(A)| \geq (1-2\gamma)\, d\, |A|$. Every check in $\Gamma^+(A)$ will flag. So $|Hy| \geq (1-2\gamma) d |y|$ unless $|y| \geq c_2 n$.

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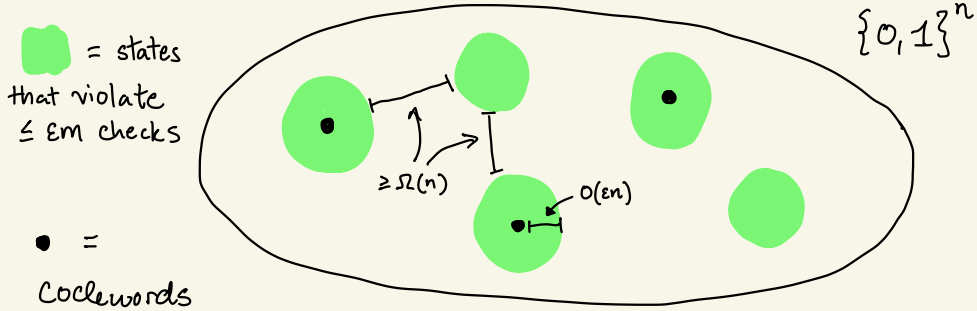
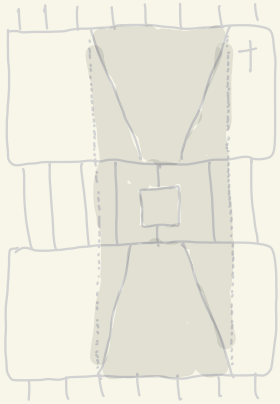


Only question is how to construct Hamiltonian with such property?

Proof sketch of the NLTS theorem

① Trivial states ⇒ Local Hamiltonians

⇒ Circuit depth lower bounds



Lightcones for low depth circuits

Error Correction Codes (ECC)

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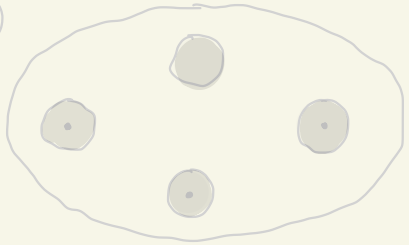

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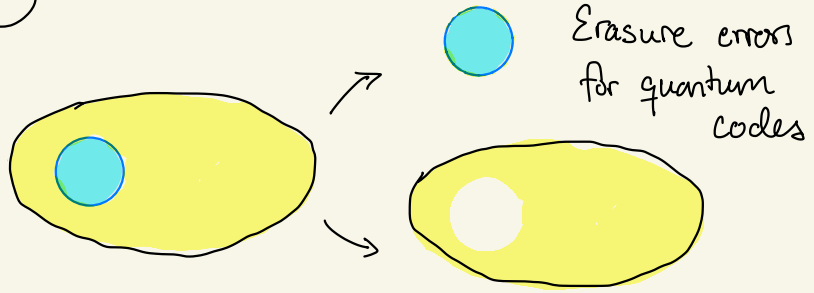
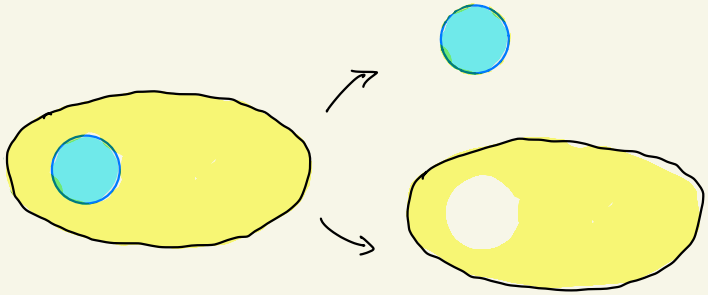


Quantum error correcting codes



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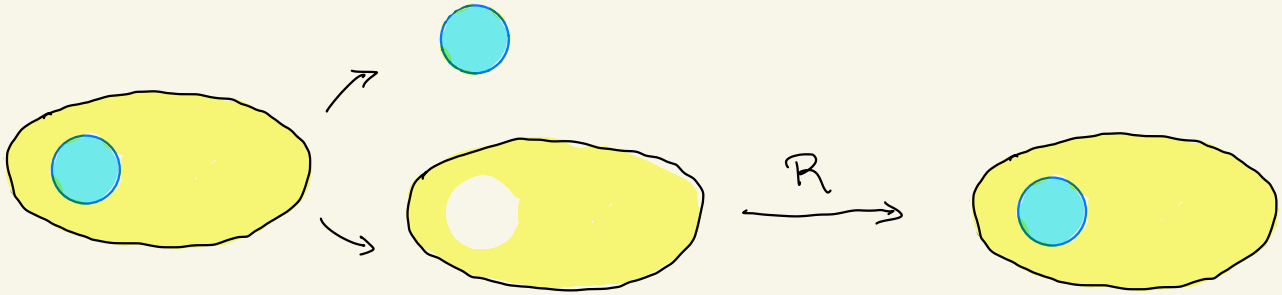

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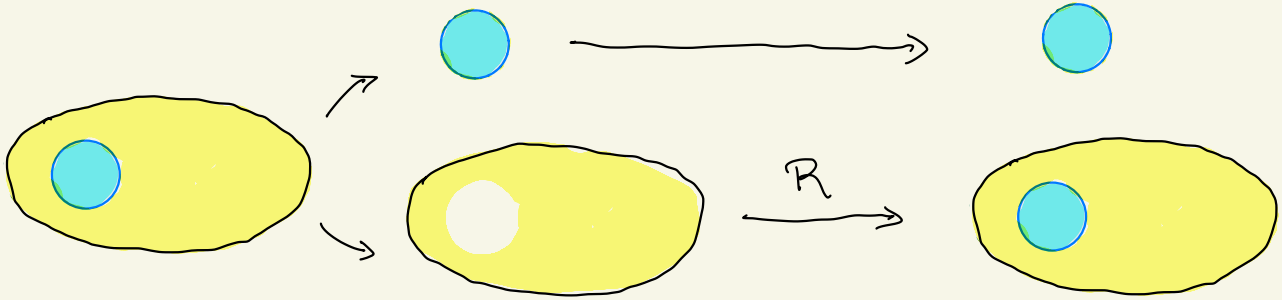

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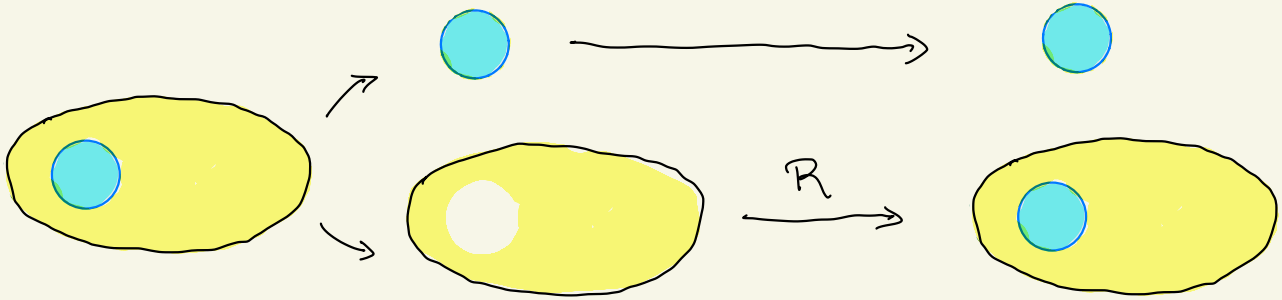

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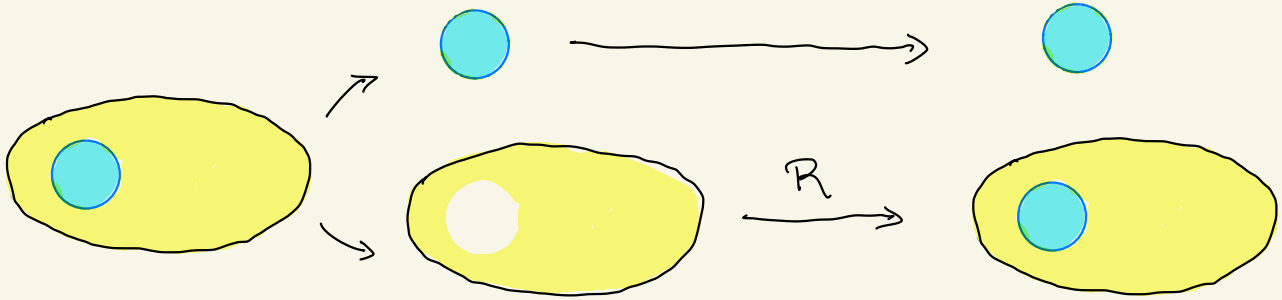

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How do we prove circuit depth lower bounds for the low-energy subspace of these code Hamiltonians?

Optimal-parameter CSS codes

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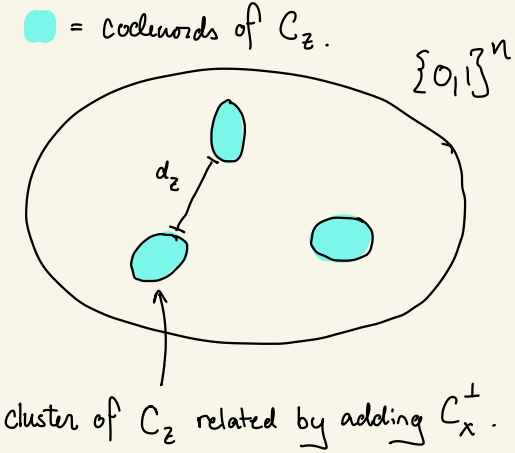

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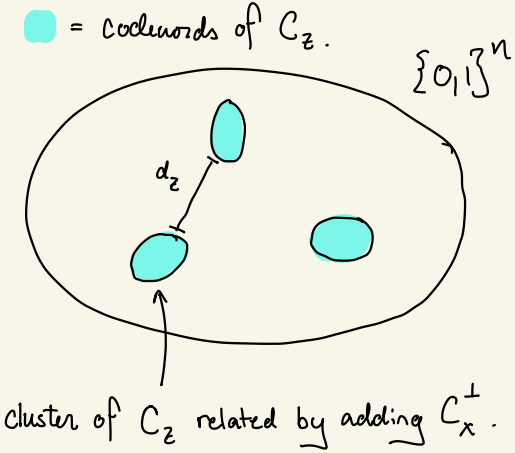

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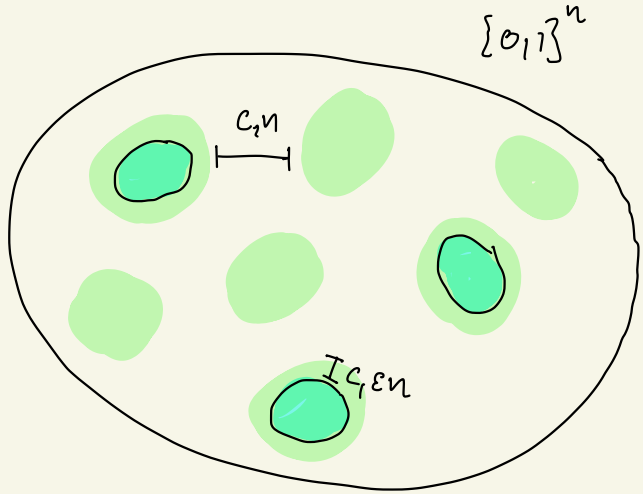

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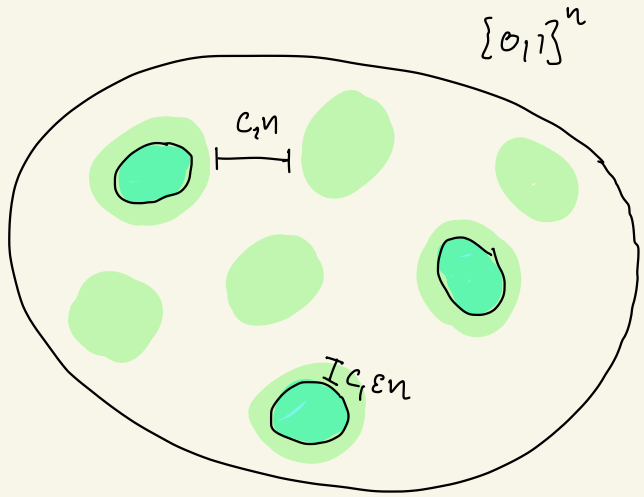
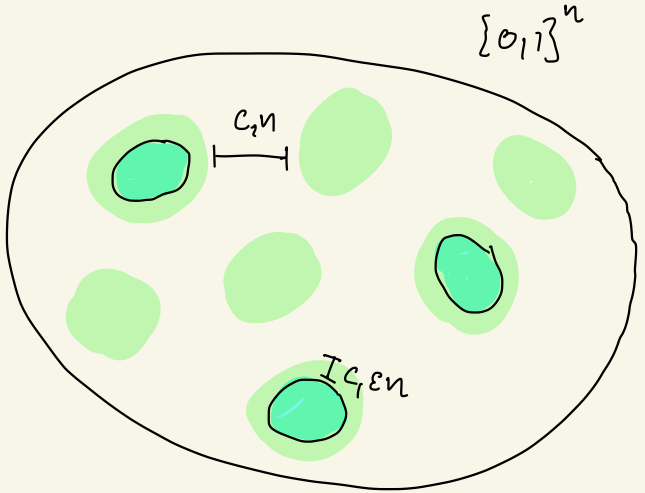


And, if we consider a $\frac{\varepsilon}{200}$-low-energy state of the code's local Hamiltonian, measuring in the Z-basis yields a dist. 99.5% supported on .

The uncertainty principle



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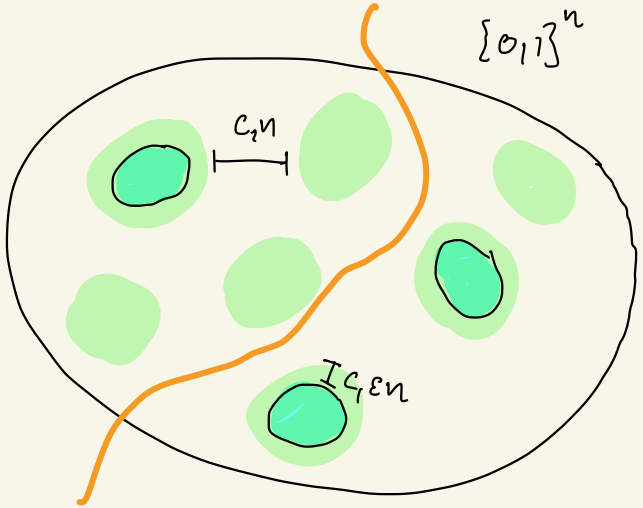

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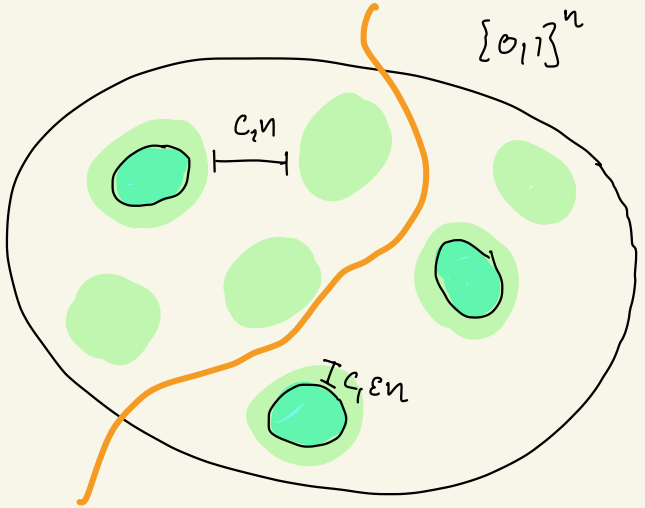

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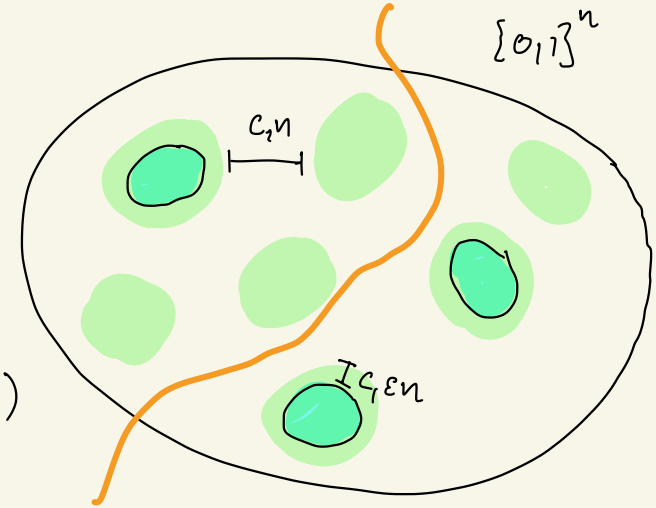

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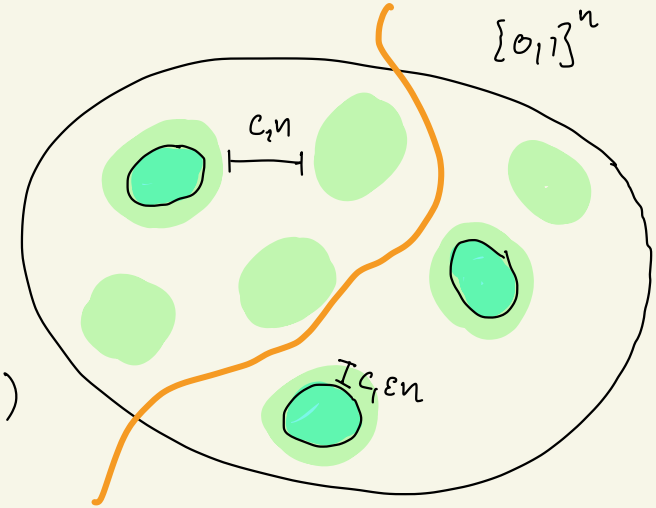

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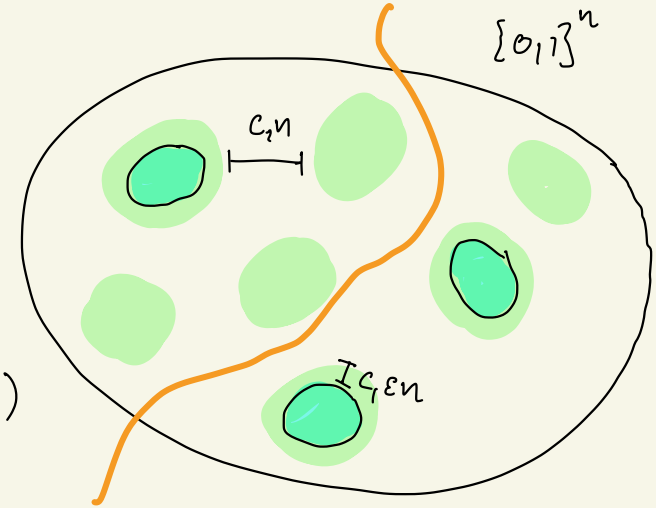

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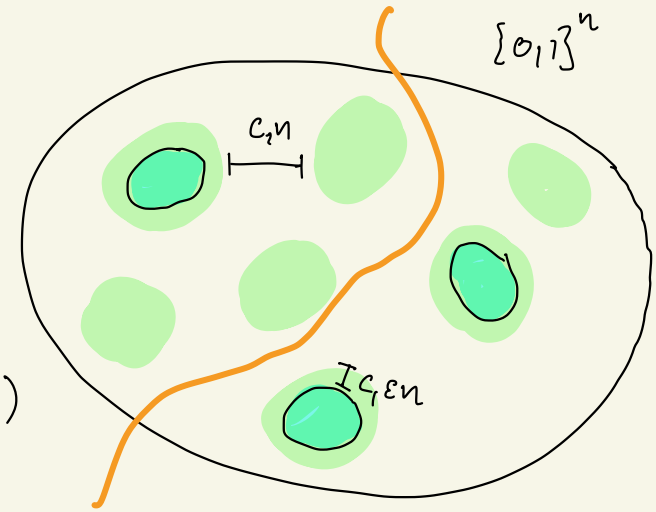

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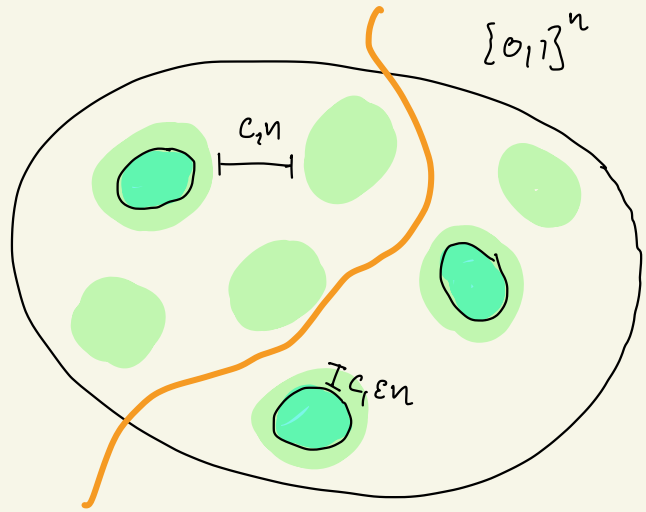

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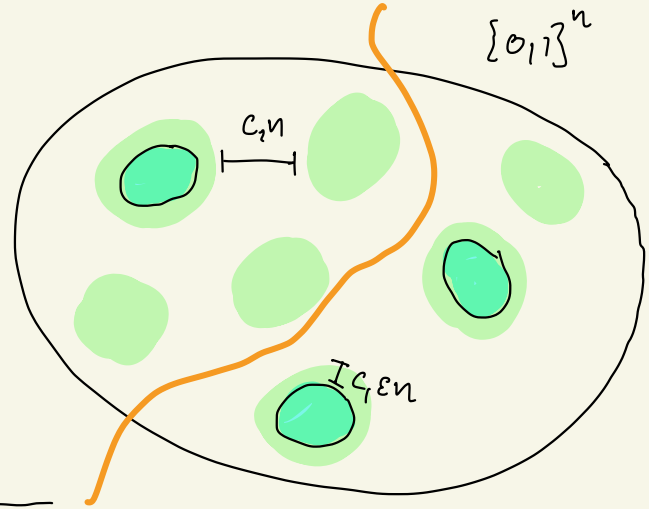

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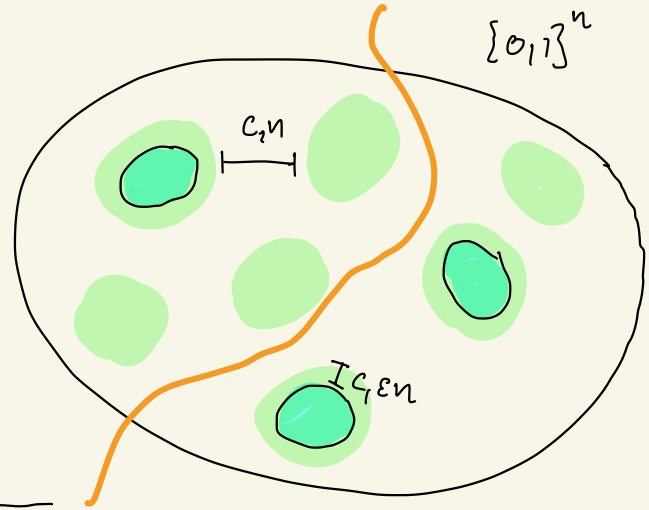

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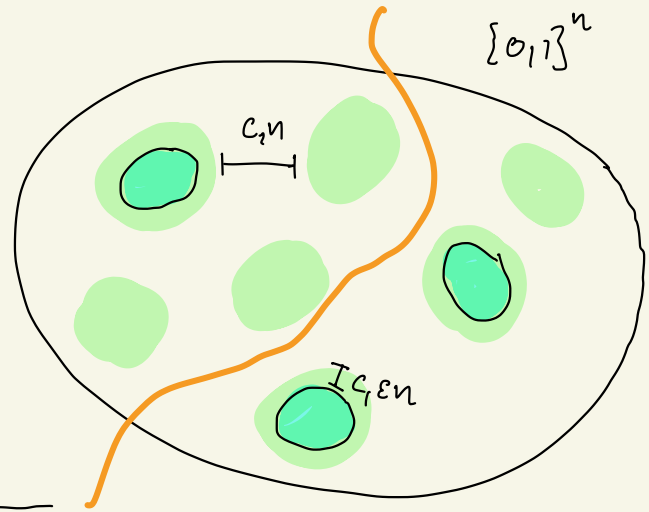

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$$D_X(T) \le 2\sqrt{\frac{1}{100}} + 2^{r_X + r_Z + O(\sqrt{\varepsilon} n) - n}$$

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↑ code rate

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k: code rate

so if $\varepsilon < O\left(\frac{k^2}{n^2}\right)$, then $D_X(T) < 0.99$.

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QPCP conjecture implications

(1) Much harder to disprove QPCP now!

(2) We need a stronger classical ansatz for classical proofs of local Hamiltonians.