

PhD Proposal:
Combined Task and Motion Planing for Multi
Robotic Arms in Pick/Place Problems

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1 Introduction

At the basis of this study we aim to express the potential of using number of robotic arms in parallel. Despite the usage of robotic arms in large applications at the industry (especially in the automotive industry) in most cases they are operating as separated units. As shown in figure 1 a car assembly line is built by a large number of robotic arms where some arms are sharing their workspaces. To avoid collision between a couple of arms a coordination method is applied, this method is based on timing principles such that the motions are priorly calculated. This means that the assembly line runs on open loop and any small changes of the sequence can cause a crush. On the contrary, having an autonomous method for coordinating motions that is based on the current state of the system may affect 3 aspects that directly connected to manufacturing costs:

1. The total time spent on manufacture a single vehicle may reduce.
2. The assembly line size may decrease.
3. A single assembly line can handle many models of vehicles.

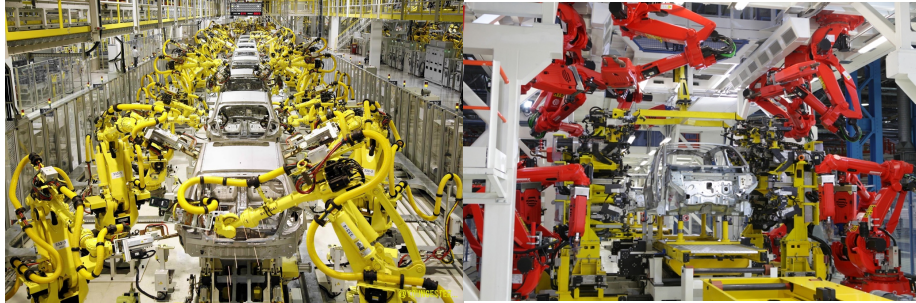


Figure 1: Assembly lines in the automotive industry

2 Literature Review

This chapter presents a literature review on integrated task and motion planning for multi robot manipulators. Although initial studies have begun since the late 80th, this issue is still a matter of research in academia due to its enormous potential. Moreover as computers getting better implementing a multi-robot planner may be a valuable opportunity for the industry. Our intention is: 1) to get the full picture of these research field 2) to know the current state of the art, and 3) formalizing a standard approach for such a problem.

2.1 Motion Planning

Todo: Motion planning for robotic arms

Todo: Collision Checking

We will start with a general description of the problem of planning for multi arms/agents. Lavalley [1] has showed that motion planning problem formulation is basically identical for single and multi robots, thus theoretically both have the same sampling based nor combinatorial search algorithms. Therefore scaling in the number of DoF is the only parameter affects the search process. For example, planning for 6 manipulators having 6 DoF each results in calculating motion plan for 36 DoF. Examine the most popular motion planners such as RRT [2], PRM [3] or their more recent variants, we found that it is computationally high to solve (exponential in time). There are two main approaches for calculating motion planning in systems consist of high number of DoF.

Centralized Motion Planning

The first (and tentative) is the Centralized Motion Planning in which the total DoF are taking into account when searching for solution. Although computational power is needed (i.e takes time), its pros comes with complete and optimal solution if any.

Decoupled Motion Planning

Alongside, the second approach Decoupled Motion Planning is more relevant in term of time consuming but lacks the completeness and optimality traits. As listed in [1,4] decoupled planning is categorized into 3 main methods:

1. *Prioritized Planning* (PrP), In this approach we sort all the robots by priority and planning an individual path for each robot. Each path is calculated based on the hierarchy obtained by the sorting procedure, such that higher rank is being calculating first. The collision free element acquired by treating the higher ranking robots as a moving obstacles.
2. *Fixed Path Coordination* (FPC), This method is divided into 2 parts. At the first stage a path is planned for each individual robot as if its the only robot in the world so that in the second stage all of the paths a getting synchronize by means of timing in order to avoid collisions.
3. *Fixed Roadmap Coordination* (FRC), This strategy extends the FPC by assuming that each robot is guided by a roadmap. This yields a wider set

of routes to achieve a single robot goal (instead of one in the FPC). Thus timing of the overall coordination may be more accessible.

Works discussing motion planning for multiple robots often adopt one of the two approaches or else suggests a hybrid solution. A comparison between those approaches is given by Sanchez and Latombe [5] using the PRM planning scheme. In this work the authors examined a 36 DoF motion planning problem with each of the approaches described above and with the same path finding technique (SBL [6]). Experimental results have shown that the decoupled planning has failed in 30 to 75% of the trials and among the successful one the time differences had no significant change.

2.2 Task and Motion Planning

High and low level planning attributes are "must to have" when designing an autonomous behaviour in robotics. Each attribute standalone has no capability of getting insights from the big picture of the world: high level plan has to be based on physical properties where low level plan has to be guided by some rule. As of writing this paper the current *state of the art* has no standard formulation for such a binding. For this reason, we aim to give our focus and effort in the area of combining task and motion planning.

Task and Motion Planning for Multi Manipulators

Motion planning for multi-agent robots is an extensive research field consist of many variants. In this work we target to combine Task and Motion Planning for Multi Manipulators (TMPMM) mission. Maybe the main difficulty in such problems is planning for continuous and abstract configuration (states), this because motion planner are not comply with abstract parameters so as task planner don't comply with continuous parameters. TMPMM assignment are separated into 2: the ordinary pick and place task and the cooperative task.

An interesting work made by Koga and Latombe [7] describes a task of moving an object in a world built with obstacles using 3 arms having 6 DoF each. The solution suggested is based on calculating a path for the object while simultaneously check whether it can be grasped by one of the arms. If it reaches a configuration where the grasp is no more valid it search to grasp the object using a different arm and continues with the object's path. This way it's ensures the object continuous motion through a calculated cartesian path while being grasped.

3 Methodology

This chapter is about defining the research problems and the ways for solving it. Since the research target (as presented in chapter 1) is general, we have to define specifically what are the problems and how to find a solution. Throughout the chapter we will discuss over *a)* the definition that is used as a basis to define a research problems *b)* the system requirements for a solution *c)* the methods for analysing a solution and *d)* the simulation/experiments that will evaluate a solution performance.

3.1 Research Problem Description

In order to define more clearly what problems are we going to solve, first we will specify the research basic definition. We will use the fact that the majority of industrial robotic arms are statically based and so a group of arms have fixed distances between their bases. This fact leads us to the next definition:

Definition 1. A configuration set of robotic arms is given such that:

- a) The arms links length, joint orientation and joint limitation is known.
- b) The arms base position is known.

To complete a problem description, we will have to specify what objects are we going to use and the action which the arms will apply. The overall package of arms, objects and actions are the building stones of the next 4 problems:

Problem 1: Pick and Place of Single object by a Set of Robotic Arms

Here, the problem description is given by a set of robotic arms together with a single object such that the task is to move the object from its initial position to a predefined goal. Knowing that only arms are able to manipulate an object we can define the next assumptions:

- The initial configuration of the arms set is known.
- The initial and final position of the object is known and located at a workspace belongs to (at list) one of the arms.
- Only a single arm can grasp the object at a time.
- There are two types of obstacles: the arms themselves and the object.

In practice the difficulty arise when a single arm is not capable of completing the task alone and the solution gets a form of series of arms performing pick and place operation (i.e transferring the object between themselves). This problem expose a number of open question:

1. What is the sequence of arms which will complete such a task.
2. What are the positions along the object path where a transfer happens.
3. What is the collision free trajectory of each arm throughout the sequence.

Problem 2: Pick and Place of Multiple objects Simultaneously

This problem expands problem 1 above such that the assignment is to move multiple objects from one place to a predefined goal using robotic arms only. This problem adds complexity to the overall system and expose the next assumptions:

- The initial configuration of the arms set is known.
- The initial and final position of all of the objects in known and located at a workspace belongs to (at list) one of the arms.
- The final configuration of all of the objects is defined without collisions.
- A single arm cannot manipulate more than one object simultaneously.
- Only a single arm can grasp an individual object at a time.
- There are two types of obstacles: the arms themselves and the objects.

At first look, this problem can be solved using *prioritize planning* principle; a solution for problem 1 is obtained for each object individually, next an execution scheduling process is maintained in order to avoid conflicts. Although it is an applicable solution it does not exploit the full capabilities of such a system which could handle a faster solution. Thus, the same open question as of problem 1 remained.

1. What is the sequence of arms that will manipulate each object.
2. What are the positions along the each object path where a transfer happens.
3. What is the collision free trajectory of each arm throughout the sequences.

Problem 3: Pick and Place of Single/Multiple objects in Environments with Obstacles

This problem is defined the same way as problem 1 and 2 (i.e as a single or multi object manipulation problem), except that it defined with a new type of obstacles (static nor dynamic). Thus, the last assumption in each problem list is changing as follows:

- The obstacles geometry, type and position are known.
- There are three types of obstacles: The arms themselves, the objects and the environmental obstacles.

The problem of cluttered environment adds complexity to the system such that it limits the motions of arms nor the positions of the transfer points. Hence, it is mainly affects the motion planning part of the total planning scheme.

Problem 4: Pick and Place of Single/Multiple objects with Non-Homogeneous Team of Robotic Arms

In the case of non-homogeneous teams of robotic arms we depict a the same problem of moving an object by a series of arms, only now there are different kind of arm. A warehouse management robot team would be a good application where heavy lifting robotic arms are needed for manipulating plates and a light lifting robotic arms are needed for loading/dispersing boxes on top of the plate. In contrast to the scenarios of problem 1-3, here we let a single arm to manipulate number of objects at a time (as long as the objects are organized in a way that enables to grasp).

3.2 Algorithm Development

The problems described in section 3.1 have both decision and motion planning attributes. Each attribute standalone has a formal approach for solving it but lacks the other skills. Still, in order to solve these problems a 2 phases solution is required:

- (phase 1) A high level task plan with a general purpose aimed to find a sequence of arms that will transfer an object from initial to goal positions. The idea behind this phase is to find a series of "arm indexes" where the object is being passed from one arm to the next such that the last arm will place the object at it's target.
- (phase 2) A low level motion plan with a general purpose aimed to actually calculate a collision free trajectory for each step of the sequence. This is a more physical phase where the joint/work spaces of all of the arms are taken into account when calculating such a trajectory. The collision free refers for an arm that is transferring an object towards the next arm in the series (or toward the object's goal).

The two phases are coupled due to the fact that any logical sequence of arms is based on geometric properties, for example any 2 following arms should share a gripping position of an object (in global coordinates) in order to transfer it between themselves. Having this fact in mind we understand that our algorithm first objective is to combine the two phases above by searching for applicable sequence it terms of kinematic feasibility. Having a feasible sequence applies for being able to grasp an object at each step by a different arm, this is literally interpreted as a state where a configuration of a single arm was set. This notion leads us to our second algorithm objective which is to calculate a transition function between those states. Here, a transition function is a motion plan as detailed in phase 2 above and this is a key feature for success in such algorithm.

As for our third objective is to be able to use this algorithm for a maximal number of arms nor objects. This is a high priority goal which makes a major contribution for such a scenarios. The trade off for large scale planning is of-course linked to computation time of the algorithm. From this perspective, We

expect the weakness of the algorithm at the motion planning phase. As shown in section 2.1 applying a *centralized* motion planning method on a system with high number of DoF will be doomed to time-out before having a solution.

3.3 Algorithm Analysis

In order to validate the capabilities of any algorithm we should assess two major specifications:

Completeness This first parameter states for knowing whether a solution exist and as long as one exist if the algorithm can find it.

Complexity This parameter has a direct link to solutions computation time. In most cases this parameter estimates the maximal operation to do in order to find a solution (if any). In practice complexity tend to over estimate the computation time of a solution which actually much more faster. Thus, to get a realistic point of view about how good is our algorithm we should compare its performance against other techniques.

3.4 Simulation and Experiments

Any design of planning algorithm that implements phase 1 and 2 of section 3.2) will be tested under simulation and experiments, this in order to present its capabilities of displacing an object using multiple arms. The simulation code will be written under MATLAB environment which later on will be re-written with c++ in order to handle the experiments under GAZEBO/ROS framework. Although GAZEBO is a 3D simulation it holds behind a dynamics engine that calculates the physics of solid parts in terms of equation of motion and interacting forces (between two parts). This means GAZEBO can be used as a experiments environment where we can test scenarios consist of as many robot arms as we need.

The objective of these simulation is to preform a parametric inquiry which aimed to check the influence of a single parameter on the overall performance. The parameters that interesting us are as follows:

1. The number of robotic arms in a setup.
2. The total percentage of the shared workspace of an arm.
3. The number of joints/links having a single arm.

Throughout the research our target is to apply this inquiry over the problems listed in 3.1, thus for each of these problems we will have to build a different setup. Table 1 shows four different setups that will be tested on each problem. As detailed, column 2 represents the number of arms in a setup, column 3 represents the percentage of workspace shared between any couple of arms and column 4 represents the number of DoF in each arm. In fact, any change of a setup parameters will change the output of a planning algorithm such that we can estimate what are the factors that influencing on:

Setup Index	Number of Arms in a Setup	Percentage of Workspace Sharing	Number of DoF in a single arm
#1	3	20	TBD
#2	3	80	TBD
#3	10	20	TBD
#4	10	80	TBD

Table 1: 4 different setups to handle a parametric inquiry over the research problems

1. The total task operation time.
2. The overall configuration size.
3. The computation time.
4. How many robotic arms can be used.
5. What type of arms can be used.
6. How "dens" the arms are located.

Also, these setups are interpreted as both two or three dimensional problem, although in general a 2D problem may be more easy to solve it is not the often case here (See Appendix.....). Still, the procedure for testing a problem in our research will be first simulation and experiments over a set of planner robotic arms and next simulation and experiments on a set of spatial robotic arms (The number of arms and the percentage of shared workspace remains the same).

ToDo: explain position controlled

4 Preliminary Results

4.1 Case Study

For our case study we want to analyse the problem of pick and place a movable object throughout a workspace containing a number of robotic arms. We represent the world using a cylinder (i.e beer can) as an object together with m grippers connected to m statically based robotic arms which allowed to move on a planar surface (i.e table). An example of a case consist of 5 arms is represented in figure 2.

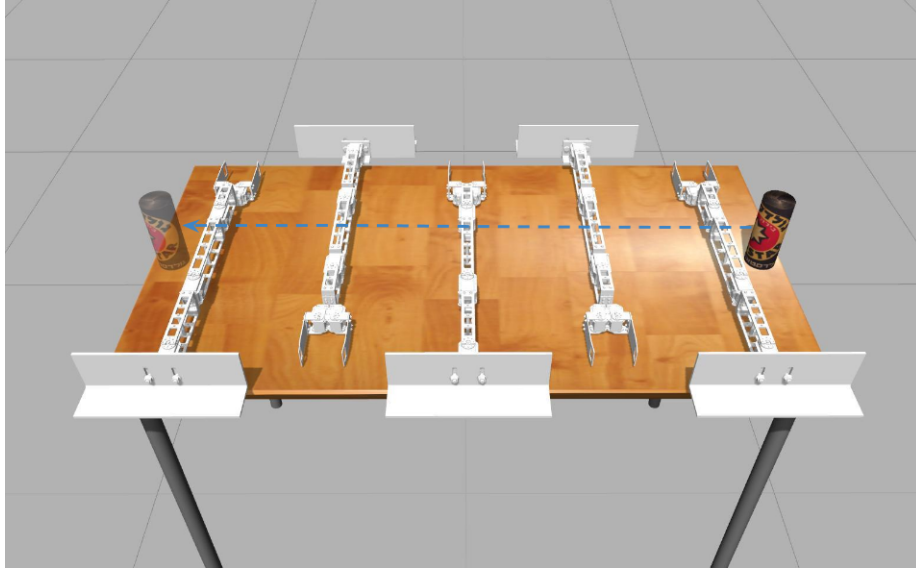


Figure 2: Case study Visualization

4.2 Problem Formulation

In this work our intention is to engage a single object pick and place problem using multi manipulators. We will handle the next symbols description for our problem statement. A manipulator will denoted as A^i (the i 'th parameter means the manipulator index) and an object as B . We assume to have an overall of m manipulators $\mathcal{A} = \{A^1, A^2, \dots, A^i \dots, A^m\}$ and a single movable object. Furthermore we assume that each A^i holds a workspace W^i and a configuration space C^i such that the Forward Kinematics (FK) is mapping W^i from C^i ($FK : C^i \rightarrow W^i$). Hence, the feasible workspace of the object will be then $\mathcal{W}^B = \{W^1 \cap W^2 \cap \dots W^i \dots \cap W^m\}$ the overall *configuration space* of the arms will be $\mathcal{C}^A = \{C^1 \times C^2 \times \dots C^i \dots \times C^m\}$ and the corresponding arm *workspace* $\mathcal{W}^A = \{W^1 \times W^2 \times \dots W^i \dots \times W^m\}$.

For the purpose of consistency with [7,8] we will presume the terminology of *transit-path*, *transfer-path* and *manipulation-path* as follows:

transit-path is a path that describes a motion of an empty arm (i.e not grasping any object). We will use this term to represent a path of an arm which avoids collision with other arms or actually moving toward an object in order to grasp it.

transfer-path is a path that describes a motion of an arm that is grasping an object within its gripper. We will use this term to represent a path of an arm that manipulates an object.

manipulation-path is a sequence of alternating *transmit* and *transfer* paths. We will use this term to represent a sequence of arms the moves an object from initial to goal positions.

Next, consider the following problem input:

- A set of m robotic arms.
- Initial and goal positions of an object $B^{init}, B^{goal} \in \mathcal{W}$

our objective is to find a *manipulation-path* along which the object could be transferred according to input requirements. By definition each manipulation path is constructed by a sequence of *Tspace* paths $\mathcal{T}^A = \langle T^1, \dots, T^k, \dots, T^n \rangle$ where $T^k \in \mathcal{W}^A$. In addition, each *Tspace* path is derived by applying the FK rule on a sequence of *Cspace* path. Therefore the corresponding *Cspace* sequence of \mathcal{T}^A resemble as $\mathcal{Q}^A = \langle Q^1, \dots, Q^k, \dots, Q^n \rangle$ where $Q^k \in \mathcal{C}^A$. This observation shows that any solution is obtained by calculating \mathcal{Q}^A such that the equivalent \mathcal{T}^A preforms a desired manipulation path. It is noteworthy that each T^k is holding a *transit* path for the entire arms (to operate in parallel) except one that holds a pair of *transit* and *transfer* paths for the actual manipulation.

4.3 Planning Logic

We let \mathcal{T}^B be the object absolute path (i.e from initial to goal positions) such that $\mathcal{T}^B \in \mathcal{W}^B$, next we consider any k element as a sub-task operation that moves the object along a portion of \mathcal{T}^B . Hence, we can reason that \mathcal{T}^B is built up by a sequence of *work-space* paths each handled on the k 'th sub-task by a different arm, thus $\mathcal{T}^B = \langle T_1^B, \dots, T_k^B, \dots, T_n^B \rangle$.

We define that object is restricted to move only when it is gripped, this aims the search to find a particular sequence of arms $S(k) \subset \mathcal{A} \mid [k = 1..n]$ that will do the feasible manipulation job. Recall that any solution is represented by a sequence of T^k 's where each holds a path for the total arms. This means that T^k is built by the set of *Tspace* path of each individual arm $T^k = \{T^{k,1}, \dots, T^{k,i}, \dots, T^{k,m}\}$. The resulted conclusion is an essential connection extracted between \mathcal{T}^A and \mathcal{T}^b such that for any k : $T_k^B = T^{k,S(k)}$.

We'll denote a *transfer point* as a position where any T_k^B starts or ends (including B^{init} and B^{goal}). Tentatively any *transfer point* located where an object is grasped or released by all of the arms in S . Any sequence S consist of n arms will have to pass through $n + 1$ *transfer points* series tp . Assuring this condition is given by forcing any $T^{k,S(k)}$ to include $tp(k)$ and $tp(k + 1)$ along their path. A summary of the general conditions for a valid \mathcal{T}^A is as follows:

$$\forall T^k : \{\exists Q^k \mid (T^k = FK(Q^k))\} \quad (1)$$

$$\forall T^{k,S(k)} : \{tp(k), tp(k + 1) \in T^{k,S(k)}\} \quad (2)$$

$$\forall T^k : \{T^{k,i} \cap T^{k,j} = \emptyset \mid i, j = 1..n ; i \neq j\} \quad (3)$$

where equation 1 referring to kinematic feasible path, equation 2 is responsible for applicable sequence in terms of pick and place positions and equation 3 imposing for having no collisions.

4.4 Planning Process

Recalling the work of Koga and Latombe [7], it is shown that the object's world trajectory is the governing law when it comes to determine which is the grasping arm at any time. On the contrary we wish to give another perspective for the problem. Our guiding rule comes from shared workspaces of the arms. We are using the fact that any *transfer point* is located only in one of the shared workspaces and consequently \mathcal{T}^B must pass through them.

Search for Arm Sequence and Transfer Points

One of the key component of our work is find any pair of arms sharing together a part of their workspace such that $W^i \cap W^j \neq \emptyset$, this fact gives an insight for any transfer that could appear. The easiest way to represent this data is by a graph where a node represent an arm workspace and an arc represents a shared workspace (between two arms/nodes). It means that a world consist of n arms is interpreted by a graph with n nodes where arcs are added on a second phase by checking any possible combination of shared workspaces. The generation of this graph will tentatively be called the *connectivity graph*.

By attributing the initial and goal position of the object to one of the arms $Tspace$ we can reason about the starting and final node in the *connectivity graph*. At this point, any graph search would give a solution with a form of a sequence of arms $S = \langle A^{first}, ..., A^k, ..., A^{last} \rangle$ connected by their shared workspaces such that:

$$B^{init} \in W(A^{first}) \mid A^{first} \equiv S(1) \quad (4)$$

$$B^{goal} \in W(A^{last}) \mid A^{last} \equiv S(n) \quad (5)$$

$$W(A^k) \cap W(A^{k+1}) \neq \emptyset \mid A^k \equiv S(k) \quad (6)$$

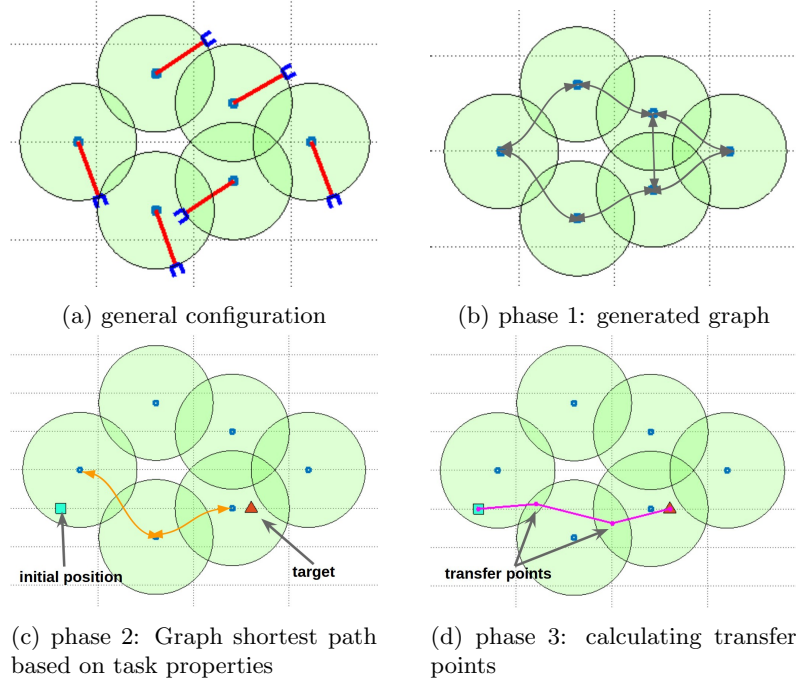


Figure 3: Generating a sequence of arms and transfer points

After generating S the next task is to determine the position of a single *transfer point* located in each shared WS. To do so a linear programming method will be used to minimize the total length accepted by connecting any pair of following *transfer point* with a straight line.

The Search Algorithm

To generate an actual sequence that will apply the condition as in equations (1)-(3) we will have to go through the steps as written in algorithm 1. The rationale behind this algorithm is first to find a sequence of arms S together with a series of *transfer points* tp that will settle with equation (2) (lines 1-5). Next, when S and tp was determined the guidelines for a motion plan was resolved, thus the second phase of the algorithm was launched (line 6) in order to calculate the sequence path that meet equations (1) and (3).

Calculating Sub-Task Path

As stated, at this stage our goal is to calculate the arms path for each phase of the *manipulation-path* represented by \mathcal{T}^A . We assume that each phase includes the following input parameters (and that the initial configuration of the total arms is known):

Algorithm 1 Main Loop

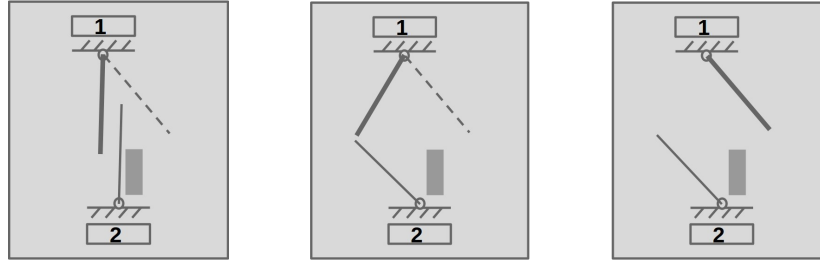
Input: $B^{init}, B^{goal}, \mathcal{A}, q^{init}$ **Output:** $traj$

- 1: $cg \leftarrow GenerateConnectivityGraph(ws)$
 - 2: $W^{first} \leftarrow \text{find}(W^{first} \text{ such that } B^{init} \in W^{first})$
 - 3: $W^{last} \leftarrow \text{find}(W^{last} \text{ such that } B^{goal} \in W^{last})$
 - 4: $S \leftarrow \text{BFS}(cg, W^{first}, W^{last})$ // generate ws sequence
 - 5: $tp \leftarrow CalcTransferPoints(S, B^{init}, B^{goal})$
 - 6: $\mathcal{T}^A \leftarrow CalcSequenceTrajectory(S, tp, q^{init})$
-

- Index of the moving arm.
- Initial and goal positions of the arms gripper

These parameters directs us to find a motion planning solution for a multi robot system. As mentioned in section 2.1 there is no formal strategy for calculating optimal nor complete trajectory for multi robots except for using *centralized* methods that suffers from exponential complexity. The pattern that we will next describe is a method that is designed to find a solution oriented for robotic arms but may still be used for other systems.

The fact that a *centralized* method includes complete and optimal attributes is that it can handle local minima's (same as A*). Lack in such feature resolved in having no reasoning when: robot 2 obstructs the path of robot 1 where actually robot 1 obstructs the path of robot 2 that should clear the path for robot 1. A simple illustration is shown in figure 4a with a case of 2 robotic arms having 1 DoF each. The task here is to move arm #1 to the dashed line. Trying to do so with applying any motion plan on arm #1 only will result with "no solution" answer. It is obvious that arm #2 needs to clear the way and the only option is to move counter-clockwise (moving clockwise is impassible due to the obstacle).



(a) Initial configuration of the arms, goal of arm #1 is dashed
 (b) A configuration of a "breaking point" from which the goal is reachable for a single arm task
 (c) Final configuration: goal achieved

Figure 4: A multi arm problem configuration consist of 2 arms with 1 DoF each together with an obstacle (grey rectangle) that limits arm #2 motion. In order to acheive the goal (figure c) a breaking point (figure b) must be acheived first.

5 Research Program

	2016						2017						2018		
	1-2	3-4	5-6	7-8	9-10	11-12	1-2	3-4	5-6	7-8	9-10	11-12	1-3	4-6	7-9
Proposal Submission															
2D Experiments															
Performing a 3D task															
Applying multi object manipulation capabilities															
Planing with non-homogeneous team of robotic arms															
Planing with dynamic constraints															
Writing a thesis															

Figure 5: Time table

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