

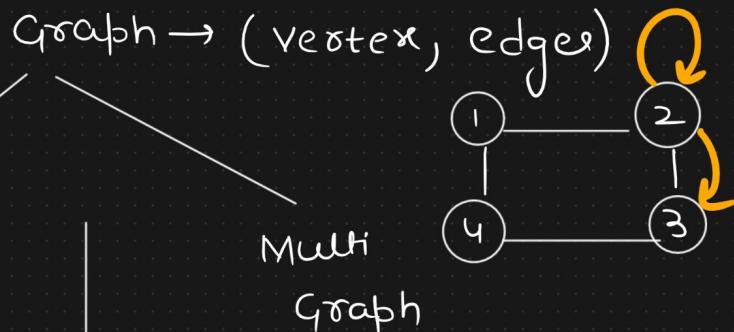
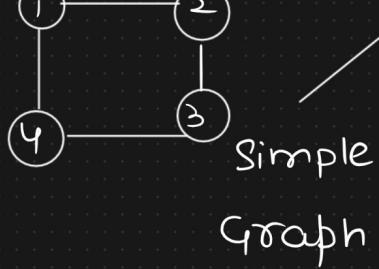
Graph



Non Linear Based Data structure
cyclic or acyclic

Tree → acyclic

↳ Every tree can be considered as graph but vice-versa
is not true.



↳ Self loops are not allowed

Self loops are allowed

↳ Parallel edges are not allowed

Parallel edges are allowed

Simple Graph

Min degree = 0

Null Graph

Complete

Max degree = Graph

V-1

Null Graph

$$|V|=4$$

(1)

(2)

(4)

(3)

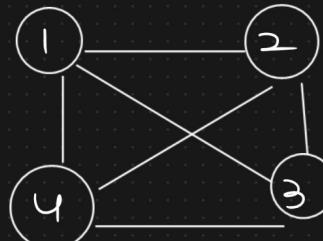
Degree of each vertex = 0

Complete Graph

$$V = n$$

$$\text{Max Degree} = V - 1$$

$$\text{Min Degree} = 0$$



$$|V|=4$$

$$|E|=6$$

Degree of each

$$\text{vertex} = 3$$

$$(V-1)$$

$$\text{Sum of Degree} = |V| * |V-1| = 4 * 3 = 12$$

$$\text{Sum of Degree} = 2 * |E|$$

$$2 * 6 = 12$$

$$\text{Sum of Degree} = 2E$$

$$\frac{\text{Sum of Degree}}{2} = E$$

(3)

$$\frac{V * (V-1)}{2} = E$$

$$V = 4$$

$$E_4 = \frac{4 * 3}{2} = 6$$

Relationship

b/w

$E \propto V$

in a

$$V \times (V-1) = 2E \quad \leftarrow$$

$$V^2 - V = 2E$$

$$|E| = \Theta(V^2)$$

$$\log E = 2 \log V$$

Complete Graph

$$\log E = \Theta(\log V) \quad \leftarrow \textcircled{4}$$

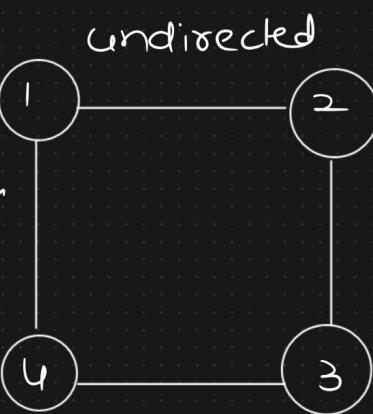
$$|E| = \# \text{ of edges}$$

Graph \rightarrow adjacency list (List)

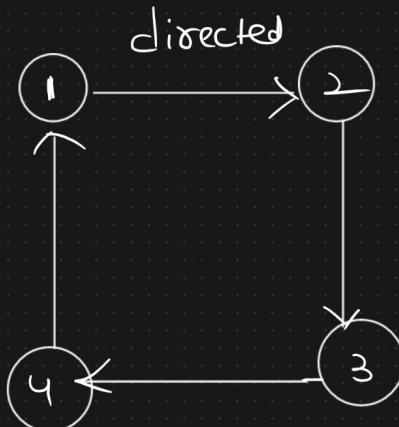
Graph \rightarrow adjacency matrix (array)

8
1 → 2 ✓
2 → 2 ✓
3 → 2 ✓
4 → 2 ✓

undirected



directed



$$\left\{ \begin{array}{l} '1': [2, 4] \\ '2': [1, 3] \\ '3': [2, 4] \\ '4': [1, 3] \end{array} \right.$$

$$\begin{aligned} \text{Sum of Degree} &= \frac{2E}{2} \\ &= \frac{2 \times 4}{2} \\ &= 8 \end{aligned}$$

$$\Theta(2E)$$

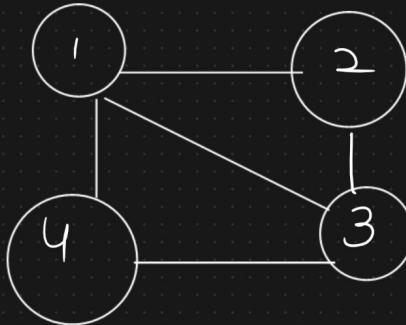
1 → Edge from
one
vertex
to
another

0 → No edge

Adjacency matrix

$$\begin{matrix} & & 1 & 2 & 3 & 4 \\ 1 & \left[\begin{array}{ccccc} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array} \right] & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{matrix} \quad V \times V$$

$\Theta(V^2)$

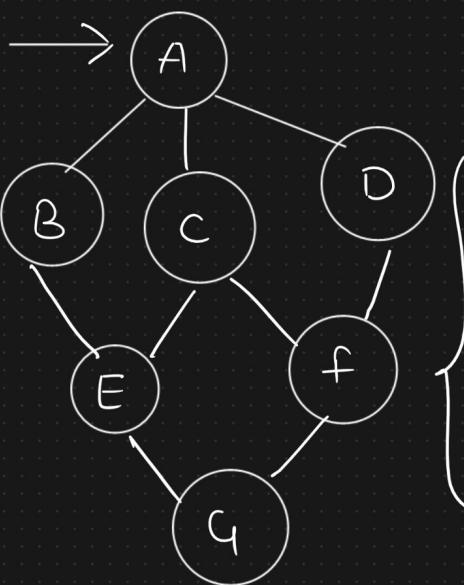


1 → 3
2 → 2
3 → 3
4 → 2

10

$$2 * E = 2 * 5 = 10$$

undirected graph



Depth first Traversal

↳ Recursion

↳ Stack Data Structure

$A \rightarrow [B, C, D]$
 $B \rightarrow (A, E)$
 $C \rightarrow (A, E, F)$
 $D \rightarrow (A, F)$
 $E \rightarrow (B, C, G)$
 $F \rightarrow (C, D, G)$
 $G \rightarrow (E, F)$

↳ take care of what
 Boolean
 vertex are already
 visited

Set → no duplicated

1	1	1	1	1	1	1
∅	∅	∅	∅	∅	∅	∅

A B C D E F G

DFT(A)

$\omega = B, C, D$

DFT(B)

$\omega = A, E$

↑ ↑

DFT(E)

$\omega = B, C, G$

$A, B, E, C,$
 F, D, G

DFT(v):

visited(v) = 1

print(v)

for all ω adj to v:

if ω is not visited:

DFT(ω)

↳ Recursion

DFT(C)

$\omega = A, E, F$
 ↑ ↑ ↑
DFT(F)

A, B, E, C, F, D, G

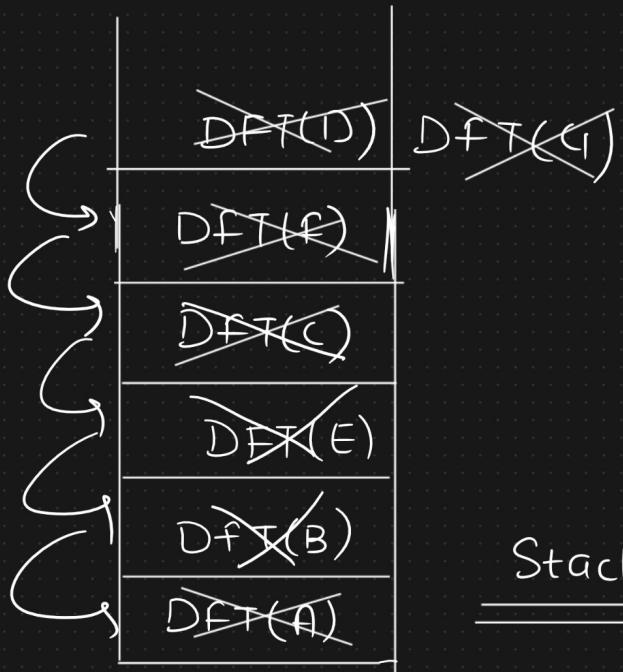
$$\omega = \{D, G\}$$

~~DFT(D)~~

$$\omega = \{A, F\}$$

~~DFT(C)~~

$$\omega = \{E, F\}$$



Stack

Visited = 5, 3, 2, 4, 8, 7

set

DFT(5)

$$\omega = 3, 7$$

~~DFT(3)~~

$$\omega = 2, 4$$

~~DFT(7)~~

$$\omega \neq \{2, 4\}$$

~~DFT(2)~~

$$\omega = \{ \}$$

5, 3, 2, 4, 8, 7

Directed graph

$$5 \rightarrow [3, 7]$$

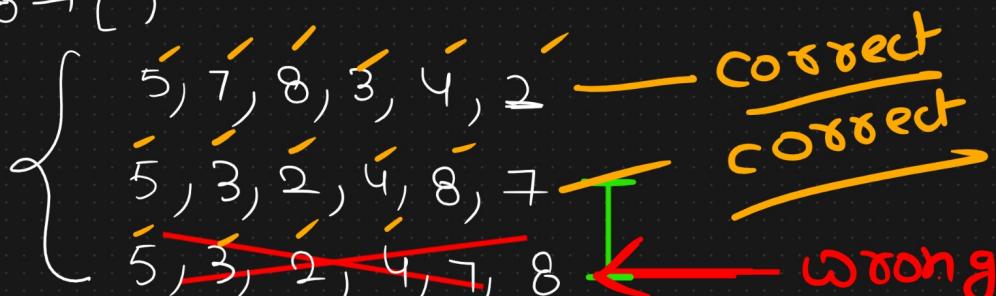
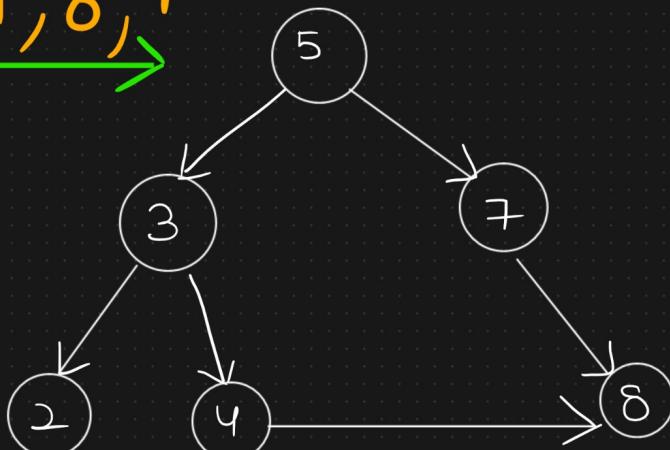
$$3 \rightarrow [2, 4]$$

$$7 \rightarrow [8]$$

$$2 \rightarrow []$$

$$4 \rightarrow [8]$$

$$8 \rightarrow []$$

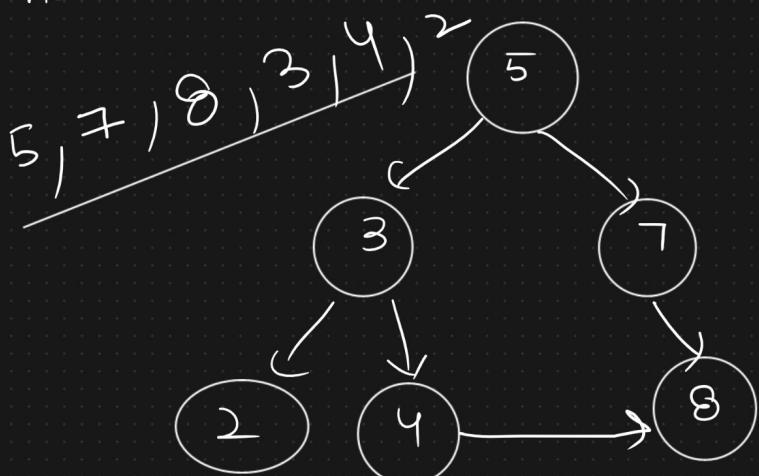
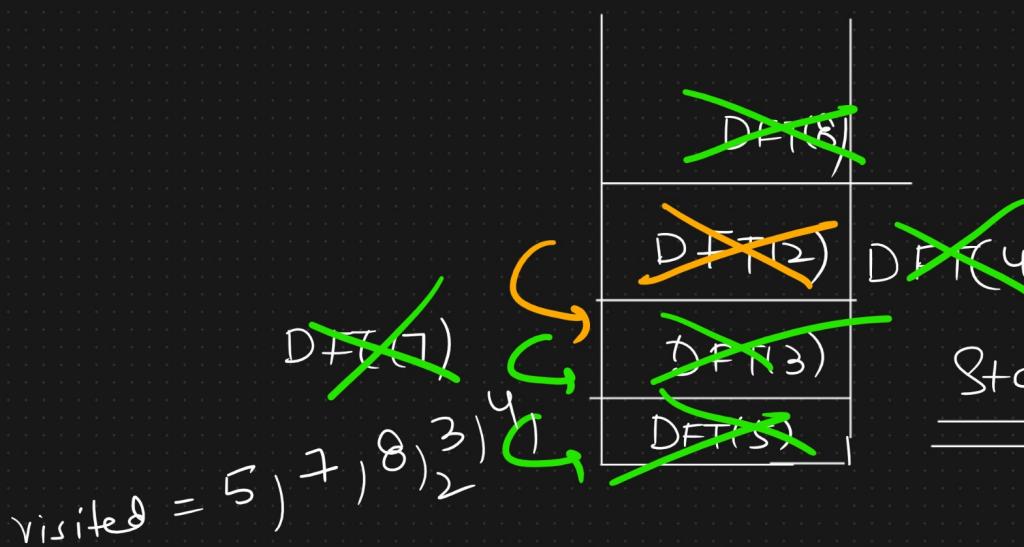


DFT(4)

$$\omega = 8$$

DFT(8)

$$\omega = []$$



~~DFT(5)~~

$\omega = 7, 3$

\uparrow

\uparrow

$DFT(7)$

$\omega = 8$

~~DFT(8)~~

$\omega = []$

$DFT(3)$

$\omega = 4, 2$

\uparrow

\uparrow

$DFT(4)$

$\omega = 6$

~~DFT(2)~~

$\omega = ()$

