Statistical Thermodynamics - HW 7 - 2D Ising Model

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Introduction

The Ising model is a mathematical model of ferromagnetism in statistical mechanics. This exercise explores the Ising model in two scenarios: a non-interaction model and a nearest-neighbor interaction model.
In this exercise you will code a simulation using the Monte-Carlo Metropolis-Hastings method for sampling from the canonical ensemble. You are free to write your simulation in any programming language of your choosing (C/Python/Matlab etc..) **we recommend that you build upon the demo file given to you in the lecture.**

Part 1: Non-Interaction Model

Questions

1.1 What is the average energy for a spin in a 2D grid with $\mu = 1, h = 1K_bT$, $T = \text{const}$? What is the average magnetization $\langle s \rangle$? **Reach the expression analytically.**

1.2 Does the simulation reach equilibrium? Show a figure that supports this claim.

1.3 How does the simulation differ when we increase the number of **spins or iterations**?

1.4 Plot the distribution of the system for different ratios $\frac{h}{K_bT}$. Explain what you observe in text.

1.5 What are the fluctuations in energy and magnetization for each ratio? Show a figure that compares them and explain their meaning from a **physical and statistical** point of view.

1.6 Change the initial conditions for the simulation (random grid, spin-up grid, spin-down grid). Does the system reach the same **macroscopic** state?

1.7 Does the simulation match the analytical result from 1.1?

Part 2: Nearest-Neighbor Interactions

Questions

2.1 Starting from different initial grids (like in 1.6), calculate the average magnetization and average energy for $J = K_bT$, assuming $h = 0$. Does the system reach the same equilibrium state?

2.2 Repeat the calculation for high temperatures, $J = \frac{1}{2}K_bT$. What changes in the equilibrium state?

2.3 What happens when an external field is added? **Must answer in text, Optional: Perform a simulation.**

2.4 The Ising Model is a classical model in statistical mechanics.

- (a) What physical phenomena does it describe?
- (b) Did you observe these phenomena in your simulations?
- (c) Can you suggest a way to calculate the temperature (ratio between field and temperature) at which this phase change happens?