Graph Theory: Depth First Search

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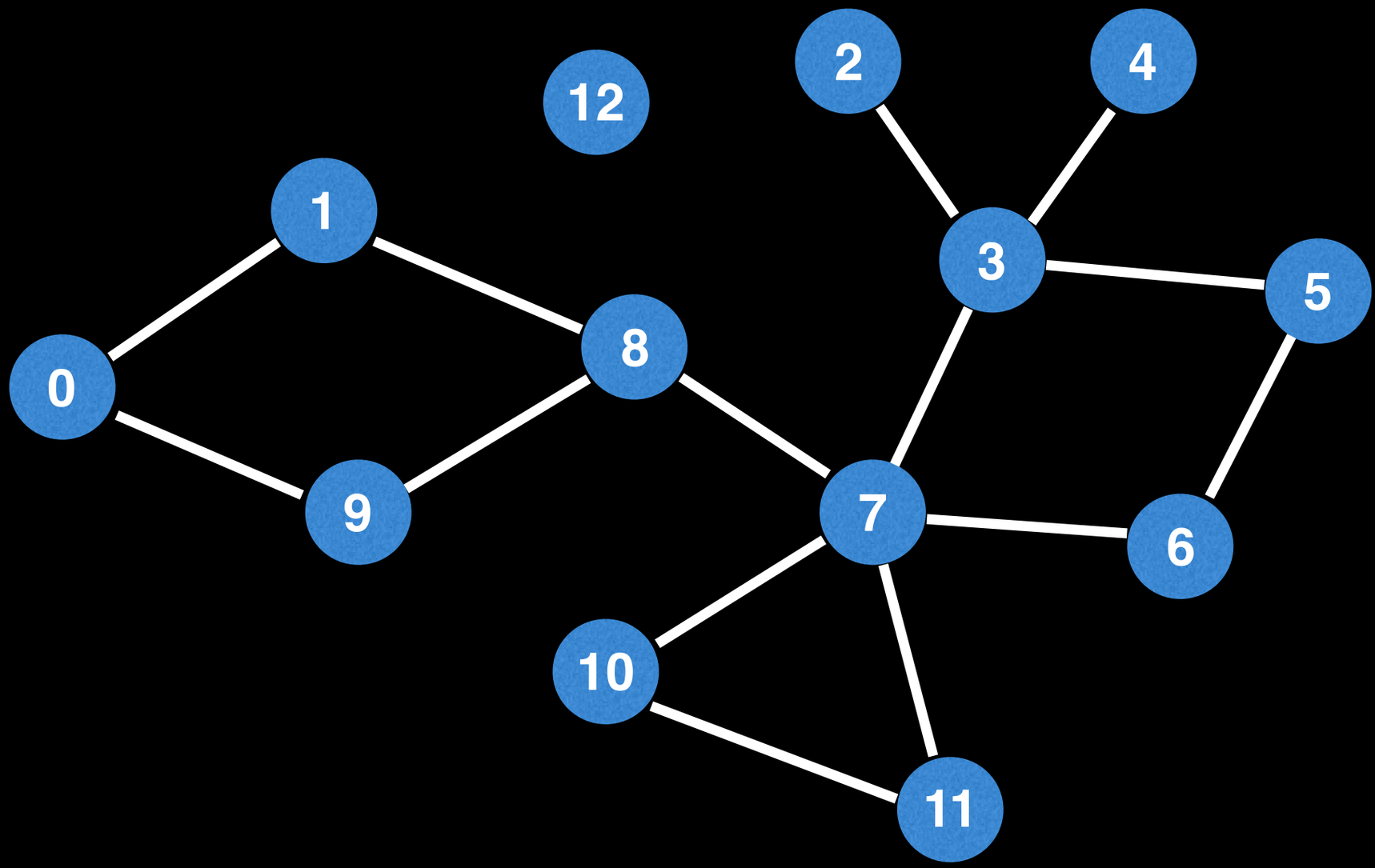
DFS overview

The **Depth First Search (DFS)** is the most fundamental search algorithm used to explore nodes and edges of a graph. It runs with a time complexity of **O(V+E)** and is often used as a building block in other algorithms.

By itself the DFS isn’t all that useful, but **when augmented** to perform other tasks such as count connected components, determine connectivity, or find bridges/articulation points then **DFS really shines**.

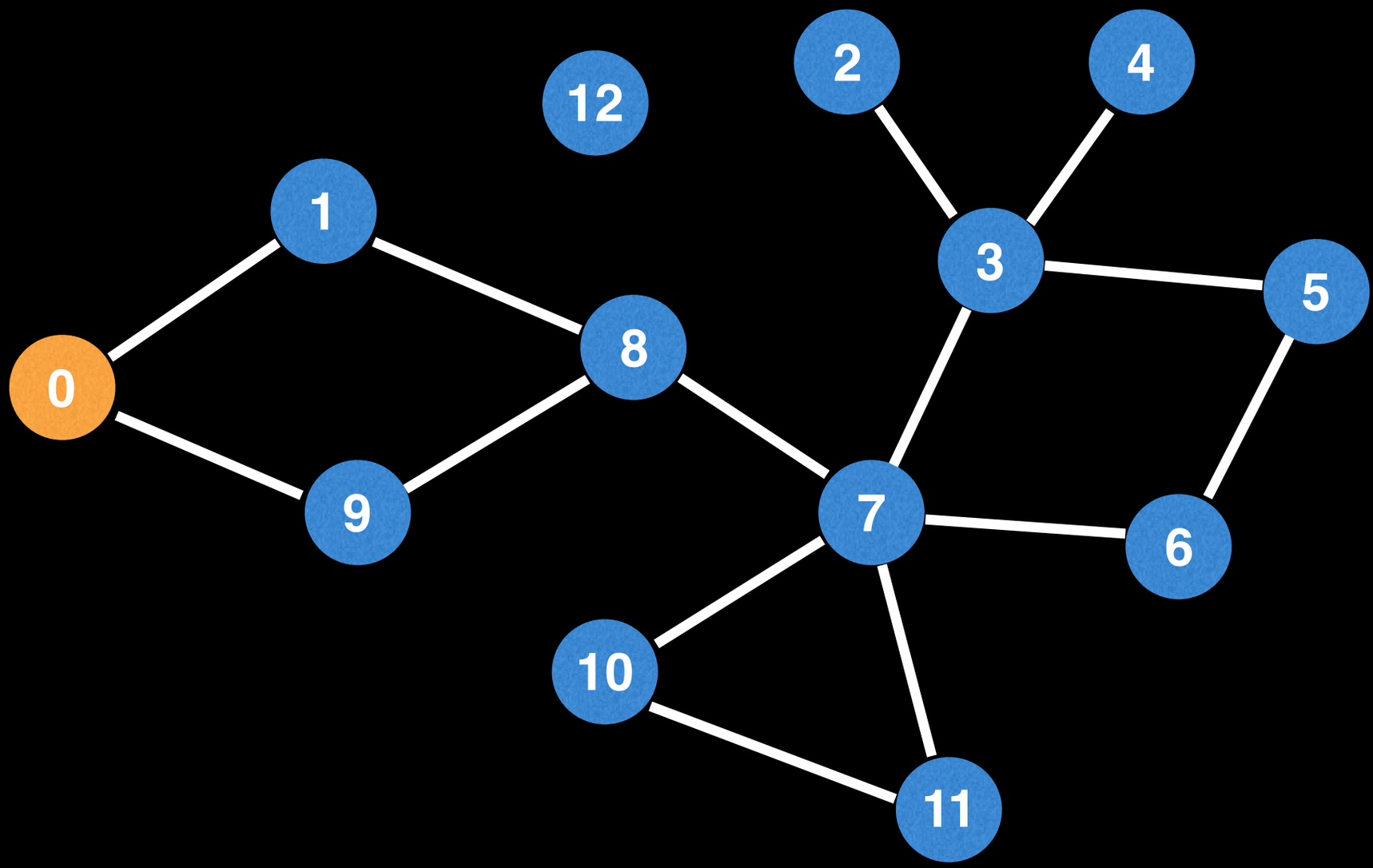
Basic DFS

As the name suggests, a DFS plunges depth first into a graph without regard for which edge it takes next until it cannot go any further at which point it backtracks and continues.



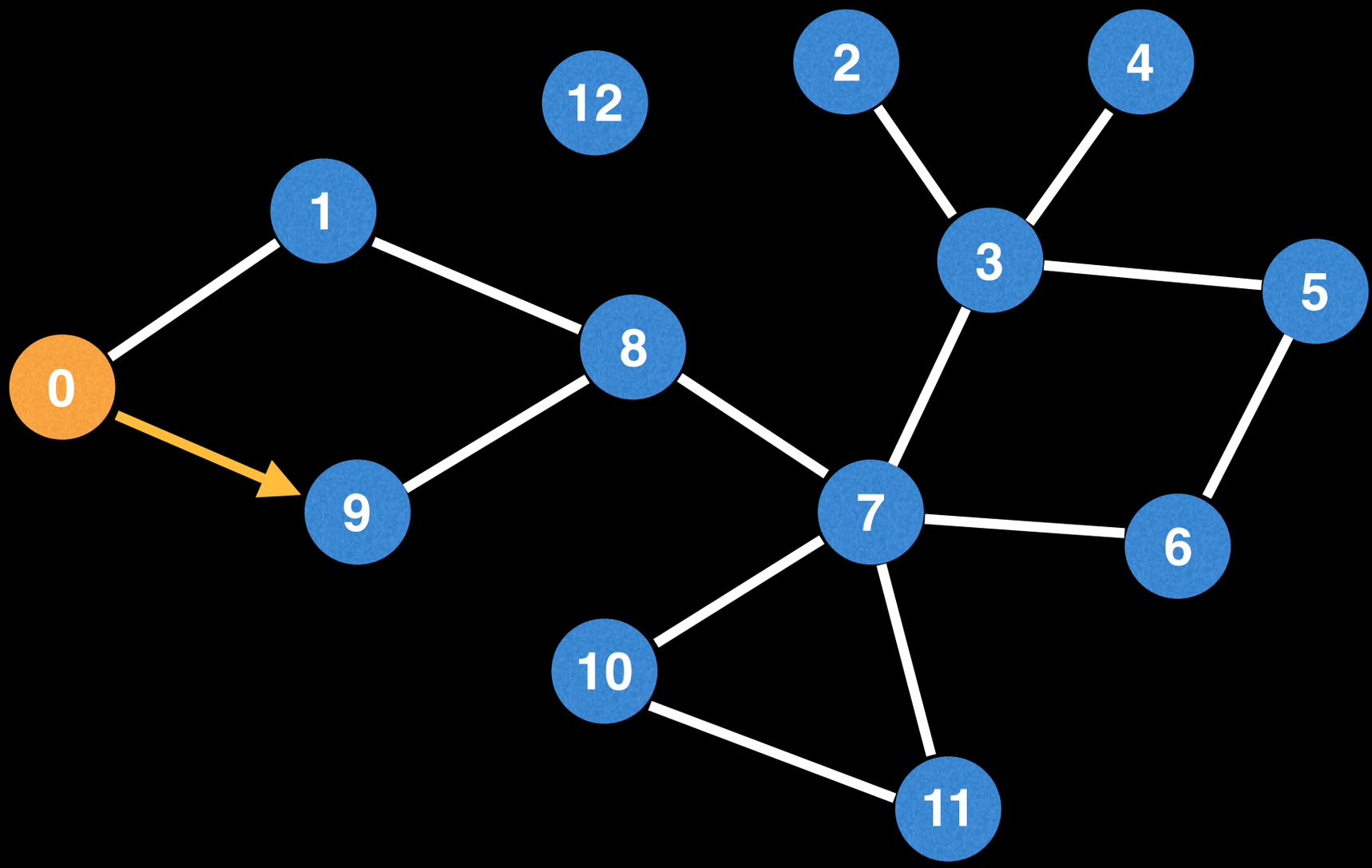
Basic DFS

Start DFS at node 0



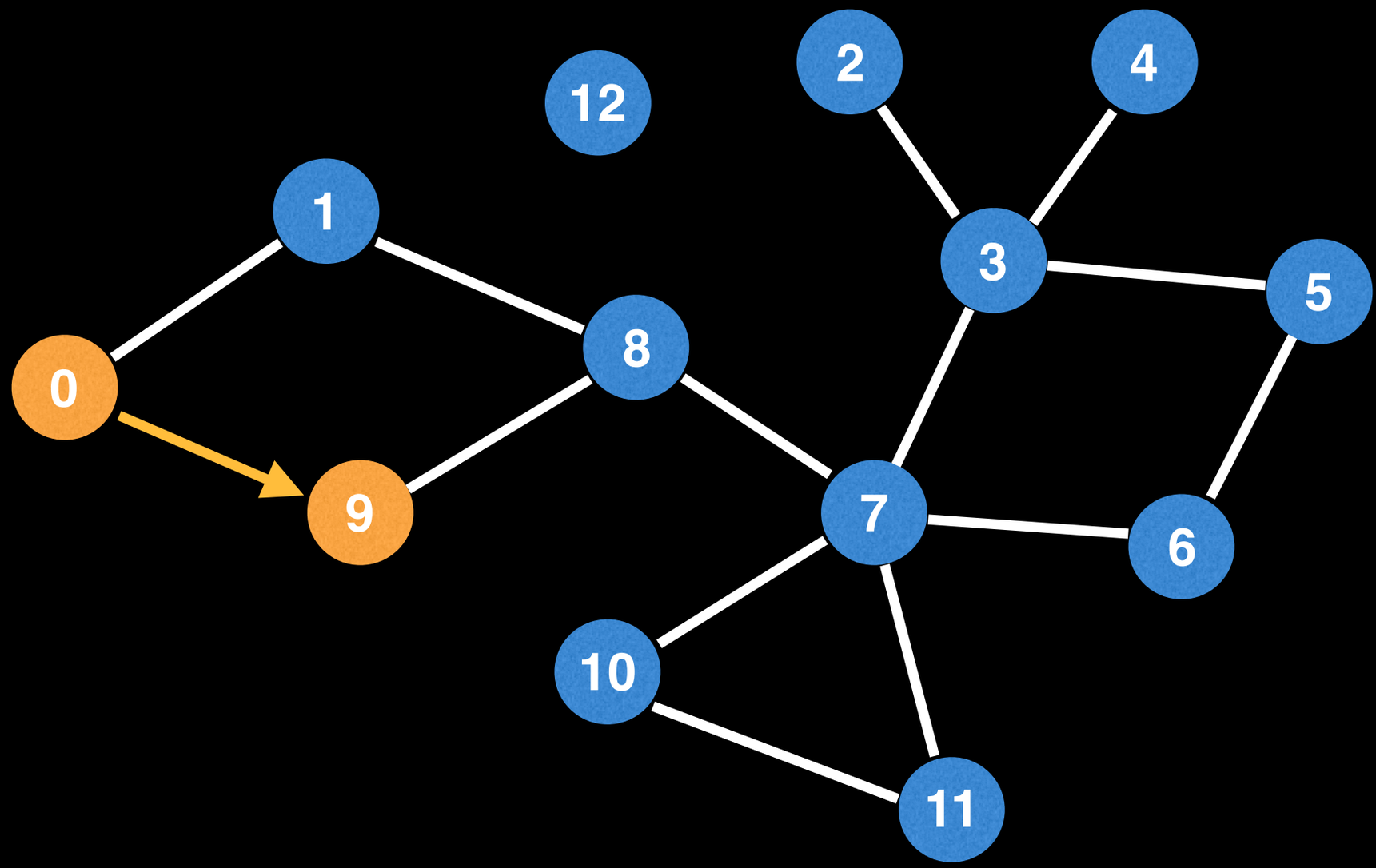
Basic DFS

Pick an edge outwards from node 0



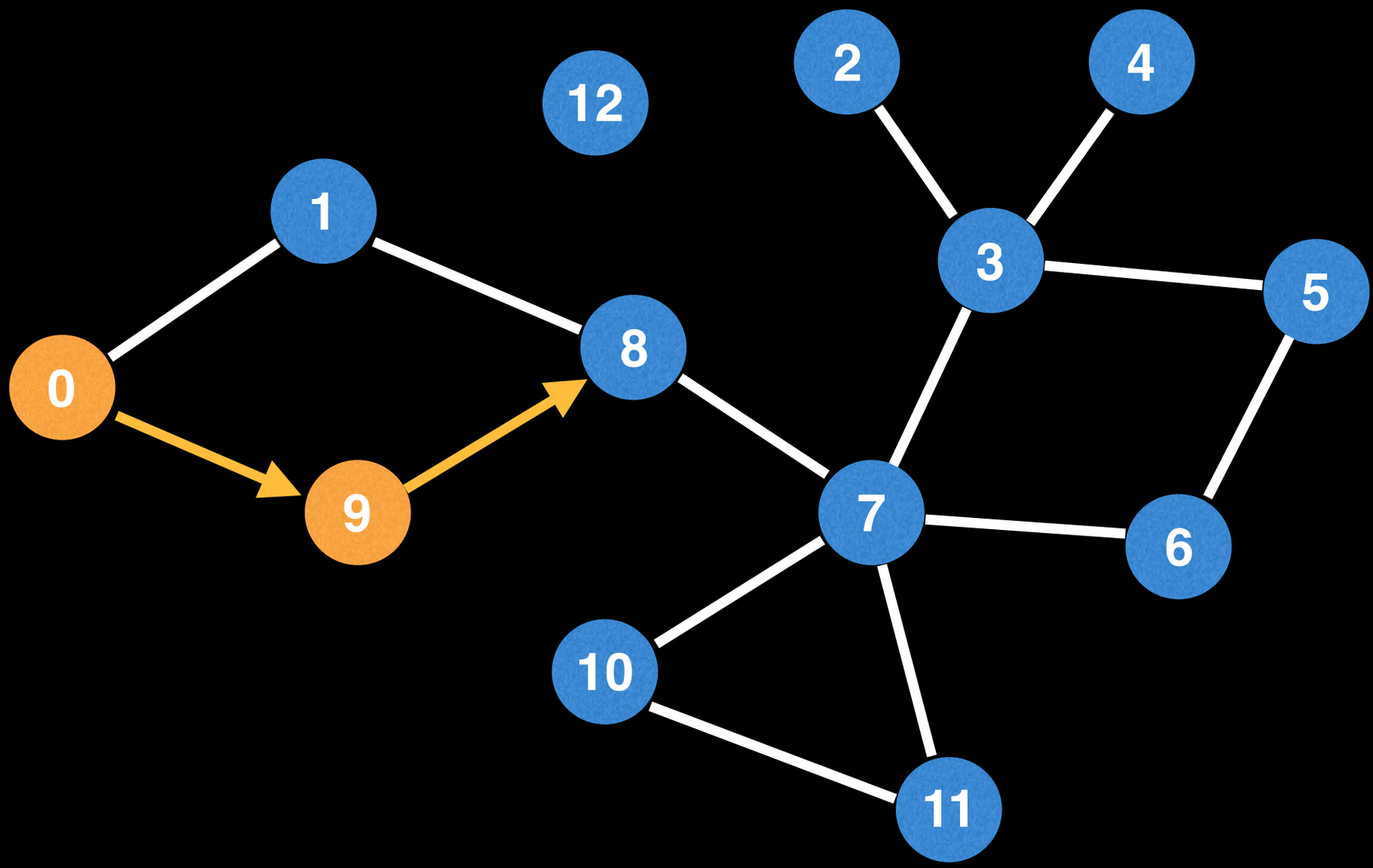
Basic DFS

Once at 9 pick an edge outwards from node 9



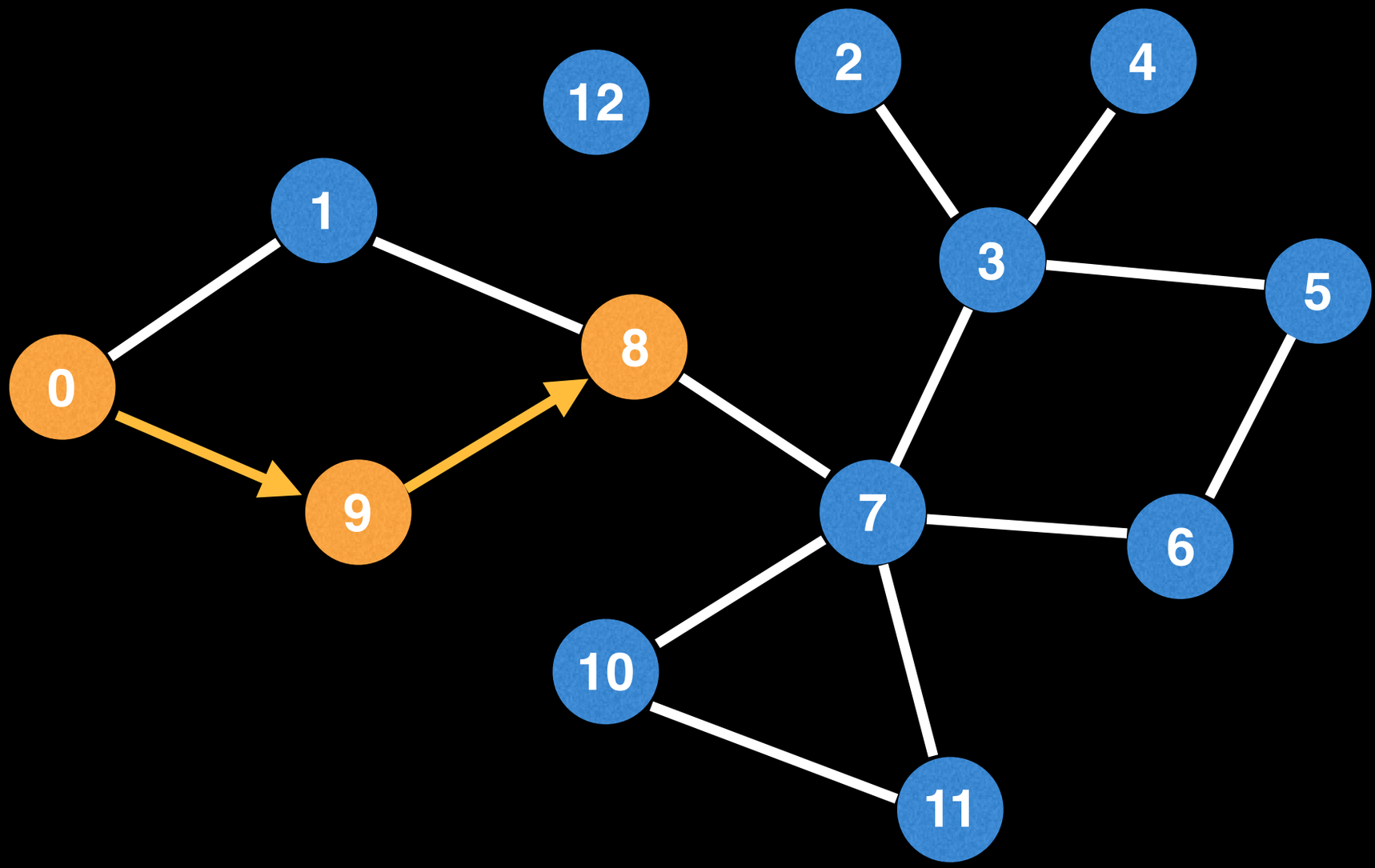
Basic DFS

Go to node 8

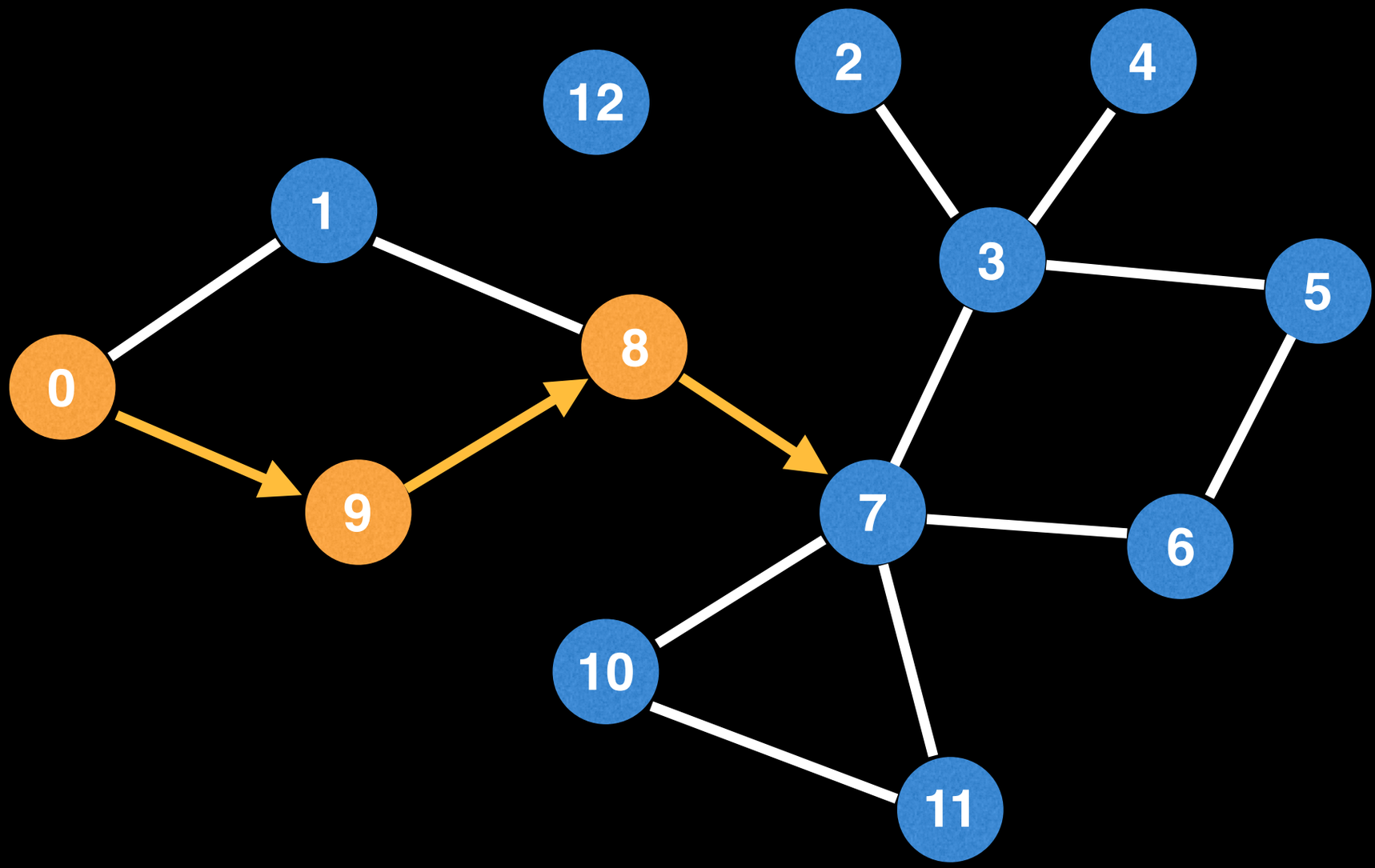


Basic DFS

Pick an edge outwards from 8…



Basic DFS



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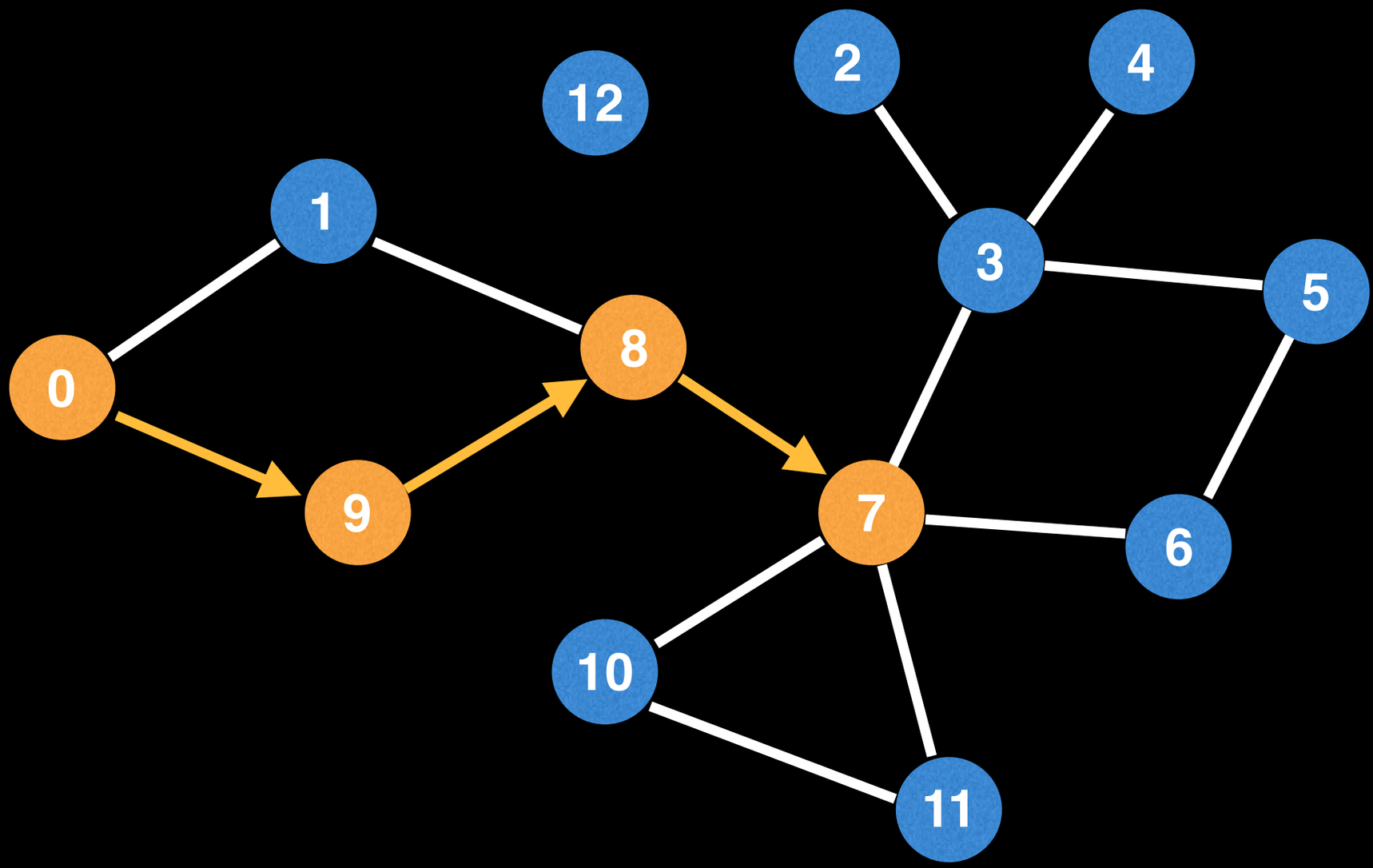

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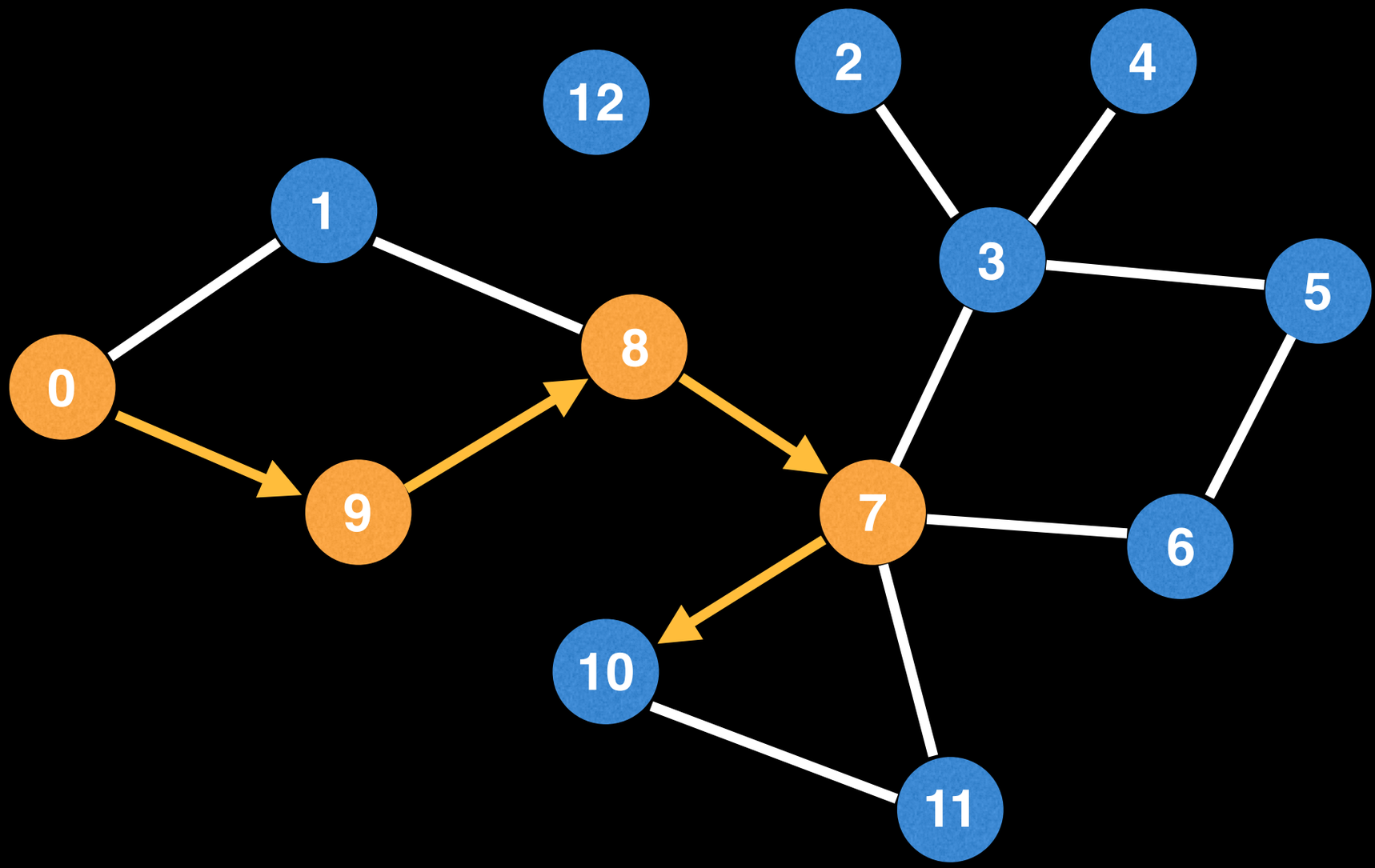

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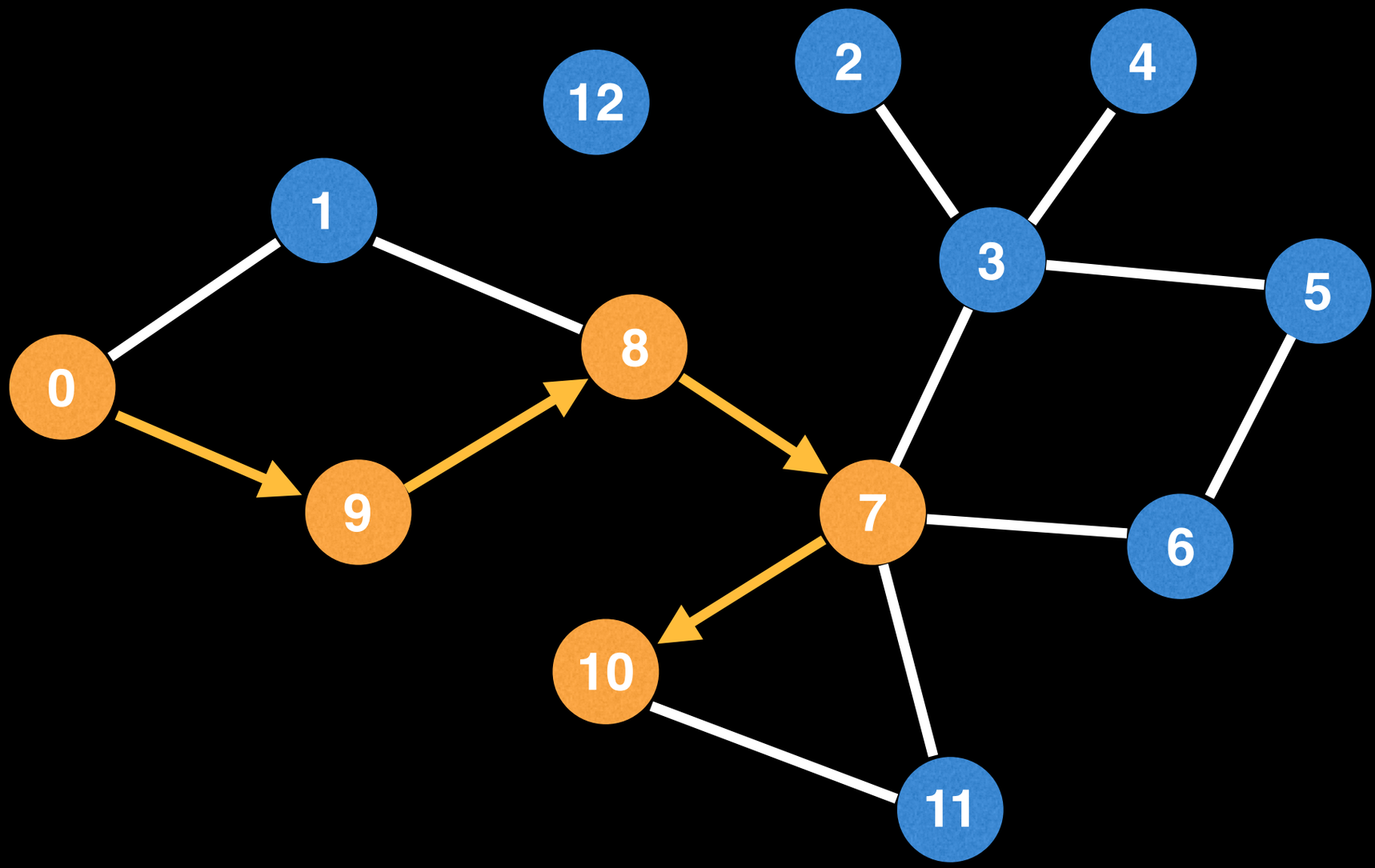

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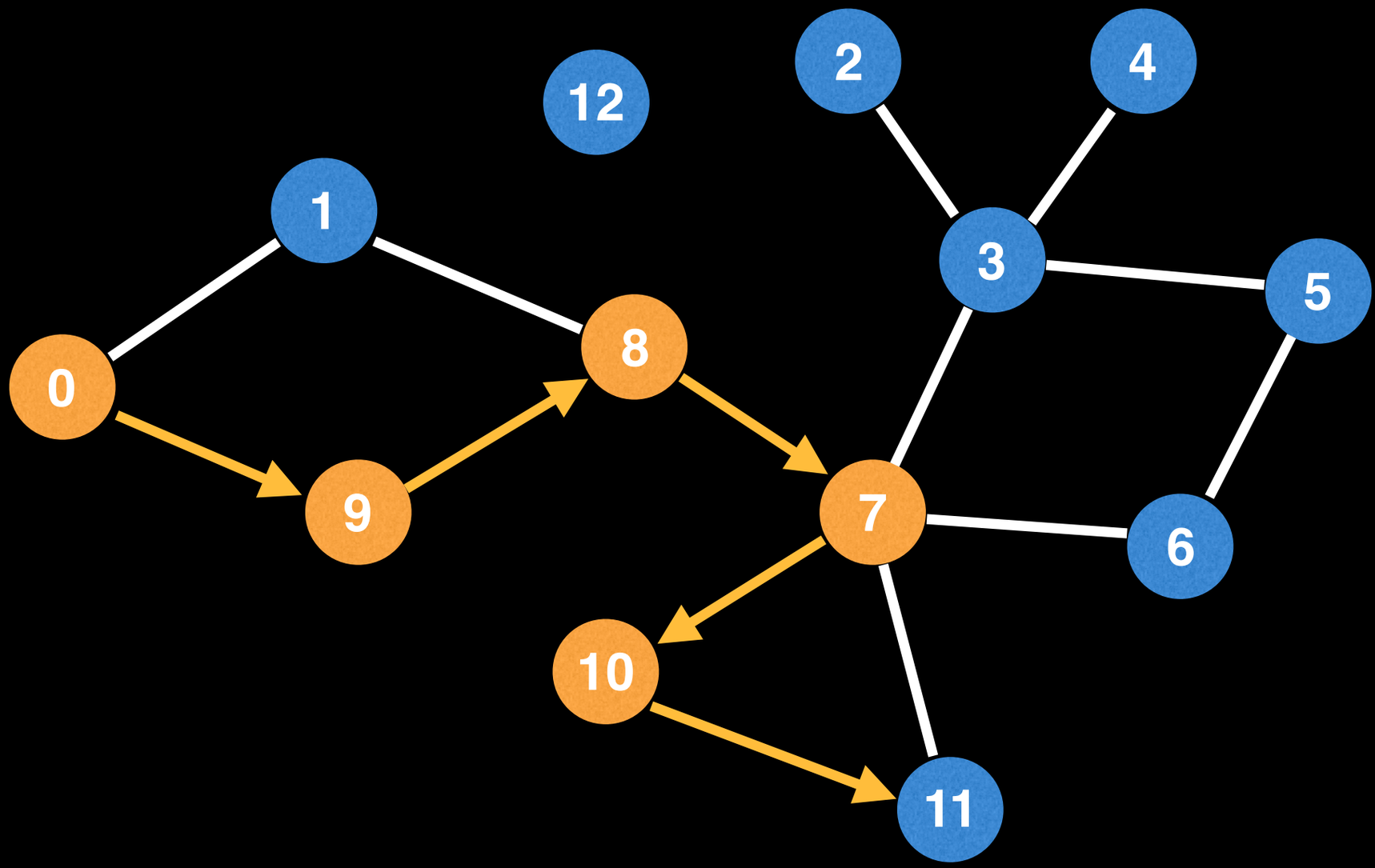

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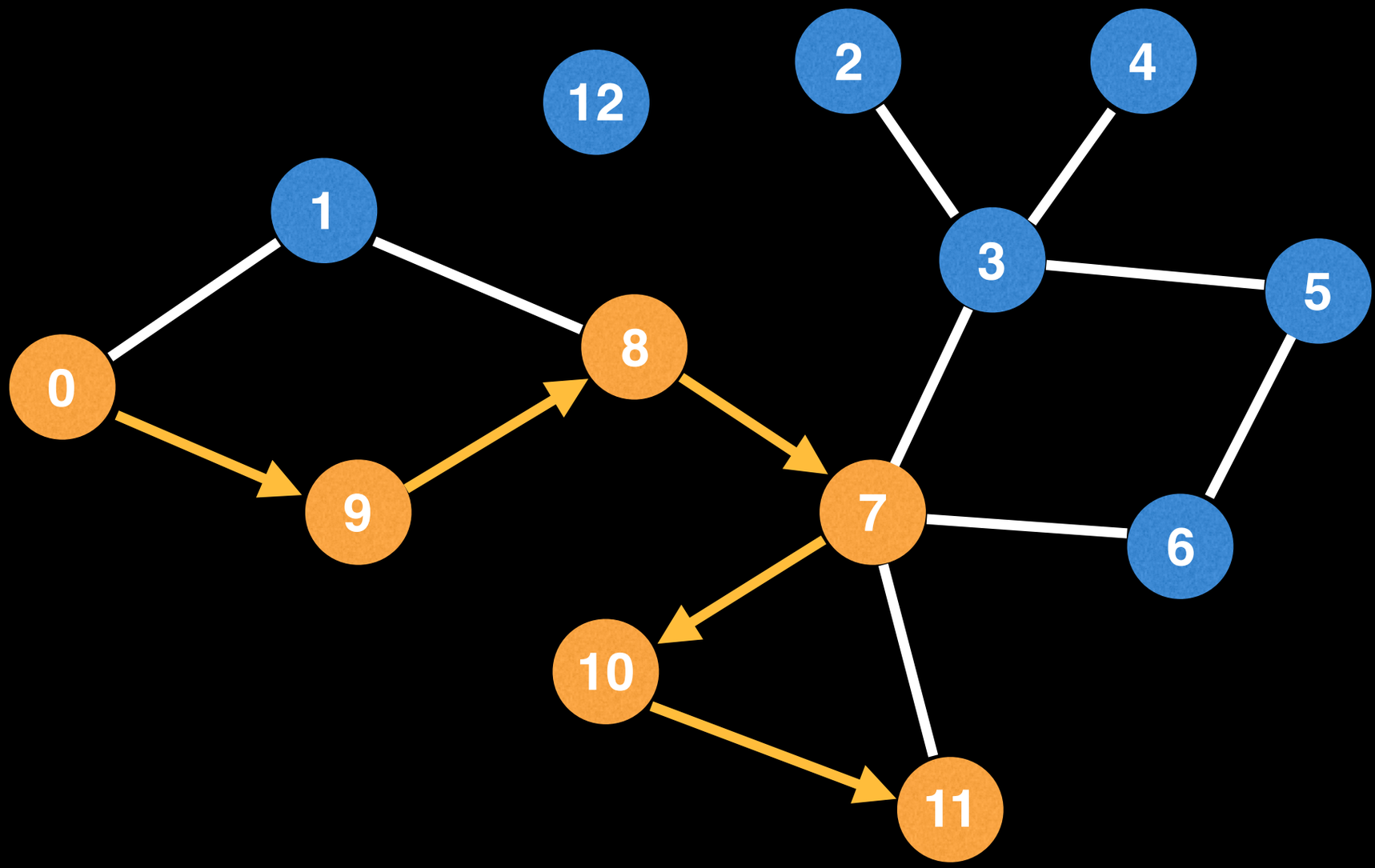
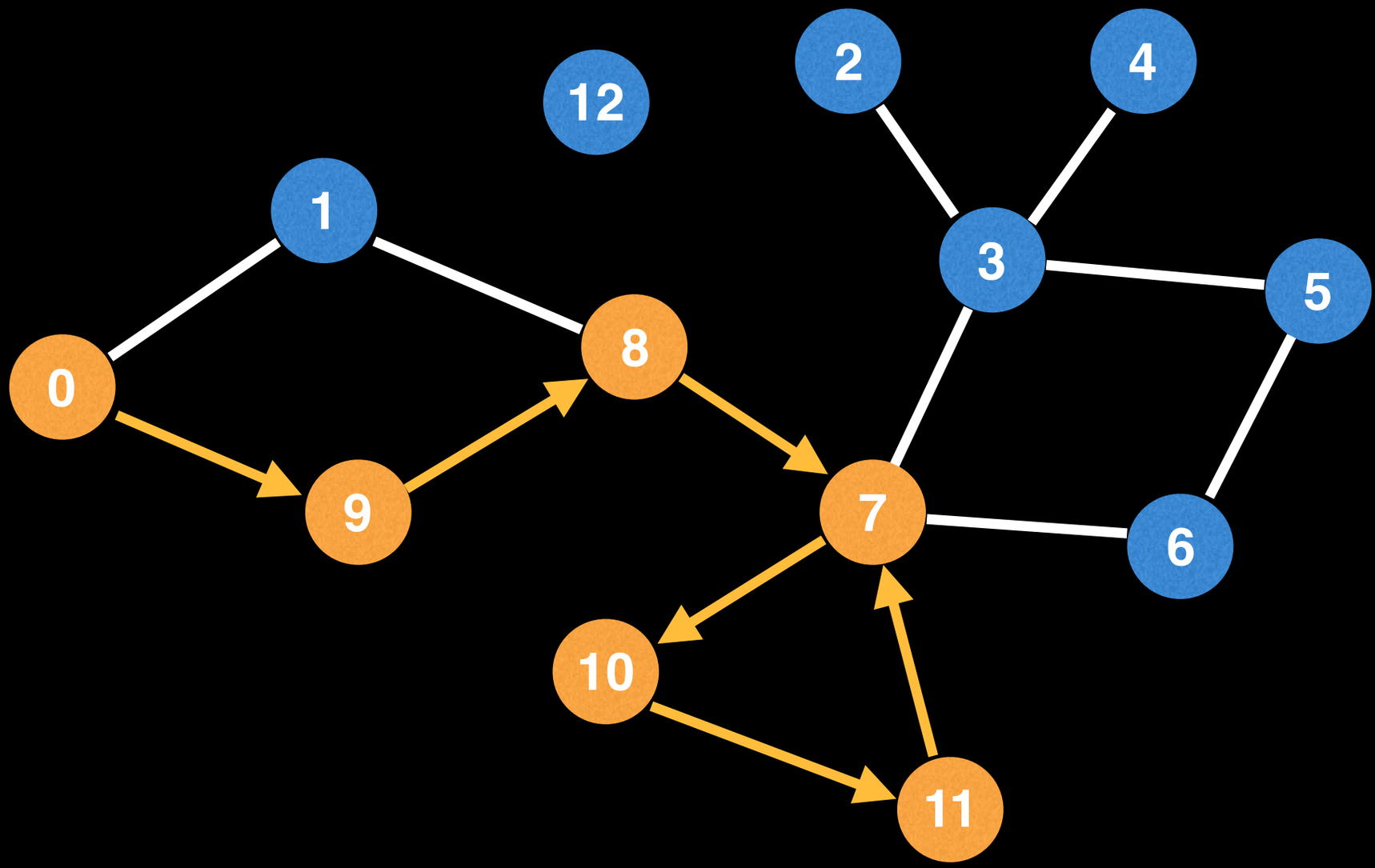


Basic DFS

Make sure you don’t re-visit visited nodes!



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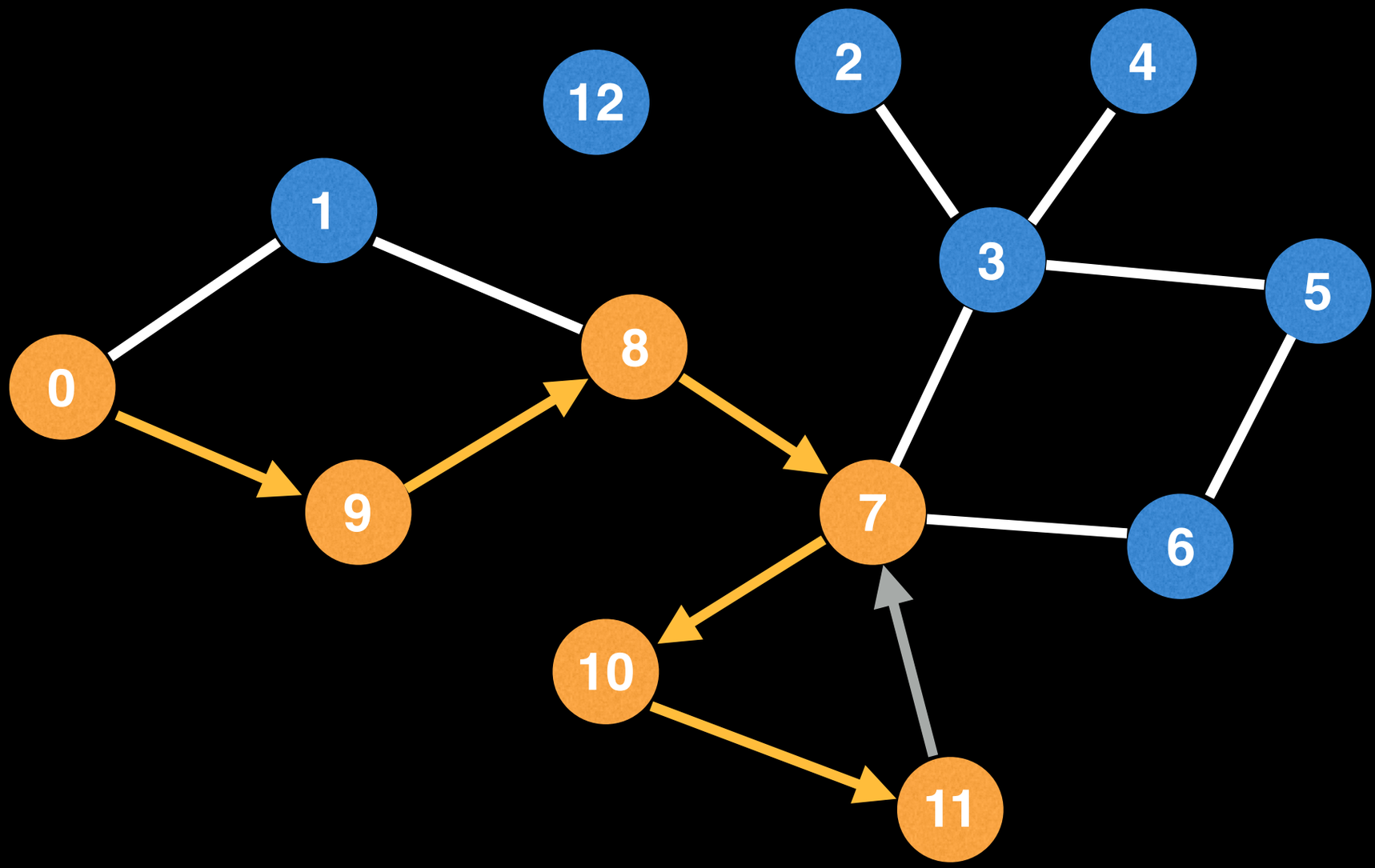

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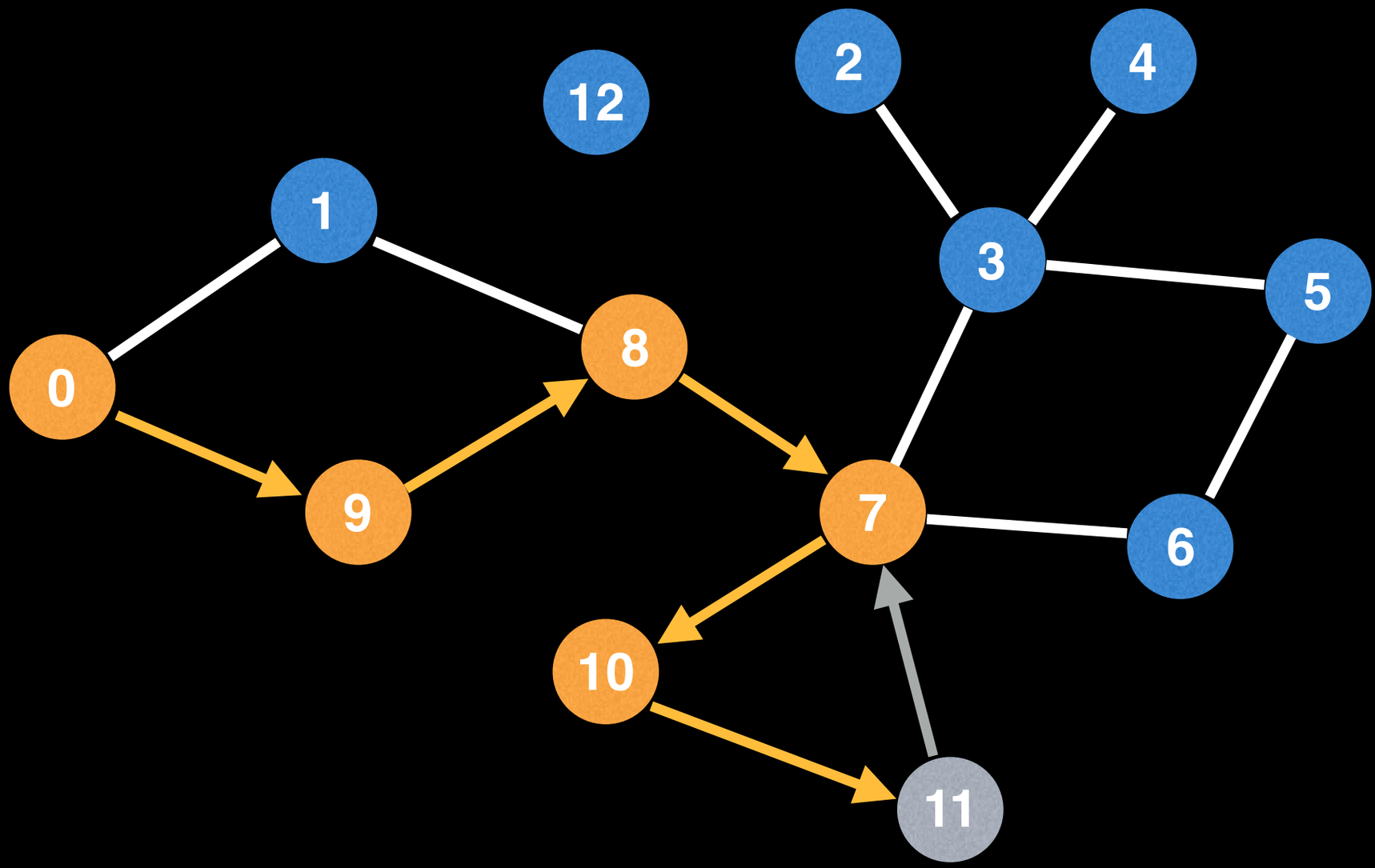

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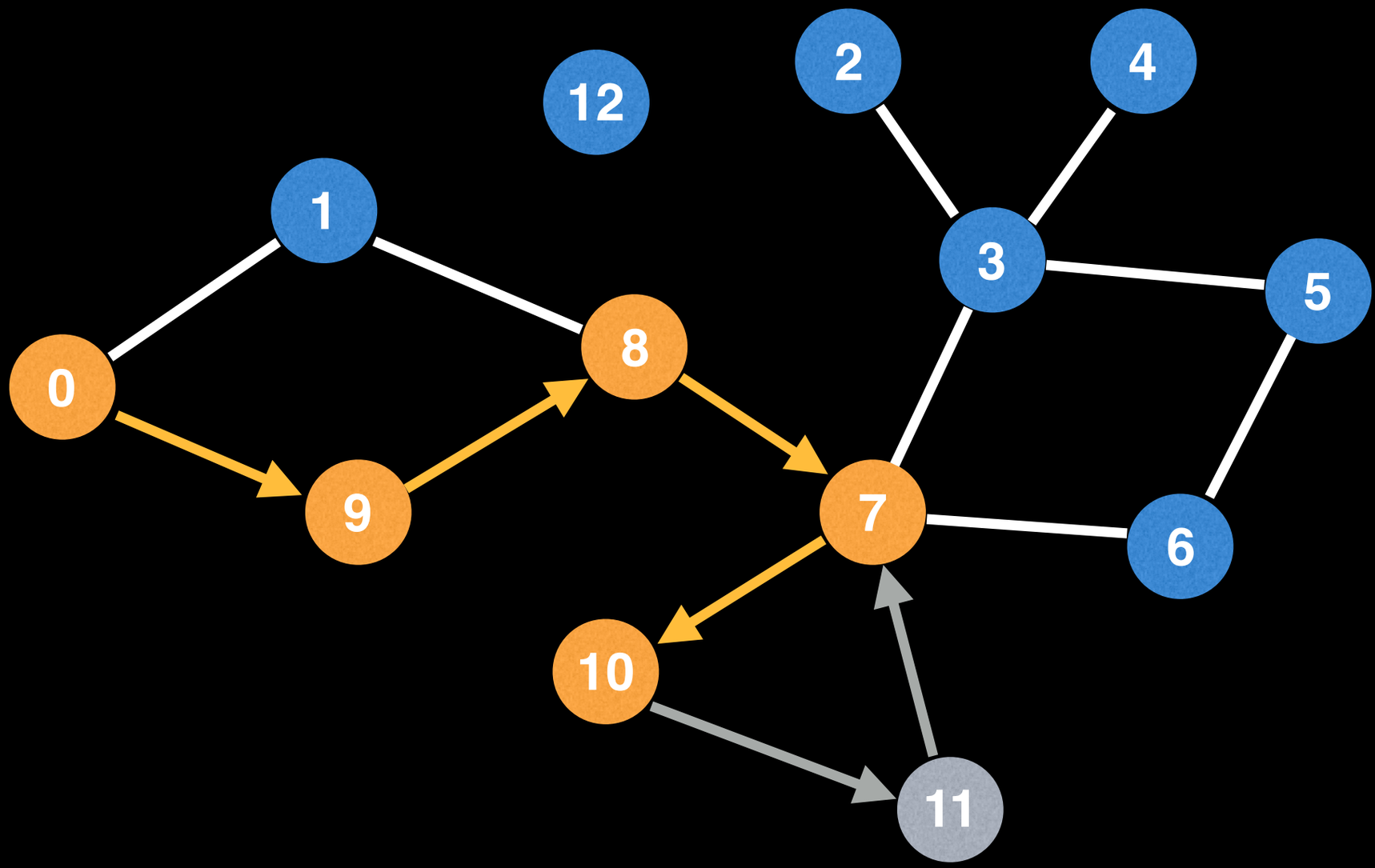

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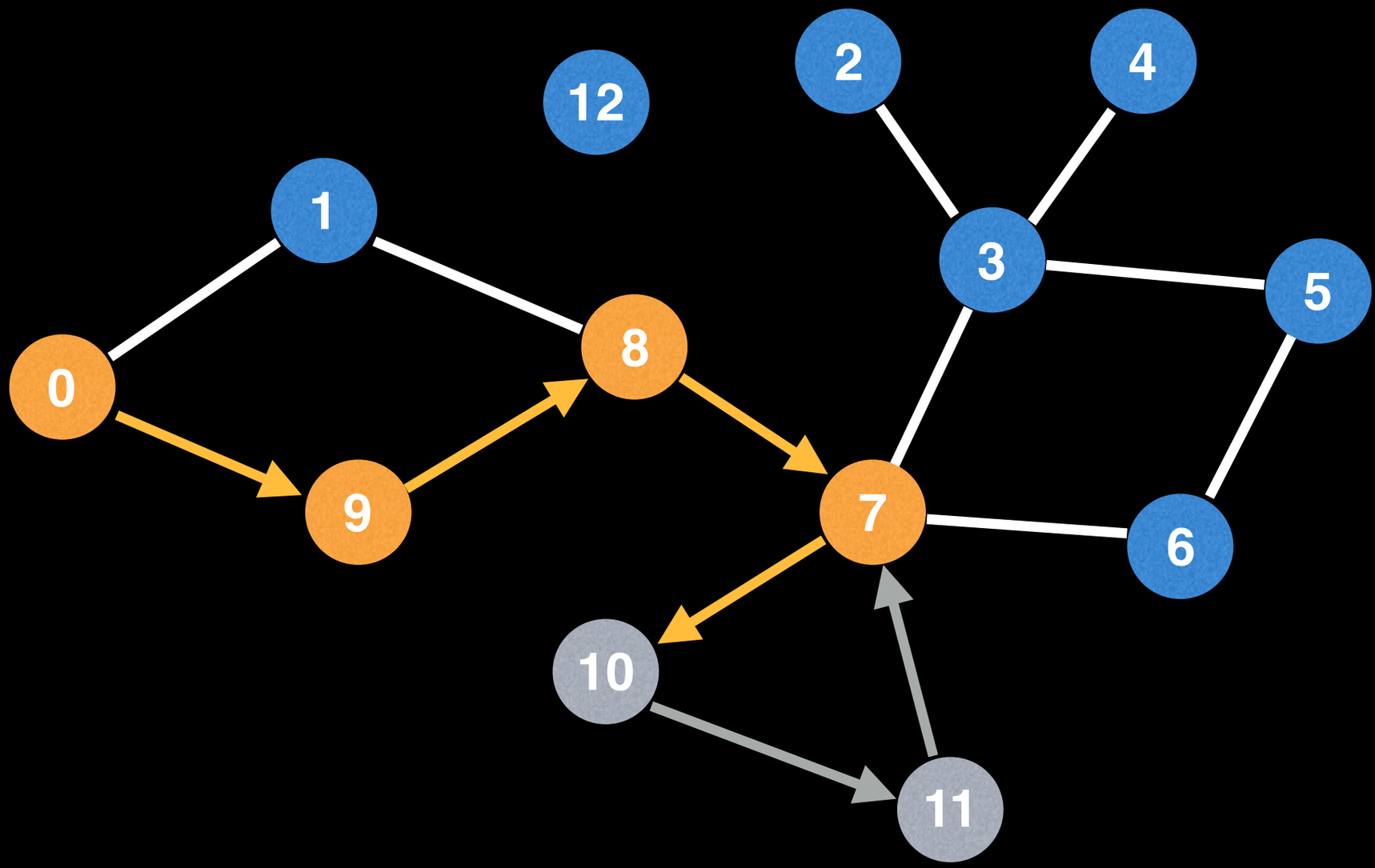

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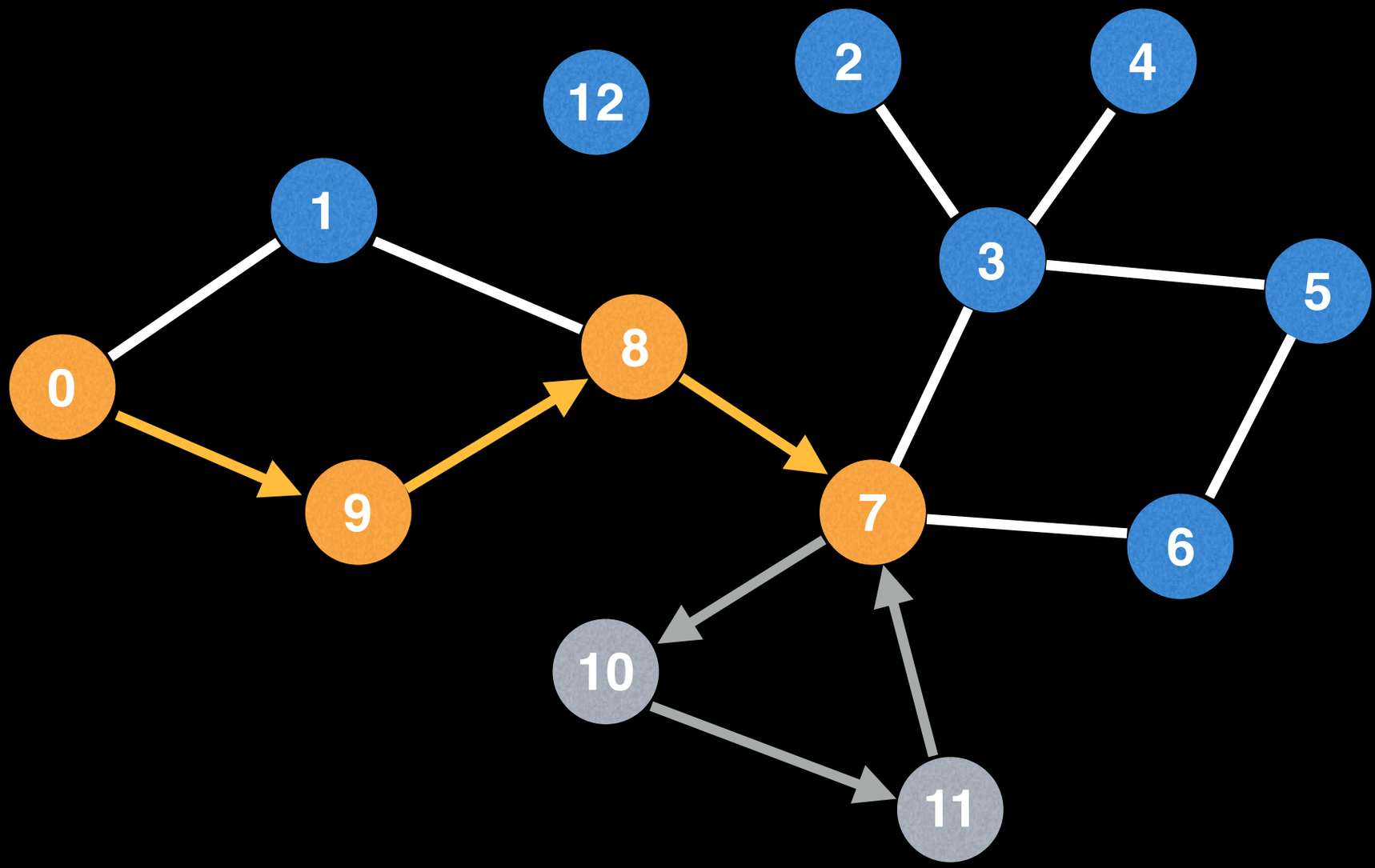
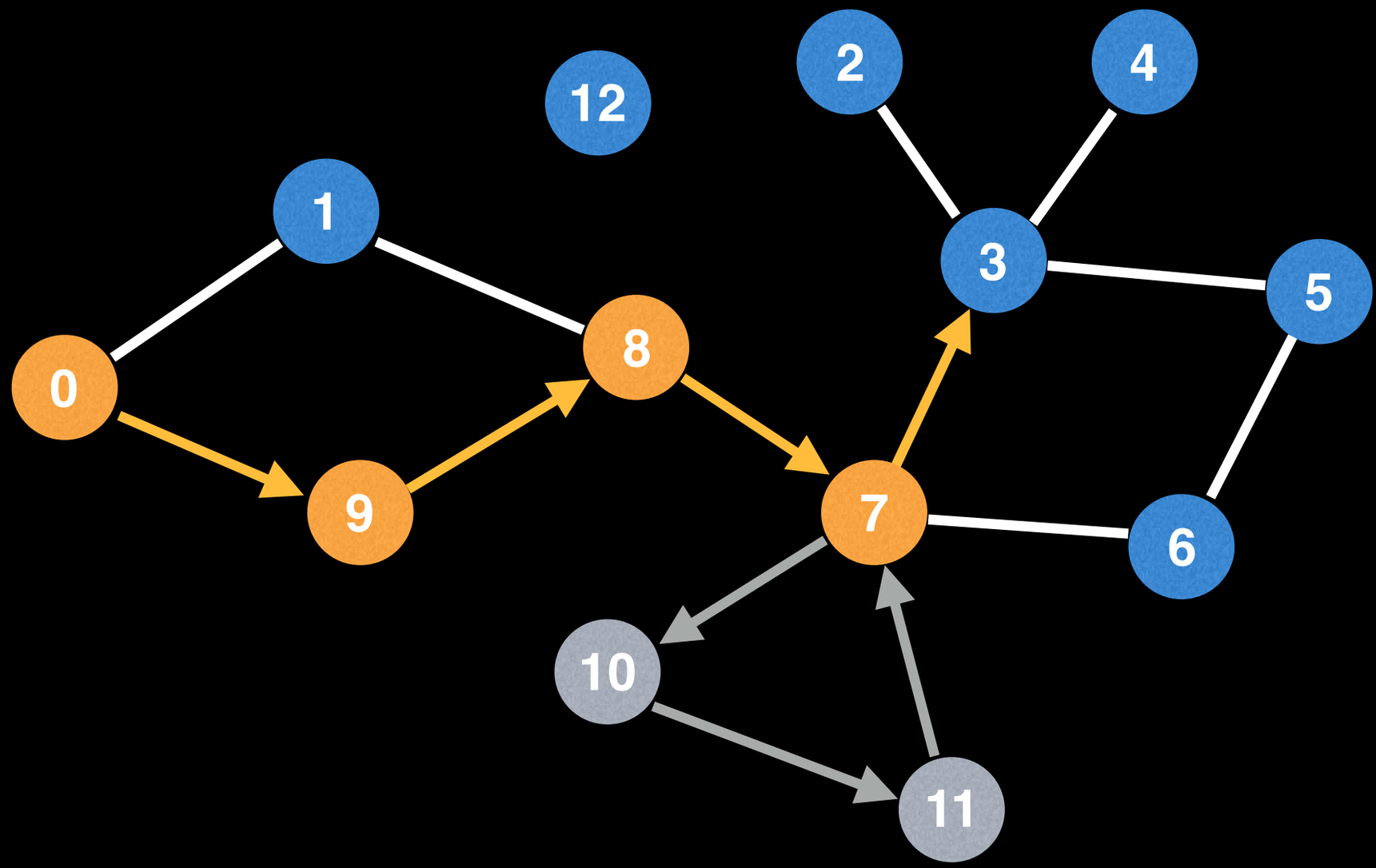


Basic DFS

We haven’t finished visiting all the neighbours of 7 so continue DFS in another direction



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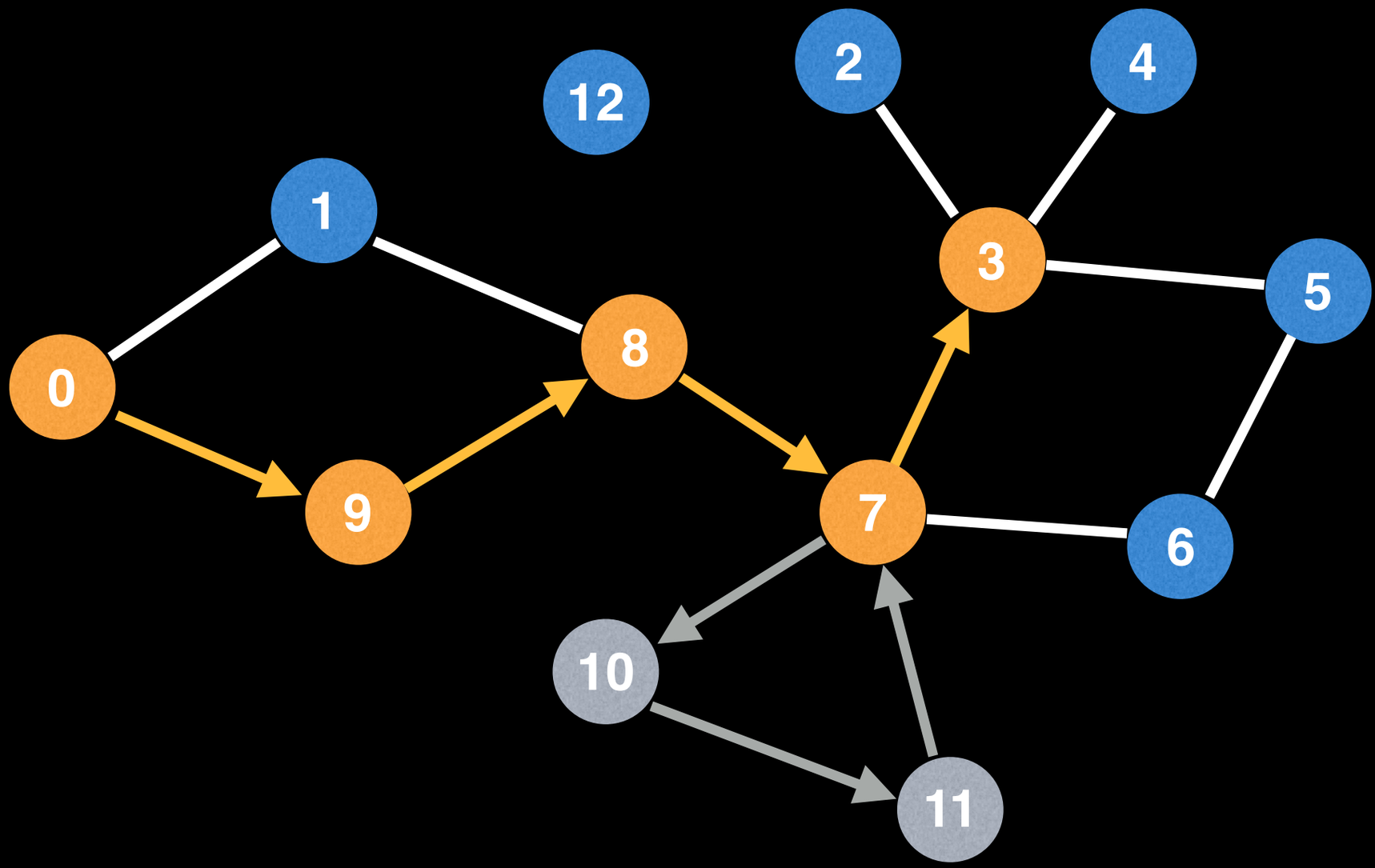

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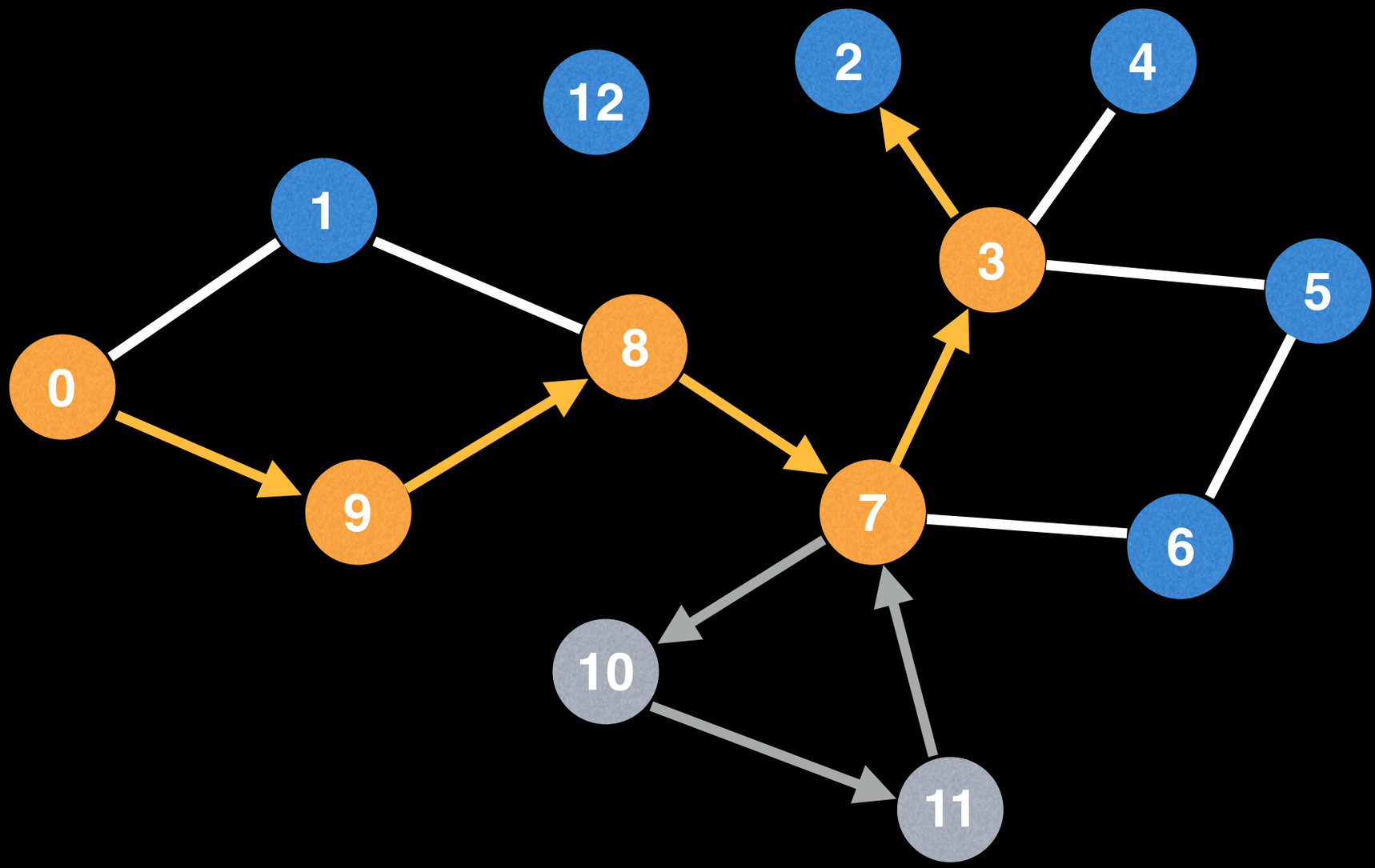

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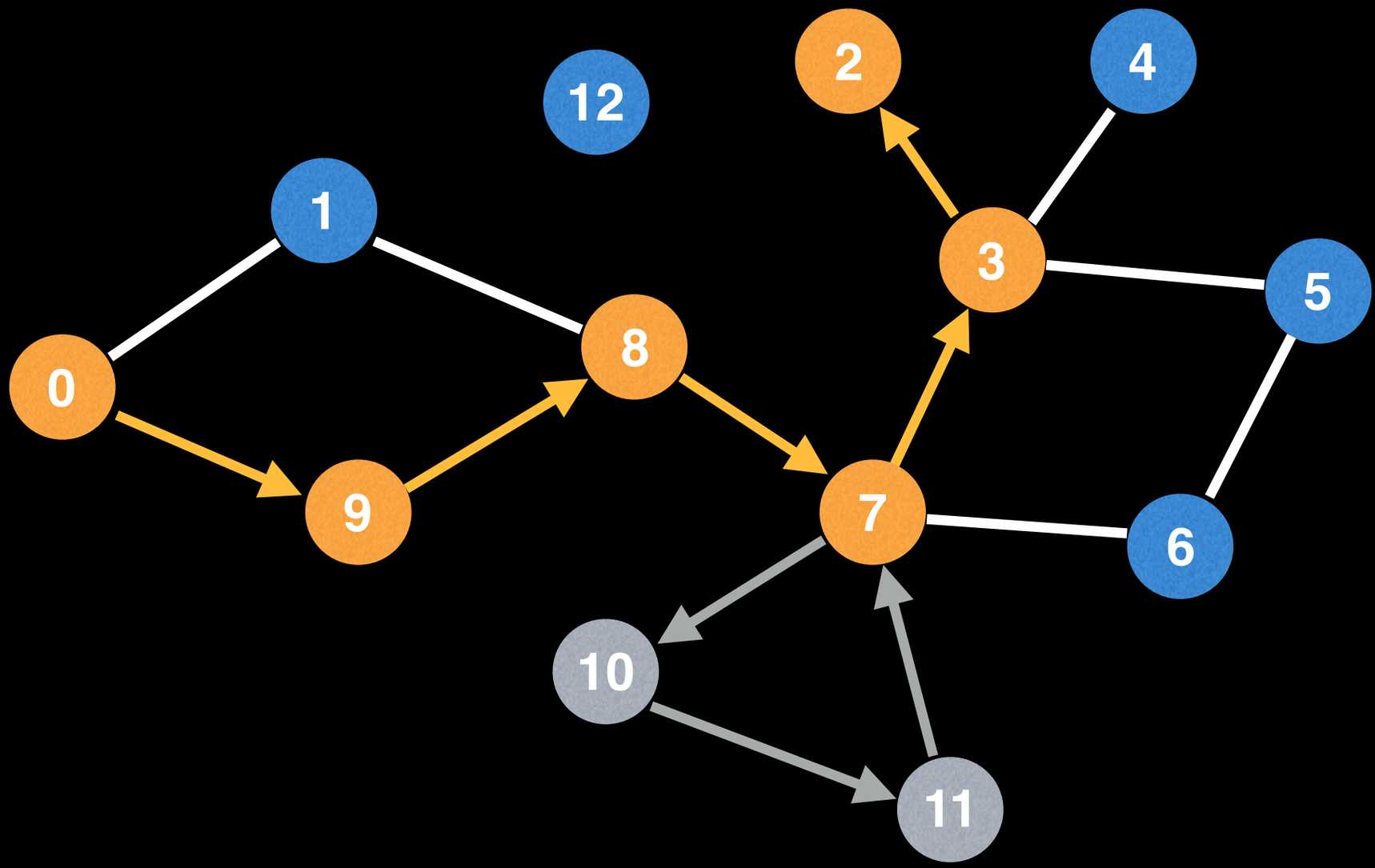
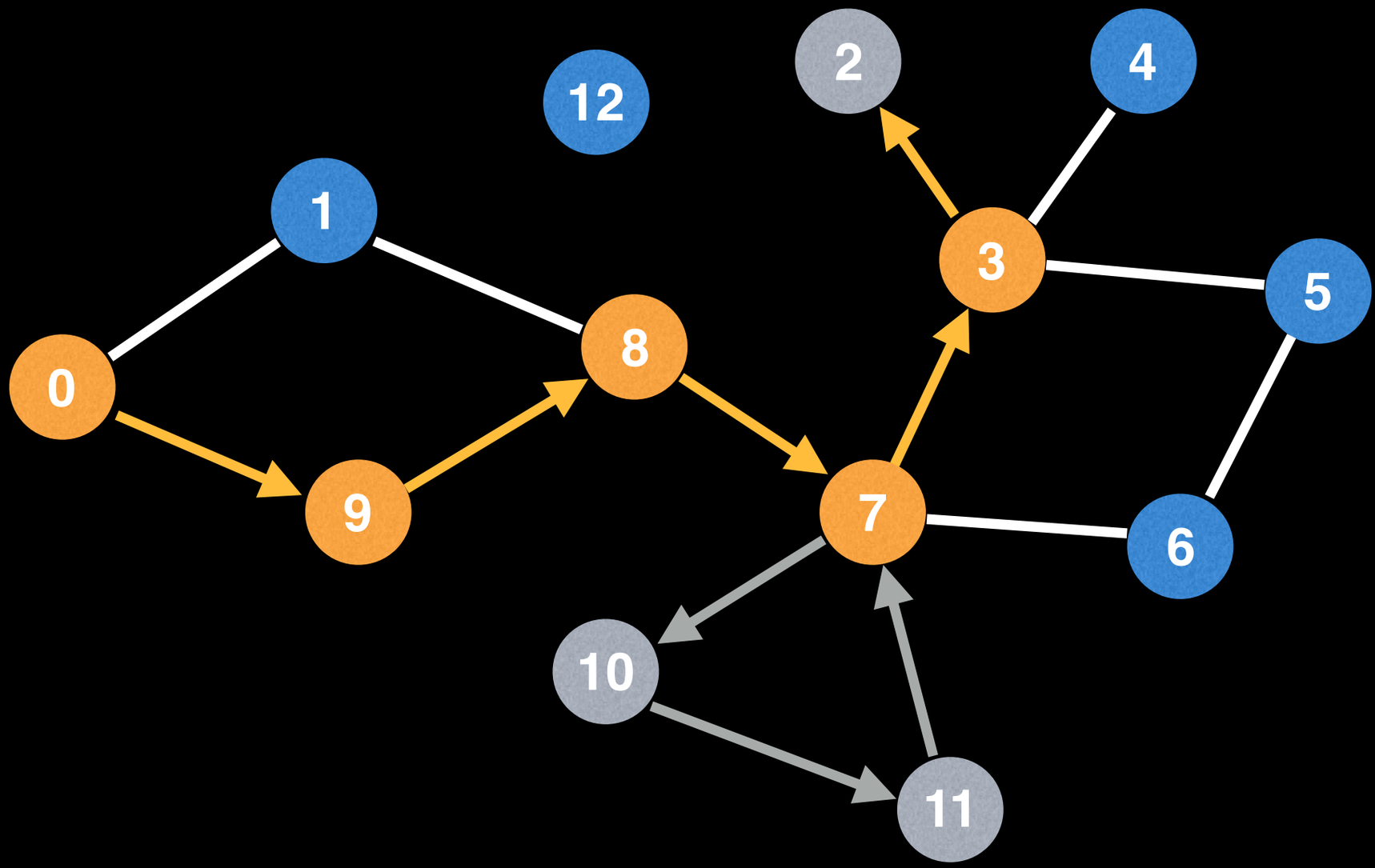


Basic DFS

Backtrack when a dead end is reached.



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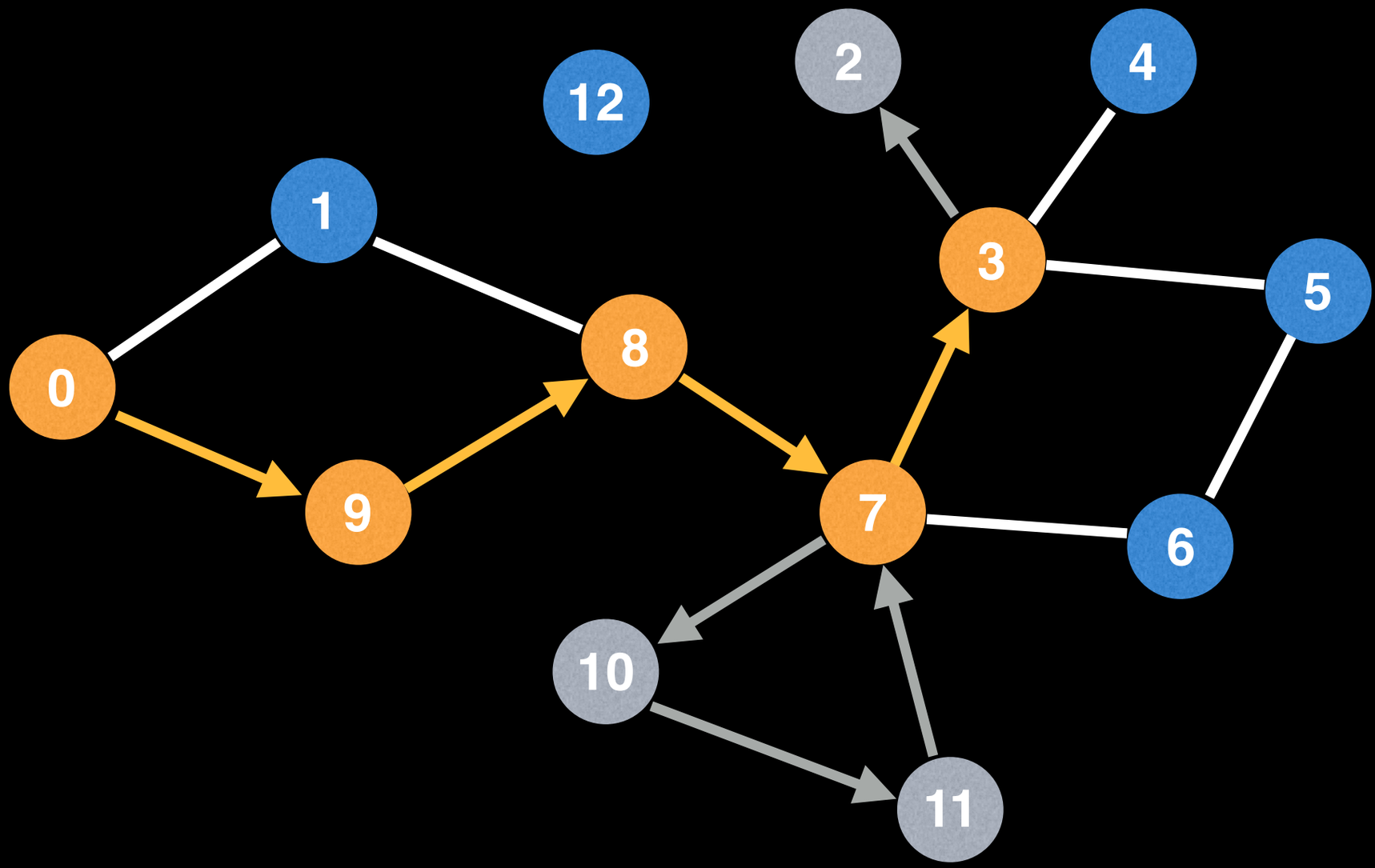

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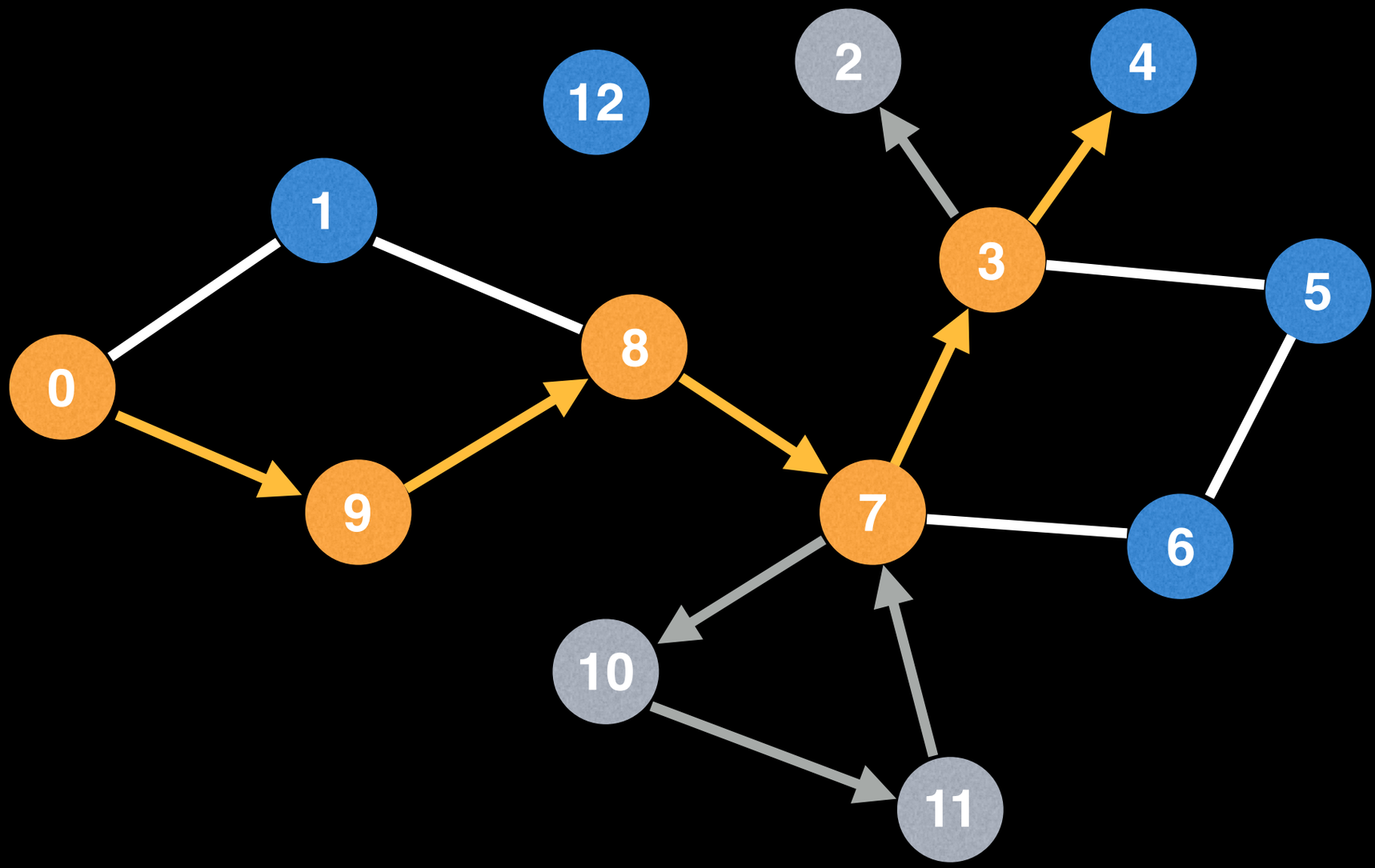

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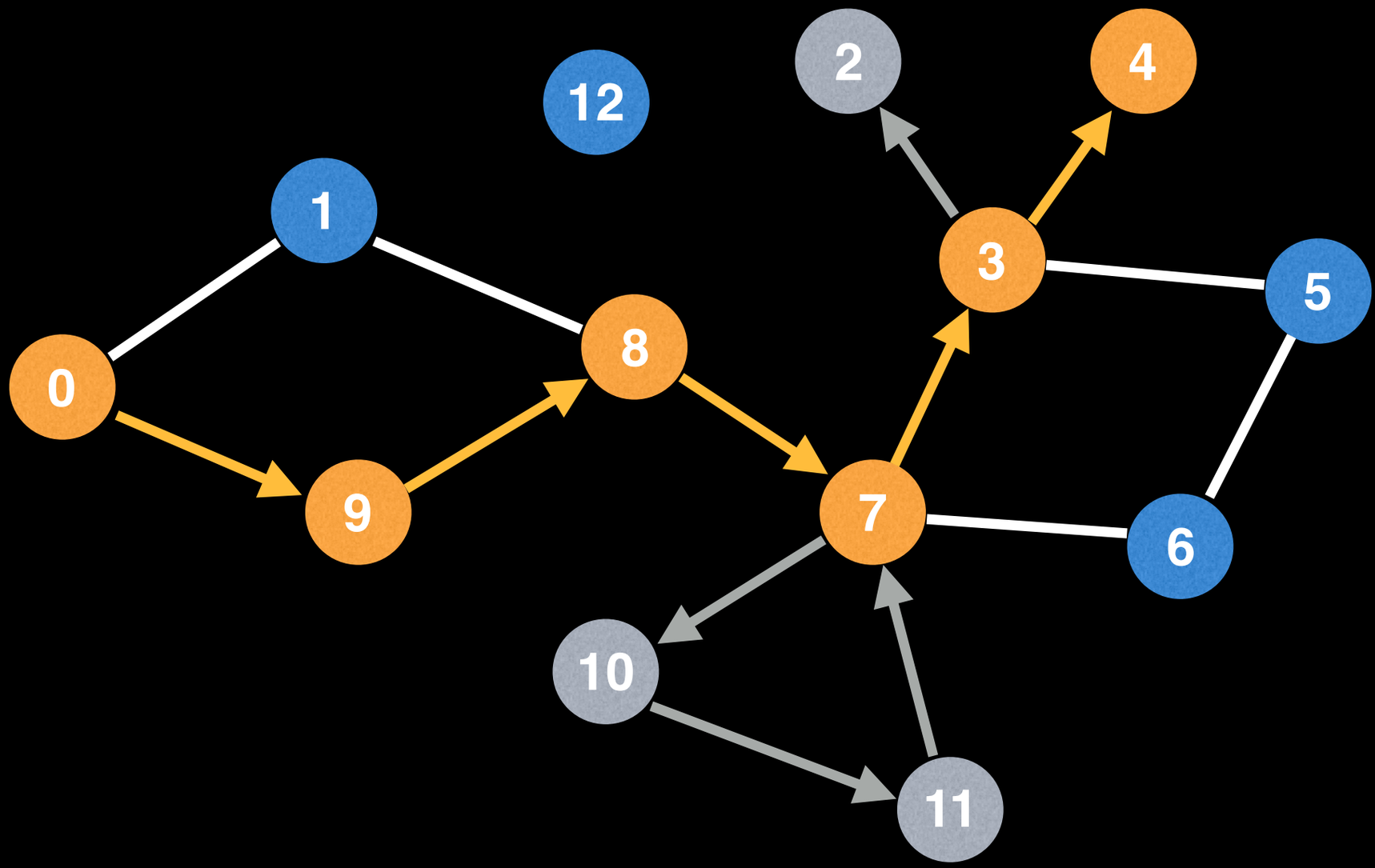

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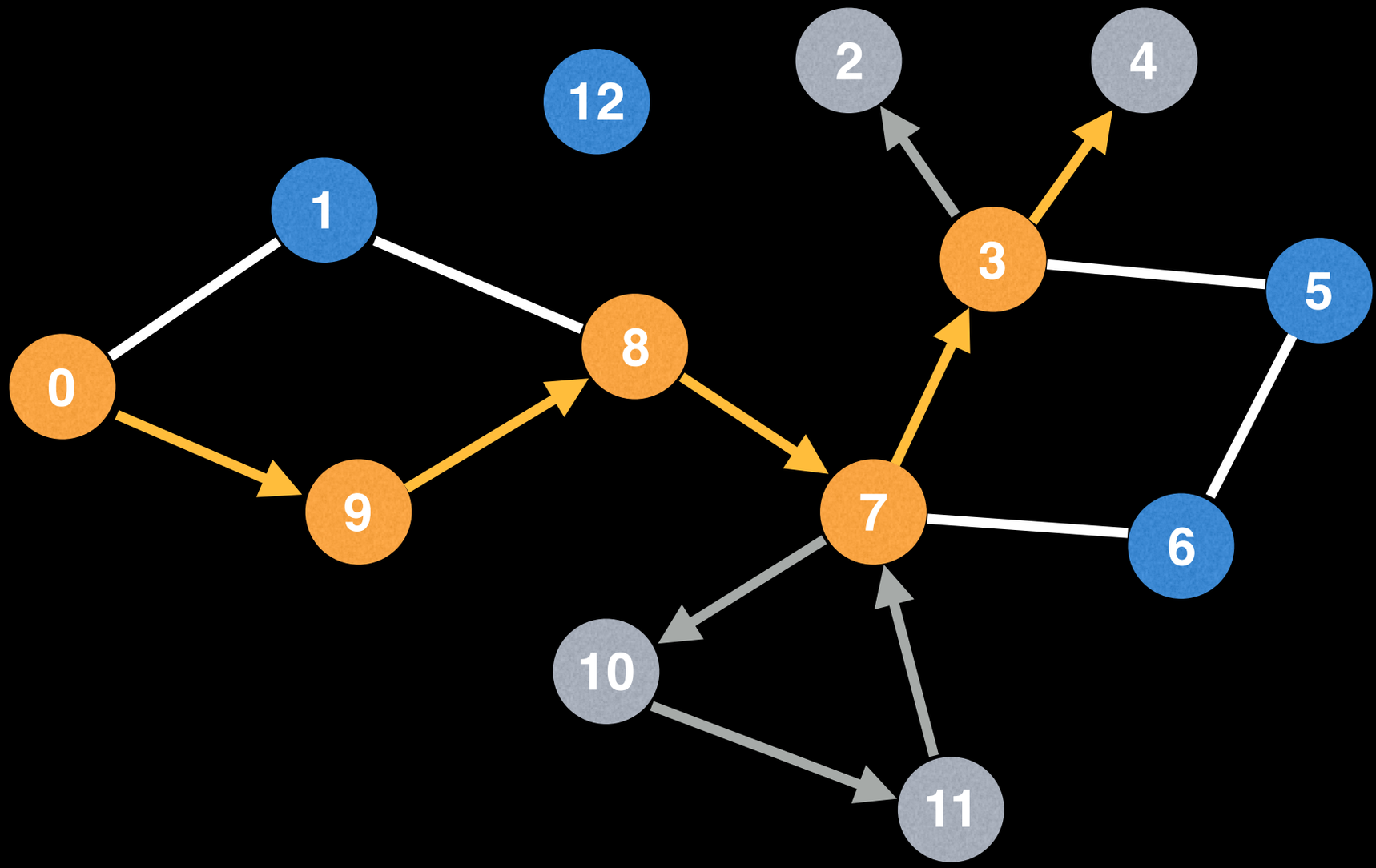

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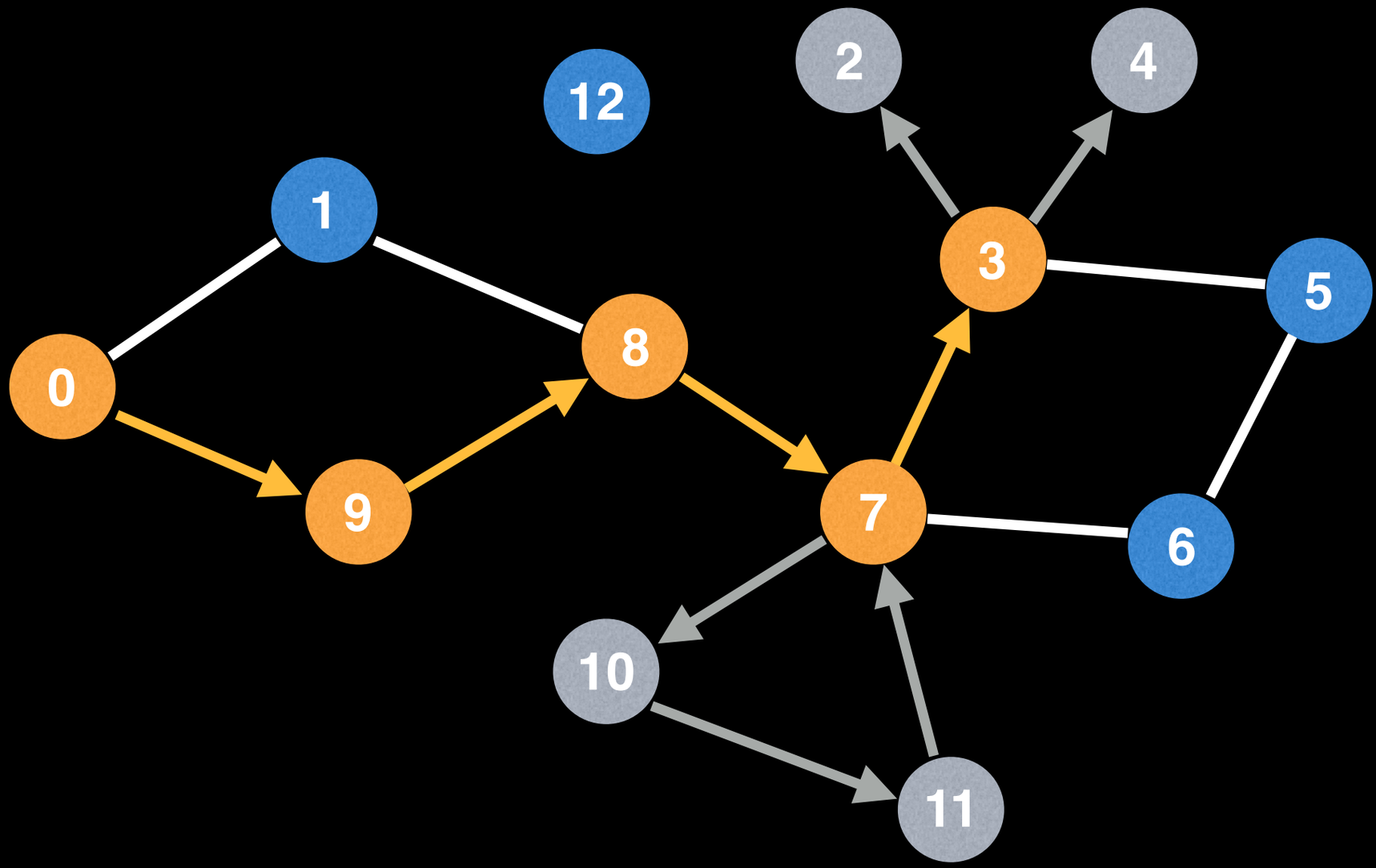

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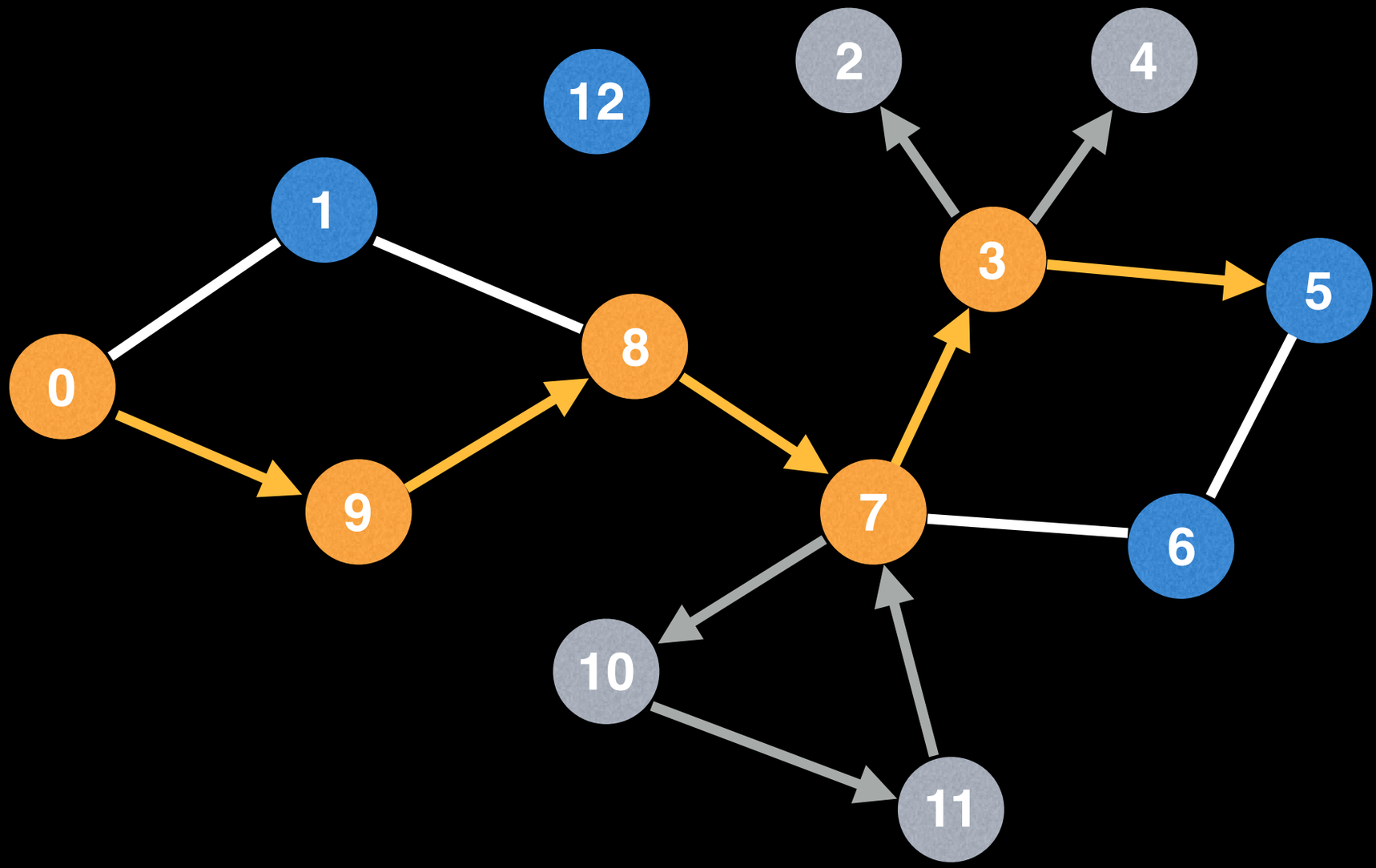

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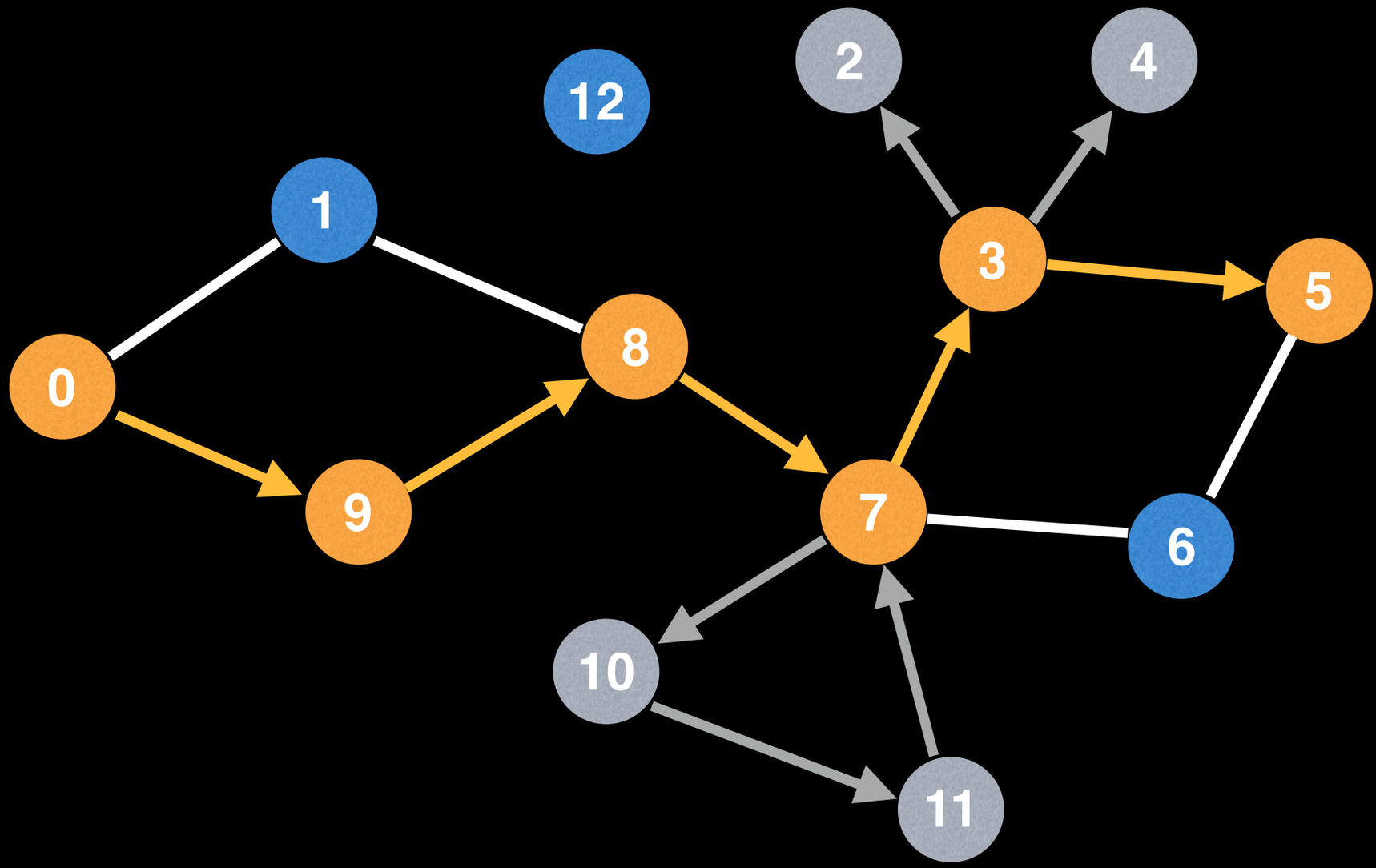

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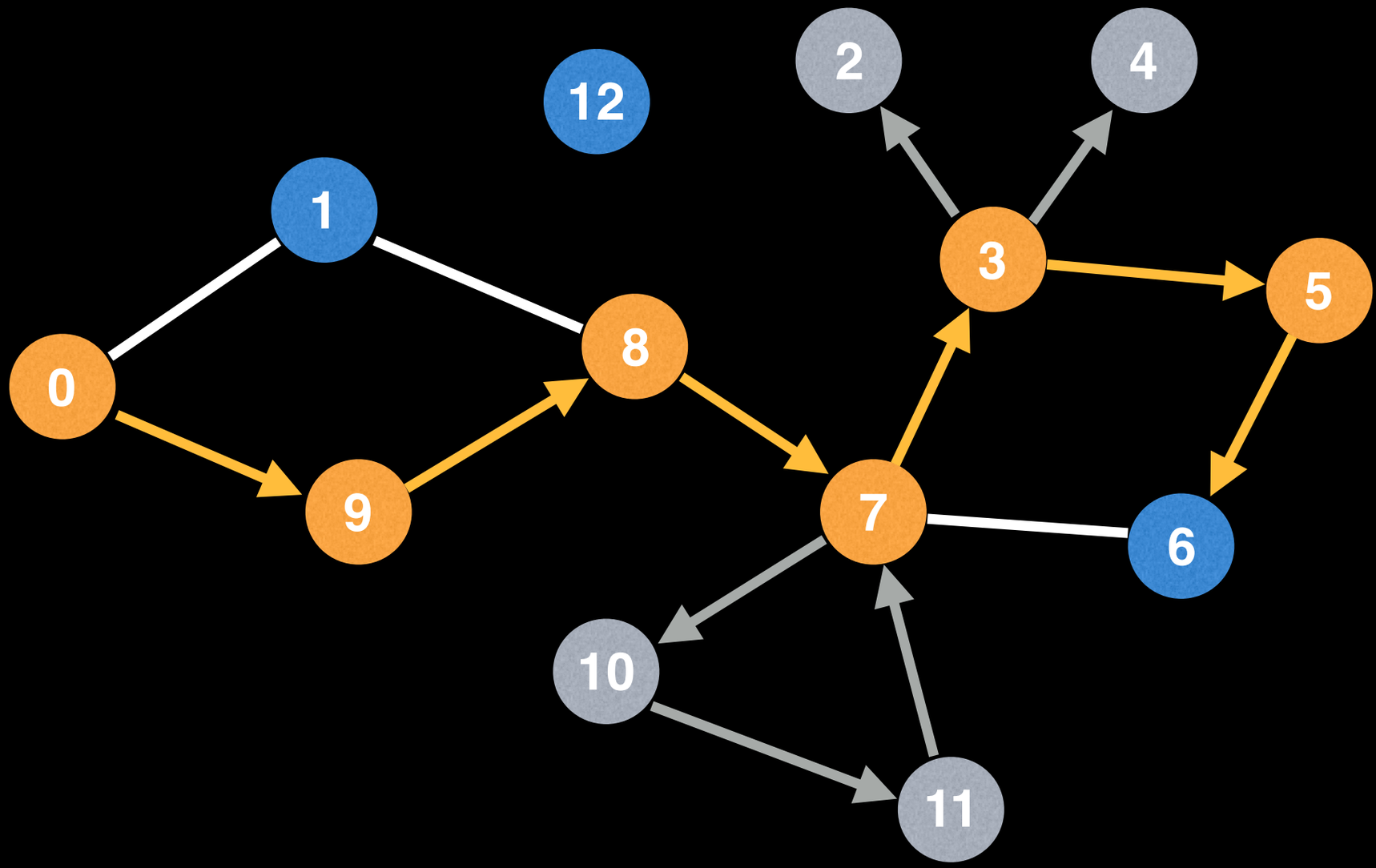

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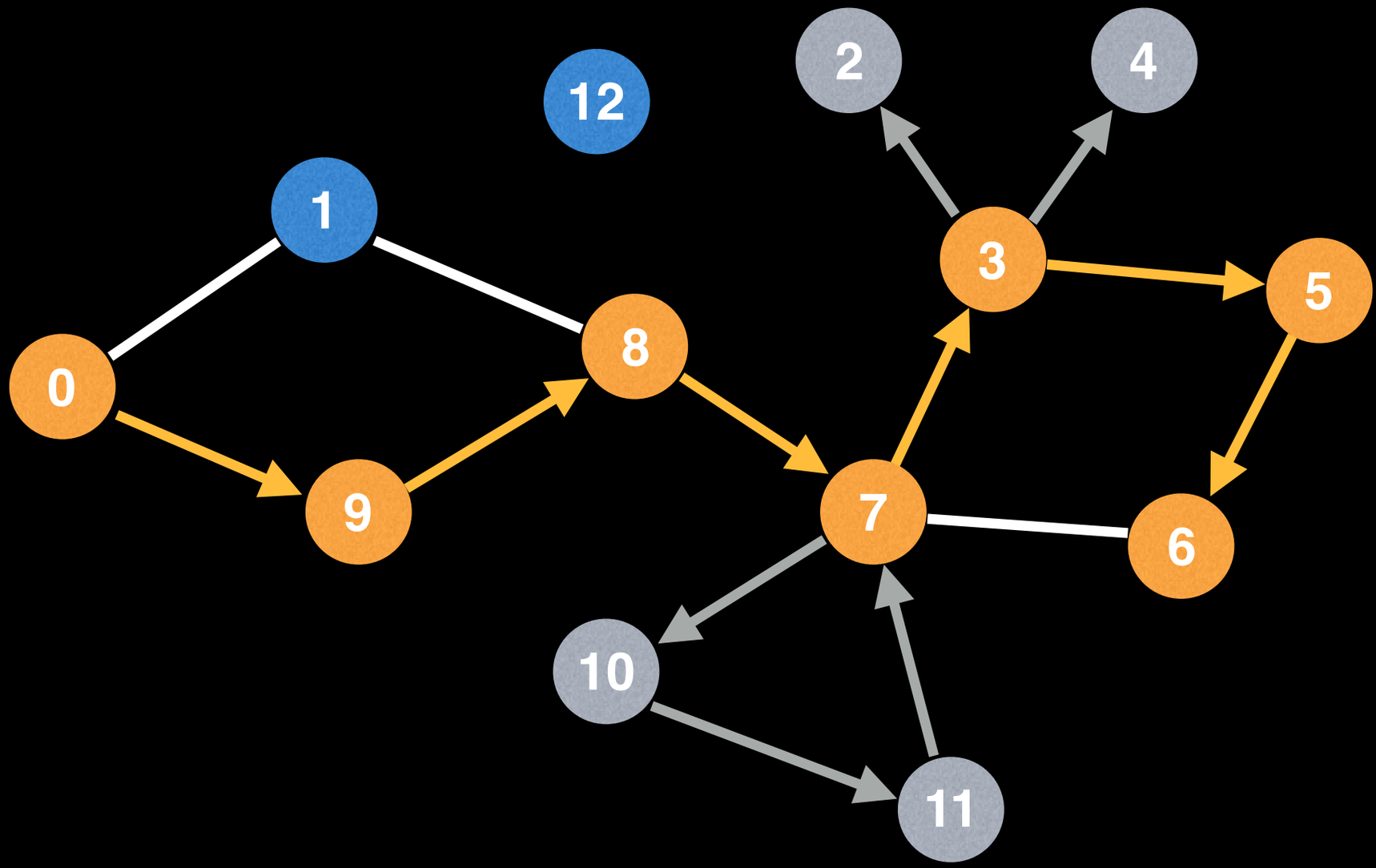

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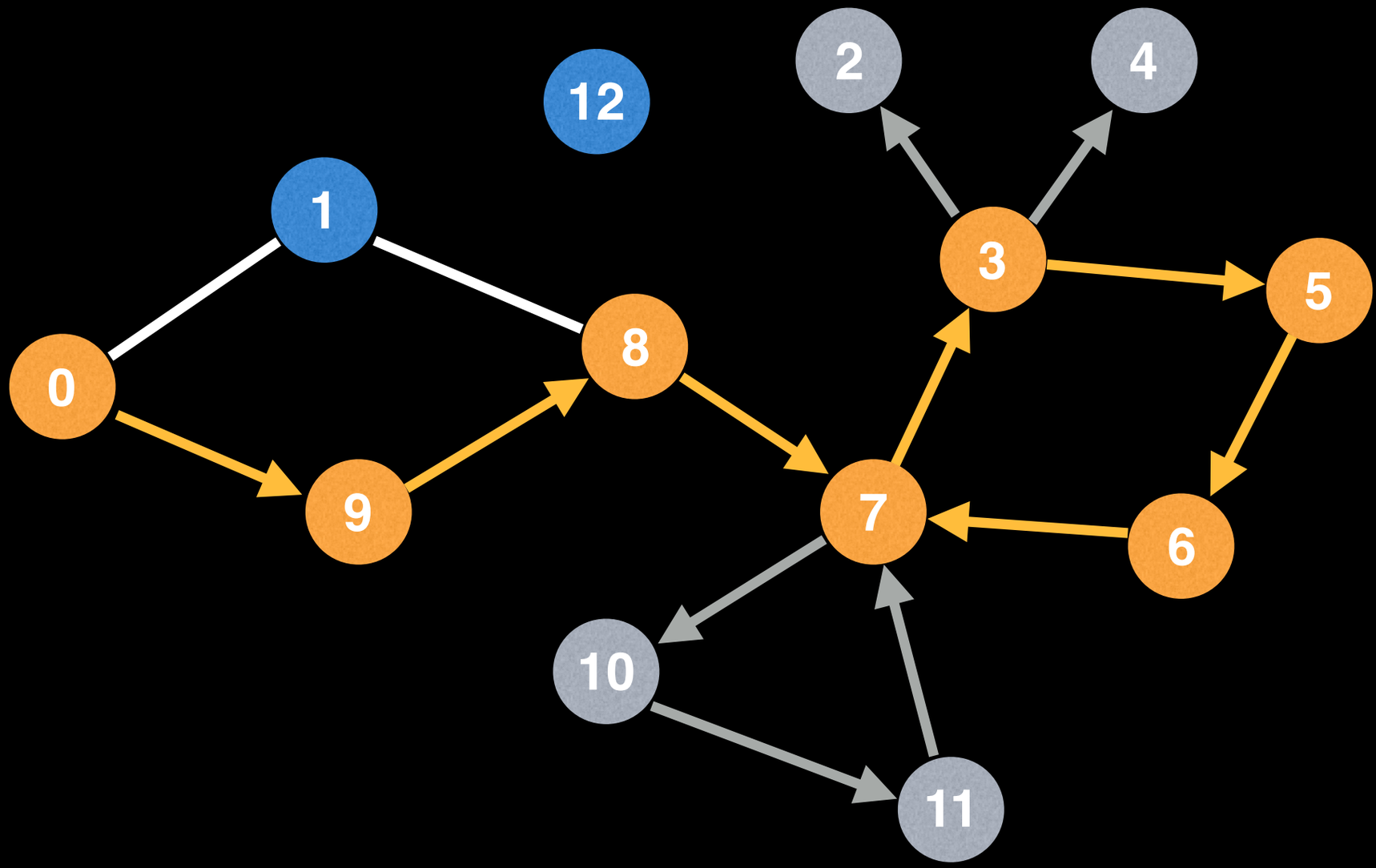

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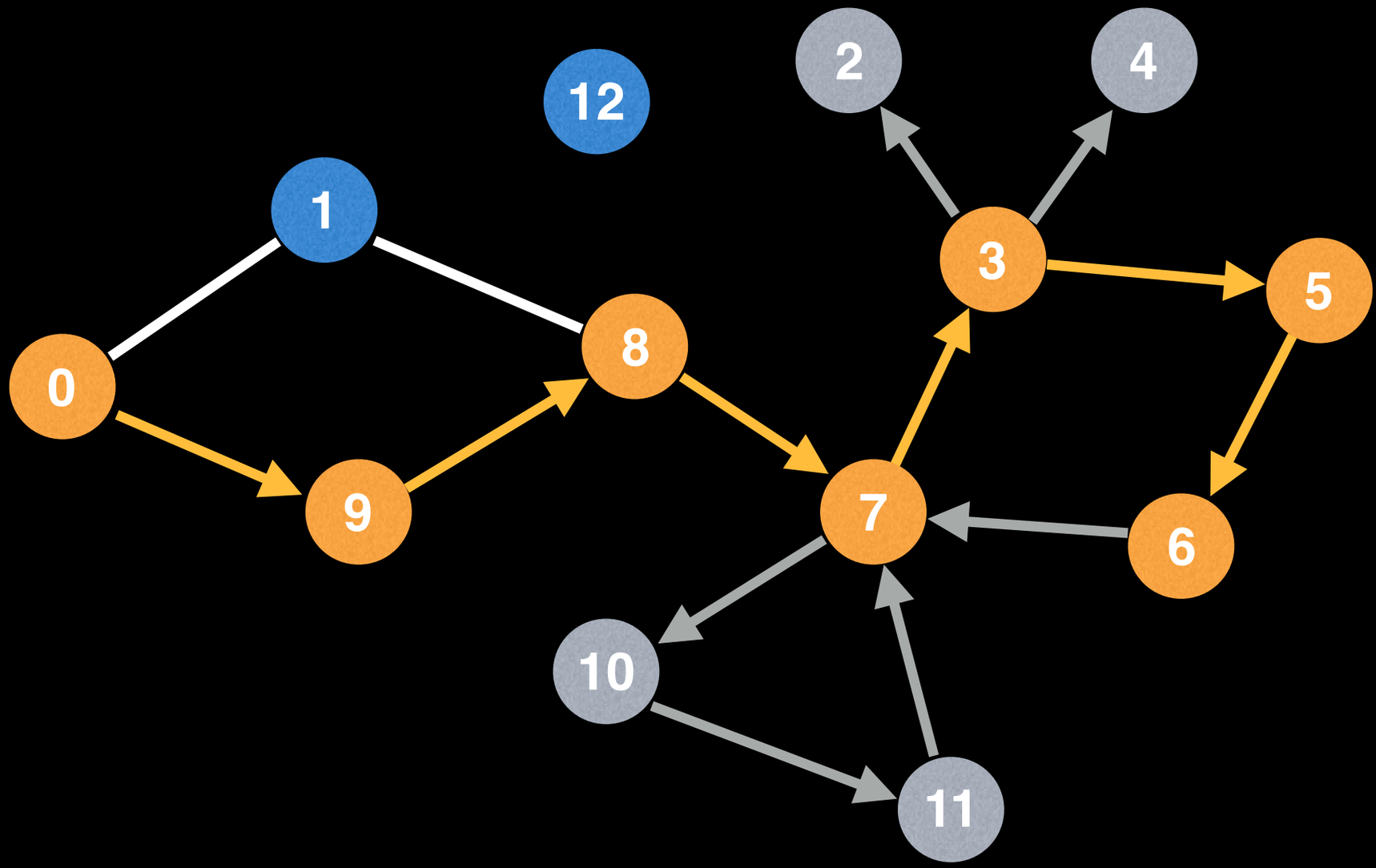

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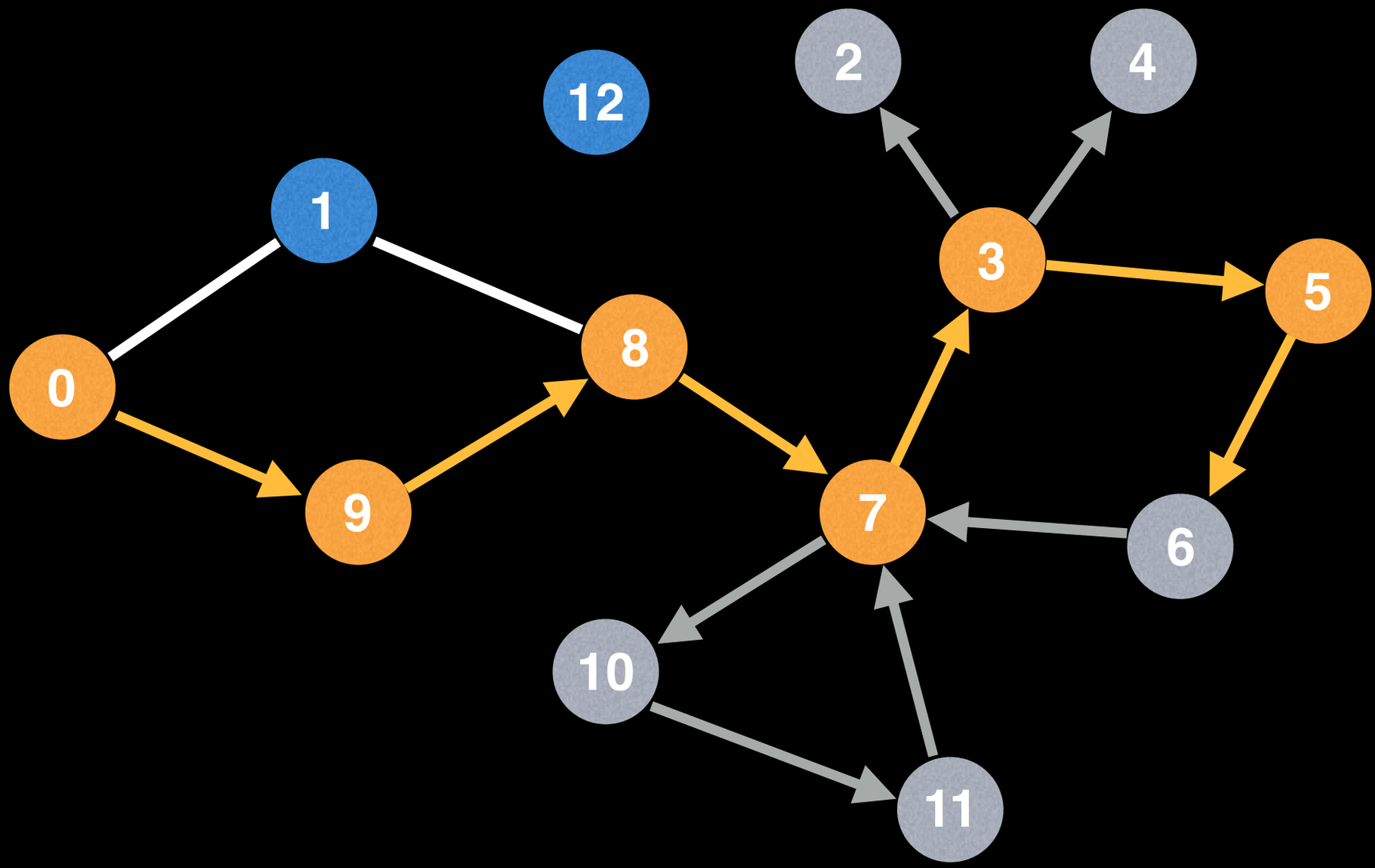

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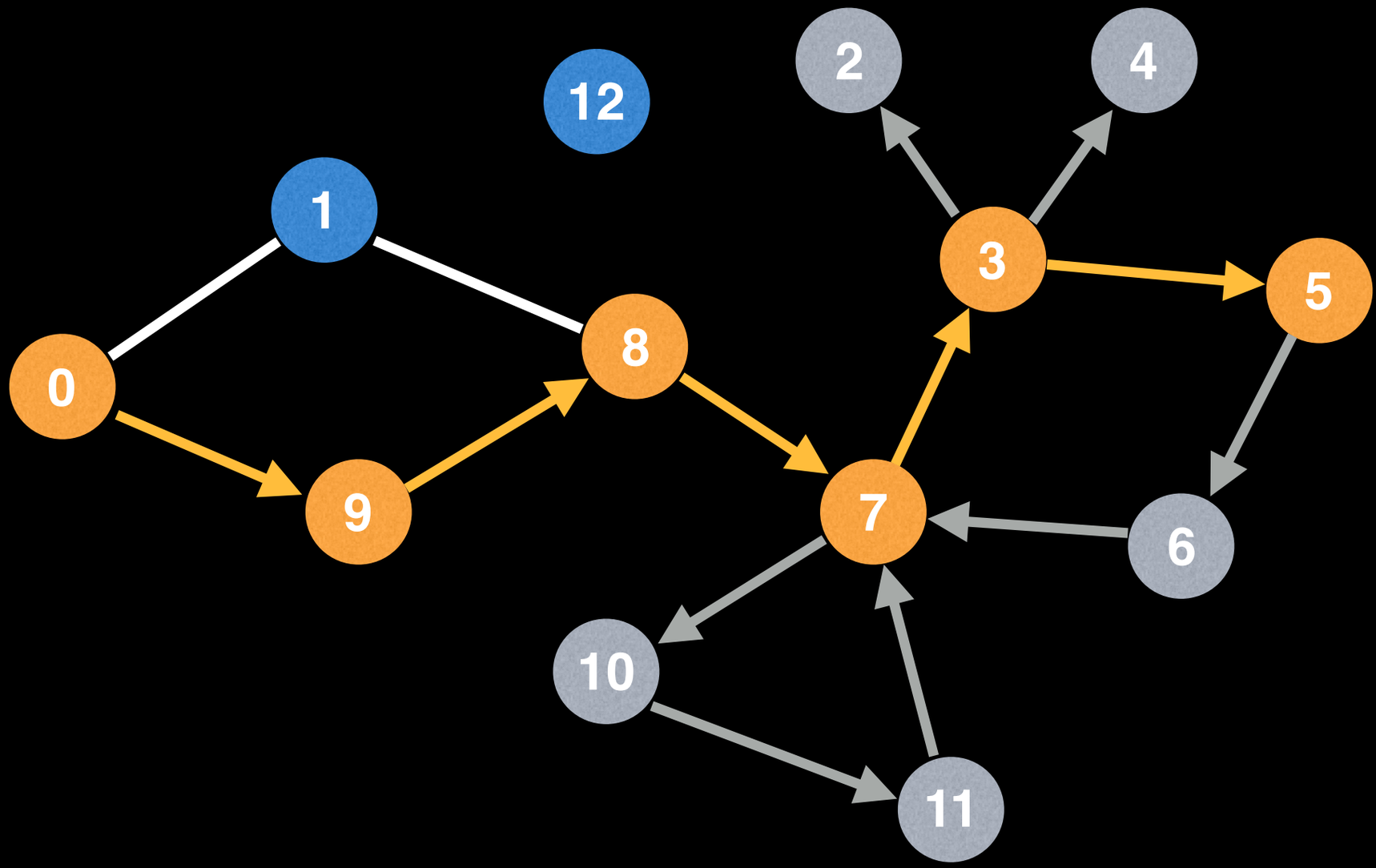

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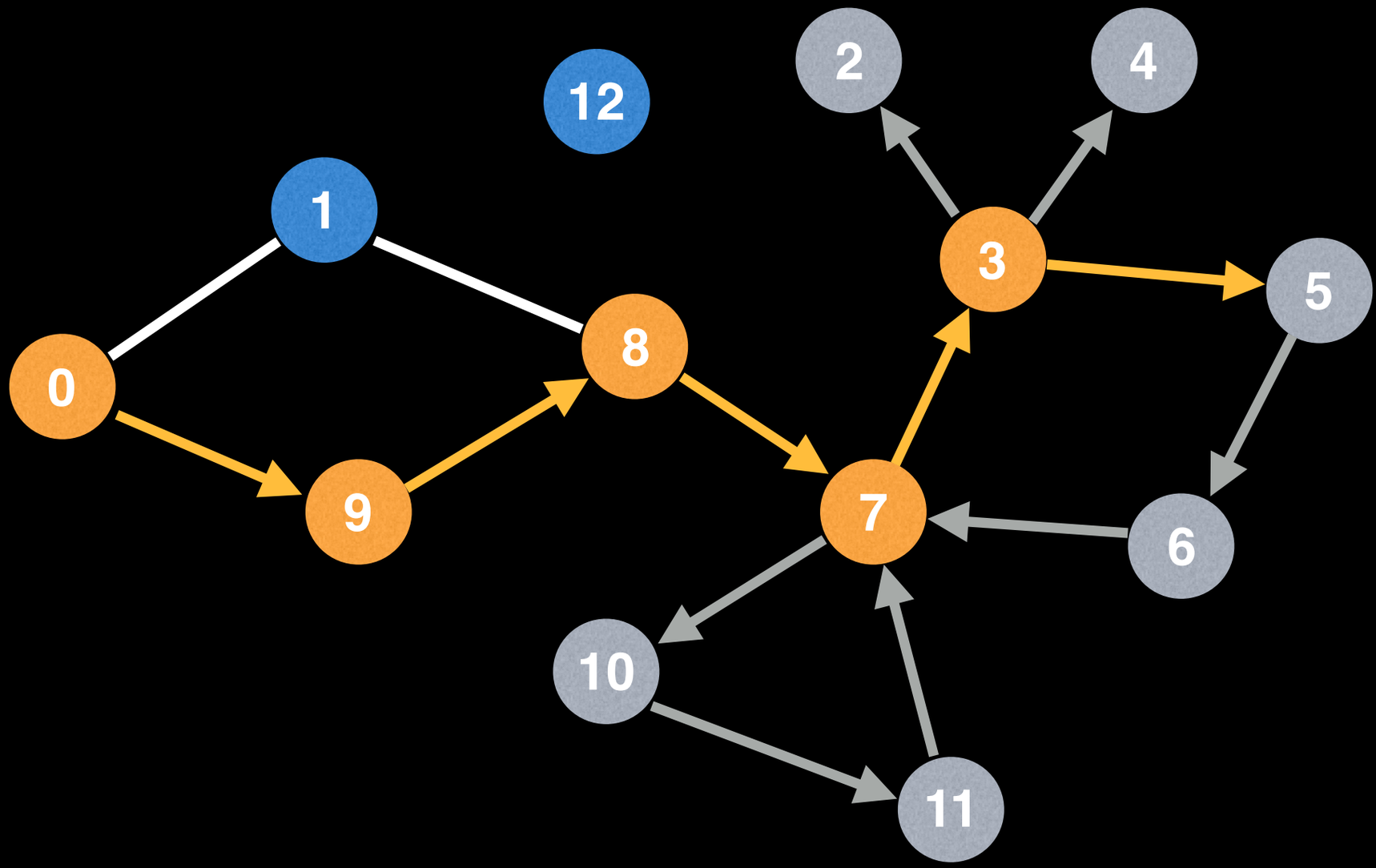

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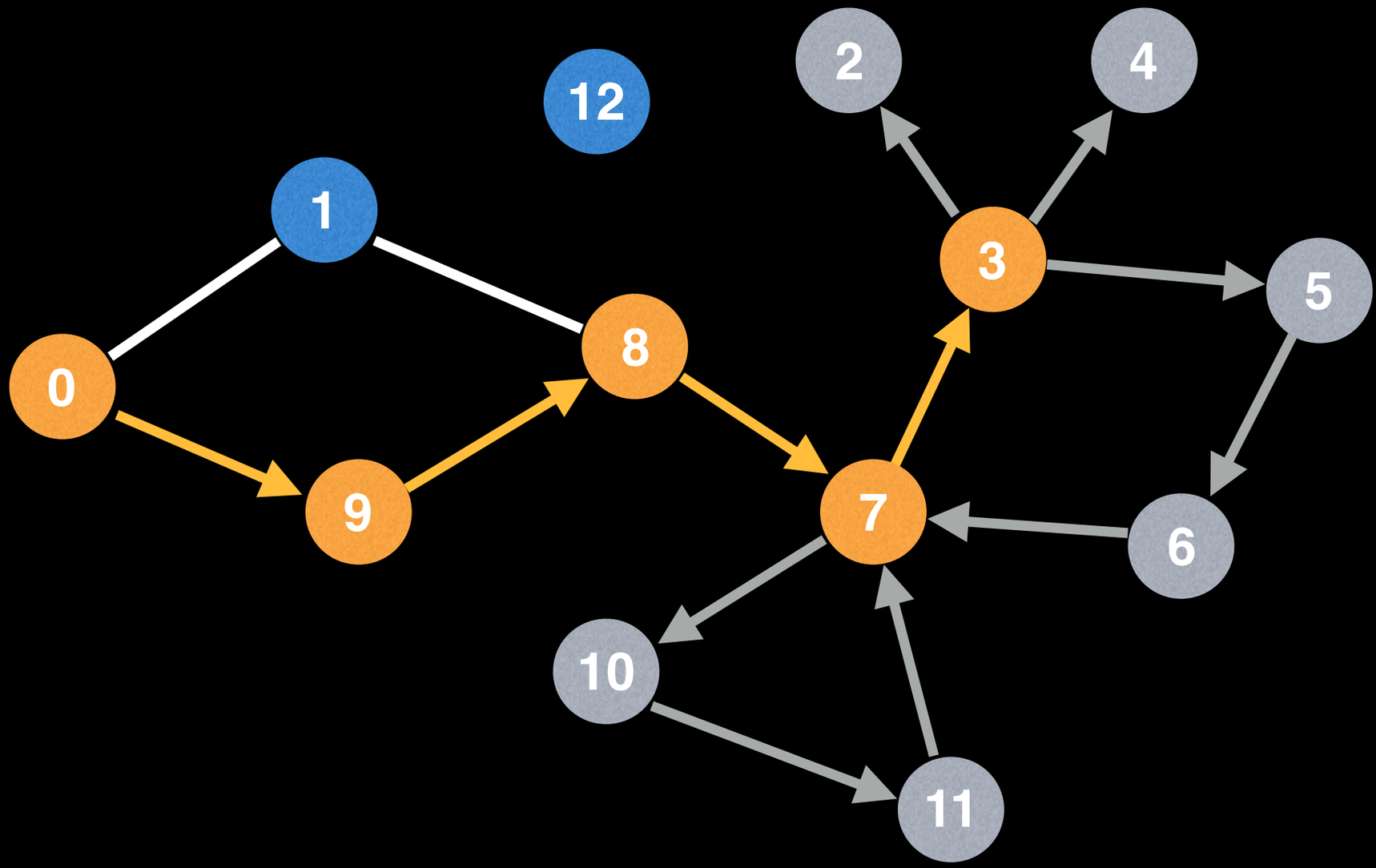

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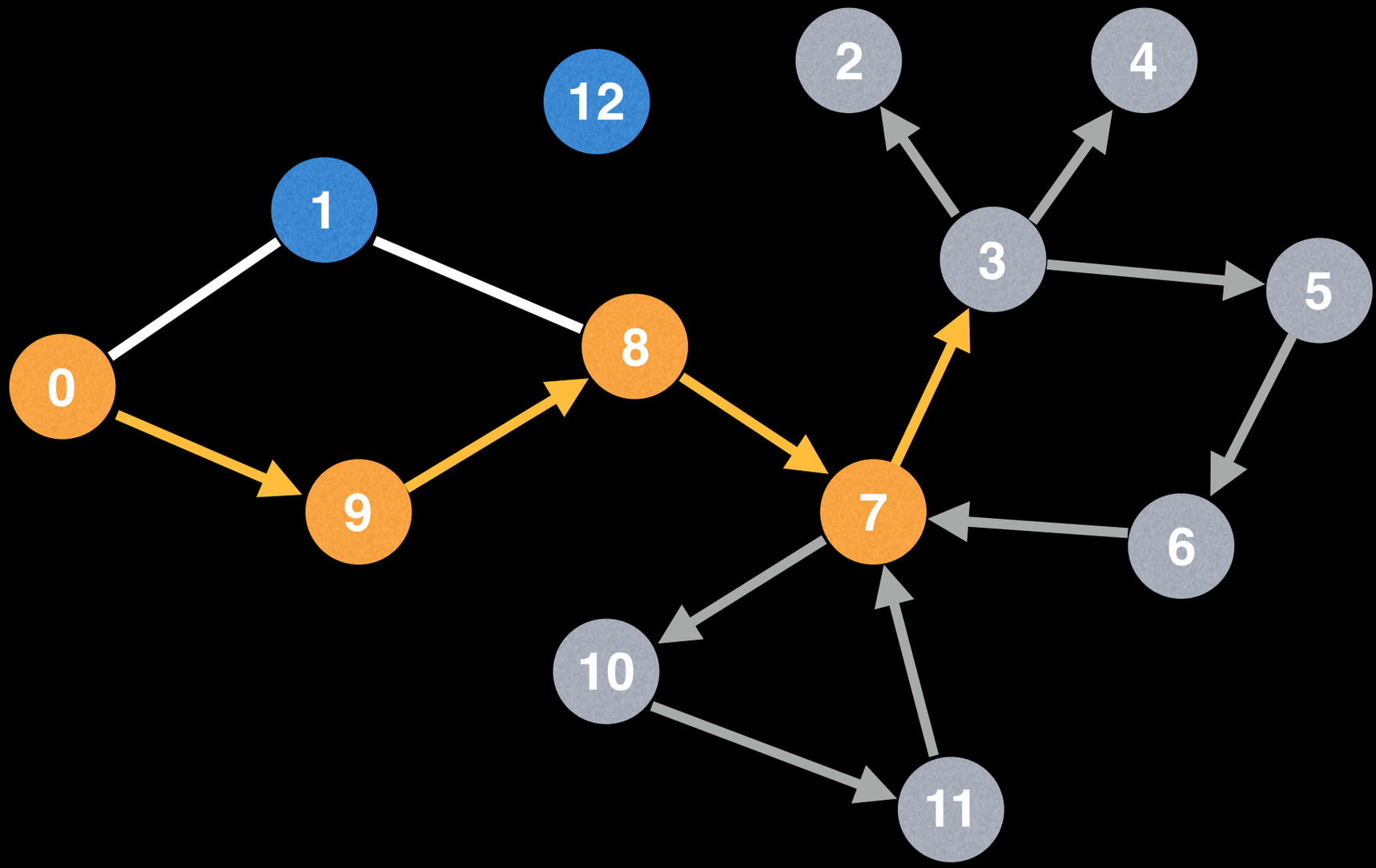

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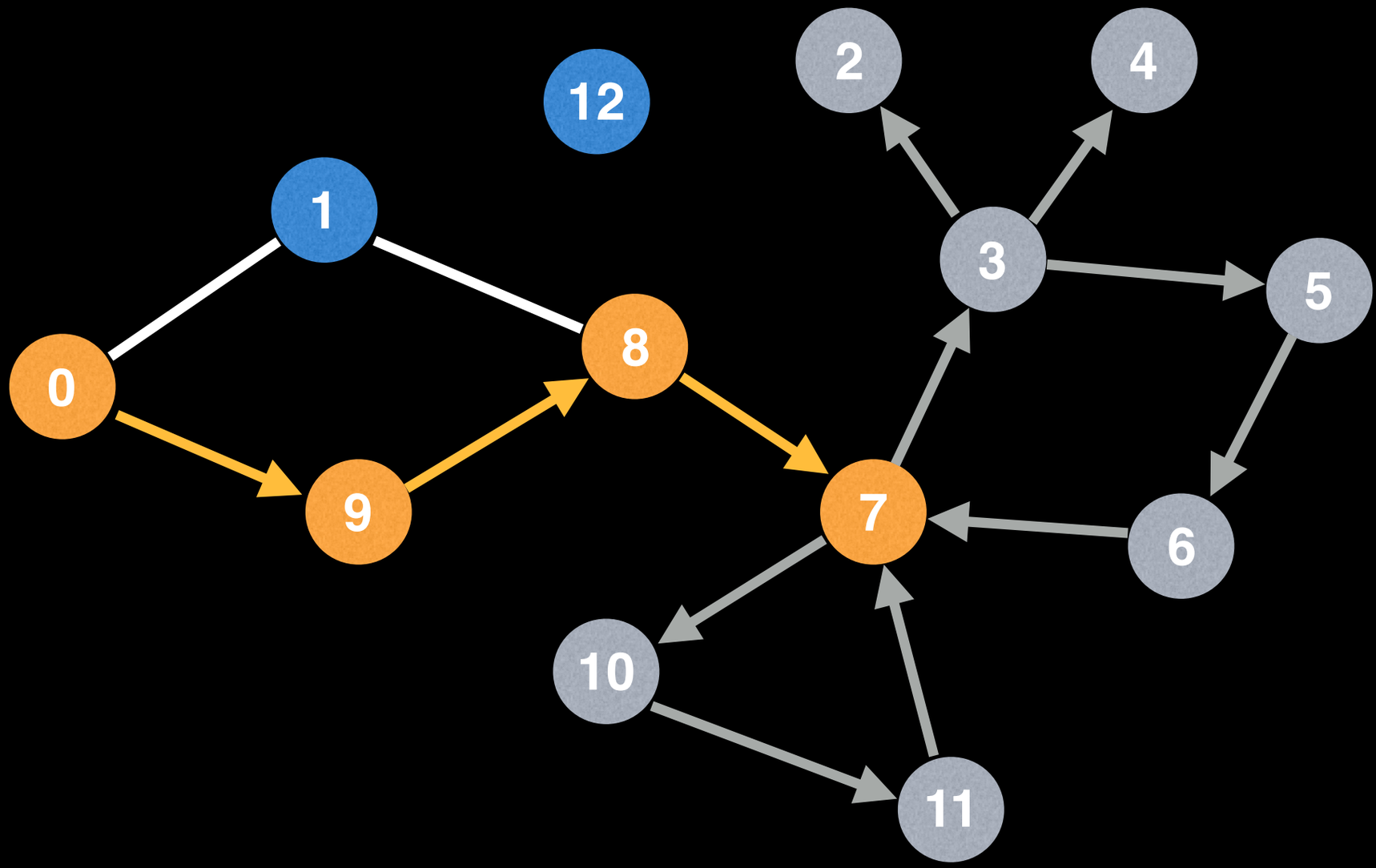

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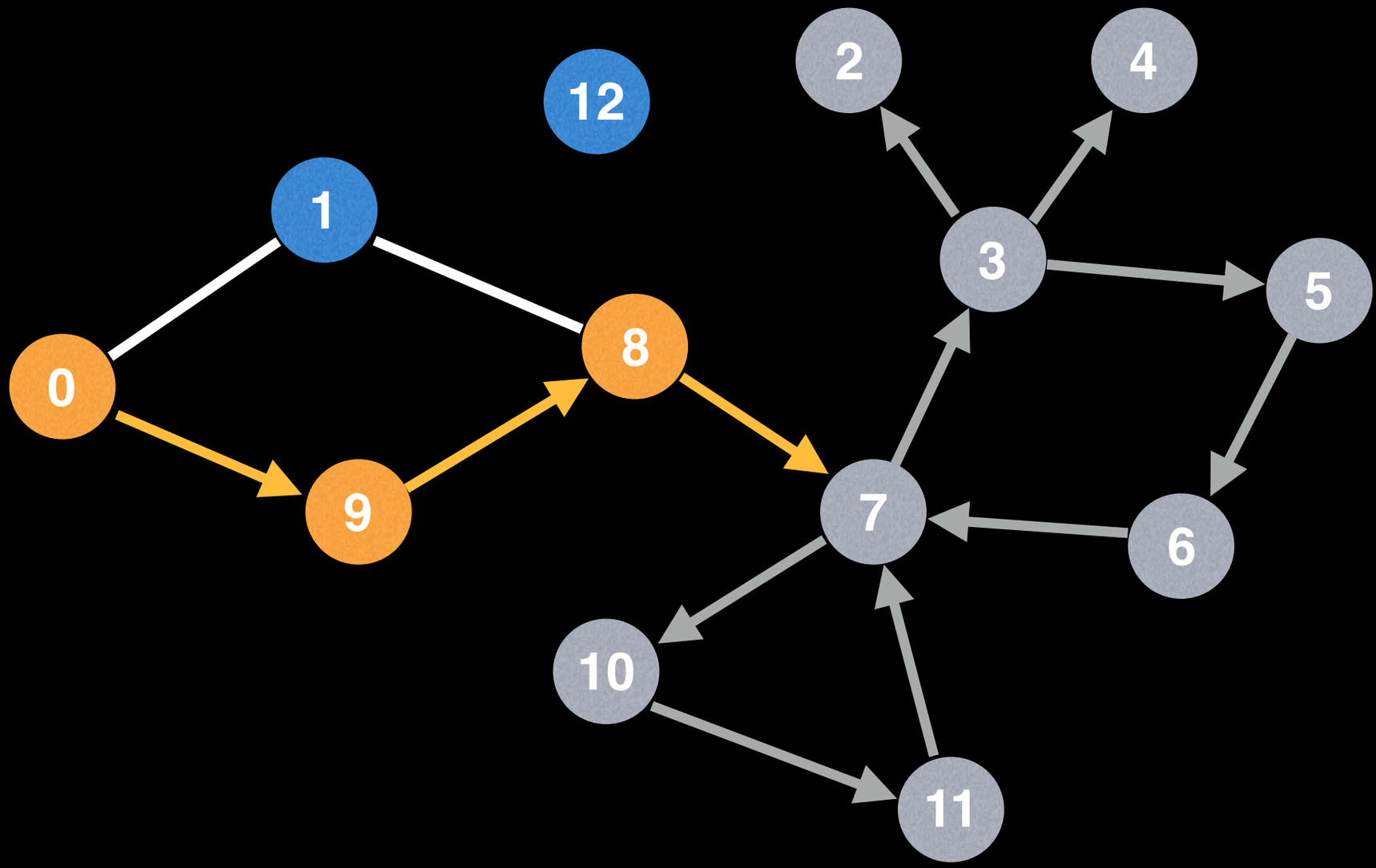

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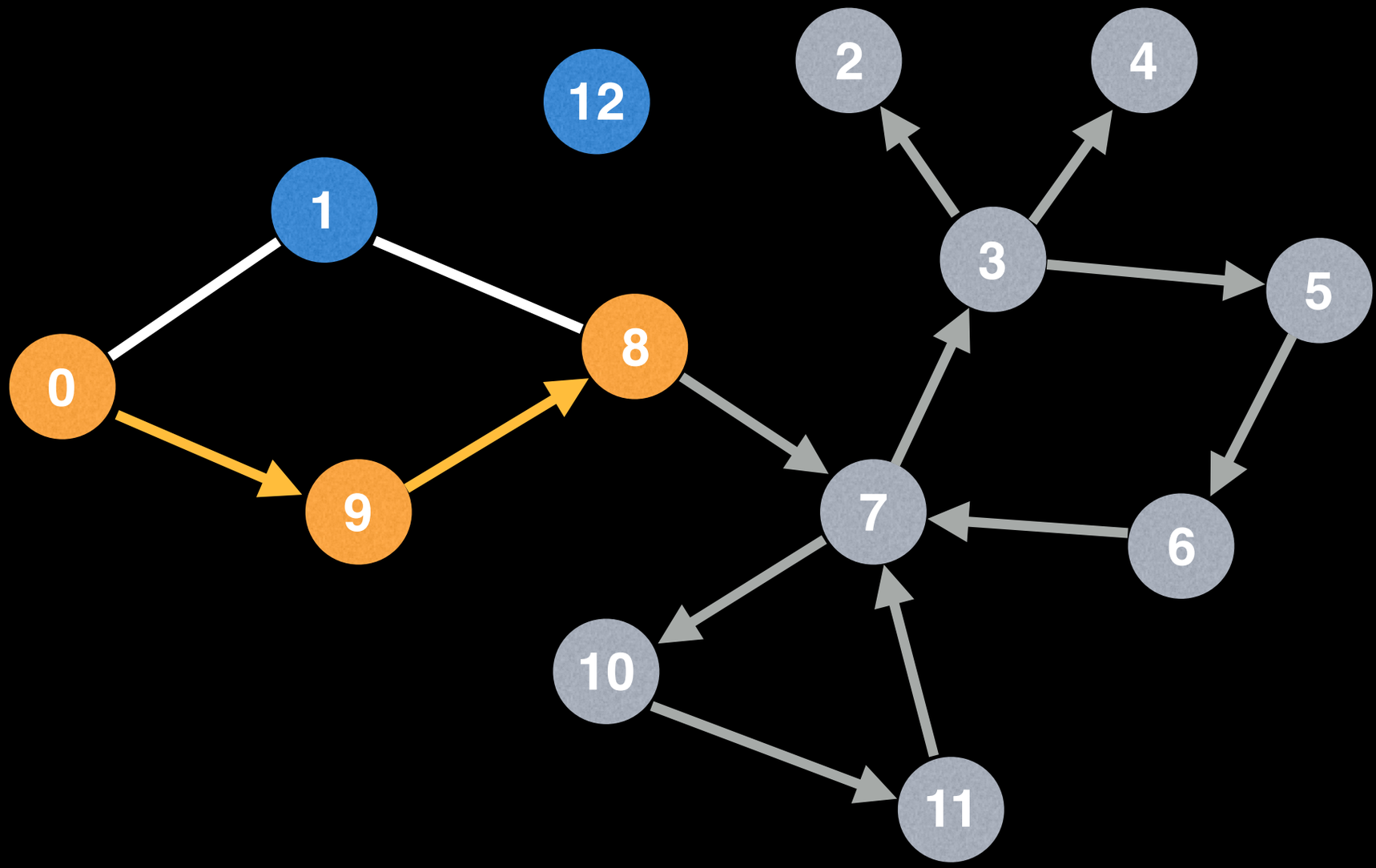

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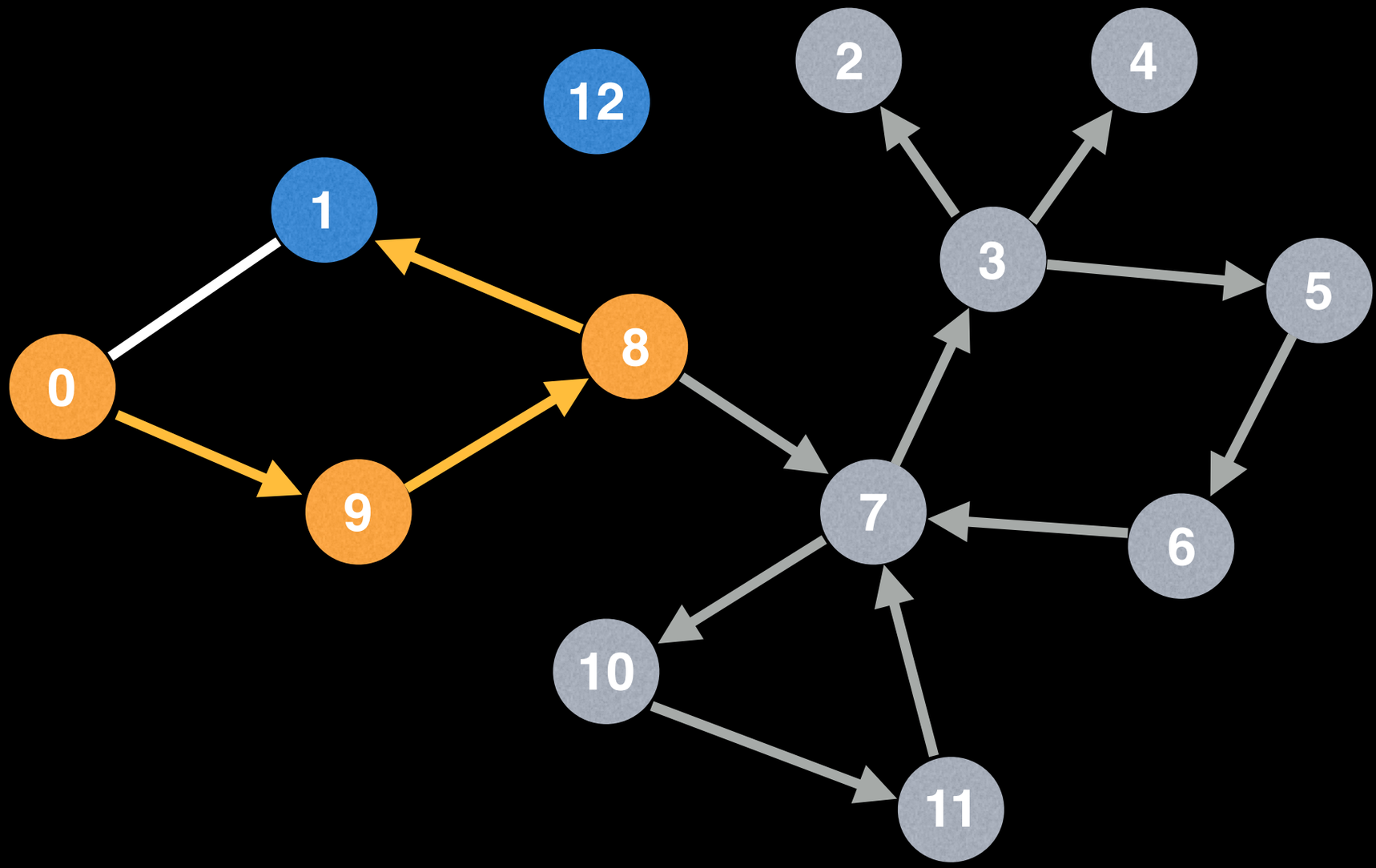

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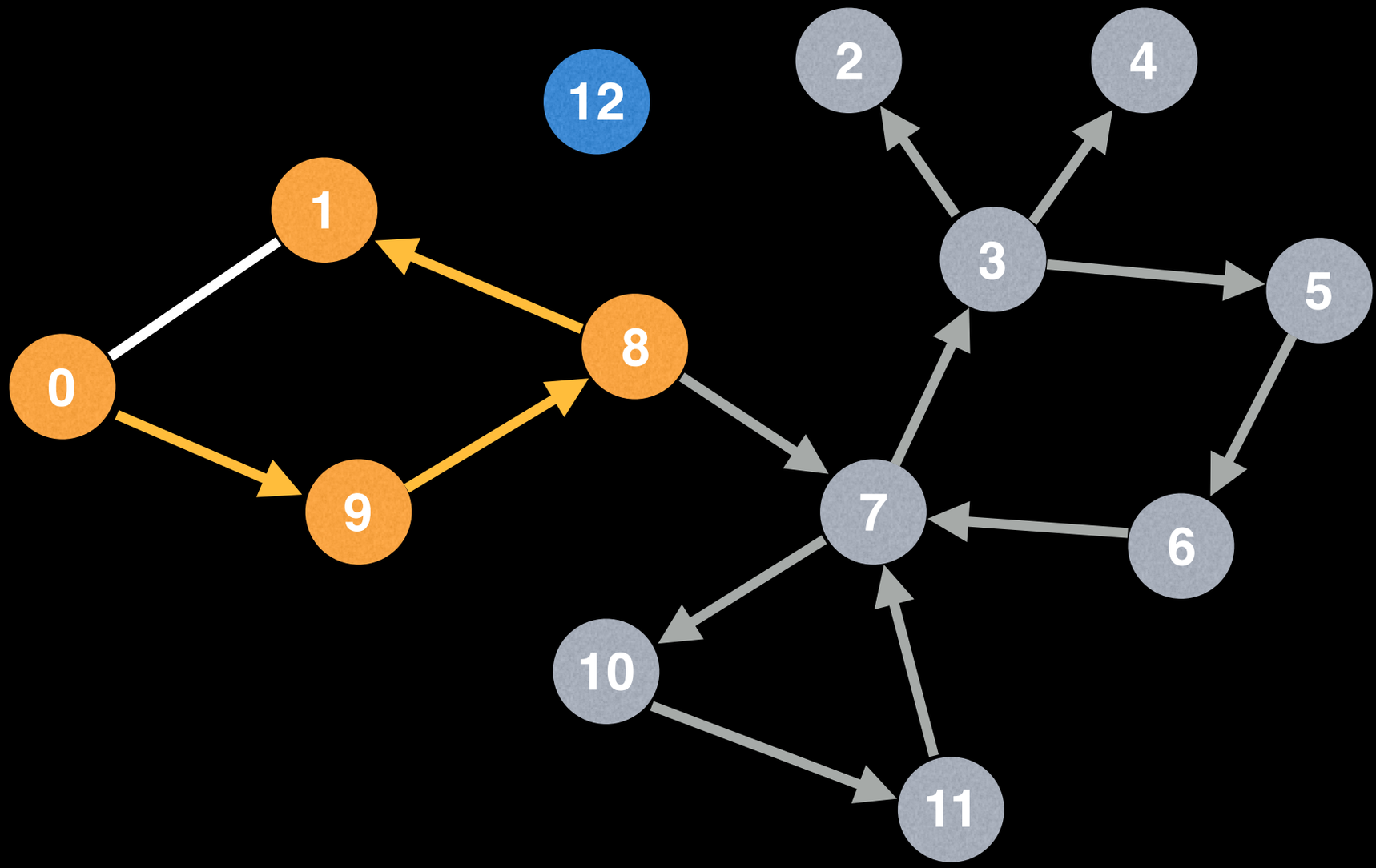

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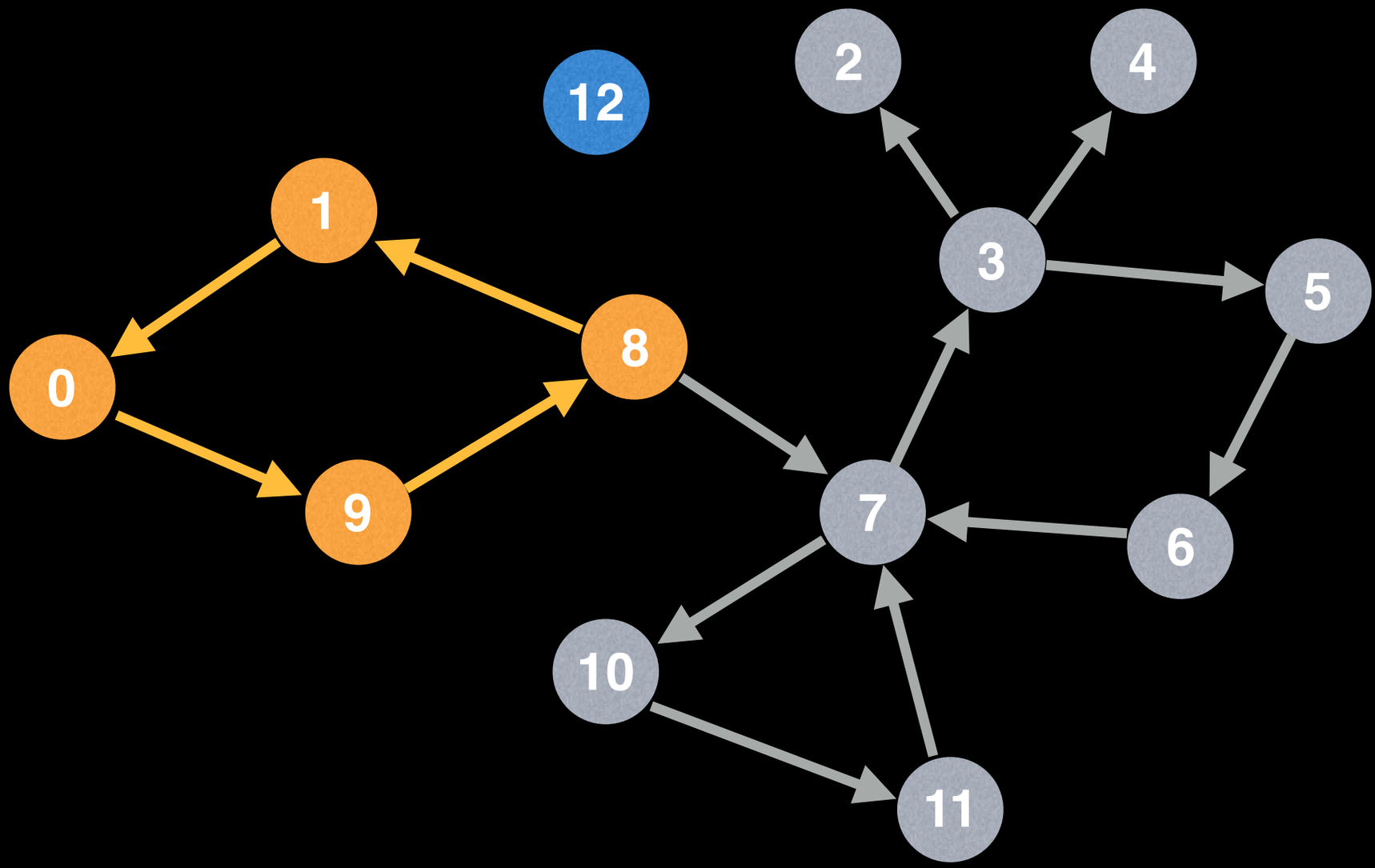

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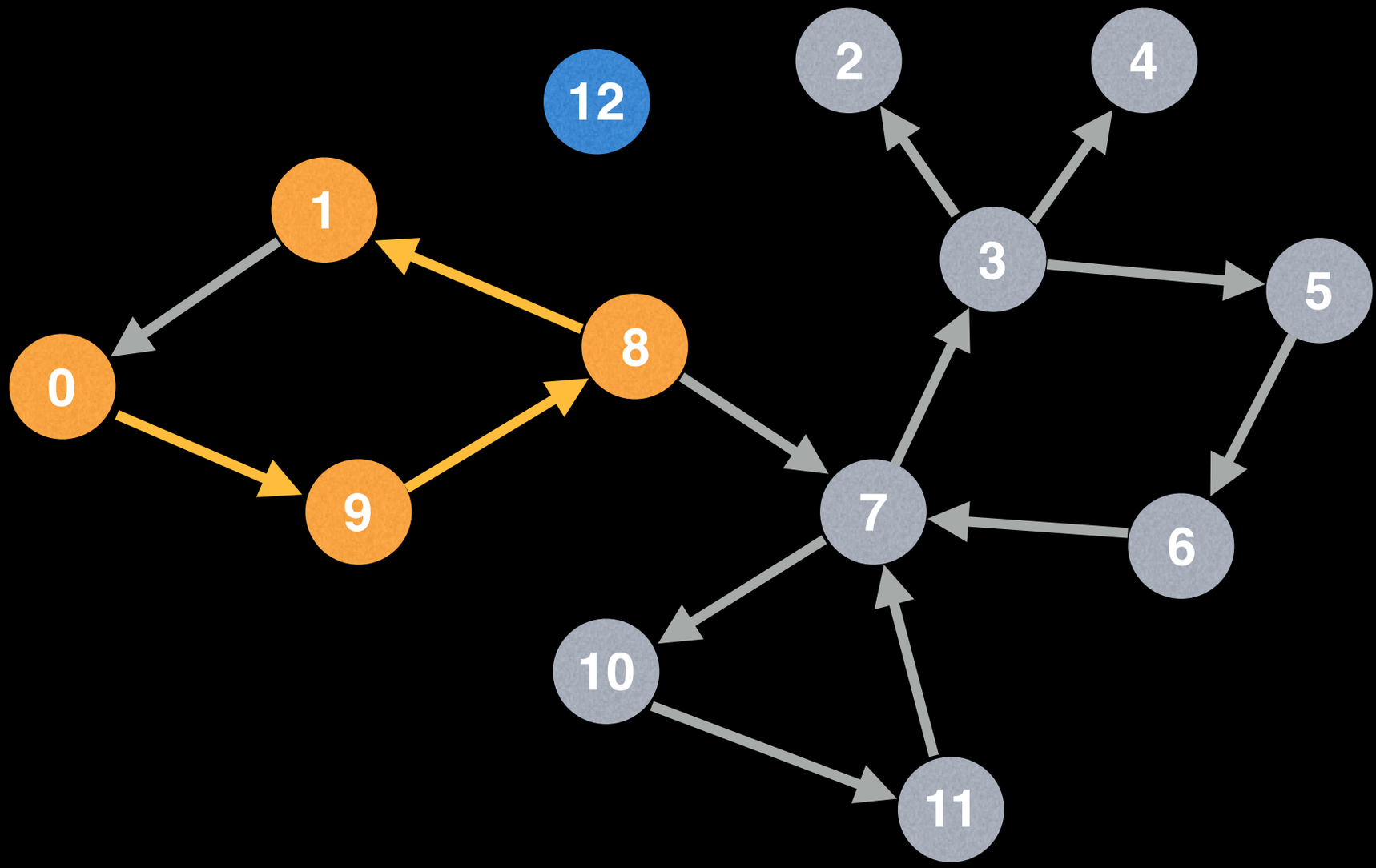

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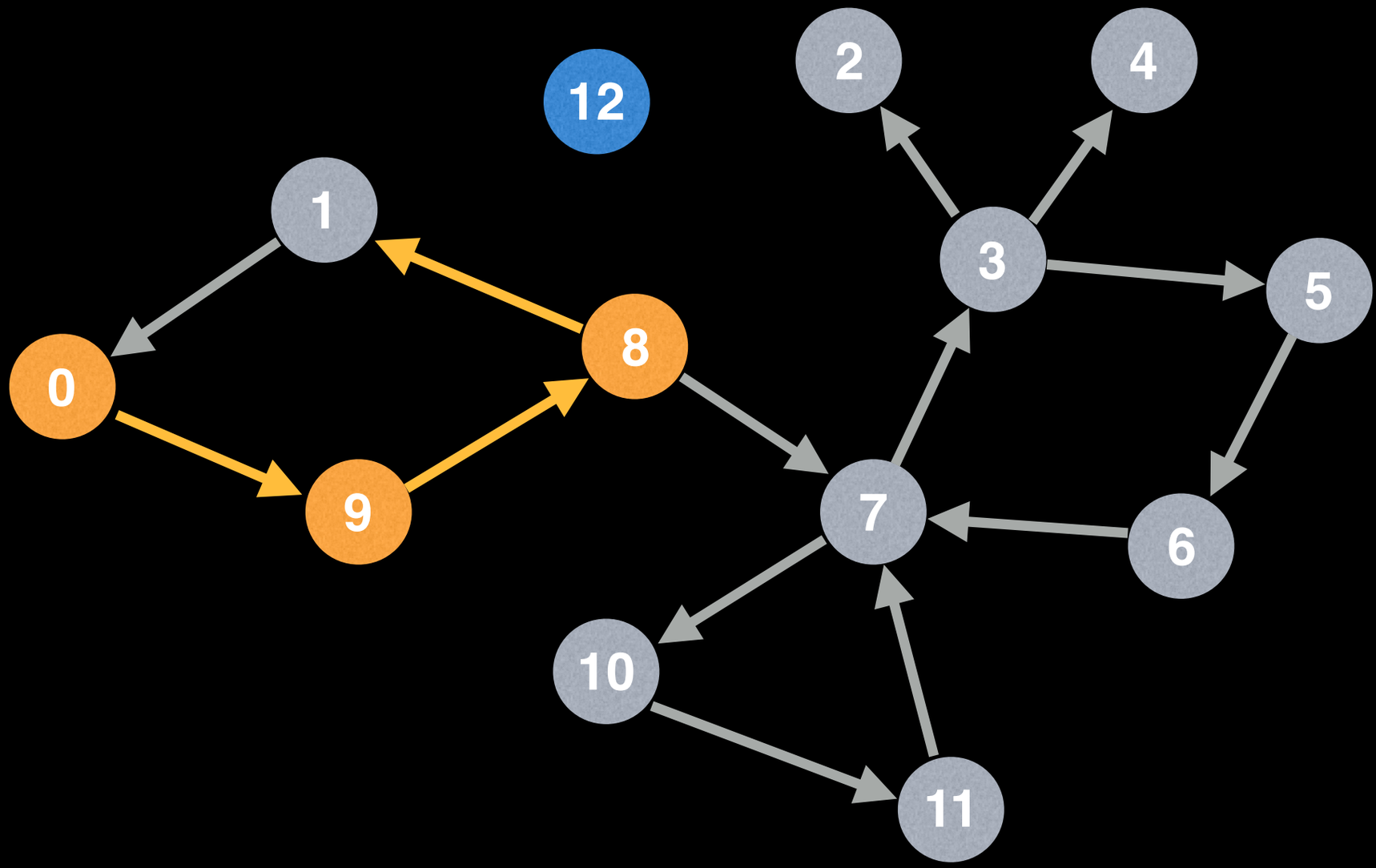
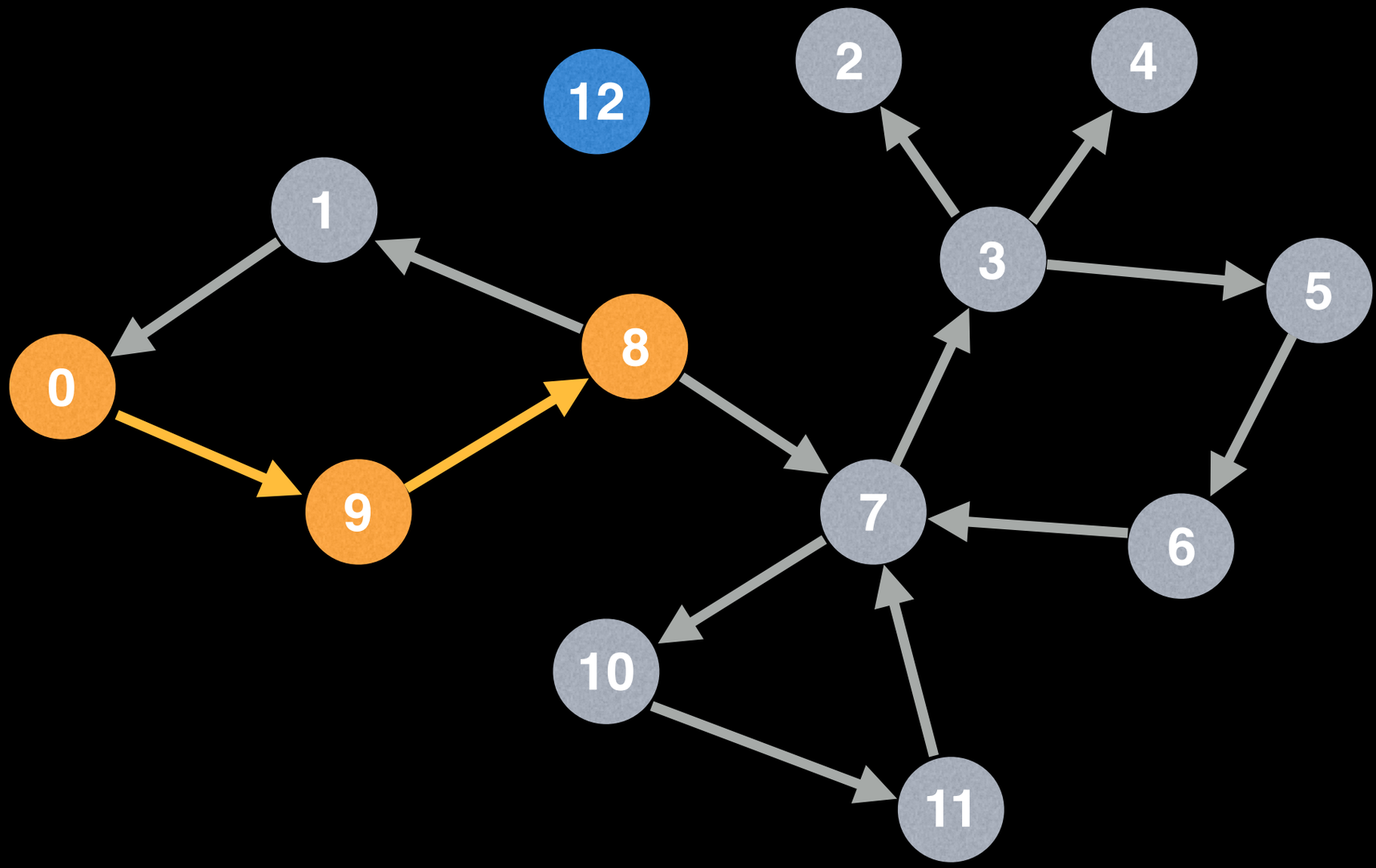


Basic DFS



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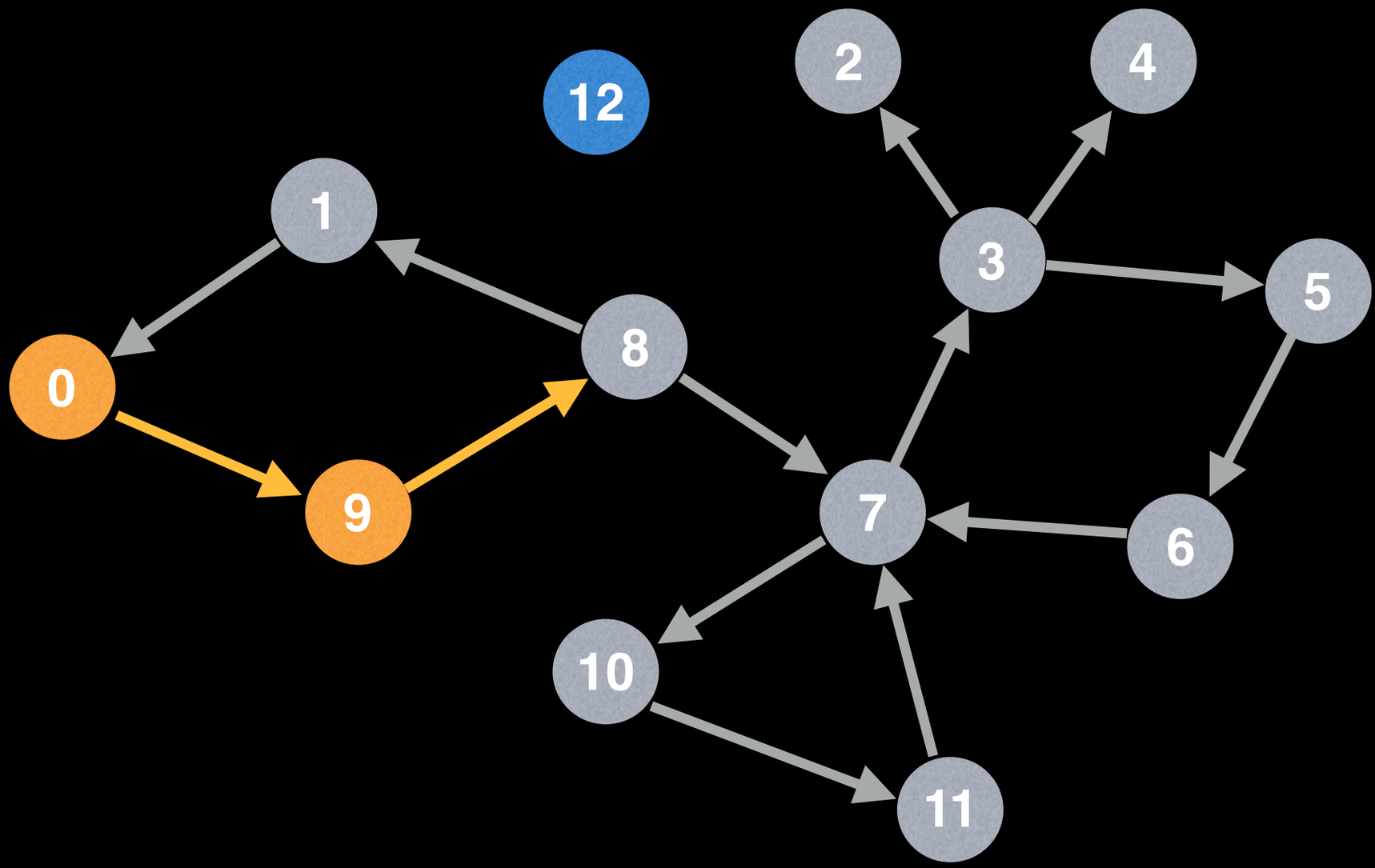

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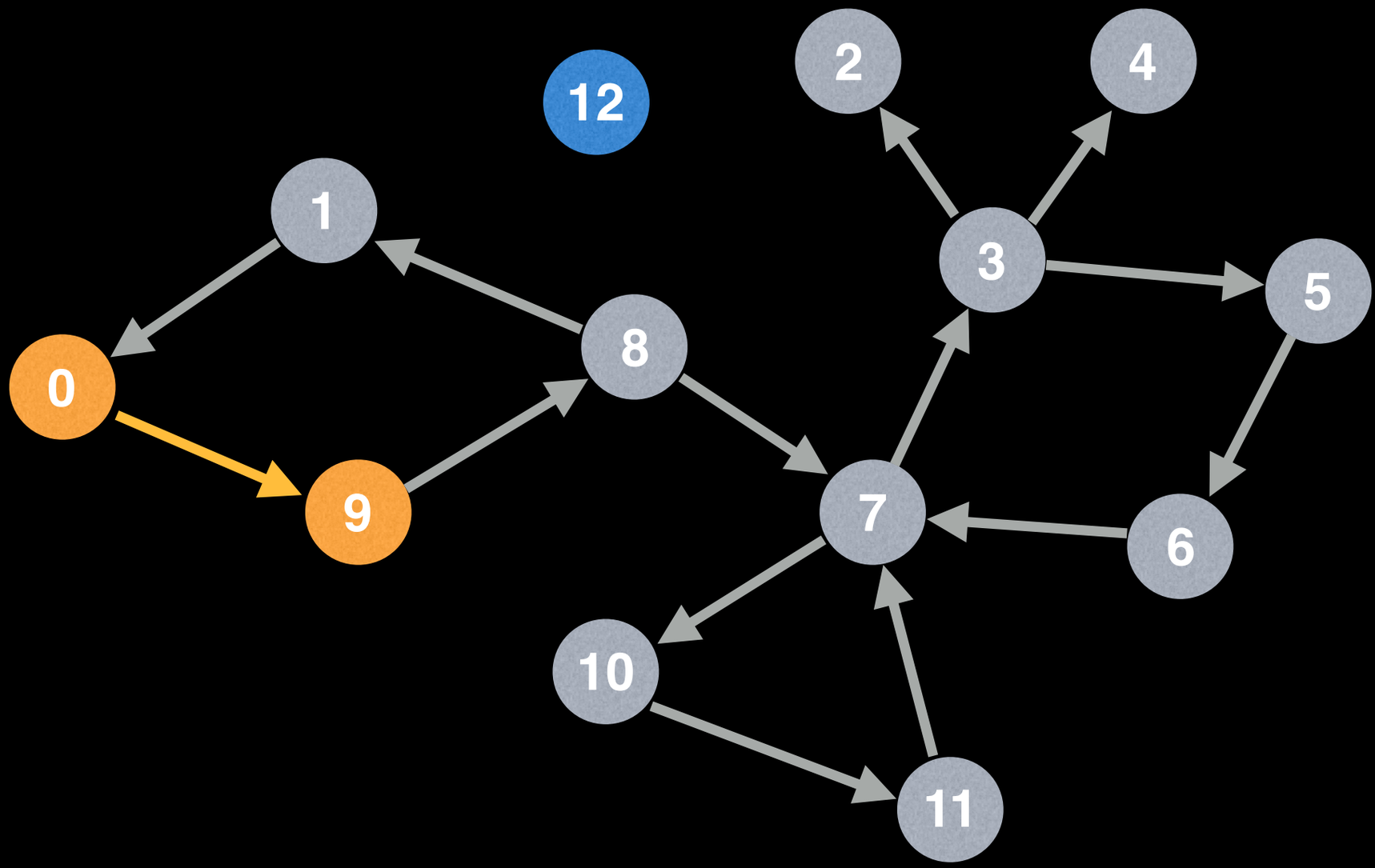

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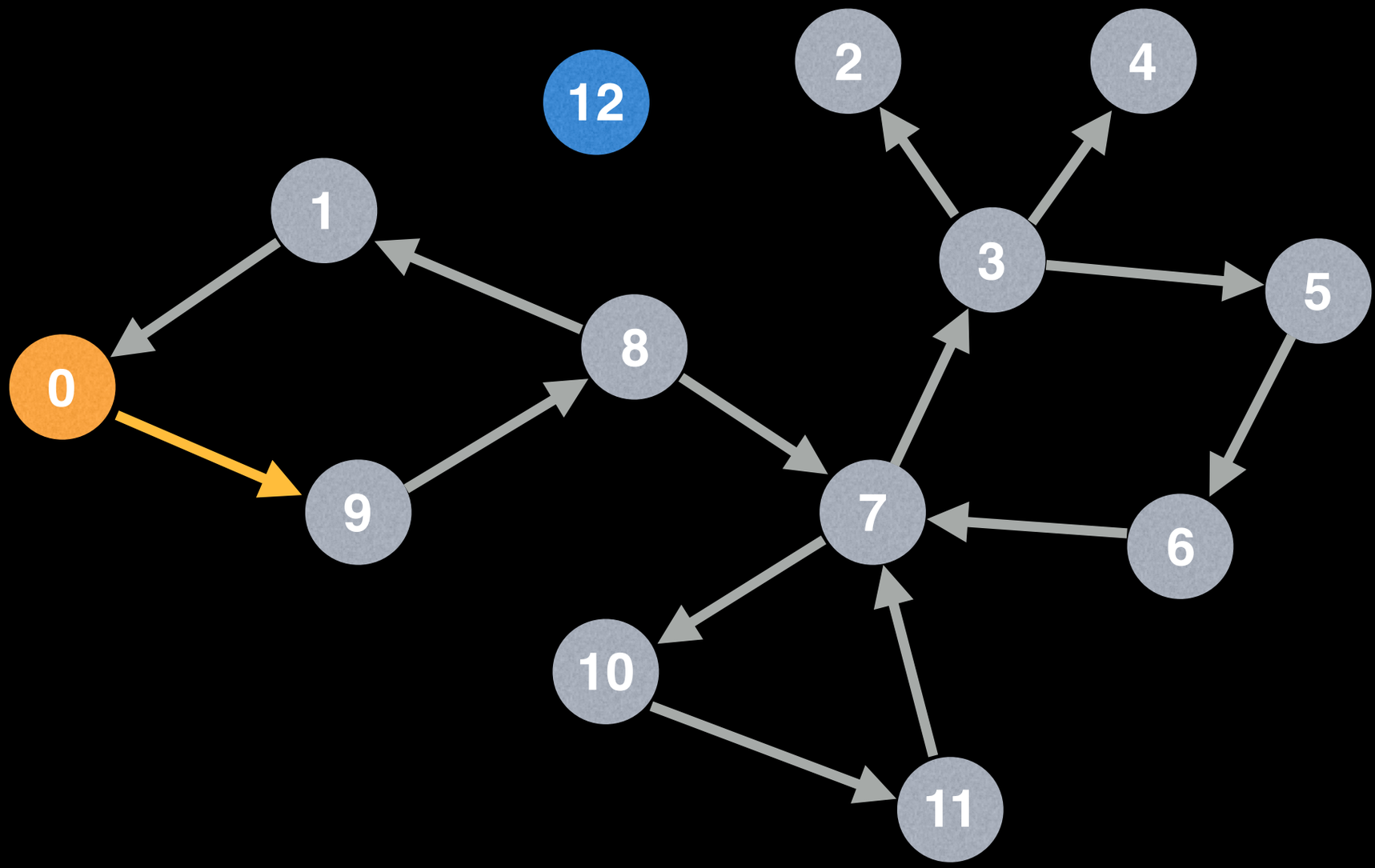

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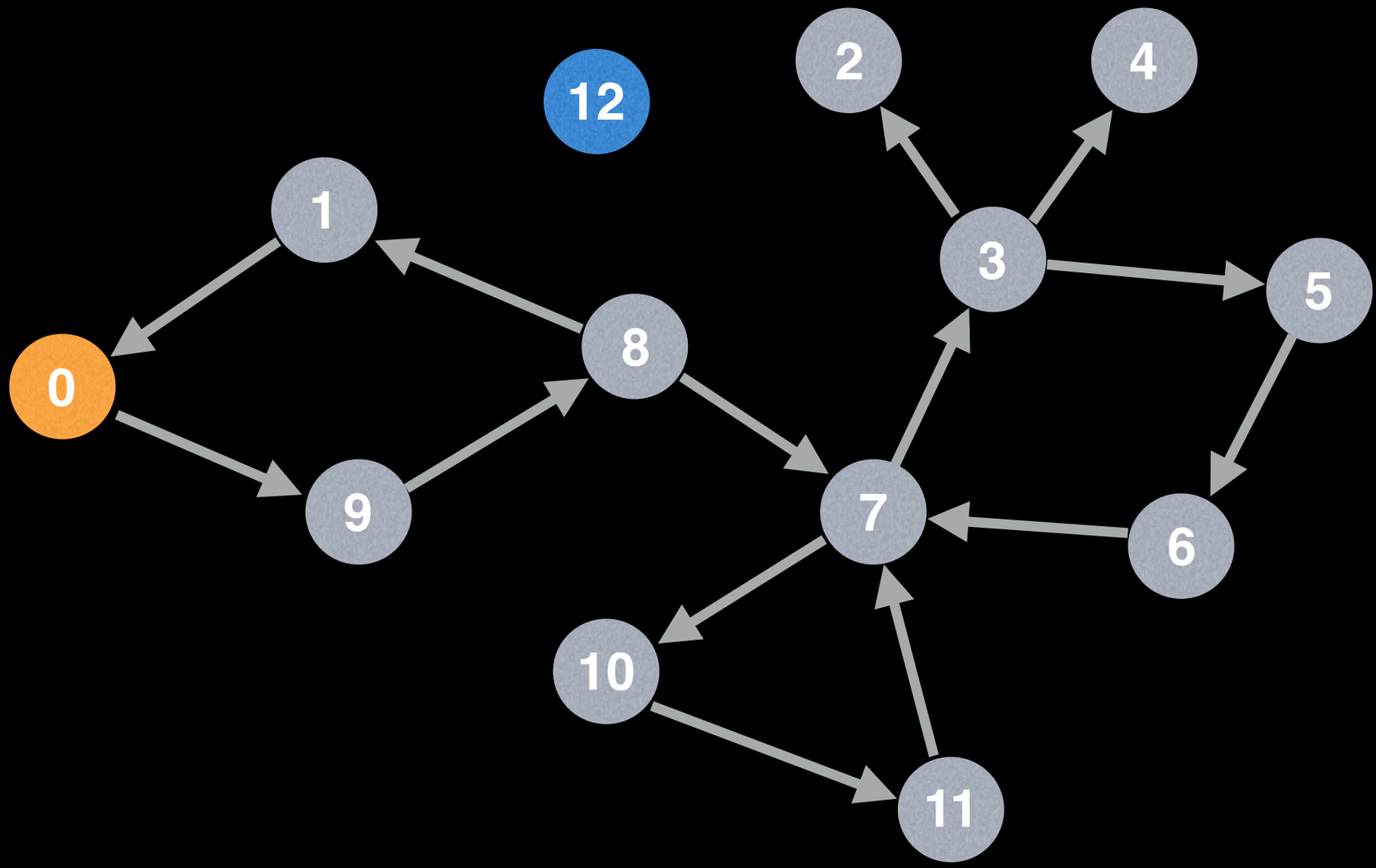

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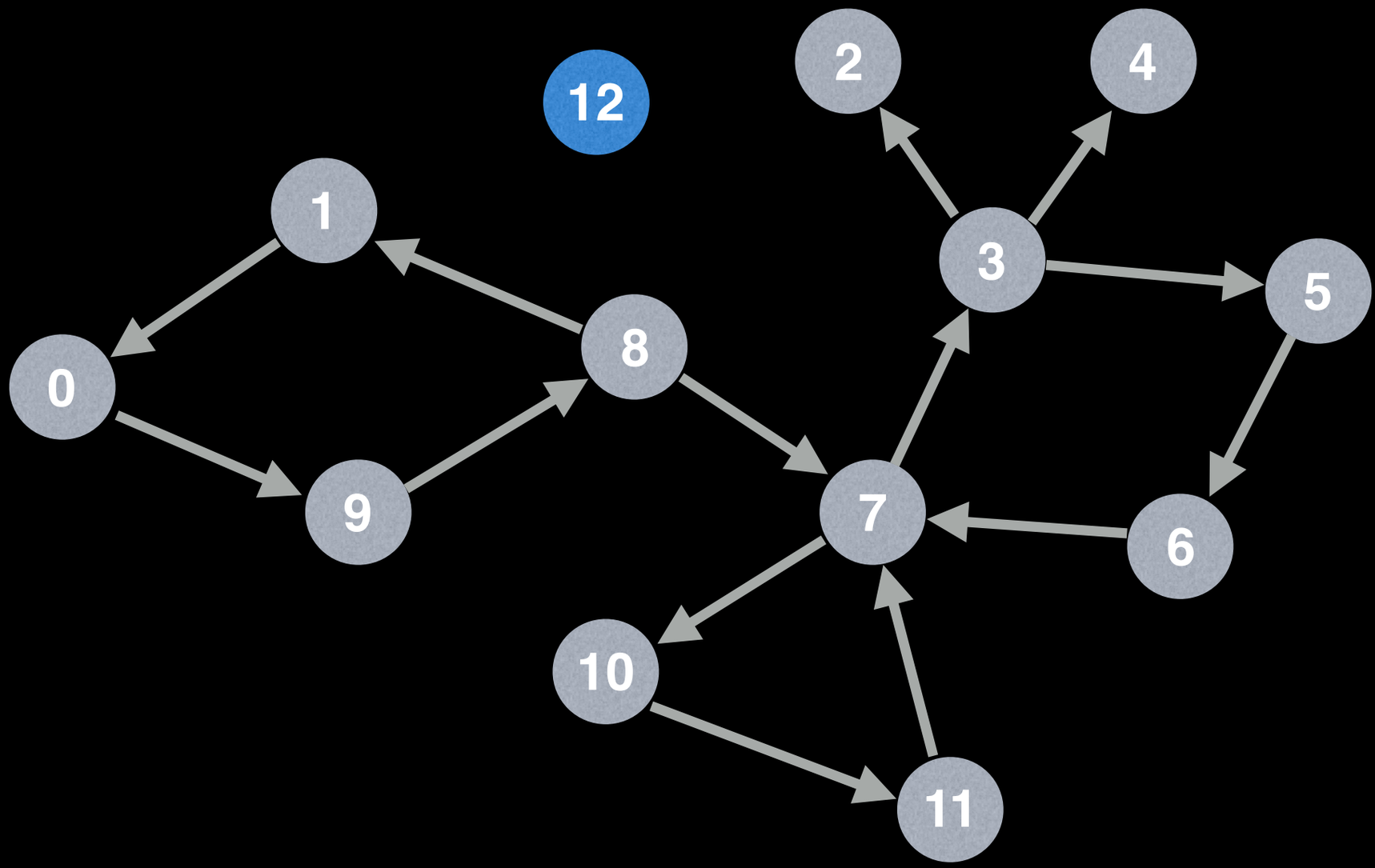

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n = number of nodes in the graph
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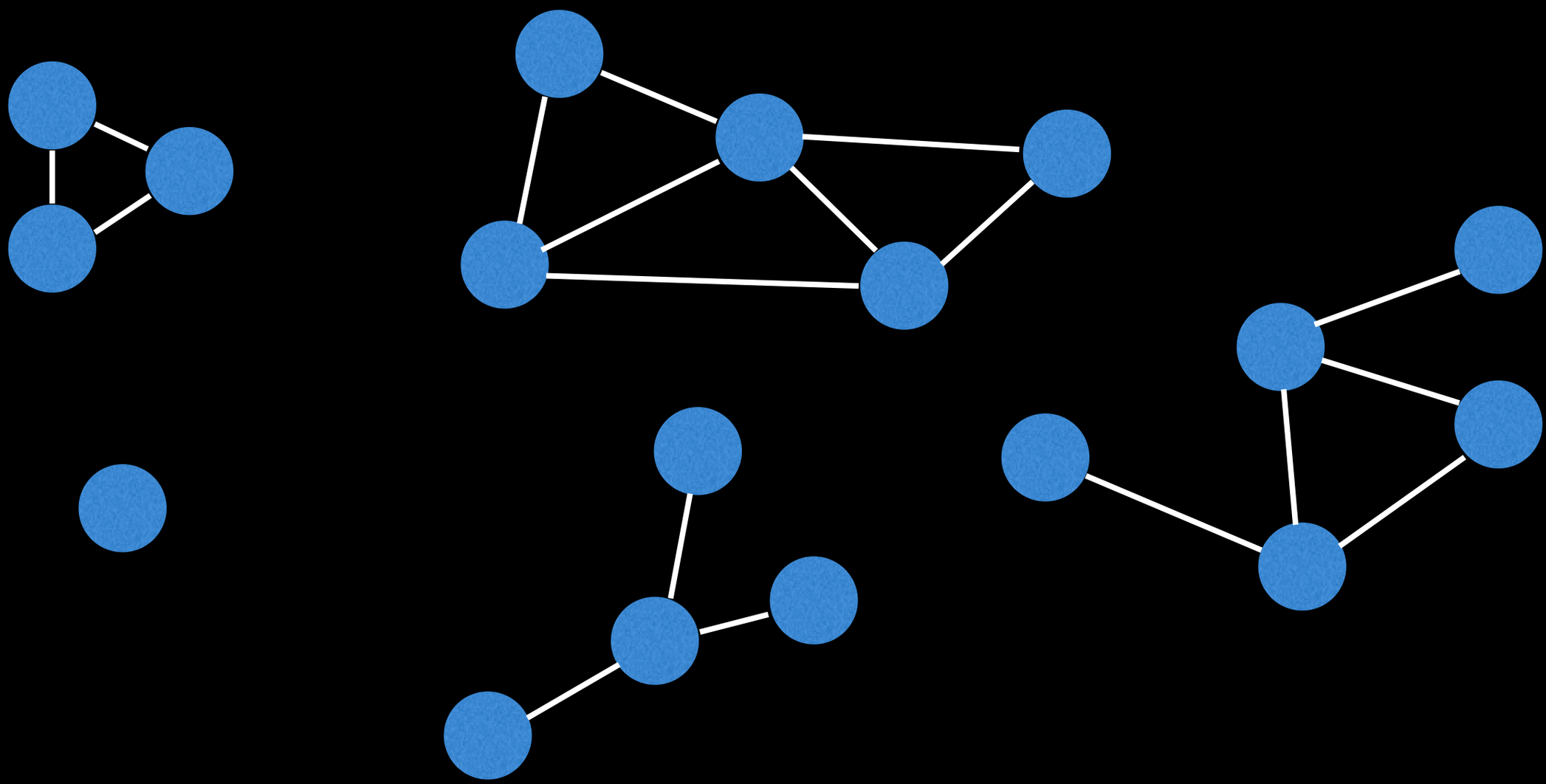

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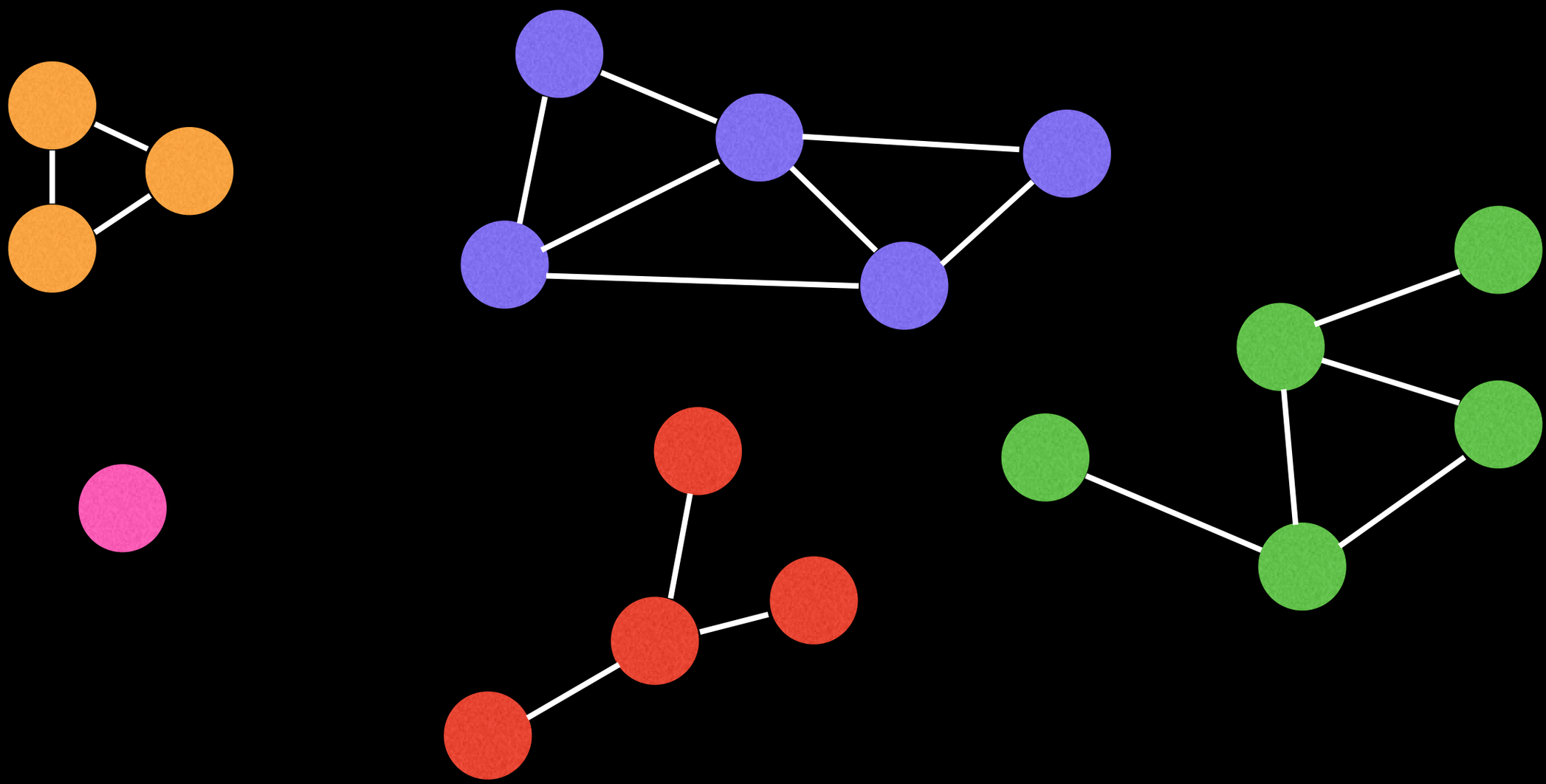
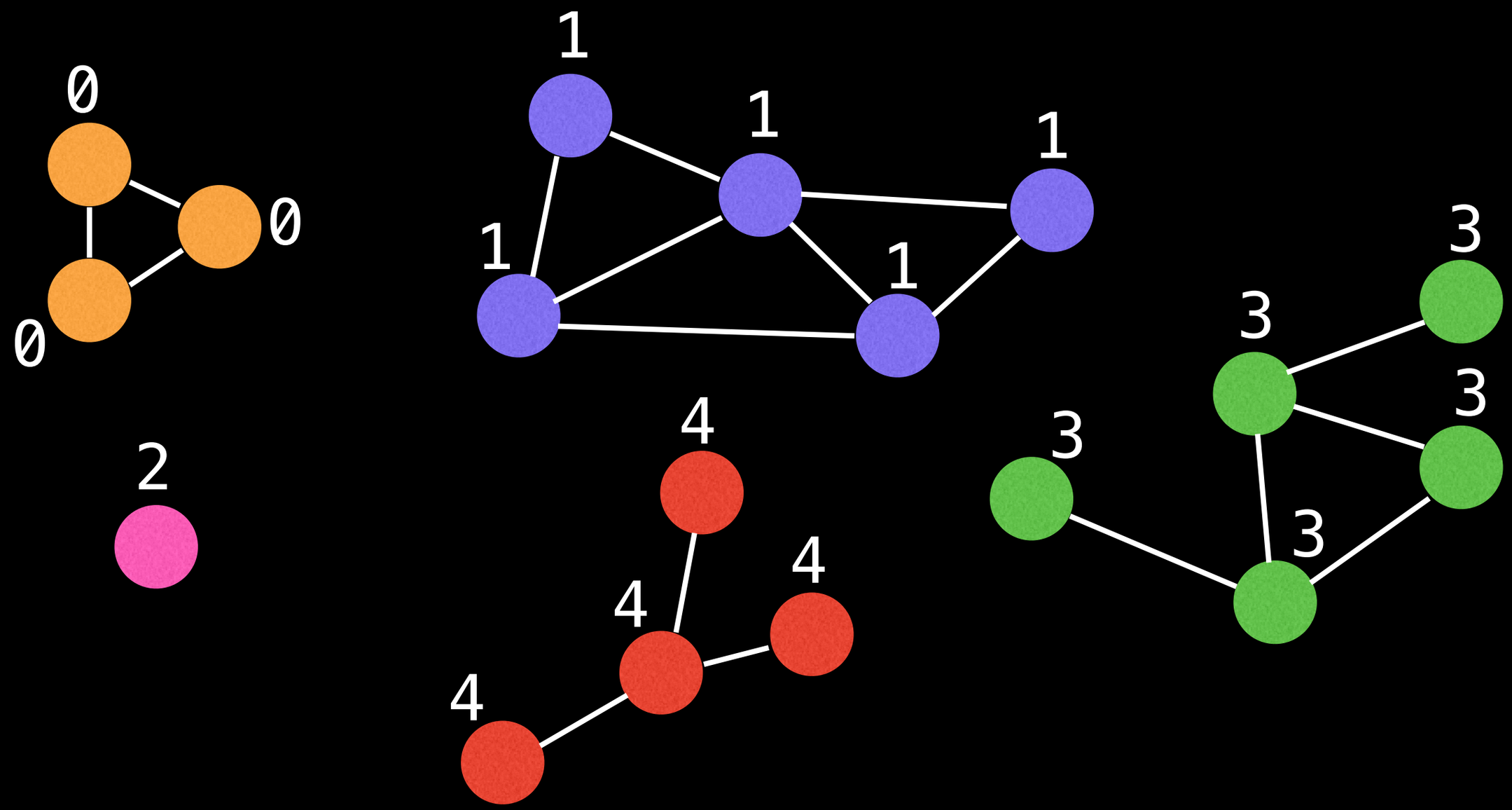
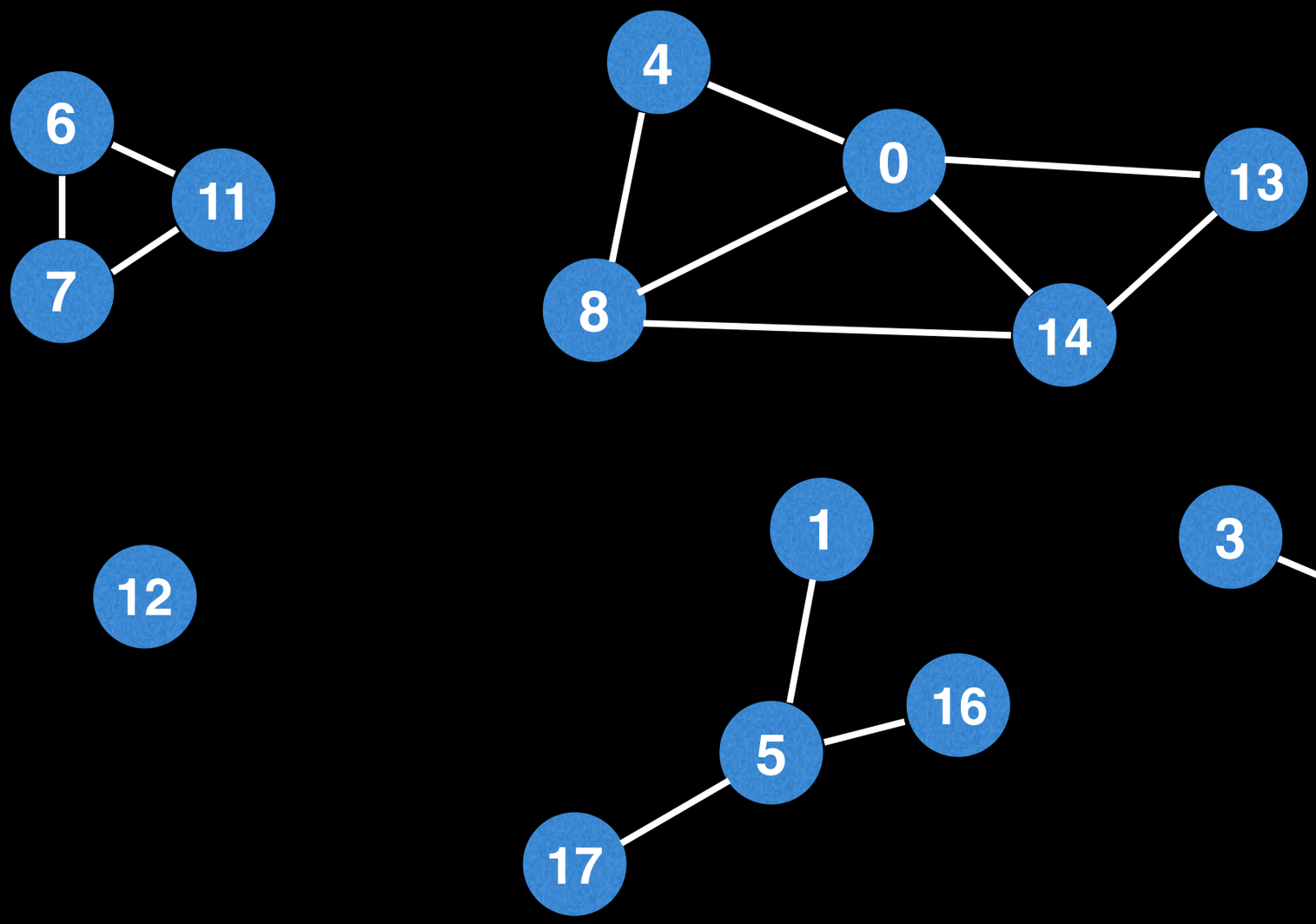


Connected Components

Assign an integer value to each group to be able to tell them apart.



We can use a DFS to identify components. First, make sure all the nodes are labeled from [0, n) where n is the number of nodes.



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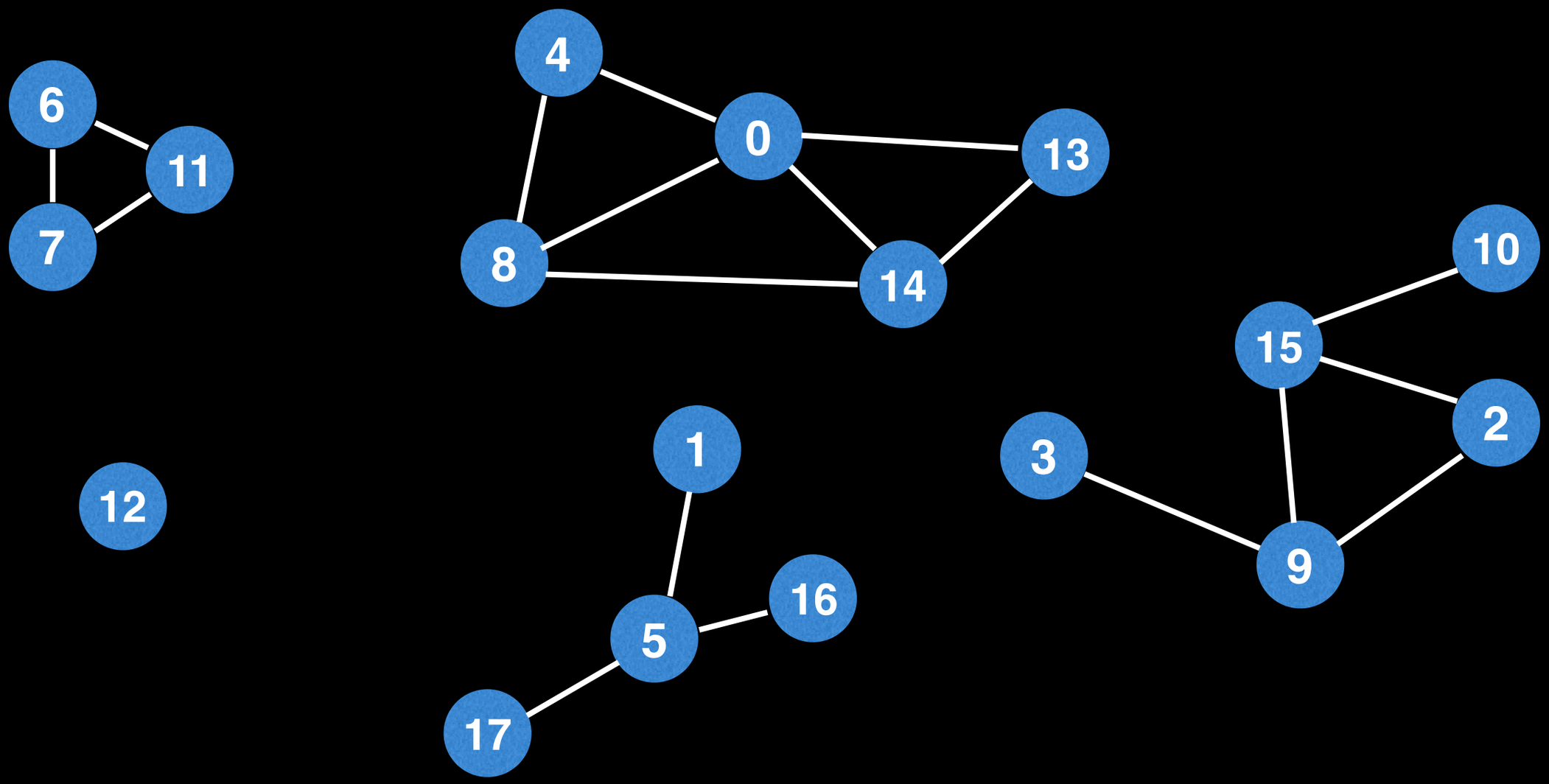

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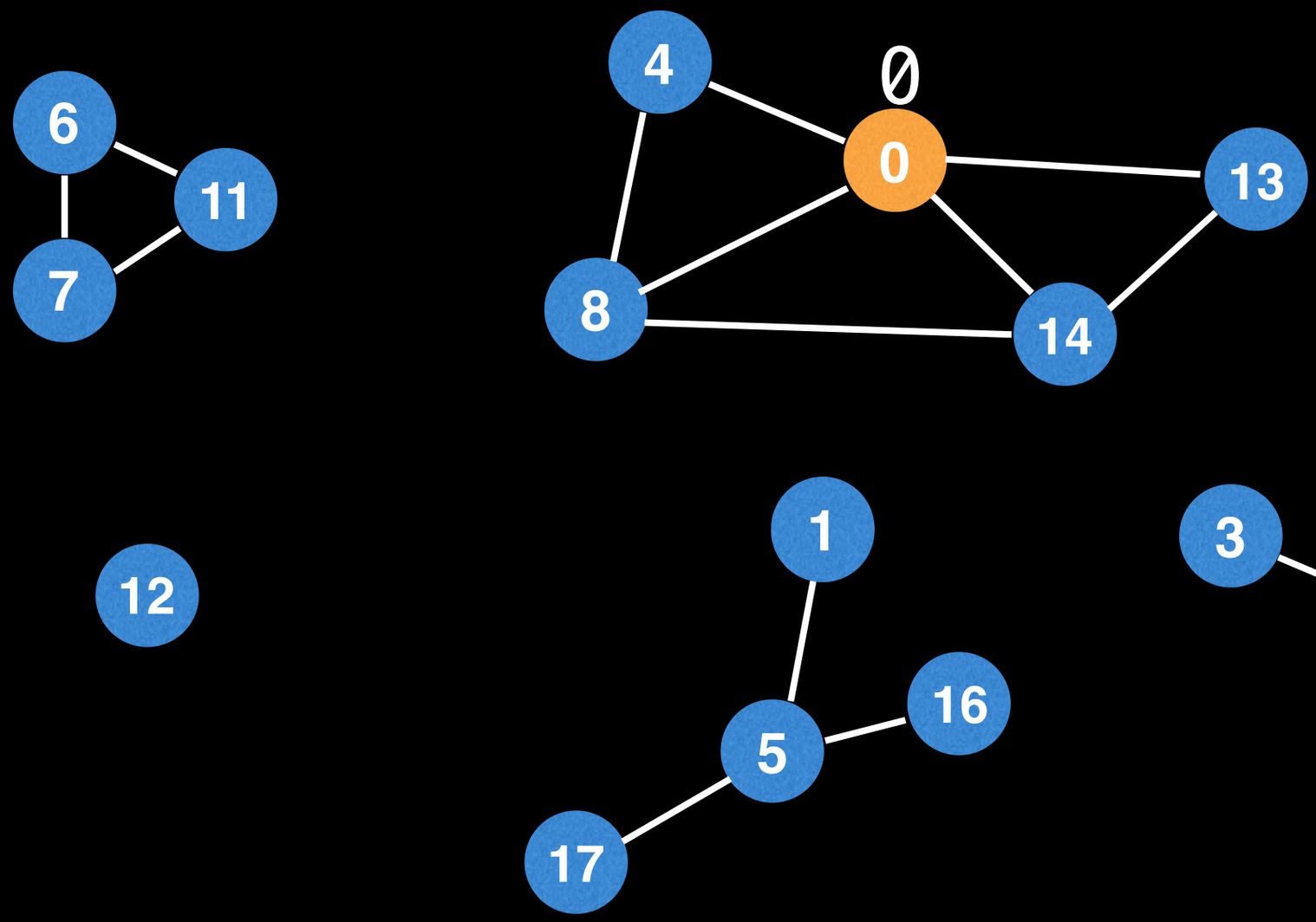

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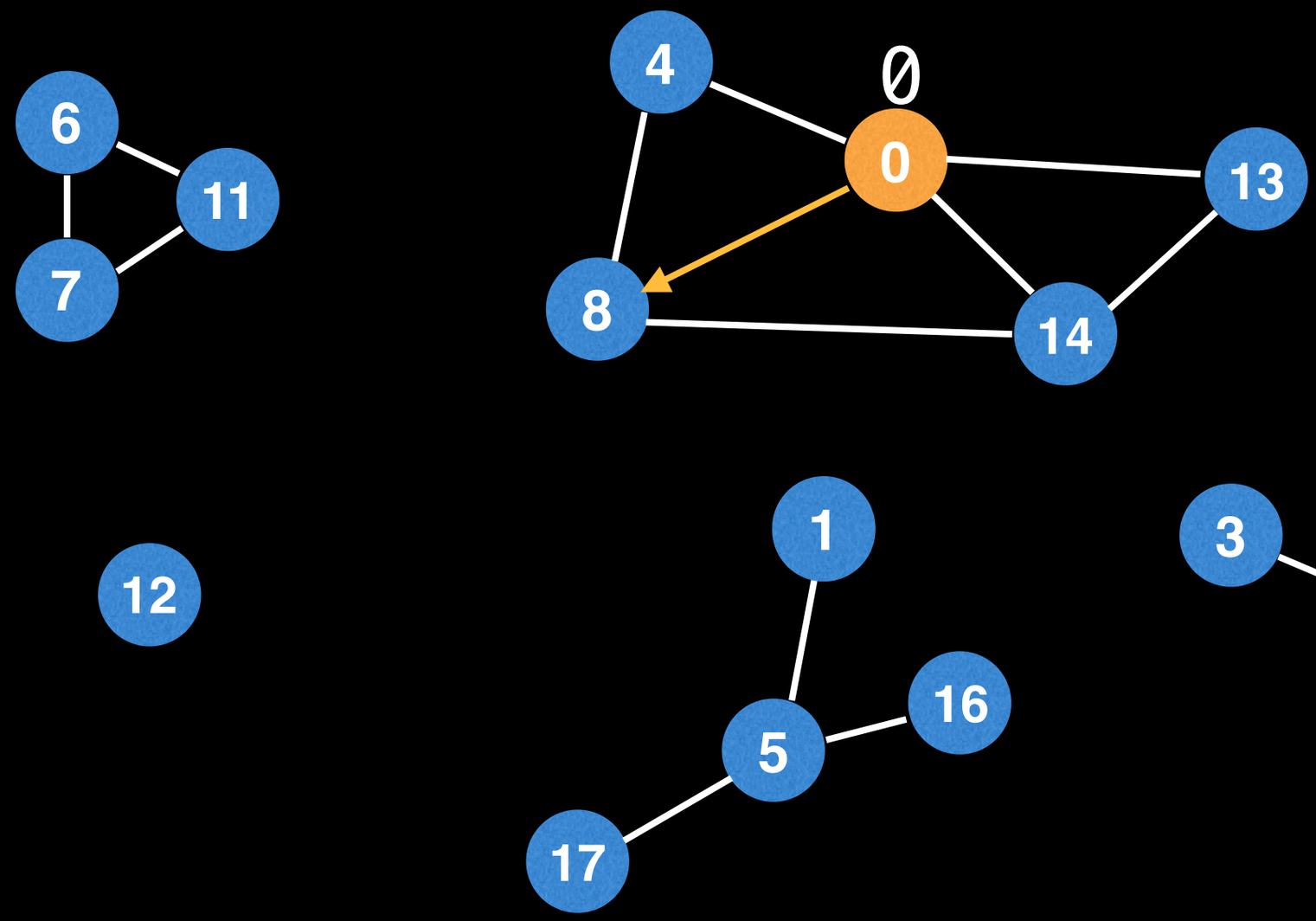

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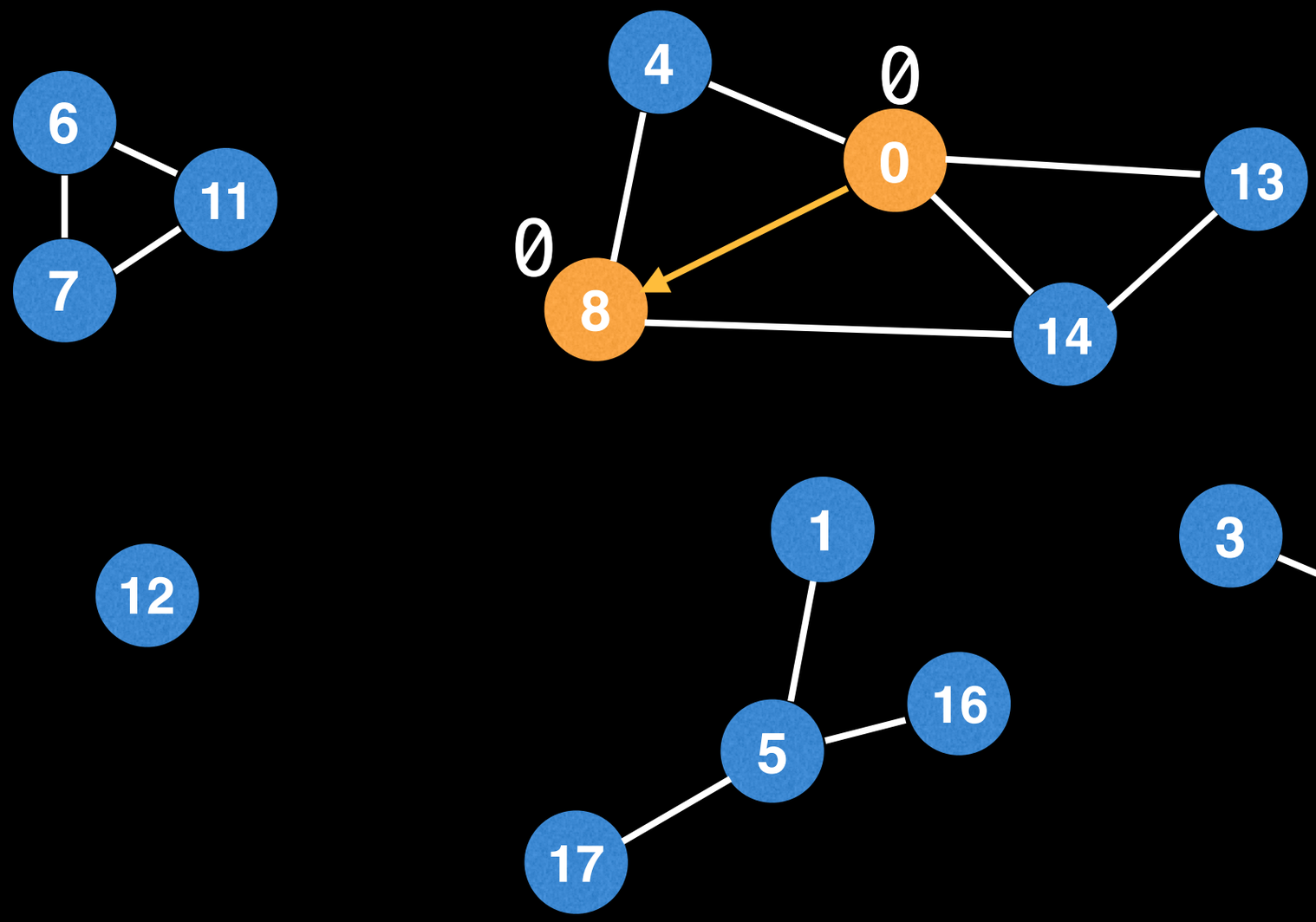

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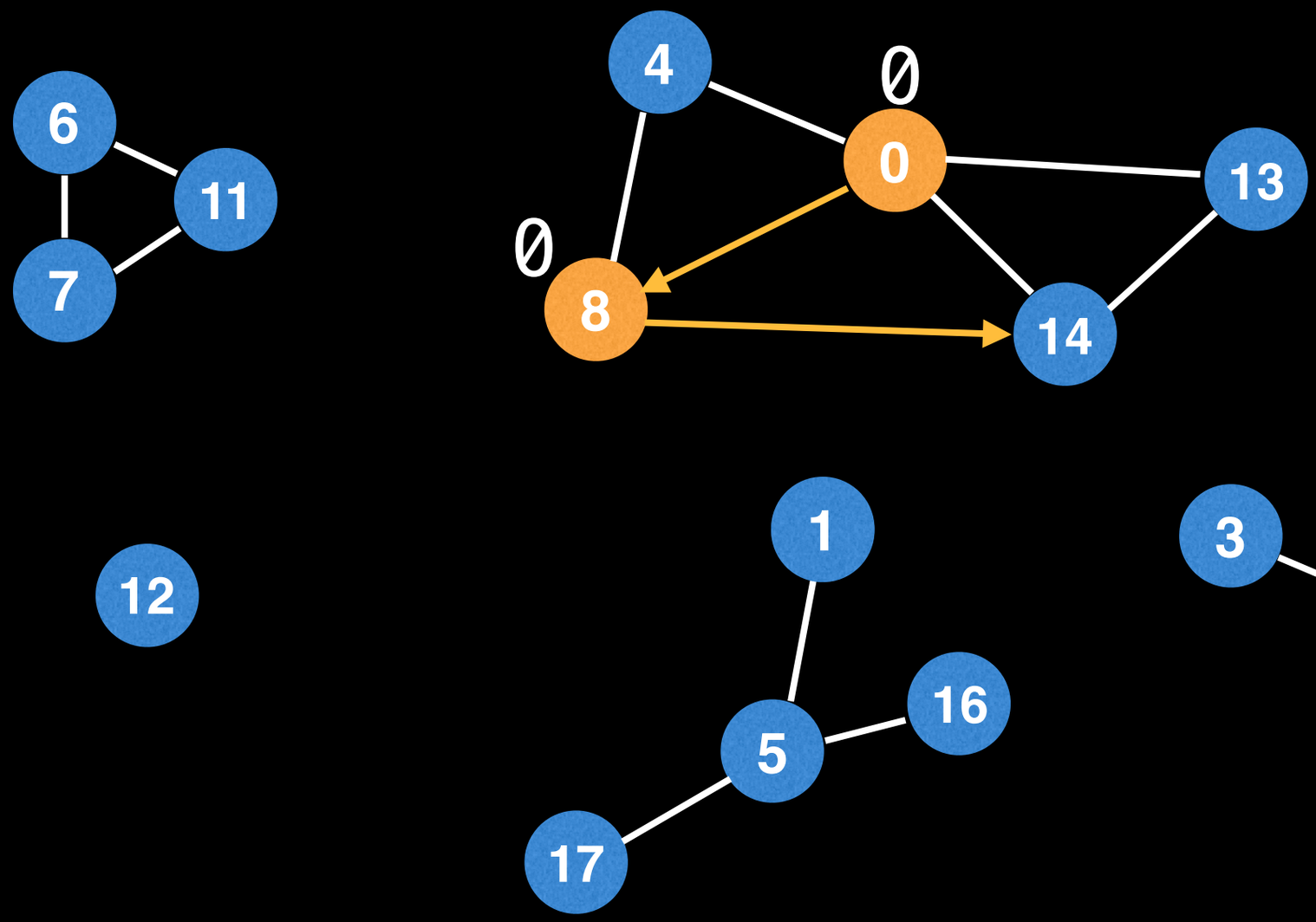

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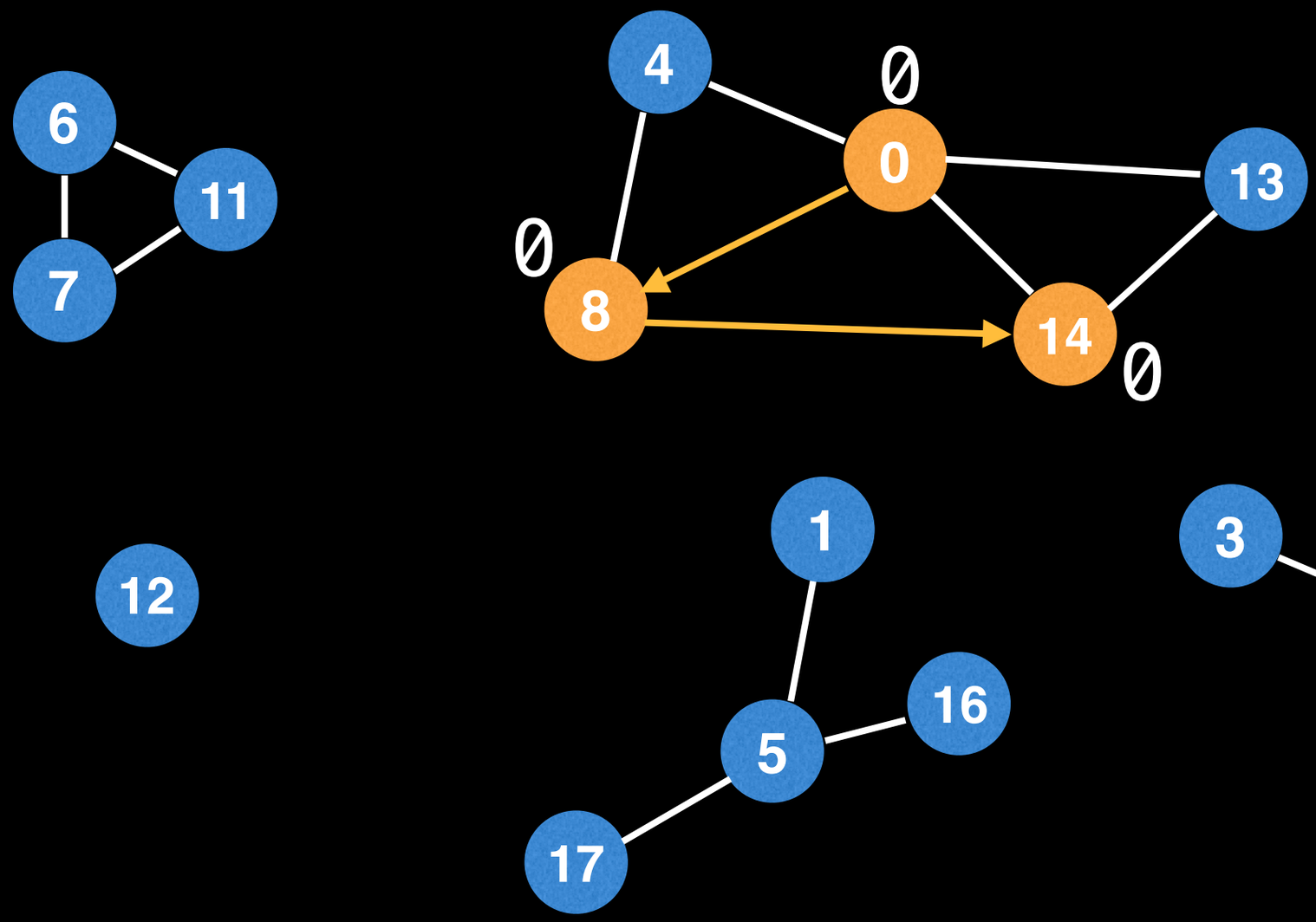

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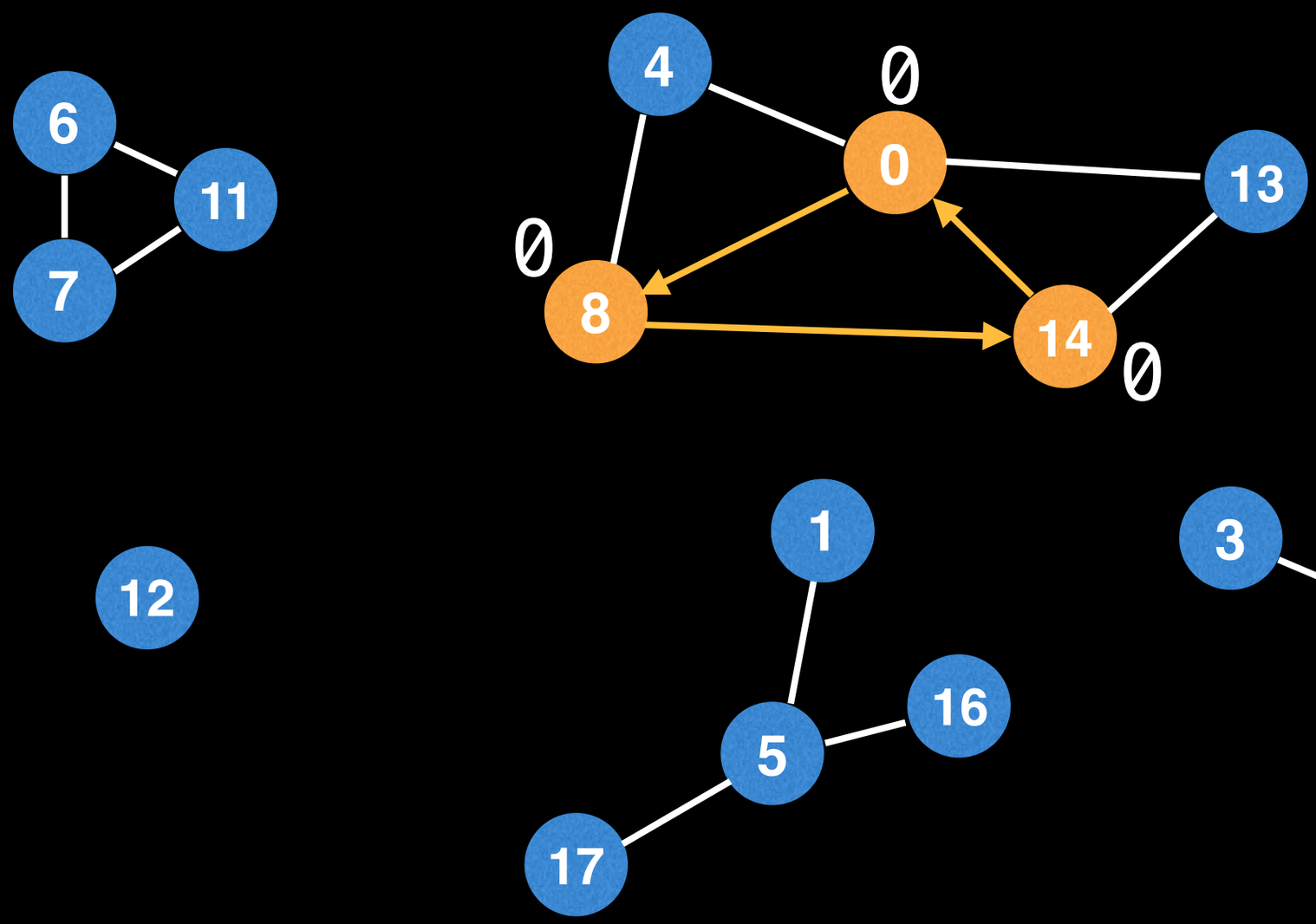

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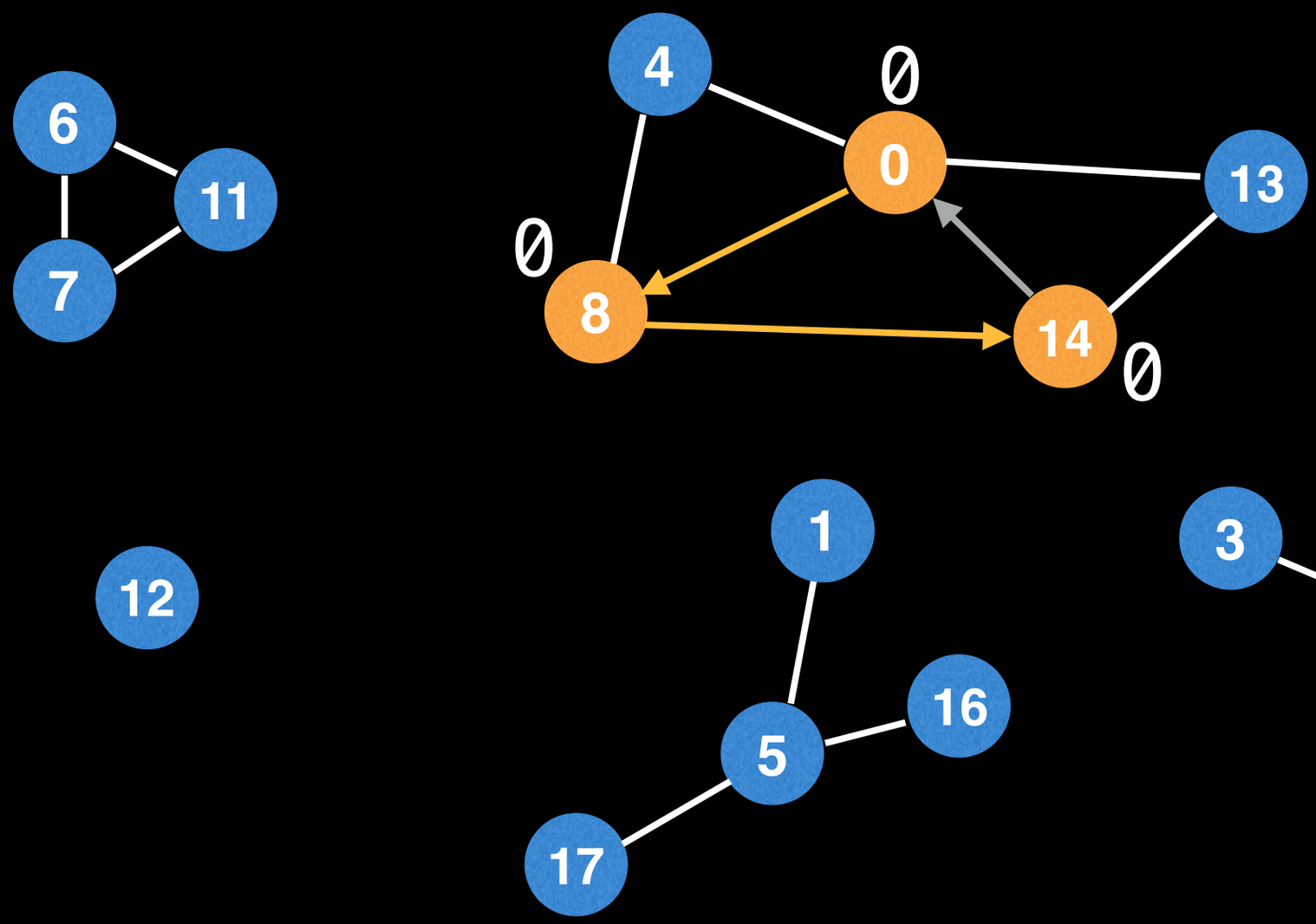

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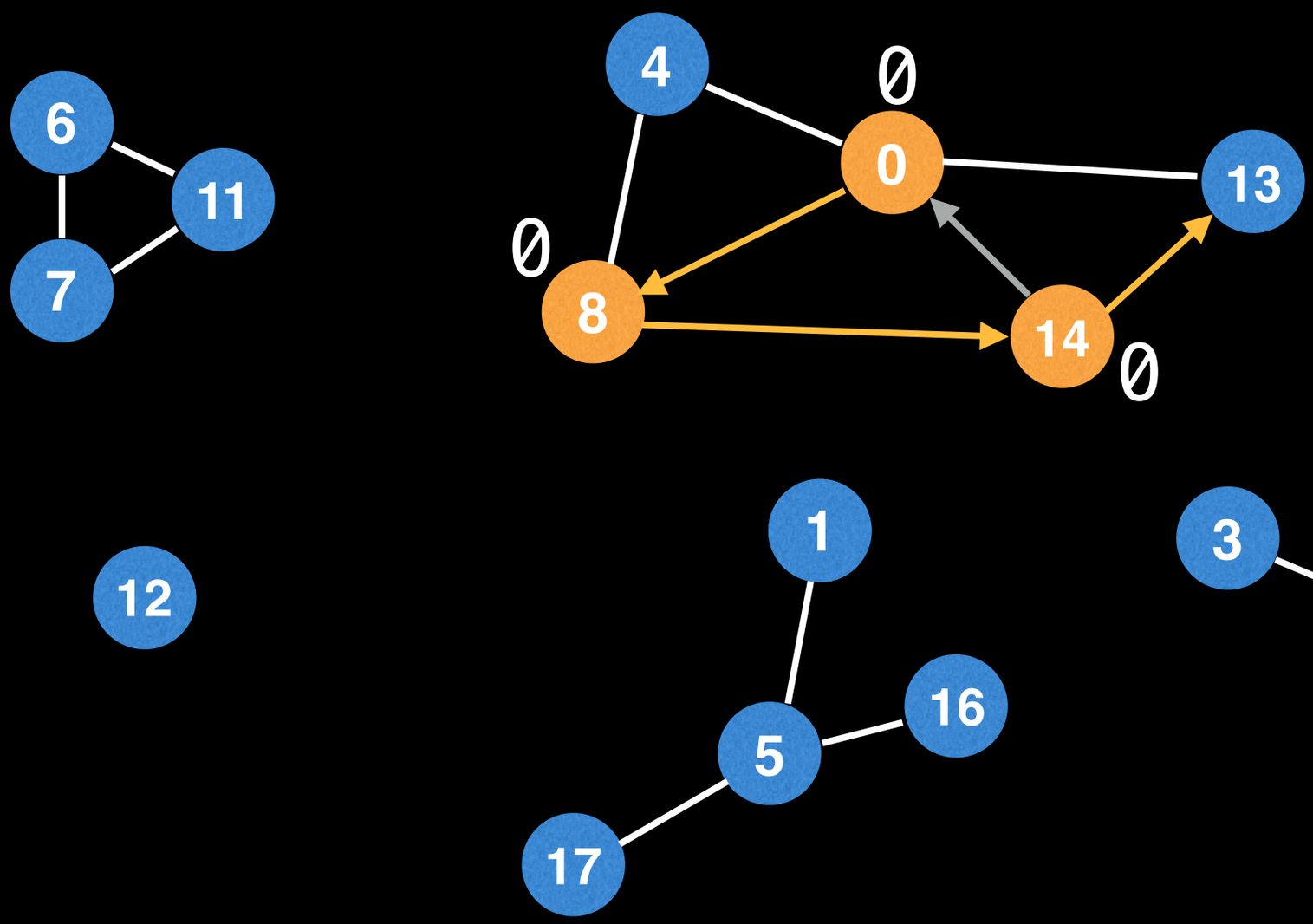

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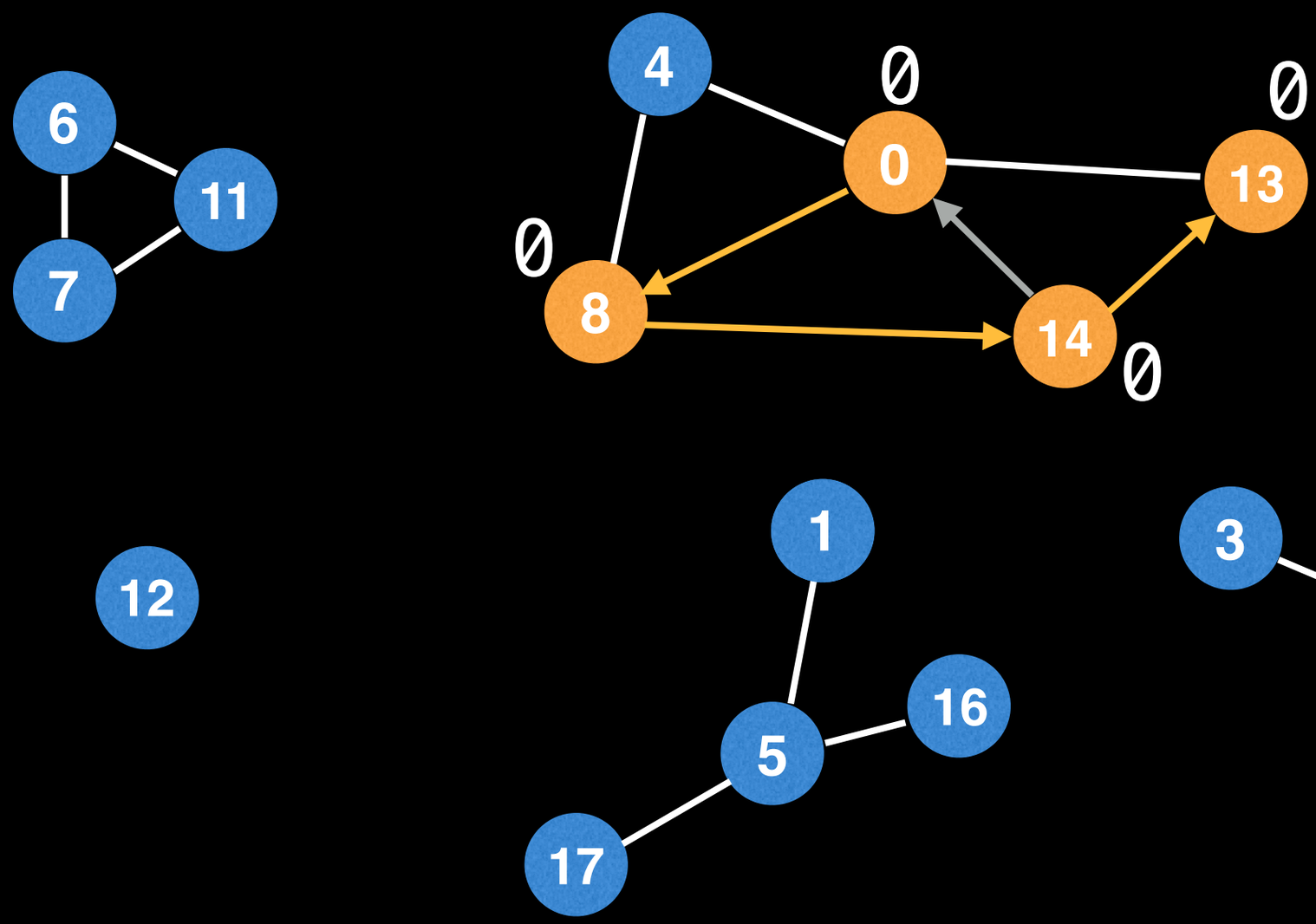

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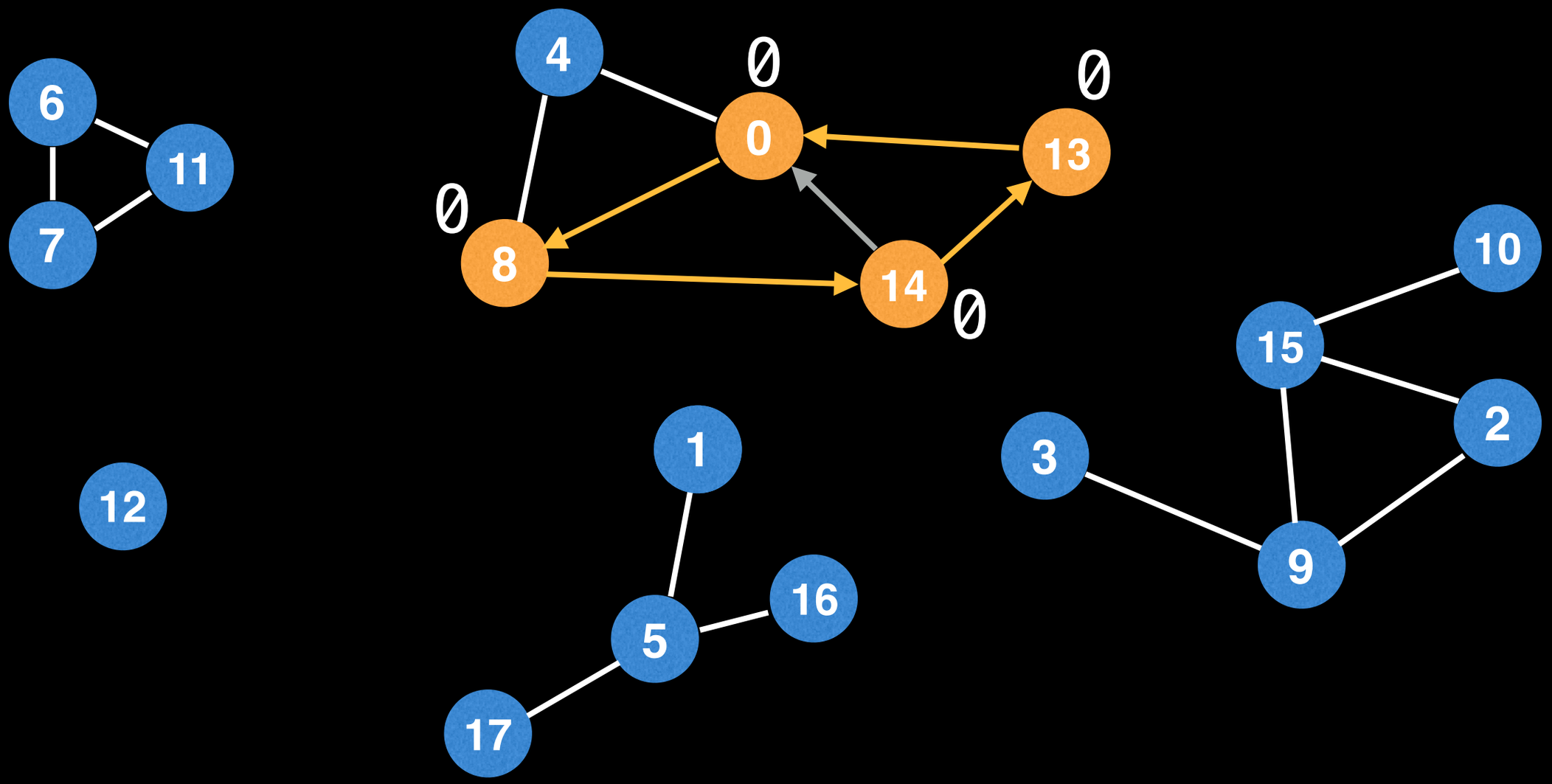

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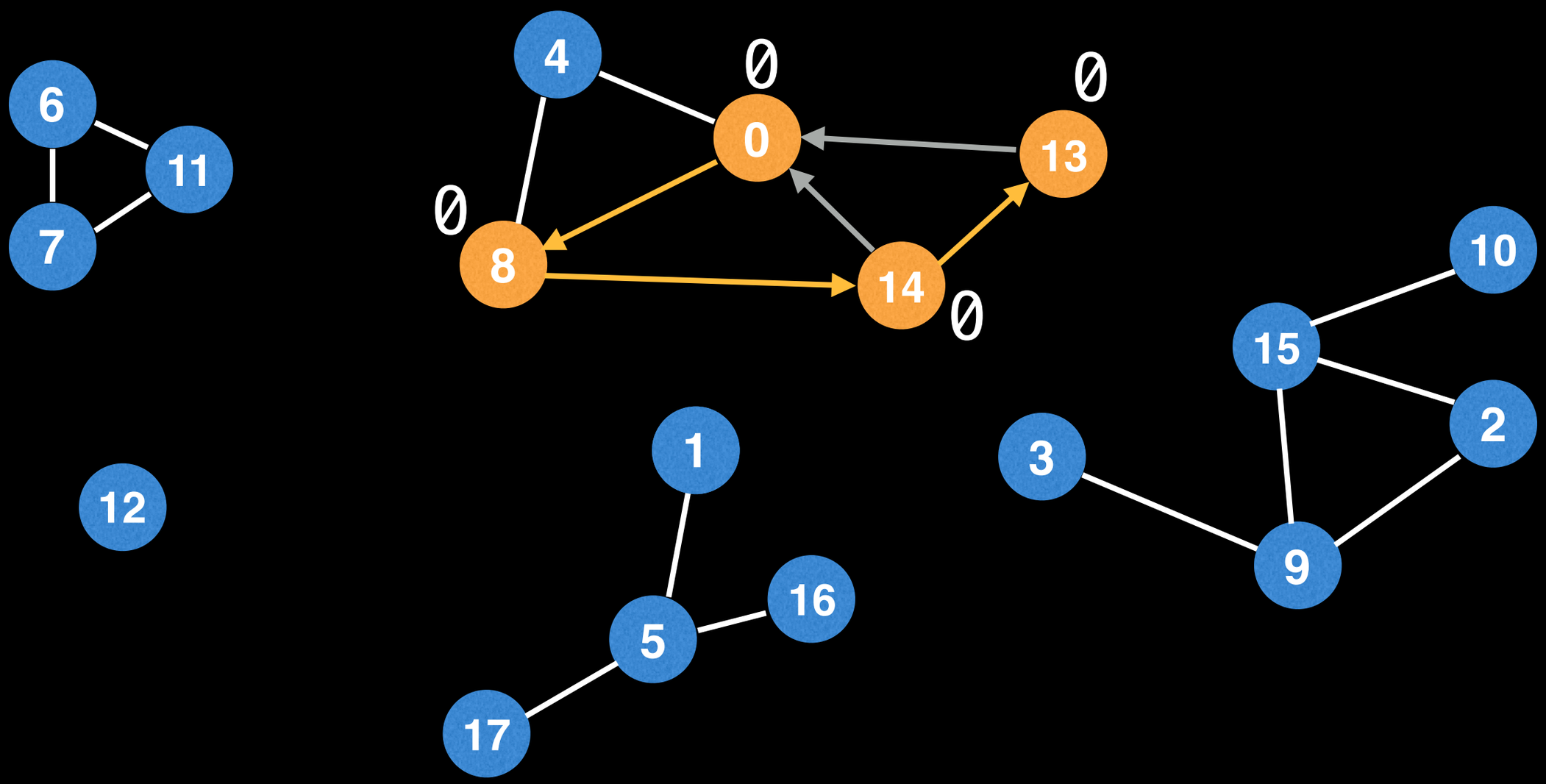

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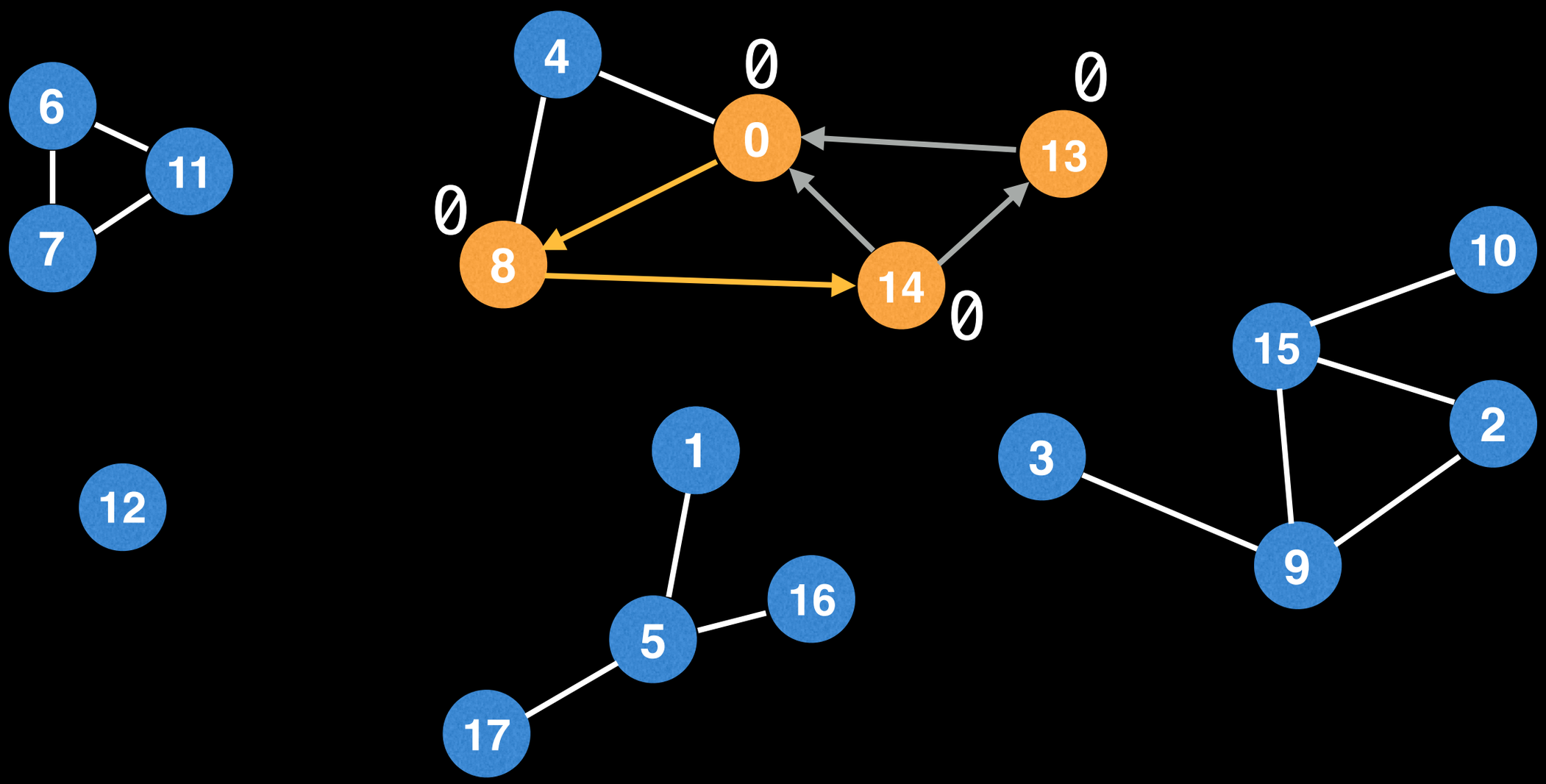

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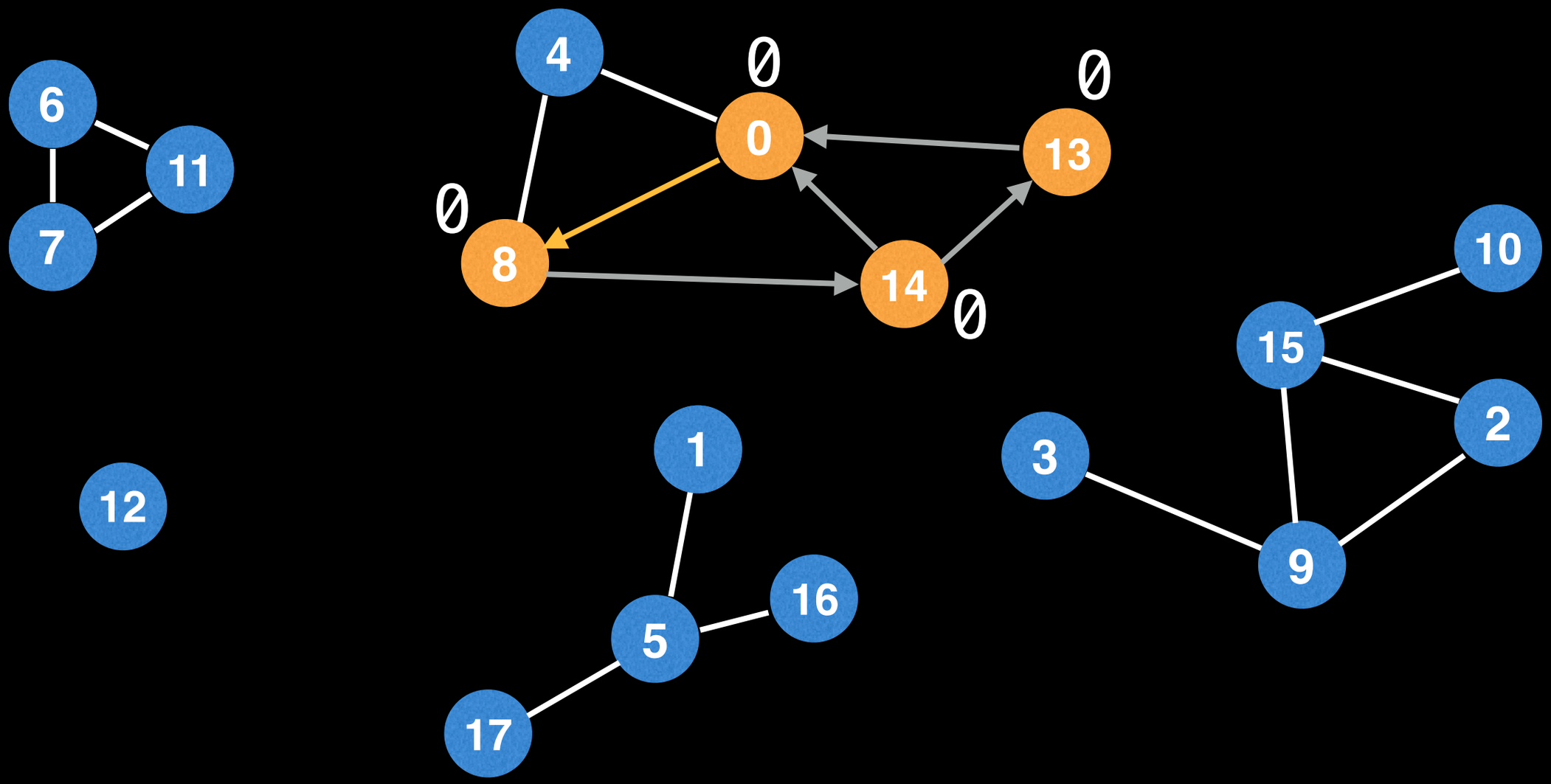

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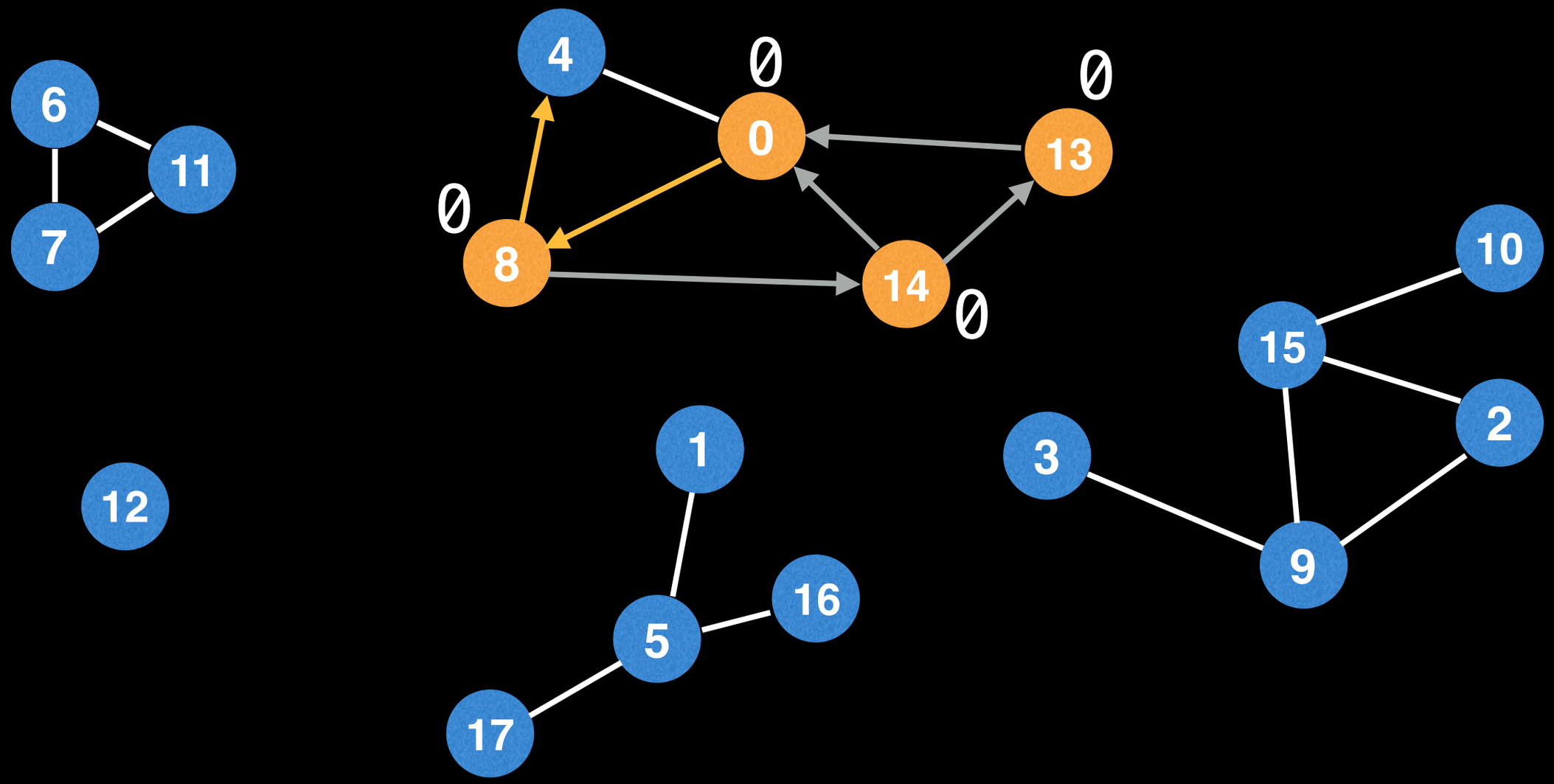

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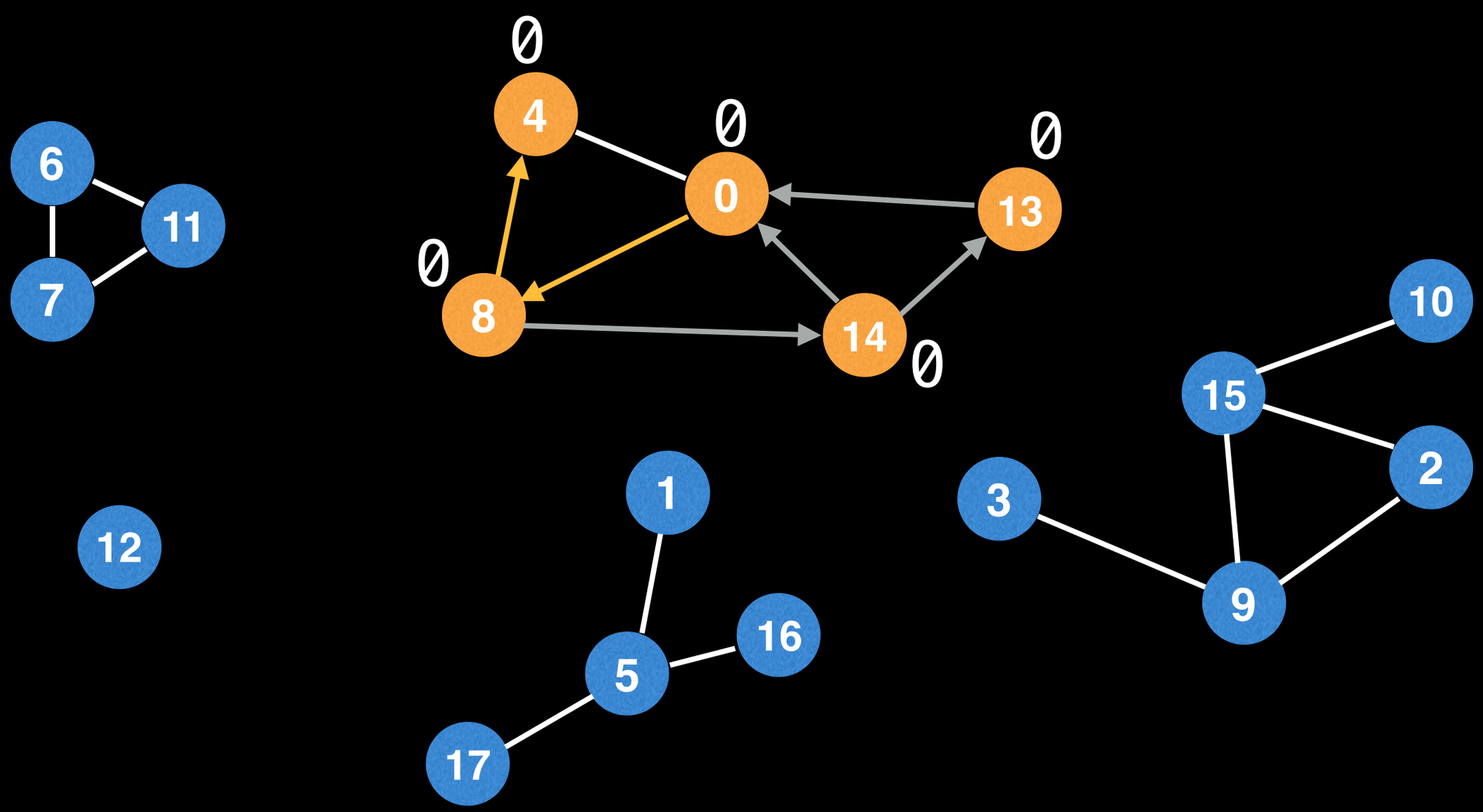

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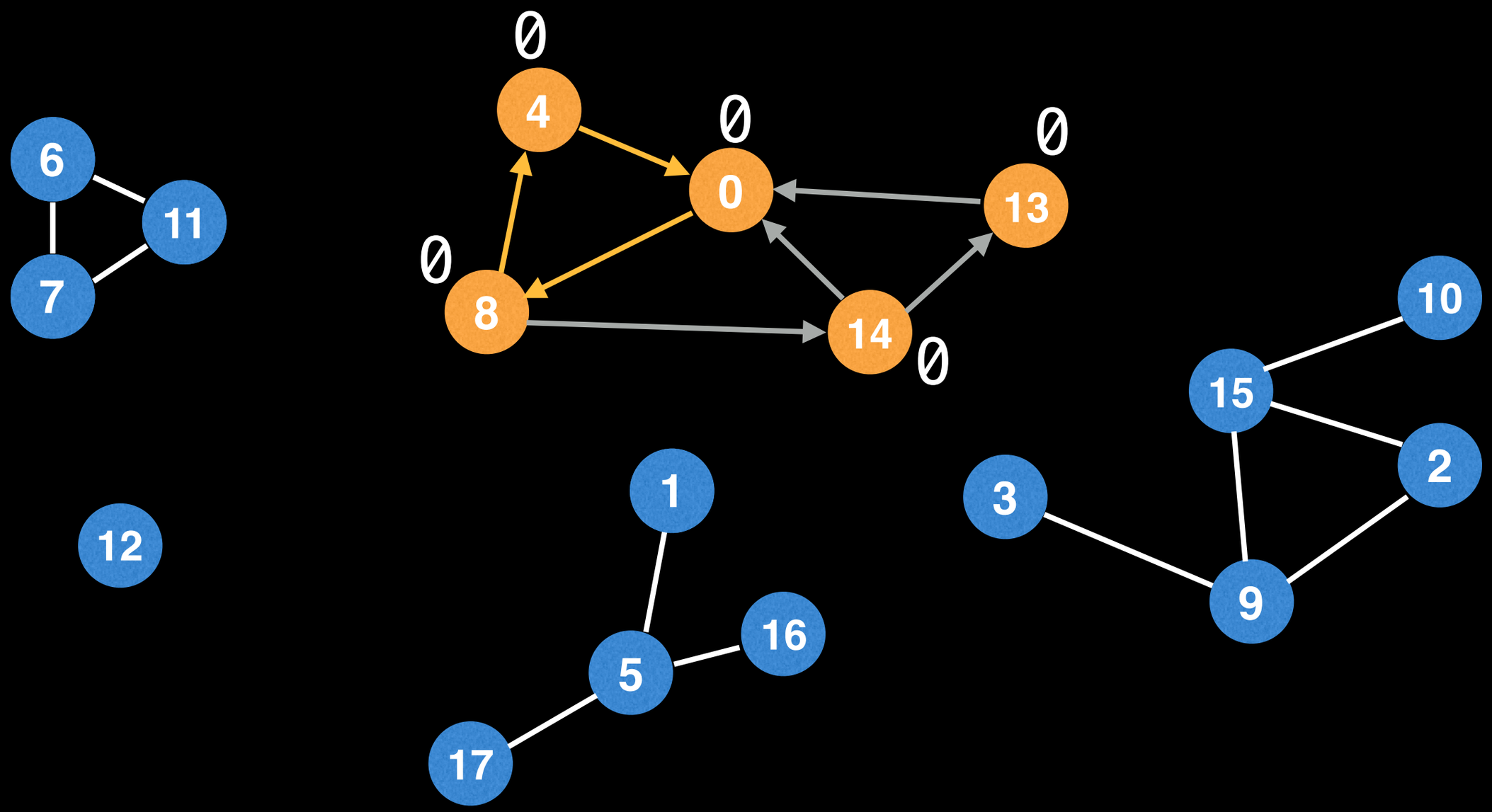

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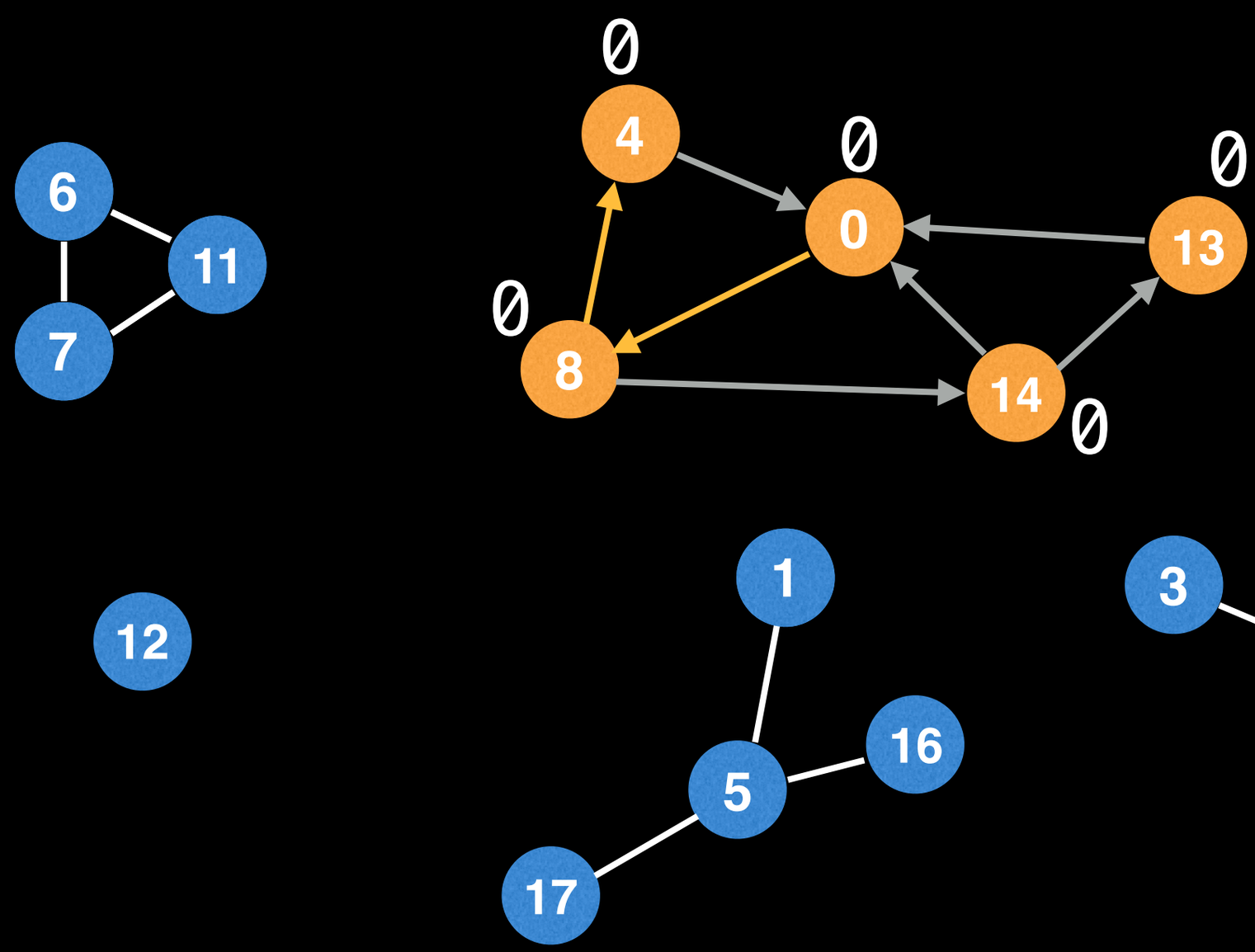

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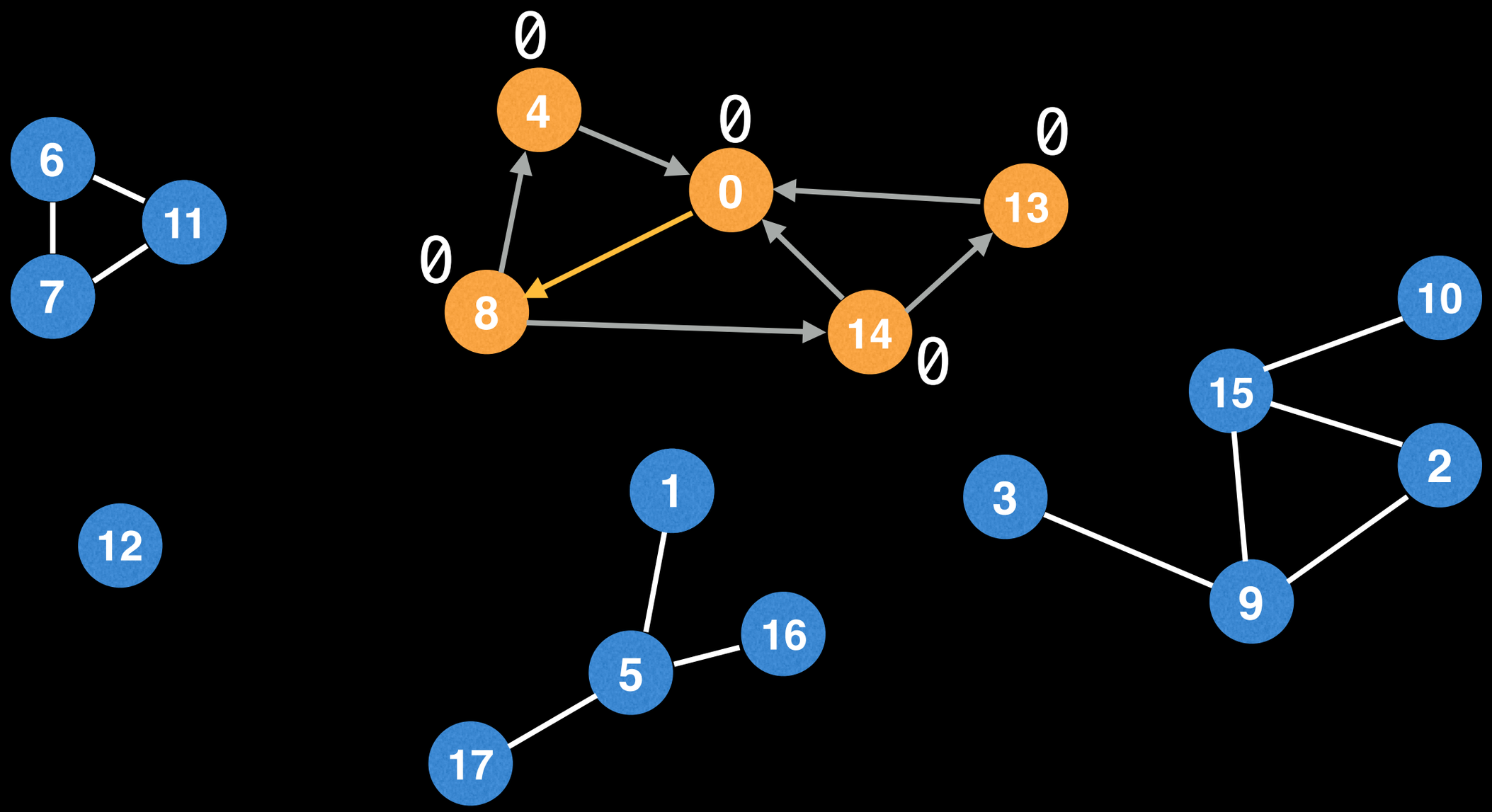

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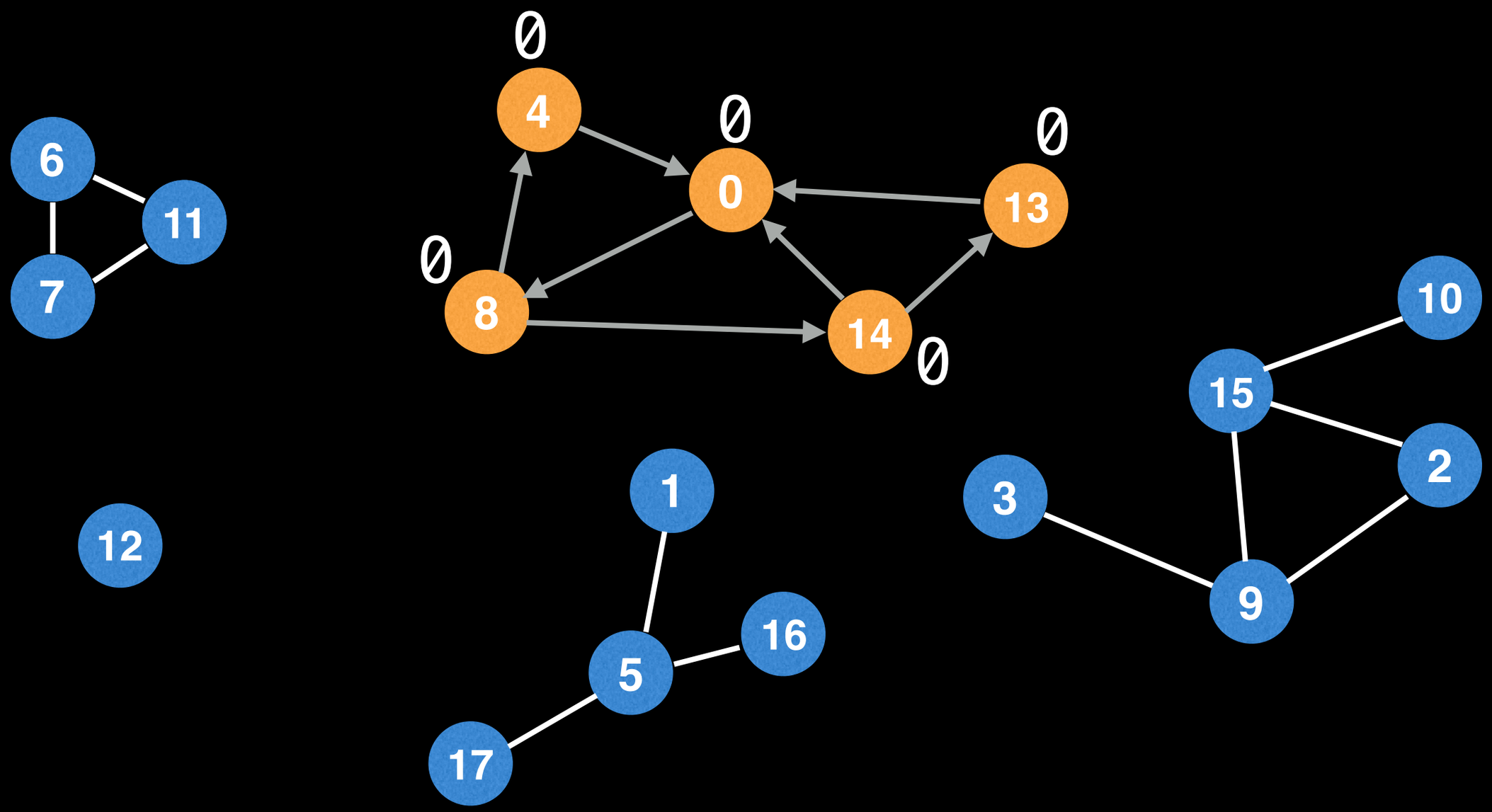

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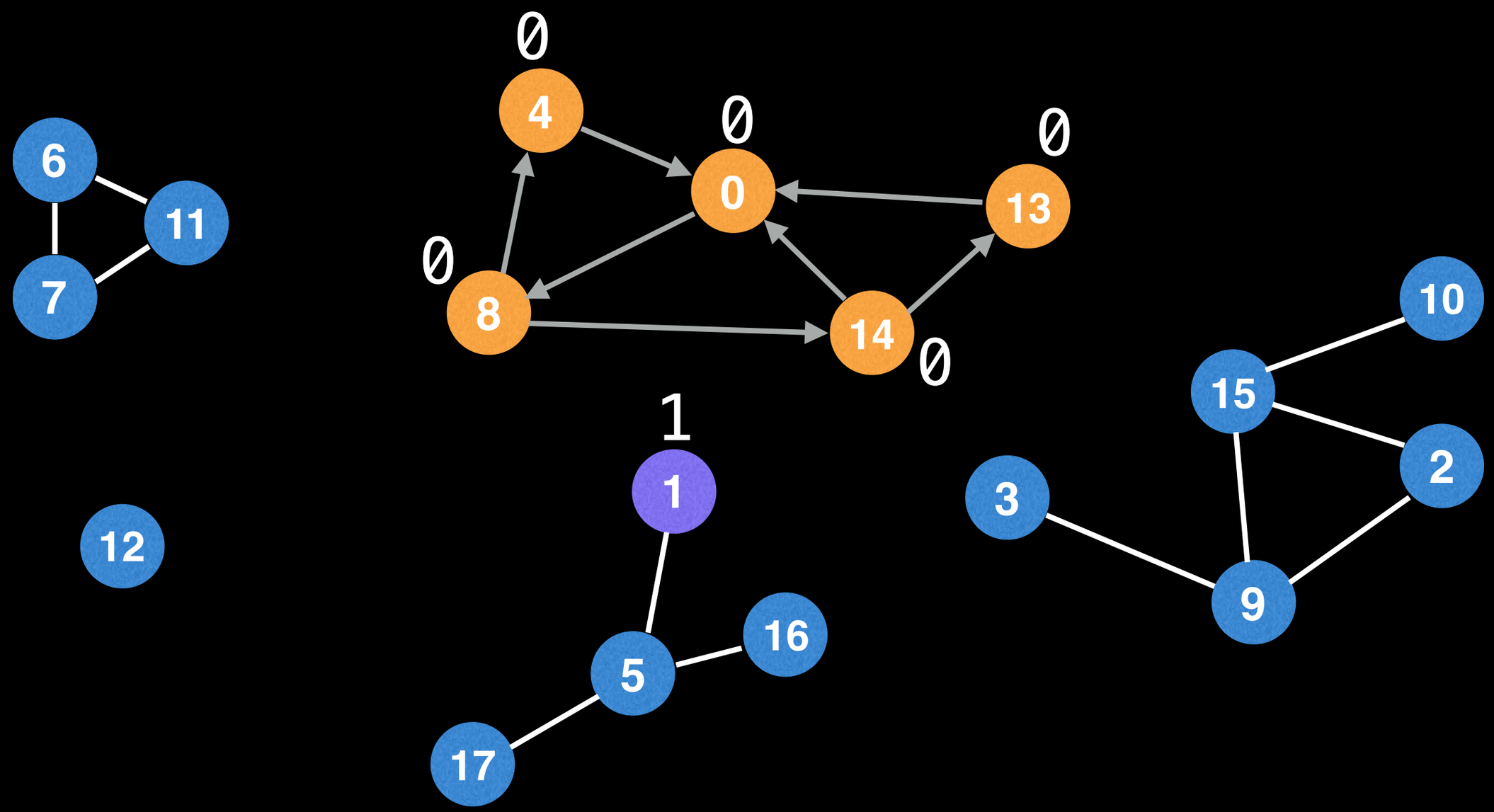

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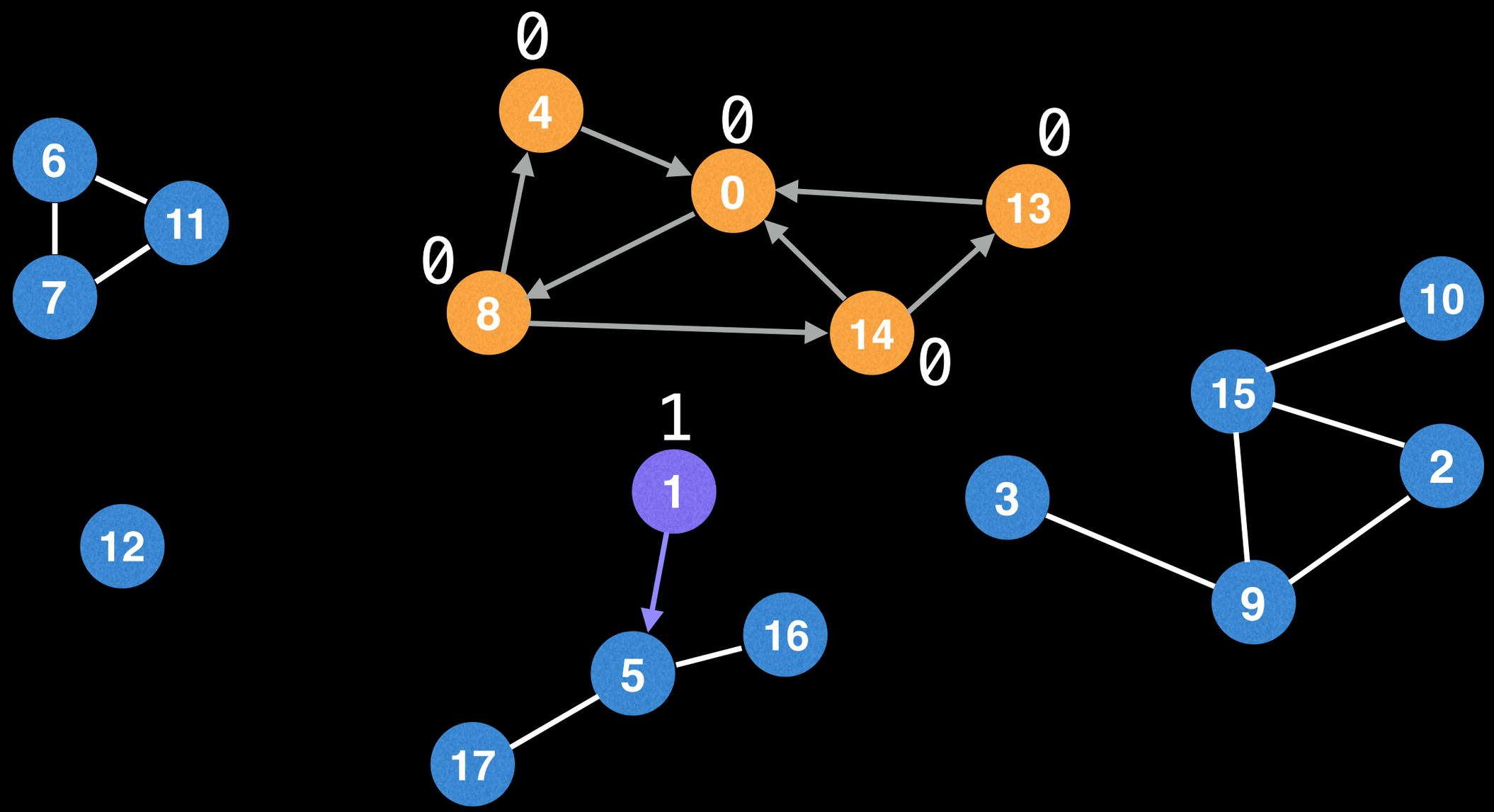

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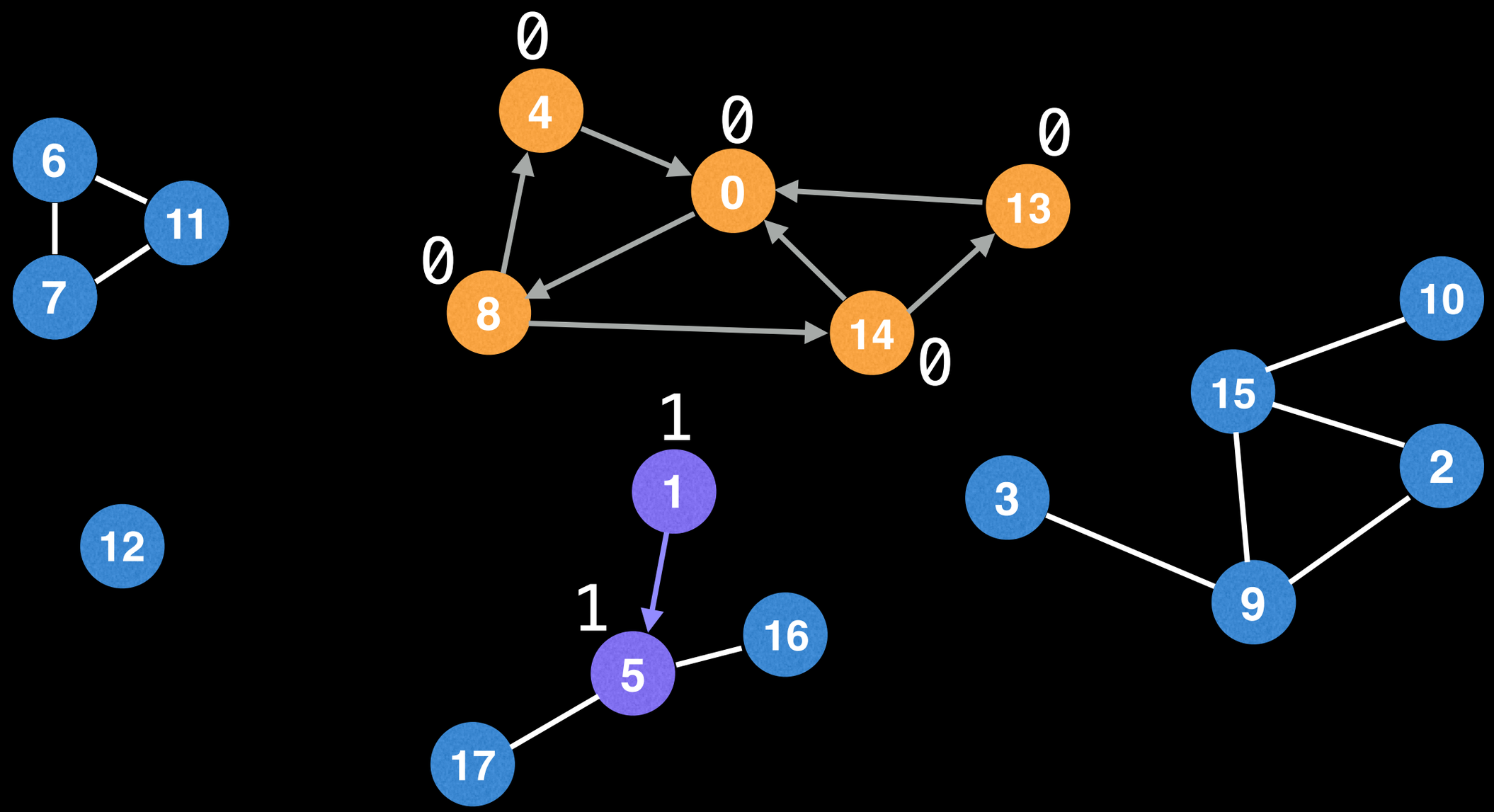

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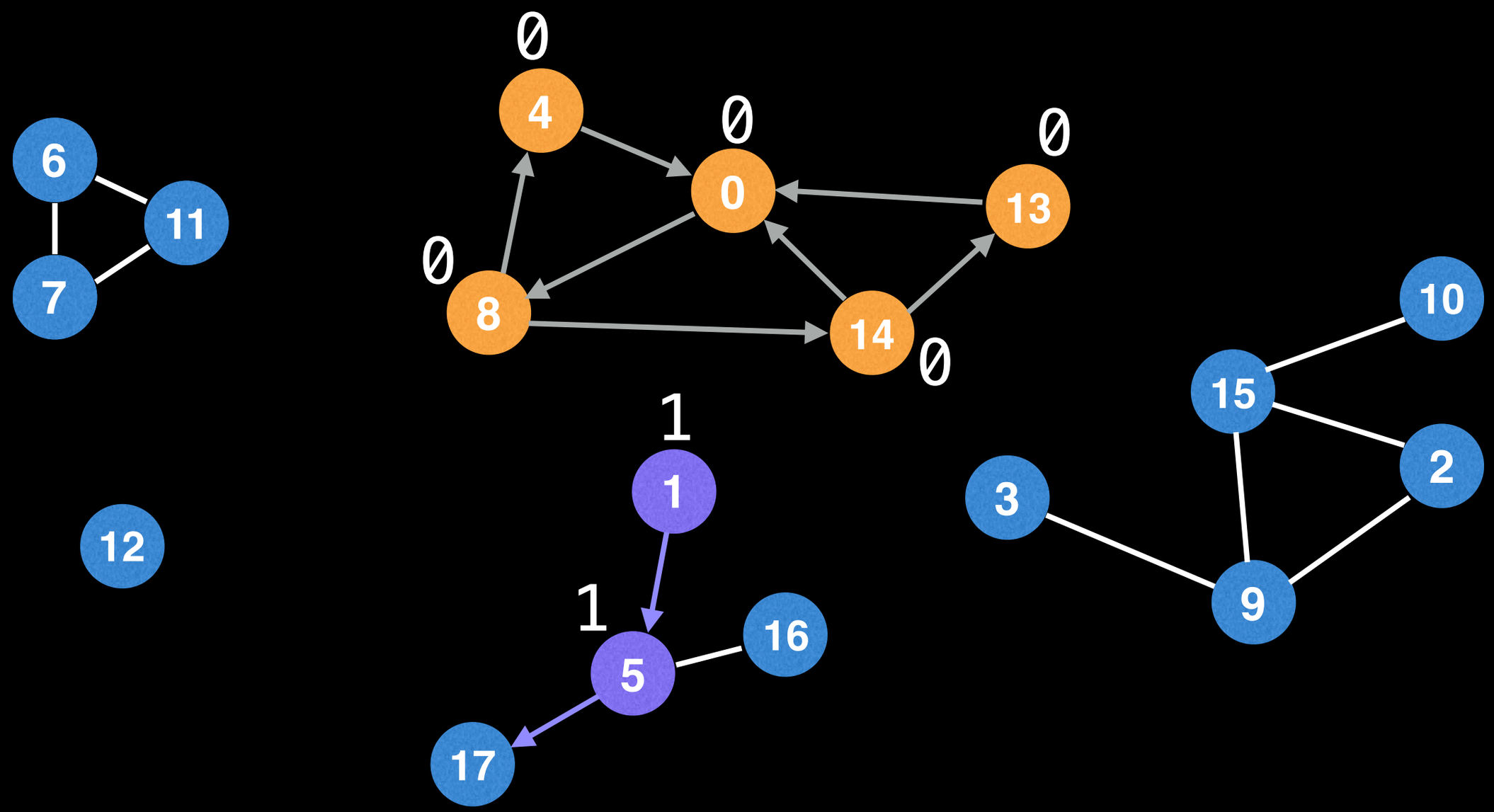

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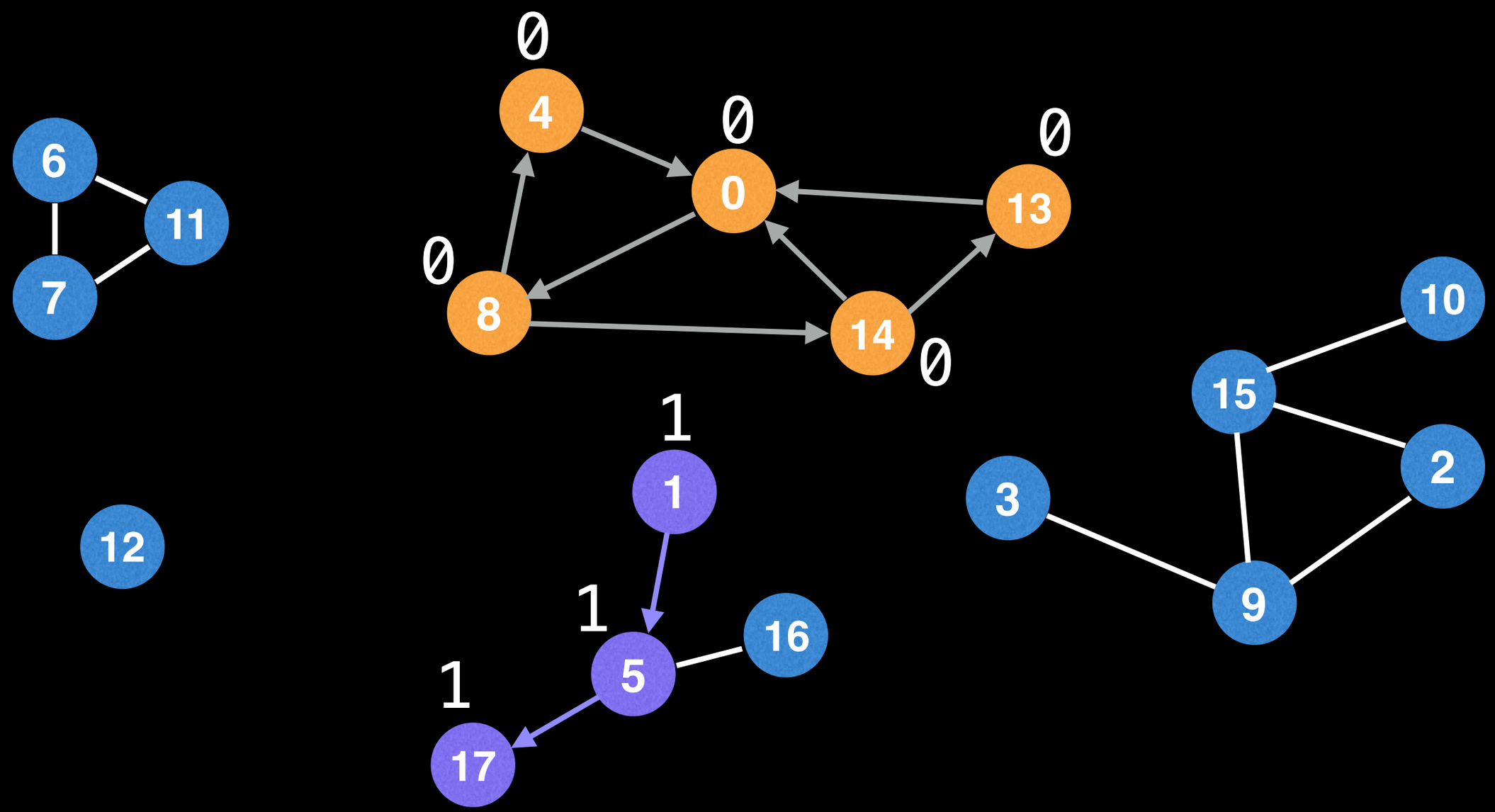

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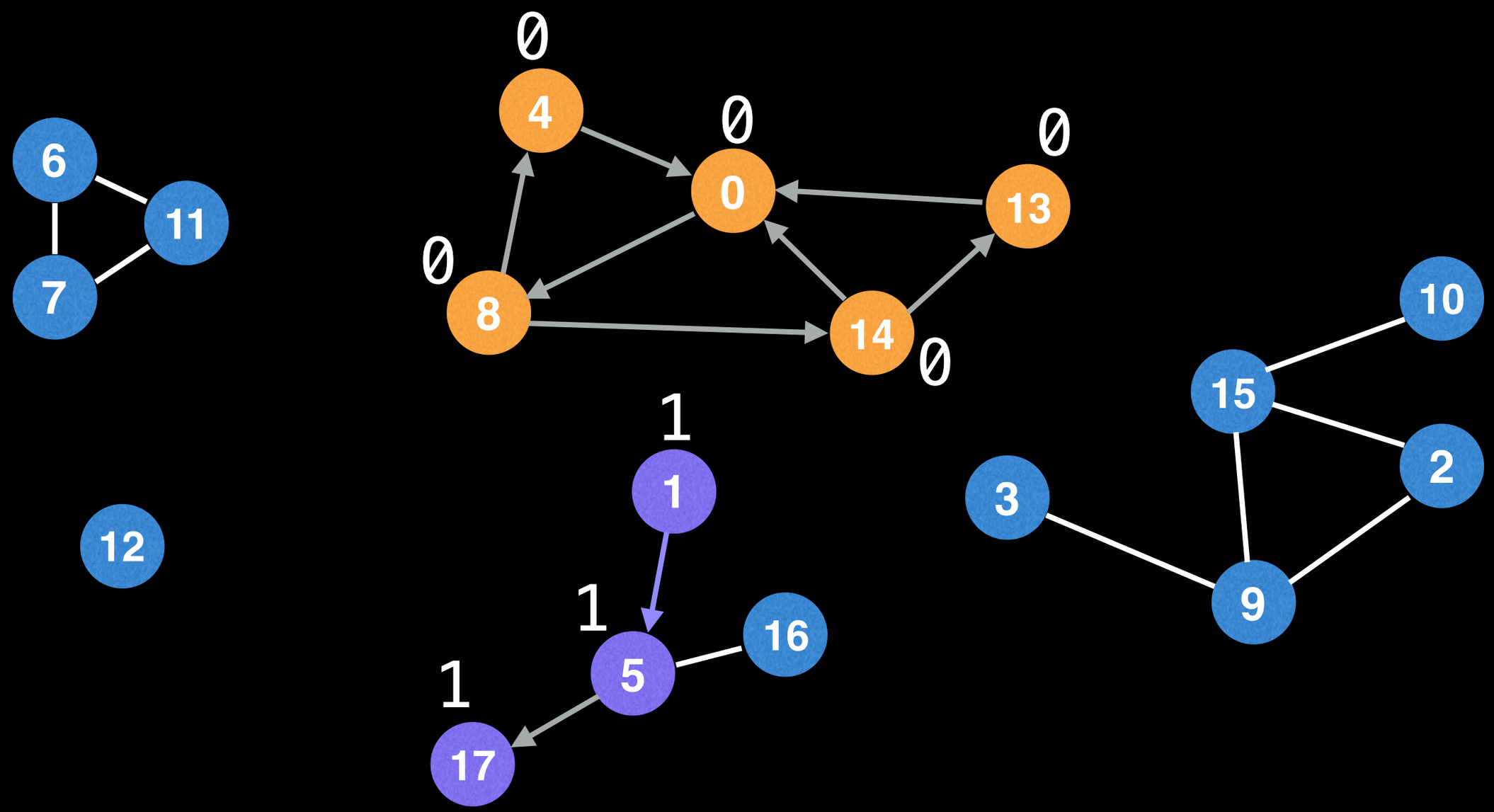

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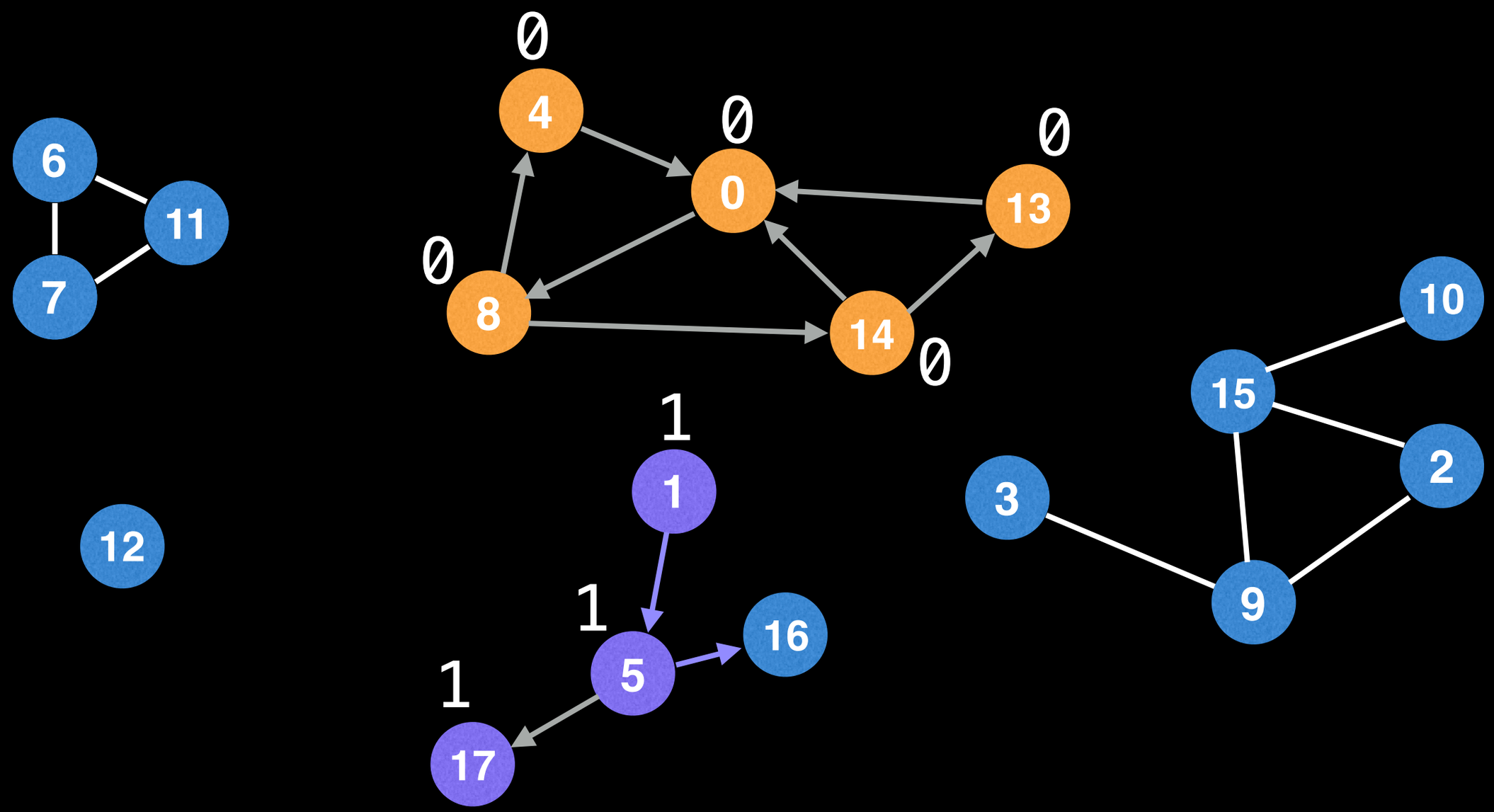

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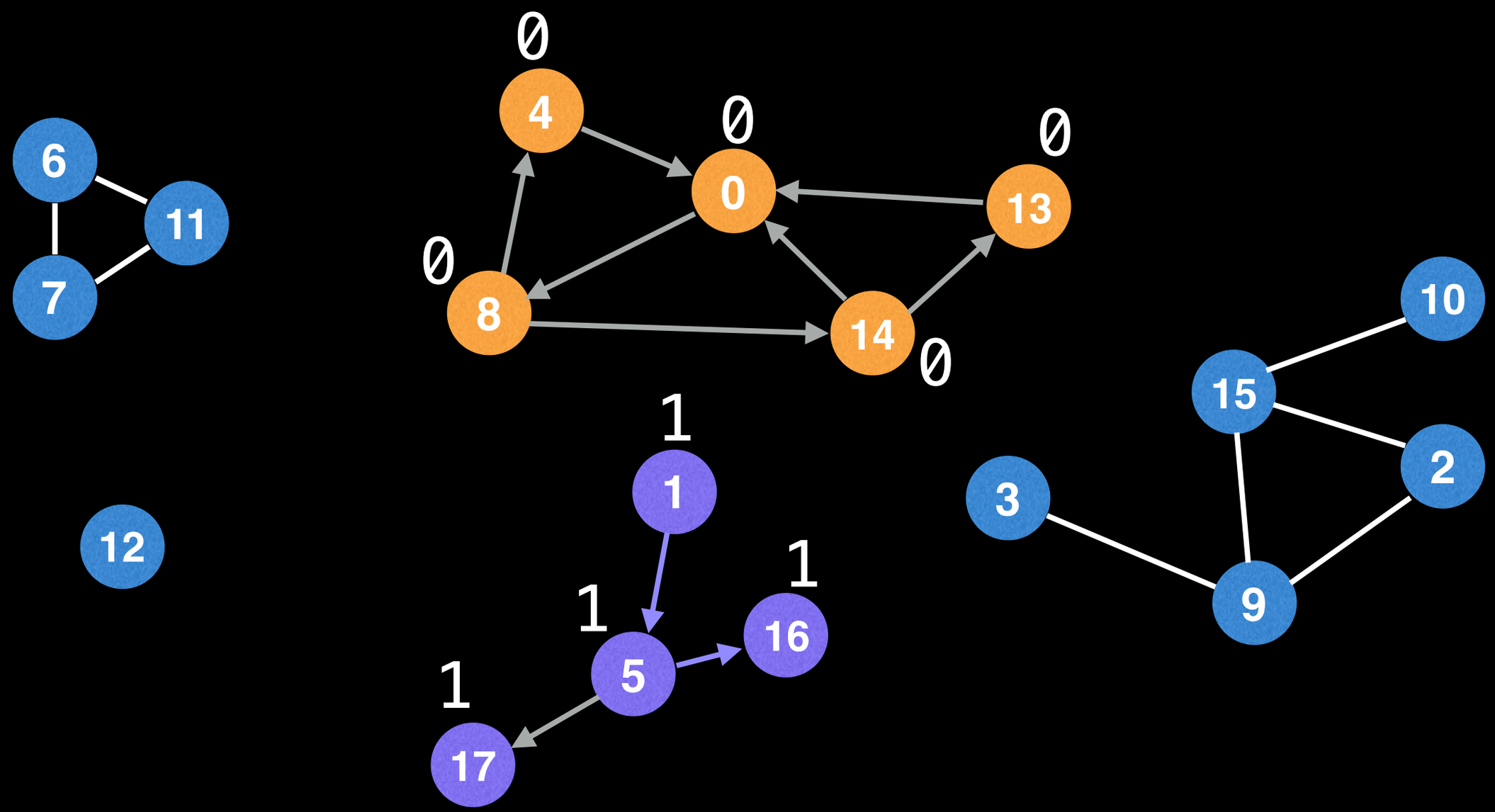

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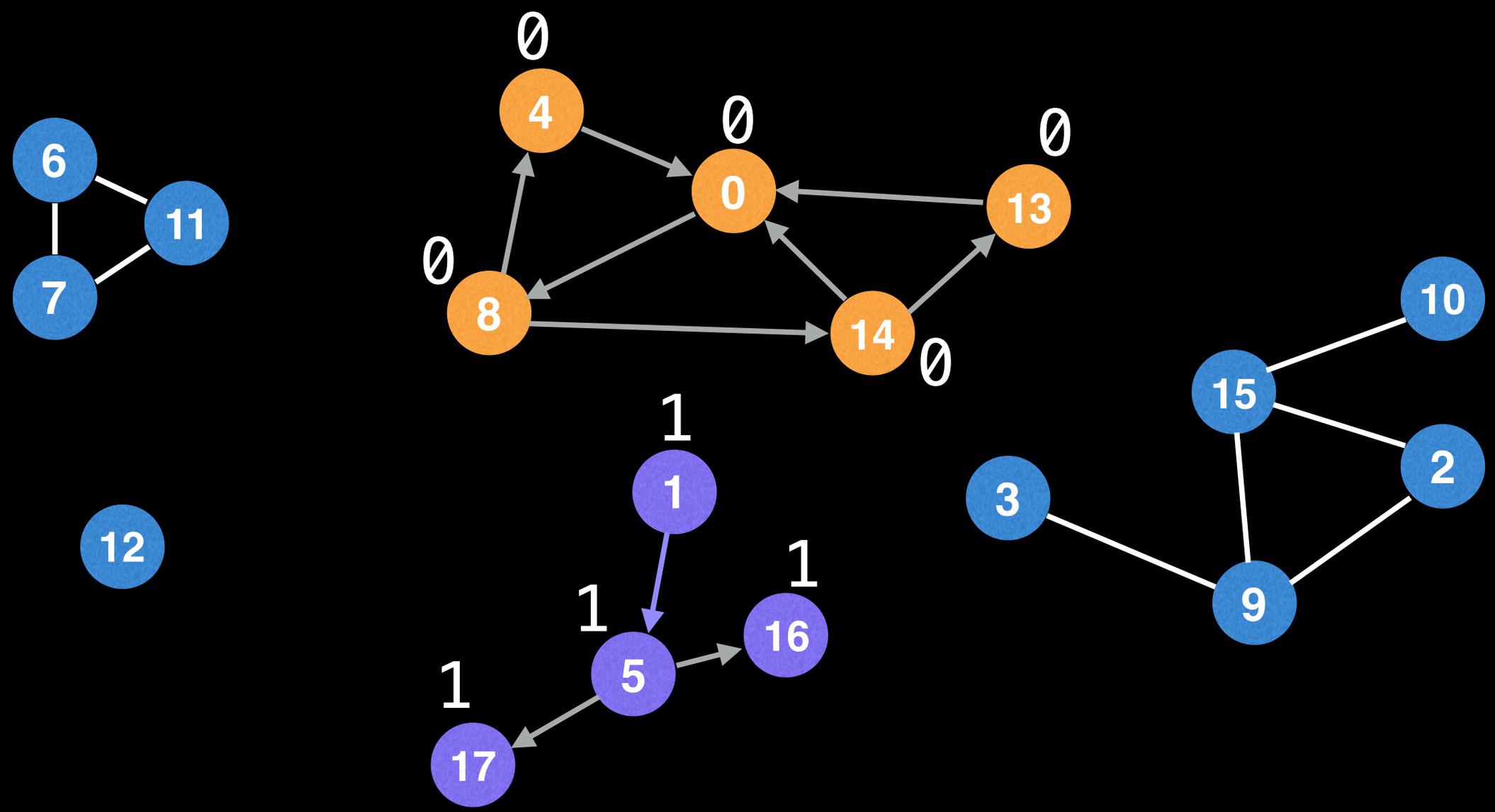

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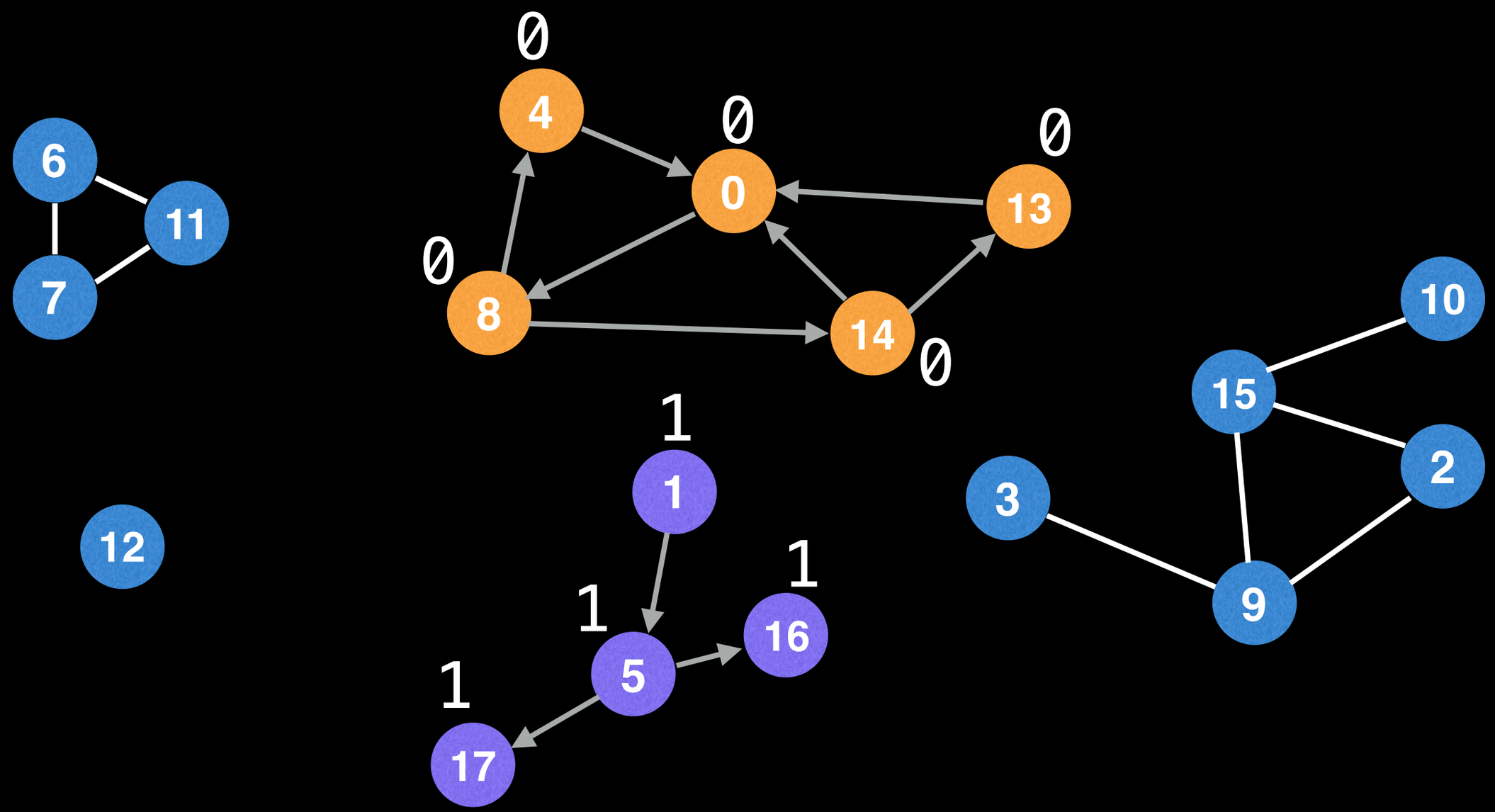
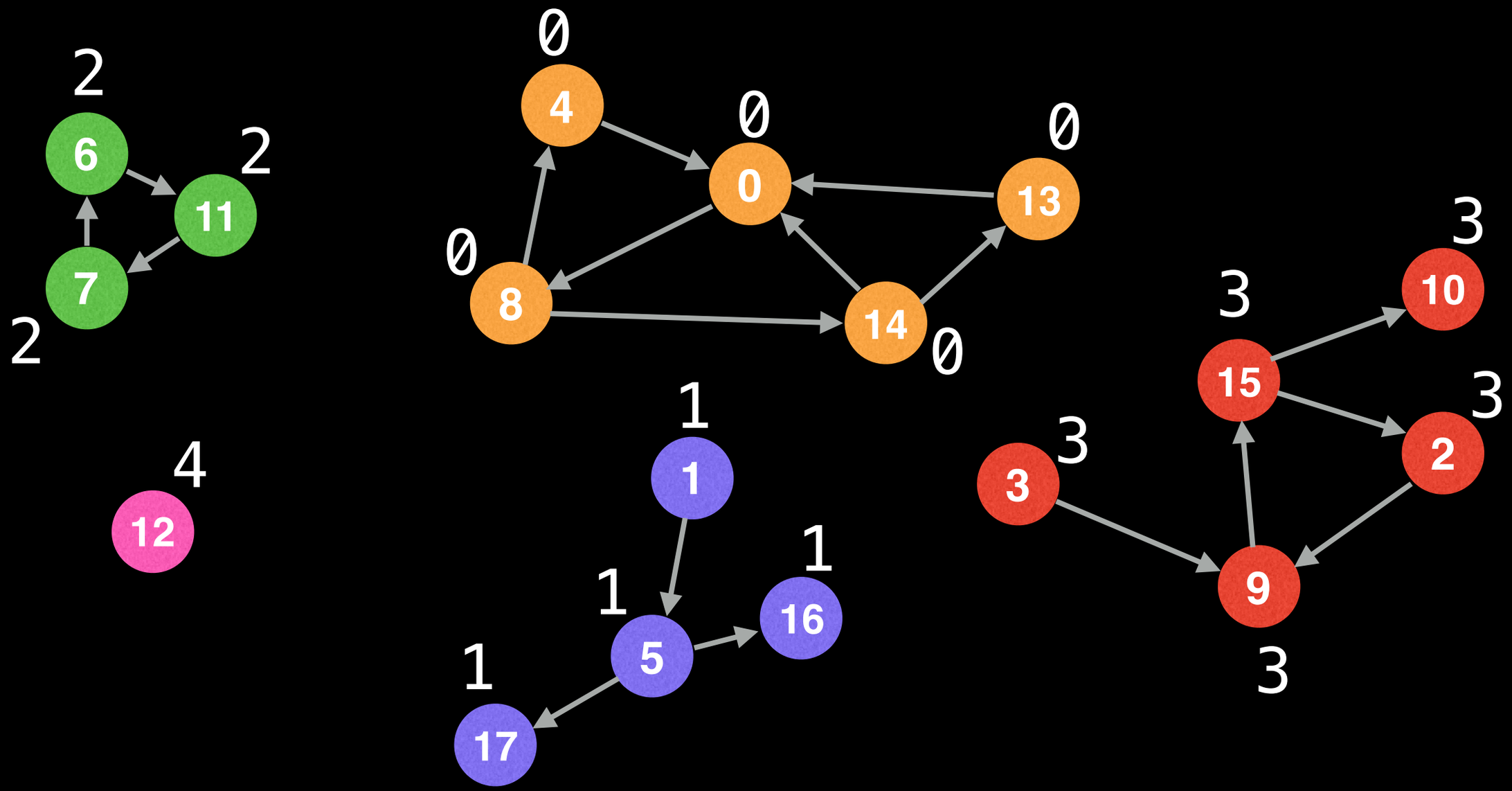


… and so on for the other components



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# Global or class scope variables
n = number of nodes in the graph
g = adjacency list representing graph
count = 0
components = empty integer array # size n
visited = [false, …, false] # size n

function findComponents():
  for (i = 0; i < n; i++):
    if !visited[i]:
      count++
      dfs(i)
  return (count, components)

function dfs(at):
  visited[at] = true
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What else can DFS do?

We can augment the DFS algorithm to:

- Compute a graph’s minimum spanning tree.
- Detect and find cycles in a graph.
- Check if a graph is bipartite.
- Find strongly connected components.
- Topologically sort the nodes of a graph.
- Find bridges and articulation points.
- Find augmenting paths in a flow network.
- Generate mazes.

Next Video: Breadth First Search