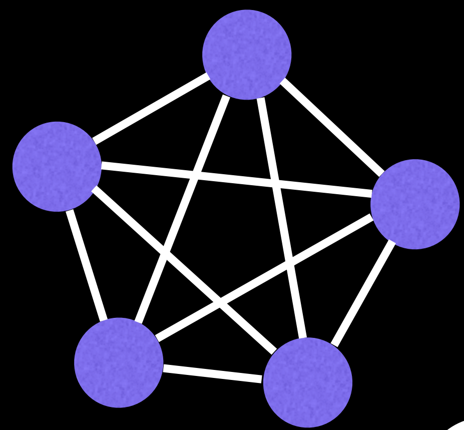
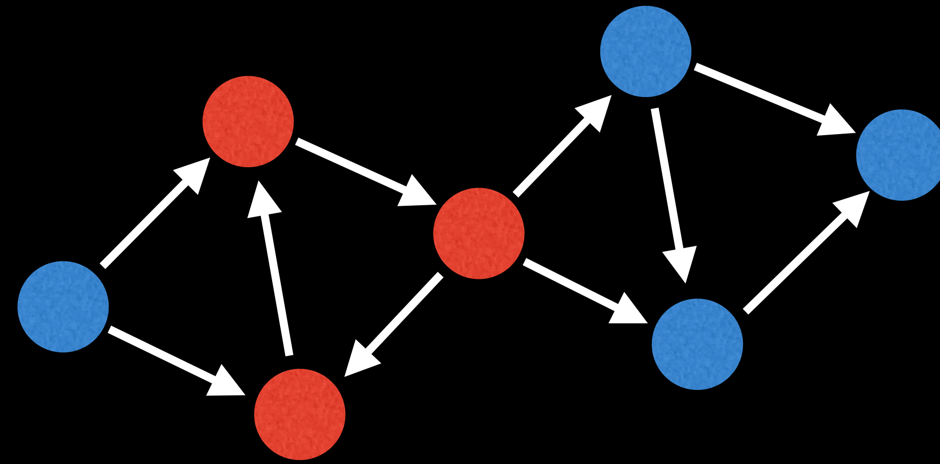
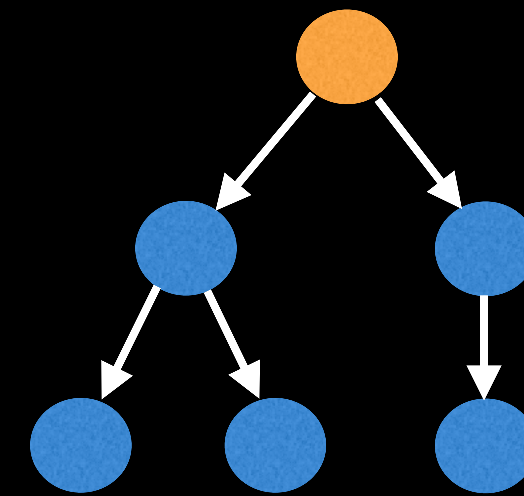
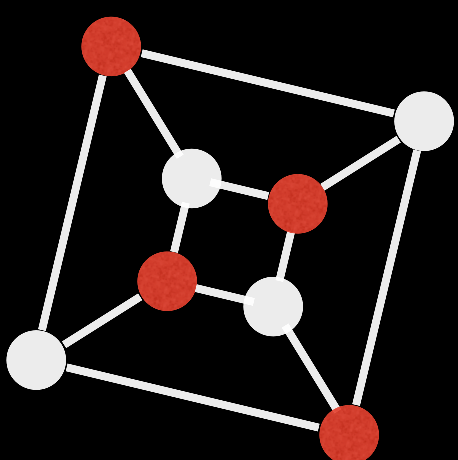
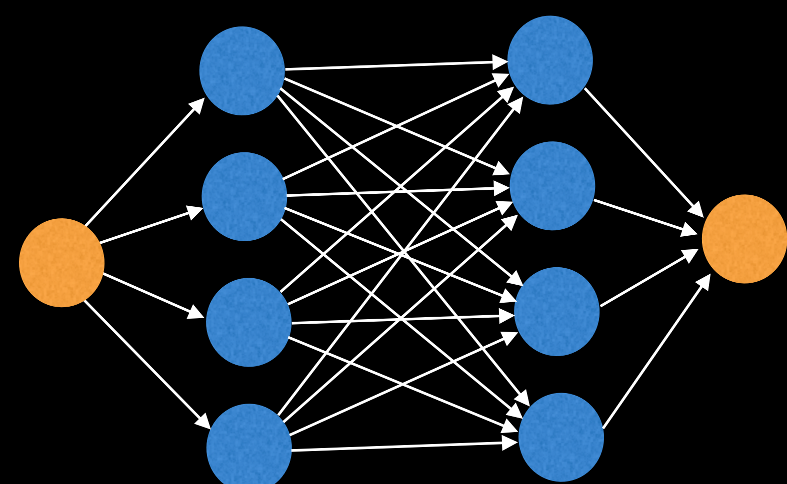


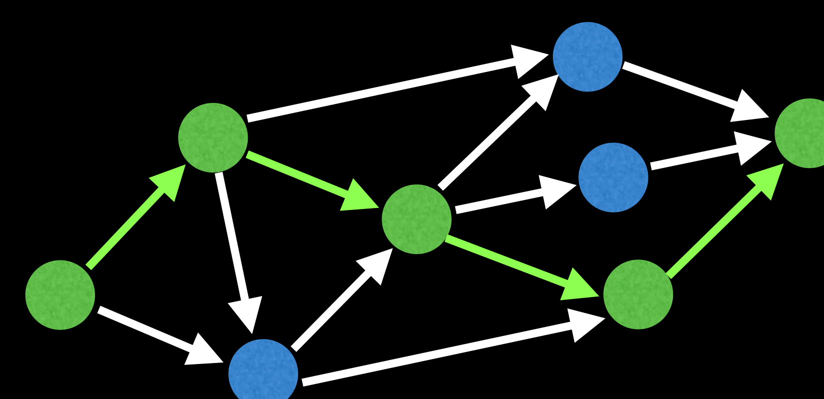




Graph Theory Video Series







Existence of Eulerian Paths and Circuits

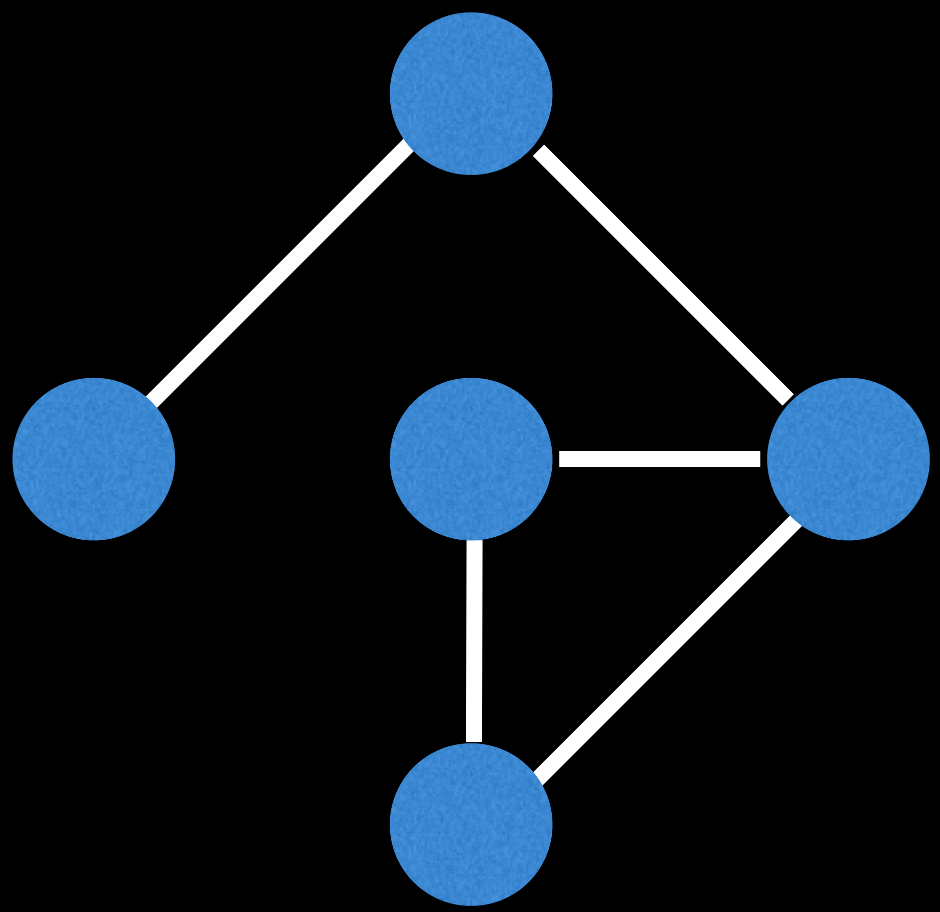
William Fiset

What is an Eulerian Path?

An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.

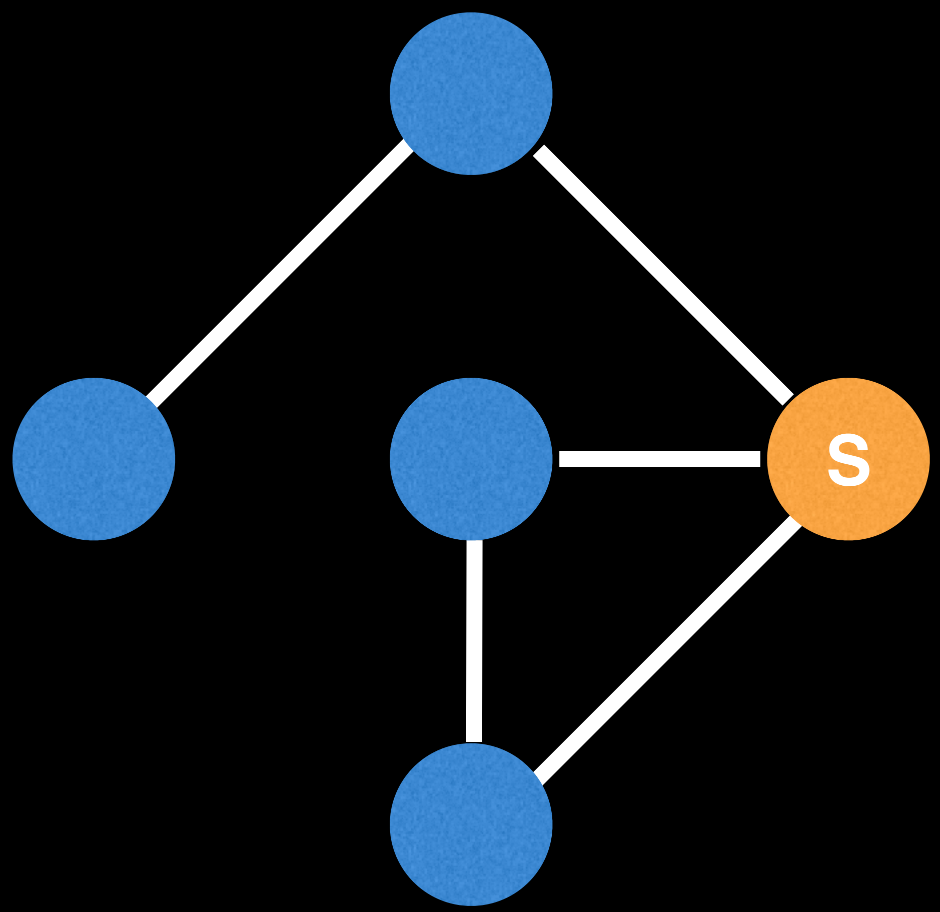
What is an Eulerian Path?

An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



What is an Eulerian Path?

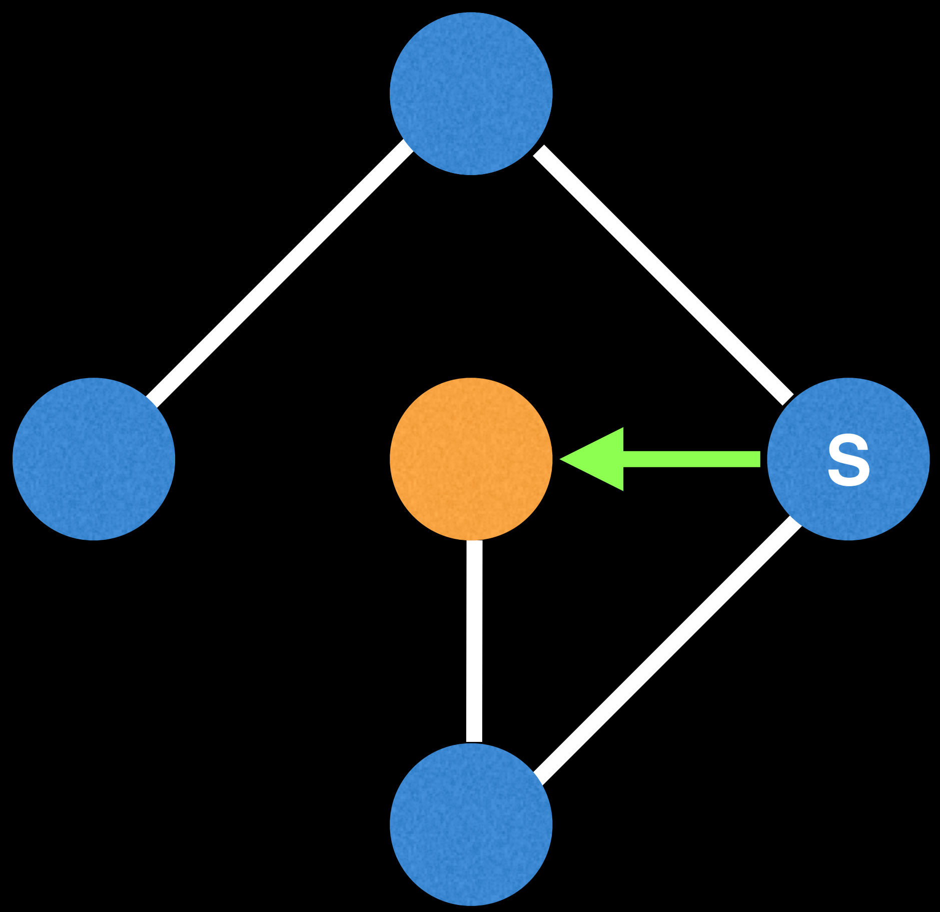
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

What is an Eulerian Path?

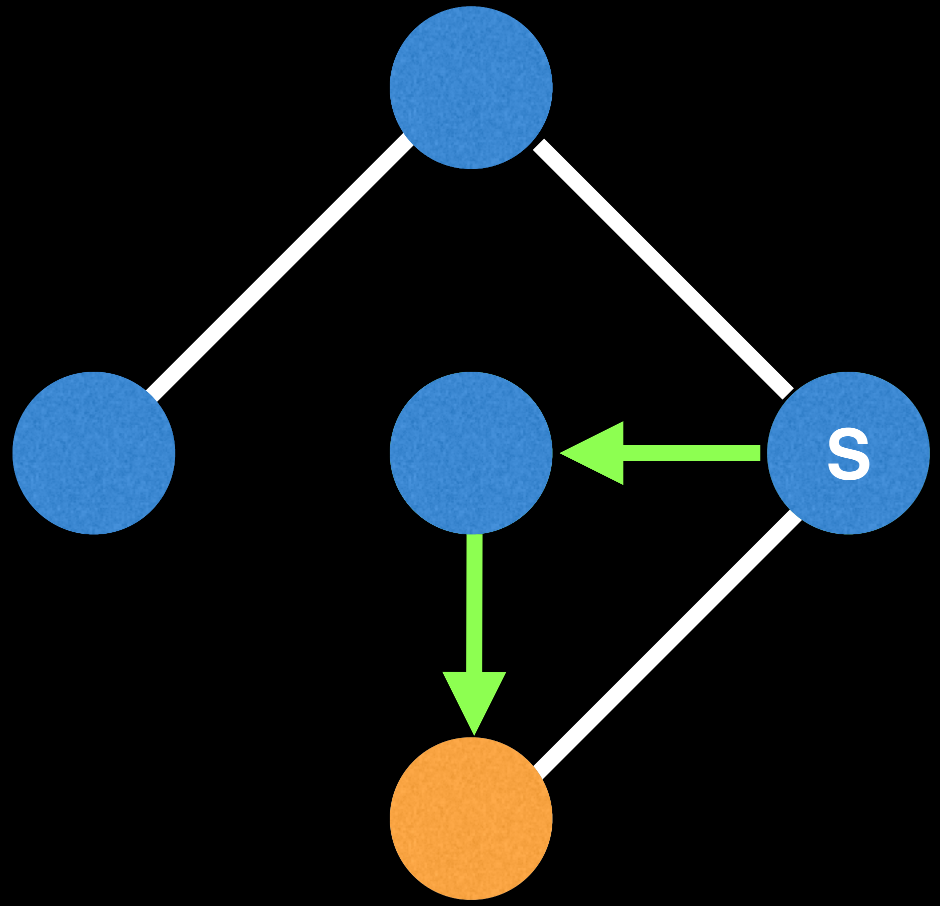
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

What is an Eulerian Path?

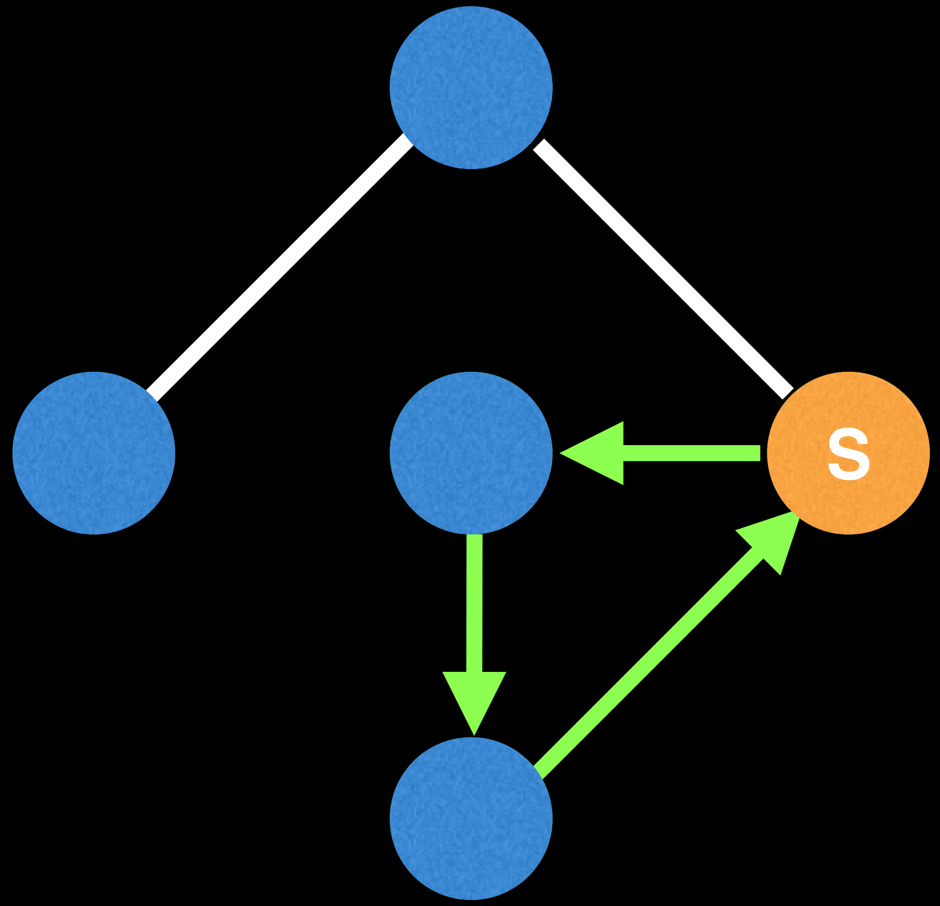
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

What is an Eulerian Path?

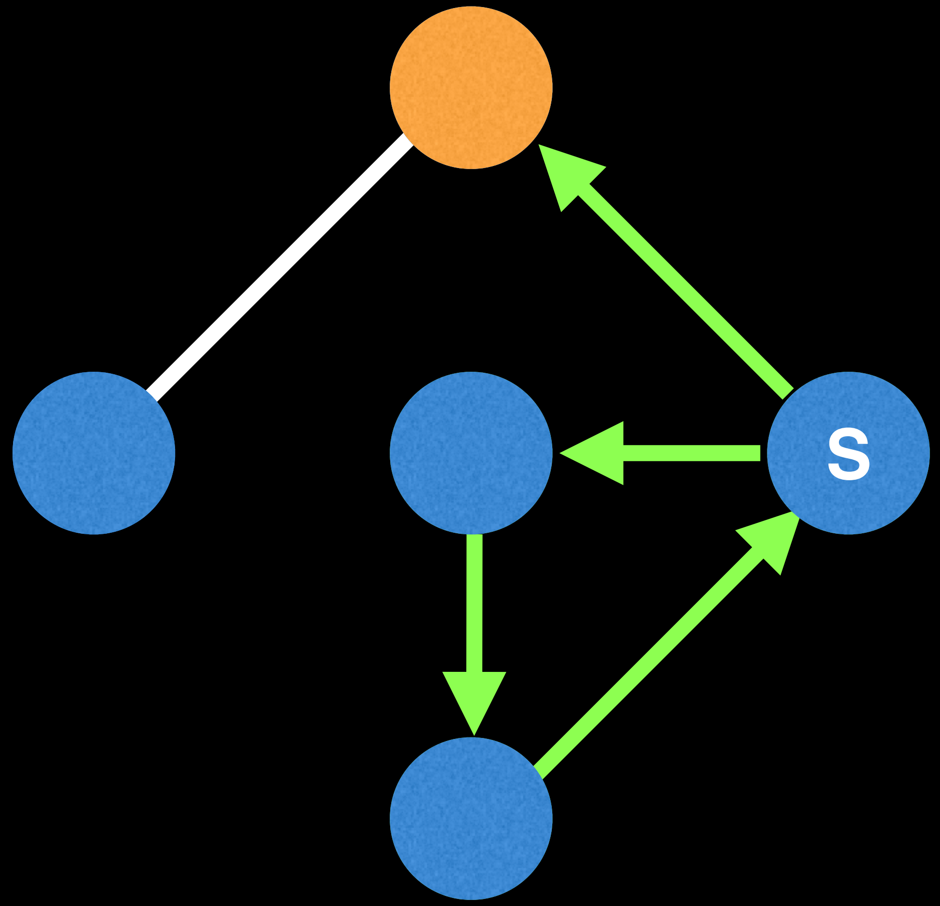
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

What is an Eulerian Path?

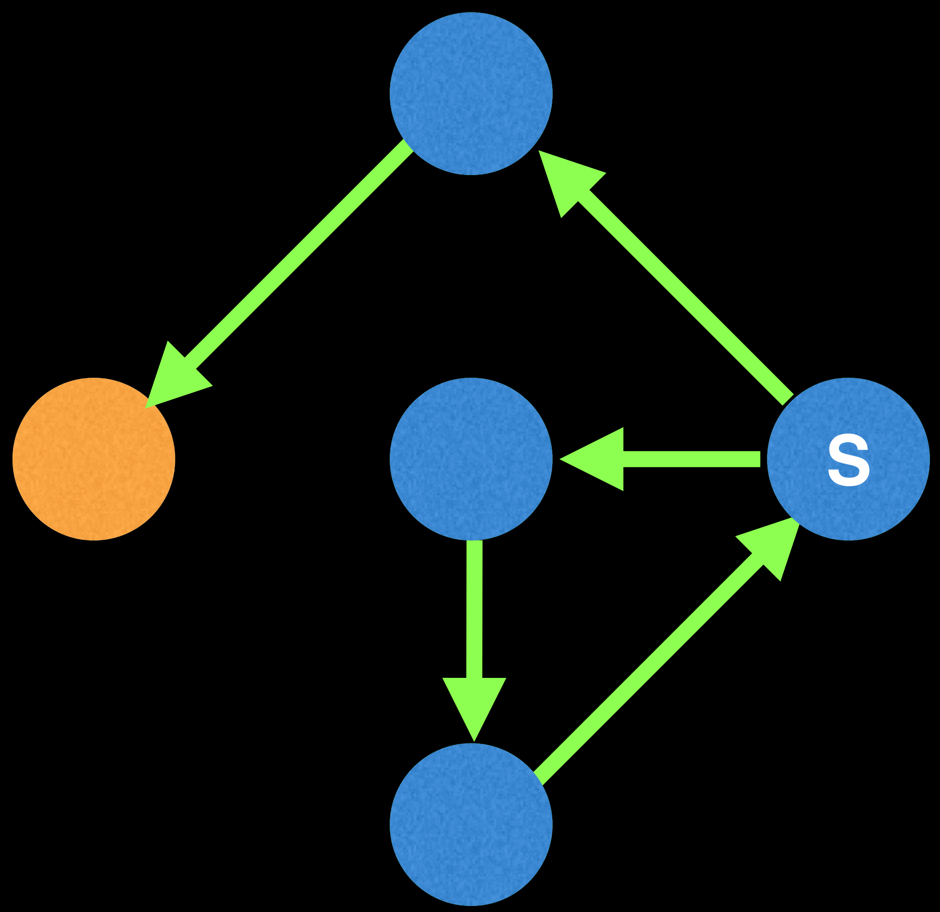
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

What is an Eulerian Path?

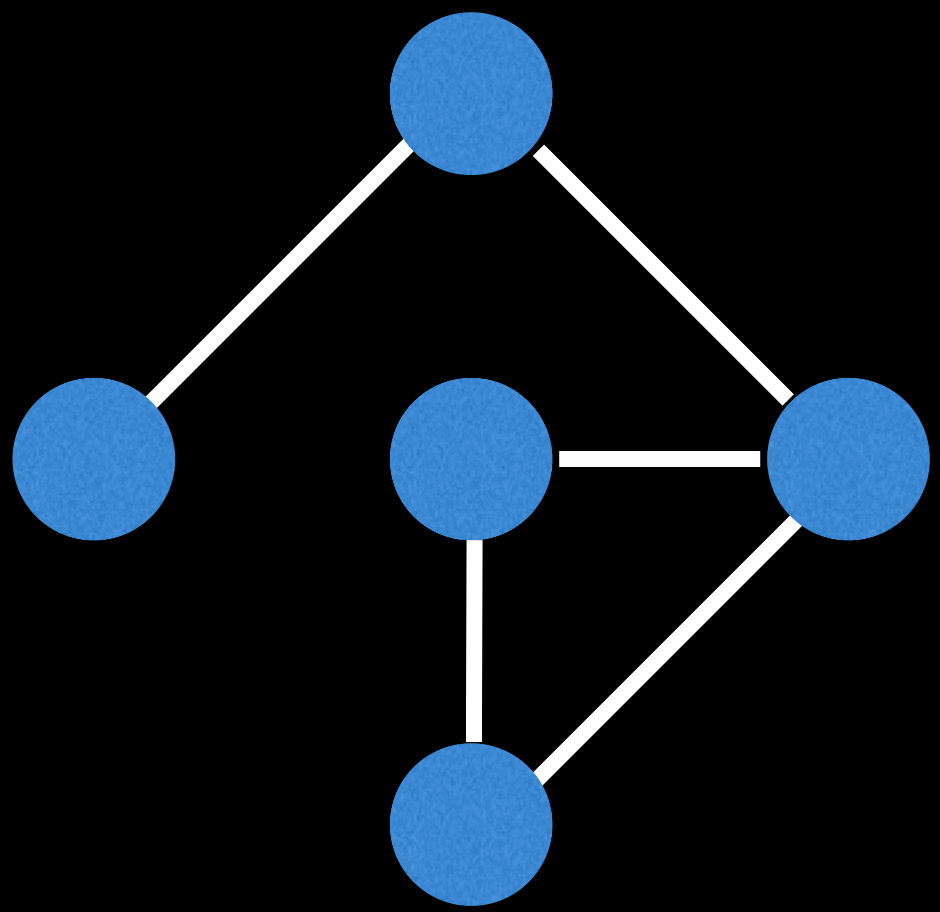
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



We can find an Eulerian path on the following graph, but only if we start at specific nodes.

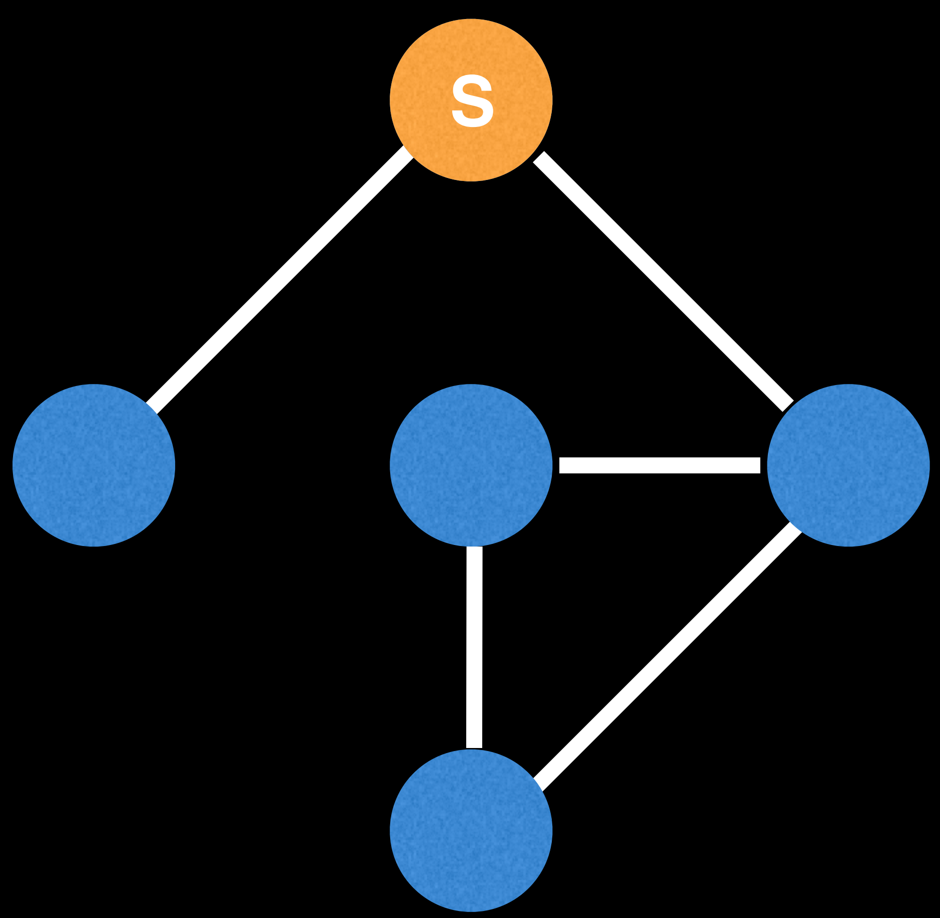
What is an Eulerian Path?

An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



What is an Eulerian Path?

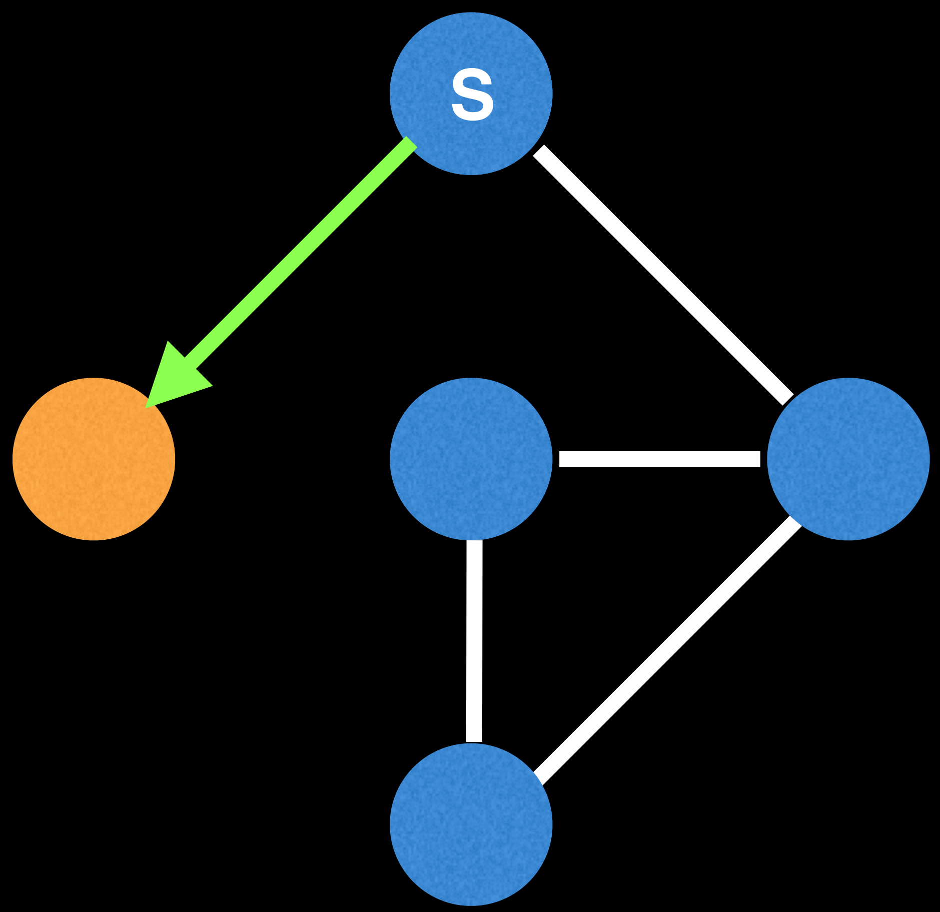
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



Suppose we start another path but this time at a different node.

What is an Eulerian Path?

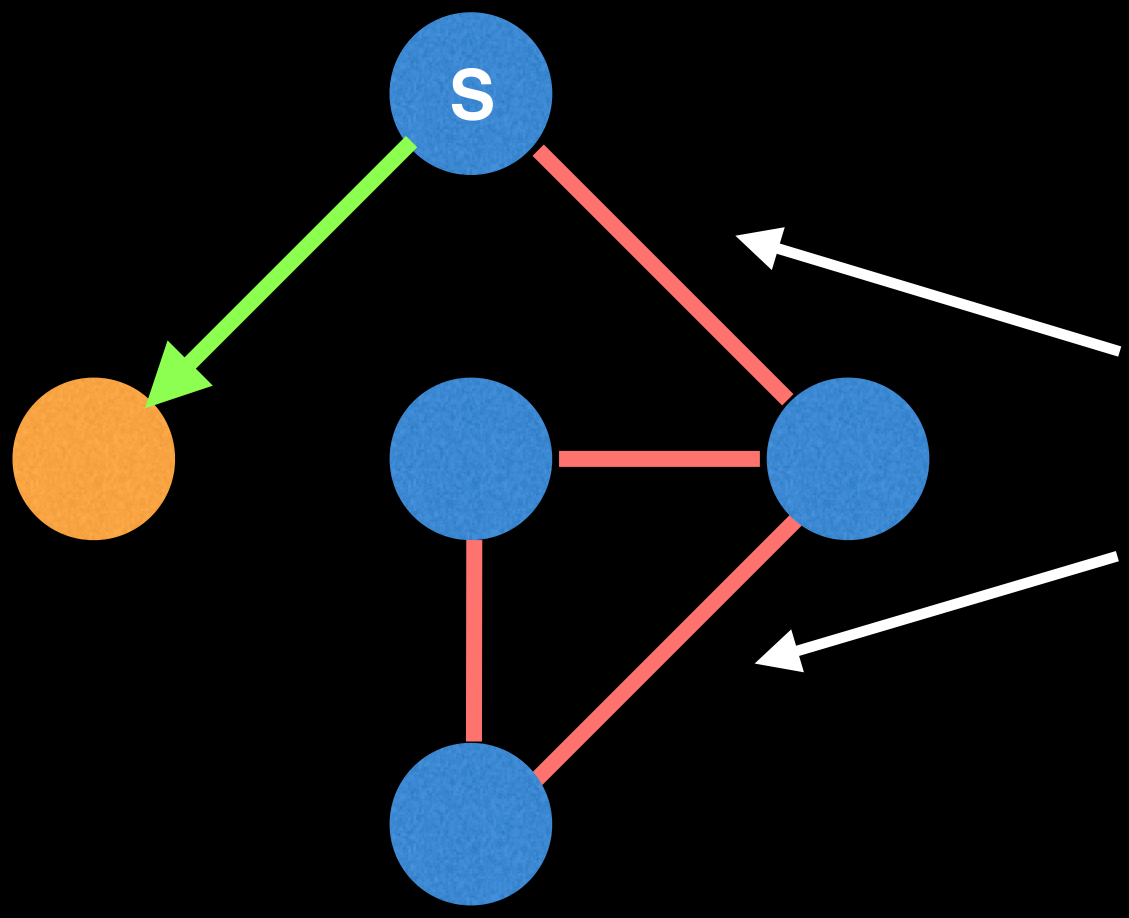
An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



Suppose we start another path but this time at a different node.

What is an Eulerian Path?

An **Eulerian Path** (or Eulerian Trail) is a path of edges that visits all the edges in a graph exactly once.



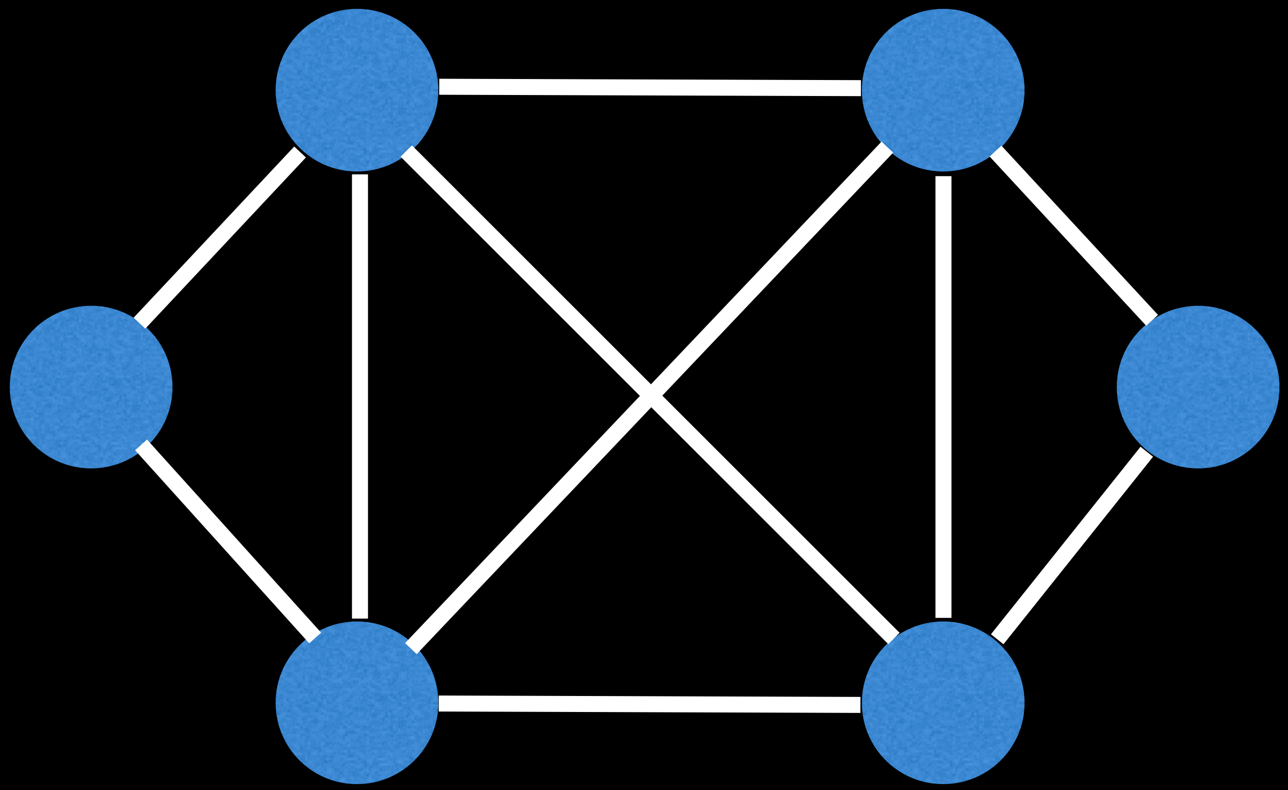
Choosing the wrong starting node can lead to having unreachable edges.

What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.

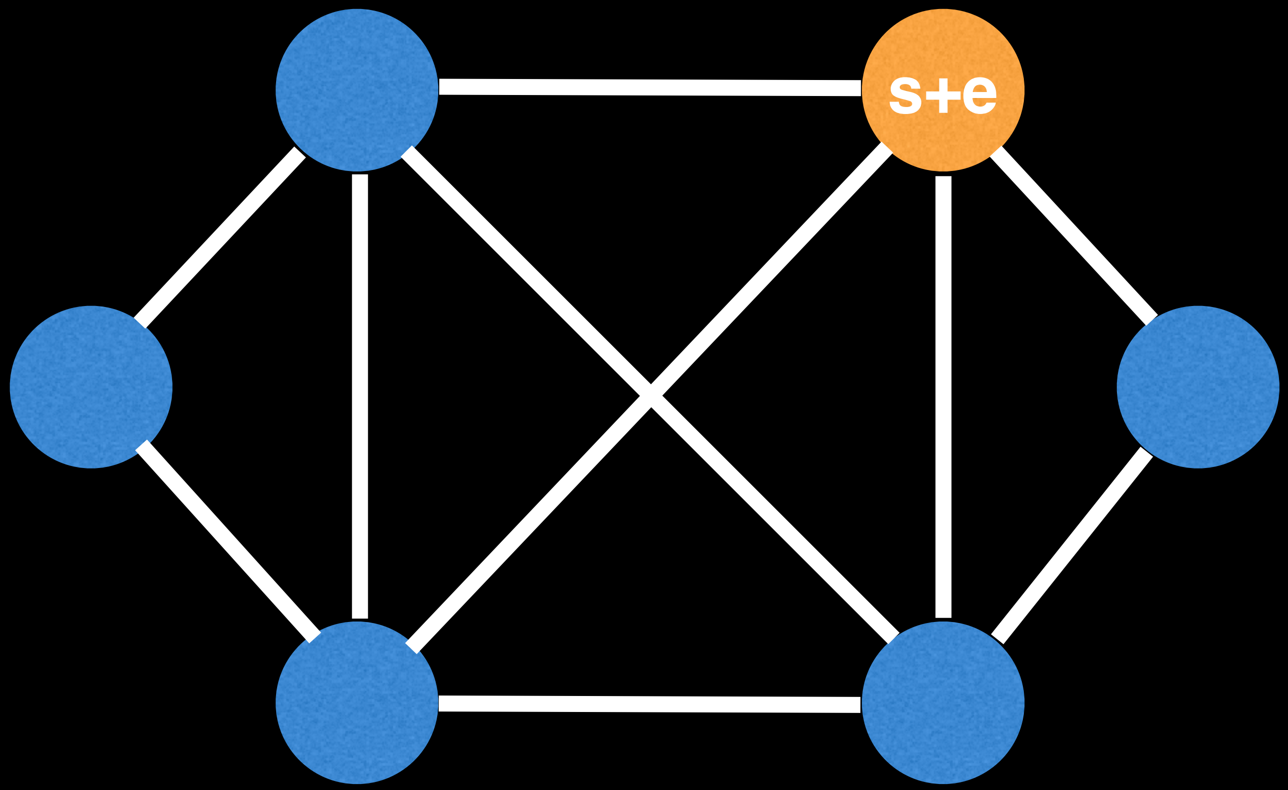
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



What is an Eulerian circuit?

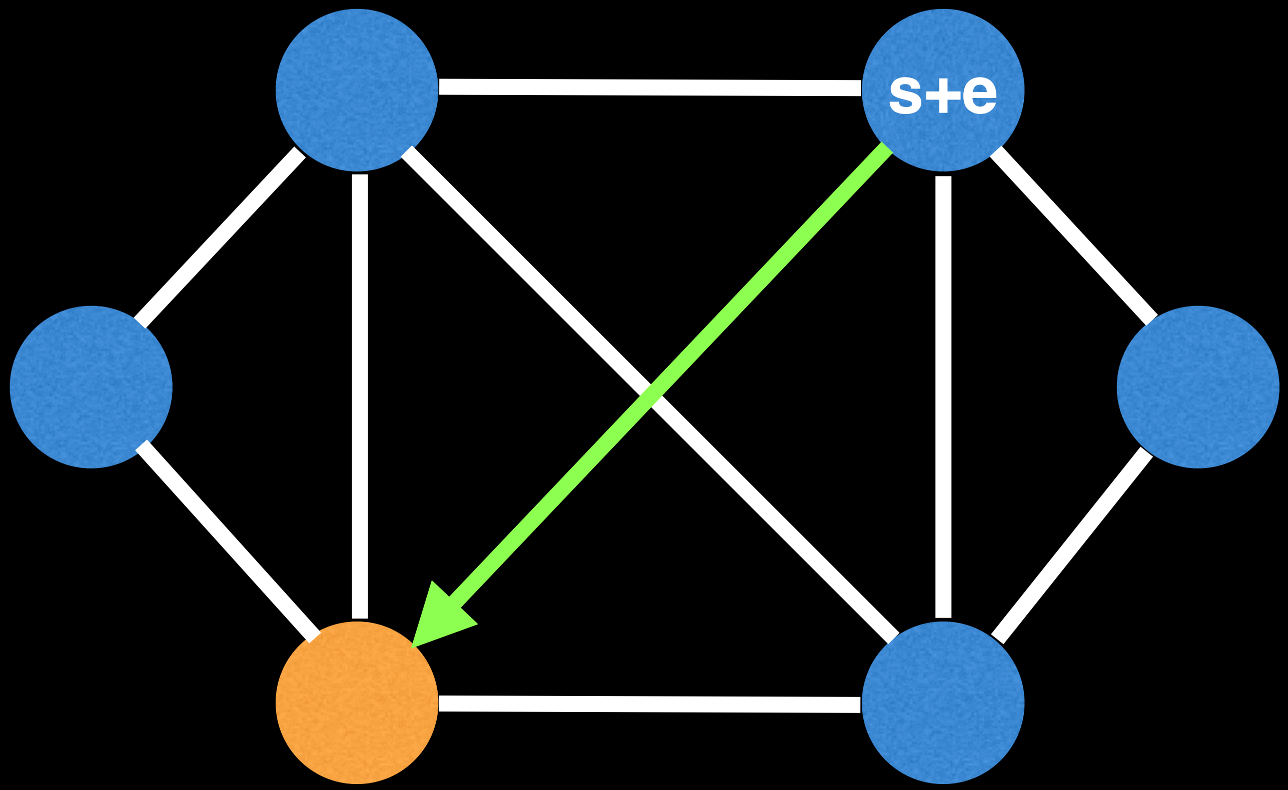
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If you know the graph contains an Eulerian cycle then you can start anywhere.

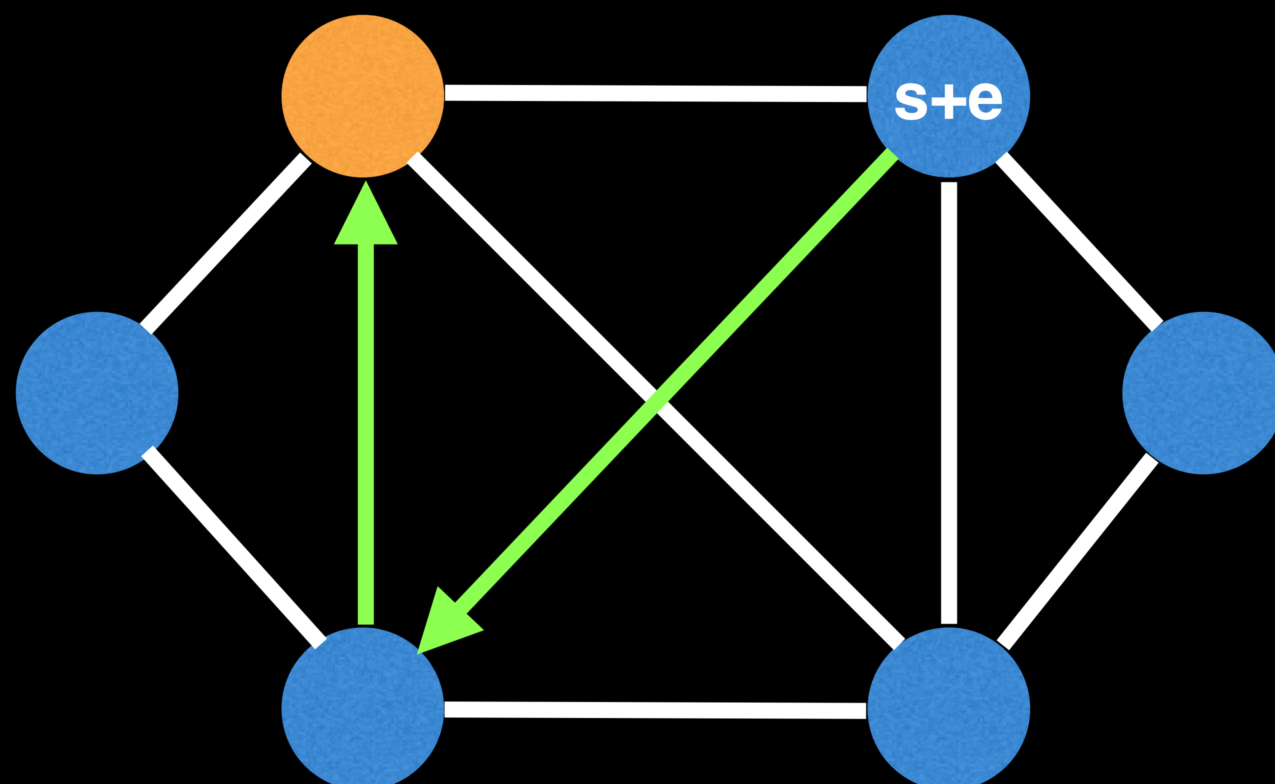
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



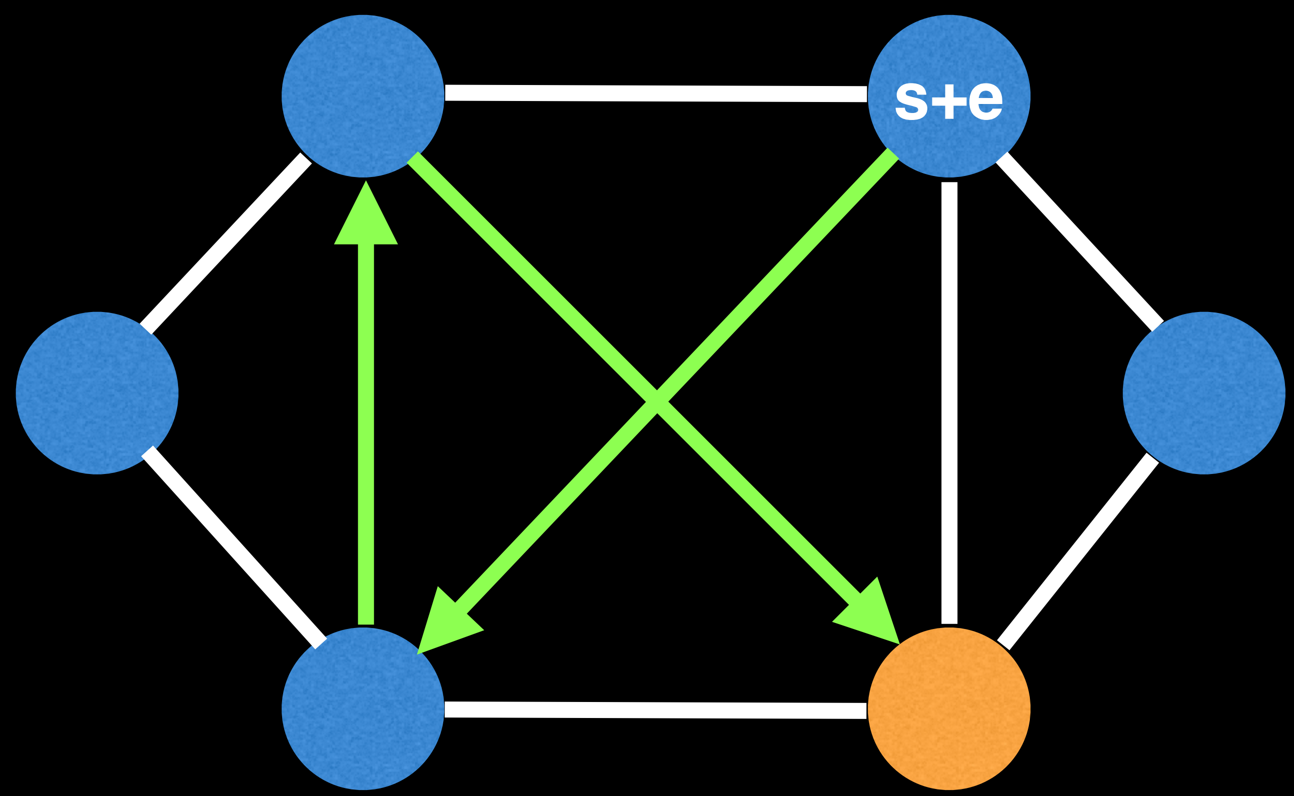
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



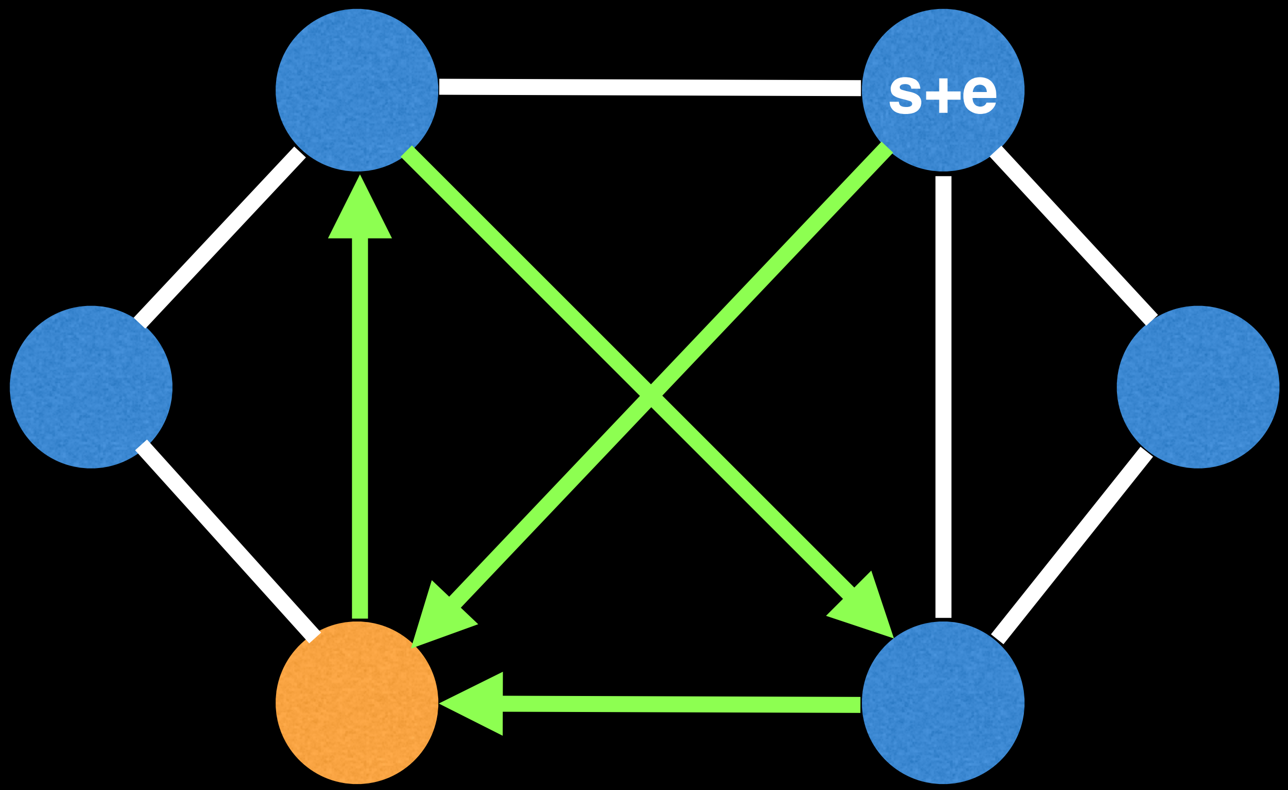
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



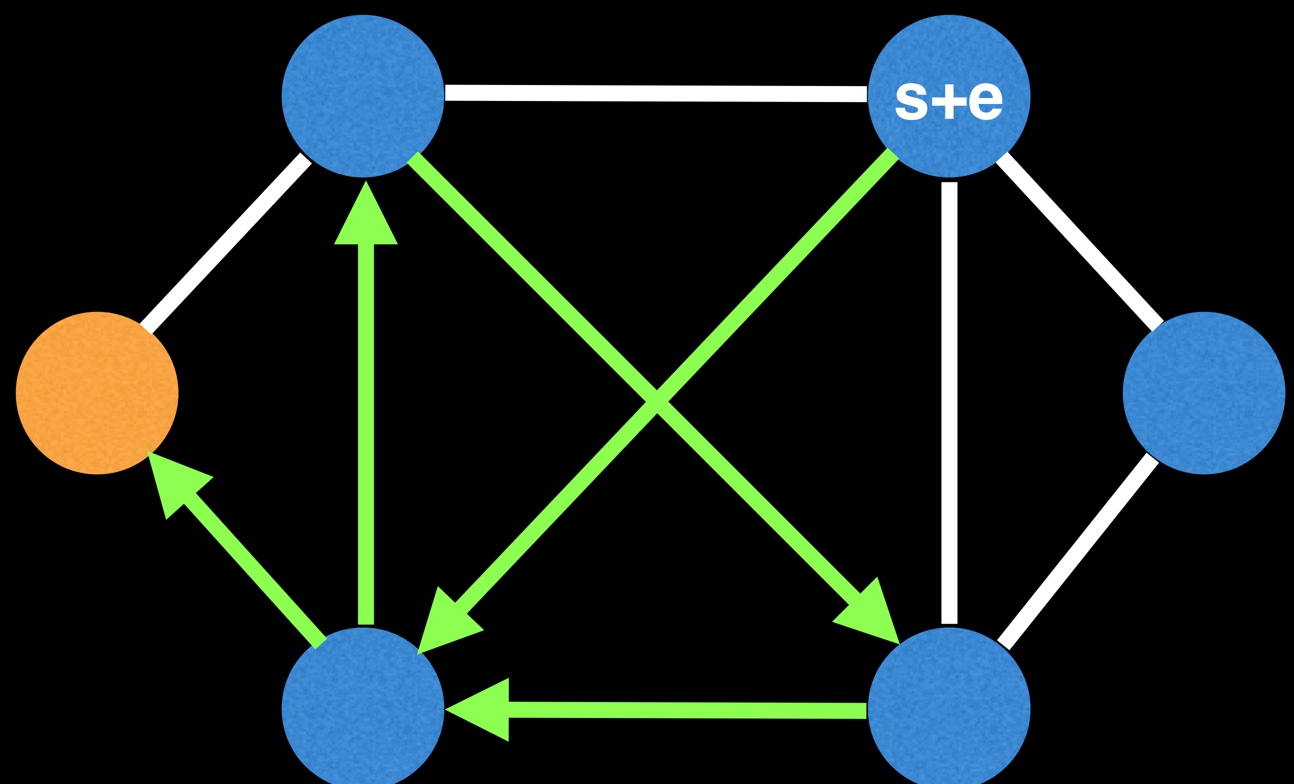
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



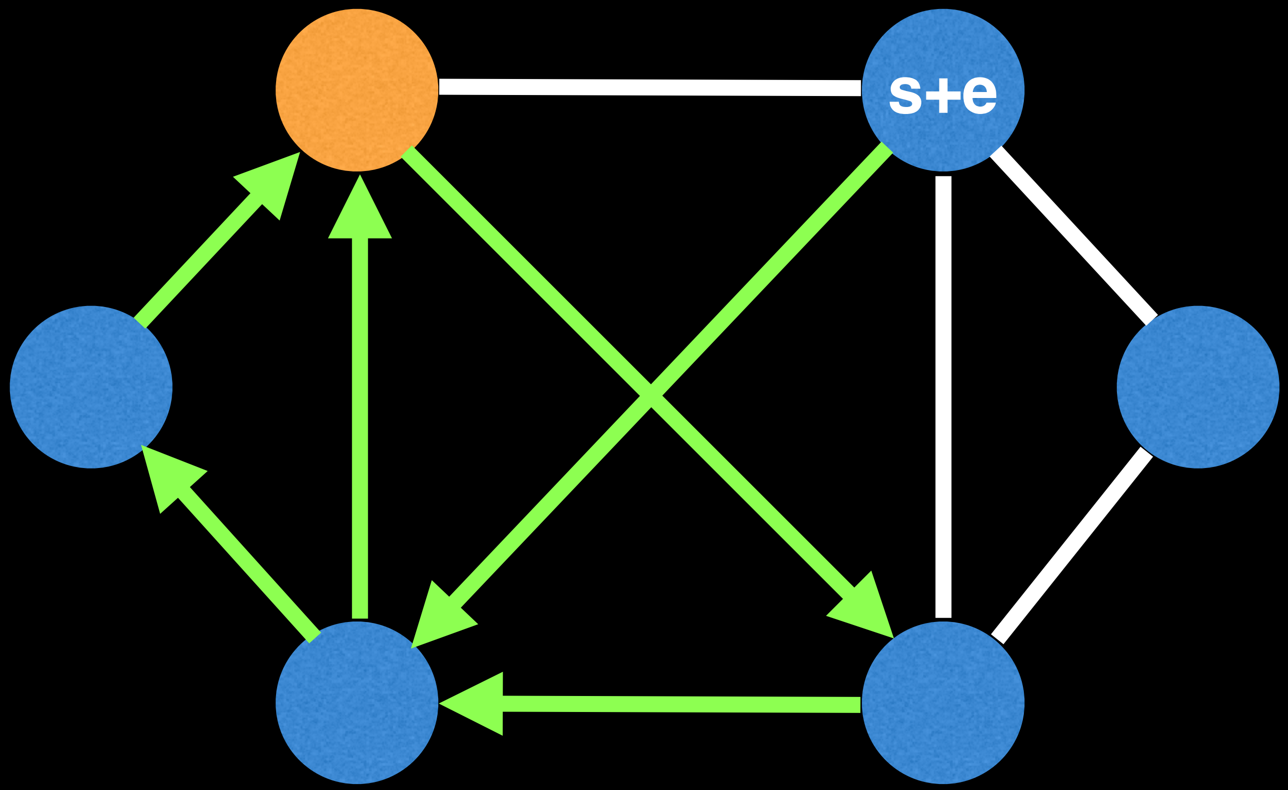
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



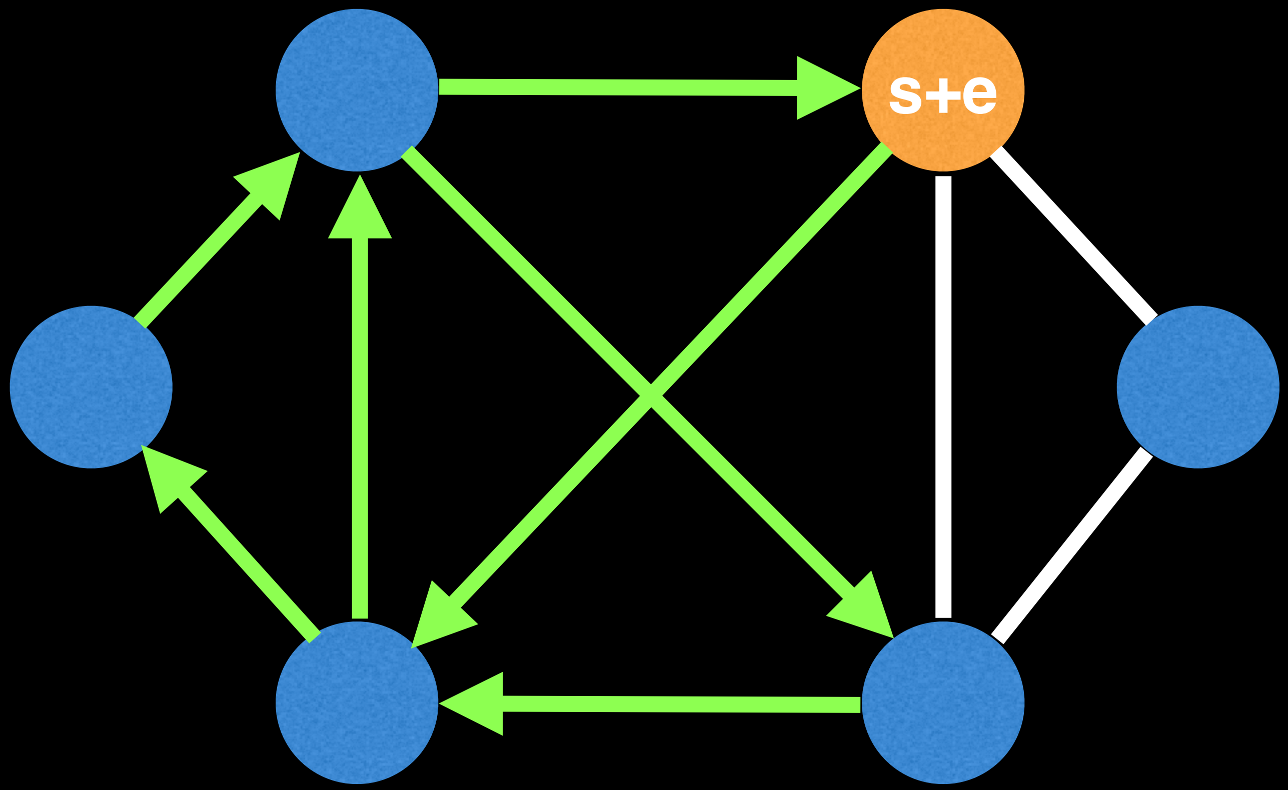
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



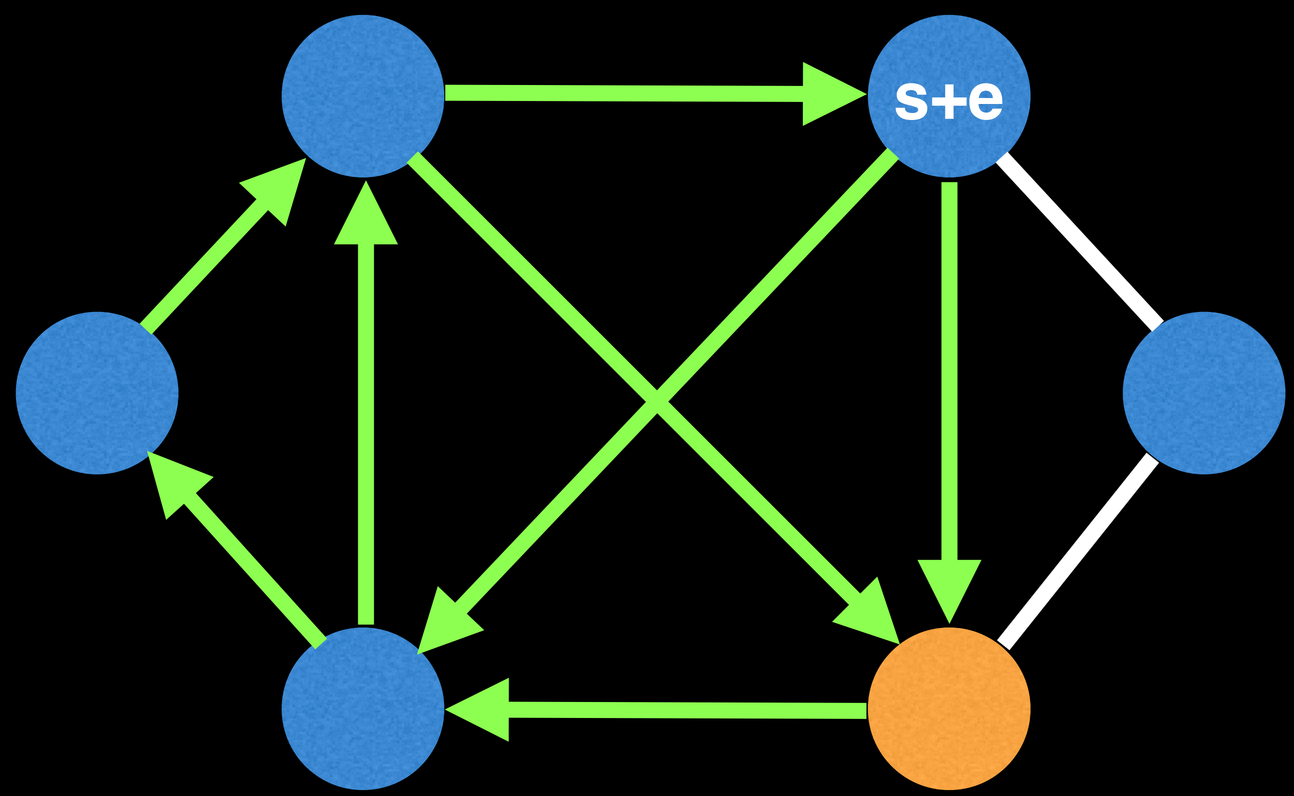
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



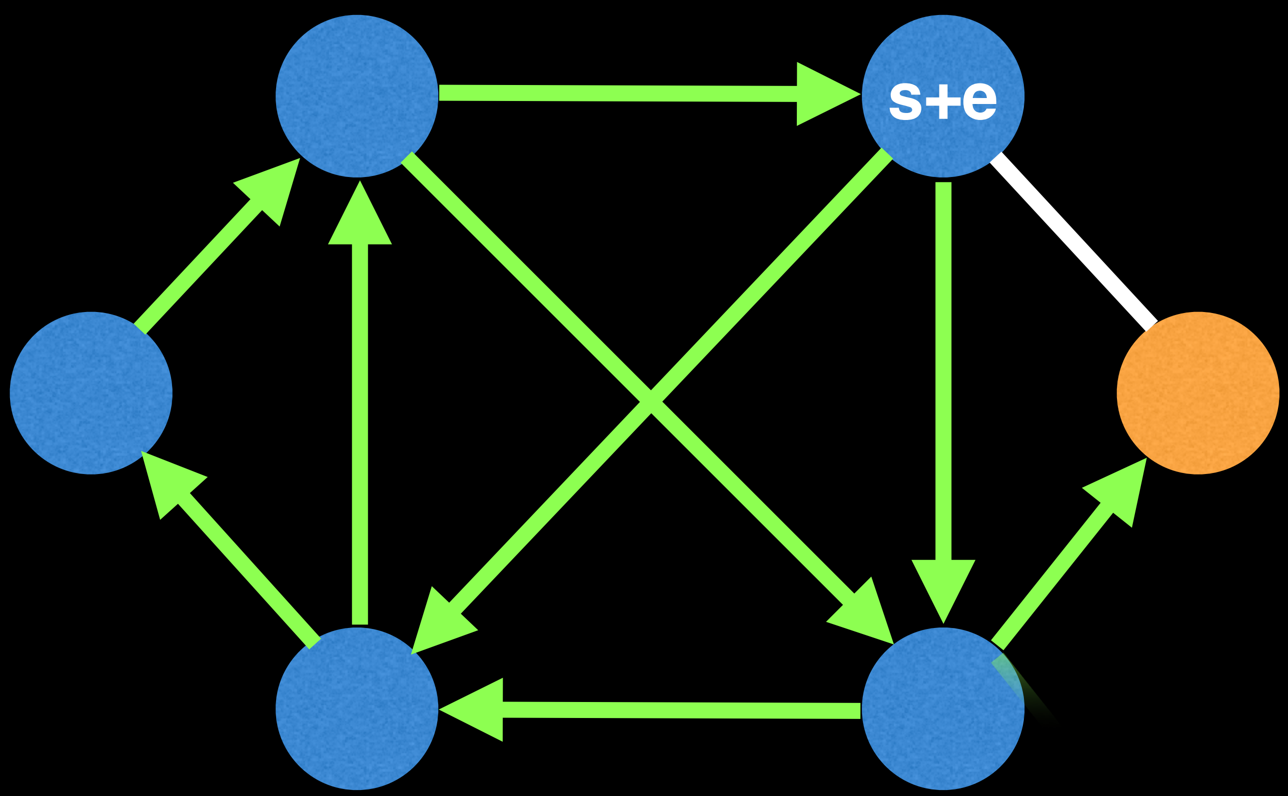
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



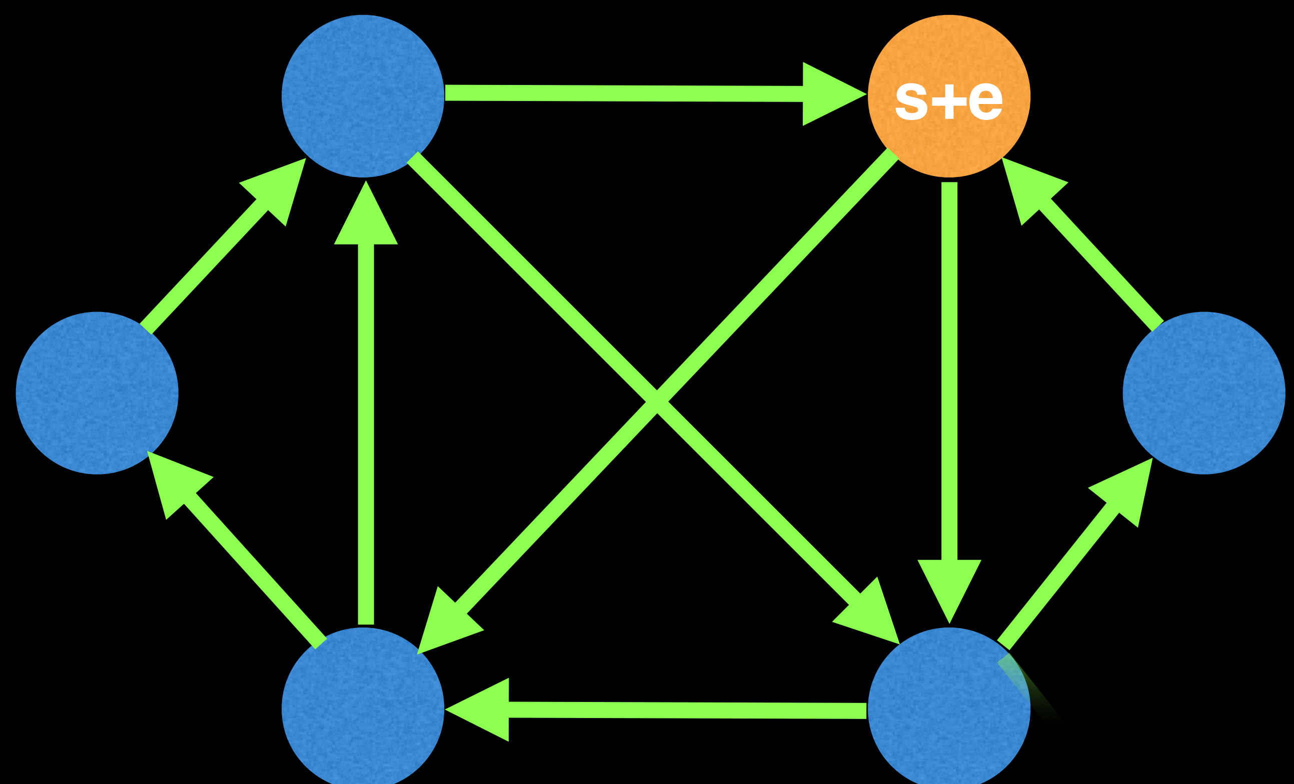
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



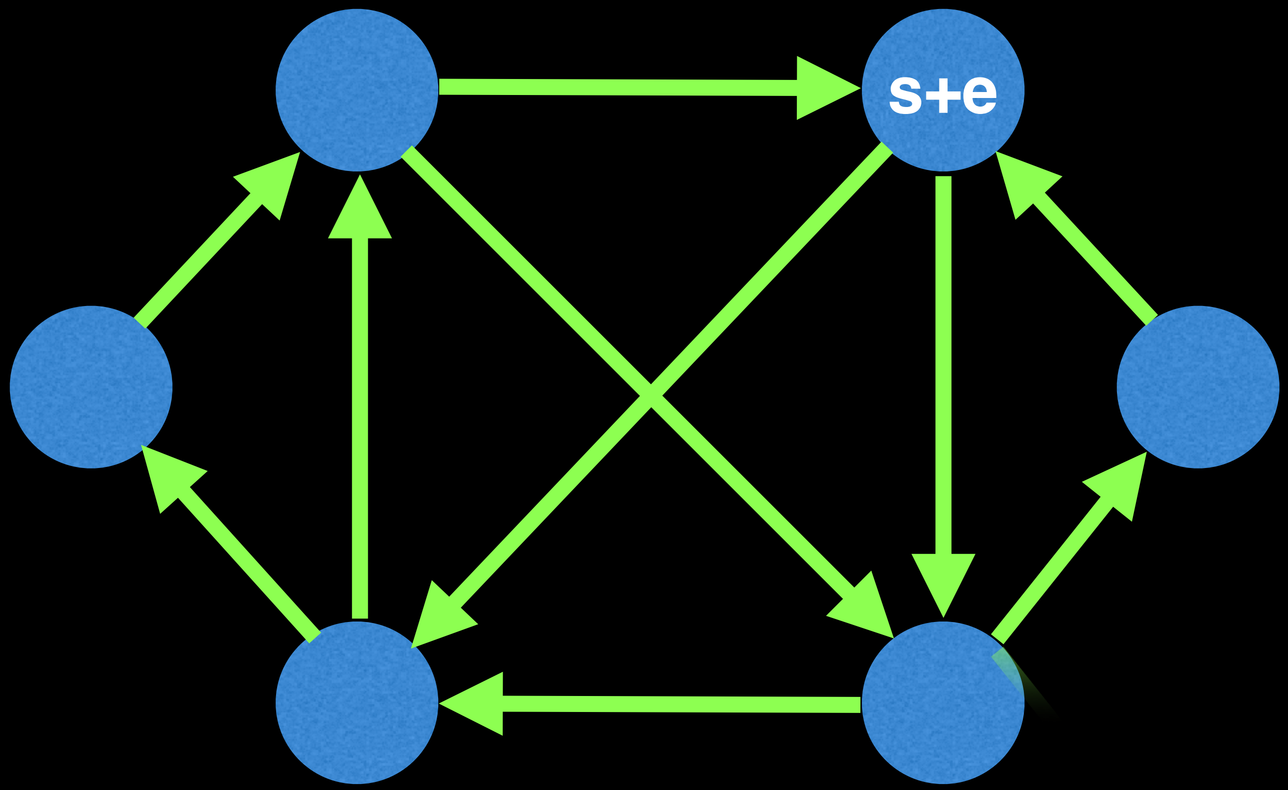
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



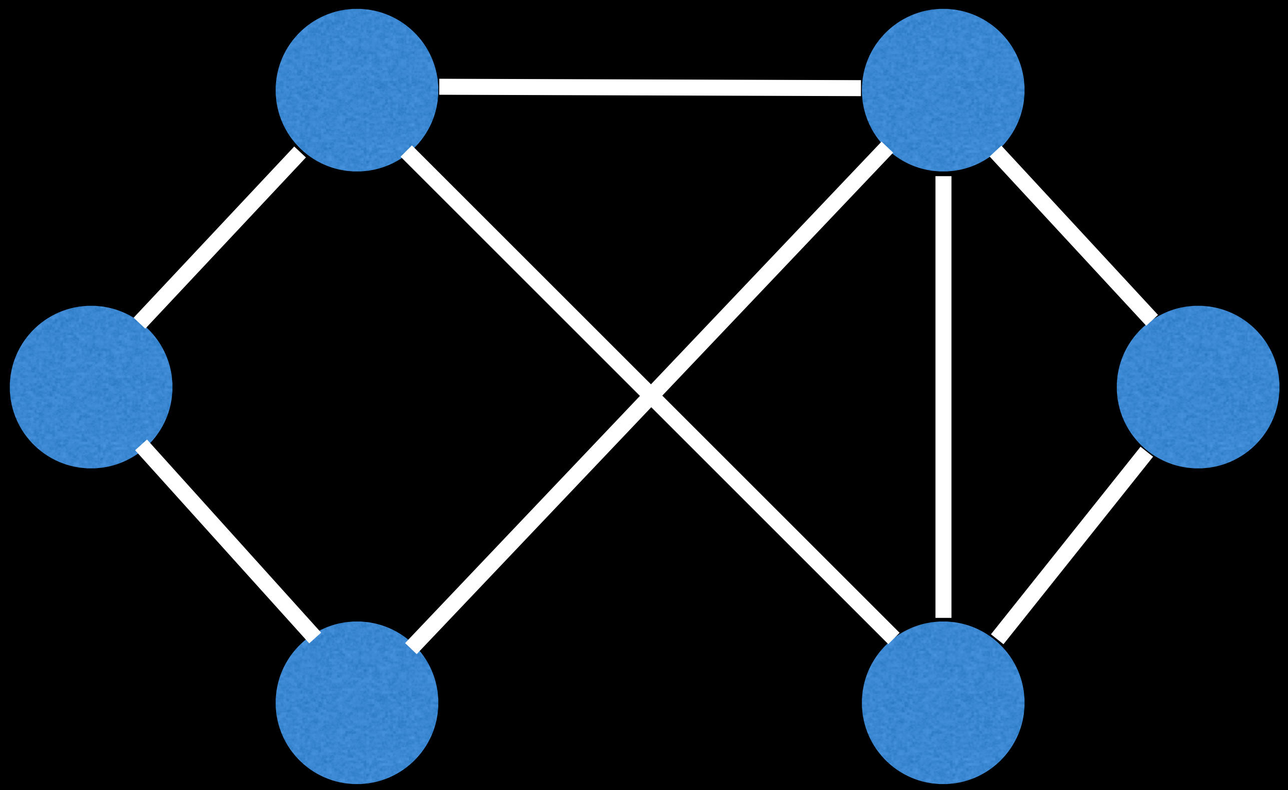
What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



What is an Eulerian circuit?

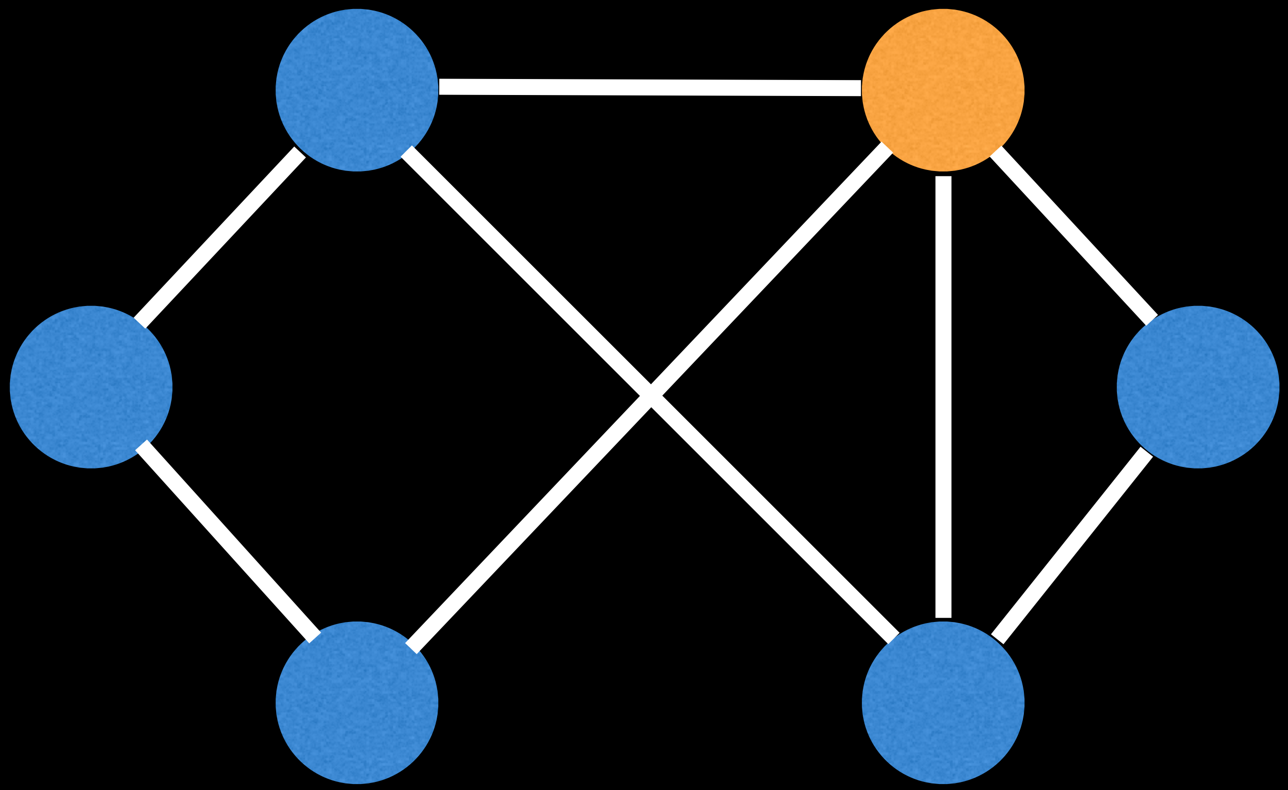
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

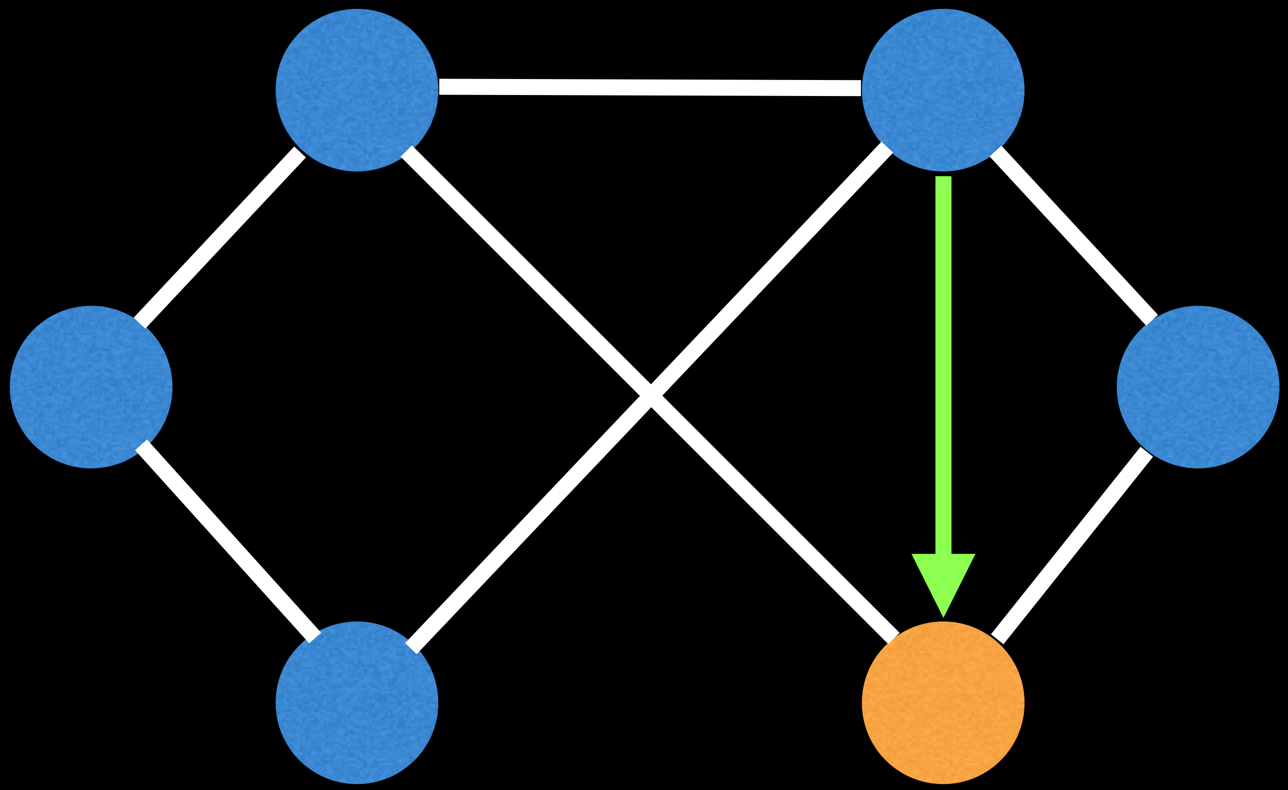
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

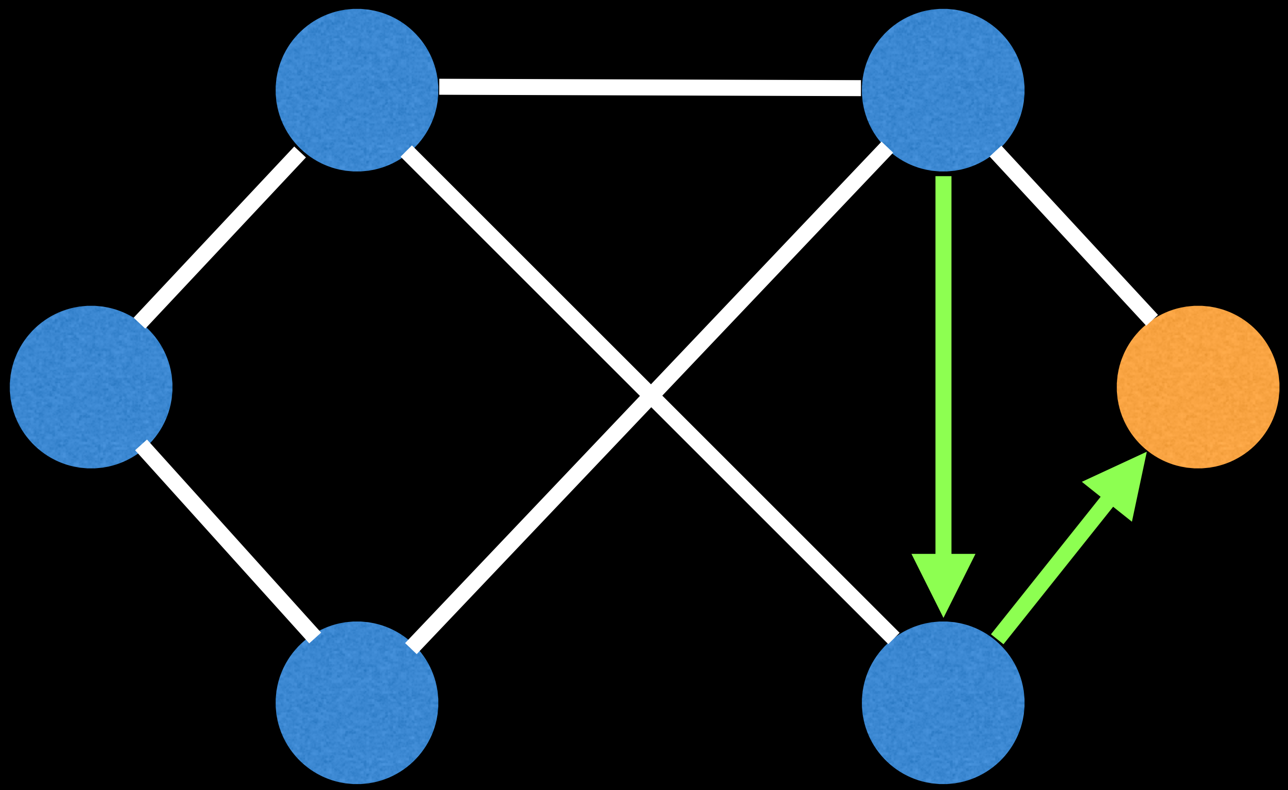
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

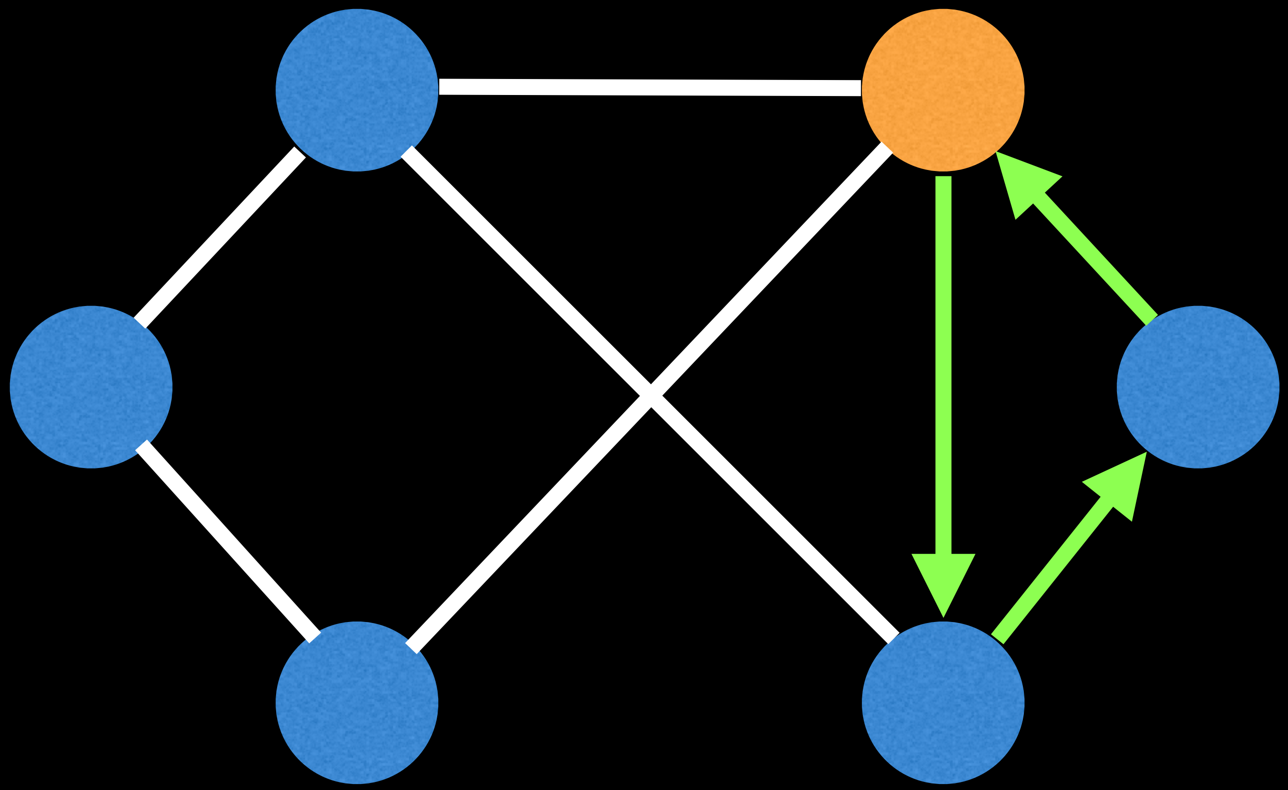
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

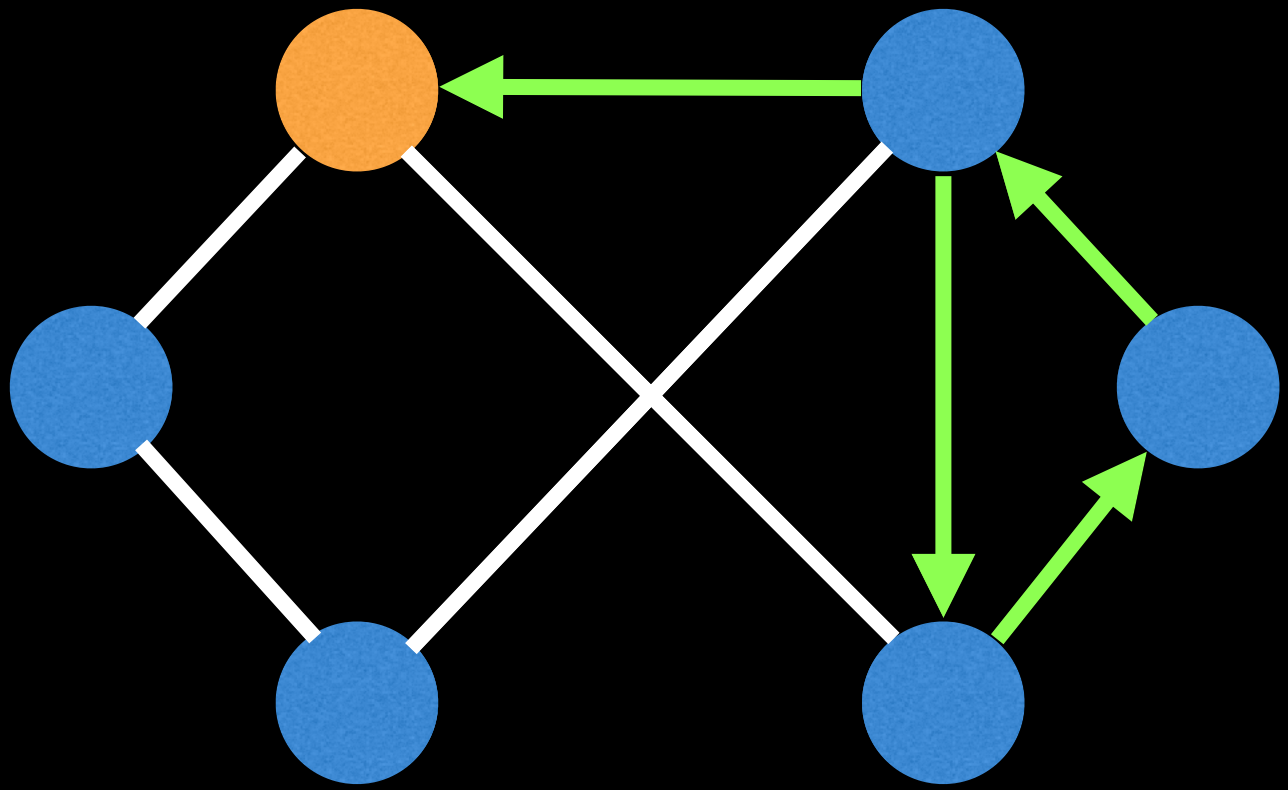
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

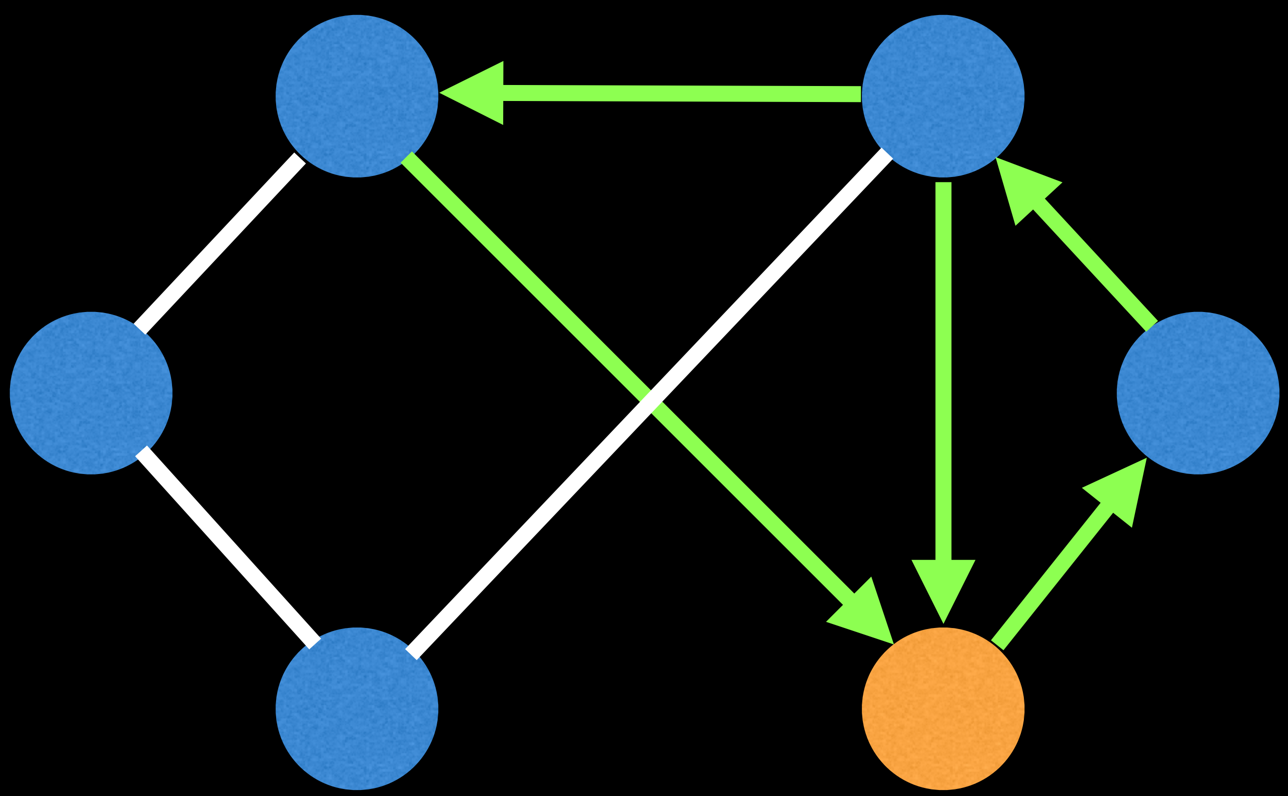
Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.



If your graph does not contain an Eulerian cycle then you may not be able to return to the start node or you will not be able to visit all edges of the graph.

What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.

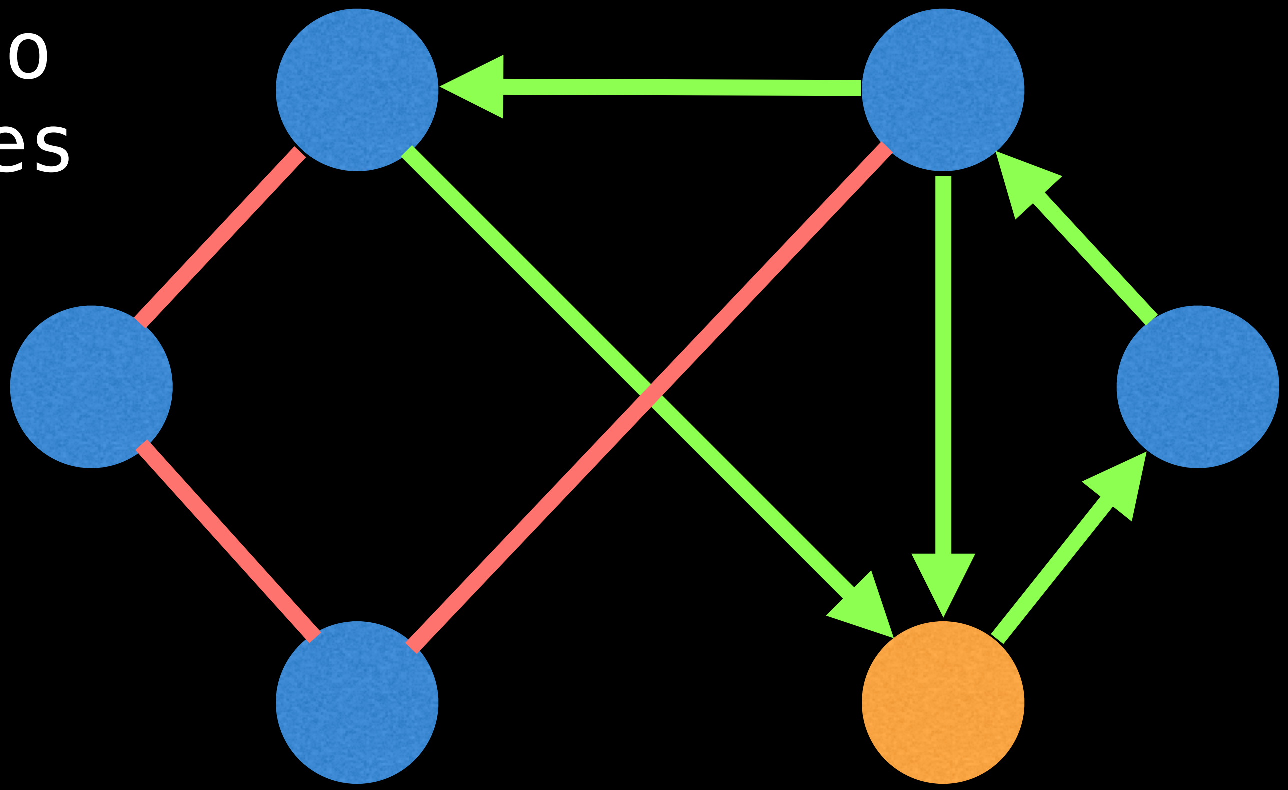


Oops, we’re stuck and can’t make it back to start node

What is an Eulerian circuit?

Similarly, an **Eulerian circuit** (or Eulerian cycle) is an Eulerian path which starts and ends on the same vertex.

There are also unvisited edges remaining



Oops, we're stuck and can't make it back to start node

What conditions are required for a valid Eulerian Path/Circuit?

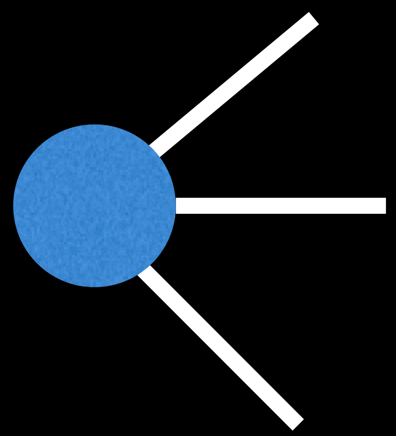
That depends on what kind of graph you're dealing with. Altogether there are four flavors of the Euler path/circuit problem we care about:

	Eulerian Circuit	Eulerian Path
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

Node Degrees

Node Degrees

<u>Undirected graph</u>

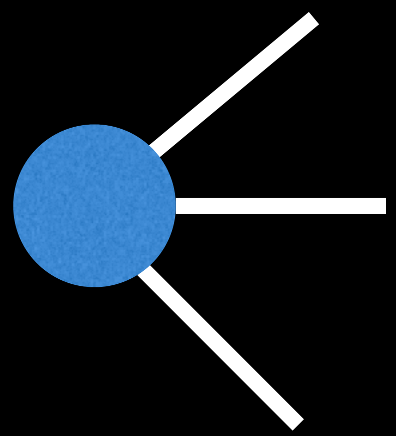


Node degree = 3

The degree of a node is how many edges are attached to it.

Node Degrees

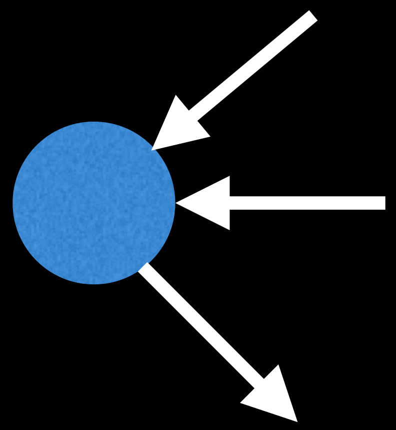
<u>Undirected graph</u>



Node degree = 3

The degree of a node is how many edges are attached to it.

<u>Directed graph</u>



In degree = 2
Out degree = 1

The indegree is the number of incoming edges and outdegree is number of outgoing edges.

What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of Euler path/circuit problem we care about:

	Eulerian Circuit	**Eulerian Path**
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of Euler path/circuit problem we care about:

	Eulerian Circuit	**Eulerian Path**
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of Euler path/circuit problem we care about:

	Eulerian Circuit	**Eulerian Path**
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of Euler path/circuit problem we care about:

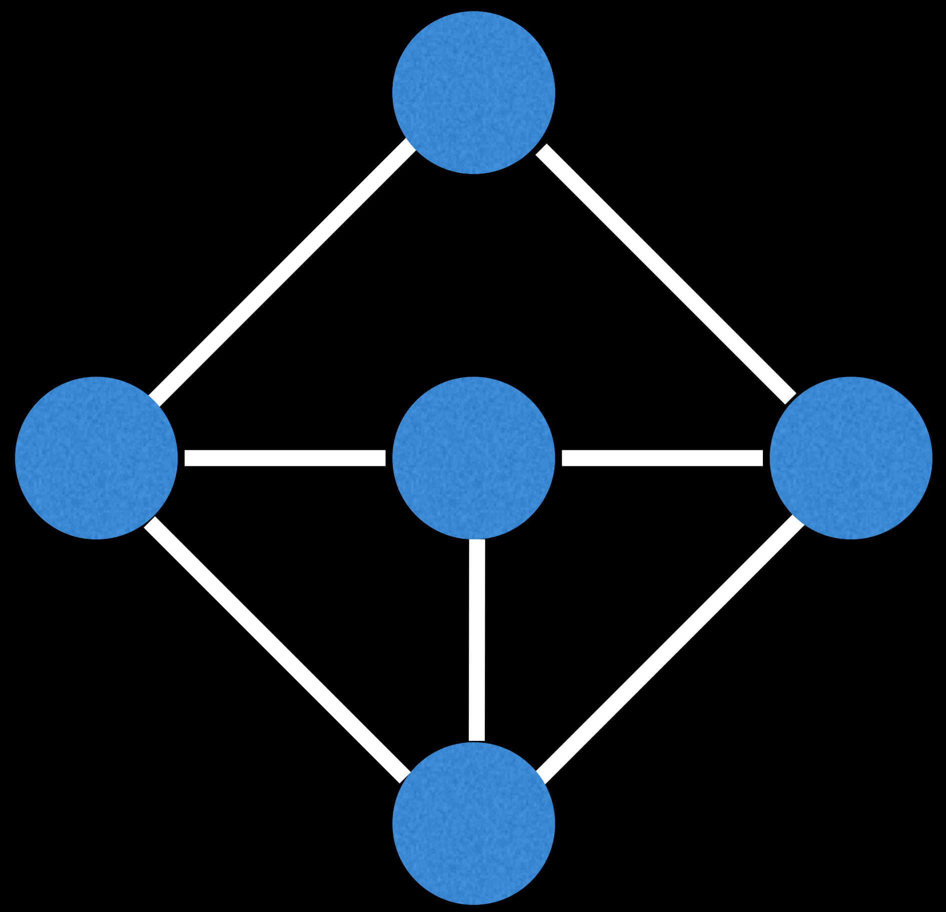
	Eulerian Circuit	Eulerian Path
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

What conditions are required for a valid Eulerian Path/Circuit?

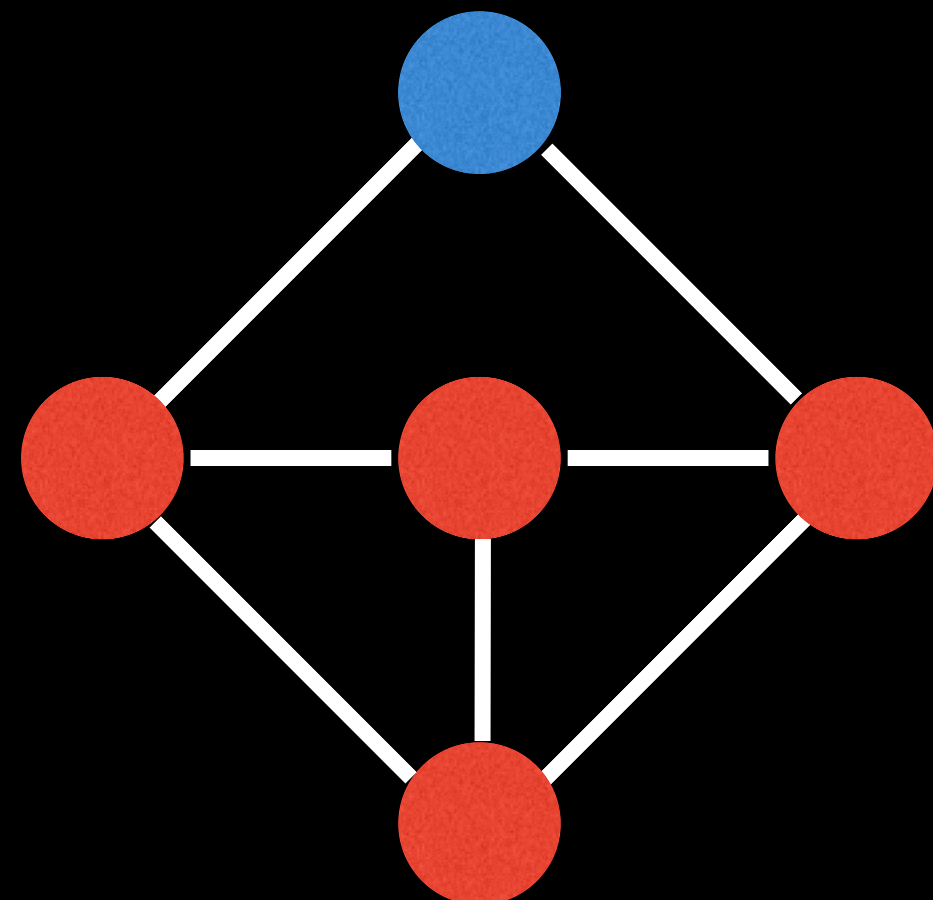
That depends on what kind of graph you're dealing with. Altogether there are four flavors of Euler path/circuit problem we care about:

	Eulerian Circuit	Eulerian Path
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

Does this undirected graph have an Eulerian path/circuit?

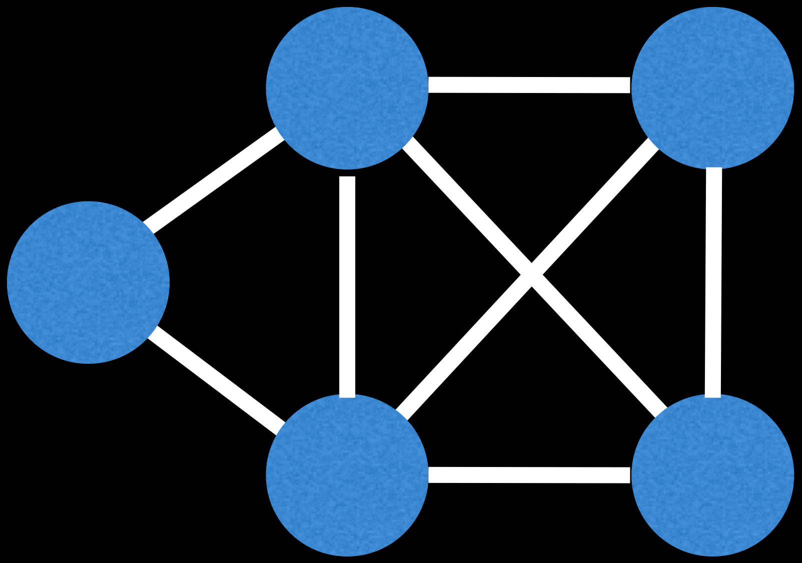


Does this undirected graph have an Eulerian path/circuit?

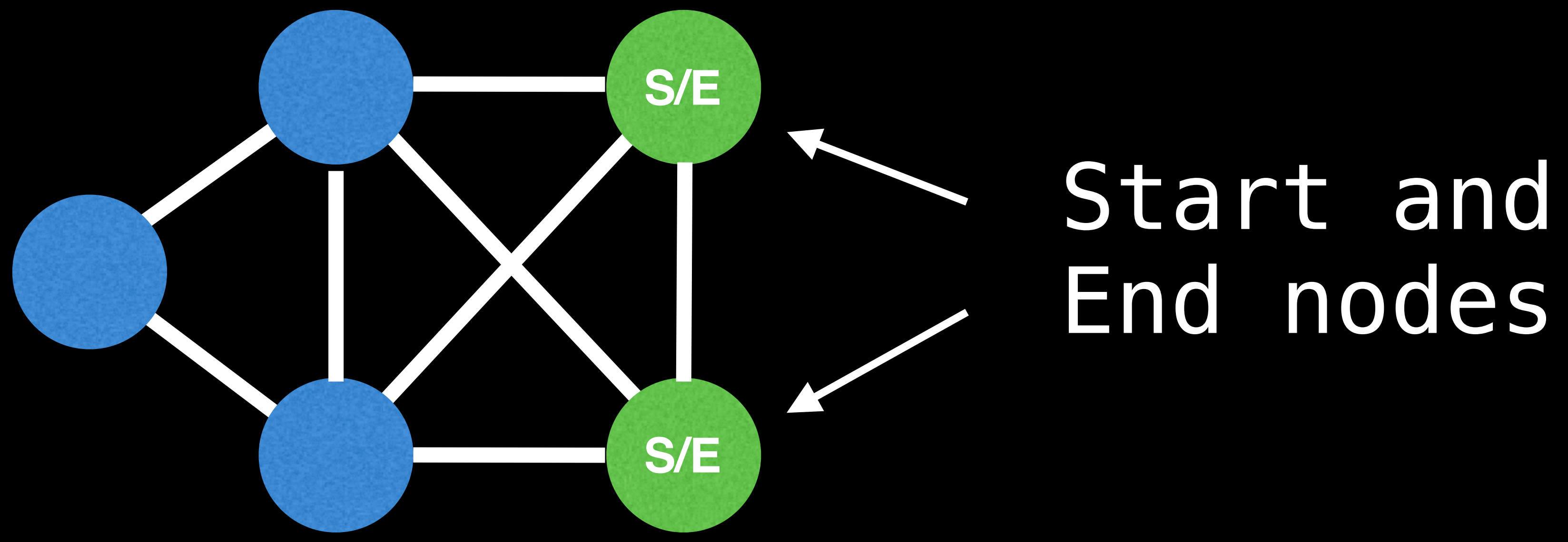


No Eulerian path or circuit.
There are too many nodes that have an odd degree.

Does this undirected graph have an Eulerian path/circuit?

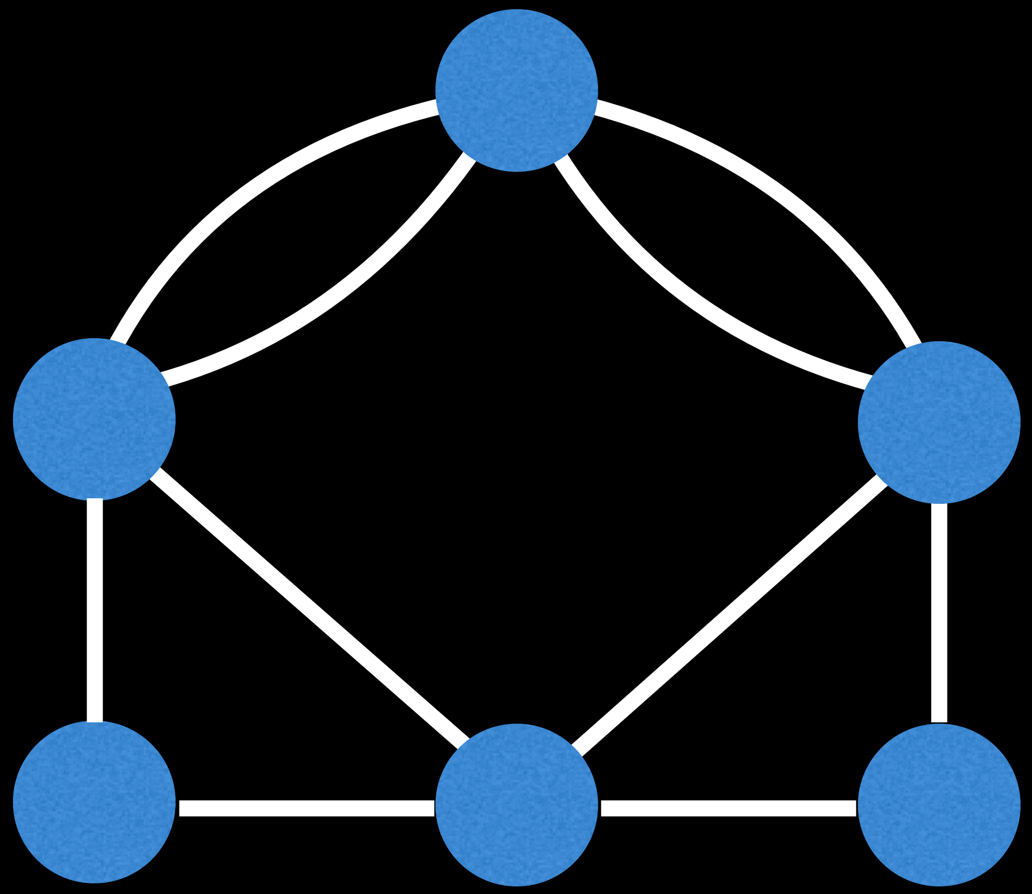


Does this undirected graph have an Eulerian path/circuit?

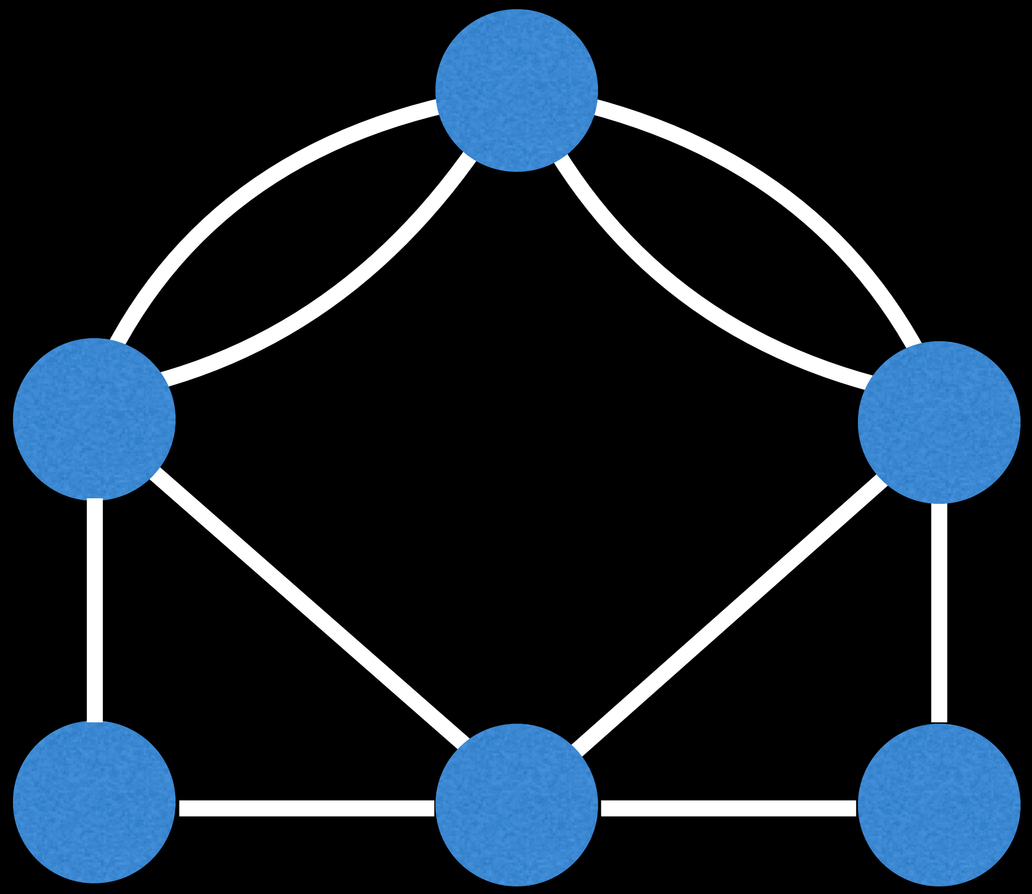


Only Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

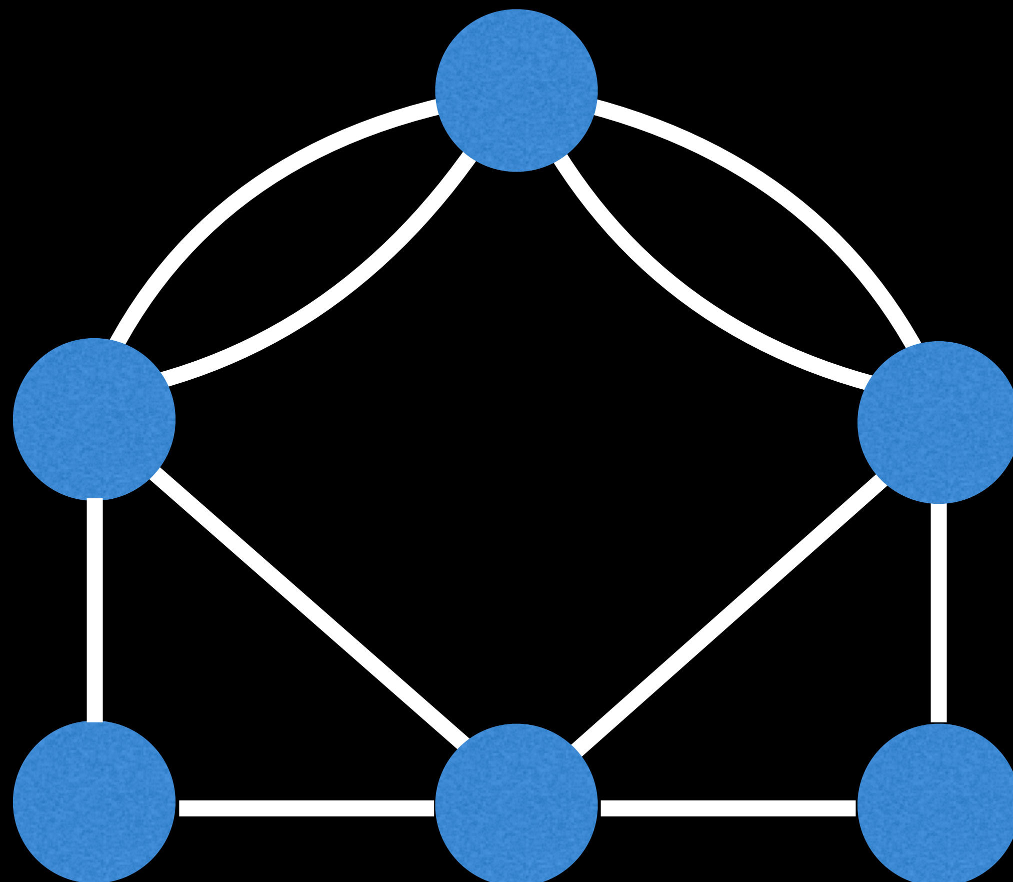


Does this undirected graph have an Eulerian path/circuit?



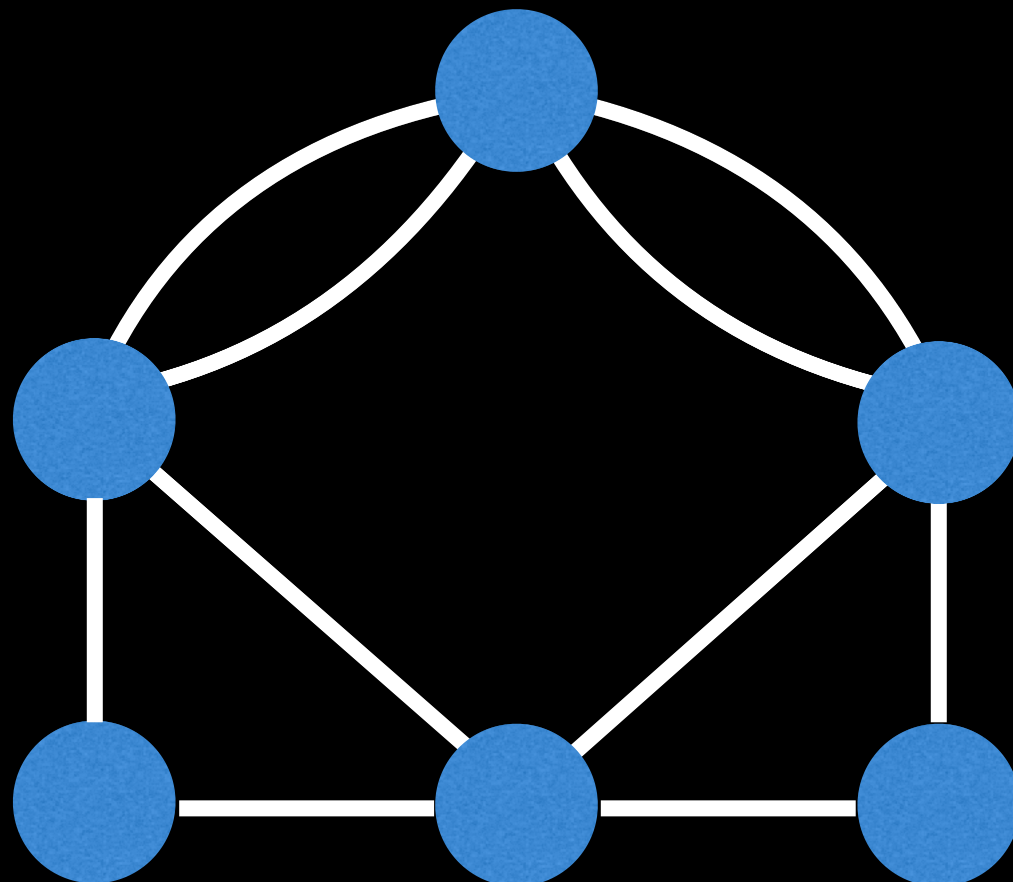
Yes! It has both an Eulerian path and circuit.

Does this undirected graph have an Eulerian path/circuit?



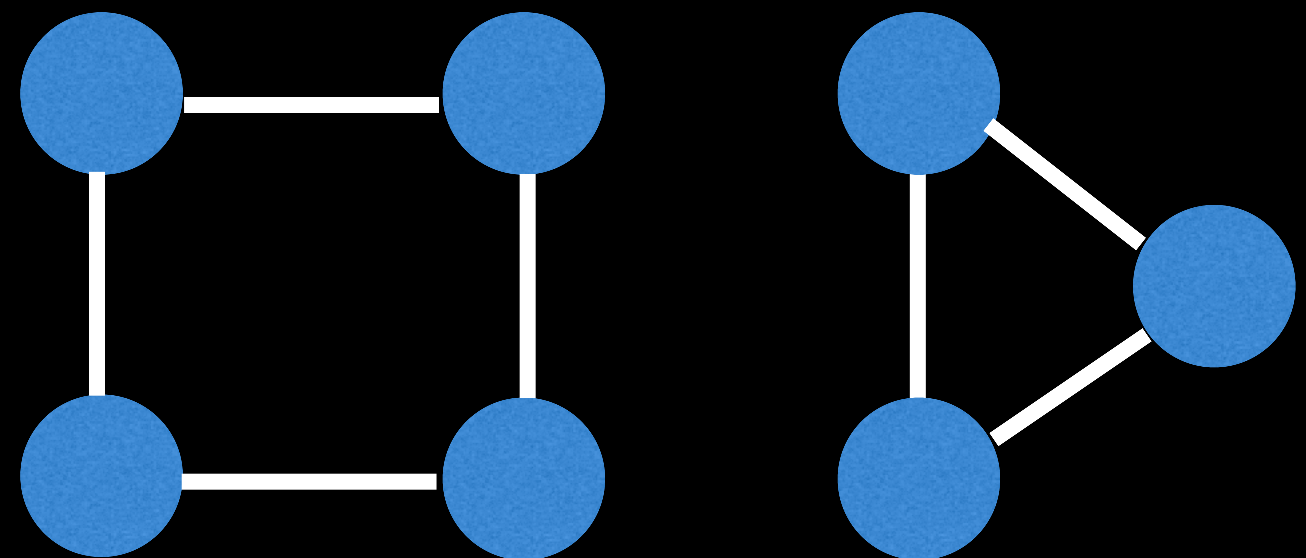
True or false: if a graph has an Eulerian circuit, it also has an Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

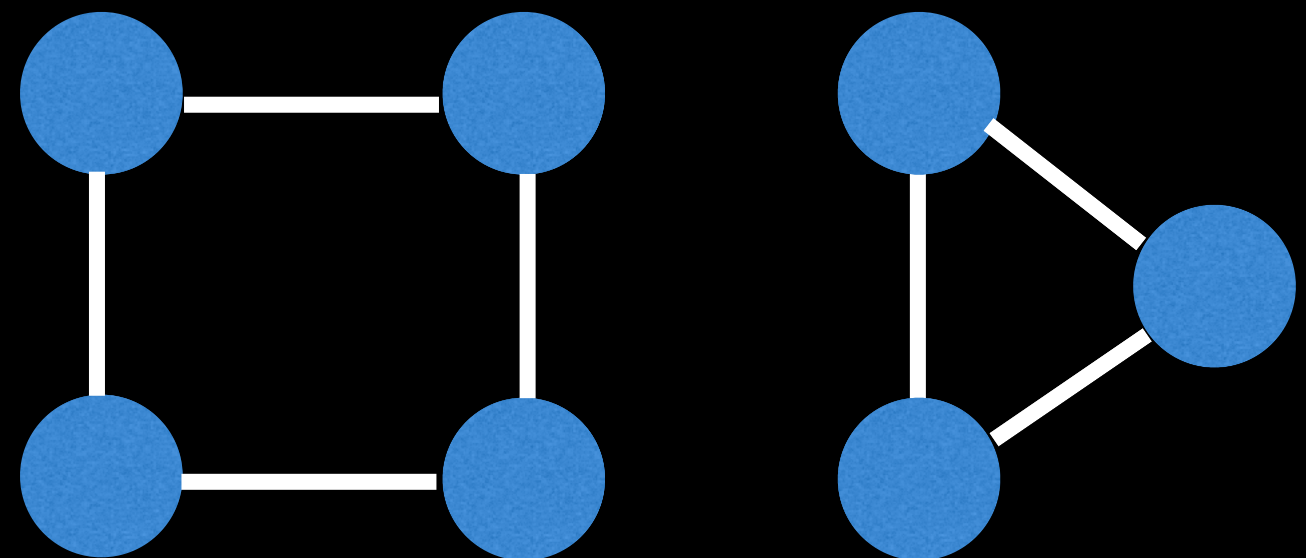


True or false: if a graph has an Eulerian circuit, it also has an Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

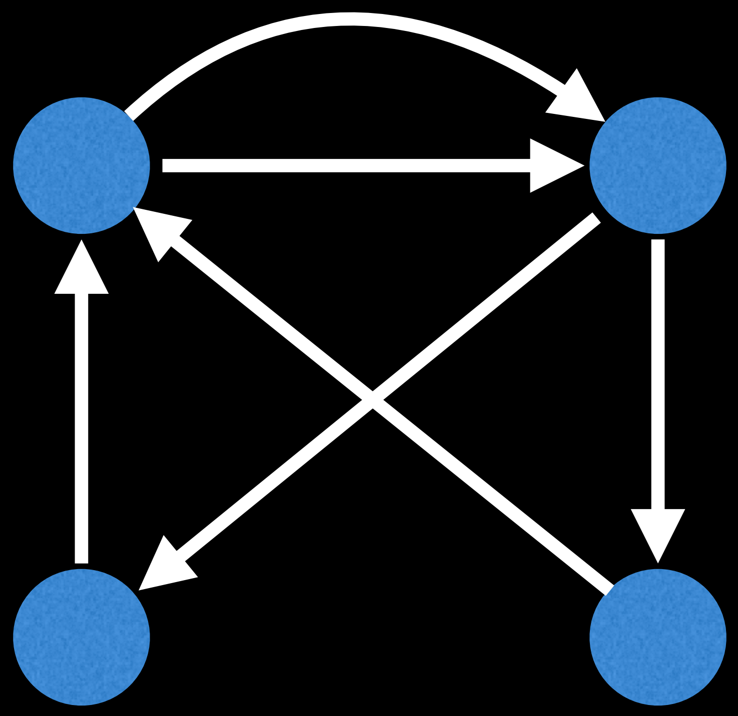


Does this undirected graph have an Eulerian path/circuit?

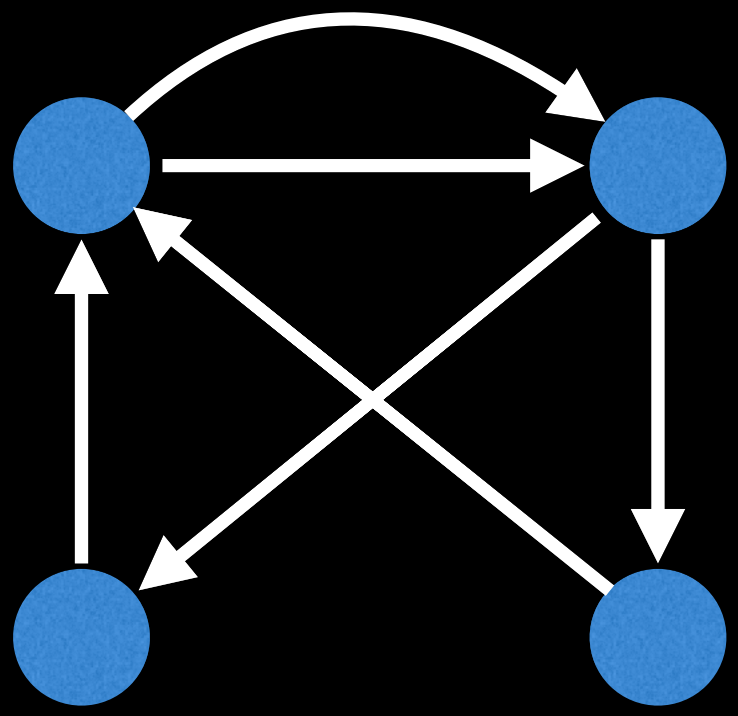


There are no Eulerian paths/circuits. An additional requirement when finding paths/circuits is that all vertices with nonzero degree need to belong to a single connected component.

Does this directed graph have an Eulerian path/circuit?

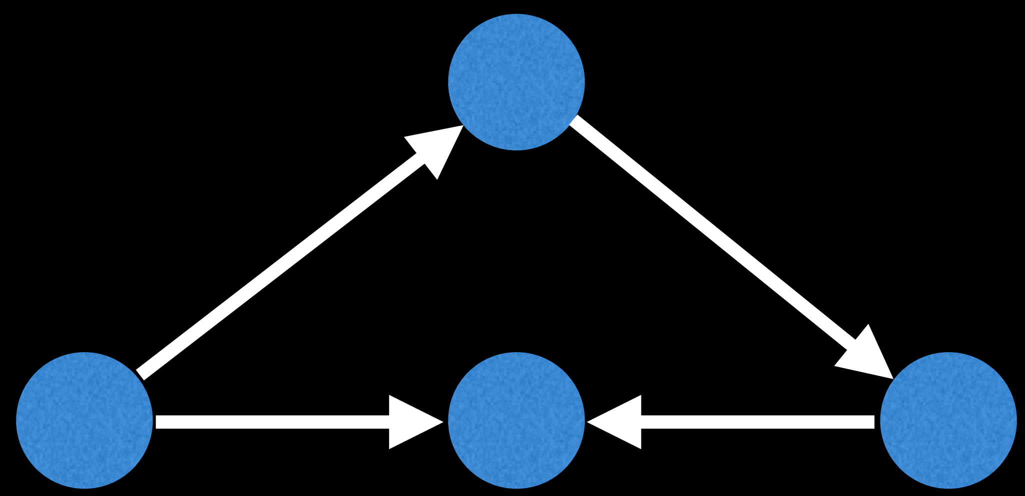


Does this directed graph have an Eulerian path/circuit?

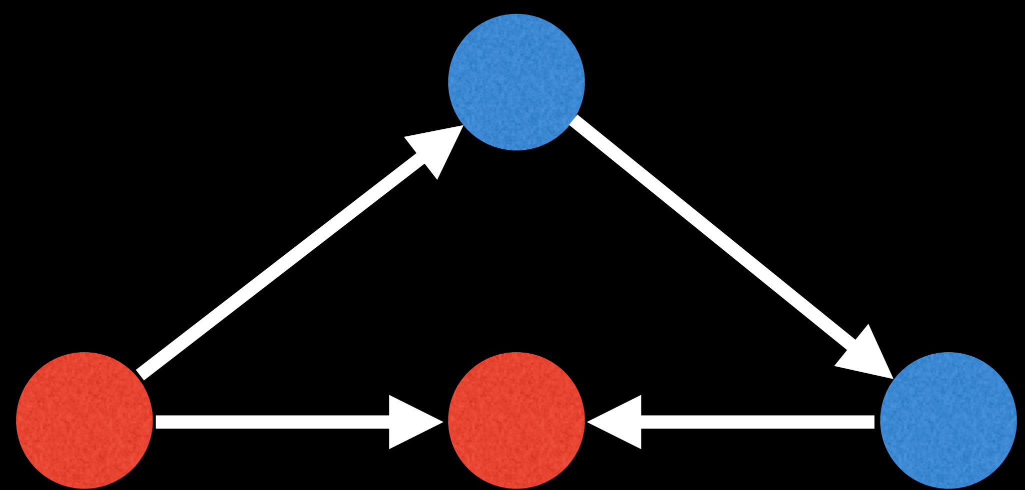


Yes, it has both an Eulerian path and an Eulerian circuit because all in/out degrees are equal.

Does this directed graph have an Eulerian path/circuit?

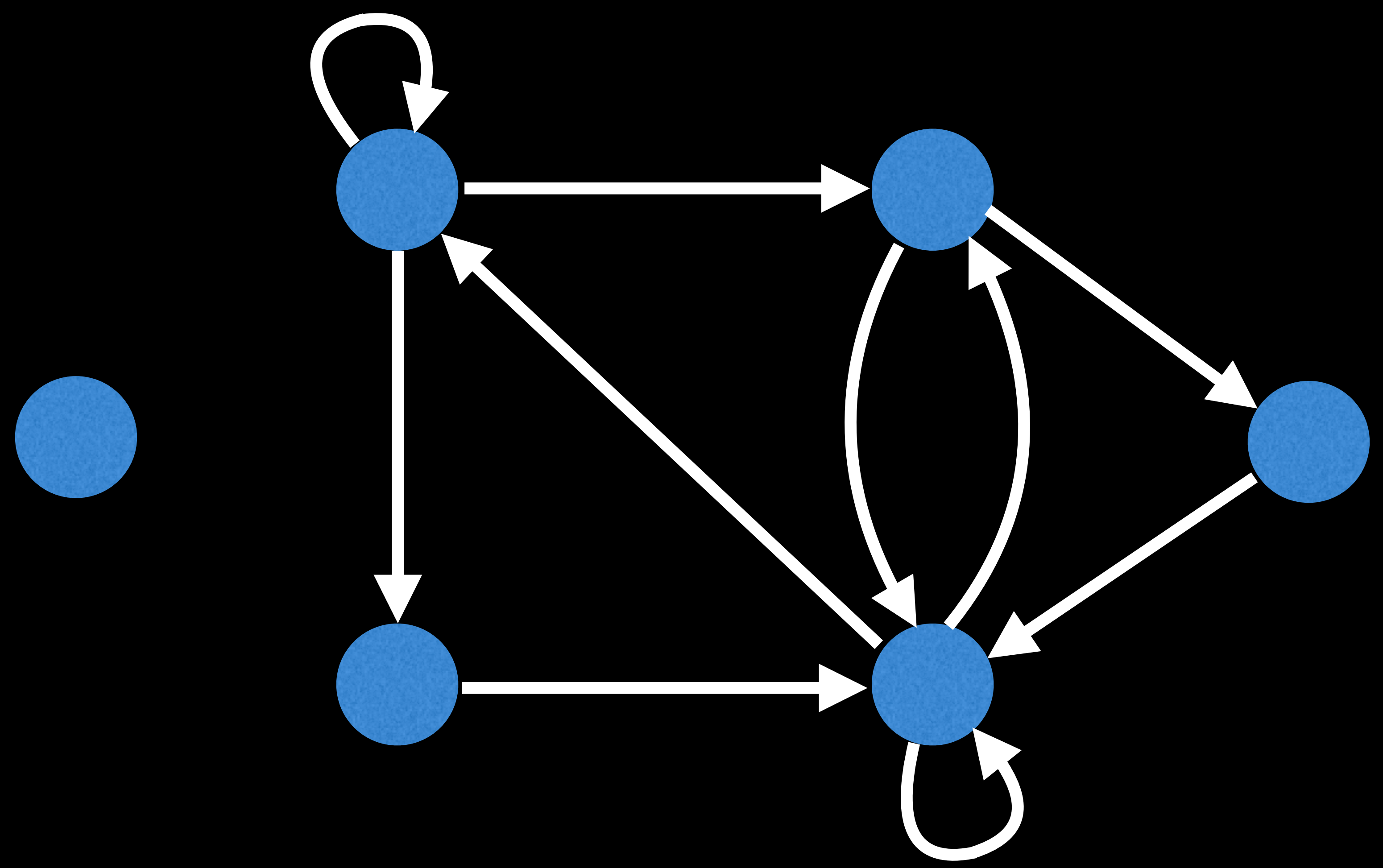


Does this directed graph have an Eulerian path/circuit?

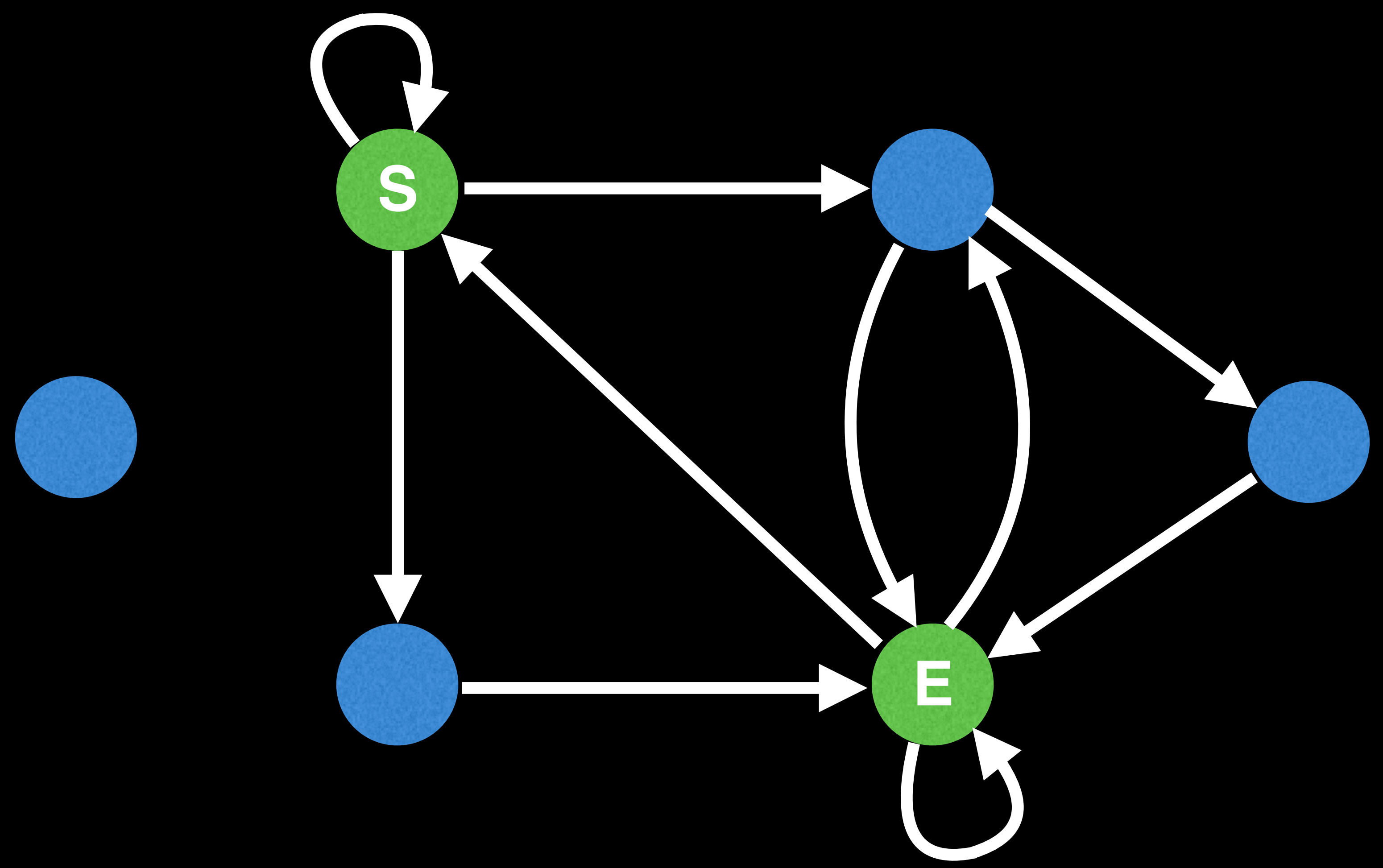


No path or circuit. The red nodes have either too many in coming or outgoing edges.

Does this directed graph have an Eulerian path/circuit?

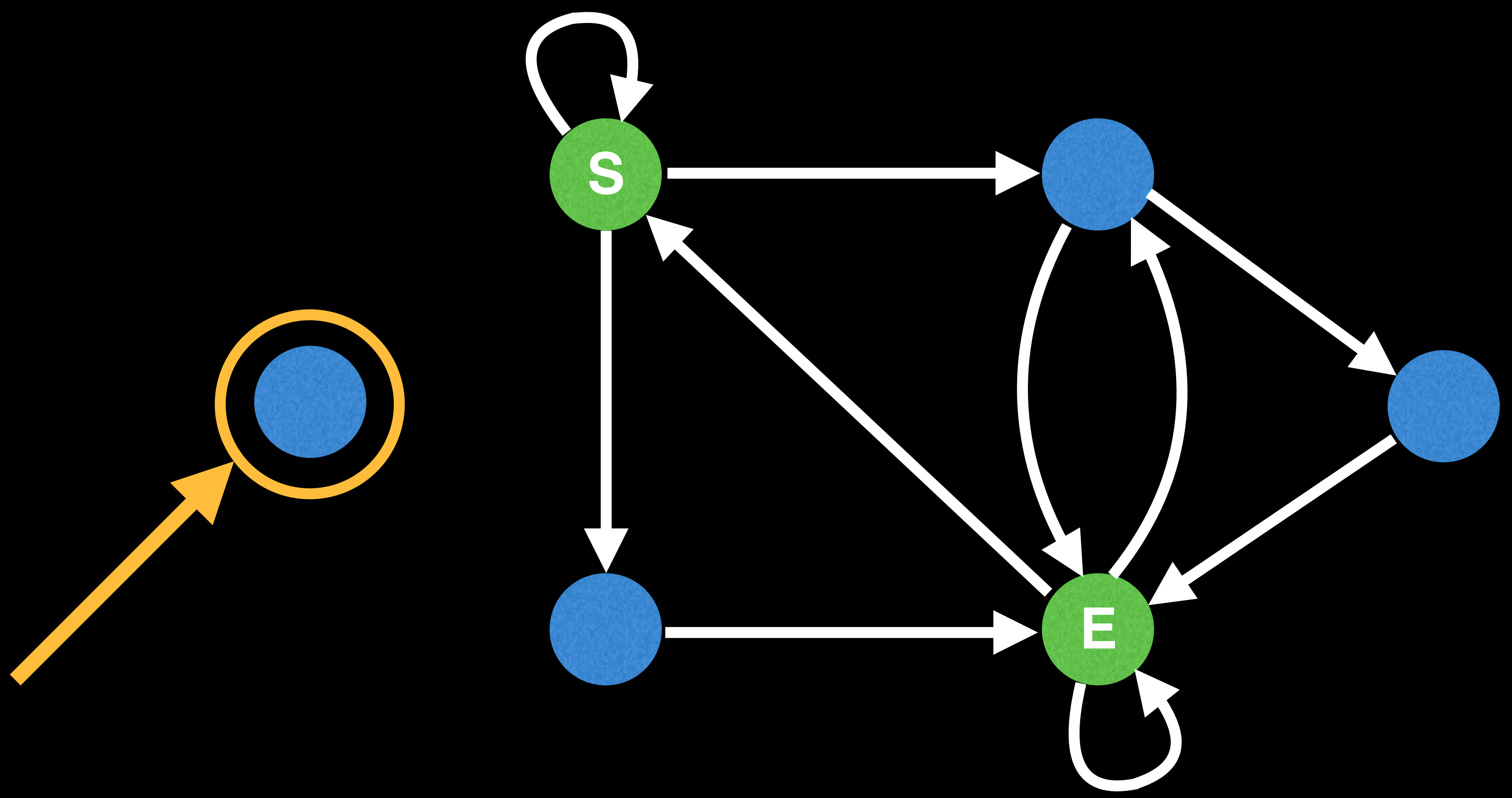


Does this directed graph have an Eulerian path/circuit?



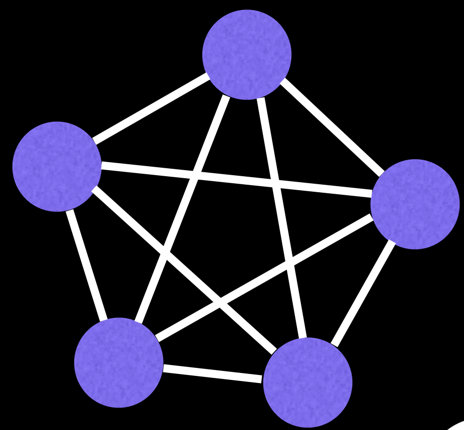
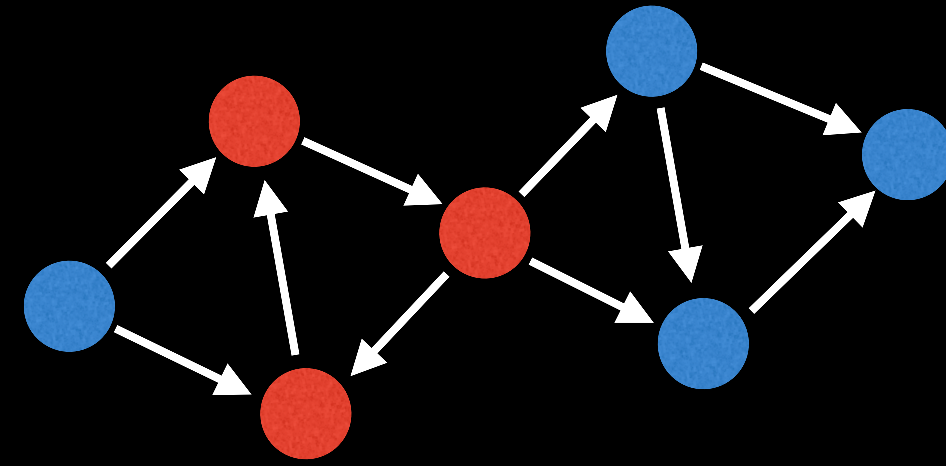
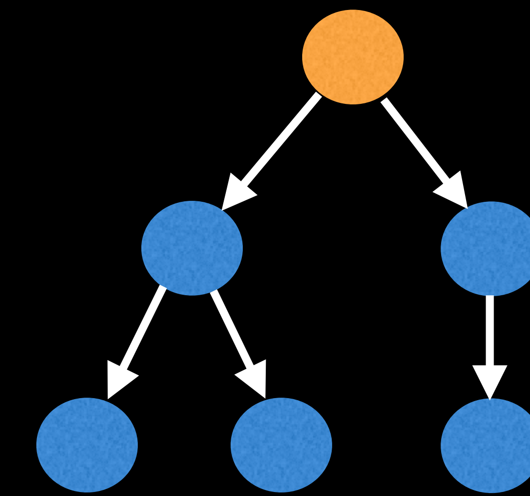
This graph has an Eulerian path, but no Eulerian circuit. It also has a unique start/end node for the path.

Does this directed graph have an Eulerian path/circuit?

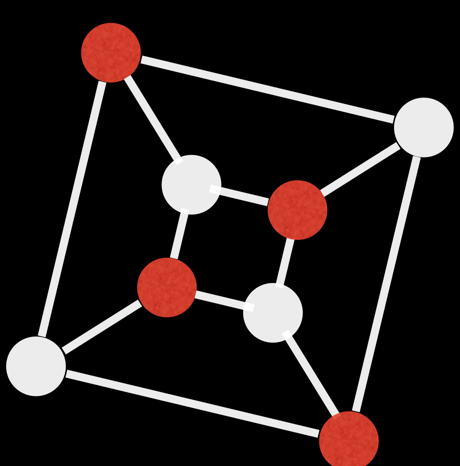
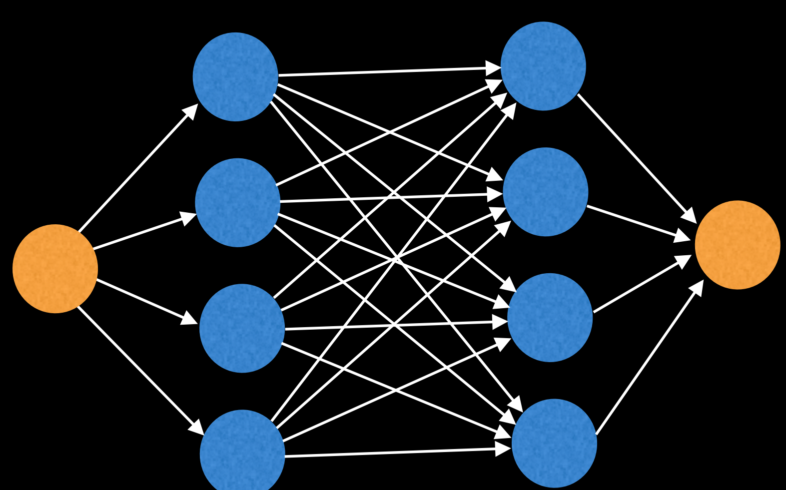
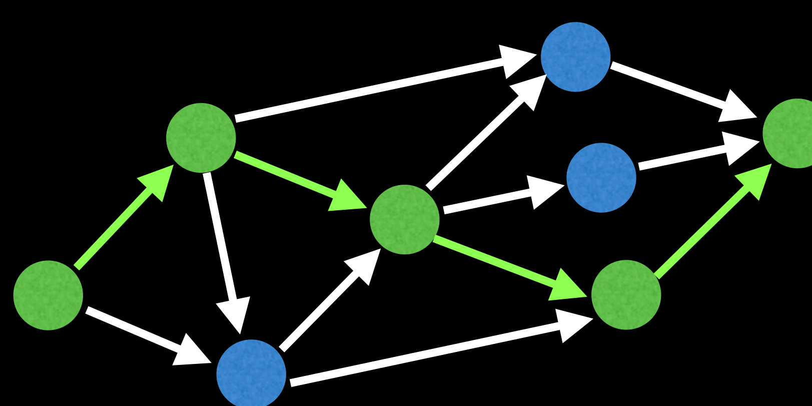


Note that the singleton node has no incoming/outgoing edges, so it doesn't impact whether or not we have an Eulerian path.

Next Video: Eulerian path algorithm

Graph Theory Video Series

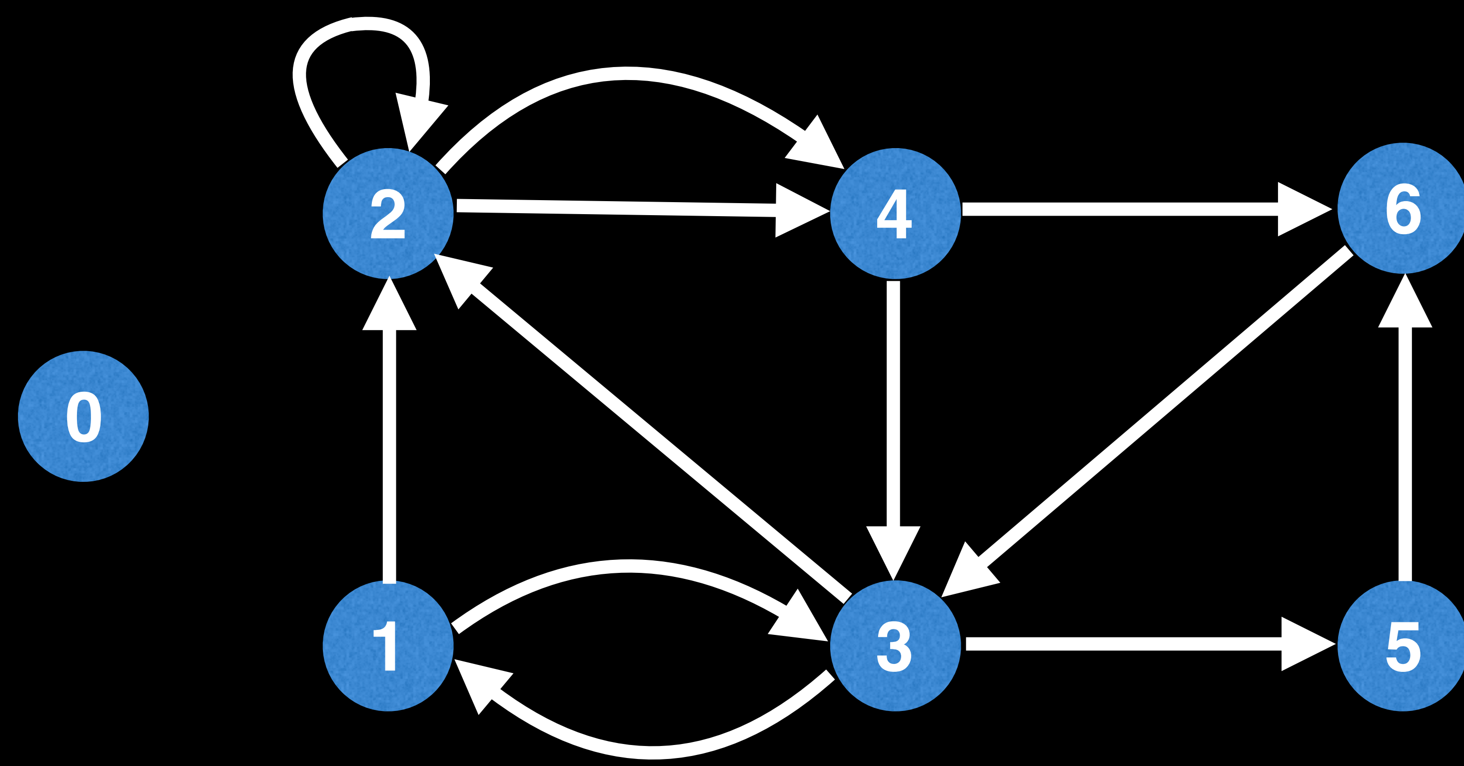




Finding Eulerian Paths and Circuits

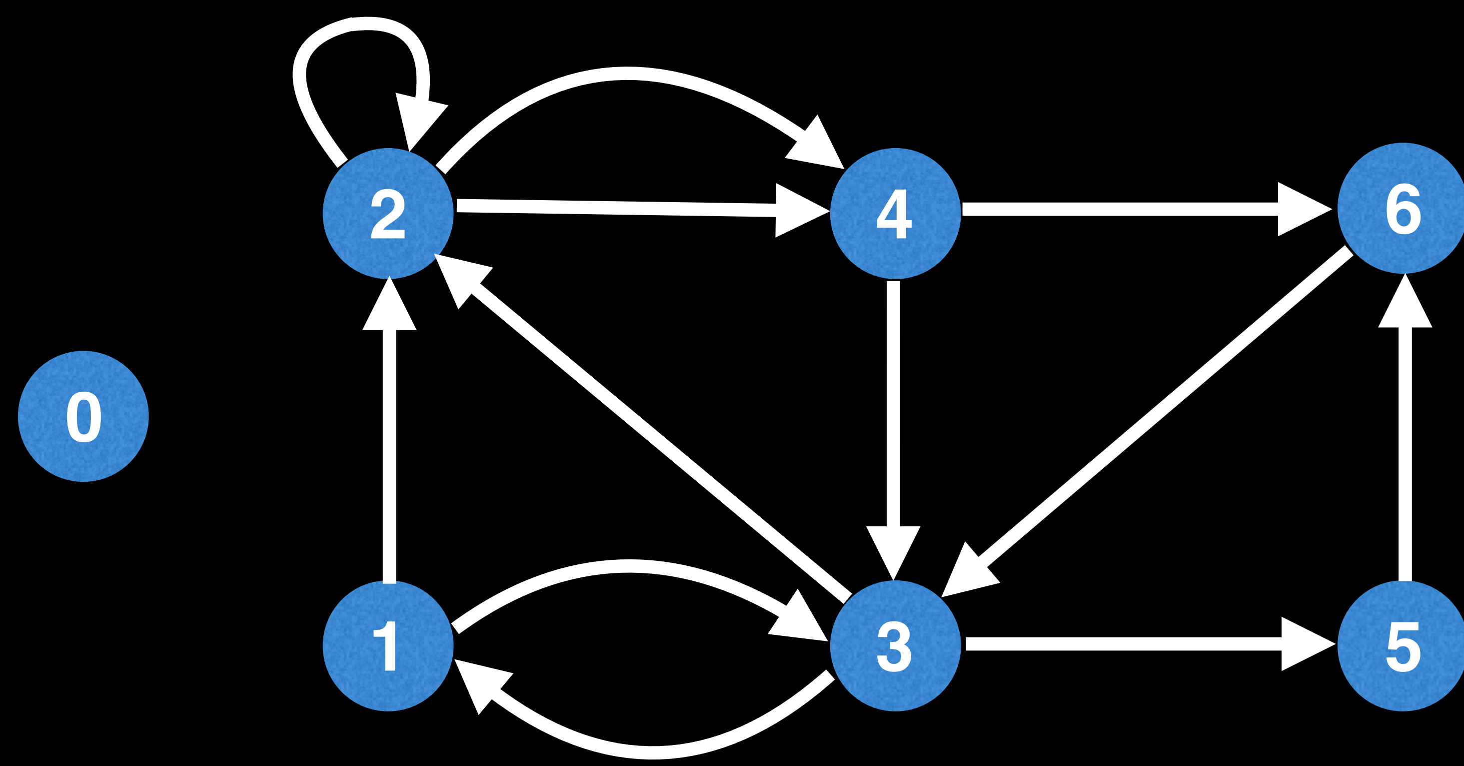
William Fiset

Previous video:

Finding an Eulerian path (directed graph)

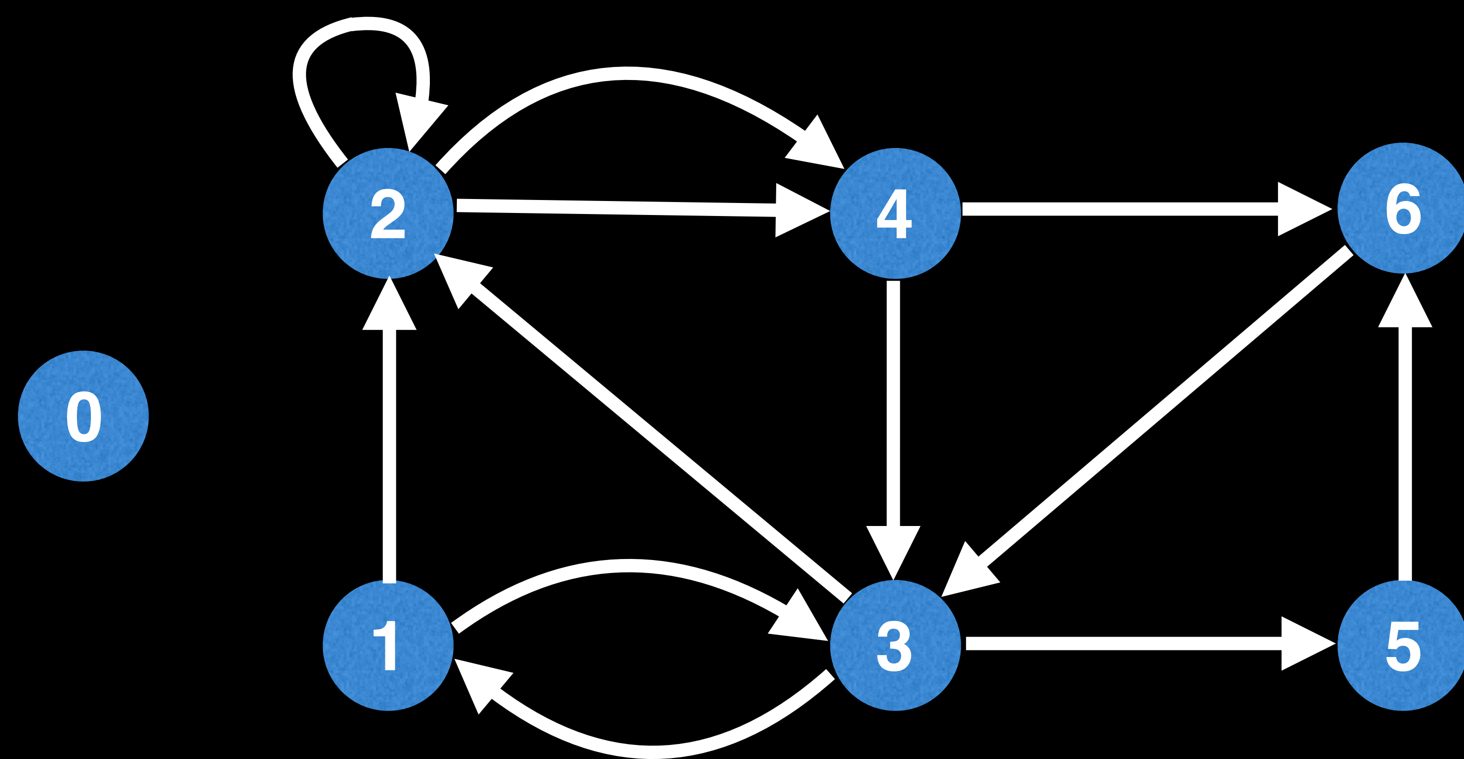


Finding an Eulerian path (directed graph)



Step 1 to finding an Eulerian path is determining if there even exists an Eulerian path.

Finding an Eulerian path (directed graph)

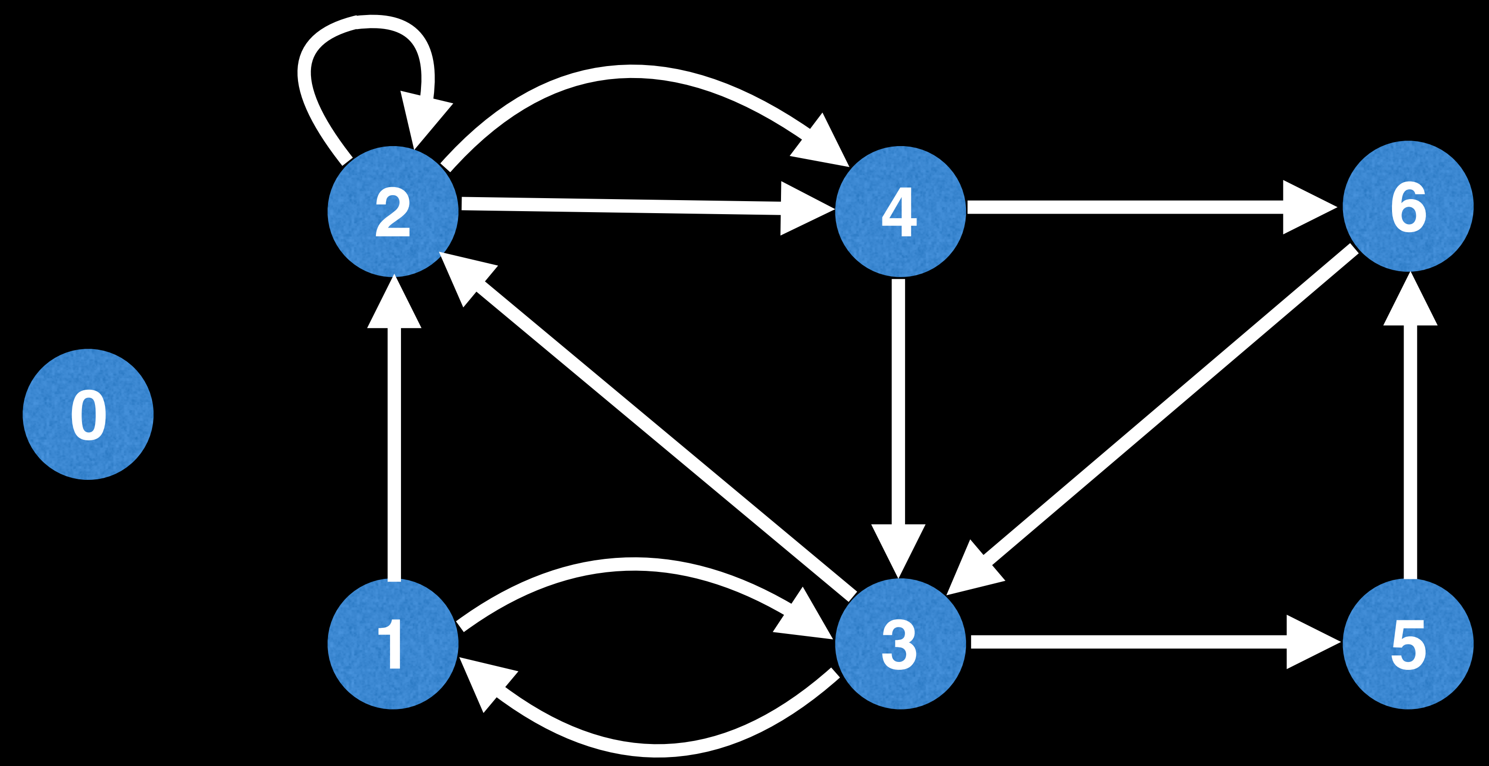


Step 1 to finding an Eulerian path is determining if there even exists an Eulerian path.

Recall that for an Eulerian path to exist at most one one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

Finding an Eulerian path (directed graph)

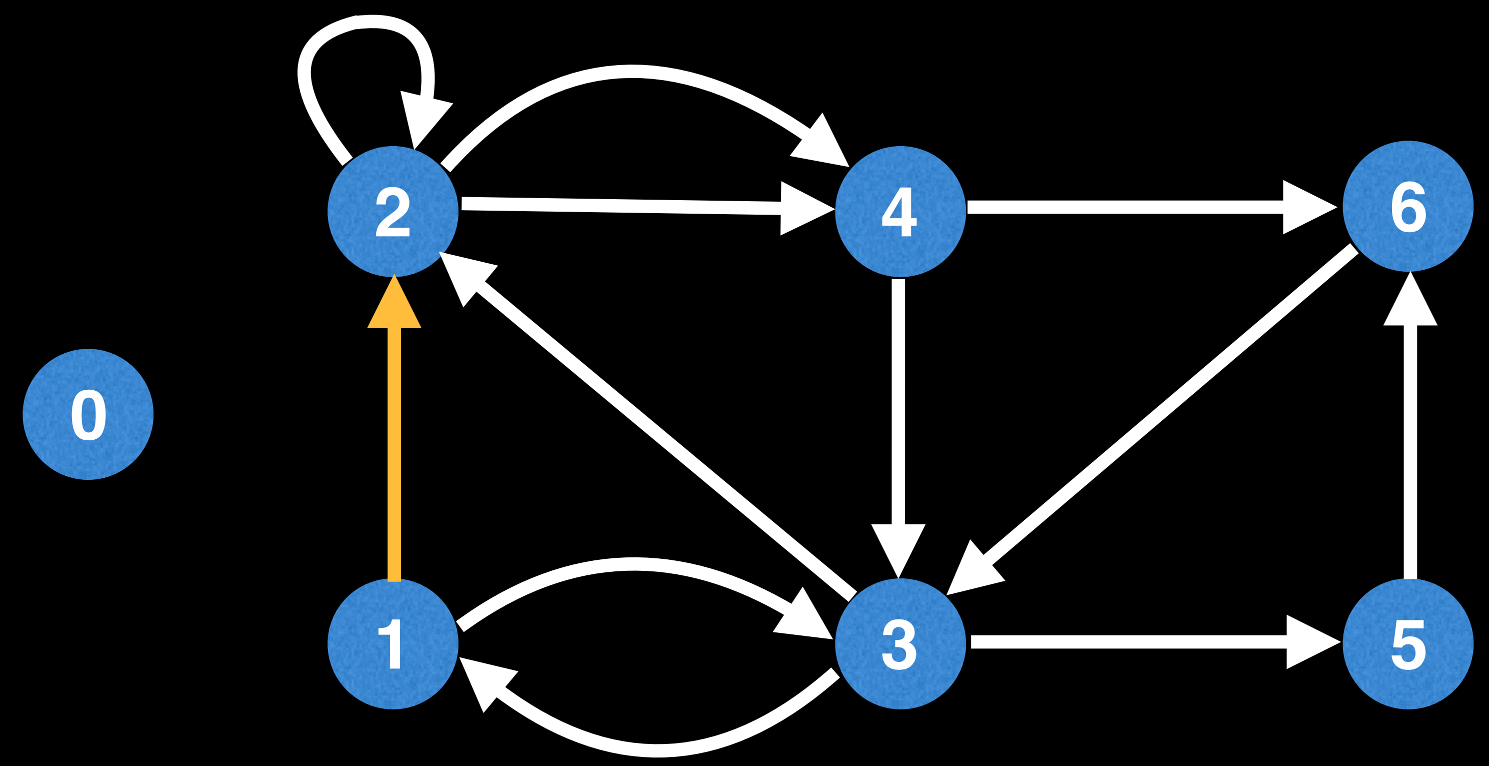
Node	In	Out
0		
1		
2		
3		
4		
5		
6		



Count the in/out degrees of each node by looping through all the edges.

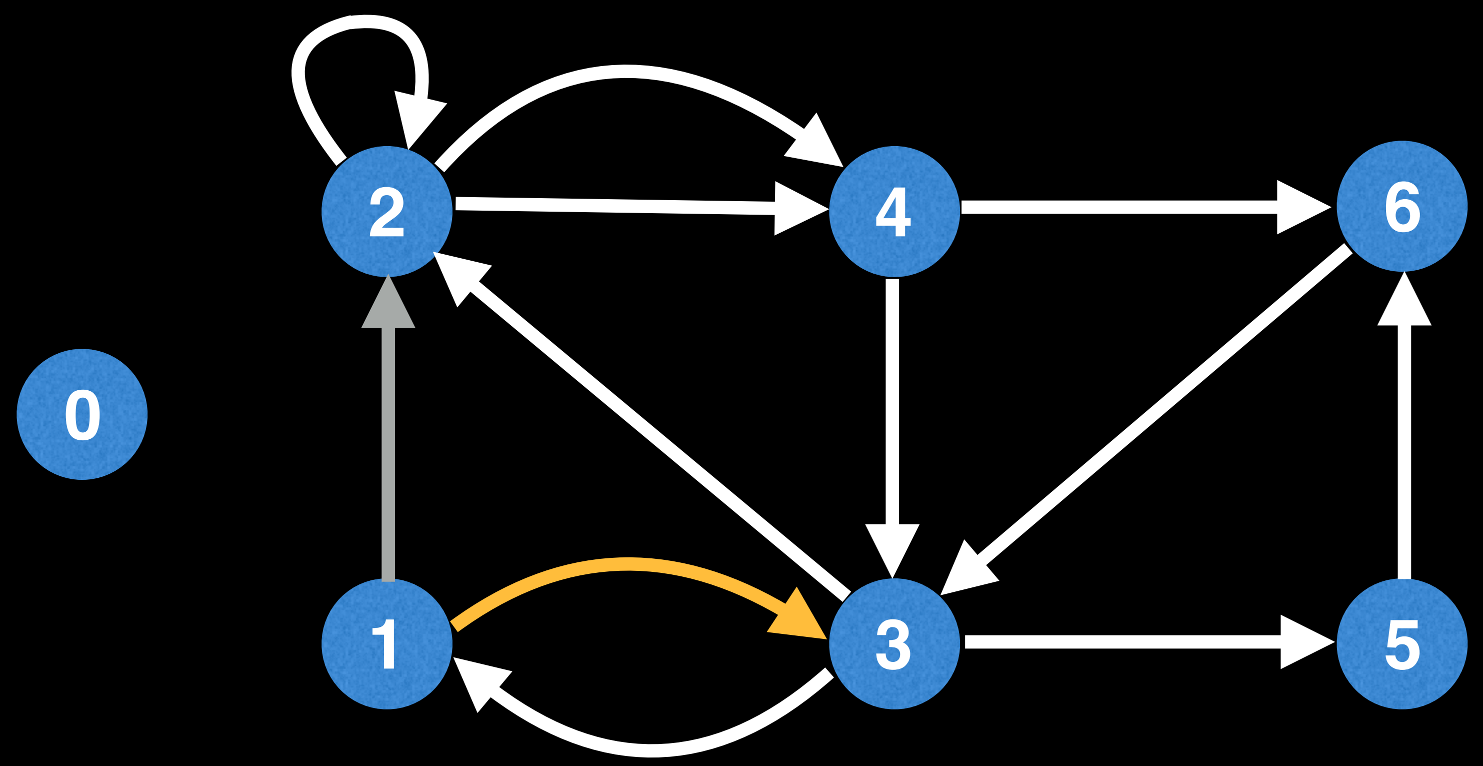
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1		1
2	1	
3		
4		
5		
6		



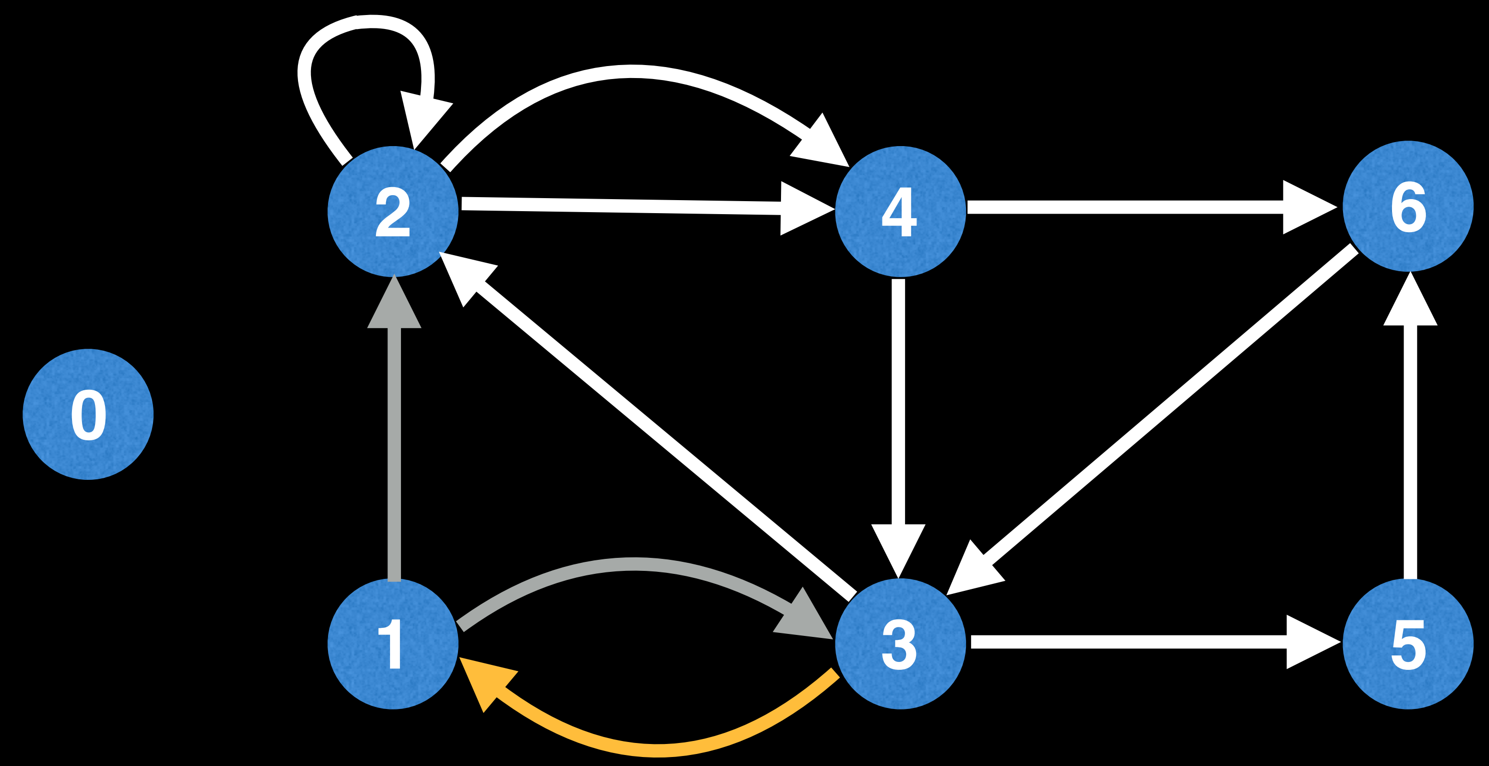
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1		2
2	1	
3	1	
4		
5		
6		



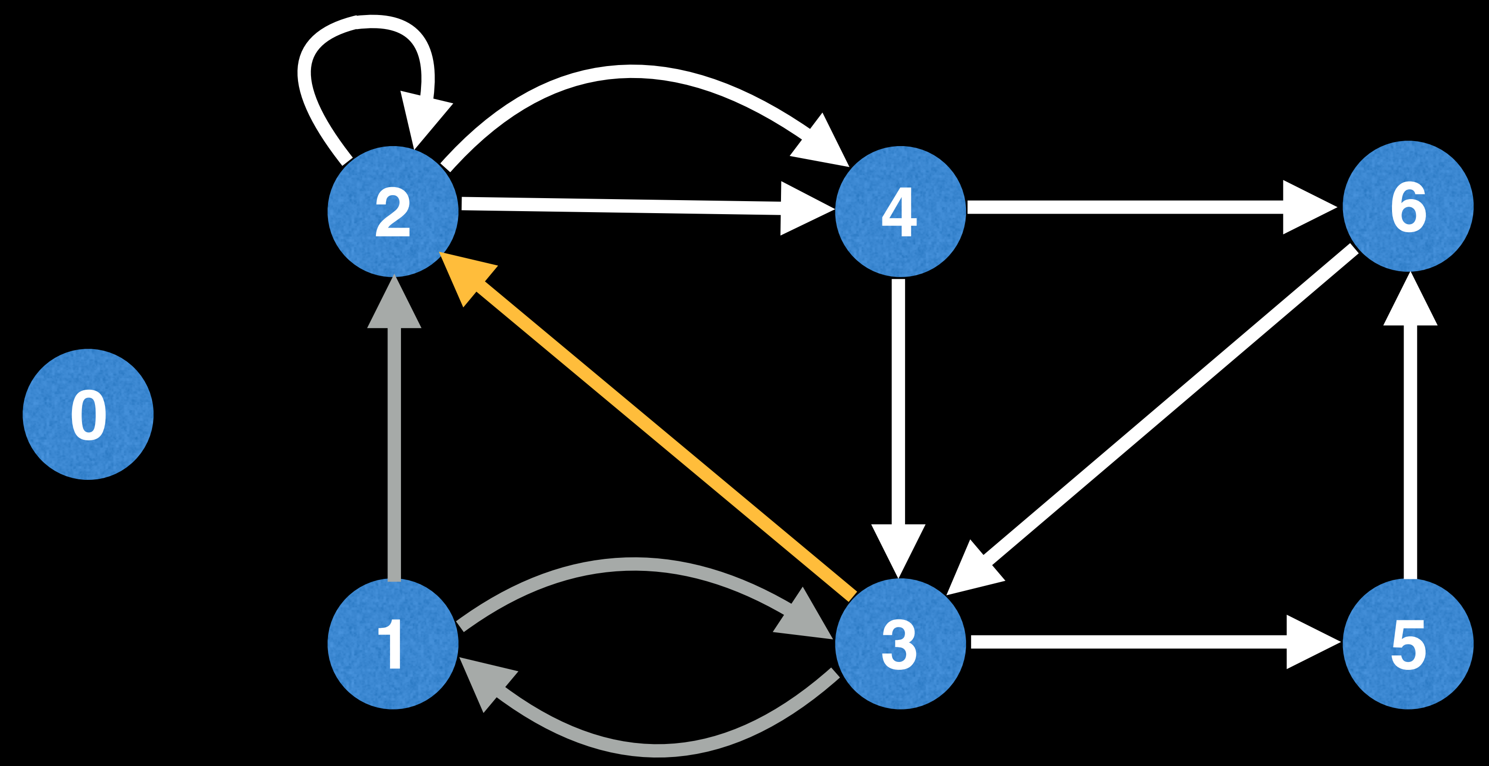
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1	1	2
2	1	
3	1	1
4		
5		
6		



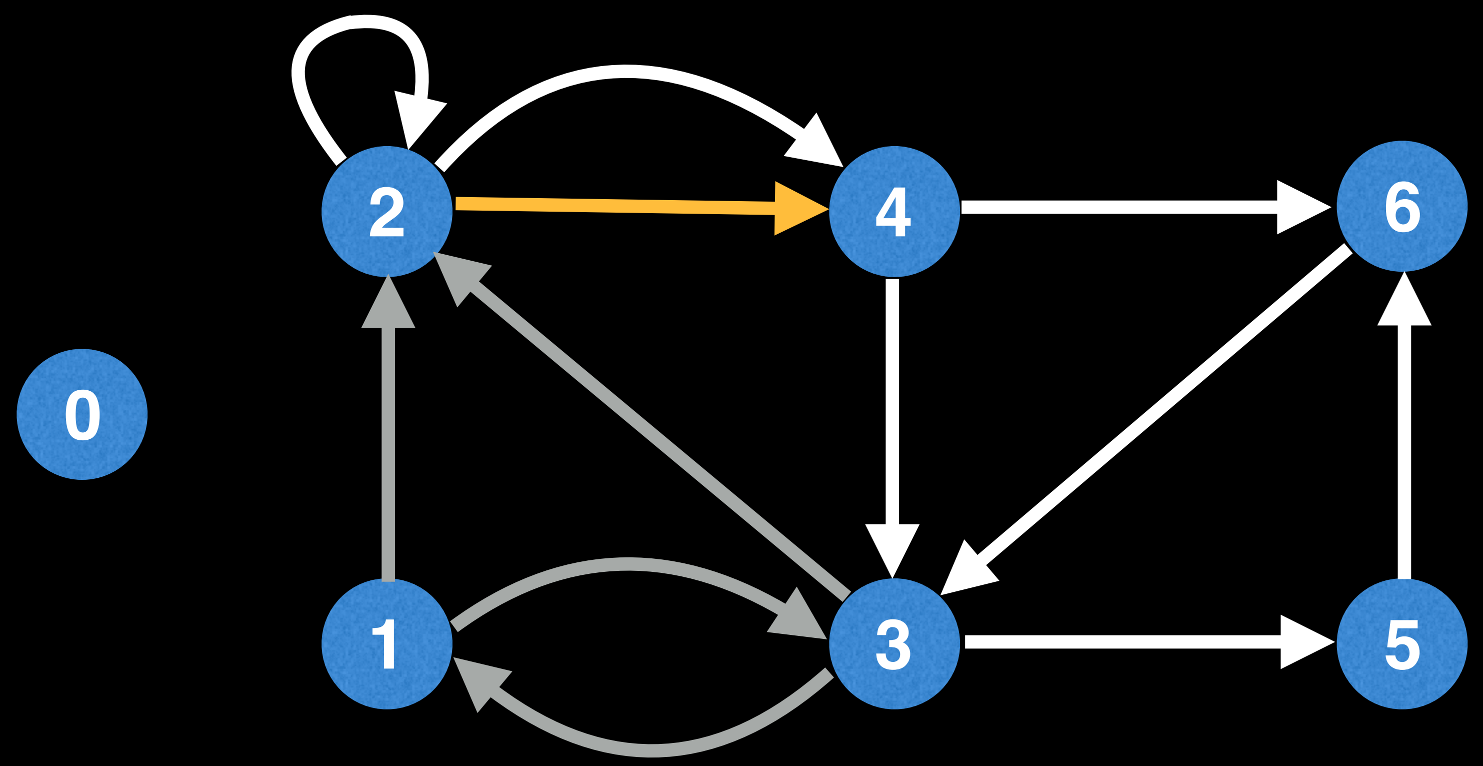
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1	1	2
2	2	
3	1	2
4		
5		
6		



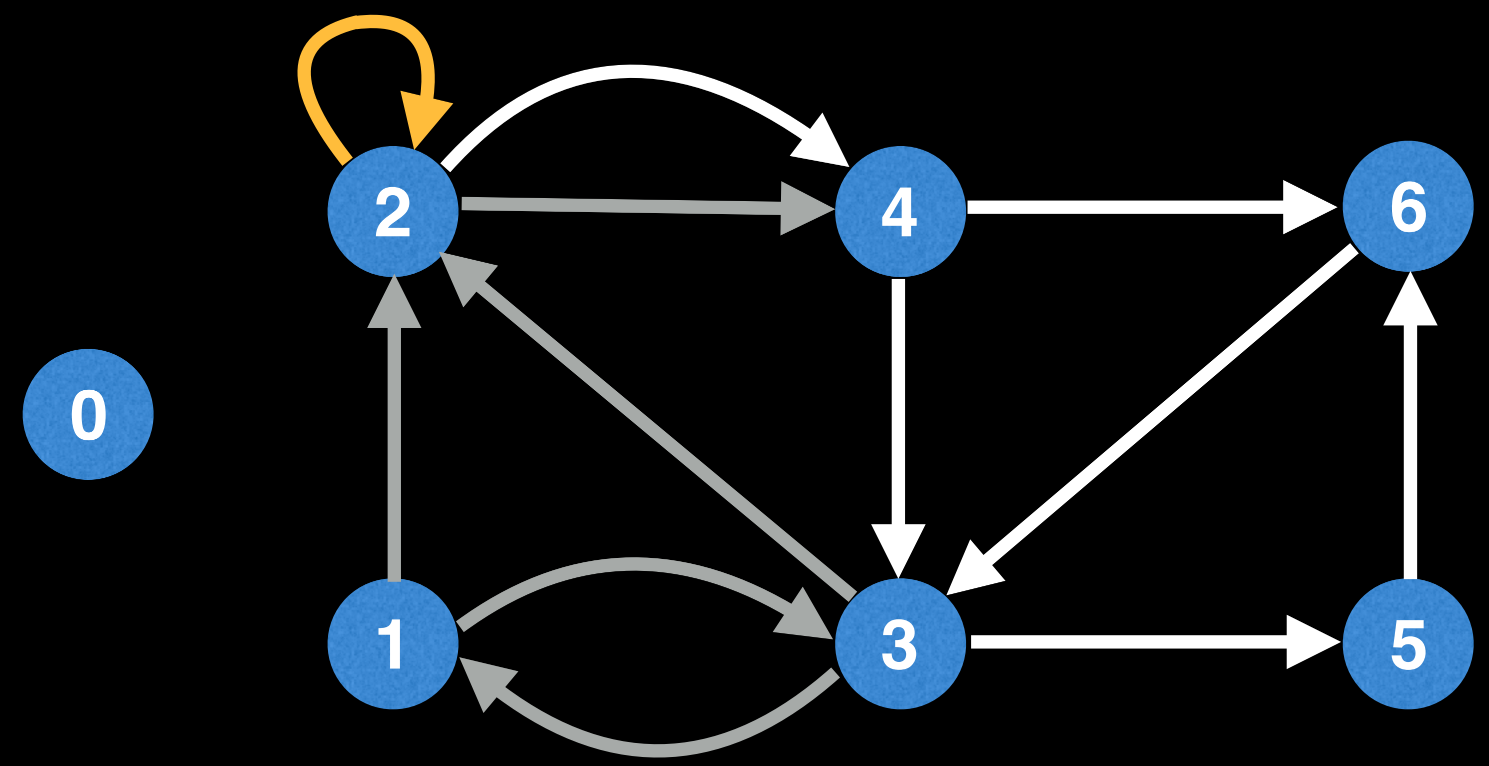
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1	1	2
2	2	1
3	1	2
4	1	
5		
6		



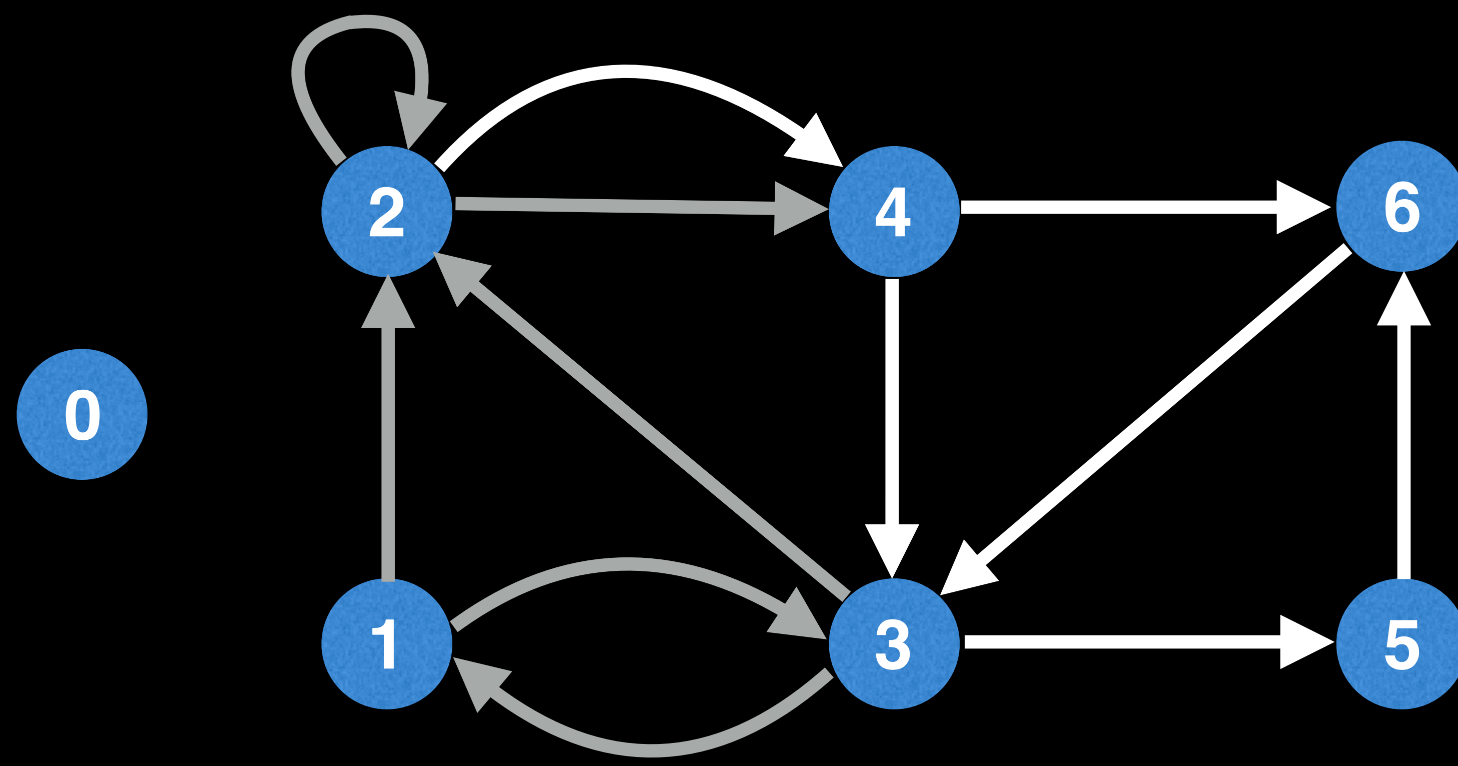
Finding an Eulerian path (directed graph)

Node	In	Out
0		
1	1	2
2	3	2
3	1	2
4	1	
5		
6		



Finding an Eulerian path (directed graph)

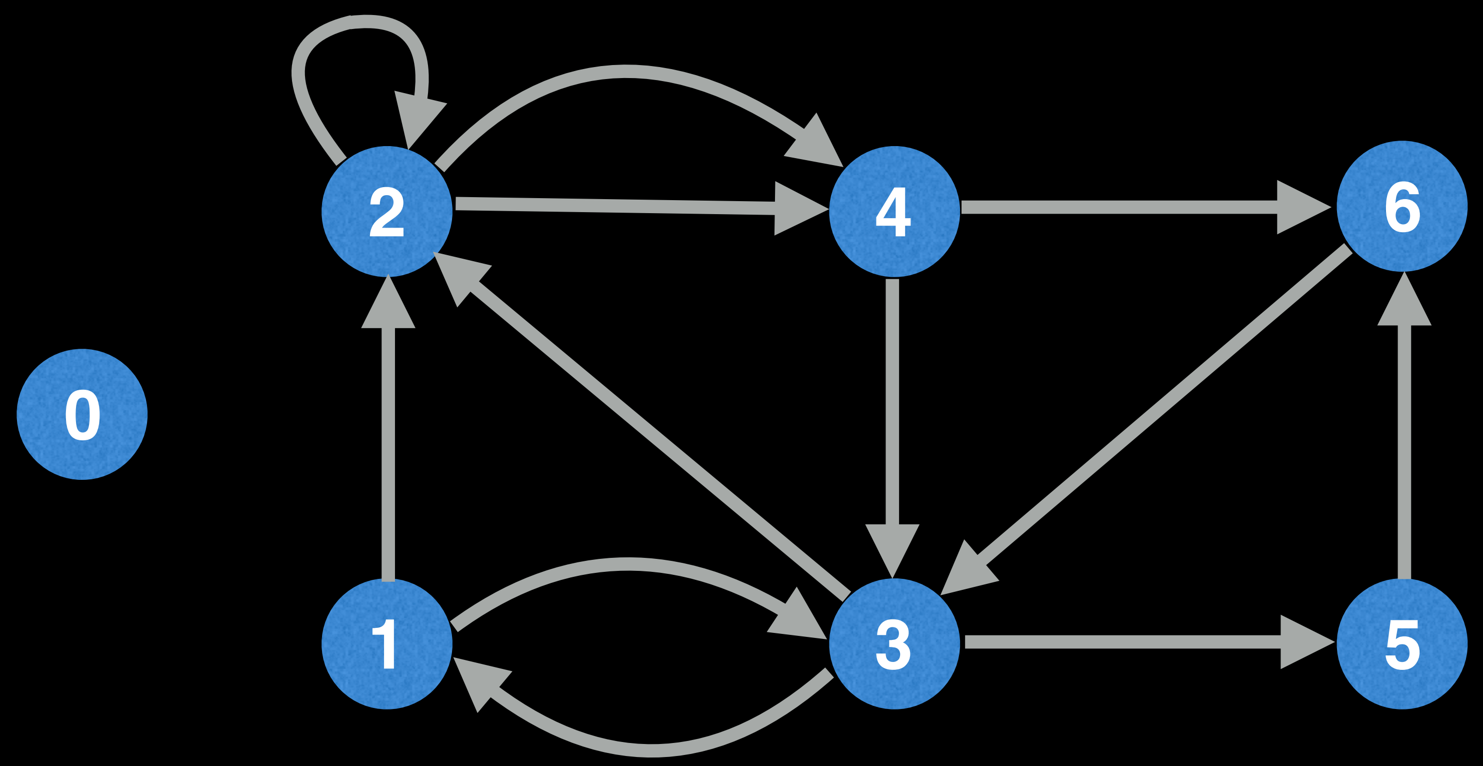
Node	In	Out
0		
1	1	2
2	3	2
3	1	2
4	1	
5		
6		



And so on for all other edges…

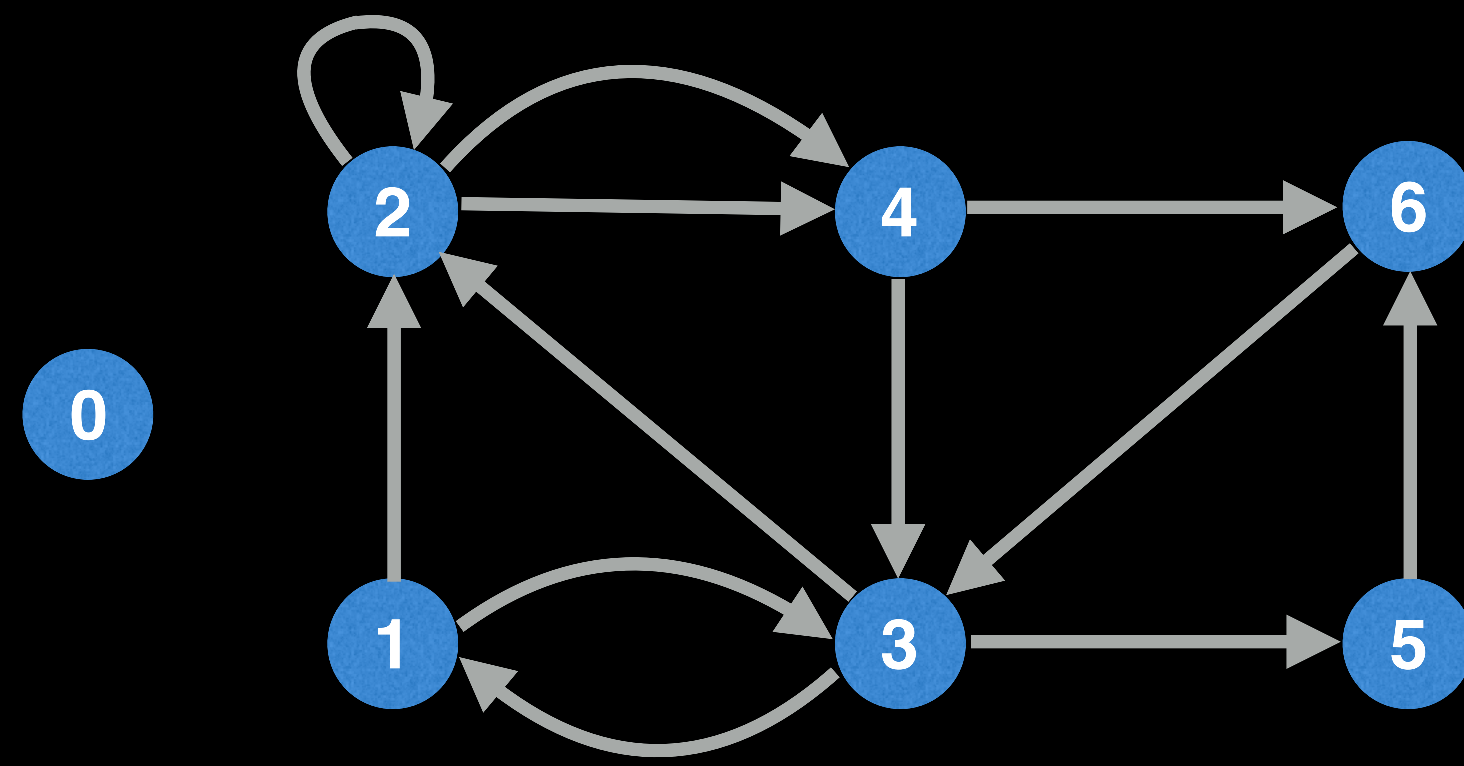
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1

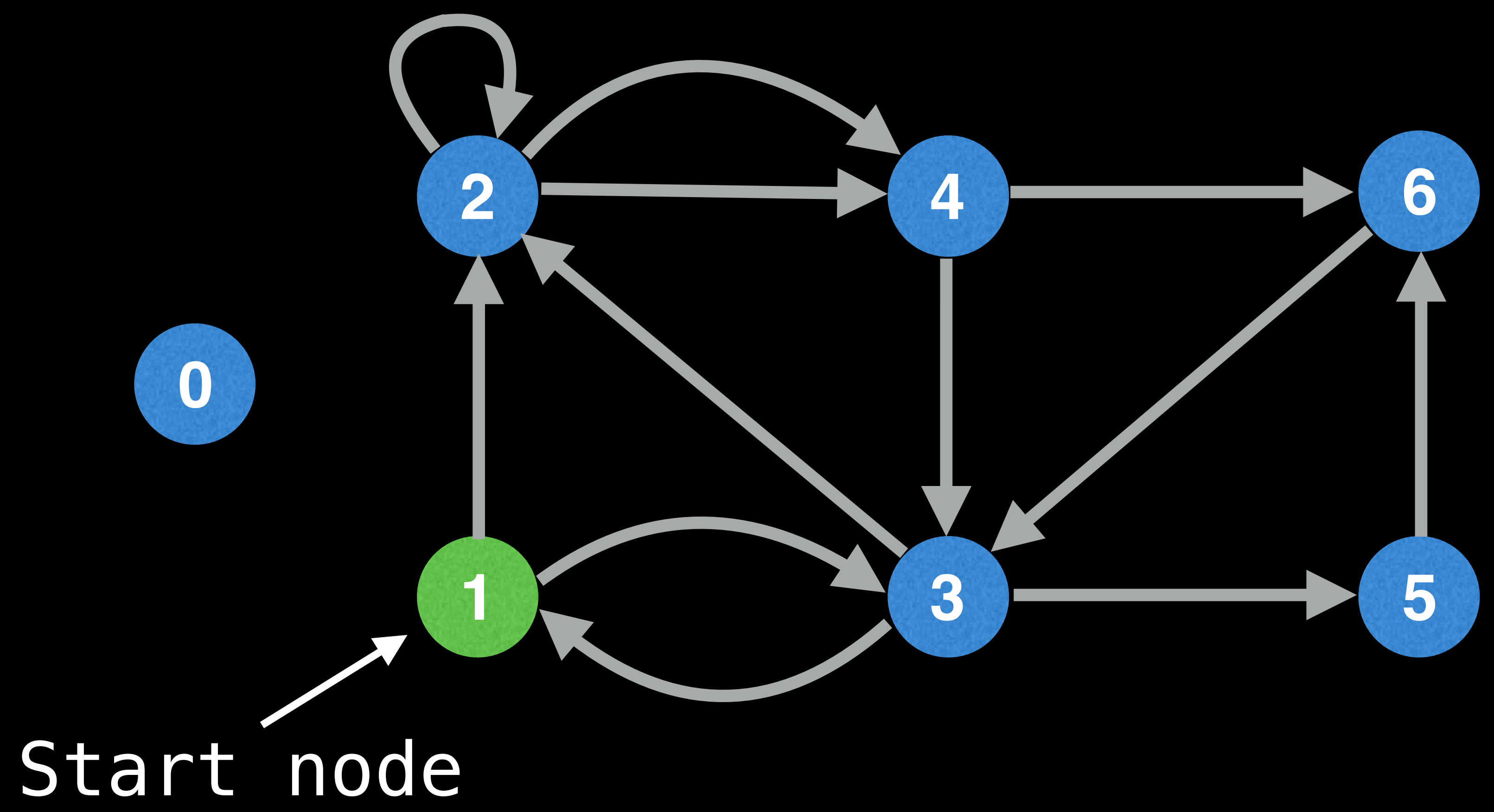


Once we've verified that no node has too many outgoing edges (out[i] – in[i] > 1) or incoming edges (in[i] – out[i] > 1) and there are just the right amount of start/end nodes we can be certain that an Eulerian path exists.

The next step is to find a valid starting node.

Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1

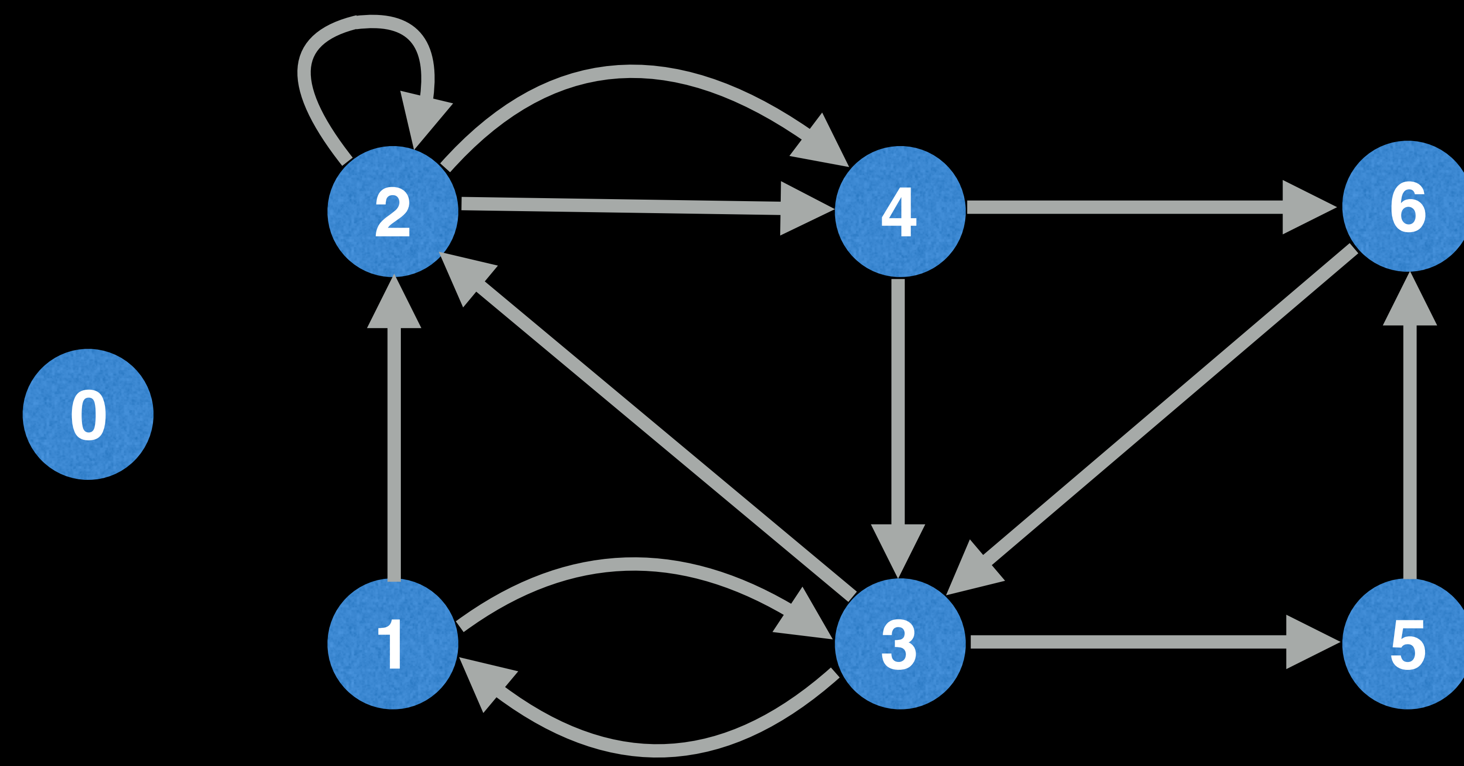


Node 1 is the only node with exactly one extra outgoing edge, so it's our only valid start node. Similarly, node 6 is the only node with exactly one extra incoming edge, so it will end up being the end node.

$$out[1] - in[1] = 2 - 1 = 1$$
$$in[6] - out[6] = 2 - 1 = 1$$

Finding an Eulerian path (directed graph)

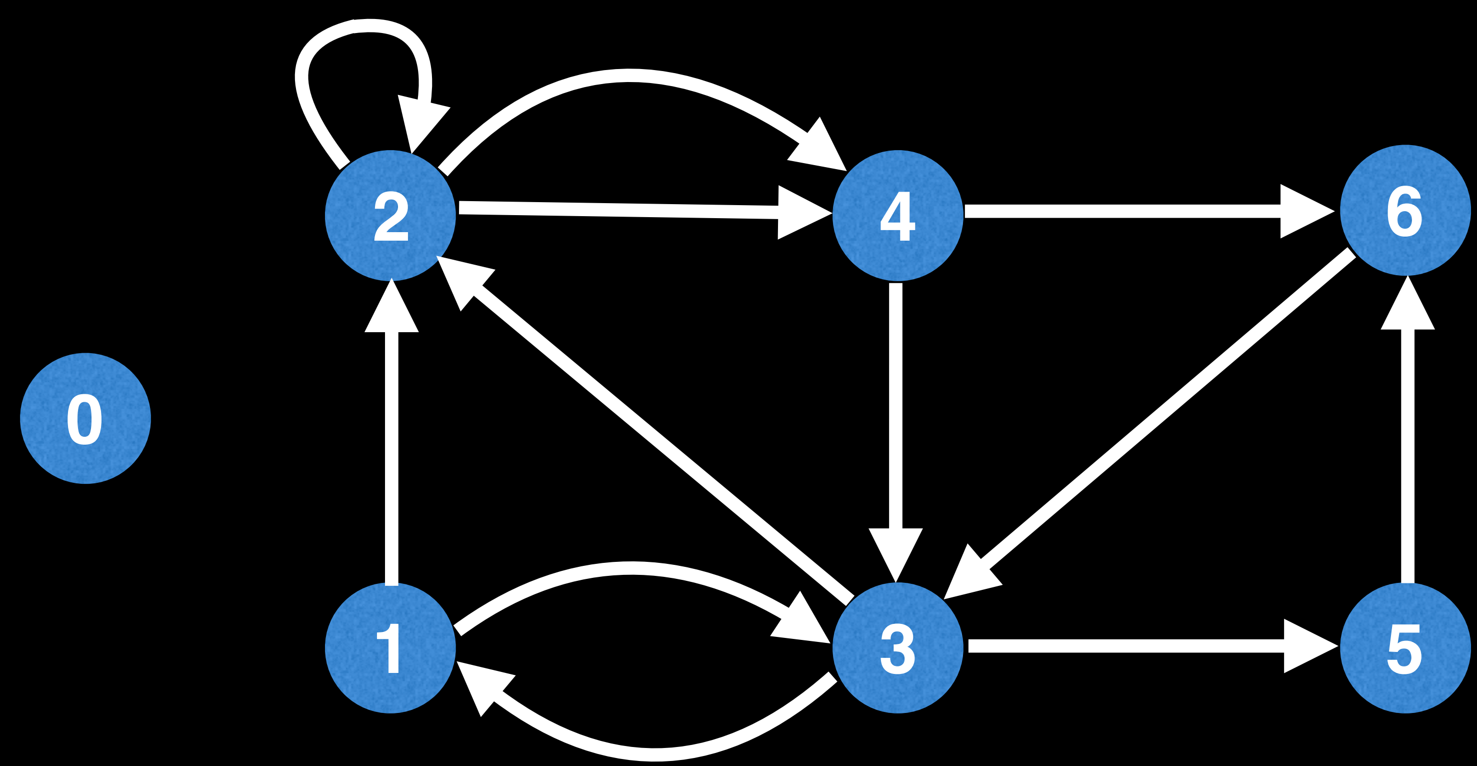
Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



NOTE: If all in and out degrees are equal (Eulerian circuit case) then any node with non-zero degree would serve as a suitable starting node.

Finding an Eulerian path (directed graph)

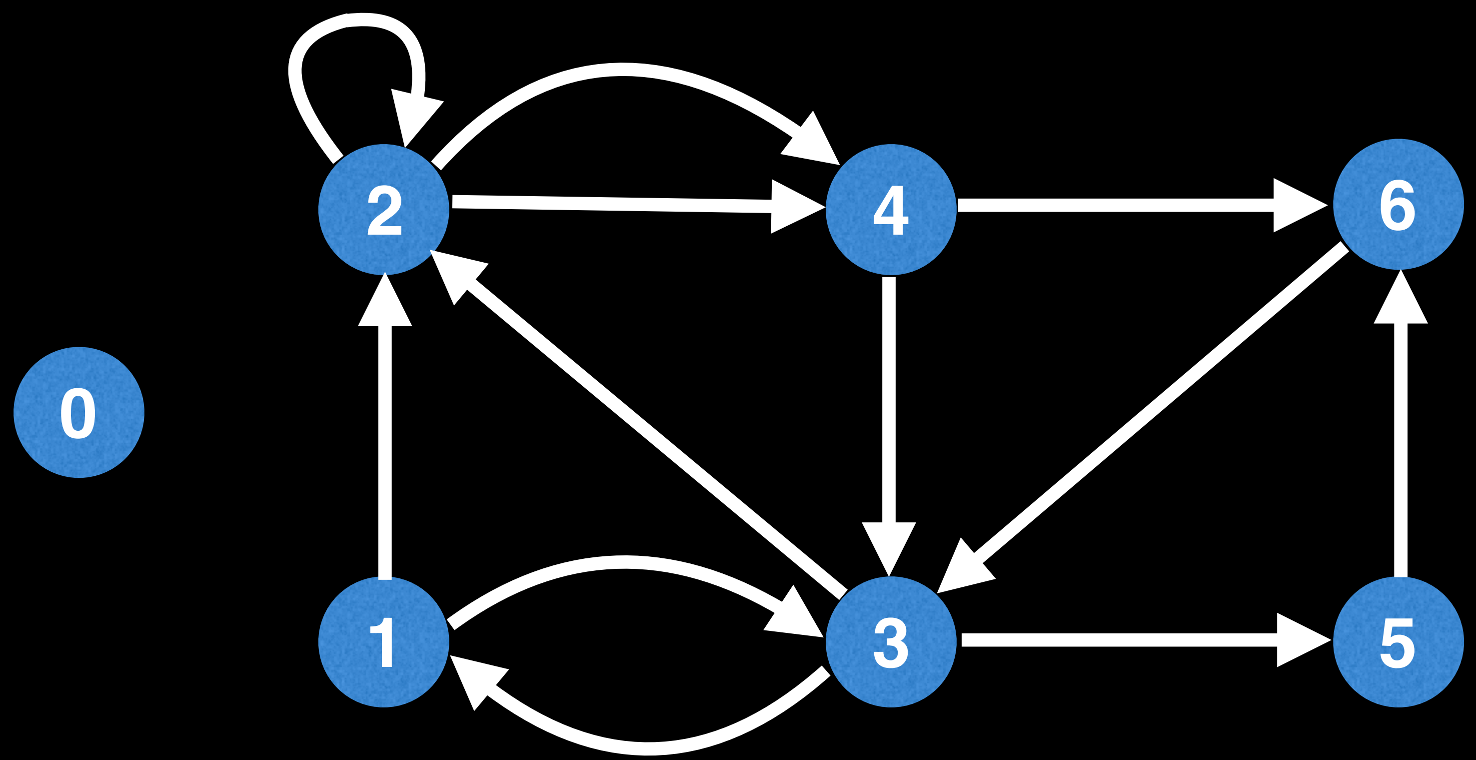
Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



Now that we know the starting node, let's find an Eulerian path!

Finding an Eulerian path (directed graph)

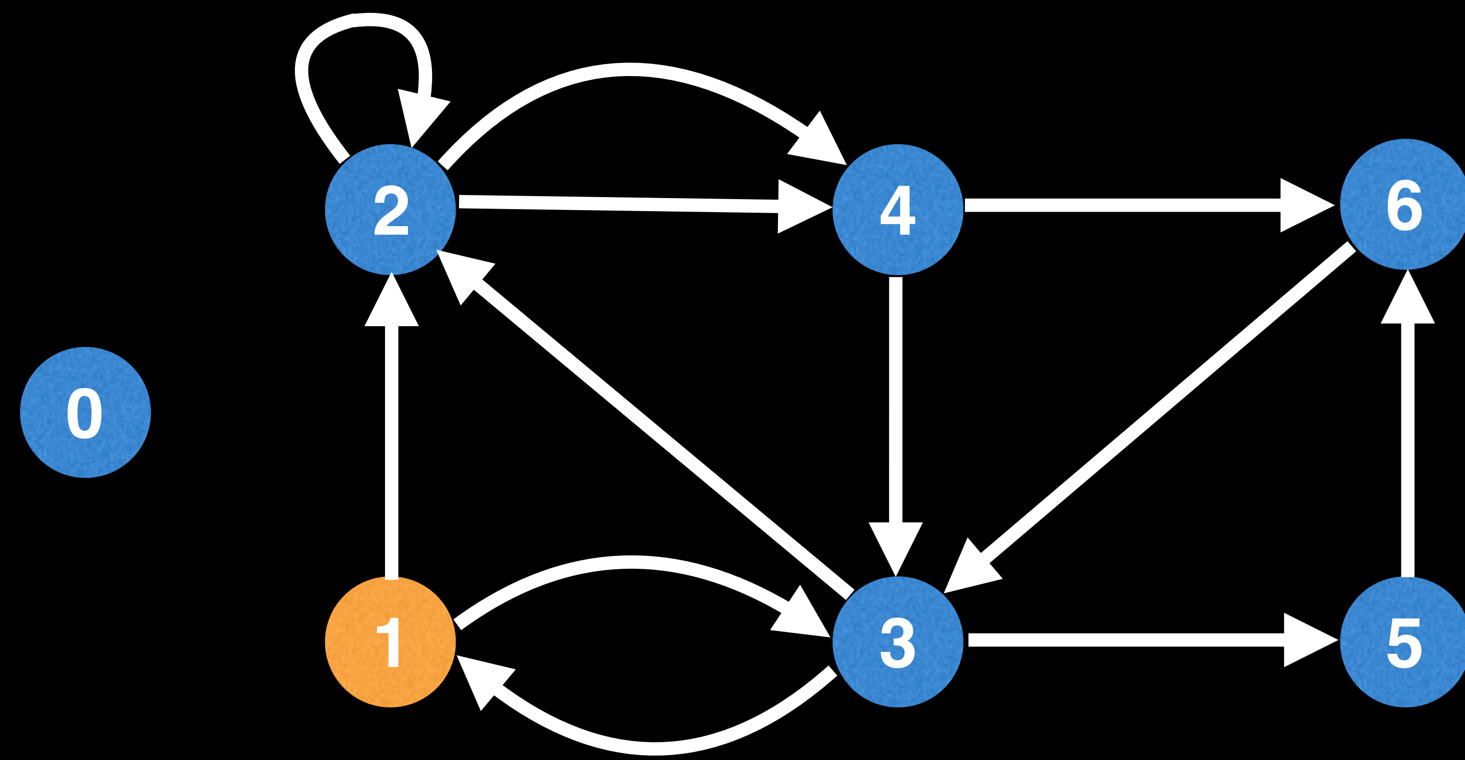
Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



Let's see what happens if we do a naive DFS, trying to traverse as many edges as possible until we get stuck.

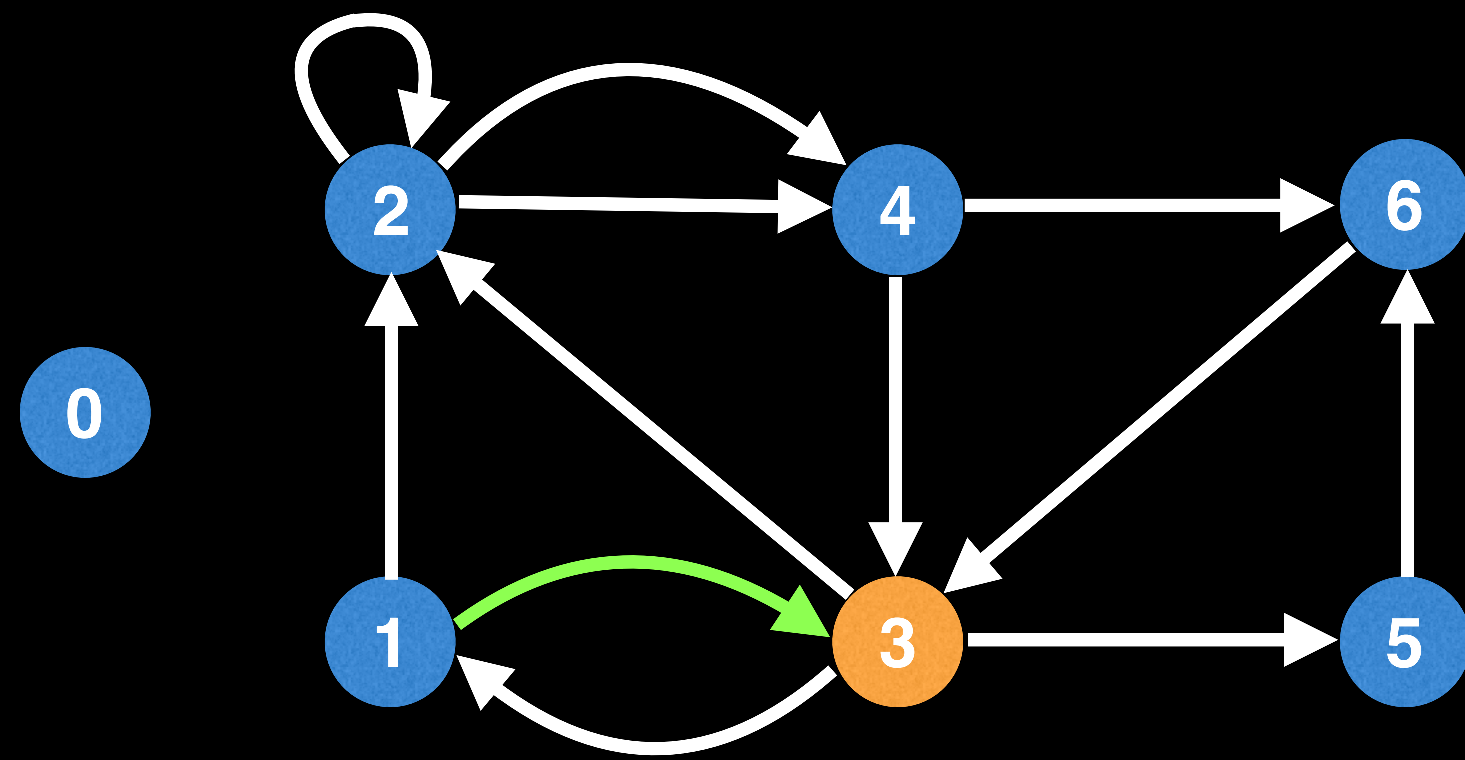
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



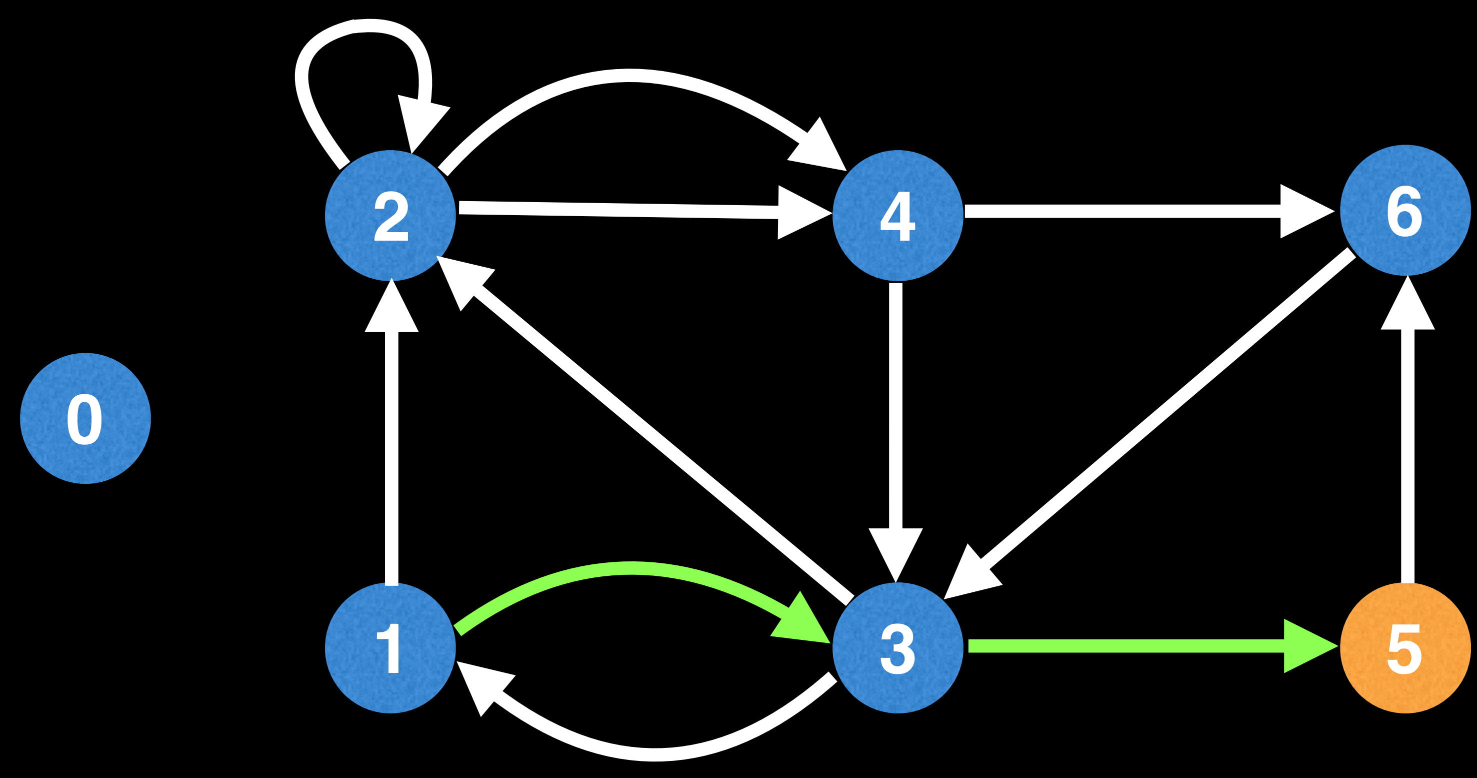
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



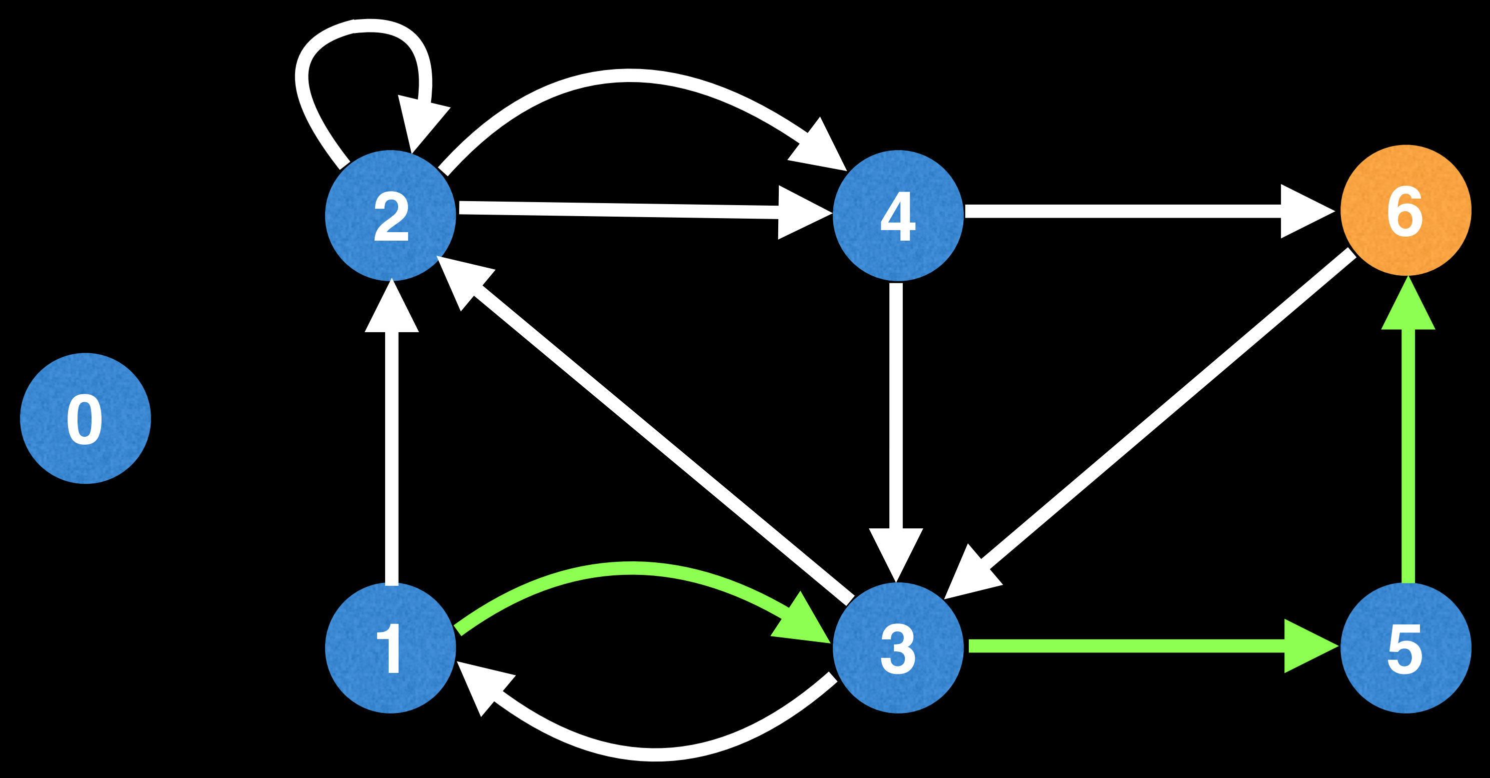
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



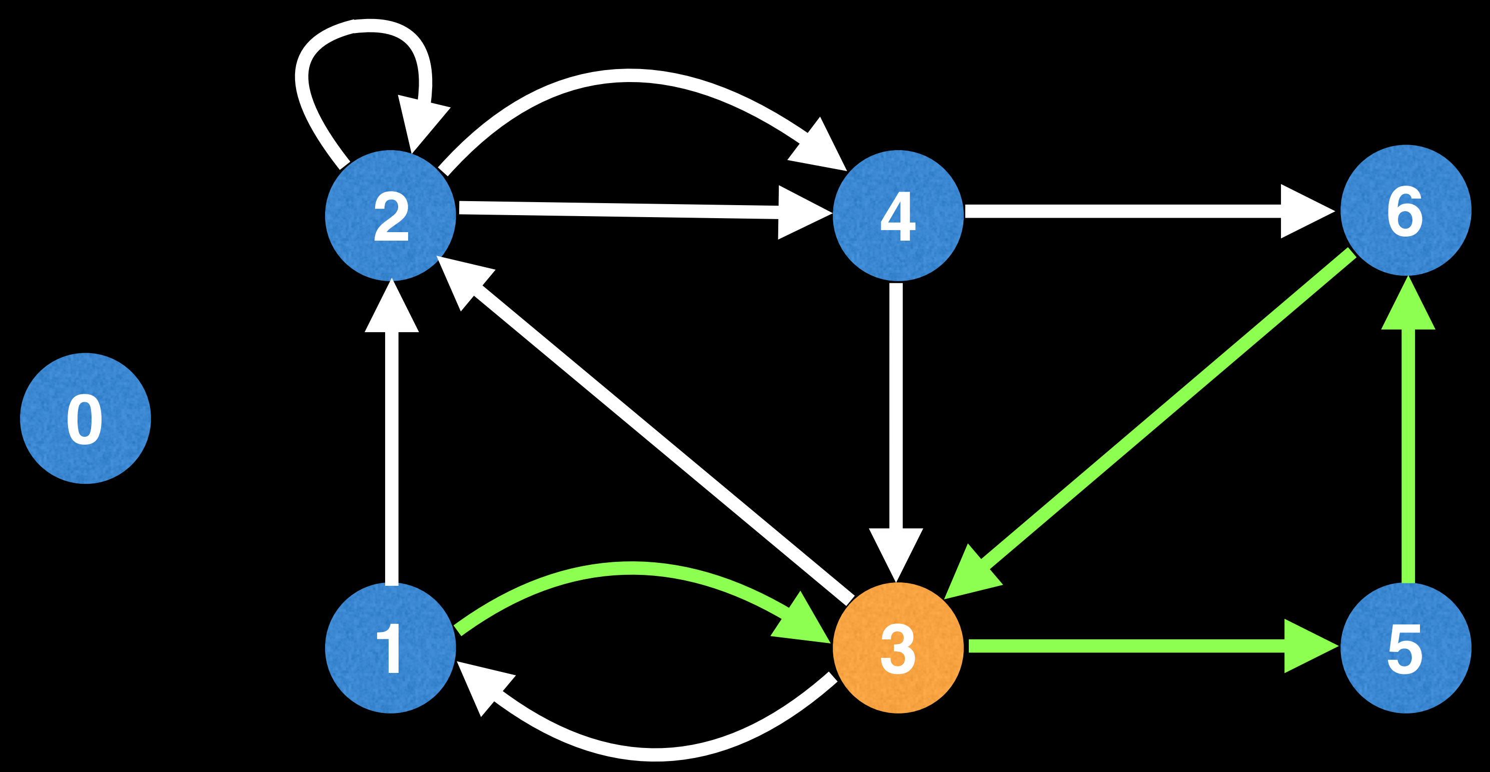
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



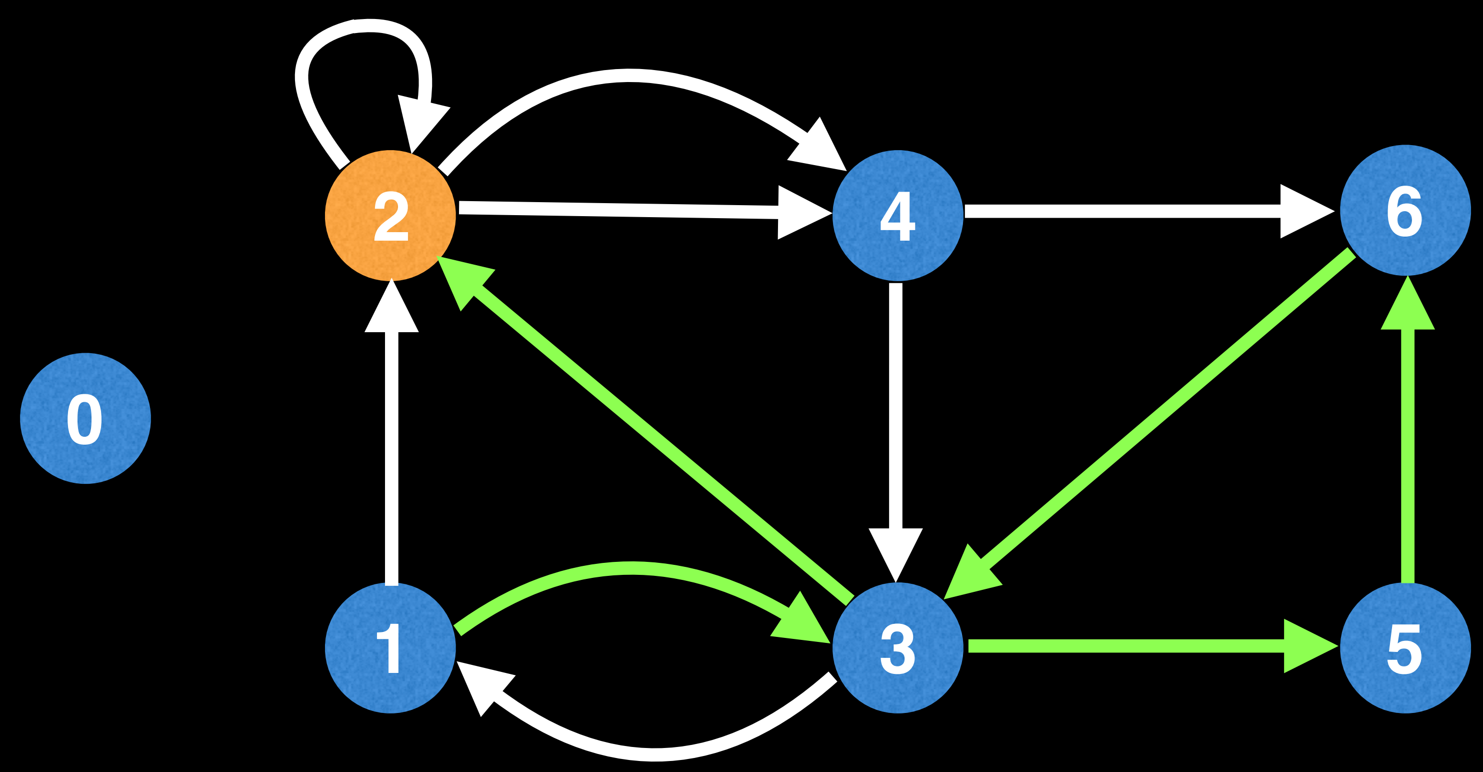
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



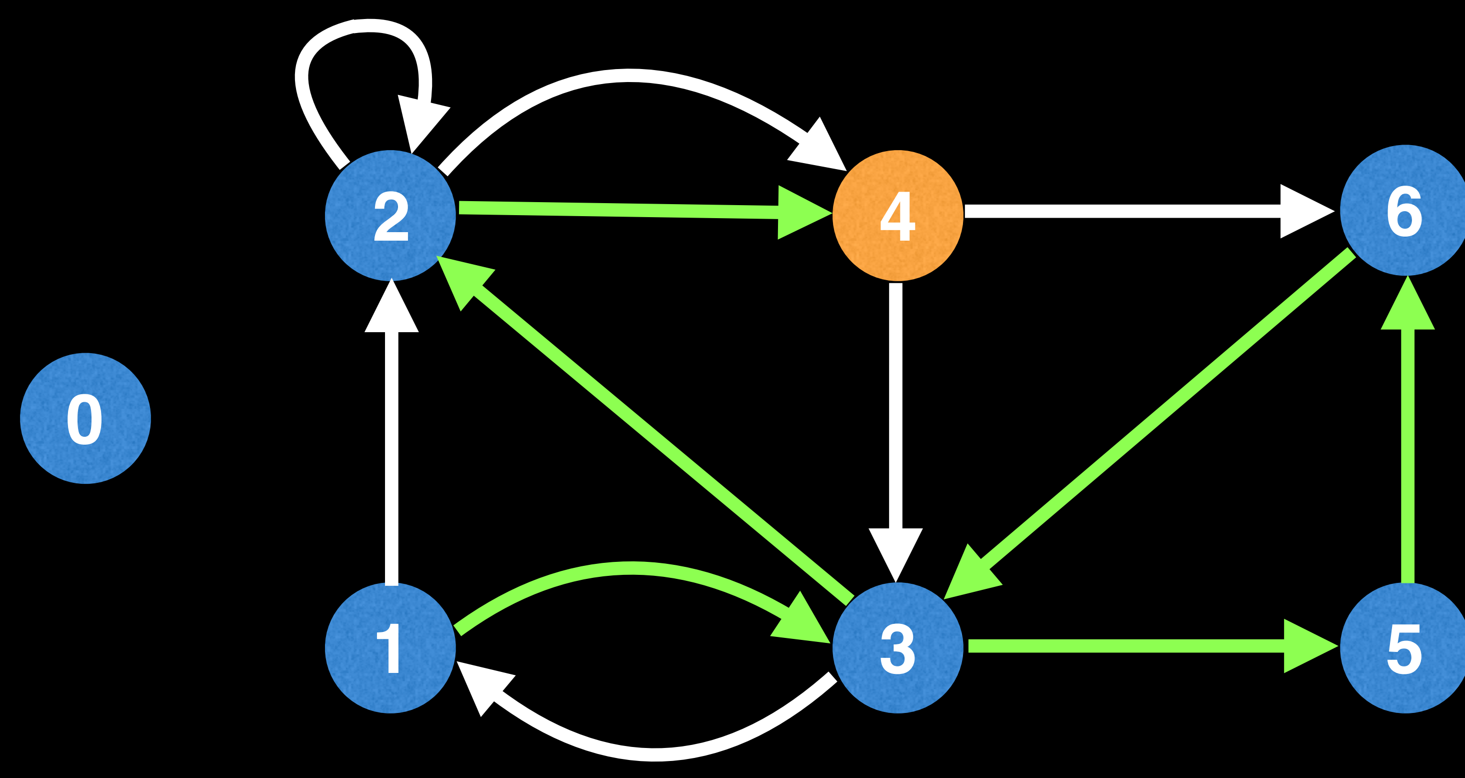
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



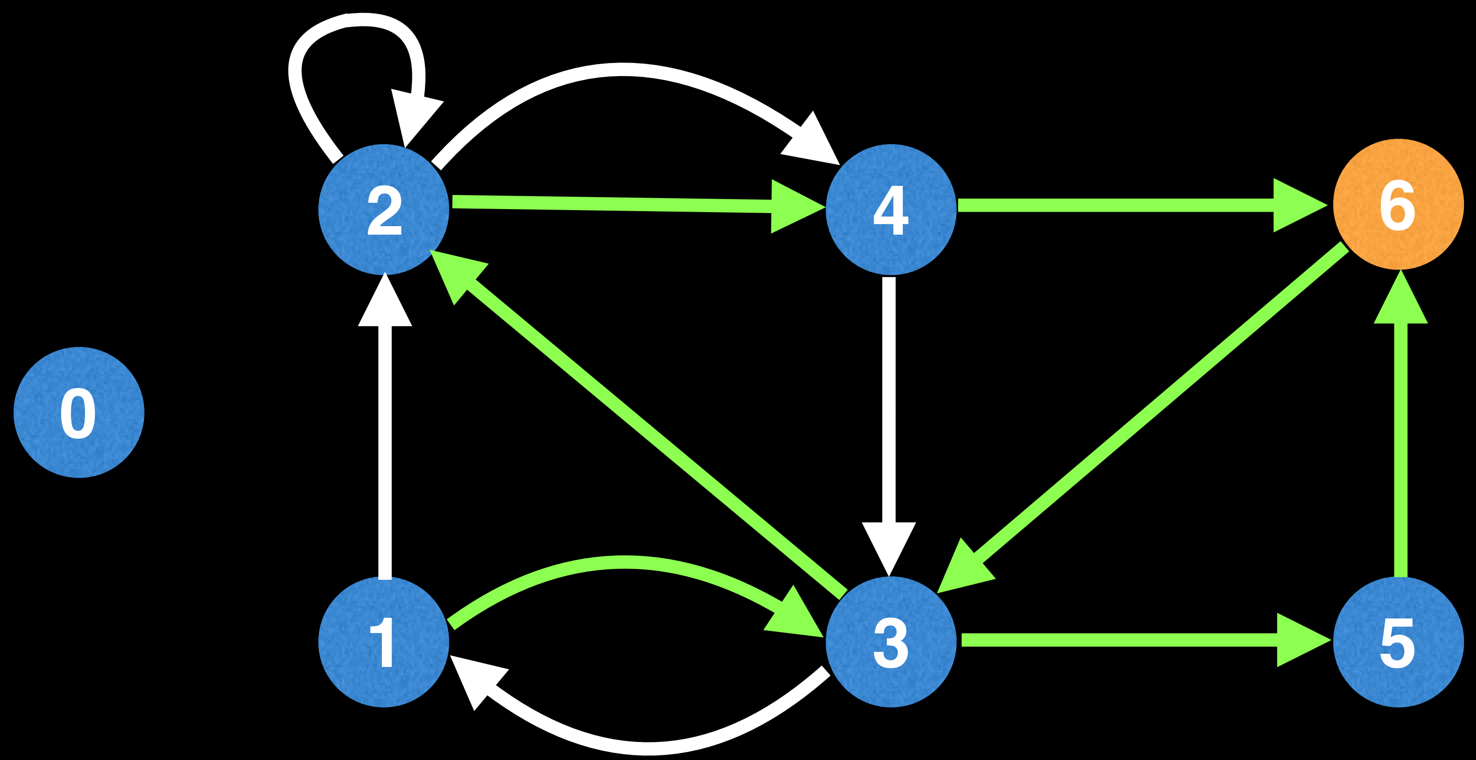
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



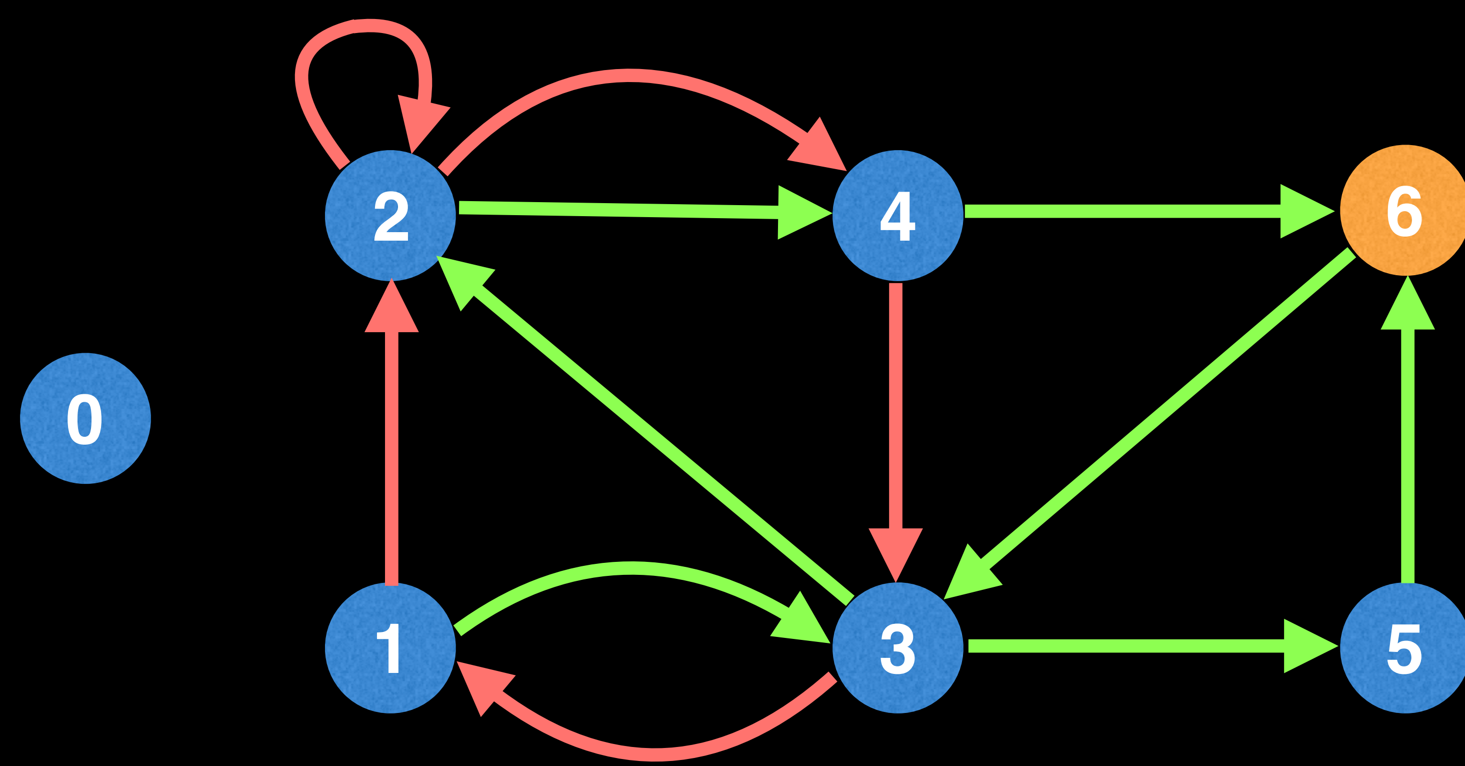
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1

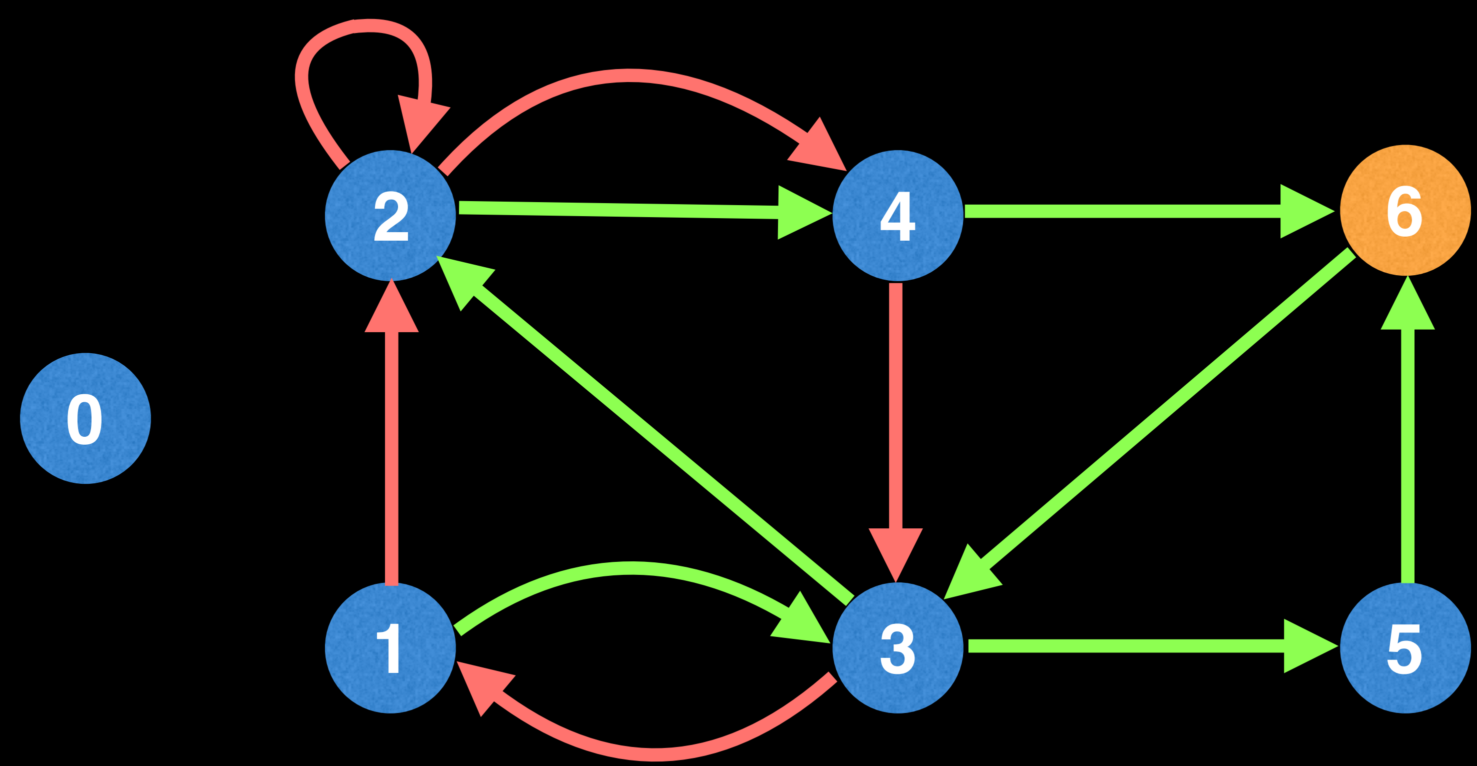


By randomly selecting edges during the DFS we made it from the start node to the end node.

However, we did not find an Eulerian path because we didn't traverse all the edges in our graph!

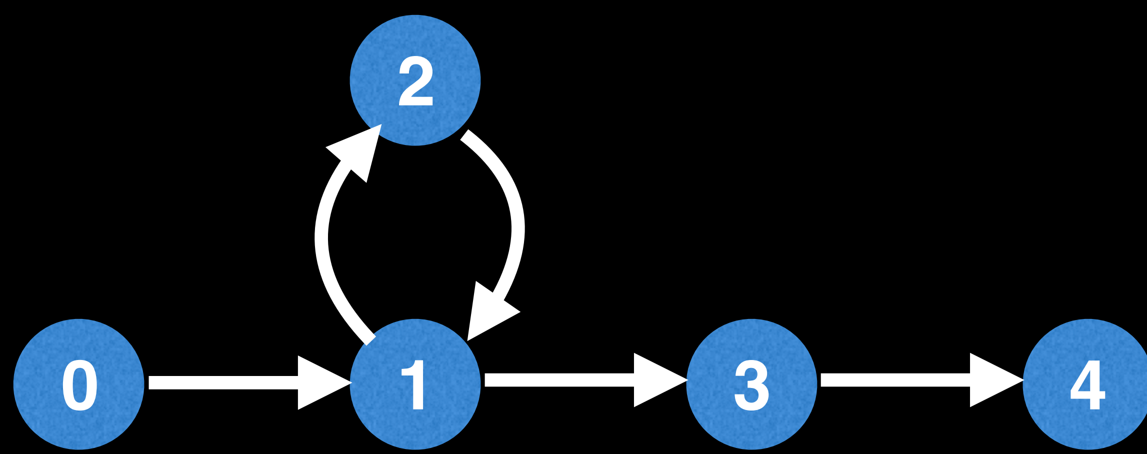
Finding an Eulerian path (directed graph)

Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



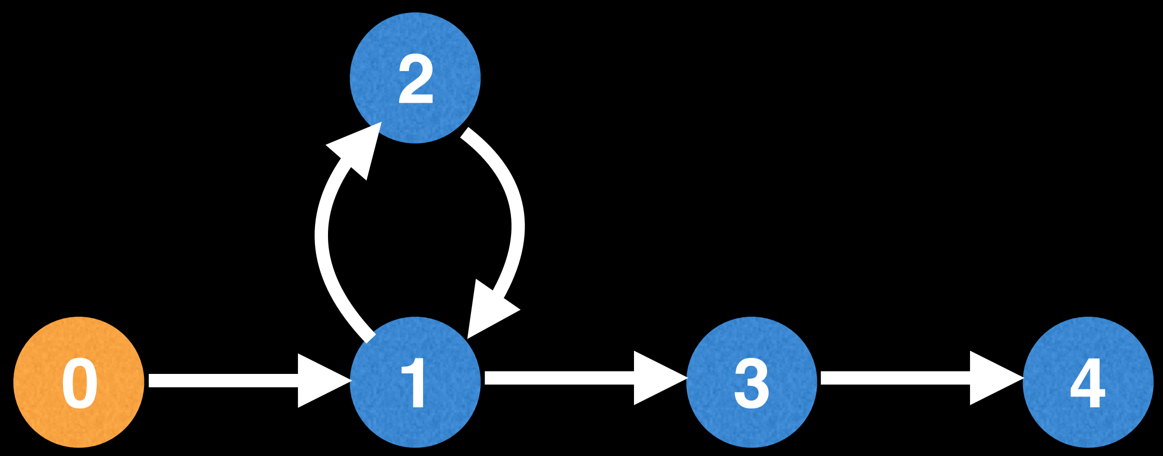
The good news is we can modify our DFS to handle forcing the traversal of all edges :)

Finding an Eulerian path (directed graph)



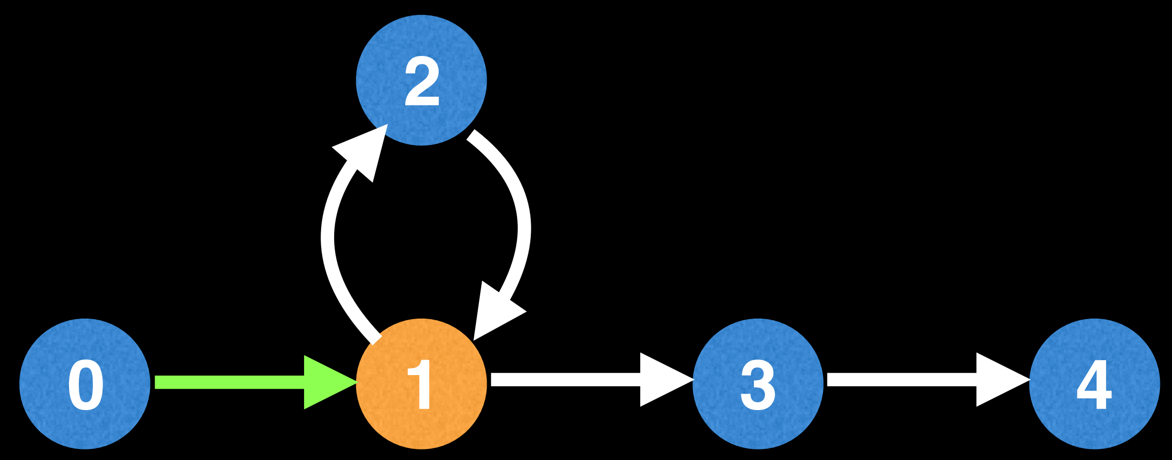
To illustrate this, consider starting at node 0 and trying to find an Eulerian path.

Finding an Eulerian path (directed graph)



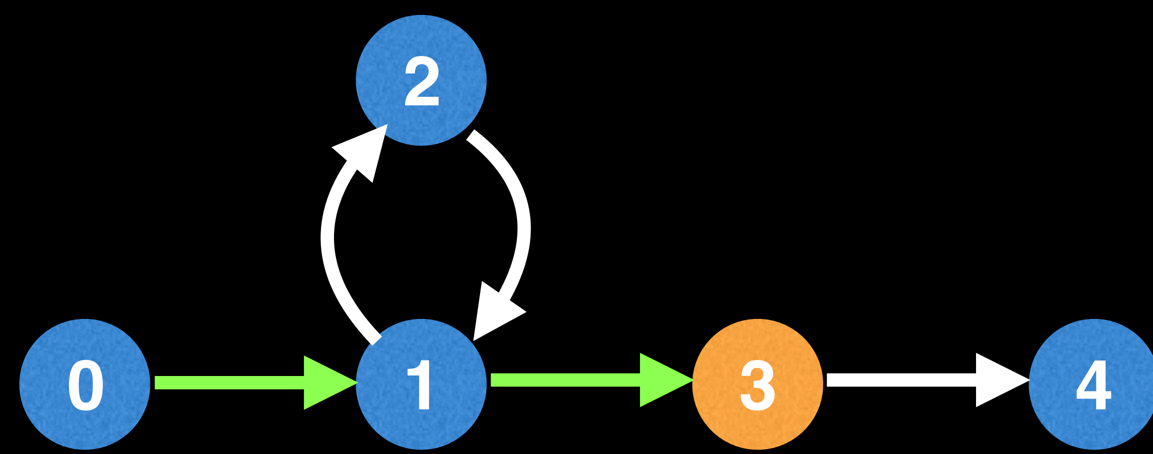
To illustrate this, consider starting at node 0 and trying to find an Eulerian path.

Finding an Eulerian path (directed graph)



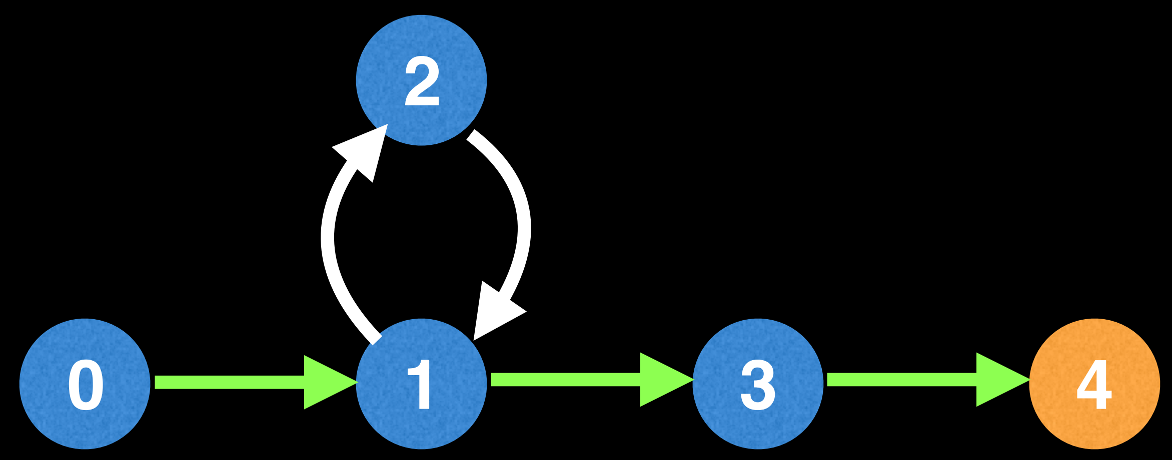
To illustrate this, consider starting at node 0 and trying to find an Eulerian path.

Finding an Eulerian path (directed graph)



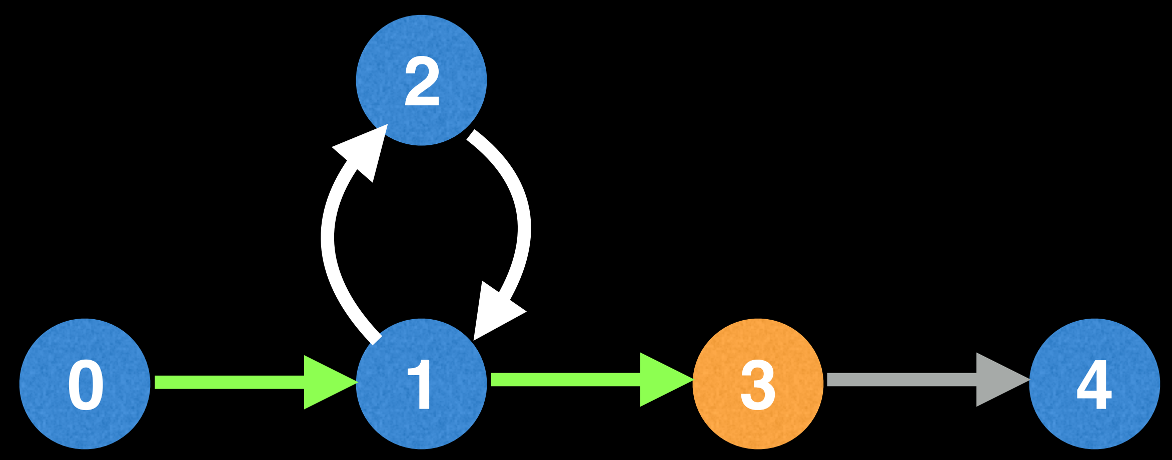
Whoops… we skipped the edges going to node 2 and back which need to be part of the solution.

Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

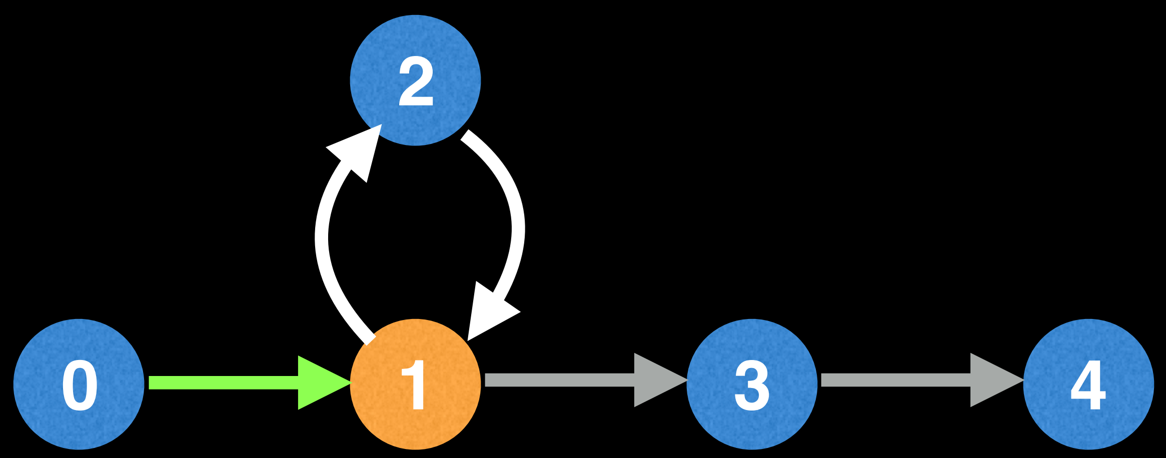
Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [4]

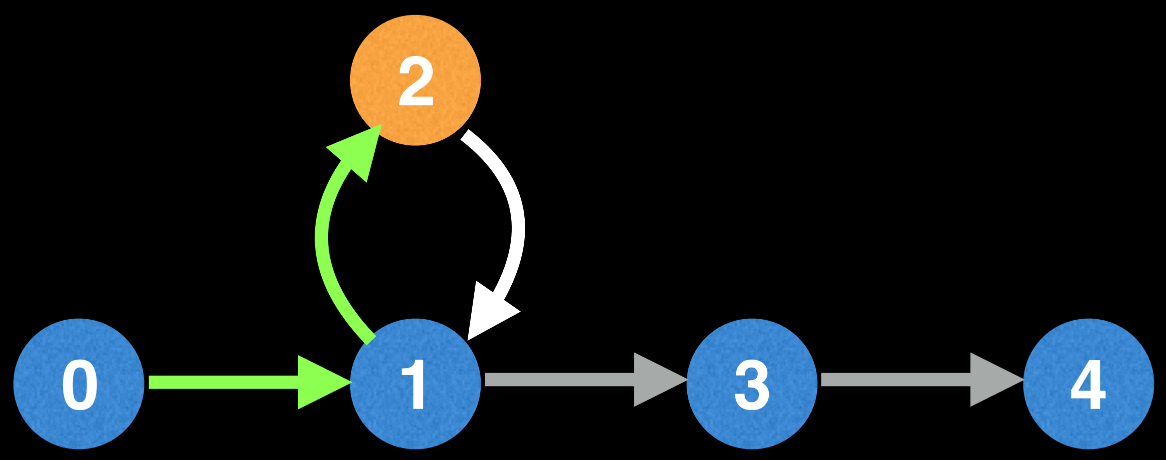
Finding an Eulerian path (directed graph)



When backtracking, if the current node has any remaining unvisited edges (white edges) we follow any of them calling our DFS method recursively to extend the Eulerian path.

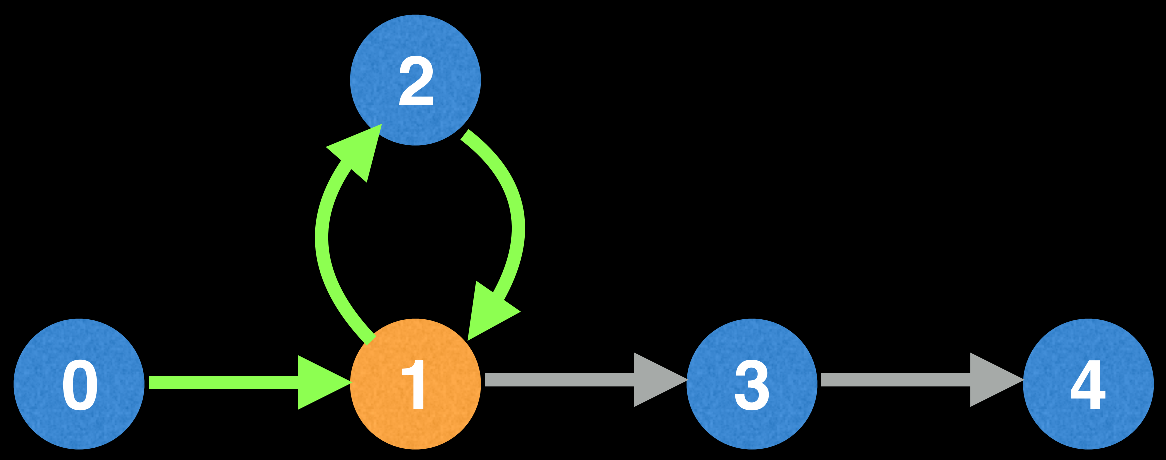
Solution: [3, 4]

Finding an Eulerian path (directed graph)



Solution: [3, 4]

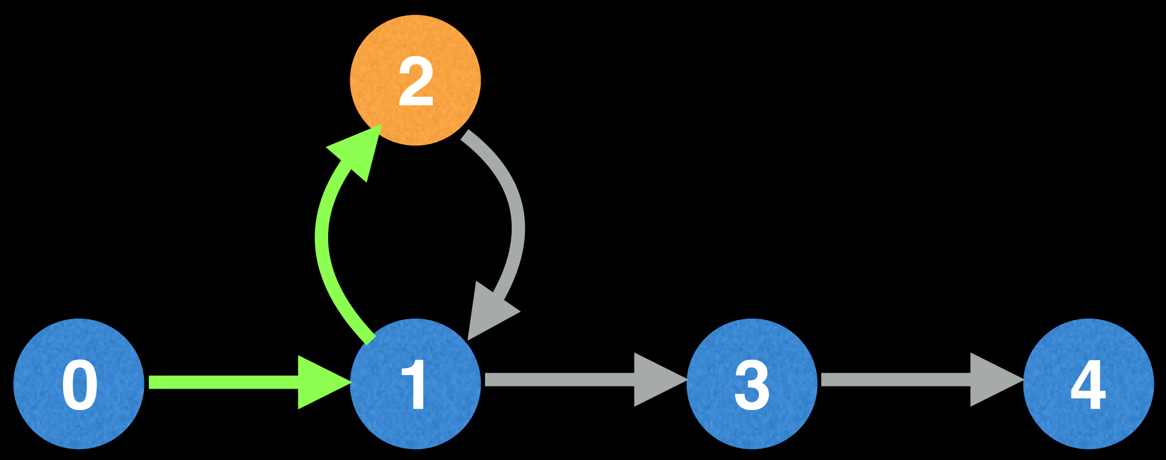
Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [3, 4]

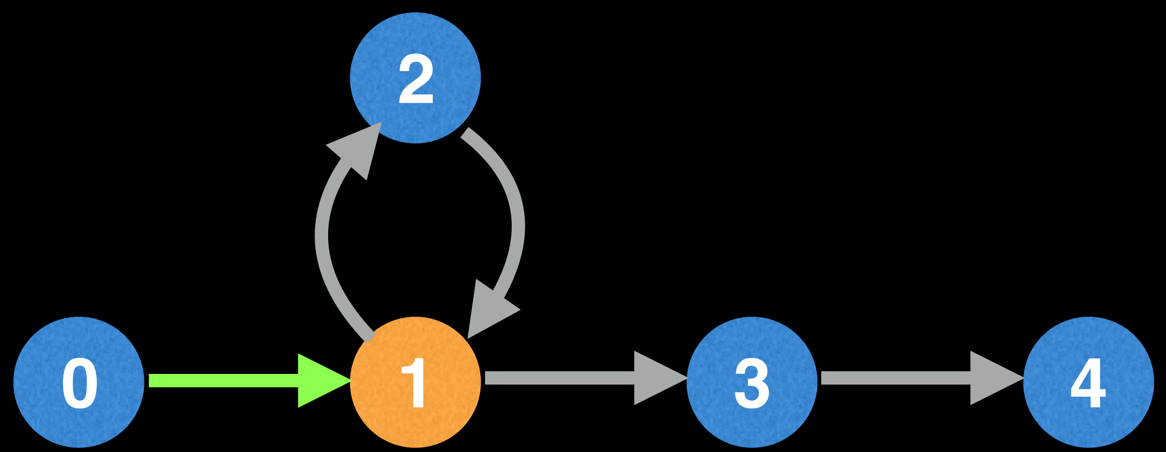
Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [1, 3, 4]

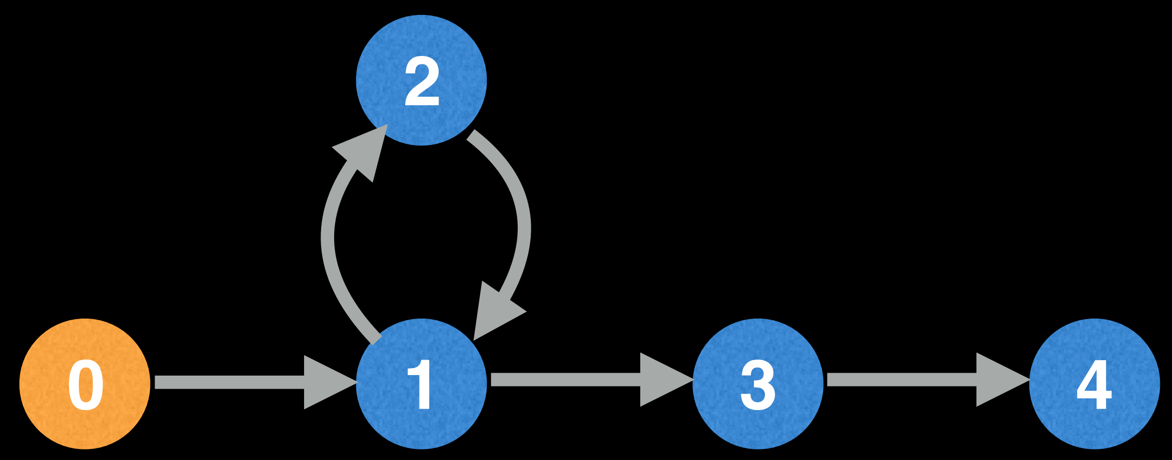
Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [2, 1, 3, 4]

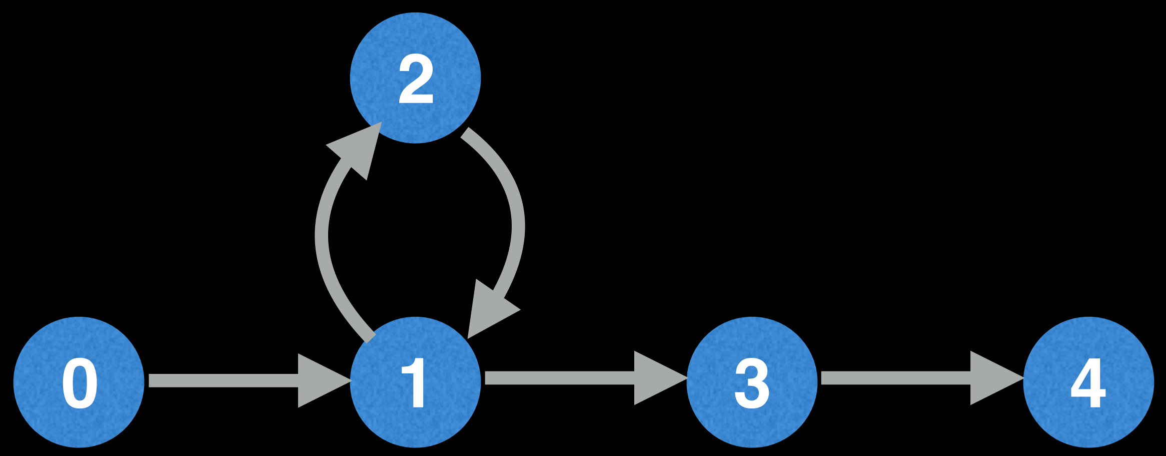
Finding an Eulerian path (directed graph)



Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [1, 2, 1, 3, 4]

Finding an Eulerian path (directed graph)

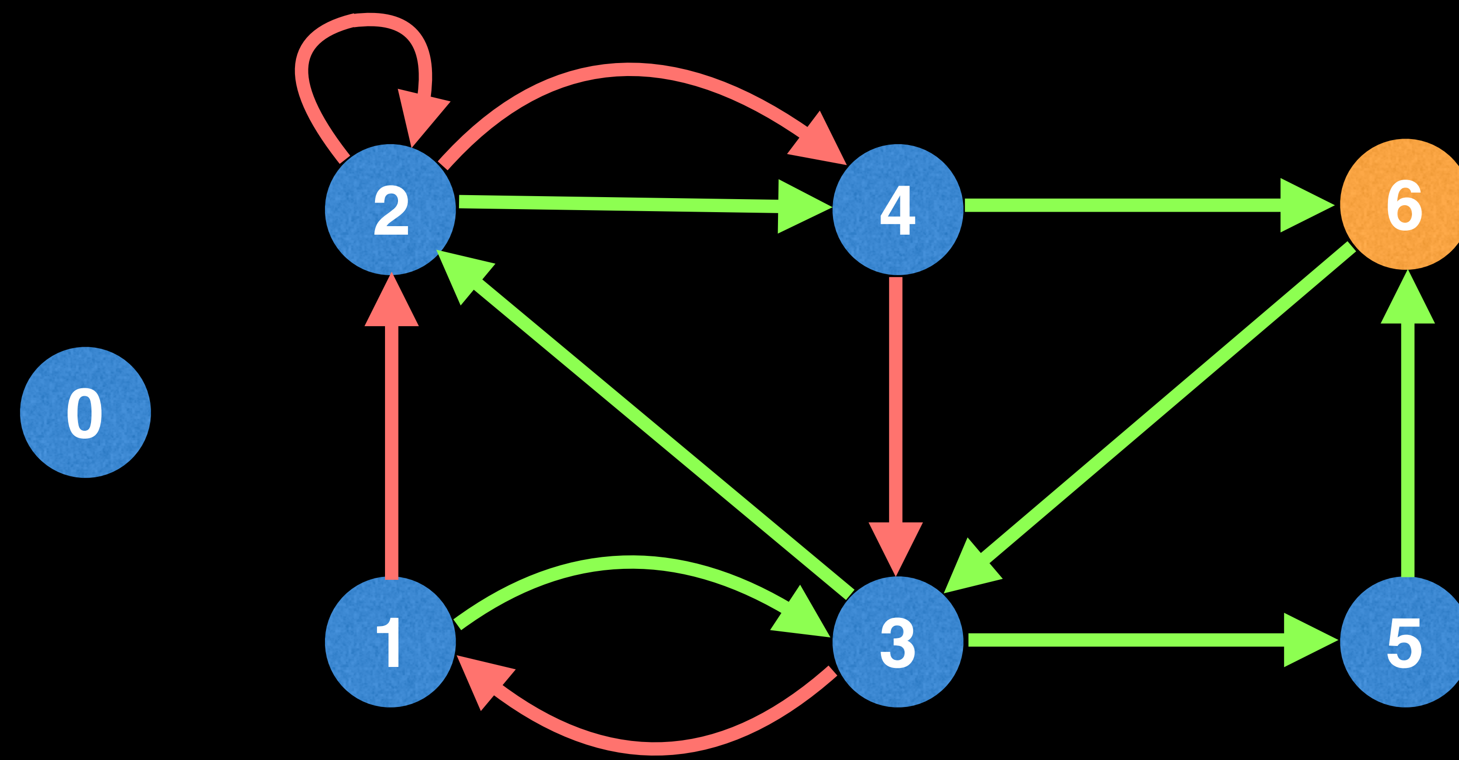


Once we get stuck (meaning the current node has no unvisited outgoing edges), we backtrack and add the current node to the solution.

Solution: [0, 1, 2, 1, 3, 4]

Finding an Eulerian path (directed graph)

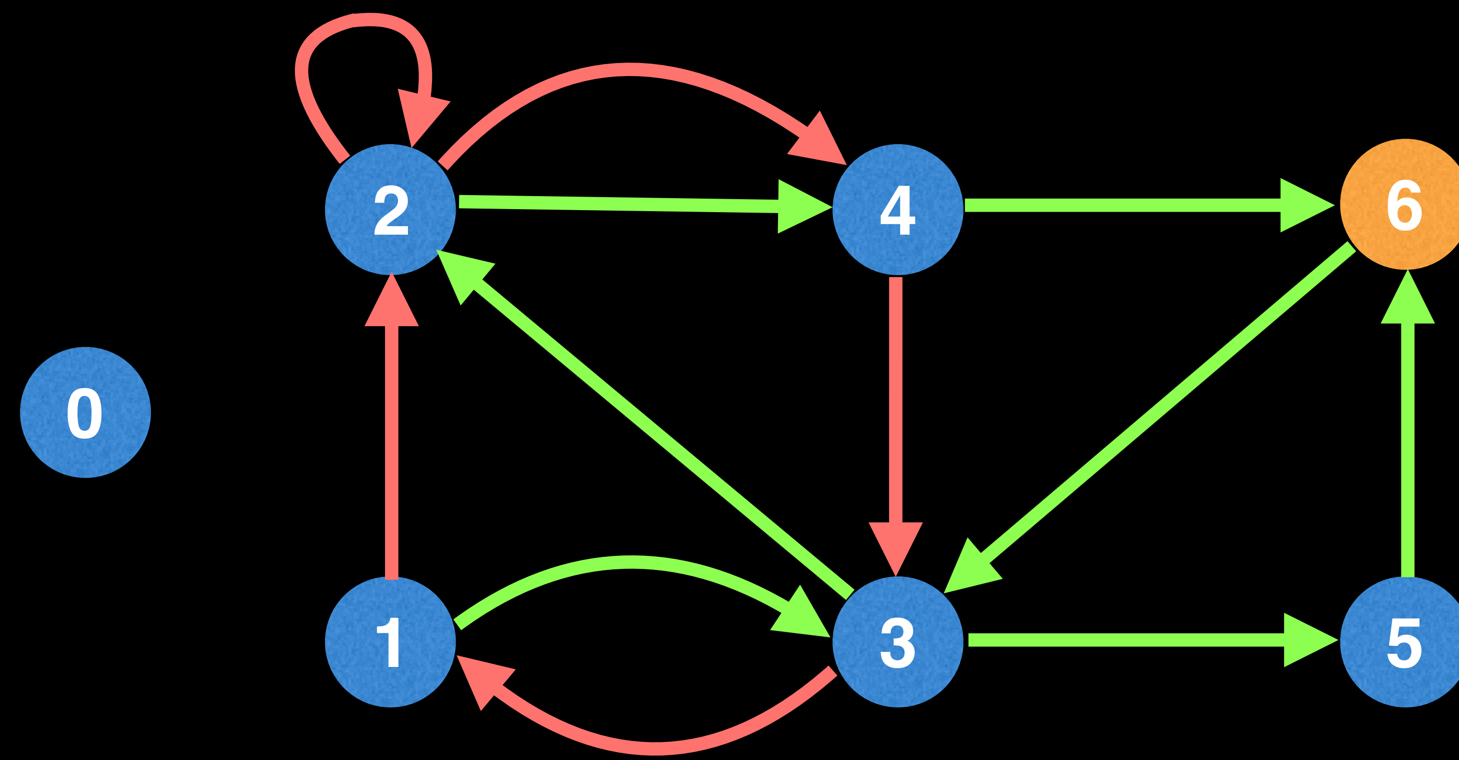
Node	In	Out
0	0	0
1	1	2
2	3	3
3	3	3
4	2	2
5	1	1
6	2	1



Coming back to the previous example, let's restart the algorithm, but this time track the number of unvisited edges we have left to take for each node.

Finding an Eulerian path (directed graph)

Node	[illegible]	Out
0	[illegible]	0
1	[illegible]	2
2	[illegible]	3
3	[illegible]	3
4	[illegible]	2
5	[illegible]	1
6	[illegible]	1

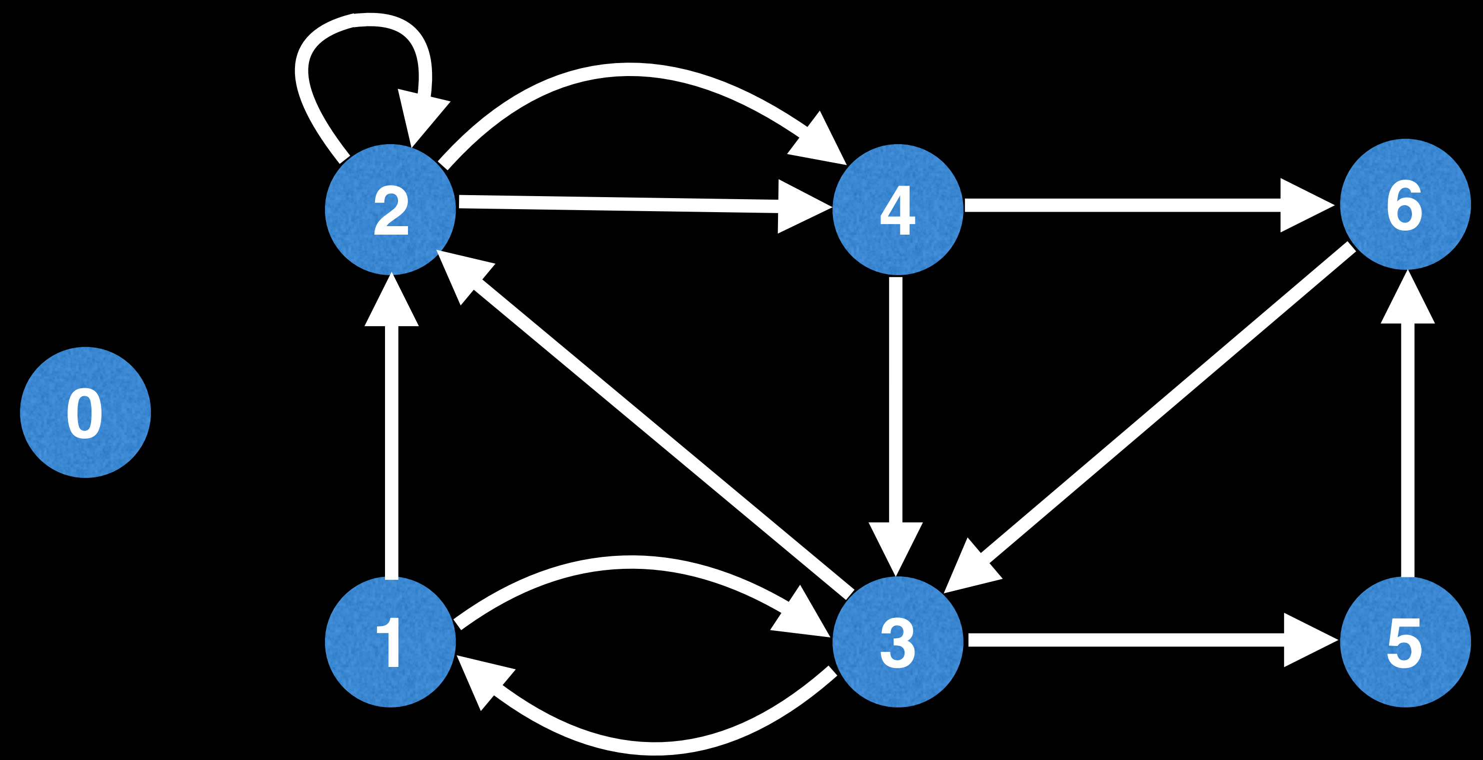


In fact, we have already computed the number of outgoing edges for each edge in the “out” array which we can reuse.

We won’t be needing the “in” array after we’ve validated that an Eulerian path exists.

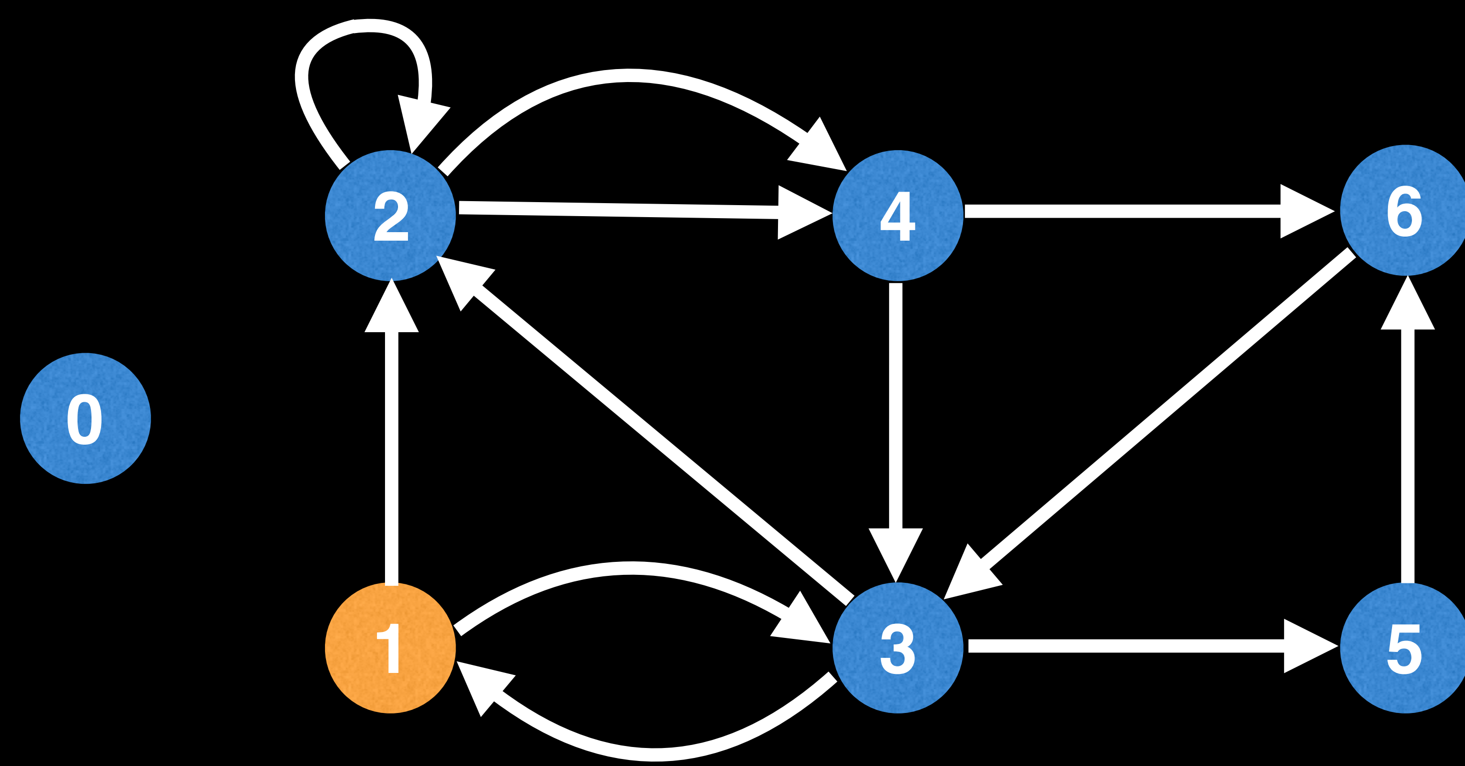
Finding an Eulerian path (directed graph)

Node	Out
0	0
1	2
2	3
3	3
4	2
5	1
6	1



Finding an Eulerian path (directed graph)

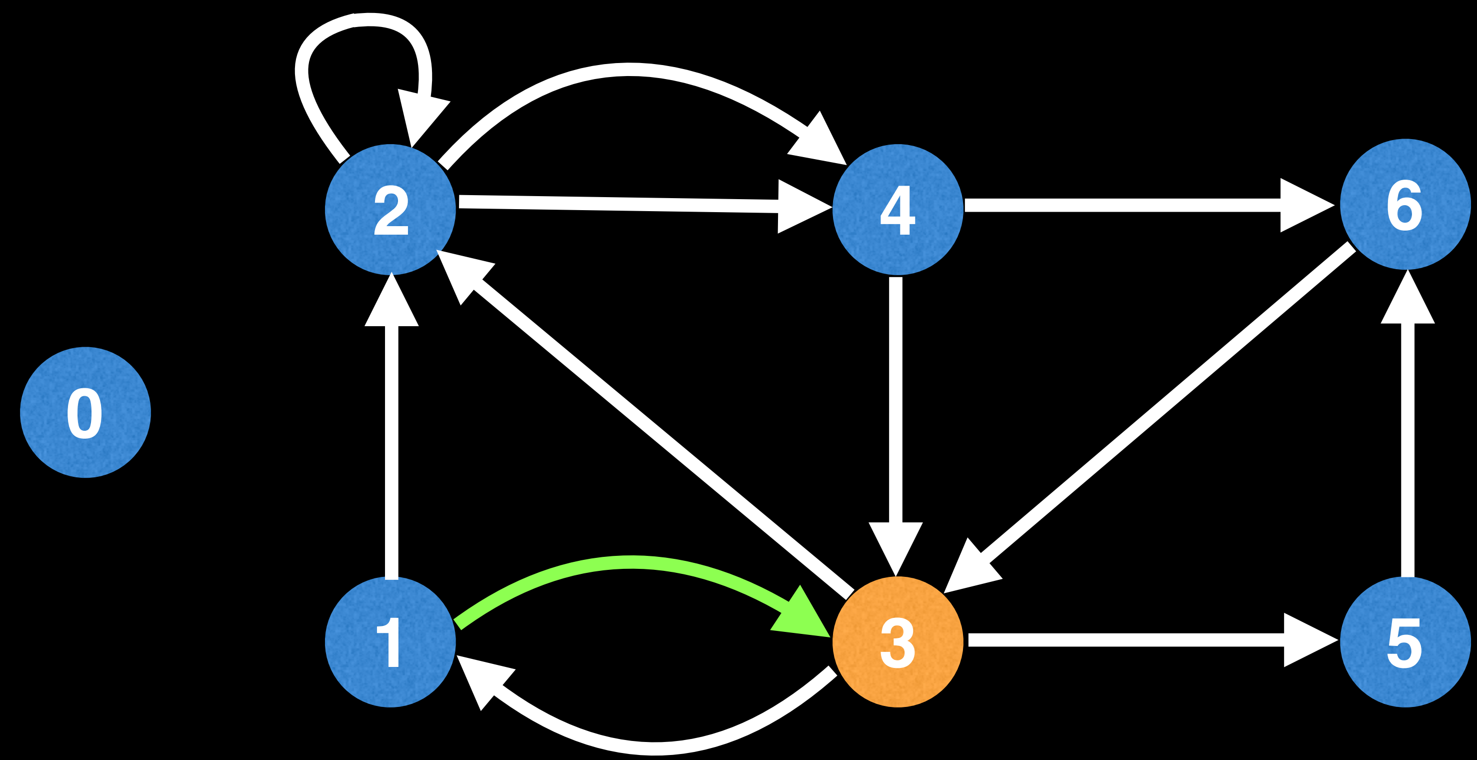
Node	Out
0	0
1	2
2	3
3	3
4	2
5	1
6	1



Solution = []

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	1
2	3
3	3
4	2
5	1
6	1

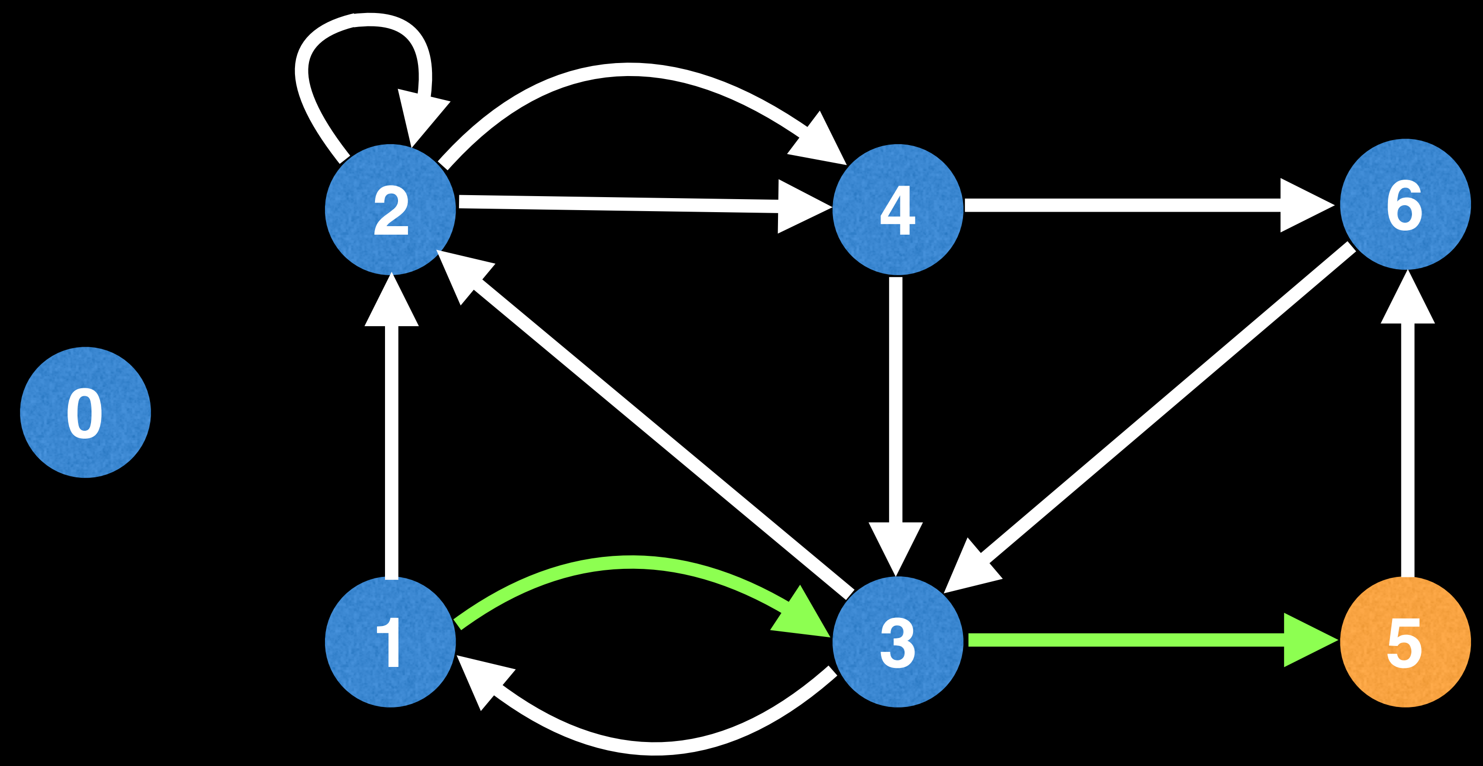


Every time an edge is taken, reduce the outgoing edge count in the out array.

Solution = []

Finding an Eulerian path (directed graph)

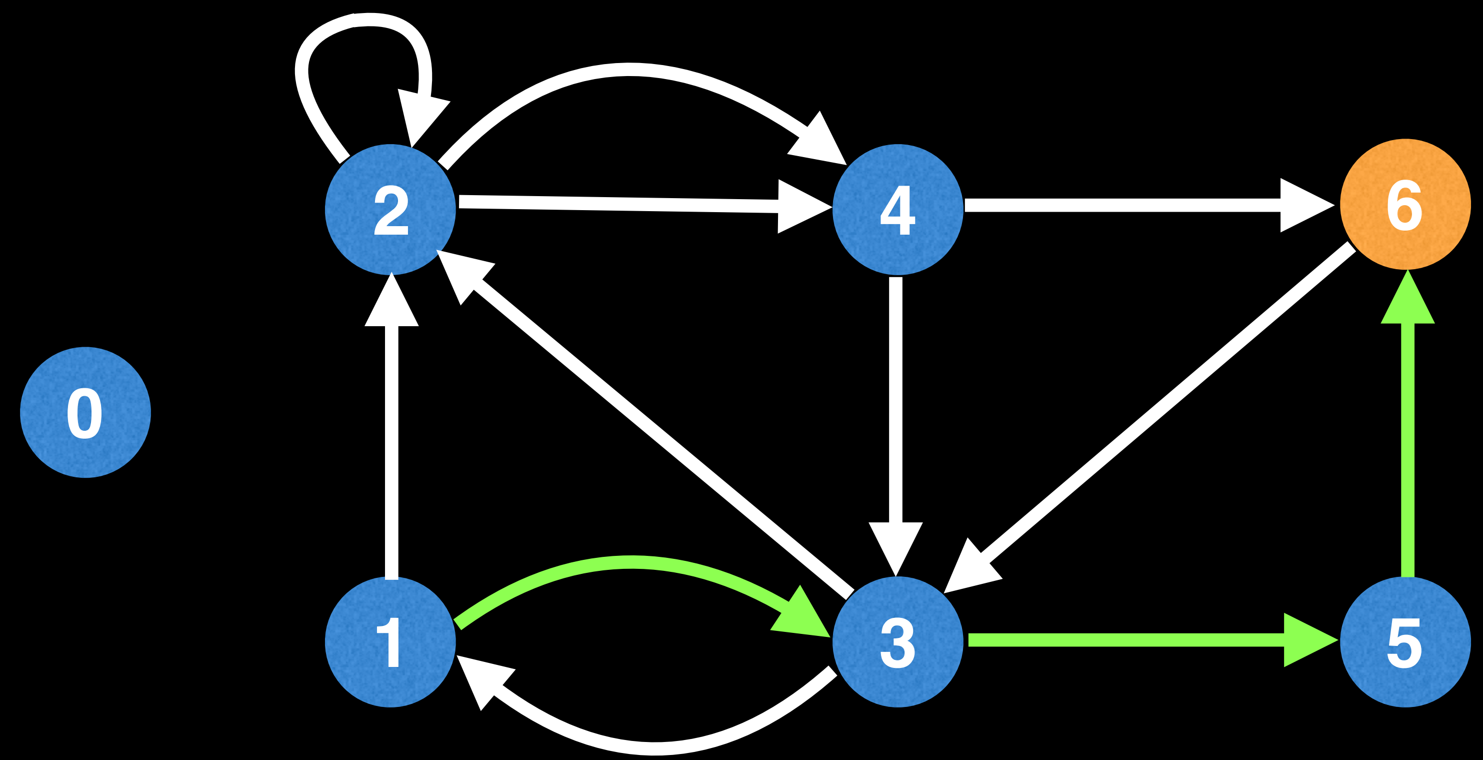
Node	Out
0	0
1	1
2	3
3	2
4	2
5	1
6	1



Solution = []

Finding an Eulerian path (directed graph)

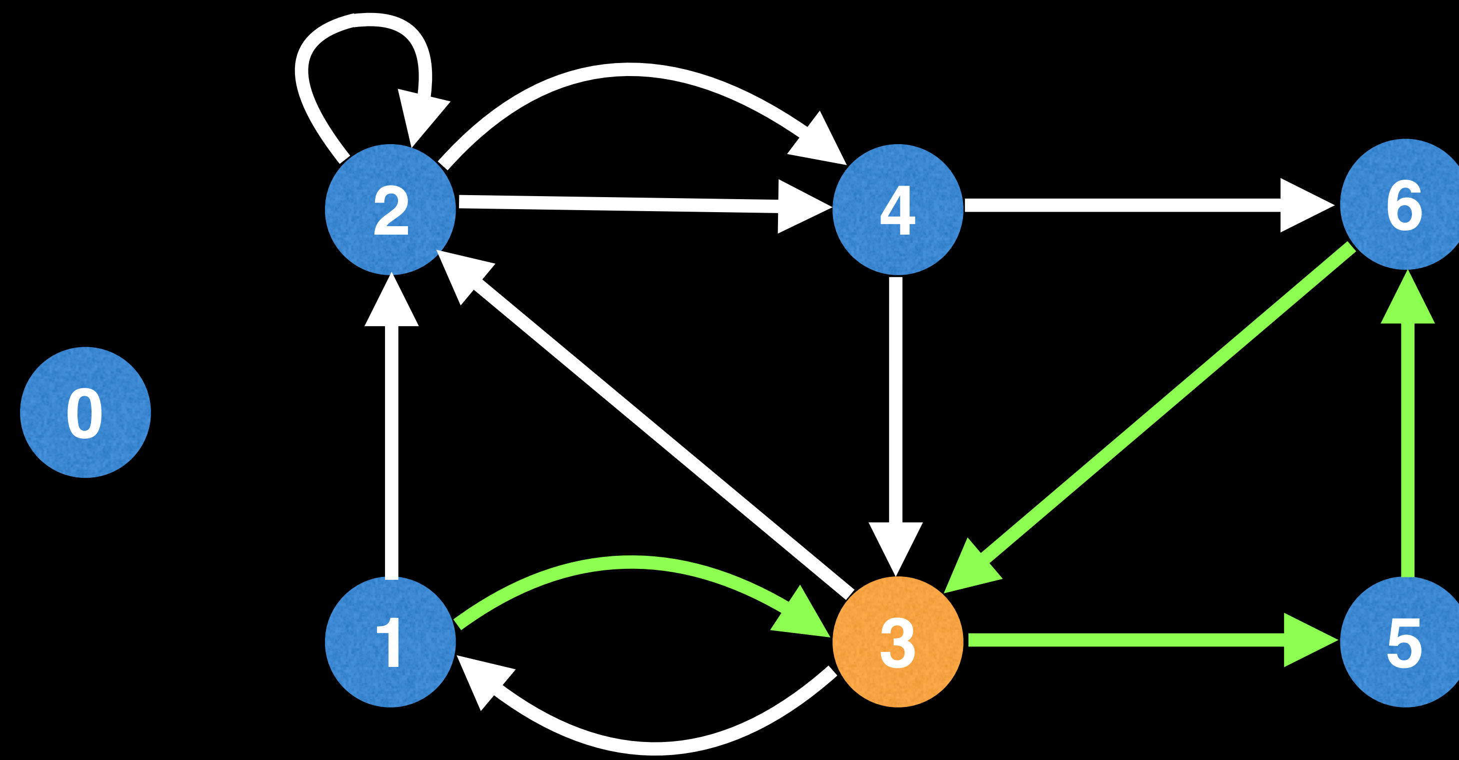
Node	Out
0	0
1	1
2	3
3	2
4	2
5	0
6	1



Solution = []

Finding an Eulerian path (directed graph)

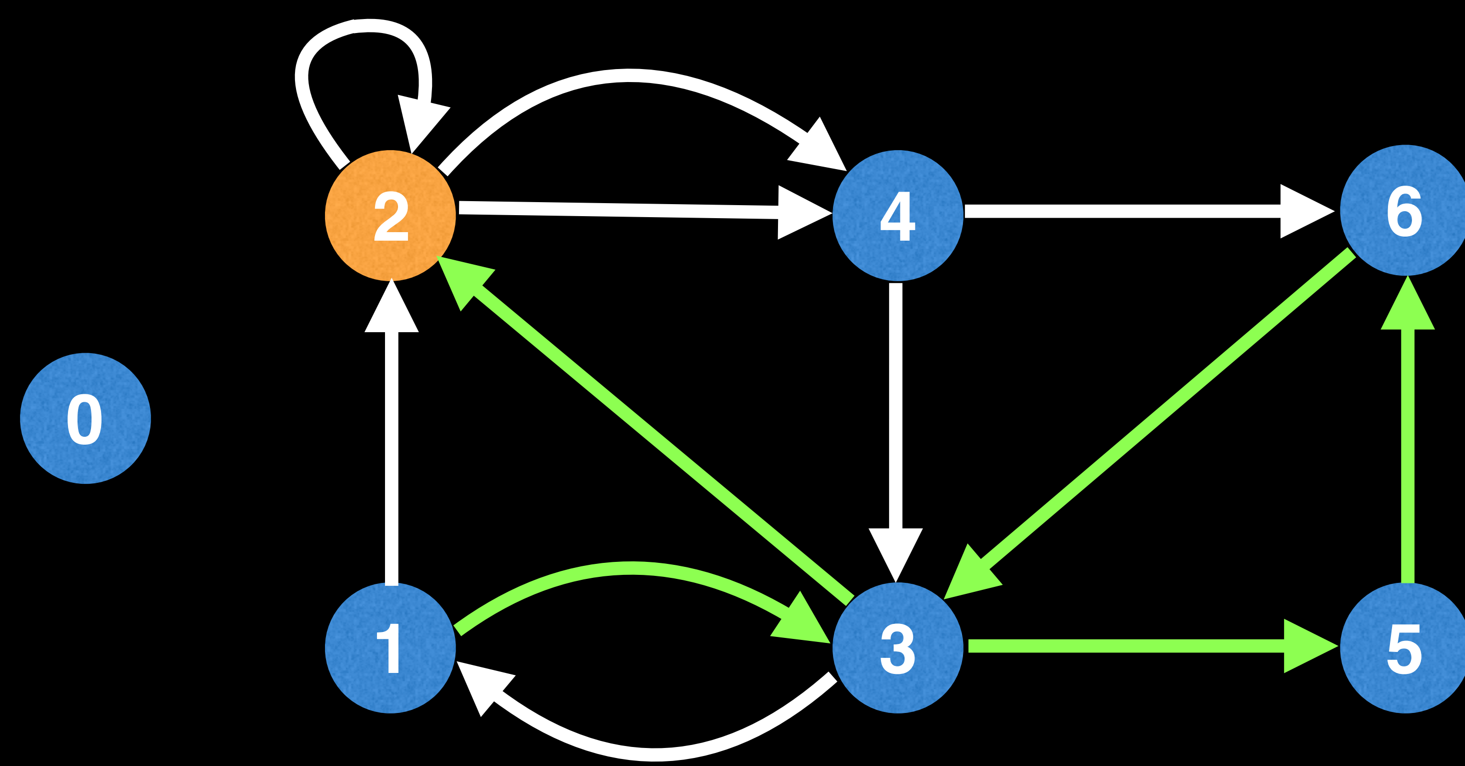
Node	Out
0	0
1	1
2	3
3	2
4	2
5	0
6	0



Solution = []

Finding an Eulerian path (directed graph)

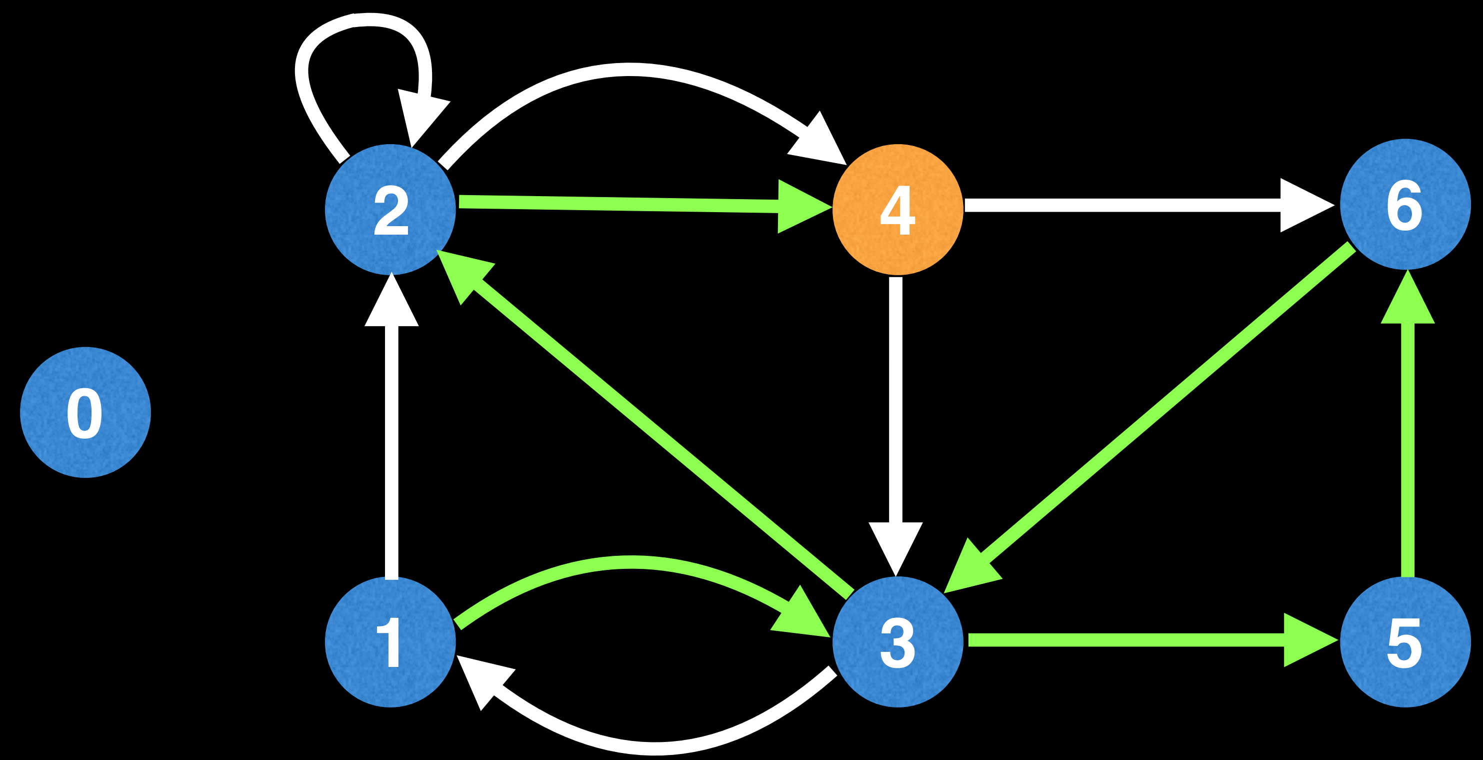
Node	Out
0	0
1	1
2	3
3	1
4	2
5	0
6	0



Solution = []

Finding an Eulerian path (directed graph)

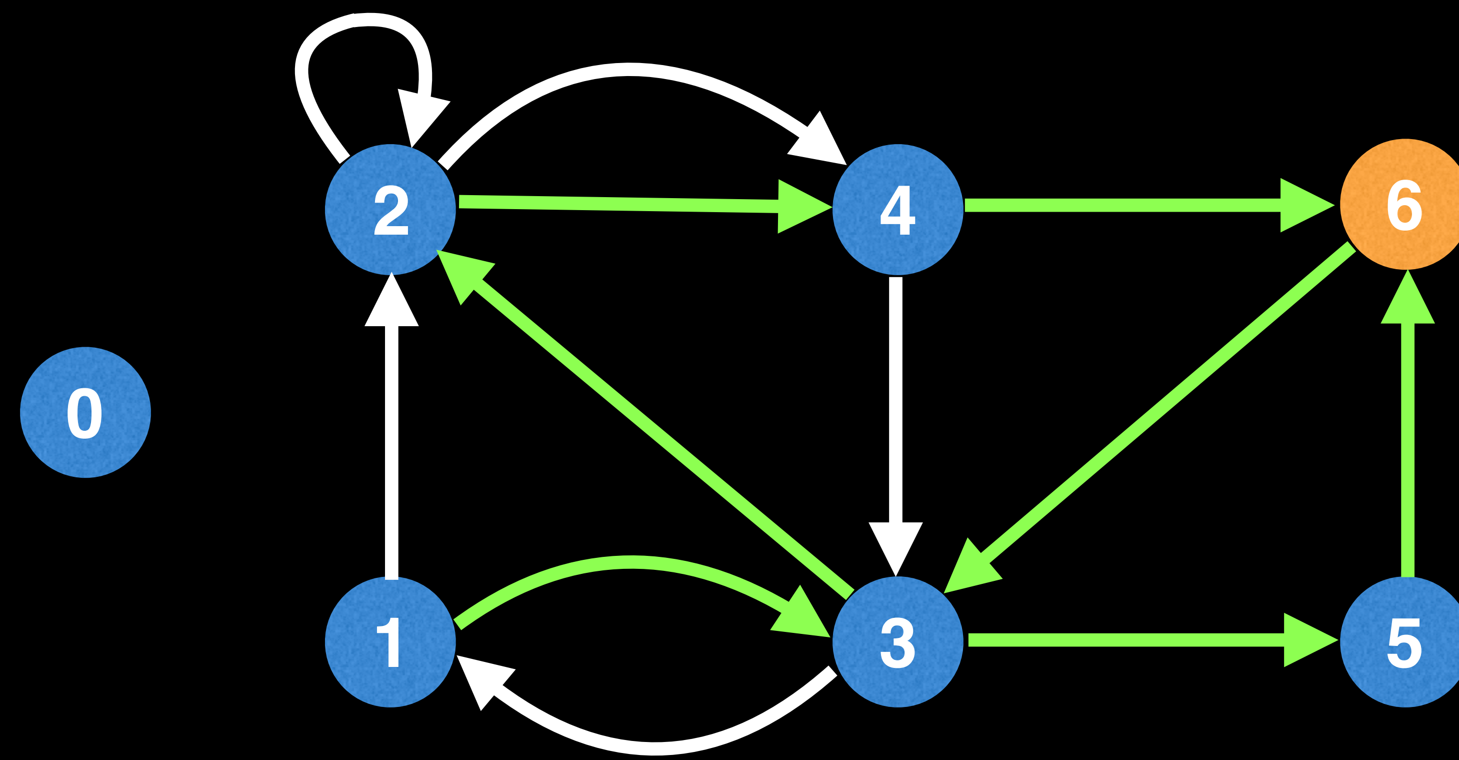
Node	Out
0	0
1	1
2	2
3	1
4	2
5	0
6	0



Solution = []

Finding an Eulerian path (directed graph)

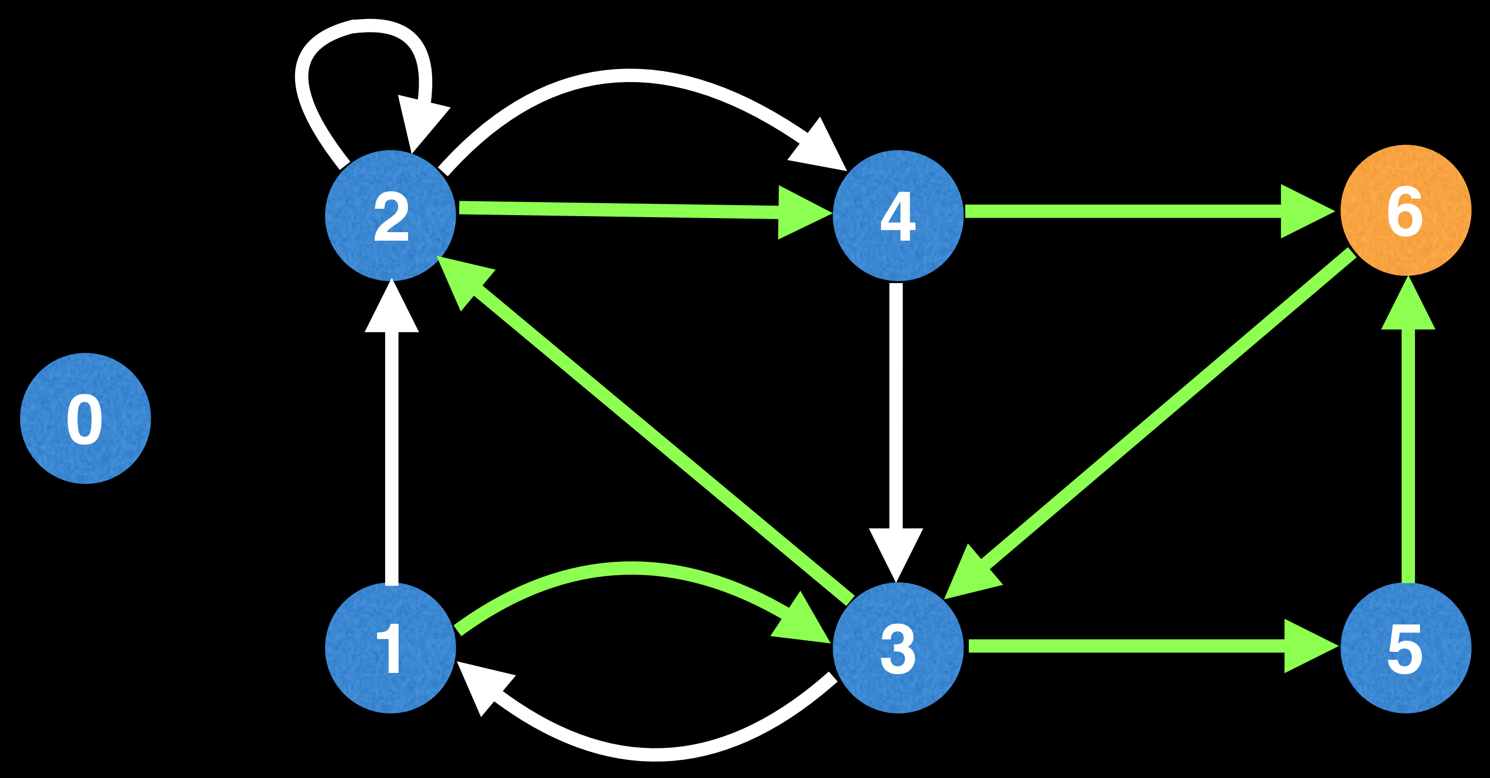
Node	Out
0	0
1	1
2	2
3	1
4	1
5	0
6	0



Solution = []

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	1
2	2
3	1
4	1
5	0
6	0

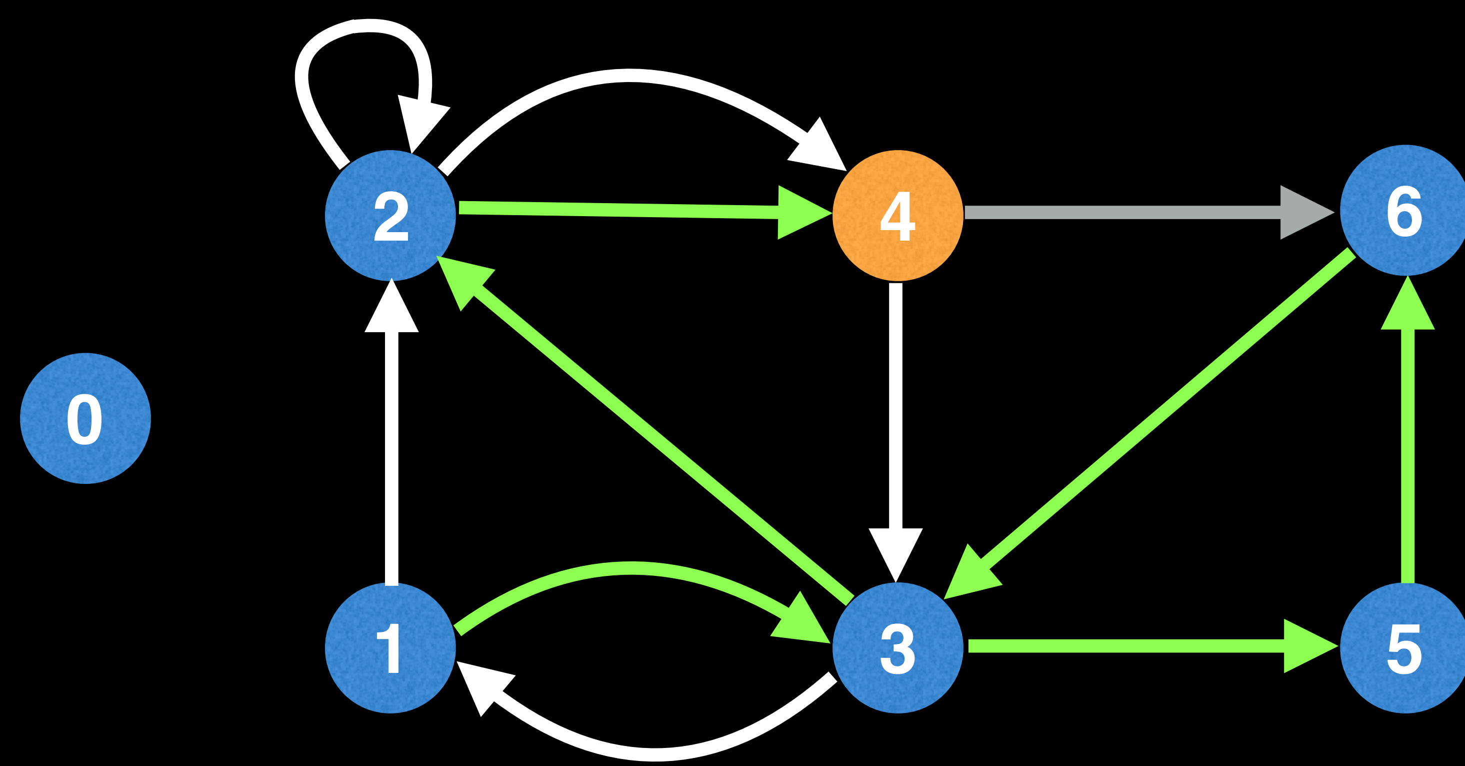


When the DFS is stuck, meaning there are no more outgoing edges (i.e out[i] = 0), then we know to backtrack and add the current node to the solution.

Solution = []

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	1
2	2
3	1
4	1
5	0
6	0

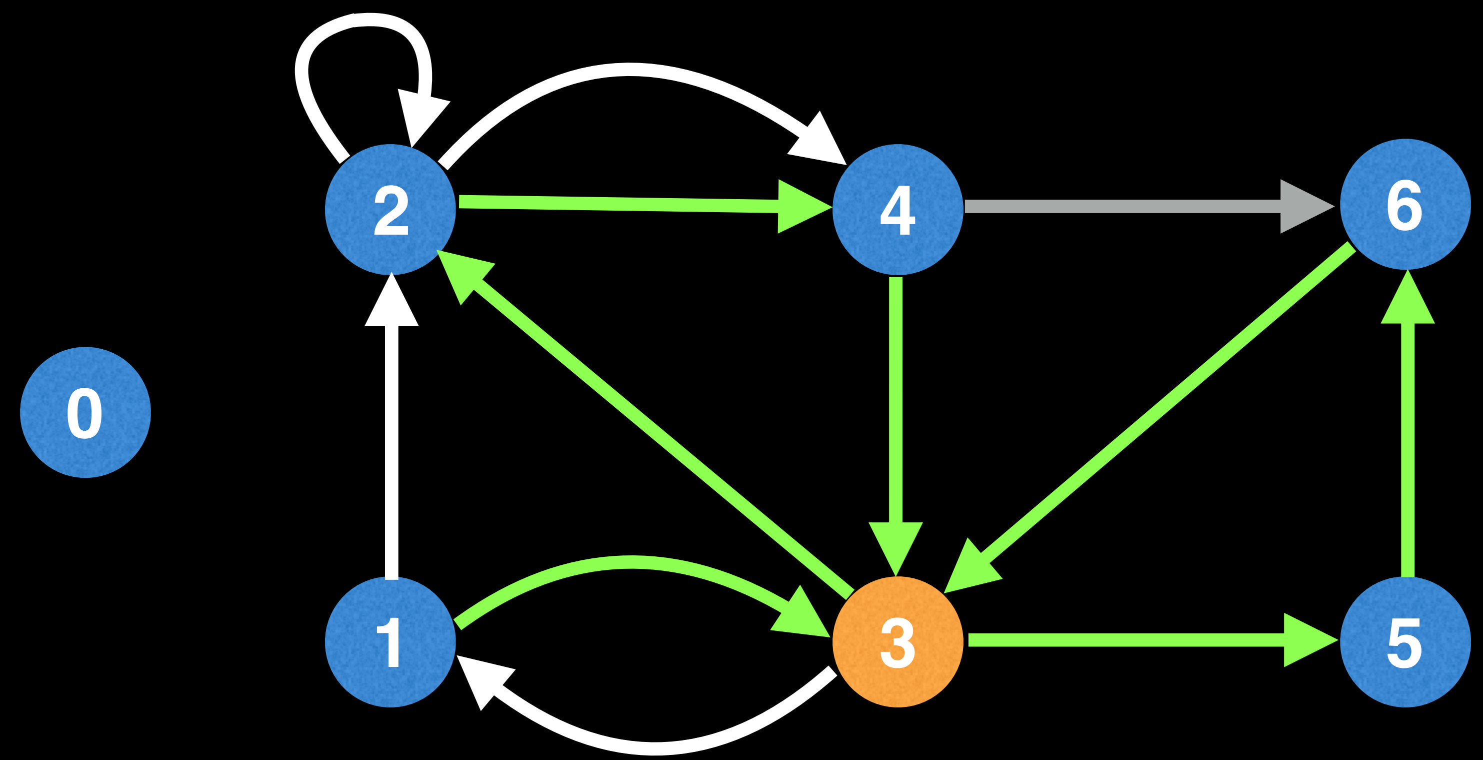


When backtracking, if the current node has any remaining unvisited edges (white edges), we follow any of them, calling our DFS method recursively to extend the Eulerian path. We can verify there still are outgoing edges by checking if out[i] != 0.

Solution = [6]

Finding an Eulerian path (directed graph)

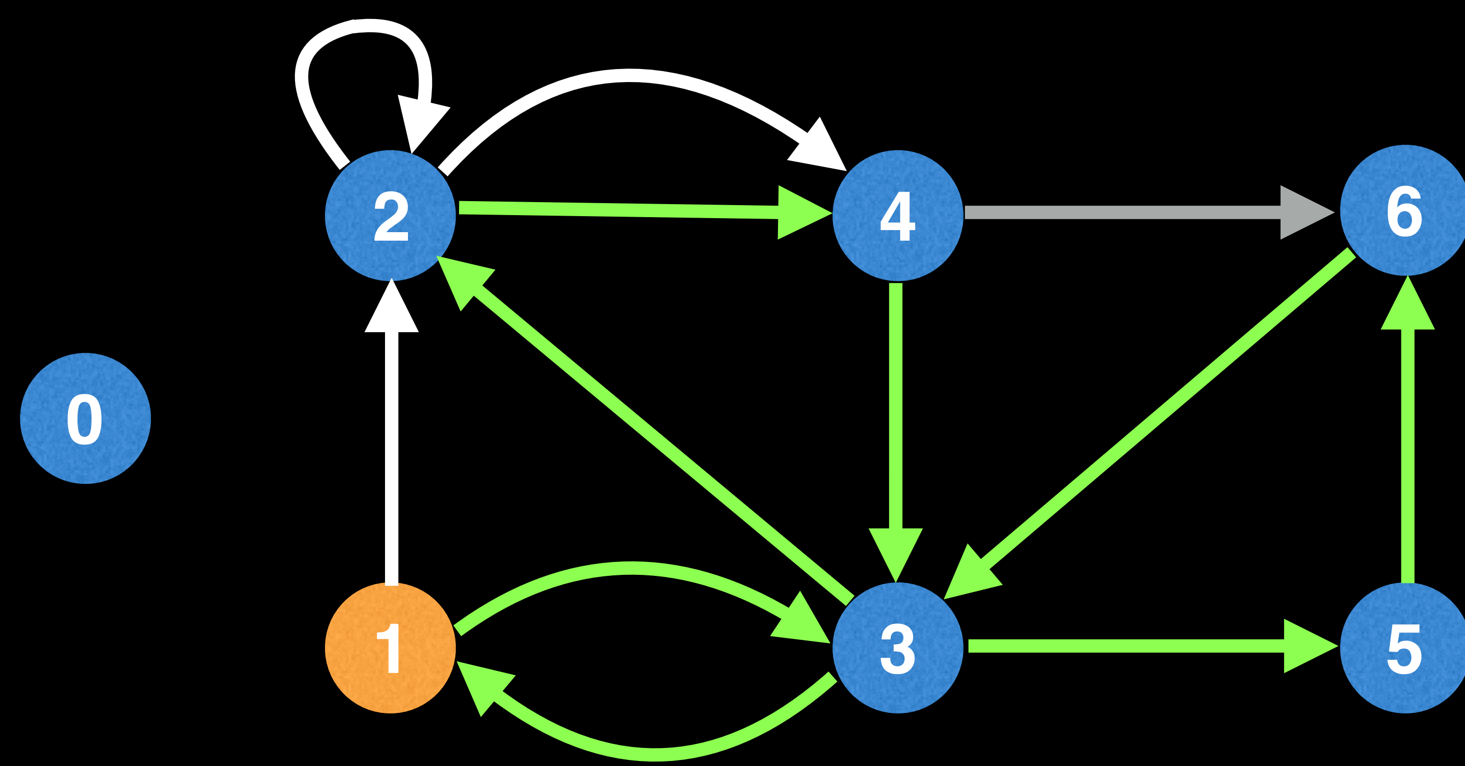
Node	Out
0	0
1	1
2	2
3	1
4	0
5	0
6	0



Solution = [6]

Finding an Eulerian path (directed graph)

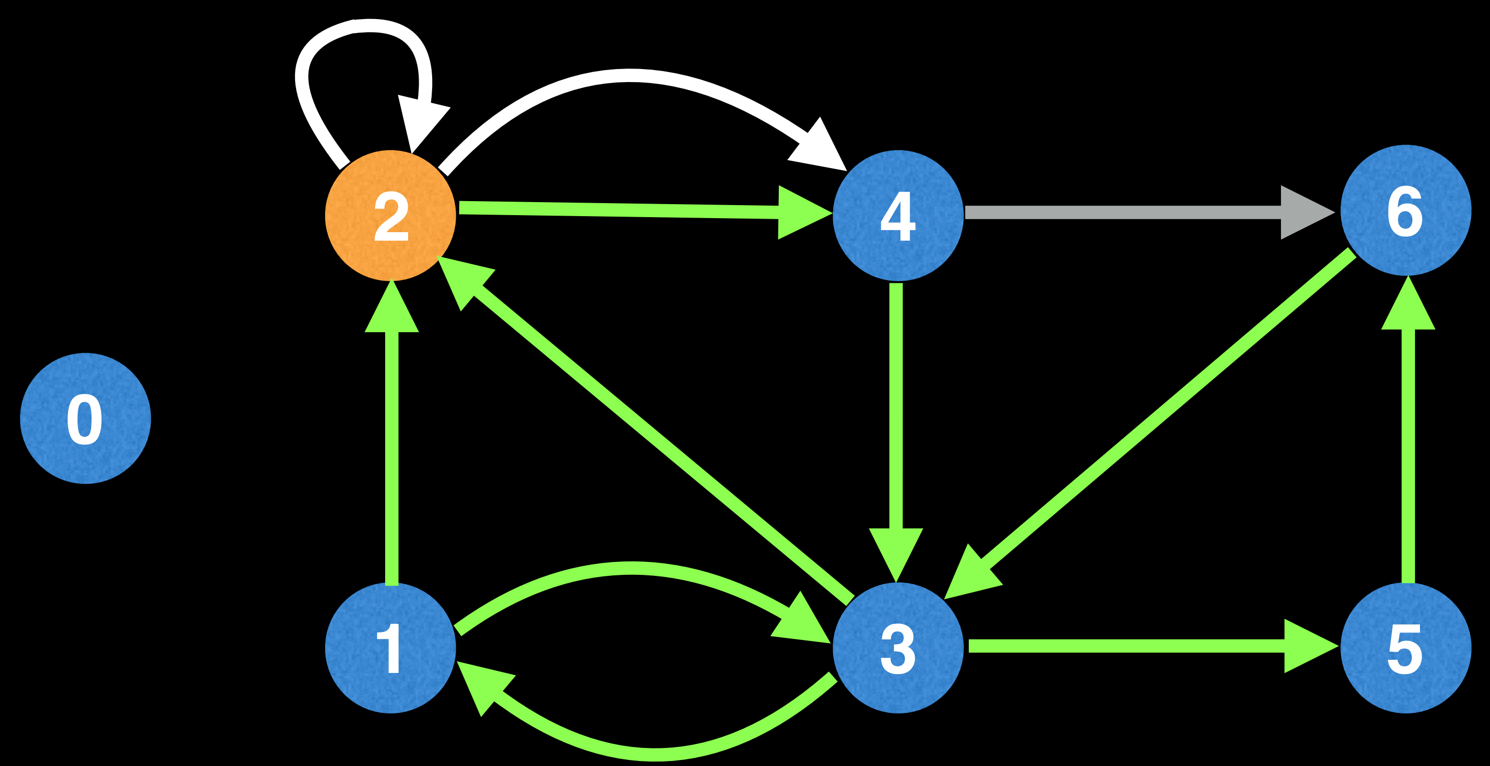
Node	Out
0	0
1	1
2	2
3	0
4	0
5	0
6	0



Solution = [6]

Finding an Eulerian path (directed graph)

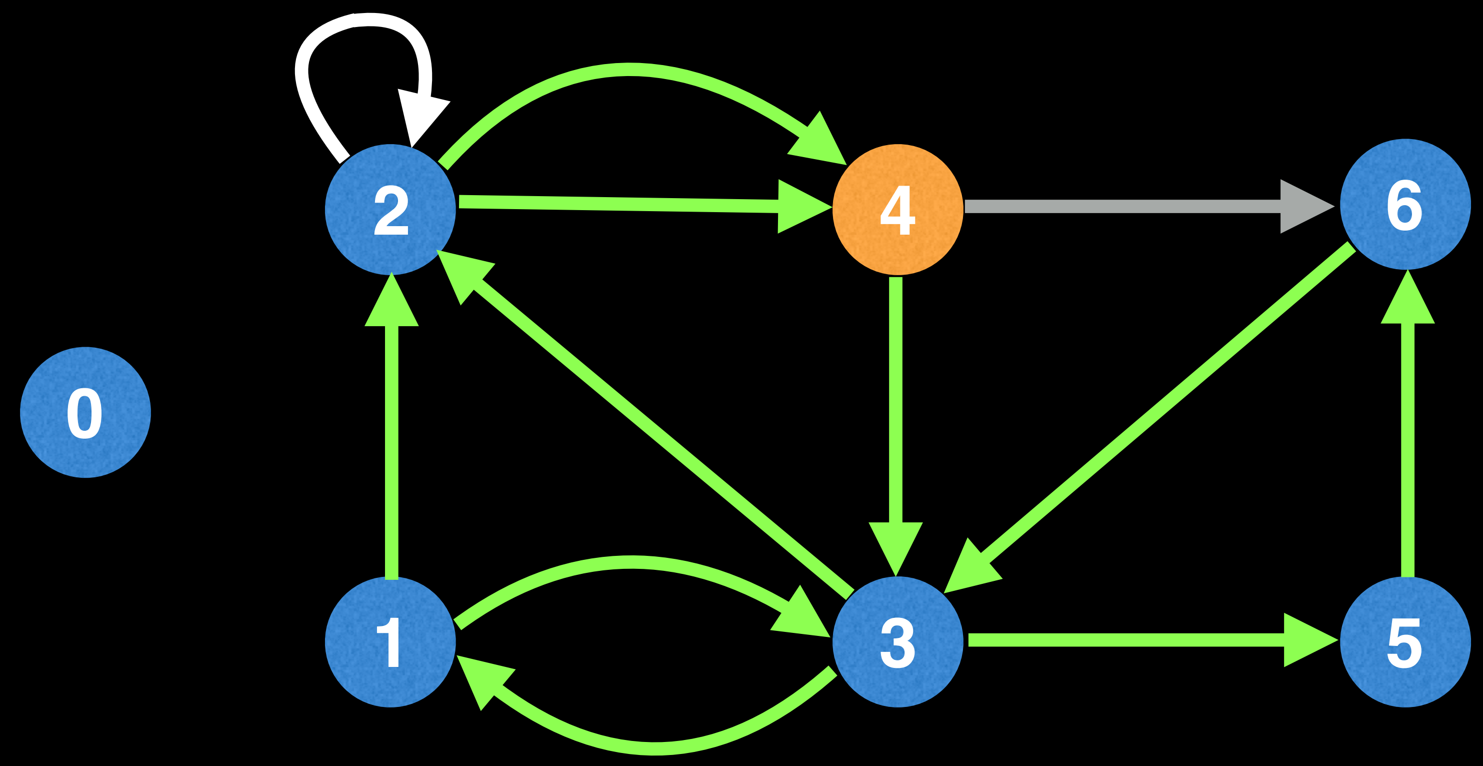
Node	Out
0	0
1	0
2	2
3	0
4	0
5	0
6	0



Solution = [6]

Finding an Eulerian path (directed graph)

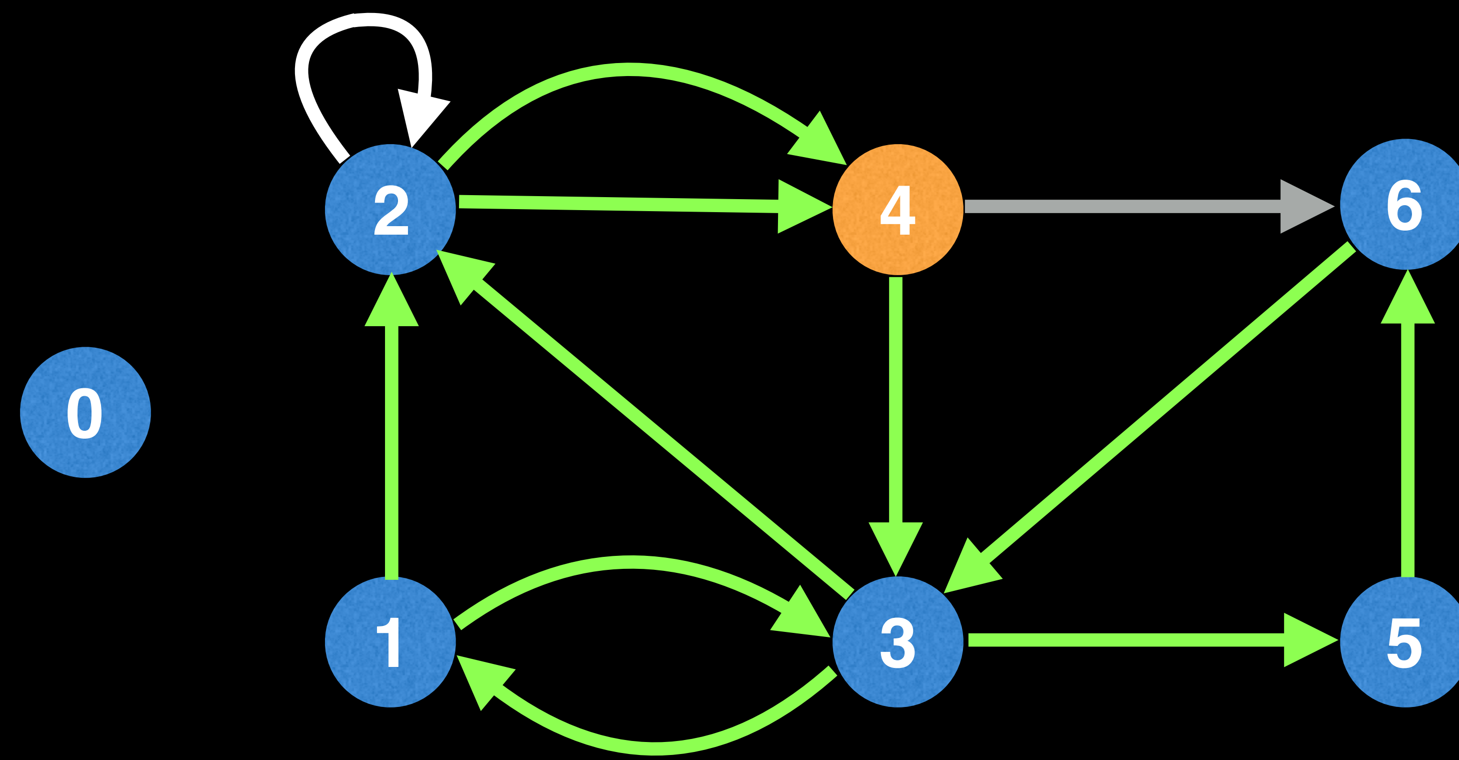
Node	Out
0	0
1	0
2	1
3	0
4	0
5	0
6	0



Solution = [6]

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	0
2	1
3	0
4	0
5	0
6	0

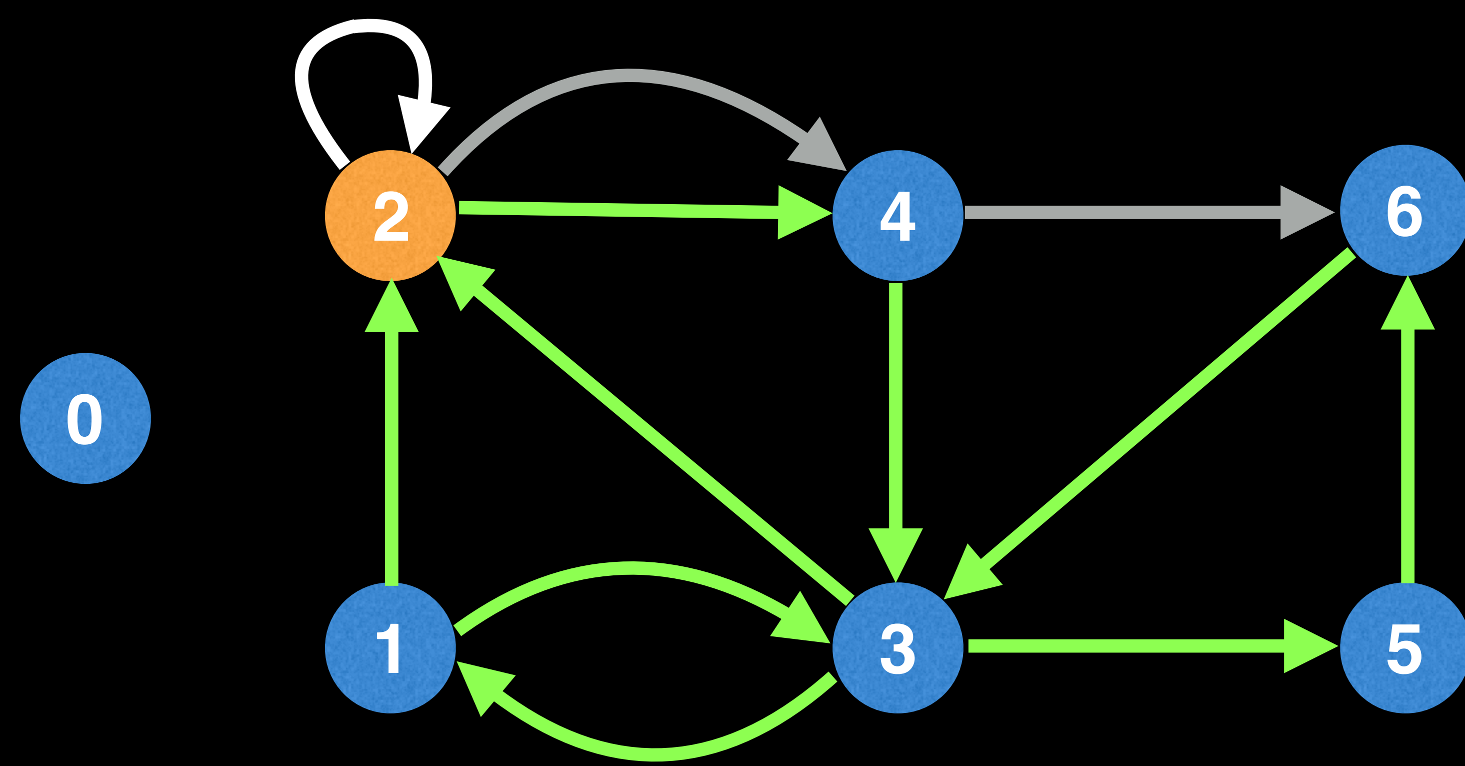


When the DFS is stuck, meaning there are no more outgoing edges (i.e out[i] = 0), then we know to backtrack and add the current node to the solution.

Solution = [6]

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	0
2	1
3	0
4	0
5	0
6	0

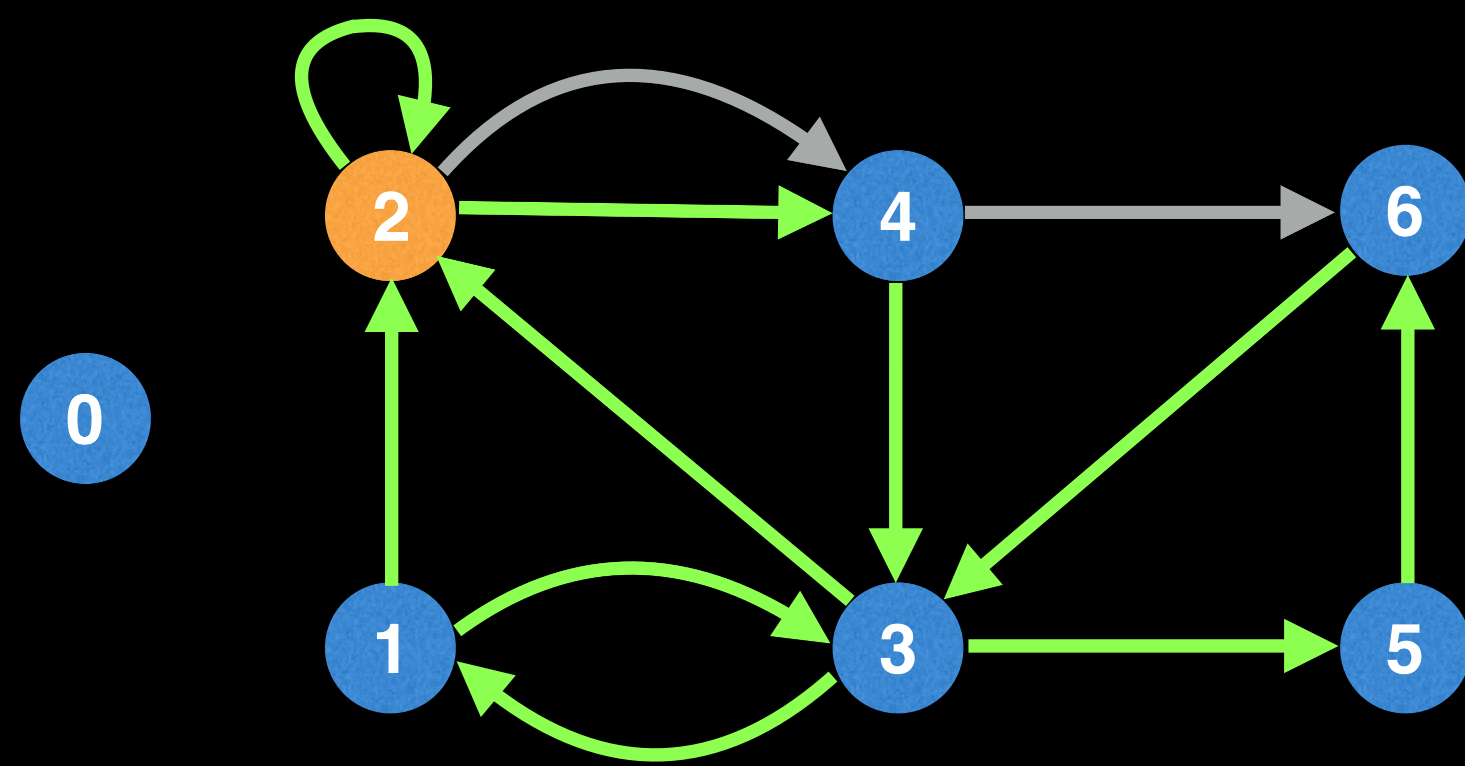


Node 2 still has an unvisited edge (since out[i] != 0) so we need to follow that edge.

Solution = [4,6]

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0

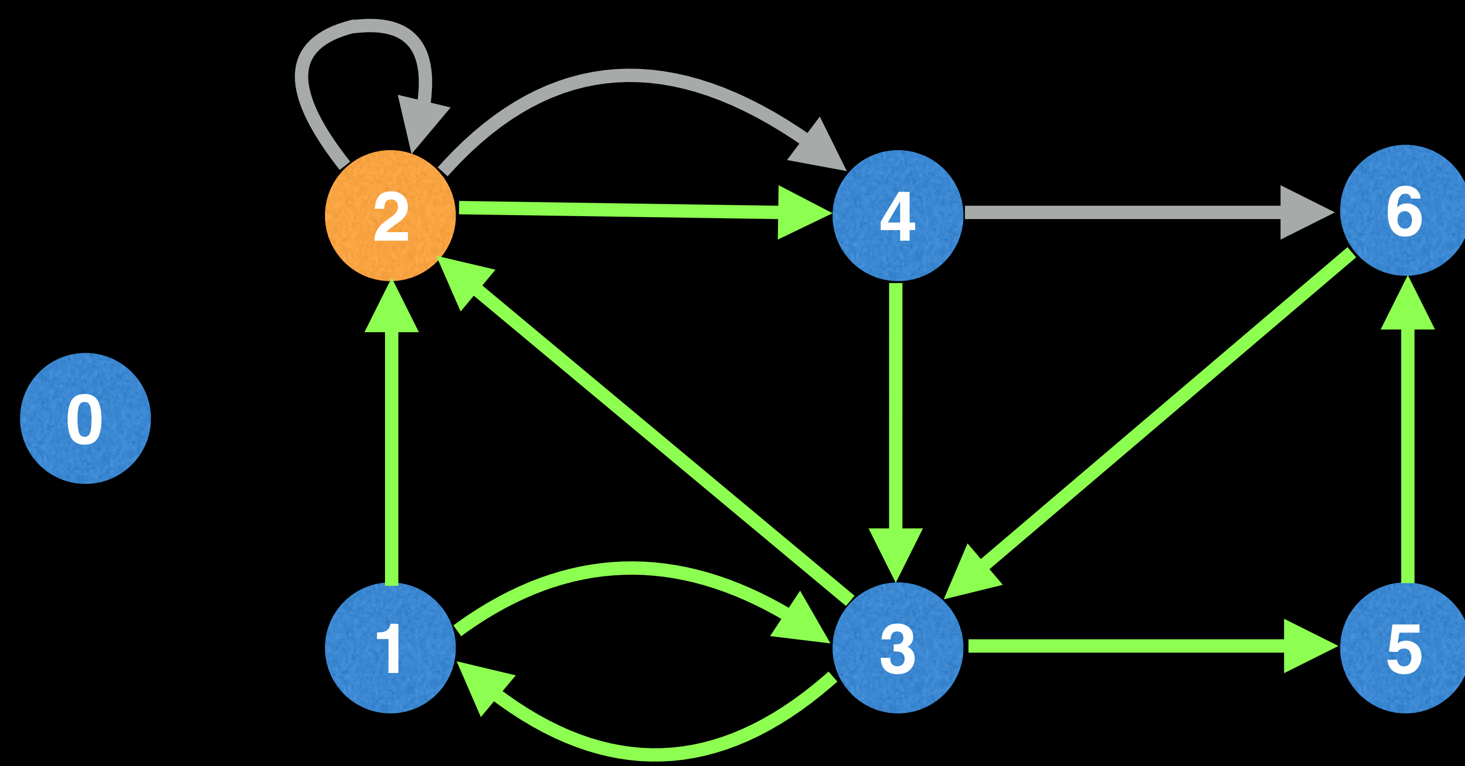


When the DFS is stuck, meaning there are no more outgoing edges (i.e out[i] = 0), then we know to backtrack and add the current node to the solution.

Solution = [4,6]

Finding an Eulerian path (directed graph)

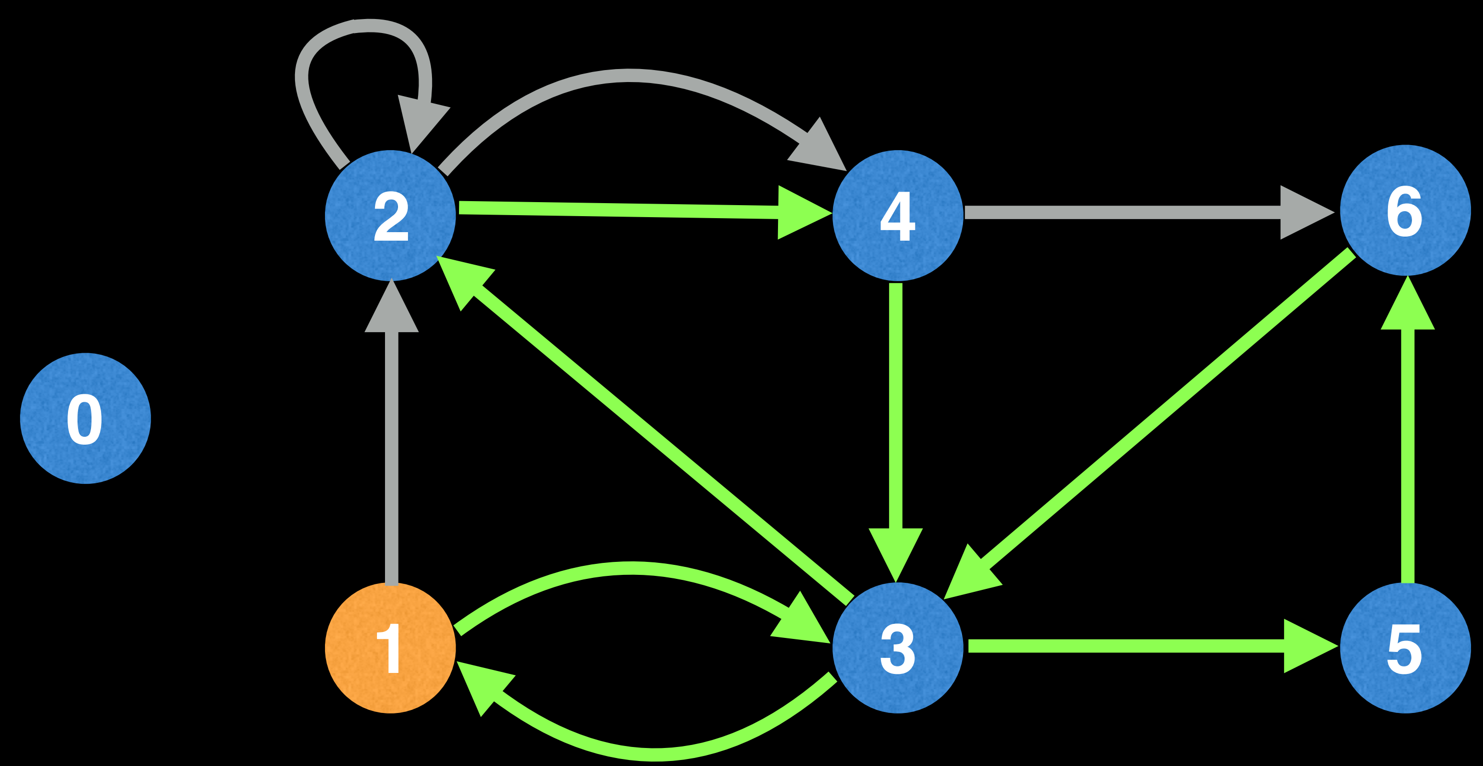
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [2,4,6]

Finding an Eulerian path (directed graph)

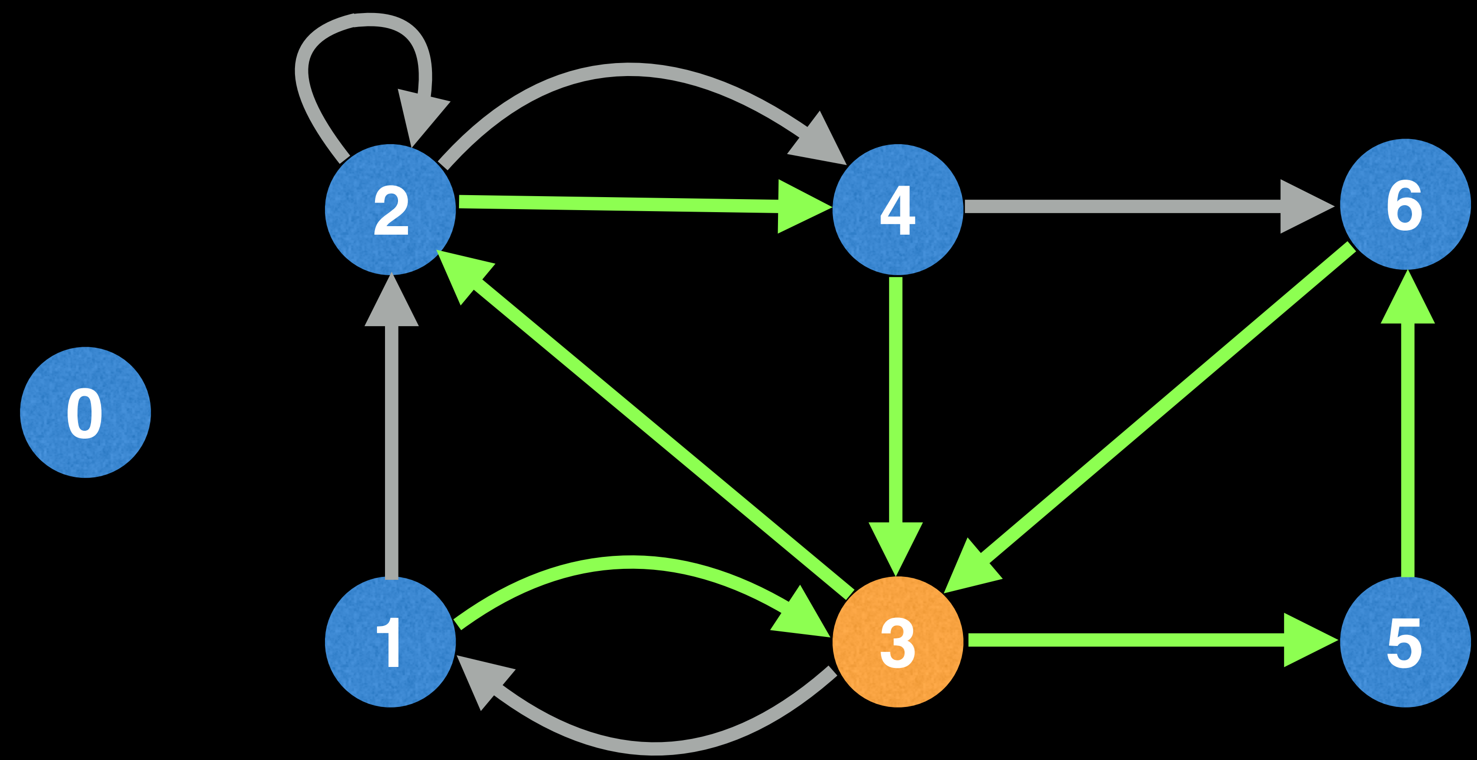
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [2,2,4,6]

Finding an Eulerian path (directed graph)

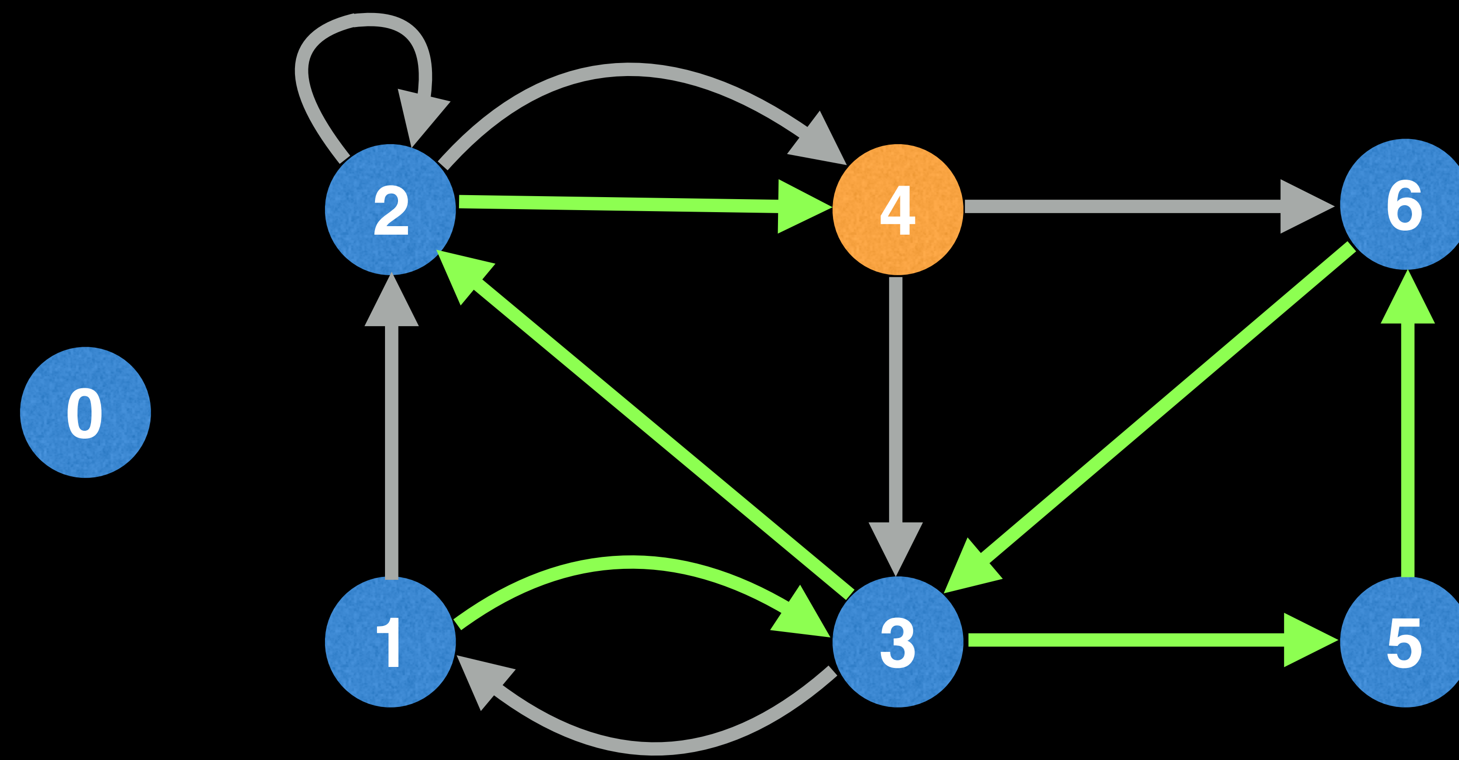
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [1,2,2,4,6]

Finding an Eulerian path (directed graph)

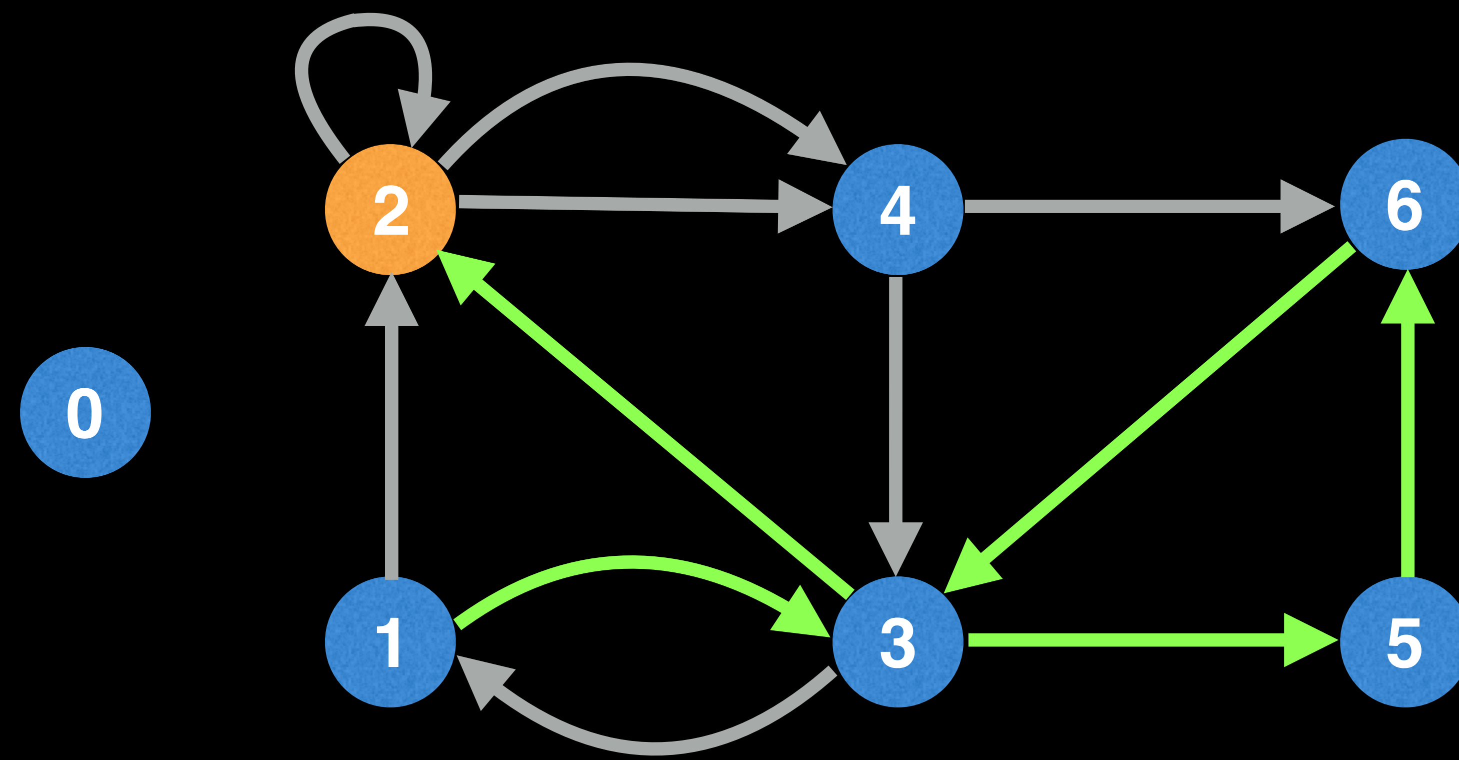
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

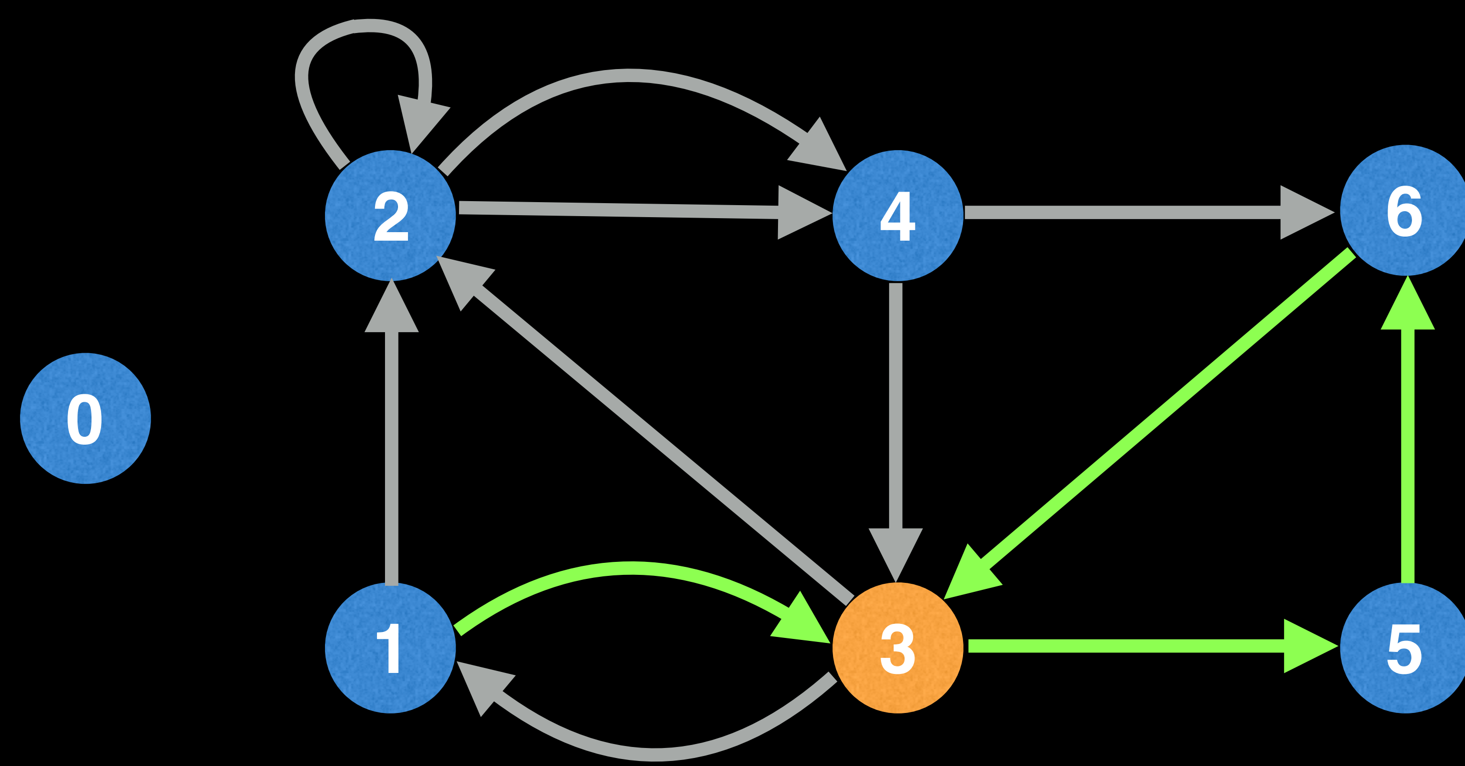
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

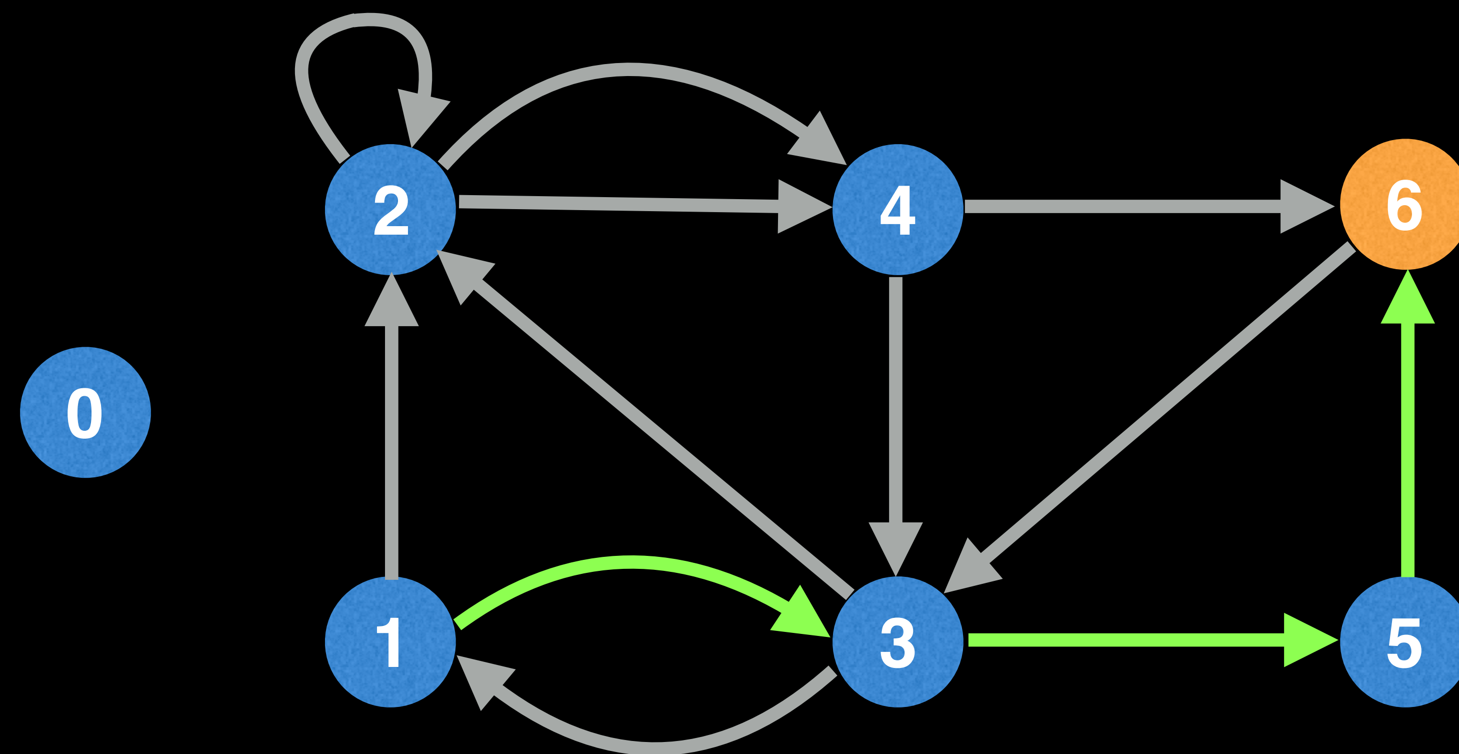
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

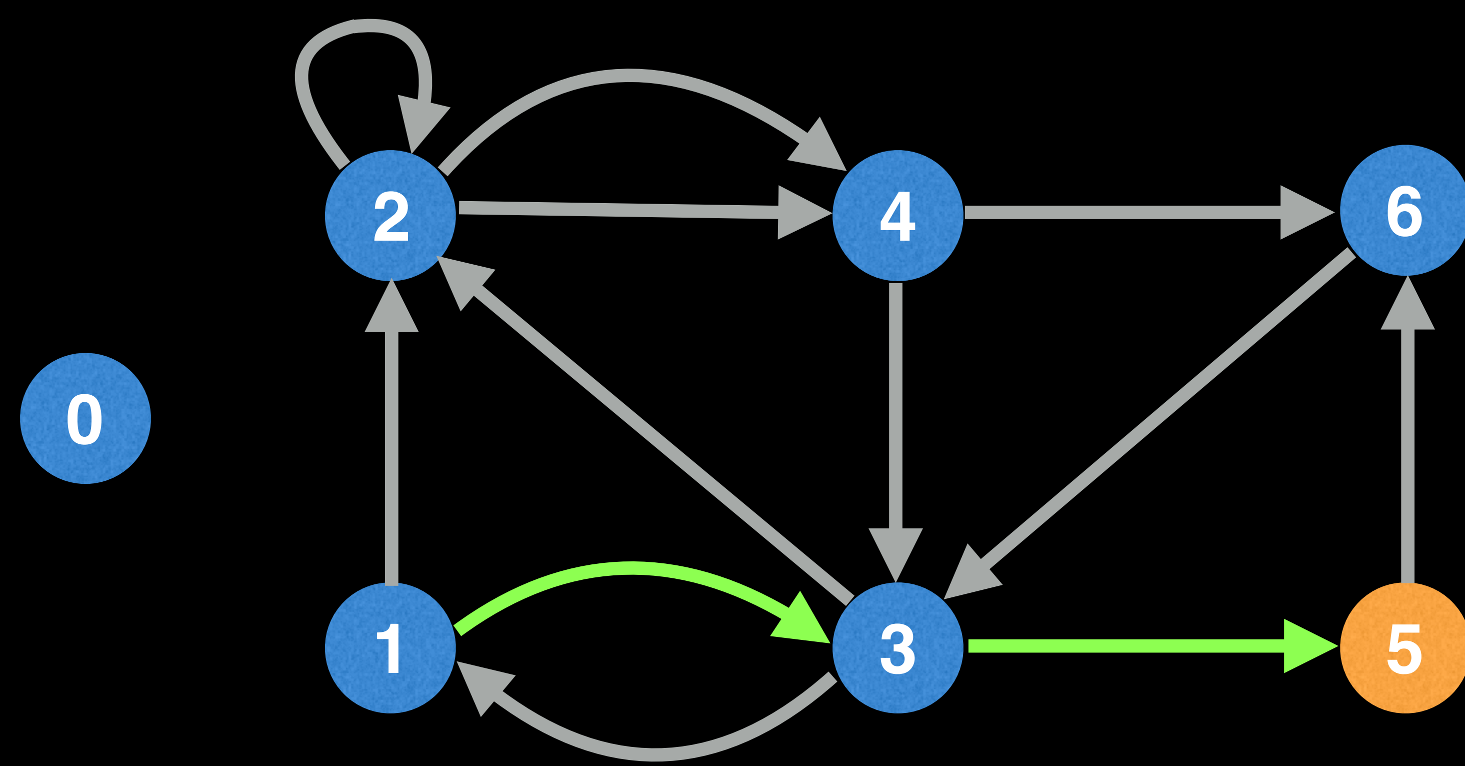
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [3,2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

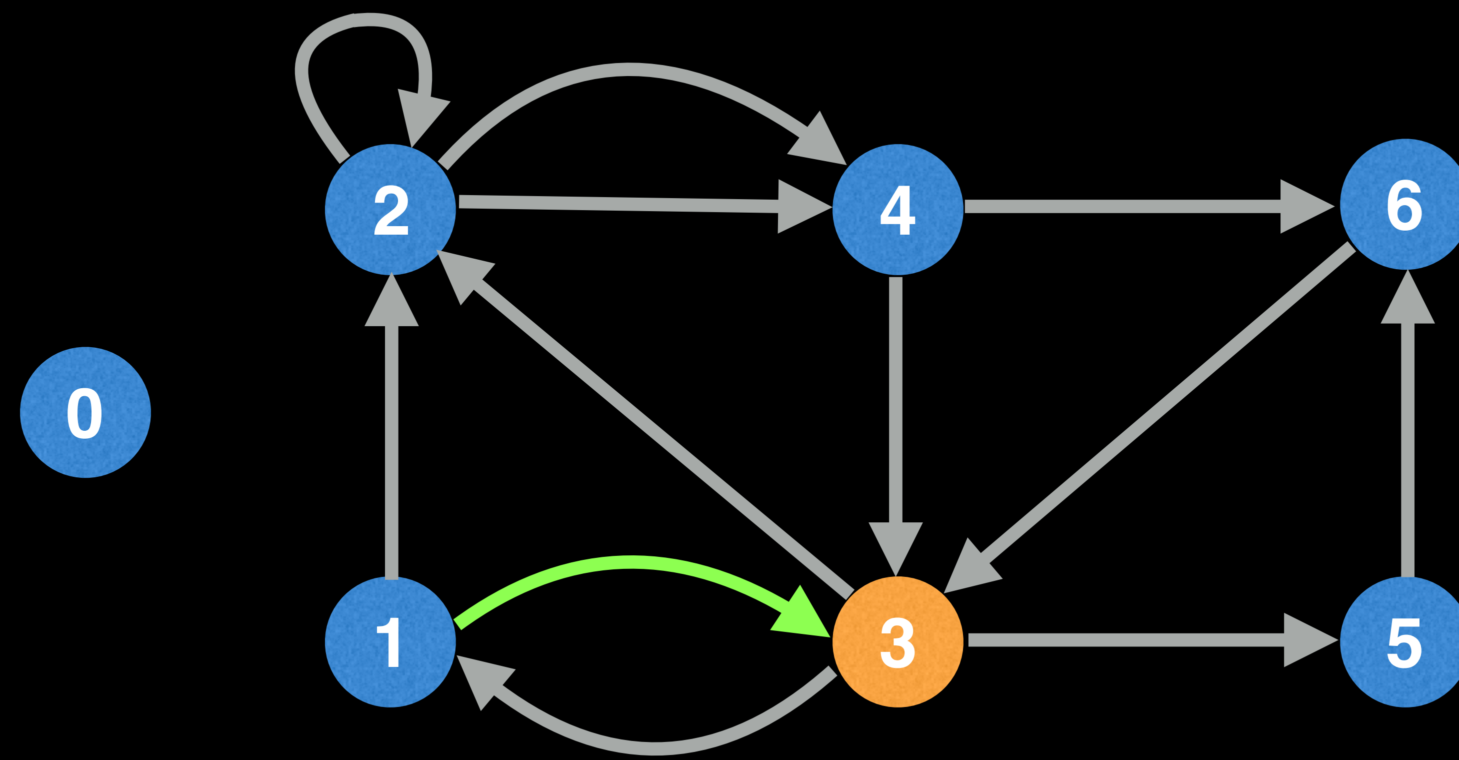
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [6,3,2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

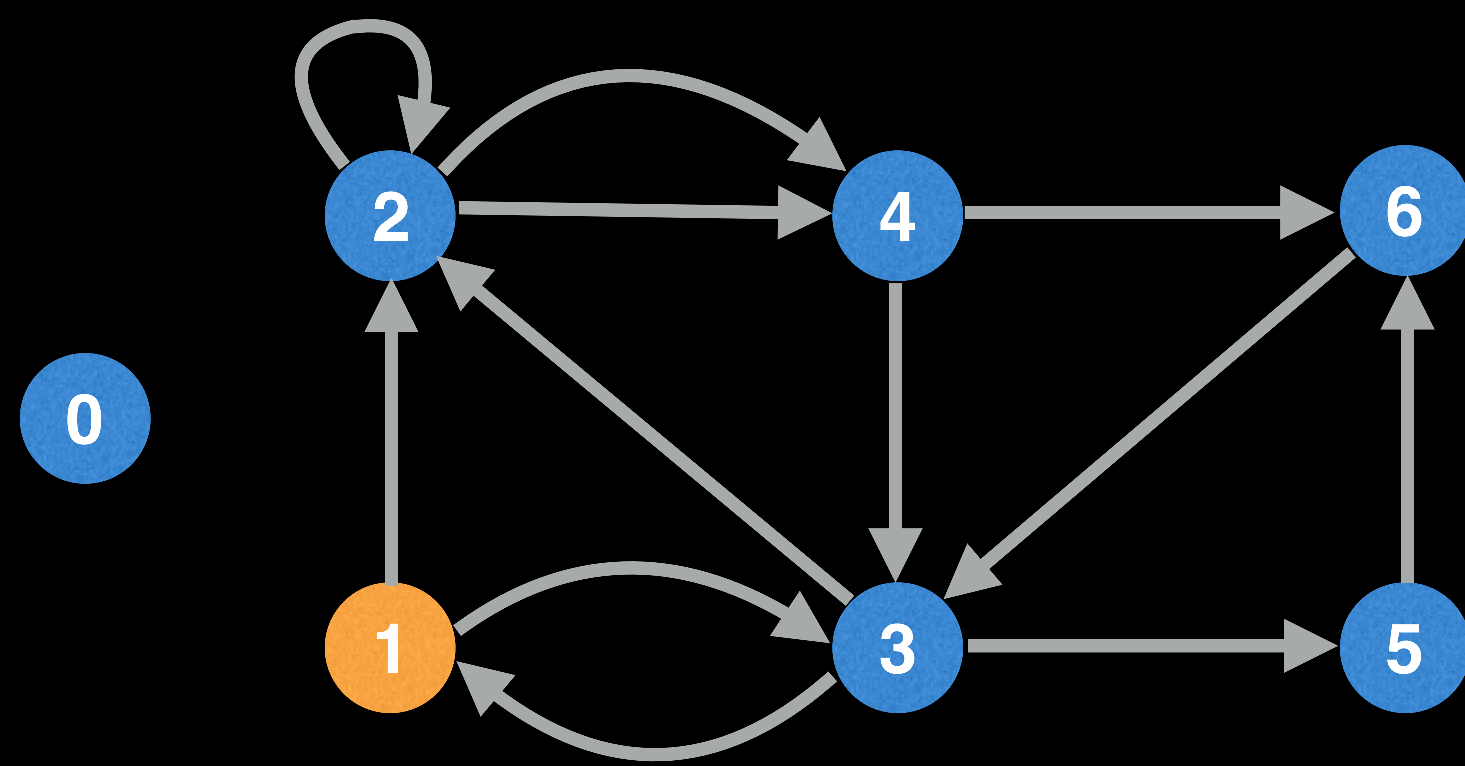
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [5,6,3,2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

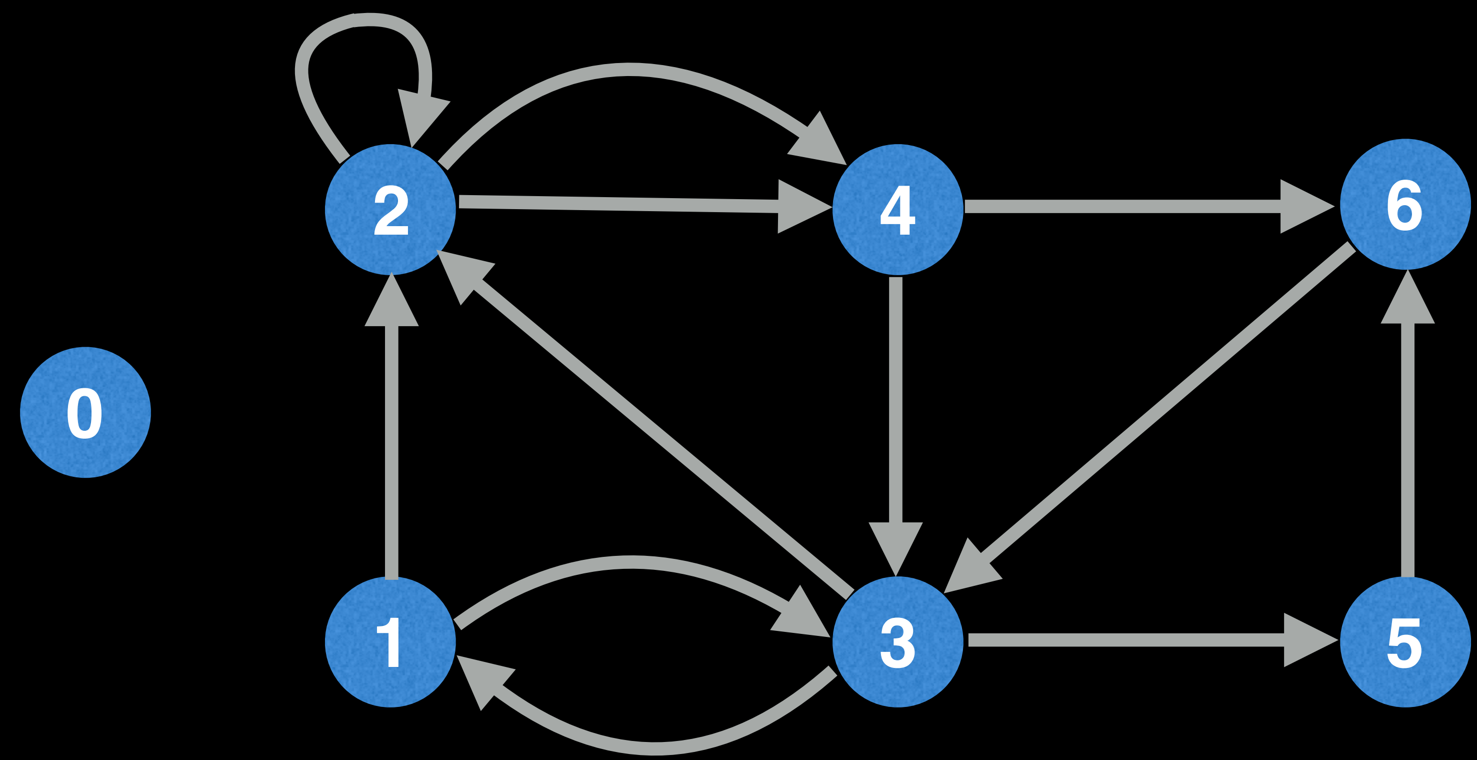
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [3,5,6,3,2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

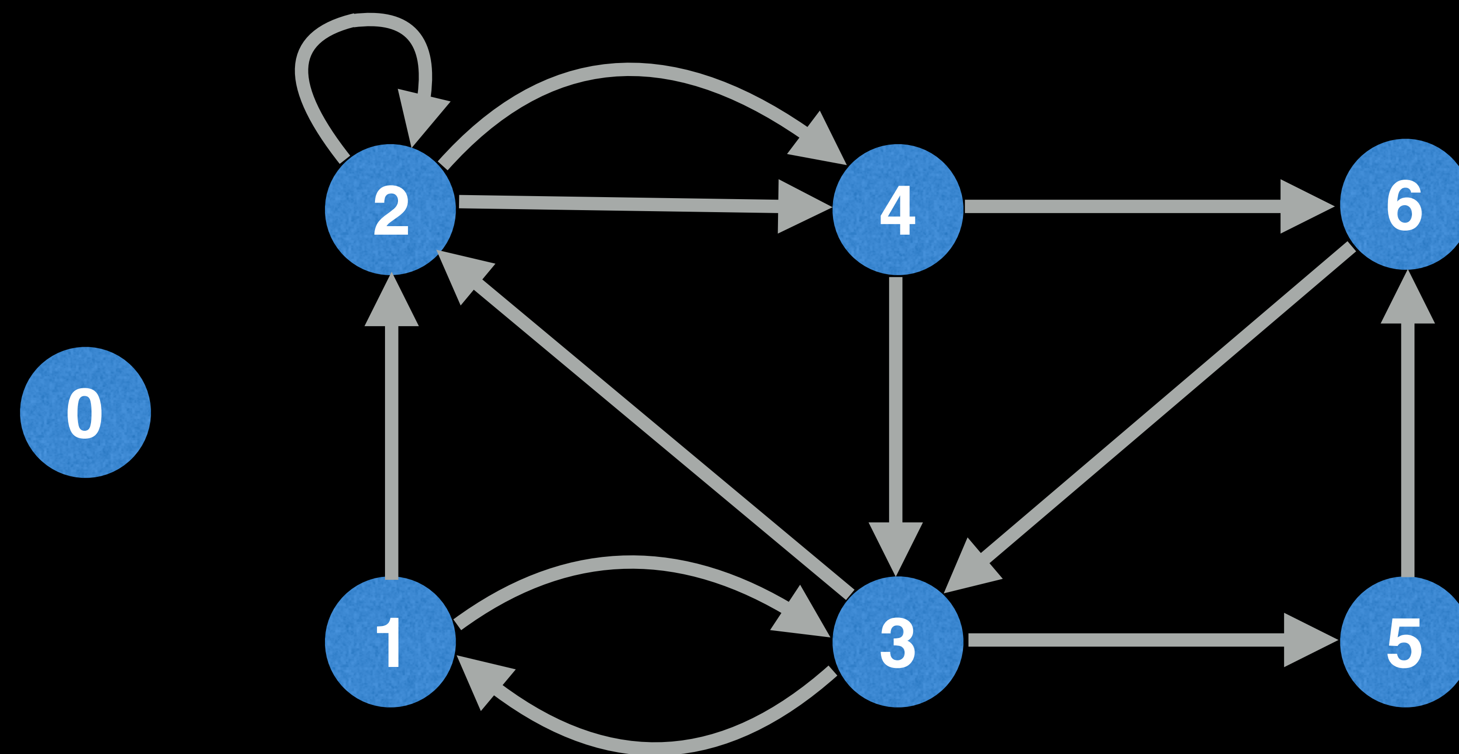
Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



Solution = [1,3,5,6,3,2,4,3,1,2,2,4,6]

Finding an Eulerian path (directed graph)

Node	Out
0	0
1	0
2	0
3	0
4	0
5	0
6	0



The time complexity to find an eulerian path with this algorithm is **O(E)**. The two calculations we're doing: computing in/out degrees + DFS are both linear in the number of edges.

Solution = [1,3,5,6,3,2,4,3,1,2,2,4,6]

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it’s impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it’s impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it's impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it's impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it’s impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it's impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
  for edges in g:
    for edge in edges:
      out[edge.from]++
      in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
function countInOutDegrees():
   for edges in g:
     for edge in edges:
       out[edge.from]++
       in[edge.to]++

function graphHasEulerianPath():
  start_nodes, end_nodes = 0, 0
  for (i = 0; i < n; i++):
    if (out[i] - in[i]) > 1 or (in[i] - out[i]) > 1:
      return false
    else if out[i] - in[i] == 1:
      start_nodes++
    else if in[i] - out[i] == 1:
      end_nodes++
  return (end_nodes == 0 and start_nodes == 0) or
         (end_nodes == 1 and start_nodes == 1)
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it’s impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it's impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

Avoids starting DFS at a singleton

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

The out array is currently serving two purposes. One purpose is to track whether or not there are still outgoing edges, and the other is to index into the adjacency list to select the next outgoing edge.

This assumes the adjacency list stores edges in a data structure that is indexable in **O(1)** (e.g stored in an array). If not (e.g a linked-list/stack/etc…), you can use an iterator to iterate over the edges.

```
function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
function findStartNode():
  start = 0
  for (i = 0; i < n; i = i + 1):
    # Unique starting node
    if out[i] - in[i] == 1: return i

    # Start at any node with an outgoing edge
    if out[i] > 0: start = i
  return start

function dfs(at):
  # While the current node still has outgoing edges
  while (out[at] != 0):

    # Select the next unvisited outgoing edge
    next_edge = g[at].get(--out[at])
    dfs(next_edge.to)

  # Add current node to solution
  path.insertFirst(at)
```

```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it's impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

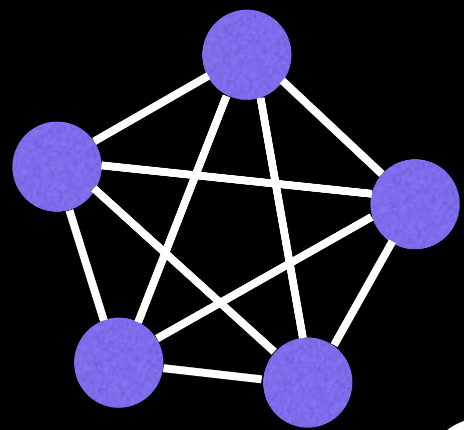
```
# Global/class scope variables
n = number of vertices in the graph
m = number of edges in the graph
g = adjacency list representing directed graph

in  = [0, 0, …, 0, 0] # Length n
out = [0, 0, …, 0, 0] # Length n

path = empty integer linked list data structure

function findEulerianPath():

  countInOutDegrees()
  if not graphHasEulerianPath(): return null

  dfs(findStartNode())

  # Return eulerian path if we traversed all the
  # edges. The graph might be disconnected, in which
  # case it’s impossible to have an euler path.
  if path.size() == m+1: return path
  return null
```

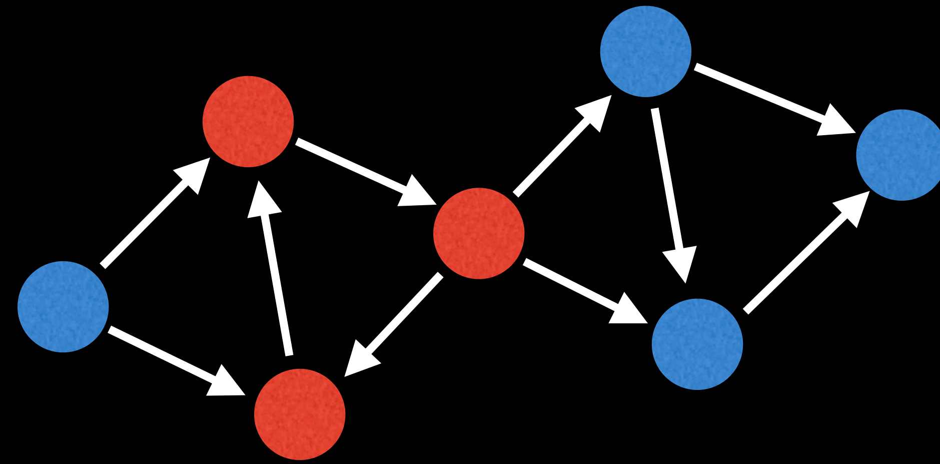
Source Code Link

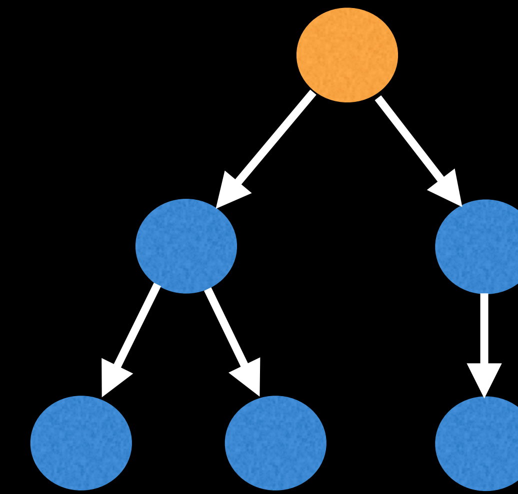
Implementation source code can be found at the following link:

github.com/williamfiset/algorithms

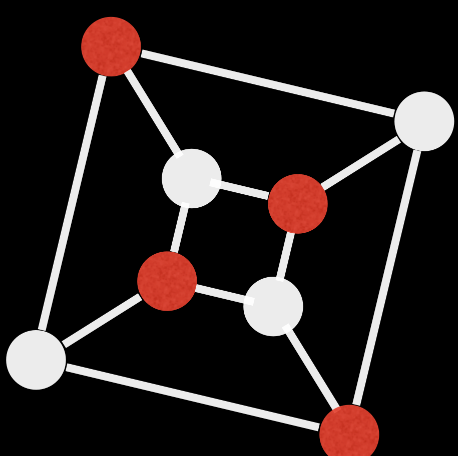
Link in the description:

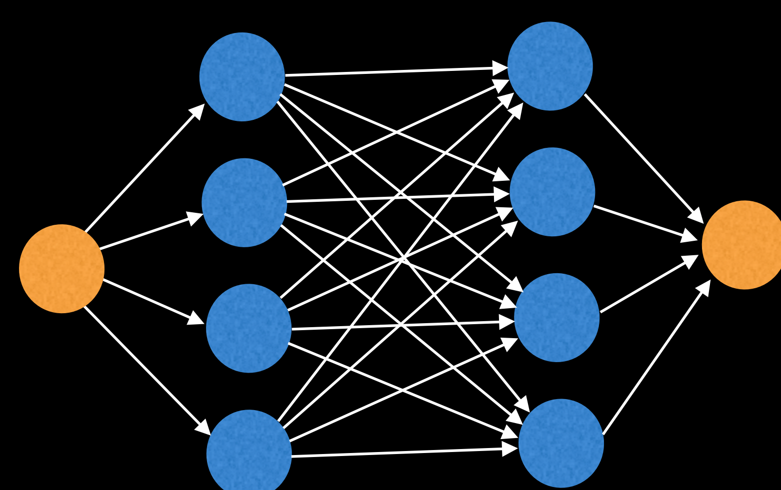


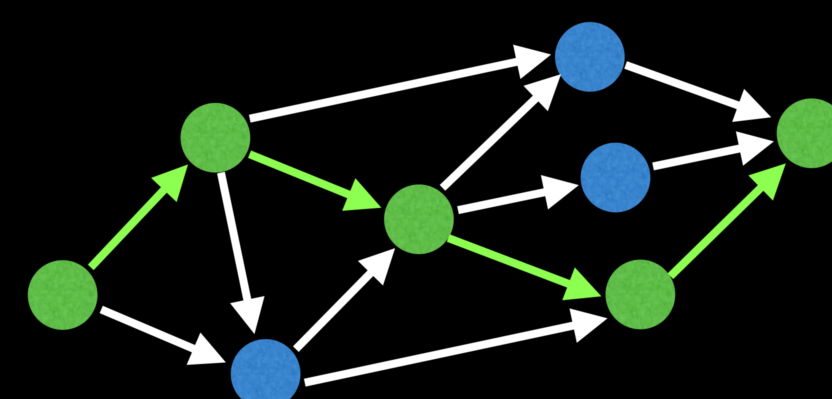




Graph Theory Video Series







Eulerian Path Source Code

William Fiset

Source Code Link

Implementation source code can be found at the following link:

github.com/williamfiset/algorithms

Link in the description: