Shortest and longest paths on DAGs

William Fiset

Directed Acyclic Graph (DAG)

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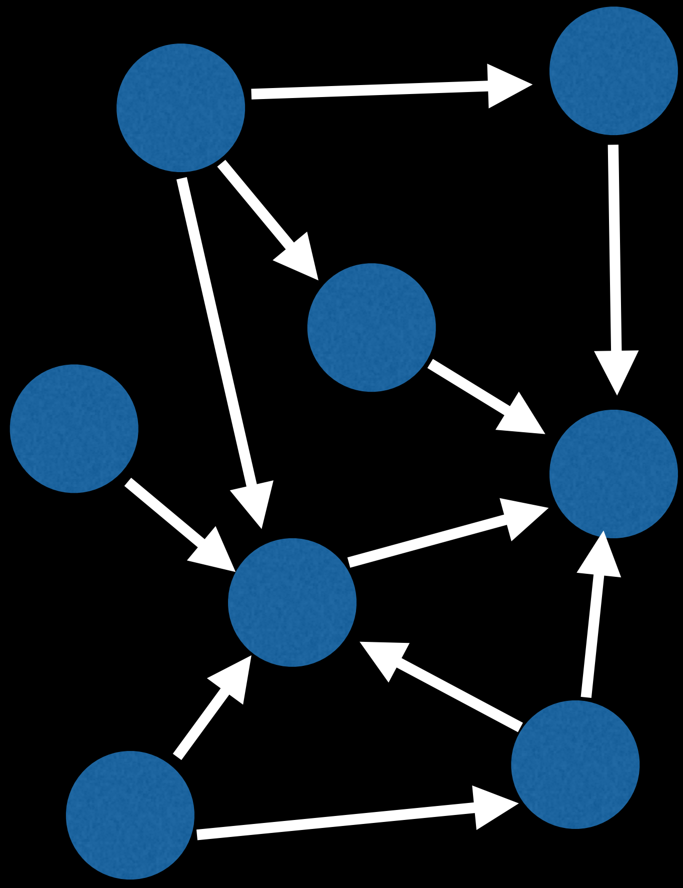

Q: Is this graph a DAG?

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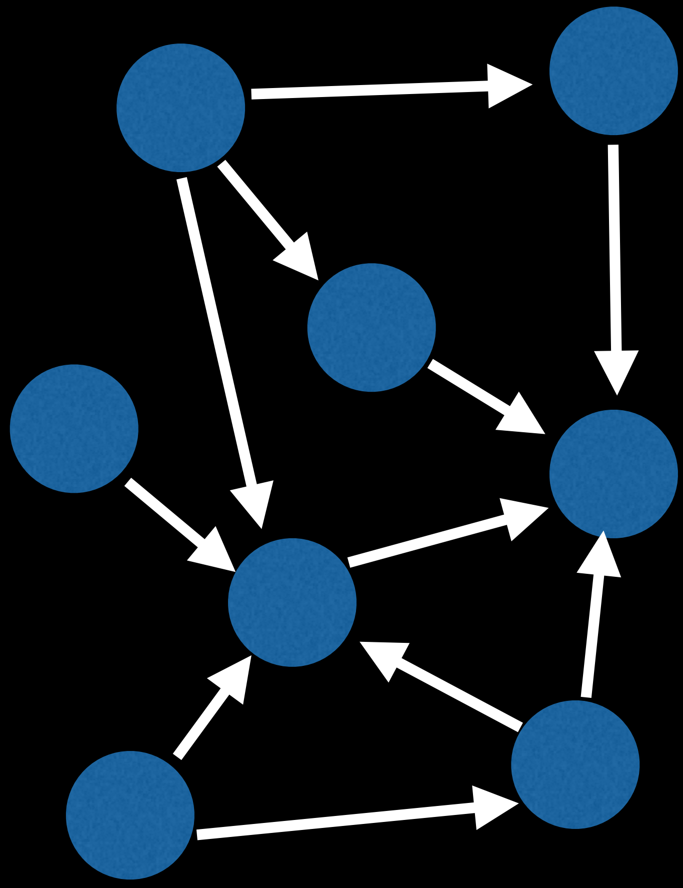

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A: Yes!

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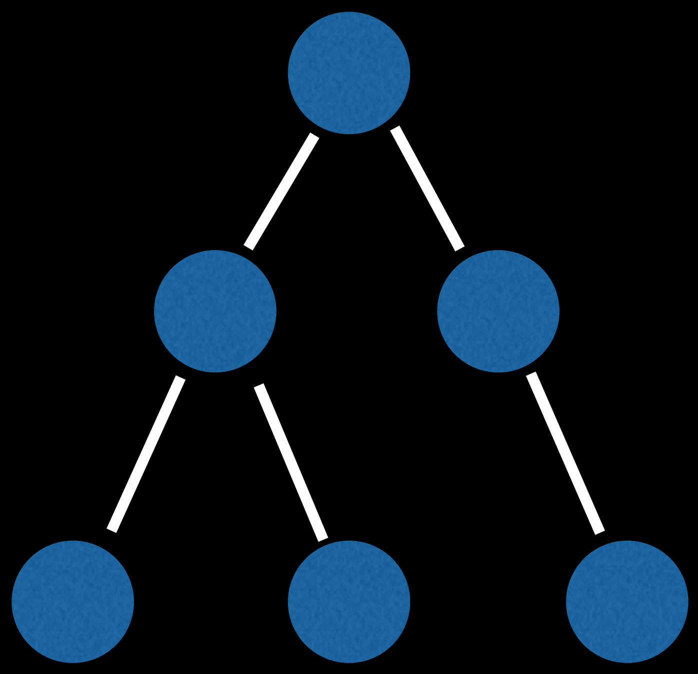

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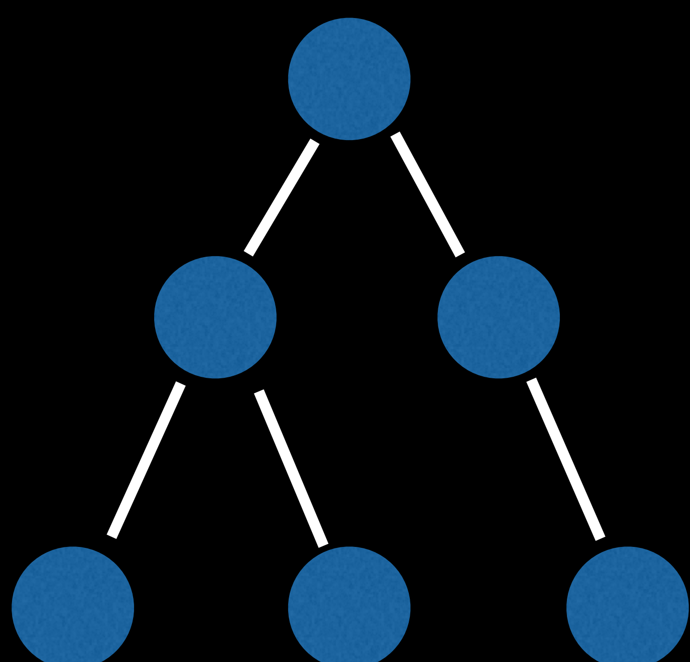

Q: Is this graph a DAG?

A: No, the structure may be a tree, but it does not have directed edges.

SSSP on DAG

The **Single Source Shortest Path (SSSP)** problem can be solved efficiently on a DAG in **O(V+E)** time. This is due to the fact that the nodes can be ordered in a **topological ordering** via topsort and processed sequentially.

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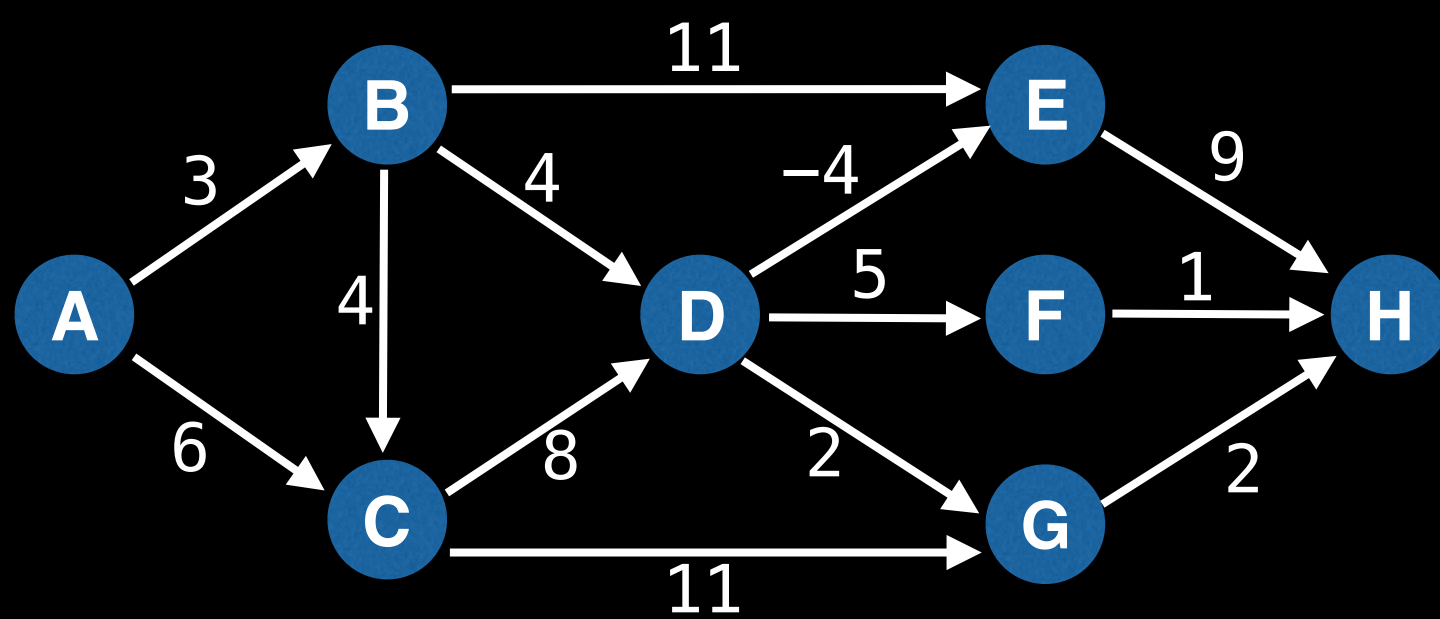

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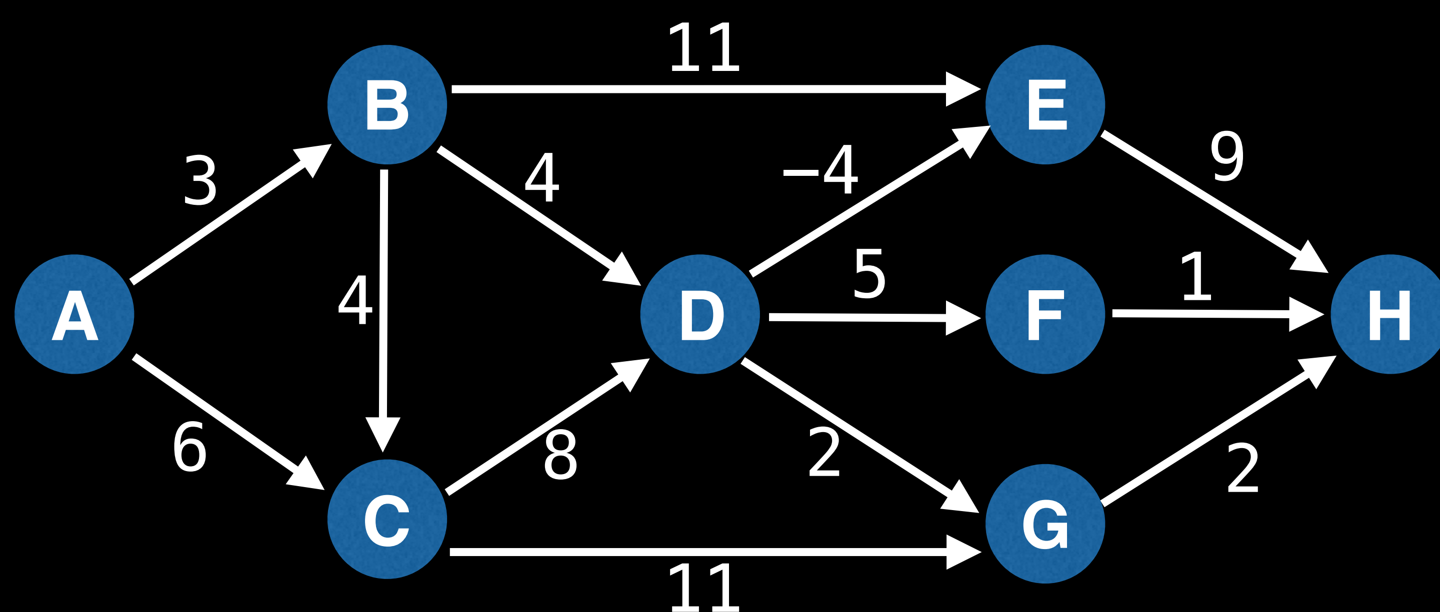

Arbitrary topological order: A, B, C, D, G, E, F, H

∞	∞	∞	∞	∞	∞	∞	∞
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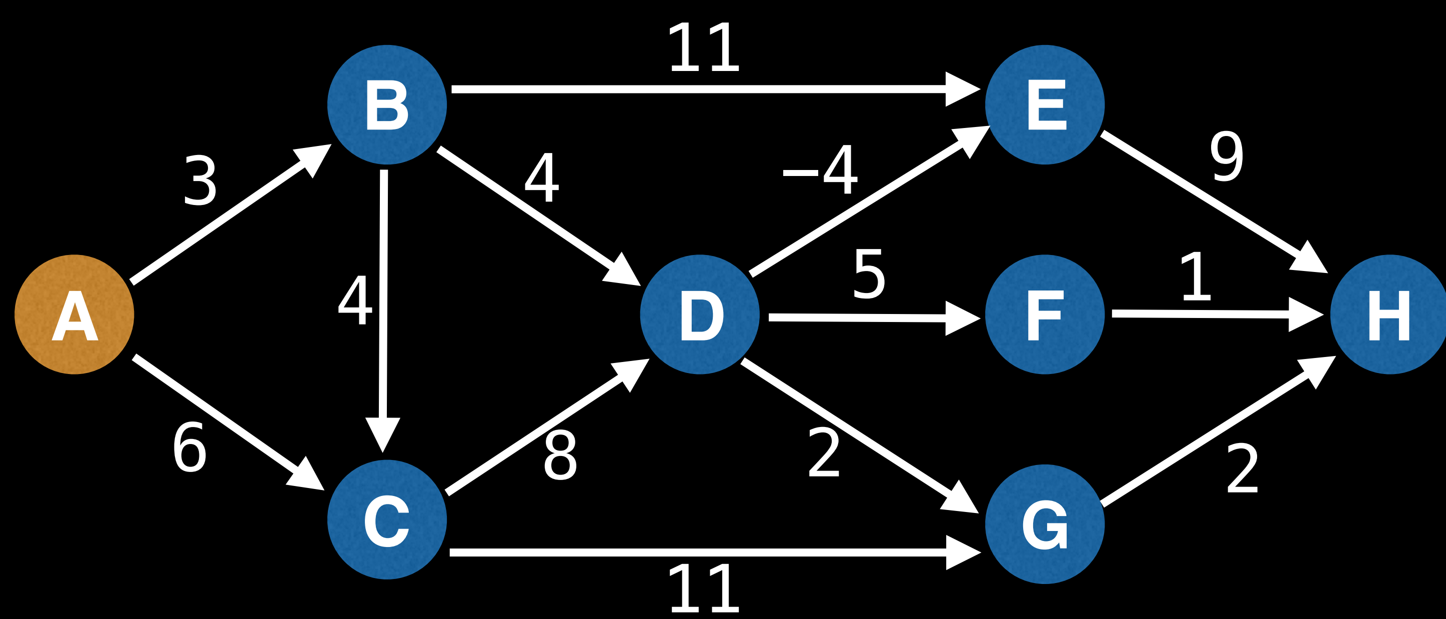

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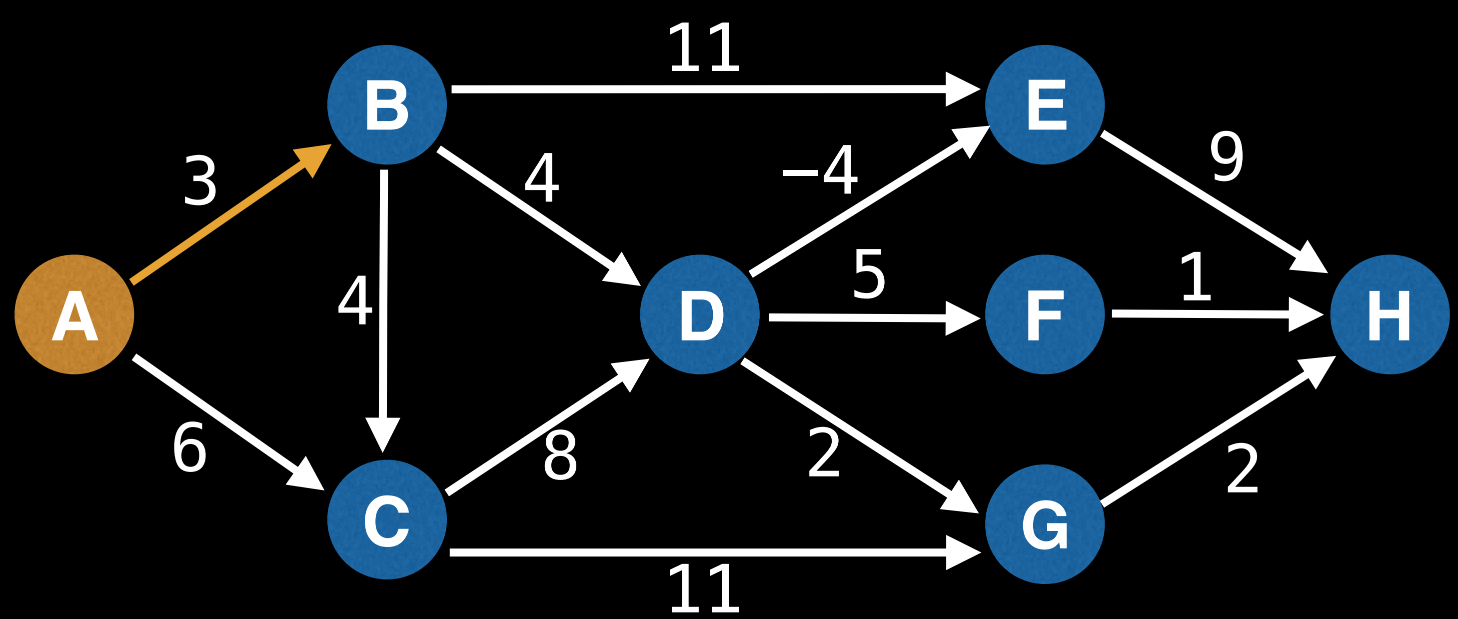

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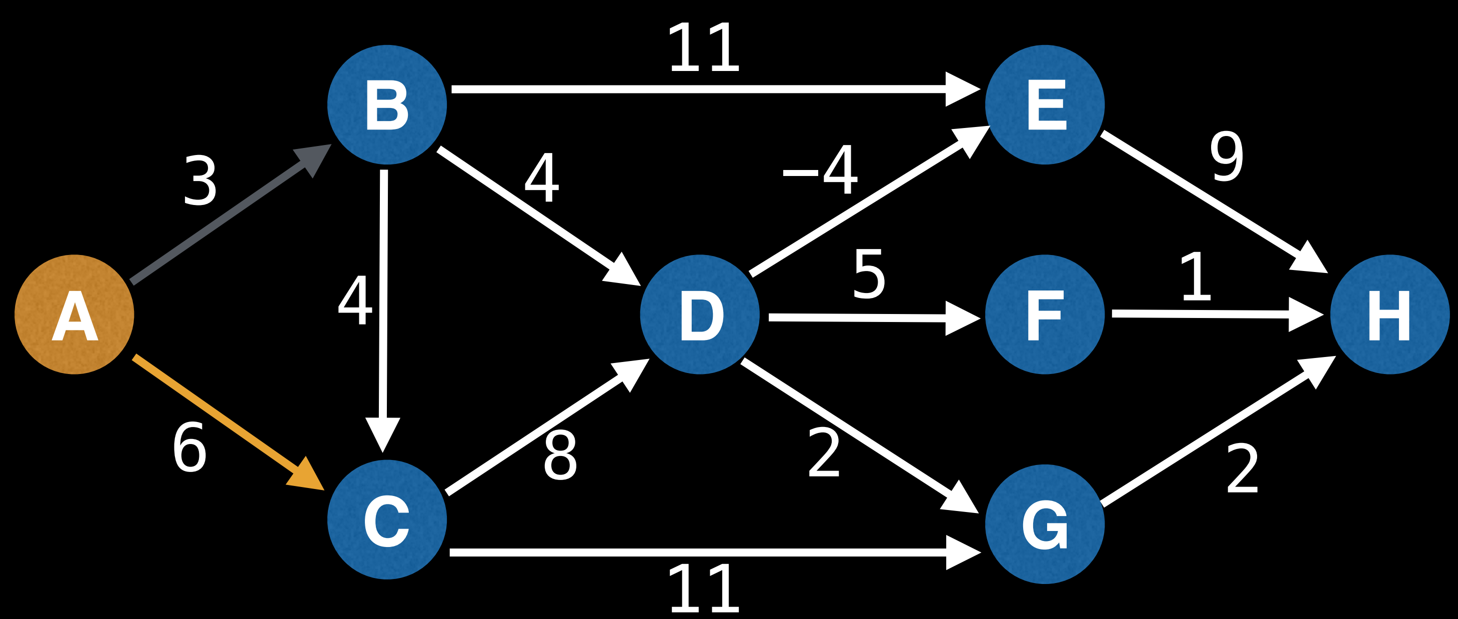

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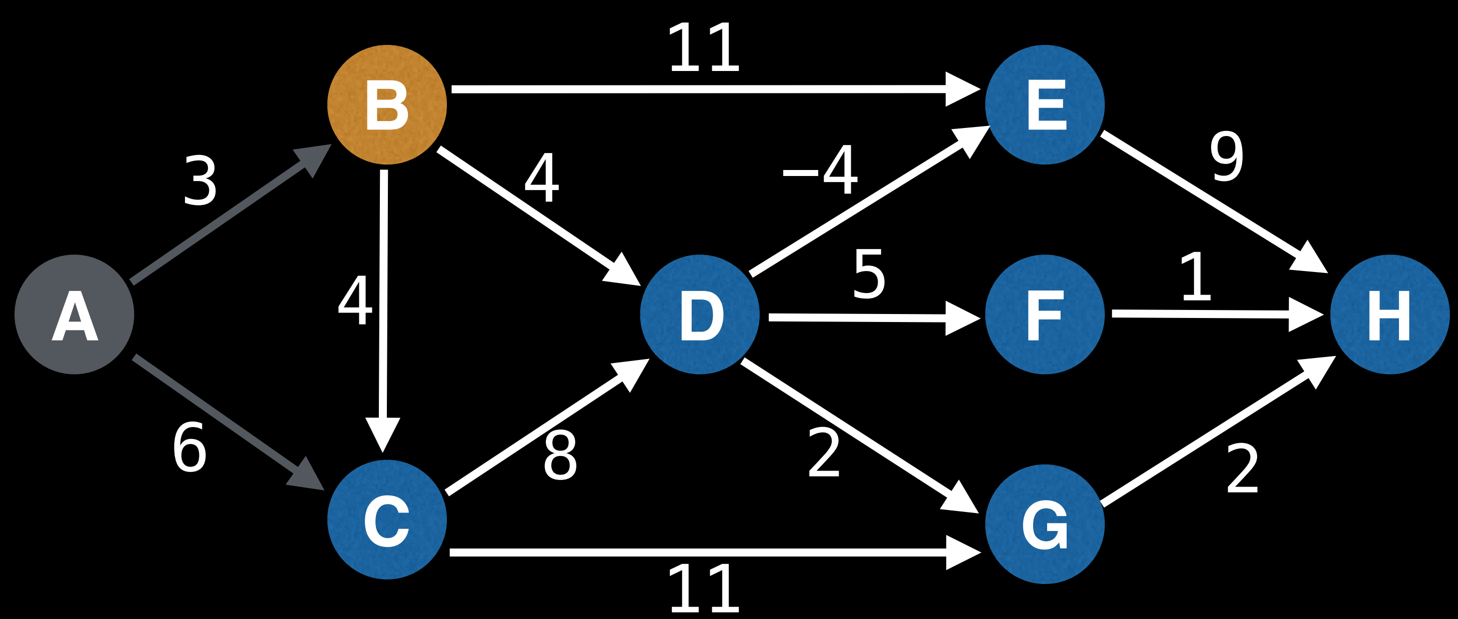

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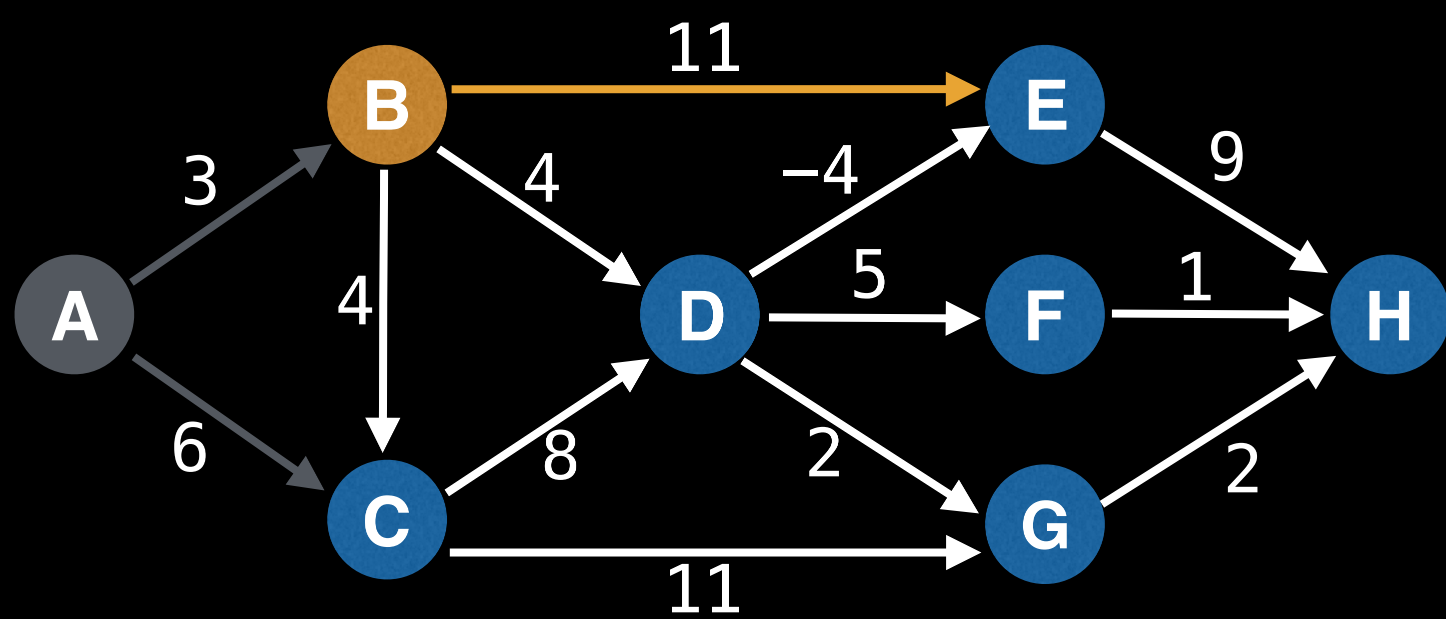

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0	3	6	∞	14	∞	∞	∞
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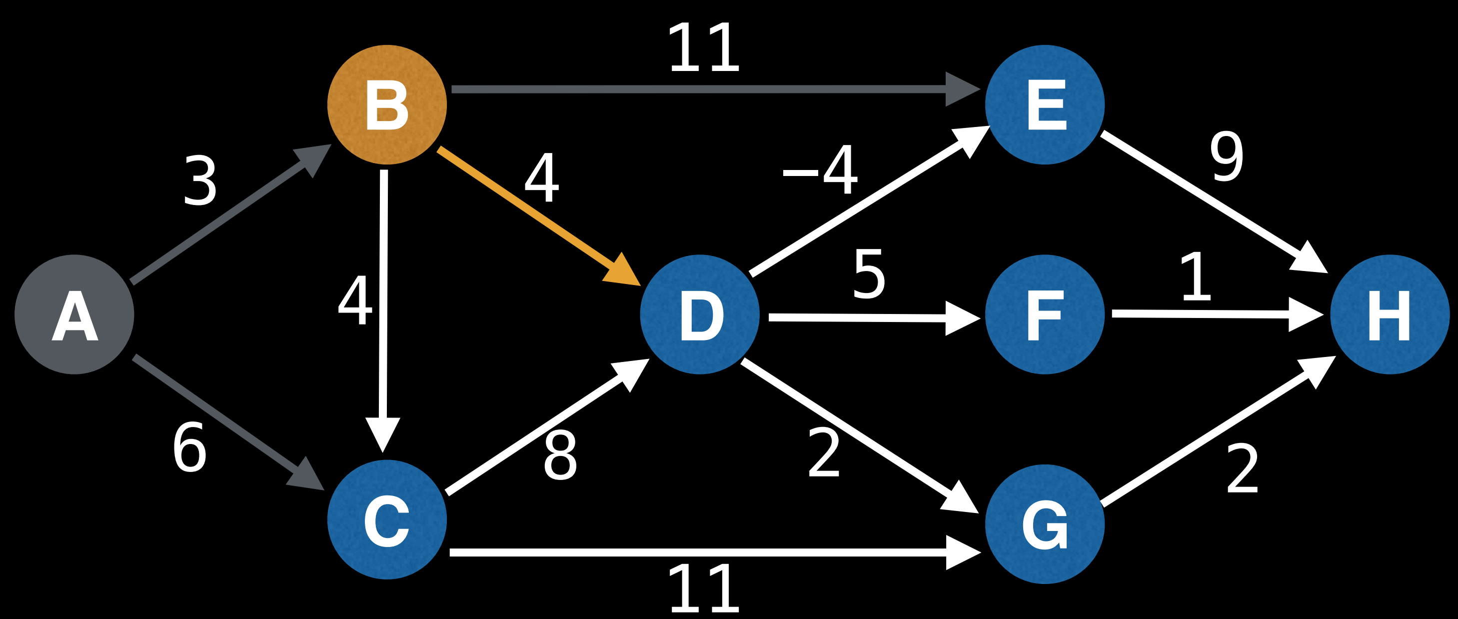

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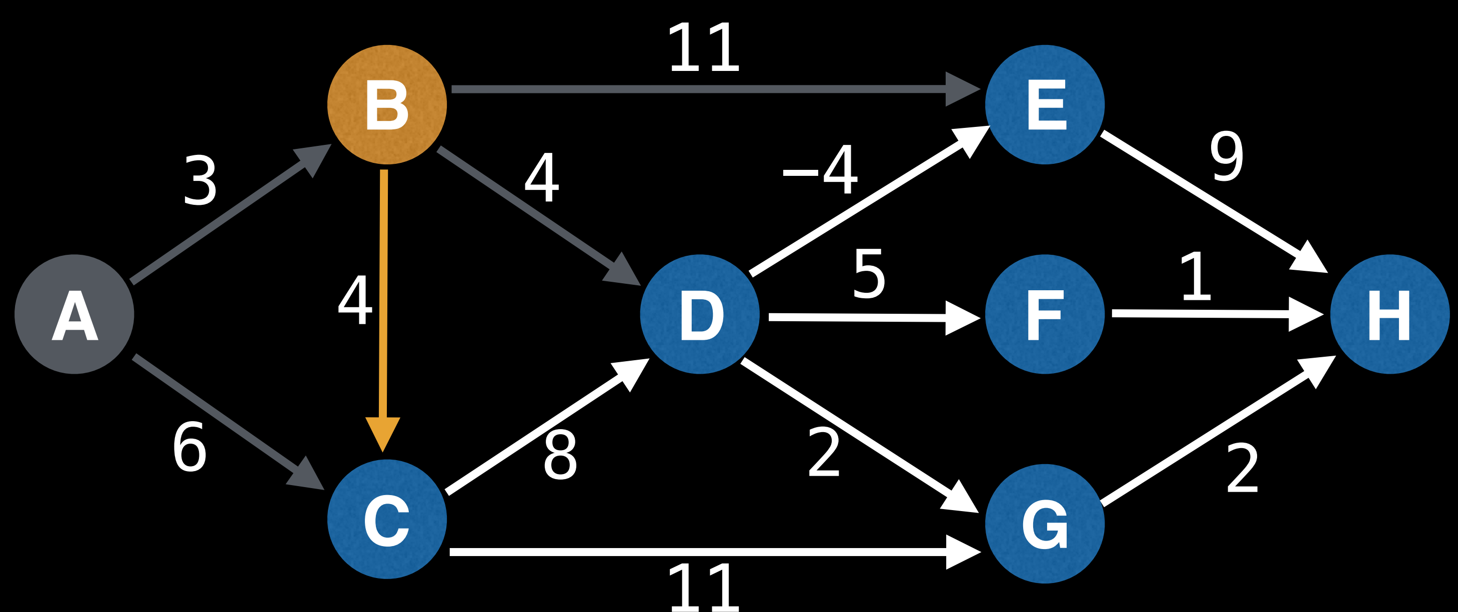

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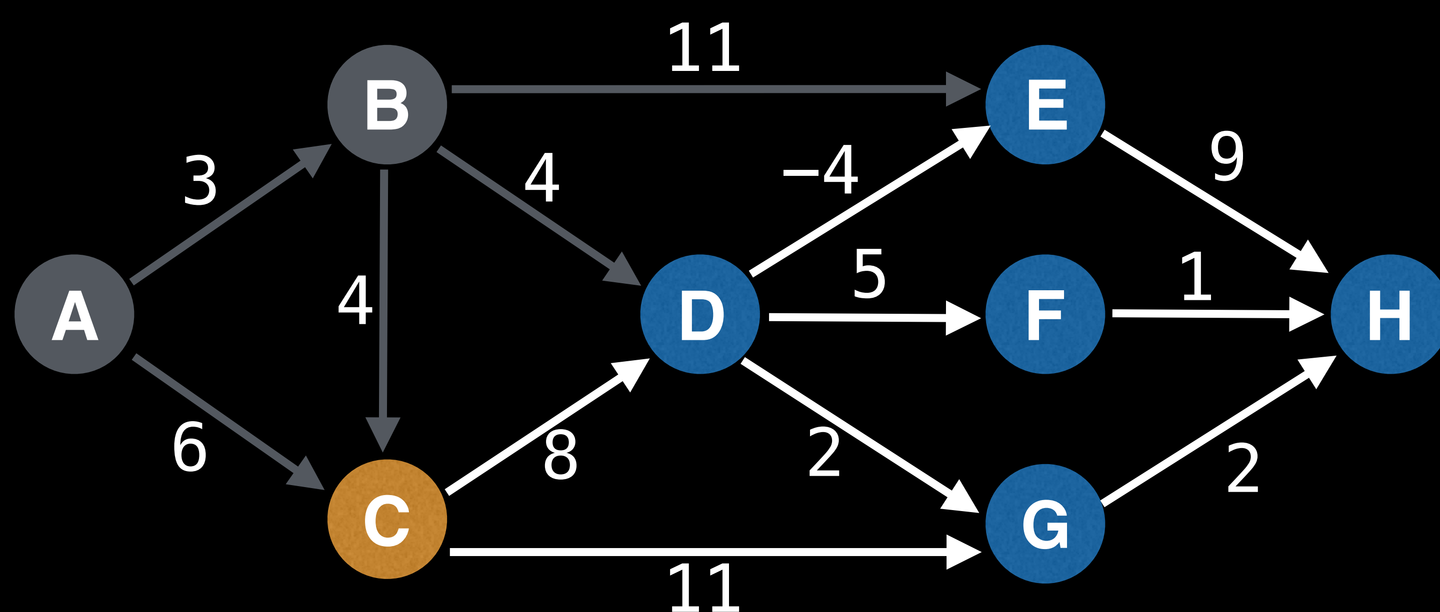

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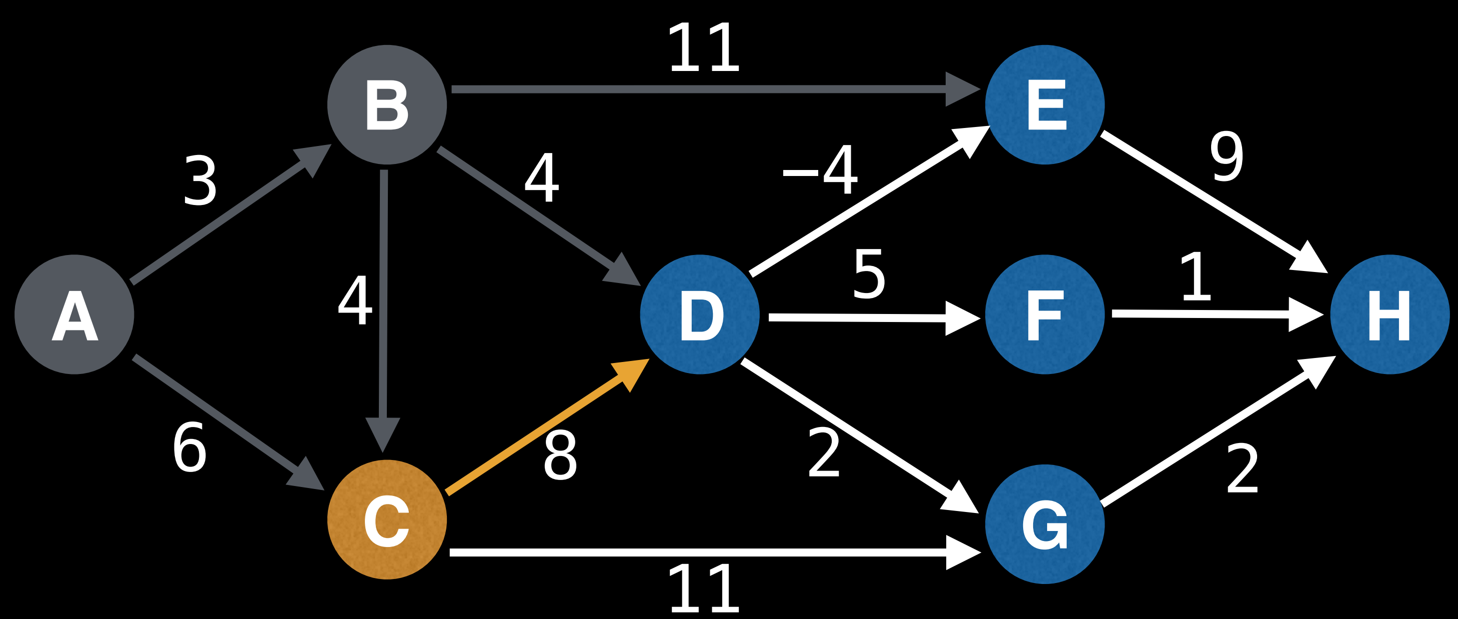

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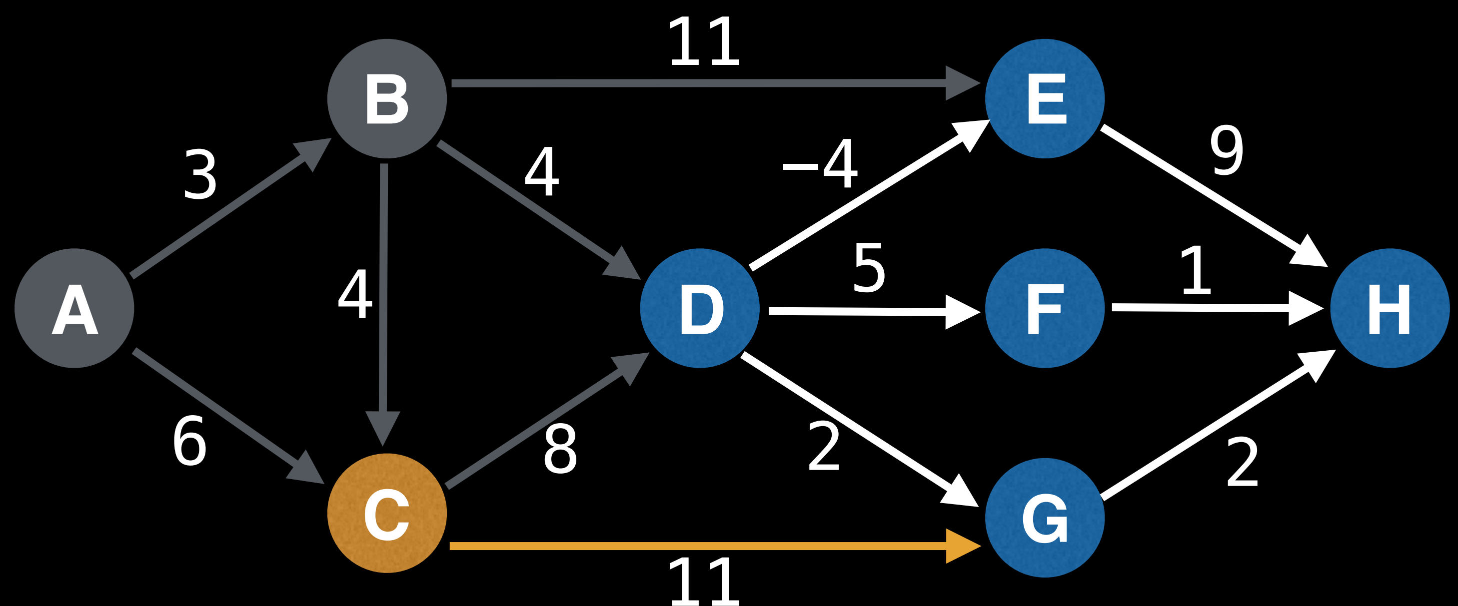

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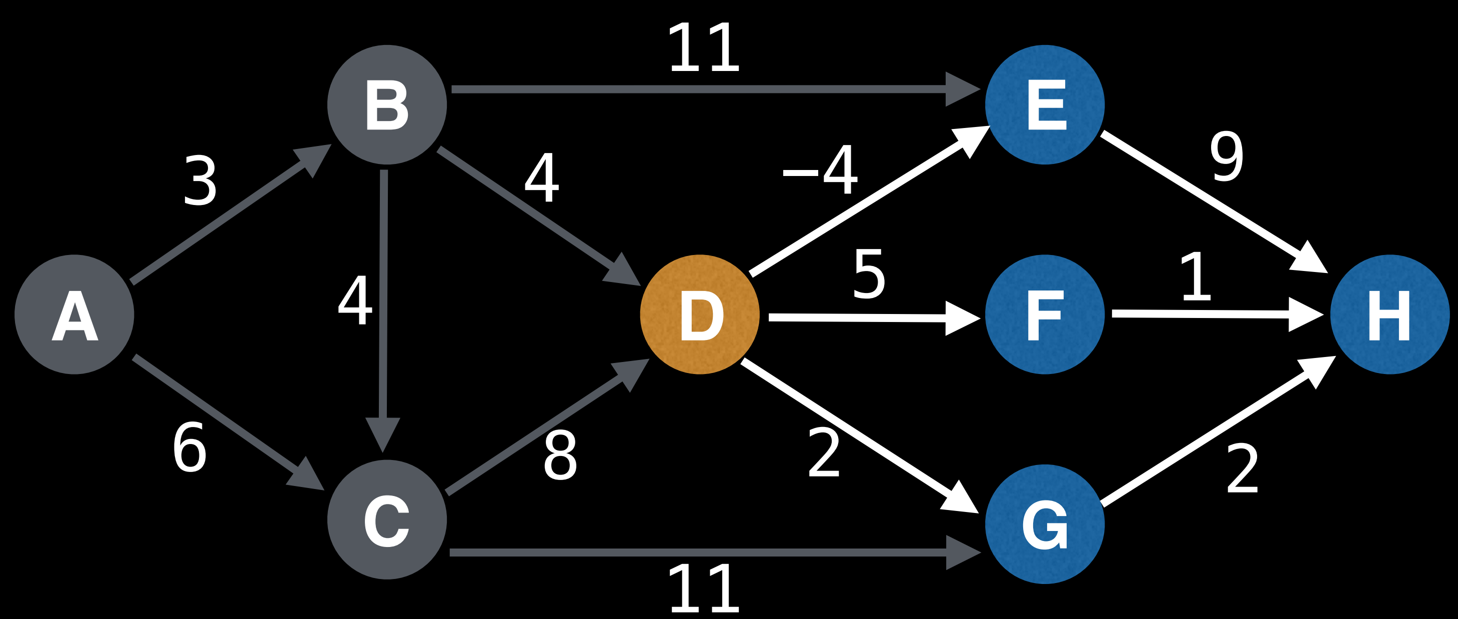

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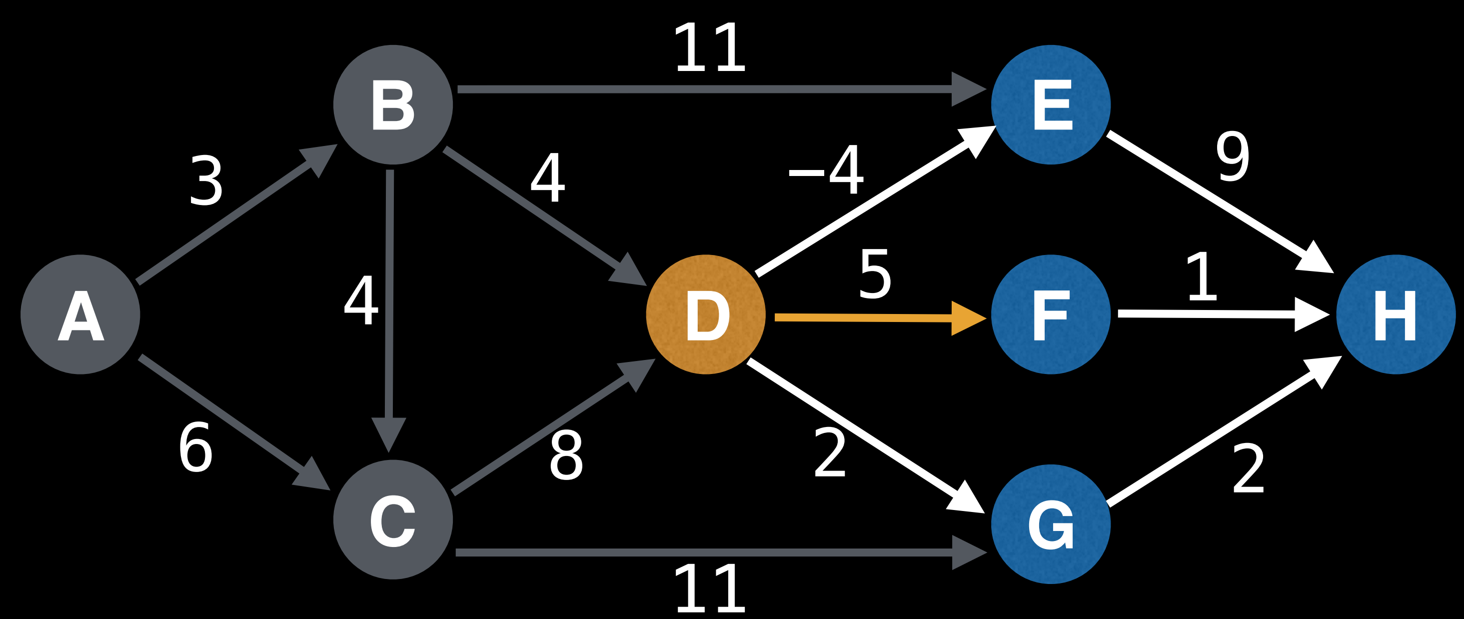

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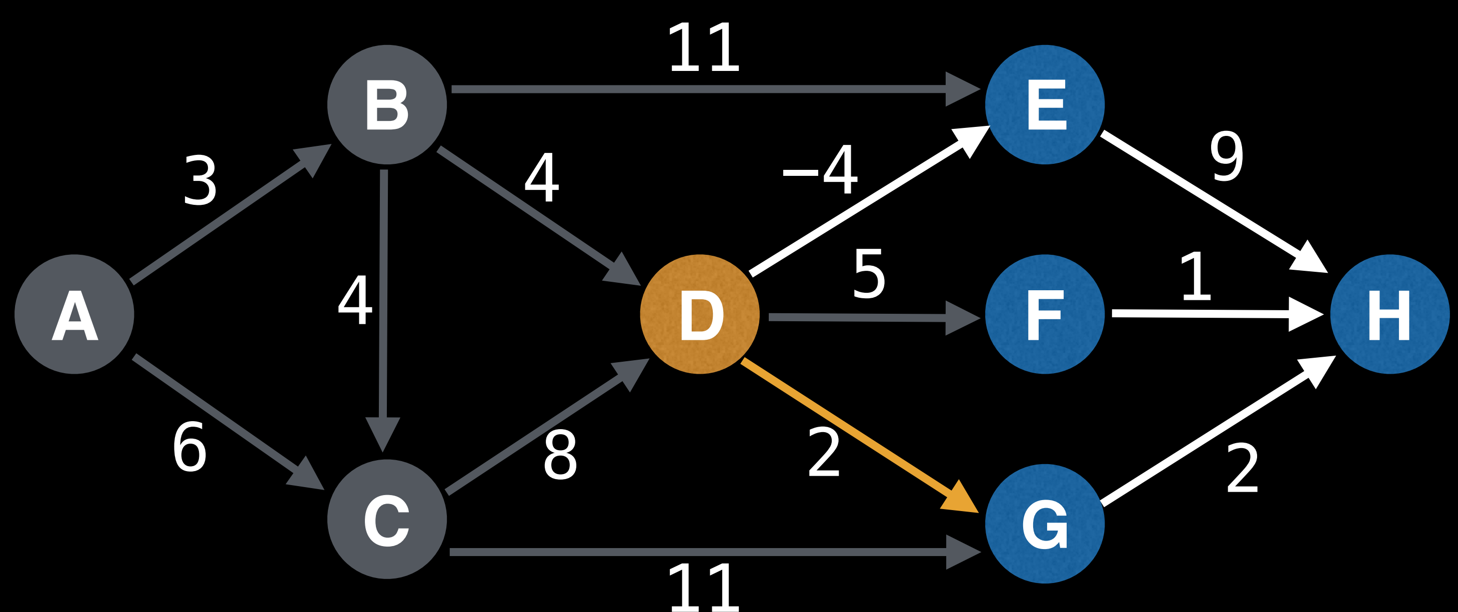

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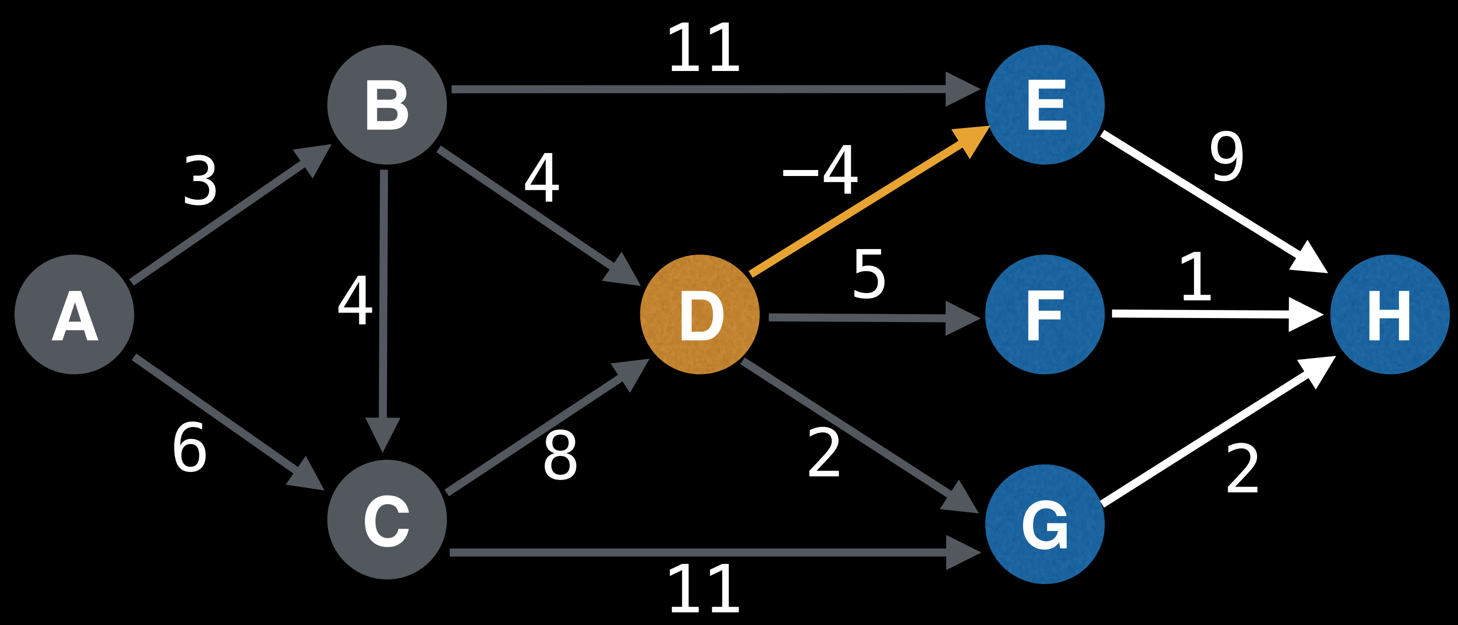

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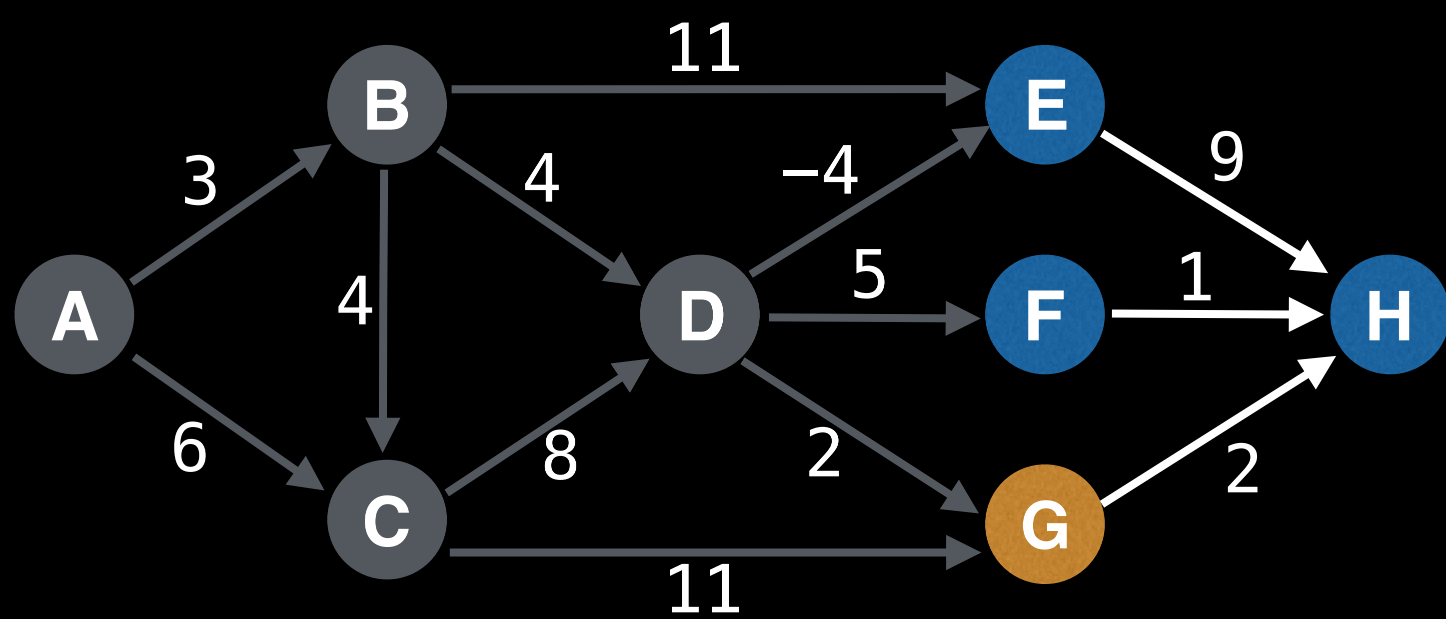

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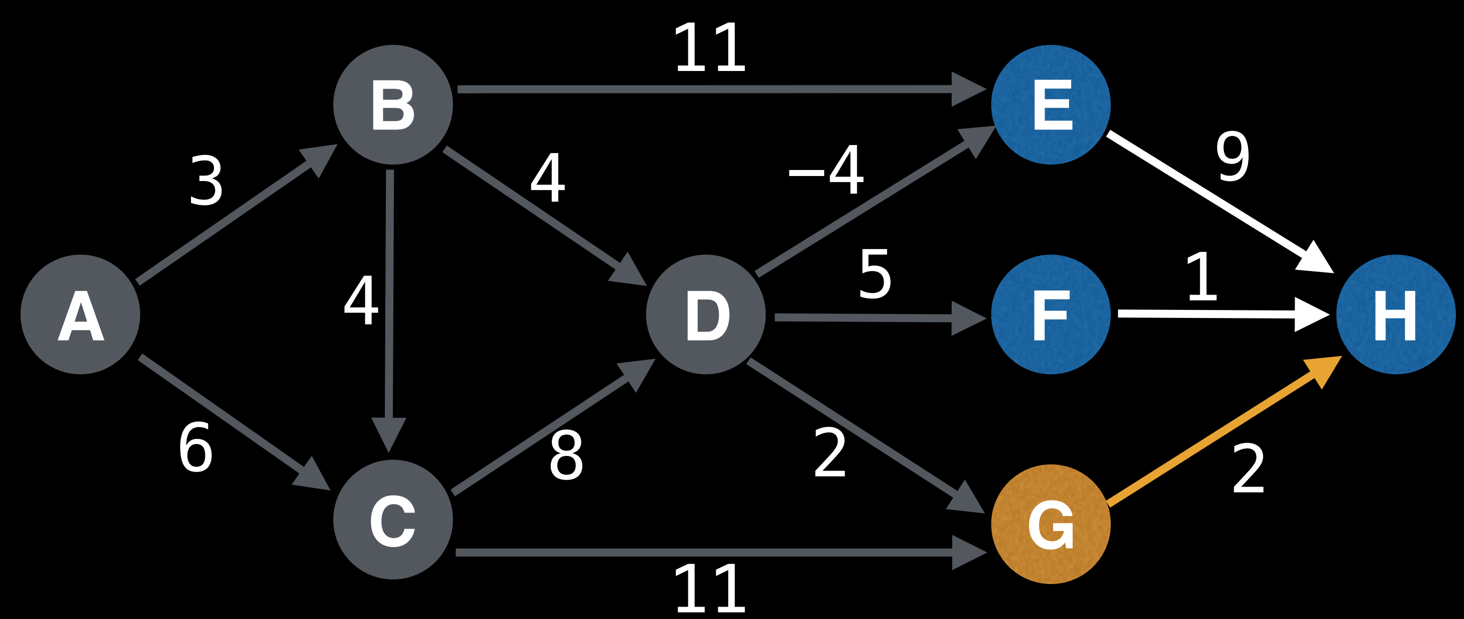

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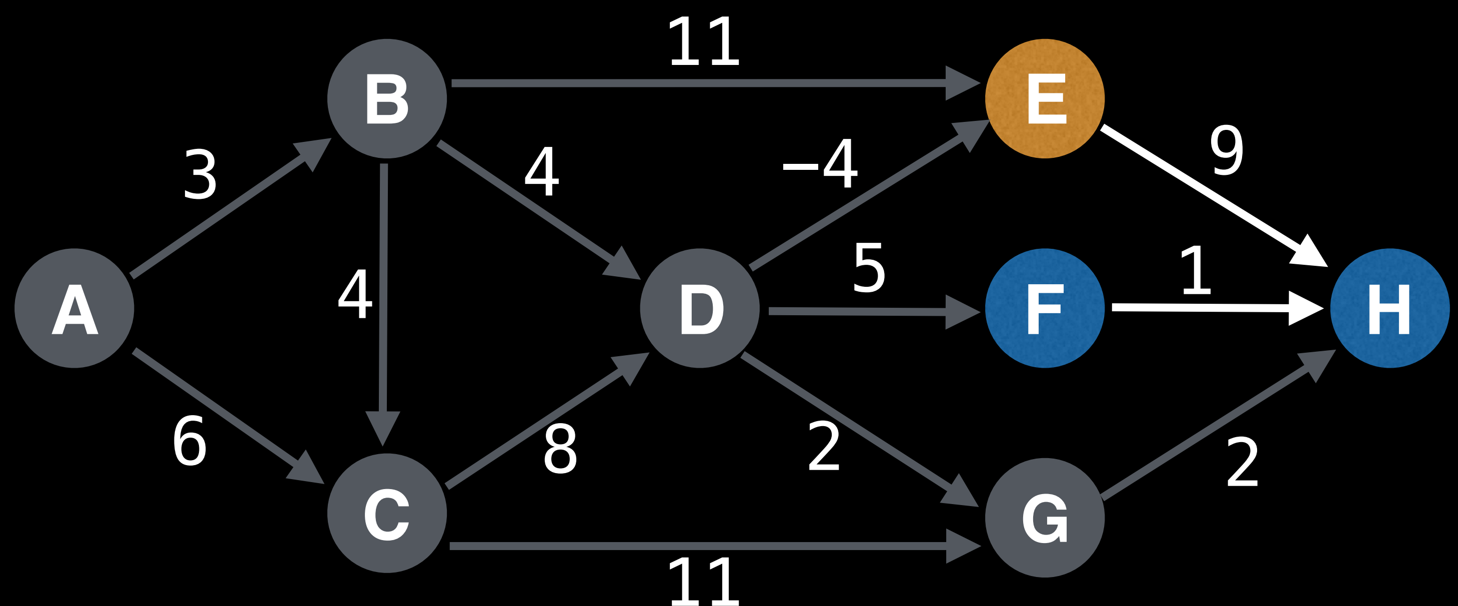

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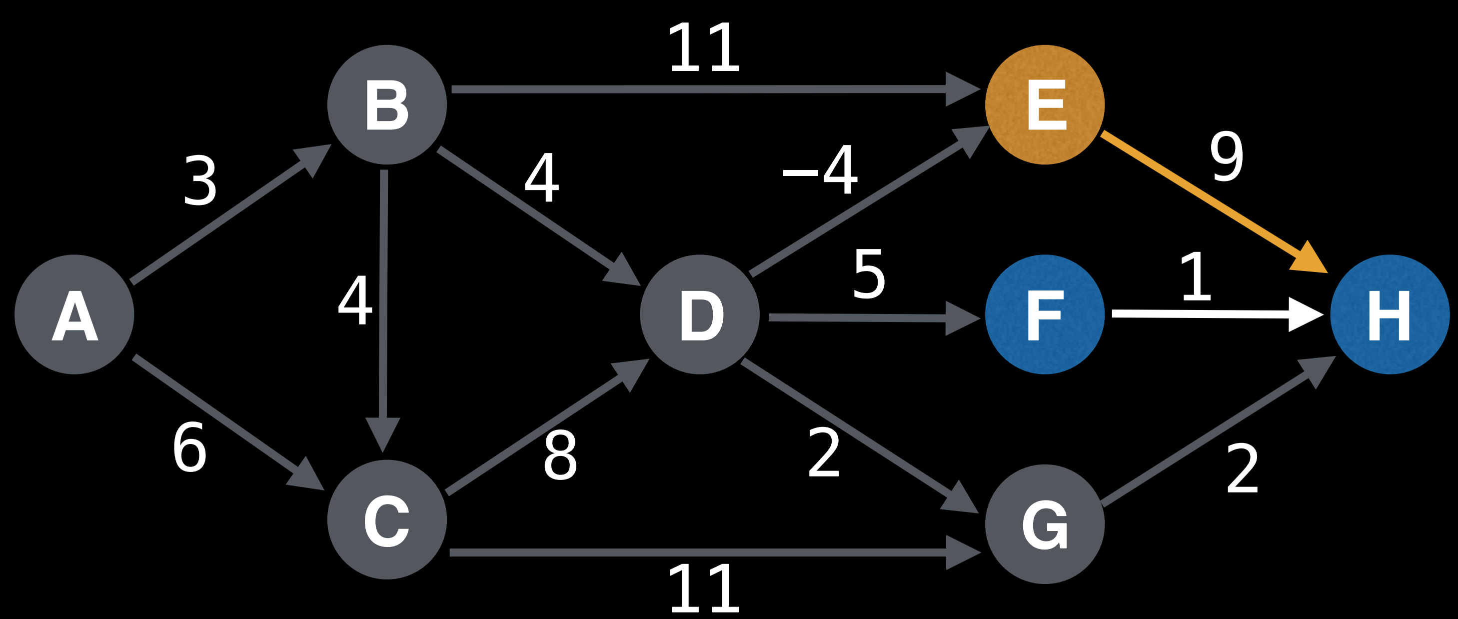

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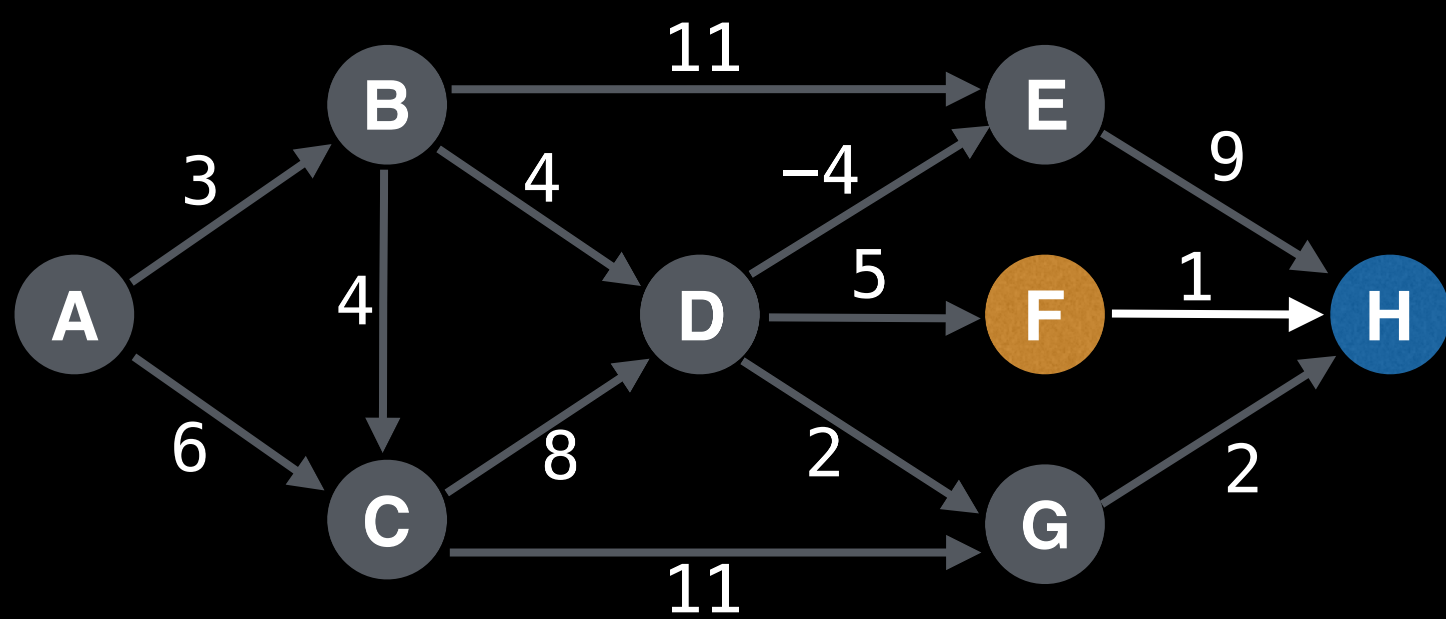

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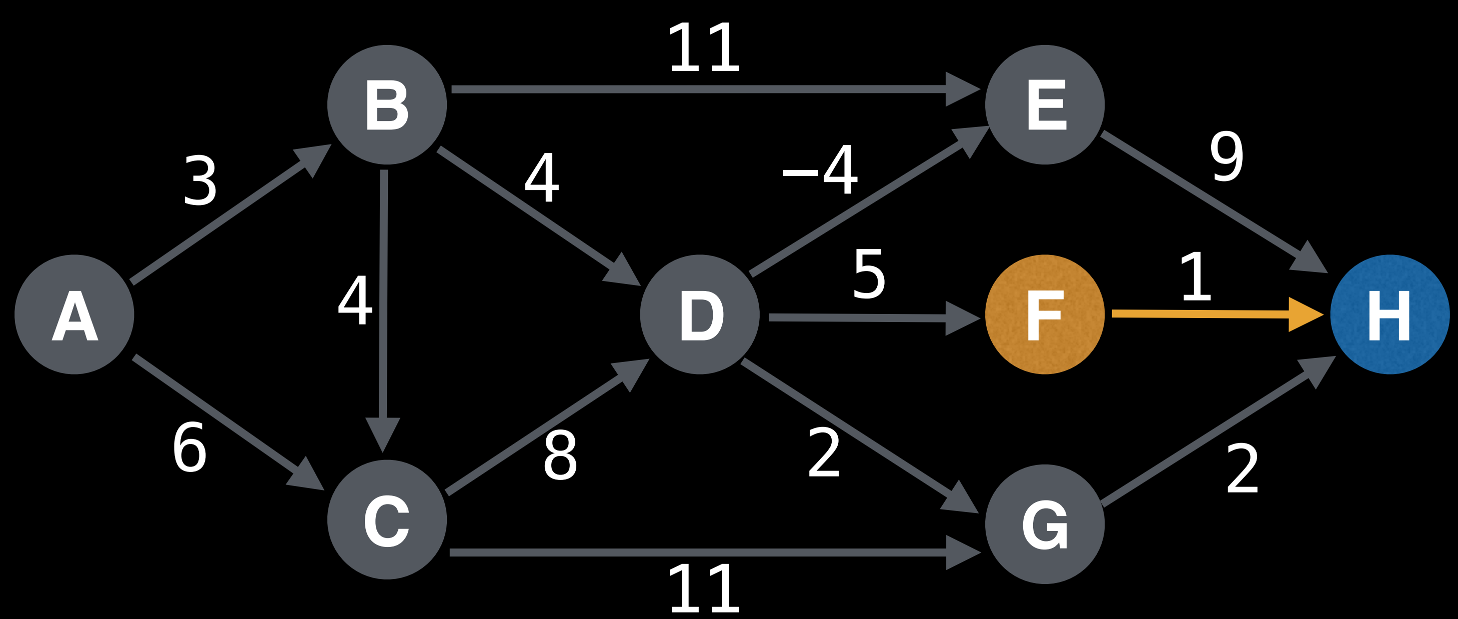

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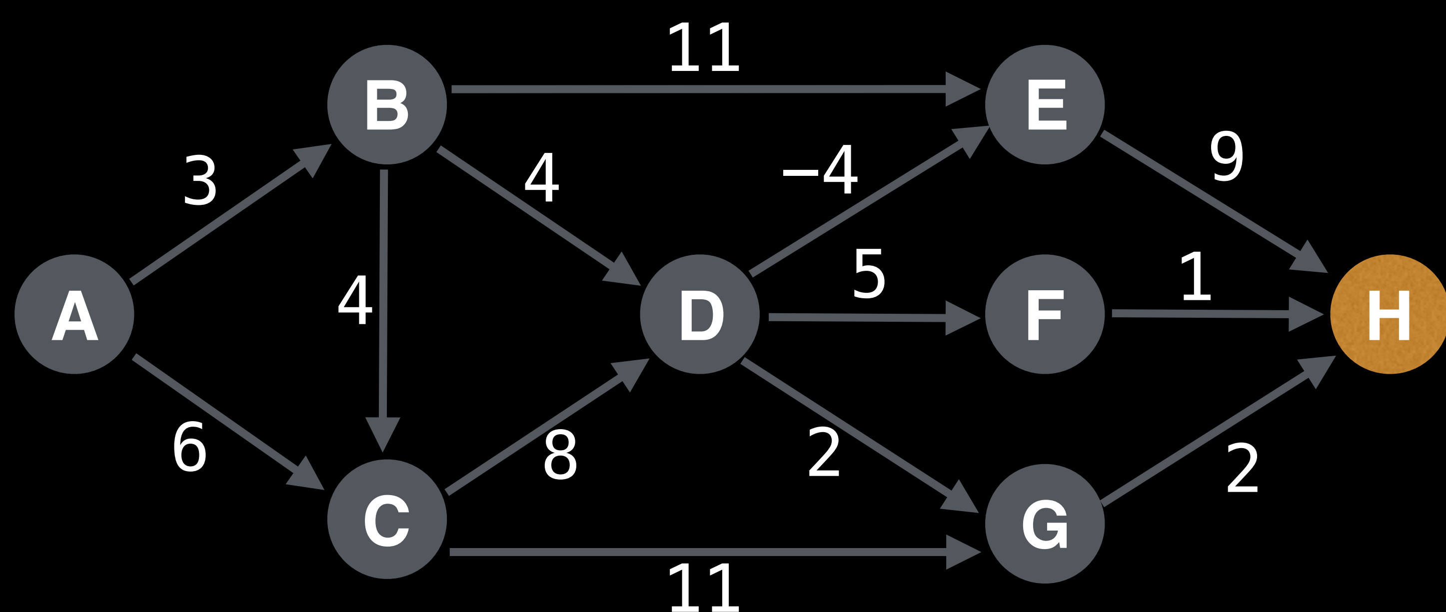

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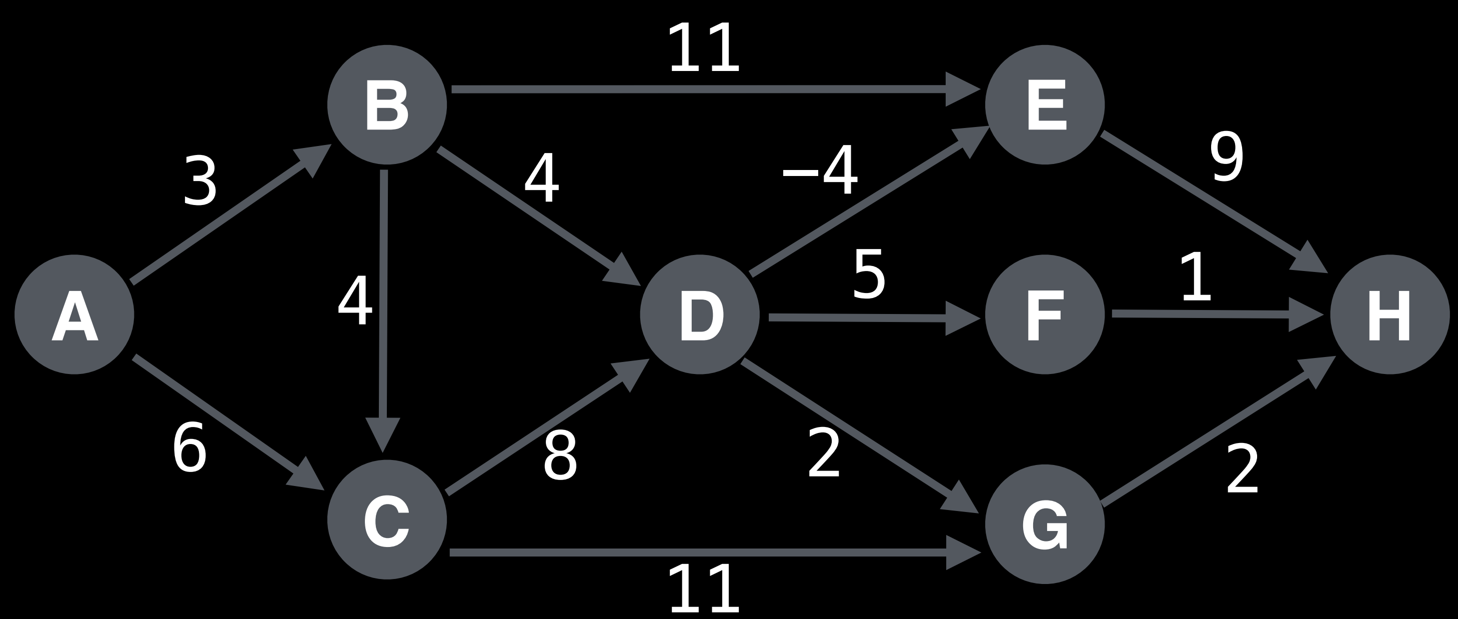

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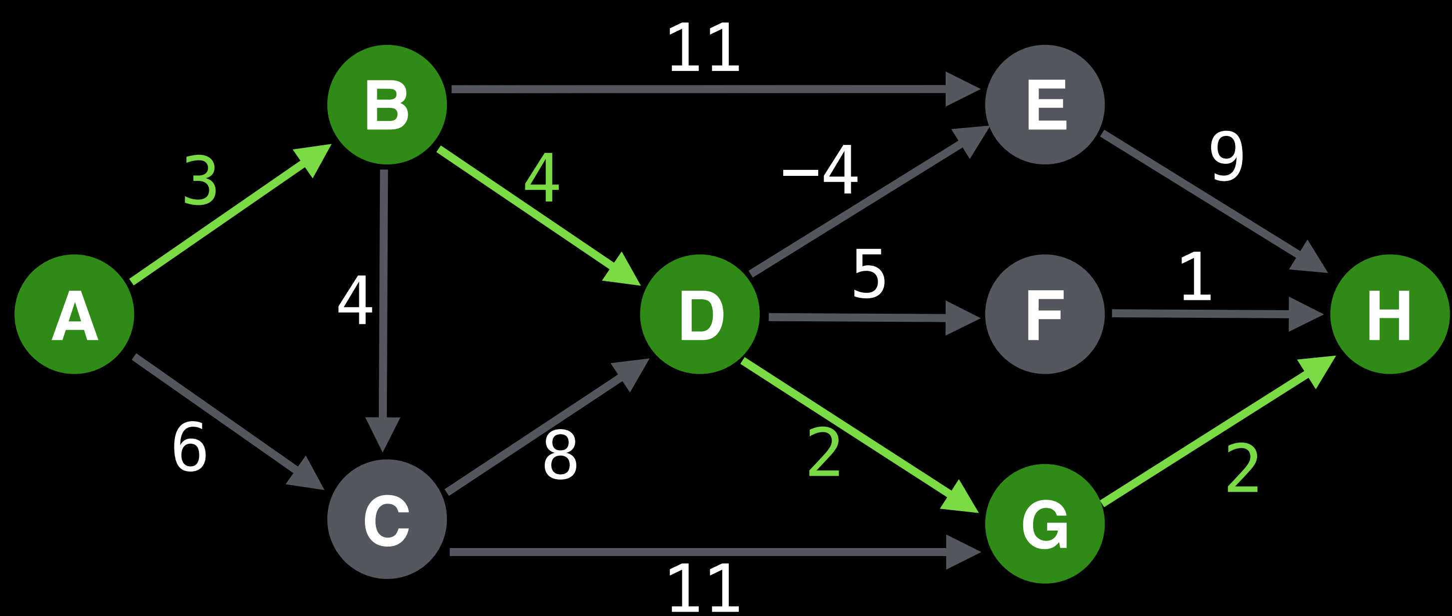

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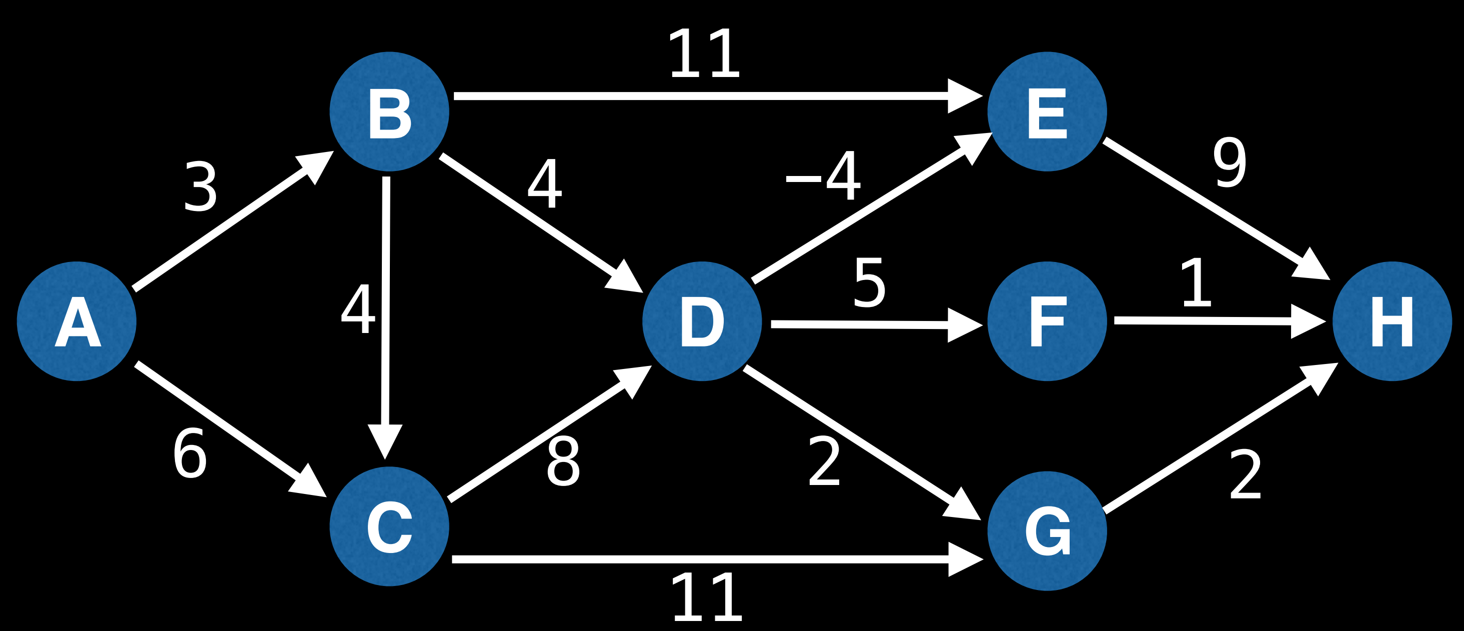

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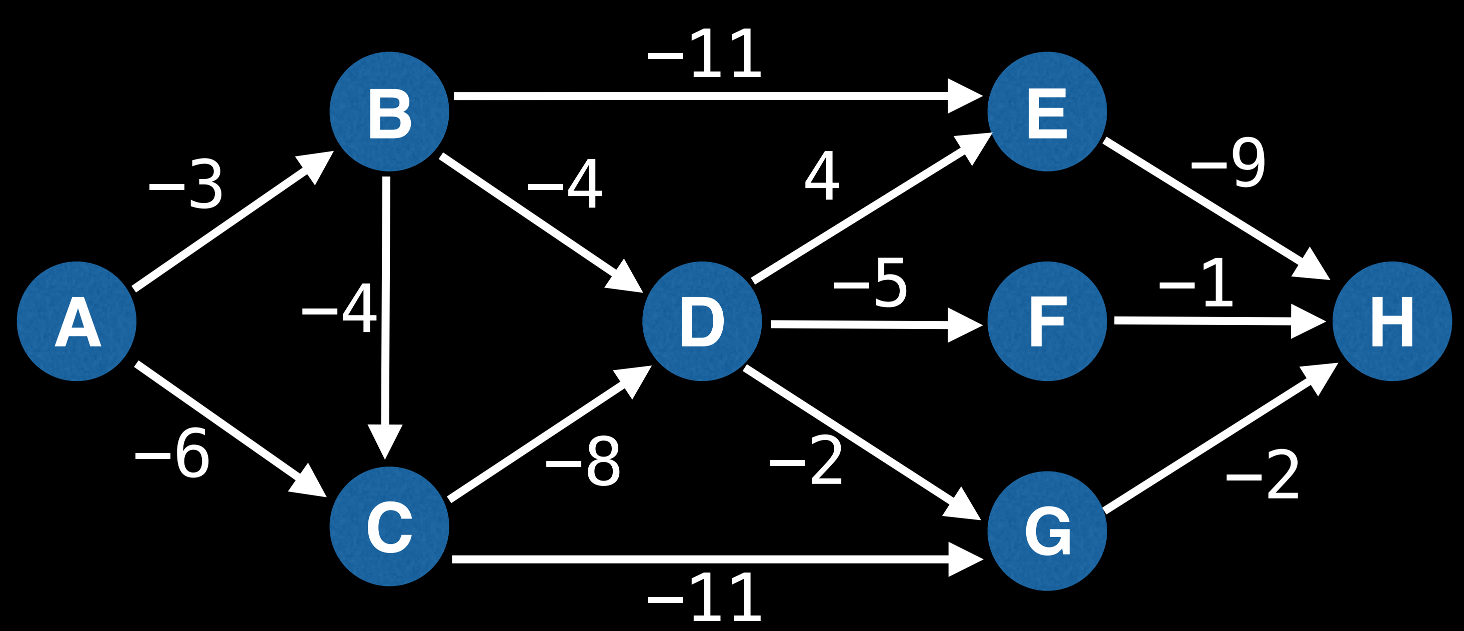

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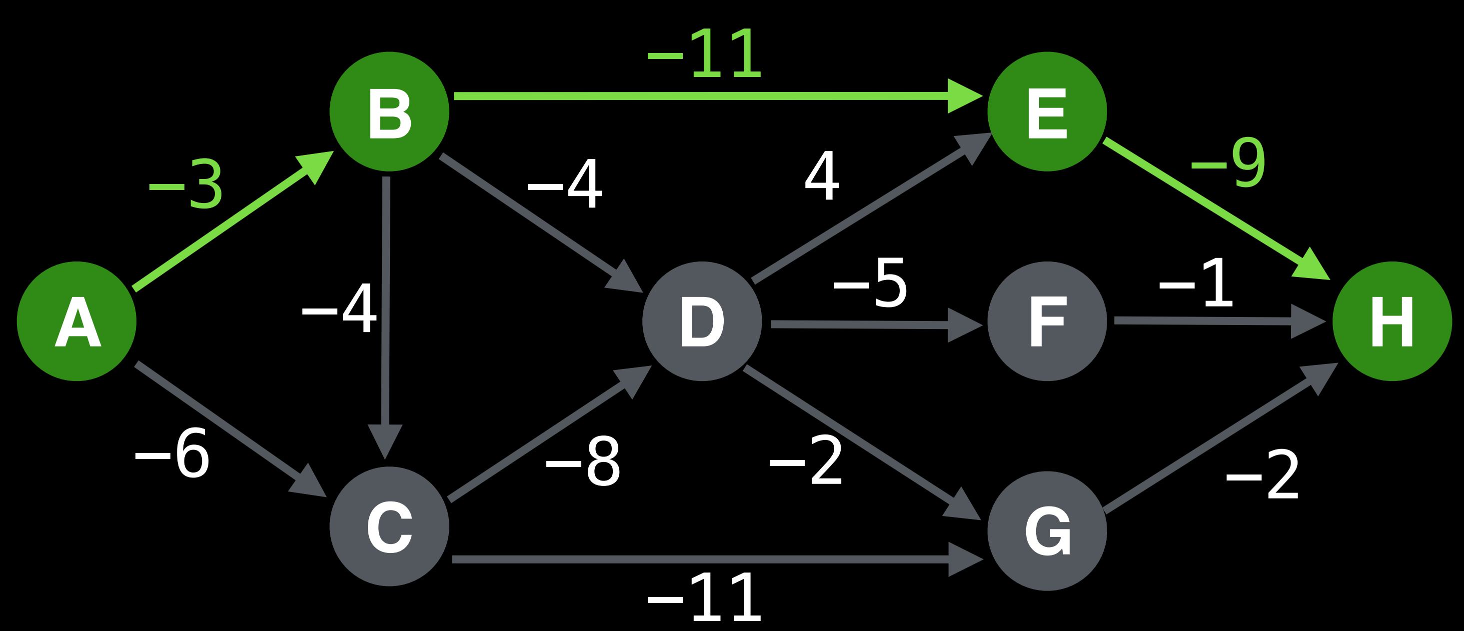

$$(-3 + -11 + -9) * -1 = 23$$

Source Code Link

Implementation source code can be found at the following link:

github.com/williamfiset/algorithms

Link in the description: