Turbulence Modelling

AM5640

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Assignment-1

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1 Introduction

Here, Option-1 (Coding task) was choosen. We numerically solve a fully-developed turbulent channel flow using $k-\varepsilon$, $k-\omega$ and RNG $k-\varepsilon$ turbulence model and compare them. Since the mean flow is one-dimensional (1D), the transport equations can be simplified into three coupled 1D-diffusion equations for the velocity and two turbulence quantities, respectively. Finite-Volume method was used.

The governing equations in a k- based eddy viscosity model (EVM) for a *stationary* ($\frac{\partial}{\partial t} <> = 0$), *spanwise homogeneous* ($\frac{\partial}{\partial z} <> = 0$), incompressible flow [comparing Eqs. 3.10, 3.12a, 3.12b, 3.35, 3.36 (Versteeg & Malalasekara [2])]

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[(\nu + \nu_t)\frac{\partial U}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial U}{\partial y}\right]$$

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left[(\nu + \nu_t)\frac{\partial V}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial V}{\partial y}\right] \quad (1)$$

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \varepsilon$$

Since the flow is fully developed we have $V = {\partial U}/{\partial x} = {\partial k}/{\partial x} = 0$. Hence, the above equations can be simplified as:-

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial V}{\partial y}\right] \quad (2)$$

$$0 = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \varepsilon \quad (3)$$

The equations reduce to simple 1D diffusion equations with some complicated source terms. The horizontal channel has a height of $y_{max} = 2\delta$ between the bottom flat plate (at $y = 0$) and the top flat plate (at $y = y_{max}$). The flow is driven by the pressure gradient ${\partial P}/{\partial x}$, as shown in the above momentum equation. Since the channel flow is fully developed, ${\partial P}/{\partial x}$ should be a constant and be balanced by the wall shear stress.

For comparison of the results with the DNS data [1], a fully developed channel flow with $Re_\tau = u_\tau \delta/\nu = 395$. Here $u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity, where $\tau_w = \mu\frac{\partial U}{\partial y}|_{wall}$. Given, $\delta = 1$ (i.e. $y_{max} = 2$), $\rho = 1$ and $-\frac{\partial P}{\partial x} = 1$.

2 Turbulence model and Boundary conditions

2.1 Turbulence Model

The turbulence models used for solving the given problem are:-

1. $k - \varepsilon$ model
2. RNG $k - \varepsilon$ model
3. $k - \omega$ model

The necessary equations are used for ε and ω. The model constant are used as provided.

2.2 Boundary conditions

The boundary conditions at the wall is as follows:

$$U = 0, k = 0, \varepsilon_{y^+ \leq 3} = 2\nu \frac{k}{y^2}, \omega_{y^+ \leq 3} = \frac{6\nu}{y^2 C_2} \tag{4}$$

The boundary conditions on the center-line of the channel is as follows:

$$\frac{\partial U}{\partial y} = 0, \frac{\partial k}{\partial y} = 0, \frac{\partial \varepsilon}{\partial y} = 0, \frac{\partial \omega}{\partial y} = 0 \tag{5}$$

3 Solver Details

3.1 Discretization

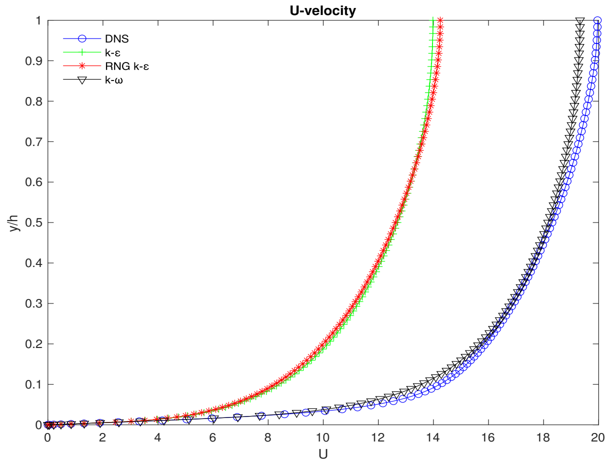
The U, k, ε and ω equations are discretized using the Finite volume method (CD). Here, domain has been discretized using DNS grid at the nodes and subsequent control volumes have been generated. Linearization of the source terms is used wherever necessary.

3.2 Solution

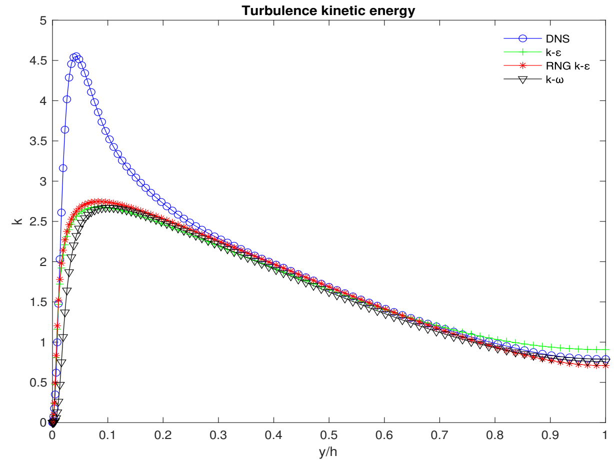
Gauss-Seidel method is used for solving the system of equations and the criteria of convergence for all the equations is taken as $\epsilon = 1 * 10^{-4}$. Where, ϵ is the residual limit. And, for finding the residue we have used $frobenius\ norm\ i.e$, $|\phi_{new} - \phi_{old}|$ and ϕ is the quantity of interest. The 3 different models have been solved $i.e$, $k - \varepsilon$, RNG $k - \varepsilon$ and $k - \omega$ model. The MATAB code is submitted for all the 3 models.

4 Results and Discussion

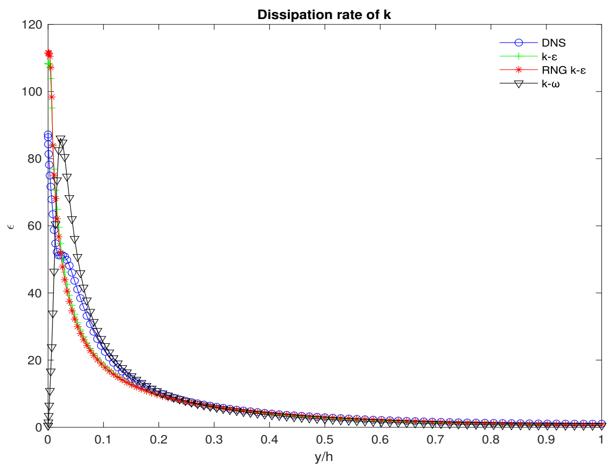
The plots for various parameters and turbulent terms are given as follows:-



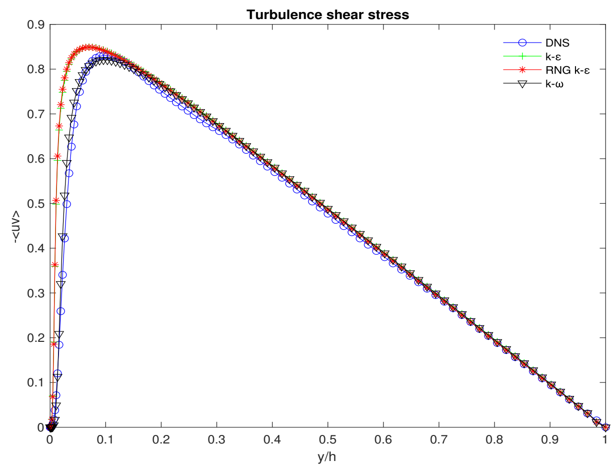
(a) Comparison for U-velocity



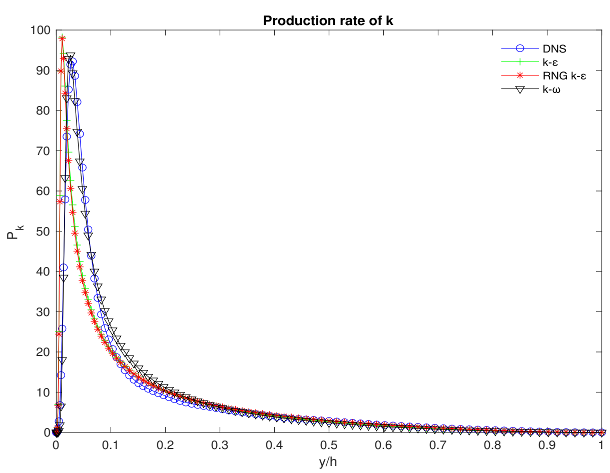
(b) Comparison for turbulence kinetic energy



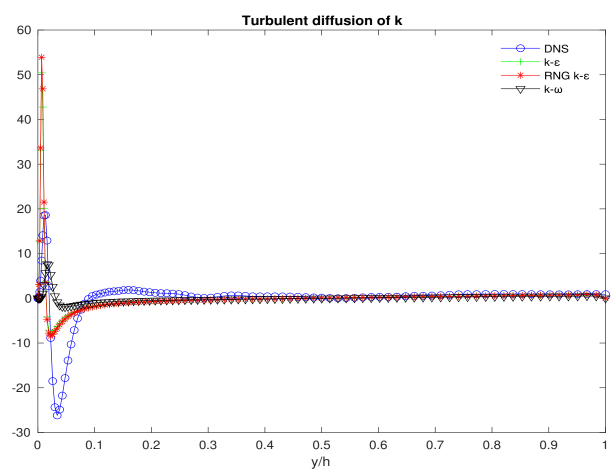
(c) Comparison for dissipation rate of k



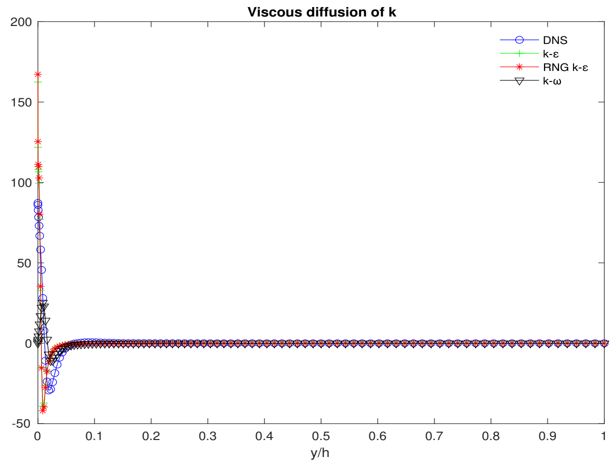
(d) Comparison for turbulence shear stress



(e) Comparison for production rate of k



(f) Comparison for turbulence diffusion of k



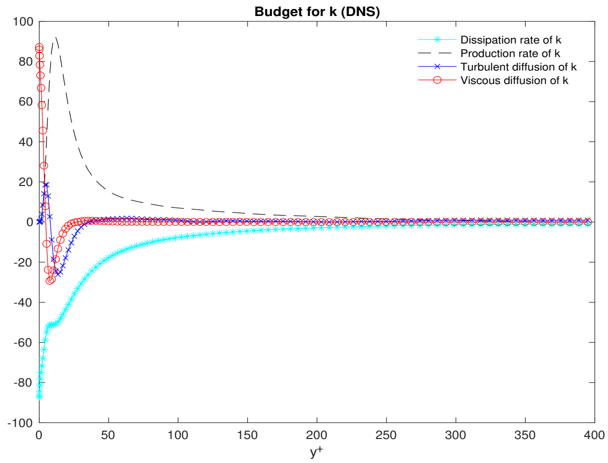
(g) Comparison for viscous diffusion of k

Figure 1: Comparison plots for turbulence quatntities between different models and DNS [1]

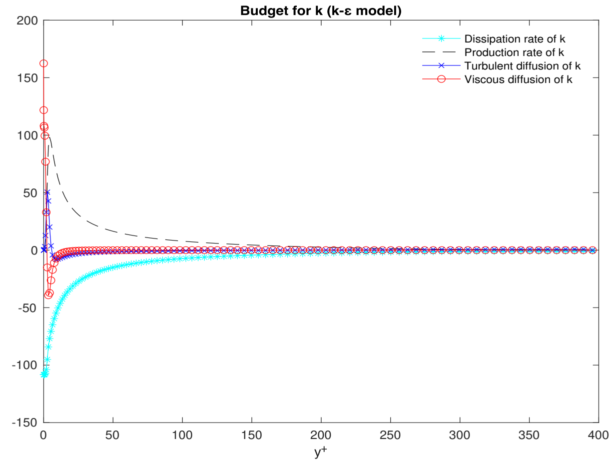
Here, from the simulated data the value of wall shear stress (τ_w) and friction velocity (u_τ) are calculated and the values are given as follows:-

Model	τ_w	u_τ
$k-\varepsilon$	1.0026	1.0013
RNG $k-\varepsilon$	1.0000	1.0000
$k-\omega$	1.0010	1.0005

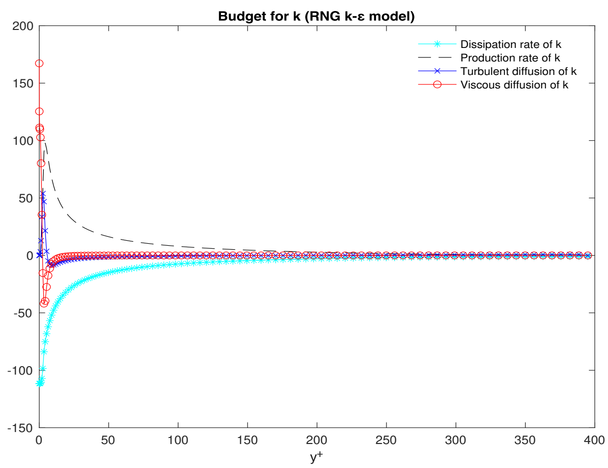
- From Fig.1a, we can see that the velocity is underpredicted for both $k-\varepsilon$ and RNG $k-\varepsilon$ model but the velocity profile predicted by $k-\omega$ model is close to the DNS data.
- From Fig.1b, it can be seen that the Turbulence kinetic energy is underpredicted by all the models.
- The dissipation rate of k as shown in Fig.1c, is overpredicted by $k-\varepsilon$ and RNG $k-\varepsilon$ model and the peak dissipation rate for $k-\omega$ model is predicted away from the wall.
- From Fig.1d, it is clear that turbulence shear stress is predicted well by all the models but, $k-\omega$ model provides the best prediction when compared with DNS data.
- As seen in Fig.1e, the production rate of k is predicted very closely when compared to DNS. But, RNG $k-\varepsilon$ overpredicted the peak production rate slightly.
- From Fig.1f, we see that the turbulence diffusion of k is not predicted well by any of the models near the wall.
- As shown in Fig.1g, the trend is well predicted but the values near the wall do not match properly with the DNS data.



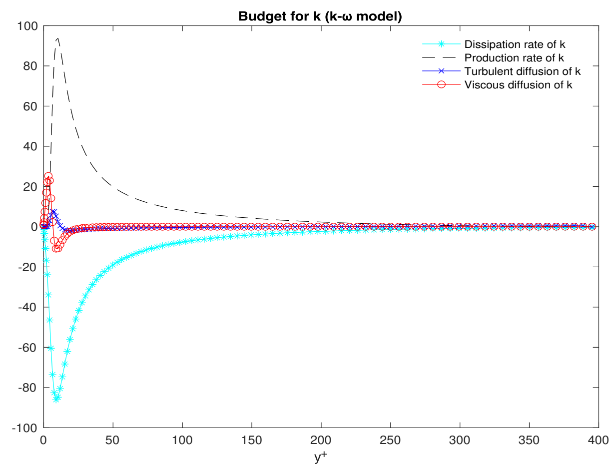
(a) Budget for k (DNS)



(b) Budget for k ($k-\varepsilon$ model)



(c) Budget for k (RNG $k-\varepsilon$ model)



(d) Budget for k ($k-\omega$ model)

Figure 2: Budget of k equation for different models and DNS [1]

From the budget plots we can see that, when compared with DNS [1] (Fig.2a) the models do predict the trends well. There is an over prediction of viscous diffusion for $k-\varepsilon$ and RNG $k-\varepsilon$ model (Fig.2b, 2c) which is due to the numerical error very close to the wall. While, for $k-\omega$ model (Fig.2d) viscous diffusion is underpredicted near the wall. But, production term is well predicted in all the models. The dissipation of k is also well predicted.

References

[1] R. D. Moser, J. Kim, and N. N. Mansour. Direct numerical simulation of turbulent channel flow up to $\mathrm{Re}_\tau = 590$. *Physics of Fluids*, 11(4):(943–945), 1999.

[2] H. Versteeg and W. Malalasekera. *An Introduction to Computational Fluid Dynamics - The Finite Volume Method*. Longman Scientific & Technical, Harlow, England, 1st edition, 1995.