DSB Portfolio 1

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2. november 2010

1 Draw the double sided spectrum of the amplitudes for the signal x(t)

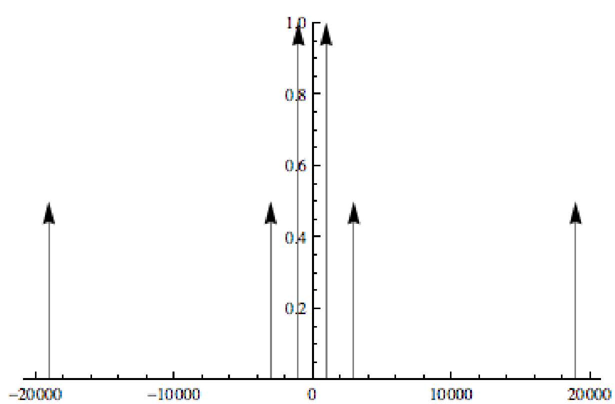
By using Eulers princible $x(t)$ can be rewritten as:

$$\begin{aligned} x(t) &= 2 \cdot \frac{e^{i2\pi \cdot 1000t} + e^{i2\pi \cdot 1000t}}{2} + \frac{e^{i2\pi \cdot 3000t} + e^{i2\pi \cdot 3000t}}{2} + \frac{e^{i2\pi \cdot 19000t} + e^{i2\pi \cdot 19000t}}{2} \\ x(t) &= e^{i2\pi \cdot 1000t} + e^{i2\pi \cdot 1000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 19000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 19000t} \end{aligned} \tag{1}$$

Which gives us the coefficients:

$$\begin{aligned} c_1 &= 1 \\ c_{-1} &= 1 \\ c_3 &= 0,5 \\ c_{-3} &= 0,5 \\ c_{19} &= 0,5 \\ c_{-19} &= 0,5 \end{aligned} \tag{2}$$

And the double sided spectrum:



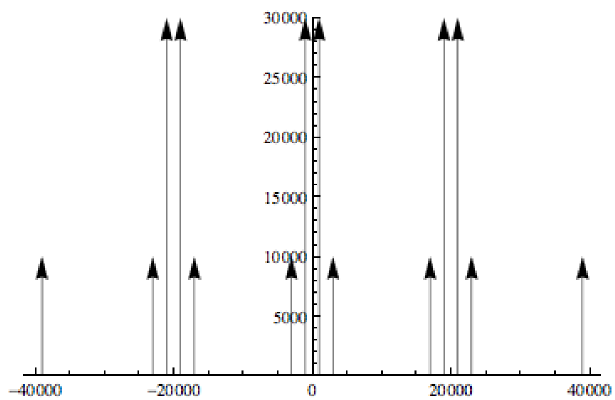
Figur 1: Double Sided Spectrum without aliasing

2 Draw the double sided spectrum xs1(t) for the sampled signal without the anti-aliasing filter

The samplerate of the A/D-converter is 20kHz. The new coefficients will be calculated by by multiplying them with the samplerate. Which gives:

$$
\begin{aligned}
c_1 &= 20000 \\
c_{-1} &= 20000 \\
c_3 &= 10000 \\
c_{-3} &= 10000 \\
c_{19} &= 10000 \\
c_{-19} &= 10000
\end{aligned}
\tag{3}
$$

And a plot of the double sided spectrum:



Figur 2: Double Sided Spectrum with aliasing

3 Calculate the A/D converter signal in dB

A/D-converter signal can be calculated by using:

$$SNR_{dB} = 10,79 + 20 \cdot Log_{10}(\frac{x_{rms}}{\Delta}) \tag{4}$$

Where:

$$\Delta = \frac{x_{max} - x_{min}}{2^m} \tag{5}$$

x_{max} and x_{min} is the maximum and minimum voltage for the A/D-converter and m is the bitsize. x_{rms} is given by:

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dT} \tag{6}$$

We can then start calcualting:

$$\begin{aligned} x_{rms} &= \sqrt{3} \\ \Delta = \frac{2-(-2)}{2^8} &= \frac{1}{64} \\ SNR_{dB} = 10,79 + 20 \cdot Log_{10}(\frac{\sqrt{3}}{\frac{1}{64}}) &= 51,6848dB \end{aligned}$$

SNR_{dB} is the strength of the signal.

4 Determine the quantization voltage for the A/D converter

Quantization is the steps in which the A/D-converter moves. It is the same as Δ which we calculated in the previous section:

$$\Delta = \frac{2-(-2)}{2^8} = \frac{1}{64}$$

5 Determine the order of the filter for the analog Butterworth

The equation can be converted to dB and solved with regards to n:

$$\begin{aligned} -48dB &= 20 \cdot log_{10}(\frac{1}{\sqrt{1 + (\frac{10000}{3000})^{2n}}}) \\ n = 4,58997 &\sim 5 \end{aligned}$$

The order of the filter is 5.

6 Determine the damping of the amplitude of the analog signal with 1000, 3000 or 10000 Hz

$$
\begin{aligned}
20 \cdot Log_{10}\left(\frac{1}{\sqrt{1+(\frac{1000}{3000})^{2\cdot5}}}\right) &= -0,00074dB \\
20 \cdot Log_{10}\left(\frac{1}{\sqrt{1+(\frac{3000}{3000})^{2\cdot5}}}\right) &= -3,0103dB \\
20 \cdot Log_{10}\left(\frac{1}{\sqrt{1+(\frac{10000}{3000})^{2\cdot5}}}\right) &= -52,2879dB
\end{aligned} \tag{7}
$$

7 Draw the double-sided spectrum xs2 for the sampled signal with the designed anti-aliasing filter

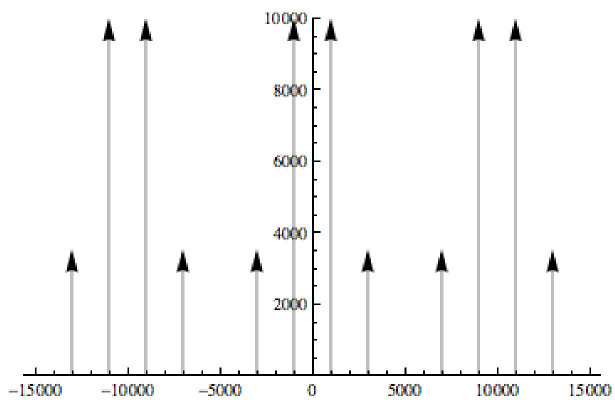
Calculating magnitudes for frequencies 1000Hz, 3000Hz and 1900Hz:

$$\begin{aligned}
\frac{1}{\sqrt{1+(\frac{1000}{3000})^{2\cdot 5}}} &= 0,999992 \\
\frac{1}{\sqrt{1+(\frac{3000}{3000})^{2\cdot 5}}} &= 0,707107 \\
\frac{1}{\sqrt{1+(\frac{19000}{3000})^{2\cdot 5}}} &= 0,000098
\end{aligned} \tag{8}$$

These magnitudes are multiplied by the coefficients and the 10kHz sample frequency to get the real magnitudes:

$$\begin{aligned}
0,999992 \cdot 1 \cdot 10kHz &= 9999,92 \\
0,707107 \cdot 0,5 \cdot 10kHz &= 3535,54 \\
0,000098 \cdot 0,5 \cdot 10kHz &= 0,49
\end{aligned} \tag{9}$$

Double sided spectrum plot. The third signal is left out as it is so small compared to the two others:



Figur 3: Double Sided Spectrum

8 Calculate the filter order n for an reconstruction filter of Butterworth type

By adapting and making two equations we can solve two variables:

$$\begin{aligned}
-0,5dB &\leq 20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{3000}{f_c})^{2n}}}) \\
20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{10000}{f_c})^{2n}}}) &= 20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{3000}{f_c})^{2n}}}) \\
n = 5,5 &\sim 6 \\
f_c &= 3636,87
\end{aligned} \tag{10}$$

The order of the filter is 6. We then find the limits of f_c:

$$\begin{aligned}
-48dB &\leq 20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{10000}{f_c})^{2 \cdot 6}}}) \\
f_c &\leq 3981,08 \\
-0,5dB &\geq 20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{3000}{f_c})^{2 \cdot 6}}}) \\
f_c &\geq 3574,81
\end{aligned} \tag{11}$$

f_c must lie between 3574,81 and 3981,08. f_c is set to 3700. And the new filter:

$$20 \cdot Log_{10}(\frac{1}{\sqrt{1+(\frac{3000}{3700})^{2 \cdot 6}}}) \tag{12}$$