

DSB Portfolio 1

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1 Draw the double sided spectrum of the amplitudes for the signal $x(t)$

By using Eulers principle $x(t)$ can be rewritten as:

$$x(t) = 2 \cdot \frac{e^{i2\pi \cdot 1000t} + e^{-i2\pi \cdot 1000t}}{2} + \frac{e^{i2\pi \cdot 3000t} + e^{-i2\pi \cdot 3000t}}{2} + \frac{e^{i2\pi \cdot 19000t} + e^{-i2\pi \cdot 19000t}}{2}$$

$$x(t) = e^{i2\pi \cdot 1000t} + e^{-i2\pi \cdot 1000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 19000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 19000t} \quad (1)$$

Which gives us the coefficients:

$$\begin{aligned} c_1 &= 1 \\ c_{-1} &= 1 \\ c_3 &= 0,5 \\ c_{-3} &= 0,5 \\ c_{19} &= 0,5 \\ c_{-19} &= 0,5 \end{aligned}$$

(2)

And the double sided spectrum:

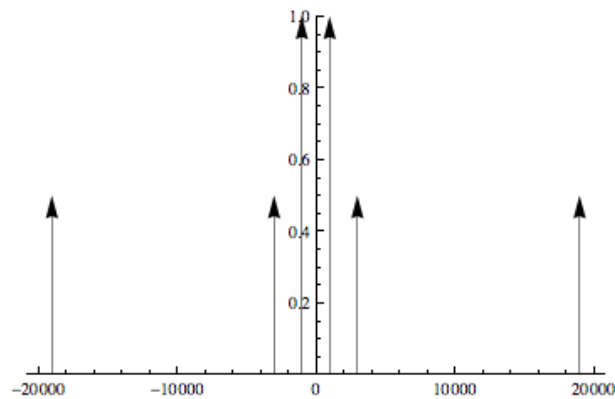


Figure 1: Double Sided Spectrum without aliasing

2 Draw the double sided spectrum $xs_1(t)$ for the sampled signal without the anti-aliasing filter

The samplerate of the A/D-converter is 20kHz. The new coefficients will be calculated by multiplying them with the samplerate. Which gives:

$$\begin{aligned} c_1 &= 20000 \\ c_{-1} &= 20000 \\ c_3 &= 10000 \\ c_{-3} &= 10000 \\ c_{19} &= 10000 \\ c_{-19} &= 10000 \end{aligned}$$

(3)

And a plot of the double sided spectrum:

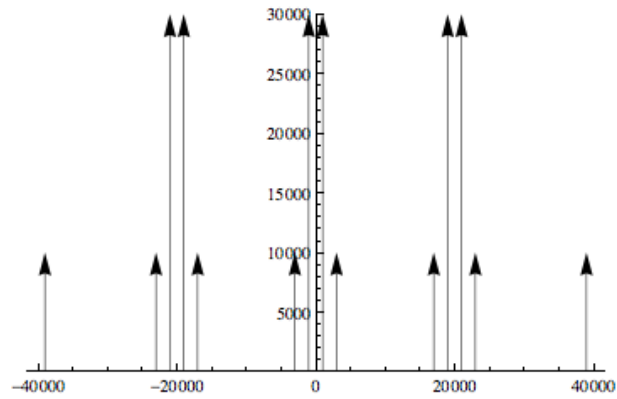


Figure 2: Double Sided Spectrum with aliasing

3 Calculate the A/D converter signal in dB

A/D-converter signal can be calculated by using:

$$SNR_{dB} = 10,79 + 20 \cdot \text{Log}_{10}\left(\frac{x_{rms}}{\Delta}\right) \quad (4)$$

Where:

$$\Delta = \frac{x_{max} - x_{min}}{2^m} \quad (5)$$

x_{max} and x_{min} is the maximum and minimum voltage for the A/D-converter and m is the bitsize. x_{rms} is given by:

$$x_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dT} \quad (6)$$

We can then start calculating:

$$\begin{aligned} x_{rms} &= \sqrt{3} \\ \Delta &= \frac{2 - (-2)}{2^8} = \frac{1}{64} \\ SNR_{dB} &= 10,79 + 20 \cdot \text{Log}_{10}\left(\frac{\sqrt{3}}{\frac{1}{64}}\right) = 51,6848dB \end{aligned}$$

SNR_{dB} is the strength of the signal.

4 Determine the quantization voltage for the A/D converter

Quantization is the steps in which the A/D-converter moves. It is the same as Δ which we calculated in the previous section:

$$\Delta = \frac{2 - (-2)}{2^8} = \frac{1}{64}$$

5 Determine the order of the filter for the analog Butterworth

The equation can be converted to dB and solved with regards to n:

$$\begin{aligned} -48dB &= 20 \cdot \log_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{10000}{3000}\right)^{2n}}}\right) \\ n &= 4,58997 \sim 5 \end{aligned}$$

The order of the filter is 5.

- 6 Determine the damping of the amplitude of the analog signal with 1000, 3000 or 10000 Hz

$$\begin{aligned}
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{1000}{3000}\right)^{2.5}}}\right) &= -0,00074dB \\
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{3000}\right)^{2.5}}}\right) &= -3,0103dB \\
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{10000}{3000}\right)^{2.5}}}\right) &= -52,2879dB
 \end{aligned}$$

(7)

7 Draw the double-sided spectrum xs2 for the sampled signal with the designed anti-aliasing filter

Calculating magnitudes for frequencies 1000Hz, 3000Hz and 1900Hz:

$$\begin{aligned} \frac{1}{\sqrt{1 + \left(\frac{1000}{3000}\right)^{2.5}}} &= 0,999992 \\ \frac{1}{\sqrt{1 + \left(\frac{3000}{3000}\right)^{2.5}}} &= 0,707107 \\ \frac{1}{\sqrt{1 + \left(\frac{19000}{3000}\right)^{2.5}}} &= 0,000098 \end{aligned} \quad (8)$$

These magnitudes are multiplied by the coefficients and the 10kHz sample frequency to get the real magnitudes:

$$\begin{aligned} 0,999992 \cdot 1 \cdot 10kHz &= 9999,92 \\ 0,707107 \cdot 0,5 \cdot 10kHz &= 3535,54 \\ 0,000098 \cdot 0,5 \cdot 10kHz &= 0,49 \end{aligned} \quad (9)$$

Double sided spectrum plot. The third signal is left out as it is so small compared to the two others:

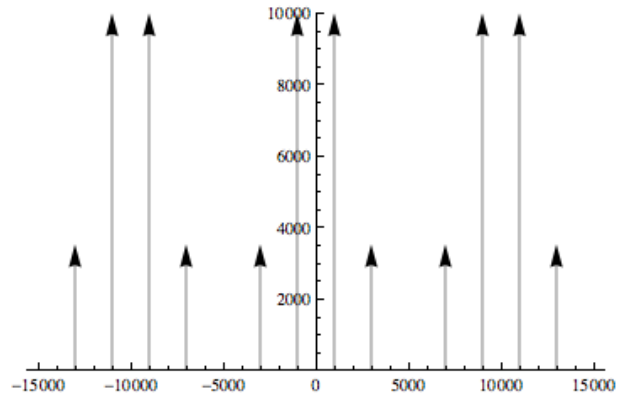


Figure 3: Double Sided Spectrum

8 Calculate the filter order n for an reconstruction filter of Butterworth type

By adapting and making two equations we can solve two variables:

$$\begin{aligned}
 -0,5dB &\leq 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{f_c}\right)^{2n}}}\right) \\
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{10000}{f_c}\right)^{2n}}}\right) &= 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{f_c}\right)^{2n}}}\right) \\
 n = 5,5 &\sim 6 \\
 f_c &= 3636,87
 \end{aligned} \tag{10}$$

The order of the filter is 6. We then find the limits of f_c :

$$\begin{aligned}
 -48dB &\leq 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{10000}{f_c}\right)^{2 \cdot 6}}}\right) \\
 f_c &\leq 3981,08 \\
 -0,5dB &\geq 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{f_c}\right)^{2 \cdot 6}}}\right) \\
 f_c &\geq 3574,81
 \end{aligned} \tag{11}$$

f_c must lie between 3574,81 and 3981,08. f_c is set to 3700. And the new filter:

$$20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{3700}\right)^{2 \cdot 6}}}\right) \tag{12}$$

9 Determine the damping of the amplitude in dB of the reconstructed signal

Damping of the frequencies 1000Hz and 3000Hz.

$$\begin{aligned}
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{1000}{3700}\right)^{2 \cdot 6}}}\right) &= -6,597^{-7}dB \\
 20 \cdot \text{Log}_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{3000}{3700}\right)^{2 \cdot 6}}}\right) &= -0,337^{-7}dB
 \end{aligned} \tag{13}$$