DSB Portfolio 1

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1 Draw the double sided spectrum of the amplitudes for the signal x(t)

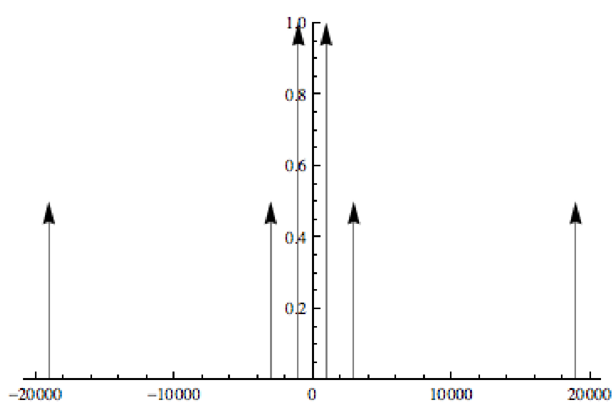
By using Eulers principle $x(t)$ can be rewritten as:

$$
\begin{aligned}
x(t) &= 2 \cdot \frac{e^{i2\pi \cdot 1000t} + e^{i2\pi \cdot 1000t}}{2} + \frac{e^{i2\pi \cdot 3000t} + e^{i2\pi \cdot 3000t}}{2} + \frac{e^{i2\pi \cdot 19000t} + e^{i2\pi \cdot 19000t}}{2} \\
x(t) &= e^{i2\pi \cdot 1000t} + e^{i2\pi \cdot 1000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 3000t} + \frac{1}{2} \cdot e^{i2\pi \cdot 19000t} + \frac{1}{2} \cdot e^{-i2\pi \cdot 19000t}
\end{aligned} \tag{1}
$$

Which gives us the coefficients:

$$
\begin{aligned}
c_1 &= 1 \\
c_{-1} &= 1 \\
c_3 &= 0,5 \\
c_{-3} &= 0,5 \\
c_{19} &= 0,5 \\
c_{-19} &= 0,5
\end{aligned} \tag{2}
$$

And the double sided spectrum:



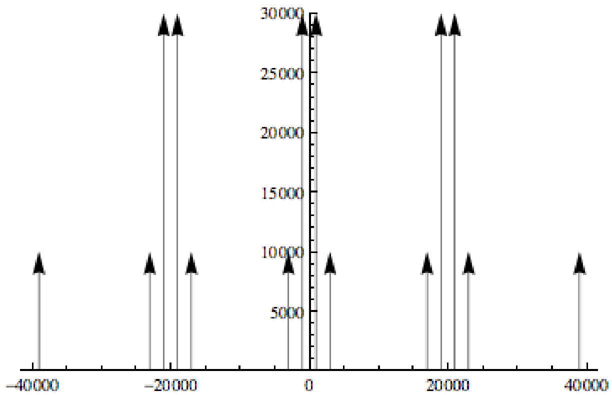
Figur 1: Double Sided Spectrum without aliasing

2 Draw the double sided spectrum xs1(t) for the sampled signal without the anti-aliasing filter

The samplerate of the A/D-converter is 20kHz. The new coefficients will be calculated by by multiplying them with the samplerate. Which gives:

$$
\begin{aligned}
c_1 &= 20000 \\
c_{-1} &= 20000 \\
c_3 &= 10000 \\
c_{-3} &= 10000 \\
c_{19} &= 10000 \\
c_{-19} &= 10000
\end{aligned}
\tag{3}
$$

And a plot of the double sided spectrum:



Figur 2: Double Sided Spectrum with aliasing

3 Calculate the A/D converter signal in dB

A/D-converter signal can be calculated by using:

$$SNR_{dB} = 10,79 + 20 \cdot Log_{10}(\frac{x_{rms}}{\Delta}) \tag{4}$$

Where:

$$\Delta = \frac{x_{max} - x_{min}}{2^m} \tag{5}$$

x_{max} and x_{min} is the maximum and minimum voltage for the A/D-converter and m is the bitsize. x_{rms} is given by:

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dT} \tag{6}$$

We can then start calcualting:

$$\begin{aligned} x_{rms} &= \sqrt{3} \\ \Delta = \frac{2-(-2)}{2^8} &= \frac{1}{64} \\ SNR_{dB} = 10,79 + 20 \cdot Log_{10}(\frac{\sqrt{3}}{\frac{1}{64}}) &= 51,6848dB \end{aligned}$$

SNR_{dB} is the strength of the signal.

4 Determine the quantization voltage for the A/D converter

Quantization is the steps in which the A/D-converter moves. It is the same as Δ which we calculated in the previous section:

$$\Delta = \frac{2-(-2)}{2^8} = \frac{1}{64}$$

5 Determine the order of the filter for the analog Butterworth

The equation can be converted to dB and solved with regards to n:

$$\begin{aligned} -48dB &= 20 \cdot log_{10}(\frac{1}{\sqrt{1+(\frac{10000}{3000})^{2n}}}) \\ n = 4,58997 &\sim 5 \end{aligned}$$

The order of the filter is 5.

6 Determine the damping of the amplitude of the analog signal with 1000, 3000 or 10000 Hz

$$
\begin{aligned}
20 \cdot Log_{10}(\frac{1}{1+(\frac{1000}{3000})^{2\cdot 5}}) &= -0,00074dB \\
20 \cdot Log_{10}(\frac{1}{1+(\frac{3000}{3000})^{2\cdot 5}}) &= -3,0103dB \\
20 \cdot Log_{10}(\frac{1}{1+(\frac{10000}{3000})^{2\cdot 5}}) &= -52,2879dB
\end{aligned}
\tag{7}
$$