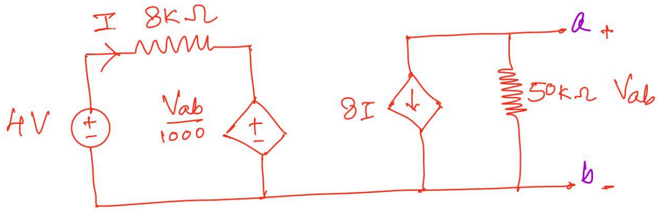
Solⁿ. 1[a.]:

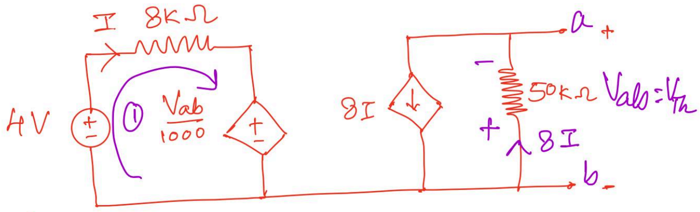
Thevenin's equivalent circuit



Calculation of V_{th}:

From loop (1);

$$I = \frac{4 - \frac{V_{ab}}{1000}}{8K} \quad \text{——} (1)$$



From outer loop;

$V_{th} = -8I \times 50K$ $(\because V_{th} = V_{ab})$

$V_{th} = -400\,I\,K$ V

from Eqⁿ (1);

$$V_{th} = \frac{-400}{8\cancel{K}}\left(4 - \frac{V_{th}}{1000}\right)\cancel{K}$$

$$V_{th} = \frac{400\,V_{th}}{8000} - \frac{1600}{8} = \frac{V_{th}}{20} - 200$$

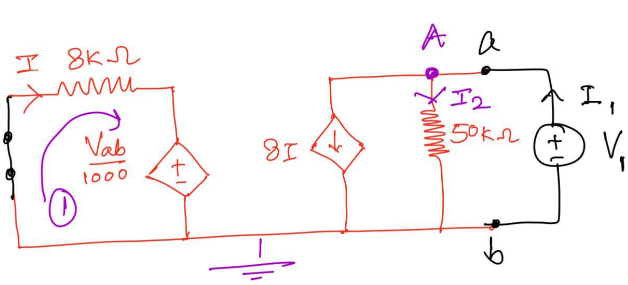
$$\frac{19V_{th}}{20} = -200$$

$$V_{th} = \frac{-200 \times 20}{19} = -210.52$$

$$\boxed{V_{th} = -210.52 \text{ Volt}}$$

Calculation of R_{Th}:

4 Volt source → short circuited



From loop (1);

$$I = \frac{-V_{ab}/1000}{8K}$$

$$I = \frac{-V_1}{8000K} \quad \text{——— (1)} \quad \left(\because V_{ab} = V_1\right)$$

Apply KCL at node A;

$$8I + I_2 - I_1 = 0$$

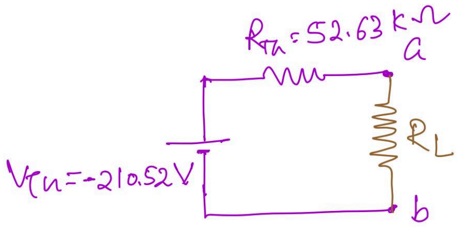
$$\frac{-8V_1}{8000K} + \frac{V_1}{50K} = I_1$$

$$V_1\left[\frac{1}{50K} - \frac{1}{1000K}\right] = I_1$$

$$R_{Th} = \frac{V_1}{I_1} = \frac{1}{\frac{20-1}{1000K}} = \frac{1000K}{19}$$

$$\boxed{R_{Th} = 52.631\ K\Omega}$$

Thevenin's Equivalent circuit:



Condition for MPT;

$$R_L = R_{Th} = 52.631K\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(-210.52)^2}{4 \times 52.631K} = 210.51\ mW$$

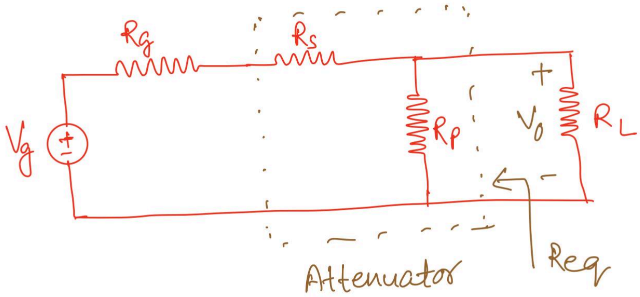
$$\boxed{P_{max} = 210.51mW}$$

Sol^n 1.[b]:

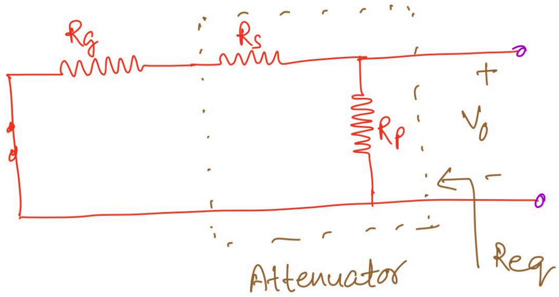
Data Given:

$$\frac{V_0}{V_g} = 0.25$$

$R_{eq} = R_g = R_{th} = 2K\Omega$



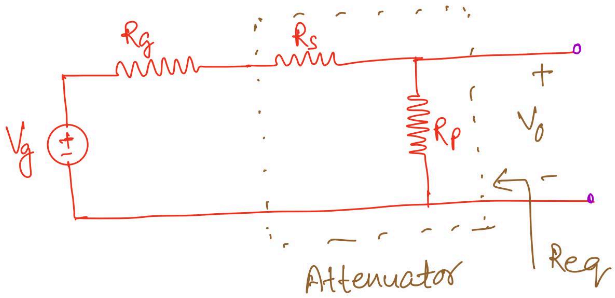
$$R_{eq} = R_p \| (R_g + R_s) = R_g$$



$$\frac{R_p \times (R_g + R_s)}{R_p + R_g + R_s} = R_g$$

$$\cancel{R_p R_g} + R_p R_s = \cancel{R_p R_g} + R_g^2 + R_g R_s$$

$$R_p R_s = R_g (R_g + R_s) \quad \text{——— (1)}$$



$$V_o = V_g \times \frac{R_p}{R_p + R_g + R_s}$$

$$\frac{V_o}{V_g} = \frac{R_p}{R_p + R_g + R_s} = 0.25 = \frac{1}{4}$$

$$4R_p = R_p + R_g + R_s$$

$$3R_p = R_g + R_s \quad \text{——— (2)}$$

from Eq^n (1) & (2);

$$\cancel{R_p} R_s = R_g \times 3 \cancel{R_p}$$

$$R_s = 2 \times 3 \, K\Omega = 6K\Omega$$

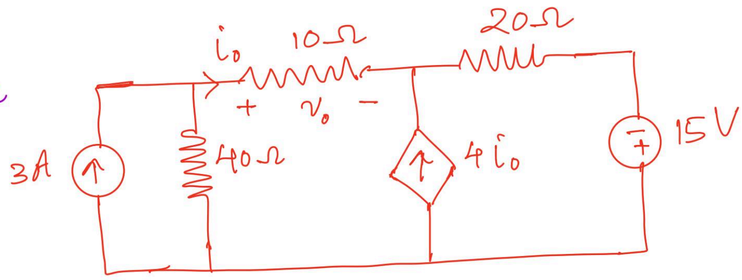
$$\boxed{R_s = 6K\Omega}$$

$$R_p = \frac{R_g + R_s}{3} = \frac{2k + 6k}{3} = \frac{8}{3}\ k\Omega$$

$$\boxed{R_p = 2.67\ k\Omega}$$

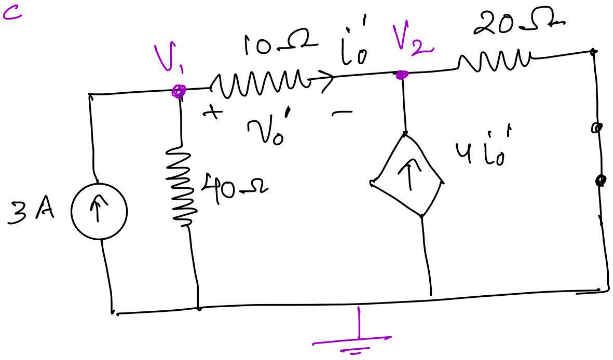
Soln. 2[a]:

i_o, v_o using superposition principle



Case 1: 3A is Activated; 15V deactivated ≅ s/c

Using Nodal Analysis;



KCL at V_1;

$$-3 + \frac{V_1}{40} + \frac{V_1 - V_2}{10} = 0$$

$$-120 + V_1 + 4V_1 - 4V_2 = 0$$

$$5V_1 - 4V_2 = 120 \quad \text{——— (1)}$$

$$i_o' = \frac{V_1 - V_2}{10}$$

KCL at V_2;

$$\frac{V_2 - V_1}{10} + \frac{V_2}{20} - 4i_o' = 0 \quad \text{——— (2)}$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{20} - \frac{4(V_1 - V_2)}{10} = 0$$

$$2V_2 - 2V_1 + V_2 - 8V_1 + 8V_2 = 0$$

$$10V_1 - 11V_2 = 0$$

$$V_1 = 1.1V_2 \quad \text{——— (3)}$$

From Equ (1) & (3);

$$5(1.1V_2) - 4V_2 = 120$$

$$5.5V_2 - 4V_2 = 120$$

$$1.5V_2 = 120$$

$$V_2 = \frac{1200}{15} = \frac{400}{5} = 80\ V$$

$$V_2 = 80\ V$$

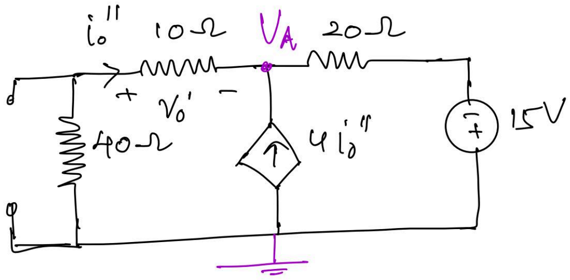
$$V_1 = 88\ V$$

$$i_0' = \frac{V_1 - V_2}{10} = \frac{88-80}{10} = 0.8\ A$$

$$i_0' = 0.8\ A$$

$$v_0' = V_1 - V_2 = 8\ V$$

Case 2: Using Nodal Analysis;



$$\frac{V_A}{50} + \frac{V_A - (-15)}{20} - 4i_0'' = 0$$

$$i_0'' = \frac{-V_A}{50}$$

$$-i_0'' + \frac{15 - 50i_0''}{20} - 4i_0'' = 0$$

$$100\ i_0'' = 15 - 50i_0''$$

$$i_0'' = \frac{15}{150} = \frac{1}{10} = 0.1\ A$$

$$\boxed{i_0'' = 0.1A}$$

$$\boxed{V_0'' = 10 \times i_0'' = 1\ V}$$

Using Superposition principle;

$$V_o = V_o' + V_o'' = 8 + 1 = 9V$$

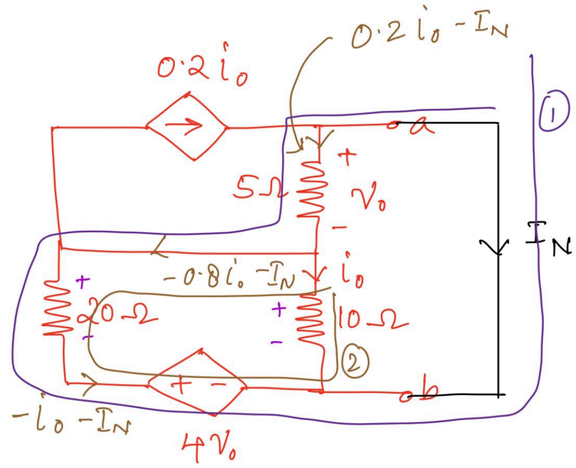
$$\boxed{V_o = 9V}$$

$$i_o = i_o' + i_o'' = 0.8 + 0.1 = 0.9A$$

$$\boxed{i_o = 0.9A}$$

Solⁿ 2[b]:

Calculation of I_N:



Apply KVL in loop ①;

$$-V_o - 20(-i_o - I_N) - 4V_o = 0$$

$$5V_o + 20i_o + 20I_N = 0 \quad \text{——— (1)}$$

$$V_o = 5(0.2i_o - I_N) \quad \text{——— (2)}$$

from eqⁿ (1) & (2);

$$25(0.2i_o - I_N) + 20i_o + 20I_N = 0$$

$$5i_o - 25I_N + 20i_o + 20I_N = 0$$

$$25i_o - 5I_N = 0$$

$$5i_o = I_N \quad \text{——— (3)}$$

Apply KVL in loop(2);

$$-20(-0.8i_o - I_N) - 4V_o + 10i_o = 0$$

$$16i_o + 20I_N - 4V_o + 10i_o = 0$$

$$26i_o + 20I_N - 20(0.2i_o - I_N) = 0$$

$$26i_o - 4i_o + 20I_N + 20I_N = 0$$

$$22i_o + 40I_N = 0$$

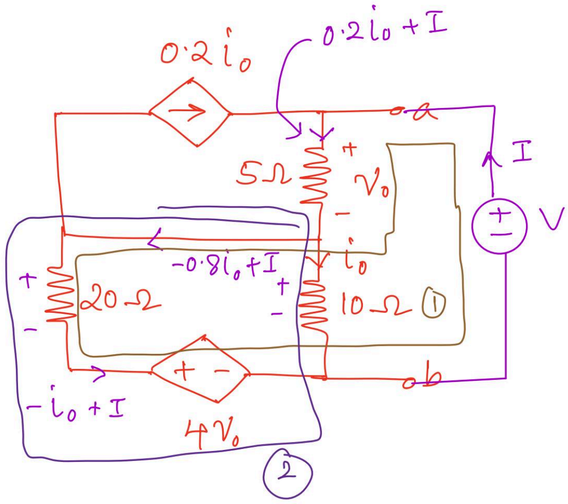
$$11i_o + 20I_N = 0$$

$$11 i_o + 100 i_o = 0$$

$$i_o = 0$$

$$\boxed{I_N = 0\ A}$$

Calculation of R_N:



Apply KVL in loop ①;

$$V - 5(0.2 i_o + I) - 20(-i_o + I)$$
$$-4 \times 5(0.2 i_o + I) = 0$$

$$V - i_o - 5I + 20 i_o - 20 I$$
$$- 4 i_o - 20 I = 0$$

$$V + 15 i_o - 45 I = 0 \quad \text{——} \quad (1)$$

Apply KVL in loop ②;

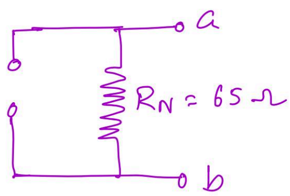
$$-20(-i_o + I) - 4 \times 5(0.2 i_o + I) + 10 i_o = 0$$

$$20 i_o - 20 I - 4 i_o - 20 I + 10 i_o = 0$$

$$26 i_o = 40 I$$

$$i_o = \frac{40}{26} I \quad \text{——} \quad (2)$$

From Eqⁿ(1) & (2);



$$V + 15 \times \frac{40}{26} I - 45 I = 0$$

$$\frac{V}{I} = 45 - \frac{600}{26} = \frac{570}{26}$$

Norton's Equivalent circuit

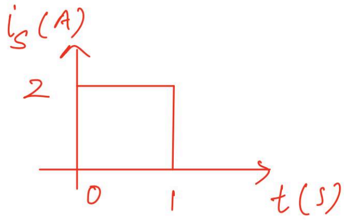
$$\frac{V}{I} = \frac{285}{13}$$

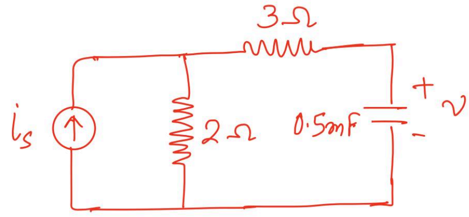
$$\boxed{R_N = \frac{V}{I} = 21.92\ \Omega}$$

Sol^n 3[a]:

Data Given:

$v(0) = 0$





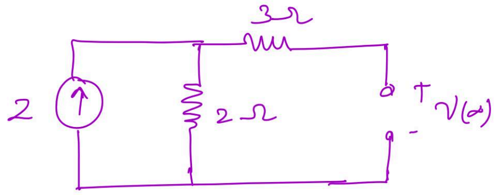
for $0 < t < 1$

at $t = 0^+$

$v(0^+) = v(0^-) = v(0) = 0\,V$

at $t \to \infty$

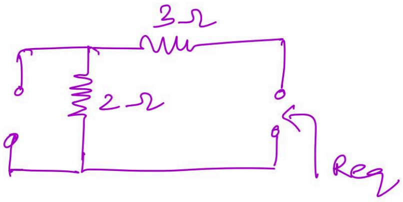
$v(\infty) = 2 \times 2 = 4\,V$



time constant $\tau = R_{eq} C$

$R_{eq} = 3 + 2 = 5\,\Omega$

$\tau = 5 \times 0.5m = 2.5\,ms$



Voltage across capacitor;

$$v(t) = v(\infty) + [v(0^-) - v(\infty)]\, e^{-t/\tau}$$

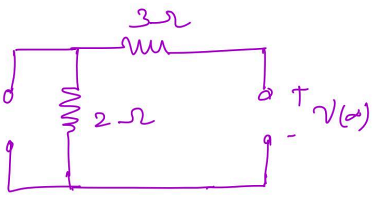
$$v(t) = 4 + (0 - 4)\, e^{-t/2.5m}$$

$$v(t) = 4(1 - e^{-400t}) \text{ volt}$$

for $t > 1$

at $t = 1^+$

$$v(1^+) = v(1^-) = v(1) = 4(1 - e^{-400}) = 4\,V$$



$v(\infty) = 0$

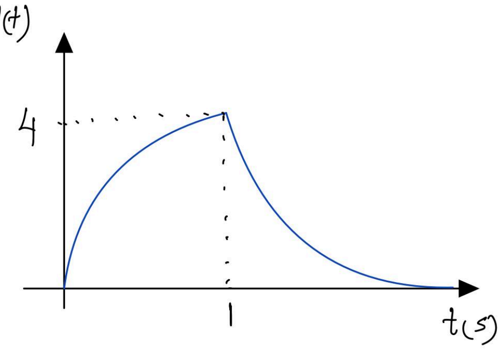
$\tau = R_{eq} C = 5 \times 0.5 = 2.5\,ms$

Voltage across capacitor,

$$v(t) = v(\infty) + [v(1) - v(\infty)]\, e^{-(t-1)/\tau}$$

$$v(t) = 0 + (4 - 0)\, e^{-(t-1)/2.5m} \text{ Volt}$$

$$v(t) = 4\,e^{-400(t-1)} \text{ Volt}$$



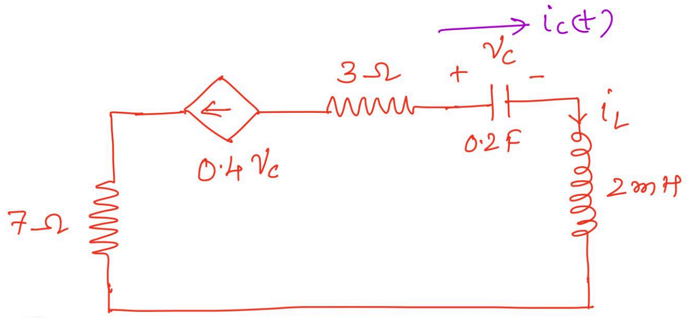
$$\boxed{v(t) = \begin{cases} 4\left(1-e^{-400t}\right), & 0<t<1 \\ 4\,e^{-400(t-1)}, & t>1 \end{cases}}$$

Solⁿ 3[b]:

Data Given:

$v_C(0^-) = 2\,V$

$i_L(0^-) = 0\,A$



$V_c(0^+) = V_c(0^-) = V_C(0) = 2\,V$

$i_L(0^+) = i_L(0^-) = i_L(0) = 0\,A$

for $t>0$

$$i_L(t) = i_C(t) = C\frac{dV_c(t)}{dt} = -0.4\,V_C \qquad \text{— (1)}$$

$$0.2\,\frac{dV_C(t)}{dt} = -0.4\,V_C$$

$$\frac{dV_C(t)}{dt} + 2\,V_C(t) = 0$$

$$dV_C(t) = -2\,V_C(t)\,dt$$

$$\int_2^{V_C(t)} \frac{dV_C(t)}{V_C(t)} = -\int_{t=0}^{t} 2\,dt$$

$$\left[\ln V_C(t)\right]_2^{V_C(t)} = -2\left[t\right]_0^t = -2t$$

$$\ln\left(\frac{V_C(t)}{2}\right) = -2t$$

$$V_c(t) = 2e^{-2t}, \quad t > 0$$

from Eqⁿ (1);

$$i_L(t) = C\frac{dV_c(t)}{dt} = 0.2 \times \frac{d}{dt}(2e^{-2t})$$

$$i_L(t) = 0.2 \times 2(-2)e^{-2t}$$

$$\boxed{i_L(t) = -0.8e^{-2t}\ A, \quad t > 0}$$

Solⁿ 4: Data Given:

$V_s = 36.3\ V$

$R_a = 2K\Omega$

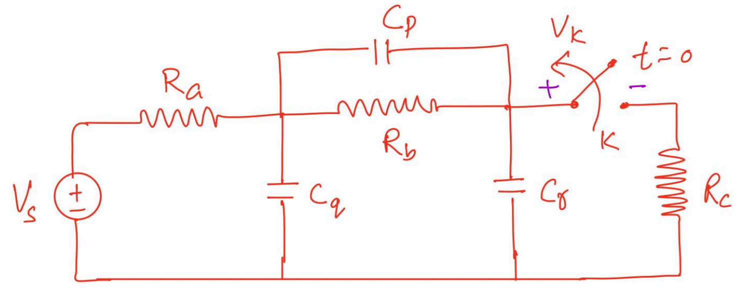
$R_b = 3K\Omega$

$R_c = 6K\Omega$

$C_p = 1\mu F$

$C_q = 2\mu F$

$C_r = 3\mu F$



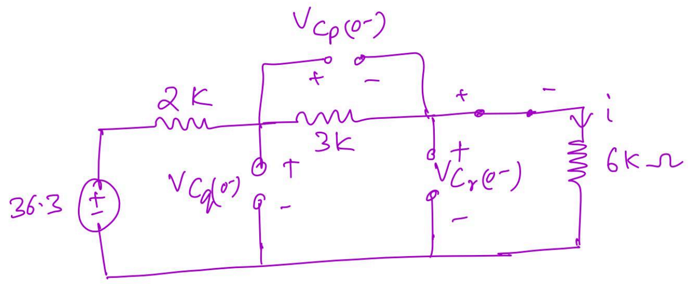
for $t < 0$

$$i = \frac{36.3}{11k} = 3.3\,mA$$

$V_{Cp}(0^-) = i \times 3k = 9.9\ V$

$V_{Cr}(0^-) = i \times 6k = 19.8\ V$

$V_{Cq}(0^-) = 36.3 - i \times 2k = 36.3 - 6.6 = 29.7\ V$

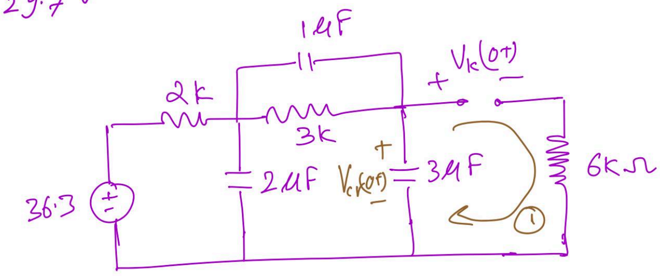


for $t > 0$

$V_{cp}(0^+) = V_{cp}(0^-) = V_{cp}(0) = 9.9V$

$V_{cr}(0^+) = V_{cr}(0^-) = V_{cr}(0) = 19.8V$

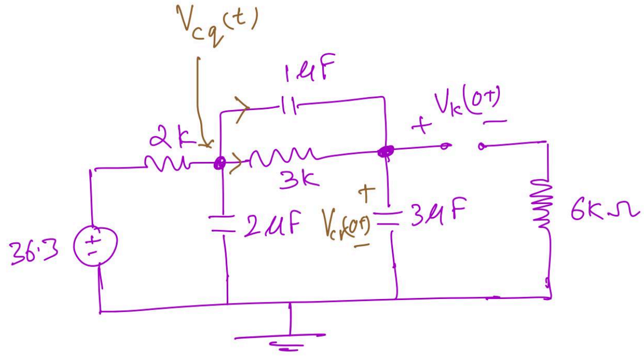
$V_{cq}(0^+) = V_{cq}(0^-) = V_{cq}(0) = 29.7\ V$



Apply KVL in loop ①;

$$-V_K(0^+) + V_{Cr}(0^+) = 0$$

$$\boxed{V_K(0^+) = V_{Cr}(0^+) = 19.8\,V}$$



Using Nodal Analysis;

KCL at node $V_{cq}(t)$;

$$\frac{V_{cq}(t) - 36.3}{2k} + 2\mu \frac{dV_{cq}(t)}{dt} + \frac{V_{cq}(t) - V_K(t)}{3k} + 1\mu \frac{d[V_{cq}(t) - V_K(t)]}{dt} = 0$$

$$1\mu \frac{dV_{cq}(t)}{dt} + 2\mu \frac{dV_{cq}(t)}{dt} = -\left[\frac{V_{cq}(t) - 36.3}{2k} + \frac{V_{cq}(t) - V_K(t)}{3k} - 1\mu \frac{dV_K(t)}{dt}\right]$$

$$\frac{dV_{cq}(t)}{dt} = \frac{-1}{3\mu}\left[\frac{V_{cq}(t) - 36.3}{2k} + \frac{V_{cq}(t) - V_K(t)}{3k} - 1\mu \frac{dV_K(t)}{dt}\right] \quad \text{—(1)}$$

KCL at node $V_K(t)$;

$$1\mu \frac{d[V_K(t) - V_{cq}(t)]}{dt} + \frac{V_K(t) - V_{cq}(t)}{3k} + 3\mu \frac{dV_K(t)}{dt} = 0$$

$$1\mu \frac{dV_K(t)}{dt} - 1\mu \frac{dV_{cq}(t)}{dt} + \frac{V_K(t) - V_{cq}(t)}{3k} + 3\mu \frac{dV_K(t)}{dt} = 0$$

$$1\mu \frac{dV_K(t)}{dt} + \frac{1}{3}\left[\frac{V_{cq}(t) - 36.3}{2k} + \frac{V_{cq}(t) - V_K(t)}{3k} - \mu \frac{dV_K(t)}{dt}\right] + \frac{V_K(t) - V_{cq}(t)}{3k}$$

$$+ 3\mu \frac{dV_K(t)}{dt} = 0$$

$$\left(4 - \frac{1}{3}\right)\mu \frac{dV_K(t)}{dt} + \frac{1}{3}\left[\frac{V_{cq}(t) - 36.3}{2k} + \frac{V_{cq}(t) - V_K(t)}{3k}\right] + \frac{V_K(t) - V_{cq}(t)}{3k} = 0$$

at $t = 0^+$

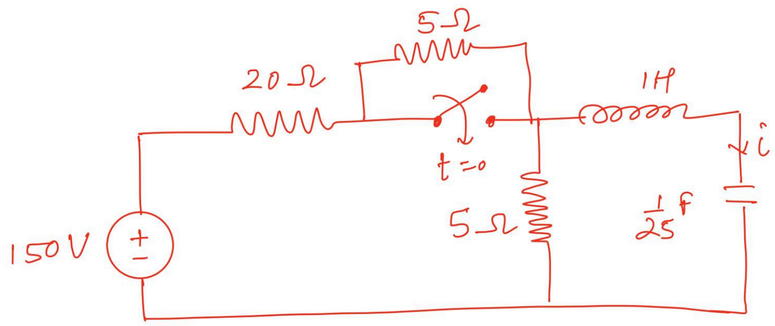
$$\frac{11}{3}\mu \frac{dV_K(0^+)}{dt} + \frac{1}{3}\left[\frac{V_{cq}(0^+) - 36.3}{2k} + \frac{V_{cq}(0^+) - V_K(0^+)}{3k}\right] + \frac{V_K(0^+) - V_{cq}(0^+)}{3k} = 0$$

$$\frac{11}{3}\mu \frac{dV_K(0^+)}{dt} + \frac{1}{3}\left[\frac{29.7 - 36.3}{2k} + \frac{29.7 - 19.8}{3k}\right] + \frac{19.8 - 29.7}{3k} = 0$$

$$\frac{11}{3}\mu \frac{dV_k(0^+)}{dt} + \frac{1}{3}\left[-\cancel{\frac{3.3}{k}} + \cancel{\frac{3.3}{k}}\right] - \frac{3.3}{k} = 0$$

$$\boxed{\frac{dV_k(0^+)}{dt} = \frac{3.3\times 3}{11\ k\ \mu} = 900\ V/s}$$

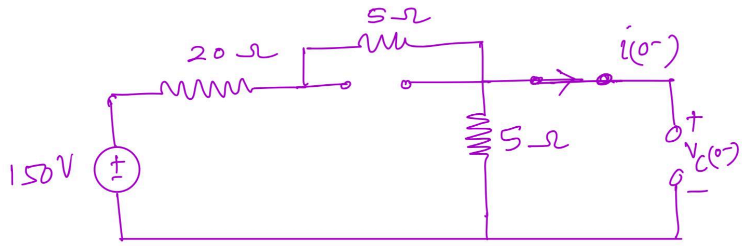
Sol^n 5:



for $t<0$

$i(0^-) = 0\ A$

$V_c(0^-) = \frac{150\times 5}{30} = 25\ V$

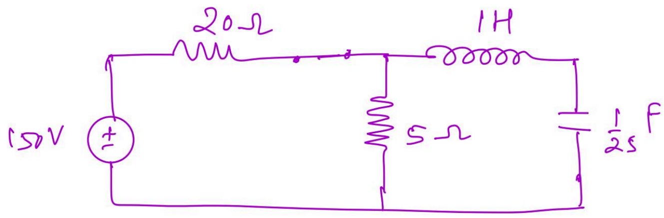


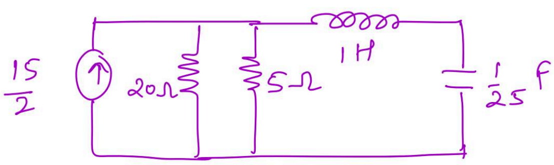
for $t>0$

$i(0^+) = i(0^-) = i(0) = 0\ A$

$V_c(0^+) = V_c(0^-) = V_c(0) = 25\ V$

Using source transformation;





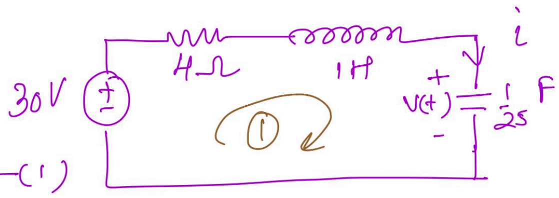
a.) Apply KVL in loop ①;

$$30 - 4i - 1\frac{di}{dt} - \frac{1}{1/25}\int_0^t i\,dt = 0$$

differentiate w.r.t t;

$$0 - 4\frac{di}{dt} - \frac{d^2i}{dt^2} - 25i = 0$$

$$\boxed{\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 25i = 0} \quad \text{—— (1)}$$



Eqⁿ (1) is the 2nd order differential Equation.

Substitute $i = ke^{st}$

$$Ks^2e^{st} + 4Kse^{st} + 25ke^{st} = 0$$

$$Ke^{st}(s^2 + 4s + 25) = 0$$

$$\boxed{s^2 + 4s + 25 = 0}$$ characteristics Equation

b.) roots of characteristics Equation;

$$S_{1,2} = \frac{-4 \pm \sqrt{16-100}}{2\times1}$$

$$S_{1,2} = -2 \pm j4.583$$

$$\boxed{\begin{aligned} S_1 &= -2 + j4.583 \\ S_2 &= -2 - j4.583 \end{aligned}}$$ roots of characteristics Eqⁿ

$\alpha = 2$, $\omega d = 4.583$

∵ roots are complex conjugate.

∴ response is underdamped in nature.

c.) $V(t) = V_{ss}(t) + V_{tr}(t)$

$V_{ss}(t) = V(t)|_{t\to\infty} = 30V$

$V_{tr}(t) = e^{-\alpha t}[A\cos\omega dt + B\sin\omega dt]$

$V(t) = 30 + e^{-2t}[A\cos 4.583t + B\sin 4.583t]$ ——— (1)

at $t=0$, $V(0) = 25V$

$V(0) = 30 + e^{-2\times0}[A\cos 0 + B\sin 0] = 25$

$$\boxed{A = -5}$$

at $t=0$, $i(0) = 0$

$$i(t) = C\frac{dV(t)}{dt}$$

$$0 = i(0) = \frac{1}{25}\Big[e^{-2t}\left(-4.583 A \sin 4.583t + 4.583 B \cos 4.583t\right)$$
$$+ \left(A\cos 4.583t + B\sin 4.583t\right) \times \left(-2e^{-2t}\right)\Big]_{\text{at } t=0}$$

$$0 = e^{0}\left[0 + 4.583 B + (A + 0)(-2)e^{0}\right]$$

$$0 = 4.583 B - 2A$$

$$B = \frac{2A}{4.583} = \frac{2\times(-5)}{4.583} = 2.182, \quad \boxed{B = 2.182}$$

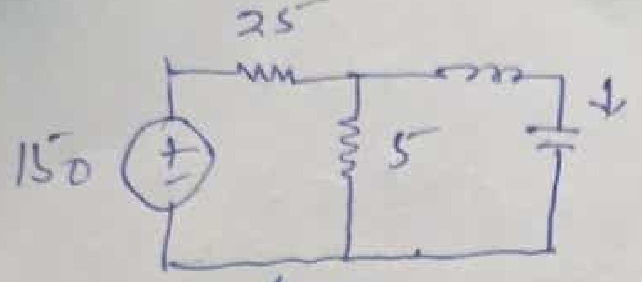
From Eq^n(1);

$$V(t) = 30 + e^{-2t}\left[-5\cos 4.583t + 2.182 \sin 4.583t\right]$$

$$i(t) = C\frac{dV(t)}{dt} = \frac{1}{25}\Big[e^{-2t}\left(-5\times 4.583 \sin 4.583t + 2.182\times 4.583 \cos 4.583t\right)$$
$$+ \left(-5\cos 4.583t + 2.182 \sin 4.583t\right)\left(-2e^{-2t}\right)\Big]$$

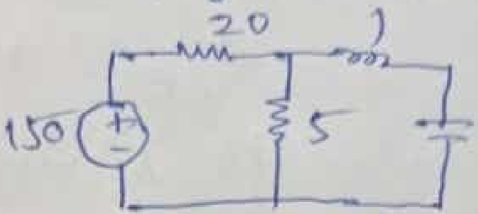
$$i(t) = \frac{1}{25}e^{-2t}\Big[-22.91 \sin 4.583t + 10\cos 4.583t + 10\cos 4.583t$$
$$- 4.364 \sin 4.583t\Big]$$

$$\boxed{i(t) = \frac{1}{25}\left[-27.27 \sin 4.583t + 20\cos 4.583t\right]e^{-2t} \ A}$$

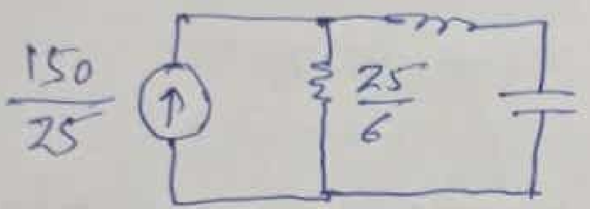


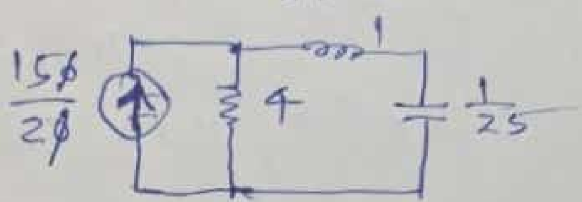
$t=0^-$

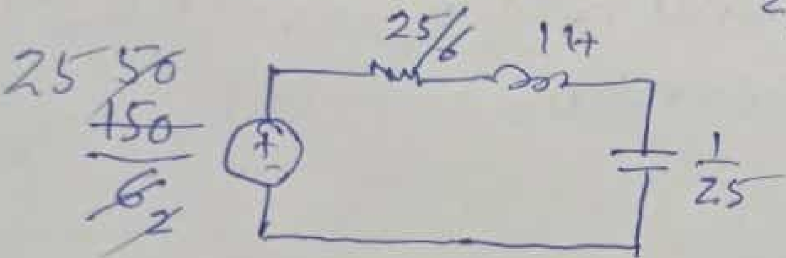
$\frac{25\times\cancel{5}}{\cancel{30}_6}=\frac{25}{6}$ $\frac{20\times\cancel{5}}{\cancel{25}_5}=4$

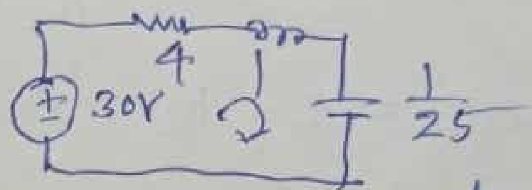


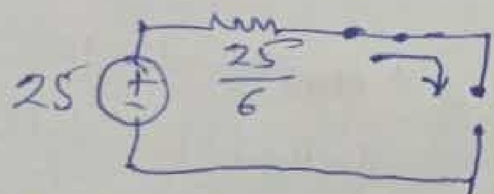
$t>0$











$4i+\frac{di}{dt}+25\int_{-\infty}^{t} i\,dt=30$

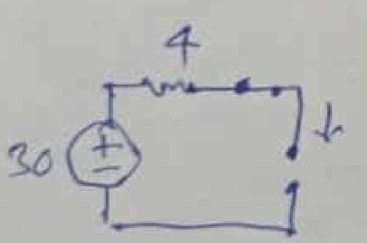
$4i+\frac{di}{dt}+25\int_{-\infty}^{0} i\,dt+25\int_{0}^{t} i\,dt=30$

$i(0^+)=i(0^-)=0$

$v_c(0^+)=v_c(0^-)=25$

$\frac{di}{dt}+4i+25+25\int_0^t i\,dt=30$

$\boxed{\frac{di}{dt}+4i+25\int_0^t i\,dt=5}$



$\therefore \frac{di}{dt}(0^+)+4i(0^+)=5$

$i(\infty)=0$

$v_c(\infty)=30$

$\frac{di}{dt}(0^+)=5$

$\frac{d^2i}{dt^2}+4\frac{di}{dt}+25i=0$

$s^2+4s+25=0$

$s_1,s_2=\frac{-4\pm\sqrt{16-4\times25}}{2}$

$s_1,s_2=-2\pm\frac{2\sqrt{-21}}{2}=-2\pm j\sqrt{21}$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= A_1 e^{(-2+j\sqrt{21})t} + A_2 e^{(-2-j\sqrt{21})t}$$
$$= A_1 e^{-2t} \cdot e^{j\sqrt{21}t} + A_2 e^{-2t} \cdot e^{-j\sqrt{3}t}$$
$$= e^{-2t}\left\{A_1 e^{j\sqrt{21}t} + A_2 e^{-j\sqrt{21}t}\right\}$$
$$= e^{-2t}\left\{A_1(\cos\sqrt{21}t + j\sin\sqrt{21}t) + A_2(\cos\sqrt{21}t - j\sin\sqrt{21}t)\right\}$$
$$= e^{-2t}\left\{(A_1+A_2)\cos\sqrt{21}t + j(A_1-A_2)\sin\sqrt{21}t\right\}$$
$$= e^{-2t}\left\{A_3\cos\sqrt{21}t + A_4\sin\sqrt{21}t\right\}$$
$$0 = i(0^+) = 1\left\{A_3 + 0\right\}$$
$$A_3 = 0$$
$$\frac{di(t)}{dt} = (-2)e^{-2t}\left\{A_3\cos\sqrt{21}t + A_4\sin\sqrt{21}t\right\} + e^{-2t}\left\{-A_3\sqrt{21}\sin\sqrt{21} + A_4\sqrt{21}\cos\sqrt{21}t\right\}$$
$$5 = \frac{di(0^+)}{dt} = (-2)\left\{A_3 + 0\right\} + 1\left\{0 + A_4\sqrt{21}\right\}$$
$$5 = -2A_3 + A_4\sqrt{21}$$
$$A_4 = \frac{5}{\sqrt{21}}$$
$$\therefore \boxed{i(t) = e^{-2t} \times \frac{5}{\sqrt{21}}\sin\sqrt{21}t}$$

Applying Laplace.

$$\left\{sI(s) - i(0^+)\right\} + 4I(s) + 25\frac{I(s)}{s} = \frac{5}{s}$$
$$sI(s) + 4I(s) + \frac{25I(s)}{s} = \frac{5}{s}$$
$$\frac{\left\{s^2 + 4s + 25\right\}I(s)}{s} = \frac{5}{s}$$

$$I(s) = \frac{5}{(s^2+4s+25)} = \frac{5}{s^2+4s+4+21}$$

$$I(s) = \frac{5\sqrt{21}}{\sqrt{21}\{(s+2)^2+(\sqrt{21})^2\}}$$

$$i(t) = \frac{5}{\sqrt{21}} e^{-2t} \sin\sqrt{21}\, t.$$