DEPARTMENT OF COMPUTER SCIENCE

The Emergence of Extreme Opinions from Social Tendencies
A Heterogeneous Opinion Dynamics Model

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Abstract

The extreme opinions of individuals cause the emergence of many problems in our society such as racism, sexism, political brawls and so on. The Opinion Dynamics models can be helpful to investigate how extreme ideas emerge in society by simulating the opinion and interaction formations of individuals by utilising networks. The emergence of extreme opinions of people can be studied and understood from the perspective of different characteristics of individuals in society. In this project, a new Opinion Dynamics model is implemented by replicating one of the most recent models and extending it to investigate the effect of heterogenous social tendencies of individuals in our society. The new model is a computational adaptive social network where the nodes in the network have opinion and social tendency attributes including conformity, homophily and attention to novelty. The edges between the nodes represent the weighted interactions of individuals. The network evolves over time by changing the edge weights and opinions based on the mathematical update rules. The results suggested that the conformity of individuals prevents extremism in the network by urging them to conform to the opinions of their neighbours. Whereas, when individuals seek sameness with both their opinions and social tendencies they tend to strengthen their connections with like-minded others who also seek sameness while losing connections with distinct others.

Dedication and Acknowledgements

I would like to express my gratitude to my supervisor, Professor Dr. Seth Bullock, for his continuous support and patience during this process. Without his guidance and assistance, I would not have been able to complete my dissertation. Additionally, I would like to extend my heartfelt thanks to my family and friends for their ongoing encouragement and care for my well-being throughout this journey.

Declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Taught Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, this work is my own work. Work done in collaboration with, or with the assistance of others, is indicated as such. I have identified all material in this dissertation which is not my own work through appropriate referencing and acknowledgement. Where I have quoted or otherwise incorporated material which is the work of others, I have included the source in the references. Any views expressed in the dissertation, other than referenced material, are those of the author.

Nisa Bayraktar, Sunday 14$^{\text{th}}$ January, 2024

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Ethics Statement

This project did not require ethical review, as determined by my supervisor, Professor Dr. Seth Bullock.

Chapter 1

Introduction

In today's social world, many problems such as racism and political brawls emerge due to the extreme opinions of people which causes society to divide into polarised groups [1, 2]. Nowadays, people's beliefs and their interactions are studied in the field of *social psychology* [3]. Social psychology emphasises both psychological and sociological concepts such as *social influence*, *phenomena* and *norms* [4]. These concepts investigate the interactions of people based on their influences on each other and their responses to the interactions [5]. However, research in social psychology mostly relied on theoretical and non-experimental objectives [6]. Therefore, many fields emerged to assess social science research from scientists by utilising natural science fields such as mathematics and psychics where experimental methods are utilised [7,8].

One of the scientific fields that emerged towards this direction is *opinion dynamics* (OD). OD investigates people's interactions and opinions based on the information exchange of individuals by utilising *social networks* [9]. Social networks are often mixed up with the nowadays online social networks commonly known as social media. However, in the field of OD, they refer to the networks that simulate the interactions and opinions of individuals [10]. In these networks, the nodes represent individuals named as agents and the edges refer to the interactions of the agents with their neighbours. To investigate the opinion and interaction formation, each node has its own idea as a numerical value called the *opinion state* and the edges can be weighted to determine the strength of the connections.

From the emergence of the OD field to now many network models have been proposed by scientists [11]. Nowadays, OD models utilise computational social networks where algorithms are used to investigate the network formation and aid the analyses of the network topology [12]. Recently, a type of computational social network called the adaptive social network widely utilised in OD research where the opinion of individuals and the structure of edges between them which demonstrate their social interactions co-evolve simultaneously [13]. This ability of the adaptive social network helps to investigate the process of opinion formation more precisely between individuals by dynamically changing their interactions and opinions. So far, the researchers that utilised adaptive social networks to investigate the emergence of extreme opinions, assume that extremism occurs as a result of specific causes and triggers hence, they are limited to assessing extremism from different perspectives [14]. However, recent research suggests that extreme ideas can arise spontaneously when individuals do not intentionally seek extreme ideas [14]. Therefore, a new adaptive social network model is proposed by Hiroki Sayama at the "*ALIFE 2020: The 2020 Conference on Artificial Life*" to study spontaneous dynamics of the emergence of extreme ideas [14]. The Sayama model assumes that individuals in the network have no intention to go extreme but they try to conform to ideas in their local neighbourhood.

In the Sayama Model, in addition to the opinion state of the agents, the concept of social tendencies is also utilised where individuals have social characteristics such as *conformity*, *homophily* and *attention to novelty* [14]. Conformity determines the tendency of agents to conform to the ideas of their neighbours. Whereas, homophily and attention to novelty refer to the edge weight change of the interactions between the agents. The agents that seek sameness with their homophily parameter strengthen their connection with like-minded others while the ones that seek difference with their attention to novelty parameter strengthen their connections with distinct others in the network. The effect of these social mechanisms on opinion formation is studied by utilising mathematical equations. The analysis of how they influence the interactions and opinions in the network is conducted with statistical methods such as regression types. The network also investigates the network science concept of community where communities refer to the clusters of individuals with similar others which can be attributed to the "*social bubbles*" in our society [1,15]. Therefore, this concept helps to understand the fragmentation in the network. The overall observation made from the Sayama model is that homophily and attention to novelty have the most significant impact on network formation with opposite directions where the former promotes extremism and fragmentation in the network, while the latter prevents them.

Even though the Sayama model offers an advanced social dynamics perspective it is still limited to asses the opinion formation close to the real-world interactions. The Sayama model assumes that everyone in the network has the same social tendencies. Therefore, the analyses of the network evolution remain at the network level where these parameters are global in the network. However, in our world people have unique characteristics and personalities [16,17].

In this research, a new model is developed to investigate the heterogenous social tendencies of individuals by utilising the Sayama model as a base network and modifying the model in a way that individuals have their unique social characteristic values. The new model is constructed by first replicating the original model. This process is then followed by an analysis of the replicated model with the regression methods used in the analysis of the original model. The regression methods are applied to the networks generated from the replicated model with the identical experimental setups used in the Sayama model. The replicated model results are validated by the original results in Sayama's paper to make sure that the model works correctly. Finally, the new model is built on the replicated model by adjusting the model parameters in a way that the individuals have their own social tendency values. To analyse the networks generated by the new model the node-level and network-level data are gathered where they represent the values of the individual nodes and their combined values in the network respectively. The node-level analyses are conducted from the perspective of individual nodes where their behaviours are investigated based on their unique social tendencies by utilising multi-linear regression which could not assess in the Sayama model due to their same social characteristic values. The network-level analysis is conducted by utilising a network science concept called *assortativity* which asses the homogeneity and heterogeneity of the network based on the correlation of nodes' unique parameter values across the network [18].

In summary, the main aim of this project is to investigate how extreme opinions arise from the perspective of unique social tendencies of individuals where they do not intentionally go extreme. More specifically, the concrete objectives are as follows:

- The deep understanding of the most recent Opinion Dynamics model by Hiroki Sayama.
- Replication of the Sayama Model to be used as a based model in this project.
- Implementing a new model to investigate the heterogenous characteristics of individuals in our society by modifying the Sayama model.
- Analysing the new model using regression techniques and network science concepts.
- Reflecting the observations made from the new model to the real world.

Chapter 2

Literature Review

This chapter offers a wide perspective of the pinion dynamics field, covering its historical roots and recent advancements in the literature. Initially, the field of social sciences and its foundational background is explored, followed by the evolution of this field and the emergence of new disciplines. The chapter then delves into the core concepts of the opinion dynamics field, including an overview of the earliest models to the most recent models. Lastly, the Sayama model utilised in this research is introduced, providing an explanation of its basic structure and objectives.

2.1 The field of Social Sciences

In the 14th century, the term science referred to *natural sciences* including physics, chemistry and mathematics, which aimed to study the physical world through experimental means [19]. Nevertheless, the social world, encompassing human beings and society, was also extensively studied by philosophers under the trend of *positivism* [20]. Positivism was committed to explaining and understanding the social world with natural science methods [21]. However, in the late 18th century, arguments against science surfaced with the emergence of new foundations and developments in natural science due to the scientific revolution, such as Newton's Physics, which caused discrepancies to emerge in the field of science [22]. This, in turn, resulted in the emergence of new science branches, such as *social sciences* [23].

In the early 19th century, the positivist philosopher Auguste Comte aimed to develop a science of society, giving rise to the emergence of *social sciences* as a scientific field of study for comprehending the concepts of society and humanity [24]. Nowadays, academic disciplines, such as psychology, economics, politics, and sociology, are studied under the umbrella of social sciences. The concept of social sciences entails understanding society and humanity in a way that other science branches alone cannot assess through non-experimental methods [25]. However, social sciences still utilise natural sciences branches in their studies [7, 8, 26].

The study of human behaviours and beliefs has been the focus of philosophers since ancient Greece [27]. With the emergence of the social science field in the 19th century, social scientists began studying these concepts under sociology and psychology. Sociology refers to the scientific field that focuses on society, groups, and social interactions, whereas psychology studies individuals in society based on human behaviours in the social world [28].

2.1.1 Social Pyschology

In 1868, the German psychologist Wilhelm Wundt proposed the term *social psychology* as a new branch situated between psychology and sociology [28]. This field aims to explore how individuals' thoughts and behaviours affect their interactions within society [29]. Social psychology employs various research methods, including data analysis techniques on data generated in laboratory experiments, which align with natural science methodologies [30]. Additionally, it incorporates *social cognition*, which is a non-experimental method that seeks to understand how individuals respond to social information in group interactions or in one-on-one relationships [31, 32].

2.1.2 Computational Social Sciences

During the first quarter of the 20th century, technological advancements throughout the years led to the emergence of the *computational social sciences* (CSS) field [33, 34]. Initially, this field was described as agent-based modelling by using computer power such as algorithms and software tools to simulate human behaviour in an artificial environment. However, with the increasing number of digital data and computational analysis techniques, the field's definition has evolved significantly. In recent research, the CSS has been defined as a means of advancing our understanding of human behaviour [34]. It is no longer restricted to merely mirroring human behaviour using novel data, but it also entails the generation of new theories and explanations of human behaviour [34]. The availability of massive amounts of digital data and the development of new analysis methods have enabled the CSS to advance existing social science disciplines such as social psychology, as well as paved the way for the emergence of new fields such as *social networks* [10].

2.2 Social Networks

The term *social networks* was first introduced by scientist Barnes and defined as structures that mimic the relationships between individuals [35]. This concept is mostly mixed up with online social networks or commonly known as social media, however, in the field of CSS, these networks refer to simulations that aid in the analysis of human behaviours based on their interactions [10]. These simulations are created by leveraging the mathematical science field, graph theory, which is why social network research is considered a part of the natural sciences [36]. The concept of graphs, also known as networks, was introduced by the Greek mathematician Euclid in the 6th century B.C. Graphs consist of sets of vertices or nodes and edges, representing the connections between them [37]. Since then, graphs have been used in various areas including social networks. With the emergence of CSS, social networks became a broad field with many sub-fileds and network types including *computational social networks* where the computational power such as algorithms are utilised [12].

In social networks, graph nodes represent individuals in the social world, while edges refer to the relationships between these individuals. Understanding the structure of the social network is critical to obtaining insights into social properties and processes [38]. However, empirical data analysis and dealing with theoretical issues can prove challenging for social scientists to make logical sense of the social world. Therefore, the field of *social network analysis* (SNA) has emerged to address this need in social network research by employing mathematical, statistical, and computational methods [38, 39].

The fundamentals of SNA are rooted in social psychology, despite both fields sharing similar subject areas and utilising similar data analytical methods in their research, they diverge in their research objectives and focus. SNA primarily seeks to comprehend the nature of social interactions between individuals, through the systematic elements of human sociology [40, 41]. The data utilised in SNA is mostly sourced from sociological questionnaires and survey data. In contrast, social psychology research concentrates on individual perceptions and behaviours instead of their interactions with others using experimental data acquired from laboratory experiments. Despite their differences, both fields intersect and complement each other. The relationship between the two fields can be further understood with the examples provided by researchers [39]:

Considering a group of people trying to make a decision and achieve a consensus, such as juries in a competition aiming to decide the winner. Traditional social psychology studies and analyse this process by focusing on the individual's traits and the resulting decision. However, this is a limited approach because it overlooks the crucial role of the influence of peers on each other when shaping the outcome in the decision-making process [39]. To gain a more complete understanding by examining the influence of members, an SNA perspective can be utilised. This approach investigates the decision-making process by focusing on the interactions among group members instead of only relying on individual attributes [39].

Moreover, SNA can also be used to examine the interactions within a group over time. For example, economists and financiers can utilise SNA to understand the stock market trends by looking at the transactions of individuals at various points in time, which allows them to study the evolution of the global economic system [39].

In essence, the SNA perspective emphasises the importance of the influence of peers on each other, their structural relationships and the evolution of their interactions over time. Thus, SNA offers a flexible set of concepts and methods as structures can appear in various forms, such as behavioural, social, political, or economic [39].

The emergence of CSS has significantly broadened the scope of network analysis. Alongside these developments, new network methodologies have arisen. One of these recent areas, *opinion dynamics* (OD), has provided a more precise understanding of the evolution of human behaviour through *social influence, phenomena and norms*, by advancing the analysis of social networks [4, 11, 42]. According to social psychology, human behaviour pertains to individual interactions in the social world [3, 43]. These connections forms based on the influence of people on each other known as a *social influence*, which then causes an action as a response to the interaction, called *social phenomena* [44]. As a result of these social dynamics, implicit social rules or *social norms*, such as common courtesy occurred in society [45].
SNA and OD diverge based on their approaches used in the field of CSS to analyse social structures and dynamics. SNA focuses on analysing the network structure and interaction patterns of individuals, while OD simulates information spread and social influence to identify factors contributing to changes in opinions over time.

The main difference between the two is that SNA is primarily data-driven, where the network is built to reflect data collected from the real world, and its structure is analysed to understand social patterns [38]. In contrast, OD is typically theoretically driven, where ideas about how people interact are captured in a model, and the model is analysed to see what these ideas imply [36]. Despite their distinct approaches, SNA and OD can be complementary and help each other make progress in understanding social dynamics.

The impact of *social influence* in various disciplines, such as politics, economy, and psychology, has become a popular subject of study by utilising OD [46]. The reason behind this was the empirical data problem which is not a concern in the OD as it is rooted in theoretical assumptions that guide the creation of experiments aimed at studying how people interact and how information spreads.

2.3 Opinion Dynamics

Understanding *social influence* is a crucial factor in comprehending human behaviour and decision-making, particularly within the field of psychology. During the 19th century, philosophers and scientists were seeking depth and qualitative understanding of individual behaviour and social influence on the emergence of regularities [11, 47, 48].

Recent OD research helps to investigate *social influence* by utilising adaptive social networks as an environment to construct *social phenomena* [49]. Unlike basic social networks, adaptive social networks have the ability to assess the dynamics of the network structure. They provide a dynamic structure to explore opinion formation in the network where the opinions and interactions co-evolve simultaneously [13, 50]. In contrast, traditional social networks assume that the social environment also known as the social context within the network is fixed, and connections do not evolve over time. OD models take advantage of computer science practices such as algorithms, therefore, there are various models within this field.

OD is a relatively new area of research, and developments in this field are ongoing. In this section, a discussion of the opinion dynamics concept from the history behind it and previous research is provided. Then the models of the opinion dynamics network are explained and the most recent research is introduced.

2.3.1 Concept

Recently, *social influence* became a popular topic in the field of complex systems [11, 42]. Society and the human brain are examples of complex systems that are difficult to study due to containing a high number of interactions, dependencies and elements.

Opinions surround us in various domains and play a significant role in addressing the current world problems that we are facing such as climate change, pandemics, migration, political elections and financial crises [42]. Thus, understanding the spread and formation of opinions is critical to investigate human behaviour and how our actions are influenced to emerge these such challenges in the social world. Opinion evaluation is affected by many factors including individuals' characteristics or information they are exposed to, and the positive and negative interactions of these individuals [11]. This process evolves over time by being exposed to new information and interactions with different opinions. Over the last decade, the answers to how opinions evaluate, reach a consensus, or polarise, leading to fragmented groups in society have been searched and scientists have been trying to measure and model human behaviour [38]. The field of OD emerged to aid the understanding of the opinion and interaction formations in society by utilising various social network models [11].

In OD models the network components consist of agents and their interactions [11]. These network structures are called agent-based models where the individuals, referred to as agents, share information and opinions to form interactions with their neighbours in a social network. Each agent holds opinion states that reflect their individual opinions, and as OD models allow for a dynamic structure, these opinion states evolve over time, influencing both the network structure also known as topology and individual opinions.

The social influence between agents occurs through bidirectional interactions, where information and opinions are exchanged. Positive interactions can strengthen connections between agents, whereas negative interactions can lead to the loss of connections. This interaction process results in changes to the topology of the network and individuals' opinion states, potentially leading to consensus or polarisation of opinions. Without the need for empirical data, OD models provide valuable insight into the complex social systems involved in opinion spread, information exchange, and group formation, surpassing the capabilities of social networks alone [42].

OD models have been improved over the years, with initial models primarily utilising basic social networks by using only physical and mathematical principles to describe the process of opinion exchange [42]. Recently, computational and adaptive social network practices have been exploring the actual dynamics of opinion expression. These models investigate social phenomena, including opinion formation and spread, using statistical physics. They offer flexibility in examining a wide range of opinion effects [46].

2.3.2 The Initial Models

From the beginning to now various OD models have been developed [11, 42]. Agent-based models are constructed with independent agents that represent individuals in the social network. Each agent has its unique opinion values represented by variables as quantitative data. In the social network, agents are the nodes and their connections with each other are represented by network edges. Based on agents' interactions with their neighbours the opinion state variable and the topology of the network change over time. There are different types of agent-based models using various social network types [11, 42]. In this section, the models are categorised based on their structures (one-dimensional, multi-dimensional and hybrid networks) and opinion data characteristics (discrete and continuous) alongside detailed explanations of their concepts. One-dimensional models are appropriate when there are limited directions that opinions can go either two of the choices, such as conservative or liberal. On the other hand, multi-dimensional models provide a wider range of agent characteristics and opinions that can be influenced by more than two aspects and directions. The hybrid models are developed to deliver a more precise and advanced investigation of complex dynamics of opinion formation and evolution in the networks by combining multiple social mechanisms referring to processes and norms that shape human behaviour, and modelling approaches. Moreover, opinion data characteristics in these models differ based on the type of their values, discrete (takes either two values -1 and +1) and continuous (takes a range of values between -1 and +1).

One-Dimensional Models with Discrete Opinion State

Basic Voter Model

The Basic Voter Model is one of the oldest models in OD proposed to study the competition of species [11, 51]. This model was constructed by using a connected directed graph where all the nodes hold each of two discrete opinions ± 1 and have edges that represent connections with some other nodes. As OD models are inherently dynamic, at each time step t the random node i and its neighbour j are selected. The node i rewrites its opinion value with the node j's opinion. Therefore, the probability of updating the opinion is equivalent to the n number of neighbours of the chosen node. After t time, the graph reaches equilibrium if every node has the same opinion. Otherwise, the final state is quantified by either calculating the opinion difference between the number of the node-neighbour pairs that hold distinct opinions, which is known as magnetization, or by taking the opinion average of all the neighbours with opposite opinions, which is referred to as energy. In subsequent years, the Basic Voter Model was adapted to the multi-dimensional structure, enabling its application to a broader range of social phenomena [52].

Ising model

Another pioneering model in OD was Ising Model constructed under statistical physics principles and provides the simplest agent-based model [52, 53]. In the Ising model, the lattice structure was used. A lattice is a regular, geometric arrangement of points or cells that represents agents in space. Lattices can take different shapes, such as square, triangular, or hexagonal, and they can be either two-dimensional or three-dimensional. The models use lattice as a network structure, the connectivity between agents is predetermined by the lattice structure and remains fixed throughout the simulation. In the Ising Model, each agent is characterised by a single variable, namely spin which refers to their opinion state can only have two directions up and down. The model incorporates one-to-one interactions between agents, referred to as spin couplings, and considers external influences in the form of a magnetic field that induces alignment of the network's opinions in one way. In other words, the magnetic field drags whole agents into the network in the same direction. Despite the complexity of human cognition and the challenge to quantify opinions, physicists quantify the spin variable by mapping it to either -1 or +1, representing the most down spin and up spin, respectively. The probability of updating the opinion is dependent exponentially on the n a number of neighbours, as the agents in the Ising model aim to interact with all of their neighbours at the same time [52].

Sznajd Model

Sznajd Model was developed inspired by social impact theory [54, 55]. This theory argues that the influence of interactions between a group and an individual relies on several factors, including the group's opinion value, distance, and strength [52]. In Sznajd model this theory is applied by considering that a group of individuals can exert more significant influence on their neighbours than a single agent if all the agents in the group hold the same opinion. However, the strength factor in the social influence theory is not taken into account, as this model is based on discrete opinions with only two possible values, ± 1. Therefore, there is no range of values to assess the strength of the opinion. In this model, the agents are connected to their immediate neighbours in the lattice. Similar to the Voter model explained previously, the opinion update rule is operated by randomly selecting an agent i and one of its neighbours j at each t. If both i and j have the same opinion value all of the neighbours of the i adopt that value. On the other hand, if they have conflicting opinion values, no updates occur in the network and the opinions state remains the same.

Findings of One-Dimensional Models with Discrete Opinions

The common features of Basic Voter, Ising and, Snazjd models are the network must reach a consensus and the findings suggest that the structure of the network is crucial in the opinion formation process [56]. For instance, the regular lattice networks where agents only interact with their immediate neighbours can support the emergence of fragmented groups in the network because they are more influenced by their closest neighbours than by agents further away in the network which might have distinct opinions [57]. Whereas, the networks where agents are connected to other random agents in the network, there is a higher probability of updating the opinions of individuals with diverse opinions [58]. This can allow for the emergence of key influential agents in the network who have more connections (not only their immediate neighbours) thus, might have a greater influence on others in the network [57]. Additionally, the time spent to reach a consensus also depends on the network structure and the distribution of opinions at the start [57]. For instance, in regular lattice networks and the even distribution of opinions initially, reaching a consensus can be slower because it will take more time for the agents to adopt the opinions with others in the networks that have the opposite opinions and further away. On the other hand, the networks where all agents have connections with each other and the distribution of the opinions is somehow biased at the start such as there are more agents with a +1 opinion state initially, reaching a consensus will be faster. The reason behind this is the high accessibility of the nodes to every opinion in the network will promote faster interactions between the agents and if there is a majority of the +1 opinion state, it will be quicker to adopt +1 for the agents who hold -1 opinion state due to their large number of interactions with the agents that have +1 opinion value.

These models can be helpful to investigate polarisation in various fields where binary choices are convenient. For instance, Ising Model is utilised in physics to investigate the polarisation of magnetic materials, whereas the Voter and Snazjd Models are used to examine political polarization [52]. However, they have some limitations due to their limited opinion state values, such as they do not have the ability to detect the emergence of extremism in the networks because there is no wide range of opinions [46]. Therefore, one-dimensional models with continuous opinion states are introduced to improve on the limitation of the models with discrete opinions.

One-Dimensional Models with Continues Opinion State

Deffuant-Weisbuch Model

The Deffuant-Weisbuch (DW) model is an agent-based model that shares similarities with the Ising model, in which each agent holds an opinion value ranging between -1 and +1 [59]. However, the DW model includes an additional parameter called bounded confidence, denoted as d, which controls the interactions among agents. Specifically, the parameter d determines whether two agents' opinion values are close enough to allow for interaction. For instance, the chosen agent i and its neighbour j hold opinions x_i and x_j [52]. If the absolute value of the difference between x_i and x_j is smaller than the determined d, the interaction is permitted. As a result, this interaction allows updating the opinion values of both agents by assigning the average opinion value moving them closer to forming groups in the network. The DW model is built on agreement dynamics meaning that agents only interact when their opinion values are sufficiently similar. Hence, if the absolute difference between i and j's opinion values is bigger than d they do not interact.

Hegselmann-Krause Model

Another model was introduced similar to the DW model in the same years by the scientists Hegselmann and Krause(HK) [60]. This model includes the bounded confidence parameter d to control the interactions based on the similarity of their opinions. However, the updating rule in the HK model differs from the DW model in that an agent interacts with all of its neighbours simultaneously instead of through pairwise interactions. The updating rule can simply be explained as an agent i interacts with all of its neighbours whose opinion value is closer to its own by comparing the absolute difference between its opinion and its neighbours' opinions with the bounded confidence parameter d. Then, agent i takes the average value of all this set of opinions as its new opinion value. This approach to opinion updating allows for more global interactions between agents compared to the pairwise interactions in the DW model.

Findings of One-Dimensional Models with Continues Opinion State

The one-dimensional models with continuous opinion states explained above, improved on the limitation of the models with discrete opinion. The key findings of these models state that continuous opinion states have the ability to detect the emergence of extremism in the network formation process. In these networks, opinions are updated by taking the average of nodes' neighbourhood opinions determined by the bounded confidence parameter d which acts as a threshold in the network to control the extremism sensitivity such as high d allows agents to interact who have a wider range of distinct opinions [59, 60]. Therefore, extremism can be detected by comparing the bounded confidence parameter d with the difference between the node's and its neighbourhood opinion states, if the node's opinion is significantly greater than d the node is considered to be extreme. This was not possible to examine in the discrete opinion state models because the agents hold one of the two opinion state values (-1 and +1) thus, the agent is either completely distinct or the same as their neighbours where nodes' opinions are updated by directly adopting from neighbours instead of assimilating to their neighbours' average opinion state. Moreover, as also found in the one-dimensional model with discrete opinions the initial distribution of the opinions also has a significant impact on the network formation in HK and DW Models [57]. For instance, a system that starts with a high level of polarisation may be more likely to remain polarised over time, while a system that starts with a diverse range of opinions may be more likely to converge to a consensus. The results of the DW model demonstrate that depending on the value d, different grouping patterns can be observed. For a large number of contrarians, the number of groups decreases due to the increase in confidence, but the groups became more distinct. Conversely, for a smaller number of contrarians, groups become closer when there are fewer of them. This finding suggests that contrarians favour a more determined fragmentation, in which the number and distance between groups increase. However, the inclusion of partial contrarians requires more time to reach a final frozen state in the system [52].

Overall, these models provided a deeper understanding of extremism emergence in the networks and were utilised to investigate real-world problems caused by extremist opinions such as the spread of hate speech in online social networks. However, multi-dimensional models are proposed to delve into this concept by considering the wider range of opinions and beliefs and their effects on the individual's interactions in the network.

Multi-Dimensional Models with Discrete Opinion State

Axelrod Model

The Axelrod model was developed as an attempt to understand how cultures persist in a society where individuals share similar beliefs and opinions under cultural dynamics [61]. This model consists of two fundamental aspects: *homophily* and *social influence*. Homophily refers to the tendency of individuals to connect with like-minded others and social influence occurs as a result of these similar connections [52]. Compared to the other models discussed in previous sections, the Axelrod model is multi-dimensional since agents possess multiple parameters, known as cultural features, that affect their interactions in the network. Cultural features refer to different traits and beliefs that individuals possess. Therefore, the update rule for this model takes into account several factors in the network, and opinion formation occurs if agents share common cultural features.

In the initial state of the network, a different random set of binary values is assigned to each agent representing different cultural features. In each time step t, the chosen agent compares its feature values to its neighbours. If the agents have at least one common trait value, then the chosen agent adopts the remaining feature values of its neighbours. Hence, this model increases the possibility of forming groups of individuals with similar characteristics and simultaneously makes it challenging to get close to agents that do not share any common traits.

When the network reaches the final stage after the t time, the cultural formation can be visualised by the number of groups in the network topology. On the one hand, if there are a small number of traits in the network, the individuals can reach a consensus where they are more likely to share common traits. On the other hand, if there are various traits in the network, the individuals might not be able to arrive at a consensus because of the low possibility of having similar traits [52].

Findings of Multi-Dimensional Models with Discrete Opinion States

The Axelrod Model introduced above enlightened the opinion formation in society by incorporating multi-dimensional opinions and traits of agents in the network which provides a more realistic reflection of people's thoughts and their interactions based on their belief in the real world. The findings of the Axelrod model show that the concept of homophily promoted community formation in the network where the agents tend to group with others who have similar opinions to them while breaking the connections and getting more isolated from those that have different opinions in the network. The communities in the network have distinct characteristics which refer to the cultural features [62]. When agents interact with others in the network who do not belong to the agent's community the i can adopt the new cultural features if the feature set is small which results in consensus [62]. Therefore, in the Axelrod model, the opinion and feature distribution also plays a significant role in the network formation like in the findings of other models explained previously.

In summary, the Axelrod model can aid to simulate the evolution of society based on cultural diffusion with discrete opinions. The multi-dimensional models with continuous states are proposed to incorporate heterogeneous opinions of agents to better reflect the real world.

Multi-Dimensional Models with Continues Opinion State

DeGroot Model

The DeGroot model was one of the first models introduced as a tool to investigate the dynamics of consensus formation in networks with multiple dimensions [63]. The model allows agents to hold multiple opinion values and is based on an undirected graph, in which the edges between agents are assigned a weight value w to determine the strength of their connections. The agents' opinions are bounded between

0 and 1. At each time step t, the update rule takes the weighted average of the opinions of the randomly chosen agent's i neighbours, which is then combined with the i's own opinion. The model includes an additional parameter, the *learning rate*, which controls the opinion formation by comparing the w given to the i based on its neighbourhood's opinions [64]. This parameter also can be helpful in situations where the consequences of a decision are uncertain or there is a lack of expertise or information among the individuals in the network [63]. The agents incorporate the opinions of their neighbours to form a collective opinion based on the available information, and they update their own opinions, which can then be propagated through the network to other individuals. This process continues iteratively until a consensus opinion is reached or until individuals are sufficiently confident in their opinions and no longer feel the need to update them based on their neighbours' opinions [63]. The speed of the consensus formation process is influenced by the initial opinions of the agents and the learning rate.

Findings of Multi-Dimensional Models with Continues Opinion States

The multi-dimensional and continuous opinions in the DeGroot model increased the diversity of ideas in the network between the agents. These various opinions resulted in forming more confident connections and reaching a consensus in the network by the learning rate parameter where individuals allow agents to change their opinions in response to new information even if they are uncertain about their opinions [63].

The hybrid models proposed to investigate both discrete and continuous opinion states effect on the network formation [52].

Hybrid Models

Frasca, Tarbouriech and Zaccarian Model

The OD models need to explain both agreement and disagreement in order to investigate the outcomes in a dynamic evolution manner. In some models, this is achieved by opinion-dependent limitation features such as bounded confidence d mentioned in HK and DW models as a threshold [59, 60]. These dynamics typically induce the group formation of opinions, that is, the population splits into separate groups of individuals having a common opinion. These limitations suppose that individuals do not influence each other if their opinions are too far apart. According to the study recent study, hybrid models offer more advantages compared to one-dimensional or multi-dimensional OD models [65]. These benefits are attributed to the incorporation of network topology as an independent (discrete) variable that interacts with the (continuous) opinion variable, resulting in the existence and completeness of solutions. Additionally, the hybrid approach permits memory or hysteresis effects within the network meaning the opinion evolution is affected by past interactions. The proposed hybrid framework explores opinion-dependent connectivity and utilises a combination of continuous flow dynamics and discrete jump dynamics, to achieve this.

The model was originally developed by reforming the classical HK model and incorporating an additional two threshold properties to ensure stability [65]. This stability is achieved by utilising a fixed threshold denoted as a confidence interval similar to the bounded confidence parameter in HK where the agent's opinion update is limited to the maximum opinion difference with its neighbour. To simplify, if the absolute difference between the chosen agent i's opinion and its neighbour j's opinion is bigger than the fixed threshold, the opinion update is not allowed. On the other hand, the adaptive threshold determines whether the agent's confidence interval should increase or decrease based on their neighbour's confidence intervals. For instance, if the majority of i's neighbours' opinions are in the same confidence interval, the i's confidence interval increases and decreases otherwise. Furthermore, the adaptive threshold takes the past interactions of i and j into account when updating the confidence interval which allows this model to illustrate memory and hysteresis effects. To clarify, the confidence level is calculated as a weighted average of the agent's own confidence level and the average confidence level of its neighbours with whom it agreed in the previous time steps. These threshold mechanism helps to prevent large opinion swings that can lead to instability.

The opinion update process consists of two main rules: dynamic flow and discrete jumps which makes this model a hybrid framework [65]. The dynamic flow demonstrates the attractive forces between the agents by utilising differential equations and involving the thresholds in this process to regulate the continuous opinion formation of the agents. Whereas, the jumps allow the discrete variations in the interaction pattern to store the network topology states. This update rule depends only on the agent's and its neighbours' state regardless of the whole network topology and size, hence the opinion formation is localised. This allows them to model be scalable and apply to large networks.

The Frasca, Tarbouriech and Zaccarian(FTZ) model opened wider perspectives in OD to analyse complex systems and encouraged the development of new hybrid models in later years.

CODA Model

The Continues Opinion and Discrete Action (CODA) model is another hybrid model proposed which combines continuous opinion dynamics with a discrete decision-making process [66]. The CODA model suggests that when agents need to choose one of two options, the belief or opinion regarding which option is superior is not always limited to a binary perspective [67]. Therefore, the model utilises probability p which represents the belief of the individuals to determine if one choice is probabilistically better than the other. To simplify, if the potential outcomes carry equal weight for both options, the alternative with a higher probability, denoted as p or 1-p, will be selected as the optimal choice [67]. The agents are only aware of the discrete opinions of their neighbours, thus it is not possible to converge to an average result as in the bounded confidence models such as DW and KH [59]. Therefore, in the CODA model agents change their probability p towards the value of their neighbours. This results in various opinion-change behaviours of agents and this process occurs continuously [52]. For instance, agents with extreme opinions only update their opinions after several interactions while less extreme others change their opinions even after one interaction.

Findings of Hybrid Models

The models explained above examined the network formation from a hybrid perspective by combining both discrete and continuous opinions (FTZ Model) and the actions of agents where the continuous opinions change by discrete actions (CODA Model). These models suggest that combining different dynamics and opinions can provide a more heterogenous structure of the OD models where they can represent the binary and the continuous opinions of people in the real world and their realistic response process by involving probabilistic behaviour in agent interactions. The finding in these models the confidence and probabilistic behaviour of agents play a crucial role when reaching a consensus and network formation because the response of the agents based on their interactions with others depends on these features of the model. For instance, the confidence parameter of the FTZ model determines the change of opinions of the agents where low-confidence agents are more likely to change their opinions in their interactions with others [65]. The CODA model suggests that changes in the probability of individuals' beliefs provide a more realistic information observation process of the real world [66].

Overall, the initial models provided in this section paved the way for the new OD models by introducing the basic concepts and different social mechanism concepts of the models such as homophily, consensus, fragmentation, extremism, and confidence. Recent models utilise these concepts in their construction process and try to provide a better explanation of real-world interactions and society.

2.3.3 Sayama Model

Computer scientist Hiroki Sayama has made significant contributions to the field of OD by developing several models. In his recent publication "*Extreme Ideas Emerging from Social Conformity and Homopily: An adaptive social network model*", the agent-based model utilised by adaptive social networks was introduced to investigate the process of extreme opinion emergence in society [14]. The model explores extremism from the perspective of individuals who conforms to the social norms of their local neighbourhood. This conforming behaviour is incorporated into the model by the conformity parameter along with other parameters including attention to novelty and homophily where these parameters are global in the network which affects all the agents with the same values defined in the initialisation. The attention to the novelty of agents determines how likely the individual interacts with their neighbours who are non-conformists and have different opinions from theirs. The concept of homophily is similar to the initial models introduced in the previous section where it determines the tendency of agents to strengthen their connections with the ones that have similar opinions to them. The opinion of each agent in the model is a continuous value randomly chosen from uniform distribution and these opinion values refer to the opinion state of the agents in the model. Opinions are updated based on the weighted interactions between the update functions where they allow agents to gradually embody their neighbours' opinions to the local social norm by taking the average of their local neighbourhoods' opinion state.

2.3.4 Findings of Sayama Model

In Sayama Model, the effects of the social behaviours of the agents on network formation are analysed through numerical experiments and methods. The findings indicated that when individuals seek sameness through homophily, the network becomes more fragmented and extremist opinions emerge [14]. Whereas, when individuals seek difference by attention to novelty the network structures in a homogenous way, where there is less diversity of ideas and individuals are highly connected. These results were obtained by analysing the communities in the network which are the clusters of agents with similar opinion states [14]. The communities determine the fragmentation in the network and they are formed from the closely interconnected node clusters that have limited connections with nodes from other clusters [68]. For instance, a network with a high number of communities suggests that the network structure is fragmented by the loss of connections between agents across communities and the strong connections of the agents inside the community. Communities can be attributed to the "social bubbles" in our society. This term is introduced by scientist Nikolov and refers to a group of people that defend the same opinions and social norms [1].

This model can be beneficial to assess opinion formation in complex dynamics and understanding how extreme opinions emerge in society based on the social tendencies of individuals. The previous models in OD were limited to asses the spontaneous emergence of extreme ideas [14]. In essence, they did not consider the conformity concept where individuals only try to conform to the social norms of their neighbourhood without any intention to go extreme [14]. Therefore, Sayama Model provides one of the most advanced models for investigating extremism emergence in real-world interactions realistically however, further study can be conducted to improve on the limitations of the Sayama Model such as homogenous nodes with the same social mechanism values. Therefore, in this thesis, the heterogeneous structure of the nodes is studied by assigning unique homophily, social conformity and attention to novelty values to each node in Sayama Model. In the following chapter, the theory and structure of the model are provided with a detailed explanation.

Chapter 3

The Sayama Model

This chapter offers a comprehensive exploration of the Sayama Model introduced in the previous chapter [14]. First, the concepts and the mathematical approaches used in Sayama Model are provided along with the experiment design and settings utilised in the analyses of the model from Sayama's paper. Secondly, the replication process of the Sayama Model is outlined, highlighting the necessary modifications or adaptations made to get the most similar results as the original model to be used as a base model in this research. Finally, the extension applied to the Sayama model is described in detail including the aim of the new model and how the analysis is conducted.

3.1 The Theory Behind the Model

The Sayama Model is introduced with the objective of exploring the emergence of extreme ideas in society based on various social mechanisms including homophily, conformity and attention to novelty [14]. The incorporation of these social mechanisms enables the assessment of the emergence of extremism and fragmentation within a social system, where individuals tend to conform to the opinions of their peers. The attention to novelty parameter in the model captures the willingness of individuals to strengthen their connections with distinct others and can be adjusted to simulate varying levels of novelty-seeking behaviour within the population. The model is also designed to investigate communities that are formed by individuals with similar opinions and beliefs within the social network which refer to social bubbles in our society [1].

The adaptive social network model is constructed by using a complete graph made of n nodes within the set of V, representing agents that have bidirectional connections with one another. These connections are modelled as weighted directed edges which control the interactions between the agents in the network. Each agent $i \in V$ possesses a unique continuous opinion value denoted as $x_i \in \mathbb{R}$. Each edge connection from neighbour j to agent i holds a unique weight value $w_{ij} \in \mathbb{R}_{\geq 0}$.

The update rule in the model is governed by mathematical equations named update functions which allow the opinion state and edge weights to evolve at each time step. These functions include the local social norm refers to the opinion state and random fluctuation in the network through social mechanisms. The mathematical representations of these functions are provided below:

$$\frac{dx_i}{dt} = c(\langle x \rangle_i - x_i) - \epsilon \tag{3.1}$$

The network utilises an opinion update rule, which is defined by the function 3.1. In this function, an agent i modifies its opinion state x_i by determining the difference between its current opinion state and the average opinion states of its neighbourhood, $\langle x \rangle_i$. This difference is then multiplied by the conformity parameter c, with epsilon ϵ denoting the stochastic fluctuation term that is randomly sampled from $N(0, 0.1^2)$ at each time interval $\Delta t = 0.1$. The c, a and h values are global for each network where the update rule affects every agent with the same value. As such, in each time step, there are ten intervals, and the update rule is executed ten times by using a simple Euler approach.

$$\langle x \rangle_i = \frac{\sum_{j \in N_i} w_{ij} x_j}{\sum_{j \in N_i} w_{ij}} \tag{3.2}$$

The computation of the average opinion state of an agent's neighbourhood is performed using function 3.2. This function is responsible for determining the sum of the weighted opinion states of an agent's neighbours. To accomplish this, the function multiplies the opinion state of agent j (x_j) by the weight w_{ij} that corresponds to the particular neighbour of the agent and then divides this value by the sum of all the weights w_{ij} that are associated with each of the agent's neighbours. This function gives rise to the division of zero problems when the sum of all the weights w_{ij} is zero indicating that there is no information conveyed from j to i. This problem will be discussed along with the proposed solution during the replication process in section 3.4.

$$\frac{dw_{ij}}{dt} = hF_h(x_i, x_j) + aF_a(\langle x \rangle_i, x_j) \tag{3.3}$$

$$F_h(x_i, x_j) = \theta_h - |x_i - x_j| \tag{3.4}$$

$$F_a(\langle x \rangle_i, x_j) = |\langle x \rangle_i - x_j| - \theta_a \tag{3.5}$$

In addition to the opinion state update, the edge weights also change in the network during each simulation. Similar to the opinion change function in 3.1, the weight update function 3.3 also takes the time interval Δt into account when calculating the weight change. However, the weight update rule is influenced by the social mechanism parameters, that relate to the attention given to novelty and homophily in the network. These parameters are represented by the functions F_h 3.4 and F_a 3.5, which are associated with parameters h and a, respectively.

On the one hand, the function F_h considers specific i, j pair at a time, and takes into account the absolute difference between their opinion states. The difference is then subtracted from a threshold parameter denoted by θ_h to assess the similarity between the agent and its neighbour. If the difference is greater than θ_h the opinion of i is distinct enough from j to lower the rate of homophilic edge weight change with a negative value. Whereas if the difference is smaller than the θ_h there is no significant opinion difference between i and j indicating that agents have similar opinions to increase the rate of homophilic edge weight change by a positive value. On the other hand, the function F_a focuses on the average opinion state of the agent's local neighbourhood that was calculated by the function 3.2. This function aims to examine whether the j's opinion is extreme relative to the average opinion in i's local neighbourhood $\langle x \rangle_i$. To examine this, the difference between the $\langle x \rangle_i$ and x_j is calculated and then subtracted from θ_a. If the difference is greater than θ_a the average opinion of i's neighbourhood $\langle x \rangle_i$ is more novel than j's opinion x_j to increase the attention to novelty by a positive value. Whereas if the difference is smaller than the θ_a the $\langle x \rangle_i$ is not significantly different than x_j which lowers the rate of edge weight change by attention to novelty with a negative value. The weight update is then calculated by multiplying each of the F_h and F_a functions with their respective global parameters (h and a) and adding the resulting values together.

3.2 Model Settings

The experimental design in Sayama's paper involved determining the appropriate settings to construct the final network for the analysis of the model.

The networks used in the analysis are generated with the following parameter values; $n \in$\{30,100,300,1000\} for the number of nodes in the graph and $c, h, a, \theta_h, \theta_a \in \{0.01, 0.03, 0.1, 0.3\}$ for the social mechanism parameters values.

For the experiments, the model was run five times for each of the parameter combinations, resulting in a total of 4096 parameter settings. Specifically, there are five social mechanism parameters, and each parameter can take four possible values, resulting in $4^5 = 1024$ unique combinations of parameter values. Combining this with the four possible values of n, the total number of parameter settings is thus, equal to $1024 \times 4 = 4096$ with the five replicates of each means 5×4096.

The initialisation of the network involves establishing directed weighted edge connections between each node and its respective neighbour pairs. The total simulation time is set to t = 100, and at each time step, every node is visited to update its opinion state and edge weights. At the start, the edge weights are randomly assigned from a uniform distribution [0,1], while the opinion states of the agents are sampled from a standard normal distribution $N(0, 1^2)$. However, during the evolution process, if an edge weight becomes negative, it is rounded up to zero. To account for the stochastic effect, a random ϵ value is sampled from $N(0, 0.1^2)$ and added to each agent's opinion state x_i at each time step.

3.3 Outcome Measures

The final network state is examined after t=100. The directed edges are converted to undirected edges by averaging the weights of the bidirectional edges. Specifically, for the edge weights w_{ij} and w_{ji} between nodes i and j, the average weight is computed as $(w_{ij} + w_{ji})/2$, resulting in a single undirected edge weight between them.

After all the configurations are done, the network is analysed using several network metrics to determine the outcome measures, which include:

1. Average of all the edge weights in the network
2. Number of communities in the network
3. Modularity of communities
4. Standard deviation of average community opinion states
5. Range of average community opinion states

As one of the objectives of the Sayama model is to analyse the fragmentation, groupings of agents with similar opinion states in the network are defined as communities, detected through the use of the Louvain modularity maximisation method to extract the community structures in complex networks [69].

This method was employed to identify the total number of communities formed in the final stage of the network (2) and analyse their structure by modularity. In particular, the modularity in 3 refers to the density of the communities which is the number of connections inside them. Furthermore, community detection also helps to calculate the community opinion state utilised in the measurement of the outcomes of 4 and 5 by identifying the nodes that belong to each community and taking the average of their opinion states. On the one hand, the range of average community opinion state 5 calculates the difference between the highest and lowest average community state value in the entire network which corresponds to the range. On the other hand, the standard deviation of average community opinion states 4 is calculated to investigate how dispersed the average community opinion states are in the network.

The outcome measures 4, and 5 are utilised to quantify the production of extreme ideas in the social network. Whereas, other outcome measures investigate the fragmentation and polarisation in the network by looking at the community structure (2 and 3) and connectivity of nodes (1) (strength of the edge weights) in the network. These five outcome measures are averaged over independent simulation runs for each combination of parameter values outlined in section 3.2, and their correlations with the model parameters are analysed using multiple linear regression methods.

3.4 Model Implementation and Replication Process

The Sayama Model is first replicated to be used as a base model for the application of the extension to investigate the social tendencies in the network with heterogeneous nodes. However, during the replication process, several obstacles are encountered due to some unclear and missing points in the Sayama paper, which are listed below:

1. Asynchronous or synchronous opinion formation process
2. Node visiting order

3. What happens to the nodes with no neighbours
4. How do social mechanism values affect the nodes with zero incoming weights
5. When the stochastic ϵ value is added to the x_i

Therefore, multiple approaches are tested to illustrate the same results in the paper. The model's update rule did not specify if the opinion formation is asynchronous or synchronous (1). In the case of an asynchronous evolution process, at each simulation, the visited node's opinion state should be updated immediately before moving to the other node in the network. Conversely, in a synchronous formation process, every node's opinion state should be updated after all the nodes have been visited. Both the asynchronous and synchronous approaches were employed on the replicated model, and a conclusion was drawn based on the comparison of the results. The generated results demonstrated that the asynchronous process produced outcomes that were more similar to the original model. Therefore, this approach was selected as the most suitable option for the extended model.

Furthermore, the Sayama paper does not mention the visiting patterns of nodes (2). Hence, the base model was first implemented with ascending order node visiting, which yielded biased results towards the nodes at the beginning because their opinion state and weight values were the first ones that were altered in each simulation. Consequently, this led to similar formation patterns of values. To address this issue and achieve a more homogeneous approach, the nodes are shuffled at each simulation in the replicated model.

Initially, the graph was complete where all the nodes are connected to each other. However, during the evolution process, the graph structure changes as the edge weights are updated. As a result, some nodes may lose interactions with all of their neighbours resulting in zero values for all edge weights entering the node. This behaviour also causes a division by zero problems in the weight update functions and the opinion update function because they incorporate the $\langle x \rangle_i$ which contains the division by the sum of all the incoming weights from j to i. Thus, nodes with zero incoming weights must be considered (4). To address this, the replicated model's algorithm includes a check before these functions prevent any update if the sum of all incoming weights to node i is less than or equal to zero. Additionally, Sayama's paper does not mention how to treat these edge weights (3) in the update functions. Therefore, in the replicated model the condition was applied to only update the node's opinion state to conform more closely to its neighbours' average opinion state. Also, the node's incoming weights are due to attention to novelty if the sum of incoming weights to the node is greater than zero. Whereas it is allowed to update the node's incoming weights due to homophily regardless of the sum of incoming weights and the stochastic effect ϵ can be added to isolated nodes' opinions.

In the original model, the stochastic effect is implemented by adding ϵ to x_i. However, the paper does not clearly state at which stage of the simulation this was done (5). Thus, this was tested first by adding the ϵ before the opinion update occurs, and then it was implemented after the opinion update functions were executed by adding the ϵ to the updated opinion value at the end of each simulation.

In the replication phase, `Python 3.7` and the `NetworkX` package were used to generate networks and conduct the same experiments as described in the Sayama paper [70, 71]. The generated networks at the end of each run[1] stored by utilising `pickle` files for later use. Subsequently, the outcome measures were calculated in different scripts by using the pickled networks. The outcome measures involve community structure, the `louvain_communities` package is utilised to detect communities within the network and the modularity is calculated by `modularity` package in `NetworkX` [72, 73]. The output data from outcome measures were then written into a `CSV` file where each row represents a different network with its specific parameter combinations and columns indicating the outcome measures. To ensure the replication of the results, the replicated model was also run five times with a total of 1024 parameter combinations, resulting in the generation of 5120 rows in a `CSV` file. The linear and multi-linear regressions were calculated using the data in `CSV` files to illustrate the replicated results.

[1] Each simulation run with the network size of 100 takes approximately two and a half hours.

3.5 The Extension

The original Sayam model has some limitations in assessing extremism in society. This research aims to address this issue by proposing a more realistic approach by introducing heterogeneous nodes to the Sayama model.

One of the limitations of the Sayama model lies in its assumption that all agents possess the same homophily, novelty, and conformity parameter values. However, in the actual social world, individuals exhibit varying degrees of susceptibility to these social mechanisms, implying that some people may be more receptive to novelty, while others may conform more to others' social norms and interact only with like-minded individuals. Additionally, the values of the social mechanism parameters were also limited only to four discrete values in the [0.01, 0.3] range. To overcome these limitations, this study extends the original Sayama model by assigning independent social mechanism parameter values to each agent by utilising both discrete and continuous values. As a result, the model can produce more realistic and generalised results that better reflect the complexities of real-world social interactions.

3.6 Extended Outcome Measures

The extension of the Sayama model aimed to shift the focus from the entire network topology to individual nodes in order to investigate the effects of various characteristics of the agents on the network dynamics. As such, specific outcome measures were determined for each node listed below:

1. Node's global eccentricity
2. Node's within community eccentricity
3. Node's community eccentricity
4. Node's community's size

In the context of network theory, eccentricity refers to the distance between node i and node j. The definition of the distance measure can change based on the study. In the extended network, eccentricity is defined as the difference between node i's and node j's *opinion states*. This measure allows us to investigate the influence of individual social mechanism parameter values on opinion states between nodes in the network.

The absolute difference between a node i's opinion state and the opinion states of all other nodes in the network is referred to as global node eccentricity (1). Additionally, the extension also encounters communities in the calculation of outcome measures 2, 3 and 4. To further investigate the eccentricity within each community, two outcome measures are defined: the difference between the node i's opinion state and the average opinion state of the community where i belongs (2), and the community's average opinion state eccentricity from the average opinion state of the whole network, to determine the extent to which the opinions of node i and its neighbours differ from the opinions in the network as a whole (3). The size of the community where node i belongs was also calculated to examine how the unique social mechanisms and values of the individual impact the formation of social bubbles in society (4).

Moreover, the extension employs the classic concept of *assortativity* in social network analysis to explain the extent of extremism among individuals within the network [18]. Assortativity is typically defined as a degree-degree correlation where the degree is the number of incoming (in-degree) and outgoing (out-degree) edges of the node, and it measures the association between any pair of node features [18]. In this study, the correlation between social mechanism values is assessed using assortativity, in order to determine the relationship between unique social mechanism parameters. Assortativity is a network-level measure defined as Network's assortativity in the extension's outcome measures thus, analysed separately from the node-level outcome measures introduced above.

There are two types of assortativity positive and negative to explain the tendency for agents to interact with others with similar properties and dissimilar properties within a network respectively. Specifically, assortativity values of the nodes can range between -1 and +1, with perfect disassortativity at -1 and perfect assortativity at +1. Perfect negative assortativity implies that agents interact with others who have dissimilar characteristics while perfectly assorted network indicates that individuals interact primarily with those who have similar properties.

This measurement is mostly calculated for undirected graphs with no edge weights. However, the Sayama model is a weighted network and thus, the approach proposed by researchers to assess the assortativity in weighted directed networks was utilised in the extended model to calculate the weighted assortativity values of the nodes [18]. This method examines the correlations between two features of the nodes (where they can be the same feature) by taking the edge weights of the network into account. The implementation of this method is carried out separately from the other outcome measurements, as it was performed on the directed version of the network before the conversion to an undirected network. The mathematical representation of the assortativity equation is stated below:

$$P_{X,Y}\left(G\right) = \frac{\sum_{i,j \in V} w_{ij}(X_i - \bar{X}_{sou})(Y_j - \bar{Y}_{tar})}{W \sigma_X \sigma_Y} \tag{3.6}$$

where $P_{X,Y}(G)$ denotes the Pearson correlation between the two features X and Y in network G.

The enumerator of this function is the sum of each visited node i's multiplication of the edge weights w_{ij} from node i to node j and $(X_i - \bar{X}_{sou})(Y_j - \bar{Y}_{tar})$, where X_i is the node i's X feature and Y_j is node j's Y feature. The $\bar{X}_{sou}$ denotes the average of feature X values in the source nodes(the vertices where the edges are going out from them to other nodes). In contrast, the $\bar{Y}_{tar}$ represents the average of Y values of the target nodes (vertices that the edges are coming into from the source). The calculation of these averages are:

$$\bar{X}_{sou} = \sum_{i,k \in V} w_{ik} \frac{X_i}{W}$$

$$\bar{Y}_{tar} = \sum_{k,j \in V} w_{kj} \frac{Y_j}{W}$$

where W is $\sum_{i,j \in V} w_{ij}$.

The denominator of the function is the multiplication of standard deviations of X (σ_X) and Y (σ_Y) with W. The standard deviations equations are as follows:

$$\sigma_X = \sqrt{\frac{\sum_{i,k \in V} w_{ik}(X_i - \bar{X}_{sou})^2}{W}}$$

$$\sigma_Y = \sqrt{\frac{\sum_{k,j \in V} w_{kj}(Y_j - \bar{Y}_{tar})^2}{W}}$$

In the extended model X and Y were replaced with the c, h and a values to investigate the bidirectional correlations between them and themselves. For instance, the possible correlations for c values are $c : c$, $c : h$, $c : a$ thus, for three social mechanisms the total possible correlations are $3 \times 3 = 9$.

3.7 Extension Settings

After resolving the uncertain points in the paper, and made sure that the replicated results are the same as Sayama's results. The social mechanism values were then assigned to agents with three methods.

The first technique was assigning c, h and a values from a random uniform distribution [0.01,0.3]. The second approach involved assigning parameter values by drawing values from a random normal distribution [0.01,0.3]. These two methods help to expand a wider range of parameter values by generating continuous variables. Finally, the third method chooses discrete values randomly from the same parameter set c, h and $a \in \{0.01, 0.03, 0.1, 0.3\}$ that was used in Sayama's paper. These processes were applied at the start of the network, where the edge weights and opinion states were initialised. The other parameter settings of the extension experiment are as follows:

- $n = 1000$ to generate a network with 1000 nodes

- $t = 100$ and the Δt=0.1 to have 1000 simulation updates
- Both threshold parameters θ_a and θ_h were globally set to 0.03 where all nodes were affected equally by these threshold parameters.

The experimental setting was conducted by executing the algorithm ten times to generate ten distinct networks for each of the three methods stated above, resulting in a total of $10 \times 3 = 30$ networks.

In the extension stage, the same experimental process was followed however, the aim of the extension is more to investigate the node characteristics and their effects instead of focusing on the whole network structure. Thus, each row in the `CSV` files represents a specific node in the networks.

As ten networks with 1000 nodes were produced for each of the three parameter-assigning methods, thus, the three `CSV` files were generated with $10 \times 1000 = 10000$ rows corresponding to each node's individual parameter values and its outcome measure values calculated from the extension's node-level outcome measures described in the previous section 3.6.

For the analysis of the assortativity outcome measure, the assortativity results for each random assigning method are written into three different `CSV` files. Each `CSV` file stores the correlation values as a column with one row representing the assortativity value of ten networks generated by each parameter-assigning method for the extension. Then, a single `CSV` file was generated by combining the uniform distribution, normal distribution and random choice methods. Therefore, the final `CSV` file has nine columns representing average correlation values and three rows corresponding to the three assigning methods.

After all the outcome measures' data for both replicated and extended networks were collected with the information provided above, the results were generated by analysing and visualising the network. These results are provided in detail in the next chapter.

Chapter 4

The Results

In this chapter, the results are presented based on the generated outcome measures during the replication and extension process along with the methodologies utilised for their visualisation. In order to analyse and visualise the results several statistical methods were used. The first section discusses the replicated results by comparing them with the original results provided in Sayama's paper. In the second section, the results of the new model including heterogeneous nodes will be represented. A detailed explanation and discussion of these results will be provided in the Critical Evaluation chapter 5.

4.1 Replicated Results

In order to conduct the analyses of the outcome measures several statistical methods were used in Sayama's paper including regression methods. These methods help to examine the trends in data to have a better understanding of the numerical values [74]. The first method utilised in the paper was linear regression (LR) on the model parameters. LR is commonly used to demonstrate the linear relation between independent and dependent variables in the simplest way. For instance, estimating the effect of one variable on another. The mathematical formula of this regression is represented below:

$$Y = a + bX \tag{4.1}$$

where Y is the dependent variable that is wanted to be predicted based on the independent variable X, a is the intercept (regression constant) value when $X = 0$ and b is the slope (regression coefficient) [74].

This method produces a line called a regression line utilising the slope b which represents the change of Y subject to the change in X [74]. Various measures exist to understand the meaning of the relation produced by LR such as p-values [75]. P-values determine the significance of the correlation of the regression values by assessing the probability of obtaining the results by chance [75]. In other words, the significance level is typically chosen as a threshold, and p-values lower than this threshold indicate a significant correlation between the variables that are not due to chance, while higher values indicate that the correlation is not significant.

In Sayama's paper, this method was utilised to investigate the influence of various settings of c, h, a, θ_a and θ_h as independent value X, on individual outcome measures stated in section 3.3 as a dependent value Y. These results are provided in figure 4.1 where each subgraph represents one outcome measure and an individual model parameter from their value set. Each blue dot represents the average of the five runs generated for the experiment where $n = 100$, $t = 100$, $\Delta t = 0.1$ and $\epsilon = 0.01$. As mentioned previously in section 3.2, the combination of social mechanism parameters settings is 1024 so with the five runs resulting in a total of $(5 \times 1024) = 5024$. Therefore, for each subgraph, $(5024/5) = 1024$ blue dots are plotted as an average of the five identical networks that hold the same parameter values.

To clarify, one of the subgraphs in figure 4.1 can be examined for a better understanding of the visualisation. In the top-left subgraph, the average weight of the network was selected as the outcome measure, and the 1024 blue dots obtained from the five identical networks, each holding different parameter values, were divided into four groups based on their values of the parameter c. Specifically, the first, second, third and fourth columns of the plot correspond to the runs where c is equal to 0.01, 0.03, 0.1 and 0.3 respectively. Thus, each column of blue dots represents one-quarter of the total $(1024/4) = 256$ blue dots. Statistically significant correlations with $p < 10^{-4}$ are demonstrated with red regression lines. Normally, these lines are linear however, to help the visualisation, horizontal axes are plotted in log scale resulting in curved trends.

The replication of the LR results was conducted by using the `CSV` file mentioned in 3.7 which stores the calculated outcome measure values and the global social mechanism parameter values of the replicated networks generated after the five runs. During this replication process, the uncertain points of the Sayama model were resolved by trying different approaches mentioned in section 3.4. These approaches and their different outcomes will be discussed in detail in Critical Evaluation chapter 5. To visualise the replicated results, the process first started with reading the `CSV` file in a `Python` script to be rearranged by taking the average of all the columns where the outcome measure values are stored. This average value indicates the average of five runs resulting in 1024 values in total. Then, the LR was implemented by using `scipy.stats` package and calculated with `linregress()` function [76, 77]. Finally, the individual subplots were generated by using the `matplotlib` library by looping through each X and Y value, and the final plot was obtained by combining each subplot [78]. This final plot is illustrated in figure 4.2. The resulting trends were consistent with the original results 4.1 where the red regression lines showed the same trends, except for the modularity θ_h line. The reason why this difference occurred will also be explained in Critical Evaluation chapter 5 in detail.

Another regression method, multi-linear regression (MLR), was employed to analyse the influence of each of the model parameters on the outcome measures. Unlike the standard LR method, MLR can assess the association between a dependent variable and two or more independent variables simultaneously [74]. The MLR model can be represented mathematically as follows:

$$Y = a + b_1 X_1 + b_2 X_2 + b_{12} X_1 X_2 .. + b_n X_n + + b_{n1n2} X_{n1} X_{n2} \tag{4.2}$$

where this time multiple independent variables X_1 and X_2 are involved along with their respective slopes b_1 and b_2 [75]. The $X_1 X_2$ represents the interactions of X_1 and X_2 variables, and b_{12} indicates this interaction's slope.

In the context of Sayama's research, MLR was employed to analyse the correlation between multiple model parameters and outcome measures by using the same network data used in LR. The results of the MLR analysis extracted from Sayama's paper are presented in Table 4.1, where, the headers represent the outcome measures, and each row presents the model parameters' coefficient values and their interactions' coefficient values (after the middle line). This table also includes the constant value a defined as *const.* in the first row. The values in this table 4.1 represent the correlation coefficients (regression coefficients b_1, b_2, $b_{12}..b_n$, b_{n1n2}), with the blue and red coefficients indicating the largest positive and negative effects, respectively. Therefore, the most influential parameters on the social network evolution process based on the outcome measures were h and a parameters which can be seen with blue and red coefficient values. The significance of the coefficients was determined by p-values, and asterisks were assigned to indicate their significance level according to the following criteria: $*** : p < 10^{-4}, ** : p < 10^{-3}; * : p < 10^{-2}$.

The replication of MLR results followed the same process by utilising the same data as the LR replication. The `statsmodels.api` package was used to perform the MLR analysis, and each outcome measure (Y) was calculated by considering every c, h, a, θ_a, and θ_h value (X) along with their interactions (after the middle line in the Table 4.2) using the function `OLS()` [79, 80]. Before this calculation, the constant value a and their pairwise interactions were added to the model parameters X. The table was generated also utilising `matplotlib` library with the same p-value significance conditions used in Sayama's results to output the asterisks. The resulting table is presented in 4.2 [78]. To aid the comparison of the original and replicated results Table 4.3 is provided where it shows the percentage difference of the coefficient values in the Tables 4.1 and 4.2. Some of the high percentage differences are highlighted with red occurred because the absolute differences of original values are close to zero. The minor percentage differences observed in the MLR and almost the same trend lines presented in the LR subplots indicated that the replication process was successful. Consequently, the implemented algorithm in the replication process served as a vanilla model for an extension by adjusting this model to investigate node-level analysis. In other words, the extension changed the way the model works by generating node-level data from heterogenous nodes where each node has its unique social mechanism parameter values unlike the same model parameter values assigned to all agents in Sayama's original model. The new outcome measures are introduced to investigate the new heterogeneous model.

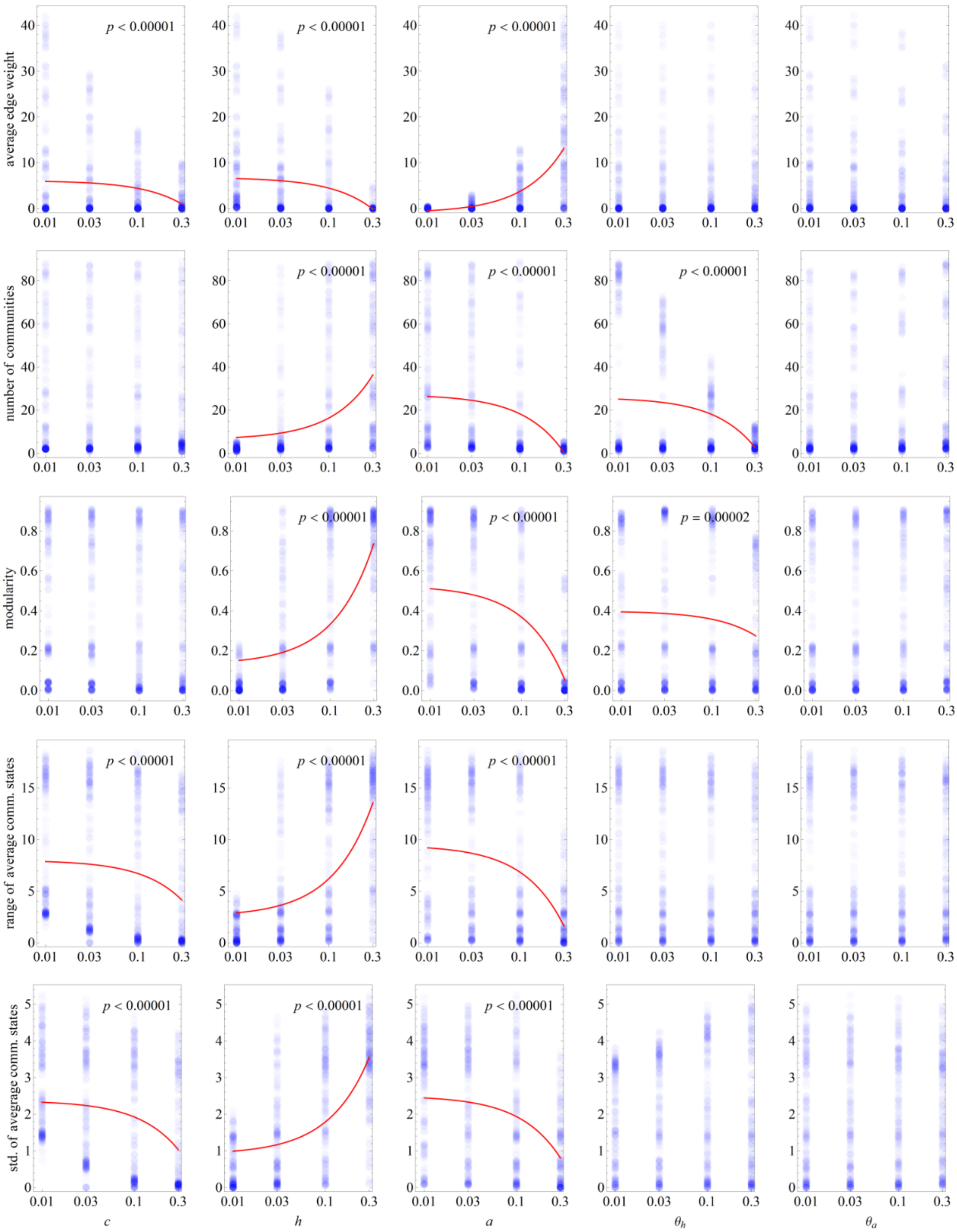

Figure 4.1: Linear regression plots from "Extreme Ideas Emerging from Social Conformity and Homophily: An Adaptive Social Network Model(2020) by Hiroki Sayama". Plots represent the distributions of outcome measures over model parameters for a network size of $n = 100$. Each dot represents an average of five simulation runs with identical experimental settings. Statistically significant correlations with $p < 10^{-1}$ are illustrated with red linear regression lines. To help the visibility the horizontal axes are in log scale thus, the linear trend lines shown in these plots are curved.

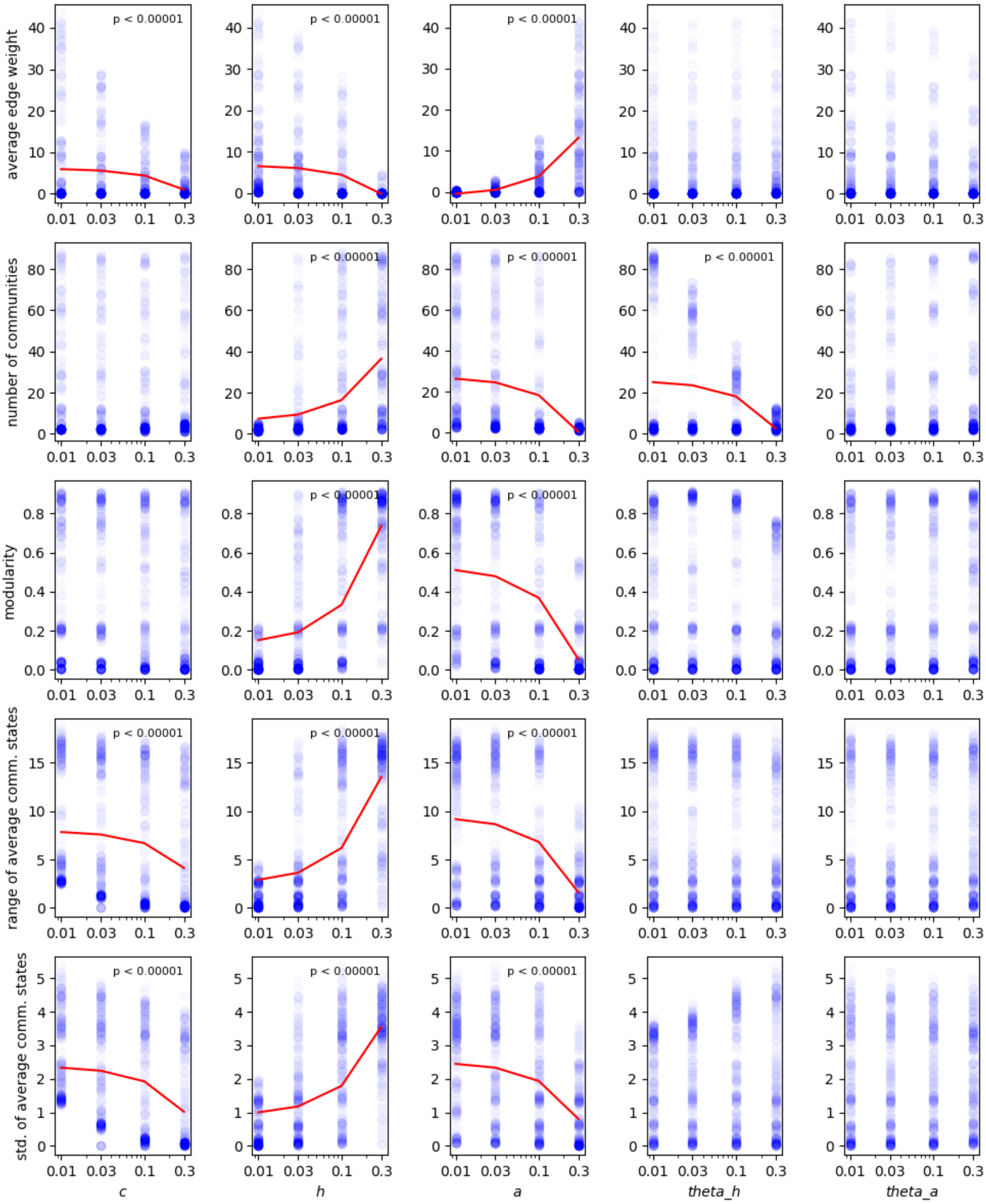

Figure 4.2: Replicated linear regression plots generated from the replicated model. Plots represent the distributions of outcome measures over model parameters for a network size of $n = 100$. Each dot represents an average of five simulation runs with identical experimental settings. Statistically significant correlations with $p < 10^{-1}$ are illustrated with red linear regression lines. To help the visibility the horizontal axes are in log scale thus, the linear trend lines shown in these plots are curved.

Outcome measure	average edge weight	number of communities	modularity	range of average comm. states	std. of average comm. states
const.	-0.336133	17.9348	0.306332	6.36077	1.92395
c	-6.32563^{***}	-19.9322	-0.311658^{***}	-14.0533^{***}	-4.52717^{***}
h	$\mathbf{-11.8516}^{***}$	$\mathbf{196.224}^{***}$	$\mathbf{2.52259}^{***}$	$\mathbf{48.7035}^{***}$	$\mathbf{9.21682}^{***}$
a	$\mathbf{100.97}^{***}$	$\mathbf{-90.2584}^{***}$	$\mathbf{-1.44367}^{***}$	$\mathbf{-23.1149}^{***}$	$\mathbf{-5.0213}^{***}$
θ_h	0.139315	-66.0141^{***}	-0.161914^{***}	-2.06788^{***}	0.98742
θ_a	-1.689908^{***}	26.8527^{***}	0.206154^{***}	0.440309^{*}	-0.482658^{*}
ch	96.777^{***}	-2.40873	0.496937	1.44904	3.96937
ca	-196.254^{***}	62.63	1.21425^{*}	32.9133^{***}	4.85103
$c\theta_h$	0.938596	23.7479	-1.42933^{*}	-31.1381^{**}	-11.4987^{***}
$c\theta_a$	1.65429	22.5155	0.415723	7.85521	3.23167
ha	-234.986^{***}	-421.495^{***}	-3.68595^{***}	-88.4928^{***}	-11.1493^{***}
$h\theta_h$	4.31408	-498.796^{***}	-1.75052^{***}	-29.9663^{**}	2.28346
$h\theta_a$	32.3537^{***}	47.8377	0.356691	8.55397	1.59301
$a\theta_h$	18.5702	447.439^{***}	1.27031^{*}	20.2363	-3.5686
$a\theta_a$	-77.7543^{***}	-79.4616	0.0335362	8.32262	4.16891
$\theta_h\theta_a$	-6.09341	-72.9776	-0.39431	-1.1345	0.91514

Table 4.1: Multi-linear regression coefficients of each outcome measure on global social mechanism parameters and their interactions (after the middle line) from "Extreme Ideas Emerging from Social Conformity and Homophily: An Adaptive Social Network Model (2020) by Hiroki Sayama". The table was generated by using the data gathered from networks with 100 nodes (n = 100) and the statistically significant coefficients are represented with the asterisks conditions (***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$). The largest magnitudes of the social mechanism parameters' coefficients are indicated in bold with red and blue colours (positive and negative effects respectively), which proves that the homophily h and attention to novelty a are the most influential parameters on network formation.

Outcome measure	average edge weight	number of communities	modularity	range of average comm. states	std. of average comm. states
const.	-0.308221	18.12595	0.307062	6.189471	1.914619
c	-6.33660^{***}	-18.42221	-0.287898^{***}	-13.9918^{***}	-4.59748^{***}
h	$\mathbf{-11.84078}^{***}$	$\mathbf{195.5679}^{***}$	$\mathbf{2.48832}^{***}$	$\mathbf{48.44288}^{***}$	$\mathbf{9.2976}^{***}$
a	$\mathbf{100.51415}^{***}$	$\mathbf{-91.3585}^{***}$	$\mathbf{-1.432859}^{***}$	$\mathbf{-22.52786}^{***}$	$\mathbf{-4.927869}^{***}$
θ_h	-0.035693	-65.490634^{***}	-0.1584	-0.240368	1.23549
θ_a	-1.6442	24.916	0.193265	1.1039	-0.31498
ch	96.69496^{***}	-0.345	0.532479	5.86	4.21
ca	-195.310859^{***}	59.0283	1.112	32.50188^{*}	4.8379
$c\theta_h$	1.0369	15.989	-1.45978	-36.2608^{**}	-11.9785^{***}
$c\theta_a$	1.0684	23.5682	0.35779	8.79	3.278
ha	-233.836^{***}	-420.30547^{***}	-3.73378^{***}	-90.6992^{***}	-11.8069^{***}
$h\theta_h$	4.56897	-500.2286^{***}	-1.6662^{*}	-33.73479^{**}	1.59413
$h\theta_a$	31.31326^{**}	46.273665	0.42533	6.222842	0.48
$a\theta_h$	17.93373	448.08503^{***}	1.24033	19.62846	-3.58759
$a\theta_a$	-77.188639^{***}	-72.8	0.02733	6.69	3.95674
$\theta_h\theta_a$	-4.64979	-68.88077	-0.31864	-6.28543	-0.208238

Table 4.2: Multi-linear regression coefficients of each outcome measure on global social mechanism parameters and their interactions (after the middle line) obtained from the replicated model. The table was generated by using the data gathered from a network with 100 nodes (n = 100) and the statistically significant coefficients are represented with the asterisks conditions (***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$). The largest magnitudes of the social mechanism parameters' coefficients are indicated in bold with red and blue colours (positive and negative effects respectively), which confirms that the homophily h and attention to novelty a are also the most influential parameters on network formation in the replicated model by showing the same trends as the original MLR tabel 4.1 .

Outcome measure	average edge weight	number of communities	modularity	range of average comm. states	std. of average comm. states
const.	8.66%	1.06%	0.23%	2.72%	0.48%
c	0.17%	7.87%	7.92%	0.43%	1.54%
h	0.09%	0.33%	1.36%	0.53%	0.87%
a	0.45%	1.21%	0.75%	2.57%	1.87%
θ_h	**118.42**%	0.79%	2.19%	**158.34**%	22.31%
θ_a	2.74%	7.48%	6.45%	**85.94**%	42.04%
ch	0.08%	**149.8**%	6.90%	**120.69**%	5.88%
ca	0.48%	5.92%	8.79%	1.25%	0.27%
$c\theta_h$	9.95%	39.05%	2.10%	15.20%	4.08%
$c\theta_a$	43.03%	4.56%	14.97%	11.23%	1.42%
ha	0.49%	0.28%	1.28%	2.46%	5.72%
$h\theta_h$	5.73%	0.28%	4.93%	11.83%	35.55%
$h\theta_a$	3.26%	3.32%	17.55%	31.55%	**107.38**%
$a\theta_h$	3.48%	0.14%	2.38%	3.04%	0.53%
$a\theta_a$	0.73%	8.75%	20.39%	21.75%	5.22%
$\theta_h\theta_a$	26.87%	5.77%	21.22%	**138.84**%	**125.85**%

Table 4.3: Percentage differences between the coefficients of multi-linear regression Table 4.1 from "Extreme Ideas Emerging from Social Conformity and Homophily: An Adaptive Social Network Model (2020) by Hiroki Sayama" and the replicated MLR Table 4.2 generated from the replicated model data. The high percentage differences were coloured with red and occurred due to the absolute difference between coefficients being close to zero.

4.2 Extension Results

The Sayama model was used as a base model to conduct a further study by assessing the effects of nodes' individual social mechanism parameters on network formation [14]. To achieve this, the replicated model was first validated by comparing the replicated results with the original results from Sayama's paper represented in the previous section 4.2 to make sure that the model works correctly. Then, the replicated model is adjusted by assigning unique social mechanism parameters to all the agents in the network and new outcome measures were introduced in order to analyse the effects of the new model with heterogenous nodes on the network evolution.

The social mechanism parameter values (c, h and a) for each node are assigned with three different methods (uniform distribution [0,03,0.3], normal distribution [0,03,0.3] and random value choice from the set $V = \{0.03, 0.01, 0.1, 0.3\}$ described in section 3.6 in the initialisation of the network. The threshold parameters θ_a and θ_h have remained as global network parameters (all the nodes have the same θ_a and θ_h values) because of their low impact on network evolution confirmed by the non-significant coefficients in both original and replicated MLR results (see Tables 4.1 and 4.1).

After constructing the new model, the four node-level outcome measures (node's global eccentricity, node's within community eccentricity, node's community eccentricity and node's community size) and one network-level outcome measure (network's assortativity) mentioned in 3.6 are calculated by utilising the ten networks data generated for each parameter-assigning policy. This data was obtained by running the new model ten times with the network size 1000 (n = 1000) under the three parameter-assigning conditions. Then, the node-level outcome measures values were calculated for each node in the network as described in section 3.6. The calculated values from the ten networks generated for every three parameter-assigning methods are written into three individual `CSV` files along with each node's corresponding attribute values (c, h and a). For instance, the `CSV` file which contains the ten networks' data where the unique model parameters of the nodes are assigned with uniform distribution has a total of 10000 rows because each network has 1000 nodes thus, the ten networks results in $10 \times 1000 = 10000$ total number of nodes. Each row in this `CSV` file represents one single node in the network which stores the data of calculated outcome measure values and the node's social mechanism parameter values from the ten networks where the unique model parameters of the nodes are assigned with uniform distribution.

The MLR is utilised to analyse the effect of heterogenous nodes on the new outcome measures. The MLR coefficients are also calculated using the same package and functions as the replicated results. The MLR is applied to the data in the three `CSV` files by assigning outcome measures to the Y and node's social mechanism parameter values to X including the constant a at the start in the MLR equation 4.2.

The results of MLR for the new model are illustrated in 4.4, 4.5 and 4.6 tables where each represents the MLR coefficient values for each uniform distribution, normal distribution and random choice methods respectively. The coefficients in the table represent the significance of the individual social parameters' effects (before the middle line) and their interactions (after the middle line) on the new outcome measures (top header). The blue and red coefficient also indicates the most influential social mechanism parameters on the outcome measures. For instance, the unique conformity values of agents had a negative significant impact on the node's community eccentricity where it decreases the difference between the agent i's opinion state and i's community's average opinion state. The significance level is demonstrated by asterisks for all coefficient values of the parameters and their interactions using the same conditions as Sayama's MLR tables where ***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$. For instance, the unique conformity parameter c value of the node i in the network has a significant negative impact on i's community eccentricity outcome measure which indicates the difference between the average opinion state of i's community and the average opinion state of the entire network. This behaviour is observed from the bold blue coefficient values of parameter c on node's community eccentricity across all the tables 4.4, 4.5 and 4.6.

In contrast, for the analysis of the network-level outcome measure, network's assortativity, which represents the association between the nodes' social mechanism parameters, the same ten networks' generated for MLR analysis are used but the calculation process was different [18]. First, the ten networks generated for each parameter-assigning method with $n = 1000$ were combined to generate three networks with $n = 10000$ nodes. Then each node's parameters' combination (i.e. cc, ch and ca) is calculated by the Pearson Correlation coefficient function introduced in section 3.6 [18]. As explained in detail in section 3.6, to calculate the weighted and directed correlation values for each interaction in the network, each node i and its neighbours j looped through along with the weight w_{ij} which corresponds to the direction of the correlation between them. Then, for the three networks, nine correlation values were calculated based on the combinations of nodes' social mechanism parameters. Therefore, the three `CSV` files for each method with one row representing one network's assortativity values were generated to illustrate the assortativity values. The row values in these three `CSV` files wrote into a single `CSV` file mentioned in 3.6. As this outcome measure is network-level, in the final `CSV` file, the total three rows correspond to the three networks' assortativity values with $n = 10000$ for every three assigning methods.

The values in this `CSV` file were used to plot the bar chart provided in 4.3. These correlation coefficients of the network based on the three assigning methods were visualised by utilising `barplot()` function in`seaborn` package to plot bar charts [81, 82]. The bar chart 4.3 represents assortativity values of each network for every parameter-assigning method which are the magnitude and the direction (negative and positive value) of the correlation between the social mechanism parameters of the nodes plotted as bars in the Y-axis, on the three assigning methods in the X-axis. The standard error of each correlation combination on each bar is calculated by using `SEM()` function in `spicy.stats` library to assess the significance of the correlations across the parameter-assigning methods [76, 83]. The error bars are plotted from the calculated error values by using `errorbar()` function in `matplotlib` library [78, 84]. The standard error formula is presented below:

$$SE = \sigma/\sqrt{n} \tag{4.3}$$

where the σ is sample standard deviation and n is number of samples [85].

In the analysis of assortativity, the σ represents the standard deviation of the individual correlation values and the sample size n is ten where it indicates the ten network correlations for each of the three parameter-assigning methods.

The standard error allows us to determine whether the sample data reflect the whole population [85]. For instance, if the standard error is high, it means that the mean of the sample is widely distributed around the population mean which can be attributed to the results gathered by chance where the sample does not represent the whole population closely. In the visualisation, the overlapping standard error bars of correlation values across the three-parameter assigning methods indicate that the correlation is non-significant with a p-value greater than 0.05 and significant otherwise. Additionally, the error bars that are far from the zero axis also suggest that the assortativity value is significantly different from zero. This can be described by the one-sample t-test difference from zero. The mathematical formula is provided below [85]:

$$t = (\bar{X} - \mu)/(s/\sqrt{n}) \tag{4.4}$$

where $\bar{X}$ is mean of the samples, μ is zero and $(s/\sqrt{n})$ is the standard error [85].

This equation of the one-sample t-test is applied to each correlation combination where the mean of the sample is the mean of the ten networks' assortativity scores. The one sample t values are calculated by `ttest1samp()` function in `spciy.stats` library [76,86]. The calculated t-test values are compared to the t-table and the t-values correspond to the $p < 0.01$ illustrated by the blue asterisks next to the error bars [87]. Therefore, the blue asterisks represent the assortativity values that are significantly different from zero.

For instance, the hh values across all the parameter-assigning methods indicate that there is a positive correlation between the nodes with similar homophily parameter values. Even though the assortativity value is low the blue asterisk suggests that the value is significantly different from zero with $p < 0.01$. Moreover, the error bars of hh do not overlap across all the parameter-assigning methods this indicates that this correlation is not due to chance.

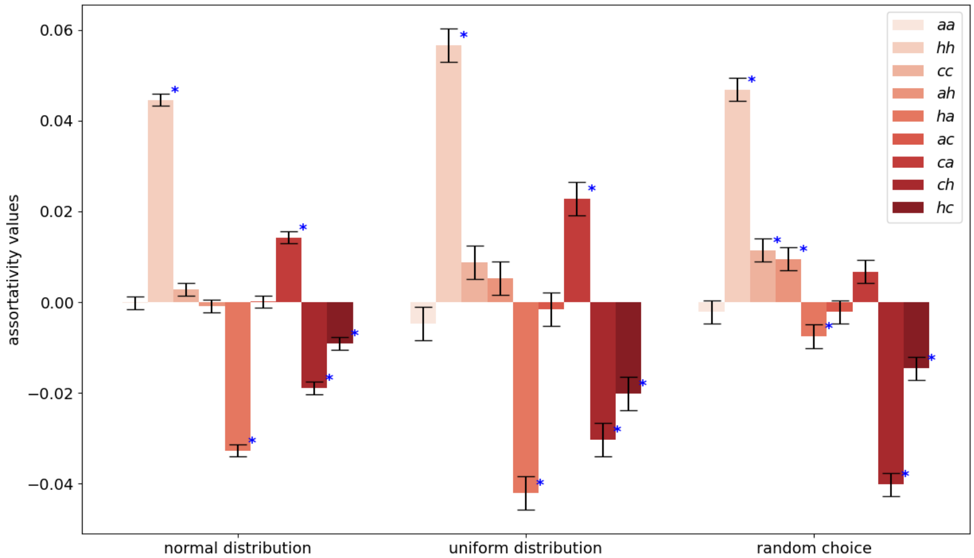

Figure 4.3: Assortativity values of three networks with size $n = 10000$ generated from the networks where the nodes' social mechanism parameters are assigned with three methods normal distribution [0.03,0.3], uniform distribution [0.03,0.3] and random value choice from the set $V \in 0.03, 0.01, 0.1, 03$. The bars represent the assortativity values (Y-axis) calculated by the Pearson correlation of nine social mechanism parameter combinations over the three-parameter assigning methods (X-axis). The error bars are illustrated by calculating the standard error of each correlation. The overlapping bars across the three-parameter assigning methods indicate that the correlation is not significant with $p > 0.05$ and significant otherwise with $p < 0.05$. The blue asterisks on the right-hand side of the error bars represent the assortativity values that are significantly different than zero with $p < 0.01$.

Outcome measure	node's global eccentricity	node's within comm. eccentricity	node's community eccentricity	node's community size
const.	1.78	1.23	1.05	337.05
c	-3.88^{***}	$\mathbf{-3.33}^{***}$	$\mathbf{-4.66}^{***}$	$\mathbf{166.26}^{***}$
h	$\mathbf{6.54}^{***}$	$\mathbf{4.29}^{***}$	-0.55	21.44
a	$\mathbf{-5.02}^{***}$	-0.18	$\mathbf{3.85}^{***}$	$\mathbf{-148.12}^{***}$
ch	7.32^{**}	6.23^{***}	16.03^{***}	-606.13^{***}
ca	0.91	8.46^{***}	2.36	-48.73
ha	-5.40^{*}	-15.96^{***}	-9.93^{***}	371.29^{***}

Table 4.4: Multi linear regression coefficients of heterogenous nodes' unique social mechanism parameters and their interaction (after the middle line) on outcome measures of the new model. The results were obtained from ten networks with size $n = 10000$ where the social mechanism parameters were assigned to each node in the network from a uniform distribution [0.03,0.3]. The statistically significant coefficients are represented with the asterisks conditions (***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$). The largest magnitudes of the social mechanism parameters' coefficients are indicated in bold with red and blue colours (positive and negative effects respectively), which represent unique c, h and a parameters of the nodes, playing a significant role in network formation.

Outcome measure	node's global eccentricity	node's within comm. eccentricity	node's community eccentricity	node's community size
const.	1.53	0.65	1.24	431.09
c	-3.07	$\mathbf{-0.96}^{**}$	$\mathbf{-1.38}^{***}$	$\mathbf{-349.70}^{**}$
h	$\mathbf{8.61}^{***}$	$\mathbf{7.86}^{***}$	-0.63	-168.16
a	$\mathbf{-6.28}^{***}$	-0.36	$\mathbf{1.19}^{**}$	$\mathbf{234.89}^{**}$
ch	19.61^{**}	-1.22	6.28^{***}	-1444.83^{**}
ca	-12.43	6.91	0.97	-453.76
ha	-7.53	-25.41^{***}	-2.97	-610.38

Table 4.5: Multi linear regression coefficients of heterogenous nodes' unique social mechanism parameters and their interaction (after the middle line) on outcome measures of the new model. The results were obtained from ten networks with size $n = 10000$ where the social mechanism parameters were assigned to each node in the network from a normal distribution [0.03,0.3]. The statistically significant coefficients are represented with the asterisks conditions (***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$). The largest magnitudes of the social mechanism parameters' coefficients are indicated in bold with red and blue colours (positive and negative effects respectively), which represent unique c, h and a parameters of the nodes, playing a significant role in network formation.

Outcome measure	node's global eccentricity	node's within comm. eccentricity	node's community eccentricity	node's community size
const.	1.79	1.29	0.92	430.18
c	$\mathbf{-3.89}^{***}$	$\mathbf{-2.67}^{***}$	$\mathbf{-3.76}^{***}$	$\mathbf{613.85}^{***}$
h	$\mathbf{3.67}^{***}$	$\mathbf{2.20}^{***}$	0.16	-25.22
a	-2.14^{***}	0.90^{***}	$\mathbf{4.40}^{***}$	$\mathbf{-727.11}^{***}$
ch	9.00^{***}	7.69^{***}	12.69^{***}	-2080.14^{**}
ca	-2.66	-0.57	-5.62^{***}	941.70^{***}
ha	2.50	-7.47^{***}	-8.52^{***}	1405.45^{***}

Table 4.6: Multi linear regression coefficients of heterogenous nodes' unique social mechanism parameters and their interaction (after the middle line) on outcome measures of the new model. The results were obtained from ten networks with size $n = 10000$ where the social mechanism parameters were assigned to each node in the network by randomly choosing a value from the set $V \in \{0.03,0.01,0.1,0.3\}$. The statistically significant coefficients are represented with the asterisks conditions (***: $p < 10^{-4}$, **: $p < 10^{-3}$; *: $p < 10^{-2}$). The largest magnitudes of the social mechanism parameters' coefficients are indicated in bold with red and blue colours (positive and negative effects respectively), which represent unique c, h and a parameters of the nodes, playing a significant role in network formation.

Chapter 5

Critical Evaluation

This chapter provides a comprehensive discussion and analysis aimed to advance the understanding of the results presented in the previous chapter, based on the research objectives. In the first two sections, both the original and replicated Sayama model results are restated to explain and interpret further. The third section explains the extension objectives and the findings of the extended experiment by comparing them with the original results. The final section provides a summary of the research's significance, along with potential directions for future work related to the recent literature.

5.1 Evaluation of The Sayama Model

In the Sayama paper, various results are presented based on the simulations of outcome measures and model parameter values to analyse the effect of each of the social mechanism parameters on the network structure [14]. In order to explain the findings and their meanings, this section first will go through linear (LR) and multi-linear (MLR) regression outcomes in Sayama's paper. Then, the replicated results will be compared with these original results.

5.1.1 Discussion of the Original Linear Regression Results

Sayama's paper employs the LR plots as a primary analysis method which is represented in Figure 4.1. LR helps to identify the most significant main effects of model parameters on each outcome measure by capturing the significant trends and plotting a regression line. Thus, these important trends in LR plots provided in Sayama's paper will be explained for each model paramater.

The first column of the LR plot 4.1 examines the impact of conformity parameter c on the outcome measures. This parameter refers to the tendency of individuals to conform to the social norms of their neighbours by gradually assimilating their opinion states to theirs. In other words, the global conformity parameter in the network determines the strength of the conformist behaviour which urges agents to adopt the opinions of their neighbourhood. As a result, the range and standard deviation of the average opinion state of the communities decrease in response to the increase in the global conformity value in the network, as demonstrated by their trend lines in the LR plot (see Figure 4.1). This behaviour occurs because when the global conformity value increases, it affects all agents in the network with high conformity value, hence, they try to conform to the opinions in their neighbourhood (i.e. the community where the agent belongs) which results in fewer variations between the opinion state values of agents in the community and the communities in the network. Additionally, the average edge weights measure also demonstrates a falling regression line because these less diverse opinions lead to a decrease in the strength of connections between agents, as individuals become more similar to one another. For instance, considering an agent i who expresses an opinion that is different from the rest of the community if the parameter c value is high in the network, it may affect all the agents in i's neighbourhood to adjust their opinions to be more similar to i.

Conversely, the number of communities and modularity do not display a significant trend line. This might be due to the low effect of the conformity parameter on network formation. Unlike the homophily h and attention to novelty a parameters, conformity parameter c does not directly participate in the weight update functions. Instead, it only affects the opinion state of the agents in the network. To simplify, the conformity parameter does not have the ability to directly weaken or strengthen the connections between its neighbours, it only tries to adopt their opinion states. Therefore, these two outcome measures may not be directly influenced by conformity because they are related to the network structure for community formation and community density, resulting in a non-significant effect of c values.

The second column of LR plots presents the influence of the homophily parameter h on the outcome measures. The plots show that there is a decreasing trend of the average edge weight while the modularity and number of communities increase. This behaviour indicates that parameter h which determines the rate of homophilic edge weight change, promoted the fragmentation of the network. Moreover, the rising regression lines of the range and standard deviation of average community states suggest that parameter h also accounted for the emergence of extreme opinions. These trends can be attributed to the homophily parameter's ability to strengthen the connections between the nodes that hold similar opinion states while weakening the connections with the ones that have distinct opinions. As the opinion state of the agents is randomly assigned in the network initialisation, there are more nodes with dissimilar opinions at the start, thus, the increase of homophilic behaviour in the network weakens the connections (detected by the decrease in average edge weight) between the nodes which leads to the emergence of a high number of small distant communities (detected by the increase in number of communities and modularity) with distinct opinion states (detected by the increase in range and standard deviation of the average community states). To further explain this behaviour, the opposite scenario can be considered where all the nodes in a network initially have the same opinion. In this case, a high level of homophily would cause nodes to form strong connections with each other, resulting in the formation of a single large community of nodes that share the same opinion. As a result, the network would not be divided into modules (i.e. communities) and no extremism will occur because of the same opinions.

Moving to the attention to novelty parameter a in the third column, which determines the rate of edge weight change by attention to novelty, exhibits an almost completely opposite impact on outcome measures to those of the homophily parameter h. Specifically, parameter a had a mitigating effect on the community formation as seen by the decreasing trend lines of the number of communities and the modularity subplots while the attention to novelty increases. Furthermore, the suppressing impact of the a on the increase of the extreme opinions in the network can be understood by the fall in range and standard deviation of average community states which is inversely proportional to the attention to novelty. These decreasing trends can be explained as, agents that seek novelty from interactions with others who have distinct opinions from them. Thus, these connections link the fragmented parts of the network, by breaking the communities and bringing back the agents inside them. The increase of various connections between the agents due to the high attention to novelty in the network, led to the increase of the edge weights resulting in a rising regression line in the average edge weight subplot.

The threshold parameters θ_h and θ_a located in the fourth and fifth columns subsequently, did not have a crucial impact on network formation compared to the homophily and attention to novelty. The reason behind this might be their less significant impact on the outcome measures after the opinions become diversified by the opinion update function because these threshold values are only involved in the helper functions F_a (see function 3.4) and F_h (see function 3.5) of the weight update.

5.1.2 Discussion of the Original Multi-linear Regression Results

Sayama's paper also utilised MLR analysis to quantify the influence levels of each model parameter on the outcome measures and their interactions representing the relationships between them. The results, as presented in Table 4.1 also highlighted the substantial impact of homophily h and attention to novelty a parameters as seen by the blue and red colours with the most significant p-values (see three asterisks condition described in 4.1). On the other hand, the MLR table revealed that the conformity parameter did not have significant coefficients, which is consistent with the observation from the LR plots where some of the trend lines do not show in the parameter c column. As found in LR trends, the MLR trends also confirm that a and h parameters had completely opposing effects on the evolution of the network shown by the negative and positive directions with blue and red colours respectively in the table. Moreover, after the middle line, some interactions of the parameters with asterisks indicate significant relations among the parameters, implying that their impact on the outcome measures is nonlinear. In essence, while each model parameter has its individual effect on opinion and community formation, their interactions and correlations also greatly influence these processes.

5.2 Evaluation of the Replicated Model

To conduct a further study on the Sayama Model from a heterogenous network perspective, the agents with unique homophily, conformity and attention to novelty attributes are introduced. The original model is replicated to be used as a base model and the results were compared to the original results to validate the replicated model. The replicated results for MLR and LR shown in 4.2 and 4.2 followed almost the same trends as the outcomes reported in the original study. These trends can be seen from almost the same regression lines in LR plots between the original 4.1 and replicated 4.2 plots. Moreover, to help the comparison of the MLR results the table 4.3 was generated by taking the percentage difference between original results and replicated ones which shows minor percentage differences in most of the rows. The values highlighted in red with the high percentage differences occurred because the absolute values of the original coefficients were too close to zero.

However, some nuances were observed in replicated results due to the sensitivity of the Sayama Model. This fragileness of the Sayama Model was noticed during the replication process where slight variations in the implementation caused different network formations. As mentioned earlier in the section 3.4, various approaches were tested to obtain the most similar results and minimise the differences.

One crucial aspect missing in Sayama's paper was the way of updating the opinions of the model (i.e. synchronously or asynchronously). Therefore, during the replication process, initially, the synchronous update process was followed, which allowed nodes to update their opinions by the opinion update rules before visiting another node-neighbour pair. However, this process caused early updates that affected the updates of other non-visited nodes in the network before completing each simulation, resulting in different outcomes than those reported by the original paper. To resolve this issue, the asynchronous update process was implemented, which updates all opinions in the network at the end of each simulation. This approach yielded similar trends in the LR and MLR results as reported in the Sayama paper.

The other unclear point in Sayama's paper relates to the node visiting order in the update process. In the replicated model, two different approaches were tested. The first approach involved visiting nodes in sequential order however, this caused biased results because the nodes were updating in the same order in each simulation hence, every simulation produced a similar network formation. The second approach was tested by shuffling the nodes before the update process, this resulted in more homogeneous outcomes where the nodes are randomised for the update process and results obtained for this process were more similar to the original results.

The nodes with zero neighbours where their edge weights between their neighbours are zero were also not mentioned as a case in the Sayama paper. These nodes caused the division to zero problems in the opinion update function which involves the division of edge weights of the nodes. Therefore, the conditions implemented in the replicated model where only the nodes that have neighbours are allowed to update their opinions and the weight updates occur only if the nodes' edge weights are greater than zero.

At what stage of the update process the stochastic value ϵ was added also did not mention in Sayama's model explanation however, the two approaches were tested where the stochastic effect added at the end of each simulation and the other one was added in each Δt, both ways did not give significant differences but the latter was chosen while constructing the replicated model.

5.2.1 Discussion of Replicated Results

After resolving the uncertain points of the implementation process in the Sayama Model, the replicated model was constructed and the network data was gathered, the LR and MLR analysis on this data were obtained using the same experimental settings as the original results generations.

One notable difference between the original and replicated results is observed in the third column of the LR table 4.2, where the θ_h did not plot a regression line on the modularity outcome measure. The reason behind this might be the low significance level of the θ_h values on the outcome measures. To clarify, as previously stated, the θ_h and θ_a values did not have a major effect on network evolution, but due to the sensitivity of the Sayama model, slight variations in the network formation could result in the complete loss of significance.

This observation can be supported by the one major distinction of the replicated MLR results in table 4.2 from original MLR outcomes, where the asterisks that represent the significance level of the model parameters on the outcome measures do not appear on θ_h and θ_a values (the fourth and fifth rows in the table respectively) except for the second column of the θ_h parameter. Therefore, just one trend line was generated in the replicated LR plots 4.2 which implies the significance of θ_h on the number of communities shown with the asterisks in the table 4.2.

In summary, the key findings of the Sayama model are social mechanism parameters h and a have the most significant influence on social network evolution with different directions where when individuals seek difference with attention to novelty the network becomes more homogenous by decreasing the fragmentation and extremism in the network. Whereas, when individuals seek sameness by homophily the network becomes more fragmented and extreme ideas emerge between the individuals. While threshold parameters are non-trivial. The replication is claimed to be successful where the main and crucial findings are the same as the replicated results' trends.

5.3 Evaluation of The Extension

In our society, every individual has unique characteristics [16, 17]. This states that the social world is heterogeneous. Thus, the extension aimed to improve on the limitations of the original model by studying the emergence of extremism in heterogeneous networks. In order to assess the heterogeneity of individuals the social mechanism parameters in the network should be distinct for every agent to have a more similar network that mirrors the actual world. This is achieved by assigning different social mechanism parameter values to agents referring to their unique conformity, homophily and attention to novelty attributes from three methods of uniform distribution [0,03,0.3], normal distribution [0.03,0.3] and random value choice from the set $V \in \{0.03, 0.01, 0.1, 0.3\}$. The threshold parameter values for θ_a and θ_h were fixed to 0.03 as mentioned previously in the experimental settings section 3.7 for the extension because of their low effects on network formation observed in the original results explained in the section above 5.1. The uniform and normal distribution methods generate continuous values for social mechanism parameters which were not assessed in the original Sayama model and were stated as future work in his paper to generalise the result trends (i.e. to examine if the continuous values will generate the same outcomes).

In the extension's analysis, node-level outcome measures are employed to investigate network formation from the perspective of individual nodes' different social tendencies. This approach offers a more detailed examination of extremism at the atomic level by analysing each individual's opinion and connections. To achieve this, the eccentricity measure is used to determine how each agent and their communities are distinct from others across the network. Besides the node-level measures, the assortativity of the network is studied as a network-level measure to assess the tendency of interactions between the agents to become strong or weak depending on their social mechanism parameter values. This allows for an investigation of the correlation between the unique social mechanism parameters of the nodes and their effects on overall network evolution. To clarify, the assortativity measures examine the interaction formations based on the nodes' individual social mechanism values instead of their opinion states.

These correlations could not assess in Sayama's paper because social mechanism parameters were the same for each node in the network. For instance, the correlation ch in the assortativity outcome measure indicates the relationship between unique conformity and homophily parameters of the nodes and can answer the question of how this correlation effect the node's positions in the network (i.e. do nodes with similar homophily and attention to novelty values end up together in the network) and how these positions affect the overall network structure. Therefore, it accounted as a network-level measure which investigates the assortativeness (i.e. homogeneity and heterogeneity) of the overall network. Assortativity can also assess the correlation of the parameters themselves (i.e. hh) such as whether agents with high homophily values tend to connect with others who also have high homophily values in the network. Furthermore, unlike the original analysis of these interactions, which was unidirectional, assortativity can evaluate both directions such as ch and hc by taking the in-coming and out-going edge weights of the node. To simplify, the assortativity measure can provide valuable insight into the network's overall homogeneity based on the correlations of unique model parameters of the nodes and their effects on the network formation. In this section, the analysis of these outcome measures is provided by first explaining the MLR tables (see Tables 4.4, 4.5 and 4.6) that represent node-level measures, followed by the observations of the bar chart (see Figure 4.3) generated for the assortativity.

5.3.1 Node-Level Analysis of the New Model

The MLR coefficient values in tables 4.4, 4.5 and 4.6 represent each parameter assigning methods (uniform distribution [0.03,0.3], normal distribution [0.03,0.1] and random value choice from the set $V \in \{0.03, 0.01, 0.1, 0.3\}$, subsequently) used for the extension process explained earlier in the section 3.7 on node-level outcome measures.

The trends across all the tables indicate that the unique conformity values of the nodes tend to decrease the extreme ideas of individuals within the network (captured by the significant negative coefficients of the within community eccentricity and node's community eccentricity highlighted with the blue) which was not the case in the original results where global conformity parameter of the nodes did not play a crucial role on the overall network evolution (see MLR table provided in Sayama's paper 4.1). These trends can be explained by the fact that agents who are more conformist tend to conform to others in their local neighbourhood by trying to adopt their social norms and become more similar to them. In these outcome measures the community where this agent belongs refers to its local neighbourhood hence, the behaviour of conformity might decrease the distinction between the opinion states of the agents, reducing the eccentricity of opinions within the community. When the opinions get less diverse in the communities by the conformist agents the opinions become more homogeneous inside them. This results in an overall decrease in the average opinion state of the network by the opinions inside the communities thus, a smaller eccentricity of the node's community with respect to the average opinion state of the network.

In contrast, unique homophily values had a positive impact on the node's within community eccentricity and node's global eccentricity, indicating that nodes with strong homophilic behaviour tend to have opinions that are farther away from their community average and the average opinion state of the whole community. This result indicates the similar homophilic behaviour detected at the network level in Sayama's MLR results (see Table 4.1) where homophily supported extremism in the network. The node-level observation of these trends can be, the agent i that shares similar general opinion as others in its community, i's homophily strength may still be greater enough to only strengthen its connections with agents in the community who are almost the same as them while weakening their connections with others who hold even slightly distinct opinions from theirs. This causes the to differentiate themselves from the community average and get more isolated from others due to the less diverse opinions in the network.

This behaviour has the potential to divide the community into different sub-communities, resulting in highly polarised networks with an increasing number of small distinct communities. However, the fragmentation in the network was not clear which is measured by the node's community size. This measure helps to investigate the effect of social tendencies on the fragmentation in the network by assessing if the social mechanism parameter values of the nodes support homogeneity by breaking their communities or promoting heterogeneity by isolating their communities. The coefficients of the node's community size varied between parameter-assigning methods and model parameters (c and a). This measure can be sensitive because the community structure remains unclear regarding the distribution of the agents with different social tendencies in the community. For instance, the community might have more homophilic agents inside it, this might cause the community to be smaller where the homophilic behaviour creates more distinct communities. Whereas the community where the majority of agents seek novelty, this might increase the community size by the ability of attention to novelty parameter breaking the communities and combining them to create a more homogenous network. Therefore, to fully understand this behaviour in community interactions, the structures of the communities in the network should be further studied such as using community-level data.

Attention to novelty parameters also consistently exhibited a positive effect across all the parameter-assigning methods by increasing the node's community eccentricity. This behaviour of parameter a indicates that nodes with higher levels of attention to novelty tend to have distinct opinions from the average opinion of their community and are more likely to explore different ideas. In other words, the nodes that seek novelty may not find the opinions in the majority of their community as novel as others in the network due to the community formation process where like-minded agents come together. Therefore novelty seeking agents be more likely to strengthen their connections with other agents in the network who have more novel ideas than others in their community while weakening their edge weights within the community. As a result, the agent might be separated from its community, and the community can be more isolated where the community state becomes more eccentric from opinions across the network.

Additionally, the variation between the conformity c and attention to novelty a parameters is also captured across the parameter-assigning methods on the node's global eccentricity. As seen on the uniform and normal distribution tables 4.4, 4.5, parameter a shows a significant negative correlation on this measure whereas, in the random choice table 4.6 the c parameter's negative effect is more significant than attention to novelty. This might be caused by the similar behaviour of these parameters which was also detected in LR plots in Sayama results (see Figure 4.1). The conformist node tends to conform with its neighbour irrespective of their opinion state (i.e. the opinion state of its neighbours can be more distinct or similar to theirs), this behaviour of c makes their opinion state closer to the others in the network. The attention to novelty parameter a also has the ability to make the network less eccentric because they tend to update their opinions with different opinions from them. Therefore both parameters can decrease the average opinion state of the network by strengthening their connections with distinct others in the network, as a result of these interactions the network becomes more homogenous thus, their opinion state becomes less eccentric from the network's average opinion state. However, the strength of their negative effect on this outcome measure might vary due to the wide range of unique c and a parameter values of the node. For instance, in the generated network there might be more agents with conformist behaviour and less attention to novelty strength due to the random parameter assigning process from a wide range of values.

The extension results also revealed certain associations between the model parameters, as evidenced by the results presented in tables 4.4, 4.5, and 4.6 after the middle line, indicating that the impact of model parameters is not linear. This finding was also reported in the original study. These are further investigated by calculating their assortativity between the model parameters.

To summarise the node-level outcome measures' results, all c, h and a parameters showed significant effects on formations in the network by having similar behaviours at the network level analysis in Sayama results. However, some uncertain trends (i.e. node's community size) and conflicts between Sayama's observations (i.e. the significance of the conformity parameter) might be caused by the ambiguousness of node-level measurements where the data varies a lot between the social mechanism parameters for each node in the network. Therefore, instead of examining the network on the atomic level, these trends can be better understood with community-level data and analysis by investigating the community characteristics and structures in the overall network.

5.3.2 Network-Level Analysis of the New Model

For the network-level analysis of the new model, the assortativity measure is utilised to investigate whether the network is homogenous or heterogenous by examining the interaction behaviours of the nodes based on the correlations between their individual social mechanism parameters.

The assortativity of the weighted directed networks generated for the new model's analysis was calculated by the Pearson Correlation [18].

The assortativity results (see Figure 4.3) suggested that the association between the model parameters in the networks generated for the extension are not highly effective on the agents' interactions as the range of assortativity values are between -0.04 and 0.06. However, the standard error bars in bar charts that are far from the zero axis indicated that these values might be different from zero. To investigate this further, one sample t-test for the difference from zero is calculated. The t-test values of the correlations are compared to the significant t-value table for the data frame size nine (the nine correlations) and $p < 0.01$. Therefore, the asterisks next to the right side of the error bars in blue are illustrated and suggest that the correlation values are significantly different from zero with $p < 0.01$. The error bars that are close to the zero axis with no asterisks indicate that the correlations of the model parameters do not affect the connections of the nodes. In other words, there is a low tendency for nodes to connect to each other based on the similar values of their unique social mechanism parameters which indicates the node interactions might just be affected by their opinion states. To further understand the assortativity measure, one can consider an example correlation value, such as ca. If the ca assortativity value for a network approaches +1 this means that the network is perfectly assortative by all agents with high conformity value where they tend to strengthen their connections with agents who have high attention to novelty.

Alternatively, the ca assortativity value for a network approaches -1 this indicates that the network is perfectly disassortative by all the agents with high conformity value tend to weaken their connections with the agents who have low attention to novelty.

Finally, if the *ca* assortativity value is close to zero this means that the network is naturally assortative where the conformist agents form connections irrespective of the attention to novelty values of other agents.

Considering two nodes i and j where i has high conformity c and j has a high attention to novelty a. The assortativity value of *ca* calculated by the Pearson correlation equation introduced in (Yuan et. al) paper determined the strength of the agent i's connection with its neighbour j who has the same attention to novelty value as i's conformity value based on the edge weight from i to j (w_{ij}). If the w_{ij} is strong, it indicates that the *ca* correlation is positive. Alternatively, i's neighbour k with low attention to novelty, this time strong w_{ik} indicates a negative correlation. If all the nodes in the network show the same behaviours as agent i, the perfect positive or negative assortativity is reached in the network by conformist agents. Therefore, this measure is related to the similarity of the unique social mechanism parameters' strengths between the agents.

Even though the magnitudes of the correlation values calculated are small, some common trends are captured across the parameter-assigning methods where the assortativity values are significantly different from zero.

The highest significant correlation values are detected in the homophily parameter where the one-way interactions, from the agents with homophily parameter values that are identical to their neighbours' homophily (*hh*), attention to novelty (*ha*) and conformity (*hc*) parameters' values. In the case of *hh*, agents with similar homophily values tend to increase the edge weights in their interactions. This correlation can be attributed to homophilic behaviour, where agents tend to strengthen their connections with those who hold similar opinions. Consequently, agents with similar homophily values end up in the same neighbourhood due to their similar opinions. These interactions of the homophilic agents might cause the emergence of fragmented communities resulting in a more assortative network.

By contrast, the negative correlation in the one-way interactions from the homophilic agents to conformist agents (*hc*) can be attributed to the opposite behaviour of the homophily which weakens the connections with its neighbours who have distinct opinions from them. In this case, conformist agents try to adopt the opinions of the agents regardless of the differences in the opinions. This causes the homophilic agent i to decrease their outgoing edge weights w_{ij} to the conformist agent j if j' neighbourhood has dissimilar opinions from i.

The explanation of the negative correlation *ha* can be attributed to the opposite behaviours of the homophily and attention to novelty parameters in the connection formations. The novelty-seeking agents are more likely to be in a neighbourhood with agents who have distinct opinions of them because they tend to strengthen their connections with the agents who have different opinions states with them. Whereas, the homophilic agents tend to weaken their connections with the agents with dissimilar ideas. Therefore, homophilic agent i might break their outgoing connections to the novelty-seeking agent j if j's neighbourhood does not have similar opinions with i, thus both agents might end up in completely different neighbourhoods.

Overall, these findings suggest that except for the homophily parameter's correlations other correlations of the model parameters do not affect the interactions of the nodes because the range of assortativity showed values that are closer to zero (captured with the error bars that have no asterisks across all the parameter-assigning methods). Additionally, most of the correlation values did not reveal statistically significant values (captured by the overlapping error bars of the correlations across all parameter-assigning methods) which indicates the correlations might be due to chance. These might be caused by the opinion states are the only values that affect the node interactions or the network-level data being too wide, thus the data varies a lot. As, also stated by researchers in their paper where the assortativity measure used in this research is proposed, if the sample size is too big (i.e. network with a lot of nodes) the assortativity values will be closer to zero [18]. Additionally, it should be noted that the correlation between the model parameters may vary depending on the distribution of attributes among nodes. In other words, assortativity is a complex measure and these correlations generated by the assortativity measure used only consider individual social parameters of the nodes, however, every node also has its unique attention to novelty, homophily and conformity values. Therefore, the correlation between the two attributes might be affected by other characteristics of the nodes. For instance, the correlation of *ch* can also depend on the conformist agent's homophily and attention to novelty characteristics because if the agent is conformist and homophilic but does not seek novelty, they might conform to the agent's where their ideas are similar to them which affects this correlation by strengthen their connections with the homophilic agents, resulting in a positive correlation. Therefore, these correlations between model parameters can be further studied by decreasing the sample size (i.e. creating new networks with fewer nodes or focusing on the smaller parts of the network such as communities) and also considering all the social mechanism parameters of the agents when calculating the assortativity.

5.4 Key Findings & Future Work

The primary conclusion drawn from this research is Sayama's model served as a useful base model by introducing the key concept of social tendencies that cause extremism and fragmentation in the overall network. However, due to the unrealistic homogenous social characteristics of the agents in the model, the addition was applied to understand the effect of unique characteristics of the individuals in the real world on the emergence of extreme ideas and polarised groups in society.

The general observation made from the new model suggests these diverse social tendencies can affect network formation significantly by influencing within-network interactions. The behaviours of the unique node parameters can be explained as when individuals only try to conform to their neighbours in the network through their unique conformity values, the network becomes more homogenous with less extreme ideas where the agent gets closer to their community states by trying to conform to the ideas inside them. This effect is supported by all the conformist agents in other communities who also mix the ideas across the communities and lower the distinction of the community states from the entire network. In contrast, when individuals seek sameness in the network their individual opinions become more isolated and extreme from the ideas in their local neighbourhood and the entire network. Whereas, when individuals seek difference and novelty, they increase the extremism of their community state in response to the opinions of the rest of the network by losing connections with like-minded others in their community and their community becomes isolated by only having agents with almost the same ideas inside it. Additionally, the network becomes more assortative (homogenous) by the correlations of the homophily parameter where the homophilic agents tend to strengthen their connections with other homophilic agents and the conformist agents. Whereas, they tend to weaken their connections with novelty-seeking agents. This behaviour of homophily promotes fragmentation in the network by connecting with not only like-minded others but also with the ones that have similar social tendencies while losing connections with distinct others.

Overall, these results suggest that when individuals try to conform to others they tend to go less extreme and prevent extremism emergence within their community and across the network. Whereas the individuals seek sameness their ideas are getting more isolated from the others and when they seek difference they affect their local neighbourhood by differentiating them from the others, urging them to form a more isolated community. Comparing the heterogenous social tendencies to the homogenous ones in the original Sayama Model, the effect of homophily remained the same which also promote extremism of individual nodes within network interactions, while attention to novelty showed the opposite effect from the original model where the agents that seek difference, promoted the extremism of the communities in the network by differentiating their community from the entire network. These conflicts between the heterogenous and homogenous nodes suggest that the unique social tendencies of agents have the ability to control extremism with their individual interactions inside the communities and across the entire network.

The individual effect of unique parameters of the nodes on the fragmentation remained uncertain. (captured with the uncertain significant coefficient values in nodes' community size outcome measure in MLR tables). However, the correlation of homophily parameters to itself and other node parameters suggests that the homophily behaviour has the ability to promote fragmentation in the network where they get closer to the homophilic agents and the conformist agents. Whereas, they weaken the connections with the novelty-seeking agents (captured with the blue asterisks next to the error bars of homophily correlations). The impact of other social mechanism correlations remained low which might be due to the sample size and the variation of the data gathered from the individuals' distinct social tendency values.

However, the extended model has certain limitations that require further investigation. The explanation of the non-linearity between the model parameters, specifically the non-significant low assortativity values, and some behaviour in node-level analysis such as the community sizes remains unclear. Therefore, alternative approaches, such as utilising community-level data and further statistical analyses, can be useful to study the correlation between the model parameters and the MLR analysis to further understand the generation of extremism and address uncertain behaviours in node-level analysis. Unfortunately, these methods are computationally expensive, thus the model can be replicated on a different platform.

As this research tries to investigate the polarisation and the emergence of extreme ideas of individuals, the model can be adjusted and new features can be added to better reflect real-world interactions.

Our society is a complex system and has a lot of interdependencies. In the real-world everyone holds opinions on various topics. The model can be improved by adding multiple opinion states to agents. This might be helpful in investigating the multi-dimensional opinions' effect on interactions between agents. For instance, considering an agent-neighbour pair i, j that has different opinion states (i.e. i is conservative and j is liberal), they might still have some common opinions such as they support the same football team which can influence their interactions in the network. Moreover, the update functions in the network model can be tailored to each agent to account for the unique responses and changes in behaviour observed in the real world [88]. Multiple homophily types can also be incorporated to investigate the impact of heterogeneity in society. Recent observations by sociologists suggest that different homophily types such as age, gender and education affect the interactions of individuals by the research results stated in the paper; in American schools, students tend to be friends with others who are the same race and age as them [89]. Another research states that people change their behaviours and feelings based on the social status of others in society [90]. Therefore, multiple homophily values can be helpful in understanding the extremism generation in the network and the correlation between similar homophily tendencies can be further investigated by using a different assortativity calculation process. Furthermore, the update functions can be tailored to each agent to account for the unique responses and changes in behaviour observed in the real world.

Validation of the results observed in the extended network can be performed using different and less sensitive base models and calculation methods such as different assortativity equations. Alternatively, as described in Chapter 2, Section 2.2, the fields of social network analysis (SNA) and opinion dynamics (OD) can complement each other, and SNA can be integrated into OD research. By using real-world data collected from online social networks such as Twitter, the process of network formation can be studied and validated, rather than relying solely on data generated in OD. SNA can be employed to investigate the formation of the network using this data.

In summary, the purpose of this research was to examine how the unique social tendencies of individuals affect the emergence of fragmentation and extreme ideas in society which causes problems such as political disputes, racism, sexism and many others. Therefore, this research tried to understand if OD networks can help identify the social mechanisms that contribute to extremism generation and develop theories to reduce polarisation in society without a need for sociologists from a mathematical and computational perspective. The results of the extended model indicate that there are certain insights that can be gathered regarding the emergence of extremism in society based on individuals' unique characteristics and interactions. The observations of the heterogenous model can be useful to determine how individuals should behave under different circumstances to achieve their goals. For instance, if a person wants to arrive at a consensus, they might need to conform more to their peers' ideas. Alternatively, a politician who wants to win an election might show less homophilic behaviour in their campaigns which may help to keep connections also with the people who do not have the same ideology as them in society. However, the field of OD is still far behind the stage to mirror the complexity of society adequately thus, further research is necessary. The first step in this direction is to improve the current model by addressing the limitations and incorporating additional features.

Chapter 6

Conclusion

In this project, a new Opinion Dynamics (OD) model is proposed by building on one of the recent models by Hiroki Sayama to improve on the limitations [14]. The objective of the new model was to investigate the emergence of extreme opinions and polarised groups in society from the perspective of heterogeneous social tendencies of individuals. In order to construct the new model, the original model is successfully replicated and validated by comparing the results gathered from the replicated model and the original model. The replicated model is modified by adding a new feature to the mode to investigate the effect of individuals' unique characteristics on the network formation. The new model has been analysed through the node and network level perspectives using statistical methods and network science concepts. The results of the new model suggest that the unique conformity behaviour of individuals prevents them to go extreme by conforming to the opinions of their local neighbourhood which has the ability to decrease the extremism among individuals and their interaction within the communities. On the other hand, homophilic agents become more extreme than others within their community and the entire network. Additionally, homophilic agents promoted fragmentation in the network by strengthening their connections with homophilic others while weakening their connections with those seeking difference. Therefore this states that homophilic agents not only promote fragmentation in their connections with agents that have similar opinion states but also in their interactions with the ones that have similar social tendencies. These trends can be attributed to the behaviour of homophily where individuals get closer to similar individuals and lose connections with the distinct ones. However, some results of the new model remained unclear such as community size and some low assortativity values. These uncertainties can be caused by the wide range of social tendency values in the network which might cause variations in the data gathered from the networks. Furthermore, the new model is still quite limited to reflect society due to its complexity and a high number of interdependencies such as individuals' multiple opinions.

OD field is still far from realistically mirroring the real-world interactions to observe the formation of society [14]. However, the new model introduced in this project can provide some insights to prevent extreme opinions and fragmentation in society by guiding individuals on how to behave in certain situations.

Bibliography

[1] D. Nikolov, D. F. Oliveira, A. Flammini, and F. Menczer, "Measuring online social bubbles," *PeerJ computer science*, vol. 1, p. e38, 2015.

[2] D. Spohr, "Fake news and ideological polarization: Filter bubbles and selective exposure on social media," *Business information review*, vol. 34, no. 3, pp. 150–160, 2017.

[3] T. M. Newcomb, R. H. Turner, and P. E. Converse, *Social psychology: The study of human interaction.* Psychology Press, 2015.

[4] G. Robins and Y. Kashima, "Social psychology and social networks: Individuals and social systems," *Asian Journal of Social Psychology*, vol. 11, no. 1, pp. 1–12, 2008.

[5] J. F. Markey, "A redefinition of social phenomena: giving a basis for comparative sociology," *American Journal of Sociology*, vol. 31, no. 6, pp. 733–743, 1926.

[6] R. M. FARR, "Experimentation: a social psychological perspective," *British Journal of Social and Clinical Psychology*, vol. 15, no. 3, pp. 225–238, 1976.

[7] R. S. Cohen, *The natural sciences and the social sciences: some critical and historical perspectives.* Springer Science & Business Media, 2013, vol. 150.

[8] K. Moon and D. Blackman, "A guide to understanding social science research for natural scientists," *Conservation biology*, vol. 28, no. 5, pp. 1167–1177, 2014.

[9] J. A. Hołyst, K. Kacperski, and F. Schweitzer, "Social impact models of opinion dynamics," *Annual Reviews Of Computational PhysicsIX*, pp. 253–273, 2001.

[10] J. C. Mitchell, "Social networks," *Annual review of anthropology*, vol. 3, no. 1, pp. 279–299, 1974.

[11] A. Sîrbu, V. Loreto, V. D. Servedio, and F. Tria, "Opinion dynamics: models, extensions and external effects," *Participatory sensing, opinions and collective awareness*, pp. 363–401, 2017.

[12] A. Abraham, *Computational social networks: Mining and visualization.* Springer Science & Business Media, 2012.

[13] H. Sayama, I. Pestov, J. Schmidt, B. J. Bush, C. Wong, J. Yamanoi, and T. Gross, "Modeling complex systems with adaptive networks," *Computers & Mathematics with Applications*, vol. 65, no. 10, pp. 1645–1664, 2013.

[14] H. Sayama, "Extreme ideas emerging from social conformity and homophily: An adaptive social network model," in *ALIFE 2020: The 2020 Conference on Artificial Life.* MIT Press, 2020, pp. 113–120.

[15] M. E. Newman and M. Girvan, "Finding and evaluating community structure in networks," *Physical review E*, vol. 69, no. 2, p. 026113, 2004.

[16] M. Dwairy, "Foundations of psychosocial dynamic personality theory of collective people," *Clinical Psychology Review*, vol. 22, no. 3, pp. 343–360, 2002.

[17] A. Furnham, "Personality and activity preference," *British Journal of Social Psychology*, vol. 20, no. 1, pp. 57–68, 1981.

[18] Y. Yuan, J. Yan, and P. Zhang, "Assortativity measures for weighted and directed networks," *Journal of Complex Networks*, vol. 9, no. 2, 2021.

[19] H. Cravens, "History of the social sciences," *Osiris*, vol. 1, pp. 183–207, 1985. [Online]. Available: http://www.jstor.org/stable/301732

[20] D. Papineau, *For science in the Social Sciences.* MacMillan, 1987.

[21] R. Outhwaite, "Positivism, sociological," *International Encyclopedia of the Social & Behavioral Sciences: Second Edition*, 2015.

[22] J. A. Schuster, *The Scientific Revolution.* Routledge, 1996, p. 26.

[23] S. E. Toulmin, "The evolutionary development of natural science," *American Scientist*, vol. 55, no. 4, pp. 456–471, 1967.

[24] A. Comte and G. Lenzer, *Auguste Comte and positivism the essential writings.* Transaction Publishers, 2010.

[25] S. Lieberson and F. B. Lynn, "Barking up the wrong branch: Scientific alternatives to the current model of sociological science," *Annual Review of Sociology*, vol. 28, no. 1, p. 1–19, Aug 2002.

[26] H. G. Small and D. Crane, "Specialties and disciplines in science and social science: An examination of their structure using citation indexes," *Scientometrics*, vol. 1, pp. 445–461, 1979.

[27] D. Konstan, *The emotions of ancient greeks: Studies in Aristotle and classical literature.* University of Toronto Press, 2006.

[28] H. P. Becker, J. Gillin, A. I. Hallowell, G. P. Murdock, T. M. Newcomb, T. Parsons, and M. B. Smith, *Psychology and Sociology.* The Macmillan Company, 1954, p. 67–101.

[29] K. J. Gergen and M. M. Gergen, *Social psychology.* Springer Science & Business Media, 2012.

[30] C. Sansone, C. C. Morf, and A. T. Panter, *The Sage handbook of methods in social psychology.* Sage Publications, 2003.

[31] J. Norris, M. Pratt, and S. Hebblethwaite, "Social cognition," 2007.

[32] R. G. Morris, L. Tarassenko, and M. Kenward, *Cognitive Systems-Information Processing Meets Brain Science.*

[33] R. Kitchin, "Big data, new epistemologies and paradigm shifts," *Big data & society*, vol. 1, no. 1, p. 2053951714528481, 2014.

[34] A. Edelmann, T. Wolff, D. Montagne, and C. A. Bail, "Computational social science and sociology," *Annual Review of Sociology*, vol. 46, pp. 61–81, 2020.

[35] J. A. Barnes, "Class and committees in a norwegian island parish," *Human relations*, vol. 7, no. 1, pp. 39–58, 1954.

[36] P. Jia, A. MirTabatabaei, N. E. Friedkin, and F. Bullo, "Opinion dynamics and the evolution of social power in influence networks," *SIAM review*, vol. 57, no. 3, pp. 367–397, 2015.

[37] R. J. Trudeau, *Introduction to graph theory.* Courier Corporation, 2013.

[38] S. Tabassum, F. S. Pereira, S. Fernandes, and J. Gama, "Social network analysis: An overview," *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*, vol. 8, no. 5, p. e1256, 2018.

[39] S. Wasserman and K. Faust, "Social network analysis: Methods and applications," 1994.

[40] S. Wasserman, *Advances in social network analysis: Research in the social and behavioral sciences.* Sage, 1994.

[41] A. Clifton and G. D. Webster, "An introduction to social network analysis for personality and social psychologists," *Social Psychological and Personality Science*, vol. 8, no. 4, pp. 442–453, 2017.

[42] A. Peralta, J. Kertész, and G. Iñiguez, "Opinion dynamics in social networks: From models to data. arxiv 2022," *arXiv preprint arXiv:2201.01322*.

[43] *American Psychological Association.* [Online]. Available: https://www.apa.org/education-career/guide/subfields/social

[44] R. B. Cialdini and N. J. Goldstein, "Social influence: Compliance and conformity," *Annu. Rev. Psychol.*, vol. 55, pp. 591–621, 2004.

[45] M. Hechter and K.-D. Opp, "Social norms," 2001.

[46] H. Xia, H. Wang, and Z. Xuan, "Opinion dynamics: A multidisciplinary review and perspective on future research," *International Journal of Knowledge and Systems Science (IJKSS)*, vol. 2, no. 4, pp. 72–91, 2011.

[47] F. Tuli, "The basis of distinction between qualitative and quantitative research in social science: Reflection on ontological, epistemological and methodological perspectives," *Ethiopian Journal of Education and Sciences*, vol. 6, no. 1, 2010.

[48] R. Ormston, L. Spencer, M. Barnard, and D. Snape, "The foundations of qualitative research," *Qualitative research practice: A guide for social science students and researchers*, vol. 2, no. 7, pp. 52–55, 2014.

[49] A. Almaatouq, A. Noriega-Campero, A. Alotaibi, P. Krafft, M. Moussaid, and A. Pentland, "Adaptive social networks promote the wisdom of crowds," *Proceedings of the National Academy of Sciences*, vol. 117, no. 21, pp. 11 379–11 386, 2020.

[50] T. Gross and H. Sayama, "Adaptive networks, adaptive networks," 2009.

[51] P. Clifford and A. Sudbury, "A model for spatial conflict." [Online]. Available: https://academic.oup.com/biomet/article-abstract/60/3/581/217208

[52] A. Sî rbu, V. Loreto, V. D. P. Servedio, and F. Tria, "Opinion dynamics: Models, extensions and external effects," in *Understanding Complex Systems*. Springer International Publishing, may 2016, pp. 363–401. [Online]. Available: https://doi.org/10.1007%2F978-3-319-25658-0_17

[53] R. J. Baxter, *Exactly solved models in statistical mechanics.* Academic Press, 1990.

[54] B. Latané, "The psychology of social impact." *American Psychologist*, vol. 36, no. 4, p. 343–356, 1981.

[55] K. SZNAJD-WERON and J. SZNAJD, "Opinion evolution in closed community," *International Journal of Modern Physics C*, vol. 11, no. 06, p. 1157–1165, 2000.

[56] M. T. Gastner, B. Oborny, and M. Gulyás, "Consensus time in a voter model with concealed and publicly expressed opinions," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2018, no. 6, p. 063401, 2018.

[57] C. Castellano, S. Fortunato, and V. Loreto, "Statistical physics of social dynamics," *Reviews of Modern Physics*, vol. 81, no. 2, p. 591–646, 2009.

[58] V. Sood and S. Redner, "Voter model on heterogeneous graphs," *Physical Review Letters*, vol. 94, no. 17, 2005.

[59] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, "Mixing beliefs among interacting agents," *Advances in Complex Systems*, vol. 03, no. 01n04, p. 87–98, 2000.

[60] R. Hegselmann and U. Krause, "Opininion dynamics and bounded confidence models," *Journal of Artifical Societies and Social Simulation (JASSS) vol.5, no. 3, 2002*, vol. 5, 2002.

[61] R. Axelrod, "The dissemination of culture: A model with local convergence and global polarization," *The Journal of Conflict Resolution*, vol. 41, no. 2, pp. 203–226, 1997. [Online]. Available: http://www.jstor.org/stable/174371

[62] P. MacCarron, P. J. Maher, S. Fennell, K. Burke, J. P. Gleeson, K. Durrheim, and M. Quayle, "Agreement threshold on axelrod's model of cultural dissemination," *PLOS ONE*, vol. 15, no. 6, 2020.

[63] M. H. Degroot, “Reaching a consensus,” *Journal of the American Statistical Association*, vol. 69, no. 345, p. 118–121, 1974.

[64] D. Acemoglu and A. E. Ozdaglar, “Opinion dynamics and learning in social networks,” *SSRN Electronic Journal*, 2010.

[65] P. Frasca, S. Tarbouriech, and L. Zaccarian, “Hybrid models of opinion dynamics with opinion-dependent connectivity,” *Automatica*, vol. 100, p. 153–161, 2019.

[66] A. C. MARTINS, “Continuous opinions and discrete actions in opinion dynamics problems,” *International Journal of Modern Physics C*, vol. 19, no. 04, p. 617–624, 2008.

[67] M. He, “Incorporating latent psychological factors and social interaction in a new generalized heterogeneous data model (ghdm),” 2020.

[68] H. Cherifi, G. Palla, B. K. Szymanski, and X. Lu, “On community structure in complex networks: challenges and opportunities,” *Applied Network Science*, vol. 4, no. 1, pp. 1–35, 2019.

[69] V. D. Blondel, J.-L. Guillaume, R. Lambiotte, and E. Lefebvre, “Fast unfolding of communities in large networks,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2008, no. 10, 2008.

[70] [Online]. Available: https://www.python.org/downloads/release/python-370/

[71] [Online]. Available: https://networkx.org/

[72] [Online]. Available: https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.louvain.louvain_communities.html

[73] [Online]. Available: https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.community.quality.modularity.html

[74] R. M. Stolzenberg, “Multiple regression analysis,” *Handbook of data analysis*, vol. 165, no. 208, pp. 175–198, 2004.

[75] S. Greenland, S. J. Senn, K. J. Rothman, J. B. Carlin, C. Poole, S. N. Goodman, and D. G. Altman, “Statistical tests, p values, confidence intervals, and power: a guide to misinterpretations,” *European journal of epidemiology*, vol. 31, pp. 337–350, 2016.

[76] [Online]. Available: https://docs.scipy.org/doc/scipy/reference/stats.html

[77] [Online]. Available: https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.linregress.html

[78] [Online]. Available: https://matplotlib.org/

[79] [Online]. Available: https://www.statsmodels.org/stable/api.html

[80] [Online]. Available: https://www.statsmodels.org/devel/generated/statsmodels.regression.linear_model.OLS.html

[81] [Online]. Available: https://seaborn.pydata.org/

[82] [Online]. Available: https://seaborn.pydata.org/generated/seaborn.barplot.html

[83] [Online]. Available: https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.sem.html

[84] [Online]. Available: https://matplotlib.org/stable/api/_as_gen/matplotlib.axes.Axes.errorbar.html

[85] T. A. Snijders, S. P. Borgatti *et al.*, “Non-parametric standard errors and tests for network statistics,” *Connections*, vol. 22, no. 2, pp. 161–170, 1999.

[86] [Online]. Available: https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.ttest_1samp.html

[87] [Online]. Available: https://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf

[88] H. C. Triandis, “The self and social behavior in differing cultural contexts.” *Psychological Review*, vol. 96, no. 3, p. 506–520, 1989.

[89] S. Currarini, M. O. Jackson, and P. Pin, "Identifying the roles of race-based choice and chance in high school friendship network formation," *Proceedings of the National Academy of Sciences*, vol. 107, no. 11, p. 4857–4861, 2010.

[90] A. S. Manstead, "The psychology of social class: How socioeconomic status impacts thought, feelings, and behaviour," *British Journal of Social Psychology*, vol. 57, no. 2, p. 267–291, 2018.