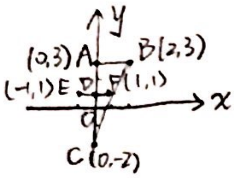
CS174 Assignment 3 - Part 1
Writing Section

1. Clipping

a. The top of the box is $y=1$ $(-1\le x\le 1)$. Apparently, line AC intersects with top of the box at $D(0,1)$ due to the answer of $\begin{cases} y=1 & (-1\le x\le 1) \\ x=0 & (-2\le y\le 3) \end{cases}$

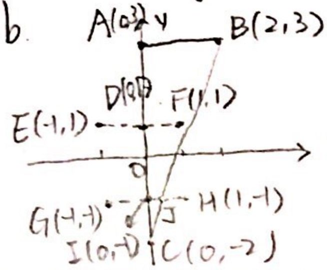


Since line BC is $y=\frac{5}{2}x-2$ $(0\le x\le 2)$

We have $\begin{cases} y=\frac{5}{2}x-2 & (0\le x\le 2) \\ y=1 & (-1\le x\le 1) \end{cases} \Rightarrow \emptyset$. There is no intersection between line BC and the top of the box, (assume E, F).

According to piazza, professor says " You do not need to provide outcodes".

b.



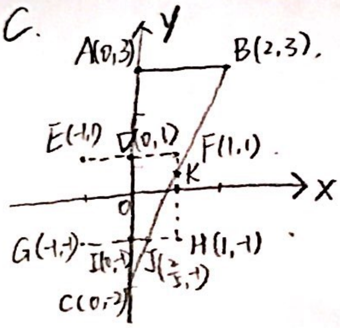
Assume the bottom edge of the box is GH with $G(-1,-1)$, $H(1,-1)$

Apparently, AC intersects with GH at $I(0,-1)$ by $\begin{cases} y=-1 & (-1\le x\le 1) \\ x=0 & (-2\le y\le 3) \end{cases}$

line BC intersects with GH at point J

$\begin{cases} y=\frac{5}{2}x-2 & (0\le x\le 2) \\ y=-1 & (-1\le x\le 1) \end{cases} \Rightarrow J(\frac{2}{5},-1)$

c.



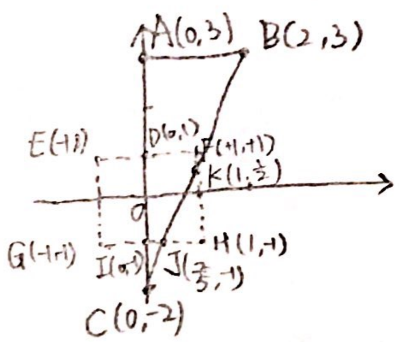
Apparently, line AC, AB don't intersect with the right edge FH, which is $x=1$ $(-1\le y\le 1)$.

Let K be the intersection point of BC and FH, we have

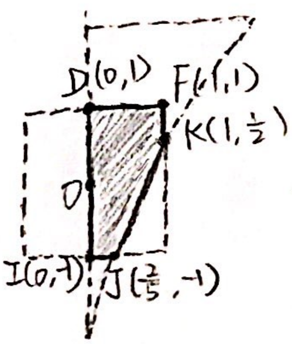
$\begin{cases} y=\frac{5}{2}x-2 & (0\le x\le 2) \\ x=1 & (-1\le y\le 1) \end{cases} \Rightarrow K(1,\frac{1}{2})$

d. the left edge EG ($x=-1$, $(-1\le y\le 1)$) does not intersect with any part of ABC.

Thus we have,



After clipping,



the shading part is what we got.

2. Rasterization / Scan-conversion

a. The implicit equation of the circle is

$$(x-x_0)^2+(y-y_0)^2=r^2, \text{ We have } F(x,y)=(x-x_0)^2+(y-y_0)^2-r^2$$

b. For the naive thought, for every x, we have $y=y_0+\sqrt{r^2-(x-x_0)^2}$

or $y=y_0-\sqrt{r^2-(x-x_0)^2}$

thus for x from round(x_0-r) to round(x_0+r) do

calculate y_{up} = round (y_0 + sqrt (r*r − (x − x_0) * (x − x_0)))

y_{down} = round (y_0 − sqrt (r*r − (x − x_0)*(x − x_0)))

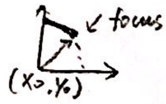
Set Pixel (x, y_{up})

Set Pixel (x, y_{down})

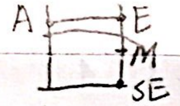
end

c. Bresenham approach. (work by octants and uses symmetry)

We first start with $x=x_0$, $y=$ round (y_0+r)



we have next candidate point



$M(x+1,\ y-0.5)$ $\quad d=F(M)=(x+1-x_0)^2+(y-0.5-y_0)^2-r^2$

if $d<0$ choose E.

$$d_{next}=F(x+1+1,\ y-0.5)=(x+1-x_0+1)^2+(y-0.5-y_0)^2-r^2=d+2(x+1-x_0)+1$$
$$=d+2(x-x_0)+3$$

if $d>0$ choose SE

$$d_{next}=F(x+1+1,\ y-1-0.5)=(x+1-x_0+1)^2+(y-y_0-1-0.5)^2-r^2=d+2(x-x_0)+3$$
$$-2(y-y_0-0.5)+1$$
$$=d+2(x-x_0)-2(y-y_0)+5$$
$$=d+2(x-y)+5$$

Start with $x=x_0$

$y=$ round (y_0+r), satisfying $x<y$, SetPixel (x,y)

$d=(x_0+1-x_0)^2+(y-0.5-y_0)^2-r^2=1+(y-0.5-y_0)^2-r^2$

while $(x<y)$

if $d<=0$

$d=d+2(x-x_0)+3$

$x=x+1$

if $d>0$

$d=d+2(x-y)+5$

$x=x+1$

$y=y-1$

SetPixel (x,y) SetPixel $(2x_0-x, y)$ SetPixel $(x, 2y_0-y)$ SetPixel $(2x_0-x, 2y_0-y)$

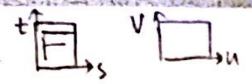
SetPixel ($y-y_0+x_0$, $x-x_0+y_0$)

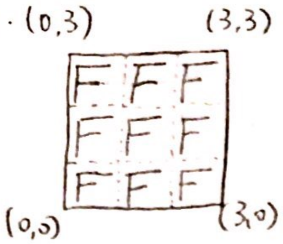
SetPixel ($y-y_0+x_0$, x_0-x+y_0)

SetPixel (y_0-y+x_0, $x-x_0+y_0$)

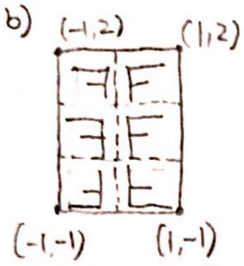
SetPixel (y_0-y+x_0, x_0-x+y_0)

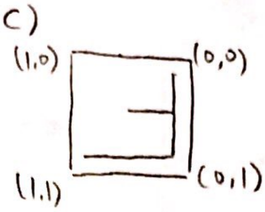
end.

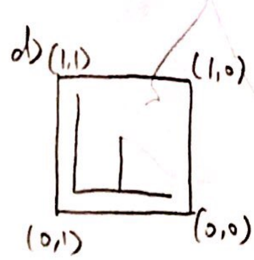
3. Textures Assume 

a) 

square → square

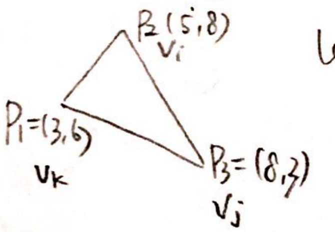
b) 

c) 

d) 

4. Interpolation

The barycentric coordinate for P is



Let $P_3=V_j$, $P_2=V_i$, $P_1=V_K$ we have,

$$\Delta t_i = \frac{(V_j-V_{new})\times(V_K-V_{new})}{(V_j-V_i)\times(V_K-V_i)} = \frac{(2,-3)\times(-3,0)}{(3,-5)\times(-2,-2)} = \frac{9}{16}$$

$$\Delta t_j = \frac{(V_i-V_{new})\times(V_K-V_{new})}{(V_i-V_j)\times(V_K-V_j)} = \frac{(-1,2)\times(-3,0)}{(-3,5)\times(-5,3)} = \frac{6}{16} = \frac{3}{8}$$

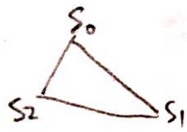
$$\Delta t_K = \frac{(V_i-V_{new})\times(V_j-V_{new})}{(V_i-V_K)\times(V_j-V_K)} = \frac{(-1,2)\times(2,-3)}{(2,2)\times(5,-3)} = \frac{1}{16}$$

thus $(\alpha,\beta,r)=(\frac{9}{16},\frac{3}{8},\frac{1}{16})$.

$$S = \frac{\frac{\alpha S_0}{W_0}+\frac{\beta S_1}{W_1}+\frac{r S_2}{W_2}}{\frac{\alpha}{W_0}+\frac{\beta}{W_1}+\frac{r}{W_2}}$$

since $W_0=W_1=W_2=1$

We have $S=\dfrac{\alpha S_0+\beta S_1+r S_2}{\alpha+\beta+r}$



As for (r.g.b)

$$r=\frac{\frac{9}{16}\times0.5+\frac{6}{16}\times0.7+\frac{1}{16}\times0.3}{1}=\frac{9}{16}=0.5625$$

$$g=\frac{\frac{9}{16}\times0.5+\frac{6}{16}\times0.2+\frac{1}{16}\times0.9}{1}=\frac{33}{80}=0.4125$$

$$b=\frac{\frac{9}{16}\times0.9+\frac{6}{16}\times0.7+\frac{1}{16}\times0.2}{1}=\frac{25}{32}=0.78125$$

, the answer is (0.5625, 0.4125, 0.78125)

approximately (0.6, 0.4, 0.8)

5. **Local Illumination** (18 pts)

a) (16 pts) Sketch the illumination that would be computed for the above scene using the Phong illumination model. The scene is lit from above using a directional light source that is coming directly from above. Use 4 sketches: one for ambient, one for diffuse, one for specular and one for the total illumination. The Phong illumination model is given by:

$$I = I_d k_d (\mathbf{n} \cdot \mathbf{l}) + I_s k_s (\mathbf{r} \cdot \mathbf{v})^n + I_a k_a$$

where $I_d = I_a = I_s = 1.0$, $k_a = 0.2$, $k_d = 0.8$, $k_s = 0.7$, $n = 100$.

Solution: For ambient,

$I = I_a K_a = 0.2$

For diffuse, when $\vec{n}$ and $\vec{l}$ has 0 angle, it reaches max $I_d K_d = 0.8$

from left to right in semicircle A→B→C

angle in $\vec{n}$, $\vec{l}$ decrease from 90° to 0°

thus $\cos\theta$ increases.

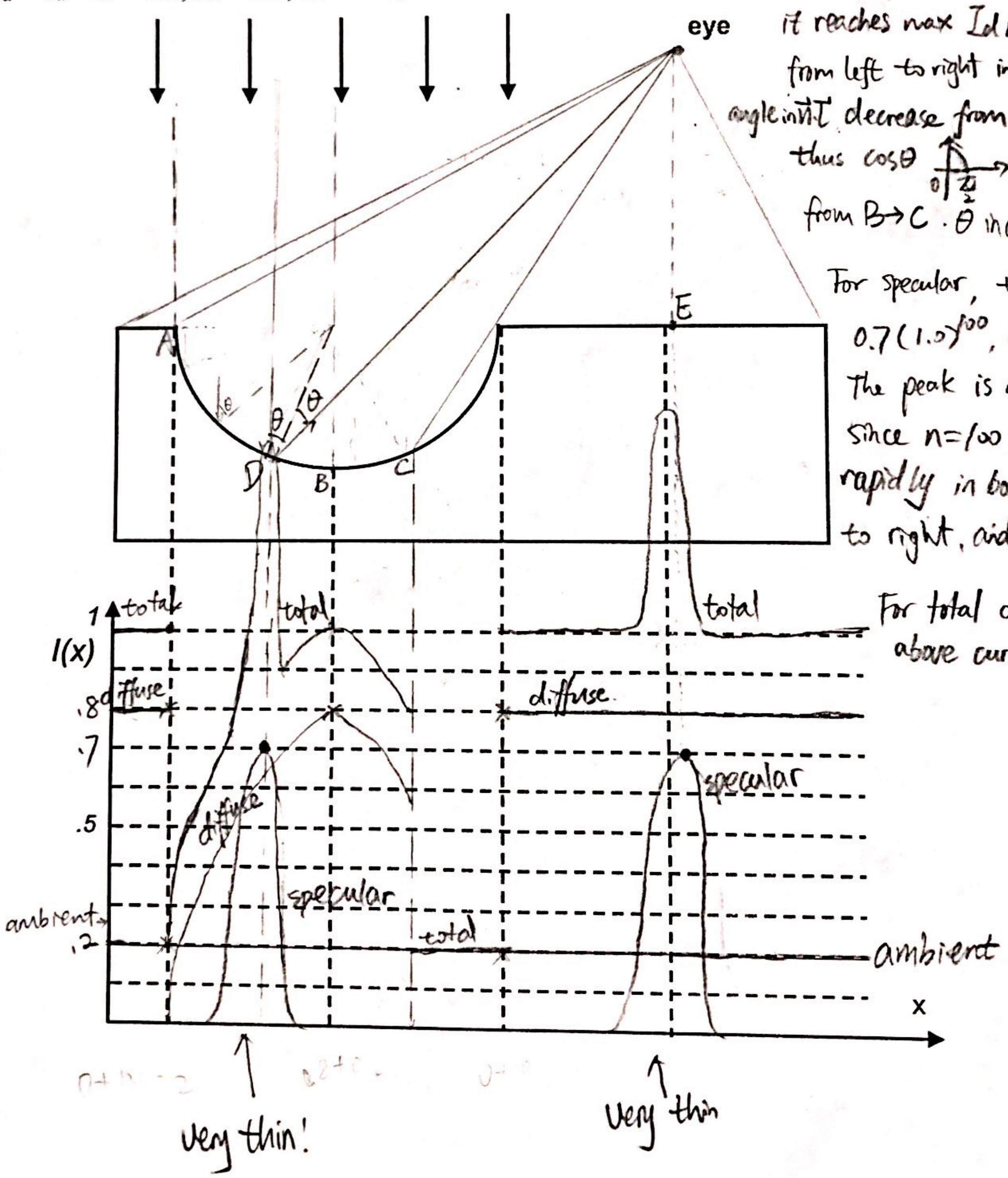
from B→C. θ increases and $\cos\theta$ decreases.

For specular, the peak value is $0.7(1.0)^{100}$, when $\vec{r}$, $\vec{v}$ has 0 angle.

The peak is at D and E.

Since $n=100$, $I_{specular}$ will decrease rapidly in both directions from peak to right, and from peak to left.

For total curves, we add the above curves together.



6. Lighting and Shading

a) We first project B on AC to get B' according to piazza.

$y_{AC} = \frac{5-2}{8-1}(x-1)+2 \Rightarrow y_{AC} = \frac{3}{7}x + \frac{11}{7}$

$y_{BB'} = -\frac{7}{3}(x-6)+3 \quad y_{BB'} = -\frac{7}{3}x+17$

$\Rightarrow B'(\frac{162}{29}, \frac{115}{29})$

$t = \frac{AB'}{AC} = \frac{\|(1,2)-(\frac{162}{29},\frac{115}{29})\|}{\|(1,2)-(8,5)\|} = \frac{19}{29}$

$N_B = (1-t)N_A + tN_C = \frac{10}{29}N_A + \frac{19}{29}N_C$

Normalize $N_A = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ $N_C(1,0)$.

$N_B = (\frac{19-5\sqrt{2}}{29}, -\frac{5\sqrt{2}}{29}) \doteq (0.41, -0.24)$

Normalize $N_B = (\frac{19-5\sqrt{2}}{\sqrt{461-190\sqrt{2}}}, -\frac{5\sqrt{2}}{\sqrt{461-190\sqrt{2}}})$

$= (0.86, -0.51)$

b) ambient illumination:

B,C,D: $I_a K_a = (.01, .02, .01)$

specular illumination:

$I_s = I_L K_s (H \cdot N)^n$, $H = \frac{L+V}{\|L+V\|}$.

$\boxed{\text{For B:}}$ $L_B = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ $V_B = (\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}})$

$H_B = (\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}}, -\frac{\sqrt{5}+2\sqrt{2}}{\sqrt{10}}) \xrightarrow{\text{Normalize}} H_B = (\sqrt{\frac{10-\sqrt{10}}{20}}, -\sqrt{\frac{10+\sqrt{10}}{20}}, 0)$

$= (0.58, -0.81, 0)$

$N_{AB} = (\frac{1}{\sqrt{26}}, \frac{-5}{\sqrt{26}})$ $N_{BC} = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$N_B = \frac{N_{AB}+N_{BC}}{2} = (\frac{\sqrt{2}+\sqrt{26}}{4\sqrt{13}}, -\frac{5\sqrt{2}+\sqrt{26}}{4\sqrt{13}}, 0)$

up

$\xrightarrow{\text{Normalize}} N_B = (\frac{1+\sqrt{13}}{2\sqrt{13+3\sqrt{13}}}, -\frac{5+\sqrt{13}}{2\sqrt{13+3\sqrt{13}}}) = (0.47, -0.88, 0)$

$I_s = (1.0, 1.0, .9)(H_B N_B)^{20} = (1.0, 1.0, .9)(0.9854)^{20} = (0.75, 0.75, 0.67)$

diffusion:

$I = I_L K_d (\vec{n}_B \cdot \vec{L}_B) = (1.0, 1.0, .9) \cdot (.3, .8, .9) [(0.47, -0.88, 0) \cdot (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)]$

$= (.3, .8, .81) \cdot 0.95 = (0.29, 0.76, 0.77)$

$I_{total} = I_a K_a + I_s + I = (1.05, 1.53, 1.45) \rightarrow I_{total} = (1.0, 1.0, 1.0)$

b) $\boxed{\text{Continue} \uparrow}$ $I_{ambient} = (.01, .02, .01)$.

For C. $N_C = (1,0,0)$.

$L_C = (0,-1)$ $V_C = (0,-1)$. $H = (0,-1)$

$I_s = I_L K_s (H \cdot N)^n = (1, 1, .9)(0)^{20} = 0$

For diffusion,

$I = I_L K_d (\vec{n}_C \cdot \vec{L}_C) = (.3, .8, .81) \cdot 0 = 0$.

$I_{total} = I_{ambient} + I_s + I = (.01, .02, .01)$.

Since it is flat shading model.
Let D be either B or C.
Here, we let D equals with C.

$I_{ambient} = (.01, .02, .01)$

$I_s \doteq I_{diffusion} = 0$

$I_{total} = I_{ambient} = (.01, .02, .01)$

c). We have

	B	C
Ambient	(.01, .02, .01)	(.01, .02, .01)
Specular	(.75, .75, .67)	(0, 0, 0)
diffuse	(.29, .76, .77)	(0, 0, 0)
total	(1.0, 1.0, 1.0)	(.01, .02, .01)

Since it is gouraud shading model BD=DC

D: Ambient (.01, .02, .01)
Specular (.375, .375, .335)
diffuse (.145, .38, .385)
total (.53, .775, .73)

d). Since it is Phong shading, BC is same as C.

$N_B = (0.47, -0.88, 0)$, $N_C = (1, 0, 0)$

linearly interpolate $N_D = \frac{N_B+N_C}{\|N_B+N_C\|} = (0.86, -0.51)$

$L_D = \frac{(8,1,0)-(7,4,0)}{\text{normalize}} = (\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}, 0)$ $V_D = (\frac{1}{\sqrt{26}}, \frac{-5}{\sqrt{26}}, 0)$

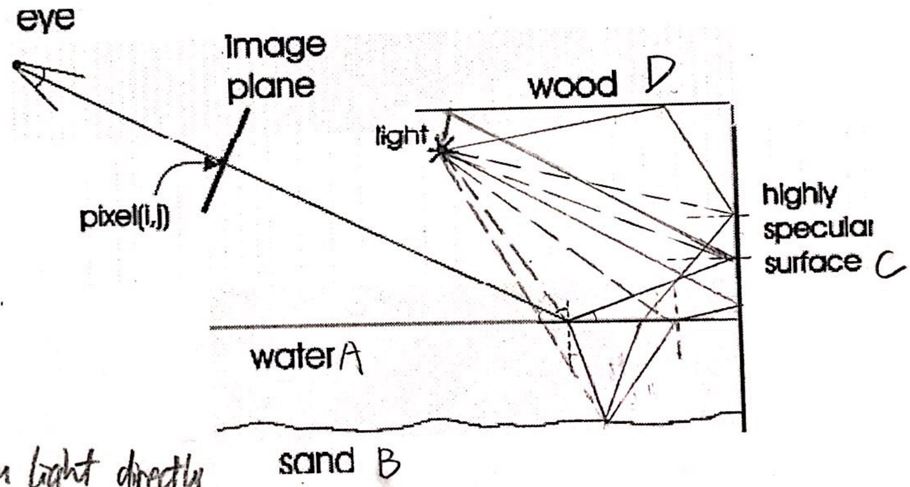
$H_D = (\frac{\sqrt{26}+\sqrt{10}}{\sqrt{260}}, -\frac{3\sqrt{26}+5\sqrt{10}}{\sqrt{260}}) \xrightarrow{\text{normalize}} H_D = (0.26, -0.97)$

① $I_s = (1.0, 1.0, .9)(H_D N_D)^{20} = (1.0, 1.0, .9) \times (0.7183)^{20} = (0.0013, 0.0013, 0.0012)$

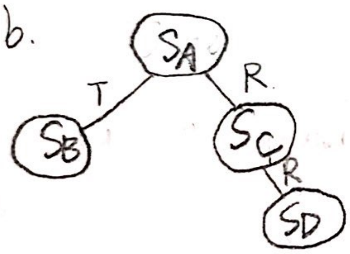
② $I_d = I_L K_d (\vec{n}_D \cdot \vec{L}_D) = (.3, .8, .81) \times 0.76$
$= (0.23, 0.6, 0.61)$ ③ $I_{ambient} = (0.01, 0.02, 0.01)$

④ $I_{total} = I_{ambient} + I_s + I_D = (0.2413, 0.6213, 0.6212)$.

7. Ray-Tracing (6 pts)



b.



Shadow rays are generated from light directly to each reflect or diffuse point as the "---" in graph indicates.

a. (3 pts) For the following scene, sketch all the ray paths and shadow rays that would be generated by a raytracer in order to compute the color for the given pixel, (i,j).

b. (3 pts) Draw the ray tree corresponding to the above ray paths. Draw the reflected paths to the right and the transmitted paths to the left. Also indicate where the

8. Parametric Curves

$P_0 = P(0)$, $T_0 = P'(0)$, $A_0 = P''(0)$, $P_1 = P(1)$

a) Give the polynomial representation for a parametric cubic curve.

$$p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

b) Compute the first and second derivative representation

$$p'(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$p''(t) = 2a_2 + 6a_3 t$$

c) Determine the basis matrix for the parametric cubic curve.

$$p^T(t) = [1 \ t \ t^2 \ t^3] \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}$$

$P_0 = P(0) = a_0$ $\quad A_0 = P''(0) = 2a_2$

$T_0 = P'(0) = a_1$ $\quad P_1 = P(1) = a_0 + a_1 + a_2 + a_3$

$$\begin{bmatrix} P_0^T \\ T_0^T \\ A_0^T \\ P_1^T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = C \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}$$

$$p^T(t) = [1 \ t \ t^2 \ t^3] \, C^{-1} \begin{bmatrix} P_0^T \\ T_0^T \\ A_0^T \\ P_1^T \end{bmatrix}$$

basis matrix $B = C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ -1 & -1 & -1/2 & 1 \end{bmatrix}$