CS174A Assignment 2 - Part 1.

1. a) Since the hint says $P_{ear\_in\_head\_coord} = D * P_{ear}$,

   We have $P_{ear\_in\_camera} = ACD\, P_{ear}$

   b). Since Tailcamera let us see from the end of the cat tail,

   $P_{tailtip} = B * P_{torso}$, According to the hint in a), we have

   $P_{torso} = KLM\, P_{tailtip}$, thus $B = M^{-1} L^{-1} K^{-1}$

2. According to the slide 6,

$$F_{total} = g - \gamma v + f_{ext}$$

$$a(t) = \frac{F_{total}}{m}$$

where γ is the damping coeff

f is the external force.

$$v(t+\Delta t) = v(t) + \Delta t\, a(t)$$

$$x(t+\Delta t) = x(t) + \Delta t\, v(t+\Delta t)$$

By computing iteratively, we get the position of a certain particle

3. According to the slide 6,

$$g_i(t) = \sum_{j \in N_i} g_{ij}$$

g_i is total force on node i due to springs connecting it to neighboring nodes $j \in N_i$

$$g_{ij} = k_{ij}\, e_{ij} \frac{d_{ij}}{\|d_{ij}\|}$$

where g_{ij} is the force spring ij exerts on node i.

$$d_{ij} = x_j - x_i$$

d_{ij} is separation of nodes

$\|d_{ij}\|$ is actual length of spring

$$e_{ij} = \|d_{ij}\| - l_{ij}$$

e_{ij} is deformation of spring

Function calls as specified in lecture:

```
void spring_forces (int num_springs, Spring *sprs)
{ int i;
  for (i=0; i<num_springs; i++) spring_force (&sprs[i]); }
```

```
void spring_force (spring *s)
{ node *node1, *node2;
  double length, extension, scale_factor;
  vector direction, force;
  node1 = s->n1; node2 = S->n2;
  vminus (node2->position, node1->position, direction);
  length = vlength (direction);
  deformation = length - s->rest_length;
  Scale_factor = ((deformation * S->spring_constant)/length);
  Vscale (scale_factor, direction, force);
  vinc (force, node1->force);
  vdec (force, node2->force); }
```

4. a. Mass-spring model with a non-zero length spring

b. Cloth - Viscoelasticity - Mass-Springs Model

c. Heating and Melting Deformable Models - Mass-Springs Model

d. Liquids - Particle Models

Solution: a. $g_{ij} = k_{ij} e_{ij} \frac{d_{ij}}{\|d_{ij}\|} = k_{ij} (\|x_j - x_i\| - l_{ij}) \frac{x_j - x_i}{\|x_j - x_i\|}$ (where $d_{ij} = x_j - x_i$ is node distance, $e_{ij} = \|d_{ij}\| - l_{ij}$ is deformation, l_{ij} is natural spring length, k_{ij} is the spring constant for ij)

(Page 10)

b. $m_i \ddot{x}_i + r_i \dot{x}_i + c_{ij} d_{ij} = f_i$

(Page 17) $m_j \ddot{x}_j + r_j \dot{x}_j - c_{ij} d_{ij} = f_j$ where $c_{ij} = \frac{k_{ij} e_{ij} + r_{ij} \dot{e}_{ij}}{\| d_{ij} \|}$

Where $d_{ij} = x_j - x_i$, $e_{ij} = \| d_{ij} \| - l_{ij}$, r_{ij} is damping coeff, k_{ij} is stiffness.

C. Since the only difference is k_{ij}.

(Page 14). $k_{ij} = \begin{cases} k_{ij}^0 & \text{if } \theta^a \le \theta^s \\ k_{ij}^0 - v(\theta^a - \theta^s) & \text{if } \theta^s < \theta^a < \theta^m \\ 0 & \text{if } \theta^a \ge \theta^m \end{cases}$ $v = k_{ij}/(\theta^m - \theta^s)$

where $\frac{\partial}{\partial t}(\mu \sigma \theta) - \nabla(C \nabla \theta) = q$ $\nabla = [\frac{\partial}{\partial u}, \frac{\partial}{\partial v}, \frac{\partial}{\partial w}]$

q is the rate of heat generation.

μ is mass density

σ is specific heat

θ is temperature

C is thermal conductivity matrix

d. The total force on a particle i,

(Page 16) $g_i(t) = \sum_{j \neq i} g_{ij}(t)$

$$g_{ij}(t) = m_i m_j (x_i - x_j) \Big(\underbrace{-\frac{\alpha}{(d_{ij}+\delta)^a}}_{\text{attraction}} + \underbrace{\frac{\beta}{(d_{ij})^b}}_{\text{repulsion}} \Big), \quad a=2, b=4$$

α and β determine the strength of the attraction & repulsion forces

$d_{ij} = \|x_j - x_i\|$. δ is minimum required separation between particles

5. According to 2 $f_{total} = g - \gamma v + f_{ext}$ where $f_{ext} = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix}$ $m=1$ $\Delta t = 1$ $g = 9.8$

$a(t) = \frac{f_{total}}{m}$

$V(t+\Delta t) = V(t) + \Delta t \cdot a(t)$ assume $\gamma = 0$ for particle.

$x(t+\Delta t) = x(t) + \Delta t \cdot V(t+\Delta t)$

thus $f_{total} = \begin{bmatrix} 2 \\ 4.9 \\ -5 \end{bmatrix}$ at $t=0$.

$a(0) = [2, 4.9, -5]^T$, $V(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $x(0) = [0,0,0]^T$

Since $\Delta t = 1$, $t = 1s$, $V(1) = V(0) + 1 \cdot a(0) = [2, 4.9, -5]^T$

$X(1) = X(0) + 1 \cdot V(1) = [2, 4.9, -5]^T$ $\quad x_y = 4.9 \neq 0$

$f_{total} = g$ and $a(1) = \frac{g}{m} = [0, -9.8, 0]^T$

$t=2s$ $\quad V(2) = V(1) + 1 \cdot a(1) = [2, -4.9, -5]^T$.

$x(2) = X(1) + 1 \cdot V(2) = [4, 0, -10]^T$ Since $x_y = 0$, it hits the ground at $\begin{bmatrix} 4 \\ 0 \\ -10 \end{bmatrix}$.

6. According to lecture 7,

$P_{eye} = [2, 10, 3]^T$, $P_{ref} = [-2, 2, 0]^T$, $V_{up} = [-1, -1, 0]^T$.

thus $K = \frac{P_{eye} - P_{ref}}{|P_{eye} - P_{ref}|} = \frac{[4, 8, 3]^T}{\sqrt{4^2+8^2+3^2}} = [\frac{4}{\sqrt{89}}, \frac{8}{\sqrt{89}}, \frac{3}{\sqrt{89}}]^T$

$i = \frac{V_{up} \times k}{|V_{up} \times k|} = \frac{(-\frac{3}{\sqrt{89}}, \frac{3}{\sqrt{89}}, -\frac{4}{\sqrt{89}})^T}{\sqrt{\frac{9}{89} + \frac{9}{89} + \frac{16}{89}}} = (-\frac{3}{\sqrt{34}}, \frac{3}{\sqrt{34}}, -\frac{4}{\sqrt{34}})^T$

$j = k \times i = (\frac{-41}{\sqrt{3026}}, \frac{7}{\sqrt{3026}}, \frac{36}{\sqrt{3026}})$ which is the camera coordinate system.

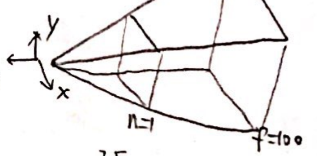
M_{cam}^{-1} transform world cordicate system to view coordinate system.

$$M_{cam}^{-1} = \begin{bmatrix} \vec{i}^T & 0 \\ \vec{j}^T & 0 \\ \vec{k}^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -P_{eye}^T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{\sqrt{34}}, & \frac{3}{\sqrt{34}}, & -\frac{4}{\sqrt{34}} & -\frac{12}{\sqrt{34}} \\ -\frac{41}{\sqrt{3026}} & \frac{7}{\sqrt{3026}} & \frac{36}{\sqrt{3026}}, & -\frac{96}{\sqrt{3026}} \\ \frac{4}{\sqrt{89}}, & \frac{8}{\sqrt{89}}, & \frac{3}{\sqrt{89}}, & -\frac{97}{\sqrt{89}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. According to Lecture 7. The projection Matrix is, $n=-1$ $f=-100$ $r=2-\sqrt{3}$ $l=-2+\sqrt{3}$

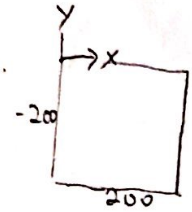
since aspect ratio $= \frac{r-l}{t-b} = \frac{1}{2}$; $b = -4+2\sqrt{3}$, $t = 4-2\sqrt{3}$

$$M_p = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -101 & -100 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$M_o = \begin{bmatrix} \frac{1}{2-\sqrt{3}}, & 0, & 0, & 0 \\ 0, & \frac{1}{4-2\sqrt{3}} & 0 & 0 \\ 0 & 0 & -\frac{2}{99} & \frac{-101}{99} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

thus $M_{proj} = M_o M_p = \begin{bmatrix} \frac{1}{\sqrt{3}-2} & 0 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{3}-4} & 0 & 0 \\ 0 & 0 & \frac{101}{99} & -\frac{200}{99} \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2+\sqrt{3} & 0 & 0 & 0 \\ 0 & -\frac{2+\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \frac{101}{99} & -\frac{200}{99} \\ 0 & 0 & 1 & 0 \end{bmatrix}$

8. 

Transform x, y to $n_x \times n_y$ (200×200) is

$$M_{vp} = \begin{bmatrix} 1 & 0 & 0 & \frac{n_x-1}{2} \\ 0 & 1 & 0 & -\frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & 0 \\ 0 & \frac{n_y}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & -\frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 100 & 0 & 0 & \frac{199}{2} \\ 0 & 100 & 0 & -\frac{199}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. $P_{VCS} = M_{cam}^{-1} P_{WCS}$

$$= \begin{bmatrix} -\frac{3}{\sqrt{34}} & \frac{3}{\sqrt{34}} & -\frac{4}{\sqrt{34}} & -\frac{12}{\sqrt{34}} \\ \frac{-41}{\sqrt{3026}} & \frac{7}{\sqrt{3026}} & \frac{36}{\sqrt{3026}} & -\frac{96}{\sqrt{3026}} \\ \frac{4}{\sqrt{89}} & \frac{8}{\sqrt{89}} & \frac{3}{\sqrt{89}} & -\frac{97}{\sqrt{89}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -2 & 1 \\ 2 & 0 & -1 & 5 \\ 1 & -3 & 2 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-19}{\sqrt{34}} & 0 & \frac{-17}{\sqrt{34}} & \frac{4}{\sqrt{34}} \\ \frac{-169}{\sqrt{3026}} & \frac{-204}{\sqrt{3026}} & \frac{51}{\sqrt{3026}} & -\frac{138}{\sqrt{3026}} \\ -\frac{66}{\sqrt{89}} & -\frac{106}{\sqrt{89}} & \frac{-107}{\sqrt{89}} & -\frac{56}{\sqrt{89}} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

thus $[\frac{-19}{\sqrt{34}}, \frac{-169}{\sqrt{3026}}, \frac{-66}{\sqrt{89}}, 1]^T$, $[0, \frac{-204}{\sqrt{3026}}, -\frac{106}{\sqrt{89}}, 1]^T$, $[\frac{-17}{\sqrt{34}}, \frac{51}{\sqrt{3026}}, \frac{-107}{\sqrt{89}}, 1]^T$ and $[\frac{4}{\sqrt{34}}, \frac{-138}{\sqrt{3026}}, \frac{-56}{\sqrt{89}}, 1]^T$.

10. $P_{clip} = M_{proj} P_{VCS}$

$$= \begin{bmatrix} 2+\sqrt{3} & 0 & 0 & 0 \\ 0 & -\frac{2+\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \frac{101}{99} & \frac{-200}{99} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{-19}{\sqrt{34}} & 0 & \frac{-17}{\sqrt{34}} & \frac{4}{\sqrt{34}} \\ -\frac{169}{\sqrt{3026}} & \frac{-204}{\sqrt{3026}} & \frac{51}{\sqrt{3026}} & -\frac{138}{\sqrt{3026}} \\ -\frac{66}{\sqrt{89}} & -\frac{106}{\sqrt{89}} & \frac{-107}{\sqrt{89}} & -\frac{56}{\sqrt{89}} \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{19}{\sqrt{34}}(2+\sqrt{3}), & 0, & \frac{-17}{\sqrt{34}}(2+\sqrt{3}), & \frac{4}{\sqrt{34}}(2+\sqrt{3}) \\ \frac{169}{2\sqrt{3026}}(2+\sqrt{3}), & \frac{102}{\sqrt{3026}}(2+\sqrt{3}), & -\frac{51}{2\sqrt{3026}}(2+\sqrt{3}), & \frac{69}{\sqrt{3026}}(2+\sqrt{3}) \\ -\frac{6666+200\sqrt{89}}{99\sqrt{89}}, & -\frac{10706+200\sqrt{89}}{99\sqrt{89}}, & -\frac{10807+200\sqrt{89}}{99\sqrt{89}}, & -\frac{13938+200\sqrt{89}}{99\sqrt{89}} \\ \frac{-66}{\sqrt{89}}, & \frac{-106}{\sqrt{89}}, & \frac{-107}{\sqrt{89}}, & -\frac{56}{\sqrt{89}} \end{bmatrix}$$

11. Normalize: divide by w.

$$P_{11} = \frac{1}{w} P_{clip} = \begin{bmatrix} \frac{19\sqrt{89}}{66\sqrt{34}}(2+\sqrt{3}) & 0 & \frac{17\sqrt{89}}{107\sqrt{34}}(2+\sqrt{3}) & -\frac{\sqrt{89}}{14\sqrt{34}}(2+\sqrt{3}) \\ \frac{169\sqrt{89}}{132\sqrt{3026}}(2+\sqrt{3}) & -\frac{51\sqrt{89}}{53\sqrt{3026}}(2+\sqrt{3}) & \frac{51\sqrt{89}}{214\sqrt{3026}}(2+\sqrt{3}) & -\frac{69\sqrt{89}}{56\sqrt{3026}}(2+\sqrt{3}) \\ \frac{6666+200\sqrt{89}}{6534} & \frac{5353+100\sqrt{89}}{5247} & \frac{10807+200\sqrt{89}}{10593} & \frac{6969+100\sqrt{89}}{2772} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

12. $P_{DCS} = M_{vp} \cdot P_{11} = \begin{bmatrix} 100 & 0 & 0 & \frac{199}{2} \\ 0 & 100 & 0 & -\frac{199}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$ (drop z coordinate)

thus we have $\left(\frac{1900\sqrt{89}(2+\sqrt{3})+6567\sqrt{34}}{66\sqrt{34}}, \frac{8450\sqrt{89}(2+\sqrt{3})-6567\sqrt{3026}}{66\sqrt{3026}}\right)$

$\left(\frac{199}{2}, \frac{-10200\sqrt{89}(2+\sqrt{3})-10547\sqrt{3026}}{106\sqrt{3026}}\right)$

$\left(\frac{3400\sqrt{89}(2+\sqrt{3})+21293\sqrt{34}}{214\sqrt{34}}, \frac{5100\sqrt{89}(2+\sqrt{3})-21293\sqrt{3026}}{214\sqrt{3026}}\right)$ and $\left(\frac{-100\sqrt{89}(2+\sqrt{3})+1393\sqrt{34}}{14\sqrt{34}}, \frac{-1725\sqrt{89}(2+\sqrt{3})-1393\sqrt{3026}}{14\sqrt{3026}}\right)$