Department of

Electrical & Electronics Engineering

Abdullah Gül University

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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CHAPTER 1

INTRODUCTION

Firstly, in the physical background part, the formulas of angular oscillation motion were given, and an impression of the subject was gained. Secondly, in the experimental setup part, an elastic lightweight suspension, and plastic radially symmetric kitchen utensil were used for the subject to be investigated in the project. The experimental setup prepared for the Math153 task was used to find the volume and the appropriate method for calculating this volume and why it was chosen was explained. For the PHYS105 task computed moment of inertia was used to find torsion modulus k and e ω and φmax were calculated to see if energy is conserved in the experiment. Also, for the MATH103 task 4 out of 9 questions were selected and linear algebra applications were explained about its applications in electrical and electronic engineering. Finally, in the EE101 task, in the C++ program, the period of the pendulum was found with the help of the class subject.

BACKGROUND

A torsional pendulum is a particular kind of pendulum in which the mass is suspended from a thin filament or wire that serves as a torsion spring. As opposed to swinging back and forth like a conventional pendulum, this kind of pendulum rotates. Dutch physicist Christiaan Huygens first described the torsional pendulum in the seventeenth century. He made the first precise clock with it and used it to research the characteristics of pendulum motion. Many scientists have since used the torsional pendulum to investigate a variety of physical phenomena. A torsional pendulum oscillates because the twisting of the wire or filament produces a restoring force when a force is applied to the mass of the pendulum. The length of the wire and the mass of the pendulum affect the period of oscillation, which can be used to calculate other physical properties such as moment of inertia and torsional constant. Numerous experiments have been conducted with the torsional pendulum to examine the effects of elasticity, gravity, and other physical characteristics. Additionally, it is frequently employed in the creation and testing of precise instruments like seismometers, gyroscopes, and clocks. The torsional pendulum has been used in engineering and technology applications in addition to scientific experiments. It is employed in the design and testing of buildings and bridges to ascertain their stability and torsion resistance.[1]

Torsion in the string that the rigid body is suspended from provides the restoring force during the oscillation of a rigid body in a torsional pendulum. The string should ideally have no mass, but in reality, it has a mass that is ignored because it is much less than that of the rigid body.

The oscillation of the rigid body can be represented as a linear angular oscillation as long as its angular displacement is small. An angle represents the oscillation's amplitude. The moment of inertia of the rigid body about the point of suspension and the axis perpendicular to it play the role of mass. The equation is shown to resemble the equation for the simple harmonic motion of a simple pendulum by using the relationship between torque and angular acceleration. This observation makes it simple to calculate the angular oscillation's period and angular frequency.[2]

$$\tau = -\kappa \cdot \theta \qquad \theta = \theta\text{max} \cdot \sin \cdot \omega \cdot t$$

$$I \cdot \alpha = -\kappa \cdot \theta$$

$$I \cdot \frac{d^2\theta}{dt^2} = -\kappa \cdot \theta \quad \rightarrow \quad \omega = \sqrt{\frac{k}{I}} \quad \rightarrow T = \frac{1}{2\pi} \cdot \sqrt{\frac{I}{k}}$$

$$\frac{d^2Q}{dt^2} = \frac{-\kappa}{I} \cdot \theta \qquad f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

CHAPTER 2

Materials

Plastic bowl	Lighter	Quilt needle	Wool rope	Ruler
Scissors	Silicone gun	Hanger	Phone camera	

ANALYTICAL AND SIMULATION PROCEDURES

1) The dimensions of the plastic bowl are measured for use in MATH153 and PHY105 TASKS.

2) A hole is made in the middle of the bottom of the plastic bowl with a quilt needle heated with a lighter.

3) The wool rope is passed through this hole and fixed by tying a small knot at the end.

4) In order for this knot not to affect the swing, the knot is glued to the base with a silicone gun.

5) The rope is cut to 0.6m in length.

6) The experimental setup is hung on one of the hangers in the Ambar building.

7) The experiment was recorded with a phone camera.

8) The necessary data is found by uploading the video to the Tracker.

CHAPTER 3

RESULTS

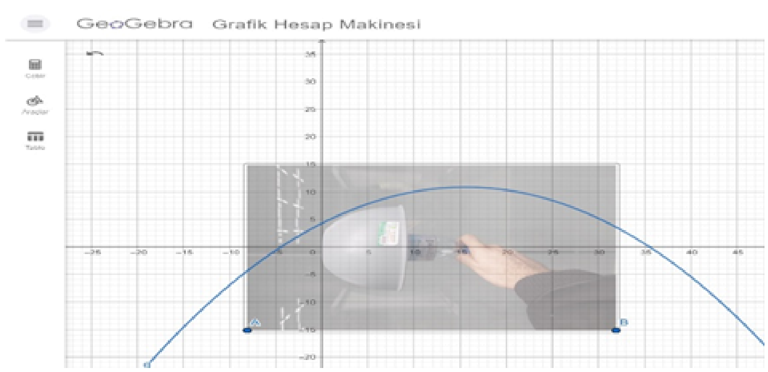
1.MATH153 TASK

1)Find the profile of a radially symmetric utensil (including its external and internal shapes), present the corresponding functions f(x) and g(x) (see Figure 3) and develop the full computation of its moment of inertia I about its center of symmetry (in the form of algebraic formulas and then with the particular numbers founded by measurements).

The function f(x) was calculated for a grey container. The equation was obtained by calculating the depth from the bottom, the diameter of the top, the radius of the bottom and the coordinates of a 3rd point for the container. the measured values are as follows: The radius measured from the ceiling is 10, the depth is 10 (10,10), the base radius is 4.25 (0,4.25) and the coordinates of the 3rd point measured are (6.9,8.8).

$$f(x) = ax^2 + bx + c$$

$$f(0) = 4.25 \rightarrow f(x) = ax^2 + bx + 4.25 \qquad f(6.9) = a\ (6.9)^2 + b(6.9) + 4.25 = 8.8$$
$$f(10) = 100a + 10b + 4.25 = 10$$



$f(x) = -0.027232351566152443\ x\hat{}(2) + 0.8473235156615244\ x + 4.25$

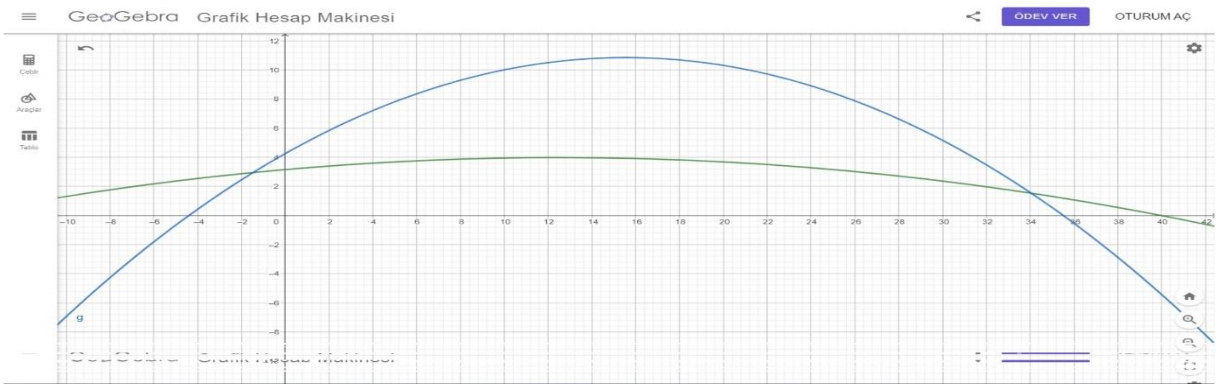
The coordinates calculated for the graph g(x) are as follows: (0, 3.15), (5.3, 3.7) and (10.4, 3.95).

$$g(x) = mx^2 + nx + k$$

$$g(0) = 3.15 \qquad g(5.3) = m(5.3)^2 + n(5.3) + 3.15 = 3.7$$

$$g(10.4) = m(10.4)^2 + n(10.4) + 3.15 = 3.95$$

$$g(x) = -0.005264805486781078\ x\hat{}(2) + 0.1316770539856001\ x + 3.15$$



Then to find calculate inertia, the following operations have been performed.

$$dm = \frac{m}{v} \cdot A\,dx$$

$$A = \pi(f(y)^2 - g(y)^2)$$

$$dm = \frac{m}{v} \cdot \pi(fy)^2 - g(y)^2)$$

$$dI = \frac{1}{2} dm(f(y)^2 + g(y)^2)$$

$$dI = \frac{\pi m}{2v}(f(y)^4 - g(y)^4)\,dx$$

$$I = \int dI = \int \frac{m\pi}{2v}(f(y)^4 - g(y)^4)\,dx$$

The volume was calculated using the Washer method and the result was 1.791liters. The mass of the container used was measured to be 0.086kg. Also π is accepted as 3.14.

1liter =1dm^3so 1.791itre equal to 0.001791m^3

$$\frac{m\pi}{2v} = \frac{0.086 \times 3.14}{2 \times 0.001791} = 0.0753880514 \sim 0.07$$

$$dI = 0.00007 \int_0^{10} (f(y)^4 - g(y)^4)\,dx$$

$$0.00007 \text{ X } \frac{f^5(x)}{5f'(x)} - \frac{g^5(x)}{5g'(x)}\Big|_0^{10}$$

$$f'(10) = 1.3919$$

$$g'(10) = 0.2369$$

(877478.49/5*1.3919) - (3103.8/5*0.2369) =120460.21

0.00007 X 120460.21 =8.4324247

$$I = 8.43$$

2) Explain why did you choose this particular method of integration (disk method, washer method, shell method, a solid with known cross-sections).

The washer method is used to calculate the volume of objects in revolution. It is a variant of the disc approach for solid objects that accommodates items with holes. It's nicknamed the "washer method" because the cross sections resemble washers. The Washer method is used to calculate the volume of two functions rotated about the x-axis. This method can only be used when there are two functions, because one function or more than three functions will not work with this method. Since two containers were used during the experiment, two different graphs were obtained. $F(x)$ (function of the outside of the plastic container) is always greater than or equal to $g(x)$ (Function of the inside of the plastic container) throughout the interval $[a, b]$. The following formula is derived. [3]

$$v = \lim_{\Delta x \to 0} \sum_{i=0}^{n-1} \pi \left\{ [f(x)]^2 - [g(x)]^2 \right\} \Delta x$$

$$v_{washer} = \pi \int_a^b \left(f(x)^2 - g(x)^2 \right) dx$$

$$f(x) = -0.027232351566152443\ x^{\wedge}(2) + 0.8473235156615244\ x + 4.25$$

$$g(x) = -0.005264805486781078x^{\wedge}(2) + 0.1316770539856001x + 3.15$$

After giving the lower limit of 0 and the upper limit of 10 to the graphs of equal 10 cm high containers, the result of the integral was found to be 1.791. $(\pi \cong 3{,}14)$

2. PHYS105 TASK.

1) Using the computed moment of inertia I, find the torsion modulus k.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$I = \int r^2\, dm = \int \rho r^2\, dV$$

$$\omega_0 = \sqrt{\frac{k}{I}} \qquad T = 2\pi\sqrt{\frac{I}{k}} \qquad \frac{I\omega^2}{2} = \frac{k\varphi^2}{2} \qquad \omega = \sqrt{\frac{k}{I}}\theta \quad = \quad \frac{2\pi}{T}\theta$$

$$T^2 k = 4\pi^2 I$$

$$k = \frac{4\pi^2 I}{T^2}$$

To find k, the value of the period from the Trecker was analysed and as a result the value of T was taken as 6.665. The inertia was taken as 8.43, as previously calculated in the math153 task. Also π is accepted as 3.14.

$$k = 4 \times (3.14)^2 \times 8.43 /(6.665)^2$$

$$k=7.4886747$$

Also, when it checks according to $2\pi\sqrt{\frac{I}{k}}$ this formula $2 \times 3.14\sqrt{8.43/7.48}$. The result is too close to what is seen on the tractor. The reason for this difference is due to external factors and approximations made during processing.

2) Measure ω and φmax and check experimentally the conservation of the energy (2).

To calculate the energy difference in theory and in experiment, the theoretical data and the values in the Trecker are compared.

$E = I\omega^2$

Theoretically, ω value was taken as 6.05 and I was taken as 8.43.

$$E = 8.43 \times (6.09)^2 = 312$$

Energy calculated experimentally using Trecker values are: ω is equal to 5.6.

$$E = 8.43 \times (5.6)^2 = 264$$

If one hundred per cent is 312, the calculation of what percentage is 264 has been made.

$100 \times 264 \div 312 = 84$ Then , energy loss is calculated at 16 percent.

EQUATION	TRACKER	ENERGY LOST
$I\omega 2$=(8,43)x(6,09)x(6,09)=312	$I\omega 2$=(8,43)x(5,6)x(5,6)=264	%16

3. **MATH103 Task.**

1 . For question 1, linear algebra is a branch of mathematics that is concerned with vector spaces and linear transformations between these spaces. When it is evaluated about its purpose and usefulness (what is necessary in real life) or relationships with other disciplines. There are several connections and information about them. Firstly, from the beginning of humanity, the world has different problems. This part focuses on how these problems can be solved using linear algebra. From ancient times to today (the first evidence of linear algebra dates from about 150 BCE in China) the use of the main methods of linear algebra (augmented matrices, elimination, and determinant-style calculations) are historically seen in Chinese sources. These methods are still essential to linear algebra. [4]

If the use of linear algebra is evaluated in the modern world, "product recommendation" can be given as the first application of linear algebra. People can shop online with several apps. According to their research, a new section can be created called "product recommendation" based on the previous orders (what the customers shopped before on this site). The main idea of this system is based on linear algebra. It collects ratings from its customers. Then, based on the product features, a new matrix is created. The result of this matrix is used to write new equations (containing unknown variables). Ultimately, these algorithms can be optimized for more effective results. Secondly, the use of GPS (Global Positioning System). It finds of position of any object in a few milliseconds and high possibility helps linear algebra. At the same time almost all modern computer science algorithms are created thank to linear algebra. [5]

When examining the relationships between electrical and electronic engineering and the area of using linear algebra, there are several roles of linear algebra in electrical and electronic engineering that can be explained. Firstly, circuit analysis and electrical engineering require calculations in this case to organise and simplify, using the benefits of linear algebra. When engineers calculate a complex system with several unknown variables efficiently on a computer, they use Gauss Elimination. Electrical devices are essential part of daily life. Does not matter What are their purposes, These devices need connecting electrical component system with each others. Engineers must be careful when they design an electrical device about the tolerance requirements for components like resistors, capacitors, and inductors. Also, they must check the voltages and currents in the circuit. Based on a resistor's voltage, current, and resistance, Ohm's law, which states V = IR, can be used to calculate the voltage drops across it. Kirchhoff's voltage law (KVL), which states that the algebraic sum of the voltages around any closed path in a circuit stays zero at all times, is a fundamental method for analyzing an electric circuit. According to Kirchhoff's voltage law, the total voltage drops surrounding any circuit path must be equal to the total voltage sources. Mesh current analysis is used to apply this law. In this method, a mesh is a single loop without any other loops, and current flows arbitrarily within each loop, ideally in a clockwise direction. In the circuit analysis, Kirchhoff's voltage law is used to calculate the current flow in each mesh. One mesh's current minus another mesh's current equals the voltage across a shared resistor, which is x (resistance). Using the matrix equation RI = V, the engineer solves for each current, determining the values of I1, I2, and I3. Then, they make a matrix of these values Because of Kirchhoff's voltage law, the matrix equation is consistent and helpful for large circuits with numerous loops.

Also, there are other areas of using linear algebra, for example, in CDMA (Code-division multiple access) using orthogonal basis vectors, error vectors, products, determinants, etc. Concepts known in algebra can be observed.[6]

2. For question 7, linear algebra has many applications, including cryptography. Cryptography is a technique that hides or scrambles data so that only authorized users can access it. It's derived from the Greek words for "to write in a hidden way" and "to transmit information securely".[7]

There is one man who should be known when analyzing the use of linear algebra in cryptology, and his name is Lester Hill. Hill created the replacement password in 1929 and it is based on the principles of matrix theory. The letters are first converted to numbers, then divided into several N-dimensional vectors, multiplied by an $n \times n$ matrix, and finally the result is mode 26. It is important to remember that the key matrix, or the ranks of the matrix and 26 interoperability, must be reversible. This method is one of many ways to create ciphertext patterns. cryptography is based on the protection of these messages except for the 2 parties that communicate with each other. There are therefore various security principles. The main idea of cryptography is based on 4 principles such as confidentiality, data integrity, authentication and non-repudiation.

First, confidentiality refers to laws and policies that ensure that only certain people or locations can access data. Data integrity ensures the accuracy and consistency of data throughout the transmission process. Authentication is the assurance that data is being claimed by the person who has a relationship with it. Non-repudiation ensures that no one involved in the transmission process can dispute the legitimacy of their signature on the data or the message sent.[8]

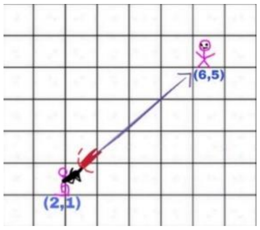
In today's technology-driven world, this system is a part of life, from people's daily messages to state affairs. It would therefore be a serious breach of security for this information to be disclosed. The prevention of this breach is one of the aims of cryptography.

3 . For question 3, linear algebra is an important component in computer graphics and image processing because it provides a mathematical basis for analyzing and enhancing visual input.

One of the basic programs for image processing applications is MATLAB. MATLAB is a program that keeps given variables in matrix format. It keeps and lists the given variables in matrix format. It uses these matrices according to the codes written in the program. RGB stands for "Red-Green-Blue" light colors; It is a method that produces other colors from red, green, and blue. [9] The RGB system automatically writes the image in the form of a matrix, the entries of which are the pixel values. As a result, any photograph is associated with a matrix, and vice versa. MATLAB can convert both images and matrices. There is an analog for any matrix storage since MATLAB reads it as a matrix. After the pieces of photographs are used, the MATLAB application writes the photographs as a matrix. After this step

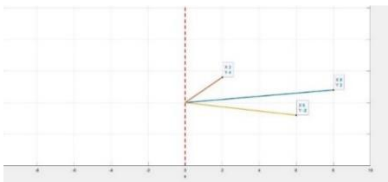
Multiplying an image by a constant, adding an image, subtracting an image, and finding the absolute value of an equation can be done. Outside of MATLAB, techniques such as singular value decomposition (SVD) use linear algebra to compress and decompress images efficiently. Image processing activities such as blurring, sharpening and edge detection rely largely on linear algebraic calculations. In image analysis and computer vision, techniques such as principal component analysis (PCA) and eigenvalue decomposition are used to differentiate object features, reduce dimensionality, and recognize objects.

Also, linear algebra serves as the foundational framework for the representation and transformation of visual elements in the field of computer graphics. Linear algebra is a field intertwined with vectors. Firstly, vector subtraction is useful to obtain a vector pointing from one location to another. [11] For example, let's say the player is standing at (2,1) with a laser rifle and an enemy is at (4,3). Subtraction is done with vectors.



$$(6,5) - (2,1) = (6-2, 5-1) = (4,4)$$

Additionally, vector addition operations are necessary for the immersion of the game and player movements.



$$a = [2,4] \; b = [6,-2] \; c = a + b \;\; c = [8,2]$$

Projection matrices created using linear algebra enable the conversion of 3D objects into 2D displays, which affects the realism of the image. Linear algebra is used for geometric modeling and 3D design in CAD systems. It allows 3D extraction of shapes, making it an important tool in architecture and engineering and healthcare. [12]

Sin and cos graphics are also one of the processes used in the game production process. It is often used to create the movement of players, to create the x and y coordinates of an object, to determine the oscillation of an object, and to reveal camera movements.[5] Additionally, linear algebraic procedures provide for more realistic experiences like anti-aliasing and shading. Linear algebra enables the design of AR and VR plans; it allows for smooth transitions between different perspectives and supports animations. [13]

Essentially, linear algebra is a versatile tool that enhances visualization, whether in the field of image processing or computer graphics.

4. For question 8, it is possible to see applications of linear algebra in the medical field, as in many other fields. Linear algebra is used in the operation of machines used to diagnose disease. An example of this is the diagnosis of skin diseases. Linear algebra consists of a matrix with columns and rows. The mathematics allows columns to be compressed into a row and a row to be compressed into a number. If the rows are a measure of skin blemishes and the columns are a ranking of melanoma risk based on biopsies, then each last compressed number is a probability of melanoma. [14] Applications of linear algebra in radiology include neural network algorithms, windowing, CT reconstruction methods, and MRI sequence algorithms. In real life, linear algebra works with vectors, matrices, and linear equations. [15] Since all digital images are matrices, most image processing in the computer is based on linear algebra. Here are some ways linear algebra can be integrated into these technologies:

In the image reconstruction phase, linear algebra techniques such as matrix inversion, least squares solutions, and Fourier transform are used to solve problems and create high-quality images. The least squares method is applied to find approximate solutions of linear equations that have no solutions. Because there may be measurement errors in the data and no adequate solution can be discovered, the least squares method is used. Another method, the Kaczmarz method, was designed to solve huge linear systems $Ax = b$. Compared to the least squares approach, Kaczmarz's method is significantly more dependable since it gives a clear image, and it is also faster. [16] Fourier transform allows the decomposition of a complex signal. In the process of creating an MR image, the Fourier transform decodes the frequencies and MR signals that form the k-space. The MR image is the 2D inverse Fourier transform of K-space. To reconstruct images from CT scans, the FBP method is utilized. This procedure entails applying a number of filters to the generated data and then reflecting it back using linear algebra methods to generate a 2D or 3D image. [17]

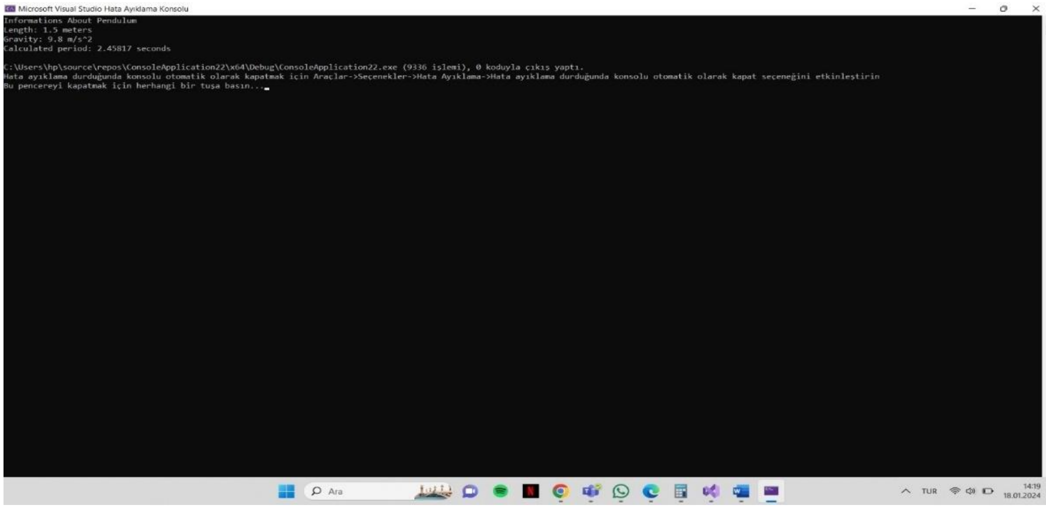
In the signal processing step, images can be represented as matrices where each element corresponds to the intensity of a pixel. Linear algebra operations on these matrices perform image-processing tasks such as filtering and enhancement. In addition, feature extraction is performed in medical images using eigenanalysis techniques. [18]

Linear transformations are required to align and compare images in the image analysis step. These transformations are often represented as matrices, and linear algebra allows these transformations to be solved.

To summarize, linear algebra underlies the basic algorithms of medical imaging systems. The field of application covers everything from reconstructing images from raw data to evaluation, processing and data security of these images

4.EE101 TASK:

```
#include <iostream>
#include <cmath>

using namespace std;
class Pendulum {
private:
    double l;
    double g;

public:

    Pendulum(double l, double g)
        : l(l), g(g) {}

    double calculatePeriod() {
        double pi = 3.14159265359;
        double T = 2 * pi * sqrt(l / g);
        return T;
    }


    void displayInfo() {
        cout << "Informations About Pendulum" << endl;
        cout << "Length: " << l << " meters" << endl;
        cout << "Gravity: " << g << " m/s^2" << endl;
        cout << "Calculated period: " << calculatePeriod() << " seconds" << endl;
    }
};

int main() {

    Pendulum call(1.5, 9.8);


    call.displayInfo();


    return 0;

}
```



DISCUSSION

Differences between real-world and theoretical data can happen for various reasons. One big reason is how accurate the model is. If the model doesn't match the theory closely, the results can turn out different from what we expected. The experimental setup must be carefully designed and controlled to minimize external influences that can lead to uncertainties. Accurate measurements of parameters such as pendulum length, moment of inertia and torsional constant contribute to a better fit. Also, the small amplitude approximation is often used in theoretical models for torsional pendulums, under the assumption that the angular displacement from the equilibrium position is small. When the angular displacement is small, this approach makes the mathematical description simpler and can improve the agreement with experimental data.

The motion of the pendulum can be affected by outside variables such as vibrations, temperature changes, and air resistance. The agreement between the theoretical model and the experiment is improved if these external factors are taken into account and adjusted for.

The damping effect, or air friction in general, is one of the negative forces acting on a pendulum. Because air friction is an unavoidable factor, strategic planning is required for its mitigation. Firstly, the best place to minimize air flow interference must be selected. The next step would be to work within this assigned surroundings to create a solitary setting for experimentation. This entails setting up an environment with the highest possible air blocking. By reducing air friction, such actions help to bring experimental results closer to theoretical values. Moreover, it is critical to recognize that the torsion pendulum effect present in the experiment's wire component makes it more difficult to achieve the theoretically predicted perfection in results.

In addition, when choosing the wool rope, care should be taken to ensure that it is not wrapped more than once because winding the wool rope more than once affects the oscillation and may cause incorrect data to be obtained. Several of these reasons may have caused loss of energy.

CONCLUSION

In conclusion, first the physics background was examined, then the experimental set up was made and the values were uploaded to the tracker application, then the MATH153 task was started and the volume of the thick-walled object was found by the washer method and the reason why this method was used was explained. Afterwards, 4 of the 9 questions given for math103 were selected and answered. Finally, c++ code was created for EE101 task.

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