Department of
Electrical & Electronics Engineering

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Project Report

EE1200 Electronic System Design (ESD) Capsule

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OBJECTIVE

The objective of this project is to design and analyze inductors to understand their properties and behavior. This project was carried out on the Helmholtz coil, which is a special case of inductors. A general B(z) formula was found for the theoretical part of the project and the required d/r ratio to be used in the experiment was calculated with the help of the Taylor series. Then, this magnetic field vector was moved to the cylindrical coordinate system, partial derivatives were found, and the inductance values of the coils were found theoretically, and finally, a circuit that could measure the inductance, such as an oscilloscope signal generator, was established with the materials in the laboratory. First of all, a disk-shaped object was taken, its radius was found, uniform windings were made with copper wire and the frequency was increased until the amplitude was 5. These coils were then placed on an insulating surface as calculated and the experimental part of the project was completed by connecting a resistor next to it.

BACKGROUND

An inductor is a passive two-terminal electrical component that, when an electric current passes through it, stores energy in a magnetic field. It is also known by the names coil, choke, and reactor. [1] According to Faraday's laws, an electric current creates a magnetic field, and a change in the magnetic field causes a current to flow through the conductor. Inducing a current in a conducting coil by use of a magnetic field also results in the creation of an opposing magnetic field by the induced current. [2] The Helmholtz coil, also known as the Helmholtz coils or the Helmholtz pair, is a pair of coils that are typically used as a precise source of magnetic field when they are driven by a precise source of electric current. This is because analytical equations can be used to calculate the value of the magnetic flux density B or magnetic field strength H at the center of the pair. The German physicist Hermann von Helmholtz (1821–1894) is the name of the inventor of the Helmholtz coil.

Real coils have a finite width w and thickness, determined by the difference between the outer diameter and the inner diameter. Assuming a uniform current distribution, the field along the axis of a thick Helmholtz coil can be derived by combining two thick solenoids. [3]

ANALYTICAL AND SIMULATION PROCEDURES

1) A fishing line assembly was provided to make the coils. The fishing line was removed from the assembly and the assembly was made empty.
2) The radius of the assembly was measured as $0.025\ m$ and its area was calculated accordingly.
3) The copper wire supplied was wrapped around the device 33 times. This process was carried out in 2 assemblies.
4) The circuit was set up and one of the coils was connected to the circuit. Waves at appropriate frequency and amplitude values were given to the circuit and inductance calculations were made.
5) The theoretically calculated inductance values were compared and analysed with the inductance values obtained in the experiment.
6) The same operations were performed by connecting the second coil to the circuit.
7) Using the obtained L value, a square wave of 5V amplitude was applied to the circuit at periods T=10L/R, T=L/R, T=L/10R.
8) At the frequencies obtained, the amplitude value was halved and a square wave was applied to the circuit again.
9) The desired circuits were realised again in Ltspice simulation.
10) The values obtained were analysed and compared with each other.

MATERIALS :

fishing line assembly	oscilloscope	resistance	calculator
copper wire	oscilloscope probes	signal generator	crocodile cables

RESULT

Preliminary task.

Determine the magnitude of the magnetic field induction B on the axis of a single circular coil of radius R with current I depending on the distance z orthogonal to its plane.

$$dB = \frac{N\mu_0}{4\pi}\frac{I\,d\vec{l} \times \vec{r}}{|r|^3}$$

$$|R| = \sqrt{R^2 + z^2}$$

$$dl = \ Rd\phi\hat{\phi}$$

$$dl \times R = \ R^2 d\phi\hat{z}$$

$$dl \times R = \left(Rd\phi\hat{\phi}\right) \times (z\hat{z} - R\hat{x}') = Rd\phi\left(R\hat{\phi} \times \hat{x}' - z\hat{\phi} \times \hat{z}\right)$$

$$dB = \frac{\mu_0}{4\pi}\frac{IR^2N\,d\phi(R\hat{z} + z\hat{x})}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$\vec{B} = \int dB$$

$$B_{(z)} = \frac{\mu_0 IR^2N}{4\pi(R^2 + z^2)^{\frac{3}{2}}}\int_0^{2\pi} (R\hat{z} + z\hat{x})\,d\phi$$

$$\int_0^{2\pi} \hat{x}'\,d\phi = 0 \;because\;of\;symmetry$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$B_{(z)} = \frac{\mu_0 IR^2N\,2\pi}{4\pi(R^2 + z^2)^{\frac{3}{2}}}$$

$$B_{(z)} = \frac{\mu_0 IR^2N}{2(R^2 + z^2)^{\frac{3}{2}}}$$

- **Question-1:** Basic model for the Helmholtz coil. Consider now two identical circular coils of radius R, the current in each of them is equal to I. The coils are arranged so that their planes are parallel and their centers lie on the same z-axis at a distance d from each other, see Figure1. Let's choose a coordinate system so that its z-axis coincides with the axis of the coils.

• Find the general equation for the magnetic field B(z).

For Helmholtz coils ,

$$B_{(z)} = B_A(z) + B_B(z)$$

For $B_A(z)$ z is equal $-\frac{d}{2}$ and for $B_B(z)$, z is equal $\frac{d}{2}$.

$$B_A(z) = \frac{\mu_0 I R^2 N}{2\left(R^2 + \left(z - \frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$B_B(z) = \frac{\mu_0 I R^2 N}{2\left(R^2 + \left(z + \frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$B_{(z)} = B_A(z) + B_B(z)$$

$$B_{(z)} = \frac{\mu_0 I R^2 N}{2\left(R^2 + \left(z - \frac{d}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{\mu_0 I R^2 N}{2\left(R^2 + \left(z + \frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

➢ **Question-2:**

Let's assuming that the currents in the coils flow in one direction. Determine at which ratio d/R the magnetic field in the center of the system on the axis of the turns will be as uniform as possible. Remark: To do that, consider the magnetic field B(x) to be expanded into the Taylor series in a neighborhood of the axis z = 0. The field in the neighborhood of z = 0 is as more homogeneous, as more derivatives of B in the Taylor expansion are equal to zero.

• Based on B'(0) and B''(0) determine d/R.

For finding the best d/r that makes the magnetic field as uniform as possible in the center of a Helmholtz coil firstly, the first and second derivatives of the magnetic fields with respect to z when z = 0

The magnetic field of a single coil formula comes from Biot-Savart law and the magnetic field formula of two coils can be found using the same formula.

B(z) = $\frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$

When two same coils are located at z=d/2 and z= - d/2 the magnetic field at a point z on the axis can be found:

$B_{total}(z)$= B(z - d/2) + B(z + d/2)

When expanding Btotal(z) around z = 0 Taylor series will be used

$B_{total}(z)$= $B_{total}(0)$ + d/dz B_{total} z + ½ d^2B_{total}/dz^2 z^2 + …

So, first of all the first derivation of $B_{total}(0)$ should be found:

$dB_{total}(0)/dz = \frac{d}{dz}\left(B\left(z-\frac{d}{2}\right)+B\left(z+\frac{d}{2}\right)\right)$

$dB_{total}/dz = B'\left(z-\frac{d}{2}\right)\cdot 1+B'\left(z+\frac{d}{2}\right)\cdot(1)$

So, at z=0;

$dB_{total}/dz = B'\left(-\frac{d}{2}\right)\cdot 1+B'\left(\frac{d}{2}\right)\cdot(1)$

For finding the single coil's formula can be used

$$B(z) = \frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$$

$$\frac{dB(z)}{dz}=\frac{d}{dz}\frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$$

$$\frac{dB(z)}{dz}=\frac{\mu_o z R^2-3z}{2\sqrt{(R^2+z^2)^5}}$$

$$dB(z)=\frac{-3\mu_o I R^2 z}{2\sqrt{(R^2+z^2)^5}}$$

Evaluating at z = d/2 and the result will be:

$$B'\left(\frac{d}{2}\right) = \frac{-3\mu_o I R^2\left(\frac{d}{2}\right)}{2\sqrt{\left(R^2 + \left(\frac{d}{2}\right)^2\right)^5}}$$

So,

$$B'\left(-\frac{d}{2}\right) = -B'\left(\frac{d}{2}\right)$$

Thus, the first derivative will be 0.

Second derivative at z = 0 ;

$$\frac{d^2}{dz^2} Btotalz(0) = B''\left(-\frac{d}{2}\right) + \left(\frac{d}{2}\right)$$

$$\frac{d^2B(z)}{dz^2} = \frac{d}{dz}\frac{3\mu_0 I R^2 z}{2\sqrt{(R^2+z^2)^5}}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 I R^2}{2}\left(\frac{1}{\sqrt{(R^2+z^2)^5}} \quad \frac{-5z^2}{\sqrt{(R^2+z^2)^7}}\right)$$

$$\frac{d^2B(z)}{dz^2} = \frac{-3\mu_0 I R^2 (R^2 - 4z^2)}{2\sqrt{(R^2+z^2)^7}}$$

Evaluating for z = d/2:

$$\frac{d^2B(z)}{dz^2} = \frac{3\mu_0 I R^2\left(R^2 - 4(\frac{d}{2})^2\right)}{2\sqrt{(R^2 + (d/2)^2)^7}}$$

This provides that the magnetic field is more uniform over a limited radius around z=0, preventing it from changing fast at the center.

So for the second derivative to be 0 it should be :

$R^2 - d^2 = 0$ and R= d so the d/r =1

- **Question 3:**

Make a reasonable guess about the homogeneity of the magnetic field in the direction perpendicular to the z-axis.

$$B(z) = \frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$$

$$B_{total}(z) = B(z - d/2) + B(z + d/2)$$

$$B_{total}(z) = \frac{\mu_0 I R^2}{2\cdot\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{\mu_0 I R^2}{2\cdot\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$B(z) = \frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$$

$$B_{total}(z) = B_{total}(0) + \left.\frac{dB_{total}}{dz}\right|_{z=0} + \left.\frac{d^2 B_{total}}{dz^2}\right|_{z=0} \frac{z^2}{2!} \cdots$$

$$\frac{dB(z)}{dz} = \frac{d}{dz}\frac{\mu_0 I R^2}{2\sqrt{(R^2+z^2)^3}}$$

$$\frac{dB(z)}{dz} = \frac{\mu_o z R^2 - 3z}{2\sqrt{(R^2+z^2)^5}}$$

$$dB(z) = \frac{-3\mu_o I R^2 z}{2\sqrt{(R^2+z^2)^5}}$$

$$\frac{dB_{total}(z)}{dz} = \frac{dB}{d^z}\left(z+\frac{d}{2}\right) + \frac{dB}{dz}\left(z-\frac{d}{2}\right)$$

$$\frac{dBtotal}{dz} = \frac{-3\mu_0 I R^2\left(z+\frac{d}{2}\right)}{2\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} - \frac{-3\mu_0 I R^2\left(z-\frac{d}{2}\right)}{2\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{\frac{5}{2}}}$$

For z=:

$$\left.\frac{dB_{total}}{dz}\right|_{z=0} = \frac{-3\mu_0 I R^2\left(0+\frac{d}{2}\right)}{2\left(R^2+\left(0+\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} - \frac{-3\mu_0 I R^2\left(0-\frac{d}{2}\right)}{2\left(R^2+\left(0-\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} = \frac{-3\mu_0 I R^2\left(\frac{d}{2}\right)}{2\left(R^2+\left(\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} - \frac{-3\mu_0 I R^2\left(\frac{d}{2}\right)}{2\left(R^2+\left(-\frac{d}{2}\right)^2\right)^{\frac{5}{2}}}$$

Since these two terms have opposite signs, the result is 0.

$$\left.\frac{dB_{total}}{dz}\right|_{z=0} = 0$$

For second derivative:

$$\left.\frac{d^2B_{total}}{dz^2}\right|_{z=0} = \frac{d^2}{dz^2}\frac{-3\mu_0 IR^2\left(0+\frac{d}{2}\right)}{2\left(R^2+\left(0+\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} - \frac{-3\mu_0 IR^2\left(0-\frac{d}{2}\right)}{2\left(R^2+\left(0-\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} = \frac{-3\mu_0 IR^2\left(\frac{d}{2}\right)}{2\left(R^2+\left(\frac{d}{2}\right)^2\right)^{\frac{5}{2}}} - \frac{-3\mu_0 IR^2\left(\frac{d}{2}\right)}{2\left(R^2+\left(-\frac{d}{2}\right)^2\right)^{\frac{5}{2}}}$$

$$\left.\frac{d^2B_{total}}{dz^2}\right|_{z=0} = 0$$

Thus, for a ratio d=r of 1, the Helmholtz coil configuration ensures that both the first and second derivatives of the magnetic field at z=0 are zero, resulting in a highly homogeneous magnetic field near the center of the coils. The coils have a symmetrical configuration along the z-axis since their centers are at z points. Therefore, the results of the singular derivatives of the magnetic field around the point z=0 are 0. As for the calculation for higher order double derivatives, the double symmetric values with respect to the z-axis will be equal to the same value, i.e. 0. Because the coils are symmetrical and identical, deviations occur due to higher order derivatives of the ideal field in the z-axis (the direction parallel to the coils), particularly in the B(z) direction. In the Taylor series, the derivatives of perpendicular components B(x) and $B(y)$ add up to zero.

- **Question 4**

Find the magnetic field on the axis of coils in the neighborhood of the center of the system as the function of μ0, I and R for the determined d/R.

When calculating the sum of the magnetic field on the axis of two coils, we assume that the distance between the centers of the coils $d = R$ and in this case $\frac{d}{R} = 1$ is assumed. The magnetic field on the axis of the coils adjacent to the center of the system can be approximated using the Taylor series. The solution is obtained by expanding the Helmholtz expression for the total magnetic field of the coil around $z = 0$.

Given formula:

$$B_{total}(z) = \text{B}(\ z - d/2) + \text{B}(\ z + d/2)$$

$$B(z) = \frac{\mu_0 IR^2}{2\sqrt{(R^2+z^2)^3}}$$

$$B\left(z-\frac{d}{2}\right) = \frac{\mu_0 IR^2}{2\cdot\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$B\left(z+\frac{d}{2}\right) = \frac{\mu_0 IR^2}{2\cdot\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

The sum of these two terms:

$$B_{total}(z) = \frac{\mu_0 IR^2}{2\cdot\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{\mu_0 IR^2}{2\cdot\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

If taylor expansion is done for z =0:

$$f(z) = f(0) + f'(0)z + \frac{f''(0)z^2}{2!} + \cdots$$

First find the derivatives of the $B\left(z-\frac{d}{2}\right)$ and $B\left(z+\frac{d}{2}\right)$ functions of the x and y functions at z=0.

$$B_{total}(0) = 2\cdot\frac{\mu_0 IR^2}{2\left(R^2+\frac{d^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 IR^2}{\left(R^2+\frac{d^2}{4}\right)^{\frac{3}{2}}}$$

The first derivative:

$$B'(z)\frac{\mu_0 IR^2(-3z)}{2(R^2+z^2)^{\frac{5}{2}}}$$

$$B'(0) = 0$$

The second derivative:

$$B''(z) = \frac{\mu_0 IR^2(-3)(R^2+3z^2)}{2(R^2+z^2)^{7/2}}$$

$$B''(0) = -\frac{3\mu_0 I}{2R^3}$$

$$B_{total}(z) \approx B_{total}(0) + \frac{B_{total}''(0)z^2}{2!}$$

$$B_{total}''(0) = 2.B''(0) = 2.-\frac{3\mu_0 I}{2R^3} = -\frac{3\mu_0 I}{R^3}$$

$$B_{total}(z) \approx \frac{\mu_0 I R^2}{\left(R^2 + \frac{d^2}{4}\right)^{\frac{3}{2}}} - \frac{3\mu_0 I}{2R^3} z^2$$

For $d = R$:

$$B_{total}(0) = \frac{\mu_0 I R^2}{\left(R^2+\frac{R^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{\left(\frac{5R^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{R^3\left(\frac{5}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{R^3\left(\frac{\sqrt{5}}{2}\right)^3} = \frac{2^3\mu_0 I}{5^{3/2}R} = \frac{8\mu_0 I}{5^{3/2}R}$$

$$B_{total}(z) = \frac{8\mu_0 I}{5^{3/2}R} - \frac{3\mu_0 I}{2R^3} z^2$$

- **Question 5:**

Now consider two identical thin coils of N turns each, placed coaxially to each other at a distance equal to their average radius. Find the magnetic field on the axis of the coils in the neighborhood of the center of the system as the function of μ0, I and R for the determined d/R.

the coils are located at z=± R/2 and the magnetic fields is:

$$B(z) = \frac{\mu_0 I R^2}{2\sqrt{\left(R^2+(\pm\frac{R^2}{2})\right)^3}}$$

For the two coils that centered at z=$\pm\frac{R}{2}$:

Btotal(z)= B(z - R/2) + B(z + R/2)

Btotal(0)= B(0 - R/2) + B(0 + d/2)

B($\pm\frac{R}{2}$) = $\frac{\mu_0 I R^2}{2\sqrt{\left(R^2+\left(\frac{R}{2}\right)^2\right)^3}}$ and $R^2 + \left(\frac{R}{2}\right)^2 = 5R^2/4$ so,

$B(+\frac{R}{2}) = \frac{\mu_0 I R^2}{2\sqrt{\left(\frac{5R^2}{4}\right)^3}} = \frac{\mu_0 I R^2}{2\left(\frac{5\sqrt{5}R^3}{8}\right)} = \frac{\mu_0 I R^2}{\left(\frac{10\sqrt{5}}{8}R^3\right)} = \frac{8\mu_0 I}{10\sqrt{5}R} = \frac{4\mu_0 I}{5\sqrt{5}R}$

$B(-\frac{R}{2})$ will have the same result because of the power 2.

$B(-\frac{R}{2}) = \frac{\mu_0 I R^2}{2\sqrt{\left(R^2+\left(-\frac{R}{2}\right)^2\right)^3}} = \frac{\mu_0 I R^2}{2\sqrt{\left(\frac{5R^2}{4}\right)^3}} = \frac{\mu_0 I R^2}{2\left(\frac{5\sqrt{5}R^3}{8}\right)} = \frac{\mu_0 I R^2}{\left(\frac{10\sqrt{5}}{8}R^3\right)} = \frac{8\mu_0 I}{10\sqrt{5}R} = \frac{4\mu_0 I}{5\sqrt{5}R}$

Btotal(0)= $\frac{8\mu_0 I}{5\sqrt{5}R}$

So, this calculation shows that the magnetic field at z=0 is quite homogeneous and added up as vectorial.

- **Question 6:**

Derive the vector function B(r,θ,z) in the cylindrical coordinate system for the field shifted from the axis z = 0 by a small distance Δ. Evaluate the radial component Br of this magnetic field for small shifts Δ as a function of r.

For a small radial displacement from the z-axis, the radial component B can be calculated by evaluating the change in the field with respect to r Br. This indicates that B is directly proportional to the rate of change of B along the z axis at the center

$$B_r \approx -r\left(\frac{\partial B_z}{\partial Z}\right)_{z=0}$$

Due to the problem's symmetry (the coils are symmetric around the z-axis and the currents flow in a circular fashion), the azimuthal component B_θ in cylindrical coordinates is:

$$B_\theta = 0$$

This is due to azimuthal symmetry, which assures that there is no net circular (azimuthal) component of the magnetic field in the middle region of the coils

This component is oriented along the z-axis and is mostly homogeneous around the center of the Helmholtz coils.

$$B_z \approx B(0) + \left(\frac{\partial B_z}{\partial Z}\right)_{z=0} z$$

$$B(r,\theta,z) = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z}$$

$$B(r,\theta,z) \approx \left(-r\left(\frac{\partial B_z}{\partial z}\right)_{z=0}\right)\hat{r} + 0\theta + \left(B(0) + \left(\frac{\partial B_z}{\partial z}\right)_{z=0} z\right)\hat{z}$$

- **Question 7:**

Evaluate the partial derivatives $\partial B/\partial r$, $\partial B/\partial z$ and $\partial B/\partial \theta$ for the whole vector B

1) $\frac{\partial B}{\partial r}$

For B_r:

$$\frac{\partial B_r}{\partial r} = -\frac{3\mu_0 I R^2\, d}{4\left(R^2 + \left(\frac{d}{z}\right)^2\right)^{\frac{5}{2}}}$$

For B_θ:

$$\frac{\partial B_\theta}{\partial r} = 0$$

For B_z:

$$\frac{\partial B_z}{\partial r} \approx 0$$

2) $\frac{\partial B}{\partial z}$

For B_r:

$$\frac{\partial B_r}{\partial z} = 0$$

For B_θ:

$$\frac{\partial B_\theta}{\partial z} = 0$$

For B_z:

$$\frac{\partial B_z}{\partial z} = \frac{3\mu_0 I R^2\, d}{4\left(R^2 + \left(\frac{d}{z}\right)^2\right)^{\frac{5}{2}}}$$

3) $\frac{\partial B}{\partial \theta}$

For B_r:

$$\frac{\partial B_r}{\partial \theta} = 0$$

For B_θ:

$$\frac{\partial B_\theta}{\partial \theta} = 0$$

For B_z:

$$\frac{\partial B_z}{\partial \theta} = 0$$

- **Question 8:**

Calculate the inductance of one of the coils theoretically.

$$L = \frac{k\mu_r\mu_0 N^2 A}{l}$$

k is Nagaoka coefficient.

(The value of 0.4 was preferred because it is compatible with experimental observations and is generally used in theoretical studies. In particular, the value 0.4 is taken for experimental studies and theoretical analyzes on the distribution of the magnetic field in systems such as cylindrical coils or Helmholtz coils.)

To compare magnetic permeability values μ_r=1 is taken as reference.

$$N = 33$$

$$A = \pi r^2$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$r = 0.025\ m$$

$$l = 0.012m$$

$$L = \frac{0.4 \times 1 \times (4\pi \times 10^{-7}) \times 33^2 \times 0.001963}{0.012}$$

$$L = 89.5\ \mu\text{H}$$

- **Question 9:**

Propose a measurement system employing oscilloscope, signal generator, and basic circuit elements to measure the inductance.

Firstly, the coil and 10ohm resistor were connected to the circuit and the sine wave with 5V amplitude, 0 offset and frequency of 100kHz was applied to the circuit and the values were recorded.

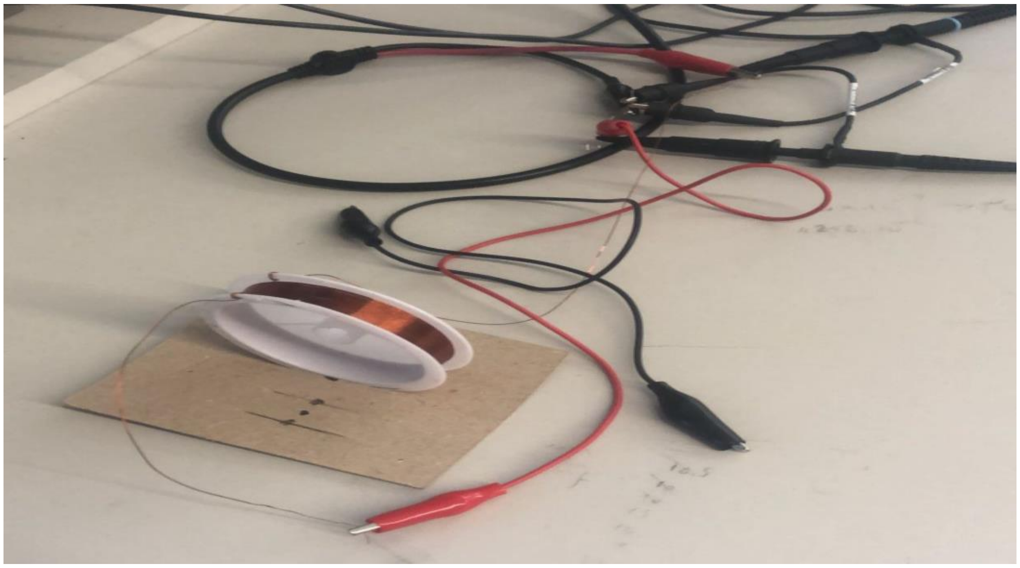

Figure 1:Connection of a single inductor to the circuit

The values of Frequency, Vin and Vout values taken from here is substituted in the formula and L is calculated.

Figure 2:The values when a single inductor is connected to the circuit.

$$Z = R + j\omega L$$

$$I = \frac{v_{in}}{Z} = \frac{v_{in}}{R + j\omega L}$$

$$v_{out} = I \cdot j\omega L = v_{\frac{in \cdot j\omega L}{R + j\omega L}}$$

$$v_{out} = v_{in} \left| \frac{\omega L}{R + j\omega L} \right|$$

$$\left| \frac{j\omega L}{R + j\omega L} \right| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$v_{out} = v_{in} \cdot \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$\frac{R^2}{(R^2 + \omega^2)L^2} = \frac{Vout^2}{Vin^2}$$

$$L^2 = \frac{R^2\, Vin^2}{Vout^2 (R^2 + \omega^2)}$$

$$L = \frac{R}{\omega} \sqrt{\frac{1 - \frac{Vout^2}{Vin^2}}{\frac{Vout^2}{Vin^2}}}$$

$$\omega = 2\pi f = 2\pi \times 100 \times 10^3 Hz \approx 628.31853 \text{ rad/s}$$

$$Vout = 520mV = 0.520V$$

$$Vin = 3.30V$$

$$\frac{Vout}{Vin} = \frac{0.520}{3.30}$$

$$\frac{Vout^2}{Vin^2} = \frac{0.2704}{10.89} \approx 0.02483$$

$$L = \frac{10}{628.31853}\sqrt{\frac{1-0.02483}{0.02483}}$$

$$\sqrt{\frac{0.97517}{0.02483}} \approx \sqrt{39.27} \approx 6.265$$

$$L = \frac{10}{628.31853} \times 6.265 = 99.7\mu H$$

The two coils are connected to the circuit by paying attention to their direction.

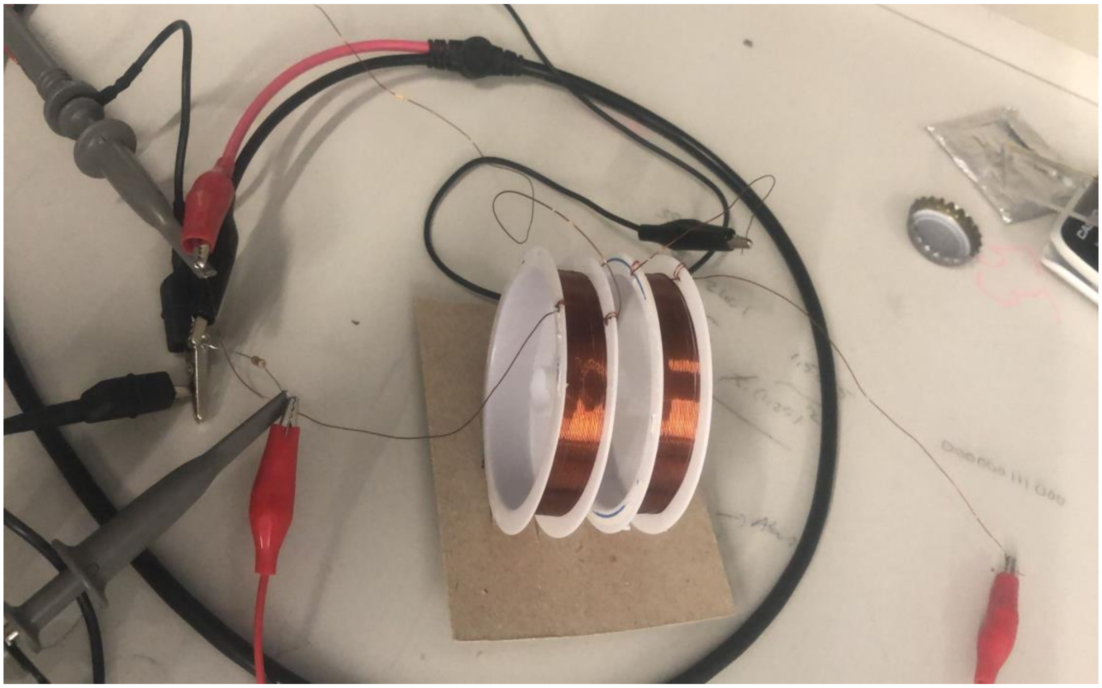

Figure 3: Connection of 2 inductor to the circuit

Then a sine wave with 5v amplitude, 0 offset and 100k frequency was given. Inductance values were calculated according to the recorded values.

Figure 4:The values when 2 inductor is connected to the circuit.,

$$L = \frac{R}{\omega}\sqrt{\frac{1 - \frac{Vout^2}{Vin^2}}{\frac{Vout^2}{Vin^2}}}$$

$$\omega = 2\pi f = 2\pi \times 100 \times 10^3 Hz \approx 628.31853 \text{ rad/s}$$

$$Vout = 304mV = 0.304V$$

$$Vin = 4.32V$$

$$\frac{Vout}{Vin} = \frac{0.304}{4.32}$$

$$\frac{Vout^2}{Vin^2} = \frac{0.092416}{18.6624} \approx 0.00495$$

$$L = \frac{10}{628.31853}\sqrt{\frac{1-0.00495}{0.00495}}$$

$$\sqrt{\frac{0.99505}{0.00495}} \approx \sqrt{201.02} \approx 14.18$$

$$L = \frac{10}{628.31853} \times 14.18$$

$$L = 225.6\mu H$$

Experiment:

Connect a 10Ω resistor in series with the designed Helmholtz coil. Apply a 0-5V symmetric square wave and investigate the voltage responses across both the coil and the resistor while varying the frequency of the square wave. Model the Helmholtz coil in SPICE and implement each step in simulation too.

- **Question 1:**

Specify the frequency of the complete charging and discharging of coil.
The amount of time required for the charging and discharging of an inductor is 5 tau. Therefore, the calculations are as follows.

$$\tau = \frac{L}{R}$$

$$\tau = \frac{225.6}{10} \times 10^{-6} = 22.56 \mu s$$

$$5\tau = 112.8\ \mu s$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{112.8 \times 10^{-6}}$$

$$f = 8867 Hz$$

In order to experimentally show the charge and discharge frequency of the coil, the values obtained by giving a 5V amplitude square wave at a frequency value of 44326Hz were taken as reference. Accordingly, the filling and discharge frequency can be obtained experimentally as follows.

$$f = \frac{1}{5} \times 44326 = 8865.2\text{Hz}$$

8865.2Hz=8.865kHz

Figure 5:The frequency of the complete charging and discharging of coil

- **Question 2:**

Change the period of the square-wave to T=10L/R, T=L/R, T=L/10R and draw the waveforms for coil and resistor.

Period values were calculated as T=10L/R, T=L/R, T=L/10R and accordingly frequency values and 5 amplitude and and 2.5 offset square wave were given to the circuit.

$$\text{For } T = \frac{10L}{R}\ ;$$

$$T = \frac{10L}{R} = \frac{225.6\times10^{-6}}{10} \times 10 \sim 0.0002256\text{s}$$

$$F = \frac{1}{0.0002256} \sim 4432.62411\text{Hz}$$

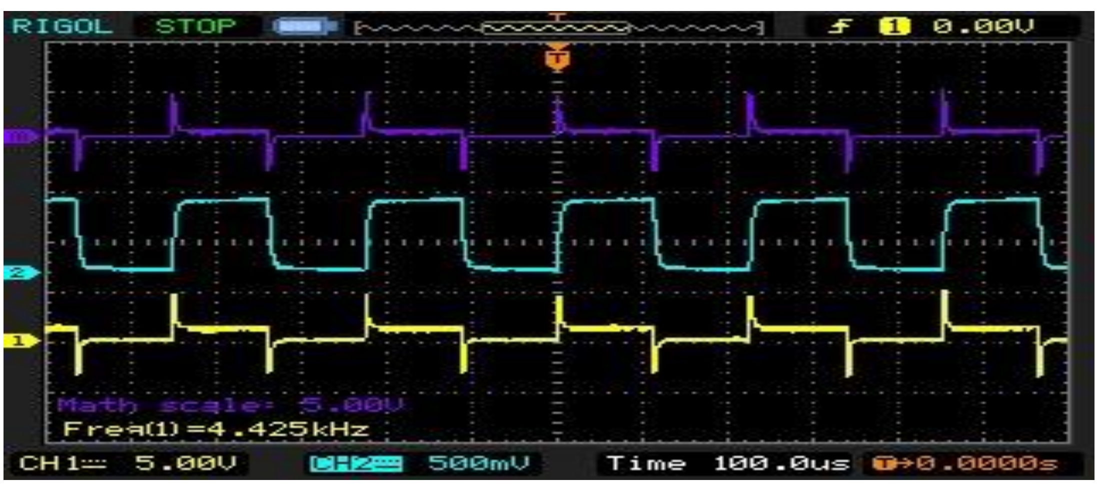

Figure 6:The values at T= 10L/R period in 5V amplitude square wave

$$\text{For} \quad T = \frac{L}{R}\ ;$$

$$T = \frac{L}{R} = \frac{225.6\times10^{-6}}{10} \sim 0.00002256\text{s} \quad \frac{1}{T} = F$$

$$F = \frac{1}{0.00002256} = 44326.2411 \text{ Hz}$$

Figure 7:The values at T=L/R period in 5V amplitude square wave

$$For \ \ T = \frac{L}{10R};$$

$$T = \frac{L}{10R} = \frac{225.6\times10^{-6}}{100} \sim 0.00000226\text{s} \quad \frac{1}{T} = F$$

$$F = \frac{1}{0.000002256} = 443262.411\text{Hz}$$

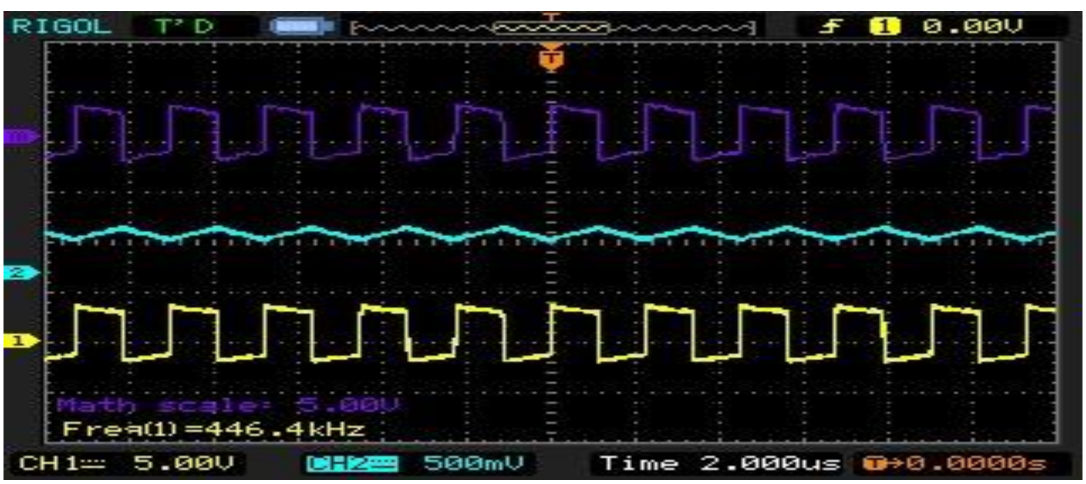

Figure 8: The values at T=L/10R;period in 5V amplitude square wave

- **Question 3:**

Reduce the amplitude of the square wave to half, repeat the measurements, and explain the observed changes.

At the calculated period values, the amplitude value was halved to 2.5V and a square wave was given at 1.25 offset.

$$\text{For } T = \frac{10L}{R} \text{ ;}$$

$$F = \frac{1}{0.0002256} \sim 4432.62411\text{Hz}$$

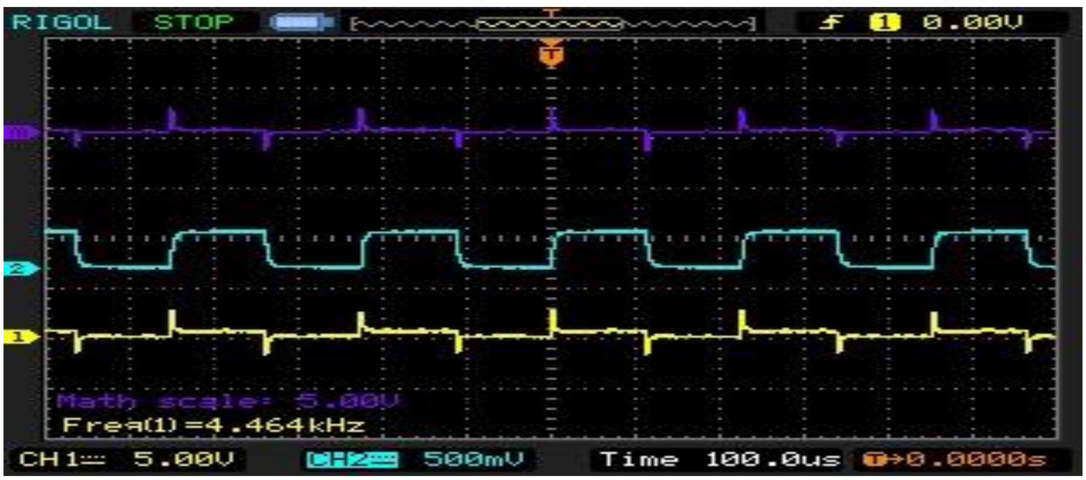

Figure 9: The values at T= 10L/R period in 2.5V amplitude square wave

$$For \quad T = \frac{L}{R} \text{ ;}$$

$$F = \frac{1}{0.00002256} = 44326.2411 \text{ Hz}$$

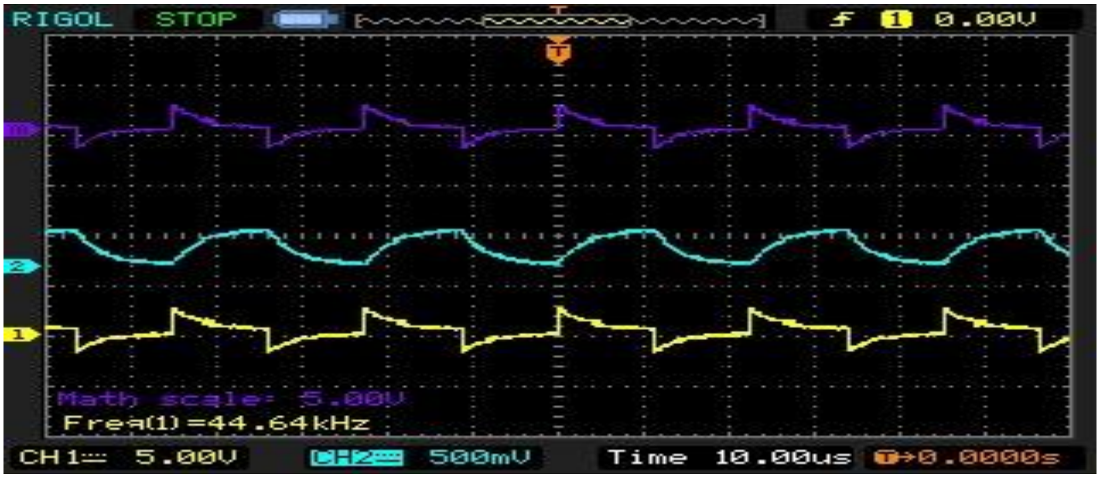

Figure 10:The values at T=L/R period in 2.5V amplitude square wave

$$For\ \ T = \frac{L}{10R};$$

$$F = \frac{1}{0.00000226} = 443262.411\text{Hz}$$

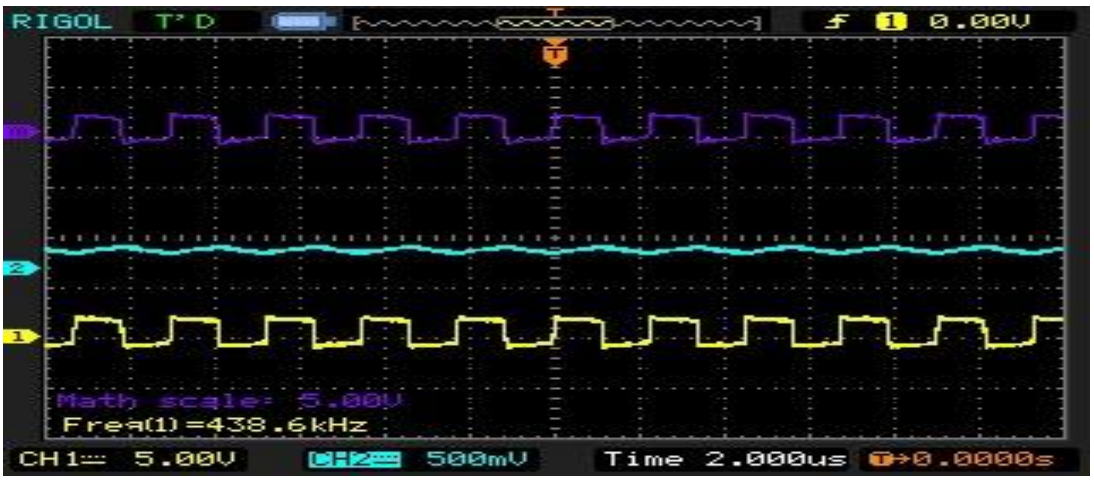

Figure 11:The values at T= 10L/R period in 2.5V amplitude square wave

- **LTSPICE:**

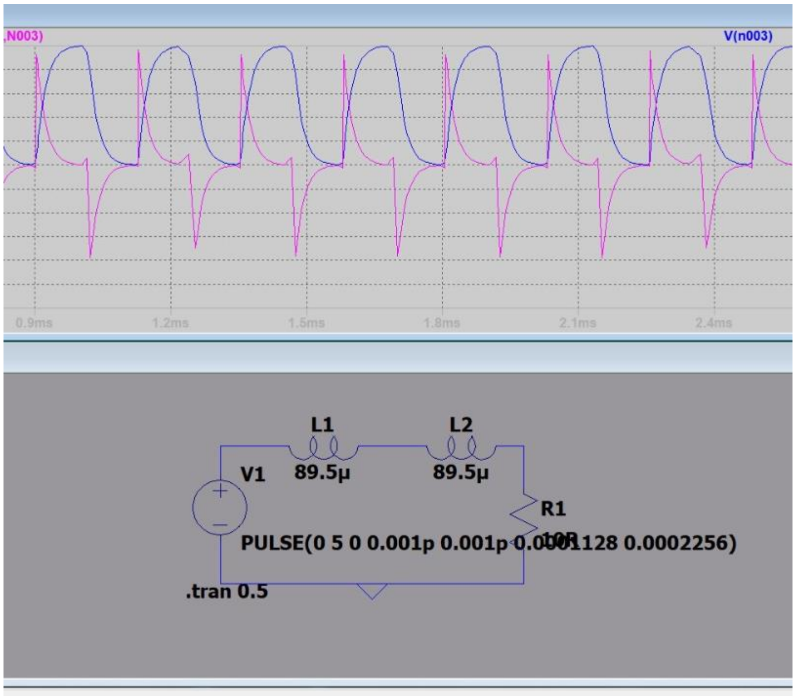

Figure 12:The square waveforms that appear when the 10 tau value is entered in the period section of ltspice

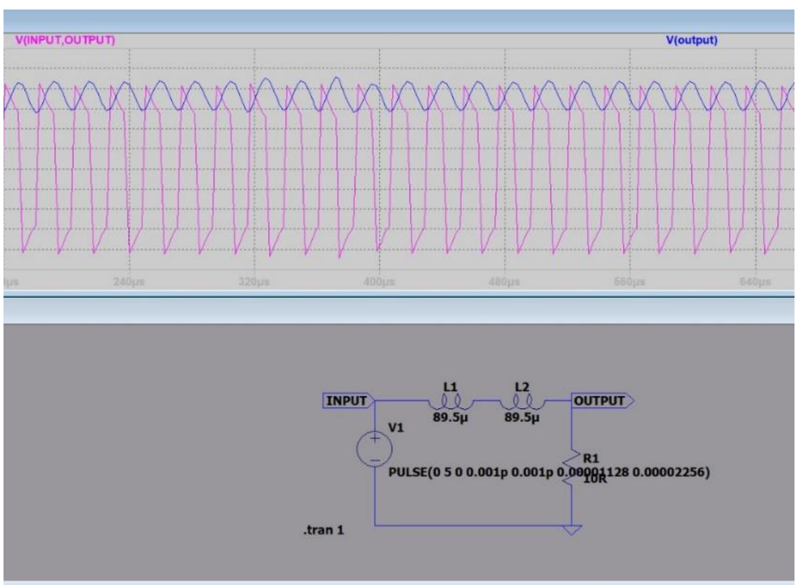

Figure 13:LTspice values at 5v amplitude for T=L/R

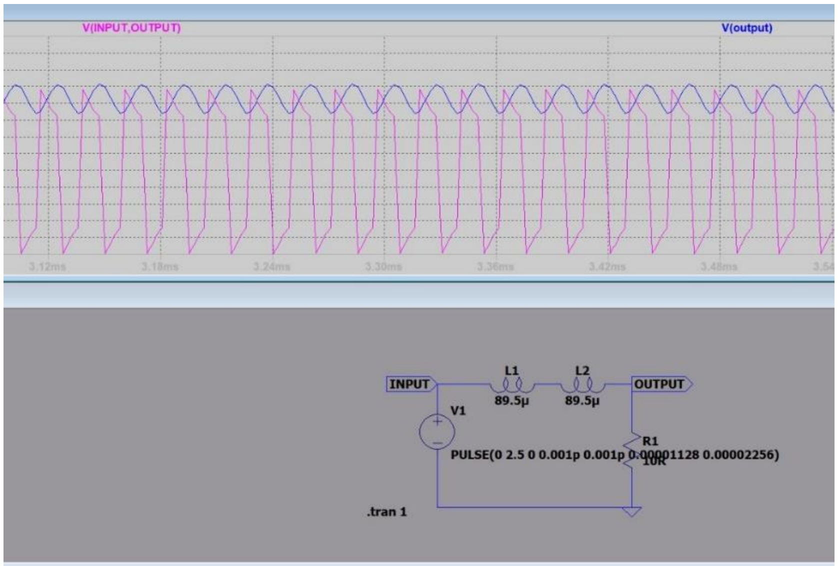

Figure 14:LTspice values at 2.5v amplitude for T=L/R

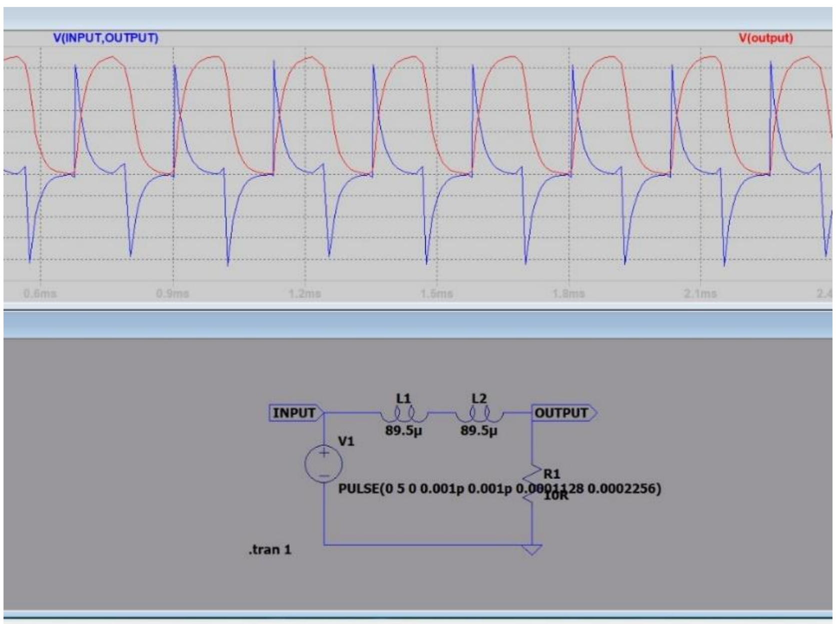

Figure 15:LTspice values at 5v amplitude for T=10L/R

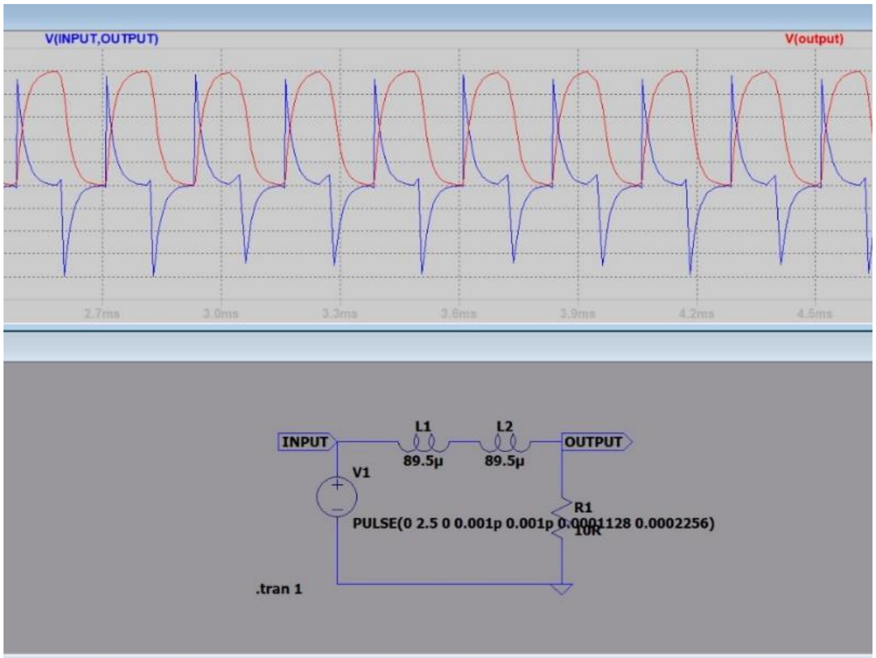

Figure 16:LTspice values at 2.5v amplitude for T=10L/R

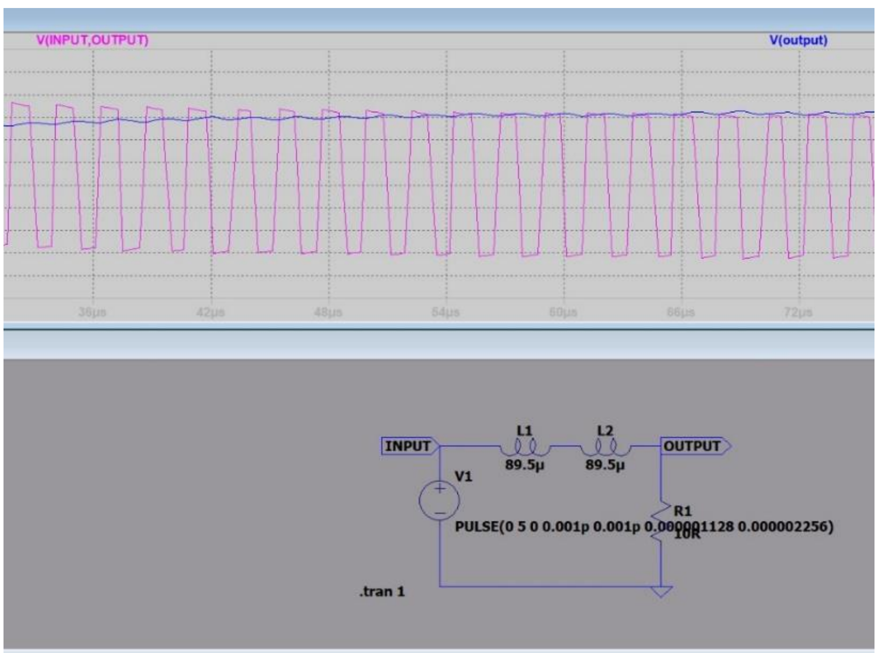

Figure 17:LTspice values at 5v amplitude for T=L/10R

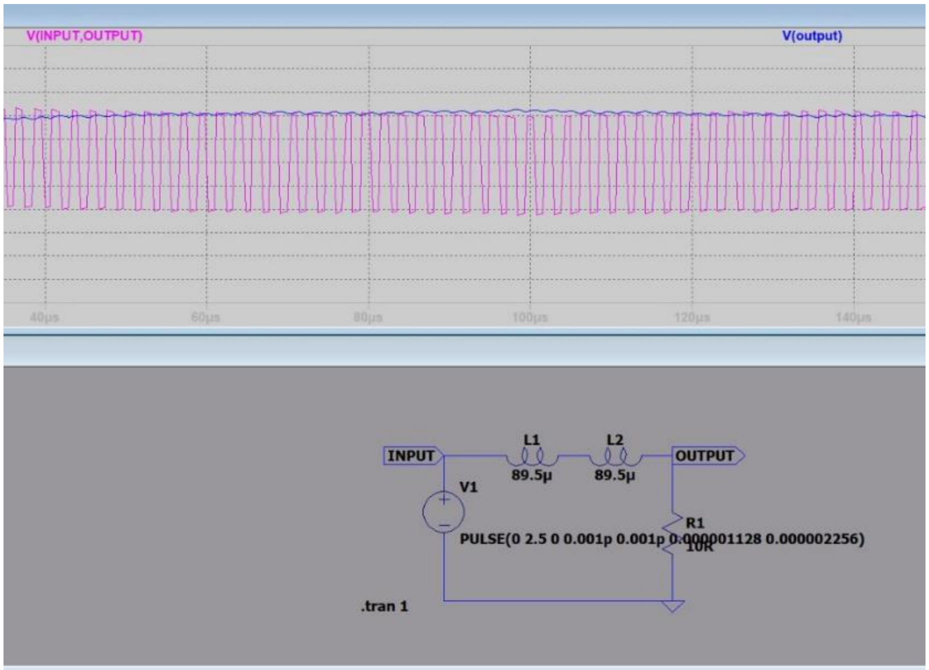

Figure 18:LTspice values at 2.5v amplitude for T=L/10R

DISCUSSION

In this experiment, a Helmholtz coil connected in series with a 10 ohm resistor was examined. First of all, a sine wave was given at different frequencies and the frequency value corresponding to 5 volts was found. The value of the coil was calculated by substituting it in the formula. By substituting the inductance of the coil and the resistors, the results T=10L/R, T=L/R, T=L/10R were calculated. Frequency values were found by dividing the period values by 1. Observations were made by using the obtained frequency values with a square wave. In the second stage of the experiment, the vpp of the square wave was halved and the steps were repeated. The Helmholtz coil was modeled in LTspice and every step performed in the laboratory was also implemented in the simulation. However, real-world results differed from LTspice simulations. For example, the material used to create a circuit with a 10 ohm resistor placed in series with the coil and the material of the module elements vary in the real world. These material differences can cause changes in cellular behavior. Since the resistor modeled in Ltspice is an ideal resistor, the difference in results is normal. In addition, the very thin and durable structure of the wrong material used in the Helmholtz coil design made winding the copper wire difficult. Copper wire with a radius of 0.2 mm has a lot of internal resistance, which affects the inductance and resistance of the coil. However, in LTspice simulations, an idealized coil model is used and the actual physical properties of the materials used in the coils may not be fully represented in these models. Another reason is that the radius of the wire in which the coil design is used is very small, which significantly affects the frequency consumption of the coil. This effect becomes especially pronounced at high frequencies. The presence of a wire then increases the capacitive presence of the coil. It has been observed that the capacitive effects observed in real-world coils are not adequately represented in LTspice simulations, especially at high frequencies such as the 440kHZ value used for a faulty part. In addition, the number of windings made during the production phase of the Helmholtz coil also affects the inductance value of the coil, and this may be possible for Ltspice and laboratory results to appear differently. . All these data and results were recorded in Ltspice records and observed by checking laboratory records.

CONCLUSION

This experiment constructed an LR circuit using a Helmholtz coil and a 10-ohm resistor. A sine wave and a 5vpp signal were sent to the circuit at a frequency value. Vin and Vout values in the circuit were observed. The values required in the formula to find the inductance were substituted. The frequency of full charge and discharge of the coil was observed from the waves observed on the oscilloscope screen. Then T=10L/R, T=L/R, and T=L/10R calculations were made. The T values found were divided by 1 to see the frequency values. A square signal was sent to the circuit with a 5vpp signal at R/10L frequency, L/R frequency, and 10R/L frequency. The same steps were repeated for a 2.5vpp signal for the second part of the experiment. The frequency of full charge and discharge of the coil was observed from the waves observed on the oscilloscope screen.

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