Department of

Electrical & Electronics Engineering

Abdullah Gül University

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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Grade: /100

CHAPTER 1

INTRODUCTION

This project aimed to study the change in the values (R, H, and time) of real life and experimental environment with the spring gun, to evaluate the equations in real life and experimental the collision to find the initial velocity of the projectile, and to be able to compare the observed results with theory, a theoretical equation was developed for tan θ by using R and H and the magnitude of v0. Also, a f(x) parabola was investigated by using the curve enveloping the points with the coordinates (x,y) which could be shut by the gun for all possible initial angles θ with the given initial velocity magnitude v0 for physics I theoretical task. Max R and H values were found for all possible initial angles θ and the previous f(x)'s maximum point and the first and second derivatives of the equation were found for the Calculus I task. Also, for the physics I experimental task, the average initial velocity v0 of the spring gun was measured, and by using theoretical equations the data found by the app Tracker were compared. Also, the mass of the ball was calculated by using an electrical balance to be able to make comments about air resistance. After that, for Math103 task matrix was obtained by selecting independent x values and found f(x) and θ depending on x values and by using that matrix inverse of matrix A by using the Gauss-Jordan method, then elimination of matrix and then LU factorization of the matrix after that, a linear system created by choosing Rmax and Hmax. As a final task, a c++ program was designed for finding the inverse of a square matrix.

BACKGROUND

This is the motion of a projectile, like a baseball, arrow, or ball, concerning the earth's mass and the force of gravity. Since ancient times, scholars have examined this topic about the beginning velocity and projection angle. Several approaches can be taken to address the non-obvious problem.[1] Galileo was the first person to study shooting motion.[2] A ball thrown above the horizontal line with a catapult, artillery, hand, or by any other means will follow the same path when falling as when rising. This shape is like that of a rope that is inverted beneath the horizontal line and is not pulled; it is formed up of both forced and natural elements and looks like both the parabola and the hyperbola in appearance.[3] In real life, there are a lot of examples of projectile motion like fireworks, and hot rock projectiles. There are a lot of formulas used to explain the mechanism of projectile motion.

y-direction data

Initial velocity = u_y	Acceleration = $a_y = -g$	Displacement = y
x-direction data		
Initial velocity = u_x	Acceleration = $a_x = 0$	Displacement = x

x-direction motion	
x-axis projectile equation $x = u_x . t$	Uniform motion
y-direction motion	
y-axis projectile equations: $y = u_y t - gt^2/2$ $v_y = u_y - gt$ $v^2{}_y = u^2{}_y - 2gy$	Non-uniform motion

Time of flight, $T = 2u\sin\theta/g$

Range of the projectile, $R = u^2\sin 2\theta/g$

Range of the projectile, $R = u^2\sin 2\theta/g$

Height of the projectile, $H = u^2\sin 2\theta/2g$ [4]

CHAPTER 2

MATERIALS

Spring gun	Spring gun balls
Protractor	Ruler
Pencil	Mobile phone

Meter	

ANALYTICAL AND SIMULATION PROCEDURES

1. A point 1 meter high was selected on the white wall and marked with a pencil.
2. The middle of the gun barrel was placed at the marked point.
3. Angles of 0, 30, 45, and 60 degrees were measured with angle measurers, and those points were marked. A 0-degree shot was made parallel to the 4-x axis.
4. The gun was aligned at a 30-degree angle with the marked point. Five shots were fired.
5. The gun was aligned at a 45-degree angle with the marked point. Seven shots were fired.
6. The gun was aligned at a 60-degree angle with the marked point. Nine shots were fired.
7. All procedures were recorded on video.
8. Videos were transferred to the tracker application.
9. The first moment when the ball left the tip of the gun and the first moment when it touched the ground was selected.
10. The x-y coordinate was correctly placed from the tracker application.
11. Shot tracking of the selected video range was performed in the Tracker application.
12. The options from which you want to get information from the columns are marked.
13. The results were recorded.

CHAPTER 3

RESULTS

Physics I theoretical tasks:

• Derive a mathematical equation for tan θ (the initial angle of v0 over the horizon) as a function of R and H and the magnitude of v0 (see Figure 1).

$$X=v_0 \cos\theta \times t$$

$$y = H + v_0 \cdot sin\,\theta - \frac{1}{2}gt^2$$

$$t = \frac{2 \cdot v_0 \cdot sin\,\theta}{g}$$

$$\tan\theta \times R - \frac{gR^2(tan^2\theta+1)}{2v_0^2} + H = 0$$

$$2v_0 R\, tan\,\theta - gR^2(tan^2\theta + 1) + 2V_0^2 H$$

$$\tan^2\theta \cdot R^2 g - \tan\theta \cdot R \cdot 2v_0^2 + gR^2 - 2HV_0^2 = 0$$

$$R^2 g \to a \quad 2Rv_0^2 \to b \quad gR^2 - 2v_0^2 H \to c$$

$$\Delta = b^2 - 4ac$$

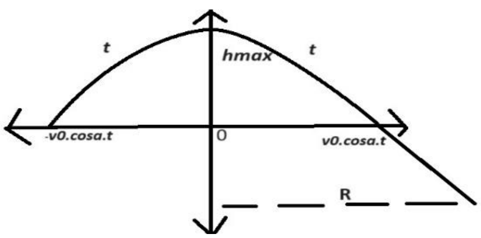
$$(2v_0^2R)^2 - 4R^2g(gR^2 - 2v_0^2H)$$

$$\Delta = 4v_0^4 \cdot R^2 - 4g^2R^4 + 8v_0HR^2g$$

$$tan\,\theta = \frac{-2Rv_0^2 \pm 2R\sqrt{v_0^4 - g^2R^2 + 2v_0Hg}}{2R^2g}$$

$$tan\,\theta = \frac{1}{gR} \cdot \left[v_0^2 \pm \sqrt{v_o^2(v_0^2 + 2gH) - g^2 \cdot R^2}\right]$$

Develop mathematical equation y = f(x) for the curve enveloping the points with the coordinates (x,y) which could be shut by the gun for all possible initial angles θ with the given initial velocity magnitude v0



$$f(x) = a(x - v_0 \cdot \cos\theta \cdot t)(x + v_0 \cdot \cos\theta \cdot t)$$

$$t = \frac{v_0 \cdot sin\,\theta}{g}$$

$$f(x) = a\left(x - \frac{v_0 \cdot cos\,\theta \cdot v_0 \cdot \sin\theta}{g}\right) \cdot \left(x + \frac{v_0 \cdot \cos\theta \cdot v_0 \cdot \sin\theta}{g}\right)$$

$$f(x) = a\left(X^2 - \frac{v_0^4 \cdot sin^2\,\vartheta \cdot cos^2\,\theta}{g^2}\right)$$

$$f(0) = h_{max} = \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

$$\frac{-a \times vo^4\,sin^2\,\theta \cdot cos^2\,\theta}{g^2} = \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

$$a = -\frac{-g}{2 \cdot v_0^2 \cdot cos^2\,\theta}$$

$$f(x) = \frac{-g}{2 \cdot v_0^2 \cdot cos^2\,\theta} \times \left(x^2 - \frac{v_0^4 \cdot sin^2\,\theta \cdot cos^2\,\theta}{\frac{2}{g}}\right)$$

$$f(x) = \frac{-gx^2}{2v_0^2 \cdot cos^2\,\theta} + \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

$$f(x) = \frac{-1}{2}\left(\frac{gx^2}{v_0^2 \cdot cos^2\,\theta} - \frac{v_0^2 \cdot sin^2\,\theta}{g}\right)$$

Calculus I tasks: • Find the maximum R and H for all possible initial angles θ

$$R^2 \cdot g^2 \cdot tan^2\,\theta - 2Rgv_0^2\,tan\,\theta + (R^2 \cdot g^2 - 2gH \cdot v_0^2) = 0$$

$$tan\,\theta = \frac{1}{Rg}\left[V_0^2 \pm \sqrt{v_0^2(v_0^2 + 2gH) - g^2 \cdot R^2}\right]$$

$\tan\theta\ exist$

$$R \le \frac{v_0\sqrt{v_0^2 + 2gH}}{g}$$

$$Rmax = \frac{v_0\sqrt{v_0^2 + 2gH}}{g}$$

$$H_{max} = H + \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

Investigate the function f(x) (dashed red line on Figure 2) and find its maximum and its derivatives df/dx and d 2 f/dx2 .

$$f(x) = \frac{-gx^2}{2v_0^2 \cdot cos^2\,\theta} + \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

$f(x)' = \frac{-gx}{v_0^2 \cdot cos^2\,\theta}$=0 x=0 $f(0) = -\frac{1}{2}\left(\frac{0 - v_0^2 \cdot sin^2\,\theta}{g}\right) = \frac{v_0^2 \cdot sin^2\,\theta}{2g}$ $max = \left(0, \frac{v_0^2 \cdot sin^2\,\theta}{2g}\right)$

$$f(x)'' = \frac{-g}{v_0^2.\,cos^2\,\theta}$$

Math103 tasks: • Obtain a matrix A as given below. You are free to choose x values but you should find f(x) and θ depending on x values. All the entries must be different from zero. ➢ Find the inverse of A by the Gauss-Jordan method. ➢ Find elimination matrices of A ➢ Find LU factorization of matrix A

$$f(x) = \frac{-gx^2}{2v_0^2 \cdot cos^2\,\theta} + \frac{v_0^2 \cdot sin^2\,\theta}{2g}$$

$$-1 \le \sin\theta \le 1$$

$$-1 \le cos\,\theta \le 1$$

For Rmax , $v_0 \sim 5$ $g \sim 10$

$$f(x) = -x^2 + \frac{5}{4}$$

$$\frac{f(x) =}{x} tan\,\theta$$

$$\theta = arctan\frac{f(x)}{x}$$

$x = -1$ $\quad f(x) = -\frac{1}{4}$ $\quad \theta_1 = 11{,}25$

$x_2 = \frac{-3}{2}$ $\quad f(x) = -1$ $\quad \theta_2 = 21{,}14$

$x = \frac{-5}{2}$ $\quad f(x) = 2$ $\quad \theta_3 = 63{,}43$

$$A = \begin{bmatrix} -1 & -1/4 & 11{,}25 \\ -3/2 & -1 & 21{,}14 \\ -5/2 & 2 & 63{,}43 \end{bmatrix}$$

Multiply the first row by -3/2 and add it to the second row

$$R2{=}R2{+}3/2\ R1 \begin{bmatrix} -1 & -1/4 & 11.25 \\ 0 & 1/2 & 24.675 \\ -5/2 & 2 & 63.43 \end{bmatrix}$$

Multiply the first row by −5/2 and add it to the third row

$$R3{=}R3{-}5/2R1 \begin{bmatrix} -1 & -1/4 & 11.25 \\ 0 & 1/2 & 24.675 \\ 0 & 15/4 & 95.675 \end{bmatrix}$$

Multiply the second row by -15 and add it to the third row:

$$R3{=}R3{-}15R2 \begin{bmatrix} -1 & -1/4 & 11.25 \\ 0 & 1/2 & 24.675 \\ 0 & 0 & -263.775 \end{bmatrix}$$

$$A^{-1} = \; =$$

$$\begin{bmatrix} -1 & -1/4 & 11{,}25 \\ -3/2 & -1 & 21{,}14 \\ -5/2 & 2 & 63{,}43 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -7688/2419 & 30686/26609 & 4772/26609 \\ 3076/2419 & -28244/26609 & 3412/26609 \\ -400/2419 & 2100/26609 & 500/26609 \end{bmatrix}$$

$$A{=}\begin{bmatrix} -1 & -1/4 & 11{,}25 \\ -3/2 & -1 & 21{,}14 \\ -5/2 & 2 & 63{,}43 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 5/2 & -21/5 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & -1/4 & 45/4 \\ 0 & -5/8 & 853/200 \\ 0 & 0 & 26609/500 \end{bmatrix}$$

• Create a linear system which has the constant vector as $b = \begin{bmatrix} Rmax \\ Hmax \end{bmatrix}$

$$hmax = \frac{v_0 2 \cdot \quad \sin^2\theta}{2g}$$

$$H_{max} = \frac{v_0^2}{2.9{,}81} + 1m$$

$v_0 = 5{,}559 \quad \frac{(5{,}559)^2}{2 \times 9.81}$

$\frac{30.902481}{19{,}62} = 1{,}57505$

$Hmax = 1 + 1{,}57505 = 2{,}57505$

$Hmax = \frac{5}{2}$

$R_{max} = \frac{v_0 \cdot sin\, 2\theta}{2g}$

$-1 \leq \sin 2\theta \leq 1$

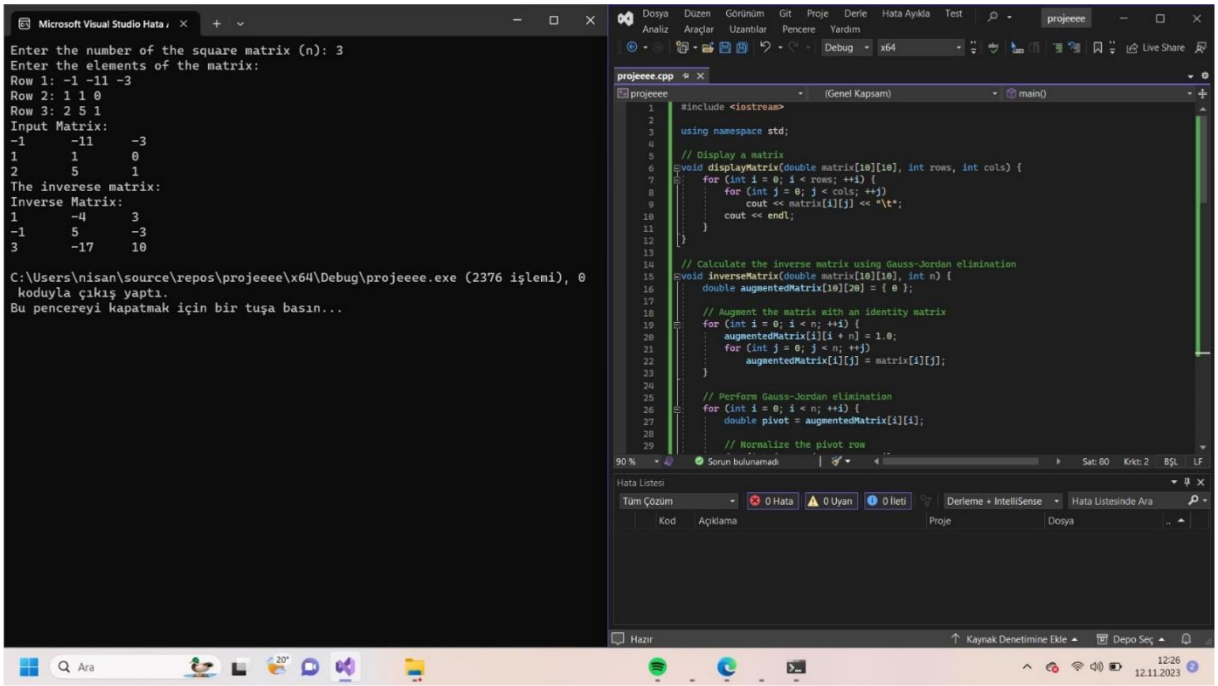
$\frac{(4{,}5776)^2}{2 \times 9{,}81} = \frac{20{,}9544218}{19{,}62} = 1{,}06801334$

$Rmax = 1$

EE101 tasks: Your task is to design the C++ program to perform these operations. 1. Create a C++ program that takes user input for the size of the square matrix and its elements. Ensure that the matrix is square (nxn). 2. Calculate the inverse matrix by using the Gauss-Jordan method and display it to the user. You should document the code and use proper commenting.

```
#include <iostream>

using namespace std;

void displayMatrix(double matrix[10][10], int rows, int cols) {
    for (int i = 0; i < rows; ++i) {
        for (int j = 0; j < cols; ++j)
            cout << matrix[i][j] << "\t";
        cout << endl;
    }
}

void inverseMatrix(double matrix[10][10], int n) {
    double augmentedMatrix[10][20] = { 0 };

    for (int i = 0; i < n; ++i) {
        augmentedMatrix[i][i + n] = 1.0;
        for (int j = 0; j < n; ++j)
            augmentedMatrix[i][j] = matrix[i][j];
    }

    for (int i = 0; i < n; ++i) {
        double pivot = augmentedMatrix[i][i];

        for (int j = 0; j < 2 * n; ++j)
            augmentedMatrix[i][j] /= pivot;

        for (int k = 0; k < n; ++k) {
            if (k != i) {
                double factor = augmentedMatrix[k][i];
```

```
                for (int j = 0; j < 2 * n; ++j)
                    augmentedMatrix[k][j] -= factor * augmentedMatrix[i][j];
            }
        }
    }

    cout << "Inverse Matrix:" << endl;
    for (int i = 0; i < n; ++i) {
        for (int j = n; j < 2 * n; ++j)
            cout << augmentedMatrix[i][j] << "\t";
        cout << endl;
    }
}

int main() {
    int n;
    cout << "Enter the number of the square matrix (n): ";
    cin >> n;

    if (n <= 0 || n > 10) {
        cout << "Invalid matrix size. Exiting program." << endl;
        return 1;
    }

    double inputMatrix[10][10] = { 0 };
    cout << "Enter the elements of the matrix:" << endl;
    for (int i = 0; i < n; ++i) {
        cout << "Row " << i + 1 << ": ";
        for (int j = 0; j < n; ++j)
            cin >> inputMatrix[i][j];
    }
    cout << "Input Matrix:" << endl;
    displayMatrix(inputMatrix, n, n);


    cout << "The inverese matrix: " << endl;
 inverseMatrix(inputMatrix, n);

    return 0;
}
```



DISCUSSION

The forces that oppose an object's relative motion as it moves through the air are referred to as air resistance. These drag forces work in the opposite direction of the oncoming flow velocity, slowing the object down.[5]

The resistance force is proportional to the speed of the object. Because the speed of the object, the number of particles it collides with, and the intensity of the crowd all rise. Resistance force is directly proportional to different exponents of speed depending on the speed of the item. For the resistance force exerted on objects moving near the earth, n = 2, that is, it is directly proportional to the square of the speed. [6]

The air resistance formula, commonly known as the drag force formula, determines the force experienced by an item traveling through a fluid medium such as air. It is given by the equation FD = 12 A CD v2, where FD is the drag force, is the density of the fluid, A is the cross-sectional area of the object, CD is the drag coefficient, and v is the velocity of the object. [7]

At the speed of gravity, all things accelerate towards Earth. This is demonstrated by the equations.The force is equal to the item's weight. This principle is based on Newton's second law, which asserts that an object's acceleration is directly connected to the force acting on it and inversely related to the object's mass. [8]

$$f_{net} = m \cdot a_{avg}$$

$$F_{net} = F_{gravity} - F_{air}$$

$$Fgravity = m \cdot g$$

$F_{air} = -\frac{1}{2} \cdot C \cdot \rho \cdot A \cdot v^2$ (C is the air resistance coefficient ρ is the density of air, A is the cross-sectional area of the object, V is the velocity)

$F_{air} = \frac{-1}{2}$ (1.225).(0.32).(0.12)$(4.5)^2$= 0.47($\rho \approx 1.225 \frac{kg}{m^3}, C \approx 0.32, A \approx 0.12$)

$Fgravity = (3.8).(9.8) = 37.24$

$F_{net} = (37.24) - (0.47) = 36.77$

If 45 and 30 degrees are given as an example;

	L (real)	H (real)	L (expected)	H (expected)
30	3.118	1.521	3.776	1.965
45	3.006	1.827	3.343	2.069

CONCLUSION

Firstly, the videos of shooting gun was recorded. Then, these videos are uploaded to Tracker to find the values of R, H, and V. As a result of observation, theoretically, values were found, and with the help of Tracker, the experimental values were also found. Equations are calculated and the results are compared. The results were different than expected because in real life there is a lot of variables such as air resistance. In conclusion, theoretical values are not the same as experimental values.

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