

Computer Organization and Architecture Laboratory

Assignment 1

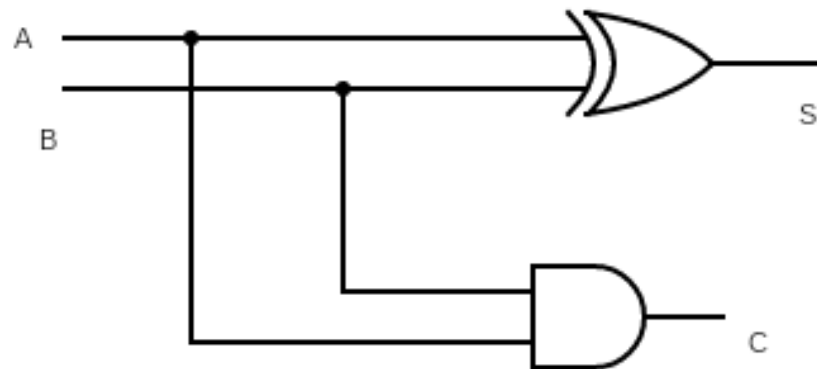
Group 1

Animesh Jha (19CS10070)

Nisarg Upadhyaya (19CS30031)

PART 1: Ripple Carry Adders

1. Half adder

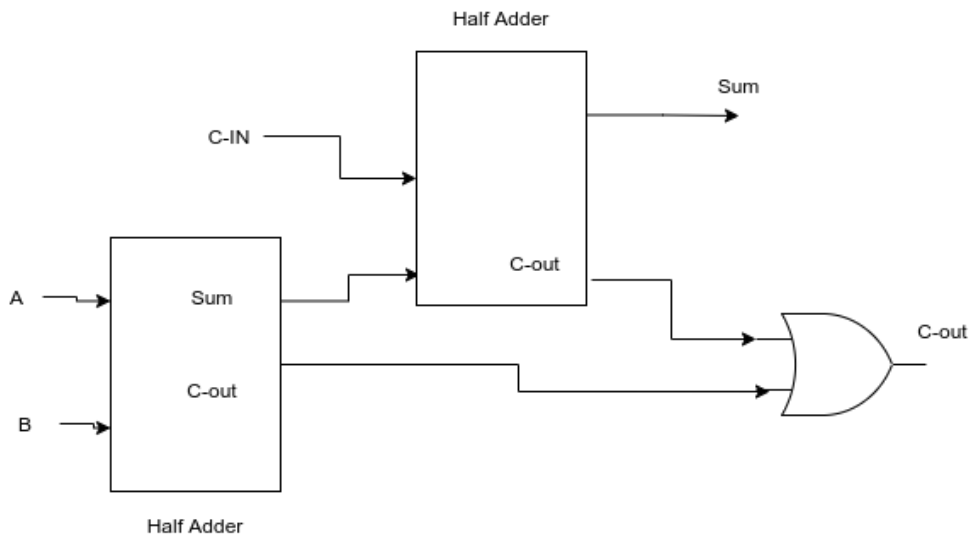


| Inputs | | Outputs | |
|--------|---|---------|-------|
| A | B | Sum | C_out |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

Logic:

```
c_out = a & b  
sum = a ^ b
```

2. Full adder

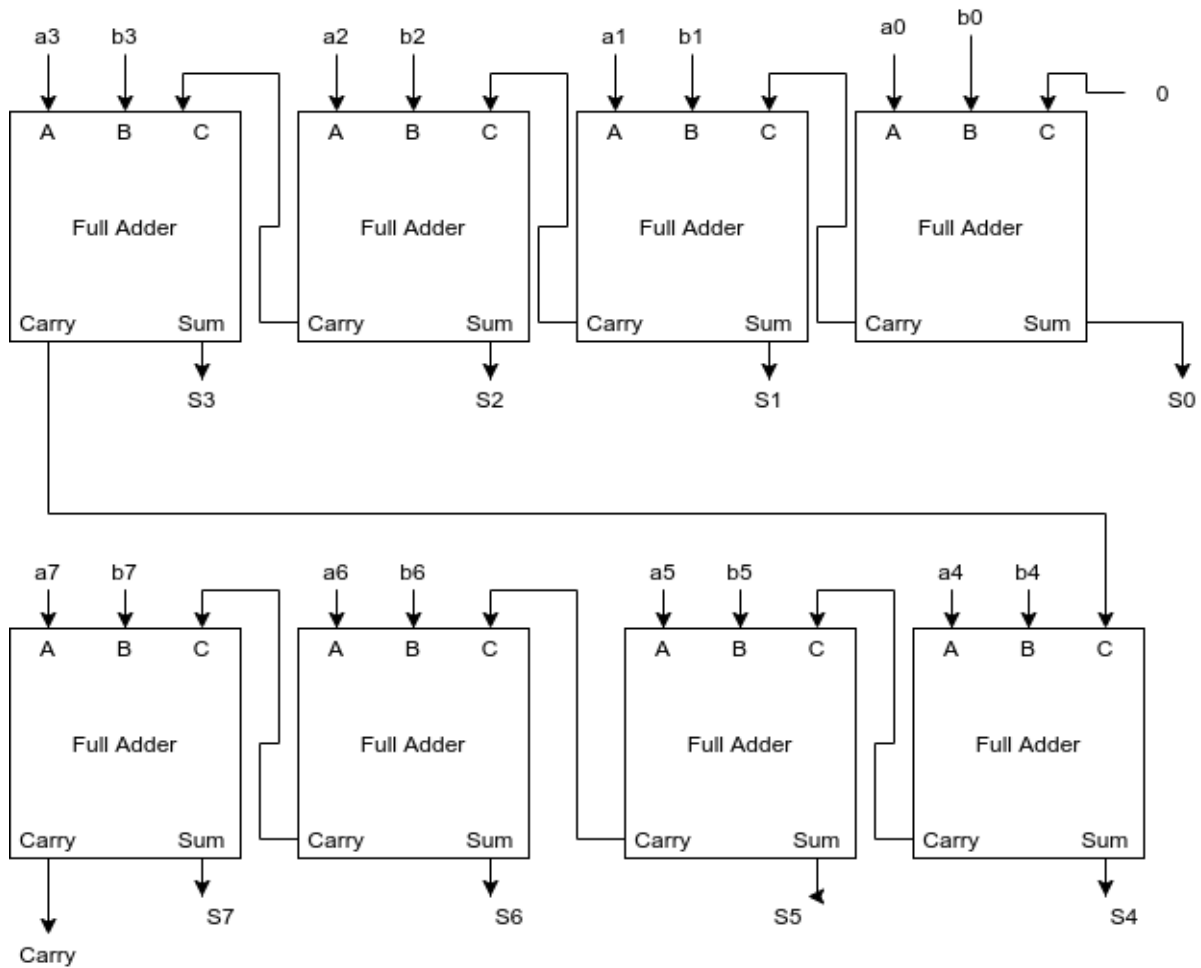


| Inputs | | | Outputs | |
|--------|---|------|---------|-------|
| A | B | C_in | Sum | C_out |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

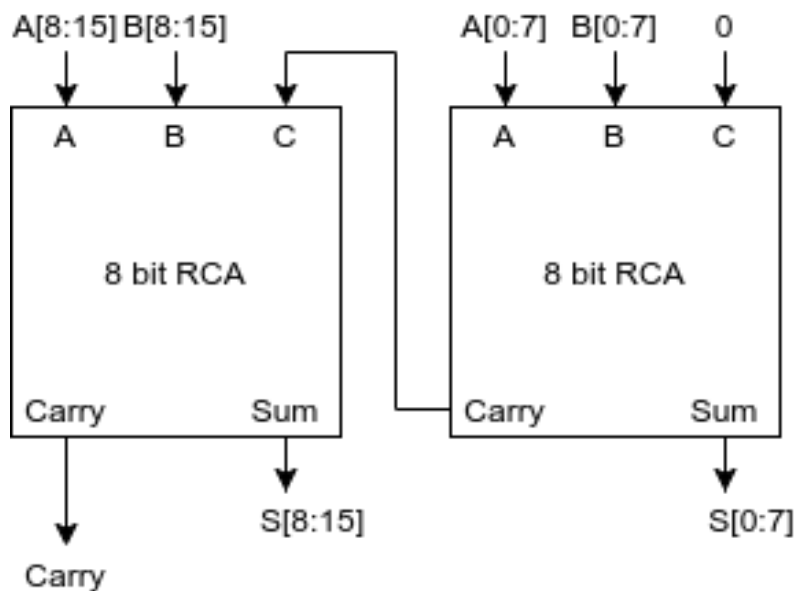
Logic:

$$\begin{aligned} \text{sum} &= a \oplus b \oplus c_{\text{in}} \\ c_{\text{out}} &= (a \& b) \vee (c \& (a \oplus b)) \end{aligned}$$

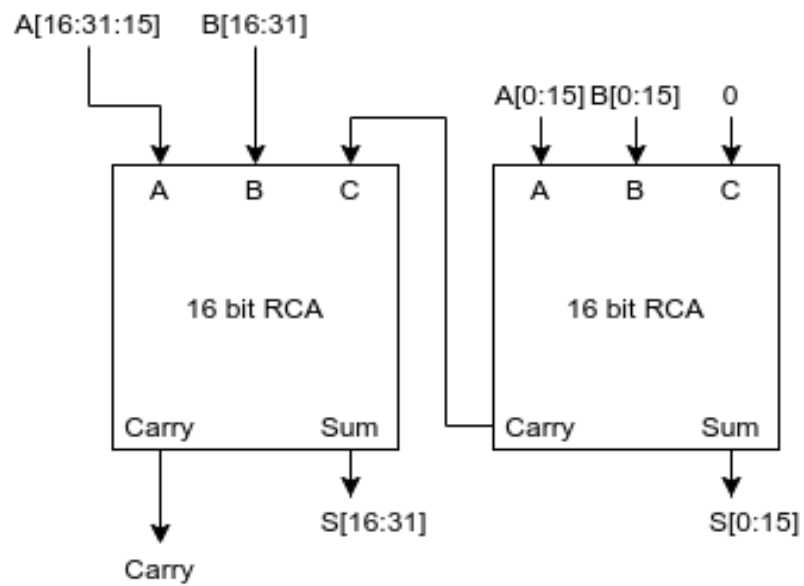
3. 8-bit adder



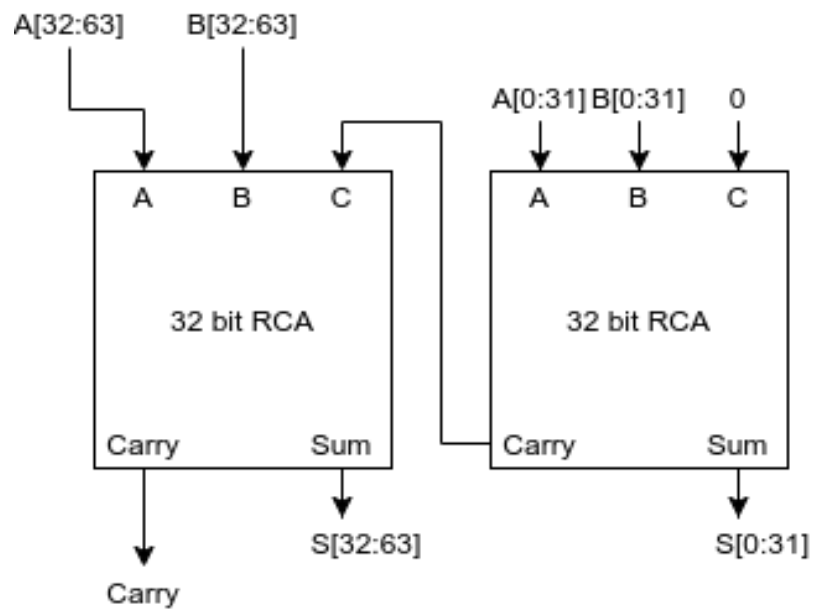
4. 16-bit adder



5. 32-bit adder



6. 64-bit adder



SYNTHESIS SUMMARY

| Circuit | Delay (in ns) | Logic Levels | Number of Slice LUTs | Number of bonded IOBs |
|------------|---------------|--------------|----------------------|-----------------------|
| 8-bit RCA | 9.949 | 36 | 40 | 26 |
| 16-bit RCA | 18.717 | 70 | 80 | 50 |
| 32-bit RCA | 36.253 | 138 | 160 | 98 |
| 64-bit RCA | 71.325 | 274 | 320 | 194 |

Q. How can you use the above circuit, to compute the difference between two n-bit numbers?

=> $a - b = a + (-b)$

=> observe that $(-b)$ is the 2's complement of b

=> $-b = \sim b + 1$, $\sim b = 1$'s complement of b

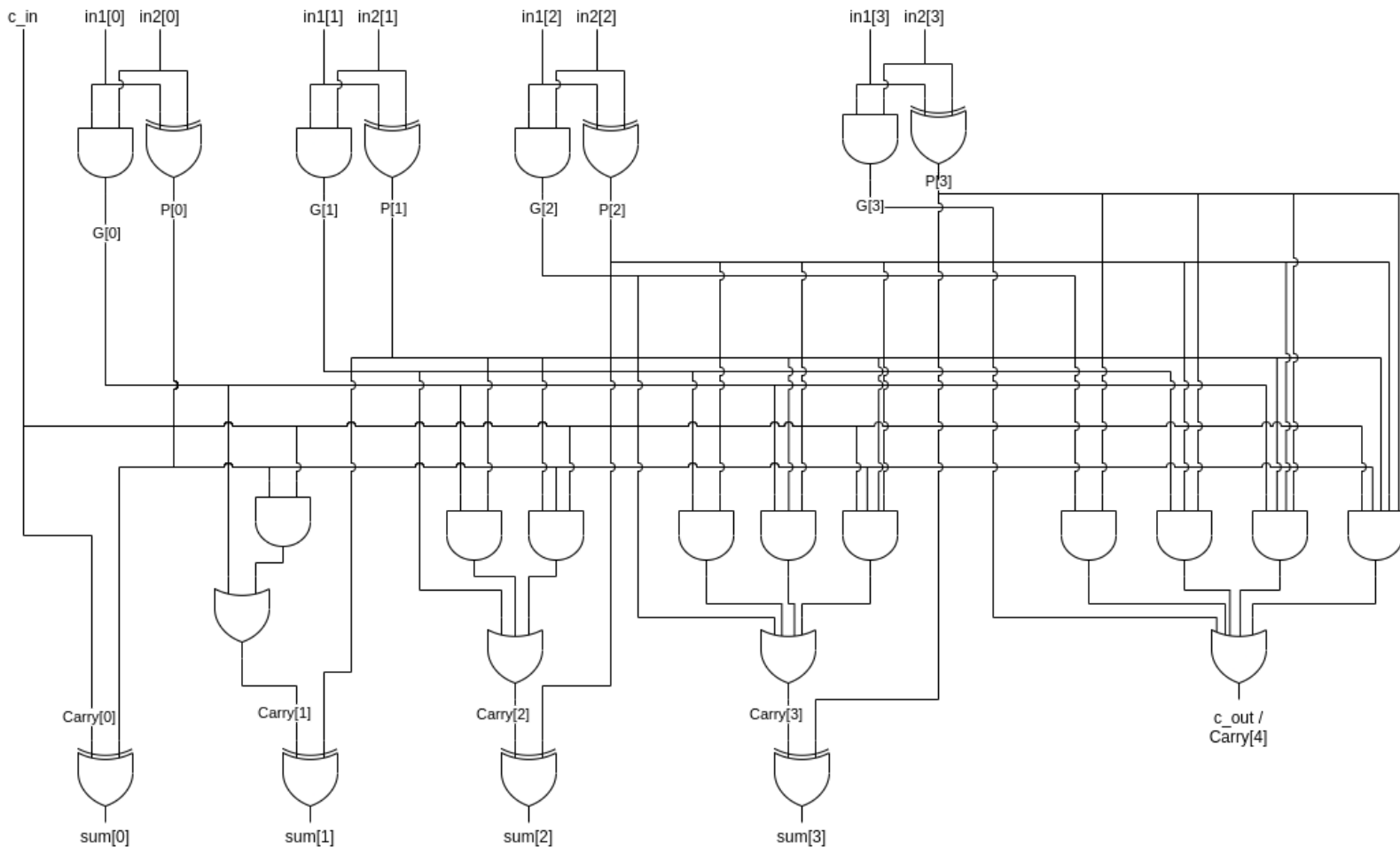
=> so $a - b = \text{RCA}(a, \sim b, 1)$ where 1 is the carry in and a and $\sim b$ are the two inputs

In the circuit we can put a switch which will be connected to the carry in of the adder and also with XOR gates with each of the input bits of b. When the switch is turned on it will make the carry in 1 and simultaneously flip all the bits of b (XOR with 1 flips the bit value). This way we will get a-b in output. When the switch is off it will not change anything and we will get a+b in output.

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PART 2: Carry Look-ahead Adders

1. 4-bit carry look-ahead adder



The carry lookahead adder reduces the delay in calculating the sum by simultaneously calculating the carries instead of waiting for the carry from the previous block to ripple in which is the case for ripple carry adder. The logic for the carry look-ahead adder is as follows:

$$G[i] = in1[i] \& in2[i], 0 \leq i \leq 3$$

$$P[i] = in1[i] \wedge in2[i], 0 \leq i \leq 3$$

Take c_in to be $carry[0]$ then:

$$sum[i] = P[i] \wedge carry[i], 0 \leq i \leq 3$$

$$carry[i] = G[i-1] \mid (P[i-1] \& carry[i-1]), 1 \leq i \leq 4$$

Recursively expanding we get:

$$\text{carry}[1] = G[0] \mid (P[0] \ \& \ \text{carry}[0]) = G[0] \mid (P[0] \ \& \ c_in)$$

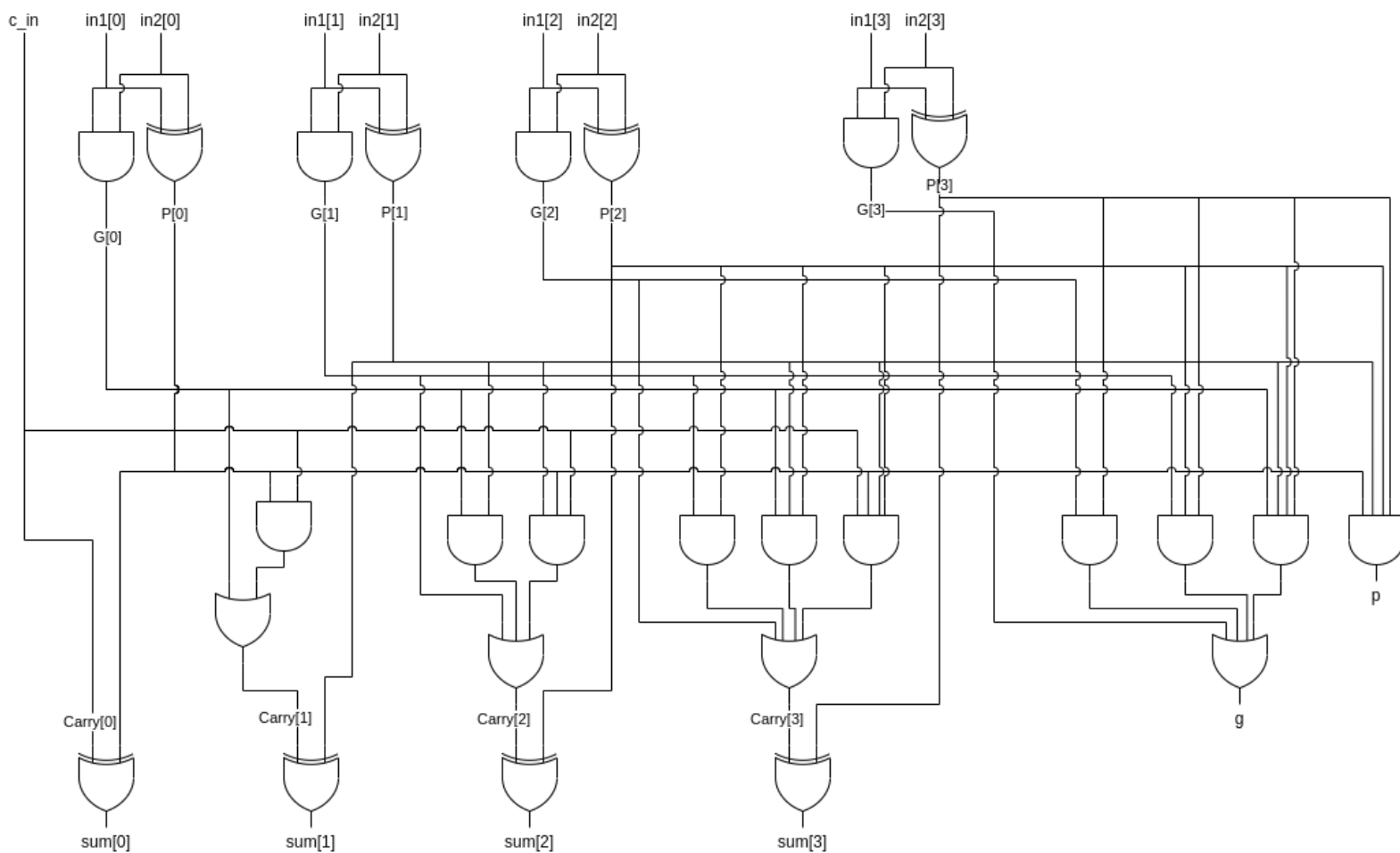
$$\text{carry}[2] = G[1] \mid (P[1] \ \& \ \text{carry}[1]) = G[1] \mid (P[1] \ \& \ G[0]) \mid (P[1] \ \& \ P[0] \ \& \ c_in)$$

$$\text{carry}[3] = G[2] \mid (P[2] \ \& \ \text{carry}[2]) = G[2] \mid (P[2] \ \& \ G[1]) \mid (P[2] \ \& \ P[1] \ \& \ G[0]) \mid (P[2] \ \& \ P[1] \ \& \ P[0] \ \& \ c_in)$$

$$\text{carry}[4] = G[3] \mid (P[3] \ \& \ \text{carry}[3]) = G[3] \mid (P[3] \ \& \ G[2]) \mid (P[3] \ \& \ P[2] \ \& \ G[1]) \mid (P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ G[0]) \mid (P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ P[0] \ \& \ c_in)$$

2. 4-bit carry look-ahead adder (augmented)

In this case instead of generating the carry out ,i.e., carry[4] we give the block propagate and generate as output which are then used by the carry lookahead unit. This leads to a modular design using which we can make 16, 32 and 64 bit adders by combining the block propagate

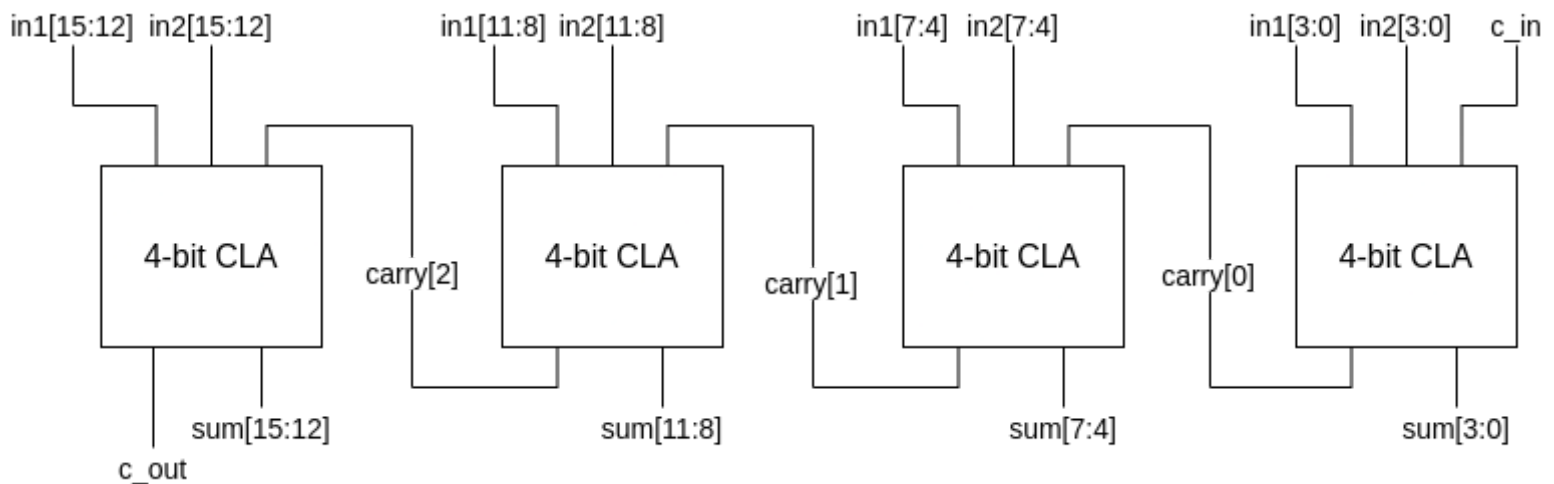


and generate from the lower levels instead of rippling the carry out every time. The other logic remains the same as the normal 4-bit CLA. The block propagate and generates are calculated as follows:

```
p = P[3] & P[2] & P[1] & P[0]
g = G[3] | (P[3] & G[2]) | (P[3] & P[2] & G[1]) | (P[3] & P[2] & P[1]
& G[0])
```

3. 16-bit carry look-ahead adder (ripple carry out)

This 16-bit adder is made by cascading four 4-bit CLAs by rippling the carry out from one block to another as shown in the figure.



Here, $carry[2:0]$ are internal carries being rippled from one block to another.

4. 16-bit carry look-ahead adder (look-ahead carry unit)

Let $P[3:0]$ and $G[3:0]$ be the block propagate and generate of the four 4-bit CLAs. Then instead of rippling the carry out from one block to another we can reduce the delay in the circuit by adding an additional lookahead unit which simultaneously calculates these carries. This way the subsequent blocks don't have to wait for the carry from the previous block. Also, this leads to a modular design where we can further calculate the block propagate and generate of this complete 16-bit CLA as a whole and use them later for higher order adders.

The logic for the lookahead carry unit is as follows:

```
Take  $c_{in}$  to be  $carry[0]$  then
 $carry[i] = G[i-1] | (P[i-1] \& carry[i-1]), 1 \leq i \leq 4$ 
```

Recursively expanding we get

$$\text{carry}[1] = G[0] \mid (P[0] \ \& \ \text{carry}[0]) = G[0] \mid (P[0] \ \& \ c_in)$$

$$\text{carry}[2] = G[1] \mid (P[1] \ \& \ \text{carry}[1]) = G[1] \mid (P[1] \ \& \ G[0]) \mid (P[1] \ \& \ P[0] \ \& \ c_in)$$

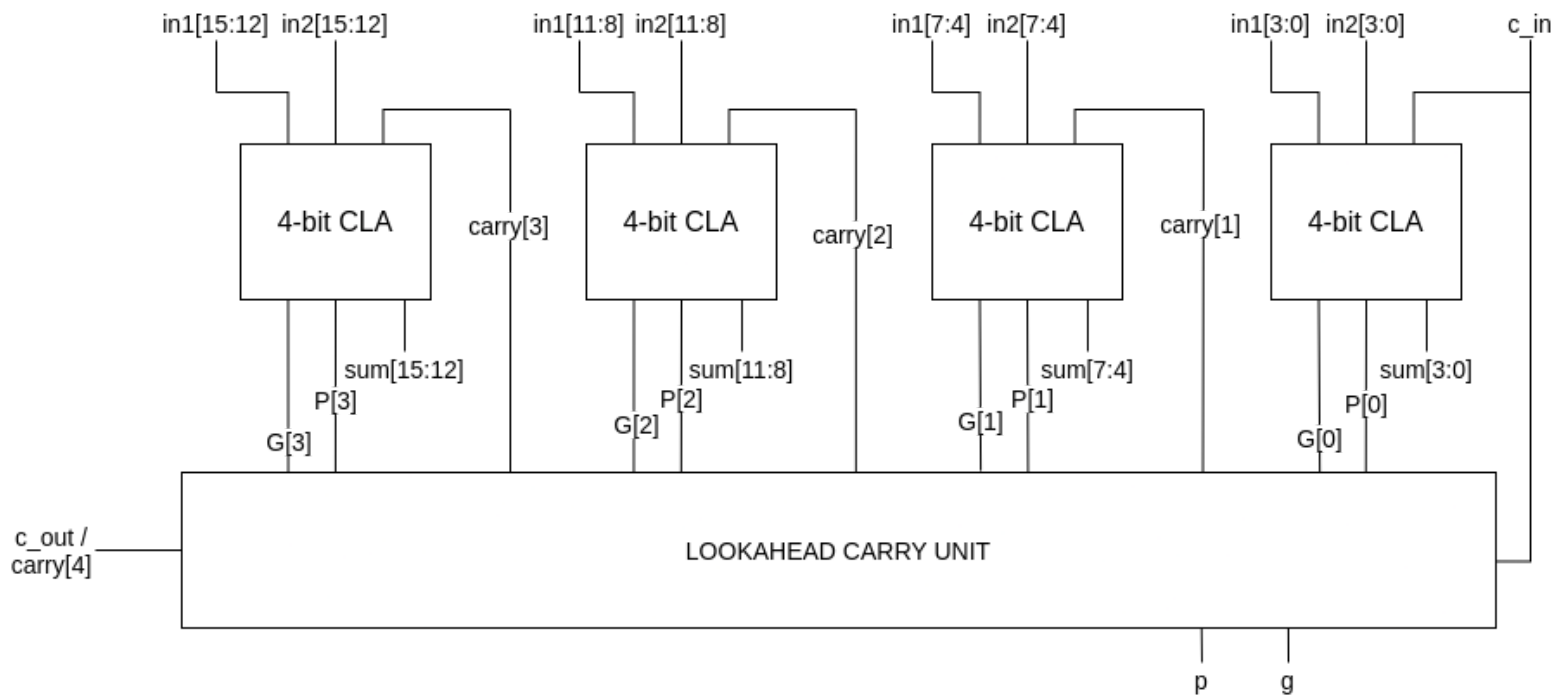
$$\text{carry}[3] = G[2] \mid (P[2] \ \& \ \text{carry}[2]) = G[2] \mid (P[2] \ \& \ G[1]) \mid (P[2] \ \& \ P[1] \ \& \ G[0]) \mid (P[2] \ \& \ P[1] \ \& \ P[0] \ \& \ c_in)$$

$$\text{carry}[4] = G[3] \mid (P[3] \ \& \ \text{carry}[3]) = G[3] \mid (P[3] \ \& \ G[2]) \mid (P[3] \ \& \ P[2] \ \& \ G[1]) \mid (P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ G[0]) \mid (P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ P[0] \ \& \ c_in)$$

Also block propagate p and generate g are calculated as:

$$p = P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ P[0]$$

$$g = G[3] \mid (P[3] \ \& \ G[2]) \mid (P[3] \ \& \ P[2] \ \& \ G[1]) \mid (P[3] \ \& \ P[2] \ \& \ P[1] \ \& \ G[0])$$



SYNTHESIS SUMMARY

| | 16-bit carry look-ahead adder (ripple carry out) | 16-bit carry look-ahead adder (look-ahead carry unit) |
|-----------------------|---|--|
| Delay (in ns) | 6.167 | 5.308 |
| Number of Slice LUTs | 24 | 43 |
| Number of bonded IOBs | 50 | 52 |
| Levels of Logic | 14 | 11 |

A CLA with an additional lookahead carry unit performs better than simply rippling the carry at the cost of additional LUTs.

- Comparison with 4-bit RCA

| 4-bit | Delay (in ns) | Number of Slice LUTs | Logic Levels |
|-------|---------------|----------------------|--------------|
| CLA | 2.363 | 9 | 4 |
| RCA | 5.565 | 20 | 20 |

- Comparison with 16-bit RCA

It can be observed that a CLA is significantly (almost 3.5x) faster than a RCA. Also the LUTs used are almost half that for a RCA.

| 16-bit | Delay (in ns) | Number of Slice LUTs | Logic Levels |
|--------|---------------|----------------------|--------------|
| CLA | 5.308 | 43 | 11 |
| RCA | 18.717 | 80 | 70 |