Game Theory – Lecture 1

AI for Economics – Module 3

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What is Game Theory?

- Analysis of strategic interaction between agents (players) where the outcome for each agent (or player) depends on not only his own action but also the actions chosen by the other players.

- Game Theory is the study of rational behaviour is interactive or inter-dependent situations.

What is Game Theory?

- Analysis of strategic interaction between agents (players) where the outcome for each agent (or player) depends on not only his own action but also the actions chosen by the other players.

- Game Theory is the study of rational behaviour in interactive or inter-dependent situations.

- Two fundamental assumptions:

➢ ***Rationality of players***

➢ ***Common Knowledge***

Rationality

- Given a set of alternatives, preferences of an agent are ***rational*** if they satisfy:

(i) *Completeness*

(ii) *Transitivity*

- ***Rationality*** implies consistency in preferences.

Common Knowledge

- A fact “X” is ***common knowledge*** if:

- Each player knows the fact “X”
- Each player knows that each player knows “X”
- Each player knows that each player knows that each player knows “X”

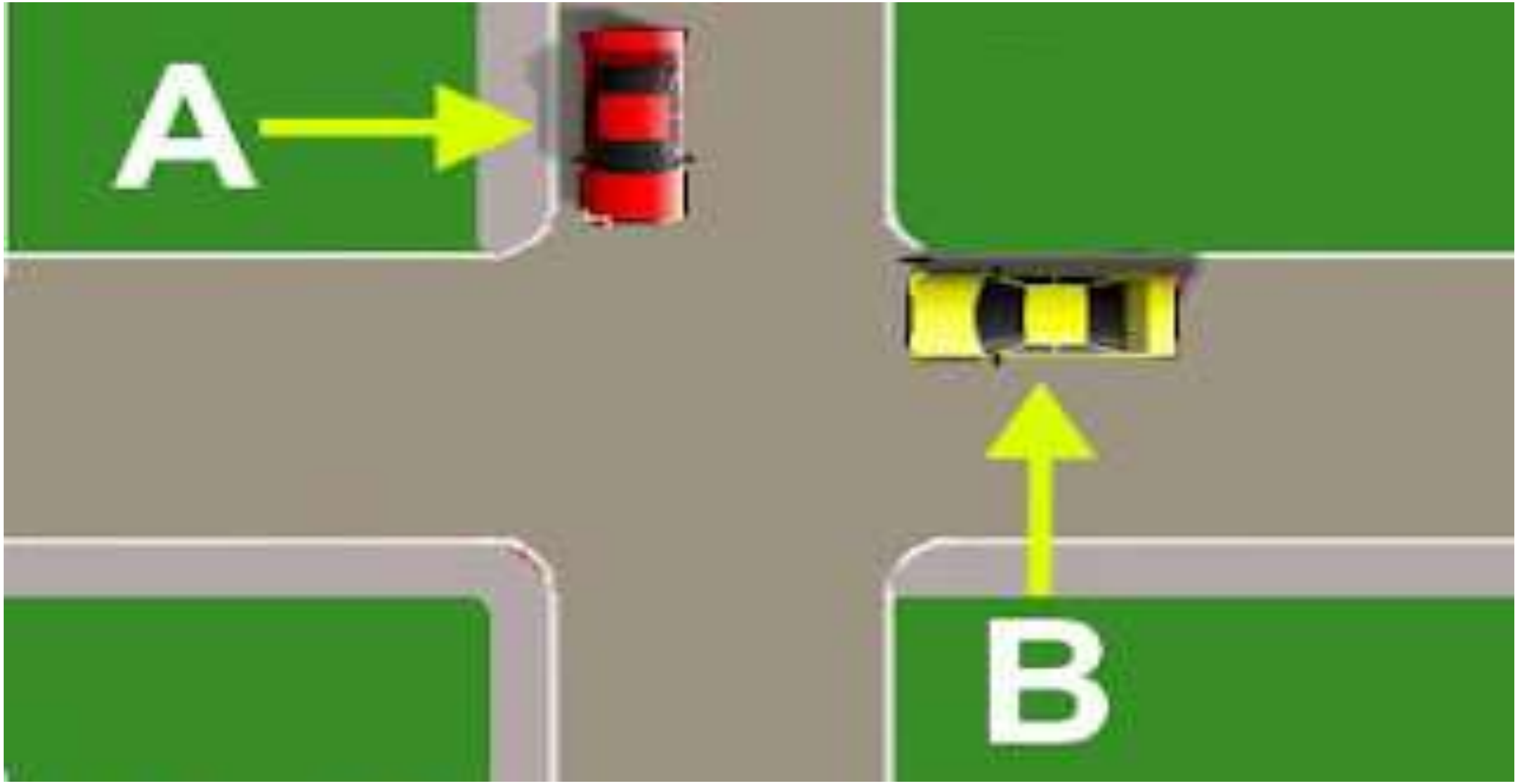
…..

- *The rules of the game are common knowledge.*

Simultaneous Move Game

- A set of ***Players***
- For each player there is a ***set of actions***
- Each player chooses an ***action,*** simultaneously**.**
- The tuple of chosen actions is called an ***"action Profile"***
- For each player there is a ***preference*** *(represented by payoffs)* over the ***set of action profiles***
- ***Complete Information:*** *Each player has all information about "set of actions" & "payoffs" of the other players.*

The “Wait – Go” Game



Strategic Form Representation

- Set of Players = { Red Car, Yellow Car}
- Each Player’s set of actions = {Go, Wait}
- Set of action profiles = {(Go,Go), (Go,Wait), (Wait,Go), (Wait,Wait)}
- Preference of each player, over the action profiles is given by the following payoff matrix.

Payoff Matrix

	Wait	Go
Wait	0,0	0,2
Go	2,0	-10, -10

Nash Equilibrium

- An *action profile* is called a *Nash Equilibrium* if **NO player has an incentive to deviate given what other players are doing**. i.e any unilateral deviation by any player is irrational.

- Thus each player is playing his / her *best response* given the actions of the other players

Best Response

	Wait	**Go**
Wait	0,0	0,2*

Best Response

	Wait	Go
Go	2, 0*	-10, -10

Best Response

	Wait	Go
Wait	0,0	0, 2^*
Go	2, 0^*	-10, -10

Best Response

	Wait	Go
Wait	0,0	0^*, 2^*
Go	2^*,0^*	-10, -10

- Thus (Go, Wait) & (Wait, Go) are two "pure strategy" Nash Equilibria of this simultaneous move game

The Penalty Kick

Strategic Form Representation

- Set of Players = { Striker, **Goalie**}
- Striker's set of actions = {Kick Left (KL) Kick , Kick Right (KR)}
- Goalie's set of actions = {Dive Left (DL) , Dive Right (DR)}
- Set of action profiles = {(KL,DL), (KL,DR), (KR,DL), (KR,DR)}

Payoff Matrix

	DL	DR
KL	0, 0	V, -V
KR	1, -1	0, 0

- V $\in (0,1)$
- The striker is right – footed
- V represents the probability of netting the ball if the striker kicks towards the left

Nash Equilibrium

- An *action profile* is called a *Nash Equilibrium* if **NO player has an incentive to deviate given what other players are doing**. i.e any unilateral deviation by any player is irrational.

- Thus each player is playing his / her *best response* given the actions of the other players

Best Responses

	DL	DR
KL	0, 0^*	V, -V

Best Responses

	DL	DR
KL	0, 0*	V, -V

	DL	DR
KR	1, -1	0, 0*

Observations & Thoughts

- Clearly this game has no pure strategy Nash Equilibrium.

- What if the players decide to randomly choose an action from his set of actions?..... Can that be a rational move ever?

- What does that even mean?

Randomization

- **Striker's** action set = {**KL** , **KR**}
- Let's say the Striker decides to choose KL & KR with probability p & (1-p) respectively.
- **Goalie's** action set = {**DL** , **DR**}
- Let's say the Goalie decides to choose DL & DR with probability q & (1-q) respectively.
- Can this be rational?

Striker's perspective

- Given that the Goalie plays DL & DR with prob. q & 1-q :

- Striker's expected payoff from playing KL = q.0 + (1-q).V = (1-q).V
- Striker's expected payoff from playing KR = q.1 + (1-q).0 = q

- Striker will choose KL if (1-q).V > q i.e q < $\frac{V}{1+V}$
- Striker will choose KR if (1-q).V < q i.e q > $\frac{V}{1+V}$

Striker's perspective

- Given that the Goalie plays DL & DR with prob. q & 1-q :

- Striker's expected payoff from playing KL = q.0 + (1-q).V = (1-q).V
- Striker's expected payoff from playing KR = q.1 + (1-q).0 = q

- Striker will choose KL if (1-q).V > q i.e q < $\frac{V}{1+V}$

(Then it's rational for Goalie to choose DL i.e q = 1)

- Striker will choose KR if (1-q).V < q i.e q > $\frac{V}{1+V}$

(Then it's rational for Goalie to choose DR i.e q = 0)

Striker's perspective

- Striker is indifferent between KL & KR if (1-q).V = q i.e q = $\frac{V}{1+V}$
- This is only when Striker will choose to randomize between KL & KR with prob. (p,1-p) where p>0.
- Expected payoff of Striker = p [(1-q).V] + (1-p) [q] = q = $\frac{V}{1+V}$; which is independent of p.
- So no matter what 'p' Striker chooses, his payoff = $\frac{V}{1+V}$

Goalie's perspective

- Given that the Striker plays KL & KR with prob. p & 1-p :

- Goalie's expected payoff from playing DL = p.0 + (1-p).(-1) = -(1-p)
- Striker's expected payoff from playing DR = p.(-V) + (1-p).0 = -p.V

- Goalie will choose DL if -(1-p) > -p. V i.e p > $\frac{1}{1+V}$
- Goalie will choose DR if -(1-p) < -p. V i.e p < $\frac{1}{1+V}$

Goalie's perspective

- Given that the Striker plays KL & KR with prob. p & 1-p :

- Goalie's expected payoff from playing DL = p.0 + (1-p).(-1) = -(1-p)
- Striker's expected payoff from playing DR = p.(-V) + (1-p).0 = -p.V

- Goalie will choose DL if -(1-p) > -p. V i.e p > $\frac{1}{1+V}$

(Then it is rational for Striker to choose KR i.e p = 0…..)

- Goalie will choose DR if -(1-p) < -p. V i.e p < $\frac{1}{1+V}$

(Then it is rational for Striker to choose KL i.e p = 1…..)

Goalie's perspective

- Goalie is indifferent between DL & DR if -(1-p)= -p.V i.e p = $\frac{1}{1+V}$

- This is only when Goalie will choose to randomize between DL & DR with prob. (q,1-q) where q>0.

- Expected payoff of Goalie = q [-(1-p)] + q [-p.V] = -p.V = - $\frac{V}{1+V}$; which is independent of q.

- So no matter what 'q' Goalie chooses, his payoff = - $\frac{V}{1+V}$

Final Solution

- It is rational for the Goalie to randomize i.e choose (q,1-q) with q>0 only if p = $\frac{1}{1+V}$

- Also in that case Goalie's payoff is independent of q.

- It is rational for the Striker to randomize i.e choose (p,1-p) with p>0 only if q = $\frac{V}{1+V}$

- Also in that case Striker's payoff is independent of p.

- Thus we infer that Striker choosing (p,1-p) with p = $\frac{1}{1+V}$ & Goalie choosing (q,1-q) with q = $\frac{V}{1+V}$ form a Nash Equilibrium