BAYESIAN GAMES

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Retro – Dating ....

Battle of Sexes

- Two Players: **Boy** (Player 1) & **Girl** (Player 2)

Battle of Sexes

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- Pure Strategy set of each player: {Cricket (C) , Movie (M)}

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- Pure Strategy set of each player: {Cricket (C) , Movie (M)}
- The pay off matrix is given by:

	Cricket (C)	**Movie (M)**
Cricket (C)	10,5	0,0
Movie (M)	0,0	5,10

Best Responses

	Cricket (C)	Movie (M)
Cricket (C)	$10^*, 5^*$	0,0
Movie (M)	0,0	$5^*, 10^*$

Pure Strategy Nash Equilibria

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Pure Strategy Nash Equilibria

- The two pure strategy Nash equilibria are: (C,C) & (M,M)
- Does there exist mixed strategy N.E?
- Let’s find them !

Mixed Strategies

- Boy (P_1) plays the mixed strategy (p, 1-p), i.e He plays C with probability p and M with probability 1-p.

- Girl (P_2) plays the mixed strategy (q, 1-q), i.e She plays C with probability q and M with probability 1-q.

Mixed Strategy Nash Equilibrium

- The MSNE of the game is given by: $((\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}))$

i.e Boy plays the mixed strategy $(\frac{2}{3}, \frac{1}{3})$ & the Girl plays the mixed strategy $(\frac{1}{3}, \frac{2}{3})$

Incomplete Information

- Now let's make it a little more realistic.
- The Boy does not know whether the girl is "Interested" or "Uninterested".
- So the Girl in this case has two "types" namely: "Interested" and "Uninterested"

The “Interested” Girl

- The pay off matrix is given by:

	Cricket (C)	Movie (M)
Cricket (C)	10,5	0,0
Movie (M)	0,0	5,10

The “Uninterested” Girl

- The pay off matrix is given by:

	Cricket (C)	Movie (M)
Cricket (C)	10,0	0,10
Movie (M)	0,5	5,0

Belief

- The boy does not know the type of the girl.
- But he has a **belief** about the type of the girl.
- The belief is a probability distribution over the set of types.
- Let’s say the girl has given no hint.
- So the **“belief”** he has that the girl is of type “Interested” is = ½.
- The **belief** is common knowledge

Bayesian Nash Equilibrium (Def.)

A **Bayesian Nash Equilibrium (BNE)** is a set of strategies, one for each "type" of a player, such that no "type" of any player has any incentive to change his/her strategy, given the beliefs (*which are common knowledge*) & what other players are doing.

Boy’s Expected Payoffs

	C , C	C , M	M , C	M , M
C	10			
M				

Boy's Expected Payoffs

	C , C	C , M	M , C	M , M
C	10			
M	0			

Boy’s Expected Payoffs

	C , C	**C , M**	M , C	M , M
C	10			
M	0	$\frac{1}{2}.0 + \frac{1}{2}.5 = 5/2$		

Boy's Expected Payoffs

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Boy’s Expected Payoffs

	C , C	C , M	M , C	M , M
C	10	$\frac{1}{2}.10 + \frac{1}{2}.0 = 5$	5	0
M	0	$\frac{1}{2}.0 + \frac{1}{2}.5 = 5/2$	5/2	5

Boy's Expected Payoffs (Best Responses)

	C , C	C , M	M , C	M , M
C	10	5	5	0
M	0	5/2	5/2	5

Boy’s Expected Payoffs (Best Responses)

	C , C	C , M	M , C	M , M
C	**10**	**5**	**5**	0
M	0	5/2	5/2	**5**

Let’s inspect:

- (C, (C,C))
- (C, (C,M))
- (C, (M,C))
- (M, (M,M))

- (C, (C,C)): If the Boy plays C it is NOT optimal for the “Uninterested girl” to play C. So (C, (C,C)) is not a BNE.

- (C, (C,C)): If the Boy plays C it is NOT optimal for the “Uninterested girl” to play C. So (C, (C,C)) is not a BNE.

- (C,(C,M)): If the Boy plays C it is optimal for the “interested” type girl to play C & it is also optimal for the “Uninterested” type girl to play M. **So (C,(C,M)) is indeed a pure strategy BNE.**

- (C, (C,C)): If the Boy plays C it is NOT optimal for the "Uninterested girl" to play C. So (C, (C,C)) is not a BNE.

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- Similarly we can argue that (C, (M,C)) & (M, (M,M)) are not BNE.

Bayesian Nash Equilibrium (Def.)

A **Bayesian Nash Equilibrium (BNE)** is a set of strategies, one for each "type" of a player, such that no "type" of any player has any incentive to change his/her strategy, given the beliefs (*which are common knowledge*) & what other players are doing.

Mixed Strategy BNE (MSBNE)

- Let's consider the following mixed strategy profile:

➢ *The Boy* (P_1) *plays (p,1-p)*

➢ *The girl of "Interested" type plays* (q_1,1- q_1)

➢ *The girl of "Un-Interested" type plays* (q_2,1- q_2)

Mixed Strategy BNE (MSBNE)

- Let’s consider the following mixed strategy profile:

➢ *The Boy* (P_1) *plays (p,1-p)*

➢ *The girl of “Interested” type plays* (q_1,*1-* q_1)

➢ *The girl of “Un-Interested” type plays* (q_2,*1-* q_2)

Is this mixed strategy profile a BNE given the beliefs??

The Boy's Payoffs

- Given that the girl of "Interested" type plays $(q_1,1- q_1)$ & the girl of "Un-Interested" type plays $(q_2,1- q_2)$

The Boy's Payoffs

- Given that the girl of "Interested" type plays $(q_1, 1- q_1)$ & the girl of "Un-Interested" type plays $(q_2, 1- q_2)$

- Expected Payoff of the "Boy" if he plays C:

$$\{10.q_1 + 0.(1- q_1)\}.\frac{1}{2} + \{10.q_2 + 0.(1- q_2)\}.\frac{1}{2}$$

The Boy's Payoffs

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- Expected Payoff of the "Boy" if he plays C:

$$\{10.q_1 + 0.(1- q_1)\}.\frac{1}{2} + \{10.q_2 + 0.(1- q_2)\}.\frac{1}{2}$$

- Expected Payoff of the "Boy" if he plays M:

$$\{0.q_1 + 5.(1- q_1)\}.\frac{1}{2} + \{0.q_2 + 5.(1- q_2)\}.\frac{1}{2}$$

- Remember in the mixed strategy profile the Boy is playing is (p,1-p)

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The Girl Connect Equation !!!!!

“Interested” Girl

- Is it optimal for the interested girl to play mixed strategy *(*q_1*,1-* q_1*)*

“Interested” Girl

- Is it optimal for the interested girl to play mixed strategy *(q_1,1- q_1)*

- Given that the boy is playing (p,1-p)

➢ Her expected payoff from playing C: 5.p + 0.(1-p)

➢ Her expected payoff from playing M: 0.p + 10.(1-p)

“Interested” Girl

- Is it optimal for the interested girl to play mixed strategy *(q_1,1- q_1)*

- Given that the boy is playing (p,1-p)

➢ Her expected payoff from playing C: 5.p + 0.(1-p)

➢ Her expected payoff from playing M: 0.p + 10.(1-p)

- If the “Interested” girl playing ***(q_1,1- q_1) with q_1> 0*** *in response* to the Boy playing (p,1-p) is a BNE:

5.p + 0.(1-p) = 0.p + 10.(1-p) i.e P = $\frac{2}{3}$

“Uninterested” Girl

- Is it optimal for the interested girl to play mixed strategy $(q_2, 1- q_2)$

“Uninterested” Girl

- Is it optimal for the interested girl to play mixed strategy *(q_2,1- q_2)*

- Given that the boy is playing (p,1-p)

➢ Her expected payoff from playing C: 0.p + 5.(1-p)

➢ Her expected payoff from playing M: 10.p + 0.(1-p)

"Uninterested" Girl

- Is it optimal for the interested girl to play mixed strategy *(q_2,1- q_2)*

- Given that the boy is playing (p,1-p)

➢ Her expected payoff from playing C: 0.p + 5.(1-p)

➢ Her expected payoff from playing M: 10.p + 0.(1-p)

- If the "Uninterested" girl playing **(*q_2*,*1*- *q_2*)** ***with*** $\boldsymbol{q_2 > 0}$ *in response* to the Boy playing (p,1-p) is a BNE:

0.p + 5.(1-p) = 10.p + 0.(1-p) i.e P = $\frac{1}{3}$

If P = 2/3 Then ?

- For the “Interested” girl:

➢ Her expected payoff from playing C: 5.p + 0.(1-p) = 5.($\frac{2}{3}$) = $\frac{10}{3}$

➢ Her expected payoff from playing M: 0.p + 10.(1-p) = 10 ($\frac{2}{3}$) = $\frac{10}{3}$

If P = 2/3 Then ?

- For the “Interested” girl:
  - Her expected payoff from playing C: 5.p + 0.(1-p) = 5.($\frac{2}{3}$) = $\frac{10}{3}$
  - Her expected payoff from playing M: 0.p + 10.(1-p) = 10 ($\frac{2}{3}$) = $\frac{10}{3}$

- For the “Uninterested girl”:
  - Her expected payoff from playing C: 0.p + 5.(1-p) = $\frac{5}{3}$
  - Her expected payoff from playing M: 10.p + 0.(1-p) = $\frac{20}{3}$
  - So it is optimal for her to play M.
  - i.e $\boldsymbol{q_2}$ **= 0**

- If p = $\frac{2}{3}$ i.e if the Boy plays the mixed strategy $(\frac{2}{3}, \frac{1}{3})$

➢ The “Interested” girl will (can?) opt for a mixed strategy.

➢ The “Uninterested” girl simply plays the pure strategy M. i.e $\boldsymbol{q_2}$ **= 0**

- If p = $\frac{2}{3}$ i.e if the Boy plays the mixed strategy ($\frac{2}{3}, \frac{1}{3}$)

➢ The "Interested" girl will (can?) opt for a mixed strategy.

➢ The "Uninterested" girl simply plays the pure strategy M. i.e $\boldsymbol{q_2}$ **= 0**

- **Recall the "girl connect Equation"**

$$\{10.q_1 + 0.(1- q_1)\}.\frac{1}{2} + \{10.q_2 + 0.(1- q_2)\}.\frac{1}{2} = \{0.q_1 + 5.(1- q_1)\}.\frac{1}{2} + \{0.q_2 + 5.(1- q_2)\}.\frac{1}{2}$$

➢ **If q_2 = 0 it implies $q_1 = \frac{2}{3}$**

- So p= $\frac{2}{3}$, $q_1 = \frac{2}{3}$, $q_2 = 0$ **seems to do the trick !**

- **So { $(\frac{2}{3}, \frac{1}{3})$, [$(\frac{2}{3}, \frac{1}{3})$, (0,1)] } is a MSBNE given the Boy's belief $(\frac{1}{2}, \frac{1}{2})$ about the Girl's type**

If P = 1/3 Then ?

- For the “Interested” girl:

➢ Her expected payoff from playing C: 5.p + 0.(1-p) = 5.($\frac{1}{3}$) = $\frac{5}{3}$

➢ Her expected payoff from playing M: 0.p + 10.(1-p) = 10 ($\frac{2}{3}$) = $\frac{20}{3}$

➢ So it is optimal for her to play M.

➢ i.e $\boldsymbol{q_1}$ **= 0**

If P = 1/3 Then ?

- For the "Interested" girl:

➢ Her expected payoff from playing C: 5.p + 0.(1-p) = $5.(\frac{1}{3}) = \frac{5}{3}$

➢ Her expected payoff from playing M: 0.p + 10.(1-p) = $10\ (\frac{2}{3}) = \frac{20}{3}$

➢ So it is optimal for her to play M.

➢ i.e $\boldsymbol{q_1}$ **= 0**

- For the "Uninterested girl":

➢ Her expected payoff from playing C: 0.p + 5.(1-p) = $\frac{10}{3}$

➢ Her expected payoff from playing M: 10.p + 0.(1-p) = $\frac{10}{3}$

- If p = $\frac{1}{3}$ i.e if the Boy plays the mixed strategy $(\frac{1}{3}, \frac{2}{3})$

- The "Interested" girl simply plays the pure strategy M. i.e $\boldsymbol{q_1}$ **= 0**
- The "UnInterested" girl will (can?) opt for a mixed strategy.

- **Recall the "girl connect Equation"**

$$\{10.q_1 + 0.(1- q_1)\}.\frac{1}{2} + \{10.q_2 + 0.(1- q_2)\}.\frac{1}{2} = \{0.q_1 + 5.(1- q_1)\}.\frac{1}{2} + \{0.q_1 + 5.(1- q_1)\}.\frac{1}{2}$$

- **If q_1 = 0 it implies q_2 = 2/3**

- So p= $\frac{1}{3}$, q_1 = 0, q_2 = $\frac{2}{3}$ **seems to do the trick !**

- **So { ($\frac{1}{3}$, $\frac{2}{3}$) , [(0,1) , ($\frac{2}{3}$, $\frac{1}{3}$)] } is a MSBNE given the Boy's belief ($\frac{1}{2}$, $\frac{1}{2}$) about the Girl's type**