Artificial Intelligence for Economics (AI60003)

Quiz 1 (Set 2) **Full Marks: 20** **12th September, 2023**

Name: **Roll:**

Q1. Suppose I take up the following positions in the market:

- Sell 2 - P_{40} (S_t, T) PUTs.
- Buy 2 - P_{60} (S_t, T) PUTs.
- Buy 3 - C_{80} (S_t, T) CALLs.
- Sell 1 - C_{120} (S_t, T) CALL.
- Sell 2 - C_{160} (S_t, T) CALLs.

$x<40$ 40

40, 60 $120-2x$

60, 80 0

80, 120 $3x-240$

120, 160 $2x-120$

160 200

i.e I have a portfolio $C^* = -2\,P_{50}(S_t, T) + 2.\,P_{70}(S_t, T) + 3.\,C_{90}(S_t, T) - C_{110}(S_t, T) - 2.\,C_{120}(S_t, T)$

How will my payoff at expiration date look like as a function of the value of the underlying asset on the expiration date? **[10 marks]**

Q2. Your earnings for your first T years of service are [x(1), x(2), ... x(T)]. Each year, you consume a part of these earnings (C(1), C(2), ... C(T)), and save the rest (S(1), ... S(T)). The interest rate on your savings is 5% per annum. The utility function for your consumption is $U(t) = C(t)/t^2$ (where t is the number of years elapsed from the start).

i) How will you plan your consumptions/savings, such that your savings after 5 years (including interest) are at least 50% of the earnings?

ii) If you don't have that constraint on the savings but want to maximize it along with the consumption utility, how will you find an optimal solution?

[Please formulate the problems and indicate how they can be solved]

iii) Suppose you save a random fraction of your saving each year, i.e. S(t)=v(t)*x(t) where v(t) is chosen uniformly between 0.2 and 0.5. What is your expected utility value and expected savings (including interest) after 3 years?

[4+3+3=10 marks]