INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination 2023-24

Date of Examination: 19th September 2023 Session: FN Duration: 2 Hrs Full Marks: 50

Subject No. : AI60003 Subject : Artificial Intelligence for Economics

Department/Center/School: Centre of Excellence in Artificial Intelligence

Specific charts, graph paper, log book etc., required

Special Instructions (if any) :

Q1. Give brief answers to the following problems: **[3 x 5 = 15 marks]**

a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

$$-[(10-a)^2 + (10-b)^2]$$

Suppose prices of both the goods are equal to 1.

i. Solve for the optimal consumption of the consumer when his income is 40.

ii. What happens to his optimal consumption when his income goes down to 10.

b) A researcher has 100 hours of work which have to be allocated between two research assistants Aditya & Gaurav. If Aditya is allocated x hours of work his utility is $-(x-20)^2$. If Gaurav is allocated x hours of work his utility is $-(x-30)^2$. The researcher is considering two proposals:

I. Aditya is given 60 hours of work & Gaurav is given 40 hours.

II. Aditya is given 90 hours of work & Gaurav is given 10 hours.

Which of the following statement is correct.

A. Proposal I is pareto optimal but Proposal II is NOT.

B. Proposal II is pareto optimal but Proposal I is NOT.

C. Both proposals are pareto optimal.

D. Neither proposal is pareto optimal.

c) In the class we saw that the greedy algorithm did NOT lead to stable match. Is it necessary that it will fail? If not, can you provide an example (with 5 boys & 5 girls) where the greedy algorithm succeeds. (Hint: Try finding an appropriate list of preferences.)

d) Two days before the expiration date Harshad wants to sell a CALL with strike price Rs. 100 i.e she wants to go short on C_{100} (S , t). The interest rate is r = 10%. And the current value of the stock is Rs. 120. Use the PUT – CALL parity equation to find a lower bound on the value of C_{100}.

e) We are trying to predict whether there will be a crop shortage or not in a given year. Several factors are recorded from past experience in earlier years and locations, based on which two decision trees are proposed. On the given dataset, which do you think is more suitable?

Government Subsidies (S)	550	437	898	480	754	604	490	725	685	815
Drought (D)	Y	N	N	Y	Y	N	N	N	Y	N
Farming workforce (W) (million)	31	18	25	22	30	28	32	35	21	33
Crop Shortage (C)	Y	Y	Y	Y	Y	N	N	N	N	N

Tree 1: If S > 500: if D=Y then predict Y, else predict N; If S < 500: predict Y
Tree 2: If D = Y: if S>600 then predict N, else predict Y; If D = N: if W<30 then predict Y, else predict N

Q2. Provide brief and clear answers to the following questions

a) Briefly discuss the idea of graph coloring & chromatic number of a graph and how it is applied in scheduling problems. **[5 marks]**

b) Explain the fundamental theorem of linear programming with respect to one linear constrained optimization problem. **[2 marks]**

c) Explain the difference between interior and exterior point methods for non-linear constrained optimization with an example. (Only formulate the problem and explain the solution approach, don't solve actually). **[3 marks]**

d) Explain how Decision Trees and Regularized Linear Regression attempt to identify important features in prediction problems. **[5 marks]**

Provide detailed answers to ANY TWO of the following problems

Q3. Consider an economy where agents are identical and they live for three periods. Suppose in the first period they invest 'e' in their education and become skilled in the second period where the level of skill h = e δ, 0 < δ < 1. The investment in education is done via borrowing from the market at a fixed rate R, i.e per unit of borrowing costs R > 1 per period. In the second period of their life they work using the acquired skill (h). The total wage of an agent with skill level h is given by w.h where w is the exogenously given wage rate. Once they earn wage income w.h in the second period, they repay their total borrowing for education. Assume that the wage income is sufficient to repay this education loan. Further, for simplicity we assume that they do not consume anything in the first period. In the second period they take their consumption and saving decision for the second and the third period. Gross return on saving is the same R per unit per period. Agents do not work in the third period and live on their saving made in the second period. Suppose the life-time utility function is given by U = $U(c_2) + \beta U(c_3)$; where c_t and c_{t+1} are the consumption levels in the second and third periods respectively. The function U(.) is assumed to be strictly positive and strictly concave with β > 0.

(a) Write down the utility maximization problem of the agents. Clearly derive the first order conditions with respect to saving 's' and investment in education 'e' in this maximization problem. **[5 marks]**

(b) Derive the optimal level of investment in education (e). Describe its relationship with the wage rate (w) and the rate of interest (R). **[5 marks]**

Q4. Covid pandemic is spreading rapidly in a country. For each month, the government can take two decisions: either declare a lockdown (which reduces disease spread but harms the economy), or let the economy function normally (which increases disease spread). Assume that initially the economy size is X0=100 and number of infections is N0=10. At the beginning of each month, let's say economy was X and number of infections was N. If lockdown is imposed for that month, the economy shrinks to 0.9*X, and the number of infections becomes 3N/2 by the end of the month. If no lockdown is imposed, the economy falls to 0.99*X but the number of infections becomes 4N by month-end. This process continues for 3 months.

(a) Draw the state space transition graph. **[1 marks]**

(b) The government's target is to keep the total number of infections below 100. Using A*, find the sequence(s) of decisions it should take for these 3 months, so that it can achieve this goal while sustaining least harm to the economy (use economic loss as edge cost). **[5 marks]**

(c) The government has two objectives: minimize the number of infections, and minimize the economic loss. Using multi-objective A*, find the pareto-optimal solutions. A solution is ruled out if the number of infections is above 300, irrespective of economic loss. **[4 marks]**

(in both cases, use any heuristic function of your choice, as long as it underestimates the actual cost)

Q5. There are two companies C1, C2. For C1, if it receives a budget of x, it generates revenue R1(x) according to Gaussian distribution N(2x, 20) and creates J1(x) jobs according to Uniform (x, 2x). For same budget, C2 generates revenue R2(x) according to N(x, 40) and creates J2(x)=2x jobs (guaranteed).

The government has a budget of 100 which it can distribute among 2 companies. Let these be denoted as X, 100-X. Denote the total revenue generated and total jobs created by R(X), J(X) respectively. Clearly, R(X)=R1(X)+R2(100-X), J(X)=J1(X)+J2(100-X).

(a) Calculate the expected revenue generated and jobs created as a function of X. **[3 marks]**

Define utility function U(X) = aR(X) + bJ(X). If an event E happens, then (a=1, b=1), but else (a=0, b=2). If event F happens, then X is chosen uniformly between (20, 50), else it is chosen uniformly between (50, 80). Probability of E happening is 60%, while that of F is 30%.

(b) Represent the situation using Bayesian Network. Mention the p(X|pa(X)) type distribution at each node X, where pa(X) are the parents of node X. **[3 marks]**

(c) Calculate the expected value of U. **[4 marks]**