Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ where C_t represents consumption or use of the resource at period t and $\delta (0 < \delta < 1)$ is the discount factor

(a) Set up the utility maximization problem of the community and interpret the first order condition.

(b) Express the optimal consumption C_t for any period t in terms of the parameters δ and X.

Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

$$-\left[(10-a)^2+(10-b)^2\right]$$

Suppose prices of both the goods are equal to 1.

i. Solve for the optimal consumption of the consumer when his income is 40

ii. What happens to his optimal consumption when his income goes down to 10.

Igor read in the press this morning that, for an expiration date of a year from now (with 5% interest) that $C_{60}(70, t) = 9$ and $P_{60}(70, t) = 4$. How can he use this information to make some money?

For this problem, assume that at this moment, $S = \$99$, $r = 0.02$, $C_{100}(99, T - \frac{1}{2}) - P_{100}(99, T - \frac{1}{2}) = \$8$, and $T = t + \frac{1}{2}$ year.

(a) Are there any arbitrage opportunities? If so, explain how to tackle this opportunity and determine how much money can be made.

(b) All sorts of events can happen in the next six months. Nevertheless, you are safe at expiration date. List the possibilities to explain how the profit you made remains made, and you can cover all debts.