INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
End-Autumn Semester 2018-19

Date of Examination 22.11.2018 **Session** FN **Duration** 3 hrs **Max. Marks** 100
Subject No. : ME 10001 **Subject:** Mechanics
Department/Center/School: Mechanical Engineering
Specific Item Required: Graph Paper may be given as per request of the student.
Instructions: Answer all questions. All parts of a question MUST be together. Figures are not to scale.

1. **(a)** Calculate the centroidal coordinates $(X_c,\ Y_c)$ of the area shown in Figure 1. **(6)**

   (b) Compute the second moments of area I_{XX}, I_{YY} and I_{ZZ}. **(10)**

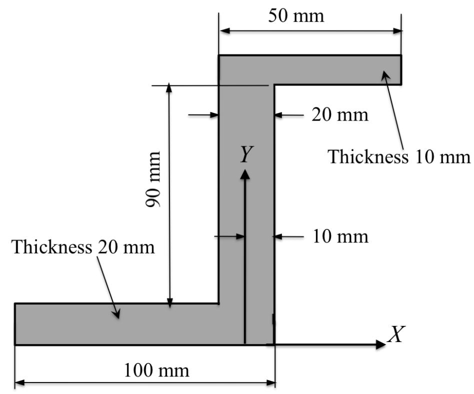


Figure 1

2. An element of a thin plate is subjected to stresses as shown in Figure 2. Find the stress state, i.e., $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$ on the element rotated by 37.5 degrees (counterclockwise) as shown.(*Stress directions in the rotated element are indicative only.*) **(12)**

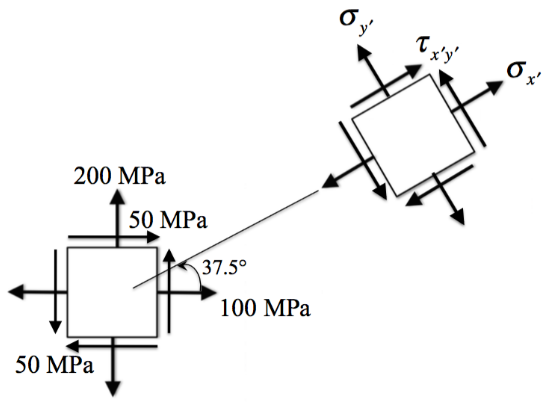


Figure 2

3. A 2 m long stepped cylinder of solid circular cross section is rigidly fixed to a vertical wall as shown in Figure 3. The cylinder is subjected to torques $T_1 = 1$ kNm and $T_2 = 2$ kNm and the shear modulus of the cylinder material is, $G = 30$ GPa.

   (a) Calculate the angles of twist for sections AB, BC and the total angle of twist of the cylinder. **(6)**

   (b) Find the maximum shear stress in the cylinder and identify the section in which it will occur. **(6)**

   (c) Determine the maximum and minimum principal stresses in section BC. **(4)**

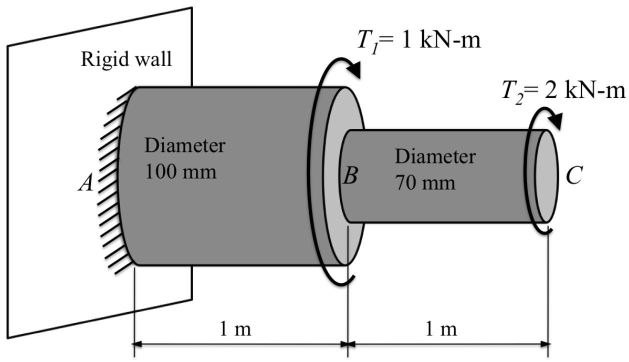


Figure 3

4. A solid circular rod is subjected to uniaxial loads 50 kN and 20 kN as shown in Figure 4. Find the elongation of the rod due to the loads. The Young's modulus and Poisson's ratio of the rod material are $E = 70$ GPa and $\nu = 0.3$, respectively. **(14)**

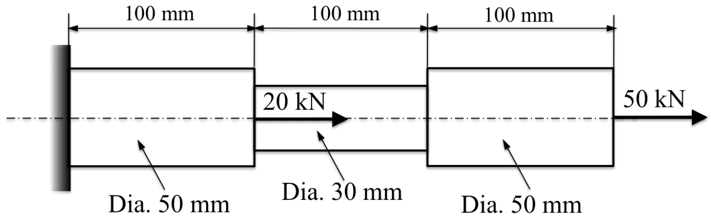


Figure 4

5. A simply supported beam is subjected to uniformly distributed load and a concentrated force as shown in Figure 5.

(a) Compute the support reactions. **(6)**

(b) Draw the shear force diagram (SFD) and bending moment diagram (BMD) of the beam. (*The diagrams must be drawn below the free body diagram of the beam on the same page. State the sign convention also.)* **(8)**

(c) Find the locations of maximum bending moment and maximum shear force. **(2)**

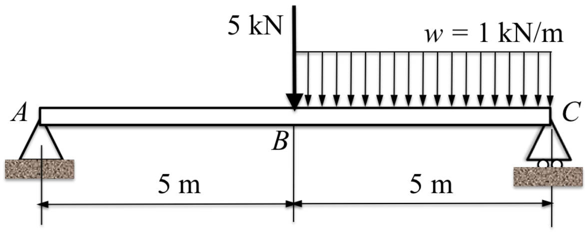


Figure 5

6. A simply supported beam having T-section is subjected to a concentrated load $P = 30$ kN at location B as shown in Figure 6.

(a) Find the magnitude of maximum bending moment M and its location in the beam. **(6)**

(b) Calculate the bending stresses at the top and bottom of the section of the beam at the location of maximum bending moment. Indicate the nature of the stresses. **(8)**

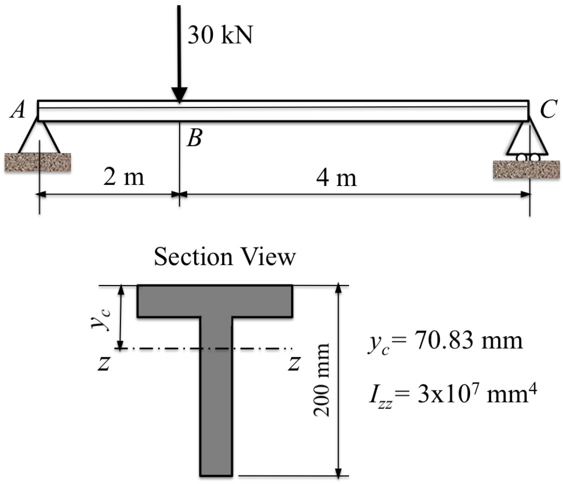


Figure 6

7. A thin spherical vessel of inner diameter 200 mm is subjected to an internal pressure $p = 10$ MPa. The maximum allowable normal stress of the vessel material is 100 MPa. The Young's modulus and Poisson's ratio of the shell material are $E = 200$ GPa and $\nu = 0.3$, respectively.

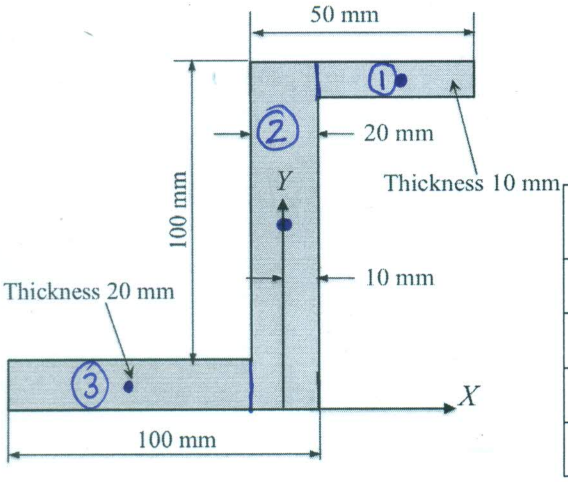
(a) Find the minimum thickness of the vessel. **(6)**

(b) Using the thickness calculated in (a) find the new volume of the vessel. **(4)**

(c) Find the maximum in-plane shear stress in the vessel. **(2)**

Mechanics (ME10001) End-Autumn 2018
Solutions

Problem1:



(a) Centroidal Coordinates:

Sl. No	A_i (mm^2)	x_i (mm)	y_i (mm)	A_ix_i (mm^3)	A_iy_i (mm^3)
1	30x10 = 300	25	115	7500	3.45x10^4
2	120x20 = 2400	0	60	0	14.4x10^4
3	80x20 = 1600	-50	10	-8x10^4	1.60x10^4
Sum	4300	-	-	-7.25x10^4	19.45x10^4

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = -16.86 \text{ mm}; \qquad \bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = 45.23 \text{ mm}$$

(b) Second Moment of Inertia:

Moment of inertia of the individual areas about their respective centroidal axes:

Area 1: $I_{x01} = \frac{1}{12}\times 30\times 10^3 = 2.5\times 10^3 \text{ mm}^4$; $\quad I_{y01} = \frac{1}{12}\times 10\times 30^3 = 2.25\times 10^4 \text{ mm}^4$;

Area 2: $I_{x02} = \frac{1}{12}\times 20\times 120^3 = 2.88\times 10^6 \text{ mm}^4$; $\quad I_{y01} = \frac{1}{12}\times 120\times 20^3 = 8\times 10^4 \text{ mm}^4$;

Area 3: $I_{x03} = \frac{1}{12}\times 80\times 20^3 = 5.33\times 10^4 \text{ mm}^4$; $\quad I_{y03} = \frac{1}{12}\times 20\times 80^3 = 8.533\times 10^5 \text{ mm}^4$;

Sl. No	A_i (mm^2)	I_{x0i} (mm^4)	d_{xi} (mm)	$A_i d^2_{xi}$ (mm^4)	I_{y0i} (mm^4)	d_{yi} (mm)	$A_i d^2_{yi}$ (mm^4)
1	300	2.5x10^3	115	39.675x10^5	2.25x10^4	25	1.875x10^5
2	2400	28.8x10^5	60	86.4x10^5	8x10^4	0	0
3	1600	5.33x10^4	10	1.6x10^5	85.33x10^4	-50	40x10^5
	Total	29.358x10^5	--	127.675x10^5	9.558x10^5	--	41.875x10^5

Moment of inertia of the composite area about its centroidal axes:

$$I_{XX} = \Sigma I_{x0i} + \Sigma(A_i d_{xi}^2) = 15.703\times 10^6 \text{ mm}^4; \qquad I_{YY} = \Sigma I_{y0i} + \Sigma(A_i d_{yi}^2) = 5.143\times 10^6 \text{ mm}^4;$$

$$I_{ZZ} = I_{XX} + I_{YY} = 20.846\times 10^6 \text{ mm}^4$$

Mechanics (ME10001) End-Autumn 2018
Solutions

Problem 2:

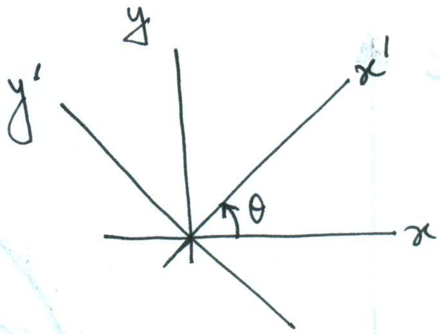
Given :

$\sigma_x = 100\,MPa$

$\sigma_y = 200\,MPa$

$\tau_{xy} = 50\,MPa$

$\theta = 37.5^\circ$



Stresses on the element in x'-y' frame:

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 150 - 50\cos 75^\circ + 50 \sin 75^\circ$$

$$\Rightarrow \boxed{\sigma_{x'} = 185.36\ MPa}$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 150 + 50\cos 75^\circ - 50\sin 75^\circ$$

$$\Rightarrow \boxed{\sigma_{y'} = 114.64\ MPa}$$

Alternatively : $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y = 300\,MPa$

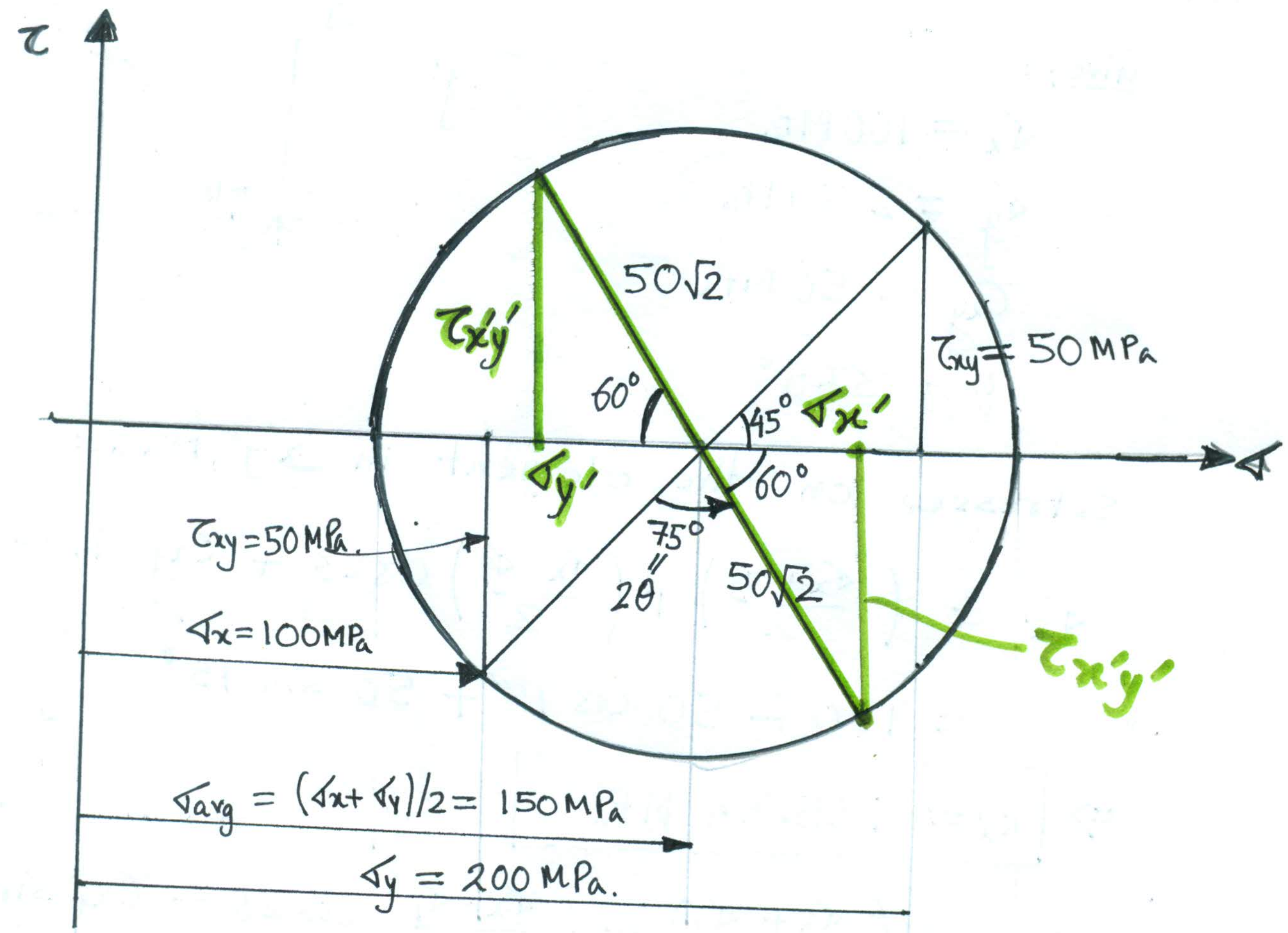
$$\Rightarrow \boxed{\sigma_{y'} = 300 - \sigma_{x'} = 114.64\,MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= 50\sin 75^\circ + 50 \cos 75^\circ$$

$$\Rightarrow \boxed{\tau_{x'y'} = 61.24\ MPa}$$

Mohr's Circle Method:



Note: With σ_x, positive τ_{xy} is plotted in the negative τ-axis.

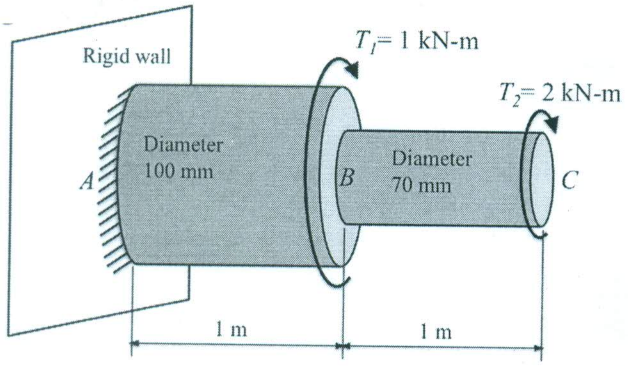
$$\Rightarrow \quad \sigma_{x'} = \sigma_{avg} + 50\sqrt{2}\cos 60^\circ = (150 + 25\sqrt{2})$$

$$\Rightarrow \quad \boxed{\sigma_{x'} = 185.36\ \text{MPa}}$$

$$\boxed{\begin{aligned} \sigma_{y'} &= \sigma_{avg} - 50\sqrt{2}\cos 60^\circ = 114.64\ \text{MPa} \\ \tau_{x'y'} &= 50\sqrt{2}\sin 60^\circ = 61.24\ \text{MPa} \end{aligned}}$$

Mechanics (ME10001) End-Autumn 2018
Solutions

Problem 3:

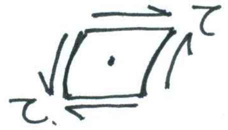


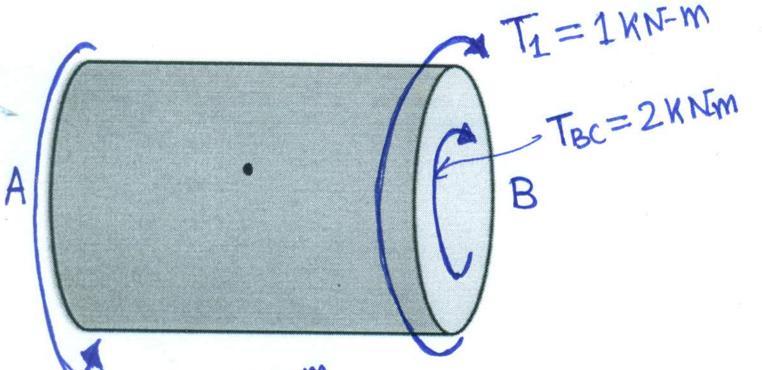
<u>Polar moment of inertia</u>

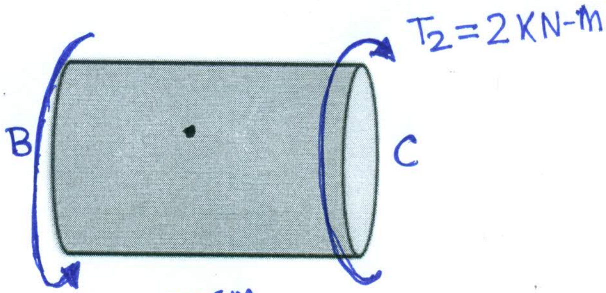
$$J_{AB} = \frac{\pi}{32} \times d_{AB}^4 = \frac{\pi}{32} \times 10^8 \text{ mm}^4$$

$$J_{BC} = \frac{\pi}{32} \times d_{BC}^4 = \frac{\pi}{32} \times 2.401 \times 10^7 \text{ mm}^4$$

<u>stress state at any point on surface:</u>







$T_{AB} = 3$ kN-m $= T_1 + T_{BC}$

$T_{BC} = 2$ kN-m.

(a) <u>Angle of twist of AB</u>: $\phi_{AB} = \dfrac{T_{AB} L}{G J_{AB}} = \dfrac{3 \times 10^6 \text{ N-mm} \times 10^3 \text{ mm}}{30 \times 10^3 \text{ MPa} \times \frac{\pi}{32} \times 10^8 \text{ mm}^4}$

$$\Rightarrow \boxed{\phi_{AB} = \frac{32}{\pi} \times 10^{-3} = 0.0102 \text{ rad}}$$

<u>Angle of twist of BC</u>:

$$\phi_{BC} = \frac{T_{BC} \cdot L}{G \cdot J_{BC}} = \frac{2 \times 10^6 \text{ N-mm} \times 10^3 \text{ mm}}{30 \times 10^3 \text{ MPa} \times \frac{\pi}{32} \times 2.401 \times 10^7 \text{ mm}^4}$$

$$\Rightarrow \boxed{\phi_{BC} = 0.0283 \text{ rad}}$$

Total angle of twist: $\boxed{\phi = \phi_A + \phi_B = 0.0385 \text{ rad}}$

(b) <u>Maximum shear stress in AB</u>:

$$\tau_{max/AB} = \frac{T_{AB} d_{AB}/2}{J_{AB}} = \frac{16 T_{AB}}{\pi d_{AB}^3} = \underline{\underline{15.28 \text{ MPa}}}$$

<u>Maximum shear in BC</u>: $\tau_{max/BC} = \dfrac{T_{BC} \times d_{BC}/2}{J_{BC}} = \dfrac{16 T_{BC}}{\pi d_{BC}^3} = \underline{\underline{29.7 \text{ MPa}}}$

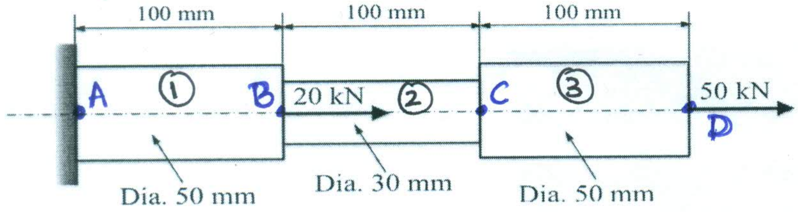
$\Rightarrow$ Maximum shear stress will occur in section BC. $\boxed{\tau_{max} = 29.7 \text{ MPa}}$

(c) Minimum principal stress in BC: $\sigma_{min} = -\tau_{max} = -29.7$ MPa

Maximum principal stress in BC: $\sigma_{max} = \tau_{max} = 29.7$ MPa

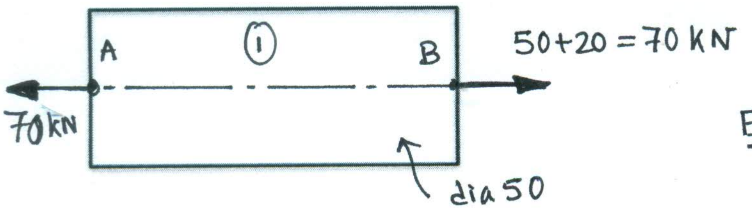
Mechanics (ME10001) End-Autumn 2018
Solutions

Problem 4:



$$E = 70\,GPa = 70\times10^{3}\ \frac{N}{mm^2}$$

Rod – AB



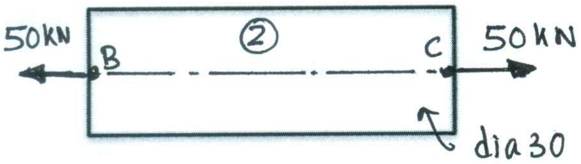
$$A_1 = \frac{\pi}{4} d_1^2 = \frac{2500\pi}{4}\ mm^2$$

$$P_1 = 70\,kN, \quad L_1 = 100\,mm$$

Elongation: $\delta_1 = \dfrac{P_1 L}{A_1 E} = \dfrac{70\times10^{3}\,N\times100\,mm}{\frac{2500}{4}\pi\,mm^2\times70\times10^{3}\,\frac{N}{mm^2}}$

$$\Rightarrow \boxed{\delta_1 = 5.093\times10^{-2}\ mm}$$

Rod - BC

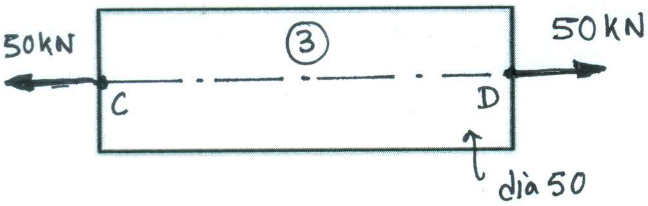


$$A_2 = \frac{\pi}{4} d_2^2 = \frac{900\pi}{4}\ mm^2, \quad P_2 = 50\,KN$$

Elongation: $\delta_2 = \dfrac{P_2 L_2}{A_2 E} = \dfrac{50\times10^{3}\times100}{\frac{900\pi}{4}\times70\times10^{3}}$

$$\Rightarrow \boxed{\delta_2 = 10.105\times10^{-2}\ mm}$$

Rod CD



$$A_3 = A_1 = \frac{2500\pi}{4}\ mm^2$$

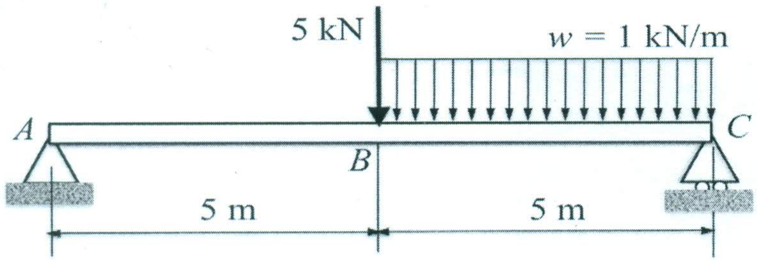
$$P_3 = 50\,KN, \quad L_3 = 100\,mm$$

Elongation: $\boxed{\delta_3 = \dfrac{P_3 L}{A_3 E} = 3.638\times10^{-2}\ mm}$

Total elongation of the rod:

$$\boxed{\delta = \delta_1 + \delta_2 + \delta_3 = 18.836\times10^{-2}\ mm = 0.188\ mm}$$

Problem 5:



(a) Support reactions:

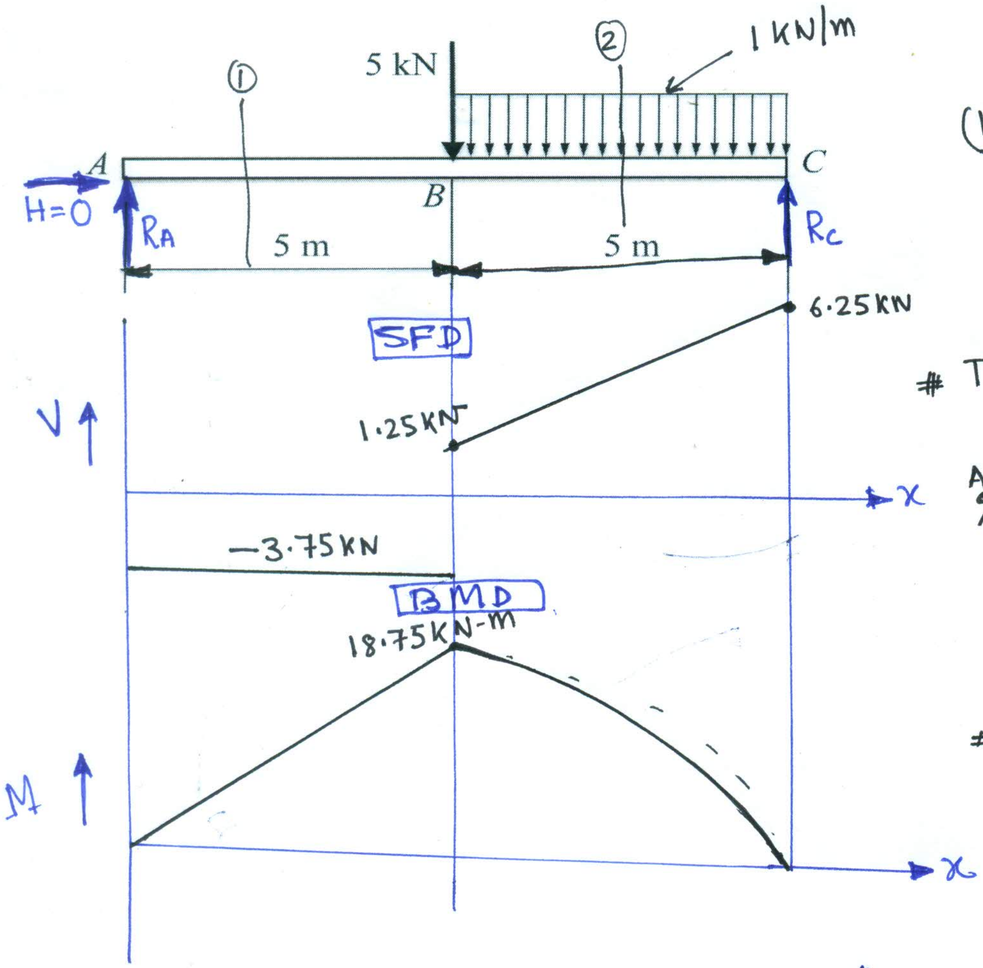
$\Sigma F = 0$: $R_A + R_C = 10\,kN$

$\Sigma M_A = 0$:

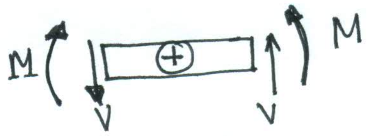
$R_C \times 10 - 5\times 5 - 5w\times 7.5 = 0$

$\Rightarrow \boxed{R_C = 6.25\,kN}$

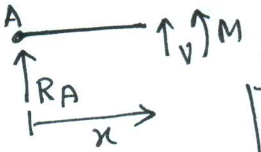
$\Rightarrow \boxed{R_A = 3.75\,kN}$



(b) Sign convention:



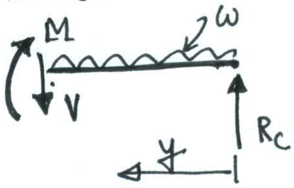
Taking section between A & B:



$\Rightarrow \boxed{V = -R_A = -3.75\,kN}$

$\boxed{M = +R_A x = 3.75x \ kN\cdot m}$

Taking section between B & C



Here $\boxed{y = (10-x)}$

$\Rightarrow V = R_c - wy = (6.25 - y)\,kN$

$\Rightarrow V = -3.75 + wx$
$(5 < x \le 10)$

$$\boxed{\begin{aligned} M &= R_c y - w y^2/2 \\ &= 6.25(10-x) - \frac{(10-x)^2}{2} \\ &\quad (5 < x \le 10 \end{aligned}}$$

at $x = 5$: $V = 1.25\,kN$
$M = 18.75\,kN$

at $x = 10$: $V = 6.25\,kN$
$M = 0$

(c) Location of maximum bending moment:

$x = 5\,m$ at B

$M_{max} = 18.75\,kN\text{-}m$

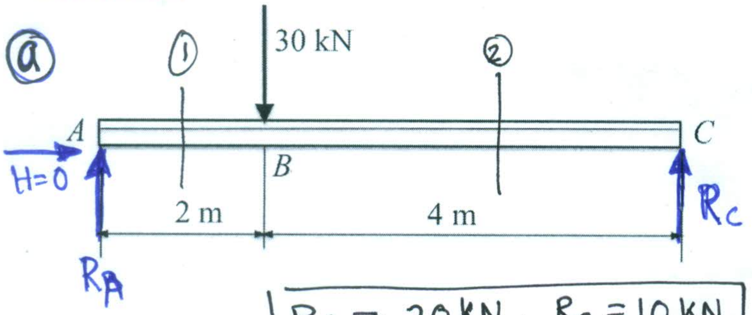
Location of maximum shear force:

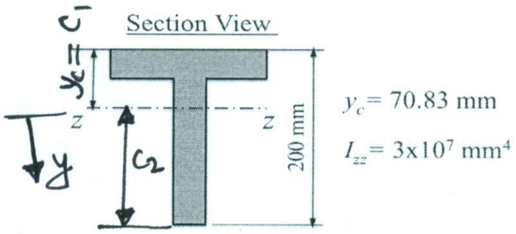
at $x = 10\,m$ (at C)

$V_{max} = 6.25\,kN$

Problem 6:

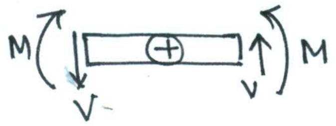
(a)



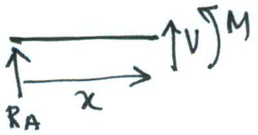


$\boxed{R_A = 20\,kN,\ R_c = 10\,kN}$ $\left(\Sigma M_A = 0:\ R_c \times 6 - 30 \times 2 = 0,\ \Sigma F = 0:\ R_A + R_c = 30\,kN\right)$

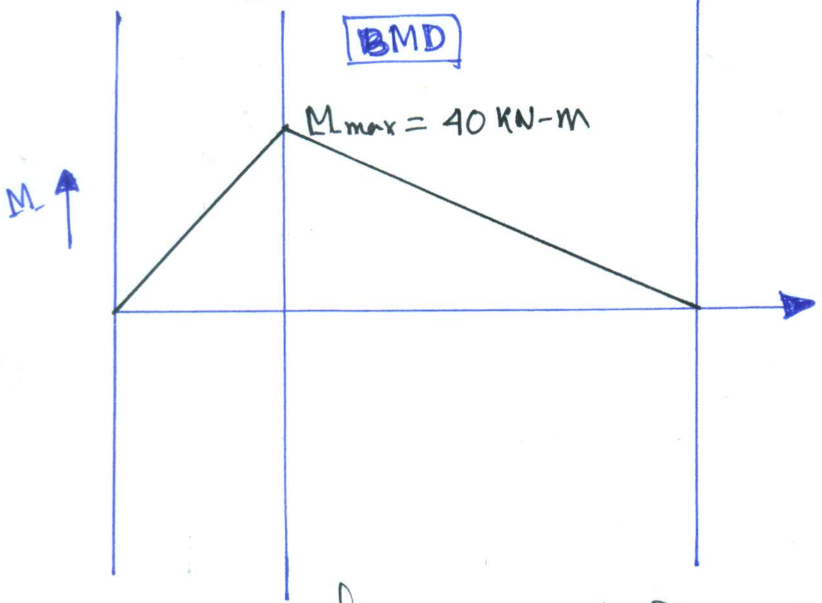
Sign convention :



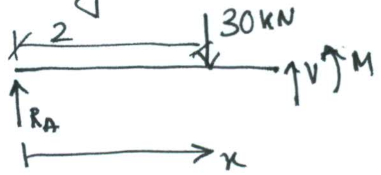
Taking section between A & B :



$\Sigma M = 0$:
$M = R_A \times x$
$= 20x$



Taking section between B & C



$\Sigma M = 0: -R_A \times x + 30 \times (x-2) + M = 0$

$\Rightarrow M = 20x - 30x + 60$

$\boxed{M = 60 - 10x}$

- Magnitude of maximum bending moment
  $M_{max} = 40\,kN\text{-}m = 40 \times 10^6\,N\text{-}mm$
- Location : at $x = 2m$ or at B.

(b) At the top: $c_1 = -y_c = -70.83\,mm$

Bending stress: $\sigma_{top} = \dfrac{M_{max} \times c_1}{I} = +\dfrac{40 \times 10^6\,N\text{-}mm\,(-70.83\,mm)}{3 \times 10^7\,mm^4}$

$\Rightarrow \boxed{\sigma_{top} = -94.44\,MPa}$ (Compressive)

At the bottom: $c_2 = 200 - y_c = 129.7\,mm$

$\Rightarrow \sigma_{bot} = \dfrac{M_{max} \times c_2}{I} = \dfrac{40 \times 10^6\,N\text{-}mm \times (129.17)}{3 \times 10^7\,mm^4}$

$\Rightarrow \boxed{\sigma_{bot} = 172.23\,MPa}$ (Tension)

Mechanics (ME10001) End-Autumn 2018
Solutions

Problem 7:

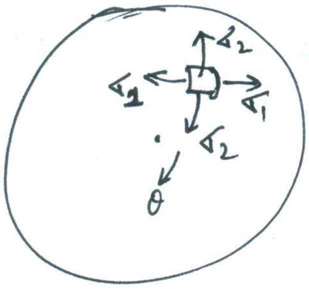
Given: Spherical vessel,

Inner radius — $r = \frac{d}{2} = 100$ mm

Internal pressure — $p = 10$ MPa

Allowable normal stress: $\sigma_{all} = 100$ MPa.

(a) Let thickness be t (in mm)



Normal stress: $\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{10^4}{2t}$ MPa.

$\leq \sigma_{all} = 100$ MPa

$$\Rightarrow \boxed{t \geq \frac{10}{2} = 5 \text{ mm}}$$

$\Rightarrow$ minimum thickness $t_{min} = 5$ mm

(b) For new volume: For $t = 5$ mm

$$\sigma_1 = \sigma_2 = 100 \text{ MPa}$$

Circumferential strain: $\varepsilon_\theta = \frac{1}{E}(\sigma_2 - \nu\sigma_1)$

$$\Rightarrow \varepsilon_\theta = \frac{1}{200 \times 10^3 \text{ MPa}} \times 100(1-\nu) \text{ MPa}$$

$$\Rightarrow \boxed{\varepsilon_\theta = 3.5 \times 10^{-4}}$$

New inner radius: $r_{new} = r(1 + \varepsilon_\theta) = 100.035$ mm

$$\boxed{V_{new} = \frac{4}{3}\pi r_{new}^3 = 4.1932 \times 10^6 \text{ mm}^3}$$

(c) Since the principal stresses $\sigma_1 = \sigma_2 = 100$ MPa (equal)

$\Rightarrow$ Maximum inplane shear stress:

$$\boxed{\tau_{max} = (\sigma_1 - \sigma_2) = 0 \text{ MPa}}$$