**Date: November 08, 2018** | **Class Test 2 (Autumn 2018-19)** | **Time: 1 hr 15 mins**

# Mechanics (ME10001)

**Full Marks: 30**

**1** A 50 mm wide and 10 mm thick plate is held between two 50 mm wide and 4 mm thick plates by a pin joint as shown in Figure 1. The pin has diameter 10 mm. The allowable shear stress in the pin material is 40 MPa and the allowable tensile stress in the plate material is 100 MPa. Also, the allowable bearing stress of the plate and pin material is 160 MPa.

(a) Determine the maximum load $F$ the joint can carry. **(7 pts)**

(b) For the load in (a), find the elongation of the portion $AB$ (100 mm long) of the middle plate. The Young's modulus of the plate material is 200 GPa **(3 pts)**

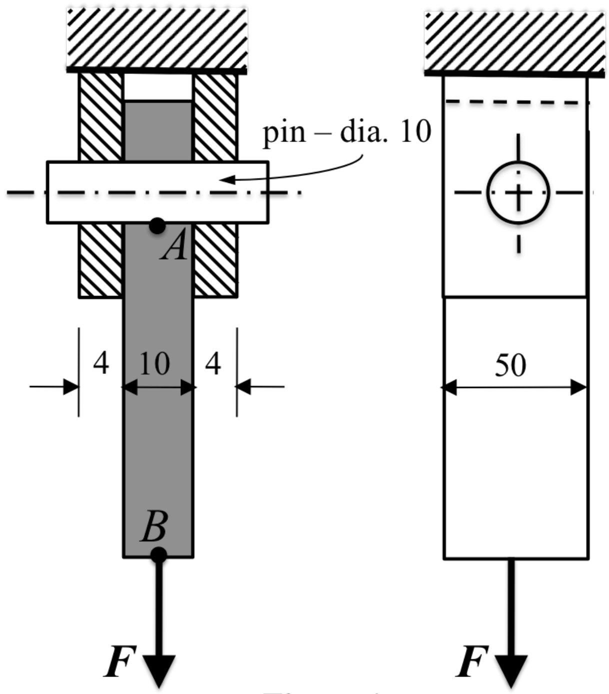



**Figure 1**

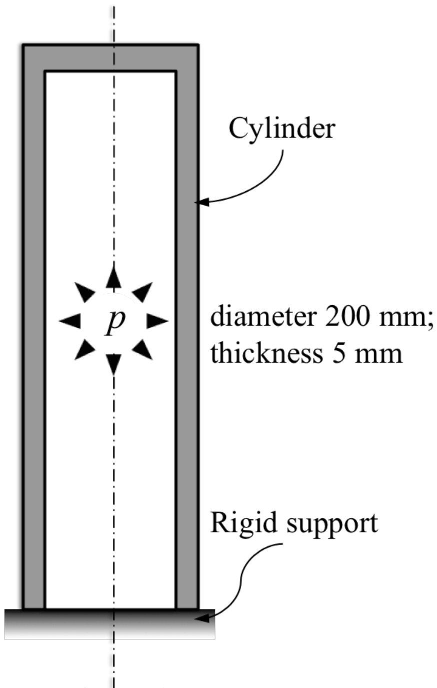



**Figure 2**

**2** The cylindrical vessel, shown in Figure 2, has diameter 200 mm and thickness 5 mm. The cylinder is welded to the rigid support and is subjected to an internal pressure (gauge) 5 MPa. The Young's modulus and the Poisson's ratio of the material are 200 GPa and 0.3, respectively.

(a) Calculate the Hoop (circumferential) and the longitudinal stresses in the cylinder. **(7 pts)**

(b) Calculate the new diameter of the cylinder. **(3 pts)**

3. A thin plate is subjected to a biaxial stress of 100 MPa (tensile) and 50 MPa (compressive) as shown in Figure 3.

(a) Find the normal and shear stresses on an element rotated by 30 degrees (counterclockwise) as shown. **(7 pts)**

(b) Also find the maximum in-plane shear stress and find the angle $\theta$ at which it occurs. **(3 pts)**

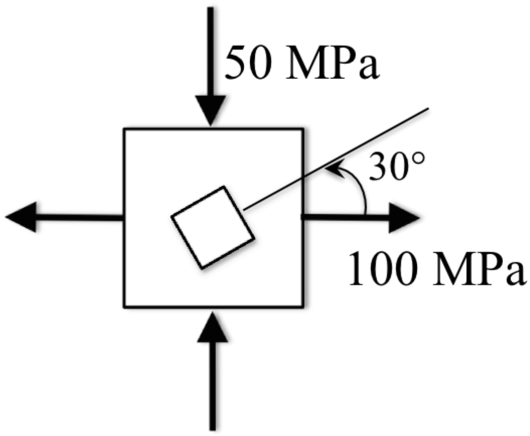


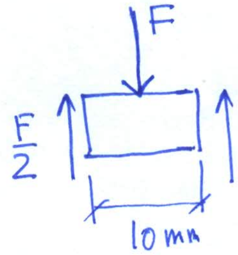

**Figure 3**

# Mechanics (ME 10001)
## Autumn 2018-19
## Class test 2 - Solutions

**Problem 1:**

(a) Checking for maximum load F that the joint can carry.

- Shearing of pin:




(pin diameter d = 10 mm)

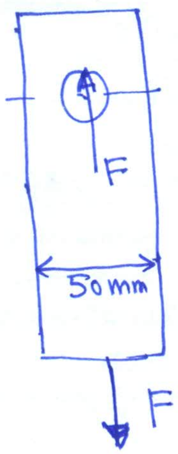
Shear stress:

$$\tau = \frac{F/2}{\pi d^2/4} = \frac{2F}{\pi d^2} = 40\,MPa$$

$$\Rightarrow \quad F = 20\pi \times (10)^2\ N$$

$$\Rightarrow \quad \boxed{F = 2\pi\ kN = 6.28\,kN}$$

- Tensile failure of middle plate:




$$\sigma = \frac{F}{(50-d)t} = \frac{F}{40 \times 10\,mm^2} = 100\,MPa$$

$$\Rightarrow \quad \boxed{F = 40\,kN}$$

- Tensile failure of 50x4 plates.

$$\sigma = \frac{F/2}{(50-d) \times t} = \frac{F}{2 \times 40 \times 4} = 100\,MPa$$

$$\Rightarrow \quad \boxed{F = 32\,kN}$$

- Bearing failure of pin / middle plate

$$\sigma_b = \frac{F}{d \times t} = \frac{F}{10 \times 10\,mm^2} = 160\,MPa$$

$$\Rightarrow \quad \boxed{\sigma_b = 16\,kN}$$

- Bearing failure of pin/ side plates

$$\sigma_b = \frac{F/2}{dt} = \frac{F}{2\times10\times4} = 160 \text{ MPa}$$

$$\Rightarrow \boxed{F = 12.8 \text{ kN}}$$

$\Rightarrow$ Comparing all the loads, we conclude that the maximum load that the joint can carry is

$$\boxed{F = 6.28 \text{ kN}}$$

(obtained from shearing of pin).

(b) Elongation of portion AB of middle plate:

$L_{AB} = 100$ mm

$$\delta_{AB} = \frac{F L_{AB}}{AE} = \frac{6.28\times10^{3}\times 100\times10^{-3} \text{ N-m}}{50\times10 \quad \times 200\times10^{3} \text{ N}}$$

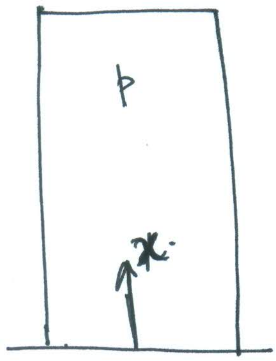
$$\Rightarrow \boxed{\delta_{AB} = \frac{6.28}{10^{6}} \text{ m} = 6.28\times10^{-3} \text{ mm}}$$

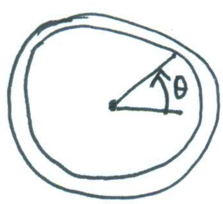
**Problem-2:**

(a) Cylindrical Pressure Vessel:

radius $- r = 100$ mm

thickness $- t = 5$ mm




Hoop stress : $\sigma_\theta = \sigma_1 = \frac{pr}{t}$

$$\Rightarrow \quad \sigma_1 = \frac{5\,\text{MPa} \times 100\,\text{mm}}{5\,\text{mm}}$$

$$\Rightarrow \quad \boxed{\sigma_1 = 100\ \text{MPa}}$$

Longitudinal Stress:

$$\boxed{\sigma_2 = \sigma_x = \frac{pr}{2t} = 50\,\text{MPa}}$$

(b) Circumferential strain $\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_x\right)$

$$\Rightarrow \quad \varepsilon_\theta = \frac{10^6}{200 \times 10^9}\left(100 - 0.3 \times 50\right)$$

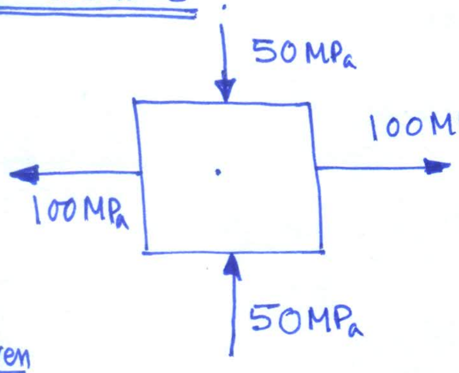
$$= \frac{50 \times (2 - 0.3)}{200 \times 10^3} = 4.25 \times 10^{-4}$$

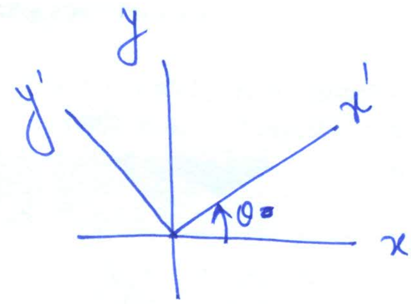
New diameter: $d_{new} = d\left(1 + \varepsilon_\theta\right)$

$$= 200\left(1 + 4.25 \times 10^{-4}\right)$$

$$\Rightarrow \quad \boxed{d_{new} = 200.085\ \text{mm}}$$

## Problem-3 :




**Given**

$\sigma_x = 100\ MPa \ (\sigma_1)$ , $\sigma_y = -50\ MPa \ (\sigma_2)$ , $\tau_{xy} = 0$ ←

$\Rightarrow \quad \theta = 0^\circ$ (or x-y) system denotes principal directions.

(a)

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \cancelto{0}{\tau_{xy}} \sin 2\theta$$

$$\theta = 30^\circ$$

$$\Rightarrow \boxed{\sigma_{x'} = 25 + 75\cos 60^\circ = 62.5\ MPa}$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\Rightarrow \boxed{\sigma_{y'} = 25 - 75\cos 60^\circ = -12.5\ MPa}$$

Alternatively, $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y = 50\ MPa$

$$\Rightarrow \boxed{\sigma_{y'} = 50 - \sigma_{x'} = -12.5\ MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = -75\sin 60^\circ\ MPa$$

$$\Rightarrow \boxed{\tau_{x'y'} = -\frac{75}{2}\sqrt{3}\ MPa = -64.95\ MPa}$$

(b) Maximum in plane shear :

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_x - \sigma_y}{2} = 75\ MPa$$

Occurs on the plane with $\theta = 45^\circ$