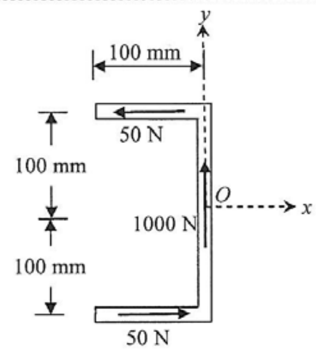
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Spring Semester 2017-18

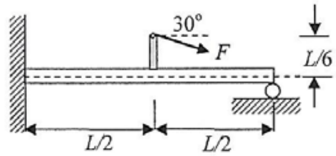
Date of Examination: 22.02.2018 Session: FN Duration: 2 hrs Maximum Marks ; 60
Subject No. : *ME10001* Subject : *Mechanics*
Department/Center/School : All departments offering B.Tech (H), DD, B.Arch, MSc degrees
Specific charts, graph paper, log book etc., required : Nil

Special Instructions (if any) : Attempt all **SEVEN** questions. Marks are indicated against each question. **Answer all the parts of one question at one place only.** Assume any data, if required, after stating it clearly. **DO NOT WRITE ANYTHING ON THE QUESTION PAPER.**

Question 1: A channel section is subjected to three forces as shown in the figure. If the force-system is to be replaced by a single force, $\vec{\mathbf{R}}$ then find (a) the magnitude and the direction of $\vec{\mathbf{R}}$ and (b) the equation of the line of action of $\vec{\mathbf{R}}$ in the given co-ordinate system.

[3+3 = 6]



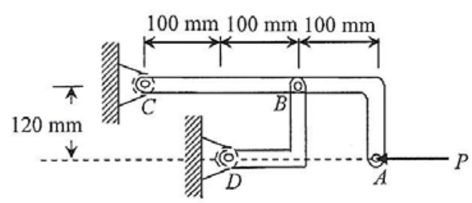


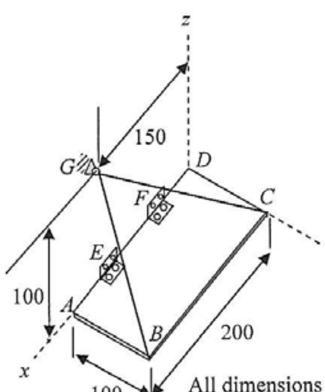
Question 2: A propped cantilever beam is loaded as shown in the figure. (a) Draw a neat Free Body Diagram (FBD) of the beam. (b) State with proper justification whether it is possible to find the reaction force at the roller by considering the equilibrium equations alone.

[3+2 = 5]

Question 3: A frame consisting of two members *ABC* and *BD* is designed to carry a force *P* as shown in the figure. It is known that the reaction force at *B* is 5 kN. (a) Draw the FBD of members *ABC* and *BD*, (b) Calculate the value of *P* when its direction is as shown in the figure and (c) Calculate the magnitude and the direction of the reaction force on the frame at *C*.

[4+4+4 = 12]



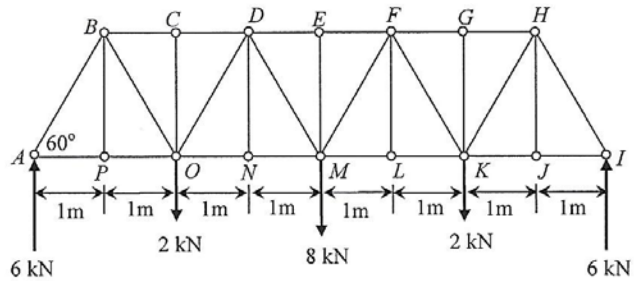


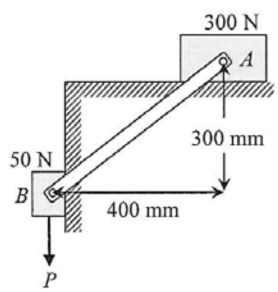
All dimensions are in mm
$AE = FD = 50$ mm

Question 4: A 100 mm × 200 mm uniform wooden panel weighing 400 N is supported by two frictionless hinges as shown in the figure. The panel is kept horizontal with the help of a single inextensible string that passes over a frictionless ring at *G*. The reaction moments at the hinges are negligible. Further the hinge at *E* allows sliding motion along its axis (i.e., *AD*). (a) Draw the FBD of the panel (b) Find the string tension.

[4+6=10]

Question 5: A plane truss is subjected to loads as shown in the figure. (a) Calculate the force in member *BO* by using *method of joints*, (b) Calculate the force in member *FM* by *method of section* and (c) Identify the zero force members in the truss (*wrong identification carries penalty*). [4+4+4 = 12]



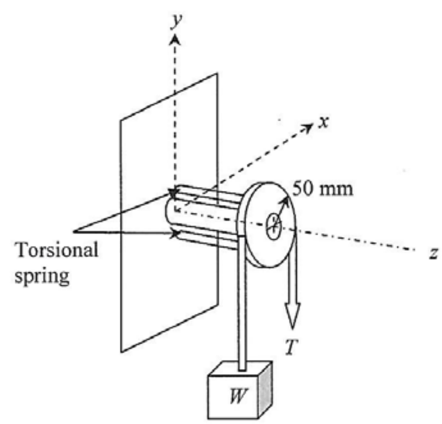


Question 6: Two rigid blocks *A* and *B* of weights 300 N and 50 N, connected by a massless rigid bar as shown in the figure, are in equilibrium. A vertical downward force *P* is then applied on block *B* to slide it down. The coefficient of static and kinetic friction between the blocks and the hatched surfaces are 0.2 and 0.18, respectively. (a) Draw the FBDs of both the blocks when $P = 0$, (b) Draw the FBDs of both the blocks when *P* is just enough to start motion. (c) Calculate the value of *P* required to start the motion.

[3+3+4 = 10]

Question 7: A massless pulley required to lift a 1 kN weight, *W*, is able to rotate freely about the smooth axle. However its rotation is restricted by a torsional spring with spring constant 100 N.m/rad (i.e., 100 N.m torque is required to rotate the pulley by 1 rad about its axis) as shown in the figure. The coefficients of static and kinetic friction between the belt and the pulley are 0.2 and 0.15, respectively. (a) Draw the FBD of the pulley and the belt as a single body, (b) Find the range of belt tension, *T*, for which the belt does not slip over the pulley and (c) Find by how many degrees the pulley rotates when the tension is large enough to lift the weight up with uniform speed.

[2+4+4 = 10]



MECHANICS

MID-SPRING SEMESTER - 2017-18

SOLUTION

Problem # 1 :

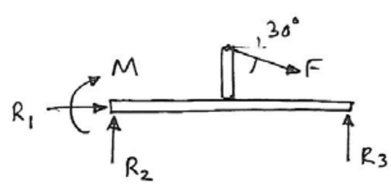
The given force system consists of a force
$$\vec{F} = 1000\,\hat{j}\ (N) \quad \text{passing through } (0,0)$$
and a couple of moment
$$\bar{C} = 0.2 \times 50\,\hat{k}\ (N\text{-}m) = 10\,\hat{k}\ (N\text{-}m).$$
Let us find the equivalent force-couple system at $(x, 0)$ where x is expressed in meter. The equivalent force couple system is a force
$$\bar{R} = 1000\,\hat{j}\ (N) \quad \text{and a couple of moment } \bar{C}_{eq} = \bar{C} - 1000\,x\,\hat{k}\ (N\text{-}m)$$
$= (10 - 1000\,x)\,\hat{k}$ N-m. If $x = \frac{10}{1000} = 0.01$ m, Then the equivalent force system consists of a single force $\bar{R}$, only. Hence,

a) The magnitude of $\bar{R}$ is $\boxed{1000\ N}$ and its direction is along $\boxed{\text{positive } y\text{-axis}}$ $(\hat{j})$.

b) The equation of the line of action is $\boxed{x = 10}$ (where x is measured in mm)

Problem #2

a) The FBD of the cantilever beam is drawn below

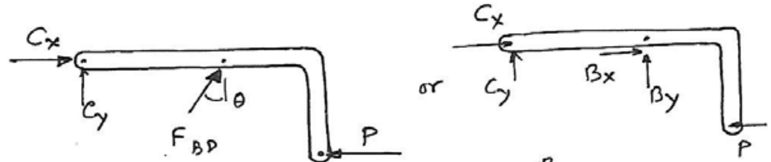


b) Only two equations of equilibrium are obtained from the above free body diagrams that relate the vertical fores and the moment. They are
$$R_2 + R_3 = F \sin 30^\circ$$
and
$$R_3 \cdot \frac{L}{2} = R_2 \cdot \frac{L}{2} + M.$$
Thus, it is not possible to get R_3, the reaction force at the roller.

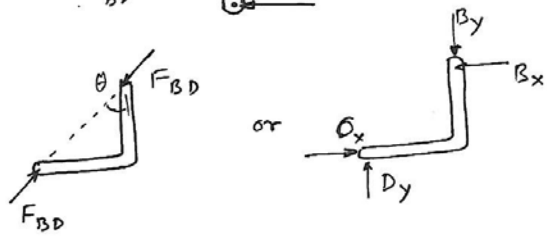
Problem # 3 :

a) The FBD's are shown below

Member ABC



Member BD



b) Given $|F_{BD}| = 5$ kN.

Considering FBD of member ABC and using the following equilibrium equation

$$M_C = 0 \Rightarrow \quad F_{BD} \cos\theta \times 0.2 - P \times 0.12 = 0 \quad \cdots (1)$$

We get $\quad P = \dfrac{F_{BD} \times 0.2 \times \cos\theta}{0.12} = 1.2804\, F_{BD}$.

Since $P > 0$ (for the direction is given), we take $F_{BD} = 5$ kN and accordingly $\boxed{P = 6.402 \text{ kN}}$

c) Consider the FBD of member ABC. The following equations of equilibrium are written

$$C_x + F_{BD} \sin\theta = P \qquad (\Sigma F_x = 0) \qquad \cdots (2)$$

$$C_y + F_{BD} \cos\theta = 0 \qquad (\Sigma F_y = 0). \qquad \cdots (3)$$

From equations (2) and (3) we get

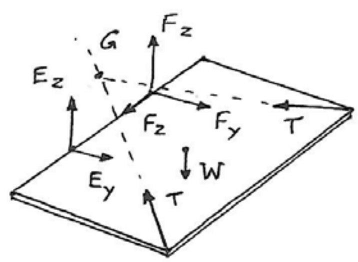
$C_x = 3.2$ kN

and $C_y = -3.84$ kN

The reaction force at C has magnitude $\sqrt{C_x^2 + C_y^2} = 5$ kN and directed along $-50.2°$ from positive x-axis. ($50.2°$ 5kN)

Problem # 4 :

a) The FBD of the panel is drawn below



b) Considering moment balance equation about the axis AD, we get

$$M_x = 0 \Rightarrow T\left(\vec{AB} \times \hat{n}_{BG}\right)\cdot\hat{i} + T\left(\vec{DC} \times \hat{n}_{CG}\right)\cdot\hat{i}$$

$$- \frac{0.1}{2} \times 400 = 0 \qquad \text{--(1)}$$

where T is in Newton, $\bar{AB} = 0.1\hat{j}$ (m), $\bar{DC} = 0.1\hat{j}$ (m),

$$\hat{n}_{BG} = \frac{\bar{r}_G - \bar{r}_B}{|\bar{r}_G - \bar{r}_B|} = \frac{-0.05\hat{i} - 0.1\hat{j} + 0.1\hat{k}}{\sqrt{(0.05)^2 + (0.1)^2 + (0.1)^2}},$$

and $$\hat{n}_{CG} = \frac{\bar{r}_G - \bar{r}_C}{|\bar{r}_G - \bar{r}_C|} = \frac{0.15\hat{i} - 0.1\hat{j} + 0.1\hat{k}}{\sqrt{(0.15)^2 + (0.1)^2 + 0.1)^2}}.$$

From equation (1) we get

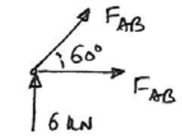
$$T\left\{\frac{0.1 \times 0.1}{\sqrt{(0.05)^2 + (0.1)^2 + (0.1)^2}} + \frac{0.1 \times 0.1}{\sqrt{(0.15)^2 + (0.1)^2 + (0.1)^2}}\right\}$$

$$= 20$$

ie $$\boxed{T = 173.65\ (N)}$$

Problem #5

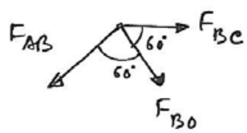
a) Consider joint - A whose FBD is shown below. From equilibrium equation ($\sum F_y = 0$) we get



$$F_{AB} \sin 60° + 6000 = 0 \qquad \cdots (1)$$

ie $$F_{AB} = - 6000 \times \frac{2}{\sqrt{3}} \text{ (N)}$$

We further note that member BP is a zero-force member. Next Consider joint B. The FBD is shown below. The following equations of equilibrium are obtained



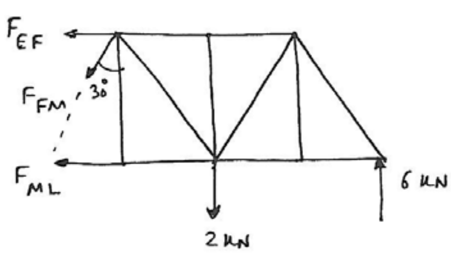
$$F_{AB} \cos 30° + F_{BO} \cos 30° = 0 \qquad \cdots (2)$$

$$F_{AB} \sin 30° = F_{BC} + F_{BO} \sin 30° \qquad \cdots (3)$$

We thus get $F_{BO} = - F_{AB} = \frac{12000}{\sqrt{3}} \text{(N)} = 6.928 \text{ kN}$,

The force in member BO is $\boxed{6.928 \text{ kN (T)}}$

b) Taking the section across members EF, FM and ML we get the following FBD.



The vertical force balance equation ($\sum F_y = 0$) gives

$$F_{FM} \cos 30° + 2000 = 6000$$

ie $$F_{FM} = \frac{4000}{\cos 30°} = \frac{8000}{\sqrt{3}} \text{ (N)} = 4.62 \text{ kN}$$

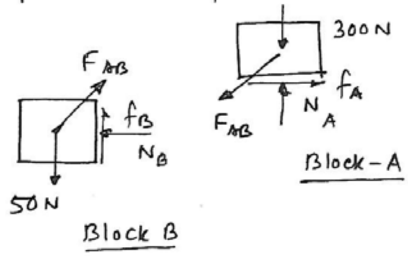
The force in member FM is $\boxed{4.62 \text{ kN (T)}}$

c) The zero force members are

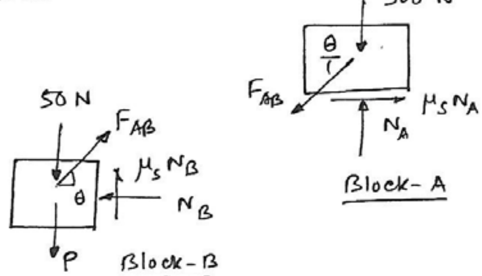
$\boxed{BP, CO, DN, EM, FL, GK, HJ}$

Problem #6 :

a) FBD s of the blocks for $P = 0$ are shown below (knowing that the blocks are in equilibrium)



b) For impending downward motion of block B the FBD s are shown below



c) The following equations of equilibrium are obtained from the free body diagrams shown in (b)

Block - A :

$$F_{AB} \cos\theta = \mu_s N_A \quad \left(\Sigma F_x = 0\right) \qquad \text{--(1)}$$

$$F_{AB} \sin\theta + 300 = N_A \quad \left(\Sigma F_y = 0\right) \qquad \text{--(2)}$$

Block - B :

$$F_{AB} \cos\theta = N_B \quad \left(\Sigma F_x = 0\right) \qquad \text{--(3)}$$

$$F_{AB} \sin\theta + \mu_s N_B = P + 50 \quad \left(\Sigma F_y = 0\right) \qquad \text{--(4)}$$

From equations (1) -(4) we get

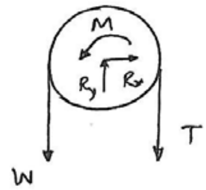
$$P = F_{AB}\left(\sin\theta + \mu_s \cos\theta\right) - 50 = 300\,\mu_s \frac{\left(\sin\theta + \mu_s \cos\theta\right)}{\left(\cos\theta - \mu_s \sin\theta\right)} - 50$$

$$= 17.06 \text{ N}.$$

The required downward force is $\boxed{17.06 \text{ N}}$.

Problem # 7

a) The FBD of the pulley + belt is shown below



, $M = K_T\theta$, where θ is measured positive in clockwise direction

b) The belt tension required for raising the weight up

$$T_1 = W e^{\mu_s \pi} = 1.8745 \text{kN} \quad \cdots (1)$$

and the belt tension required to prevent the weight from falling down

$$T_2 = W e^{-\mu_s \pi} = 533.49 \text{ N} \quad \cdots (2)$$

The range of value of T for which the belt does not slip over the pulley is

$$\boxed{533.49\text{N} < T < 1874.5\text{N}}$$

c) When the weight is moving upward with uniform speed

$$T = W e^{\mu_k \pi} = 1.602 \text{ kN}. \quad \cdots (3)$$

From the FBD shown in (a) we get the following equilibrium equation after taking moment about the centre of the pulley

$$M = (T - W)r. \quad \cdots (4)$$

Since $M = K_T\theta$, we get

$$\theta = \frac{(T-W)r}{K_T} = \frac{W(e^{\mu_k \pi} - 1)r}{K_T} = 0.301 \text{ rad}.$$

The pulley rotates by $\boxed{17.2454°}$ in the clockwise direction.

— x —