

# Hints of Problem Set - 8

SPRING 2020

## MATHEMATICS-II (MA10002)(Integral Calculus)

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1. (a) Ans.  $8xye^{2x^2}$  [Use  $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ ]
- (b) Ans.  $e^{2x}$  [ , , ]
- (c) Ans.  $-8 + \frac{2u^2}{v^2}$  [Use  $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$ ]
- (d) Ans.  $\rho^2 \sin \phi$  [Use  $J = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{vmatrix}$ ]
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2. (a) Ans. -6 [hints.  $u=y-x$ ,  $v=y+\frac{x}{3}$ ]
- (b) Ans.  $\frac{1}{3} \log \frac{3}{2}$  [hints.  $u=\frac{y^2}{x}$ ,  $v=xy$ ]
- (c) Ans.  $\frac{125}{4}$
- (d) Ans.  $\frac{1}{6}[\sqrt{2} + \log(1 + \sqrt{2})]$
- (e) Ans.  $\frac{4a^3}{9}(3\pi - 4)$  [hints. Change into polar coordinates  $x=r\cos\theta$ ,  $y=r\sin\theta$ ]
- (f) Ans. 24 [hints.  $u=\frac{2x-y}{2}$ ,  $v=\frac{y}{2}$  and  $w=\frac{z}{3}$ ]
- (g) Ans.  $\frac{4\pi}{3}(e - 1)$  [hints. change it to the spherical coordinates]
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3. (a) Ans.  $2a^2(\pi-2)$  [Surface area  $= \iint_D \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2} dx dy$ ]
- (b) Ans. 5176.8958
- (c) Ans.  $a^2$
- (d) Ans. 15 [hints. take  $u=2x-3y+2$ ,  $v=x+2y$ ,  $w=x-z$ , Calculate jacobian and use the formula  $V = \iiint \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du dv dw$ ]
- (e) Ans.  $\frac{7\pi}{6}$  [hints. Surface area  $= \iint_D \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2} dx dy$ ]
- (f) Ans.  $\frac{2\pi}{3}$  [hints Surface area  $= \iint_D \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2} dx dy$ ]
- (g) Ans.  $\frac{3}{2}$  [hints. Volume  $= \iiint dV$ ]
- (h) Ans.  $\frac{\pi}{8}(e^4 - 5)$  [hints. put  $x=r\cos \theta$ ,  $y=r\sin\theta$ ,  $z=z$ , compute Jacobian and corresponding range in the region D]
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