INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date of Examination: 26.11.2014(FN) — Time: 3 hrs
End Semester Examination (Autumn) — Full Marks:120
Subject No. ME10001 — Subject Name: MECHANICS
No. of students: 675

Instructions: ***Answer all SIX questions of equal marks. Any data, if not furnished, may be assumed with justification.***

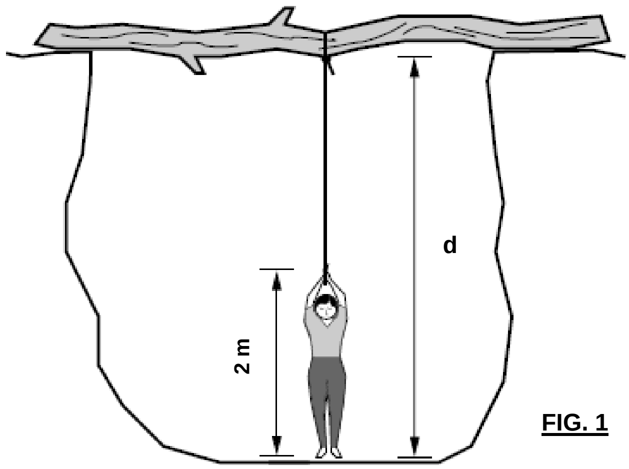
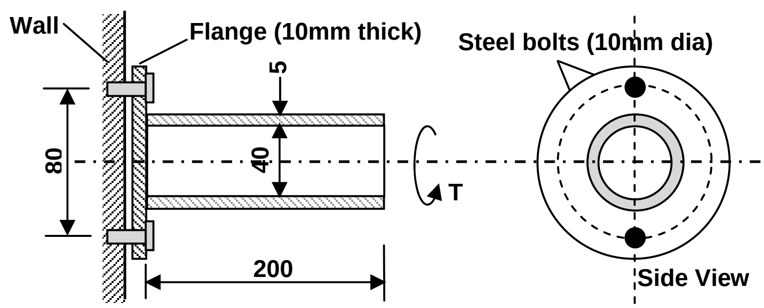


FIG. 1

1. An explorer lowers herself into a pit using the arrangement shown in Fig. 1, comprising a rigid tree trunk and a massless linearly elastic rope of undeformed hanging length of 5 m and undeformed diameter of 20 mm. When her hands reach the end of the rope, as shown in Fig. 1, her toes just reach the bottom of the pit (without touching it). If the weight of the explorer is 600 N and her stretched length is 2 m (as shown in the figure), estimate the depth, d of the pit (see figure), if the Young's modulus of the rope is known to be 10 MPa.

2. An aluminum tube of 40 mm inner diameter and 50 mm outer diameter is rigidly fixed to an aluminum circular flange. The assembly is fixed to a wall by two steel bolts of 10mm diameter as shown in Fig.2. The allowable shear stress and bearing stress of steel are 45MPa and 90MPa respectively. For aluminum, the allowable shear stress and bearing stress are 30MPa and 60MPa, respectively. (a) Determine the maximum torsion T the system can withstand and (b) the angle of twist of the tube for the T computed in part (a). $G_{Aluminum}$ =28 GPa.



All dimensions in mm

FIG. 2

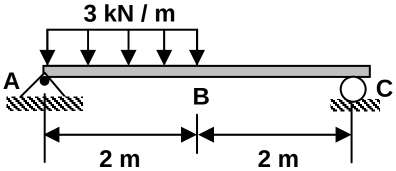


FIG. 3

3. Draw the shear force diagram and the bending moment diagram for the beam subject to the loading shown in Fig.3.

   The diagrams must be drawn below the beam diagram on a fresh page and the sign convention followed for shear force and bending moment must be indicated. The support reactions at A and C are to be calculated and all other relevant calculations must be shown.

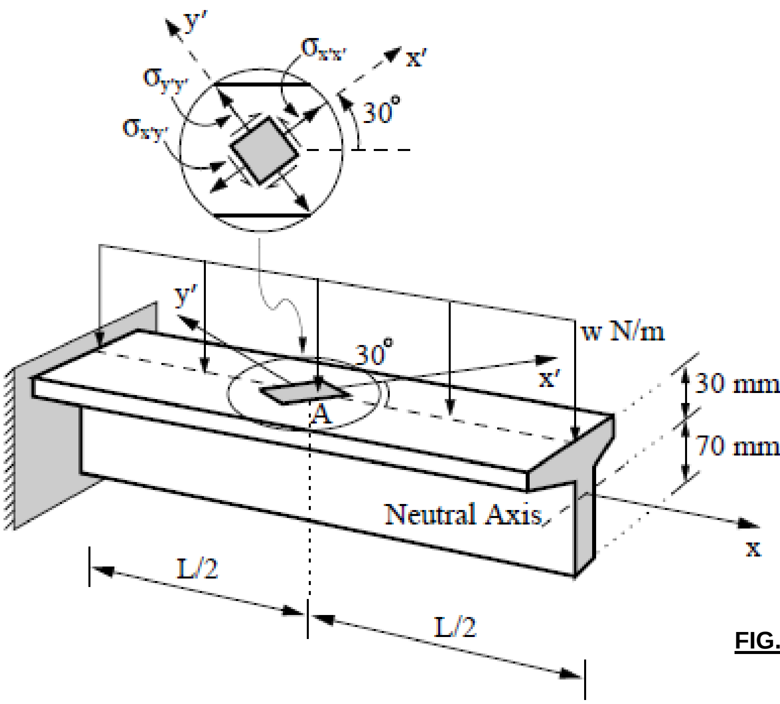


FIG. 4

4. A cantilever beam of length L = 4 m, has E=200 GPa, I = 1.5 × 10 ^{6} mm^{4}, Poisson ration ν = 1/3 and linear weight density w N/m, as shown in Fig. 4. At the top of the beam surface at A (middle of the span), a strain recorder is attached at 30^{0} with the x-axis which measures the normal strain $\boldsymbol{\varepsilon}_{x'x'}$ at the top surface along the x′-axis (inset, Fig.4). (a) Determine the expression of the beam normal stress $\boldsymbol{\sigma}_{xx}$ at the top-most fiber at A and draw the corresponding stress element (note that there is no shear stress on this element). (b) Express the stresses on the element rotated by 30^{0}, as shown in the inset in Fig. 4. (c) Taking $\boldsymbol{\varepsilon}_{x'x'} = 2 \times 10^{-4}$ mm/mm, calculate the linear weight density w of the beam.

5. A cast iron beam of rectangular cross section and carrying uniform load of 5kN/m is shown in Fig.5. Draw the bending moment diagram. If the allowable compressive and tensile bending stresses for the beam are 50MPa and 39.5MPa respectively, determine the minimum value of h of the beam cross section to withstand the given stresses.

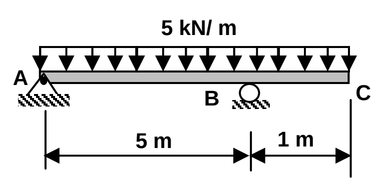
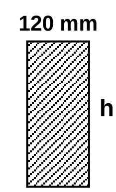


FIG. 5



Beam section

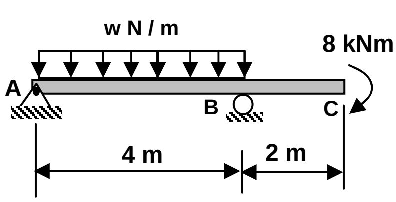


FIG. 6

6. An overhung beam of constant flexural rigidity EI, carrying uniform load of w N/m is shown in Fig.6. A moment of 8 KNm is applied at the beam end C, so that the displacement at C is zero. Calculate the magnitude of load intensity w.

You need to calculate the support reactions, state the boundary conditions and determine the equation for the beam elastic line.

SOLUTIONS FOR END SEM EXAM: MECHANICS -2014 AUTUMN

1. Elongation of the Rope $= \dfrac{600 \times 5 \times 10^3}{\frac{\pi}{4} \times 20^2 \times 10}$ mm

$= \underline{955 \text{ mm}}$

$\therefore$ Pit depth $= 5000 + 2000 + 955$ mm

$= 7955 \text{ mm} = \underline{7.955 \text{ m}}$

2. (a) Shear of the Bolts $= 2 \times \left(\frac{\pi}{4} \times 10^2\right) \times 45$ N

$= 7.07 \times 10^3$ N

Torsional resistance $= 7.07 \times 10^3 \times 40$ N-mm

$= \underline{282.74 \text{ N-m}}$

(b) Bearing of the Bolts $= 2 \times (10 \times 10) \times 60$ N

$= 12 \times 10^3$ N

Torsional resistance $= 12 \times 10^3 \times 40$ N-mm

$= \underline{480.0 \text{ N-m}}$

(c) $\tau = \dfrac{Tr}{I_p}$, on the Cylinder Surface.

or $30 = \dfrac{T \times 25}{\frac{\pi}{32}(50^4 - 40^4)}$ $\therefore 30 = \dfrac{T \times 25}{36.23 \times 10^4}$

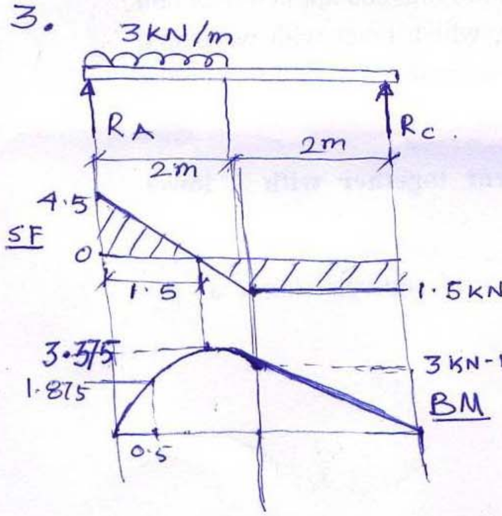
$T = \underline{434.72 \text{ N-m}}$

$\therefore T_{allowable} = \underline{282.74 \text{ N-m}}$

(d) $\phi = \dfrac{282.74 \times 10^3 \times 200}{28 \times 10^3 \times 36.23 \times 10^4} = \underline{5.57 \times 10^{-3} \text{ rad}}$

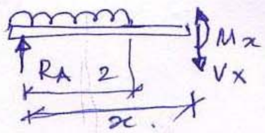
$= \underline{0.32 \text{ degree}}$

3.



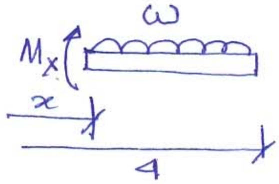
$$R_A \times 4 - 6 \times 3 = 0 \quad \therefore R_A = 4.5\,KN$$
$$\therefore R_C = 1.5\,KN$$

$$R_A - 3x - V_x = 0 \quad \therefore V_x = 4.5 - 3x$$
$$M_x + \frac{3x^2}{2} - R_A x = 0 \quad \therefore M_x = 4.5x - \frac{3x^2}{2}$$



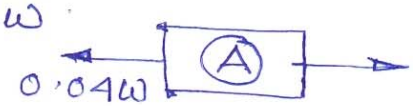
$$R_A - 6 - V_x = 0 \quad \therefore V_x = 4.5 - 6 = -1.5\,KN$$
$$M_x + 6(1 + x - 2) - R_A x = 0$$
$$M_x + 6(x-1) - 4.5x = 0$$
$$M_x = 4.5x - 6(x-1) = -1.5x + 6$$

4.

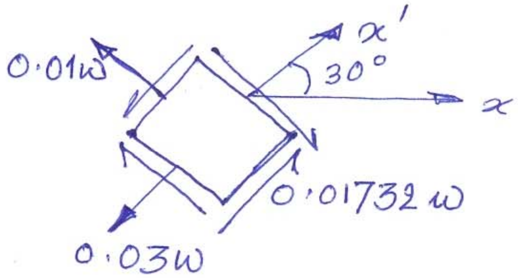


$$M_x + \frac{\omega(4-x)^2}{2} = 0, \quad M_x = -\frac{\omega}{2}(4-x)^2$$
$$\text{at } x = L/2 = 2, \quad M_x = -\frac{\omega}{2} \times 4 = -2\omega$$
$$= -2\omega \times 10^3\ N\text{-}mm$$

(a) $$\sigma_x\Big|_{top} = \frac{(+2\omega \times 10^3) \times 30}{1.5 \times 10^6} = 0.04\omega$$



(b) $$\sigma_{x'x'} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 60^\circ = 0.75\,\sigma_x = \underline{0.03\,\omega}$$
$$\tau_{x'y'} = \frac{\sigma_x}{2}\sin 60^\circ = 0.433\,\sigma_x = 0.01732\,\omega$$
$$\sigma_{y'y'} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 240^\circ = 0.25\,\sigma_x = 0.01\,\omega$$
$$\tau_{y'x'} = \frac{\sigma_x}{2}\sin 240^\circ = -0.433\,\sigma_x = -0.1732\,\omega$$

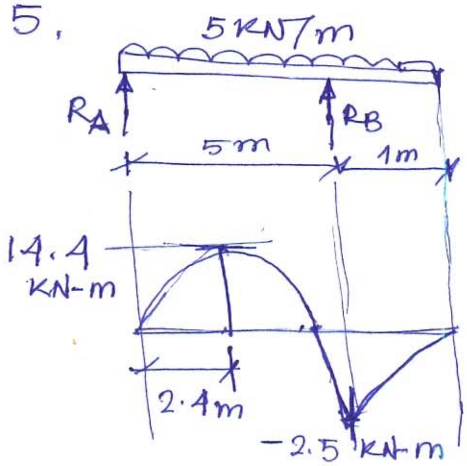


(c) $$\epsilon_{x'x'} = \frac{\sigma_{x'x'}}{E} - \nu\frac{\sigma_{y'y'}}{E} = \frac{0.03\omega}{200 \times 10^3} - \frac{0.01\omega}{3 \times 200 \times 10^3} = 2 \times 10^{-4}$$

or $$\omega(0.03 - 0.01/3) = 2 \times 10^{-4} \times 200 \times 10^3 \quad \therefore \underline{\omega = 1500\,N/m}$$

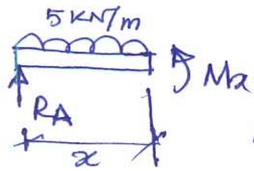
$$\therefore \sigma_{x'x'} = 45\,N, \ \sigma_{y'y'} = 15\,N, \ \tau_{x'y'} = 26\,N, \ \sigma_x = 60\,N$$

5.



$R_A \times 5 - 30 \times 2 = 0 \quad \therefore R_A = 12\text{ KN}$

$R_B = 18\text{ KN}$

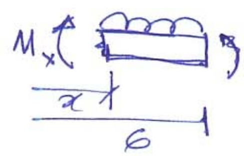


$$M_x + \frac{5x^2}{2} - R_A x$$

$$\therefore M_x = 12x - \frac{5x^2}{2}$$

$$\frac{dM_x}{dx} = 0 = 12 - 5x \quad \therefore x = 2.4\text{ m}$$

$$M_x = 0 = 12x - \frac{5x^2}{2}, \quad x = 0 \text{ or } 4.8\text{m}.$$



$$M_x + 5\frac{(6-x)^2}{2} = 0$$

$$\therefore M_x = -\frac{5(6-x)^2}{2}$$

$$I = \frac{bh^3}{12}, \quad y = \frac{h}{2}$$

$$Z = I/y = \frac{bh^2}{6}$$

$$= \frac{120h^2}{6}$$

$$= 20h^2$$

$$\sigma = \frac{M_{max}}{Z}, \quad Z = \frac{M_{max}}{\sigma}$$

$$(a)\ h^2 = \frac{14.4 \times 10^6}{20 \times \sigma_t} = \frac{14.4 \times 10^6}{20 \times 39.5}$$

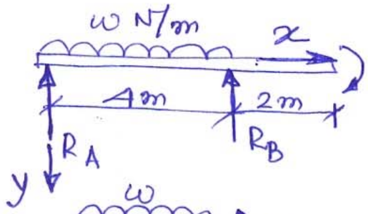
$$\underline{h = 135\text{ mm}}$$

$$(b)\ h^2 = \frac{14.4 \times 10^6}{20 \times \sigma_c} = \frac{14.4 \times 10^6}{20 \times 50}$$

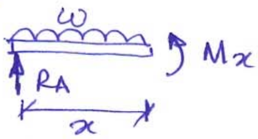
$$\therefore \underline{h = 120\text{ mm}}.$$

$$\therefore \underline{h_{minm.} = 135\text{ mm}}$$

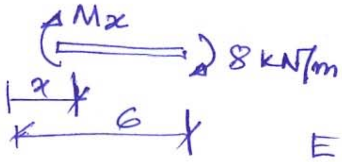
6.



$$R_A \times 4 + 8 - 4w \times 2 = 0$$
$$\therefore R_A = 2w - 2 \;,\; R_B = 2w + 2$$



$$M_x - R_A x + \frac{wx^2}{2} = 0 \;;\; M_x = R_A x - \frac{wx^2}{2}.$$
$$EI\frac{d^2y}{dx^2} = -M_x = \frac{wx^2}{2} - R_A x. \quad \text{—— (1)}$$
$$EI\frac{dy}{dx} = \frac{wx^3}{6} - (2w-2)\frac{x^2}{2} + c_1 \quad \text{—— (2)}$$
$$EI\,y = \frac{wx^4}{24} - (2w-2)\frac{x^3}{6} + c_1x + c_2 \quad \text{—— (3)}$$



$$M_x + 8 = 0 \quad \therefore M_x = -8$$
$$EI\frac{d^2y}{dx^2} = -M_x = 8 \quad \text{—— (4)}$$
$$EI\frac{dy}{dx} = 8x + D_1 \;(5), \quad EI\,y = 8\frac{x^2}{2} + D_1x + D_2 \;\text{—— (6)}$$

Boundary Conditions

$$\left(x=0, y=0 \,;\, x=4, y=0\right)\left(x=6, y=0 \,;\, x=4, y=0 \,;\, \frac{dy}{dx}\Big|_{eq(2)} = \frac{dy}{dx}\Big|_{\substack{at\ x=4 \\ eq(5)}}\right)$$

equns — (1), (2) & (3) equations (4) (5) & (6)

Eqn. (3) $x=0, y=0, \therefore C_2 = 0$

Eqn (3) $x=4, y=0$ $\quad 0 = \frac{w4^4}{24} - (2w-2)\frac{4^3}{6} + c_1 4$

$$c_1 \times 4 = (2w-2)\frac{4^3}{6} - \frac{w4^4}{24} \quad \therefore c_1 = \frac{16}{6}w - \frac{32}{6}$$

Eq (2) & Eq (5) at $x = 4$

$$\frac{w4^3}{6} - (2w-2)\frac{4^2}{2} + \left(\frac{16}{6}w - \frac{32}{6}\right) = 8\times4 + D_1$$
$$\therefore D_1 = w\frac{4^3}{6} - 2w\times\frac{4^2}{2} + \frac{16}{6}w + \frac{2\times4^2}{2} - \frac{32}{6} - 32 = \left(-\frac{16w}{6} - \frac{128}{6}\right)$$

Eq (6) at $x=6, y=0$

$$\therefore 0 = 8\times\frac{6^2}{2} + D_1 6 + D_2$$
$$\therefore D_2 = -4\times6^2 + 6\left(\frac{16}{6}w + \frac{128}{6}\right) = 16w - 16$$

Eq (6) at $x=4, y=0$

$$0 = 8\times\frac{4^2}{2} - \left(\frac{16w}{6} + \frac{128}{6}\right)\times4 + 16w - 16$$
$$\frac{64w}{6} - 16w = 4^3 - \frac{128\times4}{6} - 16 \quad \therefore \underline{w = 7\,kN/m}$$