Chapter-3: Methods of Circuit Analysis

3.0: INTRODUCTION

So far, we have analyzed relatively simple circuits by applying Kirchhoff's laws in combination with Ohm's law. This approach can be used for all circuits but as they become more complicated and involve more elements, this direct method becomes very cumbersome. In this chapter, we will apply these laws to develop two powerful techniques that aid in the analysis of complex circuit structures.

1. Nodal Analysis: Based on a systematic application of Kirchhoff's current law (KCL).

2. Mesh Analysis: Based on a systematic application of Kirchhoff's voltage law (KVL).

Using these two techniques, we can analyze any linear circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage. Cramer's rule is used for solving simultaneous equations which allows to calculate circuit variables as a quotient of determinants.

3.1: NODAL ANALYSIS

Nodal analysis gives a general technique for analyzing circuits using node voltages as the circuit variables. In this section, we shall assume that circuits do not contain voltage source. Circuits that contain voltage sources will be discussed in the next section. In nodal analysis, our interest is to find the node voltages. Consider a circuit with n nodes without voltage sources, the nodal analysis of a circuit involves the following three steps:

1. Select one node as the reference node. Assign voltages to the remaining $n-1$ nodes, i.e., $v_1, v_2, \ldots\ldots, v_{n-1}$. The voltages are referenced with respect to the reference node.

2. Apply KCL to each of the $n-1$ nonreference node. Express the branch currents in terms of node voltages by using Ohm's law.

3. Solve the simultaneous equations to obtain the unknown node voltages.

We shall now explain these three steps systematically.

The first step in the nodal analysis is to select a node as the reference or datum node. The reference node is assumed to have zero potential and is commonly known as ground. A reference node is indicated by any of the three symbols as shown in Fig.3.1. The type of ground shown in Fig. 3.1(a) is known as chassis ground and is used in devices where the chassis, case or enclosure acts ~~acts~~ as a reference point for

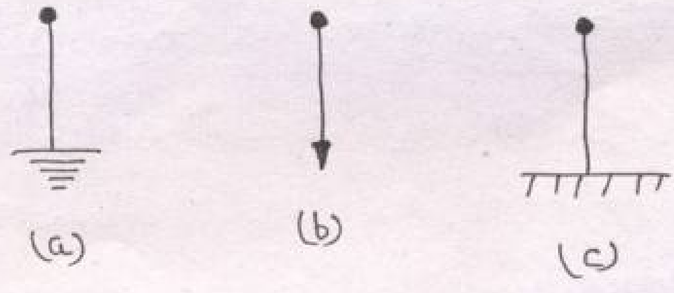

Fig. 3.1: Different symbols for indicating a reference node

(a) chassis ground (b) common ground
(c) ground

all circuits. When the potential of the earth is used as reference, we use the earth ground as shown in Fig. 3.1(b) or 3.1(c). In this book, we shall always use the symbol of Fig. 3.1(a).

After selecting a reference node, assign voltage designations to nonreference nodes. For example, consider the circuit of Fig. 3.2(a). Node o is the

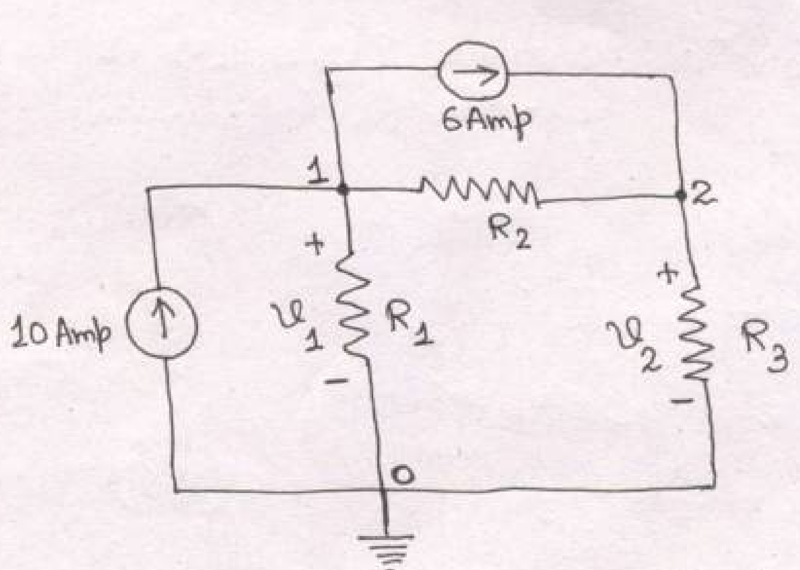

Fig. 3.2(a): Circuit with two independent current sources

reference node ($v_0 = 0$). Nodes 1 and 2 are assigned voltages v_1 and v_2, respectively. A node voltage is defined as the voltage rise from the reference node to a nonreference node.

The Second step in the nodal analysis is to apply KCL to each nonreference node in the circuit. For further explanation, circuit of Fig. 3.2(a) is redrawn in Fig 3.2(b) to avoid putting too much information on the same circuit.

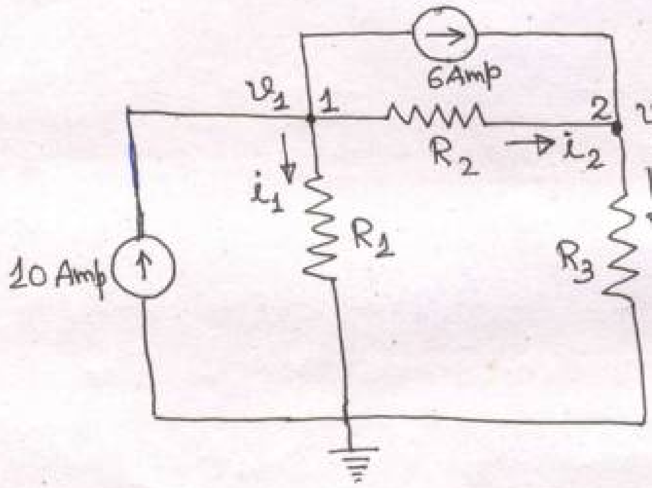

Fig. 3.2(b): Circuit of Fig. 3.2(a) is redrawn with useful information.

Applying KCL at node 1, we have,

$$10 = 6 + i_1 + i_2 \quad \text{-----} \quad (3.1)$$

At node 2

$$6 + i_2 = i_3 \quad \text{-----} \quad (3.2)$$

To obtain the unknown currents i_1, i_2 and i_3 in terms of node voltages, we apply Ohm's law.

From Fig. 3.2(b), we have,

$$i_1 = \frac{v_1 - 0}{R_1} = \frac{v_1}{R_1} \quad \text{....} \quad (3.3)$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{......} \quad (3.4)$$

$$i_3 = \frac{v_2 - 0}{R_3} = \frac{v_2}{R_3} \quad \text{....} \quad (3.5)$$

Substituting eqns. (3.3) and (3.4) in eqns. (3.1) results, in

$$10 = 6 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$\therefore \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{1}{R_2} v_2 = 4 \quad \text{....} \quad (3.6)$$

Substituting eqns. (3.4) and (3.5) in eqns. (3.2), we get,

$$6 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

$$\therefore -\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2 = 6 \quad \text{...} \quad (3.7)$$

The third step in the nodal analysis is to solve for the node voltages. If we apply KCL to $n-1$ nonreference node, we will obtain $n-1$ simultaneous equations such as eqns. (3.6) and (3.7). For the circuit of Fig. 3.2, we can easily obtain v_1 and v_2 by solving eqns. (3.6) and (3.7) using any standard method, such as the elimination method, substitution method, matrix inversion or cramer's rule.

To use the matrix inversion or Cramer's rule, we must put all the equations ~~eqns (3.6) and (3.7)~~ in matrix form. For example, eqns. (3.6) and (3.7) can be written in matrix form as

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \quad \cdots (3.8)$$

Eqn. (3.8) can be solved to get v_1 and v_2.

3.2: SIMULTANEOUS EQUATIONS AND CRAMER'S RULE

Consider a set of simultaneous equations having the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \qquad\quad &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned} \qquad (3.9)$$

Eqn. (3.9) can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \cdots (3.10)$$

Eqn. (3.10) can be put in a compact form as

$$AX = B \quad \cdots \quad (3.11)$$

Where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

A is square $(n \times n)$ matrix while X and B are column $(n \times 1)$ matrices.

There are several methods for solving eqn. (3.10). These include back substitution, Gaussian elimination, matrix inversion, Cramer's rule and numerical analysis.

Cramer's Rule

Cramer's rule can be used to solve the simultaneous equations. According to Cramer's rule, the solution of eqn. (3.10) is:

$$x_1 = \frac{\Delta_1}{\Delta}$$

$$x_2 = \frac{\Delta_2}{\Delta} \qquad \cdots (3.12)$$

$$\vdots$$

$$x_n = \frac{\Delta_n}{\Delta}$$

Where the Δ's are the determinants given by

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{vmatrix}, \ldots \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n \end{vmatrix} \qquad \cdots (3.13)$$

Note that Δ is the determinant of matrix A and Δ_K is the determinant of the matrix formed by replacing the K-th column of matrix A by B.

From eqn. (3.12), it is evident that Cramer's rule applies only when $\Delta \neq 0$. When $\Delta = 0$, the set of equations has no unique solution, because the equations are linearly dependent.

The value of the determinant Δ can be obtained by expanding along the first row:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

$$= a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13} + \cdots + (-1)^{1+n} a_{1n}M_{1n} \quad \cdots (3.14)$$

Where the minor M_{ij} is an $(n-1) \times (n-1)$ determinant of the matrix formed by striking out the i-th row and j-th column.

The value of Δ may also be obtained by expanding along the first column:

$$\Delta = a_{11}M_{11} - a_{21}M_{21} + a_{31}M_{31} + \cdots + (-1)^{n+1} a_{n1}M_{n1} \quad \cdots (3.15)$$

For a 2×2 matrix,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \quad \text{--- (3.16)}$$

For a 3×3 matrix

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\therefore \Delta = a_{11}(-1)^2 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^3 \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$+ a_{31}(-1)^4 \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$\therefore \Delta = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23}$$

$$- a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21} \quad \text{--- (3.17)}$$

A simple method of obtaining the determinant of a 3×3 matrix is by repeating the first two rows and multiplying the terms diagonally as follows:

$$\Delta = \begin{array}{c|ccc|c} & a_{11} & a_{12} & a_{13} & \\ & a_{21} & a_{22} & a_{23} & \\ & a_{31} & a_{32} & a_{33} & \\ - & a_{11} & a_{12} & a_{13} & + \\ - & a_{21} & a_{22} & a_{23} & + \\ - & & & & + \end{array}$$

$$\therefore\ \Delta = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21} \quad \cdots (3.18)$$

Ex-3.1: Determine the node voltages in the circuit shown in Fig. 3.3(a).

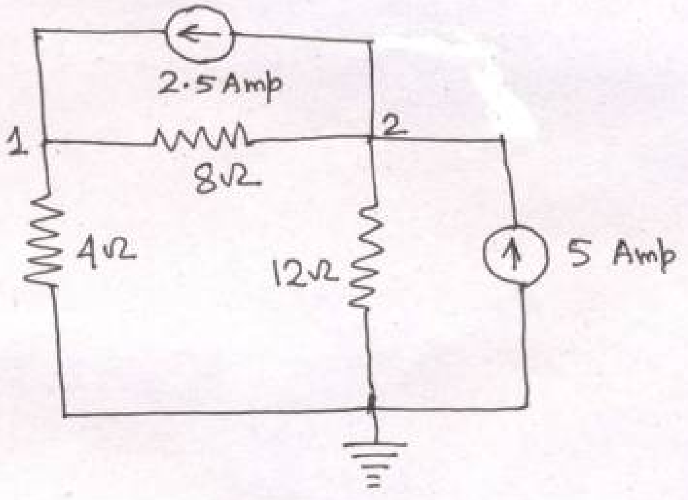

Fig. 3.3(a): Circuit for Ex-3.1

Soln.

Fig. 3.3(b) shows the circuit for analysis of Fig. 3.3(a).

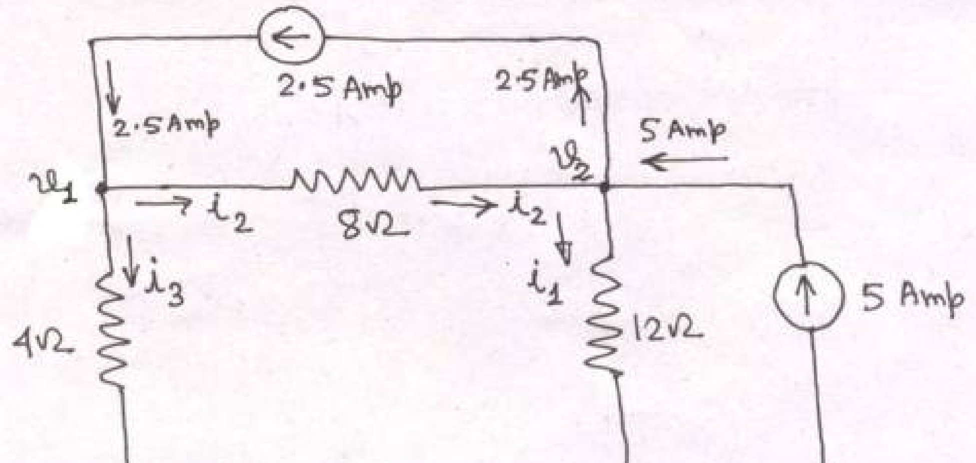

Fig. 3.3(b): Circuit for analysis of original circuit shown in Fig. 3.3(a).

At node 1, applying KCL and Ohm's law gives,

$$2.5 = i_2 + i_3 = \frac{v_1 - v_2}{8} + \frac{v_1 - 0}{4}$$

$$\therefore \quad 3v_1 - v_2 = 20 \quad \cdots (i)$$

At node 2,

$$5 + i_2 = 2.5 + i_1$$

$$\therefore \quad 2.5 + \frac{v_1 - v_2}{8} = \frac{v_2 - 0}{12}$$

$$\therefore \quad -3v_1 + 5v_2 = 60 \quad \cdots (ii)$$

By solving eqns.(i) and (ii), we get

$v_1 = 13.33$ Volt, $v_2 = 20$ Volt.

EX-3.2: Calculate the node voltages in the circuit shown in Fig.3.4(a).

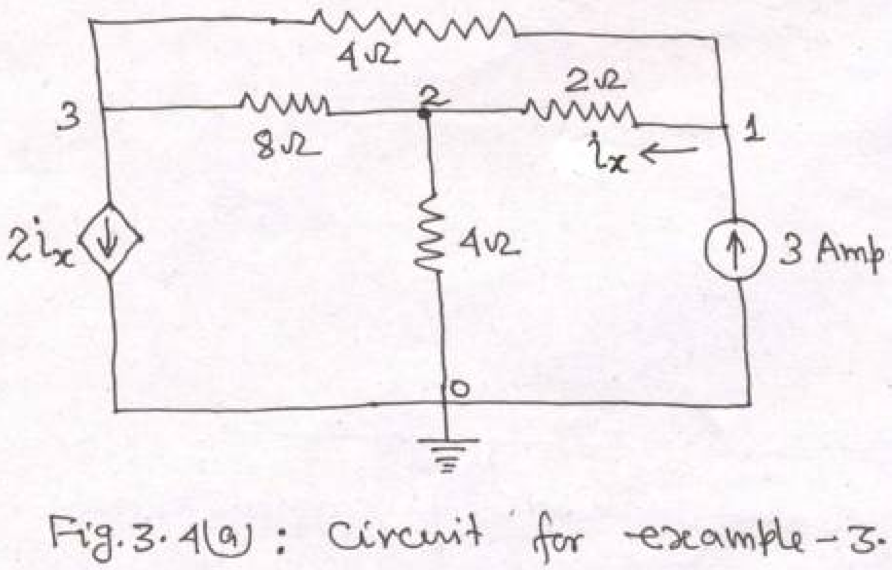

Fig.3.4(a): Circuit for example-3.2

Soln.

Fig. 3.4(b) shows the circuit for analysis of Fig. 3.4(a).

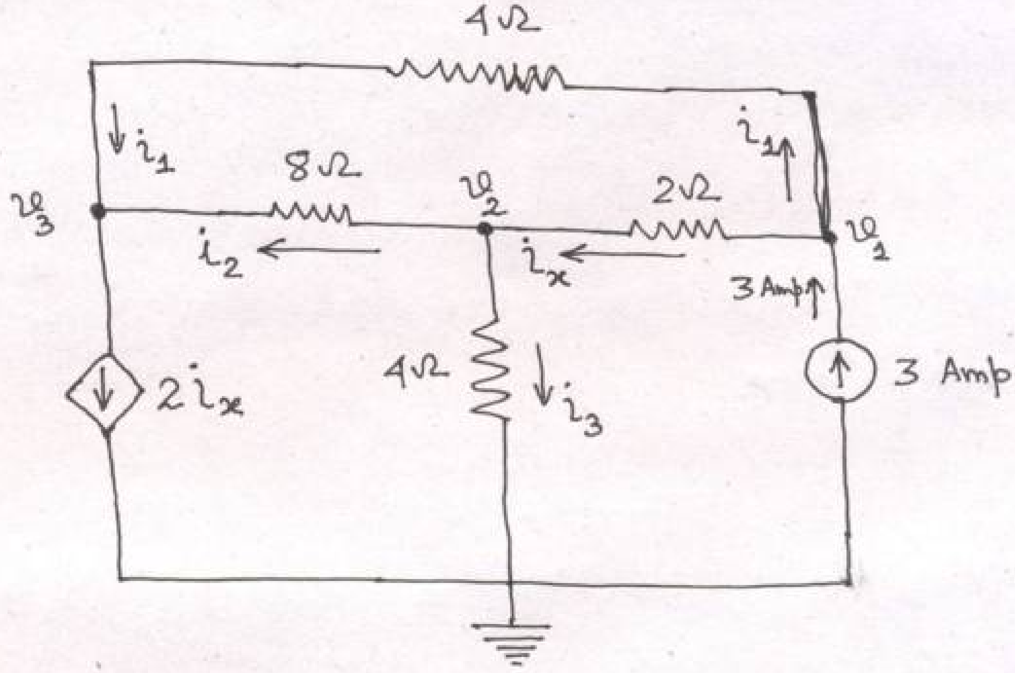

Fig. 3.4(b): Circuit for analysis of Fig. 3.4(a).

At node 1,

$$i_1 + i_x = 3$$

$$\therefore \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$\therefore 3v_1 - 2v_2 - v_3 = 12 \quad \text{--- (i)}$$

At node 2,

$$i_x = i_2 + i_3$$

$$\therefore \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$\therefore -4v_1 + 7v_2 - v_3 = 0 \quad \text{--- (ii)}$$

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At node 3,

$$i_1 + i_2 = 2 i_x$$

$$\therefore \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$\therefore 2v_1 - 3v_2 + v_3 = 0 \quad \cdots \text{(iii)}$$

To use Cramer's rule, we put eqns.(i), (ii) and (iii) in matrix form.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \quad \cdots \text{(iv)}$$

From eqn. (iv), we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{c} \\ \\ - \\ - \\ - \end{array} \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \\ 3 & -2 & -1 \\ -4 & 7 & -1 \end{vmatrix} \begin{array}{c} \\ \\ \\ + \\ + \\ + \end{array}$$

$$\therefore \quad \Delta = 21 - 12 + 4 + 14 - 9 - 8 = 10$$

Similarly

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} = \begin{array}{c} \\ \\ - \\ - \\ - \end{array} \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \\ 12 & -2 & -1 \\ 0 & 7 & -1 \end{vmatrix} \begin{array}{c} \\ \\ \\ + \\ + \\ + \end{array}$$

$\therefore \Delta_1 = 84 + 0 + 0 - 0 - 36 - 0 = 48$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \\ 3 & 12 & -1 \\ -4 & 0 & -1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48$$

$\therefore \Delta_2 = 24$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & 12 \\ -4 & 7 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0$$

$\therefore \Delta_3 = -24$

Thus, we obtain

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ Volt}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ Volt}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ Volt.}$$

3.3: NODAL ANALYSIS WITH VOLTAGE SOURCES

We will now consider how voltage sources affect nodal analysis. For the purpose of explanation consider Fig. 3.5.

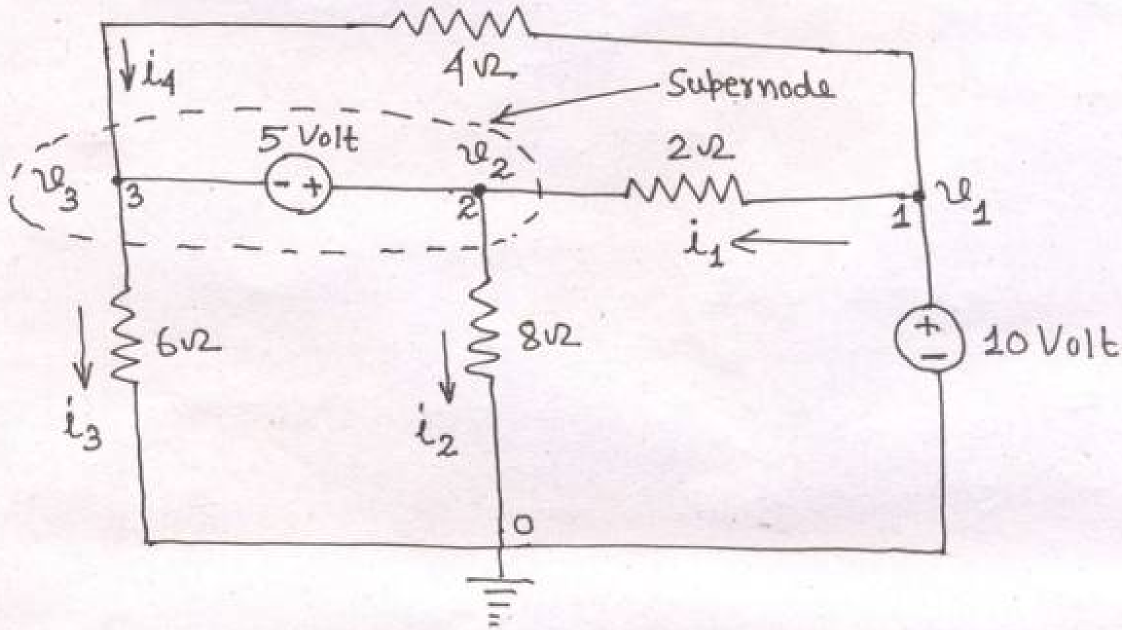


Fig. 3.5: A simple circuit with a supernode

In Fig. 3.5, a voltage source is connected between the reference node (node 0) and a nonreference node (node 1). Therefore, voltage at the nonreference node equal to the voltage of the voltage source. Hence,

$$v_1 = 10 \text{ Volt} \quad \cdots\cdots (3.19)$$

If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a <u>supernode</u> or <u>generalized node.</u> A <u>supernode</u> requires the application of both KCL and KVL to determine the node voltages.

In Fig. 3.5, nodes 2 and 3 form a supernode. An independent voltage source is connected between nonreference nodes 2 and 3.

Now we apply KCL at supernode,

$$i_1 + i_4 = i_2 + i_3 \quad \cdots \quad (3.20)$$

or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \quad \cdots \quad (3.21)$$

To apply Kirchhoff's voltage law to the supernode in Fig. 3.5, the circuit is redrawn in Fig. 3.6.

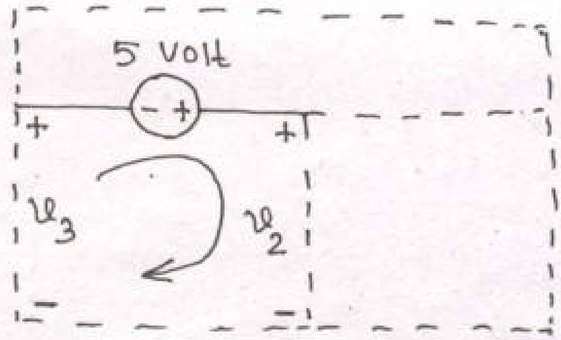


Fig. 3.6: Applying KVL to a supernode in the clockwise direction.

Going around the loops in clockwise direction, we have,

$$-5 + v_2 - v_3 = 0$$

$$\therefore v_2 - v_3 = 5 \quad \cdots\cdots \quad (3.22)$$

From eqns. (3.19), (3.21) and (3.22), we obtain the node voltages. Therefore, a supernode has the following properties;

1. A supernode requires the application of both KCL and KVL.

2. A supernode has no voltage of its own.

3. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.

Ex-3.3: Determine the node voltages of the circuit shown in Fig. 3.6.

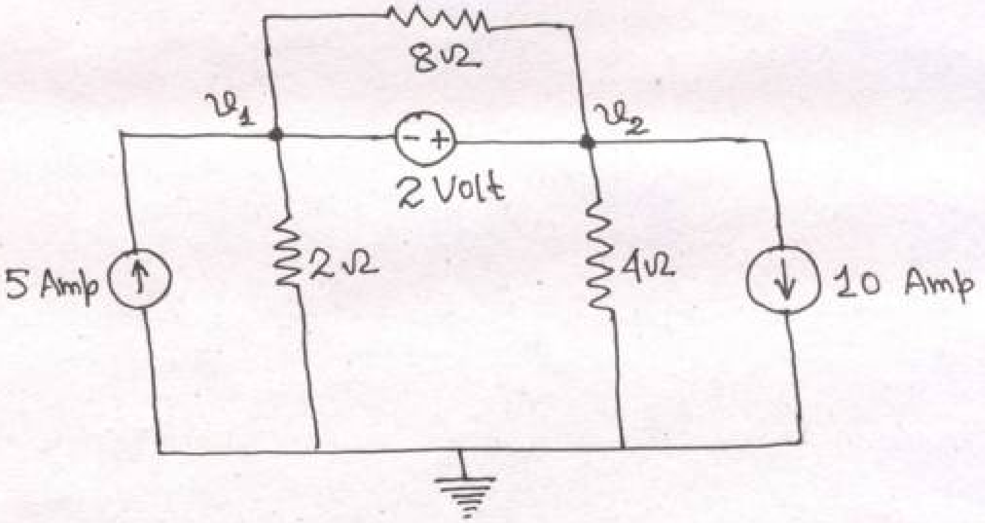
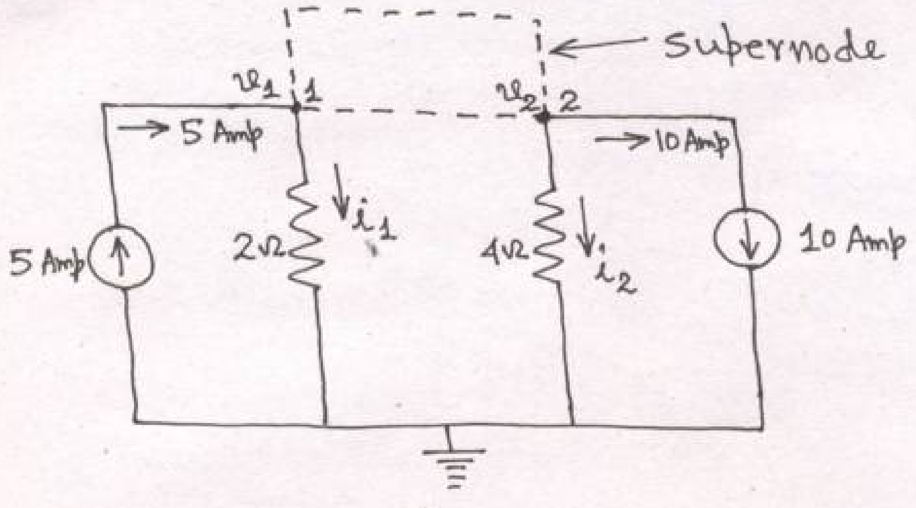


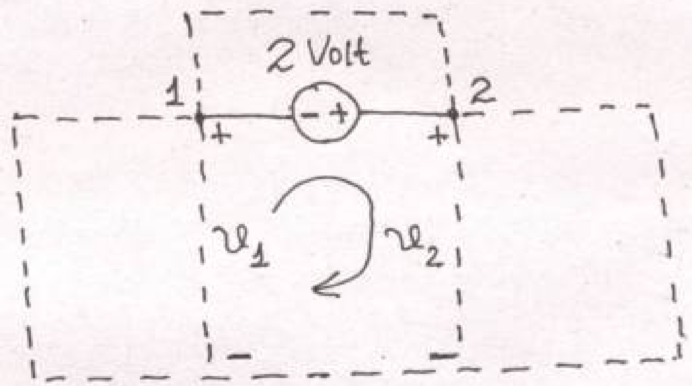
Fig. 3.6: Circuit for Ex-3.3

Soln.

For the purpose of understanding the problem, two circuits are shown in Figs 3.7(a) and (b) for applying KCL to the supernode and KVL to the loop.



(a)



(b)

Fig. 3.7: Applying (a) KCL to the supernode
(b) KVL to the loop in the clockwise direction

Applying KCL to the ~~supernode~~ supernode as shown in Fig. 3.7(a),

$$5 = i_1 + i_2 + 10$$

$$\therefore 5 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 10$$

$$\therefore \quad 2v_1 + v_2 = -20 \quad \text{....} \quad (i)$$

Applying KVL to the loop in the clockwise direction as shown in Fig. 3.7(b).

$\therefore \quad -2 + v_2 - v_1 = 0$

$\therefore \quad v_1 - v_2 = -2 \quad \text{----(ii)}$

Solving eqns.(i) and (ii), we obtain,

$v_1 = -7.33$ Volt, $v_2 = -5.33$ Volt.

Note that $8\,\Omega$ resistor does not make any difference because it is connected across the supernode.

<u>EX-3.4</u>: Determine i_1, i_2 and i_3 of the circuit as shown in Fig.3.8 using node-voltage method.

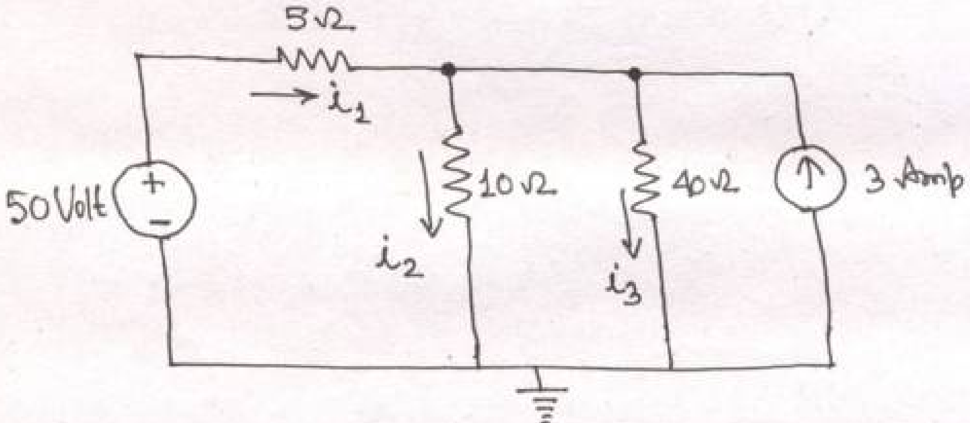


Fig.3.8: Circuit of EX-3.4

<u>Soln.</u>

Circuit of Fig.3.8 has two essential nodes: one nonreference node and one reference node. Fig.3.9 shows these decisions.

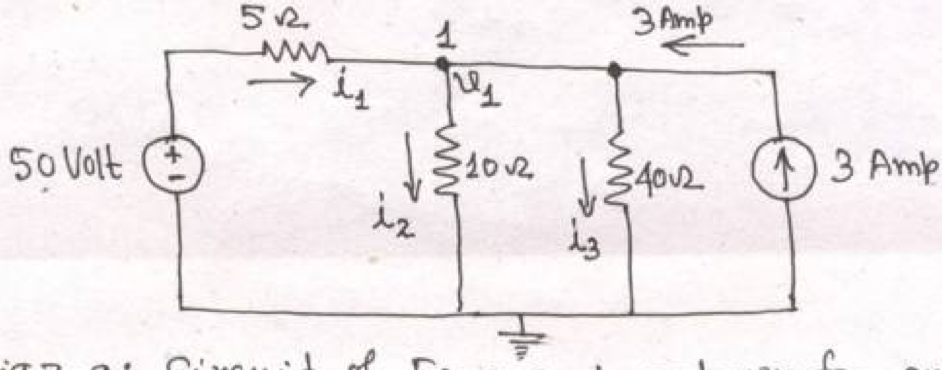


Fig.3.9: Circuit of Fig.3.8 is redrawn for analysis

At node 1,

$$\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0$$

$$\therefore v_1 = 40 \text{ Volt}$$

Hence,

$$i_1 = \frac{50-40}{5} = 2 \text{ Amp.}$$

$$i_2 = \frac{40}{10} = 4 \text{ Amp}$$

$$i_3 = \frac{40}{40} = 1 \text{ Amp.}$$

EX-3.5: Determine the node voltages and powers dissipated in 5 Ω resistor in the circuit shown in Fig. 3.10.

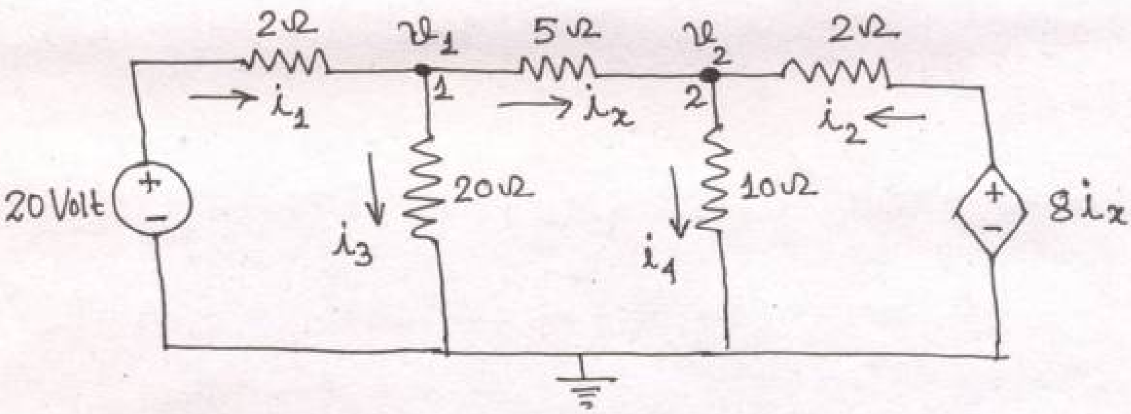


Fig. 3.10: Circuit for EX-3.5

Soln.

At node 1,

$$i_1 = i_3 + i_x$$

$$\therefore \frac{20 - v_1}{2} = \frac{v_1 - 0}{20} + \frac{v_1 - v_2}{5} \quad \text{.... (i)}$$

At node 2,

$$i_1 = i_x + i_2$$

$$\therefore \frac{v_2 - 0}{10} = \frac{v_1 - v_2}{5} + \frac{8i_x - v_2}{2} \quad \text{----(ii)}$$

Also

$$i_x = \frac{v_1 - v_2}{5} \quad \text{----(iii)}$$

From eqns. (ii) and (iii), we get

$$\frac{v_2}{10} = \frac{v_1 - v_2}{5} + 4\left(\frac{v_1 - v_2}{5}\right) - \frac{v_2}{2}$$

$$\therefore \left(\frac{1}{10} + \frac{1}{5} + \frac{4}{5} + \frac{1}{2}\right) v_2 = \left(\frac{1}{5} + \frac{4}{5}\right) v_1$$

$$\therefore \quad v_1 = 1.6\, v_2 \quad \text{----(iv)}$$

Solving eqns. (i) and (iv), we obtain,

$v_1 = 16$ Volt, and $v_2 = 10$ Volt.

$$\therefore \quad i_x = \frac{v_1 - v_2}{5} = \frac{16 - 10}{5} = 1.2 \text{ Amp}$$

$$\therefore \quad P = (1.2)^2 (5) = 7.2 \text{ Watt.}$$

Ex-3.6 : Determine the node voltages of the circuit as shown in Fig. 3.11.

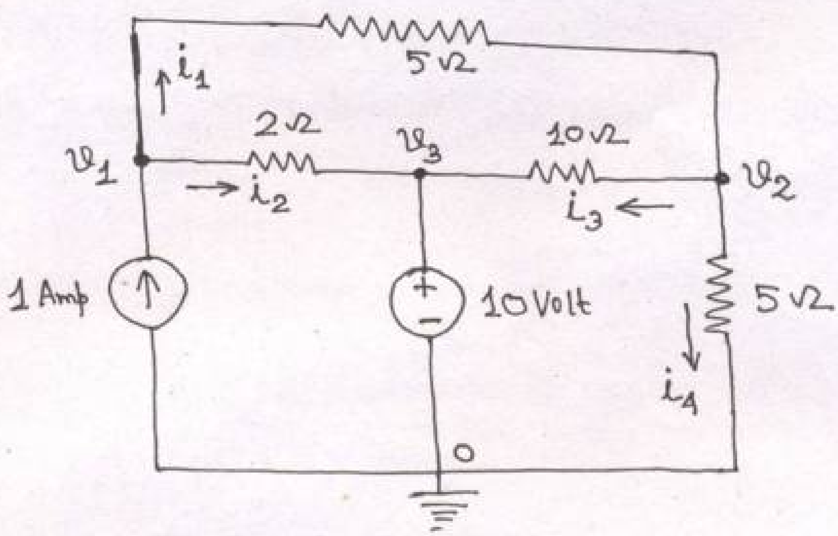


Fig. 3.11: Circuit for EX-3.6

<u>Soln.</u>

$v_3 = 10$ Volt

At node 1,

$i_1 + i_2 = 1$

$\therefore \dfrac{v_1 - v_2}{5} + \dfrac{v_1 - 10}{2} = 1$

$\therefore 0.7v_1 - 0.2v_2 = 6 \quad \cdots \text{(i)}$

At node 2,

$i_1 = i_3 + i_4$

$\therefore \dfrac{v_1 - v_2}{5} = \dfrac{v_2 - 10}{10} + \dfrac{v_2 - 0}{5}$

$\therefore -0.2v_1 + 0.5v_2 = 1 \quad \cdots \text{(ii)}$

Solving eqns. (i) and (ii), we obtain

$v_1 = 10.32$ Volt ; $v_2 = 6.13$ Volt.

Ex-3.7: Determine the node voltages of the circuit as shown in Fig. 3.12.

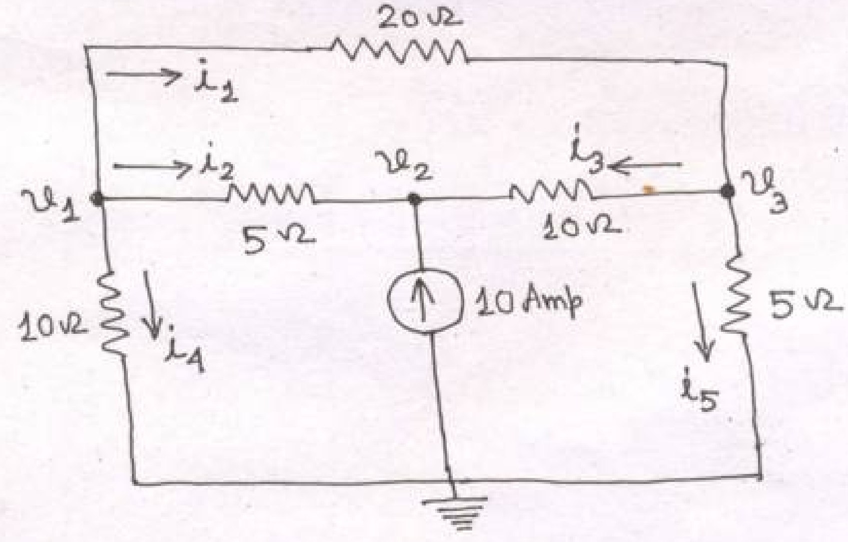


Fig. 3.12: Circuit for Ex-3.7

Soln.

At node 1,

$$i_1 + i_2 + i_4 = 0$$

$$\therefore \frac{v_1 - v_3}{20} + \frac{v_1 - v_2}{5} + \frac{v_1}{10} = 0$$

$$\therefore 0.35 v_1 - 0.2 v_2 - 0.05 v_3 = 0 \quad \text{---(i)}$$

At node 2,

$$i_2 + i_3 + 10 = 0$$

$$\therefore \frac{v_1 - v_2}{5} + \frac{v_3 - v_2}{10} + 10 = 0$$

$$\therefore -0.2 v_1 + 0.3 v_2 - 0.10 v_3 = 10 \quad \text{---(ii)}$$

At node 3,

$$i_1 = i_3 + i_5$$

$$\therefore \frac{v_1 - v_3}{20} = \frac{v_3 - v_2}{10} + \frac{v_3}{5}$$

$$\therefore -0.05 v_1 - 0.10 v_2 + 0.35 v_3 = 0 \quad \text{---(iii)}$$

Solving eqns (i), (ii) and (iii), we obtain

$v_1 = 45.45$ Volt; $v_2 = 72.73$ Volt; $v_3 = 27.27$ Volt.

EX-3.8: Using node voltage method, determine the currents of the circuit as shown in Fig. 3.13.

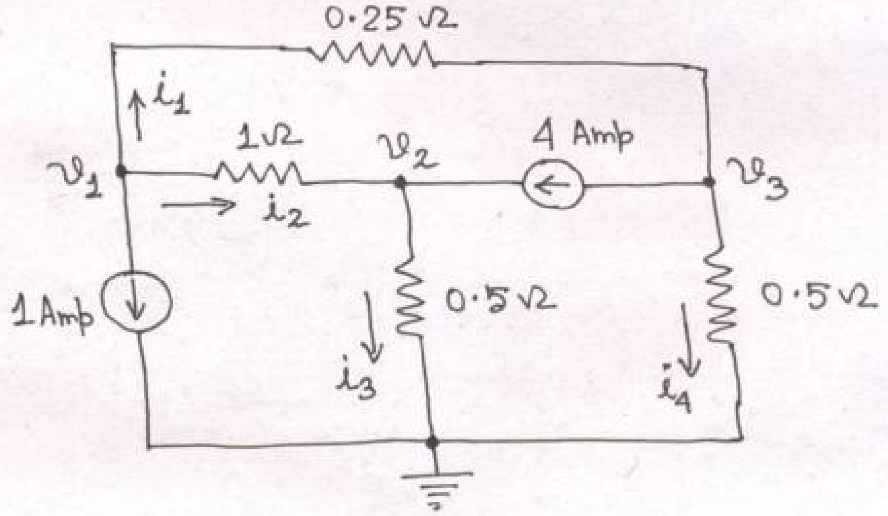


Fig. 3.13: Circuit for EX-3.8

Soln.

At node 1,

$$i_1 + i_2 + 1 = 0$$

$$\therefore \frac{v_1 - v_3}{0.25} + \frac{v_1 - v_2}{1} + 1 = 0$$

$$\therefore 5v_1 - v_2 - 4v_3 = -1 \quad \text{----(i)}$$

At node 2,

$$i_2 + 4 = i_3$$

$$\therefore \frac{v_1 - v_2}{1} + 4 = \frac{v_2}{0.5}$$

$$\therefore v_1 - 3v_2 = -4 \quad \text{----(ii)}$$

At node 3,

$$i_1 = 4 + i_4$$

$$\therefore \frac{v_1 - v_3}{0.25} = 4 + \frac{v_3}{0.5}$$

$$\therefore 4v_1 - 4v_3 - 2v_3 = 4$$

$$\therefore 2v_1 - 3v_3 = 2 \quad \text{----(iii)}$$

Solving eqns.(i), (ii) and (iii), we get

$v_1 = -\frac{7}{6}$ Volt ; $v_2 = \frac{17}{18}$ Volt ; $v_3 = -\frac{13}{9}$ Volt

$$\therefore i_1 = \frac{v_1 - v_3}{0.25} = 4\left(-\frac{7}{6} + \frac{13}{9}\right) = \frac{10}{9} \text{ Amp}$$

$$i_2 = \frac{v_1 - v_2}{1} = \left(-\frac{7}{6} - \frac{17}{18}\right) = -\frac{19}{9} \text{ Amp}$$

$$i_3 = 2v_2 = 2 \times \frac{17}{18} = \frac{17}{9} \text{ Amp}$$

$$i_4 = 2v_3 = 2 \times \left(-\frac{13}{9}\right) = -\frac{26}{9} \text{ Amp.}$$

EX-3.9: Determine the voltages v_1, v_2 and v_3 of the circuit as shown in Fig.3.14.

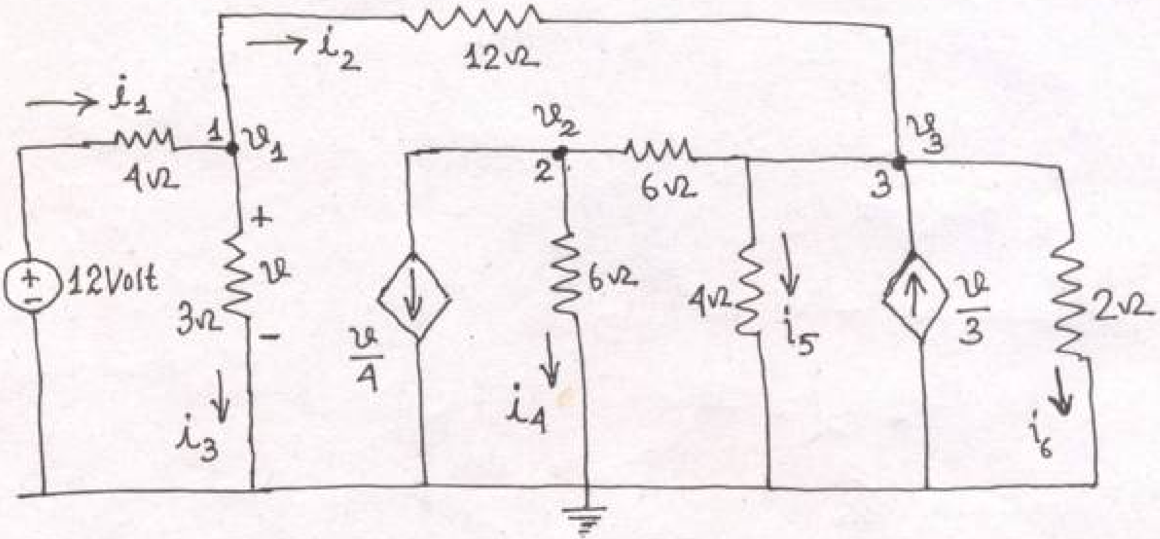


Fig.3.14: Circuit for EX-3.9

Soln.

At node 1,

$$i_1 = i_2 + i_3$$

$$\therefore \frac{12 - v_1}{4} = \frac{v_1 - v_3}{12} + \frac{v_1 - 0}{3}$$

Note that $v_1 = v$

$$\therefore \frac{12 - v}{4} = \frac{v - v_3}{12} + \frac{v}{3}$$

$$\therefore 8v - v_3 = 36 \quad \cdots (i)$$

At node 2,

$$\frac{v_3 - v_2}{6} = \frac{v}{4} + \frac{v_2}{6}$$

$$\therefore 3v + 4v_2 - 2v_3 = 0 \quad \cdots (ii)$$

At node 3,

$$\frac{v_3 - v_2}{6} + \cancel{\frac{v}{3}} + i_5 + i_6 = \frac{v}{3} + i_2$$

$$\therefore \frac{v_3 - v_2}{6} + \frac{v_3}{4} + \frac{v_3}{2} = \frac{v}{3} + \frac{v - v_3}{12}$$

$$\therefore 5v + 2v_2 - 12v_3 = 0 \quad \cdots (iii)$$

Solving eqns. (i), (ii) and (iii), we obtain

$v = v_1 = 4.686$ Volt; $v_2 = -2.769$ Volt; $v_3 = 1.491$ Volt.

Ex-3.10: Determine v_1, v_2 and v_3 of the circuit shown in Fig.3.15 using nodal analysis.

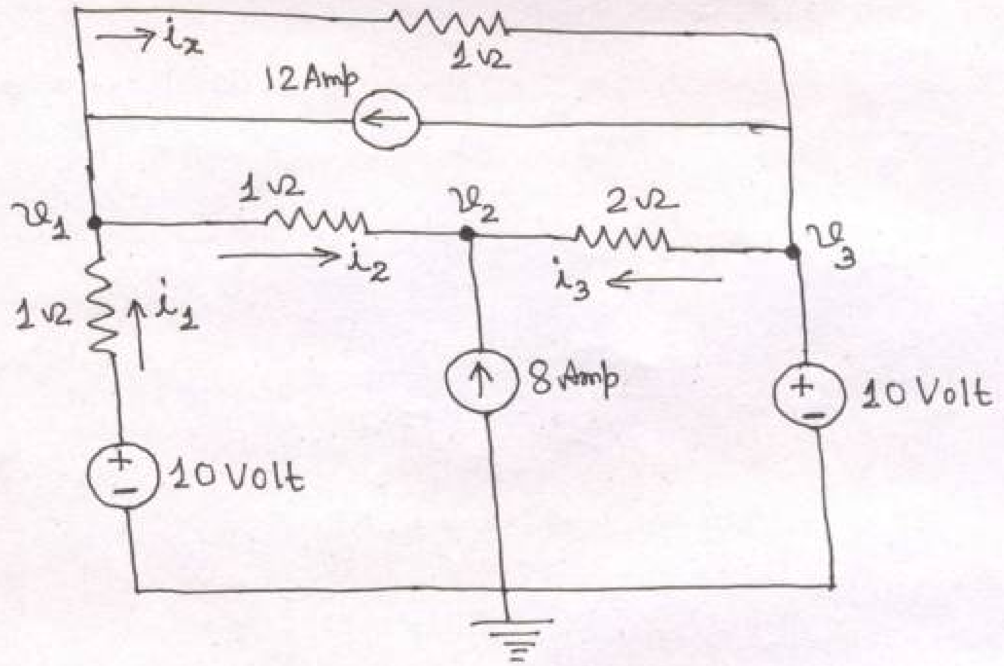


Fig.3.15: Circuit for Ex-3.10

Soln.

From Fig.3.15, $v_3 = 10$ Volt

At node 1,

$$\cancel{\frac{10 - v_1}{1}} + \cancel{\frac{v_1 - v_2}{1}}$$

$$\frac{10 - v_1}{1} + 12 = \frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{1}.$$

$$\therefore\ 10 - v_1 + 12 = v_1 - v_2 + v_1 - 10$$

$$\therefore\ 22 - v_1 = 2v_1 - v_2 - 10$$

$$\therefore\ 3v_1 - v_2 = 32 \quad \text{---(i)}$$

At node 2,

$$\frac{v_1 - v_2}{1} + 8 + \frac{v_3 - v_2}{2} = 0$$

$$\therefore\ v_1 - v_2 + 8 + \frac{10 - v_2}{2} = 0$$

$$\therefore\ 2v_1 - 3v_2 = -26 \quad \text{----(ii)}$$

Solving eqn. (i) and (ii), we obtain,

$v_1 = 17.428$ Volt; $v_2 = 20.285$ Volt.

current $i_x = \dfrac{v_1 - v_3}{1} = (17.428 - 10) = 7.428$ Amp.

$$i_2 = \frac{v_1 - v_2}{1} = (17.428 - 20.285) = -2.857 \text{ Amp.}$$

$$i_3 = \frac{v_3 - v_2}{2} = \frac{10 - 20.285}{2} = -5.142 \text{ Amp.}$$

EX-3.11: Determine v_2 using nodal analysis of the circuit shown in Fig. 3.16.

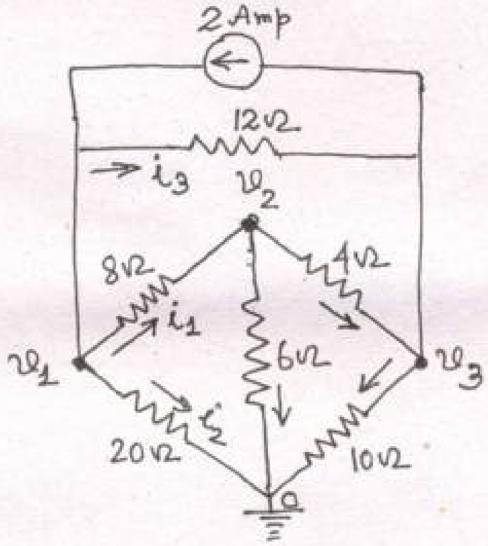


Fig. 3.16: Circuit for EX-3.11

Soln.:

At node 1,

$$\frac{v_1 - v_2}{8} + \frac{v_1 - 0}{20} + \frac{v_1 - v_3}{12} = 2$$

$$\therefore\ 31v_1 - 15v_2 - 10v_3 = 240 \quad \text{---(i)}$$

Similarly at node 2,

$$-3v_1 + 13v_2 - 6v_3 = 0 \quad \text{--(ii)}$$

Similarly at node 3,

$$5v_1 + 15v_2 - 26v_3 = 120 \quad \text{-(iii)}$$

Solving Eqns. (i) (ii) and (iii), we get,

$v_2 = 0.0$; This means bridge circuit is balanced.