

Tutorial Sheet-5
Mathematics-II(MA10002)
(Numerical Analysis)

1. Prove that

$$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right],$$

where Δ is the forward difference operator.

2. Show that

$$\Delta \binom{n}{x+1} = \binom{n}{x},$$

where the forward difference operator Δ operates on n . Hence show that

$$\sum_{n=1}^N \binom{n}{i} = \binom{N+1}{i+1} - \binom{1}{i+1}.$$

3. For any positive integer k prove that

$$\nabla^k y_n = \sum_{i=0}^k (-1)^i \binom{k}{i} y_{n-i},$$

∇ being the backward difference operator.

4. For linear interpolation, in case of equispaced tabular data, show that the error does not exceed $1/8^{\text{th}}$ of the second difference.
5. We have the following tabular values:

x	0	1	2	3
f(x)	1	0	7	28

Find the Interpolating polynomial using

- i. Newton's Forward Interpolation Formula.
 - ii. Newton's Backward Interpolation Formula.
6. From the following table find the number of students who obtained less than 45 marks :

Marks	No. of students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

7. The population of a town in West Bengal was as given below :

Year:	1891	1901	1911	1921	1931
Population (in thousands):	46	66	81	93	101

Estimate the population for the year 1925.

8. Given that $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$, $f(7) = 128$. Find the value of $f(5)$ with the help of Lagrange's interpolation formula.

9. Using Lagrange's interpolation formula to express

$$\frac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}$$

as the sum of partial fraction.

10. By means of Lagrange's formula, prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right].$$

Note: $y_x = y(x)$.

11. Compute the value of following integral

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x) dx$$

using trapezoidal rule with 6 equal subintervals.

12. A car laps a racetrack in 84 s. The speed of the car at each 6-s interval is determined using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table:

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?

Note: use trapezoidal formula

13. How big should the spacing h be so that the computation of

$$\int_0^1 e^x dx$$

by trapezoidal rule will be correct to five decimal places.

14. Let

$$y = ax^2 + bx + c$$

be the equation of the parabola passing through the point $(-h, y_0)$, $(0, y_1)$, (h, y_2) . Find the area underlying the parabola bounded by the x-axis and the two ordinates $x = -h$, $x = h$ using Simpson's $1/3^{\text{rd}}$ rule. What conclusion do you draw from the result.

15. Calculate the value of the integral

$$\int_4^{5.2} \ln x dx$$

using Simpson's $1/3^{\text{rd}}$ rule with 6 equal subdivisions of the interval.