RESONANCE

A circuit is said to be in resonance when the applied voltage V and the resulting current I are in phase.

Thus at resonance, the equivalent complex impedance of the circuit consists of only R.

Since V & I are in phase, the power factor of a resonant circuit is unity.

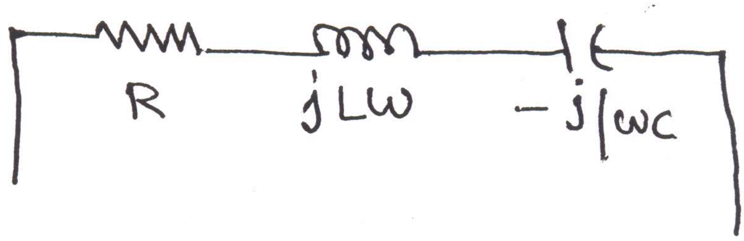
Whenever the natural frequency of oscillation of a system (could be electrical, mechanical or a civil structure or a hydraulic) coincides with the frequency of the driving force (a voltage source in an electric circuit or a wind force in civil structure etc.), the two system resonate with respect to each other and the system has maximum response to a fixed magnitude of driving force. This phenomenon is Known as resonance.

Two types of resonance in the electric circuits.

i) Series Resonance

ii) Parallel Resonance

SERIES RESONANCE



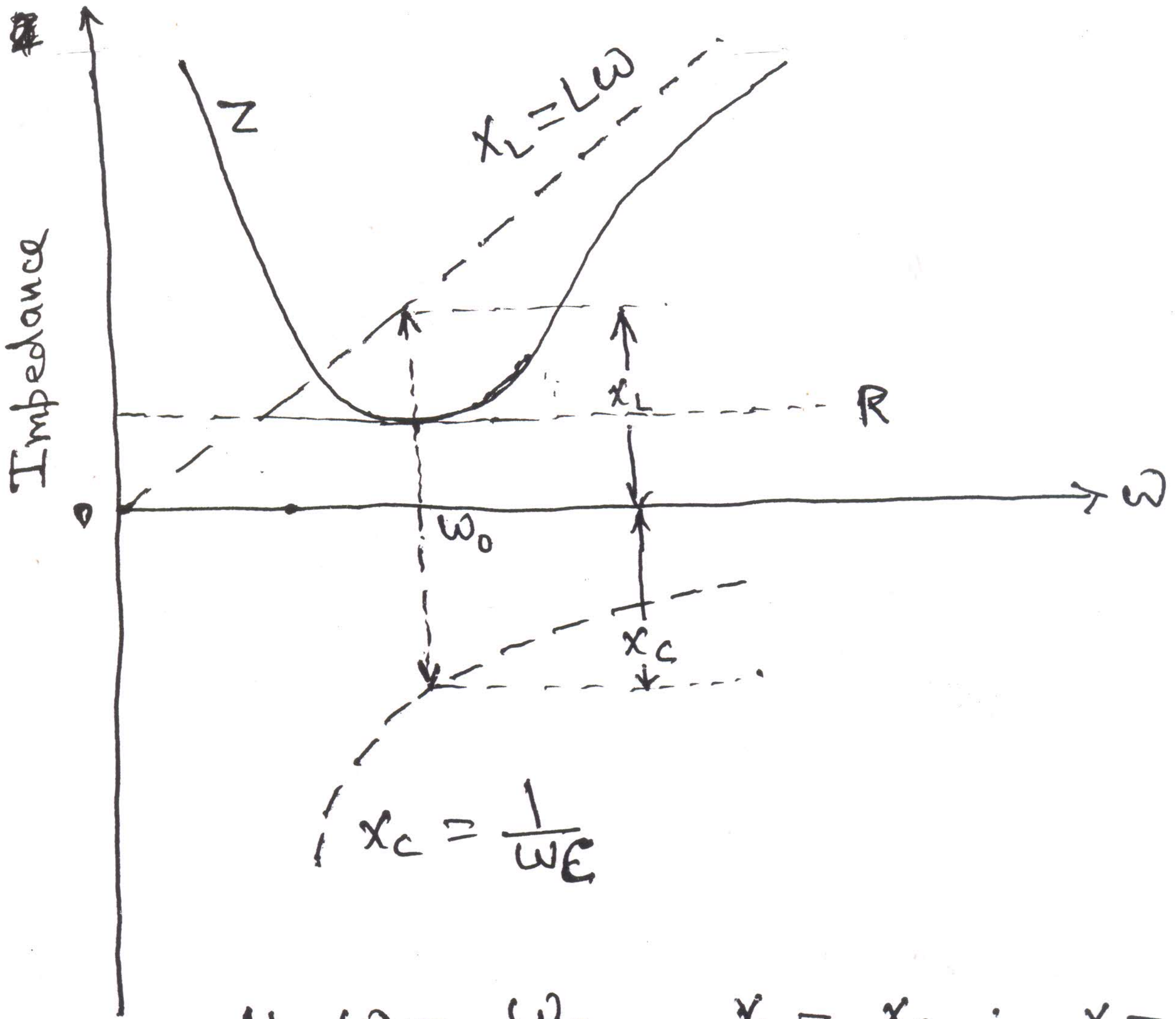
$$Z = R + j\left(L\omega - \frac{1}{\omega C}\right) = (R + jX).$$

The circuit is in resonance when $X = 0$, i.e, when $L\omega = \frac{1}{\omega C}$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}} = \omega_0$$

Then, since $\omega = 2\pi f$, the resonant frequency is given by,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}$$



At $\omega = \omega_0$, $X_L = X_C$; $X = 0$

$\therefore$ Thus, $Z = \sqrt{R^2 + X^2} = R$

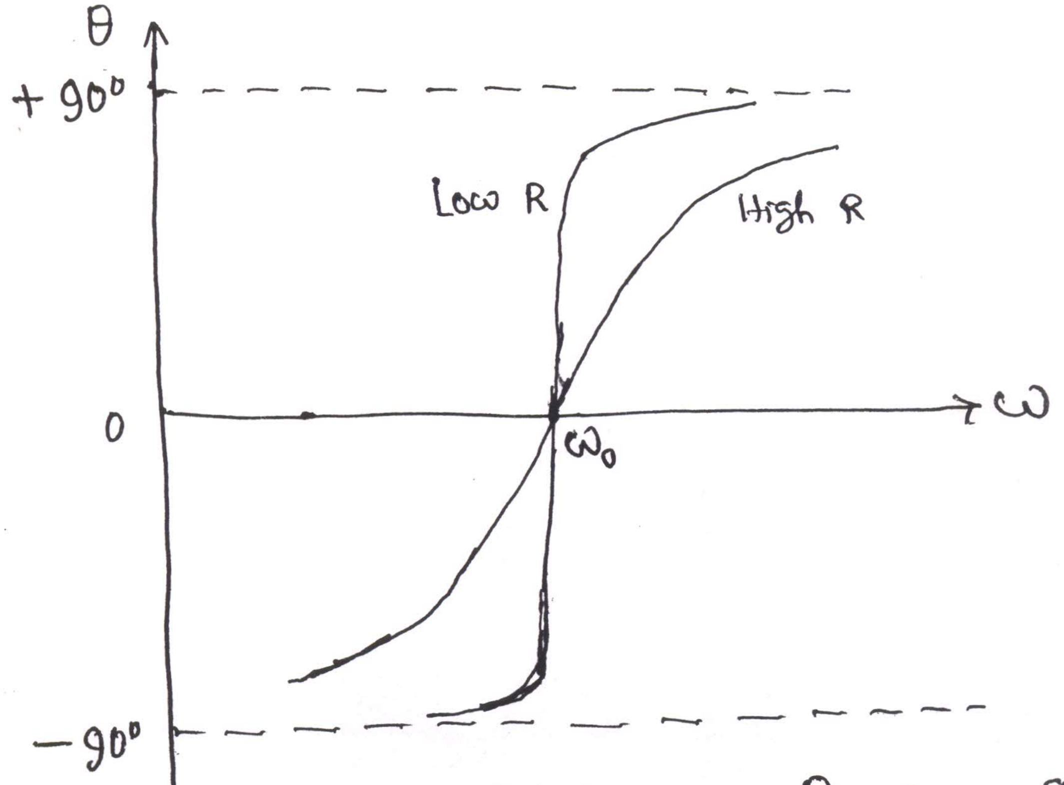
Thus at resonance, the impedance Z is a minimum. Since $I = \frac{V}{Z}$, the current is maximum.

$$\theta = \tan^{-1}\left(\frac{L\omega - \frac{1}{\omega C}}{R}\right)$$

$$L\omega - \frac{1}{\omega C} < 0 \quad \therefore \omega < \frac{1}{\sqrt{LC}}$$
$$\omega^2 < \frac{1}{LC} \quad \therefore \omega < \omega_0$$

At frequencies below ω_0 ($\omega < \omega_0$), the capacitive reactance is greater than the inductive reactance ($\frac{1}{\omega C} > L\omega$) and θ is negative.

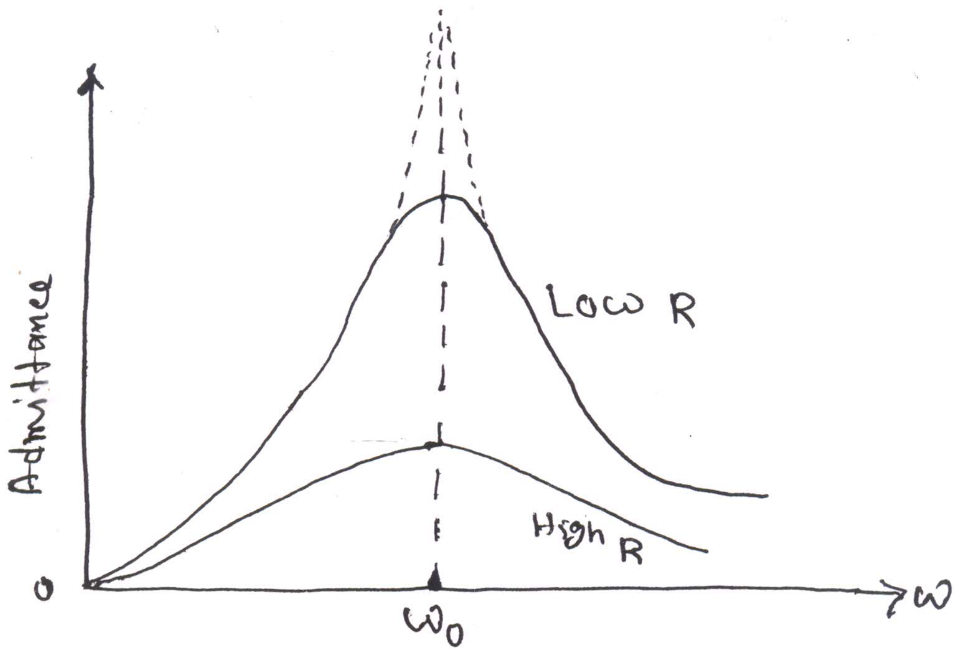
If the resistance is low, the angle changes more rapidly with frequency as shown in Figure below.



As $\omega \to 0$, $\theta \to -90^\circ$

As $\omega \gg \omega_0$, $\theta \to +90^\circ$

$$Y = \frac{1}{Z} \; ; \quad I = YV$$

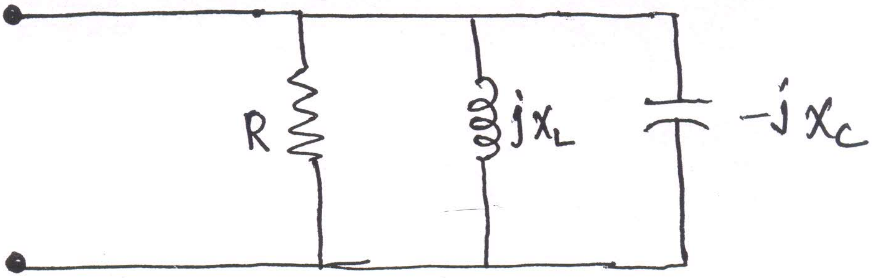


The above plot is also an indication of <u>current versus ω.</u>

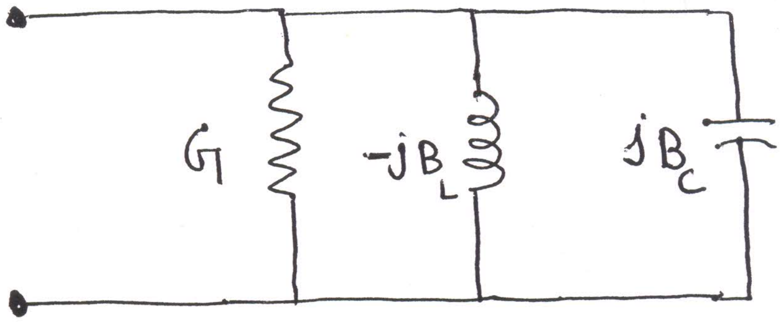
Maximum current occurs at ω_0 and that a low resistance results in a higher current.

The dotted curve shows the limiting case where $R \to 0$

PARALLEL RESONANCE, PURE RLC CIRCUIT.



⇓



$$\boxed{G = \frac{1}{R} \quad B_L = \frac{1}{X_L} \quad B_C = \frac{1}{X_C}}$$

$$G = \frac{1}{R} \; ; \quad \boxed{\frac{1}{jX_L} = -\frac{j}{X_L} = -jB_L}$$

$$\boxed{\frac{1}{-jX_C} = \frac{j}{X_C} = jB_C}$$

$$Y = G + j(B_C - B_L) = G + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$\boxed{\therefore Y = G + j\left(\omega C - \frac{1}{L\omega}\right)}$$

$\therefore Y = G + jB$

where $B = (B_C - B_L)$

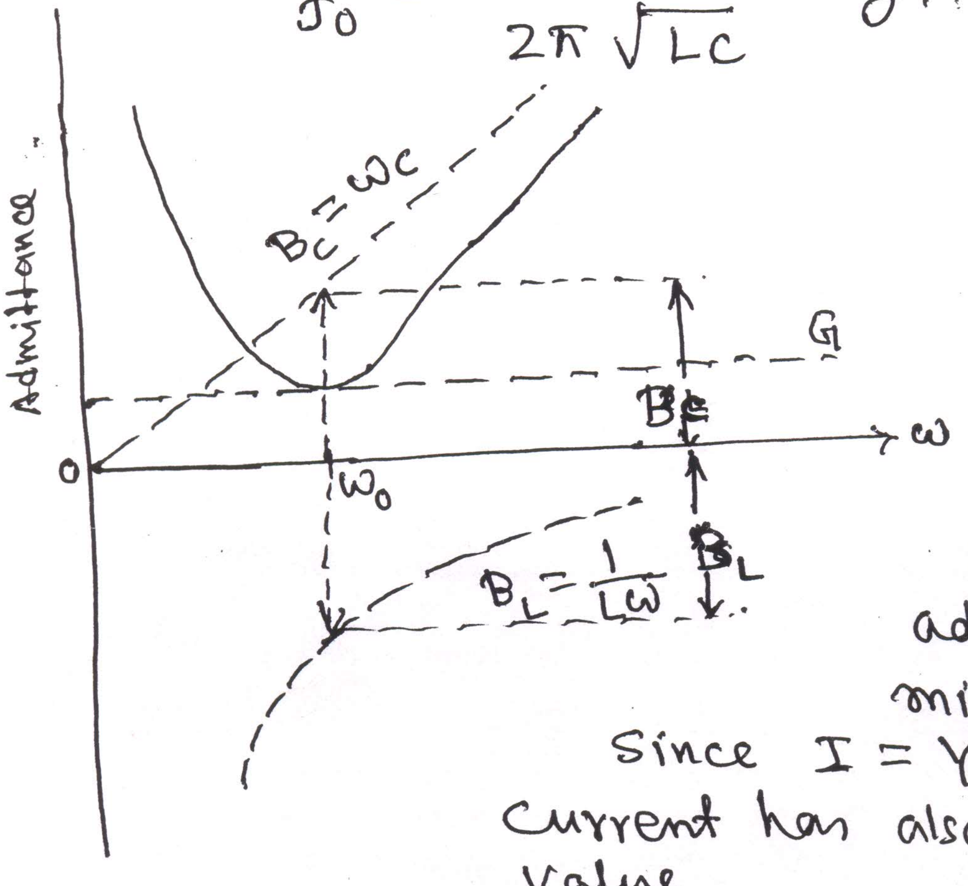
The circuit is in resonance when $B = 0$, i.e.,

$$\omega C = \frac{1}{L\omega}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}} = \omega_0$$

As in the series RLC circuit, the resonant frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

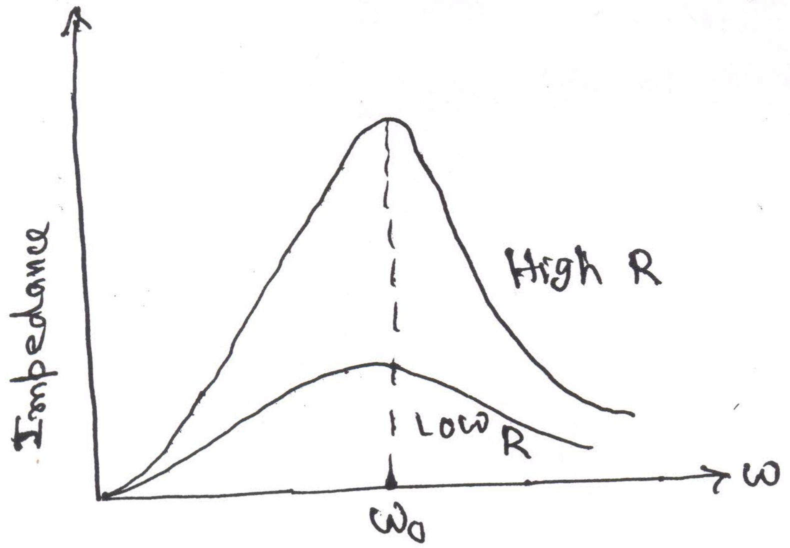


At $\omega = \omega_0$

$B_C = B_L$

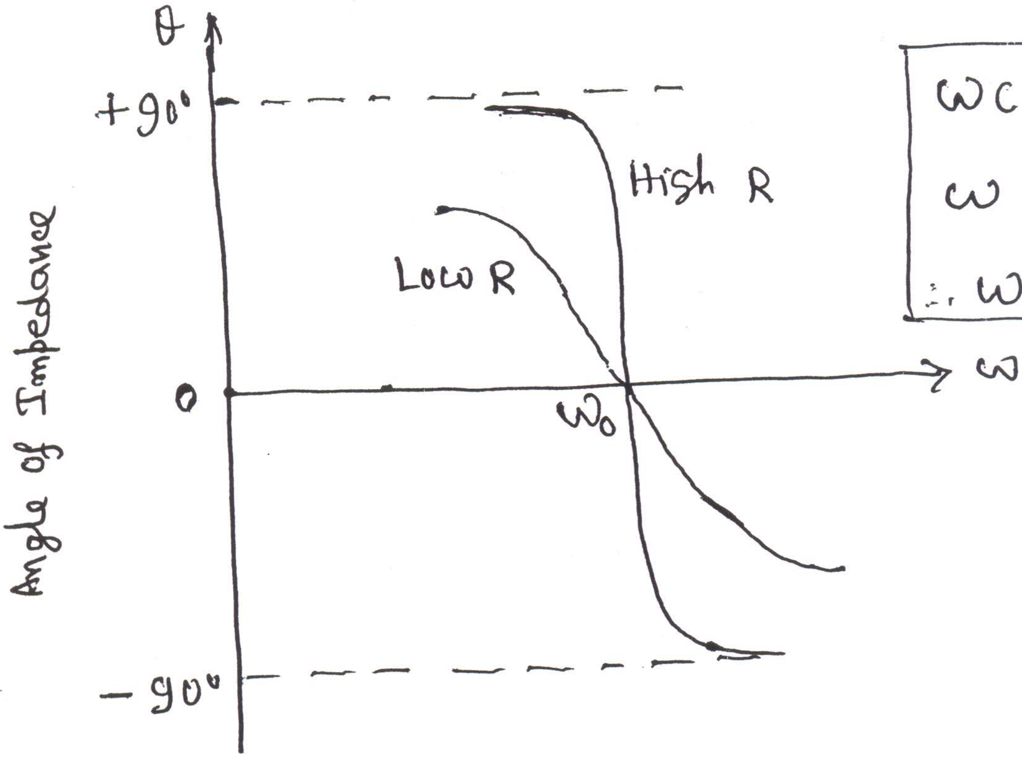
$\therefore Y = G$

At resonance, admittance is minimum and since $I = YV$, the current has also minimum value



Now, angle of admittance is given by

$$\theta_Y = \tan^{-1}\left(\frac{B_C - B_L}{G}\right) = \tan^{-1}\left(\frac{\omega C - \frac{1}{L\omega}}{G}\right)$$



$\omega C - \frac{1}{L\omega} < 0$

$\omega < \frac{1}{\sqrt{LC}}$

$\therefore \omega < \omega_0$

At $\omega < \omega_0$, $B_L > B_C$, $\theta_Y \rightarrow$ Negative.

Angle of Impedance $\theta \rightarrow$ Positive

As $\omega \rightarrow 0$, $\theta_Y \rightarrow -90°$, $\theta \rightarrow +90°$

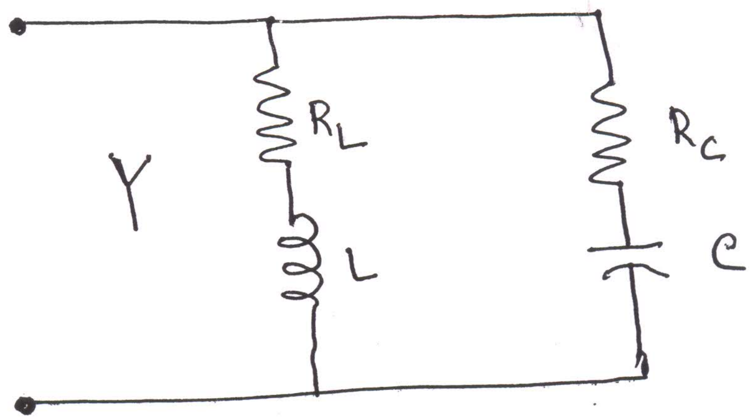
~~At $\omega < \omega$~~

At $\omega > \omega_0$, $B_C > B_L$; $\theta_Y \rightarrow$ Positive

Angle of Impedance $\theta \rightarrow$ ~~Positive~~ Negative

As $\omega \gg \omega_0$, $\theta_Y \rightarrow +90°$, $\theta \rightarrow -90°$

PARALLEL RESONANCE, TWO-BRANCH CIRCUIT



$$Y = Y_L + Y_C = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$= \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right) + j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right)$$

The circuit is at resonance when the complex admittance is a real number.

Then

$$\frac{X_C}{(R_C^2 + X_C^2)} = \frac{X_L}{(R_L^2 + X_L^2)} \quad \text{--- (1)}$$

$$\therefore \quad \frac{1}{\omega_0 C}\left(R_L^2 + L^2\omega_0^2\right) = \omega_0 L\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right) \quad \text{--- (2)}$$

Each of the five quantities in Eqn. (2) may be made variable in order to obtain resonance.

Solving eqn. (1) for ω_0, we obtain

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

Thus the resonant frequency ω_0 of the two-branch parallel circuit differs from that of the pure R, L and C in parallel by the factor

$$\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

Frequency must be a real positive number.

Hence the circuit will have a resonant frequency ω_0 when

$$R_L^2 > \frac{L}{C} \quad \text{and} \quad R_C^2 > \frac{L}{C}$$

OR

$$R_L^2 < \frac{L}{C} \quad \text{and} \quad R_C^2 < \frac{L}{C}.$$

When $R_L^2 = R_C^2 = \frac{L}{C}$

The circuit is resonant at all frequencies

Solving Eqn. (1) for L, we obtain

$$L = \frac{1}{2}C\left[Z_C^2 \pm \sqrt{Z_C^4 - 4R_L^2X_C^2}\right] \quad \text{---} (3)$$

Where

$$Z_C^2 = (R_C^2 + X_C^2); \qquad Z_C^4 = (R_C^2 + X_C^2)^2$$

Now if in Eqn. (3),

$$Z_C^4 > 4R_L^2 X_C^2,$$

We obtain two values of L for which the circuit is resonant.

If $Z_C^4 = 4R_L^2 X_C^2$,

the circuit is in resonance at

$$L = \frac{1}{2}CZ_C^2$$

When $Z_C^4 < 4R_L^2 X_C^2$,

No value of L will make the circuit resonant.

Similarly,

Solving Eqn. (1) for C, we obtain

$$C = 2L\left[\frac{1}{Z_L^2 \pm \sqrt{Z_L^4 - 4R_C^2X_L^2}}\right] \quad \text{--- (4)}$$

Here if $Z_L^4 > 4R_C^2 X_L^2$, we obtain two values of C for which the circuit is resonant.

If $Z_L^4 = 4R_C^2X_L^2$,

The circuit is in resonance at,

$$C = \frac{2L}{Z_L^2}$$

Solving Eqn. (1) for R_L, we obtain

$$R_L = \sqrt{\omega^2 LCR_C^2 - \omega^2L^2 + L/C} \quad \text{--- (5)}$$

and solving for R_C,

$$R_C = \sqrt{\frac{R_L^2}{(\omega^2 LC)} - \frac{1}{\omega^2C^2} + L/C} \quad \text{--- (6)}$$

If the radicand in Eqn. (5) or (6) is positive, then we have a value for R_L or R_C for which the two-branch circuit is in resonance.

QUALITY FACTOR Q

The quality factor of coils, capacitors and circuits is defined by

$$Q = 2\pi\left(\frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}}\right)$$

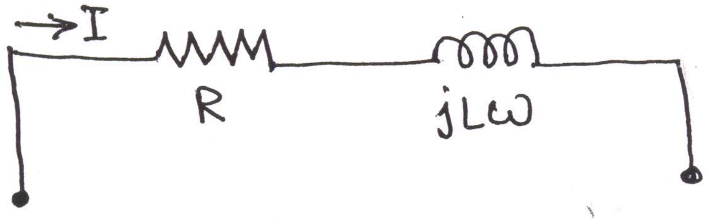


Fig.1

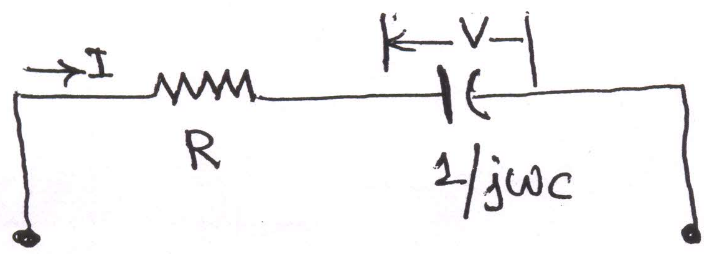


Fig.2

The energy dissipated per cycle in the circuits of Fig. 1 and Fig. 2 is given by the product of the average power in the resistor $\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$ and the period T or $\frac{1}{f}$

In the RL series circuit of Fig. 1, the maximum stored energy is $\frac{1}{2} L I_{max}^2$. Then

$$Q = 2\pi \left(\frac{\frac{1}{2} L I_{max}^2}{\left(\frac{I_{max}^2}{2}\right) R \times \left(\frac{1}{f}\right)} \right)$$

$$\therefore Q = \frac{2\pi f L}{R} = \frac{L\omega}{R}$$

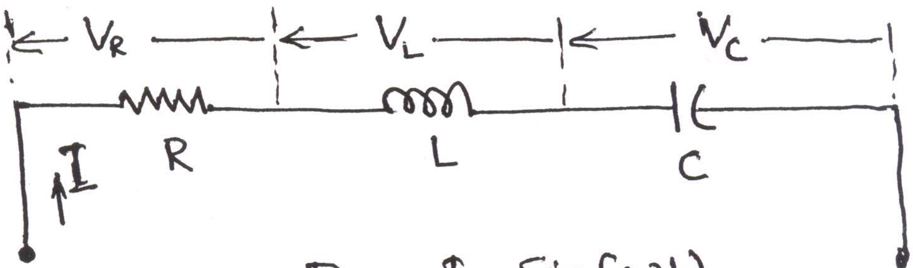
In the RC series circuit of Fig. 2, the maximum stored energy is $\frac{1}{2} C V_{max}^2$ OR $\frac{1}{2} I_{max}^2 / \omega^2 C$ $\left[V_{max} = I_{max} \cdot X_C\right]$

Then

$$Q = 2\pi \left(\frac{\frac{1}{2} I_{max}^2 / \omega^2 C}{\left(\frac{I_{max}^2}{2}\right) \cdot R \cdot \left(\frac{1}{f}\right)} \right)$$

$$\therefore \quad Q = \frac{1}{\omega C R}$$

A series RLC circuit at resonance stores a constant amount of energy.

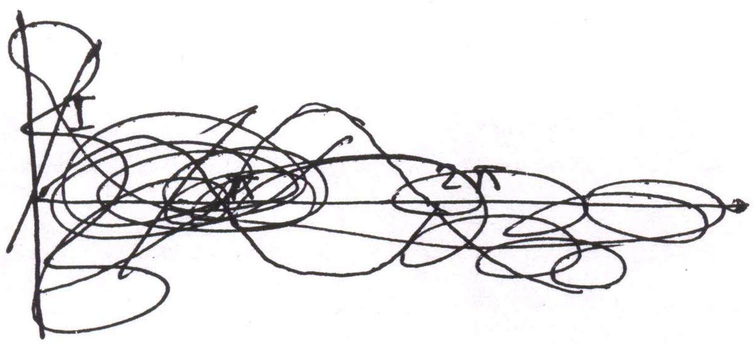


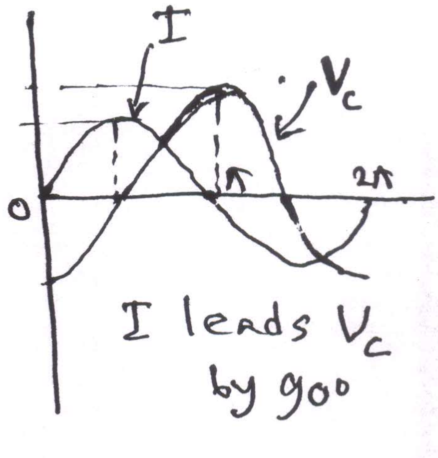
$V_C = V$

$$I = I_m \sin(\omega t)$$

$$V = V_c = \frac{1}{C} \int I_m \sin(\omega t)\, dt = \frac{I_m}{\omega C} (-\cos \omega t)$$

$$\therefore V = \frac{I_m}{\omega C} \sin(\omega t - 90^\circ).$$





I leads V_c by 90°

Since when the capacitor Voltage is maximum, the inductor current is zero, and Vice versa, $\frac{1}{2}CV_{max}^2 = \frac{1}{2}LI_{max}^2$,

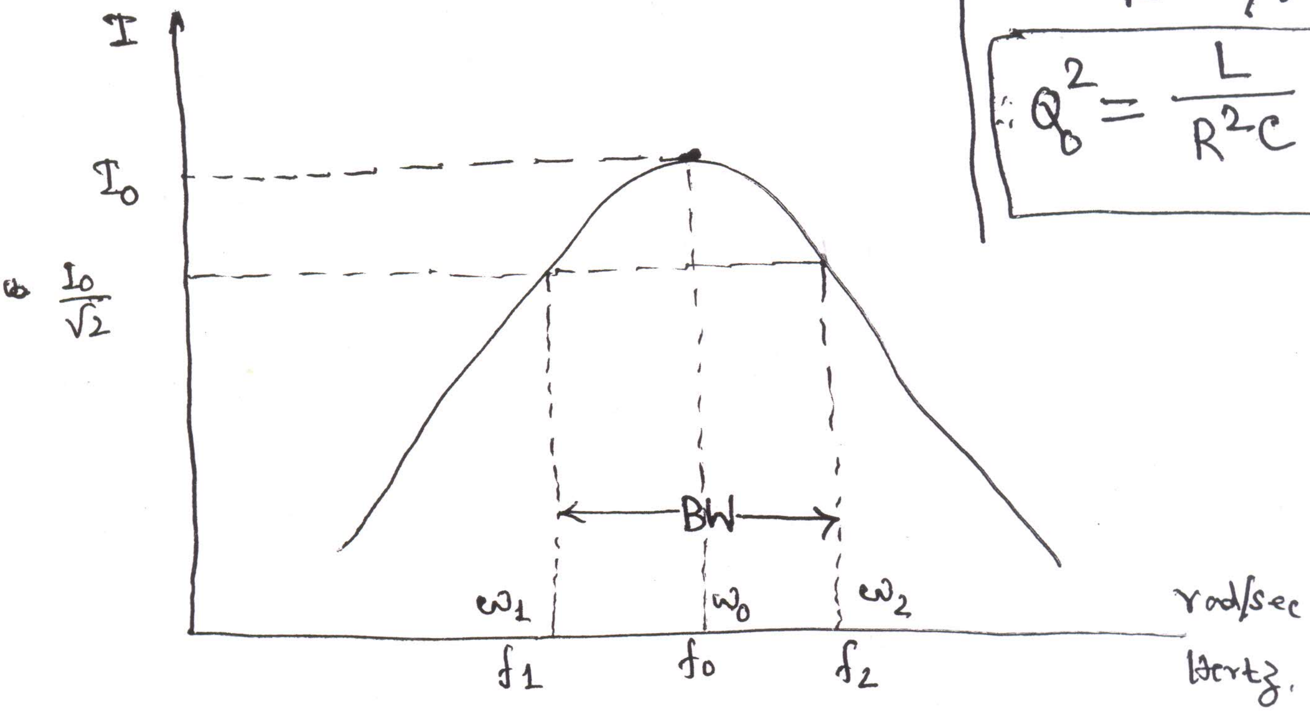
Then

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$Q_0 \times Q_0$

$= \frac{\cancel{\omega_0} L}{R} \times \frac{1}{\cancel{\omega_0} CR}$

$$\boxed{\therefore Q_0^2 = \frac{L}{R^2 C}}$$



At ω_0, current is maximum

Since the power delivered to the circuit is I^2R, at $I = \frac{I_0}{\sqrt{2}}$, the power is one-half of the maximum value which occurs at ω_0. The points corresponding to ω_1 and ω_2 are called the half-power points. The distance between these points is called the bandwidth BW.

$\boxed{\text{At half-power frequencies, the net reactance is equal to the resistance}}$

A circuit with high Q will have a very sharp current response curve as compared to one which has a low value of Q.

~~Let us assume~~ we specify two frequencies ω_1 & ω_2 at which $\rightarrow$ $(|X_L - X_C| = R)$

Since at ω_1, the circuit is capacitive ($X_C > X_L$), therefore at ω_1,

$$X_C - X_L = R \quad \text{---(i)}$$

at ω_2

$$X_L - X_C = R \quad \text{---(ii)}$$

The corresponding impedances are:

At ω_1

$$Z_1 = R + j(X_L - X_C) = R - jR$$

$$\therefore Z_1 = \sqrt{2}\,R \angle -45^\circ$$

At ω_2

$$Z_2 = \sqrt{2}\,R \angle 45^\circ$$

At ω_1, $I_1 = \dfrac{V \angle 0^\circ}{\sqrt{2}\,R \angle -45^\circ} = \dfrac{V}{\sqrt{2}\,R} \angle 45^\circ$

Similarly, at ω_2

$$I_2 = \frac{V}{\sqrt{2}\,R} \angle -45^\circ$$

Since $\dfrac{V}{R} = I_0$, the current at resonance frequency ω_0.

$$\left.\begin{aligned} \therefore I_1 &= 0.707\, I_0 \angle 45^\circ \\ I_2 &= 0.707\, I_0 \angle -45^\circ \end{aligned}\right\}$$

At ω_1,

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - L\omega_1 = R \quad \text{--- (iii)}$$

At ω_2

$$X_L - X_C = R$$

$$\therefore L\omega_2 - \frac{1}{\omega_2 C} = R \quad \text{--- (iv)}$$

Eqn. (iii) − Eqn. (iv)

$$\therefore \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} - (\omega_1 + \omega_2)L = 0$$

$$\therefore \frac{1}{\omega_1\omega_2} = LC = \frac{1}{\omega_0^2}$$

$$\therefore \boxed{\omega_0 = \sqrt{\omega_1\omega_2} \quad \text{--- (V)}}$$

Again

Eqn (iii) + Eqn. (iv)

$$\frac{1}{\omega_1 C} - L\omega_1 + L\omega_2 - \frac{1}{\omega_2 C} = 2R$$

$$\therefore \frac{\omega_2 - \omega_1}{\omega_1\omega_2 C} + (\omega_2 - \omega_1)L = 2R$$

$$(\omega_2 - \omega_1)\left(\frac{1}{\omega_0^2 C} + L\right) = 2R$$

$$\therefore \frac{(\omega_2 - \omega_1)(1 + LC\omega_0^2)}{\omega_0^2 C} = 2R$$

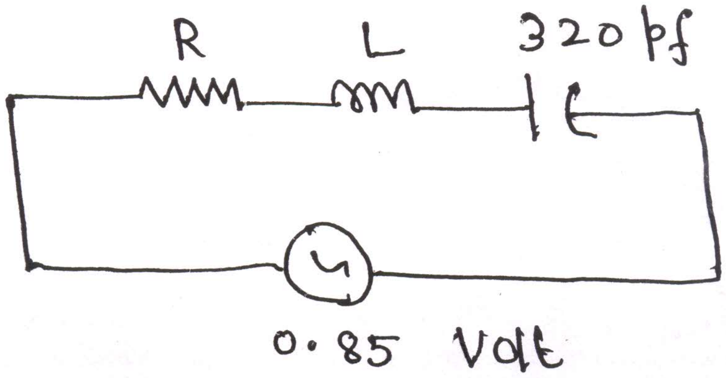
$$\therefore (\omega_2 - \omega_1)\left(\frac{2}{\omega_0^2 C}\right) = 2R$$

$$\therefore \omega_2 - \omega_1 = \omega_0^2 CR$$

$$\therefore \frac{\omega_2 - \omega_1}{\omega_0} = \omega_0 CR = \frac{1}{Q_0}$$

$$\boxed{\therefore Q_0 = \frac{\omega_0}{\omega_2 - \omega_1} \quad \text{--- (vi)}}$$

EX-1



Determine the value of L for resonance if Q = 50 and f_0 = 175 KHz. Also find the circuit current, the voltage across the capacitor and the bandwidth of the circuit.

Soln

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore 175\times10^{3} = \frac{1}{2\pi\sqrt{320\times10^{-12}\,L}}$$

$$\boxed{\therefore L = 2.58\ mH}$$

~~The reactance~~

$$X_L = L\omega = 2.58\times10^{-3}\times 2\pi\times 175\times10^{3}\ \Omega$$

$$\therefore X_L = 2840\ \Omega$$

Since $Q = \dfrac{\omega_0 L}{R}$

$$\therefore R = \frac{\omega_0 L}{Q} = \frac{175\times10^{3}\times 2.58\times10^{-3}}{50}$$

$$\therefore R = 56.8\ \Omega.$$

The impedance of the circuit at resonance is

$$Z = R = 56.8\,\Omega.$$

$$\therefore\ I_0 = \frac{V}{R} = \frac{0.85}{56.8} = 14.96\ \text{mA}.$$

Also

$$V_c = \frac{I_0}{\omega_0 C} = \frac{R I_0}{\omega_0 C R} =$$

$$\boxed{\therefore\ V_c = QV}$$

$$\boxed{\begin{array}{l}\text{Voltage across the capacitor} \\ = QV = 50 \times 0.85 = 42.5\ \text{Volt.}\end{array}}$$

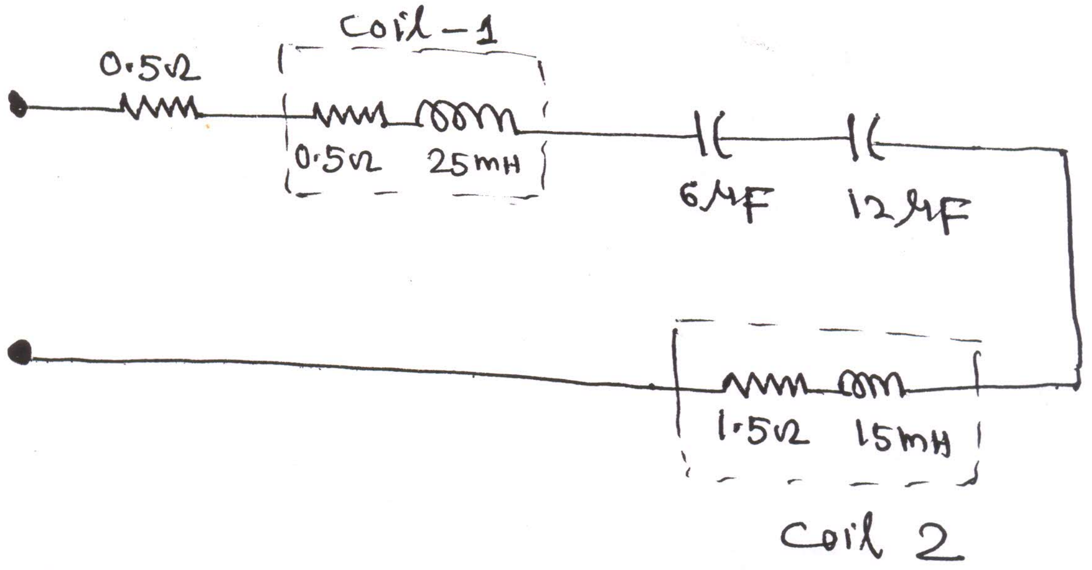
Bandwidth, $= (\omega_2 - \omega_1)$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

$$\therefore\ \boxed{BW = \frac{f_0}{Q} = \frac{175 \times 10^3}{50} = 3.5\ \text{KHz}}$$

Ex-2: For the circuit shown in Figure, $R_1 = 0.5\Omega$, $R_2 = 1.5\Omega$, $R_3 = 0.5\Omega$, $C_1 = 6\mu F$ and $C_2 = 12\mu F$, $L_1 = 25\,mH$ and $L_2 = 15\,mH$.

Determine (i) the frequency of resonance (ii) Q of the circuit (iii) Q of coil 1 and coil 2, individually.



Soln.

Total inductance of the circuit

$$L = (25+15) = 40\,mH$$

$$\text{Total } C = \frac{6 \times 12}{(6+12)} = 4\mu F.$$

$\therefore\ f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{40\times10^{-3}\times 4\times10^{-6}}}$

$\boxed{\therefore\ f_0 = \dfrac{10^4}{8\pi}\ \text{Hz.}}$ OR $\boxed{\omega_0 = 2.5\times10^{3}\ \text{rad/sec.}}$

Q of the circuit $= \dfrac{L\omega_0}{R}$

$= \dfrac{2.5\times10^{3}\times 40\times10^{-3}}{2.5} = 125$ 40 ✓

$\boxed{\therefore\ Q = 125\ 40}$ ✓

Q of coil 1 $= \dfrac{L_1\omega_0}{R_1} = \dfrac{2.5\times10^{-3}\times 2.5\times10^{3}}{0.5} = 125$

125

Q of coil 2 $= \dfrac{L_2\omega_0}{R_2} = \dfrac{15\times10^{-3}\times 2.5\times10^{3}}{1.5}$

$= 25$

$\boxed{BW = \dfrac{f_0}{Q} = \dfrac{10^4}{8\pi\times 125}\ \text{Hertz}}$ $\simeq$ 10 Hz.

40

Ex-3

A coil having a 5Ω resistor is connected in series with a 50μF capacitor. The circuit resonates at 100 Hz. (a) determine the inductance of the coil (b) If the circuit is connected across a 200 Volt, 100 Hz source, determine the power delivered to the coil (c) Voltage across the capacitor and the coil (d) BW of the circuit.

Soln.

At resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore \quad 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\therefore \quad 2\pi \times 100 = \frac{1}{\sqrt{50 \times 10^{-6} \times L}}$

$\boxed{\therefore L = 50 \text{ mH}}$

The current at resonance $\frac{V}{R} = \frac{200}{5} = 40$ Amp power dissipated $= (40)^2 \times 5 = 8000$ Watts,

$= 8KW.$

Voltage across capacitor $= I X_C$

$$= \frac{40}{50 \times 10^{-6} \times 100 \times 2\pi} = \frac{8000 \text{ Volt}}{2\pi}$$

$= 127.32$ Volt $\Rightarrow 1273.2$ V.

The Impedance of the coil

$= R + jL\omega$

$= 5 + j\, 50 \times 10^{-3} \times 2\pi \times 100 = (5 + j\,31.4)\,\Omega$

$V_L = 40 \times (5 + j\,31.4) = 1256$ Volt

Q_0 of the coil $= \frac{31.4}{5} = 6.3$

$BW = \frac{f_0}{Q_0} = \frac{100}{6.3} = 16$ Hz.

Ex-4: A Series circuit with $R = 50\,\Omega$, $L = 0.05\,H$ and $C = 20\,\mu F$ has an applied Voltage $V = 100\angle 0^\circ$ Volt with a Variable frequency. Find the maximum Voltage across the inductor as the frequency is Varied.

Soln

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}}$$

Magnitude of the voltage across L is

$$V_L = I(L\omega) = \frac{\omega L V}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}} \quad \text{---(1)}$$

Setting the derivative $\frac{dV_L}{d\omega}$ of eqn-(1) equal to zero and solving for ω, We obtain the value of ω when V_L is a maximum

$$\frac{dV_L}{d\omega} = \frac{d}{d\omega}\left[\omega L V\left(R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}\right)^{-\frac{1}{2}}\right]$$

$$\therefore \quad R^2 - \frac{2L}{C} + \frac{2}{\omega^2 C^2} = 0$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}}\sqrt{\frac{2}{2 - \frac{R^2 C}{L}}} \quad \text{--- (2)}$$

Since $Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 CR}$

$$\therefore \quad Q_0^2 = \frac{L}{R^2 C} \quad \text{---- (3)}$$

$$\therefore \quad \frac{R^2 C}{L} = \frac{1}{Q_0^2} \quad \text{---- (4)}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}}\sqrt{\frac{2}{2 - \frac{1}{Q_0^2}}}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}}\sqrt{\frac{2Q_0^2}{(2Q_0^2 - 1)}} \quad \text{-- (5)}$$

Eqn. (5) shows that for high Q, the maximum voltage across L occurs at

$$\omega_0 \simeq \frac{1}{\sqrt{LC}}$$

If Q is high, Maximum Voltage are also obtained across R and C at ω_0.

With low Q, V_C maximum occurs below ω_0 and V_L maximum occurs above ω_0.

From eqn. (2),

$$\omega = \sqrt{\frac{2}{2LC - R^2C^2}}$$

$$\therefore \omega = \left[\frac{2}{2(0.05)(20\times10^{-6}) - (50\times20\times10^{-6})^2}\right]^{\frac{1}{2}}$$

$$\therefore \omega = 1414 \text{ rad/sec.}$$

$$X_L = L\omega = 70.7\ \Omega$$

$$X_C = \frac{1}{\omega C} = 35.4\ \Omega$$

$$Z = 50 + j(70.7 - 35.4)$$

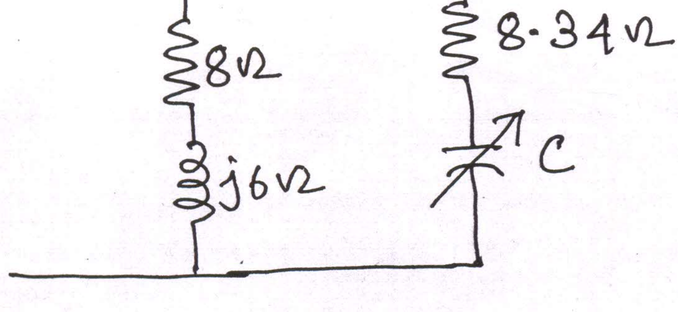
$$\therefore Z = 61.2 \angle 35.3'\ \Omega$$

$$I = \frac{V}{Z} = \frac{100}{61.2} = 1.635\ \text{Amp.}$$

$$V_L(\max) = 1.635 \times 70.7\ \text{Volt}$$

$$\therefore V_L(\max) = 115.5\ \text{Volt.}$$

<u>EX-5</u>



Find C which results in resonance when $\omega_0 = 5000$ rad/sec.

$$Y = \frac{1}{8+j6} + \frac{1}{8.34 - jX_C}$$

$$\therefore Y = \left(\frac{8}{100} + \frac{8.34}{69.5 + X_C^2}\right) + j\left(\frac{X_C}{69.5 + X_C^2} - \frac{6}{100}\right)$$

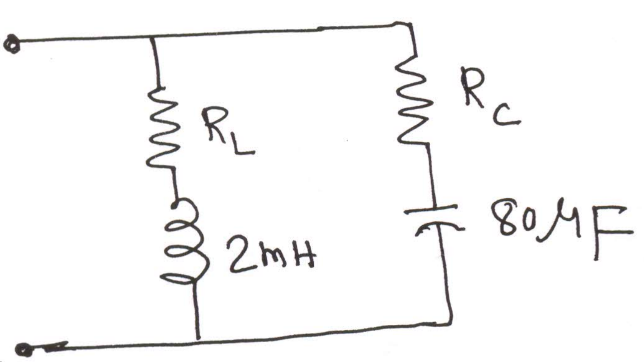
At resonance

$$\frac{X_C}{69.5 + X_C^2} - \frac{6}{100} = 0$$

$$\therefore X_C = 8.35\,\Omega = \frac{1}{\omega_0 C}$$

$$\therefore C = \frac{1}{5000 \times 8.35} = 24\,\mu F$$

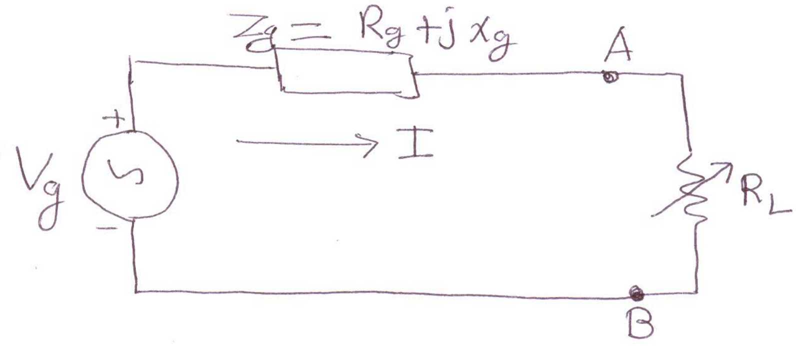
<u>Ex-6</u>



Determine R_L & R_C which cause the circuit to be resonant at all frequencies.

Maximum Power Transfer Theorems

Case-1: Load : Variable resistance R_L



$$I = \frac{V_g}{(R_g + R_L) + j\,x_g}$$

$$\therefore |I| = \frac{V_g}{\sqrt{(R_g + R_L)^2 + x_g^2}}$$

$$P = |I|^2 R_L = \frac{V_g^2\, R_L}{(R_g + R_L)^2 + x_g^2}$$

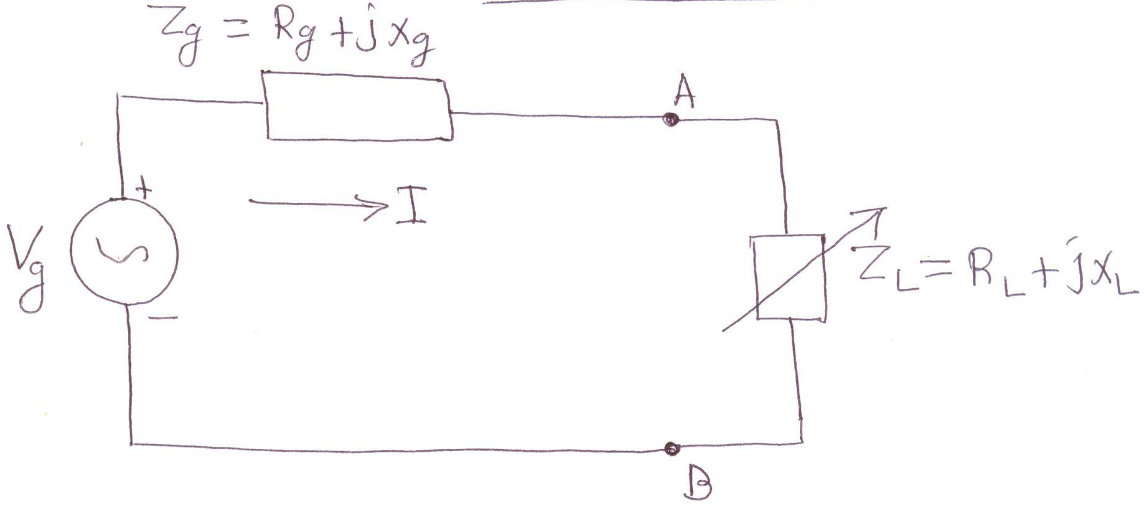
$$\frac{dP}{dR_L} = 0$$

$$\therefore R_L^2 = R_g^2 + x_g^2$$

$$\therefore R_L = \sqrt{R_g^2 + x_g^2} = |Z_g|$$

If $x_g = 0$, $\therefore R_L = R_g$.

Case 2: Load: Impedance Z_L with Variable resistance and Variable reactance.



$$I = \frac{V_g}{(R_g + R_L) + j(X_g + X_L)}$$

$$\therefore |I| = \frac{V_g^2}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$$

$$\therefore P = |I|^2 R_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2} \quad \text{---- (i)}$$

If R_L in Eqn (i) is held fixed, the Value of P is maximum when $X_g = -X_L$.

Then Eqn.(i) becomes

$$P = \frac{V_g^2 R_L}{(R_g + R_L)^2} \quad \text{---} (ii)$$

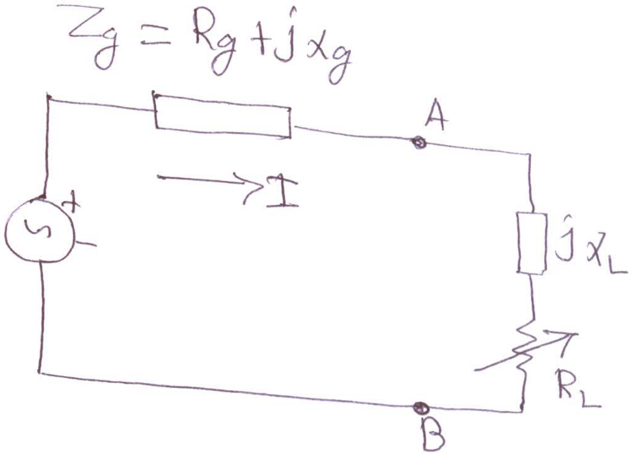
Consider now R_L to be variable.
As shown in case-1, the maximum power is delivered to the load when $R_L = R_g$.

If $R_L = R_g$ and $X_L = -X_g$,

then $Z_L = R_L + jX_L = R_g - jX_g$

$$\therefore Z_L = Z_g^* \quad \text{---} (iii)$$

<u>Case-3:</u>



$R_L = |Z_g + jx_L|$

$\therefore R_L = |R_g + j(x_g + x_L)|$

Ex-1:

In the circuit of Fig. 1, the load Z_L Consists of a pure resistance R_L. Find the value of R_L for which the source delivers maximum to the load. Determine the value of the maximum power P.

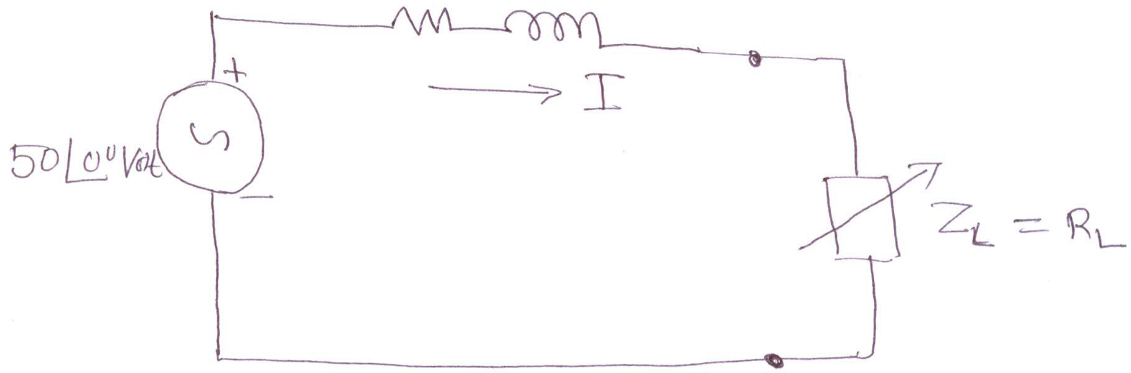


Fig. 1.

Soln

$$R_L = |Z_g| = |10+j20| = 22.4\Omega$$

$$I = \frac{V_g}{(Z_g + R_L)} = \frac{50\angle 0^\circ}{(10+j20+22.4)} = 1.31\angle -31.7^\circ \text{ Amp}$$

$$P = (1.31)^2 \times 22.4 = 38.5 \text{ Watt.}$$

Ex-2:

Repeat Ex-1, with $Z_L = R_L + jX_L$, and R_L & X_L are both variable.

Soln.

$$Z_L = Z_g^* = 10 - j20$$

Total Impedance,

$$Z_T = Z_g + Z_L = 10 + j20 + 10 - j20 = 20\Omega$$

$$I = \frac{50\angle 0^\circ}{20} = 2.5\angle 0^\circ \text{ Amp.}$$

$$P = (2.5)^2 \times 10 = 62.5 \text{ Watt.}$$

Ex-3:

In the circuit shown in Fig. 2, the resistance R_g is variable between 2 and 55 Ω. What value of R_g results in maximum power transfer across the terminals AB?

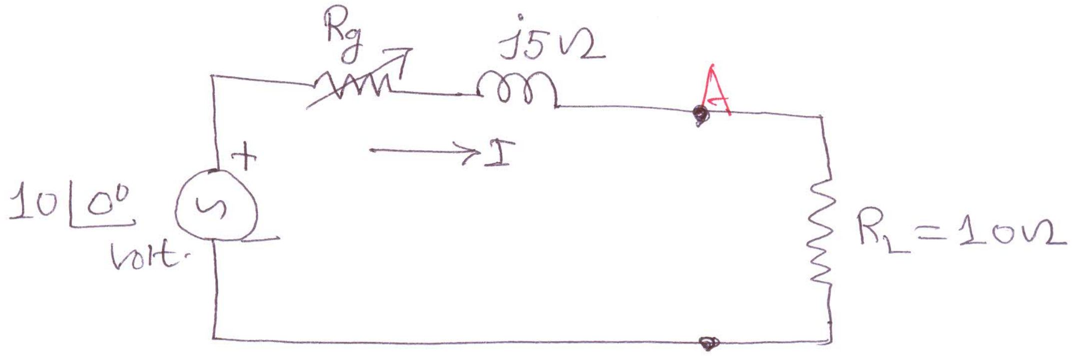


Fig. 2

Ex-4:

In the network shown in Fig. 3, the load connected across terminals AB consists of a variable resistance R_L and a capacitive reactance X_C which is variable between 2Ω and 8 Ω. Determine the values of R_L and X_C which result in maximum power transfer. Calculate the maximum power P delivered to the load.

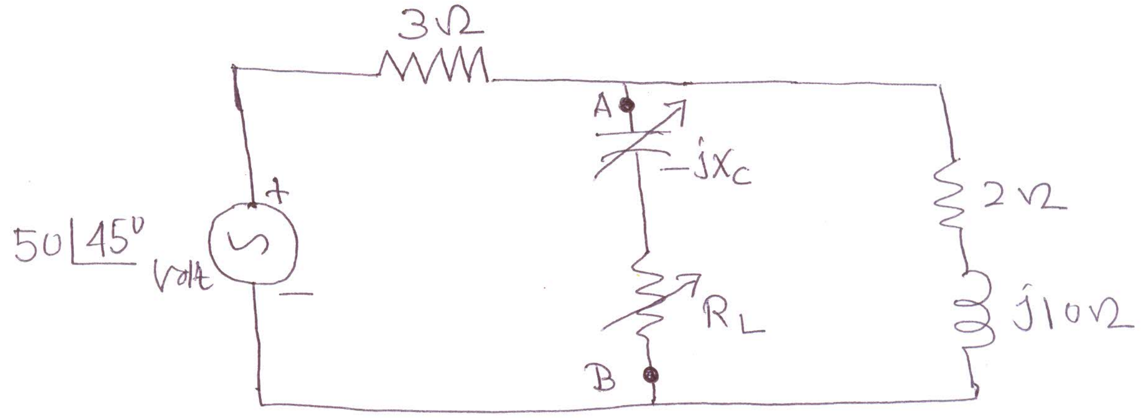


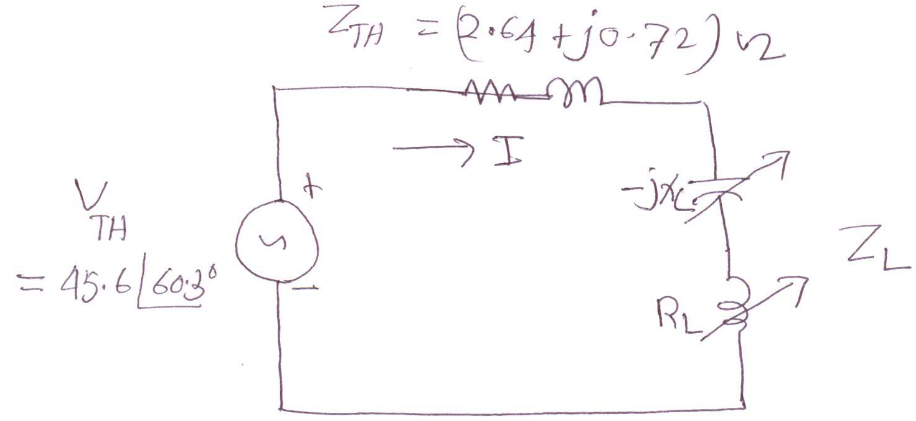
Fig. 3

Soln

$$V_{TH} = V_{AB} = \frac{50\angle 45^\circ}{(5+j10)}(2+j10)$$

$$= 45.6\angle 60.3^\circ \text{ volt}$$

$$Z_{TH} = Z_{AB} = \frac{3\times(2+j10)}{(5+j10)} = (2.64+j0.72)\,\Omega$$



Maximum power transfer occurs,

when $Z_L = Z_{TH}^{*} = (2.64 - j0.72)\,\Omega$

X_c is adjustable between 2Ω and 8Ω.

Hence the closest value of X_c is 2Ω

and

$$R_L = |Z_g - jX_c| = |2.64 + j0.72 - j2|$$

$$\therefore R_L = |2.64 - j1.28| = 2.93\,\Omega$$

$$Z_T = Z_{TH} + Z_L = \underline{2.64 + j0.72} + \underline{2.93 - j2}$$

$$\therefore Z_T = 5.57 - j1.28 = 5.7\angle -13^\circ\,\Omega$$

$$\therefore I = \frac{45.6\angle 60.3^\circ}{5.7\angle -13^\circ} = 8\angle 73.3^\circ \text{ Amp.}$$

$$P = (8)^2 \times 2.93 = 187.5 \text{ Watt.}$$