INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
Mid-Spring Semester 2019-20

Date of Examination: 19.02.2020. Session: AN Duration: 2 Hrs Full Marks : 30
Subject No. : PH11001 Subject Name: PHYSICS
Department/Center/School: PHYSICS

Special Instructions:
For the objective (Multiple Choice) questions, mention only the correct option against the question number on the first-page of your answer booklet. Answers to the objective questions, written elsewhere will not be evaluated!

Question paper is covering 4 pages.

Objective Type Questions

Answer all questions Q1 to Q6

[Marks: 6×2 = 12]

Q1) An object of mass 20 kg moves with simple harmonic motion along the x axis. Initially (at $t = 0$ sec) it is located at a distance 4 metres away from the origin $x = 0$ m (equilibrium position), and has acceleration 100 m/s^2 directed towards $x = 0$ m. The angular frequency is:

(a) 5 sec^{-1}

(b) $\frac{5}{2\pi}$ sec^{-1}

(c) 0.5 sec^{-1}

(d) $\frac{5}{\pi}$ sec^{-1}

Q2) A mass 1 kg at the end of a spring with spring constant 25 N/m is undergoing under-damped oscillations along x-axis. A damping force is acting on the mass, where the value of damping coefficient is 6 kg sec^{-1}. It, thus, follows the equation of motion, as given below –

$$\ddot{x} + 6\dot{x} + 25x = 0$$

The logarithmic-decrement [$\ln(\frac{A_{n+1}}{A_n})$, where A_{n+1} and A_n are successive amplitudes separated by a time period] for this under-damped oscillator is –

(a) 3π
(b) $(3\pi)/2$
(c) $(3\pi)/4$
(d) $\pi/2$

Q3) A simple harmonic oscillator of mass $1Kg$ experience a damping force of 2 $N.sec/m$. It undergoes amplitude resonance by the influence of an external force at angular frequency of 5 $/sec$.

$$\ddot{x} + 2\dot{x} + \omega_0^2 x = 0$$

Calculate the angular frequency at which maximum power is absorbed (round off to one decimal place).

(a) $4.2/sec$
(b) $5.2/\text{sec}$
(c) $7.2/\text{sec}$
(d) $8.2/sec$

Q4) Two spring mass systems with equal mass m and spring constant k are coupled in series via a third spring with a spring constant of $\frac{k}{2}$. Find out the ratio of the fast and slow frequency of the normal modes.

(a) $\sqrt{2}$
(b) $\sqrt{3}$
(c) 1.5
(d) 3.0

Q5) The wave equation of a harmonic wave is given by

$$\xi(x,t) = (0.01\text{ m})\ [\sin(0.5x - 10t + 0.4)]$$

If ϕ_s is the phase difference between the two points separated in space by 0.5 m at the same time and ϕ_t is the phase difference between the two points separated by 0.04 sec in time then the ratio $\left|\frac{\phi_s}{\phi_t}\right|$ is:

(a) 1/8
(b) 3/8
(c) 5/8
(d) 7/8

Q6) Consider a vector $\vec{A} = y\hat{i} - x\hat{j} + z\hat{k}$. Find the surface integral $\int \left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{S}$, over the *hemispherical* surface $x^2 + y^2 + z^2 = 1, z \geq 0$. You should consider the outward pointing unit normal to the surface ($\hat{r}$) as the direction of $d\vec{S}$.

(a) 0

(b) - π

(c) - 2π

(d) 4π

Subjective Type Questions

Answer all questions

Q7) The equation of motion of a forced damped oscillator is given by: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$. The external force amplitude is $F_0 = 5\ N$.

At very low frequency limit, such that, $\frac{\omega}{\omega_0} \ll 1, \frac{\omega}{\beta} \ll 1$, the amplitude of oscillation is $10\ cm$. **(i)** Estimate the spring constant of the oscillator.

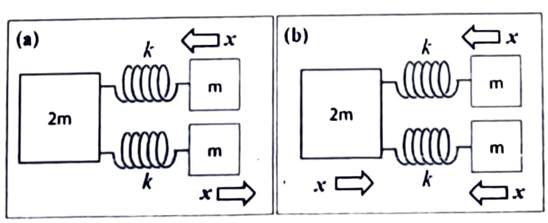
At very high frequency of $\omega = 1\ KHz$, such that $\frac{\omega_0}{\omega} \ll 1, \frac{\beta}{\omega} \ll 1$, the amplitude of the same forced damped oscillator is given by, $a = 5 \times 10^{-6} m$. **(ii)** Estimate the mass (m) of the oscillator.

[Marks: 2+2]

Q8) Three masses, one of mass "$2m$" and other two masses of mass "m" are coupled, as shown below, using ideal massless springs of spring constant "k". These masses can move only in one spatial dimension. Consider that gravity and friction plays no role in this problem.

Fig. (a) represents one normal mode motion of the coupled system, where the two masses "m" have equal and opposite displacements at any time and the mass "$2m$" is always stationary. (i) Write down the equation of motion for this normal mode motion for any of the masses "m". (ii) Using this equation, write down the corresponding normal mode frequency for this motion.

Fig. (b) represents another normal mode motion for the coupled system, where the displacements of the two masses "m" are same, both in magnitude and direction. The displacement of mass "2m" is equal and opposite to the displacement of the mass "m". (iii) Write down the equation of motion for this normal mode motion for the mass "2m". (iv) Using this equation, write down the corresponding normal-mode frequency. [Hint: For motion (b), the effective spring constant is $2k$.]



[Marks – 1+1+1+1]

Q9) The phase velocity of a surface wave on a liquid is given by $v_p = \left(\frac{g\lambda}{2\pi} + \frac{2\pi S}{\lambda\rho}\right)^{1/2}$, where g is acceleration due to gravity, S is surface tension of the liquid, λ is wavelength of the wave and ρ is density of the liquid. If $S = 4 \times 10^{-2}\ {}^{N}/_{m}$, $\rho = 1000\ {}^{kg}/_{m^3}$ and $g = 10\ {}^{m}/_{s^2}$ then find (a) the value of the wavelength for which the phase velocity is minimum (b) the value of the minimum phase velocity.

[Marks – 2+2]

Q10) Consider the following electric and magnetic fields in three dimensions (x,y,z):
$\mathrm{E}_y = \mathrm{f}(x - ct) + \mathrm{g}(x + ct), \mathrm{E}_x = 0,\ \mathrm{E}_z = 0$ and $c\mathrm{B}_z = \mathrm{f}(x - ct) - \mathrm{g}(x + ct),\ \mathrm{B}_x = 0,\ \mathrm{B}_y = 0,$
where f and g are any two arbitrary functions, c is a constant and t is the time.

a) Evaluate $\nabla \times \vec{E}$ and $\frac{\partial \vec{\mathrm{B}}}{\partial t}$ from above.

b) Using the expressions for the fields given above, find the Laplacian $\nabla^2 \vec{\mathrm{E}}$ in terms of $\frac{\partial^2 \vec{E}}{\partial t^2}$.

[Marks - 2+1]

Q11) A plane electromagnetic wave travelling ~~in vacuum~~, has electric field given by

$$\vec{E}(x, y, z, t) = \left(P\hat{j} - Q\hat{k}\right) \cos\left(\omega\left(t - \frac{x}{c}\right)\right)$$

Here P, Q and ω are all constants. What is the *average* energy flux per unit time, passing through a unit area, placed normal to the direction of propagation of this wave (average energy current density), in terms of these given constants. The average is to be taken over one time-period of the electromagnetic wave.

[Marks - 3]