Problem Set-6 **SPRING 2020**

MATHEMATICS-II (MA10002)(Integral Calculus)

Hints and Answers

1. Answers: i) Divergent ii) Convergent iii) Divergent iv) Convergent if $p < 1$ v) Convergent vi) Divergent.

2. i) Apply comparison test. Answer: Divergent.

ii) If $n = 0$ then use definition to show the divergence, and for $n - 1 \leq 0$, apply comparison test. Answer: Convergent if $n > 0$.

iii)-iv) Convergent v)Convergent if $m > 0, n > 0$ vi) Convergent if $0 < p < 1$.

vii) Here 0 is not a point of infinite discontinuity. Take the function $\frac{1}{x^2}$ for comparison test. Answer: Convergent.

ix) Apply μ test when $m - n < 0$. Answer: Convergent if $n < 1 + m$.

x) Take the function $\frac{1}{\sin x}$ and apply comparison test. Answer: Divergent.

4. f is bounded and integrable on $[\epsilon, 1]$. Divide this interval into p subintervals $[\frac{1}{2}, 1]$, $[\frac{1}{3}, \frac{1}{2}]$, $\ldots$,$[\epsilon, \frac{1}{p}]$ and then take p tends to infinity. Answer: Convergent.

6. Choose a positive real number a such that $am < \pi$, then examine the convergence of $\int_0^a \frac{\sin mx}{x^n} dx$ using comparison test and $\int_a^\infty \frac{\sin mx}{x^n} dx$ using definition.

7. Consider the integral $\int_0^{n\pi} \frac{1}{1+x^2 \sin^2 x} dx$ and use $\int_0^{n\pi} \frac{1}{1+x^2 \sin^2 x} dx = \sum_{r=1}^{n} \int_{(r-1)\pi}^{r\pi} \frac{1}{1+x^2 \sin^2 x} dx$.

8. In Beta function put $n = 1 - m$ and $x = \frac{t}{1+t}$.

10. i) $\cot^p x = \cos^p x \sin^p x$.

ii)-iv) Use Beta function.

v) $b - a = (x - a) + (a - b)$.

11. i) Ans.: $ab \log(\frac{b}{a})$, ii) $\frac{2 \times 13! \times 8!}{23!}$

13. Take the product in reverse order and use the result $\Gamma(m)\Gamma(1 - m) = \frac{\pi}{\sin m\pi}$.