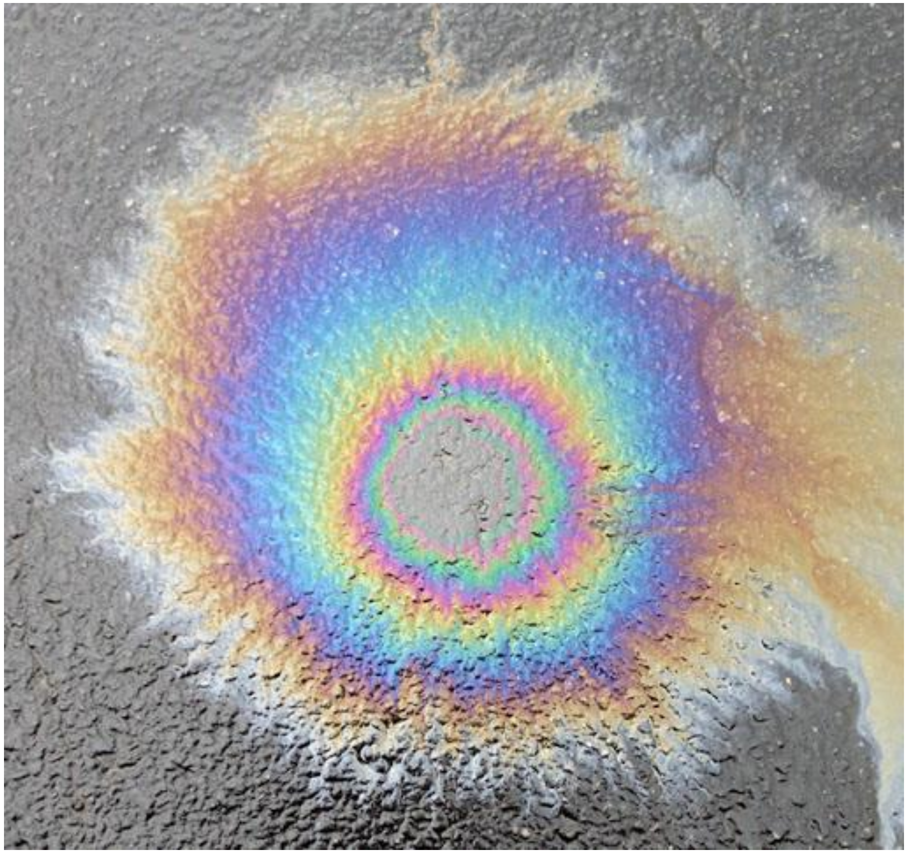
Interference
by
Division of Amplitude

Thin film Interference

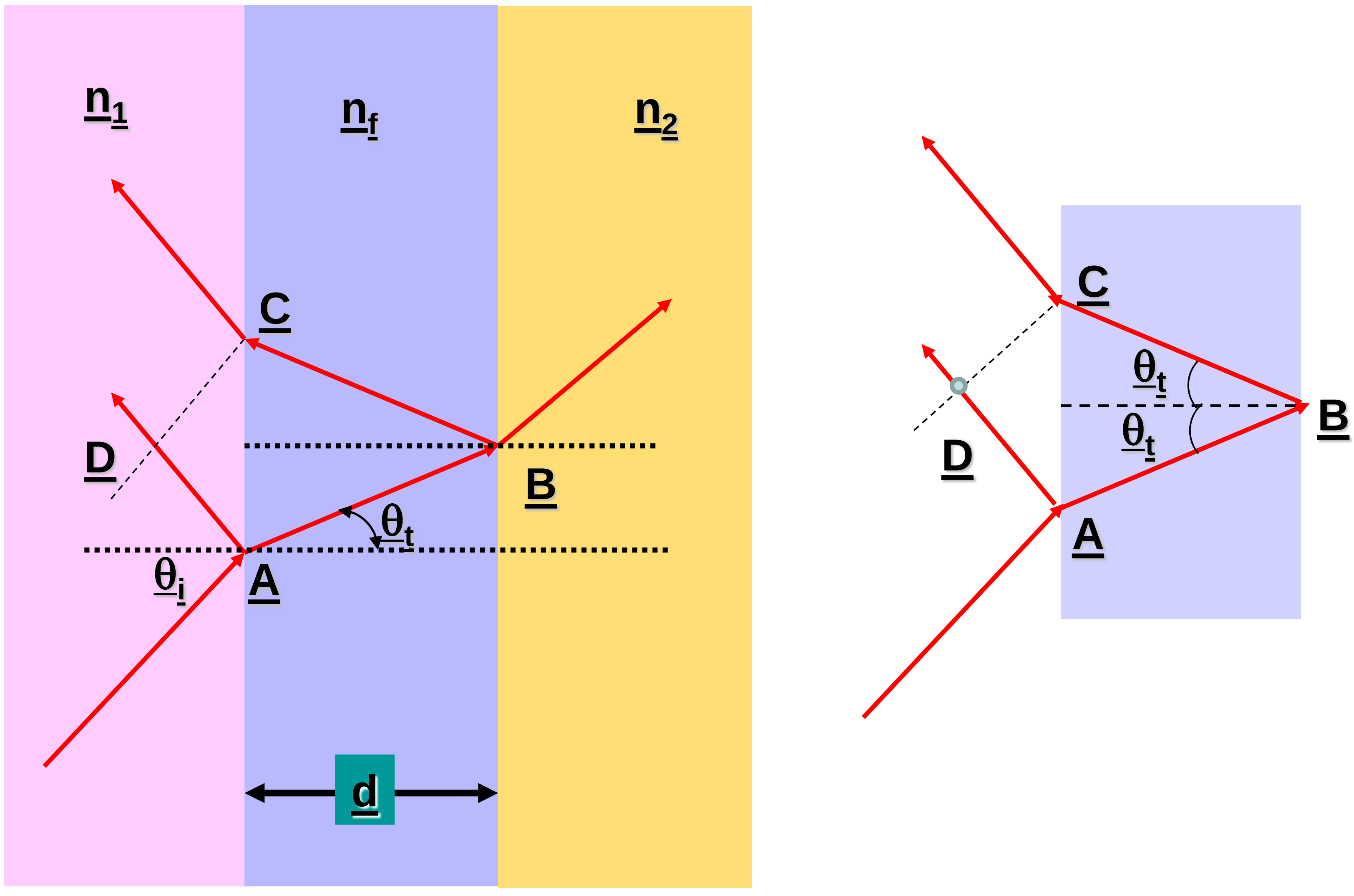
Colorful oil layer on a wet street



Soap Bubble

Source of images –
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/oilfilm.html
https://pxhere.com/en/photo/875196

Thin Film Interference



Optical Path

Path travelled by a ray is d in a medium with refractive-index n

Then phase gained by the ray due to this travel is $(\frac{2\pi}{\lambda}\, d)$.
Here λ is the wavelength of light in medium n2

The phase gained can also be written as $(\frac{2\pi}{\lambda_0}\frac{\lambda_0}{\lambda} d) = \frac{2\pi}{\lambda_0}\, nd)$
Where, $\frac{\lambda_0}{\lambda} = n$ (refractive index of medium in which ray has travelled)

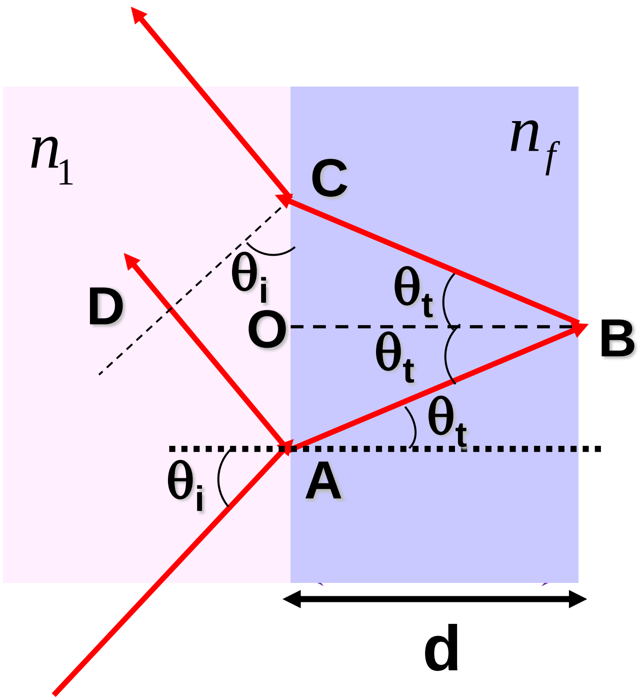
The optical path nd can be thought as the equivalent path in vacuum, where the wavelength of light is λ_0

Optical path difference for the first two reflected beams

$$\Lambda = n_f[AB + BC] - n_1(AD)$$

$$AB = BC = d/\cos\theta_t$$

$$AD = AC\sin\theta_i$$



$$AC = AO + OC$$

$$AO = OC = d\tan\theta_t$$

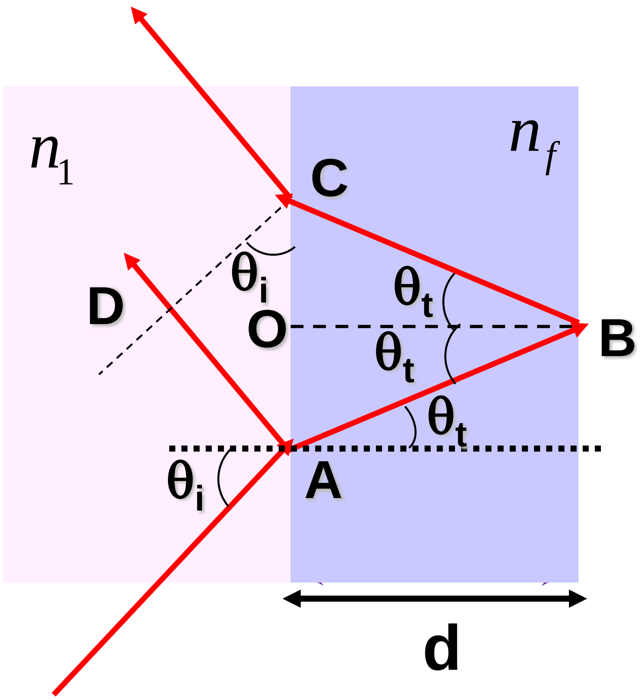
$$\text{Thus, } AD = (2d\tan\theta_t)\,\text{Sin}\theta_i$$

$$\text{Also } n_1\text{Sin}\theta_i = n_f\text{Sin}\theta_t \quad \text{(Snell's law)}$$

$$\text{Thus, } AD = (2d\tan\theta_t)\,\frac{n_f}{n_1}\text{Sin}\theta_t$$

Optical path difference for the first two reflected beams

$$\Lambda = n_f[AB + BC] - n_1(AD)$$

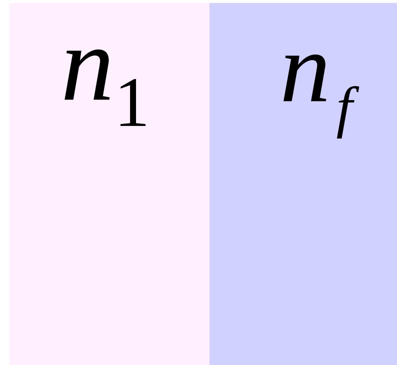


$$\Lambda = \frac{2dn_f}{\cos\theta_t} - 2dn_f\tan\theta_t\sin\theta_t$$

$$\Lambda = \frac{2dn_f}{\cos\theta_t}(1 - \sin^2\theta_t) = 2dn_f\cos\theta_t$$

Optical Path Difference

$$\Lambda = 2dn_f \cos\theta_t$$



$n_1 < n_f \Rightarrow \pi$ *phase shift*

$n_1 > n_f \Rightarrow 0$ *phase shift*

Phase shift (in the case of external reflection)

$$\delta = k_0 \Lambda \pm \pi$$

$$\delta = \frac{4\pi n_f}{\lambda_o} d \cos\theta_t \pm \pi$$

For $n_1 > n_f > n_2$, or $n_1 < n_f < n_2$,
the $\pm\pi$ phase shift will not be present

Phase shift $\Rightarrow$ $$\boxed{\delta = \frac{4\pi n_f}{\lambda_o} d\cos\theta_t \pm \pi}$$

$$\left(\lambda_f = \frac{\lambda_0}{n_f}\right)$$

Condition for maxima ($\delta = 2m\pi$)

$$d\cos\theta_t = (2m+1)\frac{\lambda_f}{4} \qquad m = 0,\ 1,\ 2,\dots$$

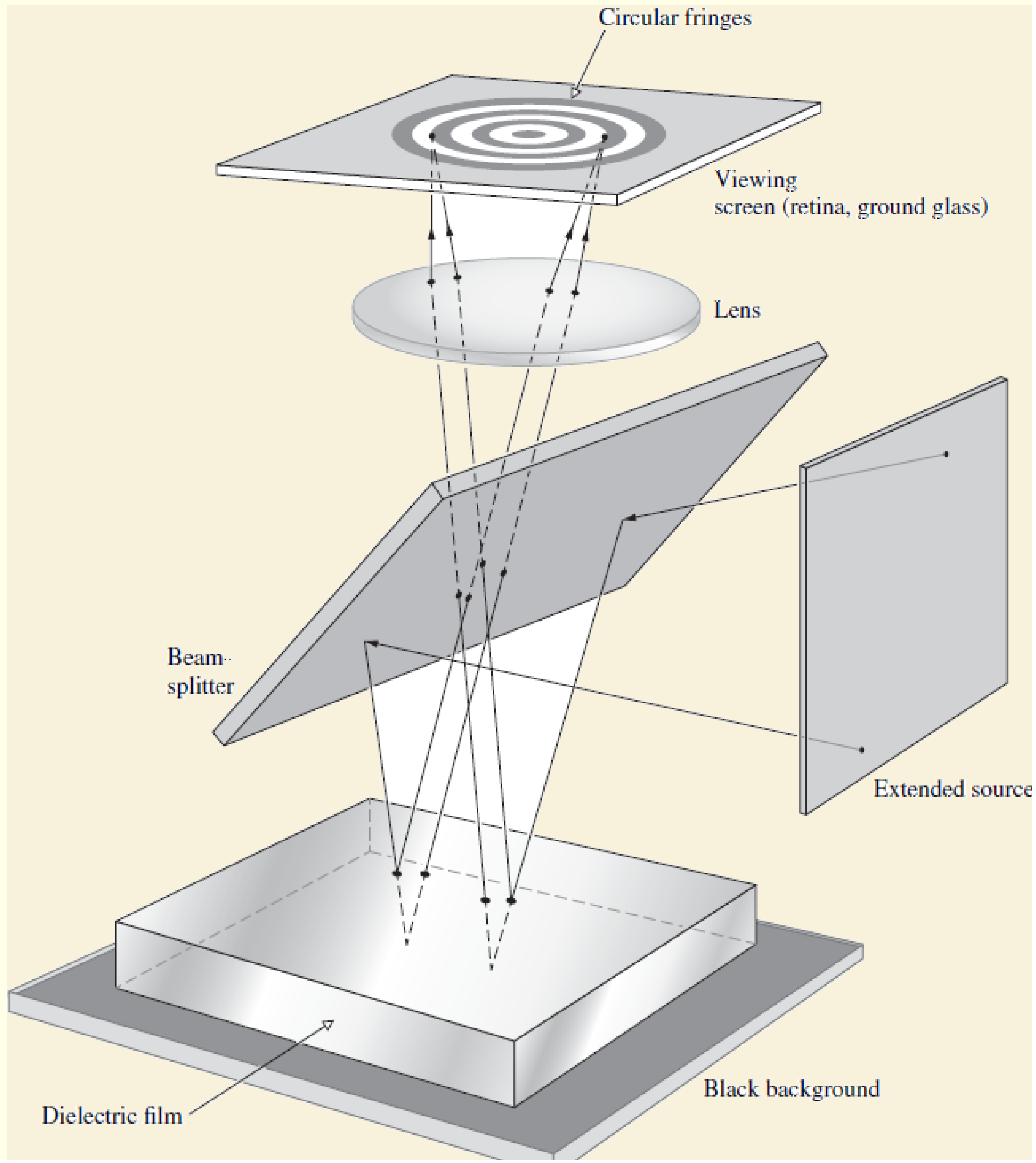
Condition for minima ($\delta = (2m+1)\pi$)

$$d\cos\theta_t = 2m\frac{\lambda_f}{4} \qquad m = 0,\ 1,\ 2,\dots$$

Note: Odd and even multiple of ($\lambda_f/4$)

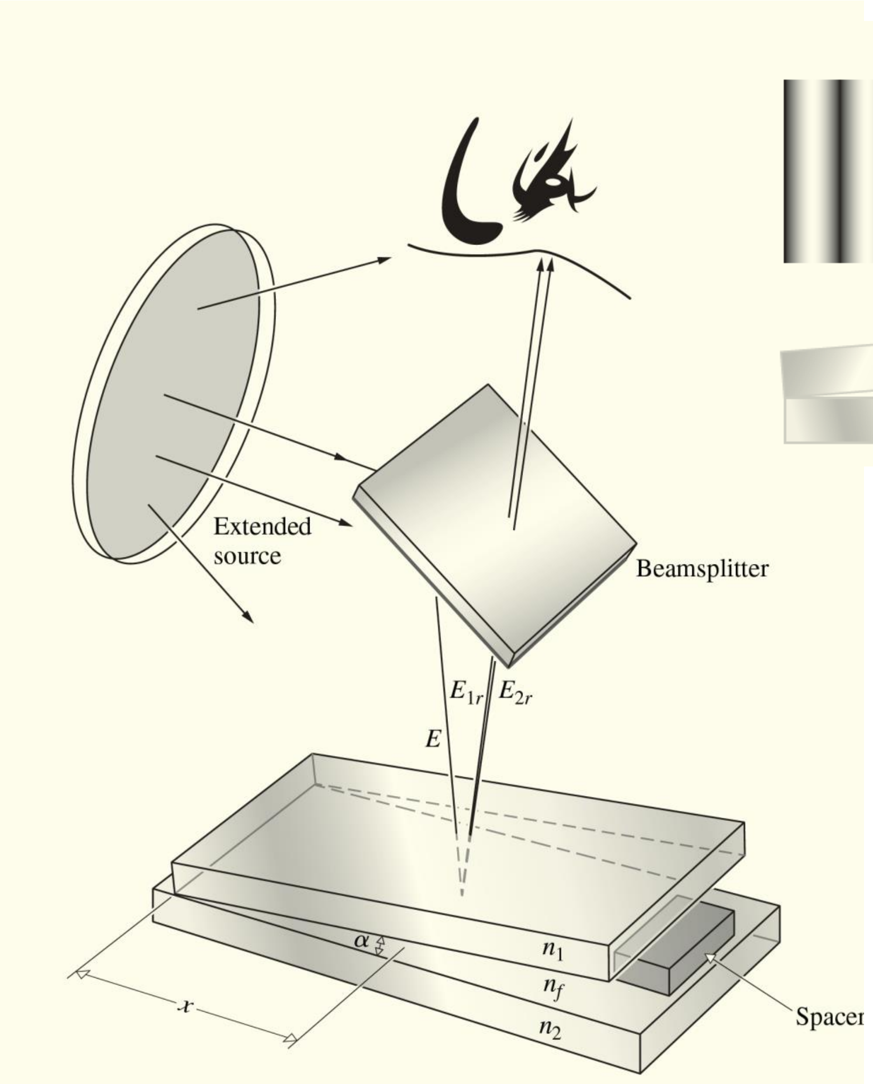
All rays incident with the same θ_i will satisfy same condition

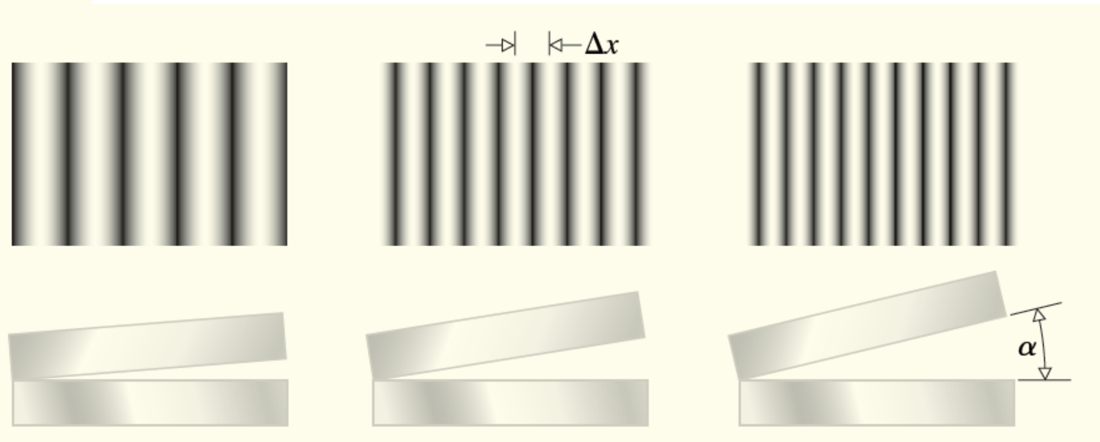
Formation of circular fringes for a uniform thickness dielectric film



Fizeau Fringes - Wedge

Fizeau Fringes (Fringes of equal thickness)

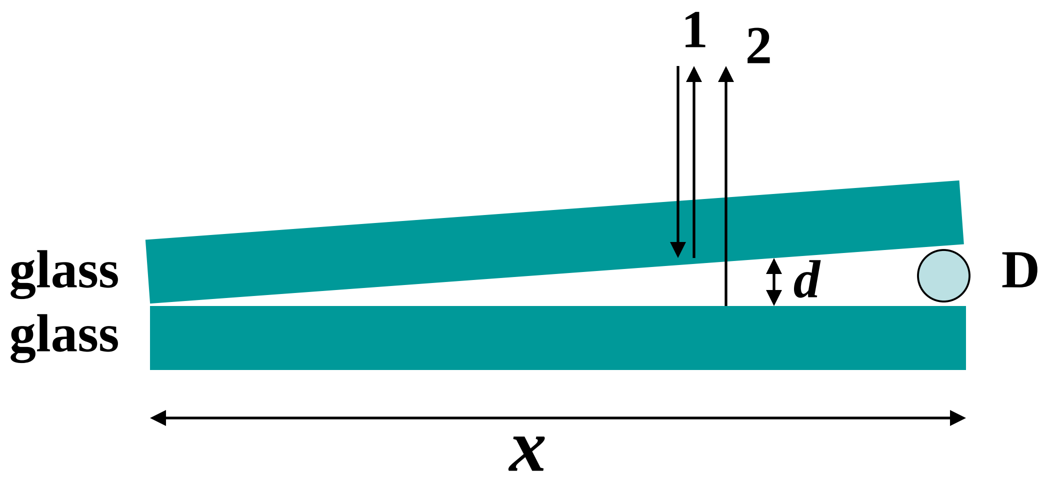




$$d = x\ \alpha$$

α: Wedge angle

Wedge between two plates



Refractive Index of wedge medium: n_f

Path difference $= 2d$

Phase difference $\delta = 2kd - \pi$

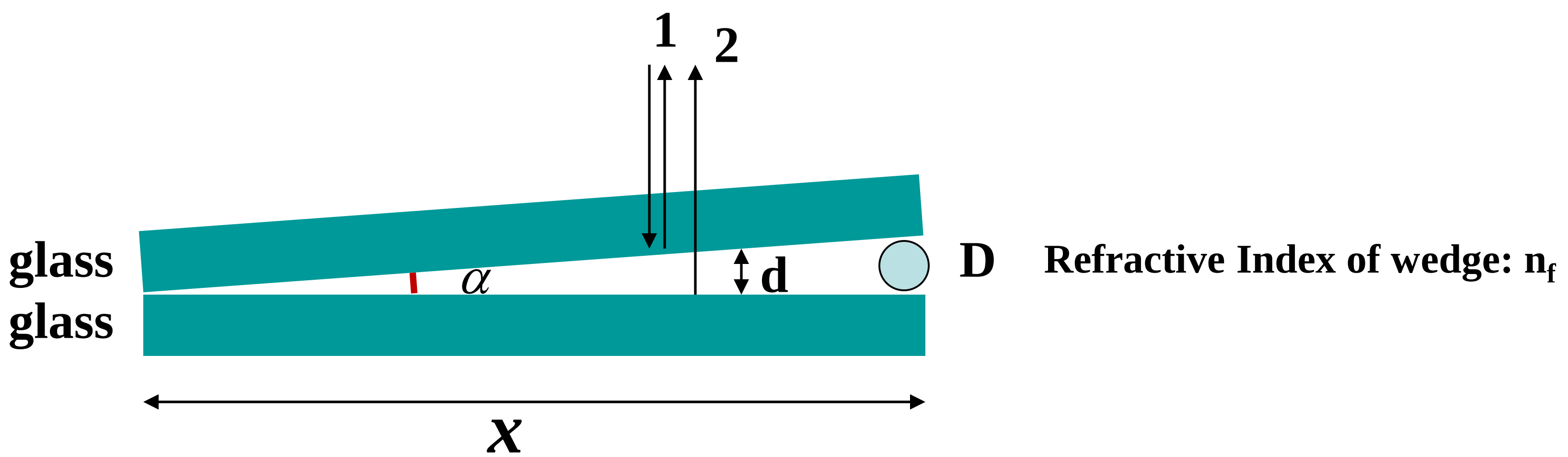
$$k = \frac{2\pi}{\lambda}$$

Maxima $2d_m = (2m + 1)\frac{\lambda}{2} = (m + ½)\, \lambda_o/n_f$

(m is an integer)

Minima $2d_m = m\lambda = m\lambda_o/n_f$

Conditions for maximum (For small values of θ_i)



$$(m+\frac{1}{2})\lambda_0 = 2n_f d_m$$

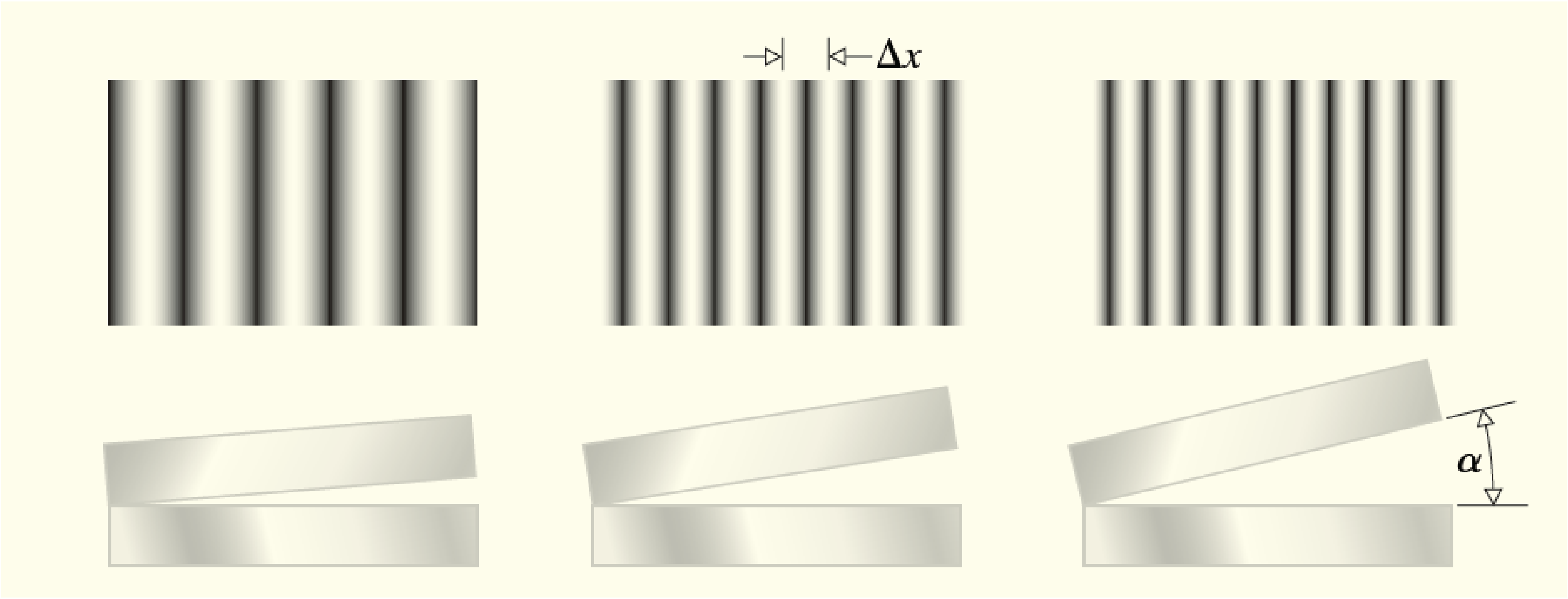
d is the thickness at a particular point

$$x_m = \left(\frac{m+1/2}{2\alpha}\right)\lambda_f$$

$$\boxed{d = x\alpha}$$

(α is a small angle)

Fringe width



Fringe width decreases with increasing wedge angle

$$x_m = \left(\frac{m + 1/2}{2\alpha}\right)\lambda_f$$

$$\Delta x = x_{m+1} - x_m$$

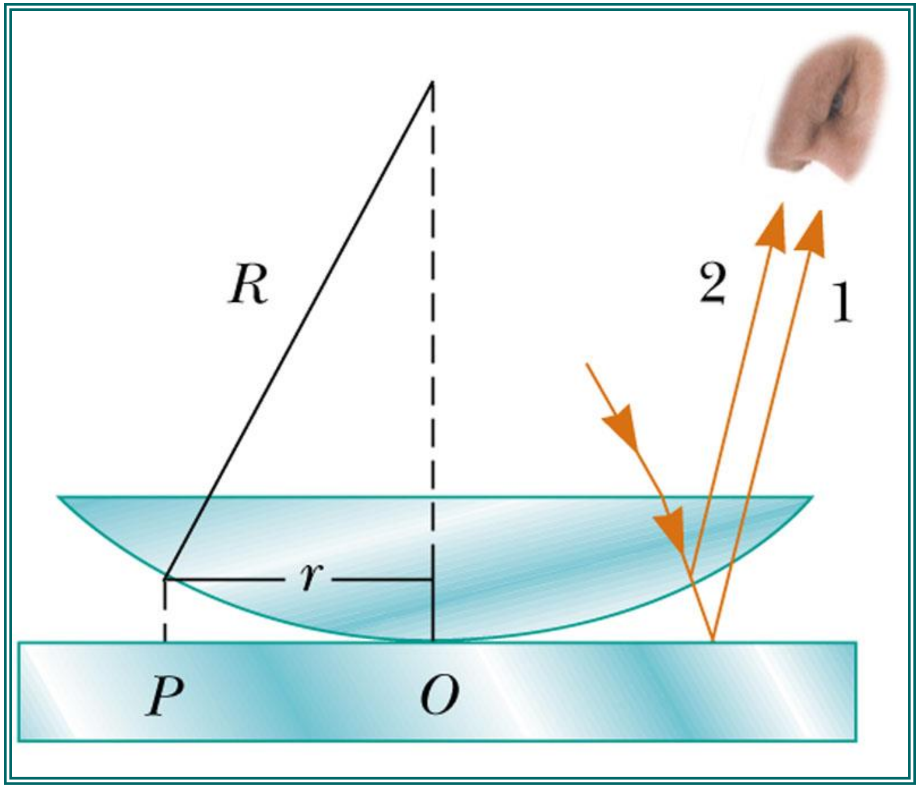
$$\Delta x = \frac{\lambda_f}{2\alpha}$$

By determining the fringe separation, one can determine α and, thus, the thickness of the spacer material can be determined

Newton's rings

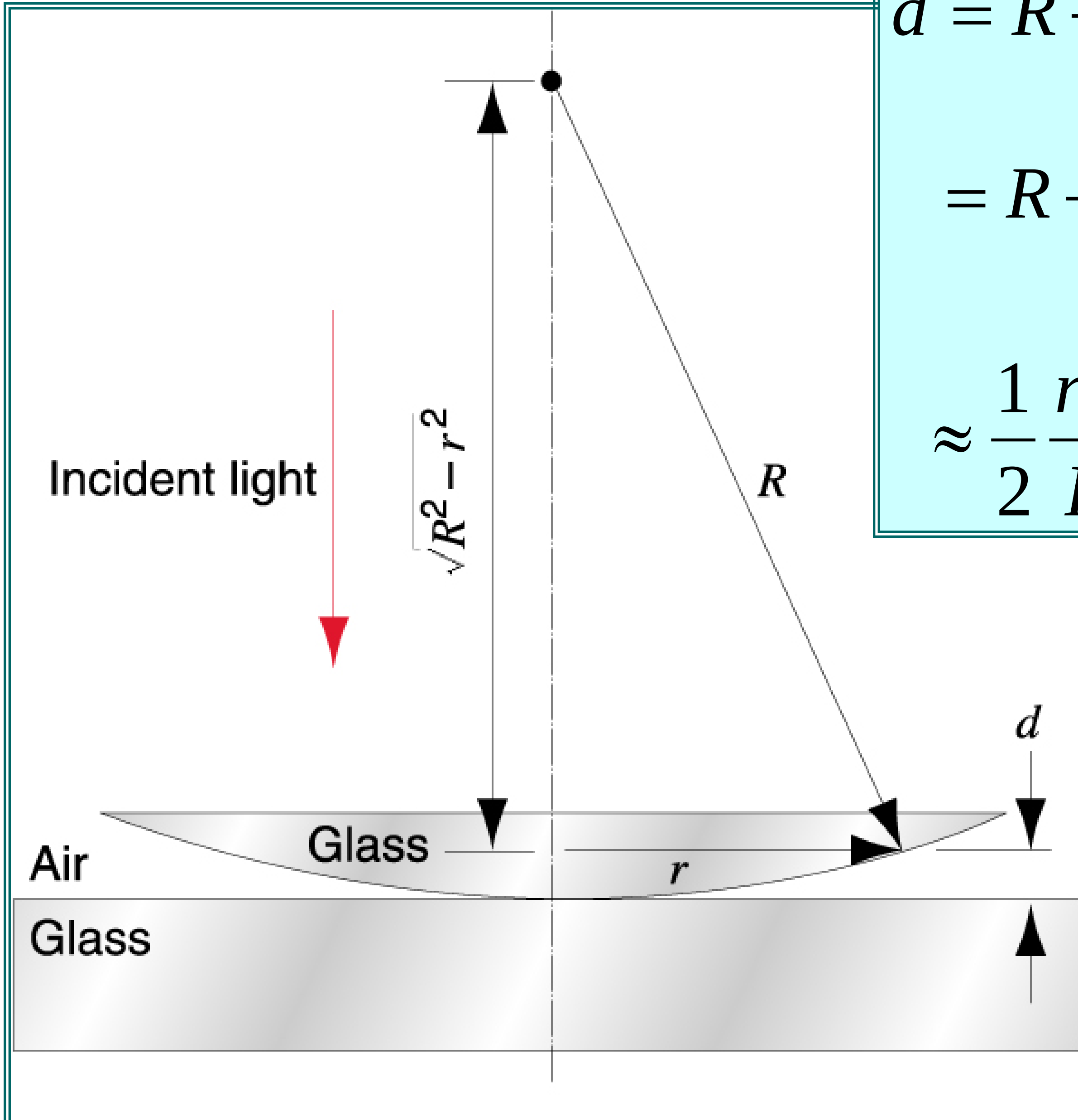
Newton's Ring

Ray 1 undergoes a phase change of 180° on reflection, whereas ray 2 undergoes no phase change



R = radius of curvature of lens

***r* = radius of Newton's ring**



$$d = R - \sqrt{R^2 - r^2}$$

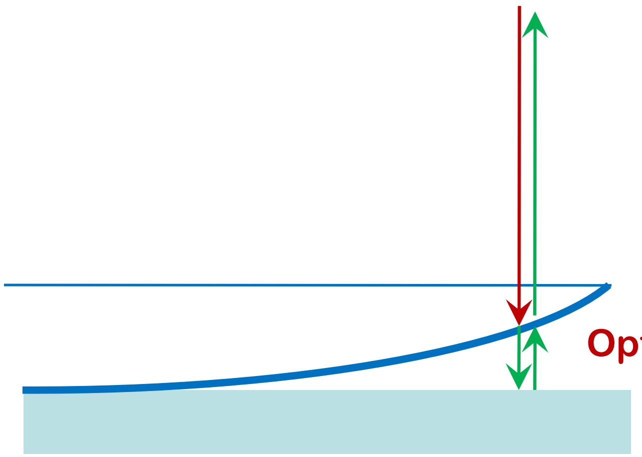
$$= R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \ldots\right]$$

$$\approx \frac{1}{2}\frac{r^2}{R}$$

For bright rings

(considering phase change of π for one of the rays)

$$2dn = (2m+1)\frac{\lambda}{2}$$



$$2dn$$

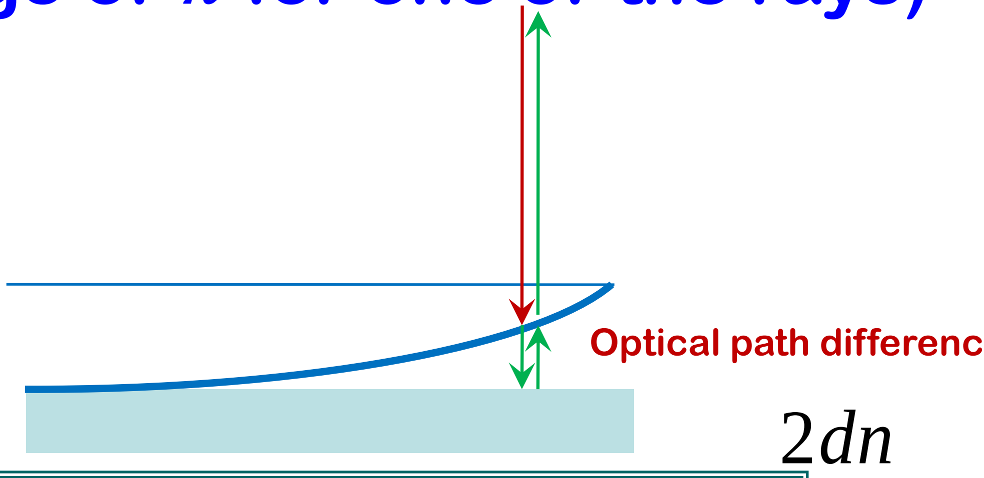
$$2\times\frac{1}{2}\frac{r^2}{R}n = (2m+1)\frac{\lambda}{2}$$

$$r_{bright} = \sqrt{(2m+1)R\frac{\lambda}{2n}} = \sqrt{(2m+1)R\frac{\lambda_n}{2n}}, \quad m = 0,1,2...$$

For dark rings

(considering phase change of π for one of the rays)

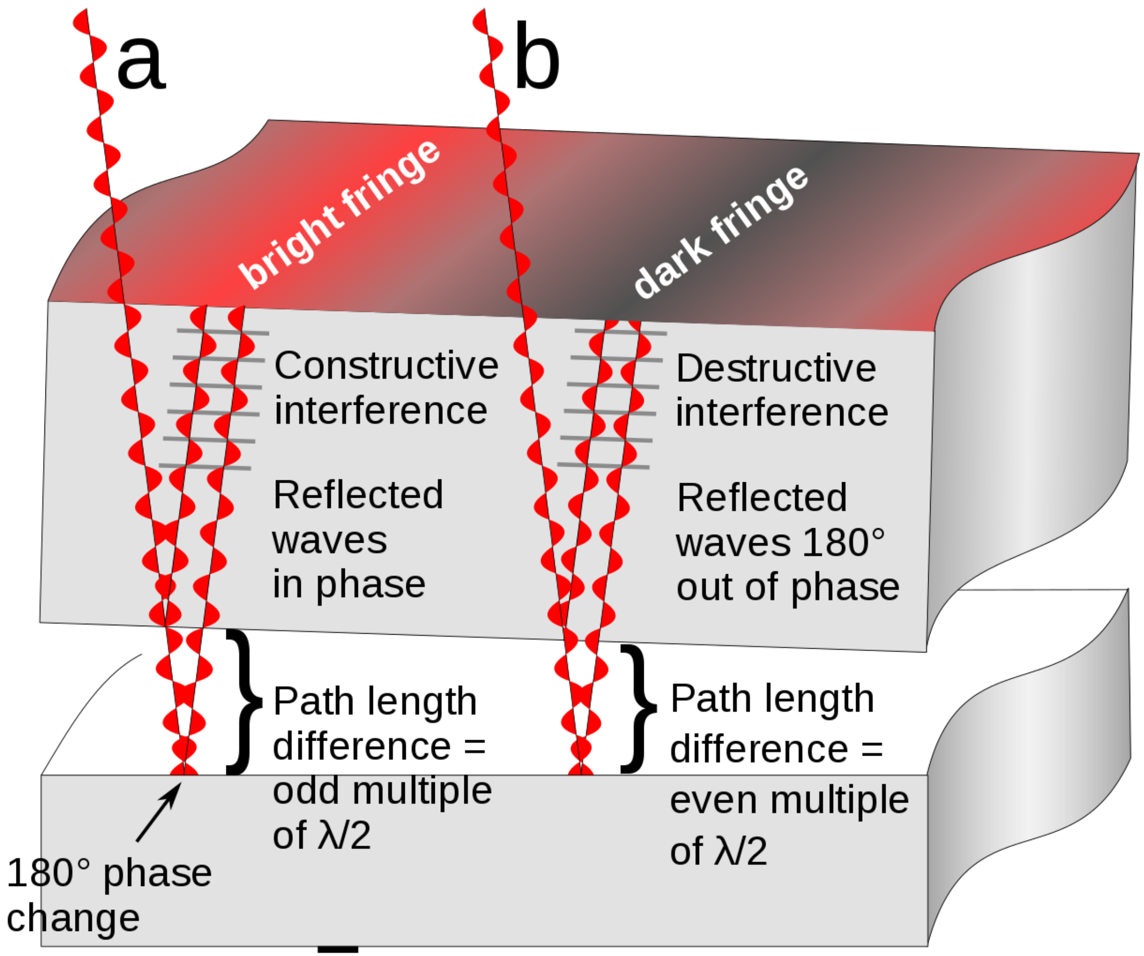
$$\boxed{d = \frac{1}{2}\frac{r^2}{R}}$$



$$2\mathrm{d}n = 2\mathrm{m}\frac{\lambda}{2}$$

$$r_{dark} = \sqrt{2mR\frac{\lambda_n}{2}},\ m = 0,1,2\ldots$$

Physical understanding of Newton's Rings



For bright fringe path difference

$$2dn = (2m+1)\frac{\lambda}{2}$$

For dark fringe path difference

$$2dn = 2m\frac{\lambda}{2}$$

Newton's Ring