Solutions of Engineering Mechanics Mid-semester Questions

1. The rigid frame shown in Fig. 1 supports a weight $W = 80$ kN. If the pulley with its centre at o is free to rotate, the self-weight of the pulley is 20 kN and its radius is 1m, gravity g acts downwards as indicated in the figure, all pin joints are frictionless and the weights of other members are neglected then find the reactions at joints A, B, C, D and E. Note that rollers at joint A cannot lift off the wall.

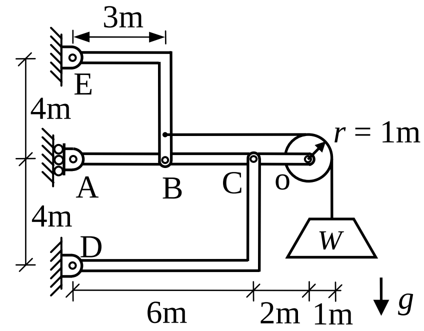
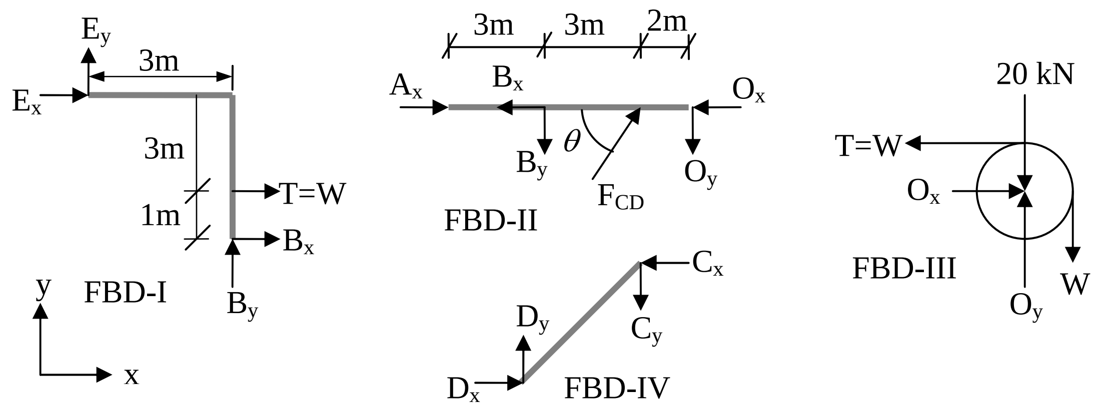


Fig. 1.

Solution:

Note that member CD is a two-force member. Then consider the following free body diagrams.



From FBD-III,

$$\sum M_z = 0 \Rightarrow T = W = 80 \text{ kN}. \sum F_x = 0 \Rightarrow O_x = T = 80 \text{ kN} \text{ and } \sum F_y = 0 \Rightarrow O_y = W + 20 \text{ kN} = 100 \text{ kN}.$$

From FBD-II,

$$\sum M_z = 0 \text{ taken at point C} \Rightarrow 3B_y - 2O_y = 0 \text{ or } B_y = \frac{2O_y}{3} = \frac{200}{3} \text{kN} = 66.67 \text{ kN}.$$

From given geometry, $\tan\theta = 4/6 = 2/3$ or $\theta = 33.7^{\circ}$.

$$\sum F_y = 0 \Rightarrow F_{CD}\sin\theta - B_y - O_y = 0 \text{ or } F_{CD} = \frac{B_y + O_y}{\sin\theta} = 300.38 \text{ kN}.$$

Then $C_x = 300.38\cos\theta$ kN $= 250$ kN and $C_y = 300.38\sin\theta$ kN $= 166.67$ kN

From FBD-IV, $D_x = C_x = 250$ kN and $D_y = C_y = 166.67$ kN

From FBD-I,

$$\sum M_z = 0 \text{ taken at point E} \Rightarrow 3T + 4B_x + 3B_y = 0 \text{ or } B_x = -\frac{3}{4}\left(B_y + W\right) = -110 \text{ kN}.$$

$$\sum F_x = 0 \Rightarrow E_x = -B_x - T = 30 \text{ kN} \text{ and } \sum F_y = 0 \Rightarrow E_y = -B_y = -66.67 \text{ kN}$$

Going back to FBD-II,

$$\sum F_x = 0 \Rightarrow A_x - B_x + C_x - O_x = 0 \Rightarrow A_x = B_x - C_x + O_x = -280 \text{ kN}$$

Validation/Cross-check:

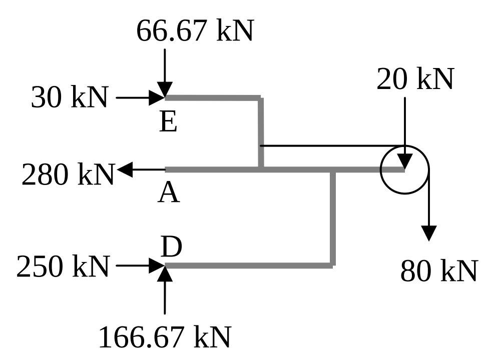
The global free body diagram is shown in the figure.

$E_y + D_y - W - 20\text{kN} = 0$ $\left(\text{validates} \sum F_y = 0\right)$

$E_x + A_x + D_x = 0$ $\left(\text{validates} \sum F_x = 0\right)$,

$4D_x - 4E_x - 6D_y - 6E_y - 2\times 20 - 3W = 0$ $\left(\text{validates} \sum M_z = 0 \text{ about C}\right)$

One can choose to take moment about any other point.



2. Find the forces in members AB, BC, CD and AD of the plane truss schematically shown in Fig. 2. The truss is loaded by a horizontal force of 2 kN and a vertical force of 5 kN. Assume all joints to be frictionless pin-joints and neglect the self-weight of all members. Note that the dashed lines in the figure are used for dimensioning purposes only.

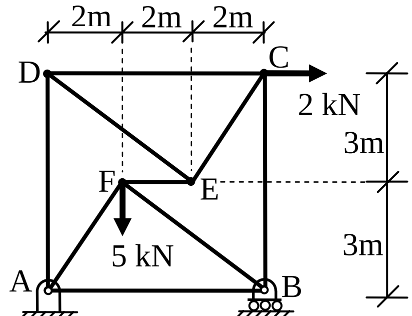
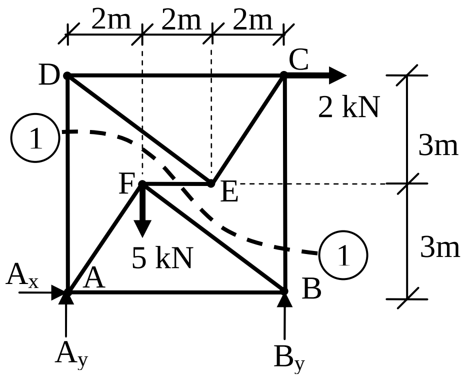


Fig. 2

Solution:

Consider the FBD of the whole truss.



From $\sum F_x = 0,\ \ A_x = -2\text{kN}$.

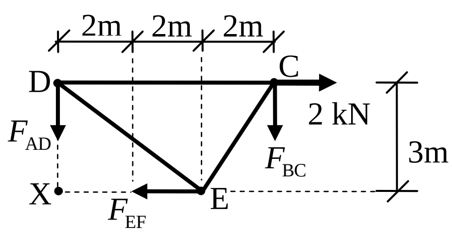
From $\sum M_z = 0$ about point A,

$$6B_y - 2\times 5 - 2\times 6 = 0 \Rightarrow B_y = \frac{11}{3}\text{kN} = 3.67\text{kN}.$$

From $\sum F_y = 0,\ \ A_y = (5-3.67)\text{kN} = 1.33\text{kN}.$

Note: For actual problem solution, one may not calculate reactions at both the joints. So, a student will not be penalized if he/she has calculated reactions at one joint, but solved the problem correctly.

Let us cut the truss into two parts through section ①—①. Now consider FBD of the top part of the section ①—① noting that all the members are assumed to be two-force members and joints are ideal.



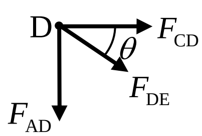
From $\sum F_x = 0,\ \ F_{EF} = 2\text{kN}$.

From $\sum M_z = 0$ about point X (See figure),

$$2\times 3 + 6F_{BC} = 0 \Rightarrow F_{BC} = -1\text{kN}.$$

From $\sum F_y = 0,\ \ F_{AD} = -F_{BC} = 1\text{kN}.$

Let us now consider equilibrium of joint D (alternatively, one can choose joint C).

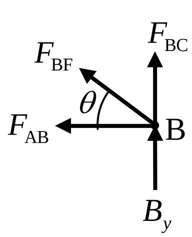


From $\sum F_x = 0,\ \ F_{DE}\cos\theta + F_{CD} = 0$.

From $\sum F_y = 0,\ \ F_{DE}\sin\theta + F_{AD} = 0$.

$$\Rightarrow \frac{F_{AD}}{F_{CD}} = \tan\theta = \frac{3}{4}(\text{from geometry}),\ \text{so}\ \ F_{CD} = \frac{4}{3}F_{AD} = \frac{4}{3}\text{kN} = 1.33\text{kN}.$$

Let us now consider equilibrium of joint B (alternatively, one can choose joint A).



From $\sum F_x = 0,\ \ F_{AB} = -F_{BF}\cos\theta$.

From $\sum F_y = 0,\ \ F_{BC} + B_y = -F_{BF}\sin\theta$.

$$\Rightarrow \frac{F_{BC}+B_y}{F_{AB}} = \tan\theta = \frac{3}{4}(\text{from geometry}),\ \text{so}\ \ F_{AB} = \frac{4}{3}(-1+3.67)\text{kN} = \frac{32}{9}\text{kN} = 3.56\text{kN}$$

Thus, the forces in the mentioned members are

$$F_{AB} = 3.56\text{kN (Tension)},\ F_{BC} = 1\text{kN (Compression)}$$

$$F_{CD} = 1.33\text{kN (Tension)},\ F_{AD} = 1\text{kN (Tension)}$$

Validation: One can check equilibrium at joint C (and/or A).

At joint C, if $\phi = \tan^{-1}(2/3)$ is angle between CE and BC then

$F_{CE}\cos\phi = -F_{BC} = 1\text{kN}$ and $F_{CE}\sin\phi = 2 - F_{CD} = 0.66\text{kN}$ which validates $\tan\phi = 2/3$.

3. A mass and pulley system is rested on a 45° wedge as shown in Fig. 3. Both the pulleys A and B are mass-less. Pulley B is free to rotate whereas pulley A is fixed (cannot rotate). The coefficient of static friction is 0.2 for all surfaces in contact including that between pulleys and the mass-less inextensible ropes passing them. Find the minimum value of mass M in terms of variable m such that the mass $3m$ does not slide down. Gravity g acts downward as indicated in the figure.

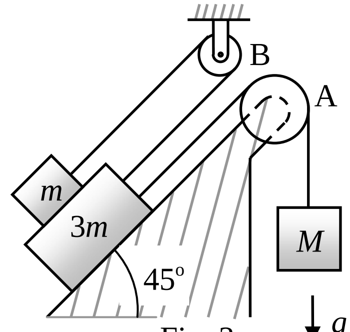
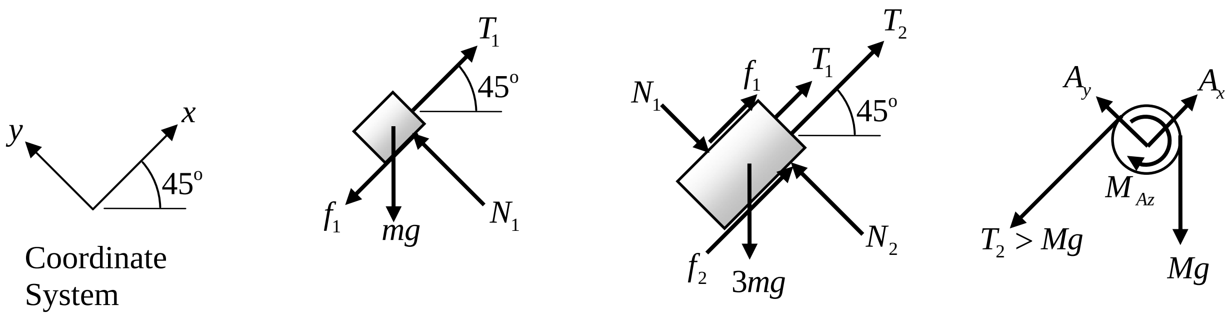


Fig. 3.

Solution:

The constraints are such that under impending motion, sliding must be simultaneous at all contacts except at pulley B where tensions on ropes at its both sides are same. For impending downward motion of the block of mass 3*m*, the following FBDs can be drawn. Note that the coordinate system (*x*-*y* frame) is chosen in the specified way in order to facilitate the calculations.



FBD-I FBD-II FBD-III

For simultaneous impending motion, we assume the following:

$f_1 = \mu_s N_1$, $f_2 = \mu_s N_2$ and $T_2 = e^{\mu_s \beta} Mg$ where $\beta = 3\pi/4$ is the angle of wrap of belt at pulley A.

Further note that moment equations are useless here because tipping conditions are not considered (bodies have wide bases and the geometric dimensions are not provided). Thus, we neglect the possibilities of tipping. The actual lines of action of the normal reaction forces are thus unknown and are thus shown arbitrarily in the FBDs. We will only use force balance equations for the solution.

From FBD-I:

$$\sum F_y = 0 \Rightarrow N_1 - mg\cos 45^\circ = 0 \text{ or } N_1 = mg/\sqrt{2}.$$

$$\sum F_x = 0 \Rightarrow T_1 - f_1 - mg\sin 45^\circ = 0 \text{ or } T_1 = \mu_s N_1 + mg/\sqrt{2} = 1.2\,mg/\sqrt{2}.$$

From FBD-II:

$$\sum F_y = 0 \Rightarrow N_2 = N_1 + 3mg\cos 45^\circ = 4\,mg/\sqrt{2}.$$

$$\sum F_x = 0 \Rightarrow T_1 + T_2 + f_1 + f_2 - 3mg\sin 45^\circ = 0$$

or

$$T_1 + T_2 + \mu_s\left(N_1 + N_2\right) - 3mg/\sqrt{2} = 0$$

$$\Rightarrow T_2 = 3mg/\sqrt{2} - 1.2mg/\sqrt{2} - 0.2\left(5mg/\sqrt{2}\right) = 0.8mg/\sqrt{2}.$$

From FBD-III:

$$\frac{T_2}{Mg} = e^{\mu_s \beta} = e^{0.2\times 3\pi/4} = 1.6$$

$$\Rightarrow \frac{0.8mg}{\sqrt{2}Mg} = 1.6 \text{ or } M = \frac{m}{2\sqrt{2}} = 0.3536m \text{ is the required minimum mass.}$$

Note: If someone uses $Mg/T_2 = 1.6$, it is obviously wrong for impending downward motion of block of mass 3*m*.

4. A front wheel driven vehicle is at rest on a horizontal plane with its rear wheels touching a step bump as shown in Fig. 4. The vehicle weighs *W*=5000N, 40% of the vehicle weight acts on the rear axles and the rest 60% acts on the front axles. The front wheels are connected to the engine and the rear wheels rotate freely. The height of the step bump is *h* = 5cm and the radius of all wheels is *r* = 30cm. If the driver starts the engine and tries to drive over the step bump at the rear wheels, find the minimum required value of static friction coefficient (μ_s) between the tyres and the road to successfully do so. If $\mu_s = 0.5$ then find the engine torque required at the front wheels. Neglect rolling friction and friction at the axles.

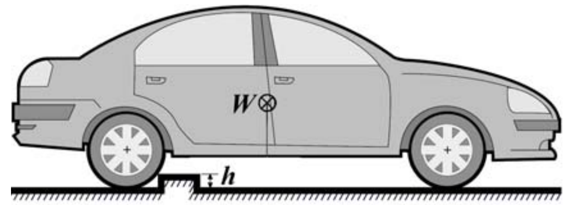
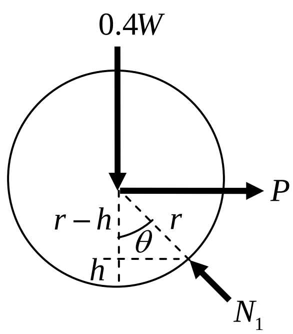


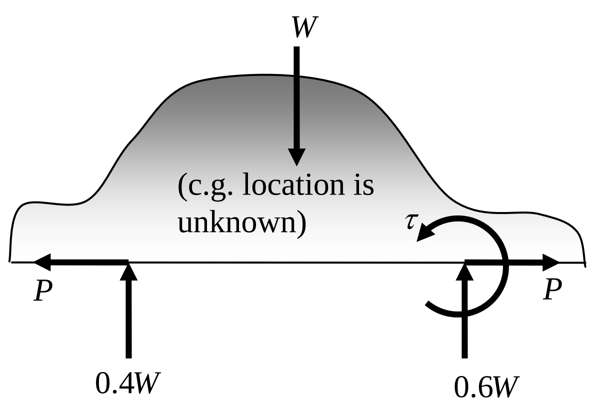
Fig. 4.

Solution:

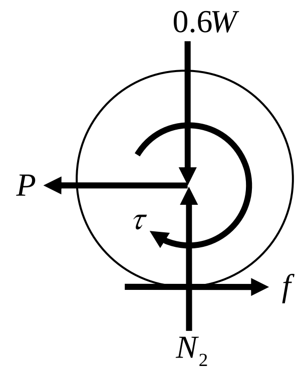
For the rear wheel to just climb over the bump, the normal reaction will exist only at the corner of the bump. Then the following FBDs can be drawn.



FBD of rear wheel(s)



FBD of car body



FBD of front wheel(s)

The traction force is *f* which is generated in the front wheels (axle) where the applied engine torque is τ .

From FBD of the rear wheel:

$$\cos\theta = \frac{r-h}{h} = \frac{30-5}{30} = \frac{5}{6}, \quad \theta = 33.56^{o}.$$

$$\sum F_y = 0 \Rightarrow N_1 \cos\theta = 0.4W \text{ or } N_1 = 0.4\frac{6}{5}W = 0.48W .$$

$$\sum F_x = 0 \Rightarrow P = N_1 \sin\theta = 0.265W$$

From FBD of the front wheel

$\sum F_x = 0 \Rightarrow f = P = 0.265W$. (maximum traction force)

For impending slip (just enough friction), $f = \mu_s N_2 = 0.6\mu_s W$.

Equating both, $0.6\mu_s W = 0.265W$ or minimum required $\mu_s = 0.265/0.6 = 0.442$.

If $\mu_s = 0.5$ then the vehicle will go over the bump. The minimum traction force required is still the same, i.e., $P = 0.265W$.

Thus, $f = P = 0.265W$ which is less than $\mu_s N_2$ (or $0.3W$).

The required driving torque is thus $\tau = fr = 0.265 \times 5000 \times 0.3$ Nm $= 397.5$Nm $\simeq 398$Nm.

Note: it is a mistake to take $f = \mu_s N_2 = 0.3W$ and then calculate corresponding torque requirement.

5. An inverted T-section of uniform thickness is hung from the ceiling through an inextensible mass-less rope as shown in Fig. 5(a). The end of the rope connected to the T-section lies on the axis of symmetry of the section. The weight of the T-section is W and a horizontal force P having the same magnitude as W acts at the location indicated in Fig. 5(a). The dimensions of the T-section are shown in Fig. 5(b) in which value of h is unknown. If the axis of symmetry of the T-section is along the line of action of the rope tension during equilibrium then **(a)** find the value of parameter h (See Fig. 5(b)), and **(b)** If A-A' is a centroidal axis then using the obtained value of h, find the second moment of area about A-A' axis.

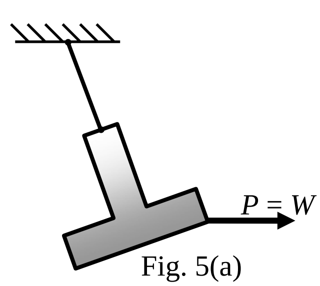


Fig. 5(a)

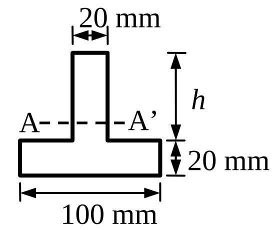
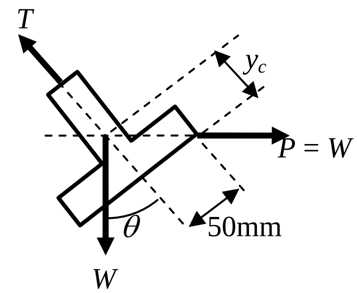


Fig. 5(b)

Solution:

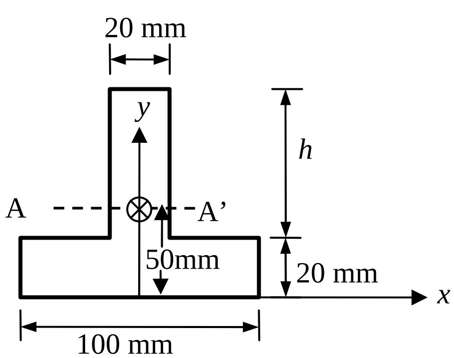


In equilibrium, it is given that the line of action of string force is along the axis of symmetry of the T-section. The centroid of the T-section also lies on the same axis of symmetry. For three forces to remain in equilibrium, they must be co-planar and concurrent (moment balance requirement).

The lines of action of string tension and the self-weight of the T-section intersect at the centroid of the T-section. Thus, force P must also pass through the centroid as shown in the FBD of the T-section.

From the FBD of the T-section, we find $\tan\theta = \dfrac{P}{W} = 1$, $\theta = 45^{\circ}$ and $T = \sqrt{2}W$. The horizontal force P (or the vertical force W) makes 45° angle with the axis of symmetry.

Thus, the centroid of the T-section lies on the axis of symmetry at a distance of $y_c = 50\text{mm}$ from the bottom of the section.



For the section with its centroid as shown, we can write

$y_c = \dfrac{\sum_{i=1,2} A_i y_{ci}}{\sum_{i=1,2} A_i}$ where A_i and y_{ci} ($i = 1,2$) refer to areas of two rectangular sections and their centroid positions along y-axis.

So, $50 = \dfrac{100\times 20\times 10 + 20\times h\times(20 + h/2)}{100\times 20 + 20\times h}$

which leads to a quadratic equation $h^2 - 60h - 8000 = 0$.

On solving, $h = \dfrac{60 \pm \sqrt{3600 + 32000}}{2}$ mm. Choosing the positive value, $h = 124.34\text{mm}$.

For calculating 2nd moment of area about A-A' axis, we use the parallel axis theorem. The distance of A-A' axis from the centroid of the lower horizontal rectangle is $50 - 10 = 40$ mm and from the centroid of the upper vertical rectangle is $20 + (124.34/2) - 50 = 32.17$ mm.

Then,

$$I_{AA'} = \frac{100\times 20^3}{12} + 100\times 20\times 40^2 + \frac{20\times 124.34^3}{12} + 20\times 124.34\times 32.17^2$$

$$= 9044195.7\text{mm}^4 \text{ or } 0.09442\text{cm}^4 \text{ or } 9.044\times 10^{-6}\text{m}^4.$$