Problem Set - 4 Autumn 2019

ANSWER/HINTS

1. (a) Put $x = r\cos\theta$, $y = r\sin\theta$

2. (a) Put $y = mx$

   (b) Put $x - y = mx^3$

3. (a) $f_x(x,y) = 2x\tan^{-1}(\frac{y}{x}) - y$
   $f_y(x,y) = x - 2y\tan^{-1}(\frac{x}{y})$

   (b) $f_x(x,y) = \frac{-1}{(x^3+y^3)}\{\sin x + \frac{3x^2(\sin y + \cos x)}{(x^3+y^3)}\}$
   $f_y(x,y) = \frac{1}{x^3+y^3}\{\cos y - \frac{3y^2(\sin y + \cos x)}{(x^3+y^3)}\}$

   (c) $f_x(x,y) = \frac{xe^{(x^2+y^2)}}{\sqrt{x^2+y^2}}\{2 - \frac{1}{(x^2+y^2)}\} - \frac{1}{x}$
   $f_y(x,y) = \frac{ye^{(x^2+y^2)}}{\sqrt{x^2+y^2}}\{2 - \frac{1}{(x^2+y^2)} + \frac{1}{y}$

   (d) $f_x(x,y) = \frac{y(x^2-10xy-5y^2)}{(x^2+xy)^2}$
   $f_y(x,y) = \frac{-x(x^2-10xy-5y^2)}{(x^2+xy)^2}$

   (e) $f_x(x,y) = \frac{y(y^2-x^2)}{(x^2+y^2)^2}\cosh(\frac{xy}{x^2+y^2})$
   $f_y(x,y) = \frac{x(x^2-y^2)}{(x^2+y^2)^2}\cosh(\frac{xy}{x^2+y^2})$

   (f) $f_x(x,y) = \frac{x}{(x^2-4y^2)^2}(x^3 - 12xy^2 - 6y^3)$
   $f_y(x,y) = \frac{y}{(x^2-4y^2)^2}(8x^3 + 9x^2y - 12y^3)$

4. (a) $f_x(0,0) = 0, f_y(0,0) = 0, f_x(0,y) = 0, f_y(x,0) = x$

   (b) $f_x(0,0) = 0, f_y(0,0) = 0, f_x(0,y) = y, f_y(x,0) = x$

   (c) $f_x(0,0) = 0, f_y(0,0) = 0, f_x(0,y)$ and $f_y(x,0)$ do not exist

   (d) $f_x(0,0) = 2, f_y(0,0) = -2, f_x(0,y) = (e^y + e^{-y}), f_y(x,0) = -(e^x + e^{-x})$

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5. (a) Differentiable

   (b) Not differentiable

   (c) Differentiable

   (d) Not differentiable

6. (a) Put $x = r\cos\theta, y = r\sin\theta, k = -h$

   (b) Put $h = k$

7. (a) Put $k = 4h, y = mx$(in case of $f_x(x, y)$,
   $x = my$(in case of $f_y(x, y)$)

8. (a) Differentiable

   (b) Differentiable

   (c) Not Differentiable

9. (a) $dw = \dfrac{2z^2dz}{(x^2+y^2)} - \dfrac{4z^3}{3(x^2+y^2)^2}(xdx + ydy)$

   (b) $dz = \dfrac{1}{x^2+y^2}\{ydx - xdy\}$

   (c) $du = \dfrac{e^{\tan^{-1}(3x+4y+5z)}}{1+\tan(3x+4y+5z)}(3dx + 4dy + 5dz)\{\dfrac{1}{1+(3x+4y+5z)^2} - \dfrac{\sec^2(3x+4y+5z)}{1+\tan(3x+4y+5z)}\}$

   (d) $dw = \dfrac{-\sin(xyz)}{y^2\ln(x^2z)+x^3y^3}(yzdx+xzdy+xydz) - \dfrac{\cos(xyz)}{(y^2\ln(x^2z)+x^3y^3)^2}\{(\dfrac{2y^2}{x} + 3x^2y^3)dx + (2y\ln(x^2z) + 3x^3y^2)dy + \dfrac{y^2}{z}dz\}$

   (e) $dw = \dfrac{(3x^2\sin y - y^3\sin x)dx + (x^3\cos y + 3y^2\cos x)dy}{e^x\ln y + \sin y\ln x} - \dfrac{(x^3\sin y + y^3\cos x)}{(e^x\ln y + \sin y\ln x)^2}\{(e^x\ln y + \dfrac{\sin y}{x})dx + (\dfrac{e^x}{y} + \cos y\ln x)dy\}$

   (f) $du = \dfrac{e^{\sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)}(xdx + ydy + zdz)\{1 - \dfrac{1}{\sqrt{x^2+y^2+z^2}}\}$

(g) $dw = \frac{1}{(x^2+y^2+z^2)}[\{1+\ln(xy)\}dx + \{\frac{x}{y} - \cos(y+2z)\}dy -$
$2\cos(y+2z)dz] - \{\frac{x\ln(xy) - \sin(y+2z)}{(x^2+y^2+z^2)^2}\}(2xdx + 2ydy + 2zdz)$

(h) $dw = \frac{1}{y}e^{\frac{x}{y}}dx - \frac{1}{y^2}(xe^{\frac{x}{y}} + ze^{\frac{z}{y}})dy + \frac{1}{y}e^{\frac{z}{y}}dz$

(i) $dz = \frac{e^{\cos(xy)}}{e^{(x^2+y^2)}}[\{1-2x^2 - xy\sin(xy)\}dx - \{x^2\sin(xy) + 2xy\}dy]$

(j) $dz = \frac{1}{2\sqrt{y^2 - \frac{x^2}{2}}\sin\sqrt{y^2-\frac{x^2}{2}}}\cos(\sqrt{y^2 - \frac{x^2}{2}})(2ydy - 2xdx)$