

Answer sheet - 11

AUTUMN 2019

MATHEMATICS-I (MA10001)(Complex Analysis)

1. (a) (i) Ans: $16\frac{1}{2}$. Parametric form $x = 2t, y = t^2 + 3$ and $0 \leq t \leq 1$.
(ii) Ans: $17\frac{1}{6}$. In the first curve $0 \leq x \leq 2, y = 3$ and in the second curve $x = 2, 3 \leq y \leq 4$.
(iii) Ans: $16\frac{1}{6}$. C is $y = \frac{x}{2} + 3, 0 \leq x \leq 2$.
(b) Ans: $1 - i$. Parametric form $x = t, y = t$ and $0 \leq t \leq 1$.
(c) Ans: $10 - 8i$. Parametric form $C_1 : x = 0, y = t$ and $0 \leq t \leq 2$ and $C_2 : x = t, y = 2$ and $0 \leq t \leq 4$.
(d) Ans: πi . Parametric form $C_1 : z = e^{it}$ and $0 \leq t \leq \pi$ and $C_2 : x = t, y = 0$ and $-1 \leq t \leq 1$.
(e) Ans: $-4 + 2\pi i$
2. (a) Hint: In all the cases value of integral is $\frac{1}{3}(5i + 1)$
(i) Parametric form $x = t, y = t$ and $0 \leq t \leq 1$.
(ii) Parametric form $C_1 : z(t) = t, 0 \leq t \leq 1$ and $C_2 : z(t) = 1 + it, 0 \leq t \leq 1$.
(iii) Parametric form $z(t) = t + it^2$ and $0 \leq t \leq 1$.
(b) Hint: Find the value of this integral along line segments OA, AB, BC and CO and then add.
Parametric form $OA : z(t) = t, 0 \leq t \leq 1$, $AB : z(t) = 1 + it, 0 \leq t \leq 1$, $BC : z(t) = 1 - t + i, 0 \leq t \leq 1$ and $CO : z(t) = i(1 - t), 0 \leq t \leq 1$.
3. (i) Ans: πi . Here $-3/2$ lies inside C and use Cauchy integral formulae.
(ii) Ans: $6\pi i$. Here $0, 1$ lies inside C , use partial fraction and Cauchy integral formulae.
(iii) Ans: $2\pi i(\frac{1}{2} - e)$. Here $0, 1$ lies inside C , use partial fraction and Cauchy integral formulae.
(iv) Ans: $13\pi i$. Here $1/2$ lies inside C and use Cauchy integral formulae.
(v) Ans: Use the polar form of z .
4. (i) Hint: Use ML inequality. Here $l = \pi$ and $|z^2 + 1| \geq 3$.
(ii) Hint: Use ML inequality. Here $l = \sqrt{2}$ and $|z| \geq 1/\sqrt{2}$.
5. (i) Ans: $-2\pi i(1 + \pi i)$. Here πi lies inside C and use Cauchy integral formulae.
(ii) Ans: $-\frac{8\pi i}{3} \cosh \frac{\pi}{2}$. Here $\pi i/4$ lies inside C and use Cauchy integral formulae.
(iii) Ans: $\frac{\pi}{16}$. Here $2i$ lies inside C and use Cauchy integral formulae.
(iv) Ans: $\frac{\pi i}{4}$. Here only 0 lies inside C and use Cauchy integral formulae.
(v) Ans: $4\pi i$. Here $1, 2$ lies inside C , use partial fraction and Cauchy integral formulae.
6. (i) Ans: $2\pi i$. Use Cauchy integral formulae.
(ii) Ans: 0 . Use Cauchy integral formulae.
(iii) Ans: $2\pi i$. Use partial fraction and Cauchy integral formulae.
7. (i) Hint: Find the value of this integral along the boundary of circle using polar form of circle.
Parametric form $C : z = e^{i\theta}$ and $0 \leq \theta \leq 2\pi$.
(ii) Hint: Find the value of this integral along the boundary of circle using polar form of circle.
Parametric form $C : z = 1 + 3e^{i\theta}$ and $0 \leq \theta \leq 2\pi$.
(iii) Hint: Find the value of this integral along the boundary of square using Parametrization of square. Parametric form $C_1 : z(t) = t, 0 \leq t \leq 1$, $C_2 : z(t) = 1 + it, 0 \leq t \leq 1$, $C_3 : z(t) = t + i, 1 \leq t \leq 0$ and $C_4 : z(t) = it, 1 \leq t \leq 0$.