FORCE SYSTEMS

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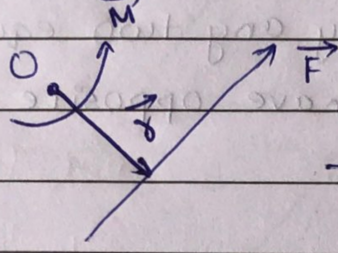
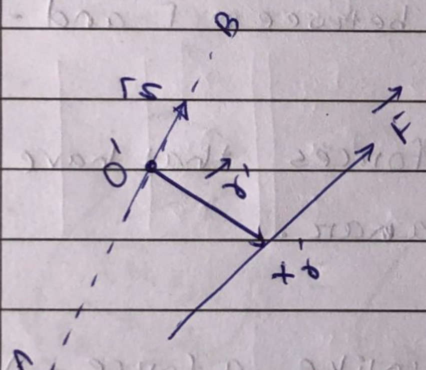
* Moment of a force about a point



$\boxed{\vec{M} = \vec{r} \times \vec{F}}$

$\longrightarrow$ $\vec{r}$ is the position vector from O to any point on the line of action of F.

* Moment of a force about a line



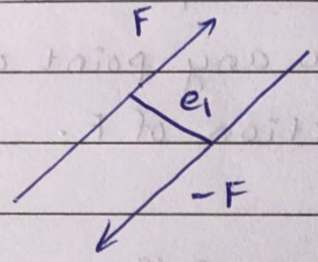
M_{AB} = moment of force F about AB

$= (\vec{r}' \times \vec{F}) \cdot \hat{n}$

To calculate moment of force F about line AB, we can choose any pair of points, so long as one of them lie on AB and the other lies on the line of action of force F.

* Couple and couple moment

→ A couple is formed by any two equal and parallel forces that have opposite sense.



$|\vec{M}|$ = couple moment

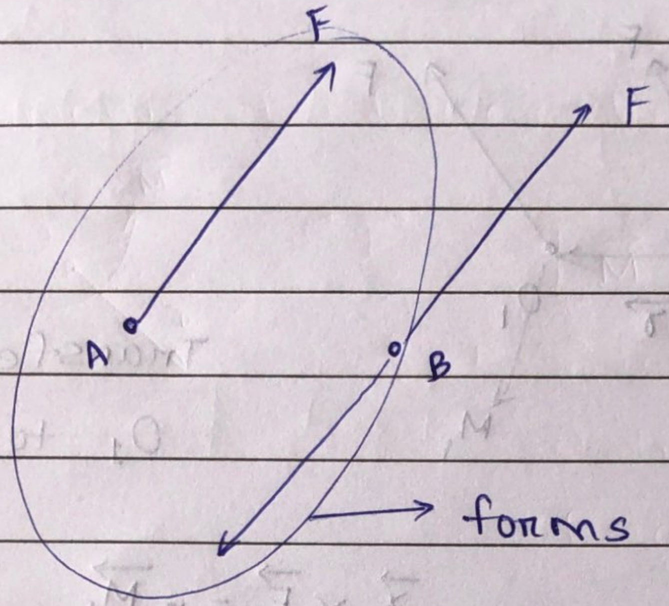
$= |e_1||F|$

⊥r distance between F and −F.

→ Two equal and parallel forces that have opposite sense are always coplanar.

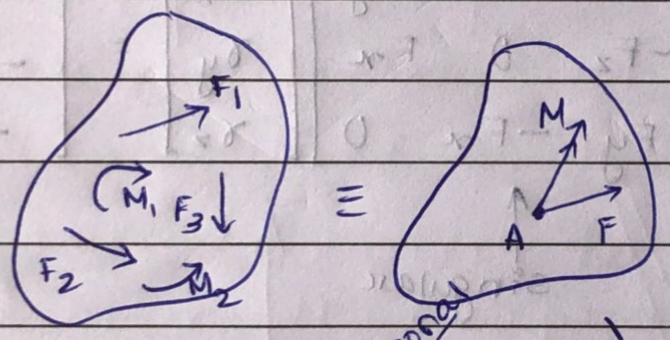
→ A couple is free vector unlike a force. A couple can always be translated in space without giving rise to any additional moments. Its point of application is arbitrary.

→ Force is not a free vector. A force can only be moved by along its line of application. If a force vector is translated parallel to its line action, so that its point of application changes, it gives rise to an additional moment.

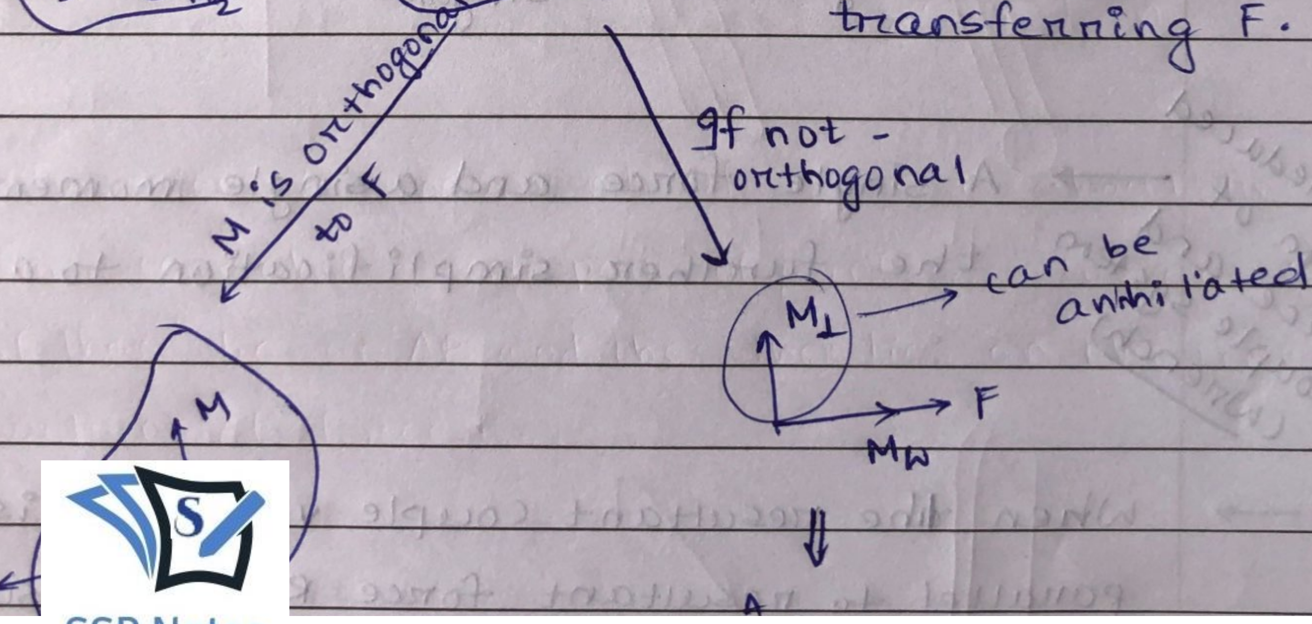


forms a couple

→ Translation of force gives rise to both a force and a moment.



Wherever M is ⊥ to F just destroy it by transferring F.



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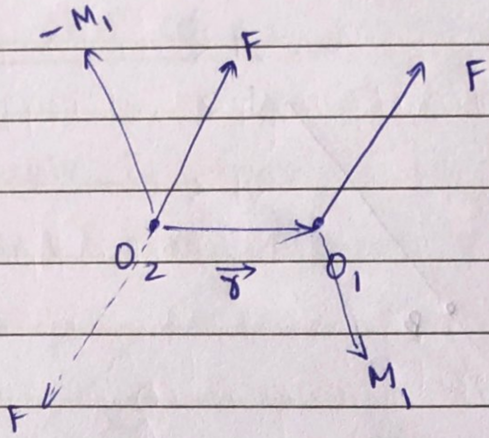
(*)



Transfer from O_1 to O_2

$$\vec{r} \times \vec{F} = -\vec{M_1}$$

In matrix vector form, this becomes :

$$\begin{bmatrix} 0 & F_z & -F_y \\ -F_z & 0 & F_x \\ F_y & -F_x & 0 \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} -M_{1x} \\ -M_{1y} \\ -M_{1z} \end{bmatrix}$$

singular

But if any one of r_x, r_y, r_z is given, then can be easily solved. ⇐

Most reduced form of force and couple system (wrench) → A single force and a single moment has the further simplification to a wrench

→ When the resultant couple vector M is parallel to resultant force R, the resultant is said to be a wrench.

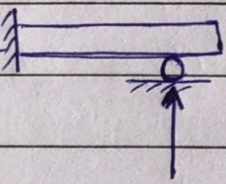
a) couple and force vectors point same dirn = +ve wrench

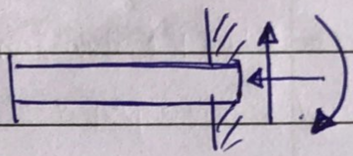
b) " " " " " opposite " -ve wrench

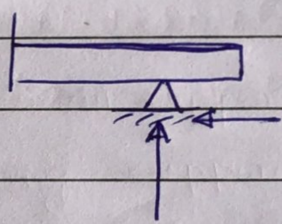
EQUILIBRIUM

→ A support is a constraint acting at a point.

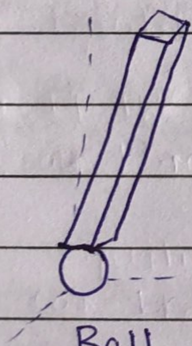
2D Supports

Roller 

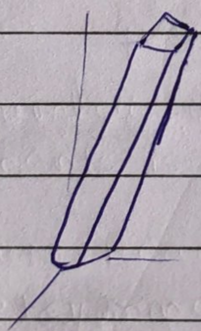
Fixed 

Pinned/ Hinge 

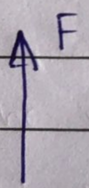
3D Supports



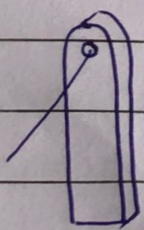
Ball

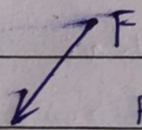


Frictionless surface

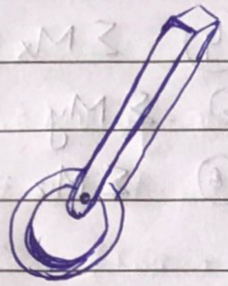


Force with known line of action

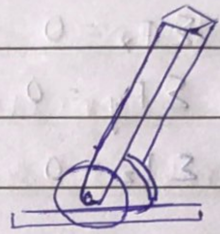




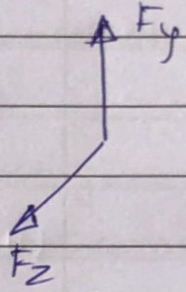
Force with known line of action.

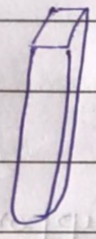


Roller on rough surface

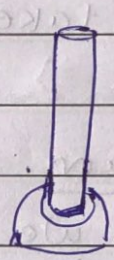


Wheel on rail

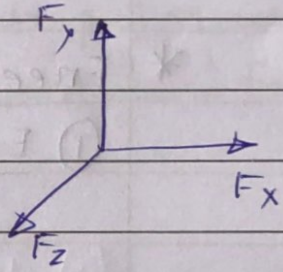




Rough surface



Ball and socket



(Others check slide)

* Equilibrium of rigid bodies

→ Resultant force and moment of the body must be zero

$$F_R = \Sigma F_i = 0$$

$$M_R = \underbrace{\Sigma M_c}_{\text{couple}} + \underbrace{\Sigma M_i}_{\text{due to forces}} = 0$$

→ In 2D :
1. $\Sigma F_x = 0$
2. $\Sigma F_y = 0$
3. $\Sigma M_z = 0$

__/__/__

In 3D: ① $\Sigma F_x = 0$ ④ $\Sigma M_x = 0$
② $\Sigma F_y = 0$ ⑤ $\Sigma M_y = 0$
③ $\Sigma F_z = 0$ ⑥ $\Sigma M_z = 0$

Calculation of moment does not depend on point about which it is taken.

* Free Body Diagram.

① External FBDs: We replace all supports by their corresponding support reactions. and also include all external forces.

- If the number of unknown reaction forces and moments is equal to the number of available equilibrium ~~reactions~~ equations, the unknowns can be determined by solving the equilibrium equations ≡ statically determinate

↓

to solve statically indeterminate structure we have to account for deformation of the body

SSP Notes

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② Internal FBD :

→ Separate at the point where connection is there and find internal forces.

→ Joining the internal FBDs must give the original structure back with same external forces acting.

(Always take moments about that point where the line of action of maximum unknown forces intersect)

* Two - force Members

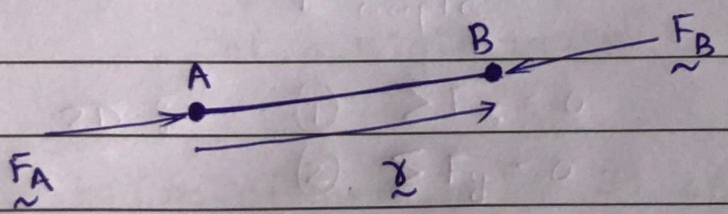
A two-force member is a structural member whose :

(I) ends are pin-jointed

(II) which is not subjected to any intermediate transverse load (or moment) between its ends.

⇒ All forces act at its joints.

and the forces F_A and F_B must be coaxial with the member :



SSP Notes

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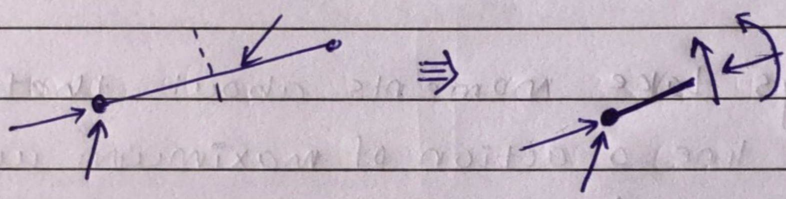
* Pin-jointed frames

→ Members connected by pin-joints.

(i) have members subjected to transverse/ intermediate loads (in general)

(ii) for such frame members, there may be a transverse force component



(a) Identify two force members

(b) Equilibrium equations for the entire structure as well as for individual members may be used.

→ Less eqn ; more unknown : Statically indeterminate

→ More eqn. ; less unknown : not rigid body.

SSP Notes

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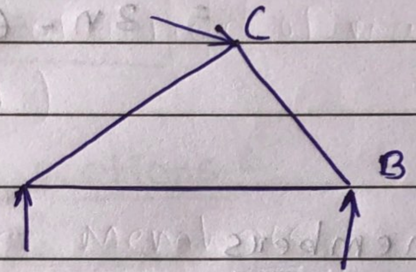
* Trusses

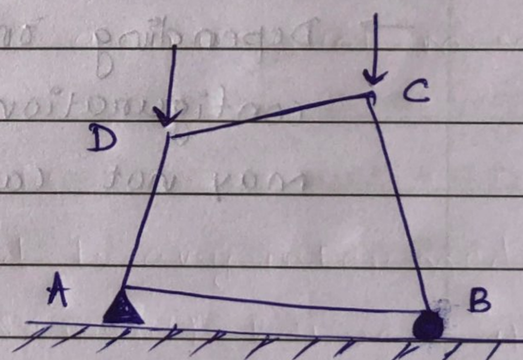
(A structure comprising of two-force members only)

- Plane truss : All members lie in a single plane
- Rigid truss : The truss will retain its shape and deform only slightly on application of load.

→ The simplest rigid truss is triangular in shape and has 3 members.



Not a rigid truss :
even if
(member > 3)



- Simple truss : A truss which is formed by repeatedly adding two members and a single joint to basic triangular unit of a rigid truss.

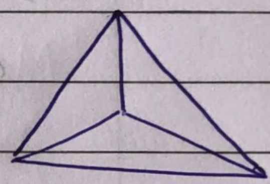
$$m \text{ (no. of members)} = 3 + 2(n-3)$$
$$= 2n - 3$$

__/__/____

- Space truss : Not all members lie in a plane.

  → The simplest space truss is tetrahedral in shape and has four joints and six members. To extend it we need to add 3 additional members for each additional joint.

m (no. of members) $= 6 + 3 \times (n-4)$
$= 3n - 6$.



* Zero force members

→ Depending on the truss geometry and local configuration, some members of the truss may not carry any force.
= zero force members.
↓
can be deleted from truss and the rest of truss can be analyzed.

SSP Notes

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MK.

(4/13) weight present

* Method of joints

→ solve for joints, mostly starting from reaction forces.

(Global FBD)

transfer wt. to joint without moment transfer.

- n joints : 2n eqns in eqm plane
- m unknown force members
- 3 unknown reaction forces.

$$\boxed{m+3=2n}$$

* Method of sections -

(If forces in few members are required)

→ Take a section, solve for unknows using discontinuity, while opening manual FBD.

SSP Notes

FRICTION

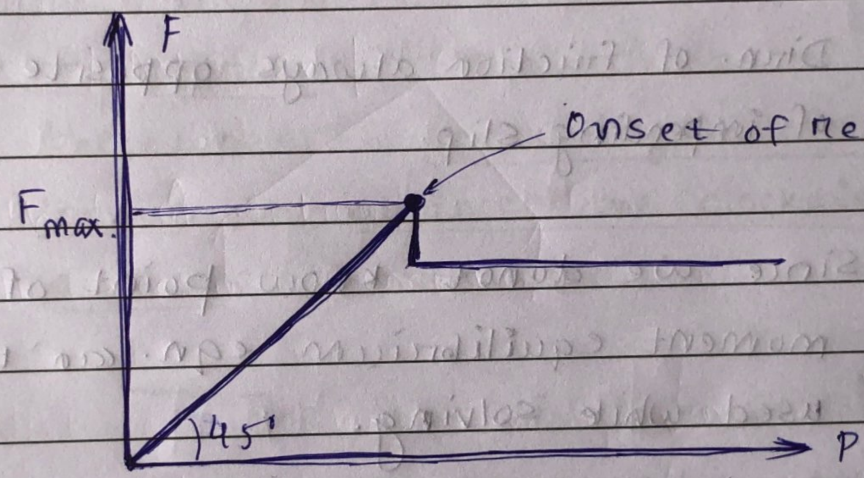
MK 6127
MK 6122

* Coulomb's Law

$F = P$ if $P < F_{max}$.

$F_{max} = \mu_s N$ if $P = F_{max}$.

$F_k = \mu_k N$ if sliding with uniform velocity.



→ ϕ = angle b/w resultant and normal.

$F = R\sin\phi$

$N = R\cos\phi$

$$\boxed{\tan\phi = \frac{F}{N}}$$

a) impending slip : $F = \mu_s \times N$.

$$\boxed{\tan\phi = \mu_s}$$

$\phi = \tan^{-1}\mu_s$ = angle of static friction

SSP Notes

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b) If sliding at constant velocity –

$\phi_k = F/N.$

$\tan\phi = F/N.$

$\Rightarrow F/N = \mu_k = \tan\phi$

$\Rightarrow \tan\phi = \mu_k$

$\Rightarrow \boxed{\phi = \tan^{-1}(\mu_k)}$

(angle of kinectic friction)

→ Dirn. of friction always opposite to dirn. of impending slip.

→ Since we donot know point of application, moment equilibrium eqn. can't be used while solving.

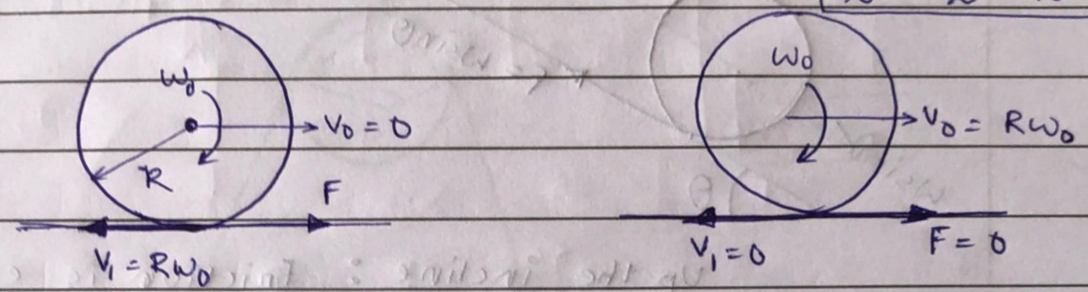
→ A wheel 'A' becomes "frozen" and does not turn its bearing → $\boxed{f_A = \mu_k N_A}$

(MK 6/22)

SSP Notes

* Wheel Friction

$$\boxed{\underset{\sim}{V_1} = \underset{\sim}{V_0} - \underset{\sim}{\omega} \times \underset{\sim}{R}}$$



Initial state　　　　　　Final state

→ Friction force 'F' can be moved to center of the wheel, generating counter clockwise torque to balance the clockwise torque applied at wheel centre.

↳ The force F at wheel centre results in acceleration at wheel centre

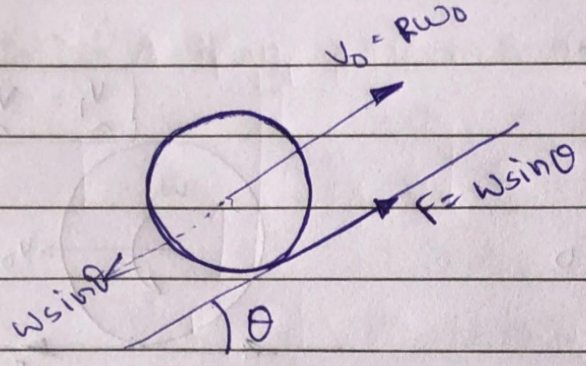
↳ the contact point has a relative velocity with respect to ground as zero

⇓

rolling without sliding

→ Thus the frictional force F is non-zero at start and rest i.e. it is necessary for accelerating or decelerating the wheel centre. However when the wheel is translating with uniform velocity of $R\omega_0$, there is no friction force active.

SSP Notes



Up the incline : Friction is essential to maintain constant velocity (against $W\sin\theta$).

ensures rolling without slipping.

→ Friction causes a moment about centre of wheel ; a torque has to be applied to balance this

$$\boxed{\theta_{max} = \tan^{-1}(\mu_s)}$$

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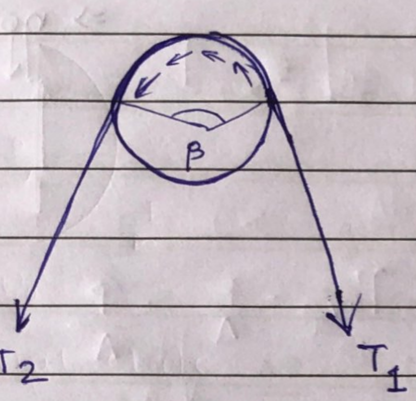
* Belt friction



$\boxed{T_1 > T_2}$

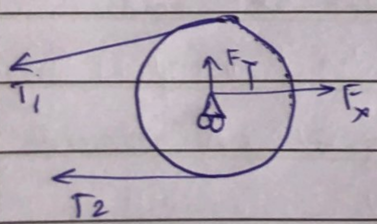
→ β = angle of wrap

$$\boxed{T_1 = T_2 e^{\mu_s \beta}} \quad (\star)$$

→ valid for stationary drum, rotating drum with impending slippage between belt and drum.

→ If the belt is slipping at a constant speed over either a rotating or a stationary drum

$$\boxed{T_1 = T_2 e^{\mu_k \beta}}$$



Addition F^{ext} 'F' is applied then T_1, T_2 increase in same ratio, such that the moment due to them developed is 'M' &

$T_{1x} + T_2 = F_x \quad T_{1y} = F_y$

SSP Notes

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→ A pulley D is free ⇒ Idler pulley

⇒ no frictional resistance

SSP Notes

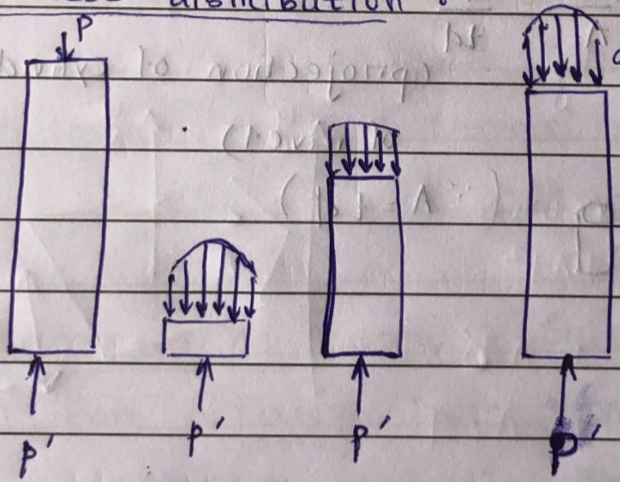
CONCEPT OF STRESS AND STRAIN

→ The force per unit area, or intensity of the forces distributed over a given section, is called the **stress** on that section.

a) $\sigma_{avg.} = \dfrac{P}{A}$ → i) Tensile stress (+ve)
ii) Compressive stress (-ve)

b) $\sigma = \lim_{\Delta A \to 0} \dfrac{\Delta P}{\Delta A}$ (for small area)

Stress distribution :

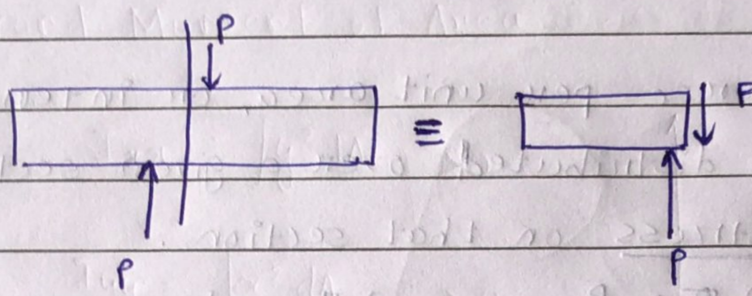


$$P = \int dF = \int_A \sigma \, dA$$

→ A uniform distribution of stress is possible only if the line of action of the concentrated loads P and P' passes through centroid of the section = centric loading

Shearing stress = $\tau_{avg.} = \dfrac{P}{A}$

(Actually τ varies from 0 to $\tau_{max.}$).

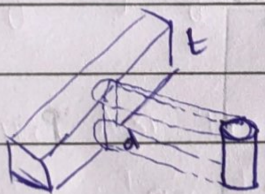


$F = P$

$$\tau_{avg} = \frac{F}{A} = \frac{P}{A}$$

* Bearing Stress

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$



(projection of cylindrical surface of rivet)

$(\because A = td)$.

PROPERTIES OF AREA

(1) First moment of Area : $\rightarrow \bar{x} = \int x\, dA$

$x_c = \dfrac{\bar{x}}{\int dA} = \dfrac{\int x\, dA}{\int dA}$ $\quad \rightarrow \bar{y} = \int y\, dA$

3D $\qquad$ 2D

$x_m = \dfrac{\int \rho\, x\, dV}{V}$ OR $\dfrac{\int \rho x\, dA}{A}$

$y_c = \dfrac{\bar{y}}{\int dA} = \dfrac{\int y\, dA}{\int dA}$ $\qquad y_m = \dfrac{\int \rho x\, dV}{V}$ OR $\dfrac{\int \rho x\, dA}{A}$

- If there is an axis of symmetry, the centroid must lie on the axis of symmetry.

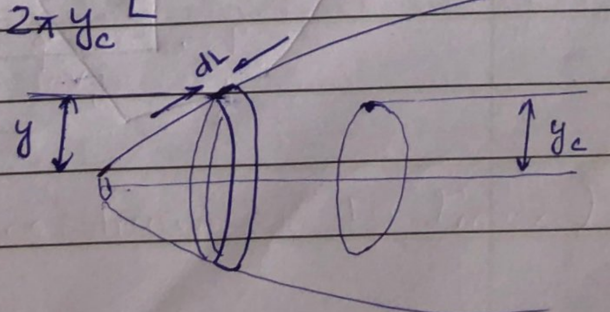
- Uniform density : centre of mass = centre of volume = centre of gravity

(2) Theorems of Pappus - Guldinus

a) & b) For a curve (area) in a plane, the area (volume) of surface obtained by rotating the curve (area) about an axis equals the length of generating curve L (area A) times the distance D travelled by centroid of curve (area).

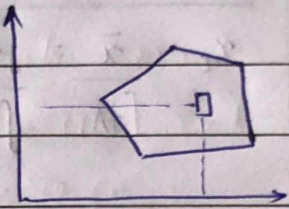
$dA = 2\pi y\, dL$

$A = 2\pi \int y\, dL = 2\pi y_c L$



③ Second Moment of Area

$$I_{xx} = \int y^2 dA \quad > 0$$



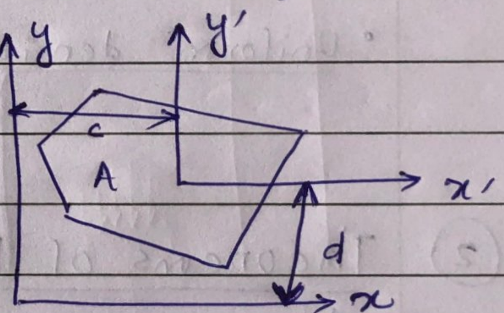
$$I_{yy} = \int x^2 dA \quad > 0$$

Radius of gyration –

$$K_x = \sqrt{\frac{\int y^2 dA}{A}} = \sqrt{\frac{I_{xx}}{A}}$$

$$K_y = \sqrt{\frac{I_{yy}}{A}}$$

$$I_{xy} = \int xy\, dy \quad \text{(product of area).}$$



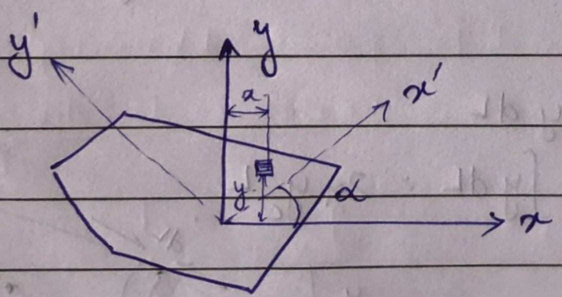
* Parallel Axis Theorem

$$I_{xx} = I_{x'x'} + Ad^2$$

$$I_{yy} = I_{y'y'} + Ac^2$$

$$I_{xy} = I_{x'y'} + Acd$$

* Rotated system



+ Chauhan   + Sin

	x	y
x'	$\cos\alpha$	$\sin\alpha$
y'	$-\sin\alpha$	$\cos\alpha$

derivative

$$I_{x'x'} = \int y'^2 \, dA = \int (-x\sin\alpha + y\cos\alpha)^2 \, dA$$

$$I_{x'x'} = \left(\frac{I_{xx} + I_{yy}}{2}\right) + \cos 2\alpha \left(\frac{I_{xx} - I_{yy}}{2}\right) - \sin 2\alpha \, I_{xy}$$

$$I_{y'y'} = \;\; " \;\; - \;\; " \;\; + \;\; "$$

$$I_{x'y'} = \sin 2\alpha \left(\frac{I_{xx} - I_{yy}}{2}\right) + \cos 2\alpha \, I_{xy}$$

* Polar moment of Inertia

$$I_z = I_{x'x'} + I_{y'y'} = I_{xx} + I_{yy} = \int r^2 \, dA$$

$\longrightarrow$ invariant w.r.t coordinate system.

At angle α, maximum second area moment –

$$\boxed{\tan 2\alpha = \left(\frac{2 I_{xy}}{I_{yy} - I_{xx}}\right)}$$

(two values of α are $\frac{\pi}{2}$ apart)

$\longrightarrow$ Two principal axes – one about which moment of inertia is max and about other moment of inertia is min.

(two values of α).

Product of inertia in principal axis system –

$$\boxed{I_{x'y'} = 0}$$

SSP Notes

MECHANICS OF DEFORMABLE BODIES

→ Rigid bodies can rotate, translate but cannot deform, but bodies of different material deform differently under same load.

→ Study of strain (ε), stress (σ), relation ($\sigma = f(\varepsilon)$) -

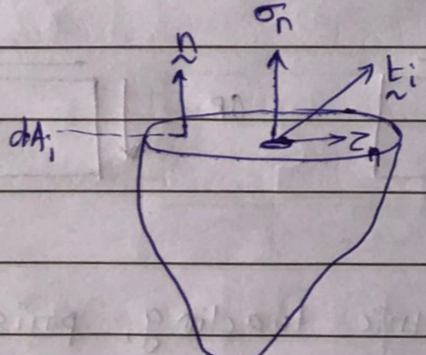
a) displacement controlled method

b) stress controlled method

STRESS

traction vector ($\underset{\sim}{t_i}$)

$$\underset{\sim}{t_i} = \lim_{dA_i \to 0} \frac{d\underset{\sim}{F_i}}{dA_i}$$ (uniform force intensity acting over the infinitesimal area)



normal stress

$$\sigma_n = \underset{\sim}{t_i} \cdot \underset{\sim}{n}$$

$$\tau_n = \sqrt{\underset{\sim}{t_i} \cdot \underset{\sim}{t_i} - (\underset{\sim}{t_i} \cdot \underset{\sim}{n})^2}$$

shear stress

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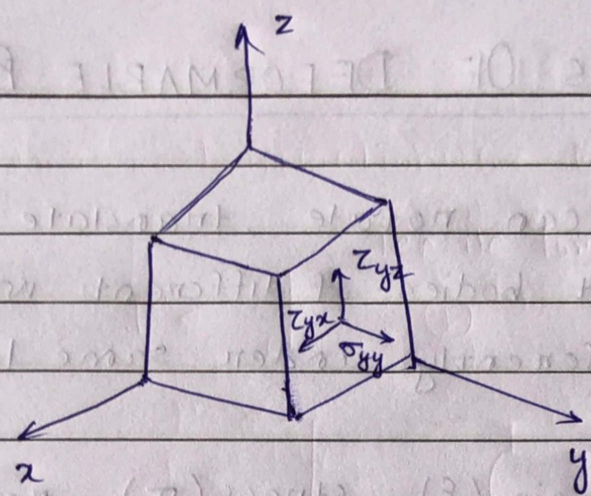
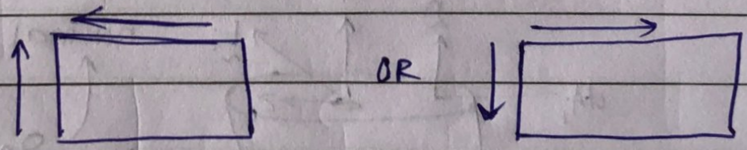


State of stress at a point - stress matrix.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$ is a symmetric matrix.

$$(\because \tau_{xy} = \tau_{yx} \; ; \; \tau_{xz} = \tau_{zx} \; ; \; \tau_{yz} = \tau_{zy})$$

→ Shearing stresses on mutually perpendicular planes only act towards or away from the intersection of such planes



<u>Axial Stress</u> (Centric loading, prismatic section, homogenous material)

$$\sigma_n = \left(\frac{P}{A}\right) \cos^2\theta$$

$$\tau_n = \left(\frac{P}{A}\right) \cos\theta \sin\theta$$

SSP Notes

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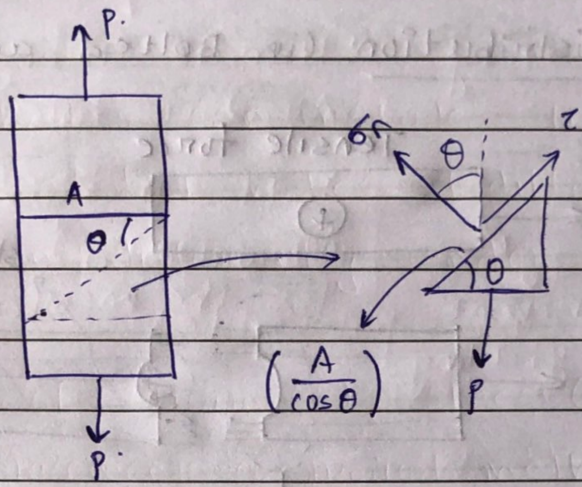
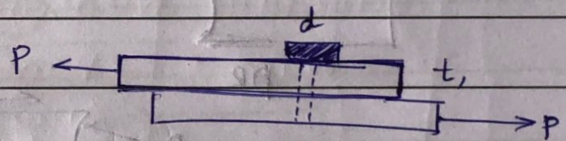
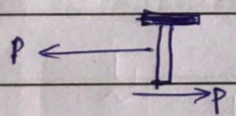


Bearing Stress : due to normal compressive force

Single Lap :

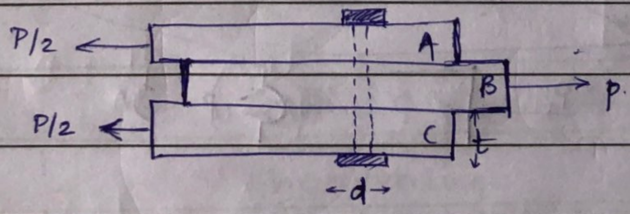




σ_b = Bearing stress $= (P/td)$

τ = shearing stress $= P/\left(\frac{\pi d^2}{4}\right)$

Double lap :



σ_b (in plate B) $= (P/td)$

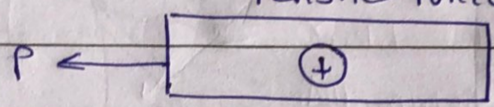
σ_b (in plate A) $= \sigma_b$ (in plate C) $= (P/2)/(td)$

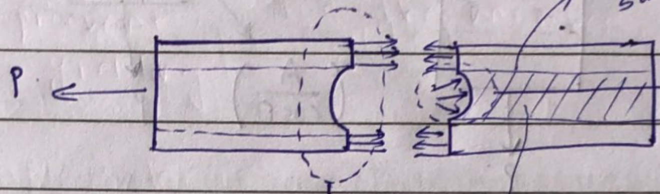
shearing stress $\tau = (P/2)/(\pi d^2/4)$

__/__/____

* Stress distribution (in Bolted and Lapped connect.)

Tensile force

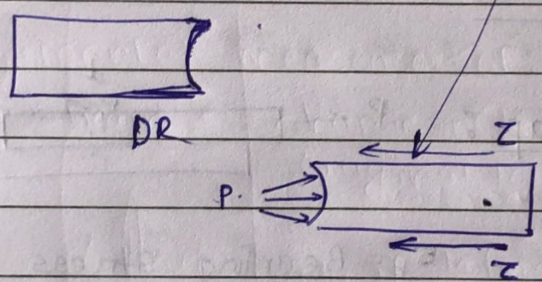




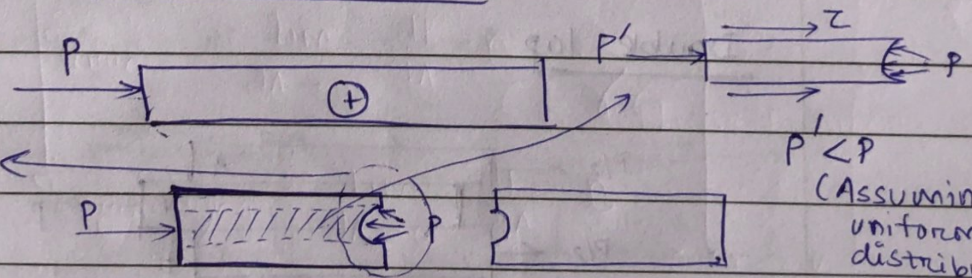
shear stress along this line

→ Bearing stress.

→ Non-contact normal stress is highest because area of contact is lowest



Compressive force



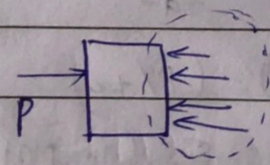
$P' < P$

(Assuming uniform distribution)

⇓

shear stress is present

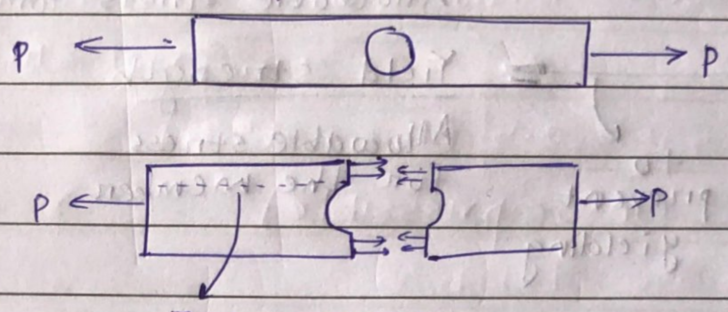
Non-contact ← normal stress is zero.



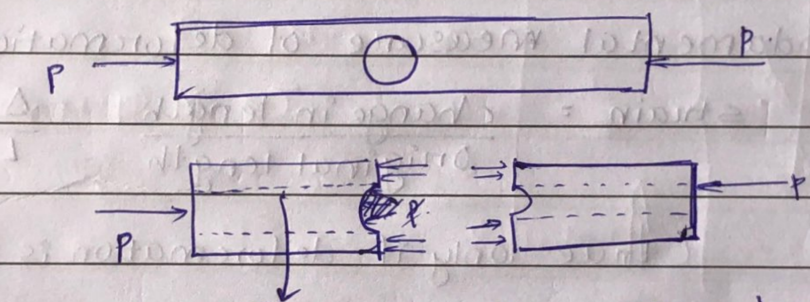
Non-contact normal stress in the plate is highest here because this is where 'P' has to be resisted entirely.

* Stress distribution in plate with a hole

Tensile

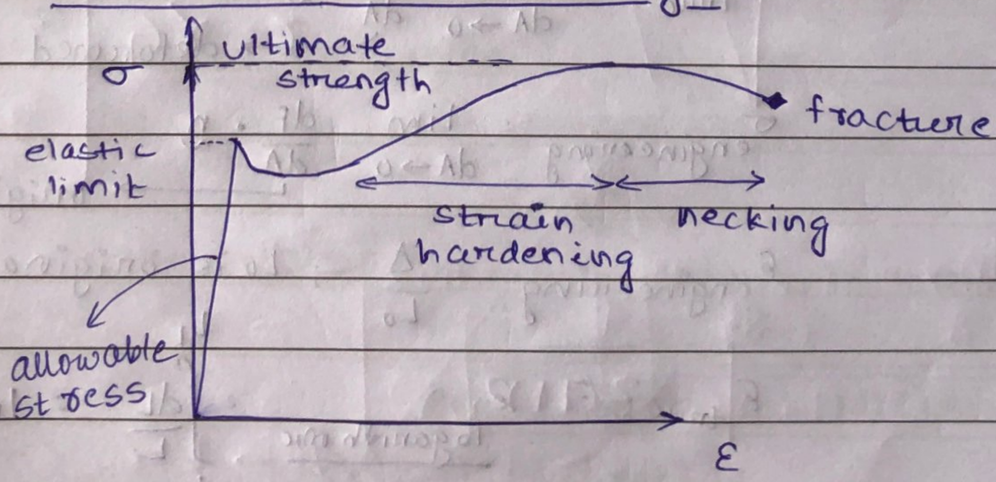


Compressive



There is shear ~~stress~~ stress 'τ'.

* Allowable Stress Design



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to prevent necking

Factor of safety = $\dfrac{\text{Ultimate strength of a member}}{\text{Allowable stress for the member}}$

$= \dfrac{\text{Yield strength}}{\text{Allowable stress for the member}}$

to prevent yielding

STRAIN

→ Fundamental measure of deformation / distortion

$$\text{strain} = \frac{\text{change in length}}{\text{Original length}} = \frac{\Delta}{L}$$

(true only if deformation is homogeneous)

a) True normal stress :

$$\sigma_{true} = \lim_{dA \to 0} \frac{dF}{dA} \cdot \underset{\sim}{n}$$

→ deformed Area

$$\sigma_{engineering} = \lim_{dA \to 0} \frac{dF}{dA} \cdot \underset{\sim}{n}$$

→ original area

$$\epsilon_{engineering} = \frac{\Delta}{L_0}$$; L_0 is original length.

$$\epsilon_{true} = \epsilon_{logarithmic} = \int_{L_0}^{L_1} \frac{dL}{L} = \ln\left(\frac{L_1}{L_0}\right)$$

SSP Notes

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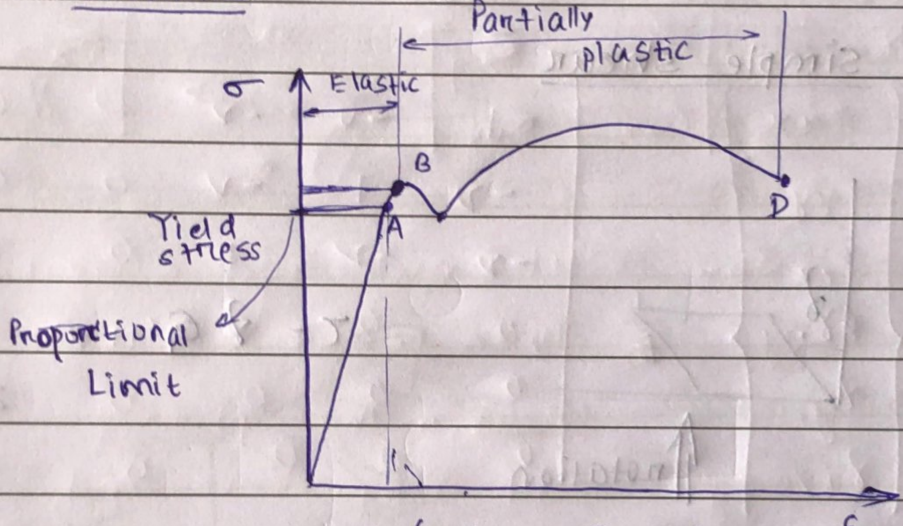
*Stress- Strain Relationship

$$\boxed{\sigma = f(\varepsilon)}$$

→ The initial linear region is related by Young's modulus.



(Mild steel(MS))

Poisson's Effect

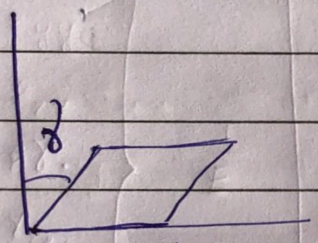
→ If a speciamen is subjected to uniaxial compression loading, laterial strain occurs

$$\text{lateral strain} = \frac{\text{change in length in laterial direction}}{\text{gage length in lateral direction.}}$$

Poisson's ratio $= \nu = - \frac{\text{lateral strain}}{\text{axial strain}}$

$\nu = 0.5$ (for rubber)
(incompressible)

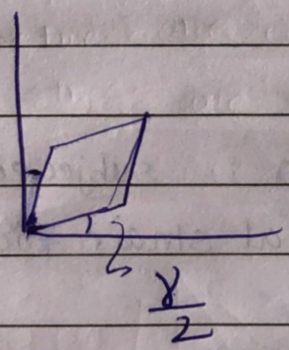
* simple shear



$\tau = \gamma G$

rotation

* Pure shear



$\tau = \gamma G$

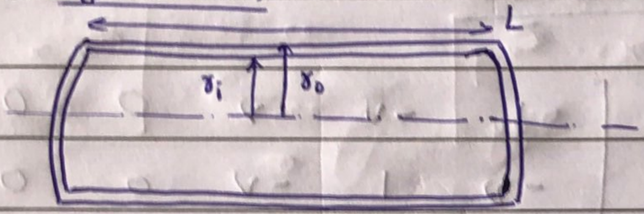
$$\boxed{G = \frac{E}{2(1+\nu)}}$$

→ strain gets added up $\int_0^L \frac{P(x)}{EA(x)} dx$.

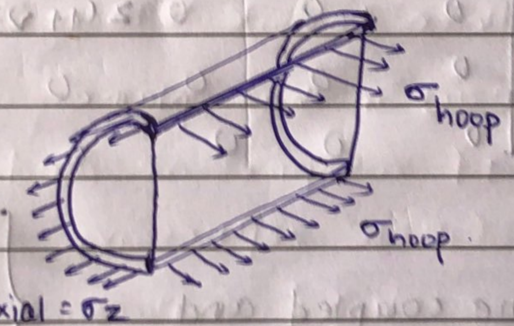
* Biaxially stress state : Thin walled Pressure Vessels

→ The walls of a thin walled pressure vessel act as a membrane i.e. under internal pressure, the walls stretch but do not bend.

ⓐ Cylindrical



(No shear because of symmetric geometry and symmetric loading)



σ_{hoop} = circumferential stress.

$$\boxed{\sigma_{hoop} = \frac{p r_i}{r_0 - r_i} = \frac{p r_i}{t}}$$

(Though σ_{hoop} varies in thickness but in thin-walled vessels, the above is correct).

$$\boxed{t < r_i (0.1)}$$

$$\boxed{\sigma_{axial} = \frac{p r_i^2}{t(r_0 + r_i)} \approx \frac{p r_i}{2t}} = \frac{\sigma_{hoop}}{2}$$

$\boxed{\text{Radial stress } \sigma_r \approx 0}$

__/__/____

(b) Spherical pressure vessels (Equibiaxial state of stress)

$$\boxed{\sigma_{hoop} = \sigma_{axial} = \frac{p\, r_i}{2t}}$$

* Constitutive Relations

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}$$

$\because$ Axial stress (strains) are coupled and shear stress are non-coupled.

Inverting the matrix –

$$\left.\begin{aligned} \lambda &= \frac{E\nu}{(1+\nu)(1-2\nu)} \\ \mu &= \frac{E}{2(1+\nu)} = G \end{aligned}\right\} \lambda, \mu \rightarrow \text{Lame's constant}$$

SSP Notes

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon\cdot\gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

* Strains in cylindrical pressure vessels

$$\begin{Bmatrix} \varepsilon_{axial} \\ \varepsilon_{hoop} \\ \varepsilon_{radial} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{axial} \\ \sigma_{hoop} \\ \sigma_{radial} \end{Bmatrix}$$

1) $\varepsilon_{hoop} = \dfrac{\delta r}{r}$

$\Rightarrow \delta r = r \cdot \varepsilon_{hoop}$ = expansion of sphere

2) $\delta C = 2\pi r \, \varepsilon_{hoop}$.

3) $\delta t = t \times \varepsilon_r$

4) $\varepsilon_{vol.}$ = Volumetric strain $\approx 2\varepsilon_{hoop} + \varepsilon_{axial}$.

(Above equations are same for spherical vessels too)

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* Transformation of stresses

$$I = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \xrightarrow[\text{Principal coordinate system}]{\text{Trasfor}^n \text{ to}} \begin{bmatrix} I_{x'x'} & 0 \\ 0 & I_{y'y'} \end{bmatrix}$$

Largest 2nd moment of inertia

smallest 2nd moment of inertia.

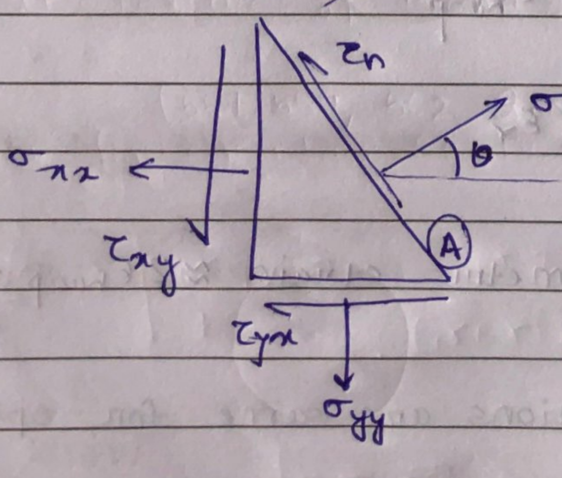
III^{ly}

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} \xrightarrow[\text{" of } \sigma]{\text{"}} \begin{bmatrix} \sigma_{x'x'} & 0 \\ 0 & \sigma_{y'y'} \end{bmatrix}$$

$\sigma_{x'x'}$ → if tensile and magnitude greater than tensile strength – initiates crack in that plane

$\sigma_{y'y'}$ → if compressive and " " " compressive strength – crushing failure.

Normal and shear stress on an inclined plane



θ → angle made by plane w.r.t x-axis

SSP Notes

$$\sigma_n = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + \tau_{xy}\sin 2\theta$$

$$\rightarrow \sigma_n = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\rightarrow \tau_n = \tau_{xy}\cos 2\theta - \frac{(\sigma_{xx}-\sigma_{yy})}{2}\sin 2\theta.$$

Now

$$\left[\sigma_n - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right]^2 + \tau_n^2 = \left\{\frac{(\sigma_{xx}-\sigma_{yy})}{2}\right\}^2 + \tau_{xy}^2$$

(Equation of Mohr's circle)

- Principal Planes

(Maxm. Normal stress in that plane)

Diagonalize the 2D stress Matrix.

$\rightarrow \frac{d\sigma_n}{d\theta} = 0 \Rightarrow \theta_1 = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{(\sigma_{xx}-\sigma_{yy})}$

$\theta_2 = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{(\sigma_{xx}-\sigma_{yy})} + \frac{\pi}{2}$

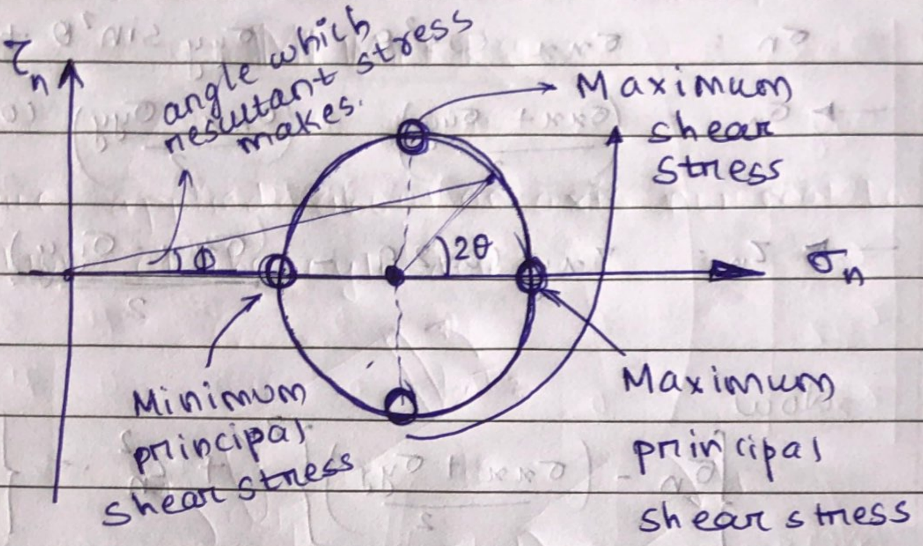
$\rightarrow$ For principal planes, $\boxed{\tau_n = 0}$

(2) Planes of maximum shear

$\rightarrow \theta_1 = -\frac{1}{2}\tan^{-1}\frac{(\sigma_{xx}-\sigma_{yy})}{2\tau_{xy}}$

$\theta_2 = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_{yy}-\sigma_{xx}}{2\tau_{xy}}\right) + \frac{\pi}{2}$

$\rightarrow \sigma_n = \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)$



Maximum shear stress $= \left(\frac{\sigma_{xx} + \sigma_{yy}}{2} , \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2} \right)$

The Mohr's Circle

→ The stress state (normal & shear stresses) on any plane whose normal makes an angle θ w.r.t x-axis will subtend 2θ (clockwise) at centre of Mohr's Circle.

(557, 559, 562 Q. No.).

SSP Notes

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* Transformation Equations for Strain

Replace σ_n by ε_n

σ_{xx} by ε_{xx}

σ_{yy} by ε_{yy}

τ_{xy} by $\frac{\gamma_{xy}}{2}$

τ_n by $\gamma_n/2$

Rest everything remains the same.

SSP Notes

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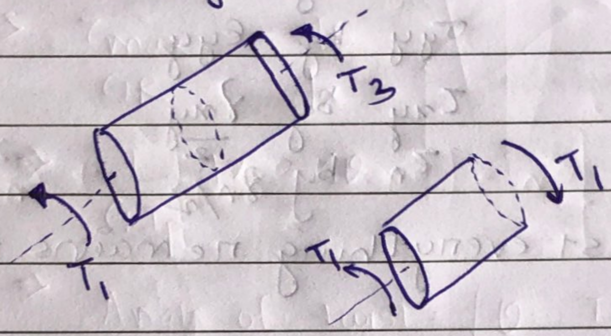
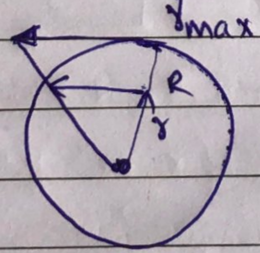
TORSION IN CYLINDRICAL/TUBULAR MEMBERS

- Torque applied about an axis normal to the cross section of the cylinder:

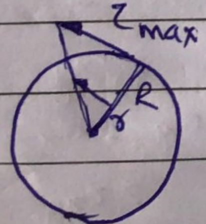


Assumption –

1) Sections perpendicular to the axis of twist remain plane after twist.
11) Shearing strain varies linearly with radial distance from axis of cylinder:



* Torsion for circular sections



$$\boxed{\tau = \tau_{max} \frac{r}{R}}$$

SSP Notes

$$\Rightarrow \int_A r(\tau\, dA) = T \Rightarrow \int_A r^2 \frac{\tau_{max}}{R}\, dA = T$$

$$\frac{\tau_{max}}{R} \int_A r^2 dA = T$$

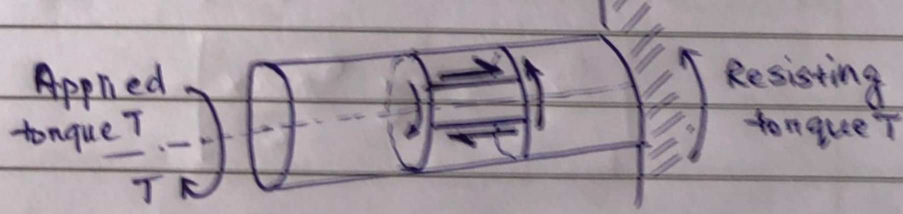
$$\Rightarrow \frac{\tau_{max}}{R} \left(\frac{\pi d^4}{32}\right) = T$$

$$\Rightarrow \boxed{\tau_{max} = \frac{TR}{I_p}} \text{ ——— } (*)$$

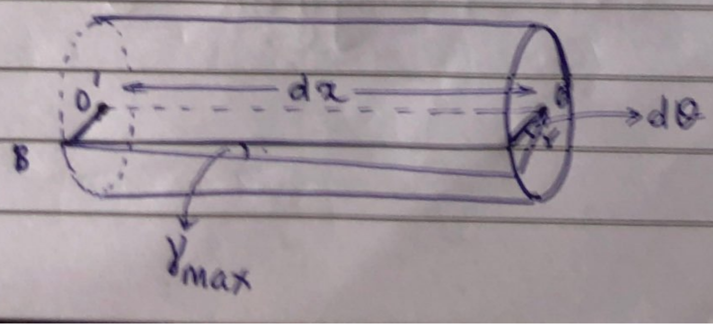
At radial distance $r \rightarrow$

$$\boxed{\tau_{max} = \frac{Tr}{I_p}}$$

→ Since shear stresses on mutually orthogonal planes are equal, a linearly varying shear stress distribution also occurs on planes orthogonal to the cross section and parallel to the axis of shaft →.



* Twist



From compatibility equation -

$$\gamma_{max}\, dx = R\, d\theta$$

$$\Rightarrow \frac{\gamma_{max}}{R} = \frac{d\theta}{dx}$$

$$\Rightarrow \frac{\tau_{max}}{GR} = \frac{d\theta}{dx}$$

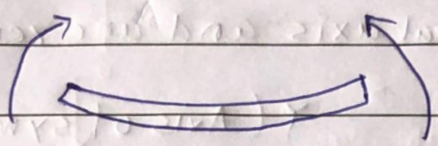
$$\Rightarrow \frac{T}{GI_p}\, dx = d\theta$$

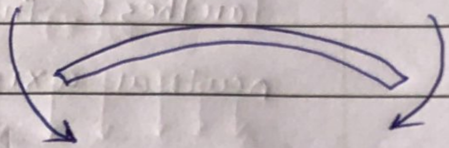
$\Rightarrow$ Angle of twist = $\boxed{\theta = \frac{TL}{GI_p}}$

For annular shaft, everything remains same, only $I_p = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$

SSP Notes

PURE BENDING OF BEAMS





→ Positive bending moment

→ Compression at top fiber

→ Tension in the bottom fiber

→ Negative bending moment

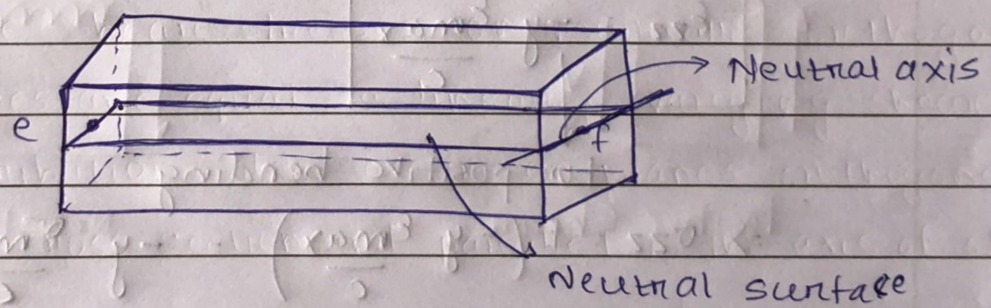
→ Compression at bottom fiber

→ Tension at top fiber.

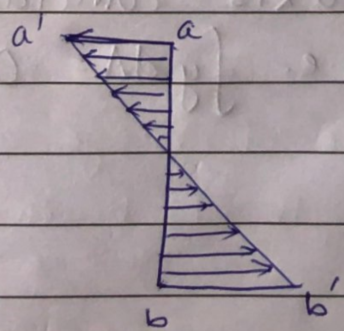
- Assumptions of Bernoulli - Euler flexure theory

  i) Plane sections remain plane after deformation

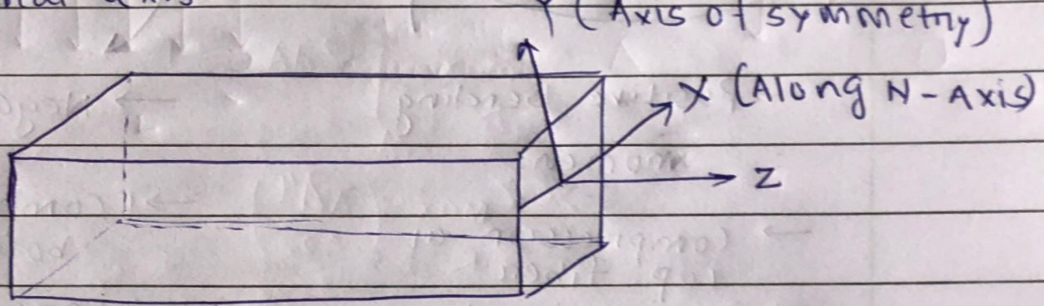
  ii) Plane sections remain orthogonal to the axis of the beam after deformation.

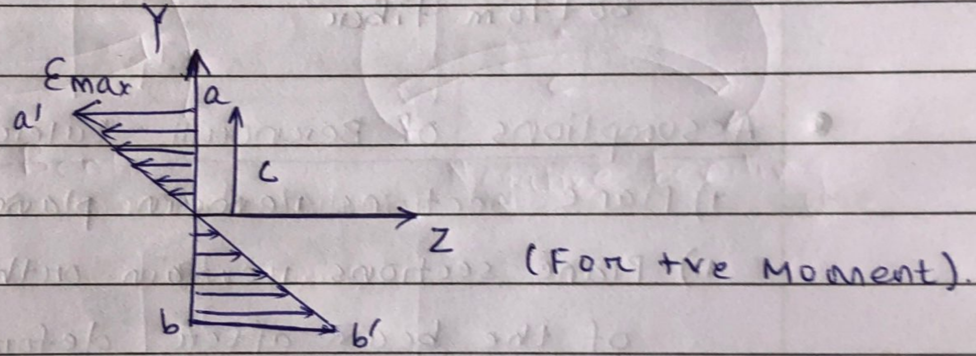


Since the plane sections remain plane, the strains are distributed linearly along depth of beam:



→ The strains are maximum at the fibre farthest from neutral axis and are zero at neutral axis.





$$\boxed{\varepsilon_{zz} = -y \frac{\varepsilon_{max}}{c}}$$

For positive bending

$$\Rightarrow \sigma_{zz} = E\left(-y \frac{\varepsilon_{max}}{c}\right) = -y \frac{\sigma_{max}}{c}$$

$$\boxed{\sigma_{zz} = -y \frac{\sigma_{max}}{c}}$$

→ Neutral axis coincides with centroidal axis i.e. $\int y \, dA = 0$

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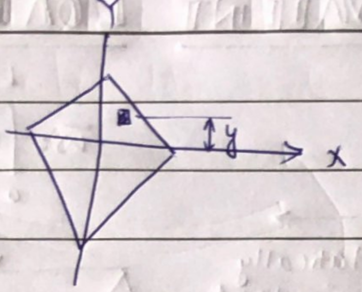


$$dM = dF_z \times y$$
$$= \sigma_{zz}\, dA \times y$$
$$= -y^2 \frac{\sigma_{max}}{c}\, dA$$

$$M = \int_A dM = -\frac{\sigma_{max}}{c} \int y^2 dA$$

$$= -\frac{\sigma_{max}\, I_{xx}}{c}$$

$$\Rightarrow M = -\frac{\sigma_{zz}\, I_{xx}}{y}$$

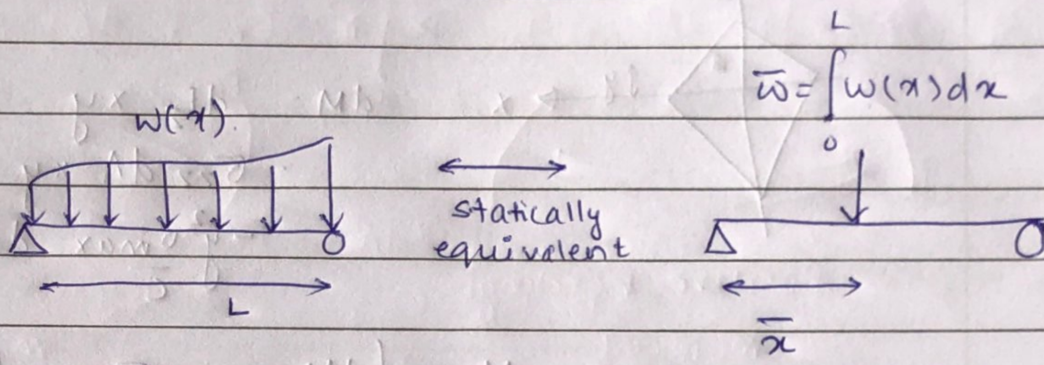
$$\Rightarrow \boxed{\sigma_{zz} = -\frac{My}{I_{xx}}}$$ (Flexure formula for beams)

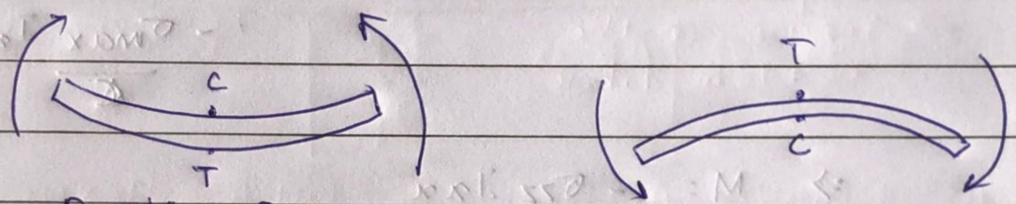
stress is axial.

$$\Rightarrow M = -EI_{xx} \frac{\varepsilon_{max}}{y_{max}}$$

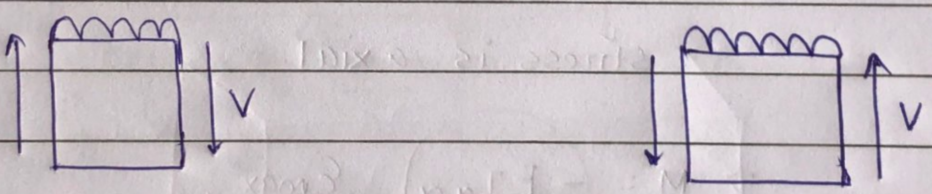
Bending rigidity
(Strength of beam in bending)

STATICALLY EQUIVALENT LOADS





Positive Bending　　　　Negative bending



Positive Shear　　　　Negative Shear

$$\boxed{\frac{dM}{dx} = V}$$

→ shear force at any cross section along the beam axis must be equal to the rate of change of bending moment along length of beam.

→ Hence if the bending moment is constant along the beam axis ⇒ shear force at any cross-section is zero ⇒ pure bending

$$\boxed{\frac{dV}{dx} = -w(x)}$$

→ Hence the rate of change of shear force is equal to the intensity of the distributed loading, but with a negative sign.

SSP Notes