INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester 2017-18

Date of Examination 28.11.2017 **Session** FN **Duration** 3 hrs **Max. Marks** 100
Subject No. : ME 10001 **Subject:** Mechanics
Department/Center/School: Mechanical Engineering
Instructions: Answer all questions. All parts of a question MUST be together. Figures are not to scale.

1. Two 40 mm wide and 15 mm thick flat plates are loaded in tension. They are joined using two rectangular splice plates of same width and thickness as the plates and two 10 mm diameter rivets as shown in Figure 1. The factor of safety against any of the ultimate load that can be carried is 2.5. The ultimate strength in tension for the plate and splice material is 400 MPa. The ultimate strength in shear of the rivet material is 170 MPa.

   (a) Calculate the tensile stress in the critical areas of the plate in terms of F. **(6)**

   (b) Calculate the shear stress in the critical rivet cross-section in terms of F. **(6)**

   (c) Find the allowable load F_{allow} considering the failure due to tension in plate and shearing of rivet. **(5)**

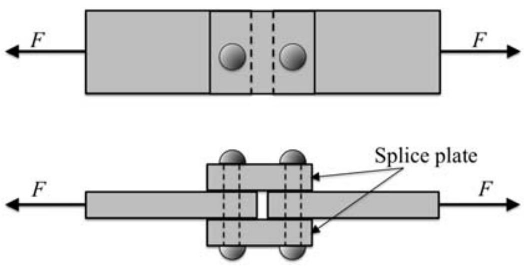


Figure 1

2. A 3 m long hollow aluminum shaft used in building structures has inner and outer diameters $d_1 = 80$ mm and $d_2 = 100$ mm, respectively. The shear modulus of aluminum is $G = 30$ GPa and the tube is subjected to pure torsion at ends.

   (a) Find the angle of twist in degrees when the maximum shear stress in the hollow shaft is 50 MPa. **(6)**

   (b) Find the diameter of a solid shaft of same material and length resisting the same torque and has the same maximum shear stress. **(8)**

   (c) Calculate that ratio of the weights of the hollow and solid shafts. **(4)**

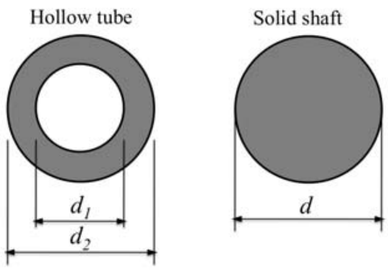


Figure 2

3. A $L = 2$ m long cantilever beam of square cross section of side b is subject to a distributed load $q = 1$ kN/m and a moment $M_1 = 1$ kN-m at the mid span as shown in Figure 3.

   (a) Draw shear force and bending moment diagrams of the beam mentioning the sign convention. **(8)**

   (b) Identify and state the location at which the bending moment is maximum in magnitude. **(2)**

   (c) Determine the minimum depth b of the beam such that the bending stress in the beam does not exceed 300 MPa. **(6)**

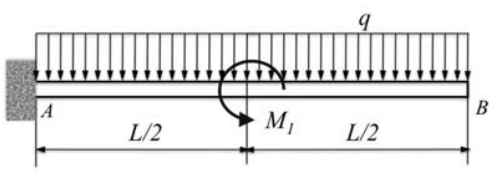


Figure 3

4. A wooden sample shown in Figure 4 was subjected to an axial load P. The sample was found to fail (break) at an angle $\theta = 60°$ when the normal stress on the oblique plane at 60° reached 50 MPa. The cross-sectional area (section $a - a$) of the sample was 100 mm^2.

   (a) Calculated the load P at failure. **(6)**

   (b) Compute the shear stress on the 60° oblique plane at failure . **(4)**

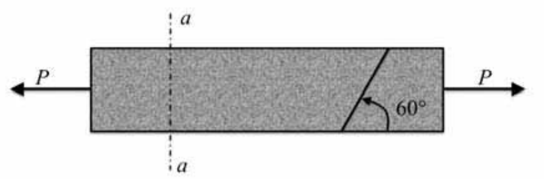


Figure 4

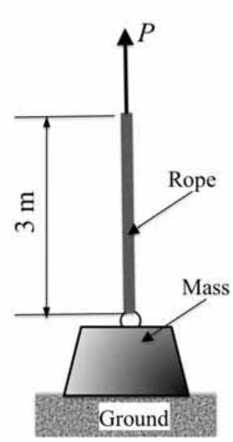


Figure 5

5. A rigid block of weight 1000 N was initially resting on a rigid ground. It is being slowly pulled upwards using a 3 m long massless rope of cross sectional area 100 mm^2 as shown in Figure 5. The material of the rope has Young's modulus $E = 1$ GPa. Determine the displacement of the top end of the rope when the mass just looses contact with the ground. **(10)**

6. A strain gauge (strain measuring device) is installed on the surface of an aluminium beverage can along its longitudinal direction as shown in Figure 6. The internal radius to thickness ratio of the can is $r/t = 100$. When the lid of the can is popped open, the strain gauge indicates a change of axial/longitudinal strain by $\epsilon = 150 \times 10^{-6}$.

   Consider the can to be cylindrical and the material has $E = 70$ GPa and $\nu = 0.33$. Neglect the pressure due to weight of the fluid in the can.

   (a) What was the internal pressure in the can before opening? **(8)**

   (b) Determine the in-plane normal and shear stresses (σ_{x1}, σ_{y1} and τ_{x1y1}) on an element rotated from longitudinal direction by $\theta = 30°$ as shown, before the can was opened. **(12)**

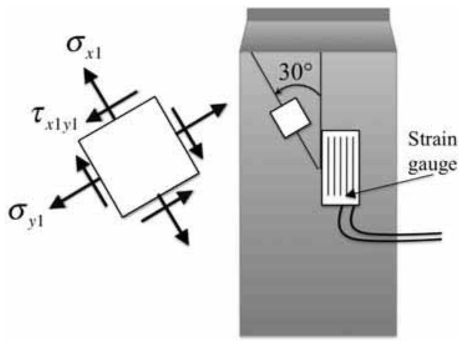
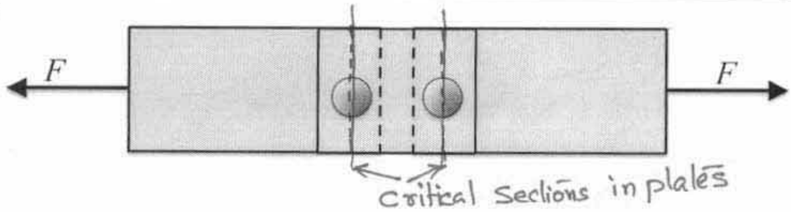


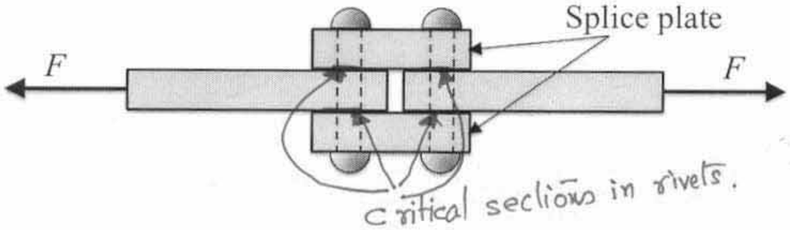
Figure 6

7. A 200 mm square plate is subjected to tensile stresses $\sigma_x = 10$ MPa and $\sigma_y = 20$ MPa. Find the percentage change in the volume of the plate if the thickness of the plate is 1 mm. The plate material has $E = 100$ GPa and $\nu = 0.3$. **(14)**

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem1:



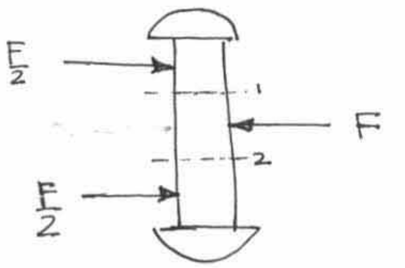


(a) Critical area in plate: $A = (40-10)\times 15 = 450\ mm^2$

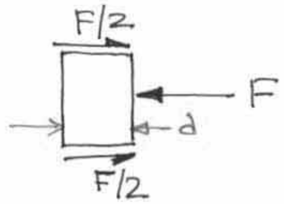
• Critical stress: $\boxed{\sigma_{crit} = \frac{F}{A} = \frac{F}{450}\ MPa}$

(F in Newton)

(b) FBD of rivet (left)



Between sections 1 & 2



Critical shear stress: $\tau_{crit} = \frac{F}{2A_{rivet}} = \frac{F}{2\times\frac{\pi d^2}{4}}$

$\Rightarrow \boxed{\tau_{crit} = \frac{2F}{\pi d^2} = \frac{F}{50\pi}\ MPa}$

(c) Factor of safety $= 2.5$, $\sigma_{allow} = \frac{\sigma_u}{2.5}$, $\tau_{allow} = \frac{\tau_u}{2.5}$

Considering failure in tension (plate):

$\sigma_{crit} \le \sigma_{allow} \Rightarrow \frac{F}{450} \le \frac{\sigma_u}{2.5} = \frac{400}{2.5}$

$\Rightarrow \boxed{F \le 72\ kN}$

Considering Failure of rivet: $\tau_{crit} \le \frac{\tau_u}{2.5} \Rightarrow F \le \frac{170\times 50\pi}{2.5} = 10.68\ kN$

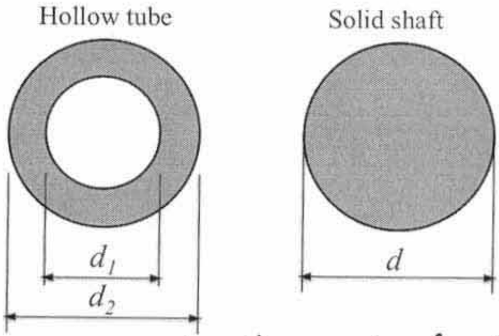
$\Rightarrow$ $\boxed{\text{Alowable load : } F_{allow} = 10.68\ KN}$

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 2:

$$\boxed{\begin{array}{l} d_1 = 80\,mm \\ d_2 = 100\,mm \end{array}}$$

$$\boxed{\begin{array}{l} G = 30\,GPa \\ \tau_{max} = 50\,MPa \end{array}}$$



Hollow shaft: $J = \frac{\pi}{32}(d_2^4 - d_1^4) = \frac{\pi}{32}(100^4 - 80^4) = 5.796 \times 10^6\ mm^4$

(a) Angle of twist:

Max. shear: $\tau_{max} = \frac{G\phi}{L} \cdot r$ $\left(\begin{array}{l} r = \text{radius of outer surface} \\ \quad = 50\,mm = d_2/2 \end{array} \right)$

Angle of twist: $L = \text{length} = 3\,m.$

$$\Rightarrow \phi = \frac{\tau_{max} L}{G r} = \frac{50 \times 10^6\,Pa \times 3\,m}{30 \times 10^9\,Pa \times 0.05\,m}$$

$$\Rightarrow \phi = \frac{15}{150} = 0.1\ rad$$

$$\Rightarrow \boxed{\phi = 5.73^\circ}$$

(b) Torque acting on hollow shaft:

Using $\tau_{max} = \frac{T r}{J}$ we get

$$\text{Torque: } T = \frac{\tau_{max} J}{r} = 50 \frac{N}{mm^2} \times \frac{5.796 \times 10^6\,mm^4}{50\,mm}$$

$$\Rightarrow \boxed{T = 5.796\ KN\text{-}m}$$

For the solid shaft: Torque, length and τ_{max} remain the same.

$$\boxed{J = \frac{\pi}{32} d^4} \quad \Rightarrow \tau_{max} = \frac{T r}{J} = \frac{T \cdot d/2}{\frac{\pi}{32} d^4} = \frac{16 T}{\pi d^3} = 50\,MPa$$

$$\Rightarrow d^3 = \frac{16 T}{\pi \tau_{max}} = \frac{16 \times 5.796\ KN\text{-}m}{\pi \times 50\ MPa}$$

$\Rightarrow \quad d = 8.39 \times 10^{-2}$ m

or $\boxed{d = 83.9 \text{ mm}}$

(c) <u>Ratio of weight</u>: $\dfrac{\text{Weight of hollow Shaft}}{\text{Weight of solid shaft}}$

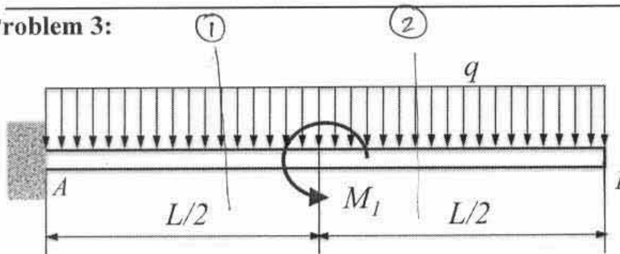
$$= \frac{\rho L A_{hollow}}{\rho L A_{solid}} = \frac{A_{hollow}}{A_{solid}}$$

$$= \frac{d_2^2 - d_1^2}{d^2} = 0.5114$$

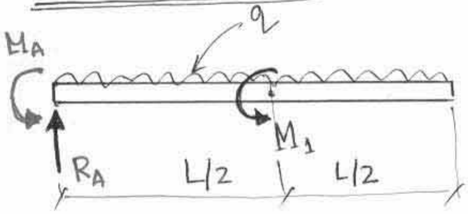
$\Rightarrow \quad \boxed{\text{Ratio of weight} = 0.51}$

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 3:



FBD of the beam



Data: $q = 1$ kN/m, $M_1 = 1$ kN-m, $L = 2$ m.

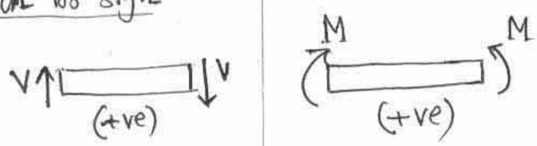
Reactions at support:

- $\boxed{R_A = qL = 2\text{ kN}}$
- $\Sigma M_A = 0$:

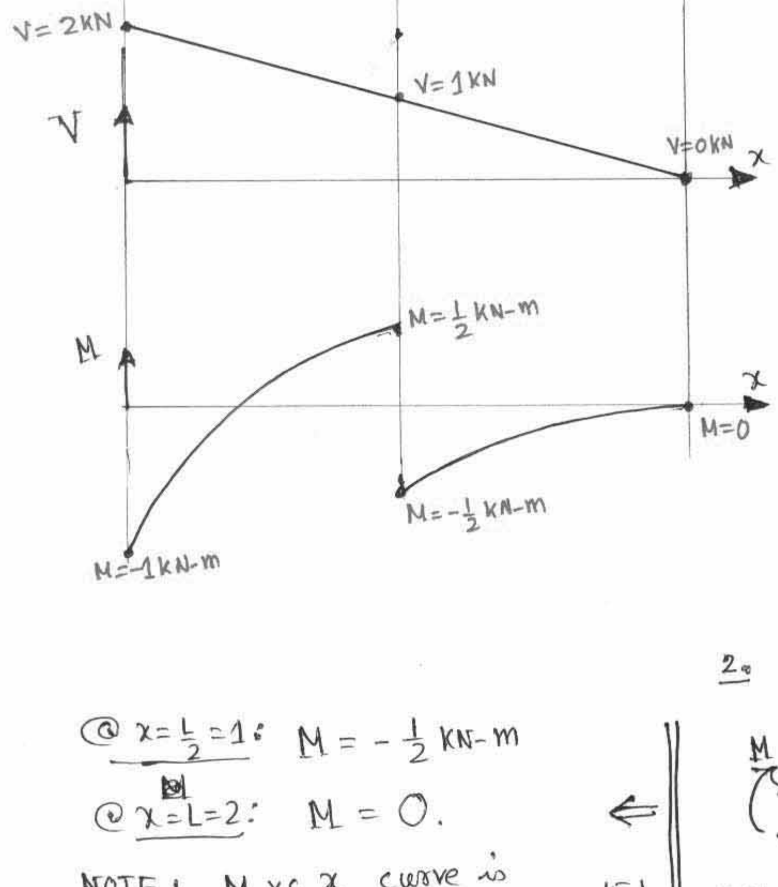
$$M_A + M_1 - \frac{qL^2}{2} = 0$$

$$\Rightarrow \boxed{M_A = 1\text{ kN-m}}$$

Convention for sign

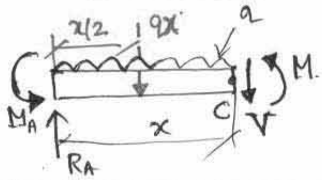


(a) SFD & BMD



Require to take two sections

1. $0 \le x \le L/2$



$$\boxed{V = R_A - qx = 2 - x \text{ (kN)}}$$

$\Sigma M_C = 0$:

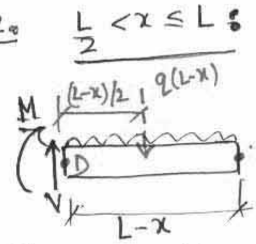
$$M + M_A + q\frac{x^2}{2} - R_A x = 0$$

$$\Rightarrow \boxed{M = 2x - \frac{x^2}{2} - 1 \text{ (kN-m)}}$$

@ $x = 0$: $M = -1$ kN-m

@ $x = \frac{L}{2}$: $M = \frac{1}{2}$ kN-m

2. $\frac{L}{2} < x \le L$: (FBD of right part)



$$\boxed{V = q(L-x) = 2 - x \text{ (kN)}}$$

$\Sigma M_D = 0$: $M = -\frac{q}{2}(L-x)^2 = -\frac{1}{2}(2-x)^2$

$\Leftarrow$

@ $x = \frac{L}{2} = 1$: $M = -\frac{1}{2}$ kN-m

@ $x = L = 2$: $M = 0$.

NOTE: M vs x curve is quadratic in x. (slope positive)

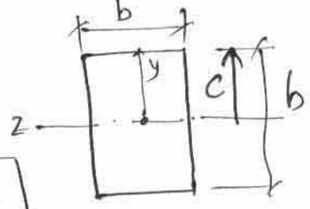
(b) From the bending moment diagram:

M has maximum magnitude at $x=0$.

$$\boxed{M_{max} = 1 \text{ kN-m}}$$

(c) Maximum bending stress:

- Corresponds to M_{max}
- Occurs at the top or bottom of the section
- Crossection – square.
- Moment of inertia of the cross-section: $\boxed{I = \frac{1}{12} b \times b^3 = \frac{b^4}{12}}$



Bending stress: $\sigma_{max} = \frac{Mc}{I}$ (c = max. distance from z-axis).

$$= \frac{M_{max} \times b/2}{b^4/12}$$

$$\Rightarrow \quad \sigma_{max} = 6 M_{max}/b^3 \le 300 \text{ MPa}$$

$$\Rightarrow \quad \frac{6 \times 10^3 \text{ N-m}}{b^3} \le 300 \times 10^6 \frac{N}{m^2}$$

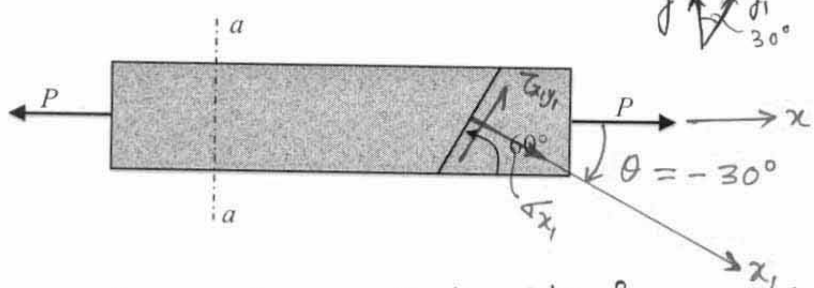
$$\Rightarrow \quad b^3 \geq \frac{6}{300} \times 10^{-3} \text{ m}^3 = 20 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow \quad \boxed{b \geq 2.714 \times 10^{-2} \text{ m} = 27.14 \text{ mm}}$$

$$\Rightarrow \quad \boxed{\text{Minimum beam depth} = 27.14 \text{ mm}}$$

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 4:



- Axis-system x_1-y_1 is rotated from x-y by $\theta = -30°$ (cw)
- Normal stress on oblique plane:

$$\sigma_{x_1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

- Shear stress on the oblique plane:

$$\tau_{x_1y_1} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta.$$

Given: $\sigma_x = \frac{P}{A} = \frac{P}{100}$ MPa (Take P in Newtons)

$\sigma_y = 0$ MPa, $\tau_{xy} = 0$ MPa.

$\theta = -30°$

(a) $$\sigma_{x_1} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos(-60°) = \frac{P}{200}(1 + \cos 60°) = \frac{3P}{400} \text{ MPa}$$

At Failure: $\sigma_{x_1} = 50$ MPa

$$\Rightarrow \frac{3P}{400} = 50 \Rightarrow P = \frac{20}{3} \times 10^3 \text{ N}$$

$$\Rightarrow \boxed{\text{Failure Load} = P_{fail} = \frac{20}{3} \text{ kN}}$$

(b) Shear Stress at failure:

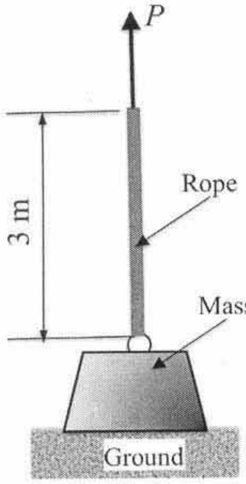
$$\tau_{x_1y_1} = -\frac{\sigma_x}{2}\sin(-60°) = \frac{P}{200} \times \sin 60°$$

$$\Rightarrow \tau_{x_1y_1} = \frac{20}{3} \times 10^3 \times \frac{1}{200} \times \frac{\sqrt{3}}{2} \text{ MPa} = \frac{50}{\sqrt{3}} \text{ MPa}$$

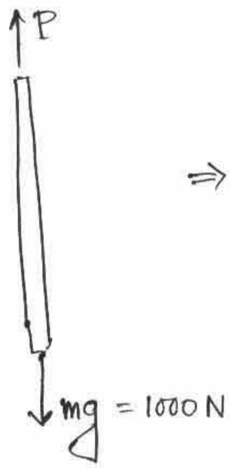
$$\Rightarrow \boxed{\tau_{x_1y_1} = 28.87 \text{ MPa}}$$

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 5:



FBD of the rope :



Axial force

$\Rightarrow$ $\boxed{P = 1000\,N}$

Cross-sectional area

$\boxed{A = 100\,mm^2}$

$\boxed{E_{rope} = 1\,GPa}$

- The rope is under uniaxial extension.
- Average axial stress:

$$\sigma = \frac{P}{A} = \frac{1000}{100}\,\frac{N}{mm^2} = 10\,MPa.$$

- $\Rightarrow$ Average axial strain:

$$\varepsilon = \frac{\sigma}{E} = \frac{10\,MPa}{1\,GPa} = 0.01 = \frac{1}{100}$$

- Elongation of the rope:

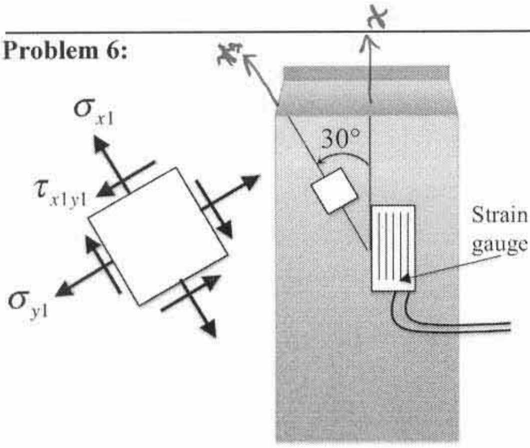
$$\delta = \varepsilon . L = \frac{1}{100} \times 3m = 0.03\,m$$

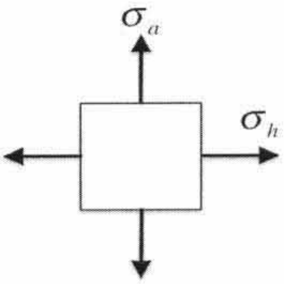
$$\Rightarrow \boxed{\delta = 3\,cm (or\ 30\,mm)}$$

$\Rightarrow$ Top end of the rope will displace by 30 mm (upward) when the mass looses contact with the ground.

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 6:





Given

$$\boxed{\frac{r}{t} = 100}$$

Hoop stress: $\sigma_h = \frac{pr}{t} = 100p$

Longitudinal stress: $\sigma_a = \frac{pr}{2t} = 50p$

(a) Assuming state of plane stress:

Axial strain: $\varepsilon_a = \frac{\sigma_a}{E} - \frac{\nu \sigma_h}{E} = \frac{1}{E}\left(50p - \nu \times 100p\right)$

$$\Rightarrow \quad \varepsilon_a = \frac{50p}{E}(1-2\nu) = 150 \times 10^{-6} \text{ (Given)}$$

$$\Rightarrow \quad p = \frac{150 \times 10^{-6} E}{50(1-2\nu)} = \frac{\overset{3}{\cancel{150}} \times 10^{-6} \times 70 \times 10^{9}}{\cancel{50} \times (1 - 2 \times 0.33)} \text{ Pa}$$

$$\Rightarrow \quad p = 617.65 \times 10^{3} \text{ Pa}$$

$\Rightarrow$ Pressure before can opening is

$$\boxed{p = 617.65 \text{ kPa}}$$

(b) $\sigma_x = \sigma_a = 50p = 30.88$ MPa $\;|\; \tau_{xy} = 0 \;|\; \theta = 30^\circ$

$\sigma_y = \sigma_h = 100p = 61.77$ MPa

$$\sigma_{x_1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \cancelto{0}{\tau_{xy}} \sin 2\theta$$

$$= 46.33 - 15.45 \cos 60^\circ = 38.61 \text{ MPa}$$

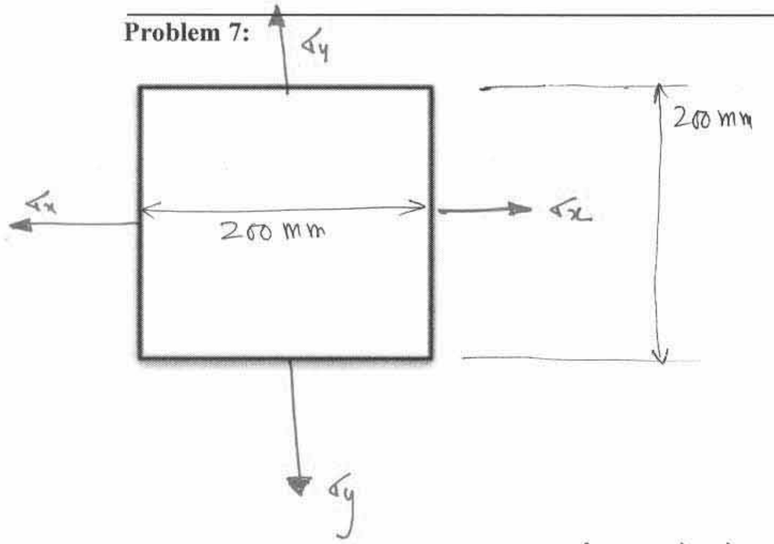
$$\sigma_{y_1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \cancelto{0}{\tau_{xy}} \sin 2\theta = 54.04 \text{ MPa}$$

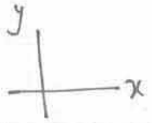
$$\tau_{x_1 y_1} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \cancelto{0}{\tau_{xy}} \cos 2\theta = 13.37 \text{ MPa}$$

$$\Rightarrow \quad \boxed{\sigma_{x_1} = 38.61 \text{ MPa}, \quad \sigma_{y_1} = 54.04 \text{ MPa}, \quad \tau_{x_1 y_1} = 13.37 \text{ MPa}}$$

Mechanics (ME10001) End Semester Exam-Autumn 2017-18
Solutions

Problem 7:





$$\boxed{\begin{aligned} E &= 100\,GPa = 100\times10^3\,MPa \\ \nu &= 0.3 \end{aligned}}$$

$$\boxed{\sigma_x = 10\,MPa, \quad \sigma_y = 20\,MPa}$$

Strains: $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{100\times10^3\,MPa}(10 - 0.3\times20)\,MPa$

$$= 4\times10^{-5}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{10^5}\times(20 - 0.3\times10) = 17\times10^{-5}$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -9\times10^{-5}$$

Elongation: $\delta_x = \varepsilon_x \times L = 4\times10^{-5}\times 200\text{ mm} = 8\times10^{-3}\text{ mm}$

$$\delta_y = \varepsilon_y \times L = 17\times10^{-5}\times 200\text{ mm} = 34\times10^{-3}\text{ mm}$$

$$\delta_z = \varepsilon_z \times t = -9\times10^{-5}\times1\text{ mm} = -9\times10^{-5}\text{ mm}$$

thickness = 1 mm

New volume: $(200+\delta_x)(200+\delta_y)(1+\delta_z)\text{ mm}^3$

$$= (200.008)\times(200.034)(0.99991)\text{ mm}^3$$

$$\boxed{V_f = 40004.8\text{ mm}^3.}$$

Original volume: $V_i = 200\times200\times1\text{ mm}^3 = 40000\text{ mm}^3.$

$\Rightarrow$ Percentage change in volume

$$\boxed{\frac{\Delta V}{V} = \frac{4.8\text{ mm}^3}{40{,}000\text{ mm}^3}\times100\% = 1.2\times10^{-2}\%}$$

Alternate method: Volume strain: $\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = 1.2\times10^{-4}$

$\Rightarrow$ % Change in volume $= \frac{\Delta V}{V}\times100\% = 1.2\times10^{-2}\%$