Chapter- 4.

CIRCUIT THEOREMS

~~4.1~~ 4.0 INTRODUCTION

In chapter 3, we have used Kirchhoff's laws and main advantage of using ~~these laws~~ ~~two laws~~ KCL and KVL is that we can analyze a circuit without tampering with its original configuration. A major drawback of this approach is that, for a large and complex circuit, tedious computation is involved. To handle the complexity of the circuits, over the years engineers have developed some circuit theorems to simplyfy circuits analysis. Such theorems include Thevenin's theorem and Norton's theorem. These theorems are applicable to linear circuits and in this chapter, we ~~will~~ first discuss the concept of circuit linearity. In this Chapter we will also discuss the concepts of superposition, source tramsformation and maximum power transfer.

4.1: LINEARITY PROPERTY

Linearity: It is the property of an element describing a linear relationship between cause and effect.

Although linearity property applies to many circuit elements, but in this chapter we shall limit its applicability to resistors only. The linearity property is a combination of both the homogeneity (scaling) property and the additivity property.

Homogeneity property: It requires that if the input (called excitation) is multiplied by constant, then the output (called response) is multiplied by the same constant.

For example, for a resistor, Ohm's law relates the input current i to the output voltage v,

$$v = iR \quad \text{-----} \quad (4.1)$$

~~If the current is increased (decreased)~~

If the current is increased (or decreased) by a constant K, then the voltage increases (or decreases) correspondingly by K, that is

$$(Ki)R = Kv \quad \text{----} \quad (4.2)$$

Additivity property: It requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

Using the voltage - current relationship of a resistor, if,

$$v_1 = i_1 R \quad \text{- - -} \quad (4.3)$$

$$v_2 = i_2 R \quad \text{- - - -} \quad (4.4)$$

then applying $(i_1 + i_2)$ gives,

$$v = (i_1 + i_2) R = i_1 R + i_2 R$$

$$\therefore v = v_1 + v_2 \quad \text{- - - -} \quad (4.5)$$

Therefore, we can say that a resistor is a linear element because its voltage-current relationship satisfies both homogeneity and additivity properties.

In general, a circuit is linear if it is both homogeneous and additive. A linear circuit consists only linear elements, linear dependent sources and independent sources. In other words, a linear circuit is one whose output is directly proportional to its input.

Note that, since power $p = i^2 R = v^2/R$, power-voltage or power-current relationship is nonlinear. In this book, we consider only linear circuits and hence theorems covered in this chapter are not applicable to power.

For the purpose of explaning the linearity principle, consider Fig. 4.1.

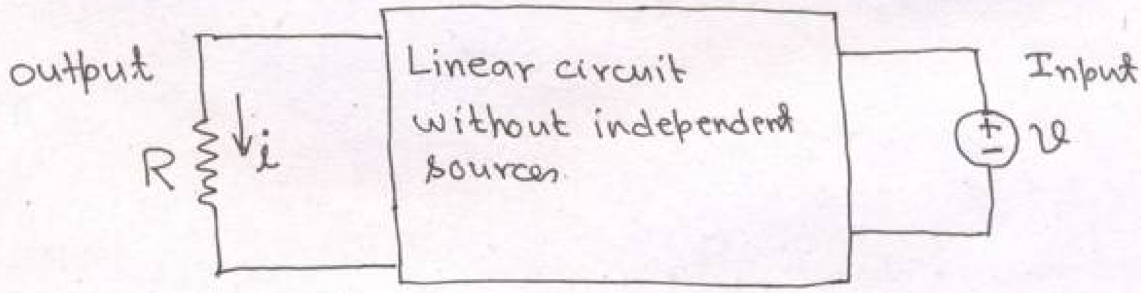


Fig. 4.1: A linear circuit

The linear circuit shown in Fig. 4.1, is excited by a voltage source v, which serves as the input. The circuit is terminated by a load resistance R. Current i flowing through R ~~can~~ may be taken as the output. ~~So~~ suppose $v = 100$ Volt gives $i = 20$ Amp. According to the linearity principle, $v = 10$ Volt will give $i = 2$ Amp.

Ex-4.1: Determine i_o when $v = 3$ Volt and $v = 6$ Volt of the circuit shown in Fig. 4.2.

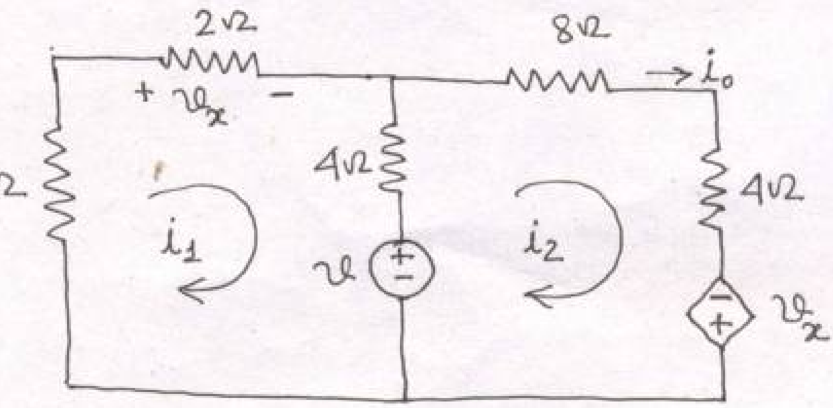


Fig. 4.2: Circuit for ~~Problem 4~~ Ex-4.1

Soln.

Applying KVL, we obtain,

$$12i_1 - 4i_2 + v = 0 \quad \text{---- (i)}$$

$$-4i_1 + 16i_2 - v - v_x = 0 \quad \text{-- (ii)}$$

But $v_x = 2i_1$, equation (ii) becomes

$$-4i_1 + 16i_2 - 2i_1 = v$$

$$\therefore \; -6i_1 + 16i_2 = v \quad \text{---- (iii)}$$

When $v = 3$ volt, solving eqns. (i) and (iii), we obtain $i_0 = i_2 = \frac{3}{28}$ Amp and when $v = 6$ Volt, $i_0 = i_2 = \frac{6}{28}$ Amp.

This clearly shows that when source voltage (input) is doubled, i_0 also doubles. Hence, the circuit is linear.

EX-4.2: Determine v_o, when $i = 5$ Amp and $i = 10$ Amp of the circuit shown in Fig.4.3.

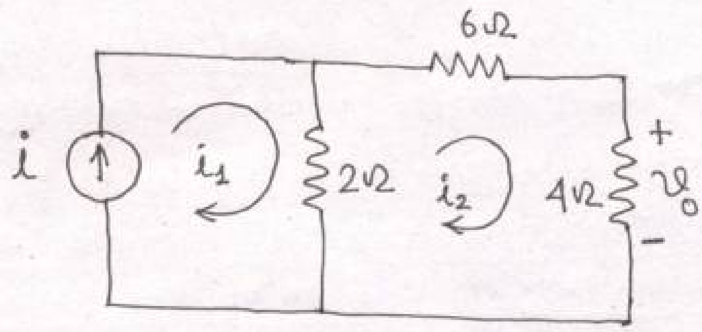


Fig. 4.3: Circuit for EX-4.2

Soln.

$i_1 = i$ - - - - (i)

$12i_2 - 2i_1 = 0$

$\therefore \quad i_2 = \frac{i_1}{6}$ - - - (ii)

Also

$v_0 = 4i_2$ - - - - (iii)

when $i = 5$ Amp; $i_1 = 5$ Amp, $i_2 = \frac{i_1}{6} = \frac{5}{6}$ Amp;

$v_0 = 4i_2 = 4 \times \frac{5}{6} = \frac{20}{6}$ Volt

Similarly, when $i = 10$ Amp; $i_1 = 10$ Amp;

$i_2 = \frac{i_1}{6} = \frac{10}{6}$ Amp; $v_0 = 4 \times \frac{10}{6} = \frac{40}{6}$ Volt.

This shows that when source current (input) is doubled, v_0 also doubles. Hence, the circuit is linear.

4.2: SUPERPOSITION PRINCIPLE

If a circuit has two or more independent sources one can determine the contribution of each independent source to the variable and then add them up. This approach is known as the superposition. The idea of superposition rests on the linearity property.

The superposition principle states that the current through (or voltage across) an element in a linear circuit is the algebraic sum of the currents through (or voltages across) that element due to each independent source acting alone.

To apply the superposition principle, two things must be kept in mind.

1. Consider one independent source at a time while all other independent sources are turned off. This means, we replace every voltage source by 0 Volt (or a short circuit) and every current source by 0 Amp (or an open circuit). Thus we obtain a simpler and more manageable circuit.

2. Dependent sources are controlled by circuit variables and hence they are left intact.

Circuit analysis using superposition may very likely involve more work and this is major disadvantage. Superposition is based on linearity and for this reason, it is not applicable to the effect on power due to each source because the power absorbed by a resistor depends on the square of the voltage or current.

EX-4.3: Using superposition theorem, determine v in the circuit shown in Fig. 4.4.

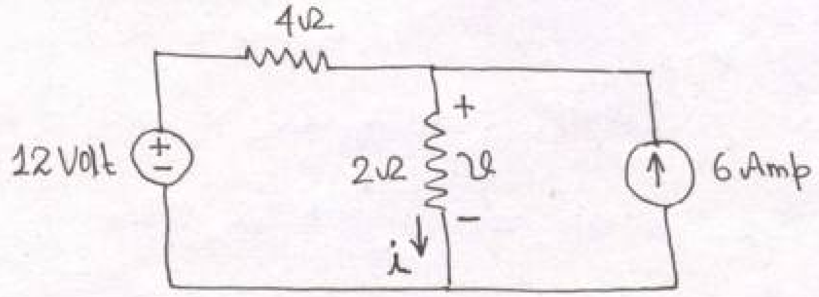


Fig. 4.4: Circuit for Ex-4.3

Soln:

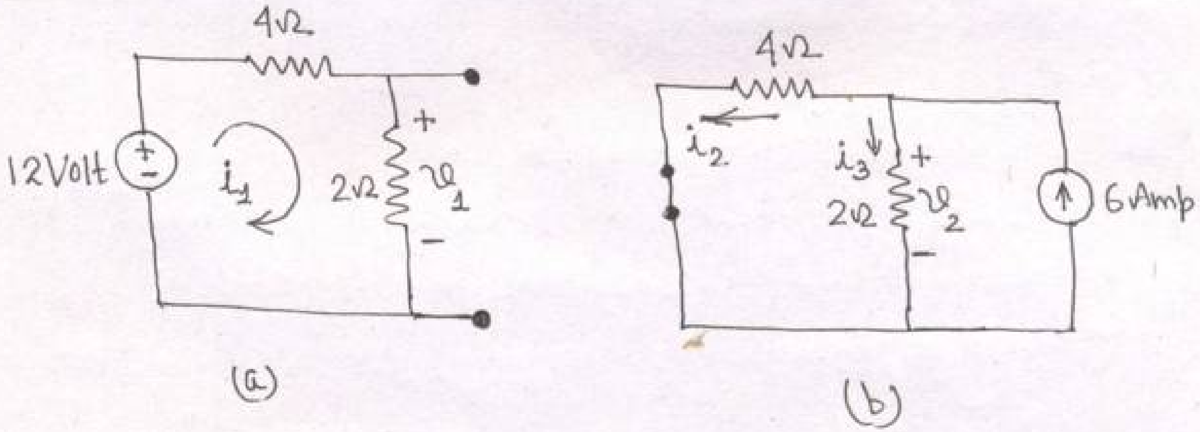


Fig. 4.5: (a) calculating v_1 (b) calculating v_2.

Let v_1 is the voltage drop across 2Ω resistor due to 12 Volt voltage source only and v_2 is voltage drop across 2Ω resistor due to 6 Amp current source only. Therefore, from the principle of superposition,

$$v = v_1 + v_2 \quad \text{---- (i)}$$

To obtain v_1, current source is set to zero as shown in Fig. 4.5(a). Applying KVL in Fig. 4.5(a), gives

$$6i_1 = 12 \therefore i_1 = 2 \text{ Amp.}$$

Thus,

$$v_1 = 2i_1 = 2\times2 = 4 \text{ Volt.}$$

To get v_2, set the voltage source to zero as shown in Fig. 4.5(b). By using current division,

$$i_3 = \frac{4}{(2+4)}\times 6 = 4 \text{ Amp}; \quad v_2 = 2i_3 = 2\times4 = 8 \text{ Volt}$$

Therefore, $v = v_1 + v_2 = 4 + 8 = 12$ Volt.

For checking the result, $i = i_1 + i_3 = 2 + 4 = 6$ Amp

$$v = 2i = 2\times6 = 12 \text{ Volt.}$$

Ex-4.4: Using superposition theorem, determine i_x in the circuit ~~shown~~ in Fig. 4.6

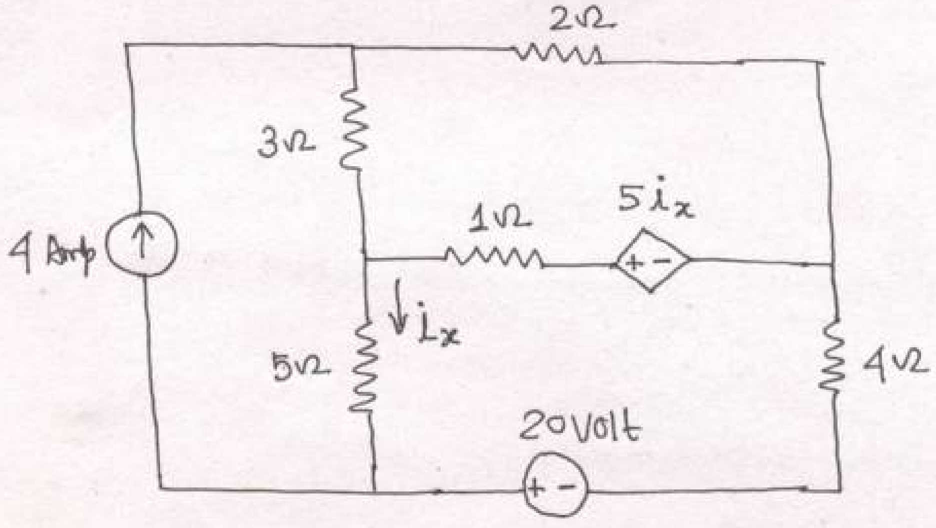


Fig. 4.6: Circuit for ~~Problem~~ Ex-4.4

Soln.

The circuit in Fig. 4.6 has a dependent voltage source, which must be left intact.

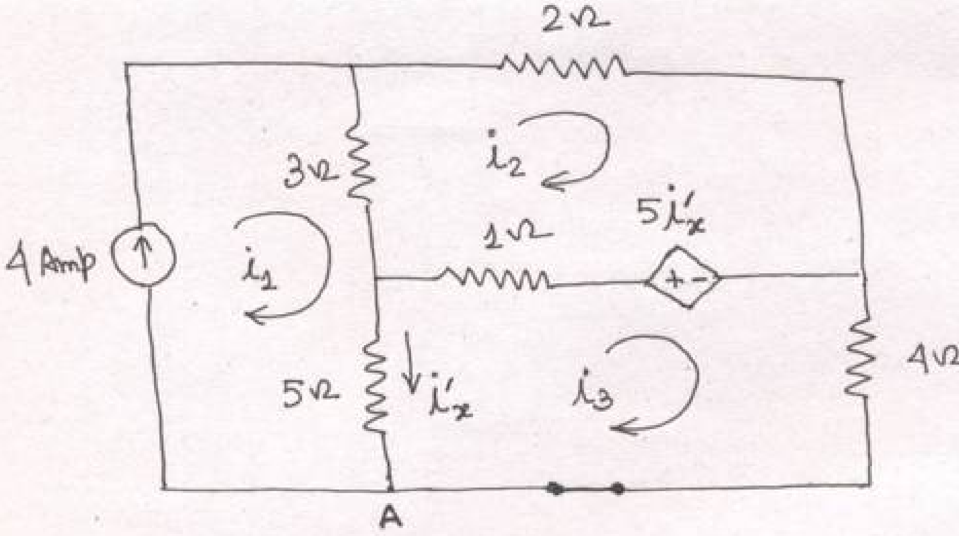
Let

$$i_x = i_x' + i_x'' \quad \cdots \quad (i)$$

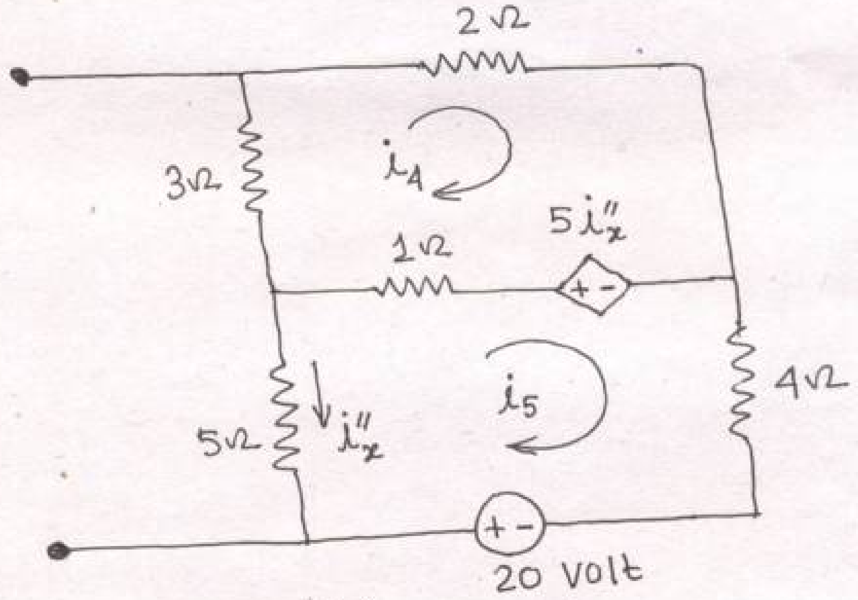
where

i_x' = current through 5Ω resistor due to 4 Amp current source only, as shown in Fig. 4.7(a)

i_x'' = current through 5 Ω resistor due to 20 Volt voltage source only, as shown in Fig. 4.7(b)



(a)



(b)

Fig. 4.7: (a) Applying superposition to obtain i'_x
(b) to obtain i''_x

For the circuit shown in Fig. 4.7(a), we apply mesh analysis to obtain i'_x.

For mesh 1,

$$i_1 = 4 \text{ Amp} \quad \cdots\cdots (ii)$$

For mesh 2,

$$-3i_1 + 6i_2 - i_3 - 5i'_x = 0 \quad \cdots\cdots (iii)$$

For mesh 3,

$$-5i_1 - i_2 + 10i_3 + 5i'_x = 0 \quad \cdots\cdots \text{(iv)}$$

At node A,

$$i_3 = i_1 - i'_x = 4 - i'_x \quad \cdots\cdots \text{(v)}$$

Substituting eqns.(ii) and and (v) in eqns.(iii) and (iv), gives two simultaneous equations

$$3i_2 - 2i'_x = 8 \quad \cdots \text{(vi)}$$

$$i_2 + 5i'_x = 20 \quad \cdots \text{(vii)}$$

Solving eqns.(vi) and (vii), we obtain

$$i'_x = \frac{52}{17} \text{ Amp} \quad \cdots \text{(viii)}$$

To obtain i''_x, we turn off the 4 Amp current source so that the circuit becomes that shown in Fig. 4.7(b).

For mesh 4, KVL gives

$$6i_4 - i_5 - 5i''_x = 0 \quad \cdots \text{(ix)}$$

for mesh 5,

$$-i_4 + 10i_5 - 20 + 5i''_x = 0 \quad \cdots \text{(x)}$$

But $i_5 = -i''_x$, substituting this in eqns (ix) and (x), we get

$$6i_4 - 4i''_x = 0 \quad \cdots \text{(xi)}$$

$$i_4 + 5i''_x = -20 \quad \cdots \text{(xii)}$$

Solving eqns.(xi) and (xii), we get,

$$i_x'' = -\frac{60}{17} \text{ Amp} \quad --- \text{(xiii)}$$

Therefore,

$$i_x = i_x' + i_x'' = \frac{52}{17} - \frac{60}{17} = \frac{-8}{17} \text{ Amp}$$

EX- 4.5: Determine i using superposition theorem of the circuit shown in Fig. 4.8

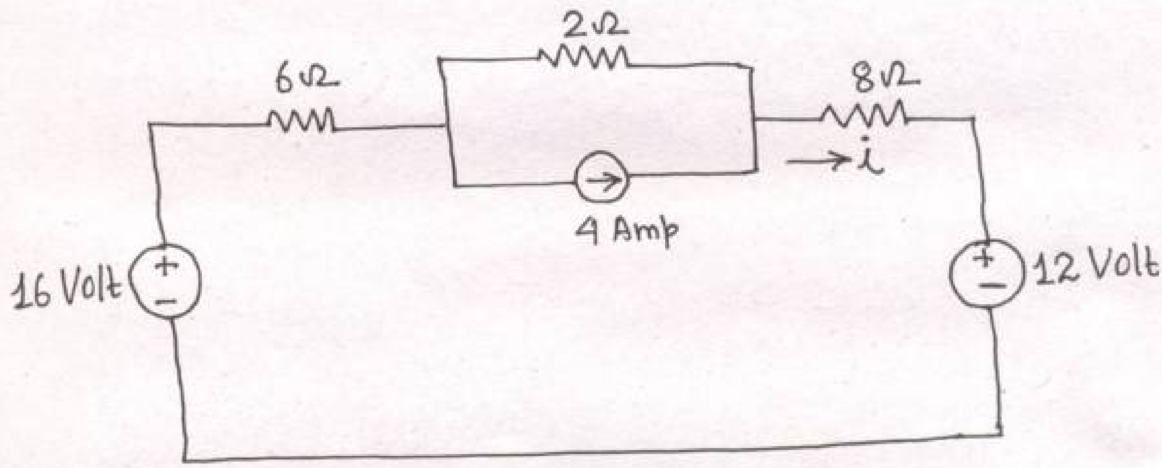
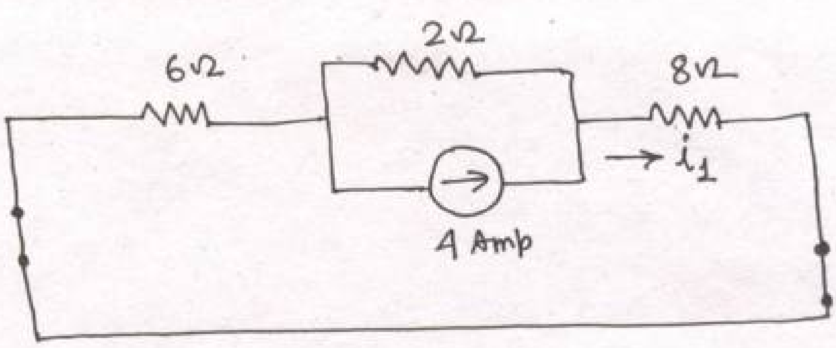
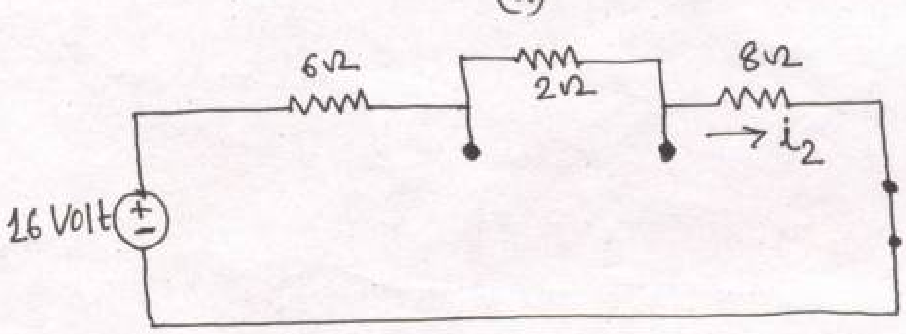


Fig. 4.8: Circuit for EX- 4.5

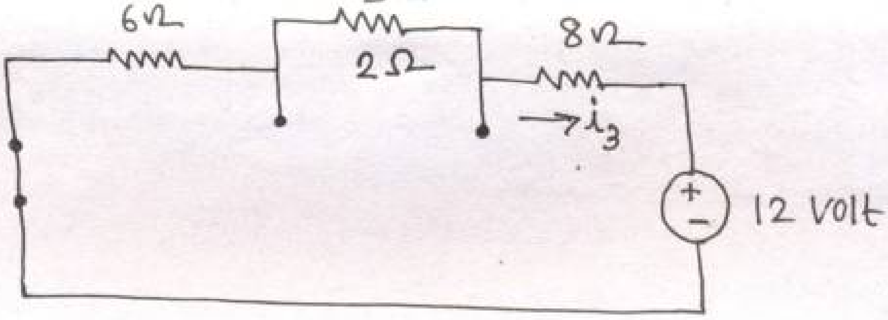
Soln.



(a)



(b)



(c)

Fig. 4.9: (a) 16 Volt and 12 Volt sources are turned-off

(b) 4 Amp current source and 12 Volt voltage source are turned off

(c) 4 Amp current source and 16 Volt voltage source are turned-off.

In Fig. 4.9(a), we apply current division principle,

$$i_1 = \frac{2}{(6+2+8)} \times 4 = 0.5 \text{ Amp}$$

In Fig. 4.9(b), we apply KVL,

$$i_2 = \frac{16}{16} = 1 \text{ Amp}$$

Similarly from Fig. 4.9(c), we obtain

$$i_3 = -\frac{12}{16} = -\frac{3}{4} \text{ Amp} = -0.75 \text{ Amp}$$

Hence

$$i = i_1 + i_2 + i_3 = 0.5 + 1 - 0.75 = 0.75 \text{ Amp.}$$

4.3: SOURCE TRANSFORMATION

Source transformation is a tool to simplify circuit analysis. Basic idea behind this is concept of equivalance. An equivalent circuit is one whose v-i characteristics are identical with the original circuit. (14)

A source transformation, shown in Fig. 4.10, allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa.

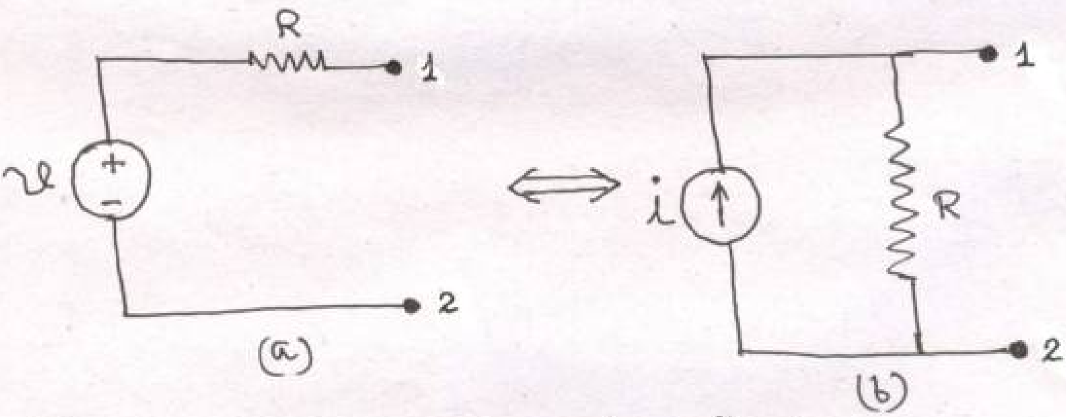


Fig. 4.10: Source transformations

Double headed arrow in Fig. 4.10 emphasizes that a source transformation is bilateral, that is we can start with either configuration and derive the other.

The two circuits in Fig. 4.10, are equivalent, provided they have same voltage-current relation at terminals 1-2. Equivalence is achieved if any resistor R_L experiences the same current flow, and thus the same voltage drop, whether connected between nodes 1, 2 in Fig. 4.10(a) or Fig. 4.10(b).

Suppose R_L is connected between nodes 1, 2 in Fig. 4.10(a). Using Ohm's law, the current in R_L is

$$i_L = \frac{v}{R + R_L} \quad \cdots\cdots \quad (4.6)$$

Now suppose the same Resistor R_L is connected between nodes 1, 2 in Fig. 4.10(b). We find the current in R_L is

$$i_L = \frac{R}{R + R_L} i \quad \cdots\cdots \quad (4.7)$$

If the two circuits in Fig 4.10(a) and Fig. 4.10(b) are equivalent, these resistor currents must be the same. Equating the right hand side of eqns (4.6) and (4.7) and simplifying, we obtain

$$i = \frac{v}{R} \quad \text{or} \quad v = iR \quad \cdots\cdots \quad (4.8)$$

When eqn. (4.8) is satisfied for the circuits in Fig. 4.10, the current i_L is the same for both circuits in the ~~figu~~ Fig. 4.10 for all values of R_L. If the current through R_L is the same in both circuits, then the voltage drop across R_L is the same in both circuits, and the circuits are equivalent at nodes 1, 2. If the polarity of v is reversed, the orientation of i must be reversed to maintain equivalence.

Source transformation also applies to dependent sources. As shown in Fig. 4.11, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa where we make sure that that eqn.(4.8) is satisfied.

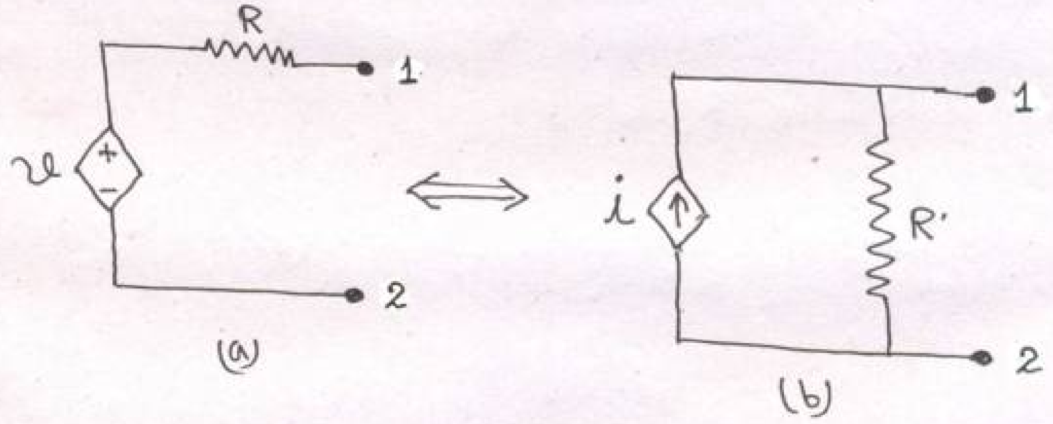


Fig. 4.11: Transformation of dependent sources.

<u>EX-4.6:</u> Using source transformation, determine v_o in the circuit shown in Fig. 4.12.

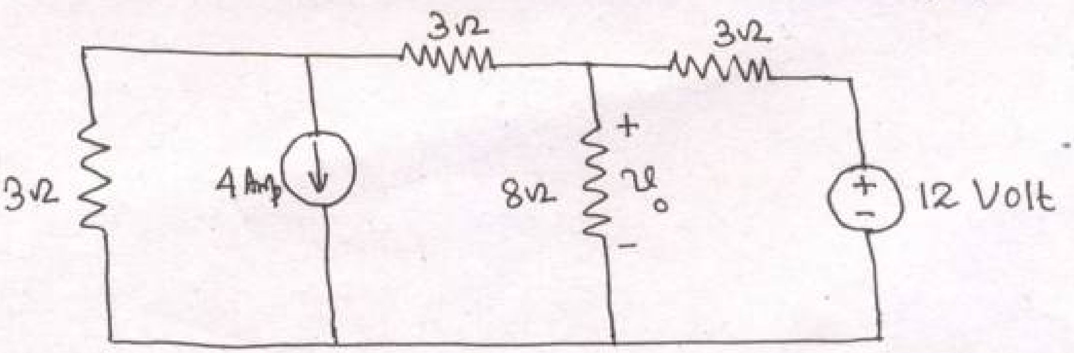
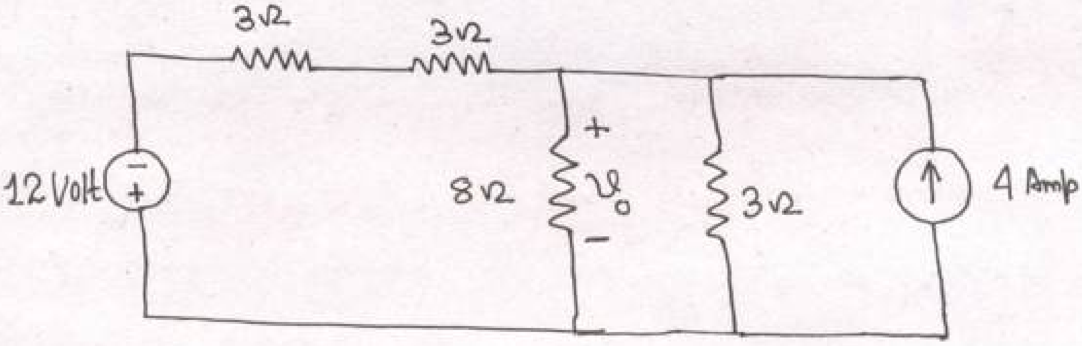


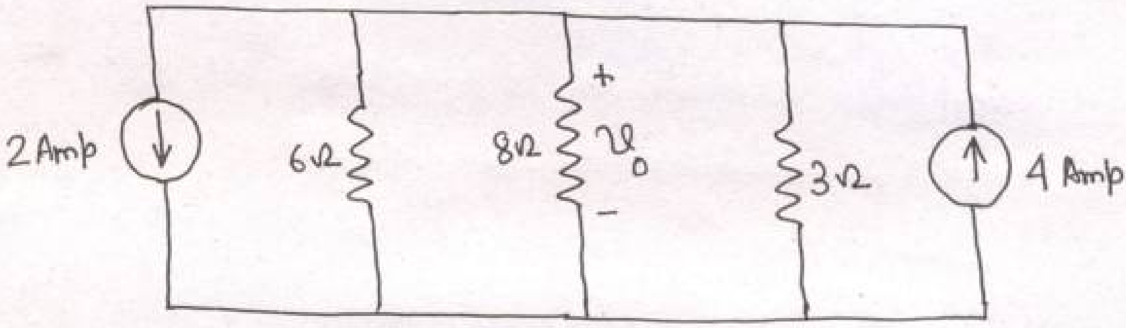
Fig. 4.12: Circuit for Ex-4.6

<u>Soln.</u>

First transform the current and voltage sources to obtain the circuit in Fig. 4.13(a)



(a)



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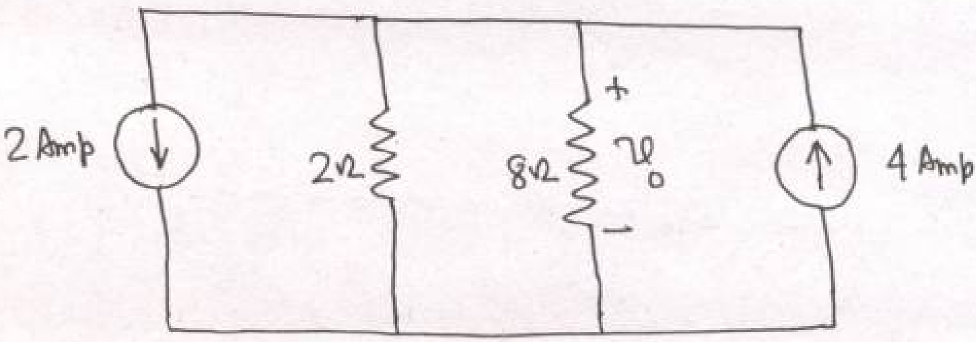

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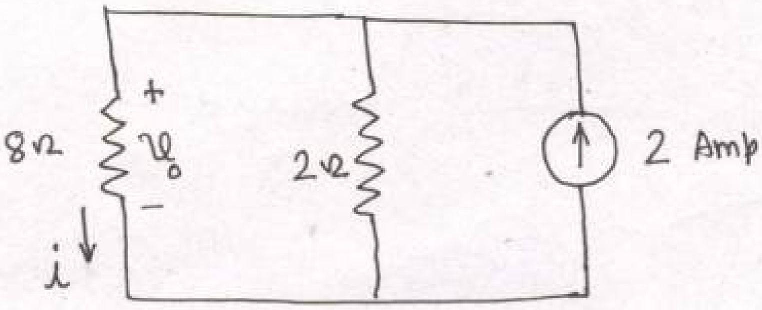


(d)

Fig. 4.13: For Ex-4.6

Combine 3Ω and 3Ω resistors in series and transforming the 12 Volt voltage source in Fig. 4.13(a) gives us Fig. 4.13(b). Now combine 6Ω and 3Ω resistors in parallel to get 2Ω and the equivalent circuit is shown in Fig. 4.13(c). Also combine the 2 Amp and 4 Amp current sources in Fig. 4.13(c) to get equivalent ~~circuit shown in Fig. 4.13(d).~~
2 Amp current source and the circuit is shown in Fig. 4.13(d).

From Fig. 4.13(d),

$$i = \frac{2}{(2+8)} \times 2 = 0.4 \text{ Amp}$$

$$\therefore \; v_0 = 8i = 8 \times 0.4 = 3.2 \text{ Volt.}$$

EX-4.7: Using source transformation, determine i_x in the circuit shown in Fig. 4.14

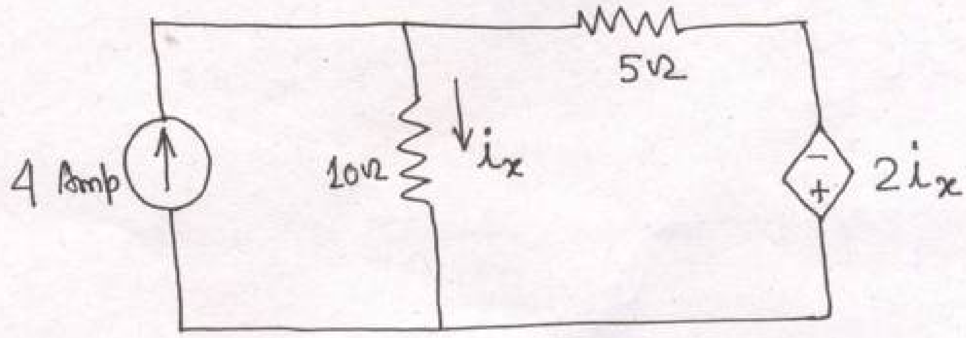


Fig. 4.14: Circuit for EX-4.7.

Soln.

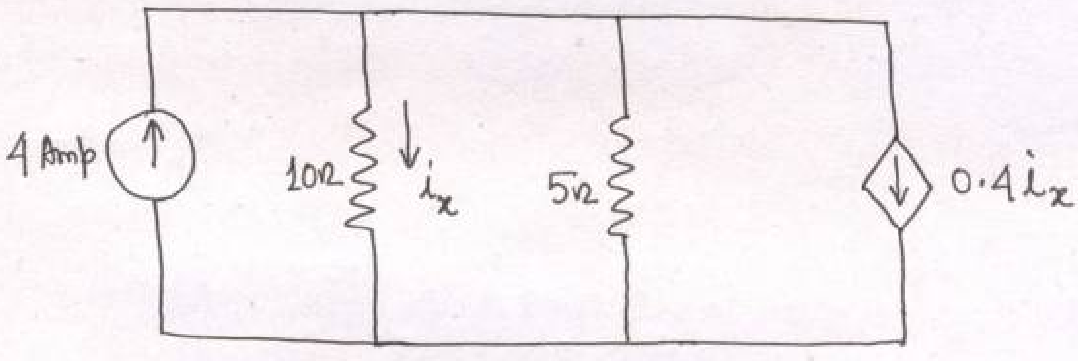


Fig. 4.15 : For EX-4.7

We convert dependent voltage source to current source shown in Fig. 4.15. From Fig. 4.15, we can easily write by inspection,

$$i_x = \frac{5}{(5+10)}(4 - 0.4 i_x)$$

$$\therefore 3.4 i_x = 4$$

$$\therefore i_x = \frac{4}{3.4} = 1.176 \text{ Amp.}$$

EX-4.8: Using source transformation technique, determine the current through load resistance $R_L = 4\Omega$ of Fig. 4.16.

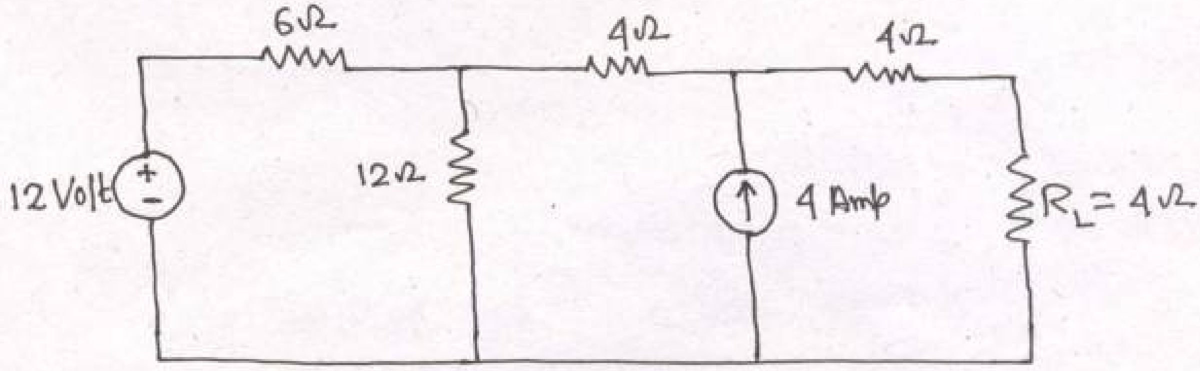


Fig. 4.16: Circuit for EX-4.8

Soln.

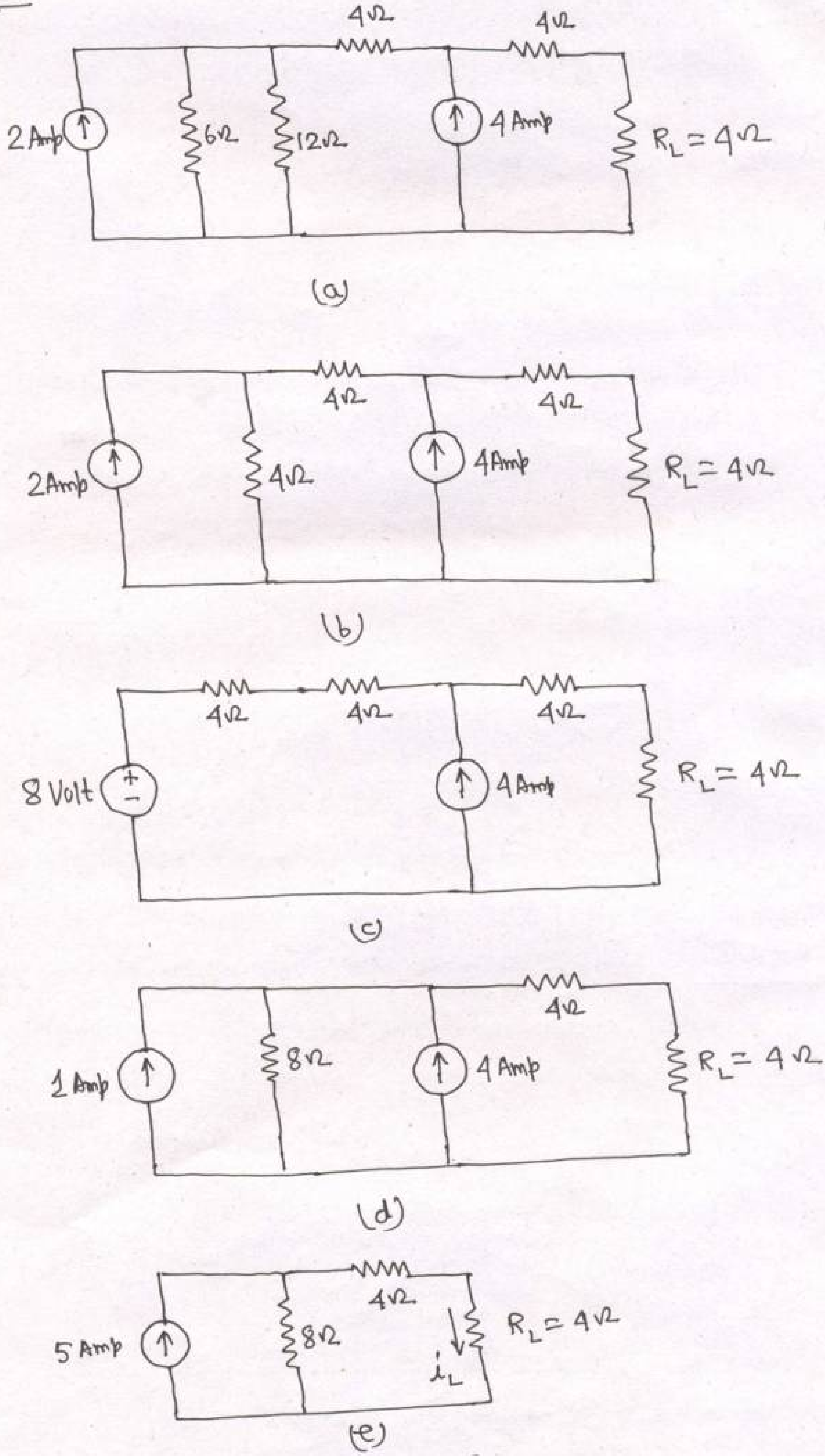


Fig. 4.17: For EX-4.8

The 12 Volt voltage Source with 6Ω Series resistor is converted to a current source ~~and~~ in parallel with 6Ω resistor as shown in Fig. 4.17(a)

6Ω and 12Ω resistors of Fig. 4.17(a) are in parallel and their equivalent is $6\times12/(6+12) = 4\Omega$ as shown in Fig. 4.17(b).

In Fig. 4.17(b), 2 Amp current Source is in parallel with 4Ω resistor and is transformed into 8 Volt voltage Source with 4Ω series resistor as shown in Fig. 4.17(c).

Next, in Fig. 4.17(c), 8 Volt voltage Source with $(4+4) = 8\Omega$ series resistor is transformed into current Source of 1 Amp with 8Ω parallel resistor as shown in ~~Fig 15~~ Fig. 4.17(d).

Finally two current Sources of 1 Amp and 4 Amp are combined to give a single current Source of 5 Amp as shown in Fig. 4.17(e).

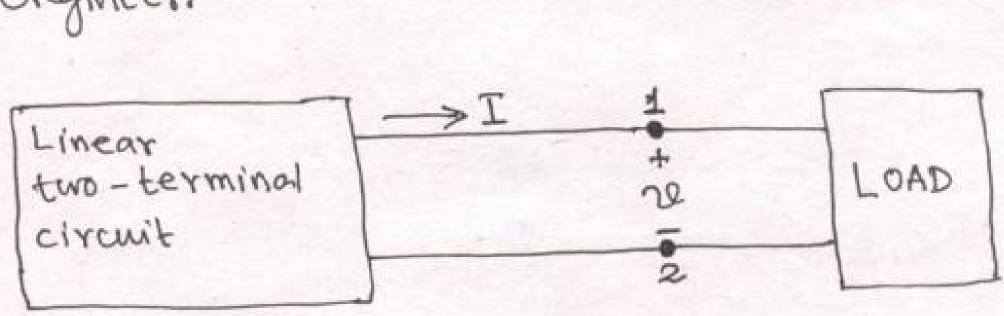
Therefore current through load resistance $R_L = 4\Omega$ is given by

$$i_4 = \frac{8}{(8+4+4)} \times 5 = 2.5 \text{ Amp.}$$

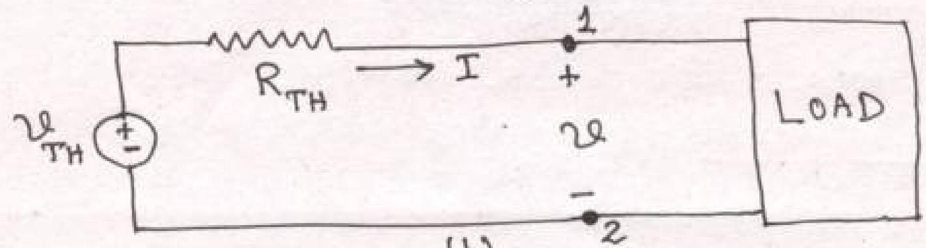
4.4: THEVENIN'S THEOREM

In this section, we Learn how to replace two-terminal circuits containing resistances and sources by simple equivalent circuits. As a typical example, a household outlet terminal may be connected to different electrical appliances constituting a variable load. Each time the variable is changed, the entire circuit has to be analyzed again. To avoid this problem, Thevenin's theorem gives a good technique by which fixed part of the circuit can be replaced by an equivalent circuit. By a two-terminal circuit, we mean that the original circuit has only two points that can be connected to other circuits. However, a restriction is that the controlling variables for any controlled sources must appear inside the original circuit.

Fig. 4.18(a) shows a linear circuit. The circuit to the left of the terminals 1-2 in Fig. 4.18(b) is known as the Thevenin equivalent circuit. It was developed by M. Leon Thevenin (1857-1926) in 1883, a French telegraph engineer.



(a)



(b)

Fig. 4.18: (a) Original circuit (b) Thevenin equivalent circuit

In Fig. 4.18, LOAD may be a single resistor or another circuit.

Thevenin's theorem states a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source v_{TH} in series with a resistor R_{TH}.

Where

v_{TH} = Open circuit voltage at the terminals

R_{TH} = Input or equivalent resistance at the terminals when the independent sources are turned off.

Our major objective is now to find v_{TH} and R_{TH}. Suppose two circuits in Fig. 4.18 are equivalent.

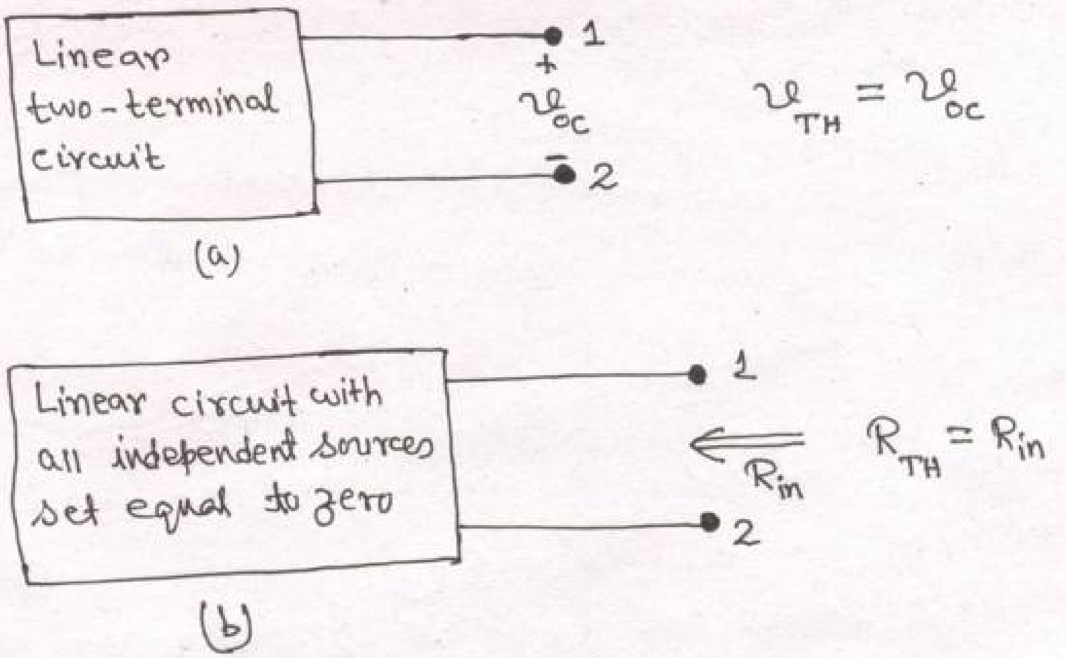


Fig. 4.19: (a) finding v_{TH} (b) finding R_{TH}.

Now let us examine the two cases.

1. If the terminals 1-2 are open circuited (by removing the LOAD), no current flows in Fig. 4.18(a), i.e. $I = 0$, so that open circuit voltage across the terminals 1-2 in Fig. 4.18(a) must be equal to the voltage source v_{TH} in Fig. 4.18(b), since the two circuits are equivalent. Thus v_{TH} is the open-circuit voltage across the terminals as shown in Fig. 4.19(a), that is, $v_{TH} = v_{oc}$.

2. Again, terminals 1-2 are open circuited with the LOAD disconnected and turn off all independent sources. The input resistance or equivalent resistance of the dead circuit at the terminals ~~of the~~ 1-2 in Fig. 4.18(a) must be equal to R_{TH} in Fig. 4.18(b) since the two circuits are equivalent. Hence R_{TH} is the input resistance at the terminals 1-2, when the independent sources are turned off, as shown in Fig. 4.19(b), that is $R_{TH} = R_{in}$.

Thevenin's theorem helps to simplify a circuit and is very important in circuit analysis. A large circuit can be replaced by a single independent voltage source and a single resistor and this replacement technique is a powerful tool in circuit design.

For finding out the Thevenin resistance R_{TH}, we need to consider two cases.

<u>Case-1:</u> If the network has no dependent sources, turn off all the independent sources. Then determine R_{TH}, which is the input resistance of the network looking between terminals 1 and 2 as shown in Fig. 4.19(b).

<u>Case-2:</u> If the network has dependent sources, turn off all independent sources. Now apply a voltage source v_o at terminals 1 and 2 and obtain the resulting current i_o. Then $R_{TH} = v_o/i_o$ as shown in Fig. 4.20(a). Alternatively, a current source i_o can be inserted at terminals 1–2 as shown in Fig. 4.20(b). Then find terminal voltage v_o and $R_{TH} = v_o/i_o$. Both the approaches give identical results. We may assume any value of v_o and i_o. For example, we may use $v_o = 10$ Volt or $i_o = 1$ Amp or any unspecified values of v_o and ~~i_o~~ i_o.

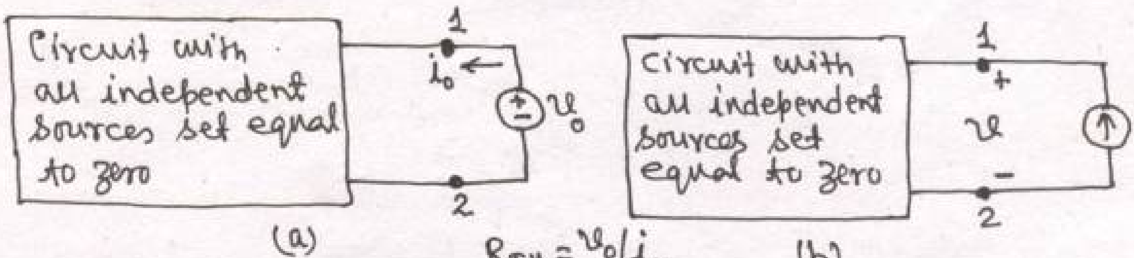


Fig. 4.20: Determination of R_{TH} when circuit has dependent sources.

It may happen that R_{TH} has a negative value. (26) The negative value ($v = -iR$) implies that the circuit is supplying power and this is possible in a circuit with dependent sources.

<u>Ex-4.9:</u> Find the Thevenin equivalent circuit of the circuit shown in Fig.4.21, to the left of the terminals 1-2. Then find current through $R_L = 8\Omega$.

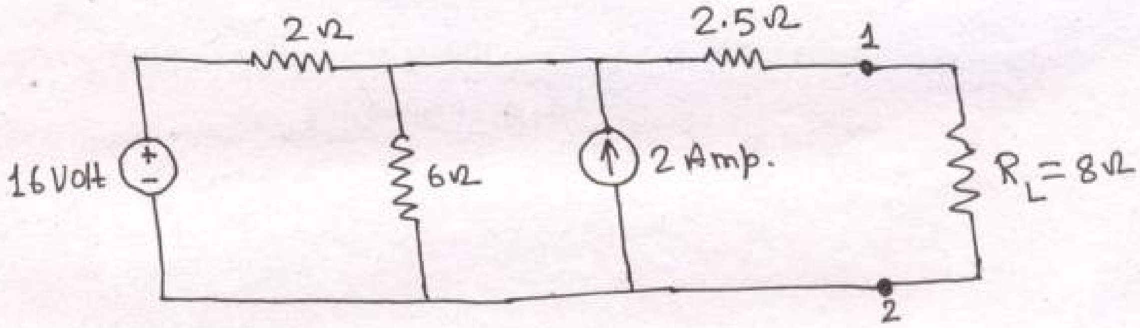


Fig. 4.21: Circuit for Ex-4.9

<u>Soln.</u>

To determine R_{TH}, we turn off the 16 volt independent voltage source (replacing it with a short circuit) and 2 Amp current source (replacing it with an open circuit). The circuit is shown in Fig. 4.22(a).

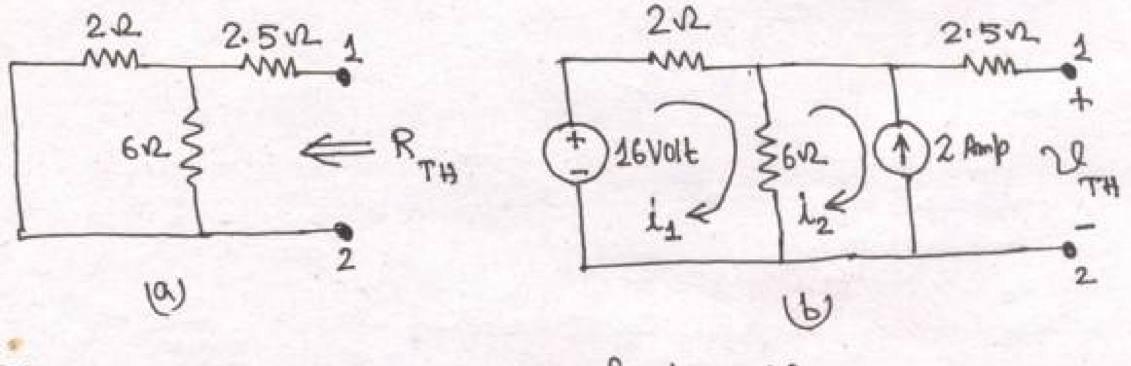


Fig. 4.21: (a) finding R_{TH} (b) finding v_{TH}.

From Fig. 4.21(a),

$$R_{TH} = \frac{2 \times 6}{2+6} + 2.5 = 4\,\Omega$$

In Fig. 4.21(b), applying mesh analysis,

$$-16 + 2i_1 + 6(i_1 - i_2) = 0 \quad \text{--- (i)}$$

and

$$i_2 = -2 \text{ Amp} \quad \text{--- (ii)}$$

$\therefore$ $i_1 = 0.5$ Amp

$\therefore$ $v_{TH} = 6(i_1 - i_2) = 6(0.5 + 2) = 15$ Volt

The Thevenin equivalent circuit is shown in Fig. 4.22.

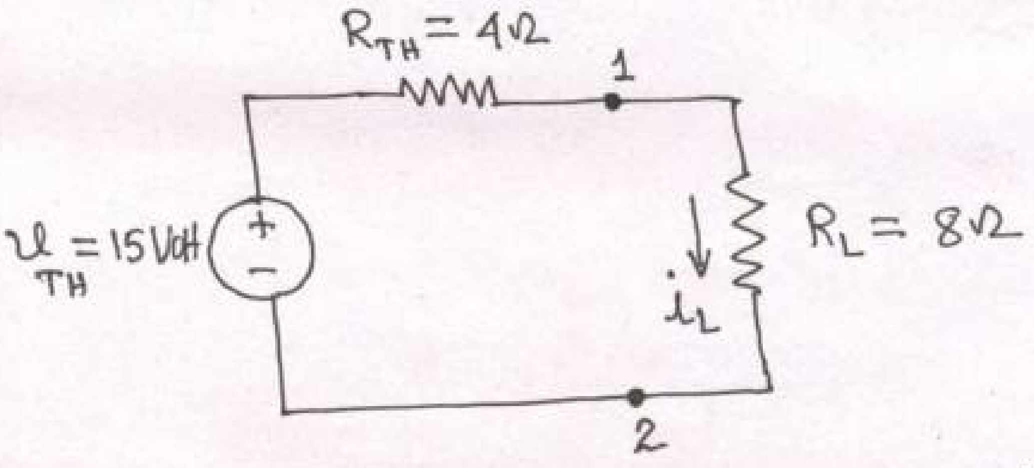


Fig.4.22: Thevenin equivalent circuit for Ex-4.9

$$i_L = \frac{v_{TH}}{R_{TH} + R_L} = \frac{15}{4 + 8} = \frac{15}{12} = 1.25 \text{ Amp.}$$

Ex-4.10: Determine v_{TH} of Ex-4.9 by using nodal analysis.

Soln.

We ignore the 2.5Ω resistor since no current flows through it. Fig. 4.23 shows the circuit of Ex-4.9 for determining v_{TH} using nodal analysis.

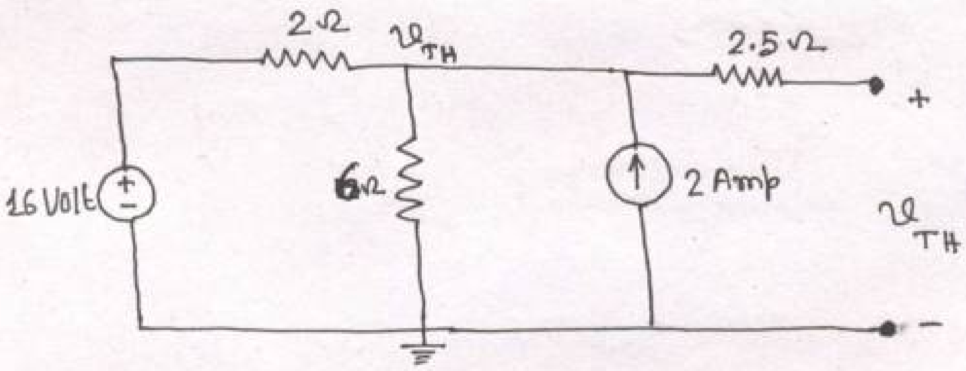


Fig. 4.23: Finding v_{TH} using nodal analysis

From Fig. 4.23, we can write,

$$\frac{16 - v_{TH}}{2} + 2 = \frac{v_{TH}}{6}$$

$$\therefore \quad 8 - \frac{v_{TH}}{2} + 2 = \frac{v_{TH}}{6}$$

$$\therefore \quad \frac{2}{3} v_{TH} = 10 \qquad \therefore v_{TH} = 15 \text{ Volt.}$$

EX- 4.11: Determine Thevenin equivalent circuit of Fig. 4.24.

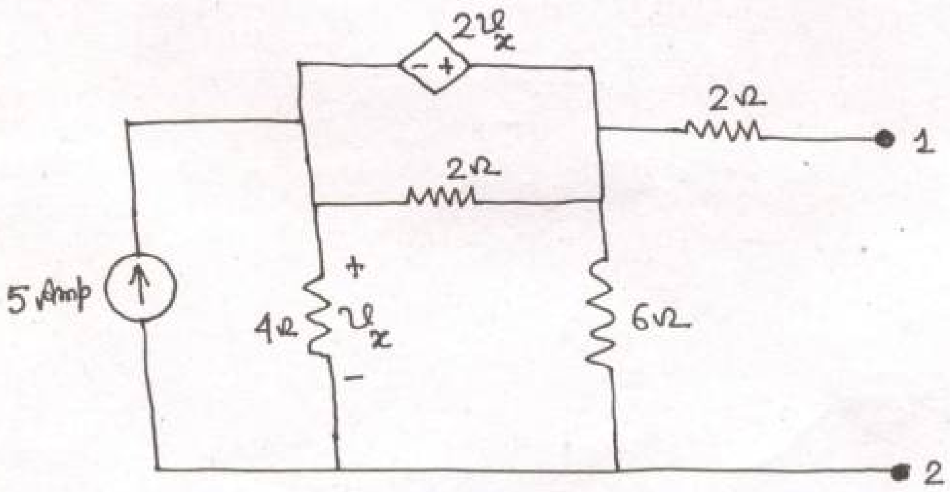


Fig. 4.24: Circuit for EX- 4.11.

Soln.

To determine R_{TH}, we leave the dependent source alone and set the independent source equals to zero.

However, circuit is excited by a voltage source v_o connected to the terminals 1-2 as shown in Fig. 4.25. For easy analysis, $v_o = 1$ Volt is chosen.

Therefore, $R_{TH} = \frac{v_o}{i_o} = \frac{1}{i_o}$

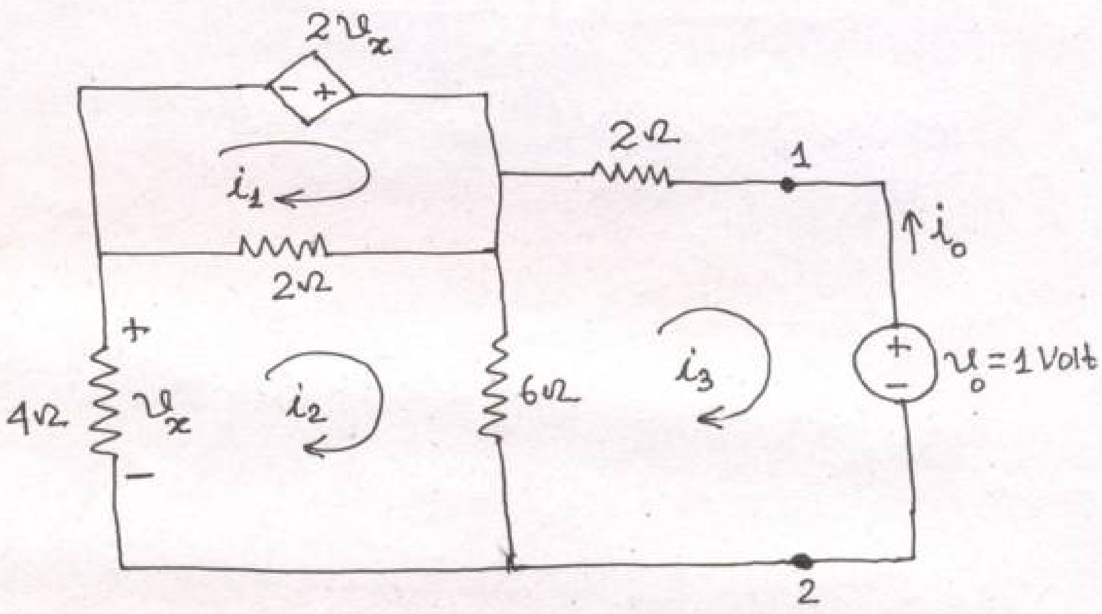


Fig. 4.25: Finding R_{TH} for Ex-4.11

Solving the circuit of Fig. 4.25, we obtain,

$i_o = \frac{1}{6}$ Amp.

$\therefore R_{TH} = \frac{v_o}{i_o} = \frac{1}{i_o} = 6\,\Omega.$

To get v_{TH}, we solve for v_{oc} of the circuit shown in Fig. 4.26. We obtain,

$v_{oc} = 20$ Volt

$\therefore v_{TH} = v_{oc} = 20$ Volt.

Thevenin equivalent circuit is shown in Fig. 4.27.

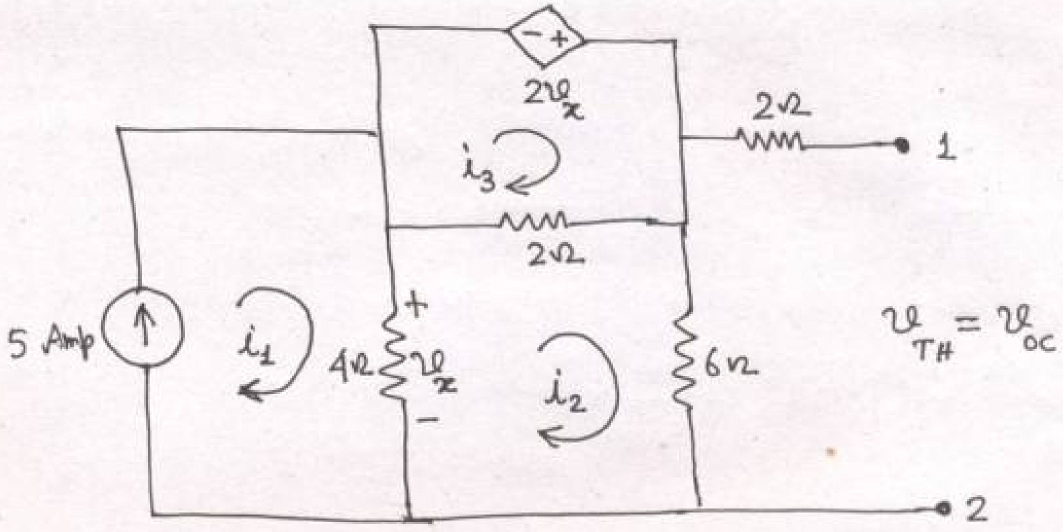


Fig. 4.26: Finding v_{TH} for Ex-4.11

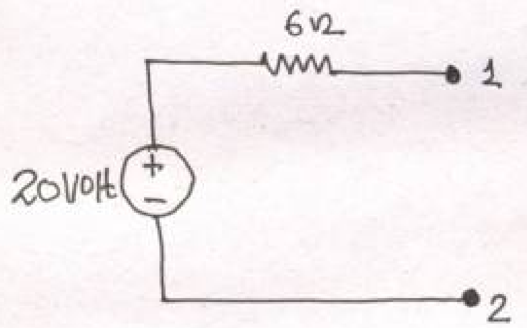


Fig. 4.27: Thevenin equivalent of the circuit in Fig. 4.24 for Ex-4.11.

Ex-4.12: Determine R_{TH} of the circuit shown in Fig. 4.28.

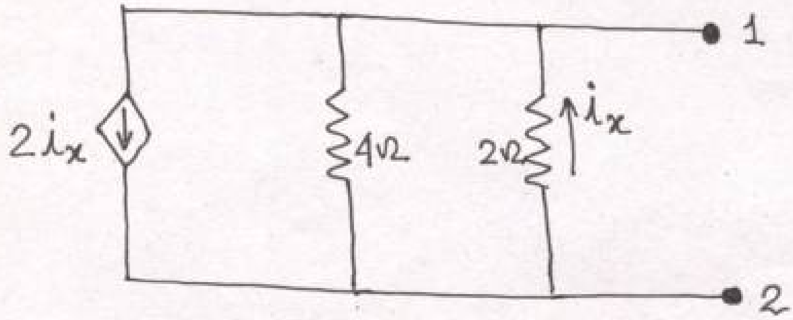


Fig. 4.28: Circuit for Ex-4.12

Soln.

In Fig.4.28, there is no independent source, hence $v_{TH} = 0.0$ Volt

To determine R_{TH}, apply a current source i_o at the terminals as shown in Fig. 4.29

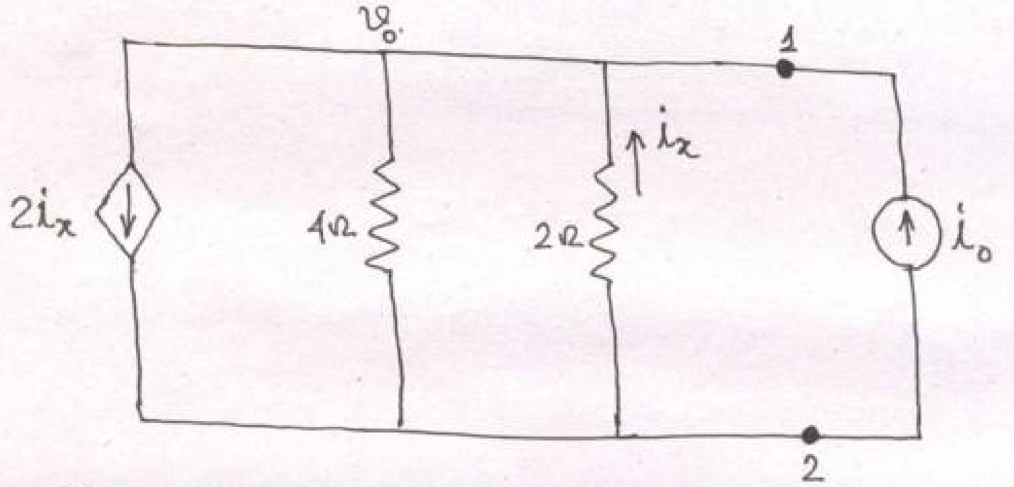


Fig. 4.29: Finding R_{TH} for EX- 4.12

Applying nodal analysis gives,

$$i_o + i_x - 2i_x = \frac{v_o}{4} \quad \text{---(i)}$$

But

$$i_x = -\frac{v_o}{2} \quad \text{-----(ii)}$$

From eqns. (i) and (ii), we get

$$v_o = -4i_o$$

Thus,

$$R_{TH} = \frac{v_o}{i_o} = -4\,\Omega.$$

Negative value of Thevenin resistance indicates that circuit of Fig. 4.28 is supplying power. It is the independent source that supplies power.

EX- 4.13: Using Thevenin's theorem, determine voltage across load resistance $R_L = 4\Omega$ of the circuit shown in Fig. 4.30.

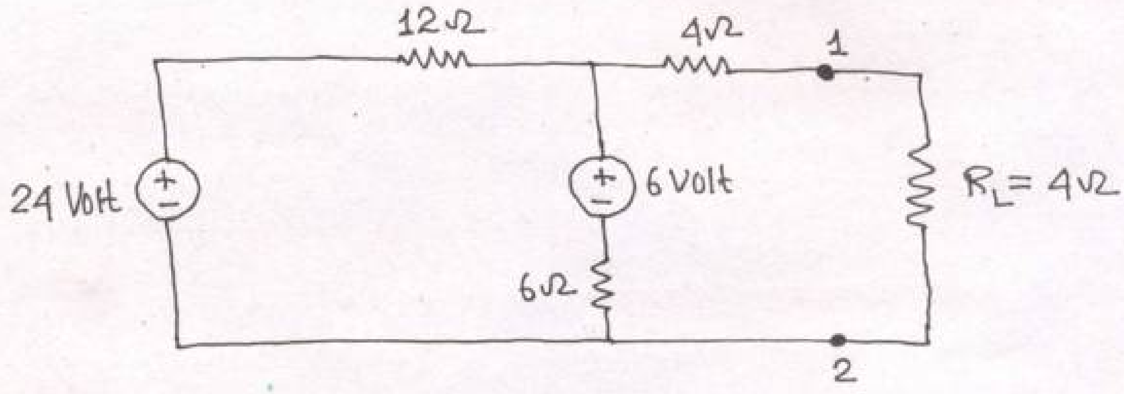


Fig. 4.30: Circuit for Ex- 4.13

Soln.

First we remove Load resistance $R_L = 4\Omega$ from terminals 1-2. Therefore terminals 1-2 are open and resulting circuit is shown in Fig. 4.31.

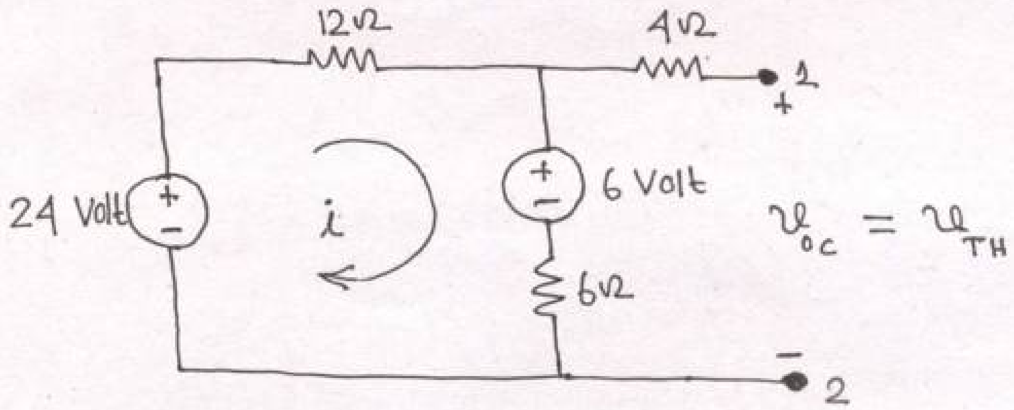


Fig. 4.31: Finding $v_{oc} = v_{TH}$ for Fig. 4.30 of Ex- 4.13

Applying KVL, we have

$$18i + 6 - 24 = 0 \qquad \therefore\ i = 1\ \text{Amp}$$

Thus

$$v_{oc} = v_{TH} = 6 + 6i = 6 + 6\times 1 = 12\ \text{Volt.}$$

To determine R_{TH}, independent sources of Fig. 4.31 are turned off (short circuited) and the circuit is shown in Fig. 4.32.

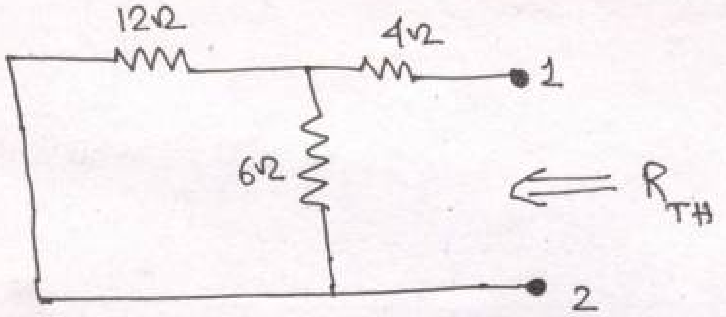


Fig. 4.32: Finding R_{TH} for Fig. 4.30 of Ex-4.13

$$\therefore R_{TH} = \frac{12 \times 6}{12+6} + 4 = 8\Omega.$$

Thevenin equivalent circuit is shown in Fig. 4.33.

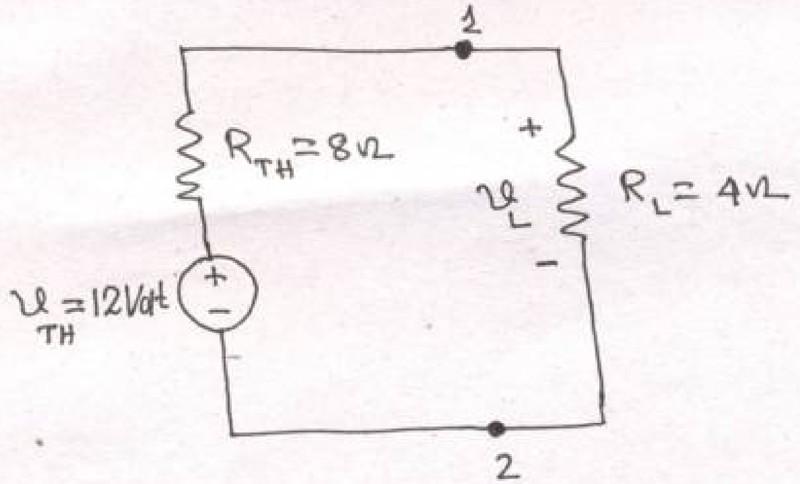


Fig. 4.33: Thevenin equivalent circuit for Ex-4.13.

Voltage across $R_L = 4\Omega$ resistance is

$$v_L = \frac{v_{TH}}{(R_{TH} + R_L)} \times R_L$$

$$\therefore v_L = \frac{12}{(8+4)} \times 4 = 4 \text{ Volt.}$$

Ex- 4.14: By using Thevenin's theorem, determine current flowing through 3Ω resistor between points 1-2- of Fig. 4.32.

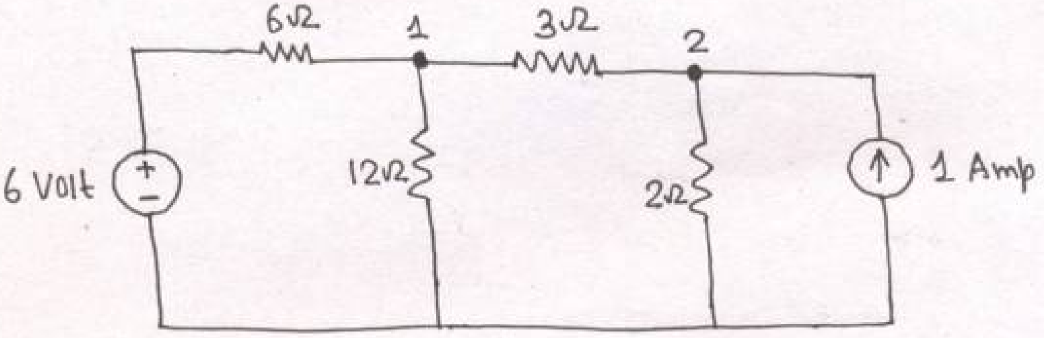


Fig. 4.32: Circuit for ~~EX-4.32~~ EX-4.14

Soln.

Opening 3Ω resistor across terminals 1-2. to determine $v_{oc} = v_{TH}$ and the circuit is shown in Fig. 4.33.

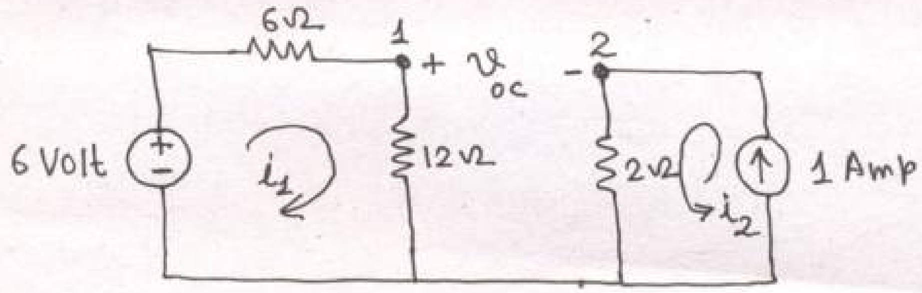


Fig. 4.33: Determining $v_{oc} = v_{TH}$ for EX-4.14

$$i_1 = \frac{6}{6+12} = \frac{1}{3} \text{ Amp}; \quad i_2 = 1 \text{ Amp}$$

$$\therefore v_{oc} = v_{TH} = 12 i_1 - 2 i_2 = 12 \times \frac{1}{3} - 2 \times 1 = 2 \text{ Volt}$$

Circuit for determining R_{TH} is shown in Fig. 4.34

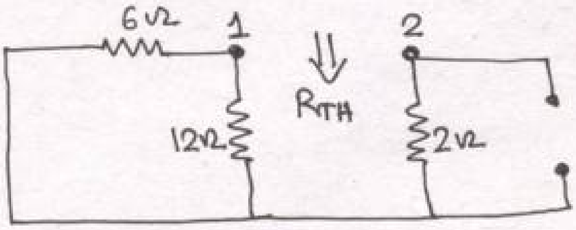


Fig. 4.34: Determining R_{TH} for EX-4.14

$$R_{TH} = \frac{6 \times 12}{(6+12)} + 2 = 4+2 = 6\Omega.$$

Thevenin equivalent circuit is shown in Fig. 4.35.

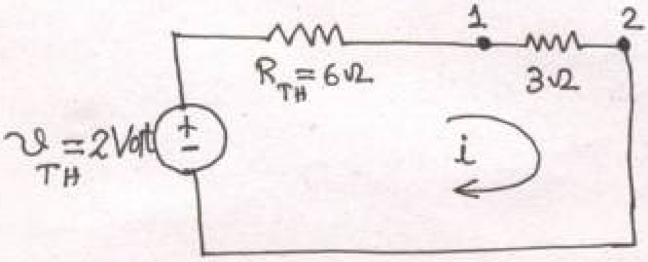


Fig. 4.35: Thevenin equivalent circuit for EX-4.14

Current through 3Ω resistor is

$$i = \frac{v_{TH}}{R_{TH}+3} = \frac{2}{6+3} = \frac{2}{9} \text{ Amp.}$$

EX-4.15: Using Thevenin's theorem, determine current through 10Ω resistor of the circuit shown in Fig. 4.36.

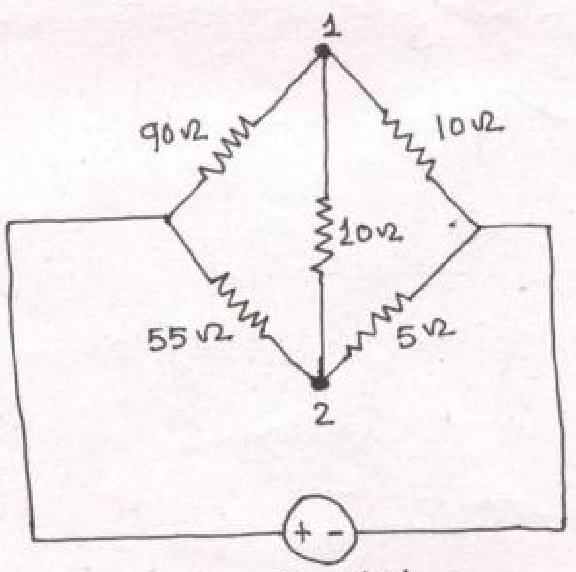


Fig. 4.36: Circuit for EX-4.15

Soln.

For determining $v_{oc} = v_{TH}$, removing 10Ω resistor across the terminals 1-2. and the resulting circuit is shown in Fig. 4.37.

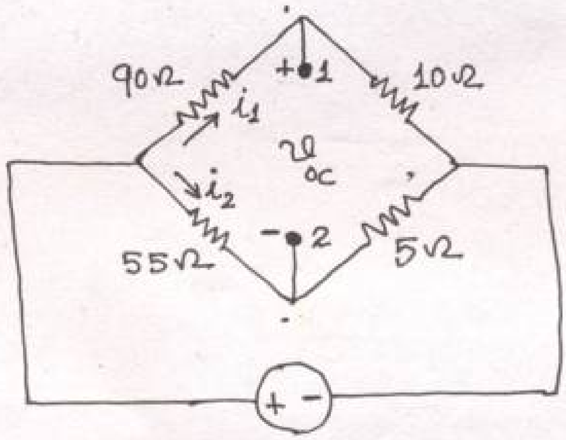


Fig. 4.37: Determing $v_{oc} = v_{TH}$ for Fig. 4.36 of Ex-4.25.

From Fig. 4.37,

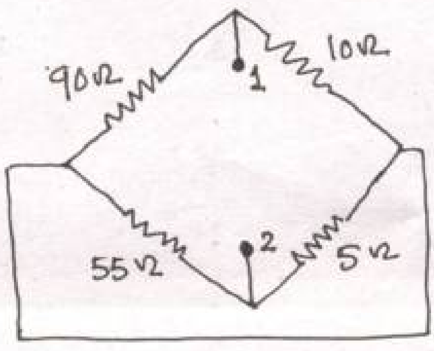
$$i_1 = \frac{20}{100} = 1/5 \text{ Amp}; \quad i_2 = \frac{20}{(55+5)} = \frac{1}{3} \text{ Amp}.$$

$$\therefore \quad 10 i_1 - 5 i_2 - v_{oc} = 0$$

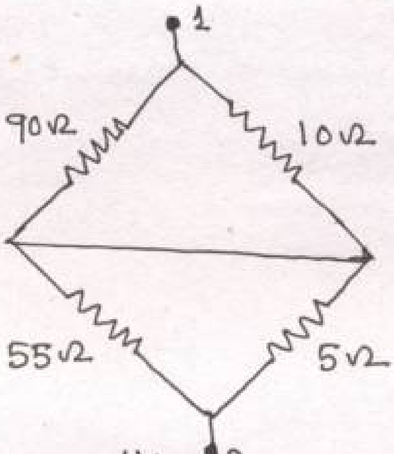
$$\therefore v_{oc} = 10 \times \frac{1}{5} - 5 \times \frac{1}{3} = 2 - \frac{5}{3} = \frac{1}{3} \text{ Volt}$$

$$\therefore \quad v_{TH} = v_{oc} = \frac{1}{3} \text{ Volt.}$$

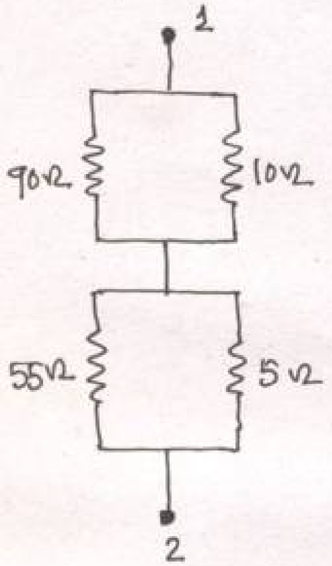
For determining R_{TH}, independent voltage source is short circuited. and the resulting circuit is shown in Fig. 4.38(a)



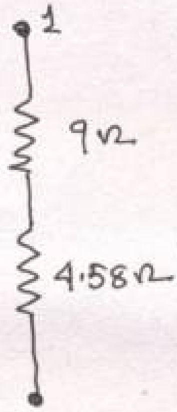
(a)



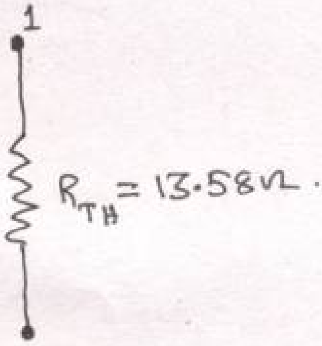
(b)



(c)



(d)



(e)

Fig.4.38: Finding R_{TH} for Fig.4.36 of Ex-4.15

Thevenin equivalent circuit is shown in Fig.4.39

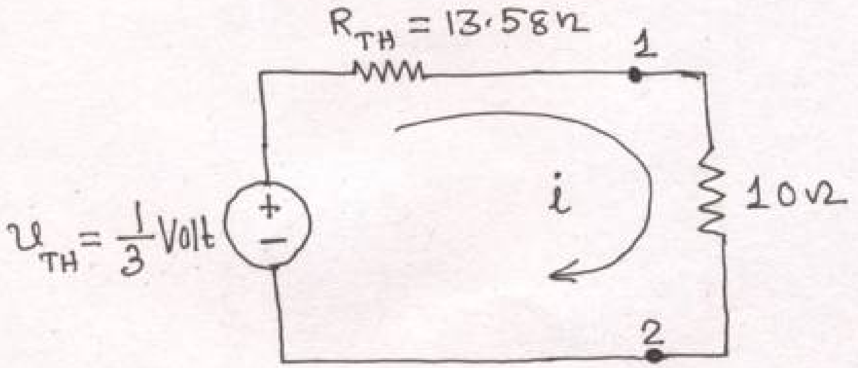


Fig.4.39: Thevenin equivalent circuit for EX-4.15

$$i = \frac{v_{TH}}{R_{TH}+10} = \frac{1/3}{(13.58+10)} = 0.0141 \text{ Amp}$$

EX-4.16: Determine current through 50Ω resistor of the circuit shown in Fig. 4.40 using Thevenin's theorem.

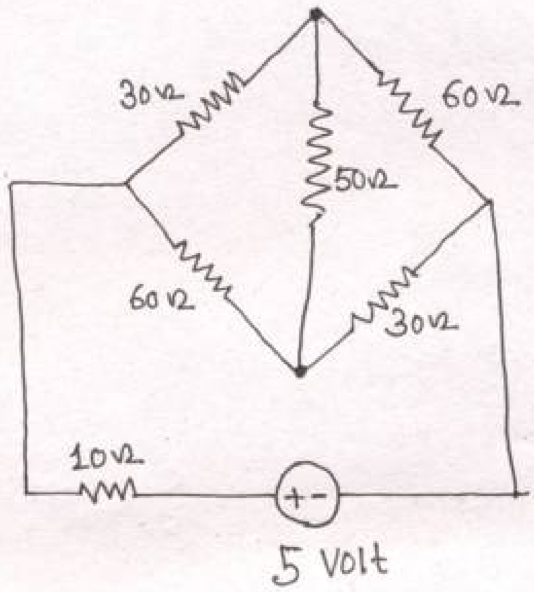


Fig. 4.40: Circuit for EX-4.16

Soln

Removing 50Ω resistor to determine $v_{oc} = v_{TH}$ and the resulting circuit is shown in Fig. 4.41

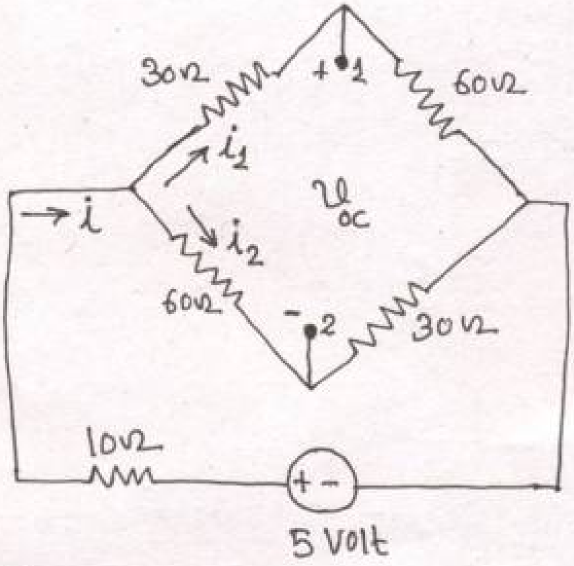


Fig. 4.41: Finding $v_{oc} = v_{TH}$

From Fig. 4.11,

$$i = i_1 + i_2 \quad \cdots (i)$$

Also $i_1 = i_2 \quad \cdots (ii)$

Applying KVL,

$$10(i_1+i_2) + i_1(30+60) - 5 = 0$$

But $i_1 = i_2$

$\therefore \quad 20i_1 + 90i_1 = 5 \quad \therefore i_1 = \frac{5}{110} = \frac{1}{22}$ Amp

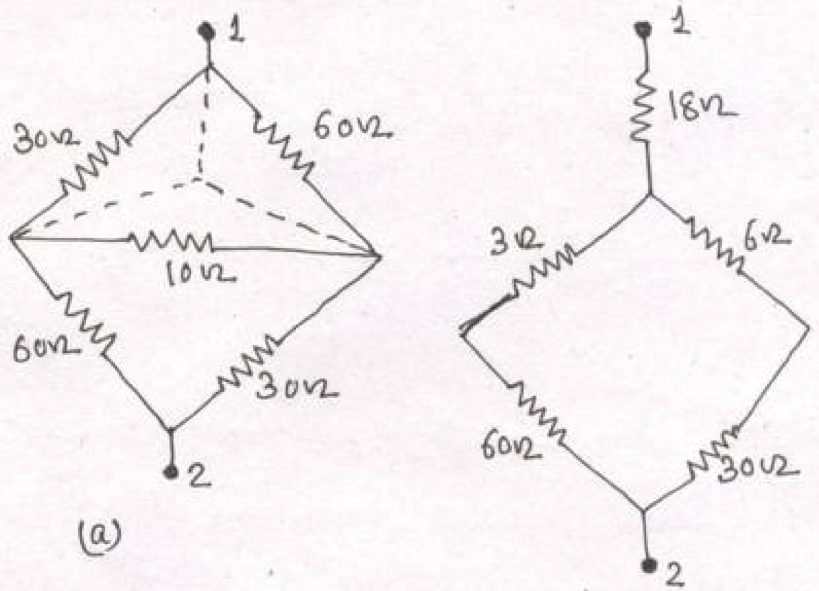
$\therefore \quad i_2 = i_1 = \frac{1}{22}$ Amp

Thus,

$$60i_1 - 30i_2 - v_{oc} = 0$$

$$\therefore \quad v_{oc} = \frac{30}{22} \text{ Volt} = 1.36 \text{ Volt}$$

To determine R_{TH}, independent voltage source is short circuited and the resulting circuits are shown in Fig. 4.42.



(a) (b)

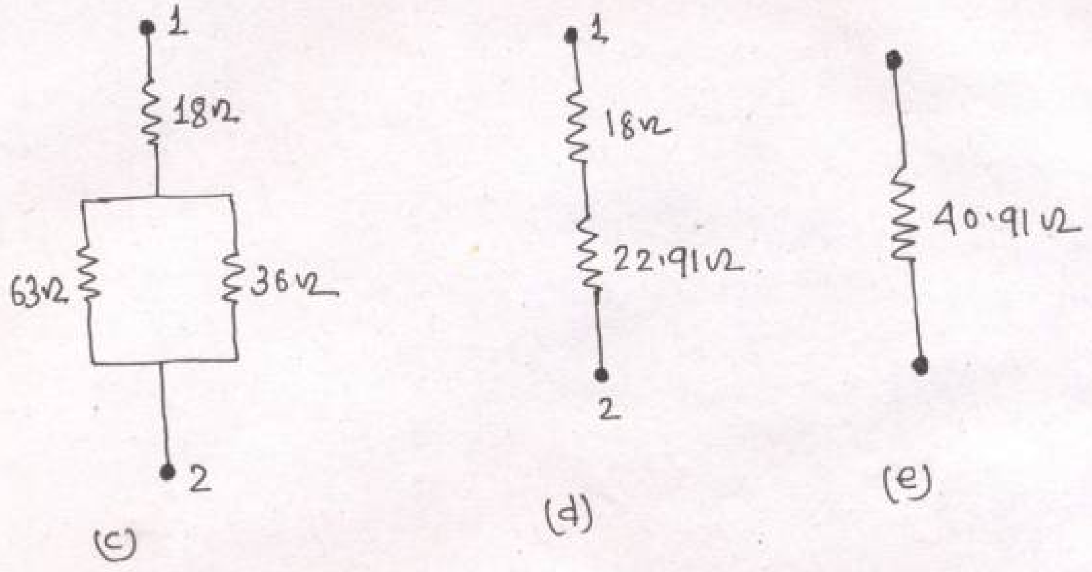


Fig. 4.42: Finding R_{TH}

$\therefore\ R_{TH} = 40.91\Omega$

Thevenin equivalent circuit is shown in Fig. 4.43

Fig. 4.43: Thevenin equivalent circuit for EX-4.16

$$\therefore\ i = \frac{1.36}{(40.91 + 50)} = 0.015 \text{ Amp.}$$

EX-4.17: Determine the input resistance R_{in} of the circuit shown in Fig. 4.44.

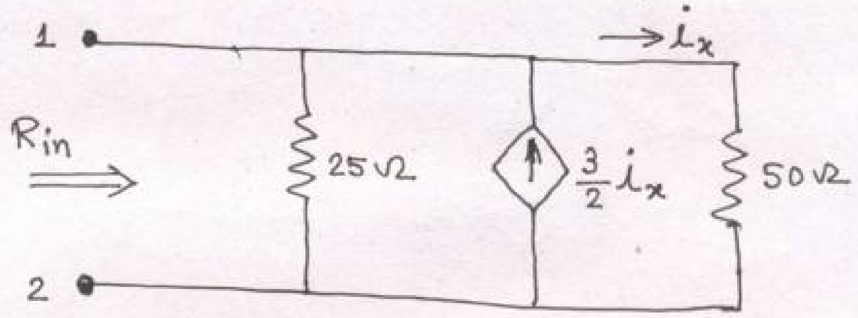


Fig. 4.44: Circuit for Ex-4.17

Soln.

Note that $R_{in} = R_{TH}$.

The circuit given in Fig. 4.44, has a dependent source but no independent source. Therefore, the approach to finding the input resistance or $R_{in} = R_{TH}$ is to apply a source at the input. A good source to apply is a 1 Amp current as shown in Fig. 4.45

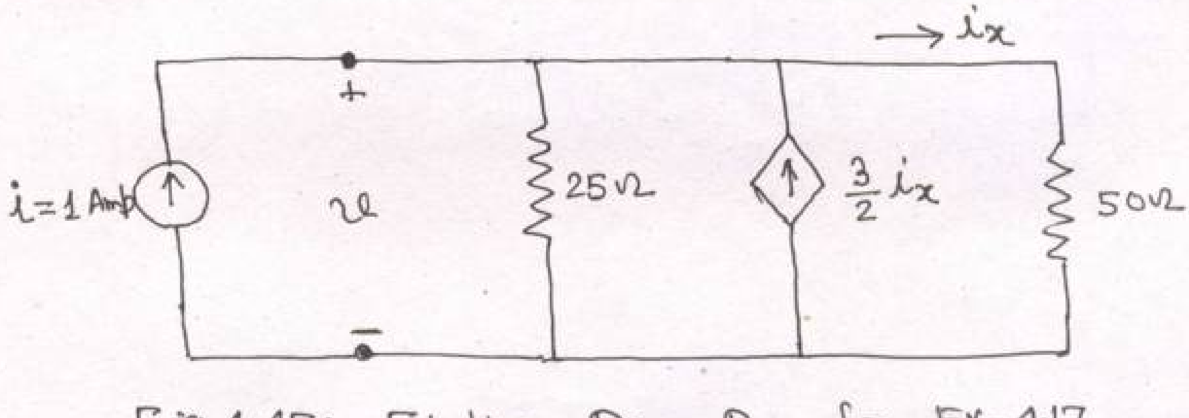


Fig. 4.45: Finding $R_{in} = R_{TH}$ for Ex-4.17.

$$R_{in} = R_{TH} = \frac{v}{i} = \frac{v}{1} = v \quad \text{---- (i)}$$

Using nodal analysis

$$\frac{v}{25} + \frac{v}{50} = 1 + \frac{3}{2} i_x \quad \text{---- (ii)}$$

Also

$$i_x = \frac{v}{50} \quad \text{----- (iii)}$$

Solving eqn.(ii') and (iii'), we get,

$$v = 33.3 \text{ Volts.}$$

$$\therefore R_{in} = R_{TH} = \frac{v}{i} = \frac{33.3}{1} = 33.3\,\Omega.$$

Ex-4.18: Find the Thevenin equivalent of the circuit shown in Fig.4.46.

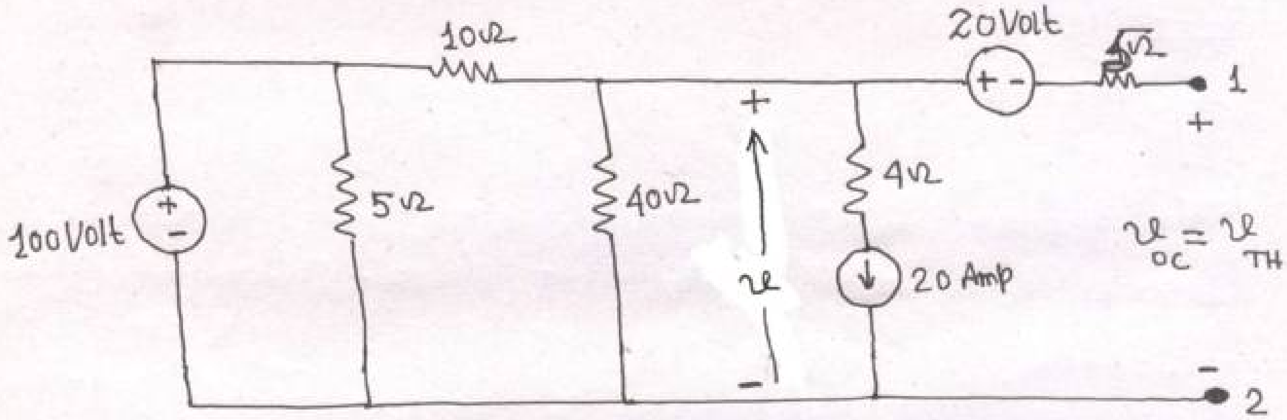


Fig.4.46: Circuit for Ex-4.18

Soln.

Applying nodal analysis, we have

$$\frac{v-100}{10} + \frac{v}{40} + 20 = 0$$

$$\therefore v = -80 \text{ Volt.}$$

Thus,

$$-20 + v - v_{oc} = 0$$

$$\therefore v_{oc} = v_{TH} = -20 - 80 = -100 \text{ Volt.}$$

Fig.4.47 shows the circuit with the voltage sources replaced by short circuits and the current source by an open circuit.

Note that 5 Ω resistor has no effect on R_{TH} because it is shorted and neither does the 4Ω resistor because it is in series with an open circuit.

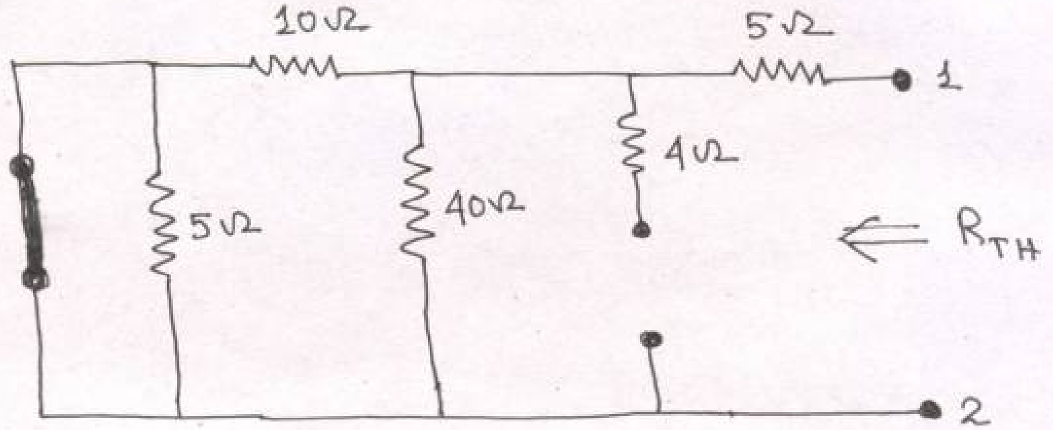


Fig. 4.47: Finding R_{TH}.

$$\therefore \quad R_{TH} = 5 + \left(\frac{40 \times 10}{40+10}\right) = 13\,\Omega$$

Thevenin equivalent circuit is shown in Fig. 4.48.

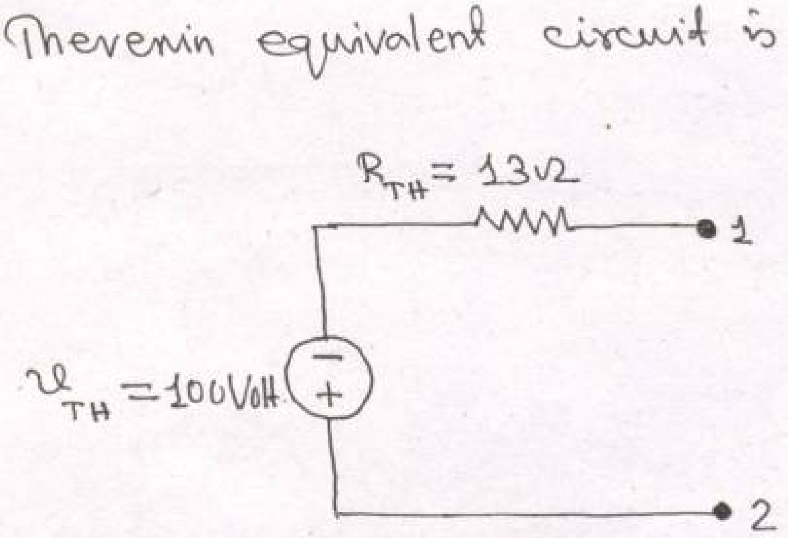


Fig. 4.48: Thevenin equivalent circuit for EX-4.18

EX-4.19 Determine the current through 10Ω resistor of the circuit shown in Fig. 4.49. Use Thevenin's Theorem.

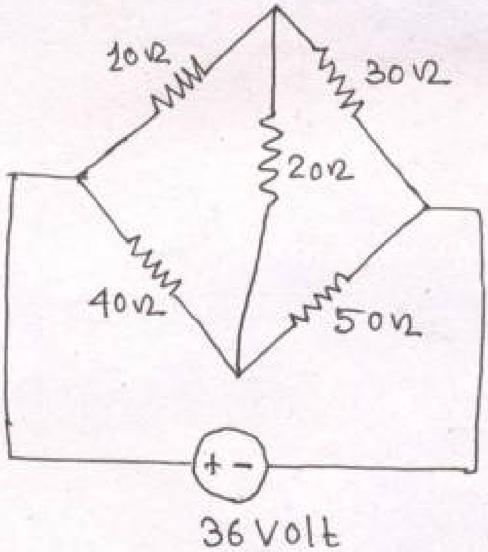


Fig. 4.49: Circuit for EX-4.19

Soln.

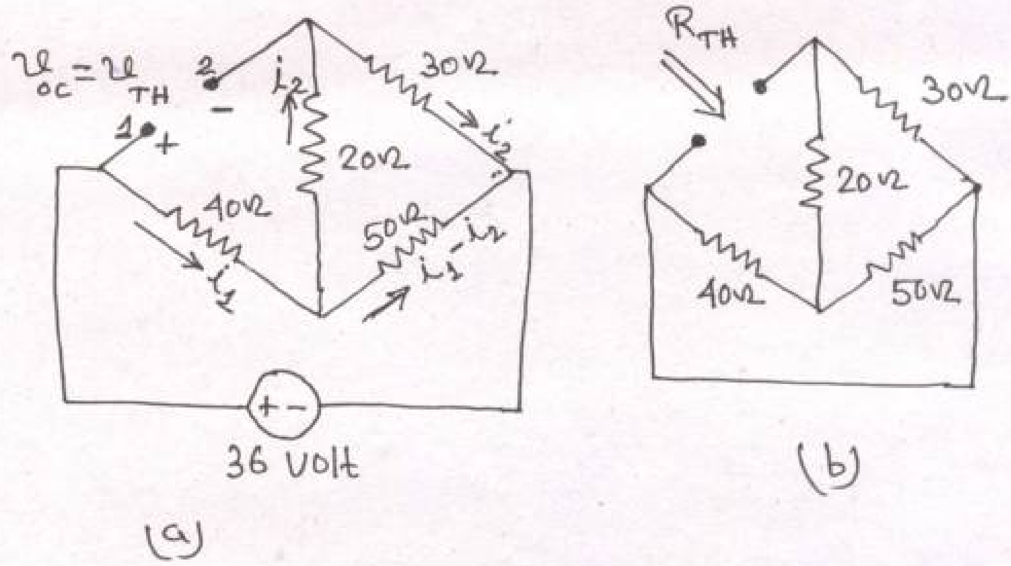


Fig. 4.50: (a) finding v_{TH} (b) finding R_{TH}.

Fig. 4.50 shows the circuit for finding v_{TH} and R_{TH}.

From Fig. 4.50(a),

$$i_1 = 2i_2 \quad \text{-- (i)}$$

$$40i_1 + 50(i_1 - i_2') - 36 = 0 \quad \text{--- (ii)}$$

Solving eqn. (i) & (ii), we get,

$i_1 = \frac{36}{65}$ Amp ; $i_2' = \frac{18}{65}$ Amp.