Problem Set - 7

Spring 2020

MATHEMATICS-II (MA10002)

1. Evaluate the following integral using differentiation under integral sign
$$I = \int_0^\infty \frac{\log(1+a^2x^2)}{1+b^2x^2}dx.$$

2. If $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ then show that $\int_0^\infty e^{-x^2}\cos ax dx = \frac{\sqrt{\pi}}{2}e^{-\frac{a^2}{4}}$.

3. Prove that
$$\frac{d}{da}\int_0^a \frac{F(x)}{\sqrt{a-x}}dx = \int_0^a \frac{F(x)+2xF'(x)}{2a\sqrt{a-x}}dx.$$
If $F(x) \in C^2[0,a]$ then under what circumstances will the integral $\int_0^a \frac{F(x)}{\sqrt{a-x}}dx$ be independent of a? $(a > 0)$

4. Find the function $f(x)$ such that for all values of a, we have
$$\int_0^a x(f(x))^2dx = \frac{a}{n}\int_0^a (f(x))^2dx.$$

5. Find the value of the integral $\int_0^\infty \frac{e^{-ax}\sin x}{x}dx$, $a > 0$ and find the value of following
   (i) $\int_0^\infty \frac{\sin x}{x}dx$
   (ii) $\int_0^\infty \frac{\sin ay}{y}dy$

6. Find the value of the following integral
   (i) $\int_0^\infty \frac{e^{-x}-e^{-tx}}{x}dx$
   (ii) $\int_0^\infty \frac{e^{-x}}{x}[a-\frac{1}{x}+\frac{1}{x}e^{-ax}]dx$, $a > -1$
   (iii) $\int_0^1 \frac{x^a-x^b}{\log x}dx$

7. Find the value of $\int_0^\pi \frac{1}{a+b\cos x}dx$, when $(a > 0, |b| < a)$ and deduce that
$$\int_0^\pi \left(\frac{1}{a+b\cos x}\right)^2 dx = \frac{\pi a}{(a^2-b^2)^{\frac{3}{2}}}.$$

8. Evaluate the following using double integrals
   (i) Find the volume of the hemisphere $x^2+y^2+z^2=a^2$, $z \geq 0$.

(ii) Find the center of gravity of a plate whose density $\rho(x, y)$ is directly proportional to x coordinate and is bounded by the curves $y = x^2$ and $y = x + 2$.

(iii) Find the area of the region bounded by the curves $y = 2x$ and $y = \dfrac{x^2}{4a}$.

9. Evaluate the following integrals by changing the order of integration

(i) $\displaystyle\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin y}{y} \, dydx$

(ii) $\displaystyle\int_0^1 \int_0^{\sqrt{2x-x^2}} dydx$

(iii) $\displaystyle\int_{-2}^1 \int_{x^2}^{2-x} xy \, dydx$

(iv) $\displaystyle\int_0^\infty \int_0^x e^{-xy} \, y \, dydx$

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