INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 24.11.2015(FN) | **Time: 3 hours**
End Semester Examination (Autumn) | **Maximum Marks: 90**
Subject No. ME10001 | **Subject Name: MECHANICS**
No. of students: 696

Instructions*: Answer all SEVEN questions. Any data, if not furnished, may be assumed with justification. Unless specified the dimensions are in mm.*

---

1. A beam subjected to the loading is shown in Fig.1.
   (a) Find the support reactions at B and C.
   (b) Draw the sign convention to be followed for shear force and bending moment diagrams.
   (c) On a fresh page, below the free body diagram of the beam, draw the shear force diagram and the bending moment diagram. All relevant calculations must be shown.
   (d) Determine the distances from A, where the bending moment (i) changes its sign and (ii) has the maximum magnitude.

(20)

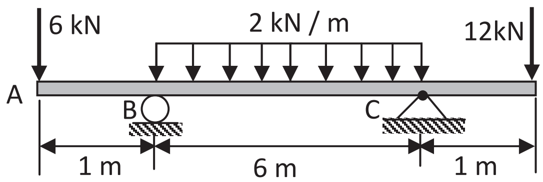


FIG. 1

2. A plate with a weld line at 120° is subjected to a shear stress of 40 MPa and a tensile normal stress σ_X, as shown in Fig.2. The plate material property values are *E*= 200 GPa, *ν*= 0.25 and *G*= 80 GPa. (a) If the normal stress along x' is to be restricted to 45 MPa, determine the maximum value of σ_X. (b) If the maximum normal stress in the plate is to be restricted to 50 MPa, determine the maximum value of σ_X. (c) If σ_X =70 MPa, determine the shear strain in the small element ABCD (oriented along x'-y') shown in the figure. (d) If σ_X = 70 MPa, determine the percentage change in the length of CD.

(15)

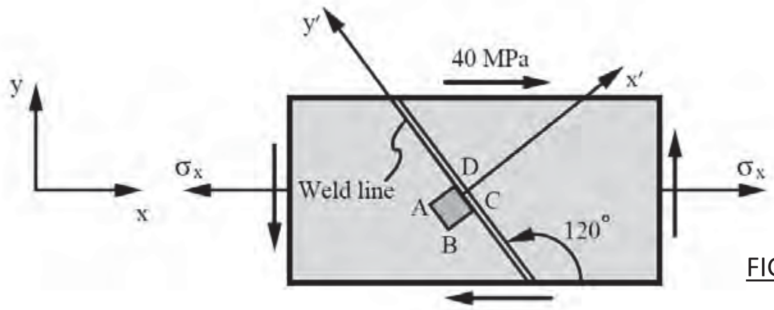


FIG. 2

3. A long cylindrical pressure vessel of 720mm diameter and 6mm thickness, shown in Fig.3, has the material properties of Young's modulus, E=200GPa and Poisson's ratio, μ=0.25. Strain of 0.00015 mm/mm was recorded in the x-direction when the pressure inside the cylinder was P.
   (a) Determine the pressure P in the vessel.
   (b) Calculate the values of the corresponding stresses on the element shown in the figure.
   (c) The corresponding change in radius of the cylinder.

(15)

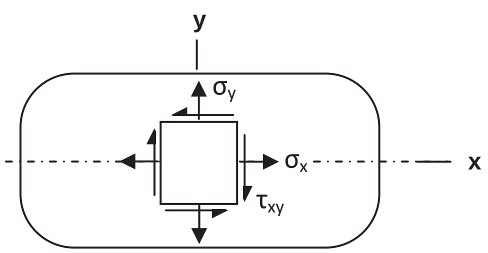


FIG. 3

4. A solid shaft of 25mm diameter is fixed at D and undergoes torsion as shown in Fig.4.
   (a) Draw the torque diagram along the axis of the shaft and determine the maximum shear stress in the shaft.
   (b) Calculate the angle of twist of A with respect to D.
   (c) Draw the angle of twist diagram along the axis of the shaft.
   Consider G = 80GPa for the shaft material.

(15)

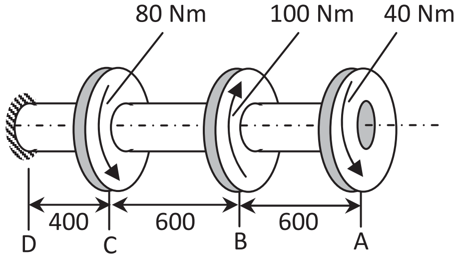


FIG. 4

5. A simply supported beam of span L is acted upon by a load P=40kN at its center (Fig.5a). The cross-section of the beam is shown in Fig.5b.
   (a) In Fig.5b, if b=160mm, t=20mm and h=160mm determine the second moment of area of the cross-section about its neutral axis.
   (b) For a similar beam cross-section, as shown in Fig.5b, for some values of b, t, h with depth of beam as 192mm, the second moment of area of the cross-section about its neutral axis is 80×10^6 mm^4. Determine the maximum span L of the beam, if the magnitude of the maximum bending stress both in tension and compression is 120MPa.

(15)

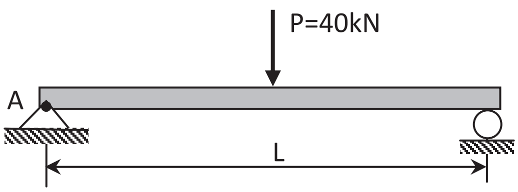


FIG. 5a

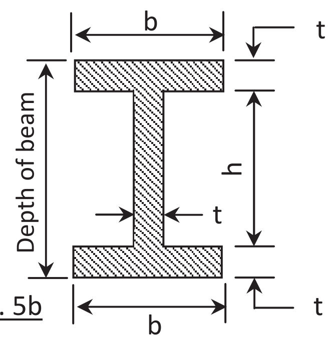


FIG. 5b

6. A simply supported beam with an overhang is shown in Fig. 6. The beam has a constant flexure rigidity of EI.
   (a) Determine the elastic curve of the beam and calculate the deflections, δ_B and δ_D at the points B and D, respectively.
   (b) Sketch the elastic curve of the beam.

(20)

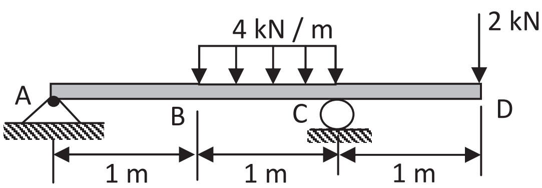
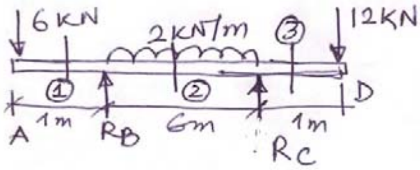


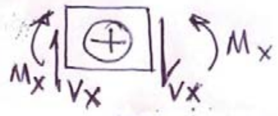
FIG. 6

A1



$R_B = 11\,KN$

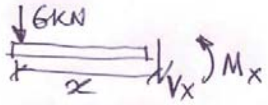
$R_C = 19\,KN$



Sign Convention

Section 1

$0 \le x \le 1$

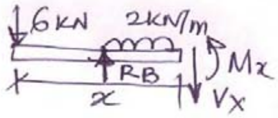


$V_x = -6\,KN$

$M_x = -6x\,KNm$

Section 2

$1 \le x \le 7$



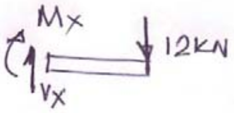
$V_x + 6 + 2(x-1) - 11 = 0 \therefore V_x = 7 - 2x$.

$M_x + 6x - 11(x-1) + \frac{2(x-1)^2}{2} = 0$

$\therefore M_x = 7x - x^2 - 12$.

Section 3

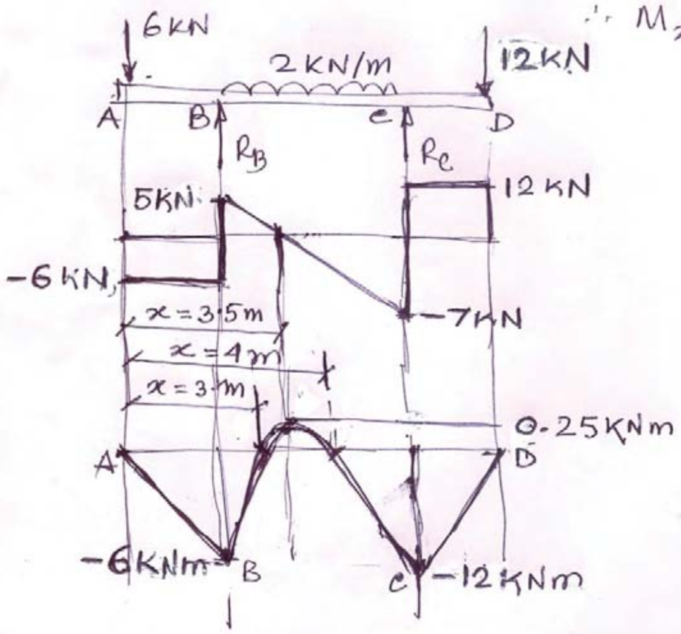
$7 \le x \le 8$



$V_x = 12\,KN$

$M_x + 12(8-x) = 0$

$\therefore M_x = -12(8-x)$



$V_x = 0 = 7 - 2x = \frac{dM}{dx}$.

$\therefore x = 3.5$

$\therefore$ Maxima at $x = 3.5m$

Span BC

$M_x = 7x - x^2 - 12$.

if $M_x = 0$,

$-x^2 + 7x - 12 = 0$

$\therefore x_1, x_2 = \frac{-7 \pm \sqrt{49-48}}{-2}$

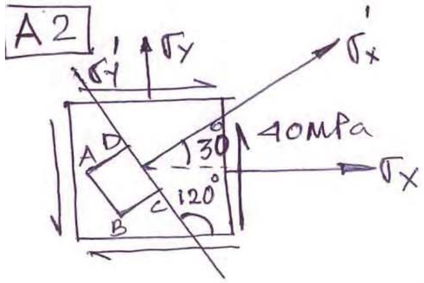
$x_1 = 3 \quad x_2 = 4$

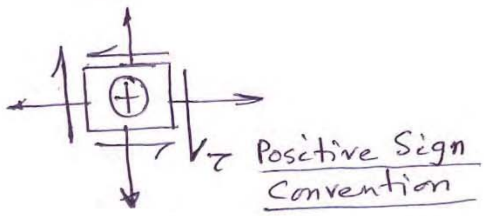
(i) changes sign at $x = 3m$ and $x = 4m$

(ii) Maximum magnitude $|12\,KNm|$ at $x = 7m$

Maximum $0.25\,KNm$ at $x = 3.5m$

A2





given, σ_x, $\sigma_y = 0$, $\tau_{xy} = -40$ MPa.

(a) $\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 60^\circ + 40\sin 60^\circ$

$\sigma_x(0.5 + 0.25) + 34.641 \leq 45$ MPa.

$\therefore$ $\underline{\sigma_x \leq 13.812 \text{ MPa}}$

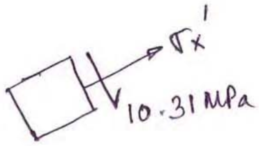
(b) $\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \leq 50$ MPa

$\frac{\sigma_x^2}{4} + \tau_{xy}^2 \leq \left(50 - \frac{\sigma_x}{2}\right)^2$ or $\frac{\sigma_x^2}{4} + 40^2 \leq 50^2 + \frac{\sigma_x^2}{4} - 50\sigma_x$

$\underline{\sigma_x \leq 18 \text{ MPa}}$

(c) $\tau_{30^\circ} = \frac{\sigma_x}{2}\sin 60^\circ - 40\cos 60^\circ$

$= 35\sin 60^\circ - 40\cos 60^\circ = \underline{10.31 \text{ MPa}}$



$\epsilon_{30} = \tau_{30}/G = \underline{1.289 \times 10^{-4} \text{ rad}}$

(d) $\sigma_x' = 35 + 35\cos 60^\circ + 40\sin 60^\circ = \underline{87.14 \text{ MPa}}$.

$\sigma_y' = 35 + 35\cos 240^\circ + 40\sin 240^\circ = \underline{-17.14 \text{ MPa}}$

$\epsilon_y' = -\frac{17.14}{E} - \mu\frac{87.14}{E} = \underline{-1.946 \times 10^{-4} \text{ mm/mm}}$

$\delta_{CD} = \epsilon_y' \times CD$

$\therefore$ % change in CD $= \frac{\delta_{CD}}{CD} \times 100 = \underline{-0.01946}$

A3

$$\sigma_x = \frac{Pr}{2t} = \frac{P.360}{2\times 6} = 30P.$$

$$\sigma_y = \frac{Pr}{t} = \frac{P\times 360}{6} = 60P$$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{30P}{E} - \frac{1}{4}\cdot\frac{60P}{E} = 0.00015 \text{ (given)}$$

or $15P = 0.00015 \times (200\times 10^3)$ $\therefore$ $\underline{P = 2\,MPa}$

$\therefore$ $\sigma_x = 60\,MPa$; $\sigma_y = 120\,MPa$; $\tau_{xy} = 0$ (as σ_x and σ_y are principal stresses)

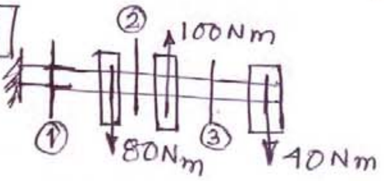
Change in circumference $= 2\pi(r+\delta r) - 2\pi r$
$= 2\pi\delta r$

Circumferencial Strain $= \epsilon_y$

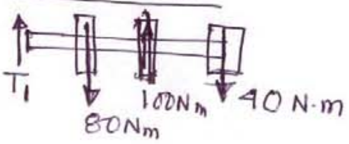
$$= \frac{120}{E} - \nu\frac{60}{E} = \frac{\delta r}{r}$$

$$\therefore \delta r = \frac{360(120 - 0.25\times 60)}{200\times 10^3} = \underline{0.189\,mm}$$

A4

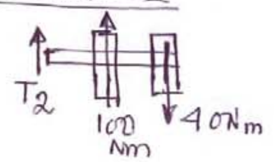


Section-1



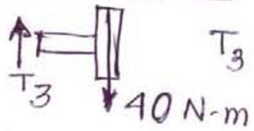
$T_1 = 20\,Nm$

Section-2

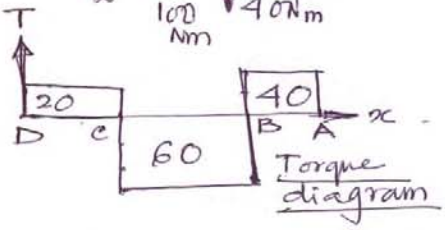


$T_2 = -60\,Nm$

Section-3



$T_3 = 40\,Nm$



Torque diagram

Magnitude of maximum torque. $= 60\,N\text{-}m$ in BC.

$\therefore$ Maximum Shear Stress

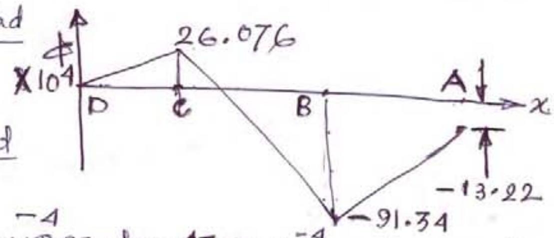
$$= \frac{Tr}{I_p} = \frac{16T}{\pi d^3} = \frac{16\times 60\times 10^3}{\pi\times 25^3}$$

$= \underline{19.56\,MPa}$ $\left[I_p = \frac{\pi d^3}{32}\right]$

$$\phi_{CD} = \frac{T_{CD}L_{CD}}{GI_p} = \frac{20\times 10^3\times 400}{GI_p}\,rad$$
$$= \underline{26.076\times 10^{-4}\,rad.}$$

$$\phi_{BC} = \frac{T_{BC}L_{BC}}{GI_p} = \frac{-60\times 10^3\times 600}{GI_p}$$
$$= \underline{-117.342\times 10^{-4}\,rad}$$

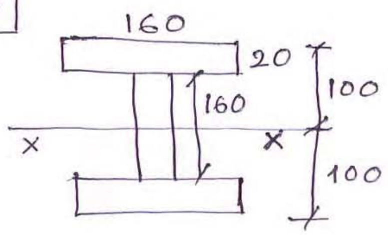
$$\phi_{AB} = \frac{T_{AB}L_{AB}}{GI_p} = \frac{40\times 10^3\times 600}{GI_p}$$
$$= \underline{78.228\times 10^{-4}\,rad}$$



$\therefore \phi_{AD} = \phi_{CD} + \phi_{BC} + \phi_{AB}$

$= (26.076 - 117.342 + 78.228)\times 10^{-4}\,rad = \underline{-13.22\times 10^{-4}\,rad = 0.076^\circ}$

A5



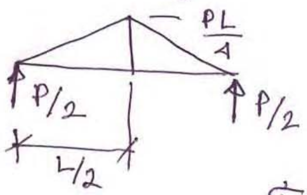
(a) Symmetric Section

$\therefore$ X – X passes through middle of the section as shown.

$$I = \left[\frac{160 \times 20^3}{12} + (160 \times 20) \times 90^2\right] \times 2 + \frac{20 \times 160^3}{12}$$

$$= \underline{58.88 \times 10^6 \text{ mm}^4}$$

(b) $\text{Bending Moment}\Big|_{max} = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$



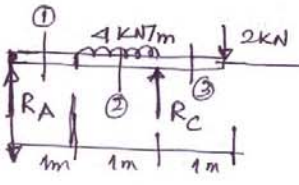
$I = 80 \times 10^6 \text{ mm}^4$ (given)

$y = 192/2 = 96 \text{ mm}.$

$$\sigma = \frac{My}{I} \quad \text{or} \quad \frac{PL}{4} \times \frac{96}{80 \times 10^6} \leq 120 \text{ MPa}.$$

$\therefore \underline{L \leq 10000 \text{ mm or } 10 \text{ m}}$

A6



$R_A = 0,\ R_C = 6\,kN$

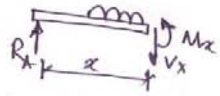
Diff Equn. $EI\dfrac{d^2y}{dx^2} = -M_x$

section 1 $(0 \le x \le 1)$

$M_x = 0.$

$EI\dfrac{d^2y}{dx^2} = 0 \quad \therefore\ EI\dfrac{dy}{dx} = A_1 \ ; \ EIy = A_1x + A_2$ — ①

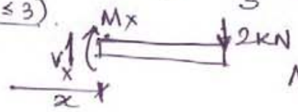
section 2 $(1 \le x \le 2)$



$M_x + \dfrac{4(x-1)^2}{2} = 0 \ \therefore M_x = -2(x-1)^2.$

$EI\dfrac{d^2y}{dx^2} = 2(x-1)^2 \ ; \ EI\dfrac{dy}{dx} = \dfrac{2x^3}{3} + 2x - 2x^2 + B_1 \ ; \ EIy = \dfrac{x^4}{6} + x^2 - \dfrac{2x^3}{3} + B_1x + B_2$ — ②

section 3 $(2 \le x \le 3)$



$M_x + 2(3-x) = 0 \ \therefore\ M_x = -2(3-x)$

$\therefore EI\dfrac{d^2y}{dx^2} = 2(3-x) \ ; \ EI\dfrac{dy}{dx} = 6x - x^2 + C_1 \ ; \ EIy = 3x^2 - \dfrac{x^3}{3} + C_1x + C_2$ — ③

Boundary Conditions.

at $x=0,\ y=0 \ \therefore A_2 = 0$ (from ①) — ⑤

at $x=2,\ y=0 \ \therefore \dfrac{2^4}{6} + 2^2 - \dfrac{2\times2^3}{3} + 2B_1 + B_2 = 0 \qquad \dfrac{4}{3} + 2B_1 + B_2 = 0$ — ⑥

at $x=1,\ \dfrac{dy}{dx}\Big|_1 = \dfrac{dy}{dx}\Big|_2 \ \therefore A_1 + A_2 = \dfrac{2}{3} + B_1$ or $A_1 = \dfrac{2}{3} + B_1$ (from 5) — ⑦

at $x=1\ \ y|_1 = y|_2 \ \therefore A_1 = \dfrac{1}{6} + 1 - \dfrac{2}{3} + B_1 + B_2$ or $A_1 = \dfrac{1}{2} + B_1 + B_2$ — ⑧

solving ⑥, ⑦ & ⑧ $\ A_1 = -\dfrac{1}{12},\ B_1 = -9/12$ and $B_2 = \dfrac{1}{6}$

at $x=2,\ y=0 \ \therefore$ from ③, $\dfrac{28}{3} + 2C_1 + C_2 = 0$ .... ⑨

at $x=2 \ \ \dfrac{dy}{dx}\Big|_2 = \dfrac{dy}{dx}\Big|_3 \ \therefore \dfrac{2}{3}\times2^3 + 2\cdot2 - 2\cdot2^2 + B_1 = 12 - 4 + C_1$

$\therefore C_1 = -89/12$ and $C_2 = \dfrac{33}{6}$ (from ⑨)

Elastic curves ① $EIy = \ \ x$; ② $EIy = \dfrac{x^4}{6} + x^2 - \dfrac{2x^3}{3} - \dfrac{9}{12}x + \dfrac{1}{6}$

③ $EIy = 3x^2 - \dfrac{x^3}{3} - \dfrac{89}{12}x + \dfrac{33}{6}.$

$\therefore \delta_B = -\dfrac{1}{12EI}$ and $\delta_D = \left(3\times9 - \dfrac{3^3}{3} - \dfrac{89}{12}\times3 + \dfrac{33}{6}\right)/EI = \dfrac{5}{4EI}$