INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date of Examination: 19. 11. 2013(FN) — **Time: 3hrs**
End Semester Examination (Autumn) — **Full Marks: 105**
Subject No. ME10001 — **All 1st year**
Subject Name: MECHANICS — **No. of students: 715**

Instructions: ***Answer all SEVEN questions; all are of equal credits. All the parts of a problem must be written consecutively. All dimensions are in mm, if not specified. Any data, if not furnished, may be assumed with justification.***

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1. Draw the shear force diagram and the bending moment diagram for the beam and loading shown in Fig.1.

The diagrams must be drawn below the beam diagram on a fresh page and the sign convention followed for shear force and bending moment must be indicated. The support reactions at A and C are to be calculated and all other relevant calculations must be shown.

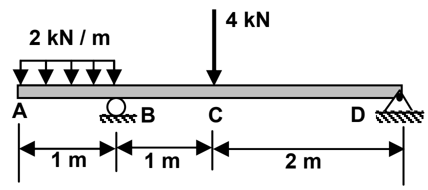


FIG. 1

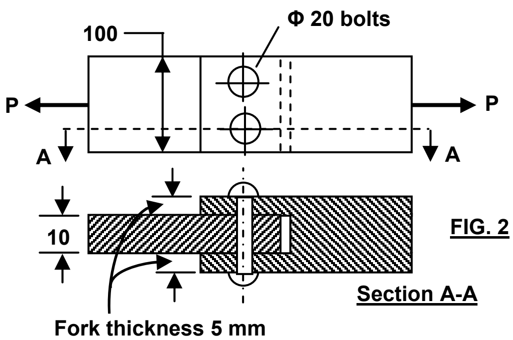


FIG. 2

2. A 100 mm wide and 10 mm thick plate is pressed inside the slot of another machine component as shown in Fig.2. The two forks of the slot are 100 mm wide and 5 mm thick. The assembly is bolted by two bolts of 20 mm diameter. If allowable stresses are 75 MPa, 60 MPa and 125 MPa in tension, shear and bearing respectively, for both the plate and the pin materials, determine the allowable load P that can be applied on the assembly.

3. A hollow tubular shaft is made by curving a properly shaped flat 4mm thick steel sheet and then welding along the joint as shown in the Fig.3. The tube is subjected to torsion T in the direction as shown.

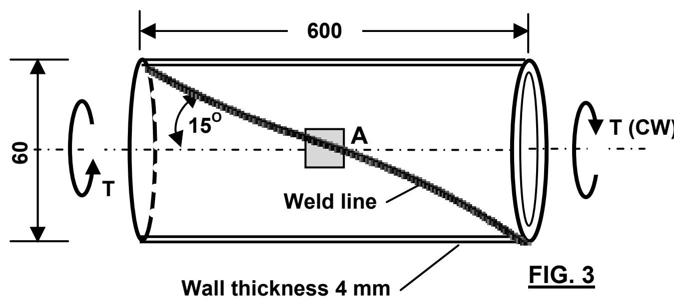


FIG. 3

(a) Determine the maximum shear stress at the outer surface of the tube in terms of torque T.

(b) Show all the stresses on an infinitesimal element A on the outer surface of the tube.

(c) Write the expression of shear stress along the weld line.

(d) If the allowable shear stress of the weld material is 60 MPa, determine the maximum value of the torque T.

(e) Determine the principal stresses corresponding to the element A using the value of torque as 1 kN-m.

(f) What is the relative angle of twist between the two ends of the shaft for 1 kN-m torque and G=80GPa?

4. A cylindrical pressure vessel made of 6mm thick plate has a mean diameter of 480 mm and is 1000 mm long as shown in Fig.4. The cylinder is pressurized with a gas to 2 MPa pressure.

   (a) Show all the stresses on an infinitesimal element A on the outer surface of the cylinder and calculate the principal stresses.

   (b) Find the change in the volume of the cylinder, assuming that the curved closed ends of the vessel do not deform. Consider E=200GPa and Poisson ratio v = 0.25.

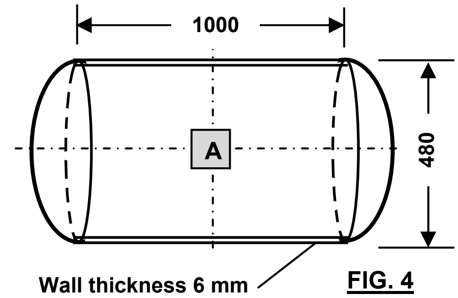


FIG. 4

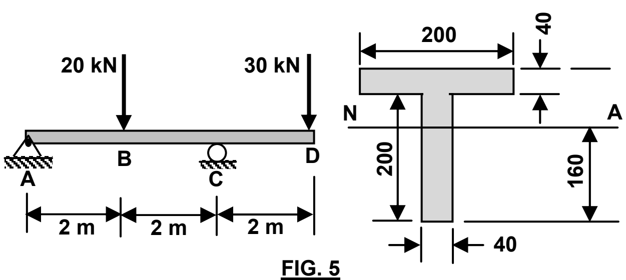


FIG. 5

5. An overhang beam of T shaped cross section is shown in Fig.5. The neutral axis (N-A) is 160 mm from the bottom of the beam. Determine,

   (a) The reactions at A and C.

   (b) The magnitudes and locations of the maximum tensile stress and the maximum compressive stress that are developed in the beam.

6. A simply supported aluminum beam of rectangular cross section is shown in Fig.6. It is observed in an experiment that the strains in the bottom most fibers at the *section 1* and *section 2* are 0.0002 mm/mm and 0.00032 mm/mm respectively.

   (a) Calculate the bending moments at the *section1* and at the *section 2*. Take E=80GPa.

   (b) Calculate the shear force at the *section 2*.

   (c) What is the horizontal shear stress at the neutral axis of the beam at the *section 2?*

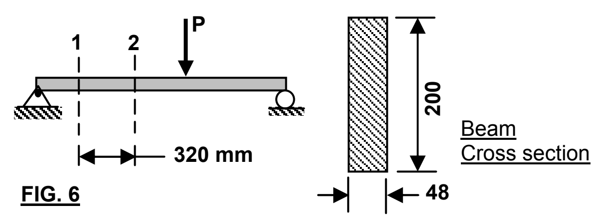


FIG. 6

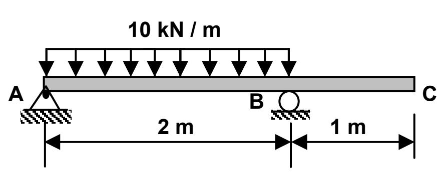


FIG. 7

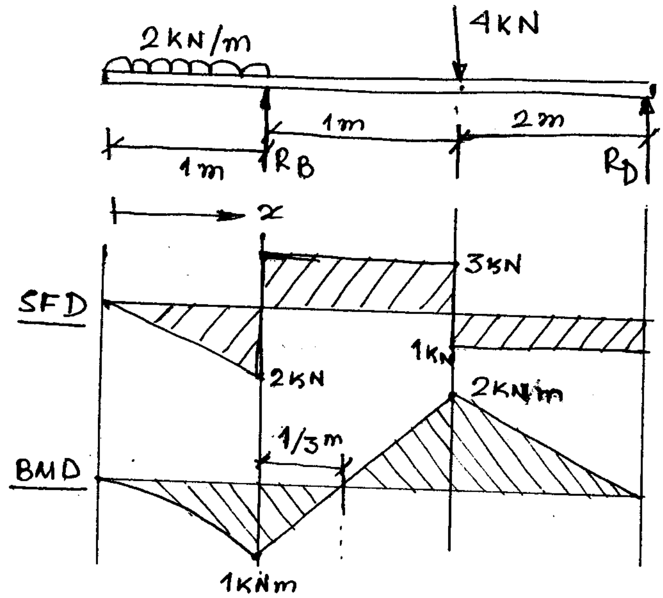
7. A beam of constant flexural rigidity (EI = 16.66×10^6 N-m^2) is loaded as shown in Fig.7. Determine,

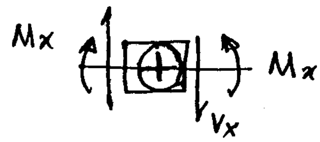
   (a) The slopes at A and B for the elastic curve of the beam segment AB.

   (b) Deflection at the free end C of the beam.

Solution End Autumn - 2013

A1.

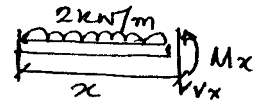




Convention

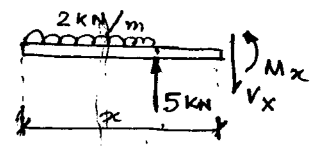
$R_B = 5\,kN$; $R_D = 1\,kN$

$0 < x \leq 1m$



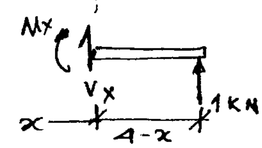
$V_x = -2x$
$M_x = -x^2$

$1 < x \leq 2m$



$V_x = 3$
$M_x = 3x - 4$

$2 < x \leq 4m$



$V_x = -1$
$M_x = 1(4-x)$

A2.

Failure in tension of the plate $= P_T = (W-2d) \times t \times \sigma_t$
$= (100-40) \times 10 \times 75 = \underline{45\,kN}$

Failure in shear of bolts $= P_s = 4 \times (\frac{\pi}{4} \times 20^2) \times 60 = \underline{75.4\,kN}$

Bearing failure of Plate/bolt $= P_B = (20 \times 10) \times 2 \times 125 = \underline{50\,kN}$

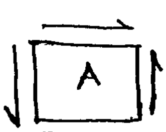
$\therefore P_{max} = \underline{45\,kN}$

A3.

(a) Shear stress on the outer surface, $\tau = \frac{Tr}{I_p}$; $I_p = \frac{\pi}{32}(60^4 - 52^4)\,mm^4$
$= \underline{55.45 \times 10^4\,mm^4}$

or $\tau = \frac{T \times 30}{55.45 \times 10^4} = \underline{5.41 \times 10^{-5}\,T}$

(b)



$\tau = 5.41 \times 10^{-5}\,T$

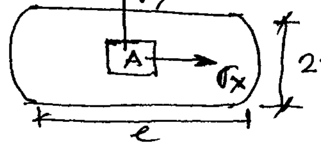
(c) $\tau_{weldline} = \left|\tau \cos(2 \times 15)^\circ\right| = \left|\tau \cos 30^\circ\right|$

(d) $(5.41 \times 10^{-5} \times T)\cos 30^\circ = 60$ $\therefore$ $\underline{T = 1.28\,kN\text{-}m}$

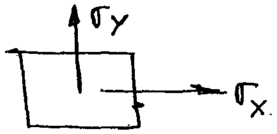
(e) $\tau = 5.41 \times 10^{-5} \times T = 5.41 \times 10^{-5} \times 1.0 \times 10^3 \times 10^3 = \underline{54.1\,MPa}$

(f) $\phi = \frac{Tl}{GI_p} = \frac{1.00 \times 10^3 \times 10^3 \times 600}{80 \times 10^3 \times 55.45 \times 10^4} = 1.35 \times 10^{-2}\,rad$
$= \underline{0.775\ degree}.$

A4



(a)



As $\tau_{xy}=0$, $\sigma_1=\sigma_x$ and $\sigma_2=\sigma_y$

$$\sigma_1 = \frac{Pr}{2t} = \frac{2\times 240}{2\times 6} = 40\,MPa$$

$$\sigma_2 = \frac{Pr}{t} = \frac{2\times 240}{6} = 80\,MPa.$$

$$\epsilon_x = \epsilon_1 = \frac{40}{E} - \frac{1}{4}\times\frac{80}{E} = 1.0\times10^{-4}\ mm/mm$$

$$\epsilon_y = \epsilon_2 = \frac{80}{E} - \frac{1}{4}\times\frac{40}{E} = 3.5\times10^{-4}\ mm/mm$$

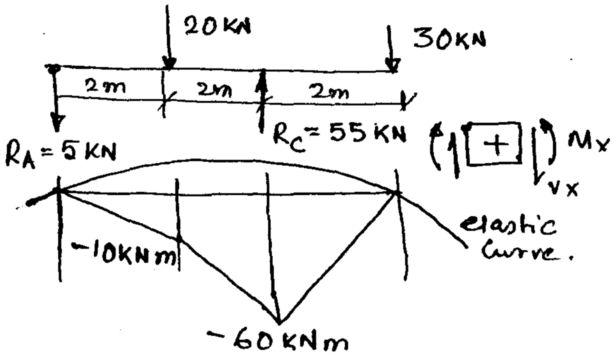
$$\epsilon_1 = \frac{dl}{l}\ ;\quad \epsilon_2 = \frac{2\pi(r+dr)-2\pi r}{2\pi r} = \frac{dr}{r}.$$

$$V = \pi r^2\times l\ ;\quad dV = 2\pi r.dr\times l + \pi r^2\times dl\ \therefore\ \frac{dV}{V} = 2\cdot\frac{dr}{r} + \frac{dl}{l}$$

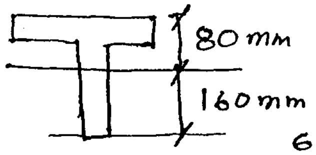
$$\therefore dv = \text{change in volume} = V\left[2\frac{dr}{r}+\frac{dl}{l}\right] = \pi\times(240)^2\times1000\left[2\times3.5\times10^{-4}+1.\times10^{-4}\right]$$

$$= \underline{1.45\times10^{5}\ mm^3}$$

A5



(a) $R_A = 5KN$
$R_c = 55KN$



$I = 85.33\times10^{6}\ mm^4$

(b) Maxm bending moment = -60 KNm
= 60 KNm (magnitude)

Top fibres are in tension and Bottom fibres are in Compression

$$\therefore \sigma_{Tensile} = \frac{My}{I} = \frac{60\times10^3\times10^3\times80}{85.33\times10^6} = \underline{56.25\ MPa}$$

$$\sigma_{Compressive} = \frac{60\times10^3\times10^3\times160}{85.33\times10^6} = \underline{112.5\ MPa}$$

A6

$I = 32\times10^{6}\ mm^4$; (a) Stress at section 1 bottom fibres $= \sigma_t = \epsilon\times E$
$= 0.0002\times80\times10^3$
$= 16$ MPa

$$\therefore M_1 = \frac{\sigma I}{y} = \frac{16\times32\times10^6}{100}$$

$$= \underline{5120\times10^3\ N\,mm}.$$

Stress at Section 2 bottom fibres $= 0.00032\times80\times10^3 = 25.6$ MPa

$$\therefore M2 = \frac{25.6\times32\times10^6}{100} = \underline{8192\times10^3\ N.mm}$$

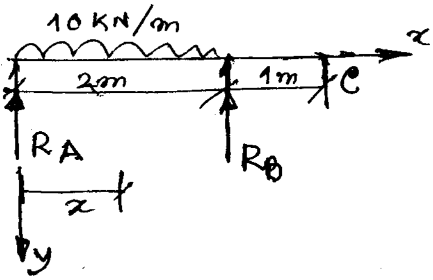
(b) $\Delta M = M2 - M1 = 3072\times10^3$ Nmm ; $V = \frac{\Delta M}{\Delta x} = \frac{3072\times10^3}{320} = 9.6$ KN

∴ Shear Stress at Section 2 = $\underline{9.6\ KN}$

(c) $Q = (48\times100)\times50\ mm^3 = 240\times10^3\ mm^3$

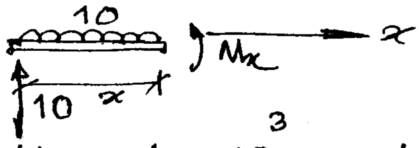
$$\tau = \frac{VQ}{Ib} = \frac{9.6\times10^3\times240\times10^3}{32\times10^6\times48} = \underline{\underline{1.5\ MPa}}$$

A7



$R_A = 5\,KN$

$R_B = 5\,KN$



$M_x = 10\times10^3 x - \frac{10\times10^3}{2} x^2$

$= 10^4 x - \frac{10^4}{2} x^2.$

$$EI \frac{d^2y}{dx^2} = -M$$

$$\therefore EI\frac{d^2y}{dx^2} = \frac{10^4}{2}x^2 - 10^4 x$$

$$EI\frac{dy}{dx} = \frac{10^4}{2}\cdot\frac{x^3}{3} - 10^4\cdot\frac{x^2}{2} + C_1$$

$$EI\,y = \frac{10^4}{6}\cdot\frac{x^4}{4} - \frac{10^4}{2}\cdot\frac{x^3}{3} + C_1x + C_2$$

Boundary Conditions: at $x=0$; $y=0$; at $x=2$, $y=0$.

$\therefore C_2 = 0$ and $0 = \frac{10^4}{6}\times\frac{2^4}{4} - \frac{10^4}{2}\cdot\frac{2^3}{3} + C_1\times 2.$

$$\therefore C_1 = \frac{10^4}{3}$$

$$\therefore \frac{dy}{dx} = \frac{10^4}{EI}\left(\frac{x^3}{6} - \frac{x^2}{2} + \frac{1}{3}\right)$$

Slope at A (θ_A) $= \frac{10^4}{3EI}$ and Slope at B (θ_B) $= \frac{10^4}{EI}\times\left(\frac{2^3}{6} - \frac{2^2}{2} + \frac{1}{3}\right)$

$= -\frac{10^4}{3EI}$

$\therefore$ Deflection at C $= \theta_B \times BC$

$= -\frac{10^4}{3EI}\times 1 = -\frac{10^4}{3\times16.66\times10^6}$

$= 2\times10^{-4}\,m = \underline{0.2\,mm}$