Problem Set - 8

SPRING 2020

MATHEMATICS-II (MA10002)(Integral Calculus)

1. Find the Jacobian of the following transformations T:
   (a) $T : u = e^{x^2-y^2}, v = e^{x^2+y^2}$
   (b) $T : u = e^x\ cosy,\ v = e^x\ siny$
   (c) $T : x = \frac{u}{v},\ \ y = u^2 - 4v^2$
   (d) $T : x = \rho sin\phi\ cos\theta,\ y = \rho sin\phi\ sin\theta,\ z = \rho cos\phi.$

2. Calculate the following double/tripple integrals (by changing the variables).
   (a) Evaluate
   $$I = \iint_R (y - x)dxdy$$
   where the region R is bounded by $y = x + 1$, $y = x - 3$, $y = -\frac{x}{3} + 2$, $y = -\frac{x}{3} + 4$.
   (b) Evaluate
   $$I = \iint_R dxdy$$
   where the region R is bounded by the parabolas $y^2 = 2x$, $y^2 = 3x$ and hyperbolas $xy = 1$,$xy = 2$.
   (c) Compuate
   $$I = \iint_R (x + y)dA$$
   where R is the trapezoidal region with vertices $(0, 0)$, $(5, 0)$,$(\frac{5}{2}, \frac{5}{2})$ and $(\frac{5}{2}, -\frac{5}{2})$ using the transformation $x = 2u + 3v$ and $y = 2u - 3v$.
   (d) Evaluate
   $$I = \int_0^1 dx \int_0^x \sqrt{x^2 + y^2}dy$$
   by tranforming to polar coordinates.
   (e) Show that
   $$\iint_R \sqrt{4a^2 - x^2 - y^2}dxdy = \frac{4}{9}(3\pi - 4)a^3$$
   , where R is the upper half of the circle $x^2 + y^2 - 2ax = 0$.
   (f) Evaluate
   $$I = \int_0^3 \int_0^4 \int_{y/2}^{y/2+1} (x + \frac{z}{3})dxdydz$$
   In xyz-space by using the transformation $u = \frac{2x-y}{2}$,$v = \frac{y}{2}$ and $w = \frac{z}{3}$. Integrate over appropiate region in uvw-sapce.
   (g) Calculate the integral
   $$\iiint_U e^{(x^2+y^2+z^2)^{\frac{3}{2}}} dxdydz$$
   where the region U is the unit ball $x^2 + y^2 + z^2 \leq 1$

3. Evaluate the following integrals.

(a) Find the surface area of that part of the sphere $z = \sqrt{a^2 - x^2 - y^2}$ which lying inside the cylinder $x^2 + y^2 = ay$. Here a is a positive constant.

(b) Evaluate

$$\iint_S 40ydS$$

where S is the portation of $y = 3x^2 + 3z^2$ that lies behind $y = 6$

(c) Find the area of the region in the xy plane bounded by the lemniscate $\rho^2 = a^2 cos2\phi$.

(d) Find the volume of the parallelepiped defined by the inequalities. $0 \leq 2x - 3y + z \leq 5$, $1 \leq x + 2y \leq 4$, $-3 \leq x - z \leq 6$.

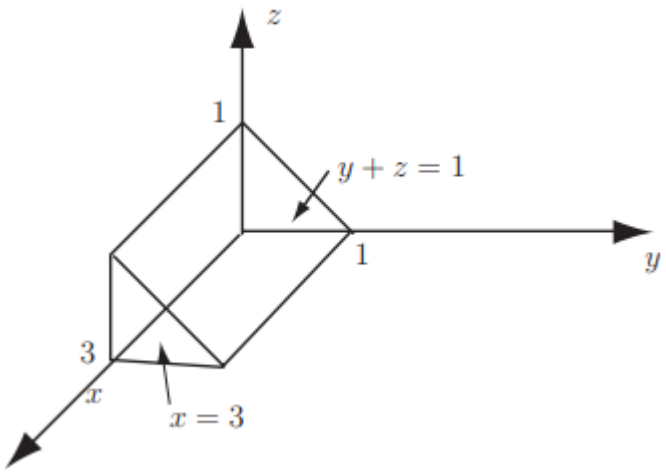
(e) Find the area of the ellipse cut on the plane $2x + 3y + 6z = 60$ by the circular cylinder $x^2 + y^2 = 2x$

(f) Evaluate

$$\iint_S z^2 dS$$

where S is the hemisphere given by $x^2 + y^2 + z^2 = 1$ with z $\geq$ 0.

(g) Find the volume of the solid prism shown in the diagram below.



(h) Determine the value of the integral

$$\iiint_D e^{x^2+y^2} dV$$

where D is the region in bounded by the planes y=0,z=0,y=x and the paraboloid $z = 4 - x^2 - y^2$.