Tutorial – 7

1. Consider a two-slit Young's interference experiment with λ= 500 nm where fringes are generated on a screen N placed at D = 1 meter apart from the slits.
(a) The fringe width decreases 1.2 times when the slit width is increased by 0.2 mm. Calculate the original fringe width.
(b) When a thin film of a transparent material is put behind one of the slits, the zero order fringe moves to the position previously occupied by the 4th order bright fringe. The index of refraction of the film is n = 1.2. Calculate the thickness (t) of the film.

Solution:

(a) We know that fringe width , $\Delta x = \frac{\lambda D}{d}$
(where d is the slit width and D distance between screen and the slits)

$$\Delta x \ \alpha \ \frac{1}{d}$$

so $\frac{\Delta x'}{\Delta x} = \frac{d}{\Delta d + 0.2\text{mm}} \quad (\Delta x' = \frac{\Delta x}{1.2})$

on solving the above equation we get slit width , d = 1 mm

original fringe width $\Delta x = \frac{\lambda D}{d} = \frac{500\times10^{-9}\times1}{1\times10^{-3}}$ = 0.5 mm

(b) Thus the fringe pattern gets shifted by a distance Δ which is given by

$$\Delta = \frac{D(n-1)t}{d}$$

Here the zero order fringe moves to the position previously occupied by the 4th order bright fringe

So we have $4\lambda = (n-1)t$

Thickness of the film, $t = \frac{4\lambda}{n-1} = \frac{4\times500\times10^{-9}}{0.2}$ = 10 μm

2. In a Lloyd's mirror experiment (see Figure 1), a bright wave emitted directly by the source S interferes with the wave reflected by the mirror M. As a result, an interference fringe pattern is formed on the screen N. The source and the screen is separated by a distance *l* = 1 m. At a certain position of the source the fringe width on the screen is equals to Δx = 0.25 mm. After the source is moved away from the plane of mirror by Δh= 0.60 mm, the fringe width decreases by a factor η= 1.5. Find the wavelength of the light.

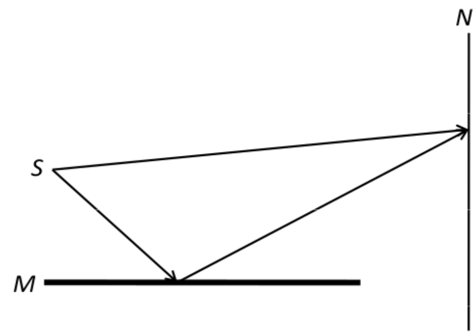


Figure 1: Lloyd's Mirror

<u>Solution:</u>

General formula is given by fringe width, $\Delta x = \frac{l\,\lambda}{d}$ (2a)

After the source moved away from the plane of the mirror,

we have $\frac{\Delta x}{\eta} = \frac{l\,\lambda}{d+2\Delta h}$ (2b)

Since d increased to d + 2Δh when source is moved away from the mirror by Δh.

Using equation (2a) and (2b) we get

$$\eta d = d + 2\Delta h$$

$$d = \frac{2\Delta h}{\eta - 1}$$

using eq. (2a), $\lambda = \frac{2\Delta h \times \Delta x}{l(\eta-1)} = \frac{2 \times 0.25 \times 0.6 \times 10^{-6}}{0.5 \times 1} = 0.6\ \mu m$

3. A plane light wave falls on a Fresnel mirrors with an angle α = 2.0′ between them. Determine the wavelength of light if the fringe-width on the screen is 0.55 mm.

<u>Solution:</u>

In this case we must let r→∞ in the formula (A plane wave is like light emitted from a point source at ∞).

So fringe width, $\Delta x = \frac{(b+r)\lambda}{2\alpha r} \approx \frac{\lambda}{2\alpha}$

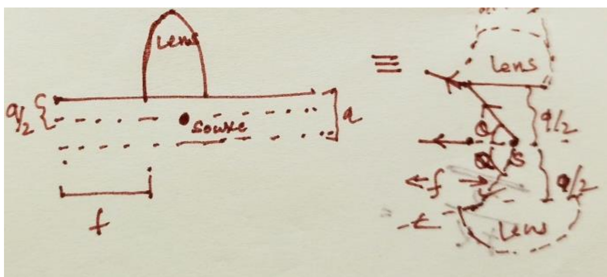
Then $\lambda = 2\alpha \,.\, \Delta x = 2\times \frac{2\times\pi}{180\times 60} \times 0.55 \times 10^{-3} = 0.64\ \mu m$

4. A lens of diameter 5 cm and focal length 25 cm is cut along its diameter into two identical halves. A layer of the lens a= 1 mm in thickness is removed and the two remaining halves of the lens are cemented together to form a composite lens. A slit is placed in the focal plane emitting monochromatic light of wavelength 0.60 µm. A screen is located behind the lens at a distance 50 cm from the lens.

(a) Find the fringe width on the screen and the number of possible maximum

(b) Find the maximum width of the slit at which the fringes on the screen will still be visible sufficiently sharp.

Solution:



(a) From the above figure the emergent light is at an angle $\tan\theta = \frac{a}{2f}$ from the axis.

Thus the divergence angle of the two incident light beams is

$\phi = 2\theta = \frac{a}{f}$ (since θ is small)

When this two light beams interfere the fringes produced on the screen have a fringe width, $\Delta x = \frac{\lambda}{\phi} = \frac{\lambda f}{a} = \frac{0.6\times10^{-6}\times25\times10^{-2}}{1\times10^{-3}} = 0.15$ mm.

Number of possible maxima is equal to $\frac{b\phi}{\Delta x} = 13$ fringes (b is distance between screen.

(b) If the slit width changes by a magnitude Δ (a/2 to a/2 +Δ)

The angle θ changes by $\Delta.\theta$ (ie. $\frac{\Delta}{2f}$)

So the fringe pattern will also shift by $\pm\frac{b.\Delta}{f}$

Equating this to $\frac{\Delta x}{2} = \frac{\lambda f}{2a}$ we obtain

$$\Delta_{max} = \frac{\lambda f^2}{2ab} = 37.5\ \mu m$$

5. Calculate frequency bandwidth for white light (frequency range 4×10^{14} Hz to $7.5 \times 10^{14}\ Hz$). Find coherence time and coherence length of white light.

Solution:

Coherence time , T= $\frac{1}{\Delta f} = \frac{1}{7.5\times 10^{14} - 4\times 10^{14}} = 2.85 \times 10^{-15}$ s

Coherence length = cT = $3\times 10^{8} \times 2.85 \times 10^{-15}$ = 0.855 µm

6. A quasi-monochromatic source emits radiation of mean wavelength 532 nm and has a frequency bandwidth of 10^9 Hz. Calculate the coherence time, coherence length and degree of monochromaticity.

Solution:

Coherence time , T= $\frac{1}{\Delta f} = 10^{-9}$s

Coherence length = cT = $3\times 10^{8} \times 10^{-9}$ = 0.3 m

Frequency corresponding to mean wavelength,

$f_0 = \frac{c}{\lambda} = \frac{3\times 10^{8}}{532\times 10^{-9}} = 5.64 \times 10^{14}$ Hz

Degree of Monochromaticity $= \frac{\Delta f}{f_0} = 1.77 \times 10^{-6}$