

### Hints and answers of tutorial sheet-1.

1. (a)  $c=0$ , (b) Not satisfied, (c)  $c=\frac{1}{2\pi+\frac{\pi}{2}}$ , (d) Not satisfied.
2. (a)  $c=\frac{\pi}{2}$ , (b)  $c=\frac{\sqrt{6}-1}{6}$ .
3. Use Rolle's th.
4. Limits in each case is  $\frac{1}{2}$
5. Use cauchy principle of convergence for sequence.
6. Apply Rolle's th. to  $g(x) = e^{-\lambda x} f(x)$
7. (a) Apply LMVT to  $f$ , (b) Apply LMVT to  $f(x) = x^{\frac{1}{3}}$  on  $[27, 28]$ , (c) Apply LMVT to  $f$  on  $[x_1, \frac{x_1+x_2}{2}]$  and  $[\frac{x_1+x_2}{2}]$  respectively.
8. Apply Rolle's th. to  $f, f'$  and  $f''$ .
9. (a) Take  $f(x) = \frac{\sin x}{x}$  on  $0 < x < \frac{\pi}{2}$  and show it is a decreasing function, (b) apply LMVT to  $f(x) = x^n - a^n$ , (c) take  $f(x) = \log(1+x) - \frac{x}{1+x}$  and  $g(x) = x - \log(1+x)$  on  $[0, \infty]$  and try to show they are increasing function.
10. (a) Take  $g(x) = \frac{1}{x} \int_1^x f(t)dt$  and apply Rolle's th,  
(b) Take  $\phi(x) = f(x) - \frac{(x-b)(x-c)}{(a-b)(a-c)}f(a) - \frac{(x-c)(x-a)}{(b-c)(b-a)}f(b) - \frac{(x-a)(x-b)}{(c-a)(c-b)}f(c)$  on  $[a, b]$  and apply Rolle's th.
11.  $\phi(x) = c_0x + c_1\frac{x^2}{2} + \dots + c_n\frac{x^n}{n+1}$  and apply Rolle's th.
12. (a) Apply Rolle's th. (b) Take  $g(x) = x^2$  and apply CMVT. (c) Apply LMVT to  $f$  on  $[a, \frac{a+b}{2}]$  and  $[\frac{a+b}{2}, b]$  respectively.
13. (a) Take  $g(x) = \begin{vmatrix} f(x) & f(b) \\ \phi(x) & \phi(b) \end{vmatrix}$  on  $[a, b]$  and apply LMVT.  
(b) Take  $g(x)$  as  $x, x^2$  and  $x^3$  and apply CMVT.
14. (a)  $f(x) = 1 - \cos x$  and  $g(x) = \frac{x^2}{2}$  (b)  $g(x) = \frac{f(x)}{x^2}$  and  $\psi(x) = \frac{1}{x^2}$  (c)  $f(x) = \ln x, x > 0$  and  $h(x) = \arcsin x, x \in \mathbb{R}$