THREE PHASE INDUCTION MOTORS

Three phase induction motors are the motors most frequently encountered in industry.

They are:

simple

rugged

low-priced

easy to maintain.

They run at essentially constant speed from zero to full load.

Three phase induction motor has two main parts:

1) a stationary stator

2) a revolving rotor.

The rotor is separated from the stator by a small air-gap which ranges from 0.4mm to 4mm, depending on the power of the motor.

The stator consists of a steel frame which encloses a hollow, cylindrical core made up of stacked laminations.
A number of evenly spaced slots, punched out of the internal circumference of the laminations, provide the space for the stator winding.

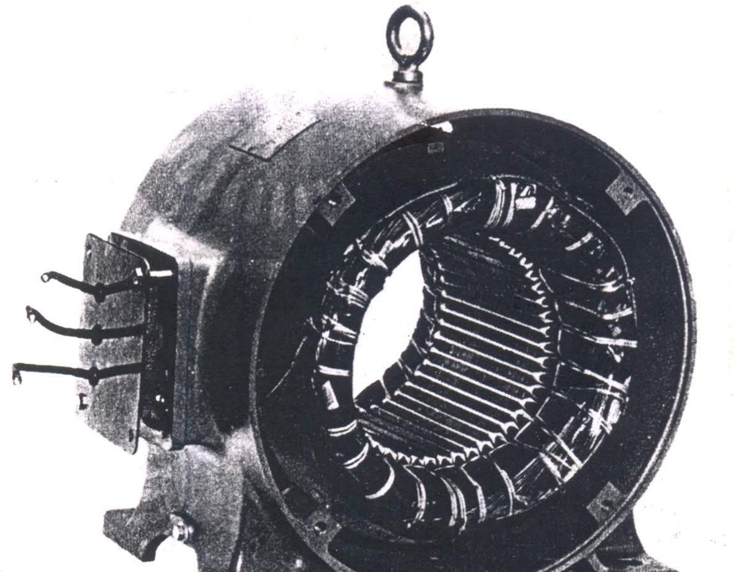


Fig.1: Stator of 3-phase induction motor.

The rotor is also composed of punched laminations. These are carefully stacked to create a series of rotor slots to provide space for the rotor winding.

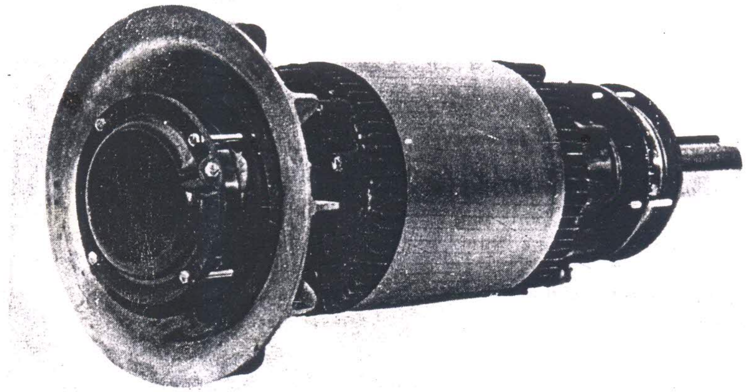


Fig. 2: Squirrel cage rotor

We use two types of rotor windings:

(a) Conventional three-phase windings made of insulated wire (Wound-rotor induction motor)

(b) Squirrel-cage windings (Squirrel-cage induction motors)

A squirrel-cage rotor is composed of bare copper bars, slightly longer than the rotor, which are pushed into the slots.

The opposite ends are welded to two copper end rings, so that all the bars are short-circuited together.

The entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

In small and medium-size motors, the bars and end-rings are made of die-cast aluminum, molded to form an integral block.

A wound rotor has a three-phase winding, similar to the one on the stator.

The winding is uniformly distributed in the slots and is usually connected in Y.

The terminals are connected to three slip-rings which turn with the rotor.

The revolving slip-rings and associated stationary brushes enable us to connect external resistors in series with the rotor windings.

The external resistors are mainly used during the start-up period; under normal running conditions, the three brushes are short-circuited.

PRINCIPLE OF THE INDUCTION MOTOR

The operation of a three-phase induction motor is based upon the application of Faraday's Law and the Lorentz force on a conductor.

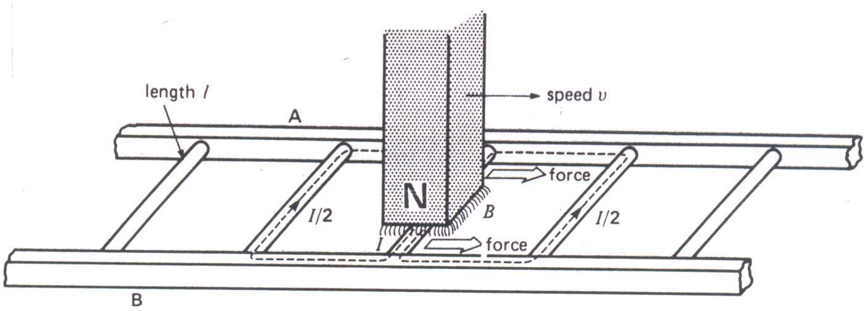


Fig. 3(a)

Consider a series of conductors of length l whose extremities are short circuited by two bars A and B.

A permanent magnet, placed above this conducting "ladder" moves rapidly to the right at a speed v, so that its magnetic field B' sweeps across the conductors. The following sequence of events then takes place:

1.) A voltage $E = Blv$ is induced in each conductor while it is being cut by the flux (Faraday's Law)

2) The induced voltage immediately produces a current I, which flows down the conductor, through the end-bars, and back through the other conductors.

3) Because the current-carrying conductor lies the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force);

4) The force always acts in a direction to drag the conductor along with the magnetic field.

If the conducting "ladder" is free to move, it will accelerate towards the right. However, as it picks up speed, the conductors will be cut less rapidly, with the result that the induced voltage E and the current I will diminish.

Consequently, the force acting on the conductors will also decrease.

If the ladder were to move at the same speed as the magnetic field, the induced voltage E, the current I and the force would all be zero.

In an induction motor, the ladder is closed upon itself to form a squirrel-cage, and the moving magnet is replaced by a rotating field.

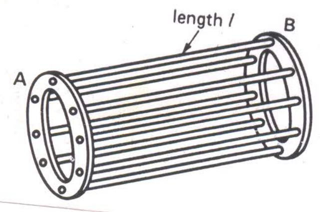


Fig. 3(b): Ladder bent upon itself to form a squirrel-cage.

The field is produced by the three-phase currents which flow in the stator windings.

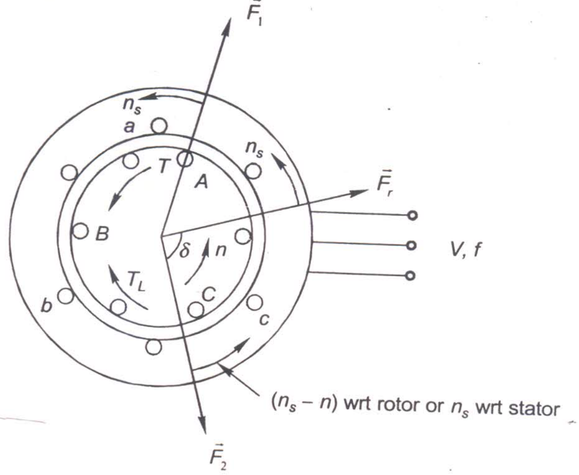


Fig. 4: *Illustrating the principle of induction machine*

- Consider a cylindrical rotor machine with both the stator and rotor wound for three phases as shown in Fig. 4.

Assume initially the rotor winding to be open-circuited and let the stator be connected to an infinite bus (V, f).

The stator currents set up a rotating magnetic field in the air-gap which runs at synchronous speed inducing emf in the stator winding which balances the terminal voltage under the assumption that the stator resistance and leakage reactance are negligible.

Also the rotating field induces emf in the rotor winding but no rotor current flows because the rotor is open-circuited.

The frequency of rotor emf's is ofcourse f.

Since the rotor mmf $F_2 = 0$, no torque is developed and the rotor continues to be stationary.

The machine acts merely as a transformer where the stator (primary) and rotor (secondary) have emf's of the same frequency induced in them by the rotating magnetic flux rather than by a stationary time-varying flux as in an ordinary transformer.

Let the rotor be now held stationary (Blocked from rotation) and the rotor winding be short-circuited.

The rotor now carries three-phase currents creating the mmf F_2 rotating in the same direction and with the same speed as the stator field.

F_2 causes reaction currents to flow into the stator from the busbar (just as in an ordinary transformer) such that the flux/pole, ϕ_r of the resultant flux density wave (rotating in the air-gap at synchronous speed) induces a stator emf to just balance the terminal voltage.

Obviously, ϕ_r must be the same as when the rotor was open-circuited.

In fact, ϕ_r will remain constant independent of the operating conditions created by load on the motor.

The interaction of ϕ_r and F_2, which are stationary with respect to each other, creates the torque tending to move the rotor in the direction of F_r

The induction motor is therefore a self-starting device.

Let the short-circuited rotor be now permitted to rotate. It runs in the direction of stator field and acquires a steady speed of n. Obviously $n < n_s$ because if $n = n_s$, the relative speed between the ~~stator~~ field and rotor winding will be zero and therefore the induced emf's and rotor currents will be zero and hence no torque is developed.

The rotor thus cannot reach the synchronous speed n_s. ~~and hence~~

With the rotor running at n, the relative speed of the stator field with respect to rotor conductors is $(n_s - n)$ in the direction of n_s. The frequency of induced emf's (and currents) in the rotor is therefore,

$$n_s - n = \frac{120 f_2}{P}$$

$$\therefore f_2 = \left(\frac{n_s - n}{n_s}\right) \cdot \frac{n_s \cdot P}{120} = sf \quad \ldots (1)$$

where

$$s = \frac{n_s - n}{n_s} = \text{slip of the rotor} \quad \cdots (2)$$

The slip "s" is the per unit speed (with respect to synchronous speed) at which the rotor slips behind the stator field.

The rotor frequency

$f_2 = sf$ is called the slip frequency

From Eqn. (2)

$$n = (1-s)n_s \quad \cdots (3)$$

From Eqn (1)

$$\frac{120sf}{P} = n_s - n \quad \cdots (4)$$

Since the rotor is running at a speed n and the rotor field at $(n_s - n)$ with respect to the rotor in the same direction, the net speed of the rotor field as seen from the stator is

$$n + (n_s - n) = n_s$$

i.e., same as the stator field.

Thus the reaction field F_2 of the rotor is always stationary with respect to the stator field F_1 or the resultant field F_r (with flux ϕ_r per pole).

Circuit diagram of a three-phase slip-ring induction motor with Δ-connected stator and Y-connected rotor is shown in Fig. 5

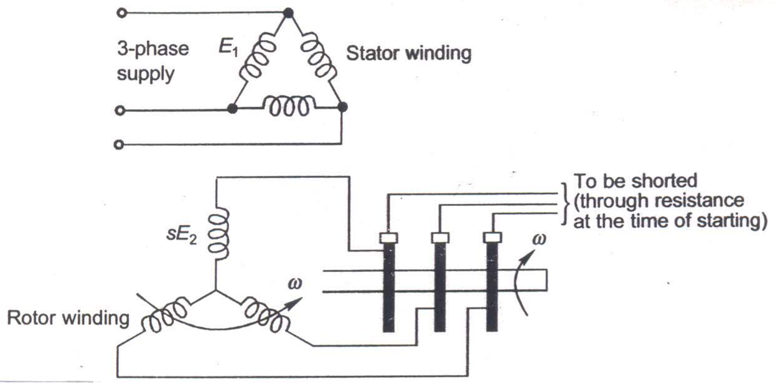


Fig. 5 3-phase, slip-ring induction motor — connection diagram

The rotor winding is connected to slip rings which are shorted through external resistances at the time of starting; the resistances are cut-out as the motor attains full speed.

The rotor of a squirrel-cage motor has permanently shorted bars. These can be replaced from a circuit point of view by an equivalent wound rotor.

From Eqn.(2)

$$s = \frac{n_s - n}{n_s} = \frac{\text{slip speed}}{\text{synchronous speed}}$$

$$\therefore \; s = \left(1 - \frac{n}{n_s}\right) \quad \text{- - - - -} \quad (5)$$

Obviously, for $n=0$, $s=1$, i.e. for the stationary rotor and $s=0$ for $n=n_s$, i.e., for the rotor running at synchronous speed.

From Eqn(1),

The frequency of currents induced in the rotor is

$$f_2 = sf \quad --- (6)$$

The normal full-load slip of the induction motor is of the order of 2% – 8%, so that the frequency of the rotor currents is as low as 1 – 4 Hz.

Per phase stator emf is given by

$$E_1 = \pi\sqrt{2}\, K_{w1} N_{ph_1} f \phi_r \quad --- (7)$$

Per phase rotor emf at s=1 (standstill rotor) is given by

$$E_2 = \pi\sqrt{2}\, K_{w2} N_{ph_2} f \phi_r \quad --- (8)$$

where

E_1 = stator induced emf/phase

E_2 = rotor induced emf/phase

K_{w1} = stator winding factor

K_{w2} = rotor winding factor

N_{ph_1} = stator turns/phase

N_{ph_2} = rotor turns/phase.

ϕ_r = resultant air-gap flux/pole.

At any slip s, the rotor frequency being sf, the rotor induced emf changes to sE_2.

Consider now the impedance of the rotor circuit

$$Z_2 = r_2 + jx_2 \quad \text{(At standstill)} \quad \ldots (9)$$

where

x_2 = Leakage reactance of rotor at standstill (rotor frequency = stator frequency, f)

When the rotor runs at slip s, its frequency being sf, its impedance changes to

$$Z_2 = r_2 + jsx_2 \quad ----(10)$$

It is, therefore, seen that the frequency of rotor currents, its induced emf and reactance all vary in direct proportion to the slip.

Fig.6 shows the rotor circuit at slip s,

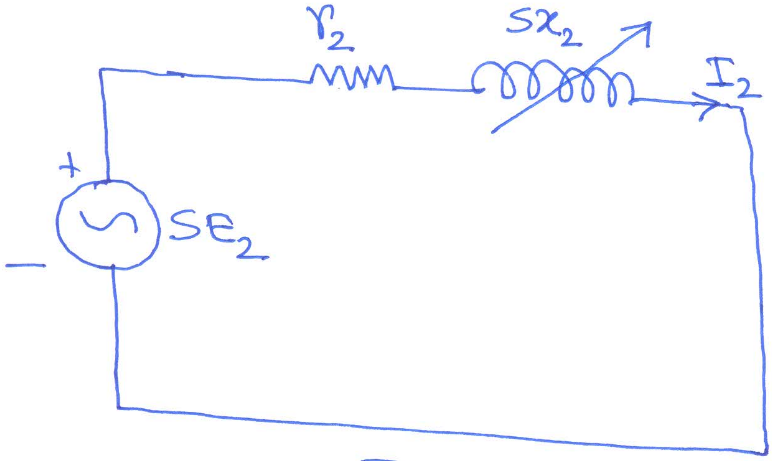


Fig. 6

The phase angle of the circuit is

$$\theta_2 = \tan^{-1}\left(\frac{sx_2}{r_2}\right) \text{ [Lagging]} \quad --(11)$$

Also

$$\frac{E_1}{E_2} = \frac{K_{w1} N_{ph1}}{K_{w2} N_{ph2}} = \frac{N_1}{N_2} = a \quad -(12)$$

where

N_1, N_2 = effective stator and rotor turns/phase

DEVELOPMENT OF CIRCUIT MODEL

1.) $\frac{E_1}{E_2} = a$; $\frac{I_2'}{I_2} = \frac{1}{a}$; $a = \frac{N_1}{N_2}$ --- (13)

where

E_2 = standstill rotor emf.

2.) Like in a transformer, the magnetizing current component I_m of the stator current lags the stator induced emf E_1 by $90°$.

3.) The induction motor is not merely a transformer which changes voltage and current levels. It in facts behaves like a generalized transformer in which the frequency is also transformed in proportion to slip such that the rotor induced emf is sE_2 and rotor reactance is sX_2.

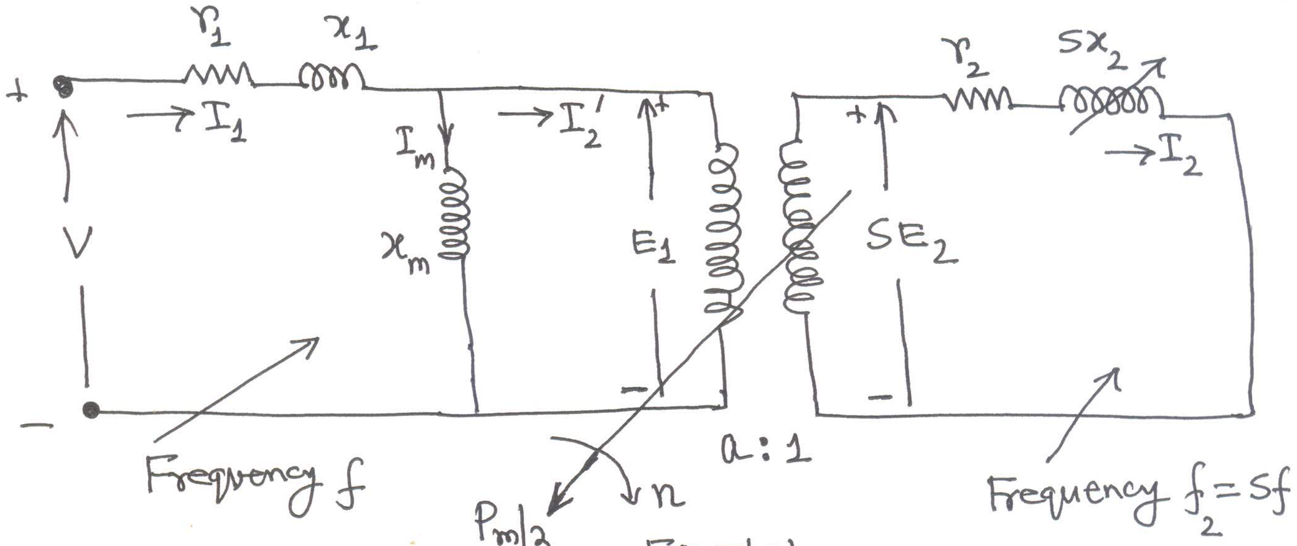


Fig. 7(a)

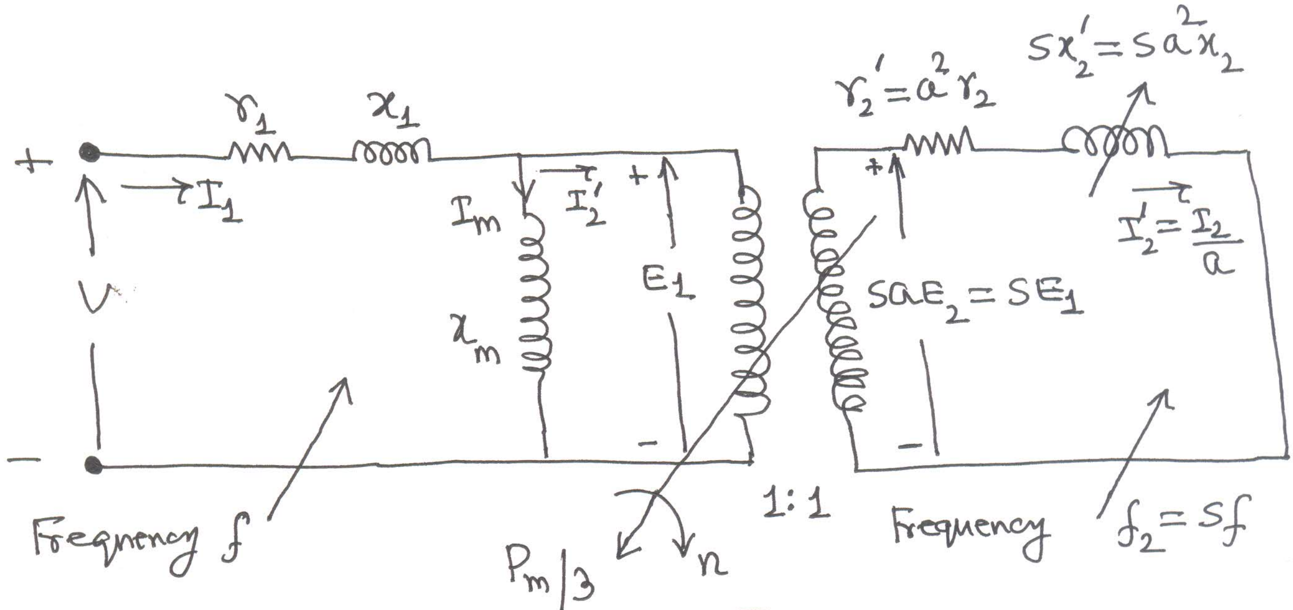


Fig. 7(b)

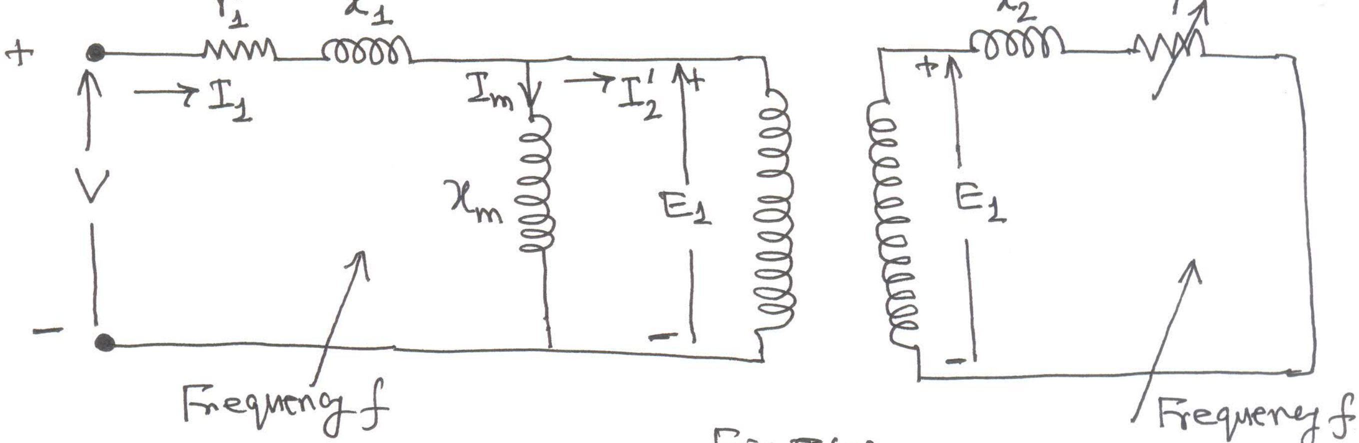


Fig. 7(c)

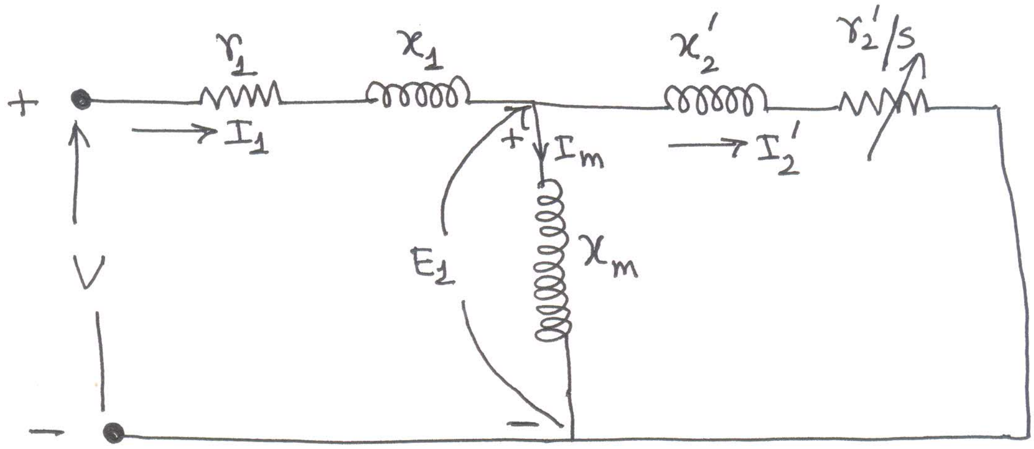


Fig. 7(d)

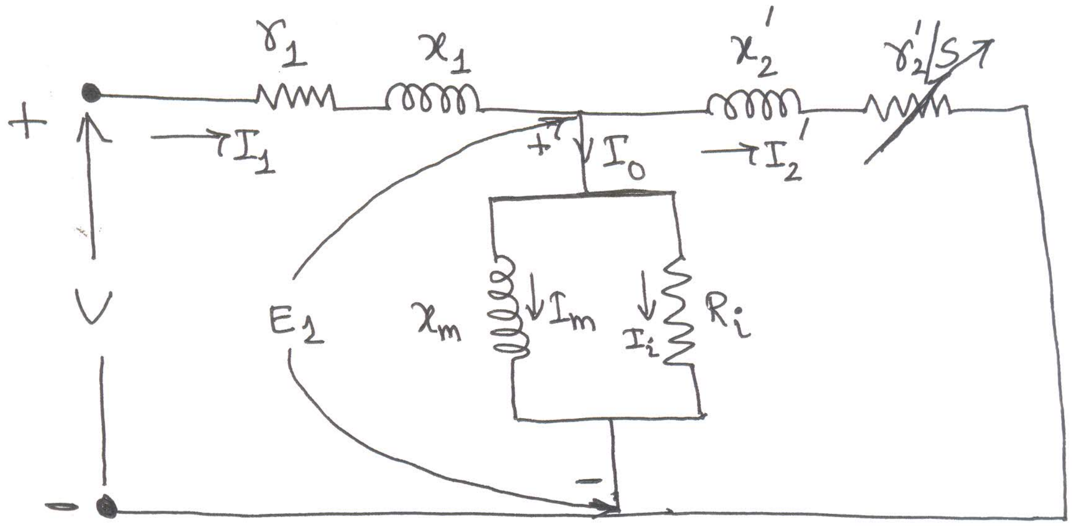


Fig. 7(e)

Fig. 7: Development of the equivalent circuit of Induction motor.

The circuit model of the induction motor can now be drawn on per phase ~~basis~~ basis as shown in Fig. 7(a)

The rotor circuit can be referred to the stator side by a two-step process —

ⓐ Modifying the rotor circuit so that the turn-ratio becomes unity. ~~and then Carrying out a frequency~~

b) Carrying out a frequency transformation resulting in an equivalent rotor circuit at the stator frequency.

From Fig 7(a),

$$I_2 = \frac{sE_2}{r_2 + jsx_2} = \frac{saE_2}{ar_2 + jsax_2}$$

$$\therefore \frac{I_2}{a} = I_2' = \frac{saE_2}{a^2r_2 + jsa^2x_2} = \frac{sE_1}{a^2r_2 + jsa^2x_2} \quad \text{--- (14)}$$

~~Where~~ Define

$$Z_2' = a^2r_2 + jsa^2x_2 = r_2' + jsx_2' \quad \text{--- (15)}$$

$$\therefore r_2' = a^2r_2\,; \quad x_2' = a^2x_2 \quad \text{--- (16)}$$

From Eqns (14) and (15)

$$I_2' = \frac{sE_1}{r_2' + jsx_2'} = \frac{E_1}{\frac{r_2'}{s} + jx_2'} \quad \cdots\cdots (17)$$

This simple trick refers the rotor circuit to the stator frequency.

If r_2' is separated from $\frac{r_2'}{s}$ to represent the rotor copper-loss as a separate entity, the circuit model is shown in Fig. 8(a) in which the variable resistance $r_2'\left(\frac{1}{s}-1\right)$ represents the mechanical output in electrical form.

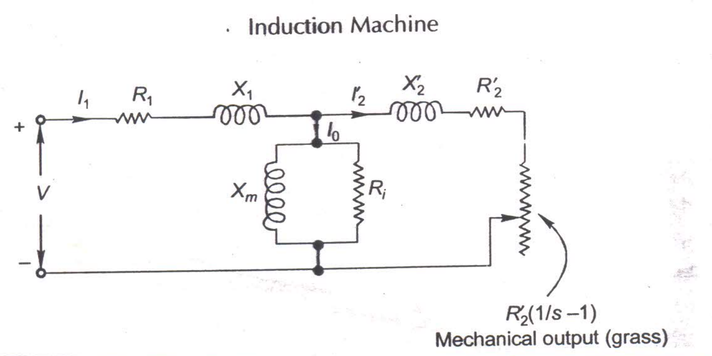


Fig. 8(a)

$R_2' = r_2'$

$X_2' = x_2'$

$R_1 = r_1$

$X_1 = x_1$

$X_m = x_m$

Alternatively the circuit model of Fig. 8(b) could be used.

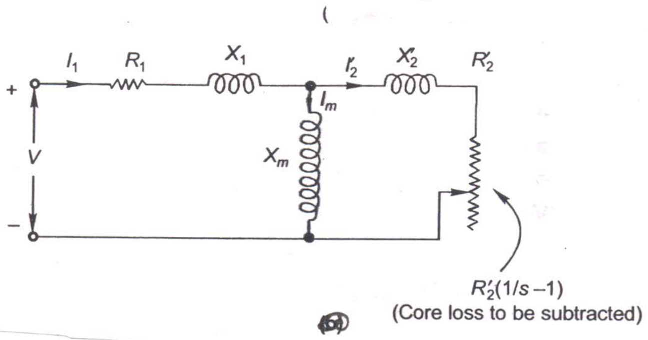


Fig. 8(b) :

This corresponds to Fig. 7(d) wherein the iron loss resistance R_i omitted and this loss would be subtracted from the gross mechanical output [power absorbed by $r_2'\left(\frac{1}{s}-1\right)$]

This amounts to certain approximation which is quite acceptable in the normal range of slip in an induction motor.

It may be noted here that the power dissipated in $r_2'\left(\frac{1}{s}-1\right)$ includes the core loss [Fig. 8(b)], which must be subtracted from it to obtain the gross mechanical power.

For getting net mechanical power output, the windage and friction loss must be further subtracted from it.

The core loss and <u>windage and friction Loss together are lumped as rotational Loss</u> as both these losses occurs when the motor is running.

The rotational Loss in an induction motor is substantially constant at constant applied voltage and motor speed varies very little from no-load to full-load.

$\boxed{\text{Note: Net mechanical power} = \text{Shaft power}}$

APPROXIMATE CIRCUIT MODEL

Approximate circuit model is shown in Fig. 9

$R = r$

$R_1 = r_1$, $R_2' = r_2'$

$R = r = r_1 + r_2'$

$X = x$

$X_1 = x_1$

$X_2' = x_2'$

$X = x = x_1 + x_2'$

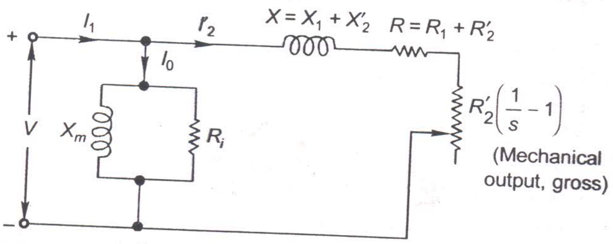


Fig. 9: Approximate circuit model of induction motor

This approximate circuit model is not so readily justified as in a transformer owing to the relative magnitude of the exciting current (magnetizing current) which, because of the presence of the air-gap, may be as large as 30% – 50% of the full-load current.

Further, the primary Leakage reactance is also necessarily higher in an induction motor compared to a transformer and so ignoring the voltage drop in primary reactance is not quite justified.

Therefore results obtained by this model are considerably Less accurate.

Magnetizing shunt branch (Fig. 9) which draws current I_0 at almost 90° lagging, the power factor at which the motor operates at full-load is low – about 0.8. At light load (Small I_2') the machine power factor is much Lower. This is the inherent problem of the induction motor because of the presence of the air gap in the magnetic circuit and the fact that the excitation current must be drawn from the mains (stator side)

POWER ACROSS AIR-GAP, TORQUE AND POWER OUTPUT

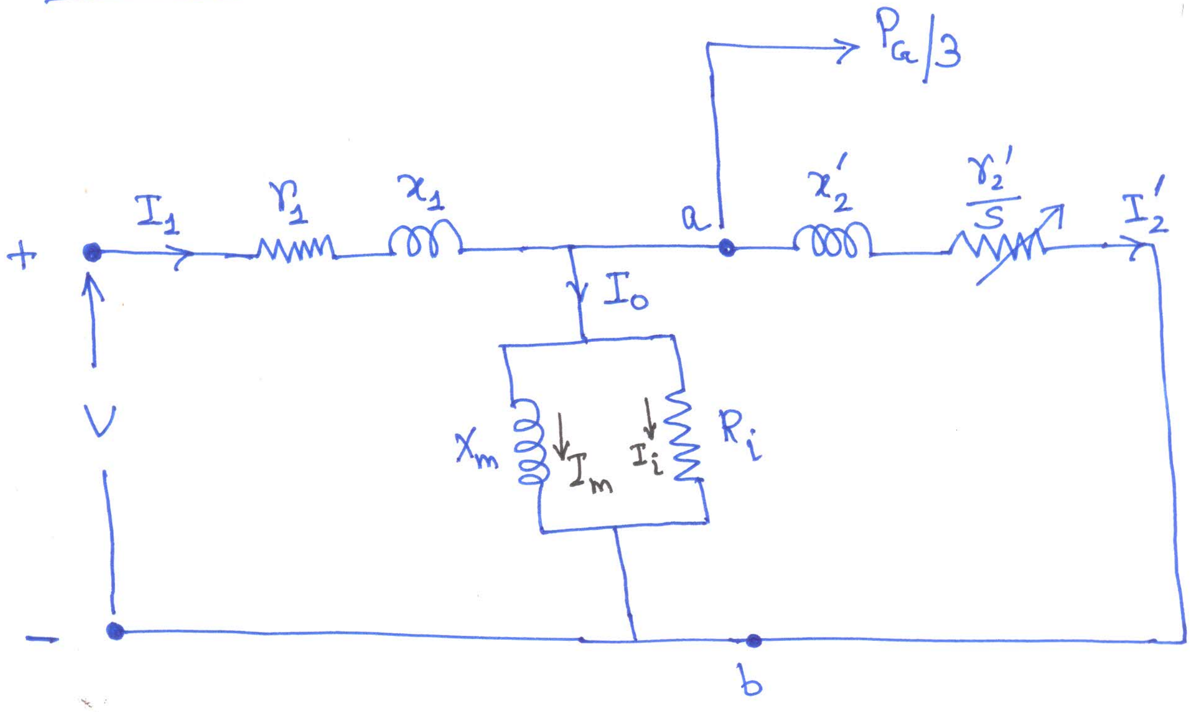


Fig. 10

The power crossing the terminals ab in Fig. 10, is the electrical power input per phase minus the stator loss (stator copper-loss and iron-loss) and hence is the power that is transferred from the stator to the rotor via the air-gap magnetic field. This is known as the power across the air-gap and its three-phase is symbolized as P_G.

From Fig. 10,

$$P_G = 3(I_2')^2\left(\frac{r_2'}{s}\right)$$

$$\therefore P_G = \frac{3(I_2')^2 r_2'}{s} \quad \text{---- (18)}$$

$\therefore$ Power across air-gap (P_G) = $\dfrac{\text{rotor copper-loss}}{\text{slip}}$ --- (19)

OR Rotor copper-loss = (slip) (P_G) = sP_G

$$\therefore P_{cr} = sP_G = 3(I_2')^2 r_2' \quad \text{--- (20)}$$

Mechanical power output (Gross),

$$P_m = P_G - 3(I_2')^2 r_2' = \frac{3(I_2')^2 r_2'}{s} - 3(I_2')^2 r_2'$$

$$\therefore P_m = \frac{3(I_2')^2 r_2'}{s}(1-s) = (1-s)P_G \quad \text{-- (21)}$$

This means that the gross mechanical power output is three times (3-phase) the electrical power absorbed in resistance $r_2'\left(\frac{1}{s}-1\right)$.

Fig.10 can therefore be drawn as in Fig.11 where $\frac{r_2'}{s}$ is represented as

$$\frac{r_2'}{s} = r_2' + \underbrace{r_2'\left(\frac{1}{s}-1\right)}_{\text{Load resistance}}$$

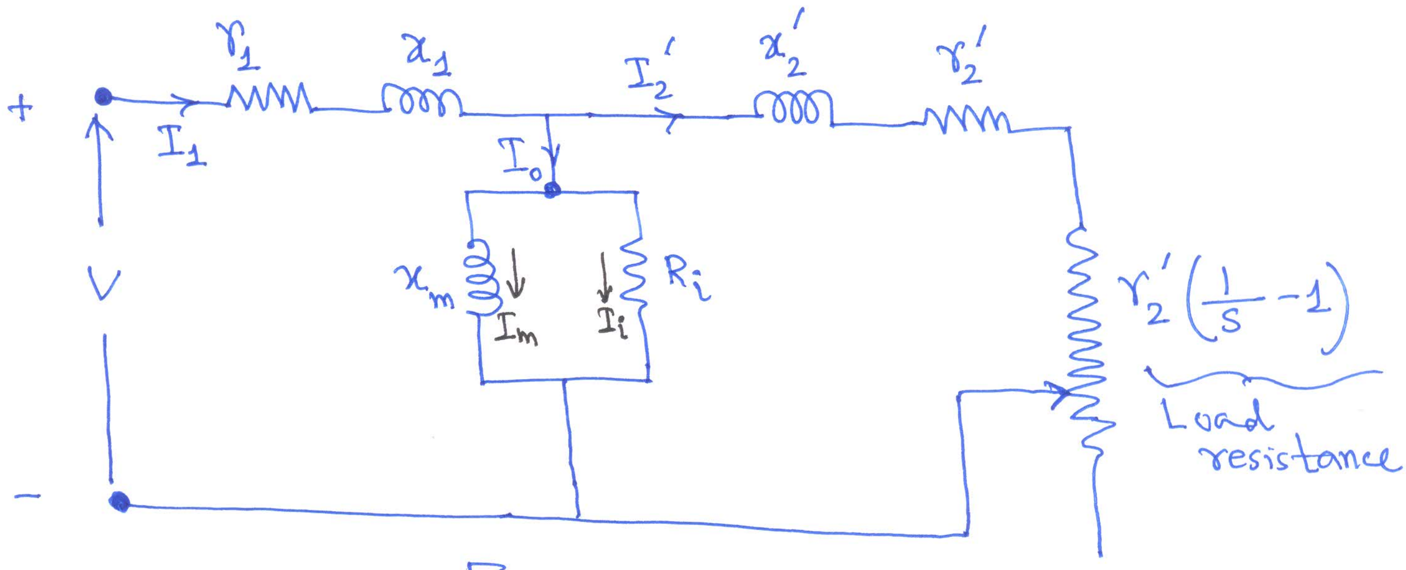


Fig.11

It is noticed from Eqn.(21) that the mechanical power output is a fraction $(1-s)$ of the total power delivered to the rotor, while as per Eqn.(20), a fraction s of it is dissipated as the rotor copper-loss.

It is then evident that high-slip operation of the induction motor would be highly inefficient.

From Eqn.(5)

$$s = \left(1 - \frac{n}{n_s}\right) = 1 - \frac{\omega}{\omega_s}$$

$$\therefore \omega = (1-s)\,\omega_s \text{ rad (mech.)/sec.} \quad \text{--- (22)}$$

Electromagnetic torque developed is then given by

$$\omega T = (1-s)\,\omega_s T = P_m = (1-s)\,P_G$$

$$\therefore T = \frac{P_G}{\omega_s} = \frac{3(I_2')^2 \cdot (r_2'/s)}{\omega_s} \text{ Nm} \quad \text{--- (23)}$$

COMPUTATIONAL PROCEDURE

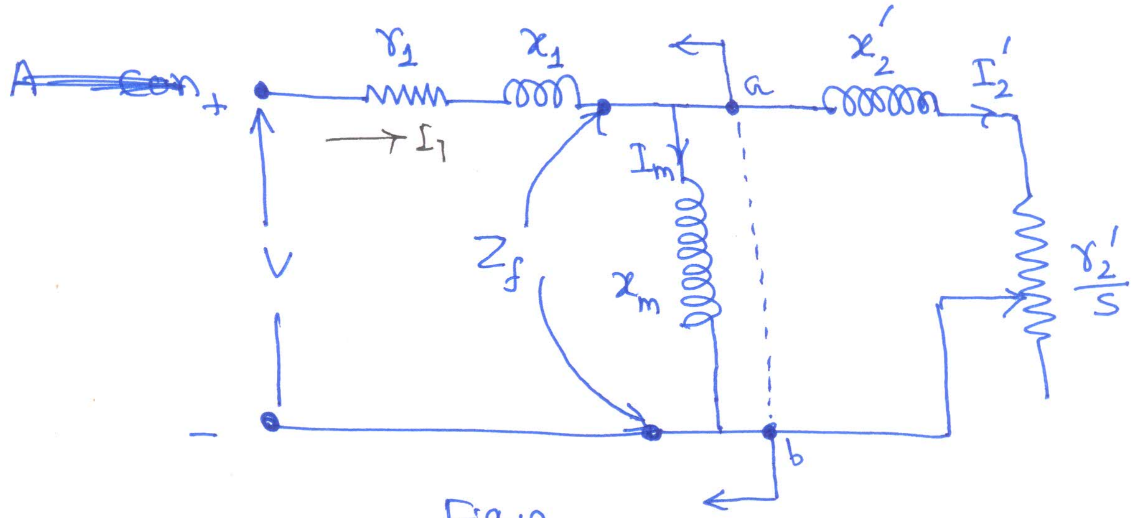


Fig. 12

$$Z_f = \left[jx_m \,\|\, \left(\frac{r_2'}{s} + jx_2' \right) \right]$$

$$= \cancel{R_f + x_f}\; r_f + jx_f$$

$$\therefore \quad P_G = 3 I_1^2 R_f$$

$$\text{and} \quad T = \frac{P_G}{\omega_s} = \frac{3 I_1^2 R_f}{\omega_s} \qquad \text{--- (24)}$$

TORQUE – SLIP CHARACTERISTIC

The expression for torque – slip characteristic is easily obtained by finding the Thevenin equivalent of the circuit to the left of ab in Fig. 12,

$$Z_{TH} = (r_1 + jx_1) \| (jx_m) = r_{th} + j x_{th}$$

$$\therefore V_{TH} = V \left[\frac{j x_m}{r_1 + j(x_1 + x_m)} \right] \quad \text{---- (25)}$$

The circuit then reduces to Fig. 13 in which it is convenient to take V_{TH} as the reference voltage.

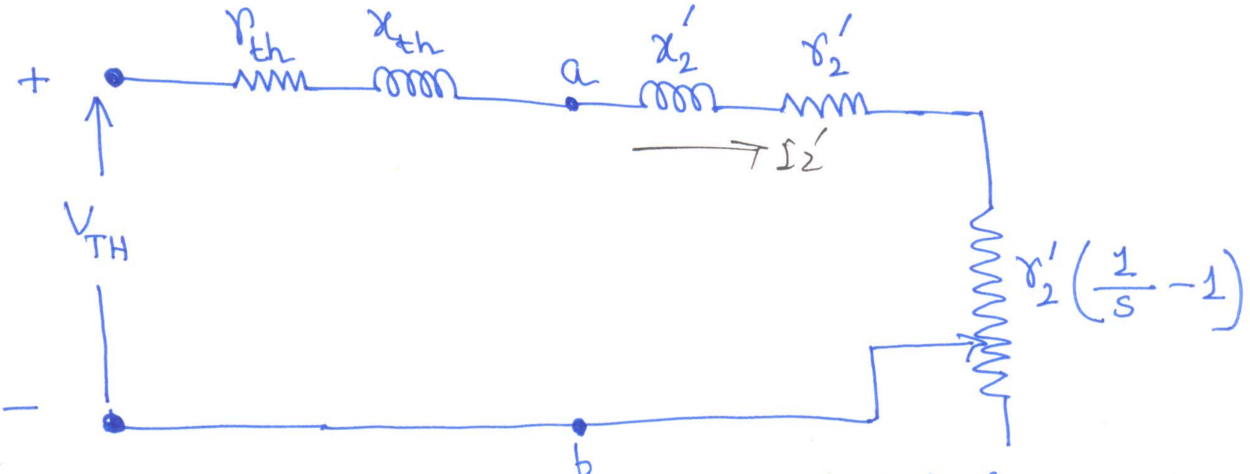


Fig. 13: Thevenin equivalent of induction motor circuit model.

From Fig. 13

$$\left(I_2'\right)^2 = \frac{V_{TH}^2}{\left(r_{th} + \frac{r_2'}{s}\right)^2 + \left(x_{th} + x_2'\right)^2} \quad \text{---(26)}$$

$$T = \frac{3}{\omega_s} \cdot \left(I_2'\right)^2 \left(\frac{r_2'}{s}\right)$$

$$\therefore T = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 \left(r_2'/s\right)}{\left[\left(r_{th} + \frac{r_2'}{s}\right)^2 + \left(x_{th} + x_2'\right)^2\right]} \quad \text{---(27)}$$

Eqn. (27) is the expression for torque developed as a function of voltage and slip. For a given value of slip, torque is proportional to the square of voltage. The torque-slip characteristic at fixed (rated) voltage is plotted in Fig. 14.

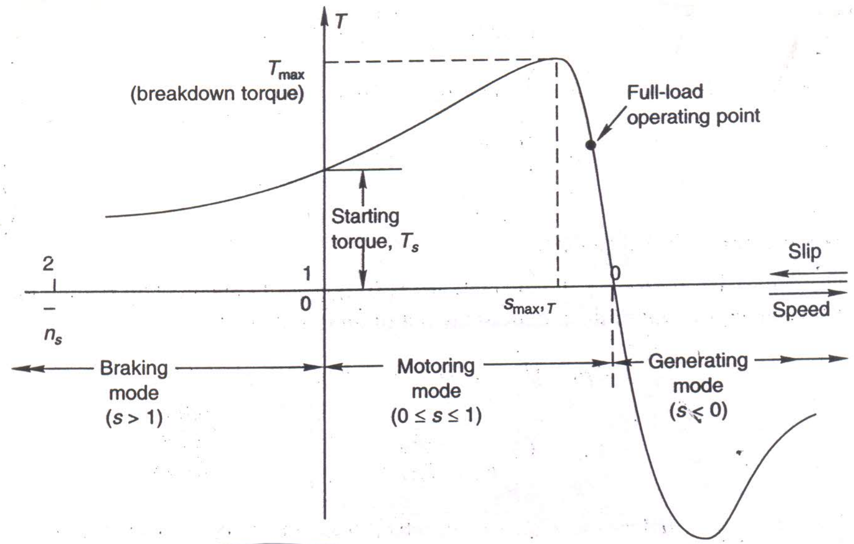


Fig.14: *Torque-slip characteristics*

Certain features of the torque-slip characteristic are given below:

1.) <u>Motoring mode</u> : $0 \leq s \leq 1$

For this range of slip, the load resistance in the circuit model of Fig. 13, is positive, i.e., mechanical power output or torque developed is in the direction in which the rotor rotates. Also:

(a) At $s = 0$, torque is zero

(b) The torque has a maximum value called the breakdown torque (T_{BD}) at slip $S_{max,T}$. The motor would decelerate to a halt if it is loaded with more than the breakdown torque.

(c) At $s = 1$, i.e. when the rotor is stationary, the torque corresponds to the starting torque, T_S. For normally designed motor $T_S < T_{BD}$.

(d) The normal operating point is located well below T_{BD}. The full-load slip is usually 2% – 8%.

(e) The torque-slip characteristic from no-load to somewhat beyond full-load is almost linear.

2) Generating Mode: $s < 0$

Negative slip implies rotor running at super-synchronous speed ($n > n_s$). The load resistance is negative in the circuit model of Fig. 13 which means that mechanical power must be put in while electrical power is put out at the machine terminals.

3.) Braking Mode: $s > 1$

The motor runs in opposite direction to the rotating field (i.e. $n < 0$), absorbing mechanical power (braking action) which is dissipated as heat in the rotor copper.

Maximum (Breakdown) Torque

From Eqn. (27),

$$T = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 \cdot \left(\frac{r_2'}{s}\right)}{\left[\left(r_{th} + \frac{r_2'}{s}\right)^2 + \left(x_{th} + x_2'\right)^2\right]}$$

For maximum torque, $\left.\frac{dT}{ds}\right|_{s = s_{max,T}} = 0$

$$\therefore \; s = s_{max,T} = \frac{r_2'}{\sqrt{r_{th}^2 + \left(x_{th} + x_2'\right)^2}} \quad \text{--- (28)}$$

From Eqns (27) & (28)

$$T = T_{max} = \frac{3}{\omega_s} \cdot \frac{0.5\, V_{TH}^2}{\left[r_{th} + \sqrt{r_{th}^2 + \left(x_{th} + x_2'\right)^2}\right]} \quad \text{--- (29)}$$

From Eqn. (29), it can be observed that the maximum torque is independent of the rotor resistance (r_2') while the slip (Eqn. 28) at which it occurs is directly proportional to it.

STARTING TORQUE

At start $s=1$,

From Eqn. (27), we get

$$T = T_{start} = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2 r_2'}{\left[(r_{th}+r_2')^2 + (x_{th}+x_2')^2\right]} \quad \text{--- (30)}$$

Starting torque increases by adding resistance in the rotor circuit.

From Eqn. (28), the maximum starting torque is achieved for $\left(S_{max,T} = 1\right)$, i.e.,

$$r_2' = \sqrt{r_{th}^2 + (x_{th}+x_2')^2} \quad \text{--- (31)}$$

From Eqn. (26)

$$I_2' = \frac{V_{TH}}{\sqrt{\left(r_{th} + \frac{r_2'}{s}\right)^2 + (x_{th}+x_2')^2}} \quad \text{--- (32)}$$

At start $s=1$,

$$\therefore I_{2\,start}' = \frac{V_{TH}}{\sqrt{(r_{th}+r_2')^2 + (x_{th}+x_2')^2}} \quad \text{--- (33)}$$

From Eqn.(33), it is clear that starting current will reduce.

This indeed is the advantage of the slip-ring induction motor in which a high starting torque is obtained at low starting current.

An Approximation

Sometimes for getting a feel (rough answer) of the operational characteristic, it is convenient to assume the stator impedance to be negligible, i.e.,

$$r_{th} = 0, \quad x_{th} = 0 \quad [\text{Fig. 13}]$$

Therefore, $V_{TH} = V$ [See Fig. 12, Fig. 13 and Eqn. (25)]

Substituting $r_{th} = 0$, $x_{th} = 0$ and $V_{TH} = V$ in Eqn. (32)

$$I_2' = \frac{V}{\sqrt{\left(\frac{r_2'}{s}\right)^2 + (x_2')^2}} \quad \ldots \quad (34)$$

Substituting $r_{th}=0$, $x_{th}=0$ and $V_{TH}=V$ in Eqn.(27), we get

$$T = \frac{3}{\omega_s} \cdot \frac{V^2\left(\frac{r_2'}{s}\right)}{\left[\left(\frac{r_2'}{s}\right)^2 + (x_2')^2\right]} \quad \text{----} \quad (35)$$

Also,

$$s = s_{max,T} = \frac{r_2'}{x_2'} = \frac{\text{rotor resistance}}{\text{standstill rotor reactance}} \quad \text{--} \quad (36)$$

$$\therefore \quad T_{max} = \frac{3}{\omega_s} \cdot \left[\frac{0.5V^2}{x_2'}\right] \quad \text{---} \quad (37)$$

$$T_{start} = \frac{3}{\omega_s} \cdot \frac{V^2 r_2'}{\left[(r_2')^2 + (x_2')^2\right]} \quad \text{--} \quad (38)$$

Maximum starting torque ($s_{max,T}=1$) is achieved under the condition,

$$r_2' = x_2'$$

and

$$T_{start(max)} = T_{max} = \frac{3}{\omega_s}\left[\frac{0.5V^2}{x_2'}\right] \quad \text{--} \quad (39)$$

Some Approximate Relationships at Low slip

Around the rated (full-load) speed, slip of the induction motor is very small such that

$$\frac{r_2'}{s} \gg x_2'$$

So that x_2' can be altogether neglected in a simplified analysis. Eqn.(34) then simplify to

$$I_2' = \frac{sV}{r_2'} \quad \text{-----} \quad (40)$$

and Eqn.(35) then simplify to,

$$T = \frac{3}{\omega_s} \cdot \frac{sV^2}{r_2'} \quad \text{-----} \quad (41)$$

From Eqn(41), it can be observed that the torque-slip relationship is nearly linear in the region of low slip.

Maximum Power Output

Since the speed of the induction motor reduces with load, the maximum mechanical power output does not correspond to the speed (slip) at which maximum torque is developed.

For maximum mechanical power output, from Fig. 13, condition is

$$r_2'\left(\frac{1}{s}-1\right) = \sqrt{\left(r_{th}+r_2'\right)^2 + \left(x_{th}+x_2'\right)^2} \quad \cdots (42)$$

The maximum power output can then be found corresponding to the slip defined by Eqn. (42). However, this condition corresponds to very low efficiency and very large current and is well beyond the normal operating region of the motor.

STARTING

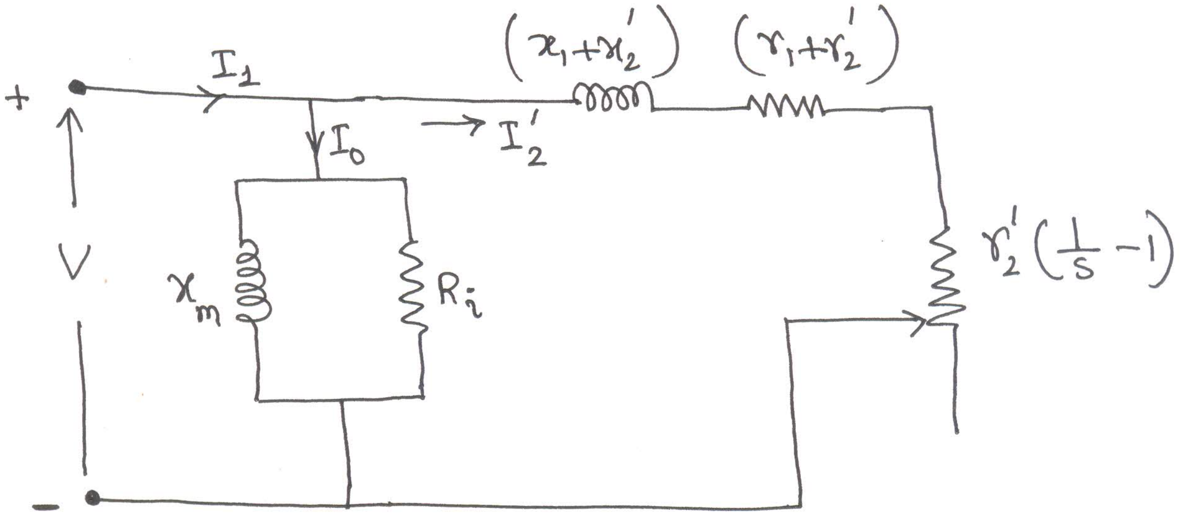


Fig. 15 (a)

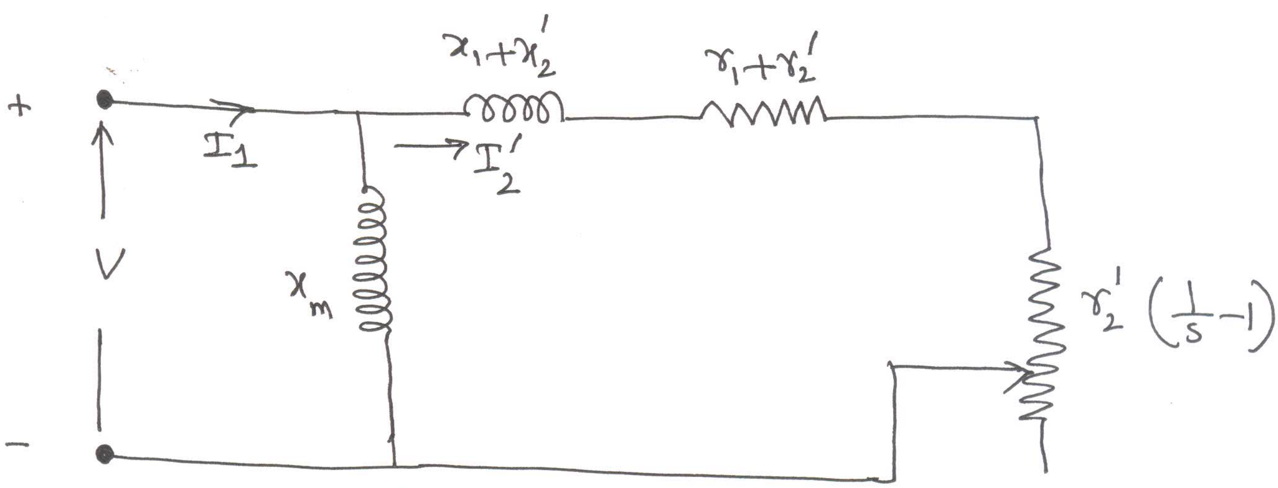
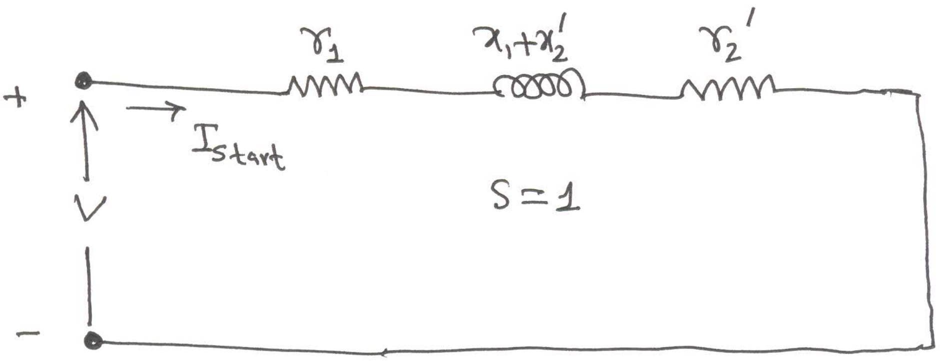


Fig. 15 (b)



$r_2'\left(\frac{1}{s}-1\right)\Big|_{s=1} = 0$

Fig. 15 (c)

At the time of starting, $s = 1$, the load resistance

$$r_2'\left(\frac{1}{s}-1\right)\Big|_{s=1} = 0. \quad \text{---} \quad (43)$$

Therefore, the motor current at starting can be as large as five to six times the full-load current.

In comparison, the exciting current in the shunt branch of the circuit model can be neglected (at start) reducing the circuit to that of Fig. 15(c).

Now, starting torque,

$$T_{start} = \frac{3}{\omega_s} \cdot I_{start}^2 \, r_2' \quad \text{----} \quad (44)$$

Assuming for simplicity (Rough approximation)

$$I_{fl} \simeq I'_{2,fl} \quad \text{----} \quad (45)$$

The magnetizing current is neglected even under full-load conditions.

Then,

full-load torque

$$T_{fl} = \frac{3}{\omega_s} \cdot I_{fl}^2 \cdot \frac{r_2'}{s_{fl}} \quad \text{----} (46)$$

Where s_{fl} = full-load slip.

Eqn. (44) ÷ Eqn. (46)

$$\frac{T_{start}}{T_{fl}} = \left(\frac{I_{start}}{I_{fl}}\right)^2 s_{fl} \quad \text{----} (47)$$

Ex-1: A 6-pole induction motor is fed from 50 Hz supply. If the frequency of rotor emf at full load is 2 Hz, find the full-load slip and speed.

Soln.

$$P = 6, \quad f = 50\text{ Hz}, \quad f_2 = 2\text{ Hz}$$

We know, $f_2 = sf \quad \therefore s = \dfrac{f_2}{f}$

$$\boxed{\therefore s = \frac{2}{50} = 0.04}$$

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\text{ rpm.}$$

$$s = \frac{n_s - n}{n_s}$$

$$\therefore 0.04 = \frac{1000 - n}{1000}$$

$$\boxed{\therefore n = 960\text{ rpm.}}$$

Ex-2: A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1% at no-load and 3~~0~~% at full-load. Find:

(a) synchronous speed (b) no-load speed
(c) full-load speed (d) frequency of rotor current at standstill
(e) Frequency of rotor current at full-load.

Soln.

$P = 6$,

No-load slip, $S_0 = 1\% = 0.01$

Full-load slip, $S_f = 3\% = 0.03$

(a) $\boxed{n_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}}$

(b) No load speed, $n_0 = ?$

We know that,

$$S = \frac{n_s - n}{n_s}$$

OR $$S_0 = \frac{n_s - n_0}{n_s}$$

$$\therefore n_0 = n_s(1 - S_0) = 1000(1 - 0.01)$$

$$\boxed{\therefore n_0 = 990 \text{ rpm}}$$

(c) Full-load speed,

$$n_{fl} = n_s(1 - S_{fl}) = 1000(1 - 0.03)$$

$$\boxed{\therefore n_{fl} = 970 \text{ rpm}}$$

(d) Frequency of rotor current at standstill, $f_2 = ?$

At standstill, slip, $s = 1$

$$\boxed{\therefore f_2' = sf = 1 \times 50 = 50\,Hz}$$

(e) Frequency of rotor current at full-load,

$$\boxed{f_2 = s_{fl} f = 0.03 \times 50 = 1.5\,Hz}$$

EX-3: A six pole, 50 Hz, 3-phase induction motor running on full load develops a useful torque of 160 Nm when the rotor emf makes 120 complete cycles per minute. Calculate the shaft power output. If the mechanical torque lost in friction and that for core-loss is 10 Nm, compute

(a) the copper-loss in the rotor windings

(b) the input to the motor

(c) the efficiency

The stator loss is given to be 800 Watt.

Soln.

$$f_2 = sf = \frac{120}{60} = 2\,Hz.$$

$$\therefore s = \frac{2}{f} = \frac{2}{50} = 0.04 = 4\%$$

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{6} \quad [\because P = 6, f = 50Hz]$$

$$\therefore n_s = 1000 \text{ rpm.}$$

Rotor speed,

$$n = (1-s)n_s = (1-0.04) \times 1000$$

$$\therefore n = 960 \text{ rpm.}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 960}{60}$$

$$\therefore \omega = 100.53 \text{ rad/sec.}$$

Shaft power output = Useful torque * rotor speed

$$= 160 * 100.53$$

$$= \underline{\underline{16.085 \text{ KW}}}$$

Mechanical power developed,

$$P_m = (\text{Useful torque} + \text{Losses}) * \text{rotor speed}$$

$$\therefore P_m = (160 + 10) * 100.53$$

$$\therefore P_m = 17.09 \text{ KW.}$$

(a)

$$P_m = 3(I_2')^2 r_2' \left(\frac{1}{s} - 1\right)$$

$$\therefore \text{Rotor-cu-loss} = 3(I_2')^2 r_2' = \frac{sP_m}{1-s}$$

$\therefore$ Rotor-cu-loss $= 17.09 * \dfrac{0.04}{(1-0.04)}$

$= 712$ W.

$= 0.712$ KW

(b) Input to the motor

$= P_m +$ rotor-cu-loss $+$ total stator loss

$= (17.09 + 0.712 + 0.80)$

$= 18.602$ KW.

(c) Efficiency, $\eta = \dfrac{\text{output}}{\text{Input}}$

$\therefore \eta = \dfrac{16.085}{18.602}$

$\therefore \eta = 86.47\%$

EX-4: A squirrel-cage induction motor has a slip of 4% at full load. Its starting current is five times the full-load current. The stator impedance and magnetising current may be neglected.

(a) Calculate the maximum torque and the slip at which it would occur.

(b) calculate the starting torque.

Soln.

(a) $$I_{start}^2 = \frac{V^2}{(r_2')^2 + (x_2')^2} \quad -(1) \quad [\because s = 1]$$

$$I_{fl}^2 = \frac{V^2}{\left(\frac{r_2'}{s_{fl}}\right)^2 + (x_2')^2} \quad -(2)$$

Eqn. (1) ÷ Eqn. (2)

$$\therefore \left(\frac{I_{start}}{I_{fl}}\right)^2 = \frac{\left(\frac{r_2'}{s_{fl}}\right)^2 + (x_2')^2}{(r_2')^2 + (x_2')^2}$$

$$\therefore \left(\frac{I_{start}}{I_{fl}}\right)^2 = \frac{\left(s_{max,T}\, x_2'/s_{fl}\right)^2 + (x_2')^2}{(s_{max,T}\, x_2')^2 + (x_2')^2}$$ $[\because r_2' = s_{max,T} \cdot x_2']$

$$\therefore \left(\frac{I_{start}}{I_{fl}}\right)^2 = \frac{(s_{max,T}^2 + s_{fl}^2)}{s_{fl}^2\,(s_{max,T}^2 + 1)} \quad -(3)$$

Given data

$$I_{start} = 5\, I_{fl} \qquad \therefore \left(\frac{I_{start}}{I_{fl}}\right)^2 = 25$$

$$s_{fl} = 0.04$$

$$\therefore 25 = \frac{(s_{max,T}^2 + (0.04)^2)}{(0.04)^2\,(s_{max,T}^2 + 1)}$$

$$\therefore S_{max,T} = 0.20$$

We Know,

$$T_{max} = \frac{3}{\omega_s} \cdot \frac{0.5 V^2}{(x_2')} \quad \text{- - - (4)}$$

[see Eqn. (37)]

$$T_{fl} = \frac{3}{\omega_s} \cdot \frac{V^2 \left(\frac{r_2'}{s_{fl}}\right)}{\left[\left(\frac{r_2'}{s_{fl}}\right)^2 + (x_2')^2\right]} \quad \text{- (5)}$$

[See Eqn (35)]

Eqn. (4) ÷ Eqn. (5)

$$\frac{T_{max}}{T_{fl}} = 0.5 * \frac{\left[(r_2')^2 + s_{fl}^2 (x_2')^2\right]}{r_2' x_2' s_{fl}}$$

$$\therefore \frac{T_{max}}{T_{fl}} = 0.5 * \frac{\left[S_{max,T}^2 + s_{fl}^2\right]}{S_{max,T} \, s_{fl}}$$

$[\therefore r_2' = S_{max,T} \cdot x_2']$

$$\therefore \frac{T_{max}}{T_{fl}} = 0.5 * \frac{\left[(0.2)^2 + (0.04)^2\right]}{0.2 * 0.04} = 2.6$$

$$\therefore T_{max} = 2.6 * T_{fl}$$

$\therefore$ Maximum torque = 2.6 times full-load torque.

From Eqn. (47),

$$\frac{T_{start}}{T_{fl}} = \left(\frac{I_{start}}{I_{fl}}\right)^2 s_{fl}.$$

$$\therefore \frac{T_{start}}{T_{fl}} = (5)^2 * 0.04$$

$$\therefore T_s = T_{fl}$$

∴ Starting torque = full-load torque.

EX-5:

The power input to a three-phase induction motor is 60KW. The stator losses total 1KW. Find the total mechanical power developed and the rotor copper losses per phase if the motor is running with a slip of 3%.

Soln.

Rotor input = stator input − stator losses

$$\therefore P_G = 60 - 1 = 59 \text{ KW}.$$

slip $s = 0.03$

Total mechanical power developed

$$P_m = (1-s) P_G = (1 - 0.03) * 59 = 57.2 \text{ KW}.$$

Rotor copper-loss $= sP_G = 0.03 \times 59$

$= \cancel{1.17\,\text{KW}}$

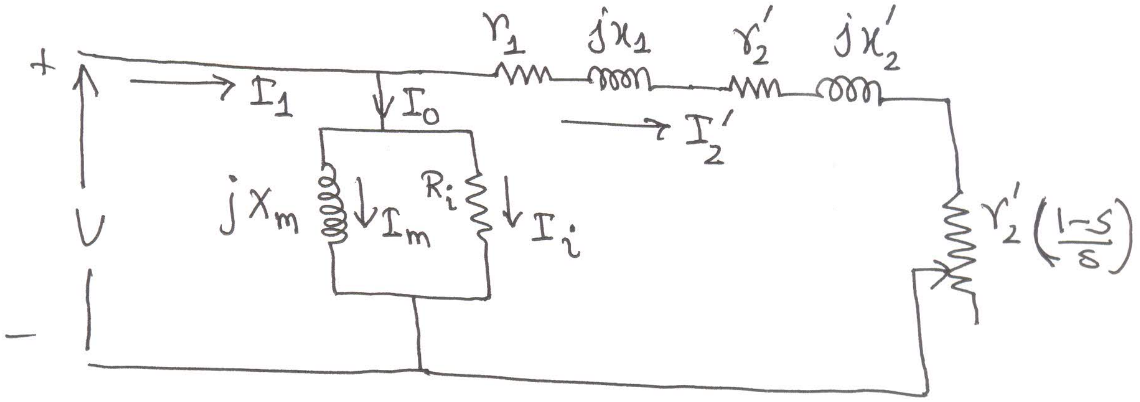
$= \cancel{1170\,\text{kW}}\ 1770\,\text{W}$

Rotor copper-loss per phase $= \dfrac{\cancel{1170}\ 1770}{3} = 590$ Watt.

Ex-6:

A 500 Volt, three-phase induction motor has a stator impedance of $(0.062 + j0.21)\,\Omega$. The equivalent rotor impedance at stand still is the same. The magnetizing current is 36 Amp, the core loss is 1500 Watt, the mechanical loss is 750 Watt. Estimate the output, efficiency and power factor at a slip of 2%.

Soln.



The phase voltage $= \frac{500}{\sqrt{3}} = 288.7$ volt.

$\therefore V = 288.7\angle 0^\circ$ volt.

slip $s = 0.02$,

$\therefore$ ~~Total~~ Given that $r_1 = r_2' = 0.062\Omega$

$x_1 = x_2' = 0.21\Omega$

$\therefore$ Total impedance

$$Z = (0.062 + j0.21) + (0.062 + j0.21) + 0.062\left(\frac{1-s}{s}\right)$$

$$= (0.124 + j0.42) + 0.062\left(\frac{1-0.02}{0.02}\right)$$

$$= 3.19\angle 7.56^\circ\ \Omega,$$

$$\therefore I_2' = \frac{V}{Z} = \frac{288.7\angle 0^\circ}{3.19\angle 7.56^\circ} = 90.5\angle -7.56^\circ$$

$$\boxed{\therefore I_2' = (89.66 - j11.9)\ \text{Amp}}$$

$$I_m = \frac{V}{jX_m} = \left(\frac{V}{X_m}\right)\angle -90^\circ$$

$$\boxed{\therefore I_m = -j36\ \text{Amp}}$$

Total core loss = 1500 Watt.

$\therefore$ Core-loss/phase = 500 Watt.

$\therefore \quad V \times I_i = 500$

$\therefore \quad I_i = \frac{500}{V} = \frac{500}{288.7}$

$$\boxed{\therefore \quad I_i = 1.73 \text{ Amp}}$$

$\therefore \quad I_0 = I_i + I_{m\phi}$

$$\boxed{\therefore \quad I_0 = (1.73 - j36) \text{ Amp}}$$

$\therefore \quad I_1 = I_0 + I_2' = (1.73 - j36 + 89.66 - j11.9)$

$\therefore \quad I_1 = (91.39 - j47.9)$

$$\boxed{\therefore \quad I_1 = 103.2 \angle -27.7^\circ \text{ Amp}}$$

$$\boxed{\therefore \text{ Power factor} = \cos(27.7^\circ) = 0.89}$$

Rotor cu-loss $= 3(I_2')^2 r_2' = 3(90.5)^2 * 0.062$ Watt

$= 1.52$ KW

Load resistance $= r_2'\left(\frac{1}{s}-1\right)$

$= r_2'\left(\frac{1-s}{s}\right)$

∴ Total mechanical power output

$= 3(I_2')^2 r_2'\left(\frac{1-s}{s}\right)$

$=$ rotor cu-loss $*\left(\frac{1-s}{s}\right)$

$= 1.52 * \left(\frac{1-0.02}{0.02}\right)$

$= 74.55\,kW$

Mechanical Loss $= 750W = 0.75\,kW$.

$\boxed{\text{Net output} = (74.55 - 0.75) = 73.75\,kW.}$

Input $= 3V I_1 \cos\phi = 3 * \frac{500}{\sqrt{3}} * 103.2 * 0.89$ Watt

$= \sqrt{3} * 500 * 103.2 * 0.89 \times 10^{-3}\,kW$

$$\boxed{\therefore \eta = \frac{73.75}{\sqrt{3} * 500 * 103.2 * 0.89 \times 10^{-3}} = 0.927}$$

Ex-7:

A 25 hp, 6-pole, 50 Hz, slip-ring induction motor runs at 960 rpm on full-load with a rotor current of 35 Amp. Allowing 250 Watt for the copper loss in the short-circuiting gear and 1000 Watt for mechanical losses, find the resistance per phase of the three phase rotor winding.

Soln.

$$n_s = \frac{120f}{p} = \frac{120 * 50}{6} = 1000 \text{ rpm.}$$

$$\therefore \text{slip} \quad s = \frac{n_s - n}{n_s} = \frac{1000-960}{1000}$$

$$\therefore s = 0.04$$

Net output $= 25 \text{ hp} = 25 \times 746 = 18.65 \text{ kW}$

Total mechanical output

$$= 18.65 + \left(\frac{250+1000}{1000}\right)$$

$$= 19.9 \text{ kW.}$$

Mechanical output = rotor cu-loss * $\left(\frac{1-s}{s}\right)$

Total rotor cu-loss = $\left(\frac{s}{1-s}\right)$ * mechanical output

$$= \left(\frac{0.04}{1-0.04}\right) * 19.9 \text{ KW}$$

829   .829

$= 812$ watt $= 0.812$ KW.

829.

$\therefore 3(I_2')^2 r_2' = (\cancel{812} - 250)$

$\therefore 3 * (35)^2 r_2' = 562$ 579.

$\boxed{\therefore r_2' = 0.153\ \Omega}$ $0.158\ \Omega$

Ex-8:

The power input to the rotor of a 440 volt, 50 Hz, 6 pole, 3-phase induction motor is 80 KW. The rotor electromotive force is observed to make 100 complete alternations per min. Calculate (a) the slip (b) the rotor speed (c) mechanical power developed (d) the rotor copper loss per phase (e) the rotor resistance per phase if the rotor current is 65 Amp.

Soln.

(a) $s = \frac{f_2}{f} = \frac{(100/60)}{50} = 0.033$

(b) $n_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$

n = rotor speed $= (1-s)n_s$

$\therefore n = (1 - 0.033) * 1000 = 967 \text{ rpm}$

(c) Mechanical power developed,

$P_m = (1-s) P_G$

$P_G = 80 \text{ KW}$

$\therefore P_m = (1 - 0.033) * 80 = 77.36 \text{ KW}$

(d) Rotor cu-loss per phase

$= \frac{1}{3} * s *$ rotor input

$\boxed{\frac{1}{3} s P_G}$

$= \frac{1}{3} * 0.033 * 80 = 0.88 \text{ KW}$

$= 880$ Watt.

(e) $(I_2')^2 r_2' = 880$

$\therefore (65)^2 r_2' = 880$; $\boxed{r_2' = 0.208 \Omega}$

Ex-9:

A 3-phase 500 Volt, 50 Hz, induction motor with 6 poles develops 20 hp at 950 rpm with a power factor of 0.86. Total mechanical losses 1 hp. Calculate for this load.

(a) the slip (b) the rotor copper loss

(c) the input if the stator losses total 1500 W

(d) the line current.

Soln.

(a)

$$n_s = \frac{120f}{P} = \frac{120 * 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{n_s - n}{n_s} = \frac{1000 - 950}{1000} = 0.05$$

(b) Total mechanical power developed,

$$P_m = (20+1)\text{ hp} = 21 * 0.746 = 15.666 \text{ kW}$$

$$P_G = \text{rotor input} = \frac{P_m}{1-s} = \frac{15.666}{1-0.05} = 16.5 \text{ kW}$$

Rotor cu-loss $= sP_G = 0.05 * 16.5 = 0.825$ kW

Stator input $= P_G +$ stator losses $= 16.5 + 1.5 = 18$ kW

$\therefore \sqrt{3}\, V I_1 \cos\phi = 18 \times 1000$,

$$\therefore I_1 = \frac{18 \times 1000}{\sqrt{3} * 500 * 0.86} = 24 \text{ Amp.}$$

Ex-10:

An 8-pole, 50 Hz, 3-phase induction motor has an equivalent rotor resistance of 0.07 Ω/phase. If its stalling speed is 630 rpm, how much resistance must be included per phase to obtain maximum torque at starting? Ignore magnetizing current.

Soln.

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm.}$$

$$S_{max,T} = S = \frac{n_s - n}{n_s} = \frac{750 - 630}{750} = 0.16$$

Since the torque is maximum (stalling)

$$\frac{r_2'}{x_2'} = S = 0.16 = S_{max,T}$$

$$\therefore x_2' = \frac{r_2'}{0.16} = \frac{0.07}{0.16} = 0.44 \, \Omega$$

At start, $S = 1$,

$$\therefore r_2' = x_2' = 0.44 \, \Omega$$

Resistance to be added $= (0.44 - 0.07) \, \Omega$

$= 0.37 \, \Omega$.

P-15

Ex-11: An induction motor has an efficiency of 0.90 when the load is 50 hp. At this load, the stator copper loss and rotor copper loss each equal to iron loss. The mechanical losses are one-third of the no-load losses. Calculate the slip.

Soln.

The mechanical losses cannot be one third of the no-load losses. This ~~should~~ should read one-third of the losses on load.

Let

x = stator copper loss = rotor copper loss = iron loss.

y = mechanical losses, then

Total losses on load $= (3x+y)$

It is given

$$y = \frac{3x+y}{3}$$

$$\therefore y = 1.5x$$

Total losses on load $= 3x+y = 3x+1.5x = 4.5x$

Input $= \frac{50 \times 746}{0.9} = 41444$ Watt.

Total Losses on load $= 0.1 * 41444$

$= 4144.4$ watt

$\therefore 4.5x = 4144.4$

$\therefore x = 921$ Watt.

Mechanical losses $= 921 * 1.5 = 1381$ Watt.

Total mechanical power developed

$= 50 * 746 + 1381$

$P_m = 38681$ Watt.

The rotor copper losses $= 921$ watt

$P_G =$ Rotor input $= \frac{\text{rotor cu loss}}{s} = \frac{921}{s}$

$P_m = (1-s) P_g = (1-s) \cdot \frac{921}{s}$

$\therefore 38681 = \left(\frac{1-s}{s}\right) * 921$

$\boxed{\therefore s = 0.023 \text{ Ans}}$