Answer sheet -10 **AUTUMN 2019**

MATHEMATICS-I (MA10001)(Complex Analysis)

1. (a) Ans: $\frac{3}{2}$
   (b) Ans: Does not exist. Put $z = x + iy$ and choose the path $y = mx$.
   (c) Ans: $\frac{a^3}{c^3}$
   (d) Ans: $-\frac{2}{5}(4 - 3i)$

2. (a) Hint: Continuous. $|\mathrm{Re}(z)| \leq |z|$ and use the definition of continuity.
   (b) Hint: Continuous. Use the polar form of z and definition of continuity.
   (c) Hint: NOT continuous. Use the polar form of z and definition of continuity.

3. (a) Hint: Nowhere differentiable. Use the definition of differentiability and choose two different path to get different value of $f'(z)$.
   (b) Hint: Nowhere differentiable. Use C-R equations.
   (c) Hint: Nowhere differentiable. Use C-R equations.

4. (a) $|\mathrm{Re}(z)| \leq |z|$ and use the definition of continuity.
   (b) Choose the path $y = mx$ to get different value of $f'(0)$.

5. Choose the path $y = mx$ to get different value of $f'(0)$.

6. (a) Use the polar form of z and definition of continuity.
   (b) Choose two different path $y = 0$ and $y = x$ to get different value of $f'(0)$.

7. (a) Use C-R equations to get $v(x,y)$ and $v(x,y) = 2y - 3x^2y + y^3 + C$.
   (b) Use C-R equations to get $v(x,y)$ and $v(x,y) = \tan^{-1}\frac{y}{x} + C$.
   (c) Use C-R equations to get $v(x,y)$ and $v(x,y) = \frac{x}{x^2+y^2} + C$.
   (d) Use C-R equations to get $v(x,y)$ and $v(x,y) = -\cos y + \cosh x + C$.
   (e) Use C-R equations to get $v(x,y)$ and $v(x,y) = e^{-x}(y \sin y + x \cos y) + C$.

8. (a) Write $f(z)$ interms of $u(x,y) + iv(x,y)$ and apply C-R equations.
   (b) Write $f(z)$ interms of $u(x,y) + iv(x,y)$ and apply C-R equations.

9. (a) $f(z) - f(z_0) = \dfrac{f(z) - f(z_0)}{z - z_0}(z - z_0)$ and use the definition of continuity.
   (b) Hint: $f(z) = |z|^2$.

10. Use C-R equations to show u and v are costant.

11. Try to show $g(z)$ satisfies the C-R equations.

12. $x = \frac{z+\bar{z}}{2}$ and $y = \frac{z-\bar{z}}{2i}$. Find partial derivative of f with respect to z and $\bar{z}$

13. Ans: Use C-R equations to find v and $f(z) = iz^4 + C$ where C is an arbitrary constant.

14. Ans: Use C-R equations to find u and v and $f(z) = e^z + \alpha$ where α is an arbitrary constant.

15. (a) Use C-R equations and $u^2 + v^2 = k$ to show $u_x = u_y = 0$ and $v_x = v_y = o$.
    (b) Find partial derivative of f with respect to $\bar{z}$ in terms of partial derivative of f with respect to x and y.
    (c) Use C-R equations to show $u_x = u_y = 0$ and $v_x = v_y = o$.
    (d) If $f(z)$ is analytic , try to show $\overline{f(\overline{z})}$ satisfies C-R equations. Similarly for the reverse direction.