GENERATION OF ALTERNATING EMF

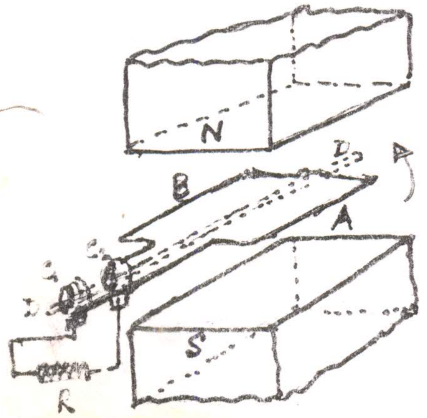


Fig. 1.

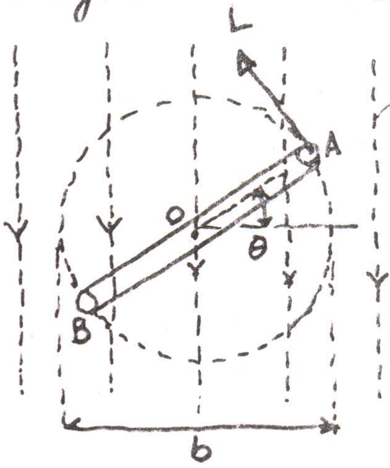


Fig. 2(a)

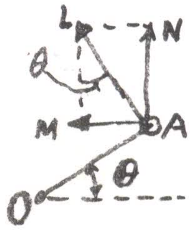


Fig. 2(b)

EMF GENERATED IN ONE SIDE OF LOOP $= B l v \sin\theta$ volts.

TOTAL EMF GENERATED IN LOOP $= 2 B l v \sin\theta$ volts.

where $v =$ tangential velocity of the conductor

$$= \frac{b}{2} \cdot n \cdot 2\pi = \pi b n \text{ metres per second.}$$

$n =$ Speed in r p s (revolutions per second)

$b =$ Breadth of the loop in metre

$l =$ Length of one side of the loop in metre

$B =$ Flux density in Tesla

Taking $A = l b =$ area of the loop in square metres

EMF GENERATED IN THE LOOP $= e = 2\pi B A n \sin\theta$ volts.

IF THE LOOP IS REPLACED BY A COIL OF N TURNS IN SERIES,

THEN EMF GENERATED IN THE COIL $= 2\pi B A N n \sin\theta$ volts.

MAXIMUM VALUE OF THE EMF GENERATED $= E_m = 2\pi B A n N$

AND INSTANTENEOUS VALUE OF EMF GENERATED IN THE COIL

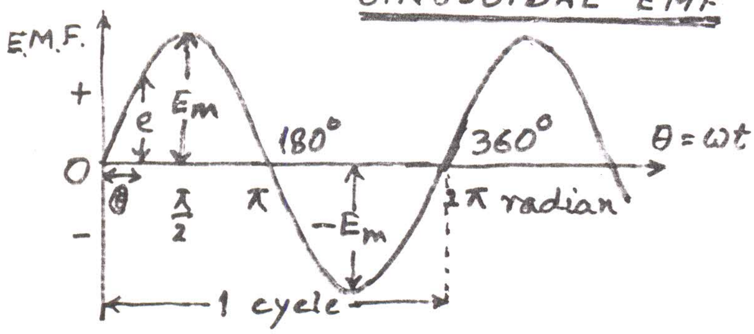
$$e = E_m \sin\theta = 2\pi B A N n \sin\theta \text{ volts.}$$

Where $\theta =$ angular displacement in time t sec.

$$= \omega t = 2\pi n t$$

ω is angular velocity in rad./sec.

SINUSOIDAL EMF



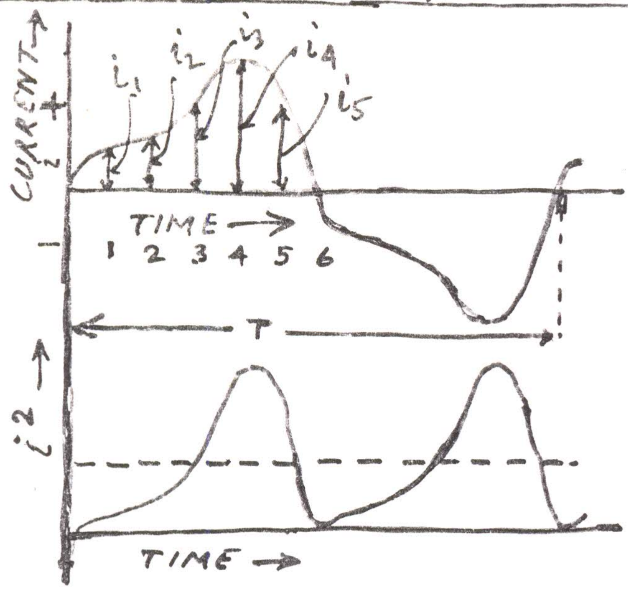
If an a.c. generator has 'p' pairs of poles & its speed is n rps, then FREQUENCY

f = No. of cycles per second
= No. of cycles per rev. × No. of rev. per second
= pn Hz.

Ex. For a 4-pole Generator to generate a voltage of 50 Hz frequency the speed $n = \frac{f}{p} = \frac{50}{2} = 25$ rps $= 25 \times 60$ rpm $= 1500$ rpm where rpm ⇒ revolutions per minute.

For 2-polar machine the speed should be 3000 rpm.

AVERAGE (MEAN,) & RMS (EFFECTIVE) VALUES OF AN ALTERNATING CURRENT.



* AVERAGE VALUE OVER A HALF CYCLE

$$I_{av} = \frac{i_1 + i_2 + i_3 + \cdots + i_n}{n}$$

$$= \frac{\text{AREA OVER HALF CYCLE}}{\text{LENGTH OF BASE OF HALF CYCLE}}$$

$$= \frac{1}{T/2}\int_0^{T/2} i\,dt$$

* ROOT MEAN SQUARE (R.M.S.) VALUE DEPENDS ON HEATING EFFECT i.e. SQUARE OF THE CURRENT.

AVERAGE HEATING EFFECT $\propto \frac{i_1^2 + i_2^2 + \cdots i_n^2}{n}$

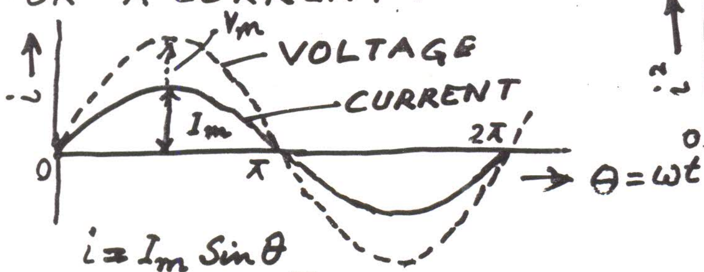
SUPPOSE I BE THE VALUE OF THE DIRECT CURRENT TO PRODUCE SAME HEATING IN THE SAME RESISTOR AS PRODUCED BY ALTERNATING CURRENT; THEN

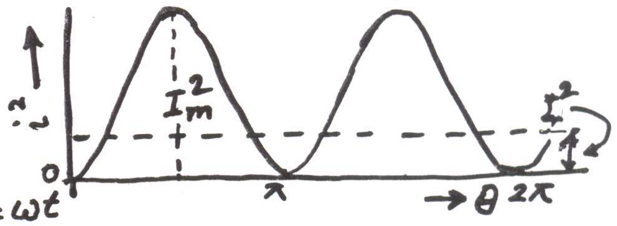
$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}} = \sqrt{\frac{\text{AREA OF } i^2 \text{ CURVE}}{\text{LENGTH OF BASE}}} \text{ OVER A HALF CYCLE.}$$

$$= \sqrt{\frac{1}{T/2}\int_0^{T/2} i^2\,dt}$$

AVERAGE AND RMS VALUES OF A SINUSOIDAL VOTAGE OR A CURRENT:





$$i = I_m \sin\theta$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{1}{\pi} \cdot I_m \left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{2}{\pi} I_m = 0.637 I_m.$$

$$i^2 = I_m^2 \sin^2\theta$$

$$I_{rms} = \left[\frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi}\int_0^{\pi} I_m^2 \frac{1}{2}(1-\cos 2\theta) \, d\theta\right]^{\frac{1}{2}}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707 I_m.$$

Similarly, $V_{av} = 0.637 V_m$ $\quad V_{r.m.s.} = 0.707 V_m.$

Average Value of a Sinusoidal current or voltage
= 0.637 × maximum value.

R.M.S. value of a sinusoidal current or voltage
= 0.707 × maximum value.

$$\text{Form Factor of a sine wave} = \frac{\text{R.M.S. Value}}{\text{Average Value}}$$

$$= \frac{0.707 \times \text{maximum value}}{0.637 \times \text{maximum value}} = \underline{\underline{1.11}}.$$

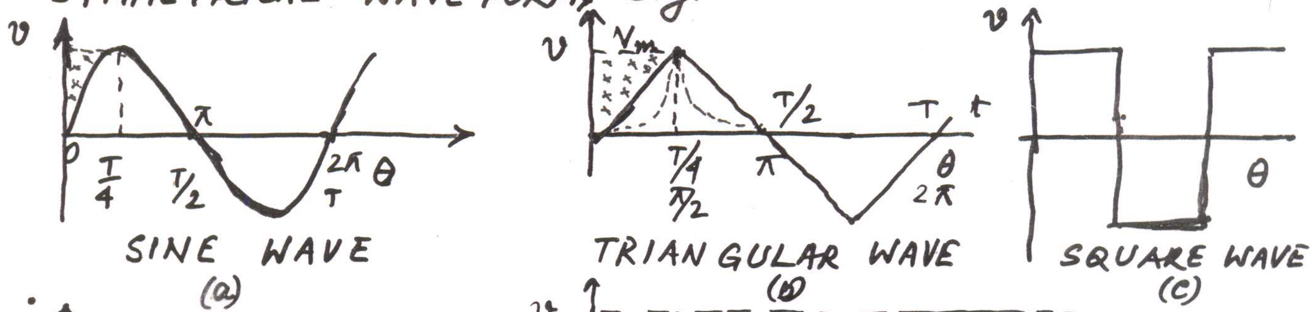
$$\text{Peak or Crest Fact of a sine wave} = \frac{\text{Maximum Value}}{\text{RMS Value}}$$

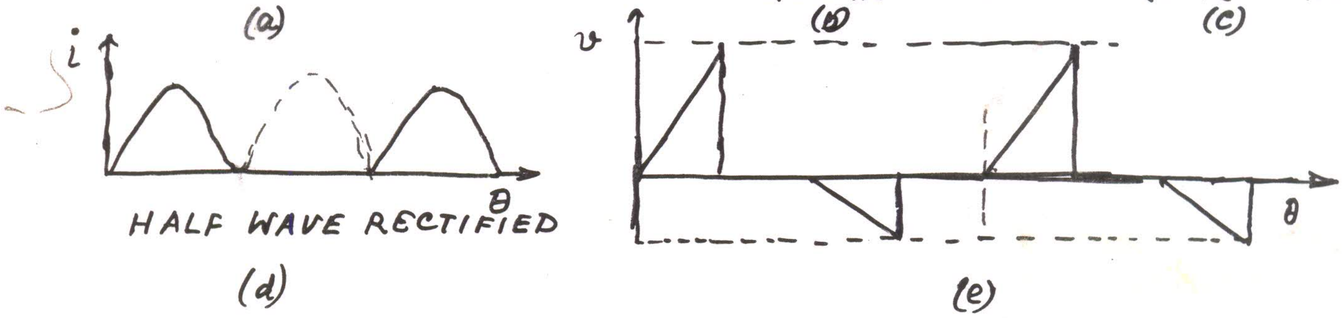
$$= \frac{\text{maximum value}}{0.707 \times \text{maximum value}} = \underline{\underline{1.414}}$$

NOTE: RMS VALUE IS ALWAYS GREATER THAN AVERAGE EXCEPT FOR A RECTANGULAR WAVE, IN WHICH CASE THE HEATING EFFECT REMAINS CONSTANT SO THAT THE AVERAGE & THE RMS VALUES ARE SAME.

SYMMETRICAL AND UNSYMMETRICAL WAVEFORMS:

THE WAVEFORMS IN WHICH +VE AND −VE HALF CYCLES ARE IDENTICAL ARE REFERRED TO AS SYMMETRICAL WAVE FORM, e.g.





THE WAVE FORMS (a), (b) & (c) ARE SYMMETRICAL WHEREAS THOSE OF (d) & (e) ARE NON-SYMMETRICAL.

NOTE: i) IN CASE OF UNSYMMETRICAL WAVEFORMS, THE AVERAGE VALUE MUST ALWAYS BE TAKEN OVER THE WHOLE CYCLE.

$$I_{av} = \frac{1}{T}\int_0^T i\,dt \quad \text{or} \quad V_{av} = \frac{1}{T}\int_0^T v\,dt .$$

IN CASE OF SYMMETRIC ALTERNATING CURRENT OR VOLTAGE THE AVERAGE VALUE OVER A COMPLETE CYCLE IS ZERO. SO THE AVERAGE VALUE IS TAKEN OVER HALF CYCLE.

ii) TO DETERMINE THE RMS VALUE OF AN ALTERNATING QUANTITY, IT IS IMMATERIAL TO TAKE THE AVERAGE OVER HALF CYCLE OR COMPLETE CYCLE.

FOR CASES (a), (b) & (c) WE MAY EVEN CONSIDER A SHORTER INTERVAL T/4 AS IT IS SYMMETRIC ABOUT THE POINT T/4. FOR THE TRIANGULAR WAVE FORM Fig. (b)

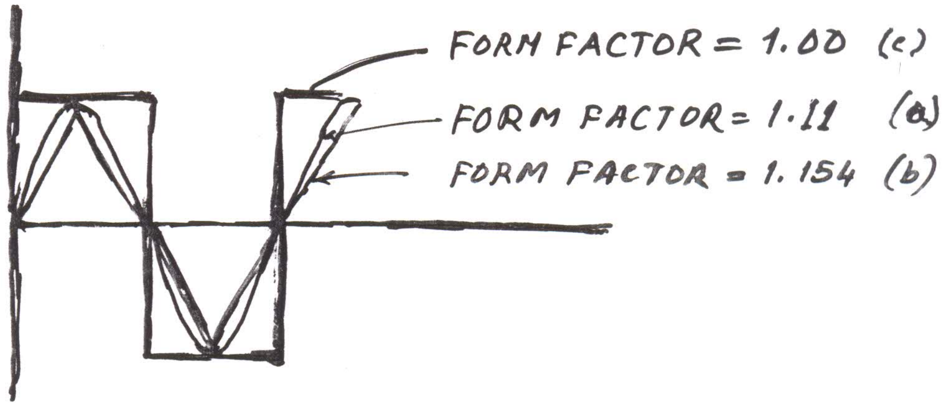
$$V_{av} = \frac{1}{T/4}\int_0^{T/4} \frac{V_m}{T/4}\cdot t\,dt = \left(\frac{4}{T}\right)^2 V_m \left.\frac{t^2}{2}\right|_0^{\frac{T}{4}} = \frac{V_m}{2}.$$

$$V_{rms} = \left[\frac{1}{T/4}\int_0^{T/4} \frac{V_m^2}{(T/4)^2}\, t^2\,dt\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{3}}.$$

FORM FACTOR $= 2/\sqrt{3} = 1.155$

PEAK FACTOR $= \sqrt{3} = 1.732$.

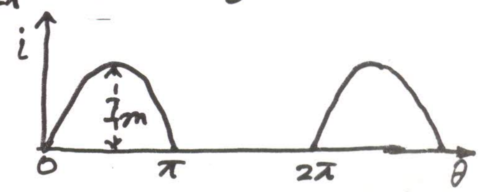
THUS THE FORM FACTORS OF WAVEFORMS SHOWN IN FIGS. (a), (b) & (c) ARE AS FOLLOWS:



FOR UNSYMMETRICAL WAVES SUCH AS IN FIGS. (d) & (e)

$$\text{AVERAGE VALUE} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta = \frac{1}{2\pi} I_m \left[-\cos\theta\right]_0^{\pi}$$

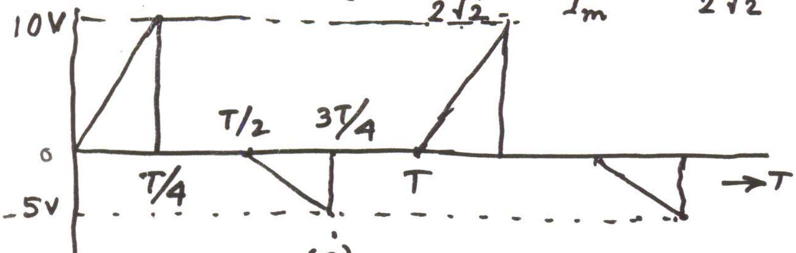
$$= \frac{1}{2\pi}\cdot 2I_m = \frac{1}{\pi}\cdot I_m.$$



(d)

$$\text{RMS VALUE} = \frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta = \frac{I_m}{2\sqrt{2}}.$$

$$\text{FORM FACTOR} = \frac{I_m}{2\sqrt{2}} \times \frac{\pi}{I_m} = \frac{\pi}{2\sqrt{2}} = 1.11.$$



(e)

$$\text{AVERAGE VALUE} = V_{av} = \frac{1}{T}\left[\int_0^{T} v \, dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T/4} \frac{10}{T/4}\cdot t \, dt + \int_0^{T/4}\left(\frac{-5}{T/4}\right) t \, dt\right]$$

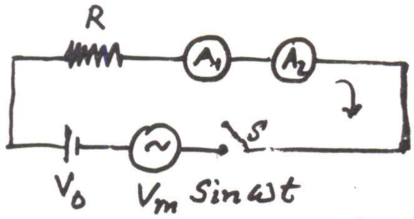
$$= 0.625 \text{ V}.$$

RMS VALUE $= V_{RMS}$

$$= \sqrt{\frac{1}{T}\int_0^{T} v^2 \, dt} = \sqrt{\frac{1}{T}\left[\int_0^{T/4}\left(\frac{10}{T/4}\right)^2 t^2 dt + \int_0^{T/4}\left(\frac{-5}{T/4}\right)^2 t^2 dt\right]}$$

$$= 3.23 \text{V}.$$

Ex. 1.



$v(t) = V_0 + V_m \sin\omega t.$

GIVEN: $V_m = 100\ V$, $V_0 = 120\ V$

$R = 20\ \Omega$.

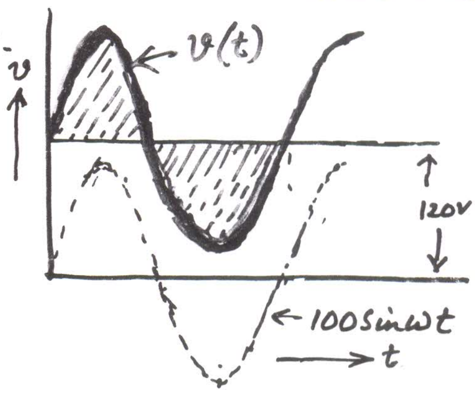
A_1 = Moving coil ammeter

A_2 = Moving iron ammeter.

WHAT WILL BE READINGS OF A_1 & A_2?

THE NET VOLTAGE ACTING IN THE CIRCUIT $= 120 + 100 \sin\omega t$ volts.

SO CURRENT $i(t) = 6 + 5 \sin\omega t$ amps.



$$I_{av} = \frac{1}{T}\int_0^T (6 + 5\sin\omega t)\, dt\ ; \quad \omega = \frac{T}{2\pi} \rightarrow \frac{2\pi}{T}$$

$= 6\ A = I_{dc}.$

= READING OF AMMETER A_1

$$I_{rms} = I = \sqrt{\frac{1}{T}\int_0^T (6 + 5\sin\omega t)^2\, dt}$$

$$= \left[\frac{1}{T}\int_0^T (36 + 60\sin\omega t + 25\sin^2\omega t)\, dt\right. = \sqrt{\left(36 + \frac{25}{2}\right)}$$

$\simeq 7\ A$ = READING OF AMMETER A_2

THUS IF DC IS SUPERIMPOSED ON AC SIGNAL, THE RESULTANT RMS VALUE $= \sqrt{I_{dc}^2 + I_{rms(ac)}^2} = \sqrt{6^2 + \left(\frac{5}{\sqrt{2}}\right)^2} \simeq 7 A.$

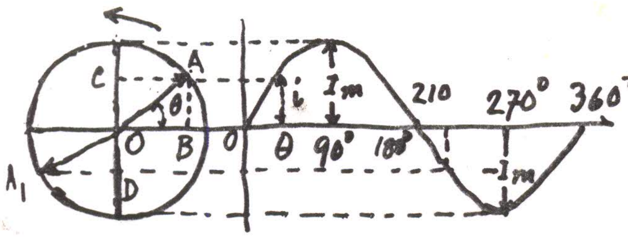
NOTE: i) MOVING COIL INSTRUMENT READS AVERAGE VALUE OVER A PERIOD. THUS FOR AC ALONE IT WILL READ ZERO.

ii) MOVING IRON, HOT WIRE, ELECTRODYNAMIC & INDUCTION TYPE INSTRUMENTS READ RMS VALUE OF THE SIGNAL.

IMPORTANT TO REMEMBER:

WHEN AN A.C. SUPPLY OF 220 V IS REFERRED, IT IS MEANT ITS RMS VALUE. THE MAXIMUM VALUE IS $220\sqrt{2} \simeq 310$ V. IN CASE OF DC 220V IT IS 220 V ONLY. THUS SHOCK LEVEL OF AC FOR SAME SPECIFIED VOLTAGE IS MORE THAN THAT OF DC VOLTAGE.

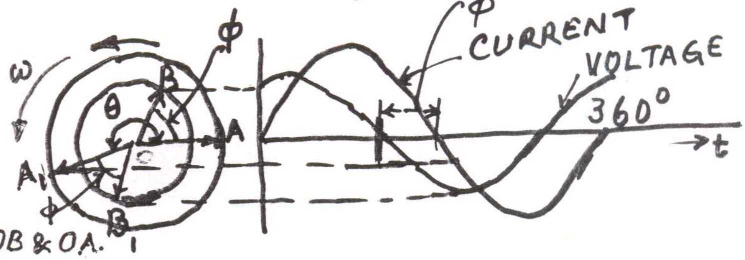
REPRESENTATION OF AN SINUSOIDAL SIGNAL BY A PHASOR



LET OA REPRESENT THE MAXIMUM VALUE OF A SINUSOIDAL QUANTITY, SAY, CURRENT, i.e. $OA = I_m$. IT IS BEING ROTATED IN THE ANTI-CLOCKWISE DIRECTION. THE VERTICAL PROJECTION OF OA IS PLOTTED WITH RESPECT TO θ.

IT WILL GENERATE A SINE WAVE. THE VERTICAL PROJECTION OF OA IS $AB = I_m \sin\theta = i$ (instanteneous value). THE ANGLE θ IS GIVEN BY $\theta = \omega t$ WHERE ω IS THE ANGULAR VELOCITY OF THE PHASOR. AGAIN ω IS GIVEN BY $\omega = 2\pi f$ RADIANS PER SECOND. THUS

$$i = I_m \sin\theta = I_m \sin\omega t = I_m \sin 2\pi f t.$$



LET US NOW CONSIDER TWO QUANTITIES SUCH AS VOLTAGE & CURRENT CAN BE REPRESENTED BY OB & OA. HERE IT IS SHOWN THAT THE VOLTAGE PHASOR OB IS <u>LEADING</u> THE CURRENT PHASOR OA. BOTH ARE ROTATING AT THE SAME ANGULAR SPEED $\omega = 2\pi f$ RADIAN PER SECOND. THUS WE CAN WRITE TAKING $OB = V_m$.

$$v = V_m \sin(\theta + \phi) = V_m \sin(\omega t + \phi) = V_m \sin(2\pi f t + \phi).$$

ϕ is KNOWN AS THE PHASE DIFFERECE BETWEEN VOLTAGE AND CURRENT. TAKING VOLTAGE AS THE REFERENCE PHASOR ONE CAN WRITE ALSO AS

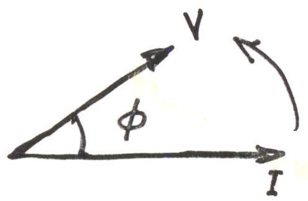
$$v = V_m \sin\theta = V_m \sin\omega t = V_m \sin 2\pi f t$$

$$i = I_m \sin(\theta - \phi) = I_m \sin(\omega t - \phi) = I_m \sin(2\pi f t - \phi)$$

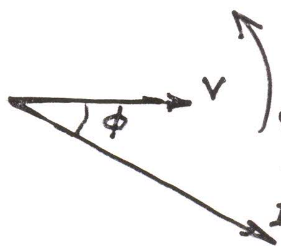
AND CAN STATE THAT THE CURRENT IS <u>LAGGING</u> BEHIND THE VOLTAGE BY AN ANGLE ϕ.

THE PHASOR DIAGRAMS ARE DRAWN WITH THE <u>RMS VALUES</u> OF SIGNALS.

THE PHASOR DIAGRAM CAN BE DRAWN AS EITHER IN(a) OR IN(b).

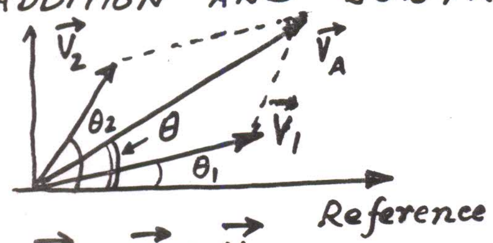


CURRENT AS REFERENCE PHASOR

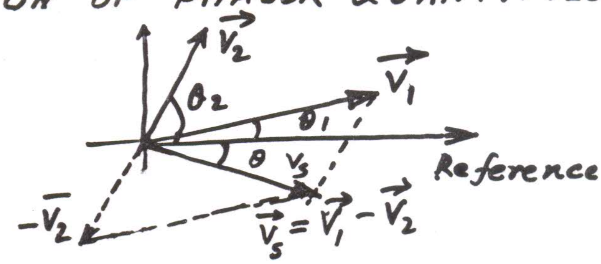


VOLTAGE AS REFERENCE PHASOR

ADDITION AND SUBTRACTION OF PHASOR QUANTITIES:



$\vec{V_A} = \vec{V_1} + \vec{V_2}$



PHASORS AS COMPLEX NUMBERS.

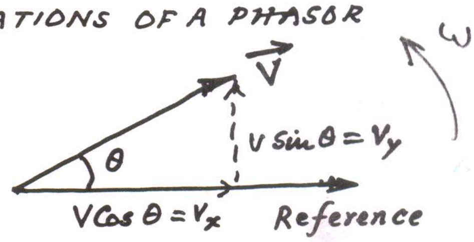
THERE ARE THREE EQUIVALENT NOTATIONS OF A PHASOR

POLAR FORM : $\vec{V} = V \angle\theta$

RECTANGULAR FORM: $\vec{V} = V\cos\theta + jV\sin\theta$

EXPONENTIAL FORM: $\vec{V} = Ve^{j\theta}$

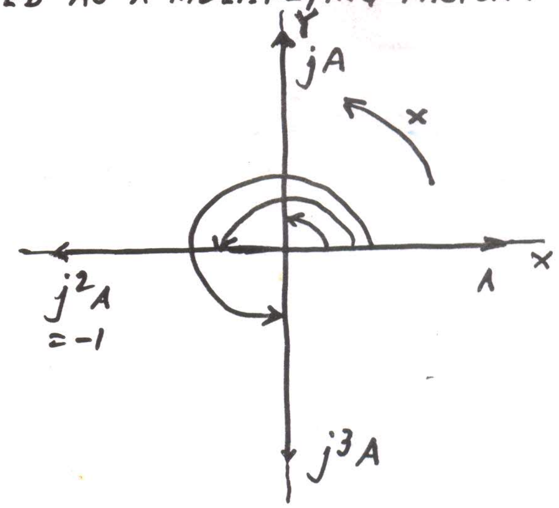
(NOTE: $e^{j\theta} = \cos\theta + j\sin\theta = 1\angle\theta$)



THE OPERATOR j PRODUCES 90° COUNTER CLOCKWISE ROTATION OF ANY PHASOR TO WHICH IT IS APPLIED AS A MULTIPLYING FACTOR.

HENCE $j^2 = -1$ OR $\boxed{j = \sqrt{-1}}$.

SO $\vec{V} = V_x + jV_y$ ALSO.



THE RECTANGULAR FORM OF EXPRESSING A PHASOR IS CONVENIENT FOR ADDITION OR SUBTRACTION OF PHASORS. THE POLAR FORM IS A CONVENIENT WAY TO DISPLAY THE TWO PARTS OF A PHASOR, NAMELY ITS MAGNITUDE AND PHASE ANGLE.

FOR EXPRESSING THE PRODUCT OR QUOTIENT OF TWO PHASORS THE EXPONENTIAL FORM IS CONVENIENT. IF

$\vec{V_1} = V_1 e^{j\theta_1}$ THEN $\vec{V_1}\vec{V_2} = V_1 V_2 e^{j(\theta_1+\theta_2)} = V_1 V_2 \angle\theta_1+\theta_2$.

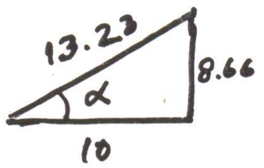
$\vec{V_2} = V_2 e^{j\theta_2}$ & $\dfrac{\vec{V_1}}{\vec{V_2}} = \dfrac{V_1}{V_2} e^{j(\theta_1-\theta_2)} = \dfrac{V_1}{V_2} \angle\theta_1-\theta_2$.

CAUTION : IN AC WORK ALL OPERATIONS OF VOLTAGE AND CURRENT SHOULD BE DONE KEEPING IN MIND THAT THOSE ARE PHASOR QUANTITIES. ALGEBRAIC MANIPULATIONS ARE POSSIBLE ONLY AFTER EXPRESSING THEM AS COMPLEX QUANTITIES.

Ex-2 Add the following currents as waves and as phasors:

$$i_1 = 5\sin\omega t \; ; \quad i_2 = 10\sin(\omega t + 60^\circ)$$

As Waves: $i = i_1 + i_2 = 5\sin\omega t + 10\sin(\omega t + 60^\circ)$



$$= 5\sin\omega t + 10\sin\omega t\cos 60^\circ + 10\cos\omega t\sin 60^\circ$$

$$= 10\sin\omega t + 8.66\cos\omega t.$$

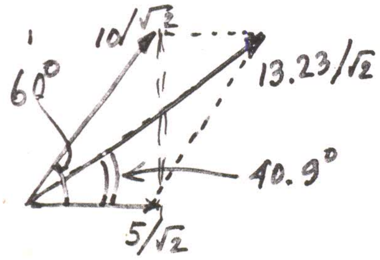
$$= \left[\frac{10}{13.23}\sin\omega t + \frac{8.66}{13.23}\cos\omega t\right] \times 13.23$$

$\underline{\alpha = 40.9^\circ}$

$$= 13.23\left[\sin\omega t\cos\alpha + \cos\omega t\sin\alpha\right]$$

$$= 13.23\sin(\omega t + \alpha) = 13.23\sin(\omega t + 40.9^\circ).$$

AS PHASORS:



$$\Sigma x = \tfrac{1}{\sqrt{2}}(5 + 10\cos 60^\circ) = \tfrac{1}{\sqrt{2}}10$$

$$\Sigma y = \tfrac{1}{\sqrt{2}}(0 + 10\sin 60^\circ) = \tfrac{1}{\sqrt{2}}8.66$$

$$\text{SUM} = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \tfrac{1}{\sqrt{2}}\sqrt{10^2 + (8.66)^2} = 13.23\tfrac{1}{\sqrt{2}}$$

$$\alpha = \tan^{-1}\frac{\Sigma y}{\Sigma x} = \tan^{-1}\frac{\frac{1}{\sqrt{2}}8.66}{\frac{1}{\sqrt{2}}\cdot 10} = 40.9^\circ.$$

SO $\quad i = 13.23\sin(\omega t + 40.9^\circ)$

WORK OUT THE SUBTRACTION OF $(i_1 - i_2)$ BY BOTH METHODS

Ans: $8.66\sin(\omega t - 90^\circ)$.

EXPRESSING THE CURRENTS IN RECTANGULAR & POLAR FORMS:

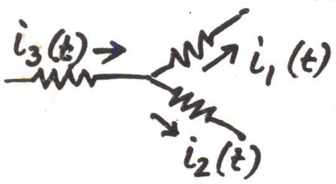
$$\vec{I}_1 = \frac{5}{\sqrt{2}}(1 + j0) = \frac{5}{\sqrt{2}}\angle 0^\circ$$

$$\vec{I}_2 \quad \frac{10}{\sqrt{2}}(\cos 60^\circ + j\sin 60^\circ) = \frac{10}{\sqrt{2}}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{2}}\angle 60^\circ.$$

MULTIPLICATION OF $\vec{I}_1\vec{I}_2 = \frac{5}{\sqrt{2}}\angle 0^\circ \cdot \frac{10}{\sqrt{2}}\angle 60^\circ = 25\angle 60^\circ$.

QUOTIENT OF $\vec{I}_1/\vec{I}_2 = \frac{5}{\sqrt{2}}\angle 0^\circ \Big/ \frac{10}{\sqrt{2}}\angle 60^\circ = 0.5\angle -60^\circ$

Ex.



GIVEN: $i_1(t) = 71\cos\omega t$

$i_2(t) = 100\sin\left(\omega t - \frac{\pi}{4}\right)$

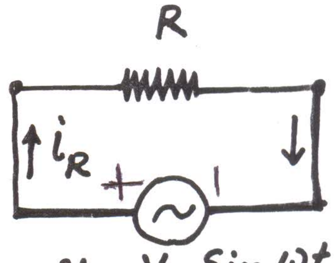
FIND $i_3(t)$.

SOLVE IT BY PHASOR DIAGRAM. Ans: $\underline{50\sqrt{2}\sin\omega t}$

STEADY STATE RESPONSE OF R-L-C CIRCUITS TO SINUSOIDAL INPUTS.

ELEMENTARY CIRCUITS:

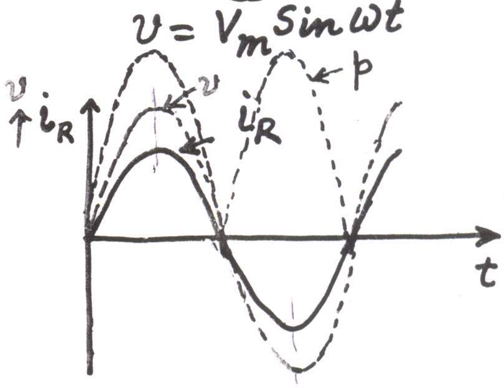
i) PURELY RESISTIVE CIRCUIT



THE INSTANTANEOUS CURRENT THROUGH THE CIRCUIT

$$i_R = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

I_m IS THE MAXIMUM VALUE OF THE CURRENT GIVEN BY $I_m = \frac{V_m}{R}$.



RMS VALUE OF THE CURRENT

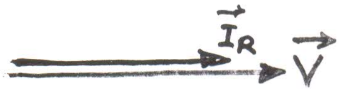
$$\vec{I}_R = \frac{\text{RMS VALUE OF VOLTAGE}}{\text{RESISTANCE}} = \frac{V}{R}$$

$$= \frac{V_m/\sqrt{2}}{R} = \frac{I_m}{\sqrt{2}}$$

IN PHASOR NOTATIONS

$$\vec{V} = V\angle 0^\circ = V(1+j0) = V+j0$$

$$\vec{I}_R = I_R \angle 0^\circ = I_R(1+j0) = I_R + j0$$



CURRENT WILL BE IN PHASE WITH VOLTAGE.

THE VOLTAGE AND CURRENT IN AC CIRCUIT IS RELATED BY IMPEDANCE FUNCTION. THE IMPEDANCE FUNCTION MUST TELL TWO IMPORTANT FACTS:

a) THE RATIO OF V_m to I_m (OR $\vec{V}$ to $\vec{I}$) AND

b) THE PHASE ANGLE BETWEEN THE WAVES OF VOLTAGE AND CURRENT (PHASORS $\vec{V}$ AND $\vec{I}$).

A SPECIAL TYPE OF NOTATION IS REQUIRED TO SIGNIFY THE TWO PROPERTIES OF IMPEDANCE FUNCTION e.g. $Z\angle\text{ANGLE}$. Z SIGNIFIES THE MAGNITUDE OF IMPEDANCE AND ANGLE GIVES THE PHASE RELATION OF VOLTAGE & CURRENT.

THE IMPEDANCE FOR A PURE RESISTANCE IS $\vec{Z}_R = R\angle 0^\circ$.

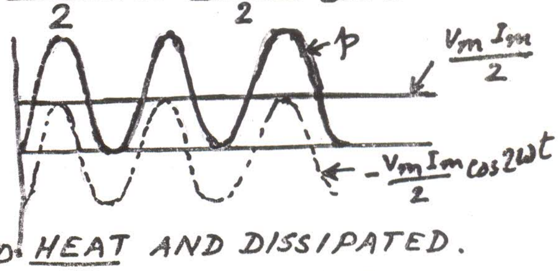
POWER: INSTANTENEOUS POWER $p = vi = V_m I_m \sin^2 \omega t$

Since $\sin^2 \omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$, $p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$

SO AVERAGE POWER $P = \frac{1}{T}\int_0^T p\, dt$

$$P = \frac{1}{T}\int_0^T \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right) dt$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V \cdot I$$



ENERGY PRODUCED IS CONVERTED INTO HEAT AND DISSIPATED.

ii) PURELY INDUCTIVE CIRCUIT

Since $v = L\dfrac{di_L}{dt} = V_m \sin\omega t$

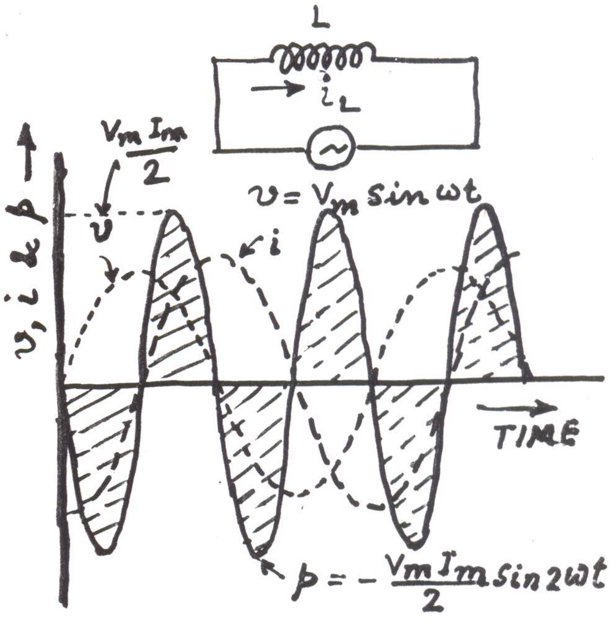
$$di_L = \frac{V_m}{L}\sin\omega t\, dt$$

$$i_L = -\frac{V_m}{\omega L}\cos\omega t$$

(Constant of integration is zero in the steady state solution as it is symmetrical about x-axis)

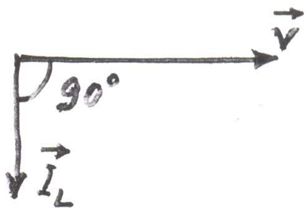
$$i_L = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ).$$



$$I_m = \frac{V_m}{\omega L} \Rightarrow \vec{I}_L = \frac{\vec{V}}{\omega L} = I_L\angle{-90^\circ}$$

$$\vec{V} = V + j0 \qquad \vec{I}_L = 0 - jI_L \qquad \vec{Z}_L = \frac{\vec{V}}{\vec{I}_L} = \omega L\,\angle 90^\circ$$



THE MAGNITUDE OF THE ABOVE IMPEDANCE ωL IS CALLED INDUCTIVE REACTANCE AND REPRESENTED BY $X_L = \omega L = 2\pi f L$.

IN A PURELY INDUCTIVE CIRCUIT CURRENT LAGS BEHIND THE VOLTAGE BY 90°.

POWER & ENERGY :

INSTANTENEOUS POWER $= p = vi = [V_m \sin\omega t][I_m \sin(\omega t - 90^\circ)]$

$$= V_m I_m(-\sin\omega t \cos\omega t).$$

$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

AVERAGE POWER

$$P = \frac{1}{T}\int_0^T p\, dt = 0$$

NOTE :
POWER VARIATION IS A DOUBLE FREQUENCY VARIATION (2ω). THE AVERAGE POWER ABSORBED IS EQUAL TO ZERO. THE IMPLICATION IS THAT THE INDUCTIVE ELEMENT RECEIVES ENERGY FROM THE SOURCE DURING ONE QUARTER OF A CYCLE OF THE APPLIED VOLTAGE AND RETURNS EXACTLY THE SAME AMOUNT ENERGY TO THE DRIVING SOURCE DURING THE NEXT ONE-QUARTER OF A CYCLE.

THE AMOUNT OF ENERGY DELIVERED TO THE CIRCUIT DURING A QUARTER OF A CYCLE

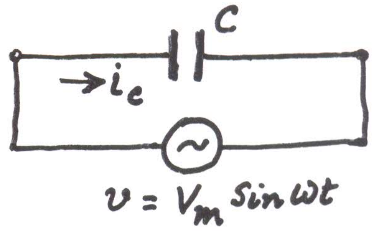
$$W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2}\sin 2\omega t\, dt$$

$$= \frac{V_m I_m}{2\left(\frac{4\pi}{T}\right)}\left[\cos\frac{4\pi}{T}t\right]_{T/4}^{T/2}$$

$$= \frac{V_m I_m}{2\omega} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= \frac{(\omega L I_m) I_m}{2\omega} = \frac{1}{2}L I_m^2$$

iii) PURELY CAPACITIVE CIRCUIT.



Since $v = \frac{q}{C} = V_m \sin\omega t$

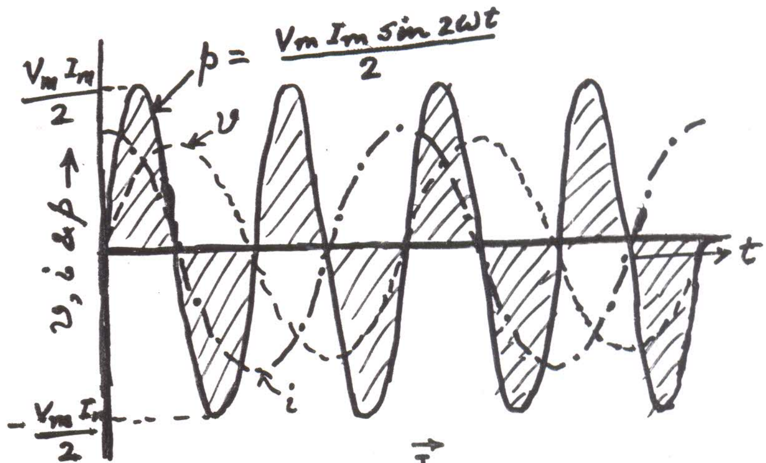
$$i_c = \frac{dq}{dt} = V_m \omega C \cos\omega t$$

$$= \frac{V_m}{(1/\omega C)} \sin(\omega t + 90^\circ)$$

$$= I_m \sin(\omega t + 90^\circ)$$

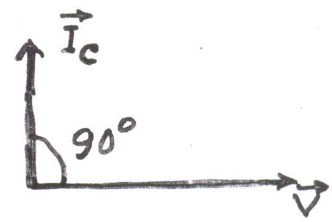
$$I_m = \frac{V_m}{(1/\omega C)}$$

$$I_c = \frac{I_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{V_m}{(1/\omega C)}.$$



$$\vec{I_c} = \frac{\vec{V}}{(1/\omega C)} \angle 90^\circ$$

$$\vec{I_c} = I_c \angle 90^\circ$$



IN PHASOR NOTATIONS

$$\vec{V} = V \angle 0^\circ = V + j0$$

$$\vec{I_c} = I_c \angle 90^\circ = 0 + j I_c$$

$$\bar{Z}_c = \frac{\vec{V}}{\vec{I_c}} = \frac{V}{I_c} \angle -90^\circ = Z_c \angle -90^\circ$$

$$= \frac{1}{\omega C} \angle -90^\circ$$

THE MAGNITUDE OF THE ABOVE IMPEDANCE $\frac{1}{\omega C}$ IS CALLED CAPACITIVE REACTANCE AND REPRESENTED BY $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$.

IN A PURELY CAPACITIVE CIRCUIT THE CURRENT LEADS THE VOLTAGE BY 90°.

POWER & ENERGY:

INSTANTENEOUS POWER

$$p = v\,i = [V_m \sin\omega t][I_m \sin(\omega t + 90^\circ)]$$

$$= V_m I_m \sin\omega t \cos\omega t = \frac{1}{2} V_m I_m \sin 2\omega t.$$

POWER VARIATION IS AGAIN A DOUBLE FREQUENCY VARIATION (2ω) AND HENCE AVERAGE POWER ABSORBED IS EQUAL TO ZERO.

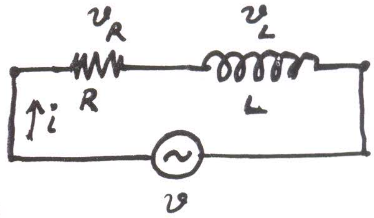
$$P = \frac{1}{T}\int_0^T p\, dt = 0$$

HERE, ALSO, THE CAPACITOR RECEIVES ENERGY FROM THE SOURCE DURING THE FIRST QUARTER OF A CYCLE AND RETURNS THE SAME AMOUNT DURING THE SECOND QUARTER CYCLE, etc.

$$W_c = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t\, dt$$

$$= \frac{1}{2} \frac{V_m I_m}{\omega} = \frac{1}{2} C V_m^2.$$

iv) SERIES R-L CIRCUIT:

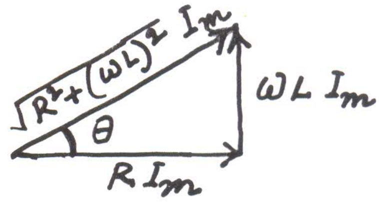


Let $i = I_m \sin \omega t$

Instantaneous voltage equation

$$v_R + v_L = v \Rightarrow Ri + L\frac{di}{dt} = v$$

i.e., $R I_m \sin \omega t + \omega L I_m \cos \omega t = v$



i.e., $I_m \left[\sin \omega t \dfrac{R}{\sqrt{R^2+(\omega L)^2}} + \cos \omega t \dfrac{\omega L}{\sqrt{R^2+(\omega L)^2}}\right] = \dfrac{v}{\sqrt{R^2+(\omega L)^2}}$

$$\cos\theta = \frac{R}{\sqrt{R^2+(\omega L)^2}}$$

$$\sin\theta = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$

Then

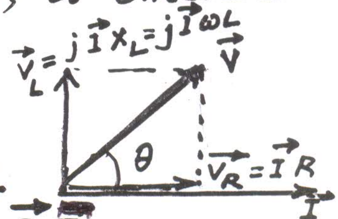
$$I_m\left[\sin \omega t \cos\theta + \cos \omega t \sin\theta\right] = \frac{v}{\sqrt{R^2+(\omega L)^2}}$$

from which

$$v = I_m \sqrt{R^2+(\omega L)^2} \sin(\omega t + \theta)$$
$$= I_m Z \sin(\omega t + \theta) = V_m \sin(\omega t + \theta)$$

Thus we get a) $Z = \sqrt{R^2+(\omega L)^2} = \dfrac{V_m}{I_m}$, b) $\theta = \tan^{-1}\dfrac{\omega L}{R}$.
and c) v leads i in R-L circuit by $\theta°$; or otherwise i lags behind v by $\theta°$

In Phasor Notations:



$$\vec{V} = \vec{V}_R + \vec{V}_L$$
$$= \vec{I}R + \vec{I}j\omega L$$
$$= \vec{I}(R + j\omega L) = \vec{I}\bar{Z}$$

Impedance:

$$\bar{Z}_{R\text{-}L\ circuit} = R + j\omega L = \sqrt{R^2+(\omega L)^2}\ \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$
$$= \sqrt{R^2 + X_L^2}\ \angle \tan^{-1}\left(\frac{X_L}{R}\right)$$

The instantaneous value of the power is given by

$$p = vi = [V_m \sin(\omega t + \theta)][I_m \sin \omega t].$$

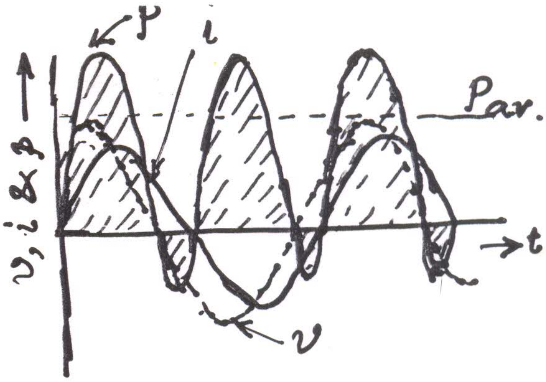
Expanding $\sin(\omega t + \theta)$ and multiplying

$$p = \frac{V_m I_m}{2}\cos\theta - \frac{V_m I_m}{2}[\cos 2\omega t]\cos\theta + \frac{V_m I_m}{2}[\sin 2\omega t]\sin\theta$$

The Average value of the power

$$P_{av} = \frac{1}{T}\int_0^T V_m \sin(\omega t + \theta) I_m \sin \omega t \, dt = \frac{V_m I_m}{2}\cos\theta.$$

The expression for average power may also be derived as follows:



$$P = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t + \theta)\sin\omega t \, dt$$

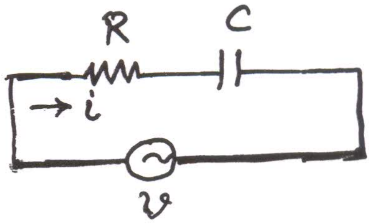
$$= \frac{V_m I_m}{T}\int_0^T \frac{1}{2}\left[\cos\theta - \cos(2\omega t+\theta)\right]dt$$

The term $\frac{V_m I_m}{2\,T}\int_0^T \cos\theta \, dt$

$$= \frac{1}{2} V_m I_m \cos\theta = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\theta = V\, I \cos\theta .$$

Other term contains $2\omega t$, the average value of which over a complete cycle is zero.

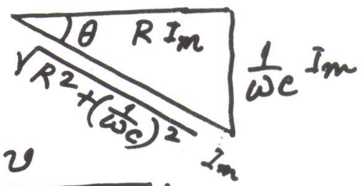
v) SERIES R–C CIRCUIT.



Let $i = I_m \sin\omega t$, then

$$v_R + v_C = v$$

i.e. $R\, i + \frac{1}{C}\int i\, dt = v$



i.e. $R\, I_m \sin\omega t - \frac{1}{\omega C} I_m \cos\omega t = v$

i.e. $I_m\left[\sin\omega t \frac{R}{\sqrt{R^2+(\frac{1}{\omega C})^2}} - \cos\omega t \frac{1/\omega C}{\sqrt{R^2+(\frac{1}{\omega C})^2}}\right] = \frac{v}{\sqrt{R^2+(1/\omega C)^2}}$.

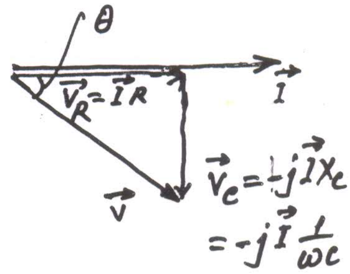
from which $I_m\sqrt{R^2+(1/\omega C)^2}\left[\sin\omega t\cos\theta - \cos\omega t \sin\theta\right] = v$

i.e. $v = I_m Z \sin(\omega t - \theta) = V_m \sin(\omega t - \theta)$

Thus we get a) $Z = \sqrt{R^2+(1/\omega C)^2} = \frac{V_m}{I_m}$, (b) $\theta = \tan^{-1}\left(\frac{+1/\omega C}{R}\right)$

and c) v lags i in R–C circuit by θ°, or otherwise the current i __leads__ the voltage v by θ°.

In phasor notations.



$$\vec{V} = \vec{V} + \vec{V}_C = \vec{I}R - \vec{I}\left(j\frac{1}{\omega C}\right) = \vec{I}\left(R - j\frac{1}{\omega C}\right)$$

$$= \vec{I}\,\vec{Z}.$$

Impedance: $\vec{Z}_{R\text{-}C\ circuit} = R - j\frac{1}{\omega C}$

$$= \sqrt{R^2+(1/\omega C)^2}\ \angle -\tan^{-1}\left(\frac{1/\omega C}{R}\right)$$

$$= \sqrt{R^2+X_C^2}\ \angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

The instanteneous value of the power is given by

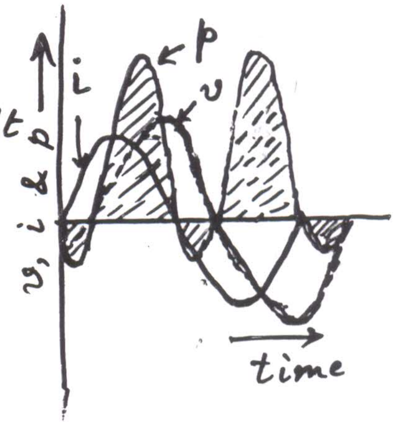
$$p = vi = [V_m \sin(\omega t - \theta)][I_m \sin \omega t]$$

The average value of the power

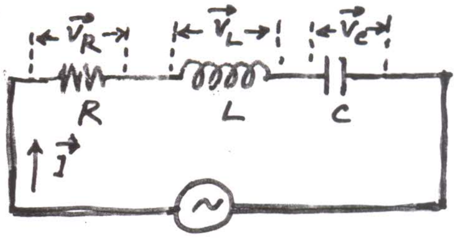
$$P = \frac{1}{T}\int_0^T V_m \sin(\omega t - \theta)\, I_m \sin \omega t \, dt$$

$$= \frac{V_m I_m}{2T}\int_0^T [\cos\theta - \cos(2\omega t - \theta)]\, dt$$

$$= \frac{1}{2} V_m I_m \cos\theta = V I \cos\theta.$$



vi) SERIES R-L-C CIRCUIT.



IN PHASOR NOTATION

$$\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$

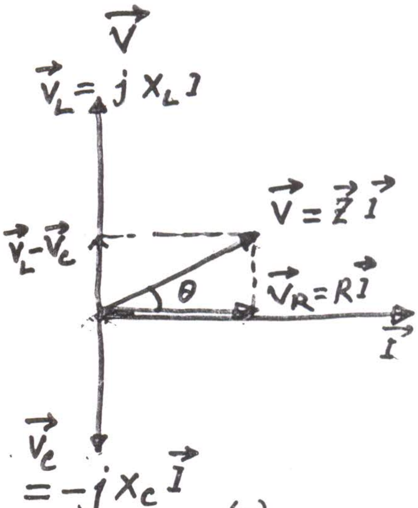
$$= \vec{I}R + j\vec{I}X_L - j\vec{I}X_C$$

$$= \vec{I}[R + j(X_L - X_C)]$$

$$\vec{Z} = R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

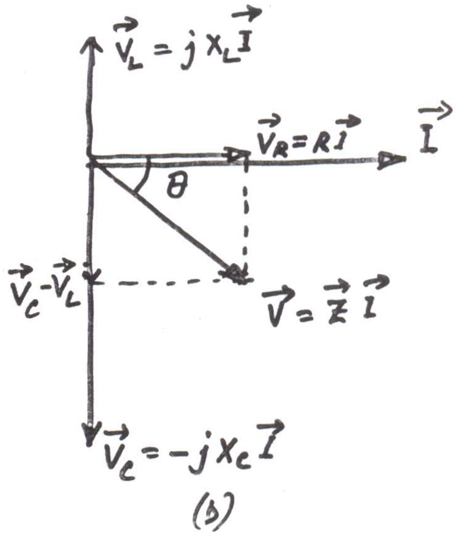
$$\vec{Z} = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \;\Big/ \tan^{-1}\frac{(\omega L - \frac{1}{\omega C})}{R}$$



(a)

If $V_L > V_C$ the circuit will behave like an R-L circuit and its phasor diagram is shown in Fig (a)

On the otherhand if $V_L < V_C$, The circuit will behave like an R-C circuit and its phasor diagram is shown in Fig. (b).



(b)

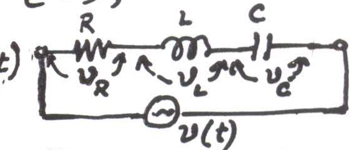
In case (a) the current I drawn will lag the supply voltage V and in case (b) the current I will lead the supply voltage V.

Ex-3. Given $v(t) = 100 \sin 40t$; $R = 10\Omega$, $L = 0.2\,H$, $C = 0.0014\,F$.

i) Determine $i(t)$, $v_R(t)$, $v_L(t)$, $v_c(t)$,

ii) Calculate the power loss,

iii) Show the phasor diagram.



Sol. Here $\bar{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) = 10 + j\left[40 \times 0.2 - \frac{1}{40 \times 0.0014}\right]\Omega$

$= (10 - j\,10)\ \Omega = 14.14 \angle -45^\circ$.

So i) $i(t) = \frac{100}{14.14 \angle -45^\circ} \sin 40t = 7.1 \sin(40t + 45^\circ)$.

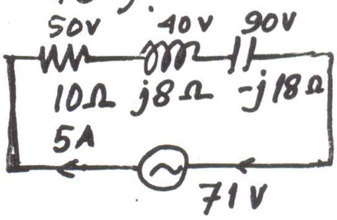
$v_R(t) = i(t) \cdot R = 71 \sin(40t + 45^\circ)$

$v_L(t) = i(t) \cdot (jx_L) = 56.8 \sin(40t + 135^\circ)$.

$v_c(t) = i(t)(-jx_c) = 127.8 \sin(40t - 45^\circ)$.

In terms of phasors

$\vec{I} = \frac{\vec{V}}{\vec{Z}} = \frac{(100/\sqrt{2})\angle 0^\circ}{14.14 \angle -45^\circ} = 5 \angle 45^\circ$.



$\vec{V}_R = R\vec{I} = 50 \angle 45^\circ$; $\vec{V}_L = jX_L \vec{I} = 40 \angle 135^\circ$

$\vec{V}_C = -jX_c \vec{I} = 90 \angle -45^\circ$

NOTE: THE VALUES IN TERMS OF PHASOR QUANTITIES ARE ALL RMS VALUES WHEREAS AS TIME FUCTIONS OR IN INSTANTENEOUS VALUES ARE EXPRESSED WITH MAXIMUM VALUES.

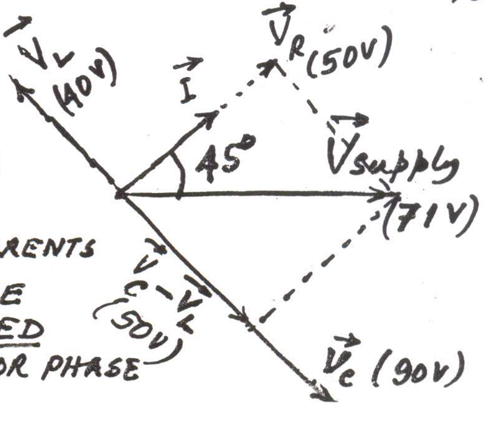
ii) Power loss $= I^2 R = (5)^2 \times 10\ W = 250\,W$. No power loss in inductance or capacitance.

Power factor $\cos\theta = \cos 45^\circ = 0.707$ leading.
CIRCUIT IS BEHAVING LIKE A R-C CIRCUIT.

iii) Phasor diagram:

NOTE CAREFULLY

THE ARITHMETIC SUM OF V_R, V_L & V_c IS MUCH GREATER THAN THE APPLIED VOLTAGE. ALTERNATING VOLTAGES OR CURRENTS OF THE SAME FREQUENCY CAN BE ADDED BUT THEY MUST BE ADDED VECTORIALLY WITH DUE REGARD FOR PHASE RELATION.



VOLT AMPERE, ACTIVE AND REACTIVE POWER

Product of r.m.s. values of current and applied voltage is called APPARENT POWER OR VOLT-AMPERE (VA). A larger unit is kVA or MVA. (Apparent power = VI)

ACTIVE POWER

$$P = VI\cos\theta \quad (\text{Watt})$$

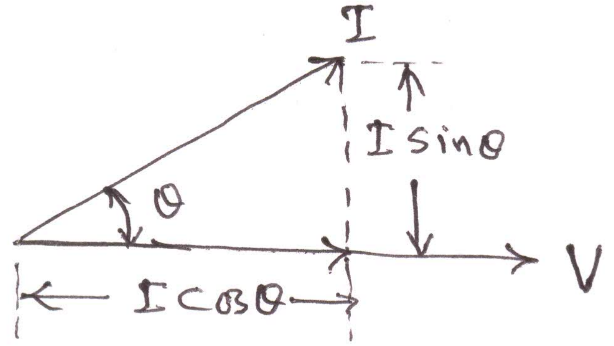
$$\therefore \cos\theta = \frac{P}{VI} = \frac{\text{Active Power}}{\text{Volt-Ampere}}$$

REACTIVE POWER

$$Q = VI\sin\theta \quad (\text{VAr})$$

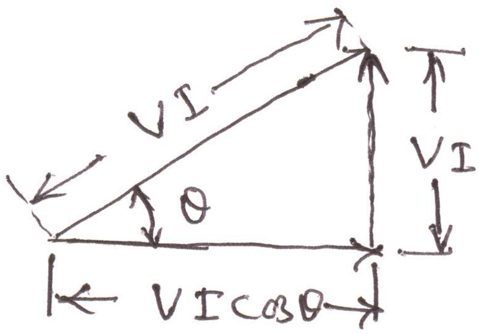
Reactive Power Factor = $\sin\theta$

$$= \frac{\text{Reactive Volt-Ampere}}{\text{Volt-Ampere}} = \frac{Q}{VI}$$



$I\cos\theta \Rightarrow$ active or power component of the current

$I\sin\theta \Rightarrow$ reactive or wattless component of current.



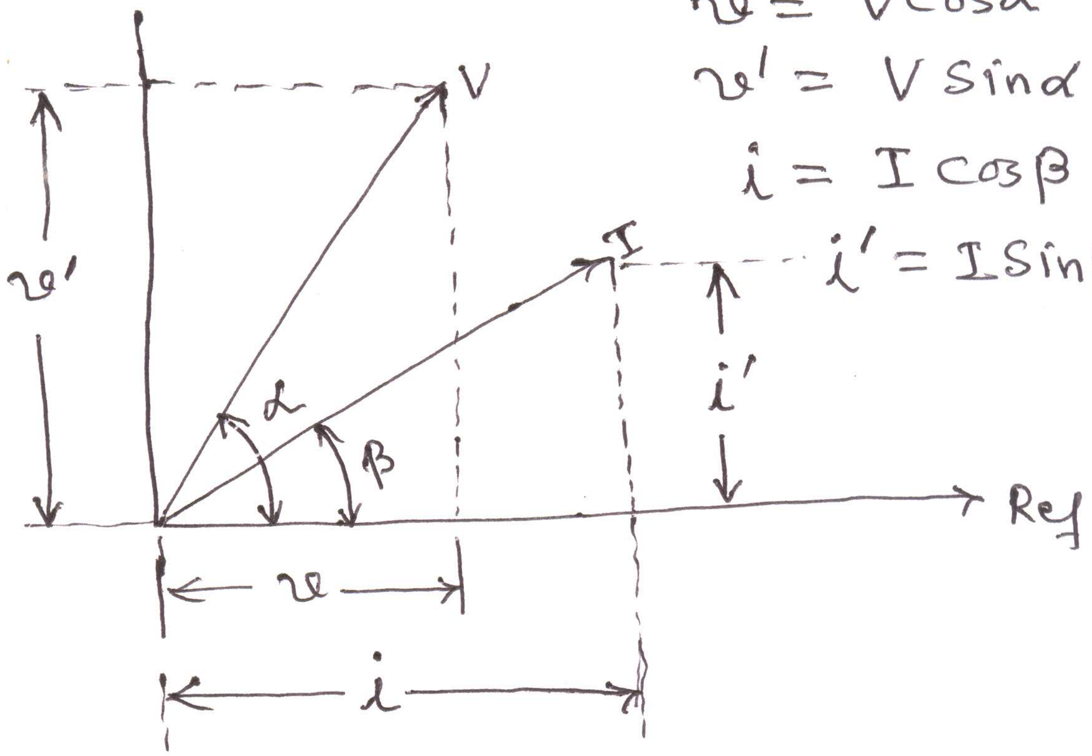
$VI\sin\theta$ = Reactive Volt-Ampere = Reactive Power

$VI\cos\theta$ ⇓ Real power

$\therefore$ Volt-Ampere $= VI$

$$= \sqrt{(\text{Real Power})^2 + (\text{Reactive Power})^2}$$

CALCULATION OF POWER USING COMPLEX NOTATION



$v = V\cos\alpha$

$v' = V\sin\alpha$

$i = I\cos\beta$

$i' = I\sin\beta$

$$\text{Voltage} = V\angle\alpha = V(\cos\alpha + j\sin\alpha)$$
$$= (v + jv')$$

$$\text{current} = I\angle\beta = I(\cos\beta + j\sin\beta)$$
$$= (i + ji')$$

Phase difference between Voltage and current $= (\alpha - \beta)$

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Therefore,

$$\text{Power Factor} = \cos(\alpha - \beta)$$

$\therefore$ Active Power $= P = VI\cos(\alpha-\beta)$.

$$\therefore P = VI(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$\boxed{\therefore P = (vi + v'i')}$$

Reactive Power $= Q = VI\sin(\alpha-\beta)$

$$\therefore Q = VI(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$$

$$\boxed{\therefore Q = (v'i - vi')}$$

Now

$$\text{current} = I\angle\beta$$

$$\therefore (\text{current})^* = I\angle -\beta$$

$$\therefore P + jQ = (\text{Voltage}) \times (\text{current})^*$$

$$\therefore P+jQ = (v+jv')(i+ji')^*$$

$$\therefore P+jQ = (v+jv')(i-ji')$$

$$\therefore P+jQ = (vi+v'i')+j(v'i-vi')$$

$$\boxed{\begin{aligned} \therefore\ P &= (vi+v'i') \\ Q &= (v'i-vi') \end{aligned}}$$

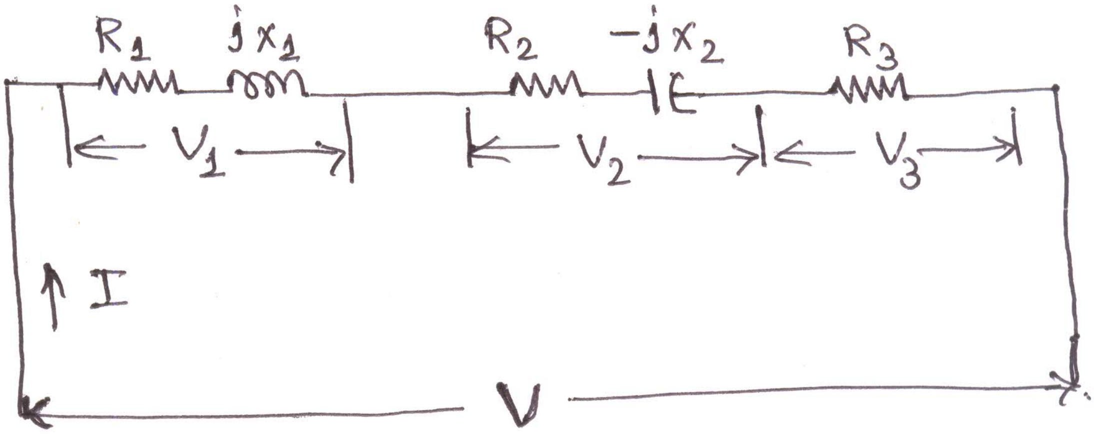
EXAMPLE -4.

$$V = (100+j200) \text{ Volt}$$
$$I = (10+j5) \text{ Amp}$$

$$\therefore P+jQ = VI^* = (100+j200)(10-j5)$$
$$= (2000+j1500)$$

$$\therefore P = 2000 \text{ Watt} = 2 \text{ kW}$$
$$Q = 1500 \text{ VAr} = 1.5 \text{ kVAr}$$

SINGLE PHASE CIRCUIT ANALYSIS



$$Z_1 = R_1 + jx_1$$

$$Z_2 = R_2 - jx_2$$

$$Z_3 = R_3$$

$$V_1 = IZ_1; \quad V_2 = IZ_2; \quad V_3 = IZ_3$$

$$V = V_1 + V_2 + V_3 = I(Z_1 + Z_2 + Z_3)$$

$$\therefore V = I\left[(R_1 + R_2 + R_3) + j(x_1 - x_2)\right]$$

$$\therefore V = IZ = I(R + jx)$$

Where, $R = (R_1 + R_2 + R_3)$ & $x = (x_1 - x_2)$

$$Z = \left(\sqrt{R^2 + X^2}\right) \angle \theta$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{X_1 - X_2}{R_1 + R_2 + R_3}\right)$$

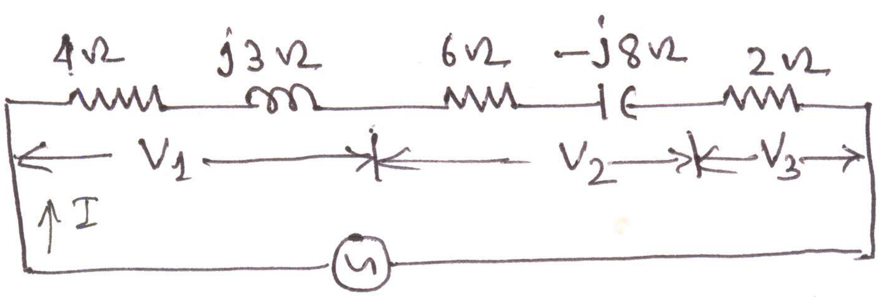
$$I = \frac{V\angle 0^\circ}{\left(\sqrt{R^2+X^2}\right)\angle\theta} = \frac{V\angle 0^\circ - \theta}{\left(\sqrt{R^2+X^2}\right)} = \frac{V\angle -\theta}{\left(\sqrt{R^2+X^2}\right)}$$

$\therefore$ Power factor $= \cos\theta$

$\tan\theta = \frac{X}{R}$; $\therefore \frac{\sin\theta}{X} = \frac{\cos\theta}{R} = \frac{1}{\sqrt{R^2+X^2}}$

$$\boxed{\therefore \cos\theta = \frac{R}{\sqrt{R^2+X^2}}}$$

EXAMPLE - 5



V = 100 volt

25

$$I = \frac{V}{Z} = \frac{100\angle 0^\circ}{(4+j3)+(6-j8)+2}$$

$$\boxed{\therefore I = (7.1 + j2.96)\text{ Amp.} = 7.69\angle 22.6^\circ \text{ Amp}}$$

$$V_1 = IZ_1 = (7.1 + j2.96)(4+j3)$$

$$\boxed{\therefore V_1 = (19.53 + j33.14)\text{ Volt} = 38.47\angle 59.5^\circ \text{ Volt}}$$

$$V_2 = IZ_2 = (7.1 + j2.96)(6-j8)$$

$$\boxed{\begin{aligned}\therefore V_2 &= (66.27 - j39.06)\text{ Volt}\\ &= 76.92\angle -30.5^\circ\end{aligned}}$$

$$V_3 = IZ_3 = (7.1 + j2.96)\times 2$$

$$\boxed{\begin{aligned}&\therefore V_3 = (14.2 + j5.92)\text{ Volt.}\\ &\therefore V_3 = 15.38\angle 22.6^\circ \text{ Volt}\end{aligned}}$$

Check

$$V_1 + V_2 + V_3 = (100 + j0)\text{ Volt.}$$

$P_1 = I^2 R_1 = (7.69)^2 \times 4 = 237$ Watt

$P_2 = I^2 R_2 = (7.69)^2 \times 6 = 355$ Watt

$P_3 = I^2 R_3 = (7.69)^2 \times 2 = 118$ Watt.

Total power

$P = P_1 + P_2 + P_3 = (237 + 355 + 118)$ Watt.

$\therefore P = 710$ Watt.

$V = 100 \angle 0^\circ$ Volt

$I = 7.69 \angle 22.6^\circ$ Amp.

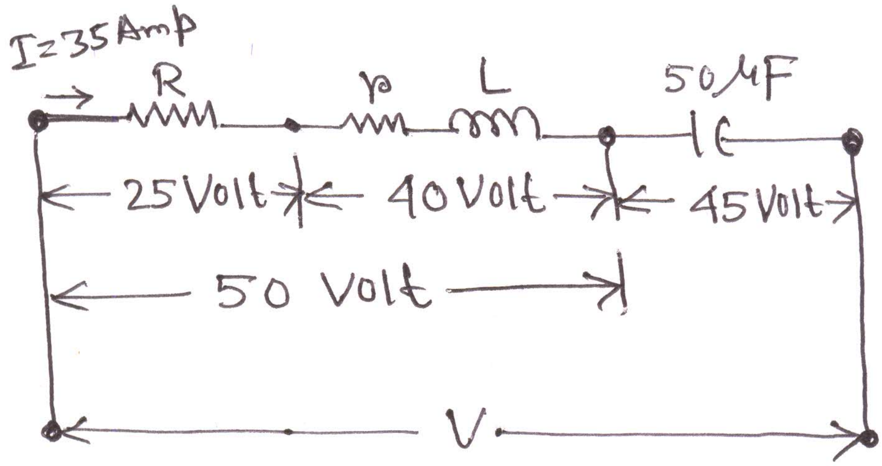
Power factor $= \cos(22.6^\circ) = 0.9232$ (leading)

$P = VI\cos\theta = 100 \times 7.69 \times 0.9232$ Watt

$\therefore P \simeq 710$ Watt.

DRAW PHASOR DIAGRAM

EXAMPLE – 6



Determine R, r, L, f and V

Soln. $V_1 = 25$ Volt; $V_2 = 40$ Volt;

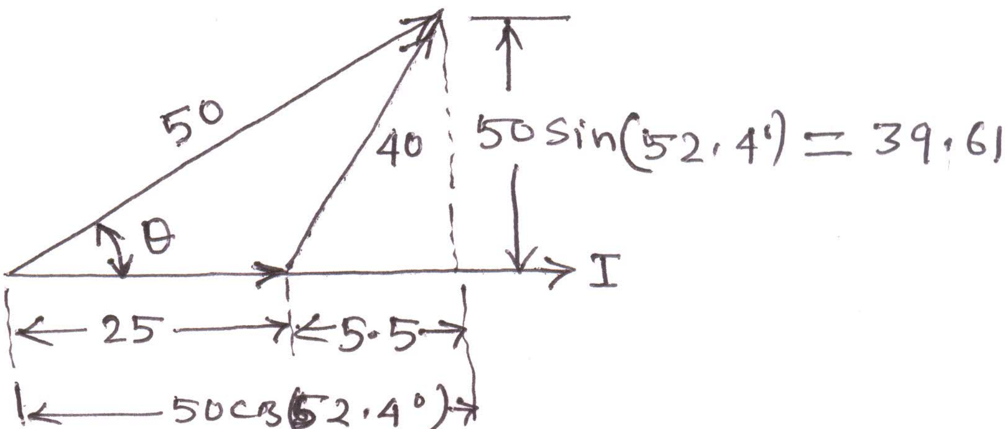
$V_3 = 45$ Volt.

$$\frac{V_3}{X_C} = I = 35$$

$$\therefore \frac{45}{1/\omega C} = 35 \Rightarrow \frac{45}{1/2\pi f C} = 35$$

$$\therefore f = 2.475 \text{ KHz}.$$

$$R = \frac{V_1}{I} = \frac{25}{35} = 0.714\,\Omega$$



$$(40)^2 = (50)^2 + (25)^2 - 2 \times 50 \times 25 \cos\theta$$

$$\therefore \cos\theta = \frac{1525}{2500} = 0.61$$

$$\therefore \theta = 52.4^\circ$$

$50\cos(52.4^\circ)$

$= 50 \times 0.61$

$= 30.5$

Voltage drop across r

$= 5.5$ Volt

Voltage drop across $L = 39.61$ Volt.

$\therefore \quad Ir = 5.5 \quad \therefore 35r = 5.5 \quad \boxed{\therefore r = 0.157\,\Omega}$

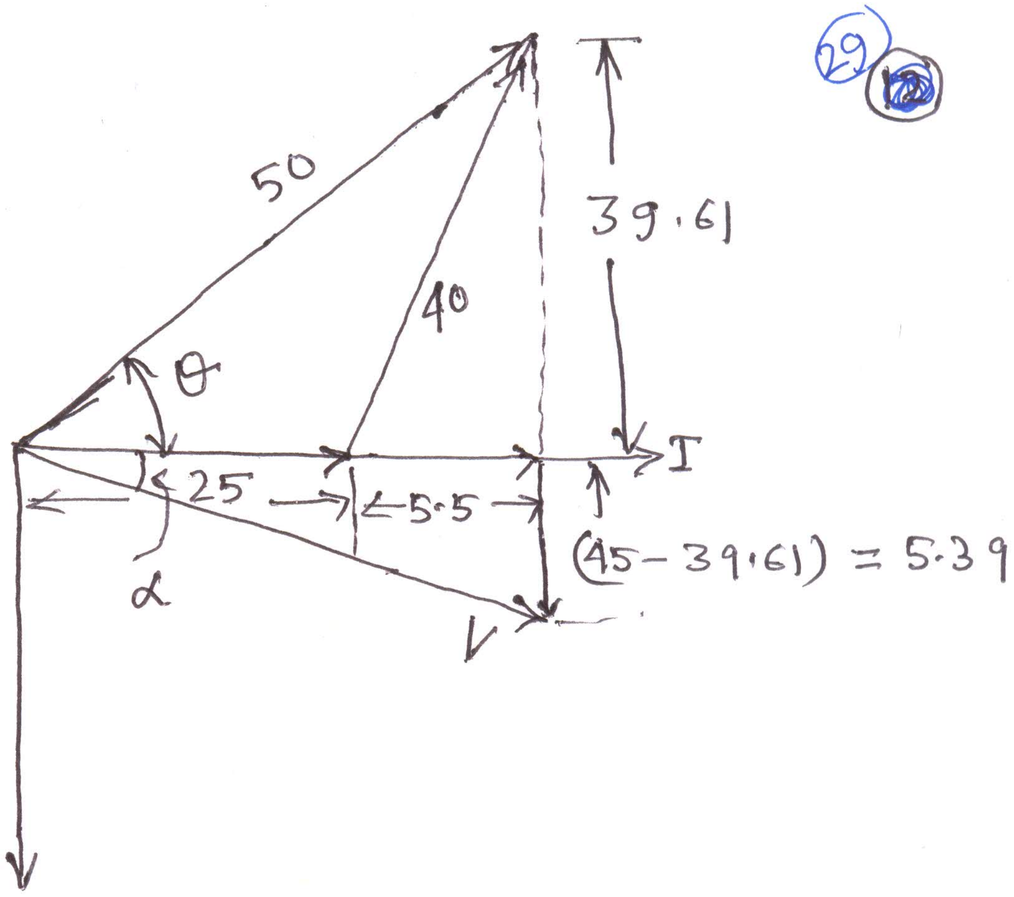
$I X_L = 39.61 \quad \therefore$ ~~EL~~

$\therefore 35 \times L \times 2\pi \times \underline{2.475 \times 1000} = 39.61$

$\therefore \boxed{L = 0.073 \text{ mH}}$

29

12



$V_3 = 45$ Volt

$$\therefore V = \sqrt{(25+5.5)^2 + (5.39)^2}$$

$$\therefore V = 31 \text{ Volt} \quad (\simeq 30.972 \text{ Volt})$$

$$\alpha = \tan^{-1}\left(\frac{5.39}{30.5}\right) = 10^\circ$$

power factor $= \cos\alpha = \cos(10^\circ) = 0.9848$

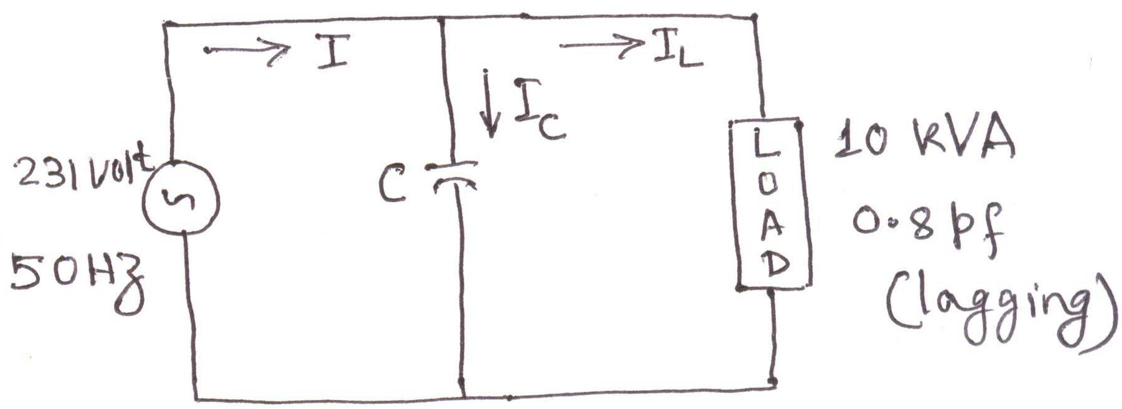
$$P = (35)^2 (0.714 + 0.157) = 1.0669 \text{ kW}$$

Also

$$P = VI\cos\alpha = 31 \times 35 \times 0.9848 \simeq 1.0669 \text{kW}$$

$=$

EXAMPLE – 7



i) C = ? to make the overall power factor (i.e., load + shunt capacitor) 0.95 lagging.

ii) Current drawn before and after installing the capacitor

Soln.

Before installing the capacitor, the current is only drawn by the load.,

i.e, $I_L = \dfrac{\text{Apparent Power of Load}}{\text{Voltage}}$

$\therefore I_L = \dfrac{10 \times 10^3}{231} = 43.29$ Amp and $\cos\theta = 0.8$

$\therefore \theta = \cos^{-1}(0.8) = 36.9^\circ$ (lagging)

To make overall power factor 0.95 lagging,

$\cos\delta = 0.95$

$\therefore \delta = 18.2^\circ$ (lagging)

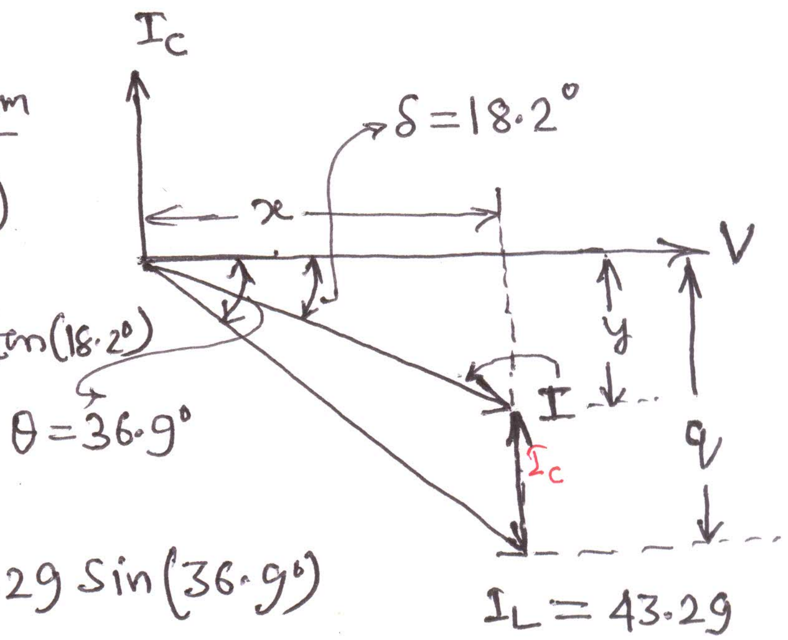
Since the pure capacitor draws a leading current at 90° with V, I_c is shown $\perp$ to V and leading V by 90°

<u>From Phasor diagram</u>

$x = 43.29 \cos(36.9^\circ)$

$= 34.63$

$y = x \tan\delta = 34.63 \tan(18.2^\circ)$



$\therefore y = 11.39$

$q = I_L \sin\theta = 43.29 \sin(36.9^\circ)$

$\therefore q = 26.0$

Supply current

$$I = \sqrt{x^2 + y^2} = \sqrt{(34.63)^2 + (11.39)^2}$$

$$\therefore \boxed{I = 36.46 \text{ Amp.}}$$

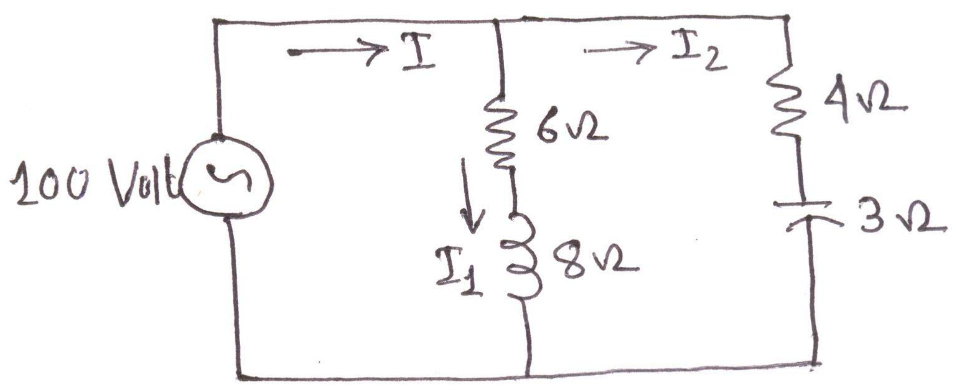
$$I_C = q - y = (26 - 11.39) = 14.61 \text{ Amp}$$

Now $I_C = \dfrac{V}{(1/\omega C)} = \omega C V$

$$\therefore C = \frac{I_C}{\omega V} = \frac{14.61}{2\pi \times 50 \times 231}$$

$$\therefore \boxed{C = 201 \, \mu F}$$

EXAMPLE — 8



Soln.

$$V = 100\angle 0^\circ \text{ Volt}$$

$$I_1 = \frac{100\angle 0^\circ}{6+j8} = (6-j8) = 10\angle -53.2^\circ \text{ Amp}$$

$$I_2 = \frac{100\angle 0^\circ}{4-j3} = (16+j12) = 20\angle 36.8^\circ \text{ Amp}$$

$$I = I_1 + I_2 = (6-j8) + (16+j12) = (22+j4) \text{ Amp}$$

$$\therefore I = 22.35\angle 10.3^\circ \text{ Amp}$$

$$\boxed{Y = \frac{1}{Z} = \text{Admittance}}$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{(6+j8)} = (0.06 - j0.08) \text{ mho}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{(4-j3)} = (0.16 + j0.12) \text{ mho}$$

$$Y_1 = 0.06 - j0.08 = g_1 - jb_1$$

$$\boxed{\therefore g_1 = 0.06 \; ; \quad b_1 = 0.08}$$

$$Y_2 = 0.16 + j0.12 = g_2 - jb_2$$

$$\boxed{\therefore g_2 = 0.16 \; ; \quad b_2 = -0.12}$$

$$\boxed{\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}}$$

$$\therefore Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 - jb_2)$$

$$\therefore Y = (g_1 + g_2) - j(b_1 + b_2)$$

$$\boxed{\therefore Y = g - jb}$$

$$\boxed{\begin{array}{l} g \Rightarrow \text{Conductance} \\ b \Rightarrow \text{Susceptance} \end{array}}$$

$$g = g_1 + g_2 = (0.06 + 0.16) = 0.22$$

$$b = b_1 + b_2 = 0.08 - 0.12 = -0.04$$

$$\therefore Y = g - jb = (0.22 - j(-0.04))$$

$$\therefore Y = (0.22 + j0.04) \text{ mho}$$

Now

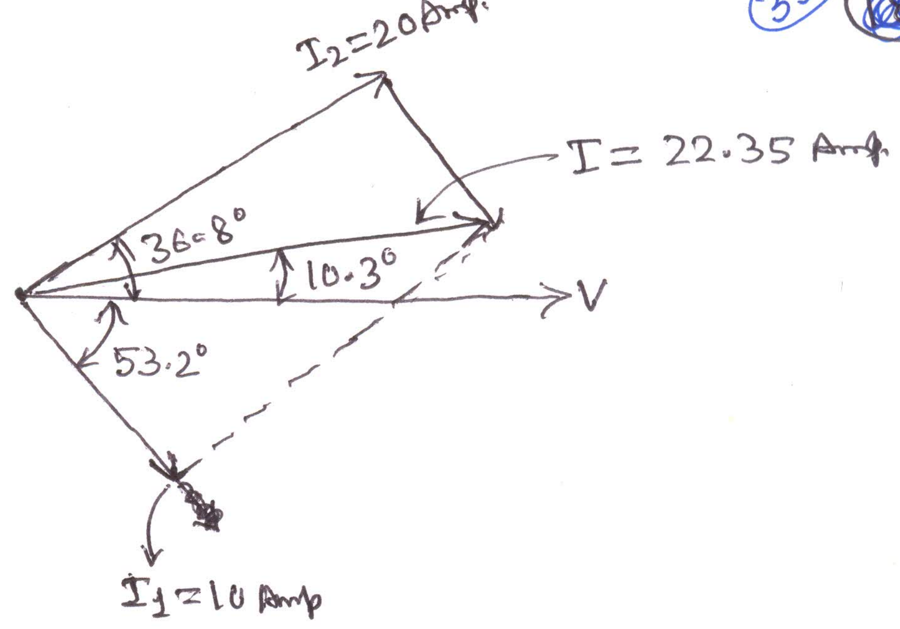
$$I = \frac{V}{Z} = YV = (100\angle 0^\circ)(0.22 + j0.04)$$

$$\therefore I = 22.35\angle 10.3^\circ \text{ Amp}$$

$$I_1 = (100\angle 0^\circ)(0.06 - j0.08) = (6 - j8) = 10\angle -53.2^\circ \text{ Amp}$$

$$I_2 = (100\angle 0^\circ)(0.16 + j0.12) = (16 + j12) = 20\angle 36.8^\circ \text{ Amp}$$

Phasor diagram



Power factor $= \cos(10.3^\circ) = 0.9838$

$$\boxed{\begin{aligned} \text{Input Power} &= VI\cos\theta \\ &= 100 \times 22.35 \times 0.9838 \\ &= 2198.793 \text{ Watt} \end{aligned}}$$

Also

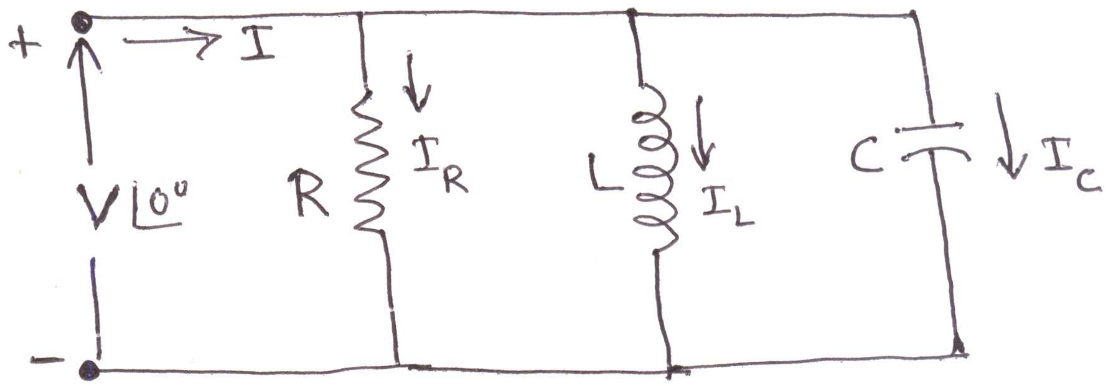
$$P_1 = I_1^2 R_1 = (10)^2 \times 6 = 600 \text{ Watt}$$

$$P_2 = I_2^2 R_2 = (20)^2 \times 4 = 1600 \text{ Watt.}$$

$$\boxed{\therefore P = P_1 + P_2 = (600 + 1600) = 2200 \text{ Watt}}$$

$$\boxed{\begin{aligned} VI\cos\theta &= 100 \times 22.36 \times 0.98388 \\ &= 2199.96 \simeq 2200 \text{ Watt} \end{aligned}}$$

PARALLEL R–L–C CIRCUIT



$$I_R = \frac{V\angle 0^\circ}{R} = \frac{V}{R}\angle 0^\circ$$

$$I_L = \frac{V\angle 0^\circ}{jX_L} = \frac{V\angle 0^\circ}{X_L\angle 90^\circ} = \frac{V}{X_L}\angle -90^\circ$$

$$I_C = \frac{V\angle 0^\circ}{-jX_C} = \frac{V\angle 0^\circ}{X_C\angle -90^\circ} = \frac{V}{X_C}\angle 90^\circ$$

$$I = I_R + I_L + I_C$$

$I_R = i_R\angle 0^\circ$, i.e, $i_R = \frac{V}{R}$

$I_L = i_L\angle -90^\circ$, i.e., $i_L = \frac{V}{X_L}$

$I_C = i_C\angle 90^\circ$, i.e., $i_C = \frac{V}{X_C}$

(37)

$$\therefore I = i_R \angle 0^\circ + i_L \angle -90^\circ + i_C \angle 90^\circ$$

$$\therefore I = i_R - j i_L + j i_C = i_R - j(i_L - i_C)$$

$$\therefore I = \left[\sqrt{i_R^2 + (i_L - i_C)^2}\right] \angle -\tan^{-1}\frac{(i_L - i_C)}{i_R}$$

~~tan~~

$$\therefore I = \left[\sqrt{i_R^2 + (i_L - i_C)^2}\right] \angle \theta$$

Where $\theta = -\tan^{-1}\left(\frac{i_L - i_C}{i_R}\right)$

If

$\theta \Rightarrow$ Negative

Resultant current lags the supply voltage

$\theta \Rightarrow$ Positive

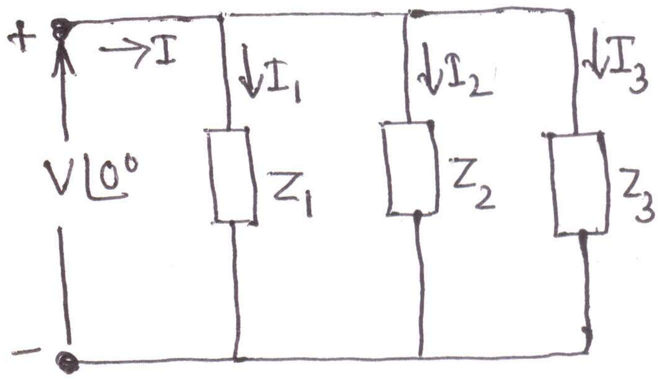
Resultant current leads the supply voltage.

In general

$$I_1 = \frac{V}{Z_1} = Y_1 V$$

$$I_2 = \frac{V}{Z_2} = Y_2 V$$

$$I_3 = \frac{V}{Z_3} = Y_3 V$$



$$I = I_1 + I_2 + I_3 = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

$$\boxed{\therefore I = (Y_1 + Y_2 + Y_3) V}$$

OR

$$\boxed{I = Y V}$$

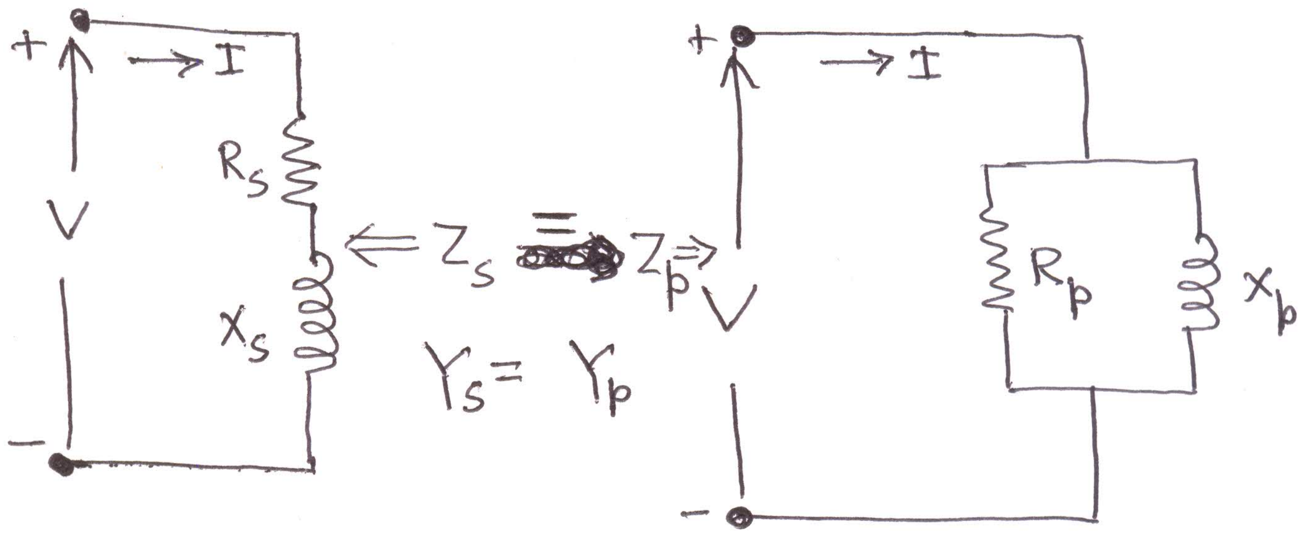
$Y \Rightarrow$ Reciprocal of Impedance

$\Rightarrow$ Admittance

> Admittances are added for parallel branches

> For branches in series, Impedances are added

PARALLEL EQUIVALENT OF A SERIES IMPEDANCE



Equivalent Circuit

$$Y = \frac{1}{Z} = \underbrace{\frac{1}{R_s + jX_s}}_{Y_s} = \underbrace{\frac{1}{R_p} + \frac{1}{jX_p}}_{Y_p}$$

$$\therefore \quad \frac{R_s}{R_s^2 + X_s^2} - j\frac{X_s}{R_s^2 + X_s^2} = \frac{1}{R_p} - j\frac{1}{X_p} = g - jb$$

$$\therefore \quad g = \text{conductance} = \frac{1}{R_p} = \frac{R_s}{(R_s^2 + X_s^2)}$$

$$b = \text{susceptance} = \frac{1}{X_p} = \frac{X_s}{(R_s^2 + X_s^2)}$$

Also

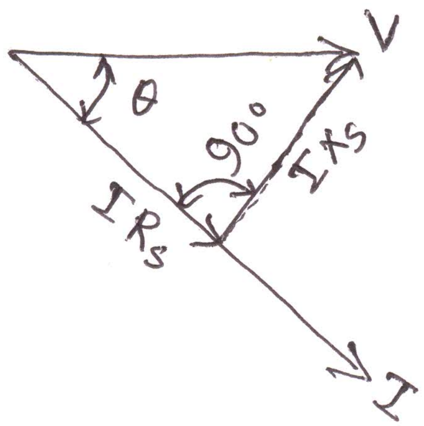
$$R_p = \frac{(R_s^2 + x_s^2)}{R_s}$$

$$x_p = \frac{(R_s^2 + x_s^2)}{x_s}$$

Now

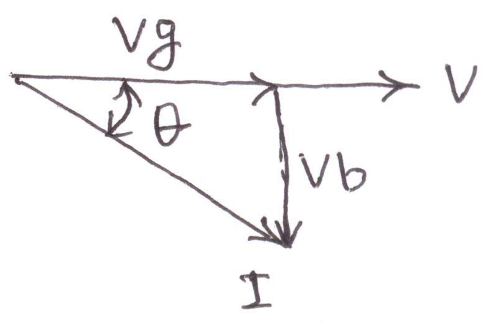
$$\frac{V}{I} = Z_s = R_s + jx_s$$

$$\therefore \boxed{V = IR_s + jIx_s}$$



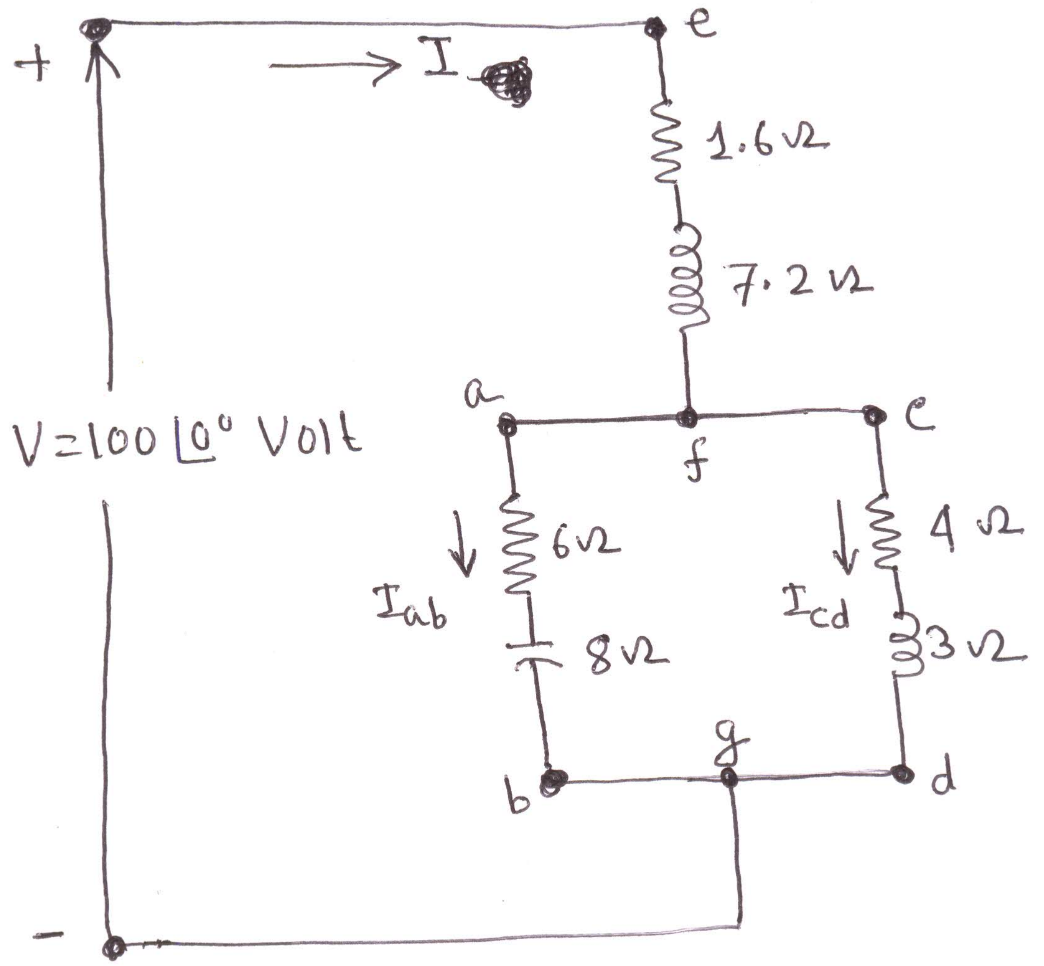
$$\frac{I}{V} = Y_p = \frac{1}{R_p} - j\frac{1}{x_p} = g - jb$$

$$\therefore \boxed{I = Vg - jVb}$$



SERIES- PARALLEL CIRCUIT

EX-9



Determine I_{ef}, I_{ab}, I_{cd}, Power and Power Factor.

Soln.

$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{6-j8} = (0.06+j0.08)\ \text{mho}$$

$$Y_{cd} = \frac{1}{4+j3} = (0.16-j0.12)\ \text{mho}$$

$$Y_{fg} = Y_{ab} + Y_{cd} = (0.22-j0.04)\ \text{mho}$$

42 25

$$Z_{fg} = \frac{1}{Y_{fg}} = \frac{1}{(0.22 - j0.04)} = (4.4 + j0.8)\ \Omega$$

$$Z_{eg} = Z_{ef} + Z_{fg} = (1.6 + j7.2) + (4.4 + j0.8)$$

$$\therefore Z_{eg} = (6 + j8)\ \Omega$$

$$\therefore I = \frac{V}{Z_{eg}} = \frac{100\angle 0^\circ}{6 + j8} = 10\angle -53.2^\circ \text{ Amp}$$

power Factor $= \cos\theta = \cos(53.2^\circ) = 0.60$

$$P = VI\cos\theta = 100 \times 10 \cos(53.2^\circ)$$

$$\boxed{\therefore P = 600 \text{ Watt}}$$

$$V_{ef} = I.Z_{ef} = I_{ef} Z_{ef} = (6 - j8)(1.6 + j7.2)$$

$$\boxed{\therefore V_{ef} = 73.8\angle 24.4^\circ \text{ Volt}}$$

$$\therefore V_{fg} = V - V_{ef} = 100\angle 0^\circ - 73.8\angle 24.4^\circ$$

$$\boxed{\therefore V_{fg} = 44.7\angle -42.8^\circ \text{ Volt}}$$

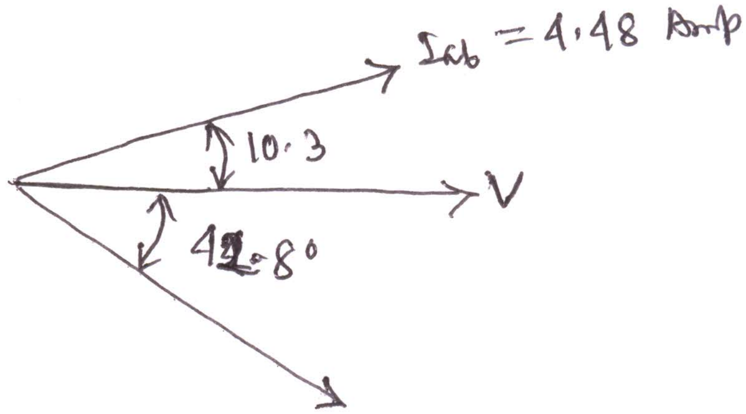
$$I_{ab} = Y_{ab} V_{fg} = (0.06 + j0.08) \times 44.7 \angle -42.8^\circ$$

$$\therefore I_{ab} = 4.48 \angle 10.3^\circ \text{ Amp}$$

$$I_{cd} = V_{fg} \cdot Y_{cd} = 44.7 \angle -42.8^\circ \times (0.16 - j0.12)$$

$$\therefore I_{cd} = 8.95 \angle -79.7^\circ \text{ Amp.}$$

$$P_{ab} = (I_{ab})^2 \cdot R_{ab} = (4.48)^2 \times 6 \cong 120 \text{ Watt.}$$



~~Pab = 4.4~~

$$V_{fg} = V_{ab} = V_{cd} = 44.7 \text{ volt}$$

$$\boxed{\begin{aligned} \therefore P_{ab} &= V_{ab} I_{ab} \cos\theta \\ &= 44.7 \times 4.48 \cos(42.8^\circ + 10.3^\circ) \\ &= \underline{120 \text{ Watt.}} \end{aligned}}$$

Similarly

$$P_{cd} = (I_{cd})^2 \cdot R_{cd} = (8.95)^2 \times 4 = 320 \text{ Watt.}$$

$$P_{ef} = (I_{ef})^2 R_{ef}$$

$$= (I)^2 R_{ef} = (10)^2 \times 1.6 = 160 \text{ Watt.}$$

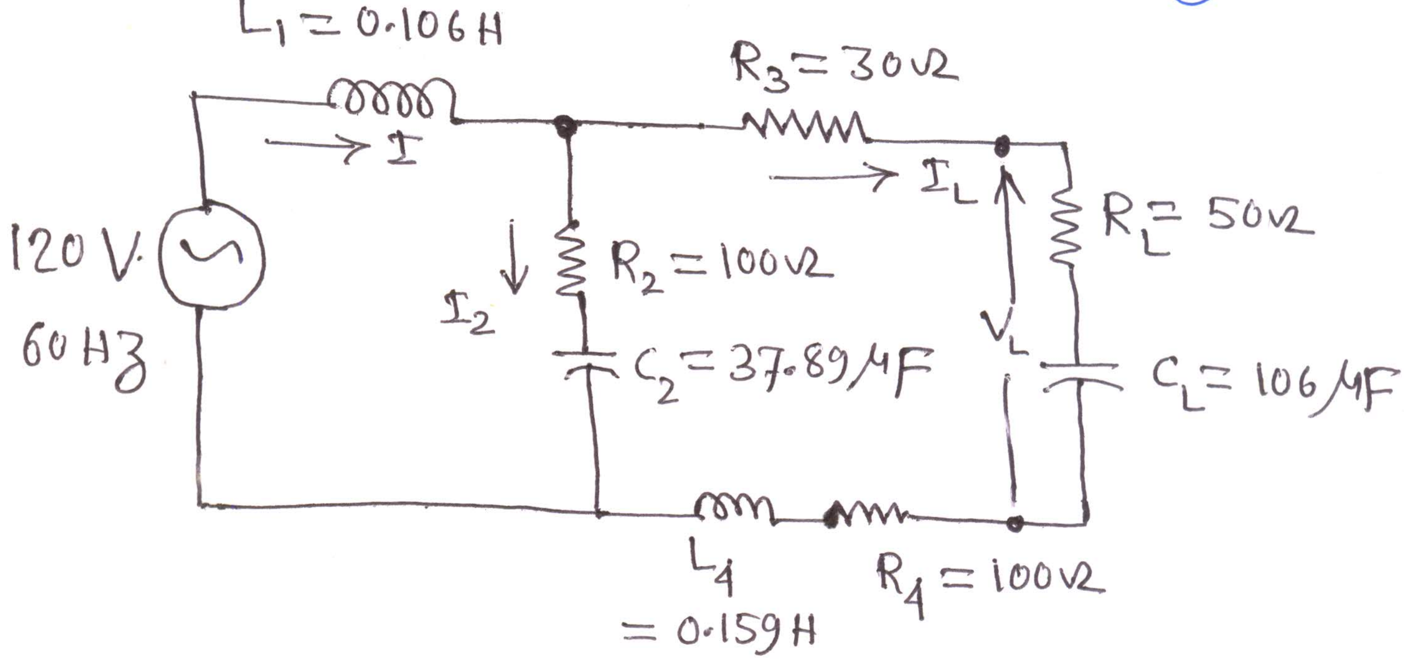
$$P = P_{ef} + P_{ab} + P_{cd} = (160 + 320 + 120)$$

$$\boxed{\therefore P = 600 \text{ Watt}}$$

$$PF_{(ab)} = \frac{6}{\sqrt{6^2+8^2}} = 0.6 \text{ (leading)}$$

$$PF_{(cd)} = \frac{4}{\sqrt{4^2+3^2}} = 0.8 \text{ (lagging)}$$

DRAW PHASOR DIAGRAM.



Find

(i) I, I_L & V_L

(ii) Find active and reactive power delivered to the load

(iii) Find apparent, active and reactive power of the entire circuit.

Ans: (i) $I = 1.5426 \angle -12.614^\circ$ Amp

$I_L = 0.6672 \angle -40.48^\circ$ Amp

$V_L = 37.3 \angle -67.05^\circ$ Volt

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(ii) $P_L = 22.25$ Watt

$Q_L = -11.126$ VAr (Capacitive)

(iii) Apparent Power = ~~122.554~~ 185.112 VA

$P =$ ~~100.4 Watt~~ 180.64 Watt

$Q =$ ~~70.27 VAr (capacitive)~~

40.425 VAr (Inductive)