Tutorial Sheet-12 **Autumn 2019**

MATHEMATICS-I(MA10001)

1. Find the singularity and classify them:

a) $\cos\frac{1}{z}$,

b) $\frac{1}{z^2\sin z}$

c) $\frac{z^2+1}{(z+1)(z-1)^2}$

d) $\frac{\sin z}{z^4}$

e) $\frac{e^z-1}{z}$

f) $\frac{z}{e^{z^2}-1}$

g) $e^{1/z}$

h) $\frac{1}{z(z^2+1)}$

i) $\frac{e^z\sinh z}{z^3}$

2. Find the residue at all singular points:

a) $\frac{z^4}{(z^2+1)^2}$

b) $\frac{e^{1/z}}{1-z}$

c) $\frac{\sin z}{z}$

d) $e^{\frac{1}{z^2}}$.

3. Find the Taylor series expansion of the following functions:

a) $\frac{1}{(z-2)(z-i)}$ about $z=0$ inside the solid disc $\mid z \mid < 1$,

b) $\frac{1}{1-z}$ about $z=2i$ and specify the region of convergence.

4. Find the laurent series of the function $f(z)=\frac{z^2-2z+3}{z-2}$ about $z=1$ in the region $\mid z-1 \mid > 1$.

5. Find Laurent series of the following function and specify the region of convergence

a) $(z-3)\sin\left(\frac{1}{z+2}\right)$ about $z=-2$,

b) $\dfrac{e^{2z}}{(z-1)^3}$ about $z = 1$.

6. Find the Laurent series about $z = 0$ of the function $f(z) = z^2 e^{1/z}$ defined on $\mathbb{C} \setminus \{0\}$.

7. Find all posiible Lauret series expansion of the function $f(z) = \dfrac{1}{z(1-z)(2-z)}$ in the region

   a) $0 <\mid z \mid< 1$,

   b) $1 <\mid z \mid< 2$,

   c) $2 <\mid z \mid$.

8. Find the Laurent series of the function $f(z) = \dfrac{e^z}{(z-1)^2}$ about $z = 1$ in the region $0 <\mid z-1 \mid< \infty$.

9. Evaluate

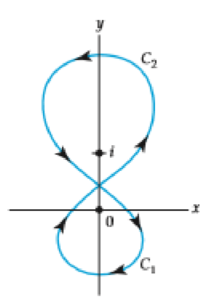
   a) $\displaystyle\oint_{|z|=2} \frac{5z-2}{z(z-1)} dz$,

   b) $\displaystyle\oint_{|z|=1} z^2 \sin\frac{1}{z} dz$,

   c) $\displaystyle\oint_{|z|=1} \frac{\sin z}{z^6} dz$,

   d) $\displaystyle\oint_{|z|=2} \frac{e^z}{z^2-2z-3} dz$.

10. Evaluate $\displaystyle\oint_{\mathbf{C}} \frac{z^3+3}{z(z-i)^2} dz$, where **C** is the contour shown in the figure



11. Evaluate

a) $\displaystyle\oint_{|z|=3} \frac{\sin \pi z^2 + \cos \pi z^2}{z^2 - 3z + 2} dz,$

b) $\displaystyle\oint_{\mathbf{C}} \frac{dz}{(z-a)^n}$, $n \in \mathbb{N}$ where **C** is a circle of radius r center at $z = a$.

12. Evaluate $\displaystyle\oint_{\mathrm{C}} \frac{z}{z^2+4} dz$ where **C** is the contour shown in the figure