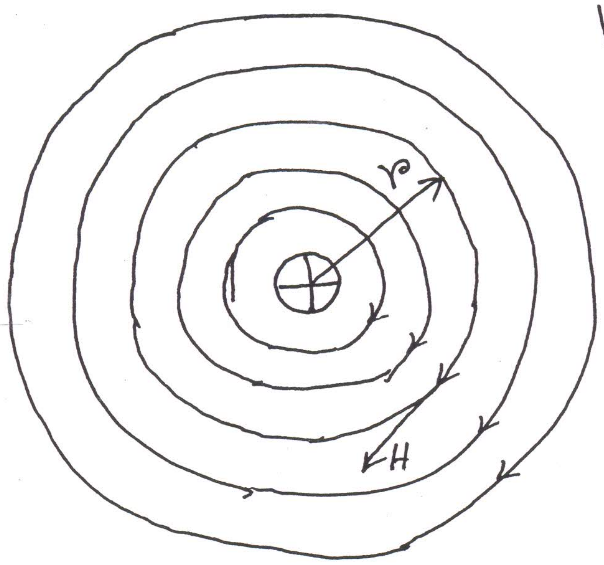
MAGNETIC CIRCUITS

The area around a magnet is called the magnetic field and it is in this area that the effects of the magnetic force produced by the magnet can be detected.

Electromagnetic system is an essential element of all rotating electric machines and electromechanical devices as well as static devices like transformers.

Magnetic field $\Rightarrow$ coupling medium allowing interchange of energy in either direction between electrical and mechanical systems.

MAGNETIC EFFECTS OF ELECTRIC CURRENT



H = Magnetic Intensity
OR
Magnetizing Force

Fig.1: Flux Surrounding current

A long straight conductor carrying current (into the plane of paper). The current causes a magnetic field to be established in the space surrounding it.

A line of flux is a closed path around the current such that the magnetic force is tangential to it all points around the line

The direction of flux is given by the **right hand rule**, which states that if the conductor is grasped by the right hand such that the thumb points in the direction of current, the flux is established in the direction in which the fingers curl.

$H \Rightarrow$ defined as the causative current per unit length of flux line enclosing the current

In this case, because of symmetry H is uniform along each flux line.

Since, H is tangential all along a flux line, for any flux line of radius r,

$$H = \frac{I}{2\pi r} \text{ A/m} \quad \text{(Amperes law)}$$

The flux density B is established by this field intensity is a property of the medium.

For air, or any non-magnetic medium, the ratio of magnetic flux density to magnetizing force is a constant, i.e., B/H = constant. This constant is μ_0, the permeability of free space (or the magnetic space constant) and ~~is equal to~~,

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-m or H/m}$$

$$\therefore \boxed{B = \mu_0 \cdot H \quad \text{Wb/m}^2 \text{ or Tesla}}$$

For all media, other than free space

$$\boxed{B = \mu_0 . \mu_r . H.}$$

where μ_r is the relative permeability and is defined as,

$$\mu_r = \frac{\text{flux density in material}}{\text{flux density in a vacuum}}$$

μ_r varies with the type of magnetic material and since it is a ratio of flux densities, it has no unit.

Also, we can write,

$$\boxed{B = \mu . H.}$$

where $\mu = \mu_0 \mu_r$ = absolute permeability.

The flux passing through the area (A) (For uniform flux density),

$$\boxed{\phi = B . A. \quad Wb.}$$

MAGNETIC CIRCUITS

Consider a toroidal ring of ferromagnetic material ($\mu > 1$, Iron, cobalt and nickel etc.).

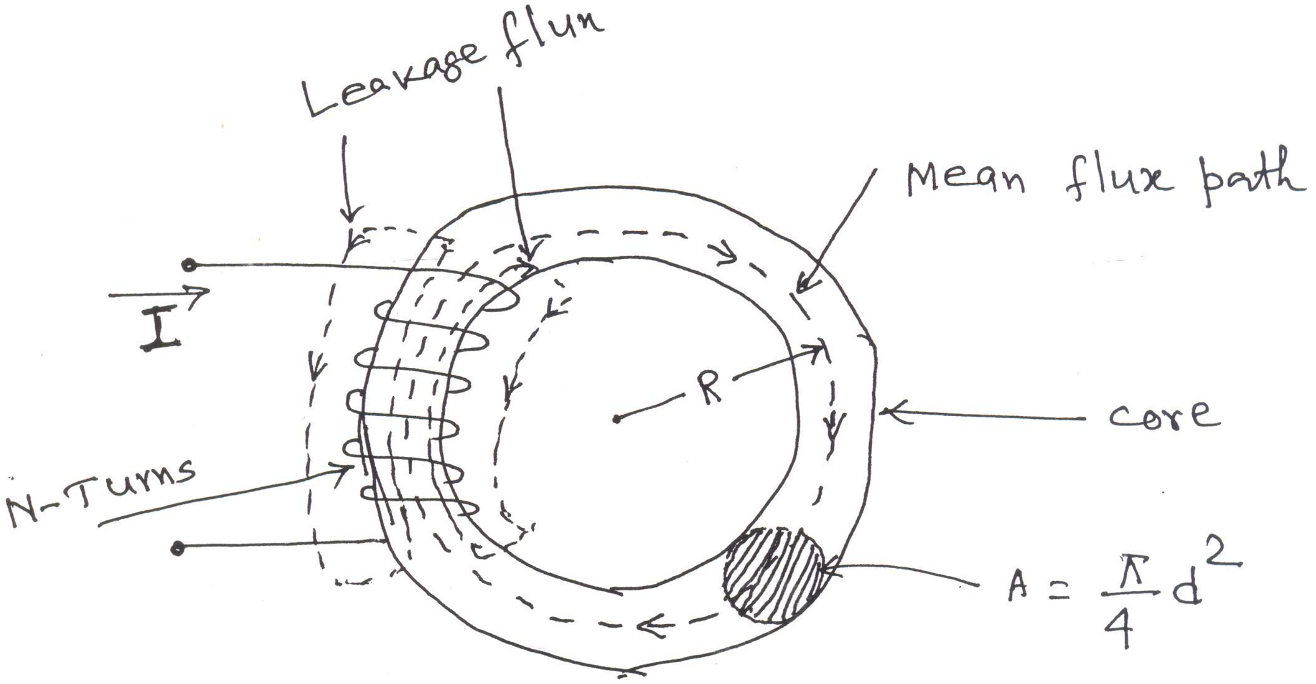


Fig. 2: Toroidal ring of ferromagnetic material with exciting coil

Mean Radius = R

Circular cross-section of diameter = d.

The ring termed as core is excited by a coil wound round it with N turns carrying a current I.

All the flux lines in the core enclose a a current of

$$\boxed{F_m = NI \ \text{A-T}}$$

which is the causative current (cause of the existence of a magnetic flux in a magnetic circuit) establishing the flux.

This is known as <u>magnetomotive force</u>, i.e.,

$F_m \Rightarrow$ magnetomotive force.

By Symmetry, H in this core is constant round each flux line and for the mean flux line of radius R, the magnetizing force (or magnetic intensity),

$$\boxed{\begin{aligned} H &= \frac{NI}{2\pi R} \ \text{A-T/m} = \frac{F_m}{2\pi R} \ \text{A-T/m} \\ \text{or} \quad H &= \frac{F_m}{l} \ \text{A-T/m} \end{aligned}}$$

where $l =$ length of the mean flux path $= 2\pi R$.

The mean flux density

$$B = \mu . H$$

$$\therefore B = \frac{\mu . F_m}{l} \text{ Wb/m}^2 \text{ or } \underline{\underline{\text{Tesla}}}$$

$$\phi = A.B = \frac{A.\mu.F_m}{l}$$

$$\therefore \phi = \frac{F_m}{\left(\frac{l}{A\mu}\right)} = \frac{F_m}{\mathcal{R}} = \mathcal{P} F_m .$$

where

$$\mathcal{R} = \frac{l}{A\mu} \text{ A-T/Wb} = \text{reluctance of the magnetic circuit}$$

$$\mathcal{P} = \frac{1}{\mathcal{R}} = \text{ Wb/A-T} = \text{permeance of the magnetic circuit.}$$

Electrical circuits	Magnetic circuits
emf, E (volt)	mmf F_m (A-T)
current, I (Amp)	flux ϕ (Wb)
resistance R (Ω)	reluctance $\mathcal{R}$ (A-T/Wb or H^{-1})
$I = \frac{E}{R} = \frac{emf}{R}$	$\phi = \frac{F_m}{\mathcal{R}} = \frac{mmf}{\mathcal{R}}$
$R = \frac{\rho l}{A}$	$\mathcal{R} = \frac{l}{A\mu} = \frac{l}{A\mu_0\mu_r}$

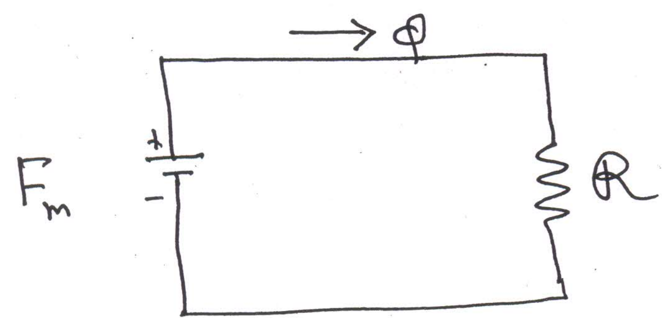


Fig. 3: DC circuit analog of magnetic system.

Fringing

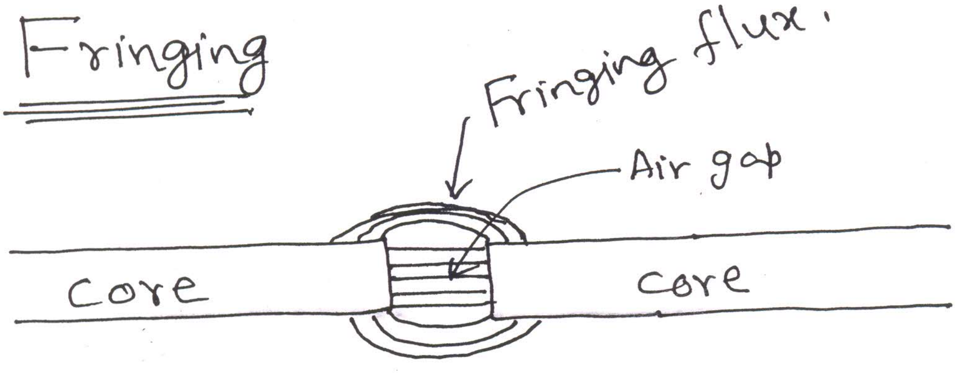


Fig. 4: Flux fringing at air-gap.

At an air-gap in a magnetic core, the flux fringes out into neighbouring air paths as shown in Fig. 4 These being of reluctance comparable to that of the gap. The result is nonuniform flux density in the air-gap (decreasing outward), enlargement of the effective air-gap area and a decrease in the average gap flux density. The fringing effect also disturbs the core flux pattern to some depth near the gap.

The effect of fringing increases with the air-gap length.

HYSTERESIS LOOP

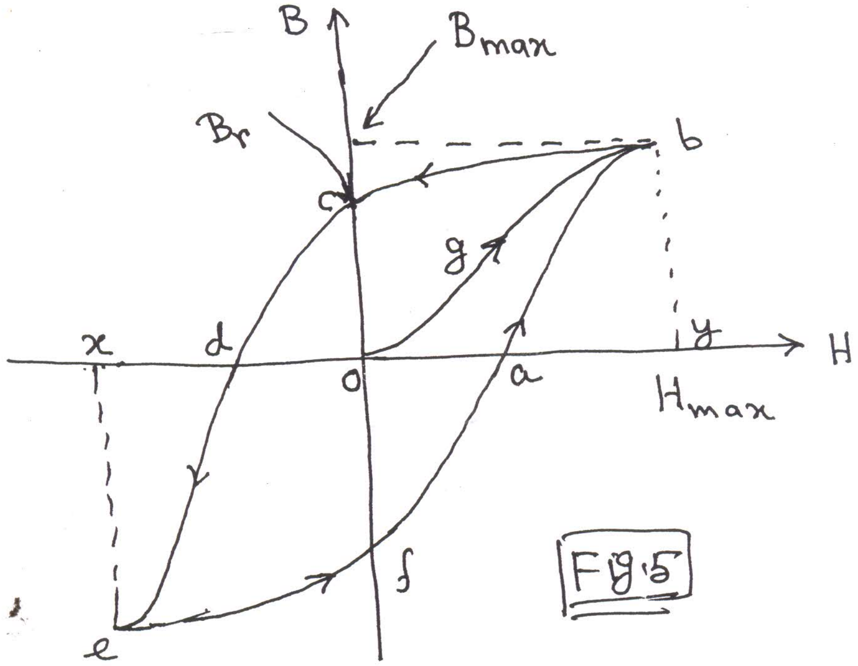


Fig 5

Let a ferromagnetic material which is completely demagnetized, i.e., one in which B = H = 0 (By reversing the magnetizing current a large number of times while at the same time gradually reducing the current to zero) be subjected to increasing values of magnetic field strength H and the corresponding flux density B measured.

The domains begin to align and the resulting relationship between B and H is shown by the curve ogb, Fig.5.

At a particular value of H, shown as oy, most of the domains will be aligned and it becomes difficult to increase the flux density any further. The material is said to be saturated. Thus by is the saturation flux density.

If the value of H is now reduced, it is found that flux density follows curve bc. When H = 0, flux remains in the iron. This remanent flux density or remanance is shown as ~~oc~~ oc = B_r = residual flux density

When H is increased in the opposite direction (~~H = ox~~), flux density decreases and when H = od, B = 0.

Magnetic field strength od required to remove the residual magnetism is called the coercive force.

HYSTERESIS AND EDDY-CURRENT LOSSES

When magnetic materials undergo cyclic variations of flux density, hysteresis and eddy-current power losses occur in them, which are together known as core loss and appear in the form of heat.

Core Loss is important in determining temperature rise, rating and efficiency of transformers, machines and other ac-operated electromagnetic devices.

Power Loss on account of hysteresis is

$$P_h = K_h \cdot f \cdot B_{max}^{n} \cdot V \quad \text{Watt.}$$

where

K_h = characteristic constant of core material

n = Steinmetz exponent, range 1.5 – 2.0, typical value ~~=~~ 1.6

V = Volume of the material (m^3)

f = supply frequency (Hz).

Eddy current power loss is given by

$$P_e = K_e f^2 B_{max}^2 V. \text{ Watt.}$$

K_e = characteristic constant of the core.

MAGNETICALLY COUPLED CIRCUITS

When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled.

MUTUAL INDUCTANCE

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as mutual inductance.

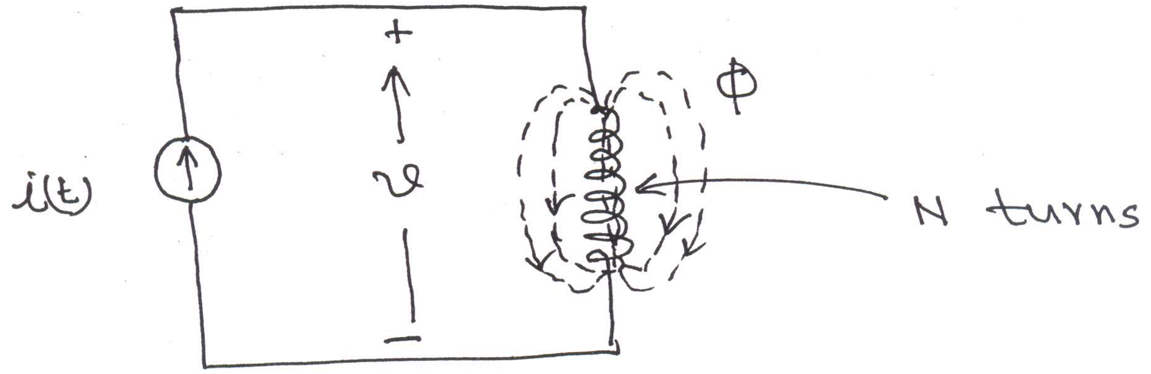


Fig. 6: magnetic flux produced by a single coil with N turns

According to Faraday's Law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of the magnetic flux; that is,

$$\boxed{v = N \cdot \frac{d\phi}{dt}}$$

But the flux ϕ is produced by current i so that any change in ϕ is caused by a change in the current. Hence,

$$v = N \cdot \frac{d\phi}{di} \cdot \frac{di}{dt}$$

$$\boxed{\therefore \quad v = L \cdot \frac{di}{dt}}$$

Thus, $\boxed{L = N \cdot \frac{d\phi}{di}} \Longrightarrow$ inductance of the coil commonly called as <u>self-inductance</u>.

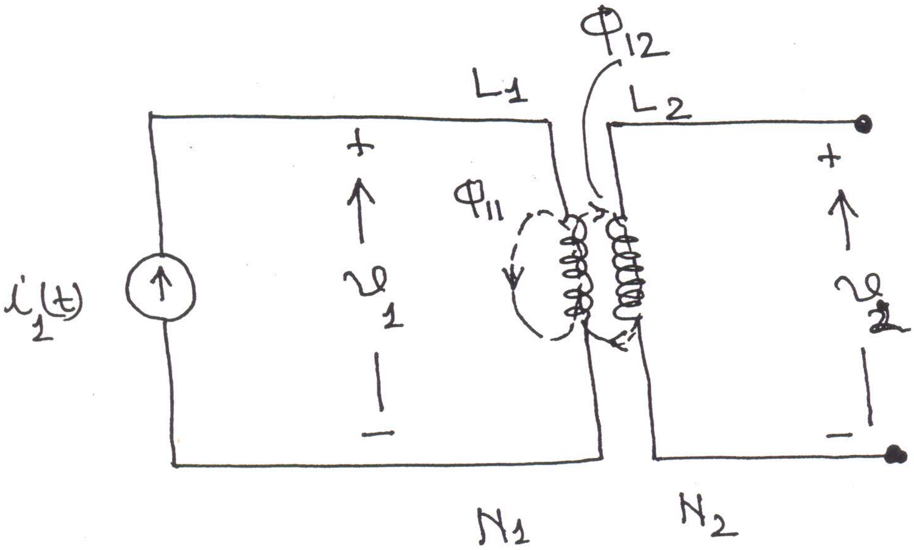


Fig. 7: Mutual inductance M_{21} of coil 2 with respect to coil 1.

L_1 = self-inductance of coil 1

L_2 = self-inductance of coil 2

N_1 = Number of turns of coil 1

N_2 = Number of turns of coil 2

ϕ_1 = magnetic flux emanating from coil 1

ϕ_{11} = one component of ϕ_1 links coil 1 only

ϕ_{12} = another component of ϕ_1 links both coil 1 and coil 2.

Hence,

$$\boxed{\phi_1 = \phi_{11} + \phi_{12}}$$

Although the two coils are physically separated, they are said to be **magnetically coupled.**

Since the entire flux ϕ_1 links coil 1, the voltage induced in coil 1 is,

$$\boxed{v_1 = N_1 \frac{d\phi_1}{dt}}$$

only flux ϕ_{12} links coil 2, so the voltage induced in coil 2 is

$$\boxed{v_2 = N_2 \cdot \frac{d\phi_{12}}{dt}}$$

Again, as the fluxes are caused by the current i_1 flowing in coil 1, we can write

$$\boxed{v_1 = N_1 \cdot \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} = L_1 \cdot \frac{di_1}{dt}}$$

where

$$L_1 = N_1 \frac{d\phi_1}{di_1} = \text{self-inductance of coil 1}$$

Similarly,

$$\boxed{v_2 = N_2 \cdot \frac{d\phi_{12}}{di_1} \cdot \frac{di_1}{dt} = M_{21} \cdot \frac{di_1}{dt}}$$

where,

$$M_{21} = N_2 \cdot \frac{d\phi_{12}}{di_1}$$

M_{21} = <u>mutual inductance</u> of coil 2 with ~~with~~ respect to coil 1.

Subscript 21 indicates that inductance M_{21} relates the voltage induced in coil 2 to the current in coil 1

Thus, the open-circuit mutual voltage (or induced voltage) across coil 2 is

$$\boxed{v_2 = M_{21} \frac{di_1}{dt}}$$

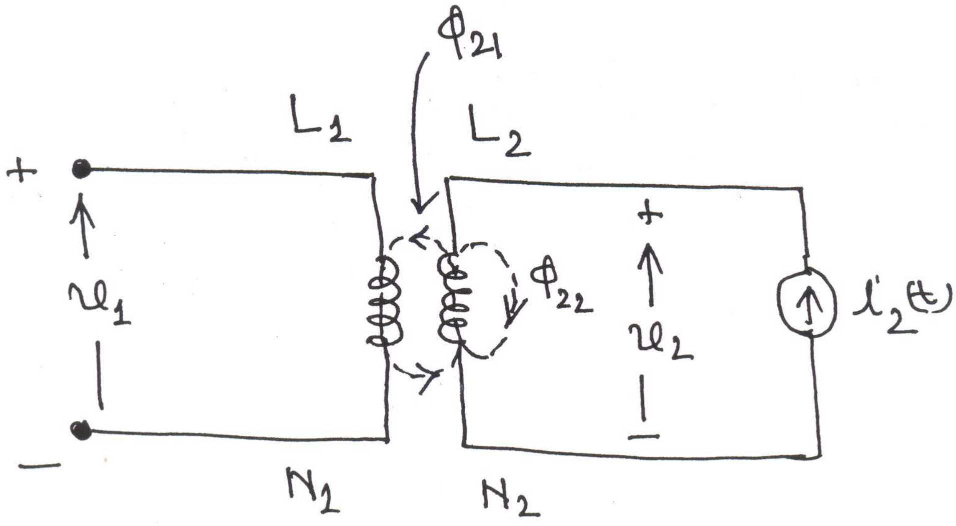


Fig. 8: Mutual inductance M_{12} of coil 1 with respect to coil 2.

Summary

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = \left(N_2 \frac{d\phi_2}{di_2}\right) \frac{di_2}{dt} = L_2 \cdot \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = \left(N_1 \frac{d\phi_{21}}{di_2}\right) \cdot \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$\therefore \quad \boxed{v_1 = M_{12} \frac{di_2}{dt}}$$

M_{12} and M_{21} are equal, that is

$$\boxed{M_{12} = M_{21} = M}$$

Keep in mind that mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources.

Recall that inductors act like short circuit to dc.

The polarity of mutual voltage $\left(M \cdot \frac{di}{dt}\right)$ is not easy to determine, because four terminals are involved.

The choice of the correct polarity for $M\frac{di}{dt}$ is made by examining the orientation or particular way in which both coils are physically wound and applying Lenz's law in conjunction with the right hand rule.

We will apply the <u>dot convention</u> in circuit analysis.

By this convention, a dot is placed in the circuit at one end of each of the two magnetically coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.

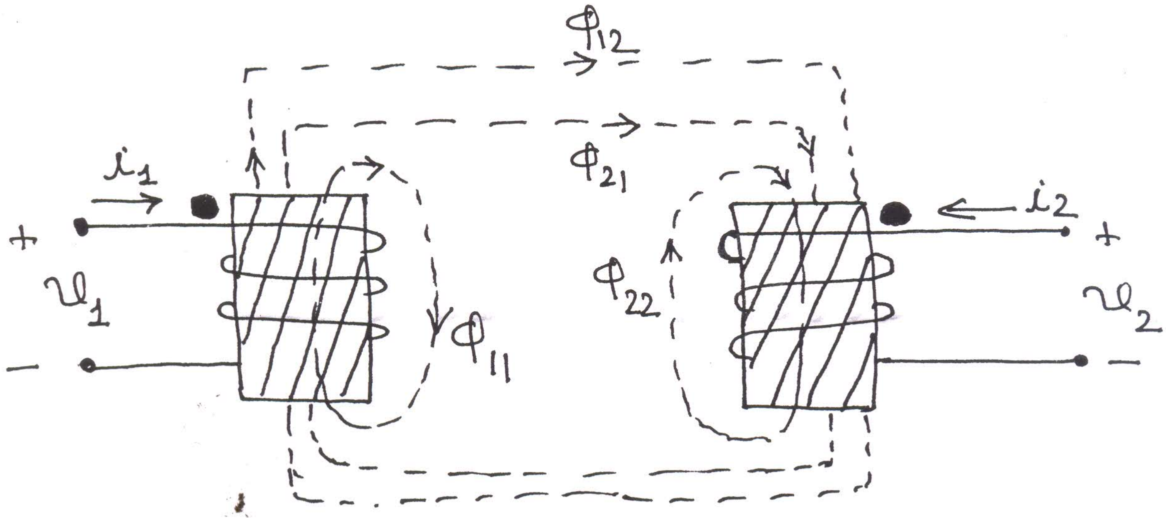


Fig. 9: Illustration of dot convention.

Dots are used to determine the polarity of the mutual voltage.

Dot convention is stated as follows:

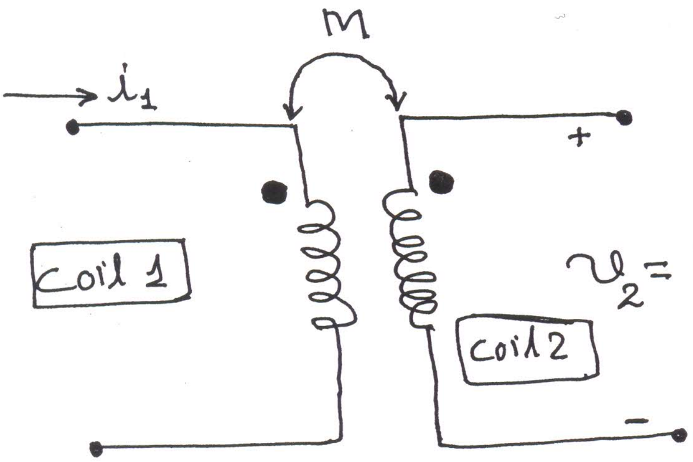
⇓

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

ALTERNATIVELY

⇓

If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.



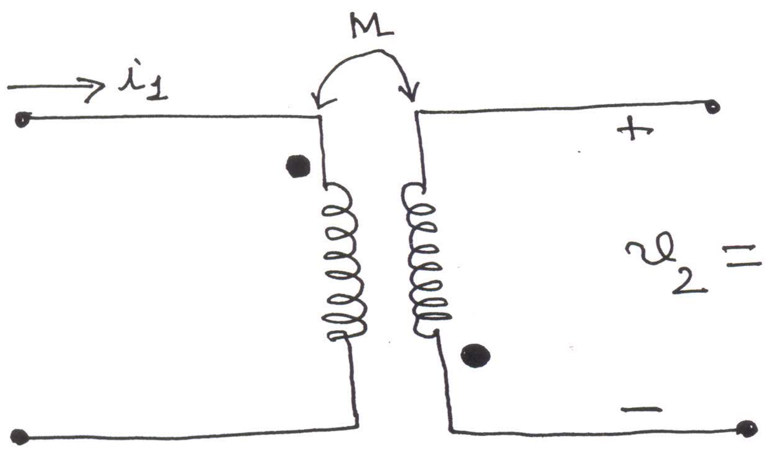
$$v_2 = M.\frac{di_1}{dt}$$

Fig.10

In Fig.10, the sign of the mutual voltage v_2 is determined by the reference polarity for v_2 and direction of i_1.

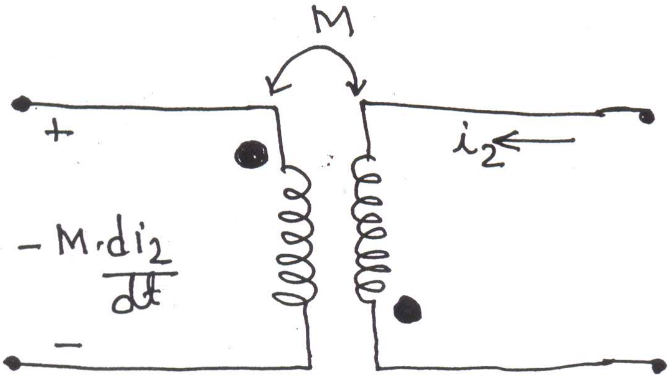
$\Rightarrow$ i_1 enters the dotted terminal of coil 1 and v_2 is positive at the dotted terminal of coil 2.

Mutual voltage is $+ M.\frac{di_1}{dt}$.



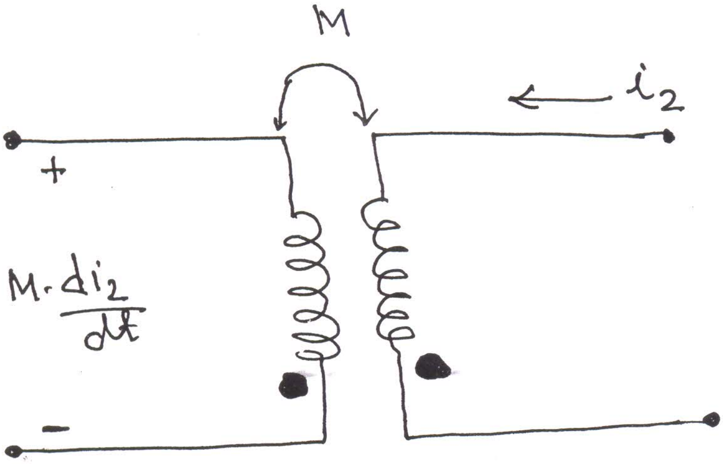
$$v_2 = - M.\frac{di_1}{dt}.$$

Fig.11



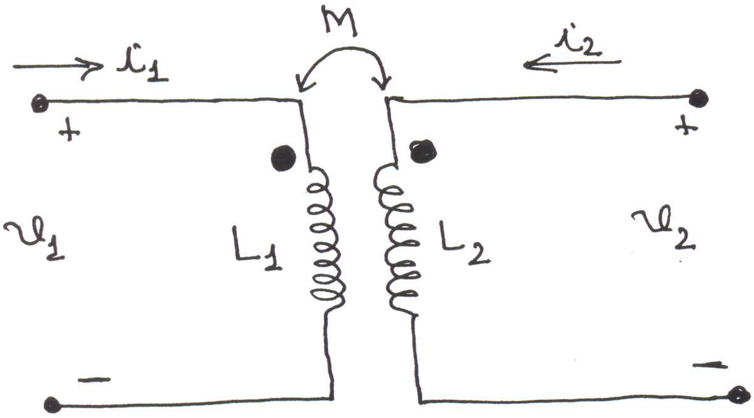
$$v_1 = -M \cdot \frac{di_2}{dt}$$

Fig. 12



$$v_1 = M \cdot \frac{di_2}{dt}$$

Fig. 13



$$\left.\begin{aligned} v_1 &= L_1 \cdot \frac{di_1}{dt} + M \frac{di_2}{dt} \\ v_2 &= L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned}\right\} \quad (1)$$

Fig. 14(a) Time-domain circuit.

Reversing the dots,
OR
Reversing the assumed direction of currents or voltages in either winding, will change the sign of mutual terms in Eqn. (1)

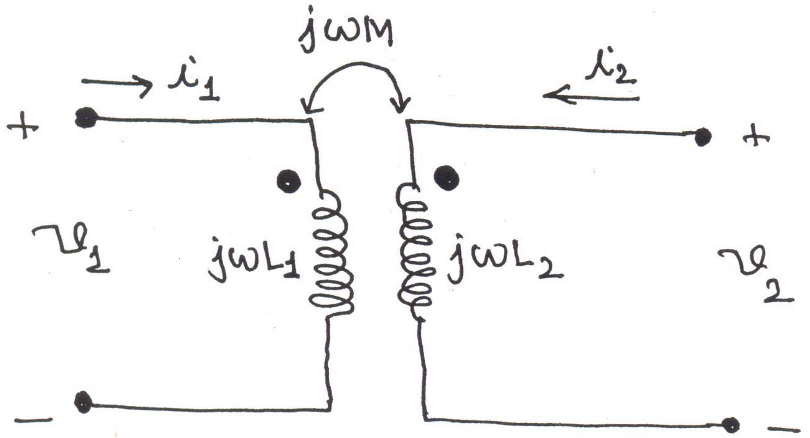


Fig. 14 b Frequency-domain circuit

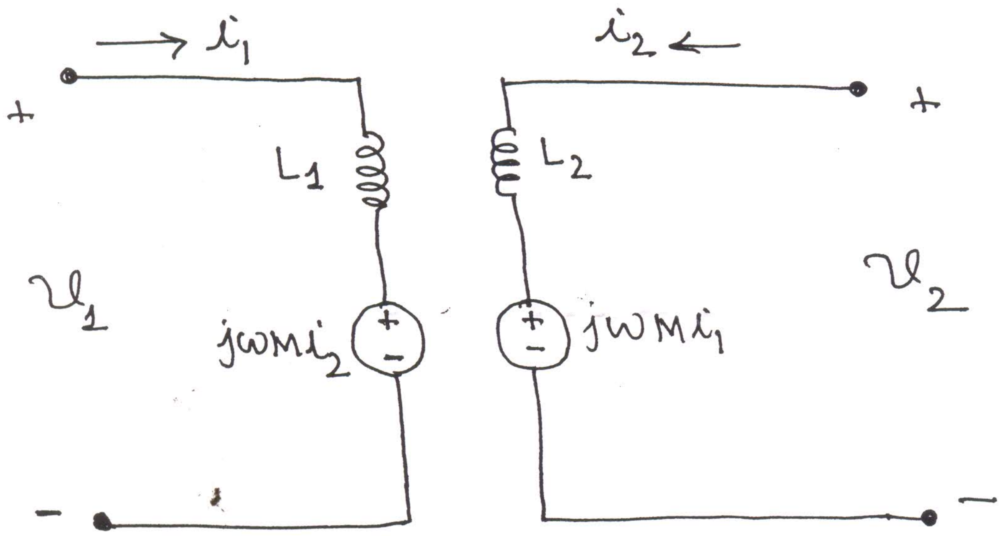
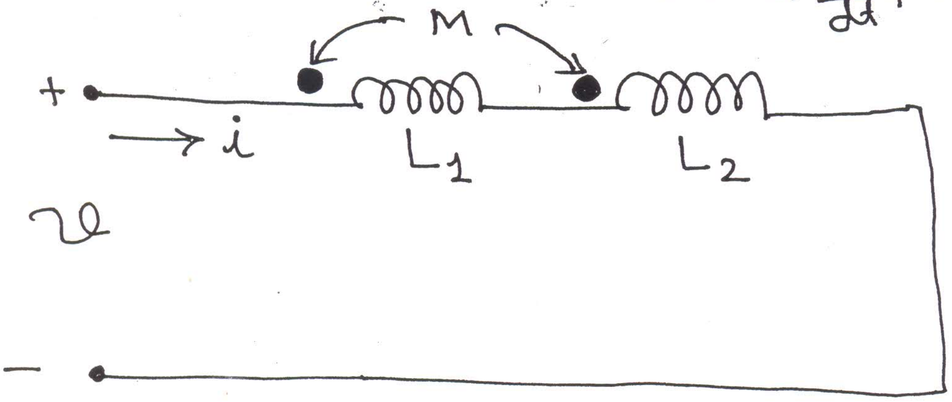


Fig. 14c Mutual inductance as Voltage generators

TWO COILS ARE IN SERIES

$$v = L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt}$$



$$\therefore V = (L_1 + L_2 + 2M)\frac{di}{dt} \quad \text{---- (2)}$$

Fig. 15a : Time domain circuit

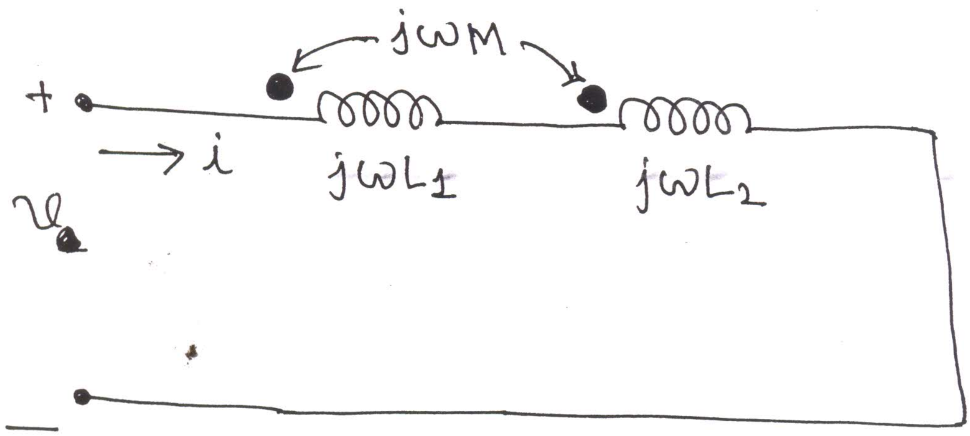


Fig. 15b Frequency domain circuit

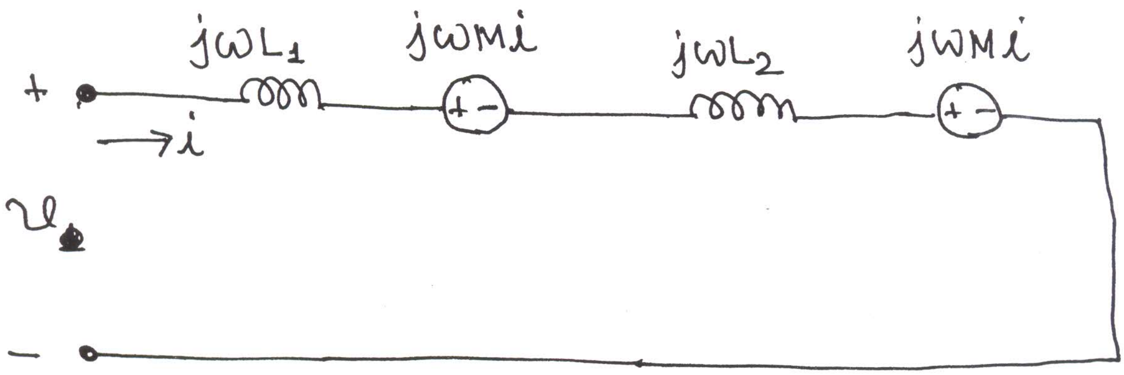


Fig. 15c Circuit with mutual-inductance Voltage generators.

From Fig. 15(c),

$$\cancel{i = \frac{v}{j\omega(L_1+L_2+2M)}} \quad \cdots \cancel{(3)}$$

$$i(j\omega L_1 + j\omega L_2) + j\omega(2M)i = v \quad \cdots (3)$$

$$\therefore \quad i = \frac{v}{j\omega(L_1+L_2+2M)} \quad \cdots (4)$$

If the dot on one coil were reversed, the sign of the mutual term either in Eqn. (2) or in Eqn. (3) would be minus.

Therefore, equivalent inductance of two mutually coupled coils connected in series is

$$L_{eq} = L_1 + L_2 \pm 2M \quad \cdots (5)$$

Since the net inductance must be positive,

$$L_1 + L_2 \geq 2M \quad \cdots (6)$$

TWO COILS ARE IN PARALLEL

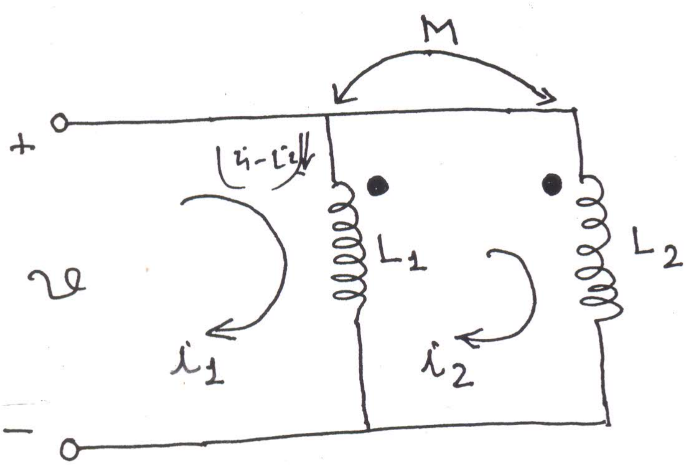


Fig.16(a) : Time-domain circuit

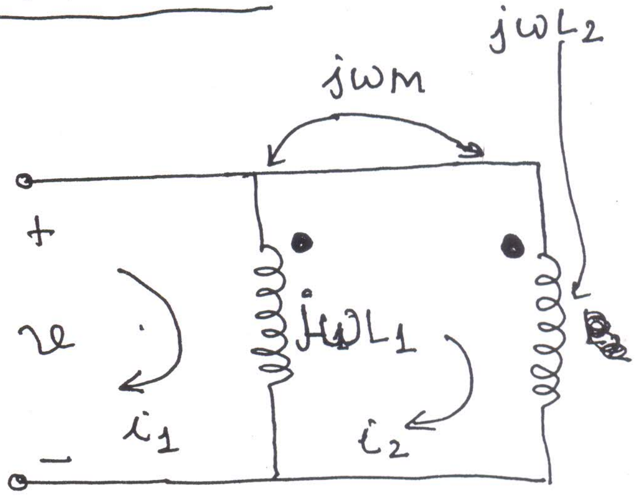


Fig.16(b) Frequency-domain circuit

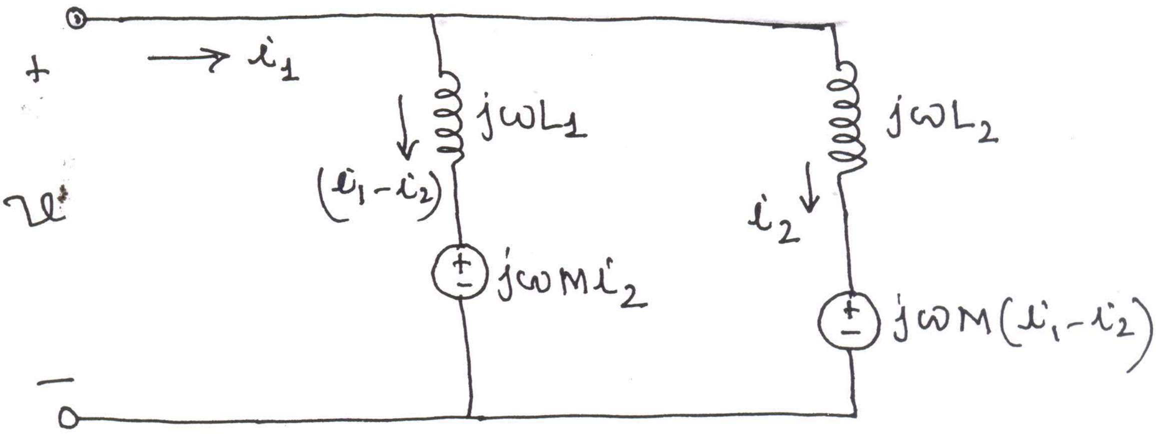


Fig.16(c) : Circuit with mutual-inductance voltage generators.

Equivalent impedance of the parallel combination is given by

$$Z_{eq} = \frac{v}{I_1} = \frac{j\omega(L_1L_2 - M^2)}{(L_1+L_2-2M)} \quad \cdots (7)$$

Equivalent inductance,

$$L_{eq} = \frac{(L_1L_2 - M^2)}{(L_1+L_2-2M)} \quad \cdots (8)$$

The sign of M in the denominator of Eqn.(8) changes with the dot polarity, but the numerator does not change since M is squared.

In Eqn.(6), we have established that the ~~expression in the~~

$$\boxed{L_1 + L_2 \geq 2M}$$

Since overall inductance must be positive

$$L_1L_2 - M^2 \geq 0 \quad \cdots (9)$$

From Eqn.(6),

$$M \leq \left(\frac{L_1+L_2}{2}\right) \quad \cdots (10)$$

From eqn.(9),

$$M \leq \sqrt{L_1 L_2} \quad \text{--- (11)}$$

Eqn.(10) states that the mutual inductance must be less than the arithmetic mean of L_1 and L_2.

While, Eqn.(11) states that the mutual inductance must be less than the geometric mean of L_1 and L_2.

But

(geometric mean of L_1 and L_2) Less than (the arithmetic mean of L_1 and L_2) except when the two inductances are equal.

Maximum value of mutual inductance is

$$M_{max} = \sqrt{L_1 L_2} \quad \text{--- (12)}$$

From eqn.(11)

$$\frac{M}{\sqrt{L_1 L_2}} \leq 1 \quad \text{--- (13)}.$$

Let us define,

$$K = \frac{M}{\sqrt{L_1 L_2}} = \text{coupling coefficient}$$

$$0 \leq K \leq 1$$

Physical meaning of $K = 1$ is that all the flux produced by the current in one of the coils links the other.

Iron-core transformer, $\Rightarrow K \simeq 1.0$

Air-core coils $\Rightarrow K << 1.0$ (very small).

REFLECTED IMPEDANCE

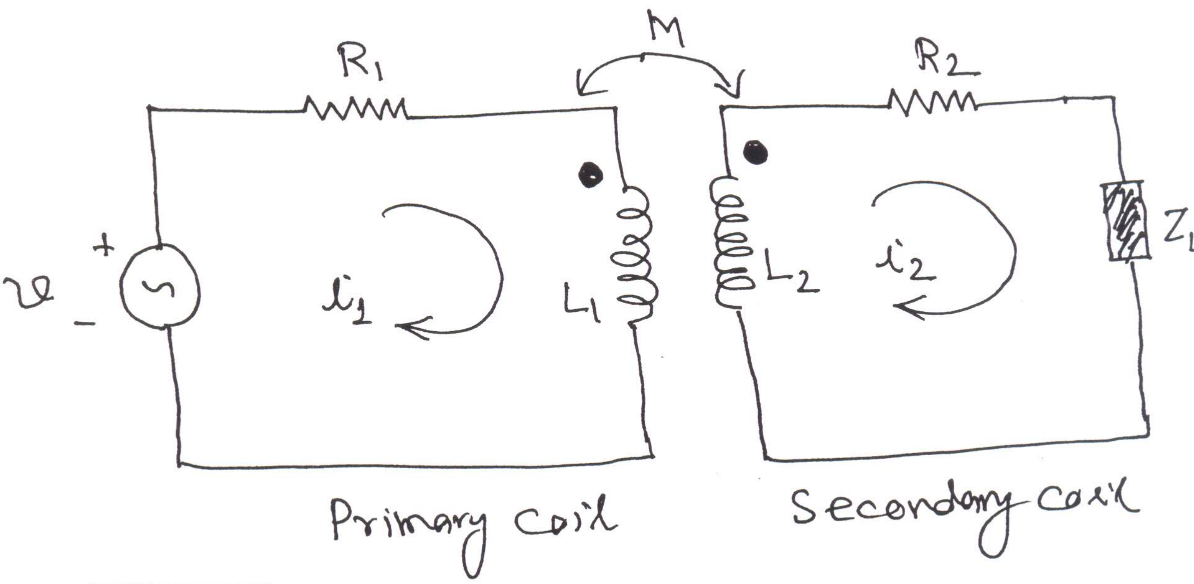


Fig. 17

$$v = (R_1 + jL_1\omega)i_1 - j\omega M i_2 \quad \cdots (14)$$

$$0 = (R_2 + j\omega L_2 + Z_L)i_2 - j\omega M i_1 \quad \cdots (15)$$

$$\Downarrow$$

$$i_2 = \frac{j\omega M i_1}{(R_2 + j\omega L_2 + Z_L)} \quad \cdots (16)$$

From eqns. (14) & (15), we get

$$\frac{v}{i_1} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \quad \cdots (17)$$

We get the input impedance as

$$Z_{in} = \frac{v}{i_1} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \quad \cdots (18)$$

INDUCED EMF

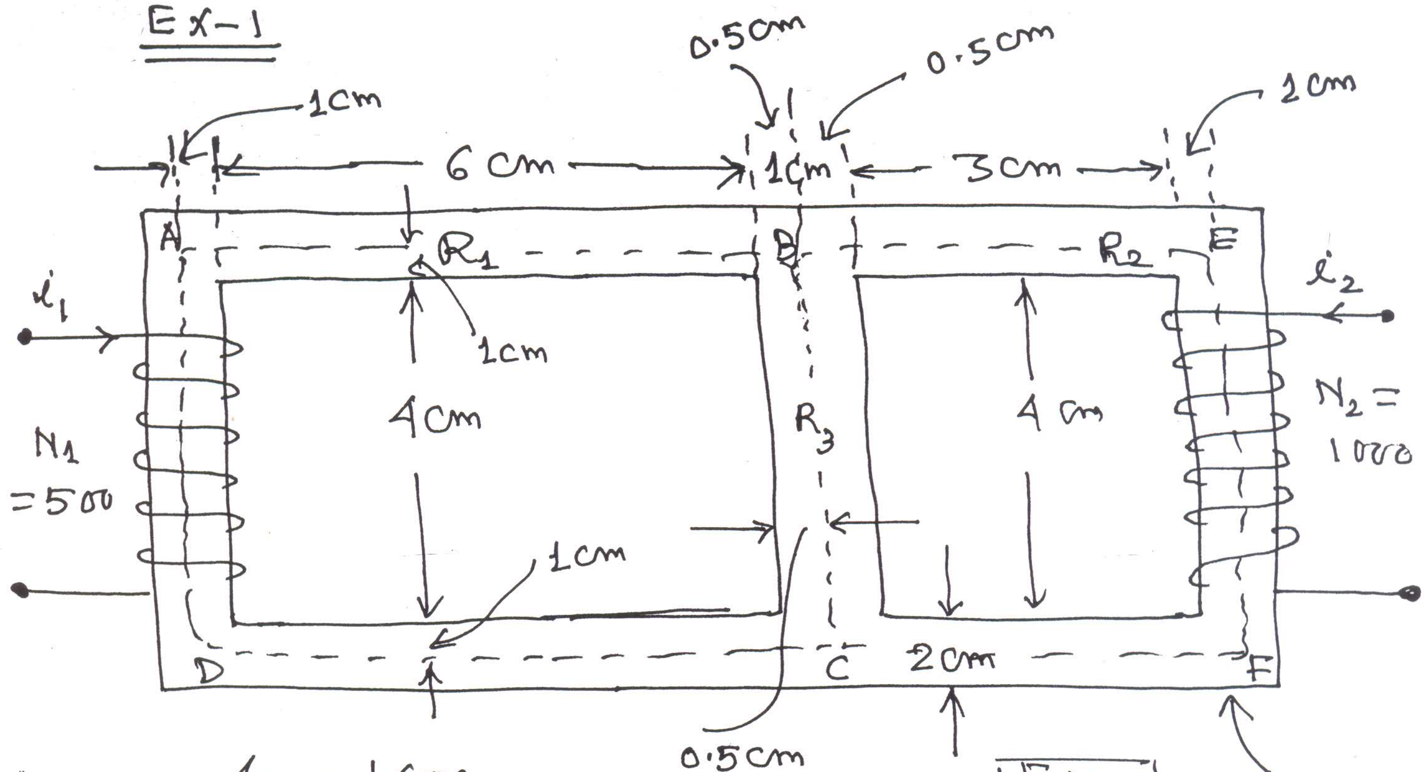
$\phi = \phi_{max} \sin(\omega t)$.

~~$v_0 = N_p$~~ $v = N \cdot \frac{d\phi}{dt} = N\phi_{max}\,\omega \cos(\omega t)$

$\therefore \quad v = 2\pi f N \phi_{max} \cos(\omega t)$

$$\boxed{\therefore \quad v_{r.m.s} = \sqrt{2}\,\pi f N \phi_{max} = 4.44 f N \phi_{max}.}$$

Ex-1



$\mu_r = 1600$

Fig. 18

Determine L_1, L_2, M_{12} and M_{21}

Each coil is excited with 1 Amp current.

Soln.

$$\mathcal{R} = \frac{l}{A\mu} = \frac{l}{A\mu_0\mu_r} \quad A\text{-}T/Wb$$

$$\phi = \frac{F_m}{\mathcal{R}} \quad Wb$$

$$l_1 = (6+1+0.5)\times 2 + (4+2) = 21 \text{ cm} = 0.21 \text{ mt}$$

$$l_2 = (3+1+0.5)\times 2 + (4+2) = 15 \text{ cm} = 0.15 \text{ mt}$$

$$l_3 = (4+2) = 6 \text{ cm} = 0.06 \text{ mt.}$$

$$A = 2\times 2 = 4 \text{ cm}^2 = 4\times 10^{-4} \text{ mt}^2$$

$$R_1 = \frac{l_1}{A\mu_0\mu_r} = \frac{0.21}{4\times10^{-4}\times 4\pi\times10^{-7}\times 1600} \text{ A-T/Wb.}$$

$$\therefore R_1 = 261113.5785 \text{ A-T/Wb}$$

$$R_2 = \frac{l_2}{A\mu_0\mu_r} = \frac{0.15}{4\times10^{-4}\times 4\pi\times10^{-7}\times 1600}$$

$$\therefore R_2 = 186509.7 \text{ A-T/Wb}$$

$$R_3 = \frac{l_3}{A\mu_0\mu_r} = \frac{0.06}{\cancel{4\times10^{-4}}\,(2\times1\times10^{-4})\times 4\pi\times10^{-7}\times 1600}$$

$$\therefore R_3 = 149207.76 \text{ A-T/Wb}$$

$$F_{m1} = N_1 I_1 = 500\times1 = 500 \text{ A-T}$$

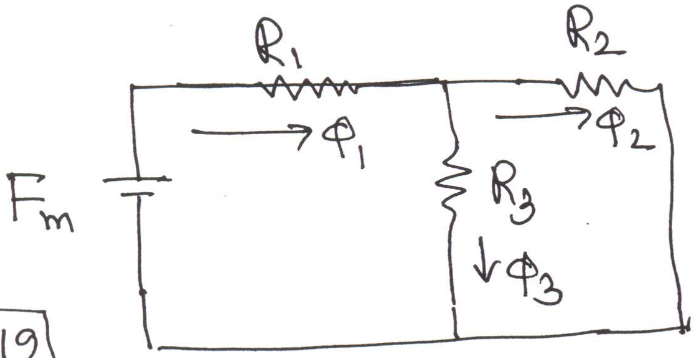


Fig. 19

$$R = R_1 + \frac{R_2\times R_3}{(R_2+R_3)}$$

$$\therefore R = 344006.78 \text{ A-T/Wb}$$

$$\Phi_1 = \frac{F_{m1}}{R} = \frac{500}{344006.78} \text{ Wb} = 1.453 \text{ mWb}$$

$$\Phi_2 = \Phi_{21} = \Phi_1 \times \frac{R_3}{(R_3+R_2)} = 0.646 \text{ mWb}$$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{500 \times 1.453}{1} \text{ mH}$$

$$\therefore L_1 = 0.7265 \text{ H}$$

$$M_{21} = \frac{N_2 \phi_{21} (= \phi_2)}{I_1} = \frac{1000 \times 0.646}{1} \text{ mH}$$

$$\therefore M_{21} = 0.646 \text{ H}$$

Similarly compute L_2, M_{12} by exciting coil-2 ($i_2 = 1$ amp, $i_1 = 0$).

EX-2

When both the coils are excited by 1 Amp current, ~~determine~~ determine ϕ_3.

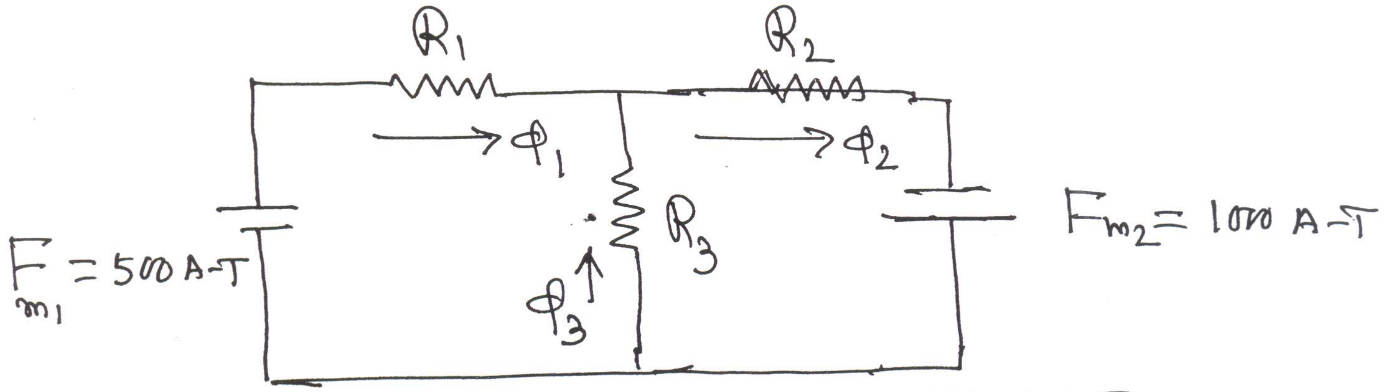


Fig. 20

$$\phi_1 = 2.745 \text{ mWb}$$
$$\phi_2 = 4.198 \text{ mWb}$$
$$\phi_3 = 1.453 \text{ mWb.}$$

Ex-3

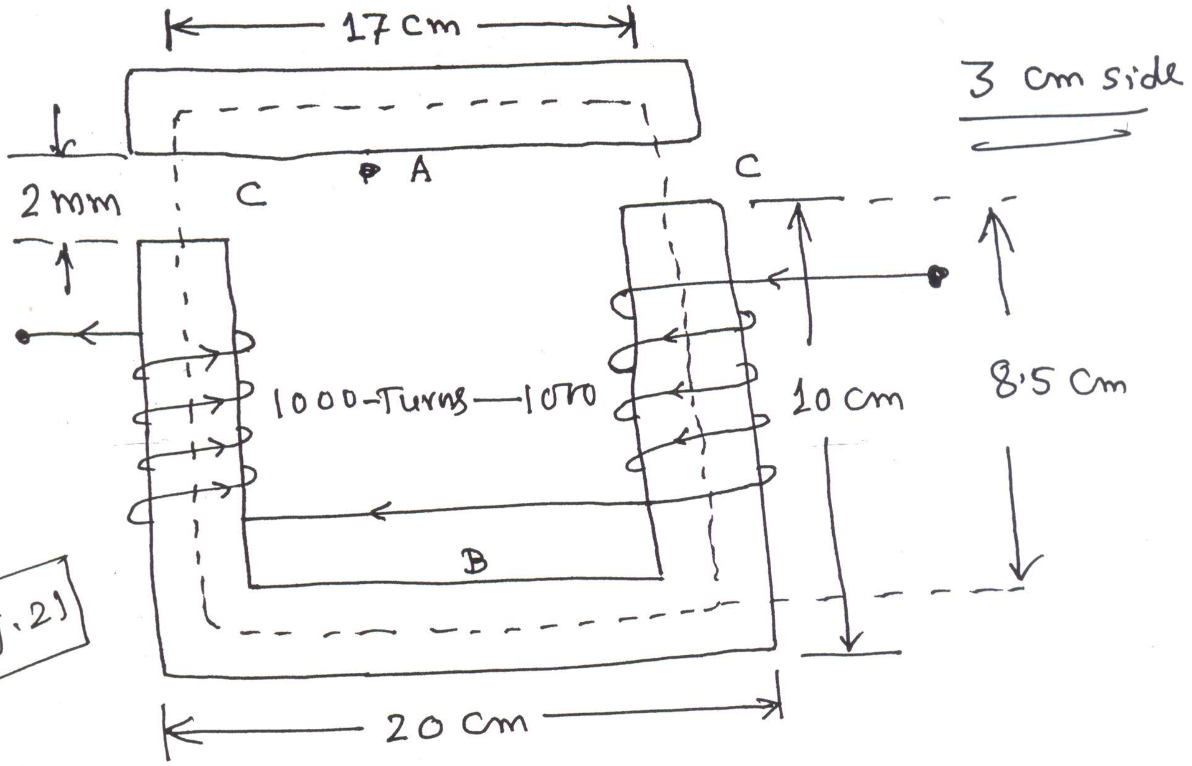


Fig. 21

$\mu_{rA} = 1000, \quad \mu_{rB} = 1200$

Soln.

$A = 3 \times 3 \text{ cm}^2 = 9 \times 10^{-4} \text{ mt}^2$

$l_A = 17 \text{ cm} = 0.17 \text{ mt}$

$l_B = (17 + 8.5 + 8.5) = 34 \text{ cm} = 0.34 \text{ mt}$

$R_A = \dfrac{l_A}{A \mu_0 \mu_{rA}} = 15.03 \times 10^4 \text{ AT/Wb}$

$R_B = \dfrac{l_B}{A \mu_0 \mu_{rB}} = 25.04 \times 10^4 \text{ AT/Wb}$

$R_C = \dfrac{2 \times l_c}{A \mu_0} = \dfrac{2 \times 0.002}{9 \times 10^{-4} \times 4\pi \times 10^{-7}} = 353.5 \times 10^4 \text{ AT/Wb}$

$\therefore R = (R_A + R_B + R_C) = 393.57 \times 10^4 \text{ AT/Wb.}$

$$mmf = (1000 \times 1 + 1000 \times 1) = 2000\ AT$$

$$\phi = \frac{mmf}{R} = \frac{2000}{393.57 \times 10^4}$$

$$\therefore \phi = 5.08 \times 10^{-4}\ Wb.$$

$$B = \frac{\phi}{A} = 0.564\ Wb/mt^2$$

EX-4

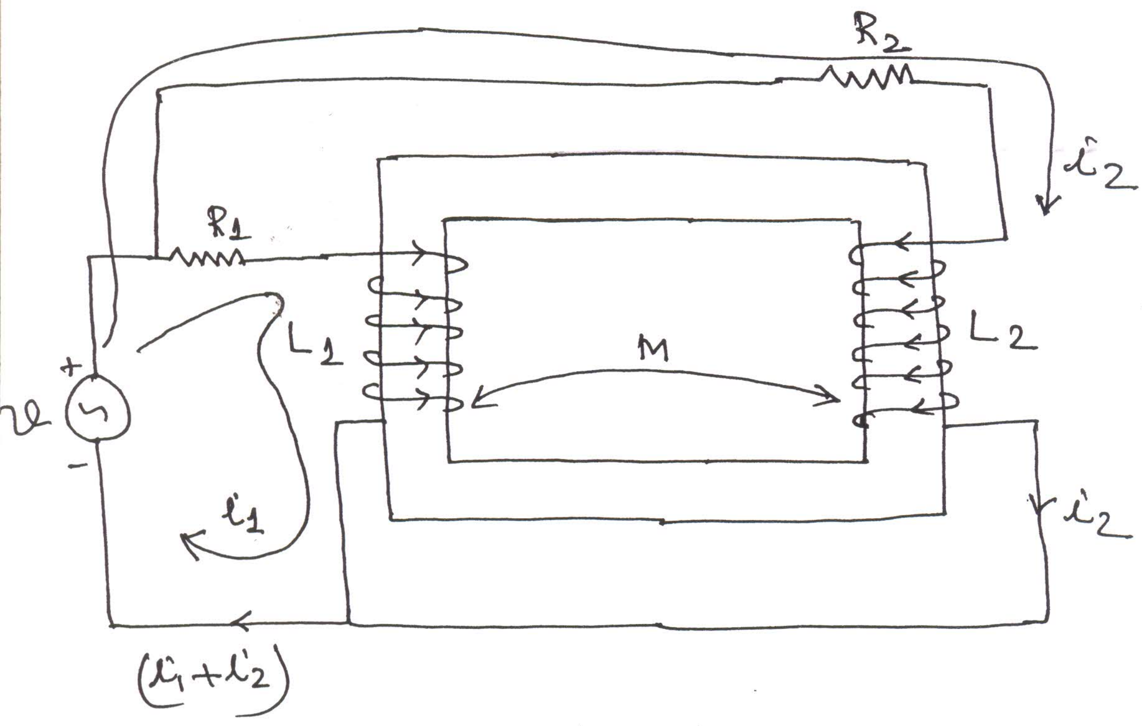
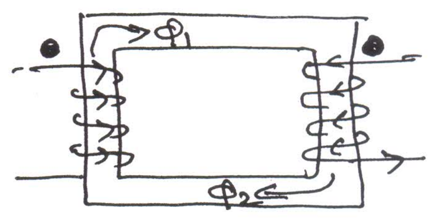


Fig. 2



$$\left.\begin{aligned} R_1 i_1 + L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} &= v \\ R_2 i_2 + L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} &= v \end{aligned}\right\}$$

Ex-5

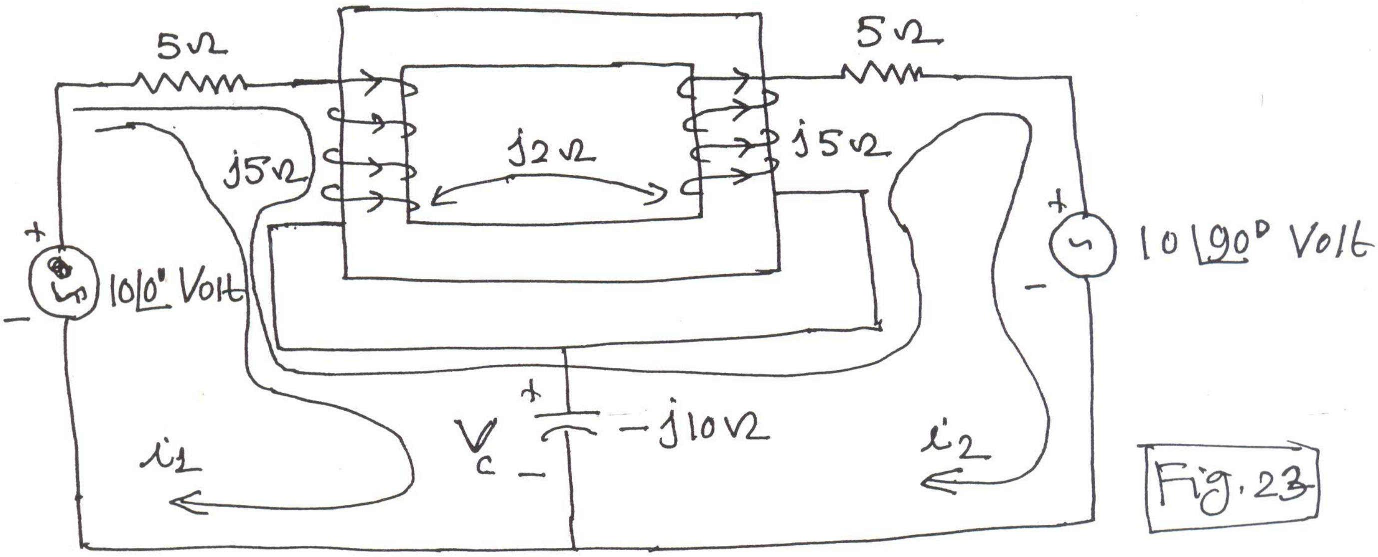
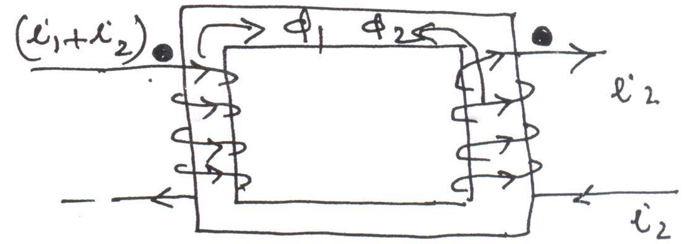


Fig. 23

$$\boxed{V_c = ?}$$



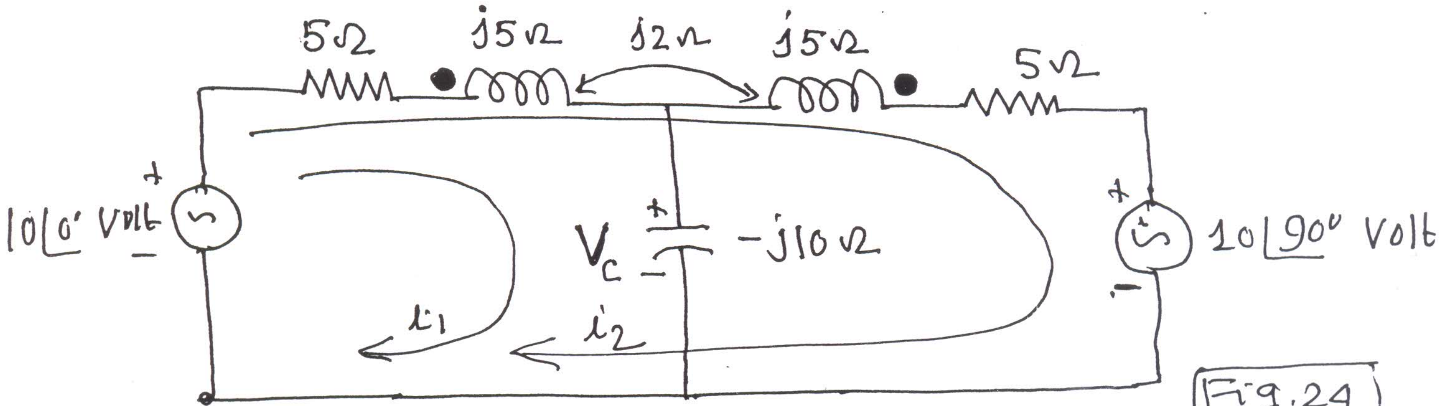


Fig. 24

$$(5+j5)(i_1+i_2) - j10\, i_1 - j2\, i_2 = 10\angle 0^\circ$$

$$\boxed{\therefore \quad (5-j5)\, i_1 + (5+j3)\, i_2 = 10} \quad \text{--- (1)}$$

$$(5+j5)(i_1+i_2) + (5+j5)\, i_2 - j2(i_1+i_2) - j2 i_2 + 10\angle 90^\circ - 10\angle 0^\circ = 0$$

$$\boxed{\therefore \quad (5+j3)\, i_1 + (10+j6)\, i_2 = 10 - j10} \quad \text{--- (2)}$$

Solving Eqns.(1) & (2), we get

$$i_1 = 1.015 \angle 113.96^\circ \text{ Amp}$$

$$\therefore\ v_c = i_1(-j10) = 10.15 \angle 23.96^\circ \text{ Volt.}$$

EX-6

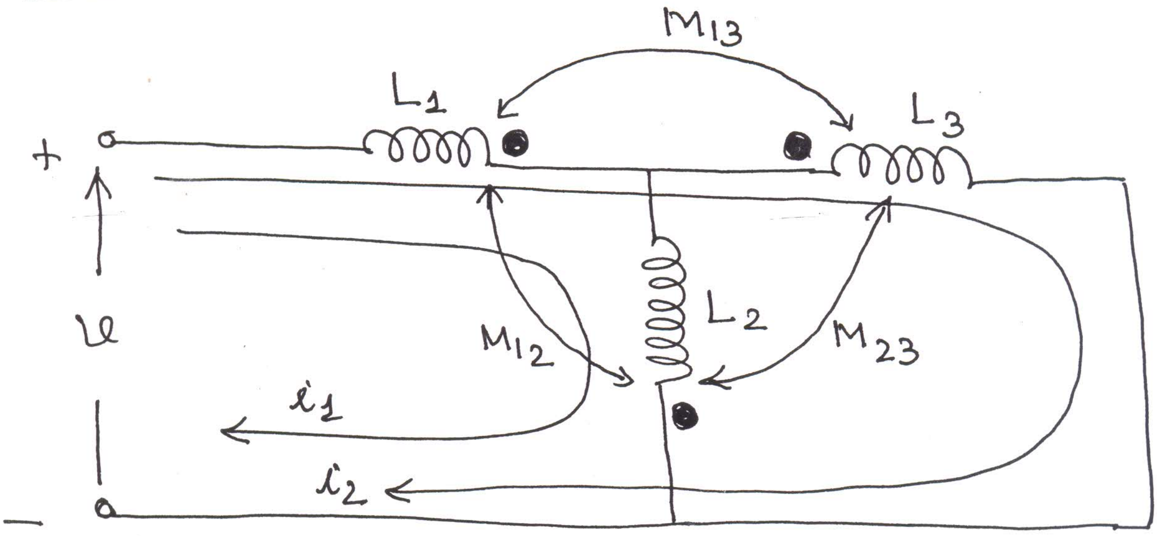


Fig. 25

$$L_1 \frac{d}{dt}(i_1+i_2) + L_2 \frac{di_1}{dt} - M_{13}\frac{di_2}{dt} - M_{23}\frac{di_2}{dt}$$
$$+ M_{12}\frac{di_1}{dt} + M_{12}\frac{d}{dt}(i_1+i_2) - v = 0$$

$$\boxed{\therefore\ (L_1+L_2+2M_{12})\frac{di_1}{dt} + (L_1+M_{12}-M_{13}-M_{23})\frac{di_2}{dt} = v \quad \text{--- (1)}}$$

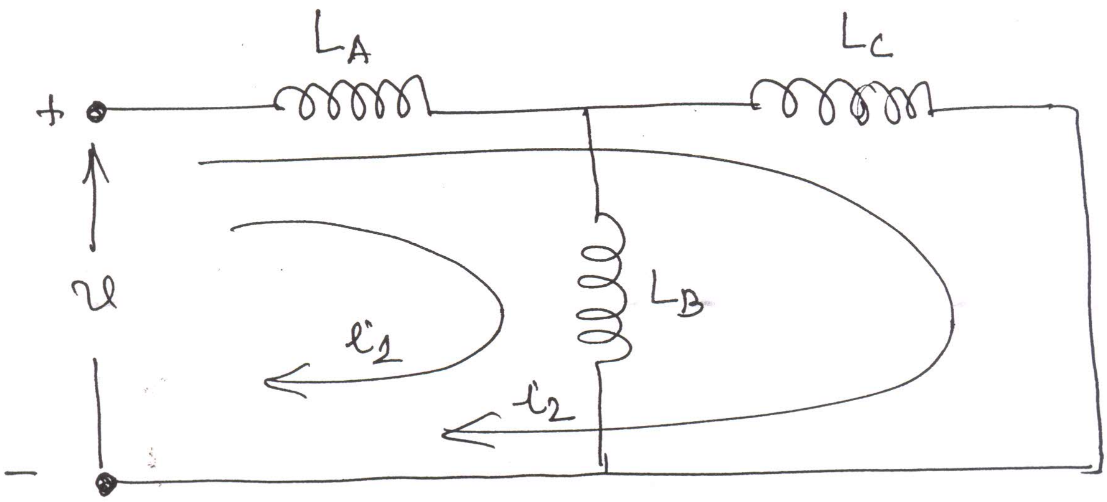
$$L_1 \frac{d}{dt}(i_1+i_2) + L_3 \frac{di_2}{dt} - M_{13}\frac{di_2}{dt} - M_{13}\frac{d}{dt}(i_1+i_2)$$
$$+ M_{12}\frac{di_1}{dt} - M_{23}\frac{di_1}{dt} = v$$

$$\boxed{\therefore\ (L_1+M_{12}-M_{13}-M_{23})\frac{di_1}{dt} + (L_1+L_3-2M_{13})\frac{di_2}{dt} = v \quad \text{--- (2)}}$$

In frequency domain, replace $\frac{d}{dt}$ by $j\omega$

$$\therefore \quad j\omega(L_1+L_2+2M_{12})\,i_1 + j\omega(L_1+M_{12}-M_{13}-M_{23})\,i_2 = v \quad \text{--- (3)}$$

$$j\omega(L_1+M_{12}-M_{13}-M_{23})\,i_1 + j\omega(L_1+L_3-2M_{13})\,i_2 = v \quad \text{--- (4)}$$



$$\boxed{\begin{aligned} L_A &= L_1 + M_{12} - M_{13} - M_{23} \\ L_B &= L_2 + M_{12} + M_{13} + M_{23} \\ L_C &= L_3 - M_{12} - M_{13} + M_{23} \end{aligned}}$$

Ex-7

Two coupled coils with respective self-inductances $L_1 = 0.05H$ and $L_2 = 0.2\,H$ have a coupling coefficient $K = 0.5$. Coil 2 has 1000 turns. If the current in coil 1 is $i_1 = 5\sin(400t)$ Amp, determine the voltage at coil 2 and the maximum flux set up by coil 1.

Soln , we know

$$\frac{M}{\sqrt{L_1 L_2}} = K \quad \therefore \quad M = K\sqrt{L_1 L_2} = 0.5\sqrt{(0.05)(0.2)}$$

$$\therefore \quad M = 0.05\,H.$$

Voltage at coil 2 is given by

$$v_2 = M.\frac{di_1}{dt} = 0.05\,\frac{d}{dt}\left(5\sin(400t)\right)$$

$$\boxed{\therefore \quad v_2 = 100\cos(400t)}$$

Also

$$v_2 = N_2.\frac{d\phi_{12}}{dt}$$

$$\therefore\ 100\cos(400t) = 1000\,\frac{d\phi_{12}}{dt}$$

$$\therefore\ \phi_{12} = 10^{-3}\int 100\cos(400t)\,dt$$

$$\therefore\ \phi_{12} = (0.25\times10^{-3})\sin(400t)\ \text{Wb.}$$

$$\therefore\ \phi_{12}^{max} = 0.25\times10^{-3}\ \text{Wb} = 0.25\ \text{mWb.}$$

Also

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} = \text{coupling coefficient}$$

$$\therefore\ \phi_1^{max} = \frac{\phi_{12}^{max}}{k} = \frac{0.25}{0.5}\ \text{mWb}$$

$$\boxed{\therefore\ \phi_1^{max} = 0.5\ \text{mWb}}$$

Ex-8

Determine the Voltage across the $5\,\Omega$ resistor for the dots given in the diagram. Then reverse the polarity in one coil and repeat

$\boxed{j5.66\Omega}$

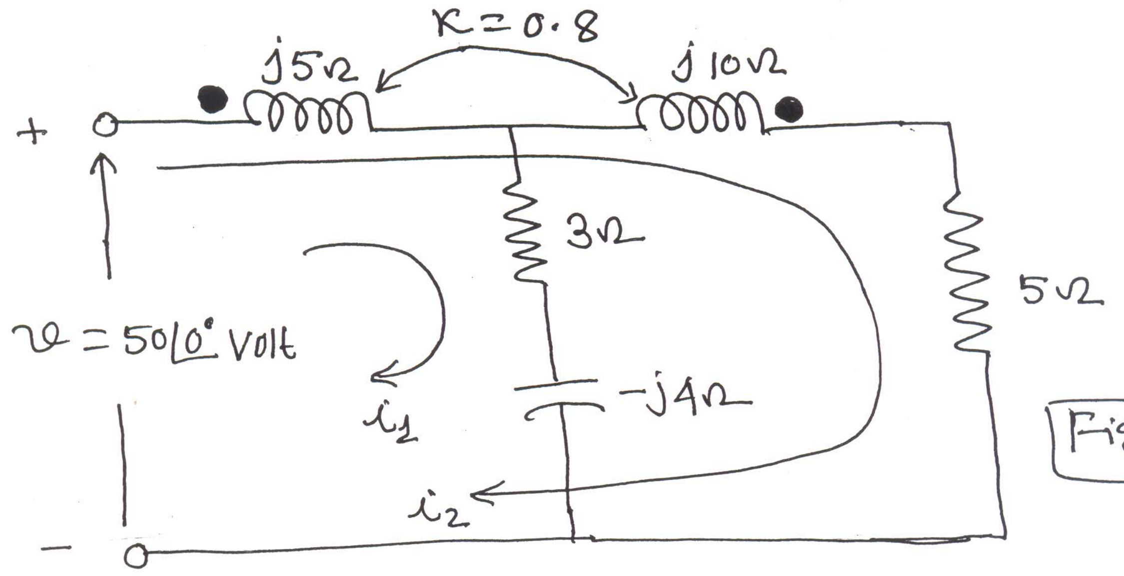


Fig. 26

Soln.

We know

$$X_{L1} = 5\Omega, \quad X_{L2} = 10\Omega$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$\therefore \quad M = K\sqrt{L_1 L_2}$$

$$\therefore \quad \omega M = K\sqrt{(\omega L_1)(\omega L_2)} = K\sqrt{X_{L1} X_{L2}}$$

$$\therefore \quad X_m = 0.8\sqrt{5 \times 10} = 5.66\Omega$$

$$\therefore \quad jX_m = j5.66\Omega.$$

Loop-1

$$(j5 + 3 - j4)\, i_1 + j5\, i_2 - j5.66\, i_2 = 50\angle 0^\circ$$

$$\boxed{\therefore \quad (3 + j1)\, i_1 - j0.66\, i_2 = 50\angle 0^\circ \quad \text{---- (1)}}$$

Loop-2

$$j5i_1 + j5i_2 + j10i_2 + 5i_2 - j5.66i_2$$
$$- j5.66(i_1+i_2) = 50\angle 0^\circ$$

$$\therefore\ j5i_1 + j9.34i_2 - j5.66i_1 - j5.66i_2 + 5i_2 = 50\angle 0^\circ$$

$$\boxed{\begin{array}{l} \therefore\ \cancel{-j0.66i_2 + j8.68i_2 = 50\angle 0^\circ} \quad \cdots \cancel{(2)} \\ -j0.66i_1 + (5+j3.68)i_2 = 50 \quad \cdots (2) \end{array}}$$

Eqn-(1) = Eqn-(2)

$$\cancel{(3+ji_1)}\ (3+j1)i_1 - j0.66i_2 = -j0.66i_1 + \cancel{j8.68i_2}\ (5+j3.68)i_2$$

$$\therefore\ (3+j1.66)i_1 = \cancel{j9.34i_2}\ (5+j4.34)i_2$$

$$\therefore\ i_1 = \cancel{\frac{(9.34i_2)\angle 90^\circ}{3.4286\angle 28.95^\circ}}\ \frac{6.62\angle 40.95^\circ}{3.4286\angle 28.95^\circ} i_2$$

$$\therefore\ i_1 = \cancel{(2.724\angle 61.05^\circ)i_2}\ 1.93\angle 12^\circ\, i_2$$

$$\cancel{(3.4286\angle 28.95^\circ) \times 2.724\angle 61.05^\circ i_2 - j0.66i_2 = 50}$$

$$\therefore\ \cancel{j9.34i_2 - j0.66i_2} = 50$$

$$3.23\angle 18.4^\circ \times 1.93\angle 12^\circ\, i_2 - j0.66i_2 = 50$$

$$\therefore\ \cancel{8.8\angle 79.45^\circ i_2 - j0.66i_2 = 50}$$

$$6.234\angle 30.4^\circ\, i_2 - j0.66i_2 = 50$$

$$\therefore\ (1\cancel{.6112 + j7.99})i_2 \cancel{= 50}$$

$\therefore$ ~~i_2~~

$$(5.377 + j\,2.494)\, i_2 = 50\angle 0^\circ$$

$$\therefore \quad i_2 = 8.43\angle -24.88^\circ \text{ Amp.}$$

$$\therefore \quad v_5 = 5 \times 8.43\angle -24.88^\circ \text{ volt}$$

$$\boxed{\therefore \quad v_5 = 42.15\angle -24.88^\circ \text{ volt.}}$$

When polarity of one coil has changed,

$$\boxed{v_5 = 19.1\angle -112.1^\circ \text{ volt}}$$