

Searching Elements in an Array:

Linear and Binary Search

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Searching

- Check if a given element (called *key*) occurs in the array.
 - Example: array of student records; *rollno* can be the key.
- Two methods to be discussed:
 - If the array elements are unsorted.
 - Linear search
 - If the array elements are sorted.
 - Binary search

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Linear Search

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Basic Concept

- **Basic idea:**
 - Start at the beginning of the array.
 - Inspect elements one by one to see if it matches the key.
- **Time complexity:**
 - A measure of how long an algorithm takes to run.
 - If there are n elements in the array:
 - **Best case:** match found in first element (**1** search operation)
 - **Worst case:** no match found, or match found in the last element (**n** search operations)
 - **Average case:** **$(n + 1) / 2$** search operations

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```

#include <stdio.h>

int linear_search (int a[], int size, int key)
{
    for (int i=0; i<size; i++)
        if (a[i] == key) return i;
    return -1;
}

int main()
{
    int x[]={12,-3,78,67,6,50,19,10}, val;
    printf ("\nEnter number to search: ");
    scanf ("%d", &val);
    printf ("\nValue returned: %d \n", linear_search (x,8,val));
}

```

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- What does the function **linear_search** do?
 - It searches the array for the number to be searched element by element.
 - If a match is found, it returns the array index.
 - If not found, it returns -1.

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Contd.

```
int x[] = {12, -3, 78, 67, 6, 50, 19, 10};
```

- Trace the following calls :

```
search (x, 8, 6) ;
```

Returns 4

```
search (x, 8, 5) ;
```

Returns -1

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Binary Search

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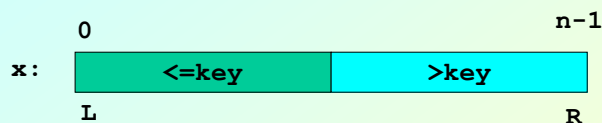
Basic Concept

- Binary search works if the array is **sorted**.
 - Look for the target in the middle.
 - If you don't find it, you can ignore half of the array, and repeat the process with the other half.
- In every step, we reduce the number of elements to search by half.

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The Basic Strategy

- What we want?
 - Find split between values larger and smaller than **key**:



- Situation while searching:
 - Initially L and R contains the indices of first and last elements.
- Look at the element at index $[(L+R)/2]$.
 - Move L or R to the middle depending on the outcome of test.

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Iterative Version

```
#include <stdio.h>

int bin_search (int a[], int size, int key)
{
    int L, R, mid;
    L = 0; R = size - 1;

    while (L <= R) {
        mid = (L + R) / 2;
        if (a[mid] < key) L = mid + 1;
        else if (a[mid] > key) R = mid - 1;
        else return mid; /* FOUND AT INDEX mid */
    }

    return -1; /* NOT FOUND */
}
```

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```
int main()
{
    int x[]={10,20,30,40,50,60,70,80}, val;

    printf ("\nEnter number to search: ");
    scanf ("%d", &val);

    printf ("\nValue returned: %d \n", bin_search (x,8,val));
}
```

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Recursive Version

```
#include <stdio.h>

int bin_search (int a[], int L, int R, int key)
{
    int mid;

    if (R < L) return -1;    /* NOT FOUND */
    mid = (L + R) / 2;
    if (a[mid] < key) return (bin_search(a,mid+1,R,key));
    else if (a[mid] > key) return (bin_search(a,L,mid-1,key));
    else return mid;    /* FOUND AT INDEX mid */
}
```

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```
int main()
{
    int x[]={10,20,30,40,50,60,70,80}, val;

    printf ("\nEnter number to search: ");
    scanf ("%d", &val);

    printf ("\nValue returned: %d \n", bin_search (x,0,7,val));
}
```

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Is it worth the trouble ?

- Suppose that the array x has 1000 elements.
- Ordinary search
 - If **key** is present in x , it would require 500 comparisons on the average.
- Binary search
 - After 1st compare, left with 500 elements.
 - After 2nd compare, left with 250 elements.
 - After 3rd compare, left with 125 elements.
 - After at most 10 steps, you are done.

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Time Complexity

- If there are n elements in the array.
 - Number of searches required in the worst case: $\log_2 n$
- For $n = 64$ (say).
 - Initially, list size = 64.
 - After first compare, list size = 32.
 - After second compare, list size = 16.
 - After third compare, list size = 8.
 -
 - After sixth compare, list size = 1.

$2^k = n$, where k is the number of steps.

$$k = \log_2 n$$

$$\log_2 64 = 6$$

$$\log_2 1024 = 10$$

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Sorting

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The Basic Problem

- Given an array
 $x[0], x[1], \dots, x[\text{size}-1]$
- reorder entries so that
 $x[0] \leq x[1] \leq \dots \leq x[\text{size}-1]$
 - List is in non-decreasing order.
- We can also sort a list of elements in non-increasing order.

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Example

- **Original list:**

10, 30, 20, 80, 70, 10, 60, 40, 70

- **Sorted in non-decreasing order:**

10, 10, 20, 30, 40, 60, 70, 70, 80

- **Sorted in non-increasing order:**

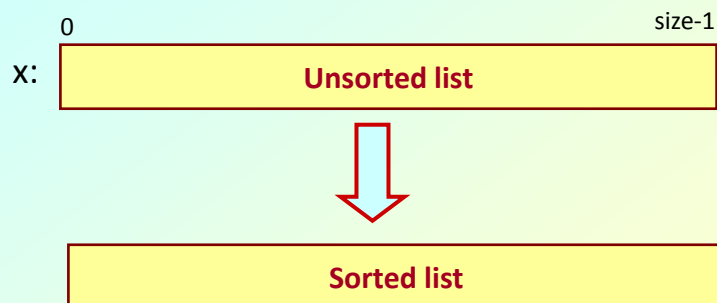
80, 70, 70, 60, 40, 30, 20, 10, 10

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Sorting Problem

- **What we want ?**

- Sort the given data sorted in the specified order



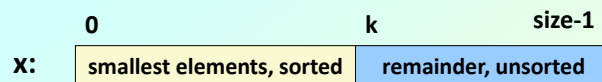
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Selection Sort

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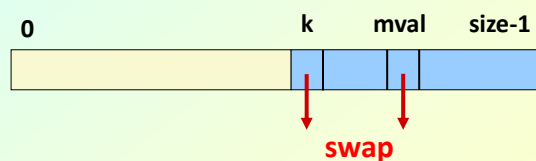
How it works?

- General situation :



- Steps :

- Find smallest element, `mval`, in `x[k..size-1]`
- Swap smallest element with `x[k]`, then increase `k` by 1



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An example worked out

PASS 1:

10	5	17	11	-3	12	Find the minimum
10	5	17	11	-3	12	Exchange with 0th
<u>-3</u>	5	17	11	10	12	element

PASS 2:

<u>-3</u>	5	17	11	10	12	Find the minimum
<u>-3</u>	5	17	11	10	12	Exchange with 1st
<u>-3</u>	<u>5</u>	17	11	10	12	element

PASS 3:

<u>-3</u>	<u>5</u>	17	11	10	12	Find the minimum
<u>-3</u>	<u>5</u>	17	11	10	12	Exchange with 2nd
<u>-3</u>	<u>5</u>	<u>10</u>	11	17	12	element

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PASS 4:

<u>-3</u>	<u>5</u>	<u>10</u>	11	17	12	Find the minimum
<u>-3</u>	<u>5</u>	<u>10</u>	11	17	12	Exchange with 3rd
<u>-3</u>	<u>5</u>	<u>10</u>	<u>11</u>	17	12	element

PASS 5:

<u>-3</u>	<u>5</u>	<u>10</u>	<u>11</u>	17	12	Find the minimum
<u>-3</u>	<u>5</u>	<u>10</u>	<u>11</u>	17	12	Exchange with 4th
<u>-3</u>	<u>5</u>	<u>10</u>	<u>11</u>	<u>12</u>	17	element

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Subproblem

```

/* Yield index of smallest element in x[k..size-1];*/

int min_loc (int x[], int k, int size)
{
    int j, pos;

    pos = k;
    for (j=k+1; j<size; j++)
        if (x[j] < x[pos])
            pos = j;
    return pos;
}

```

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The main sorting function

```

/* Sort x[0..size-1] in non-decreasing order */

int sel_sort (int x[], int size)
{
    int k, m;

    for (k=0; k<size-1; k++)
    {
        m = min_loc (x, k, size);
        temp = a[k];
        a[k] = a[m];
        a[m] = temp;
    }
}

```

} **SWAP**

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```

int main()
{
    int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
    int i;
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
    sel_sort(x,12);
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
}

```

```

-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89

```

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Analysis

- How many steps are needed to sort n items ?

- Total number of steps proportional to n^2 .
- No. of comparisons?

$$(n-1) + (n-2) + \dots + 1 = n(n-1)/2$$

Of the order of n^2

- Worst Case? Best Case? Average Case?

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Insertion Sort

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Basic Idea

- Insert elements one at a time, and create a partial sorted list.
 - Sorted list of 2 elements, 3 elements, 4 elements, and so on.
- In general, in every iteration an element is compared with all the elements before it.
- After finding the position of insertion, space is created for it by shifting the other elements and the desired element is then inserted at the suitable position.
- This procedure is repeated for all the elements in the list.

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An example worked out

PASS 1:

<u>10</u>	5	17	11	-3	12	item = 5
<u>? 10</u>	17	11	-3	12	compare 5 and 10	
<u>5 10</u>	17	11	-3	12	insert 5	

PASS 2:

<u>5 10</u>	17	11	-3	12	item = 17
<u>5 10 17</u>	11	-3	12	compare 17 and 10	

PASS 3:

<u>5 10 17</u>	11	-3	12	item = 11
<u>5 10 ? 17</u>	-3	12	compare 11 and 17	
<u>5 10 ? 17</u>	-3	12	compare 11 and 10	
<u>5 10 11 17</u>	-3	12	insert 11	

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PASS 4:

<u>5 10 11 17</u>	-3	12	item = -3
<u>5 10 11 ? 17</u>	12	compare -3 and 17	
<u>5 10 ? 11 17</u>	12	compare -3 and 11	
<u>5 ? 10 11 17</u>	12	compare -3 and 10	
<u>? 5 10 11 17</u>	12	compare -3 and 5	
<u>-3 5 10 11 17</u>	12	insert -3	

PASS 5:

<u>-3 5 10 11 17</u>	12	item = 12
<u>-3 5 10 11 ? 17</u>	12	compare 12 and 17
<u>-3 5 10 11 ? 17</u>	12	compare 12 and 11
<u>-3 5 10 11 12 17</u>		insert 12

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Insertion Sort

```
void insert_sort (int list[], int size)
{
    int i,j,item;

    for (i=1; i<size; i++)
    {
        item = list[i] ;
        j = i - 1;
        while ((item < list[j]) && (j >= 0))
        {
            list[j+1] = list[j];
            j--;
        }
        list[j+1] = item ;
    }
}
```

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```
int main()
{
    int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
    int i;
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
    insert_sort (x,12);
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
}
```

```
-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89
```

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Time Complexity

- Number of comparisons and shifting:

- Worst case?

$$1 + 2 + 3 + \dots + (n-1) = n(n-1)/2$$

- Best case?

$$1 + 1 + \dots \text{ to } (n-1) \text{ terms} = (n-1)$$

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Bubble Sort

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How it works?

- The sorting process proceeds in several passes.
 - In every pass we go on comparing neighboring pairs, and swap them if out of order.
 - In every pass, the largest of the elements under considering will *bubble* to the top (i.e., the right).

10	5	17	11	-3	12
5	10	17	11	-3	12
5	10	17	11	-3	12
5	10	11	17	-3	12
5	10	11	-3	17	12
5	10	11	-3	12	<u>17</u>

← Largest

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An example worked out

PASS 1:

10	5	17	11	-3	12
5	10	17	11	-3	12
5	10	17	11	-3	12
5	10	11	17	-3	12
5	10	11	-3	17	12
5	10	11	-3	12	<u>17</u>

PASS 2:

5	10	11	-3	12	<u>17</u>
5	10	11	-3	12	<u>17</u>
5	10	11	-3	12	<u>17</u>
5	10	-3	11	12	<u>17</u>
5	10	-3	11	<u>12</u>	<u>17</u>

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PASS 3:

5	10	-3	11	<u>12</u>	<u>17</u>
5	10	-3	11	<u>12</u>	<u>17</u>
5	-3	10	11	<u>12</u>	<u>17</u>
5	-3	10	<u>11</u>	<u>12</u>	<u>17</u>

PASS 4:

5	-3	10	<u>11</u>	<u>12</u>	<u>17</u>
-3	5	10	<u>11</u>	<u>12</u>	<u>17</u>
-3	5	<u>10</u>	<u>11</u>	<u>12</u>	<u>17</u>

PASS 5:

-3	5	<u>10</u>	<u>11</u>	<u>12</u>	<u>17</u>
-3	5	<u>10</u>	<u>11</u>	<u>12</u>	<u>17</u>

Sorted

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- How the passes proceed?

- In pass 1, we consider index 0 to n-1.
- In pass 2, we consider index 0 to n-2.
- In pass 3, we consider index 0 to n-3.
-
-
- In pass n-1, we consider index 0 to 1.

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Bubble Sort

```
void swap(int *x, int *y)
{
    int tmp = *x;
    *x = *y;
    *y = tmp;
}
```

```
void bubble_sort
    (int x[], int n)
{
    int i,j;

    for (i=n-1; i>0; i--)
        for (j=0; j<i; j++)
            if (x[j] > x[j+1])
                swap(&x[j], &x[j+1]);
}
```

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```
int main()
{
    int x[ ]={-45,89,-65,87,0,3,-23,19,56,21,76,-50};
    int i;
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
    bubble_sort (x,12);
    for(i=0;i<12;i++)
        printf("%d ",x[i]);
    printf("\n");
}
```

```
-45 89 -65 87 0 3 -23 19 56 21 76 -50
-65 -50 -45 -23 0 3 19 21 56 76 87 89
```

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Time Complexity

- Number of comparisons :
 - Worst case?
$$1 + 2 + 3 + \dots + (n-1) = n(n-1)/2$$
 - Best case?
 - Same

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- How do you make best case with $(n-1)$ comparisons only?
 - By maintaining a variable **flag**, to check if there has been any swaps in a given pass.
 - If no swaps during a pass, the array is already sorted.

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```
void bubble_sort (int x[], int n)
{
    int i, j, flag;

    for (i=n-1; i>0; i--)
    {
        flag = 0;
        for (j=0; j<i; j++)
            if (x[j] > x[j+1])
            {
                swap(&x[j], &x[j+1]);
                flag = 1;
            }
        if (flag == 0) return;
    }
}
```

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Some Efficient Sorting Algorithms

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- Two of the most popular sorting algorithms are based on **divide-and-conquer** approach.
 - Quick sort
 - Merge sort
- Basic concept (divide-and-conquer method):

```
sort (list)
{
  if the list has greater than 1 elements
  {
    Partition the list into lowlist and highlist;
    sort (lowlist);
    sort (highlist);
    combine (lowlist, highlist);
  }
}
```

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Quick Sort

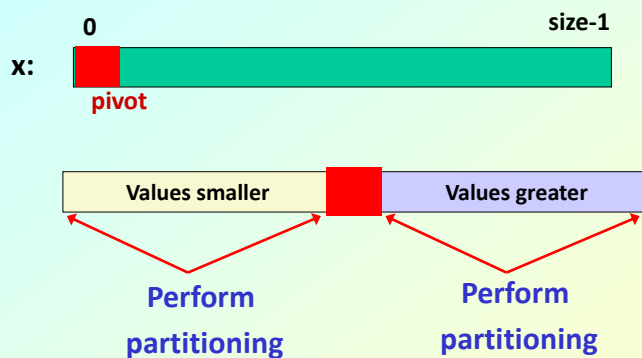
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How it works?

- At every step, we select a **pivot element** in the list (usually the **first** element).
 - We put the pivot element in the final position of the sorted list.
 - All the elements less than or equal to the pivot element are to the left.
 - All the elements greater than the pivot element are to the right.

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Partitioning



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```

void print (int x[], int low, int high)
{
    int i;

    for(i=low; i<=high; i++)
        printf(" %d", x[i]);
    printf("\n");
}

```

```

void swap (int *a, int *b)
{
    int tmp=*a;
    *a=*b;
    *b=tmp;
}

```

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```

void partition (int x[], int low, int high)
{
    int i = low+1, j = high;
    int pivot = x[low];
    if (low >= high) return;
    while (i<j) {
        while ((x[i]<pivot) && (i<high)) i++;
        while ((x[j]>=pivot) && (j>low)) j--;
        if (i<j) swap (&x[i], &x[j]);
    }
    if (j==high) {
        swap (&x[j], &x[low]);
        partition (x, low, high-1);
    }
    else
        if (i==low+1)
            partition (x, low+1, high);
        else {
            swap (&x[j], &x[low]);
            partition (x, low, j-1);
            partition (x, j+1, high);
        }
}

```

```

int main (int argc, char *argv[])
{
    int x[] = {-56,23,43,-5,-3,0,123,-35,87,56,75,80};
    int i=0;
    int num;

    num = 12;    /* Number of elements */

    partition(x,0,num-1);

    printf("Sorted list: ");
    print (x,0,num-1);
}

```

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Trace of Partitioning: an example

45 -56 78 90 -3 -6 123 0 -3 45 69 68

45 -56 78 90 -3 -6 123 0 -3 45 69 68

-6 -56 -3 0 -3 45 123 90 78 45 69 68

-56 -6 -3 0 -3 68 90 78 45 69 123

-3 0 -3 45 68 78 90 69

-3 0 69 78 90

Sorted list: -56 -6 -3 -3 0 45 45 68 69 78 90 123

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Time Complexity

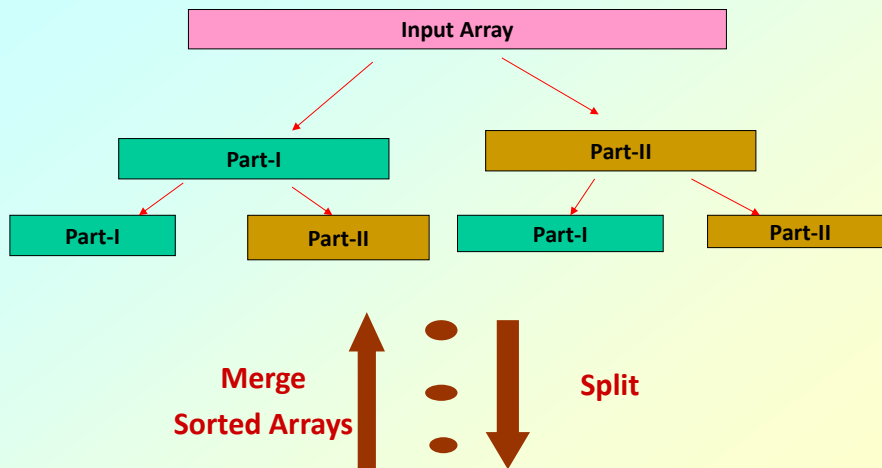
- **Worst case:**
 $n^2 \implies$ list is already sorted
- **Average case:**
 $n \log_2 n$
- **Statistically, quick sort has been found to be one of the fastest algorithms.**

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Merge Sort

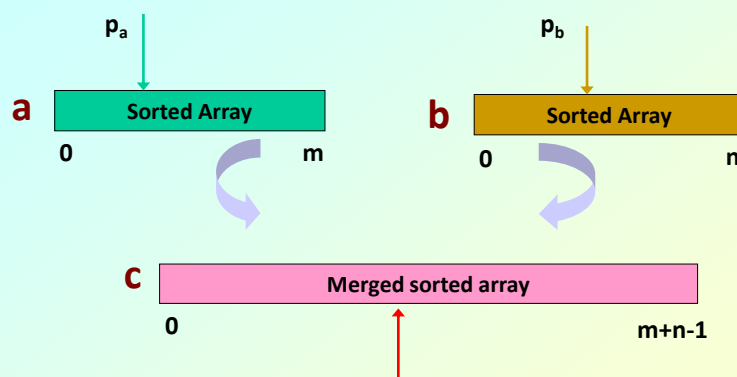
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Merge Sort



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Merging two sorted arrays



Move and copy elements pointed by p_a if its value is smaller than the element pointed by p_b in $(m+n-1)$ operations; otherwise, copy elements pointed by p_b .

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Example

- Initial array A contains 14 elements:
 - 66, 33, 40, 22, 55, 88, 60, 11, 80, 20, 50, 44, 77, 30
- Pass 1 :: Merge each pair of elements
 - (33, 66) (22, 40) (55, 88) (11, 60) (20, 80) (44, 50) (30, 70)
- Pass 2 :: Merge each pair of pairs
 - (22, 33, 40, 66) (11, 55, 60, 88) (20, 44, 50, 80) (30, 77)
- Pass 3 :: Merge each pair of sorted quadruplets
 - (11, 22, 33, 40, 55, 60, 66, 88) (20, 30, 44, 50, 77, 80)
- Pass 4 :: Merge the two sorted subarrays to get the final list
 - (11, 20, 22, 30, 33, 40, 44, 50, 55, 60, 66, 77, 80, 88)

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```
void merge_sort (int *a, int n)
{
    int i, j, k, m;
    int *b, *c;

    if (n>1) {
        k = n/2;
        m = n-k;
        b = (int *) malloc(k*sizeof(int));
        c = (int *) malloc(m*sizeof(int));

        for (i=0; i<k; i++)
            b[i]=a[i];
        for (j=k; j<n; j++)
            c[j-k]=a[j];

        merge_sort (b, k);
        merge_sort (c, m);
        merge (b, c, a, k, m);
        free(b); free(c);
    }
}
```

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```

void merge (int *a, int *b, int *c, int m, int n)
{
    int i, j, k, p;
    i = j = k = 0;
    do {
        if (a[i] < b[j]) {
            c[k]=a[i]; i++;
        }
        else {
            c[k]=b[j]; j++;
        }
        k++;
    } while ((i<m) && (j<n));

    if (i == m) {
        for (p=j; p<n; p++) { c[k]=b[p]; k++; }
    }
    else {
        for (p=i; p<m; p++) { c[k]=a[p]; k++; }
    }
}

```

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```

main()
{
    int i, num;
    int a[ ] = {-56,23,43,-5,-3,0,123,-35,87,56,75,80};

    num = 12;

    printf ("\n Original list: ");
    print (a, 0, num-1);

    merge_sort (a, 12);

    printf ("\n Sorted list: ");
    print (a, 0, num-1);
}

```

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Time Complexity

- Best/Worst/Average case:

$n \log_2 n$

- Drawback:

- Needs double amount of space for storage.
- For sorting n elements, requires another array of size n to carry out merge.

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Example :: sorting arrays of structures (bubble sort)

```
#include <stdio.h>
struct stud
{
    int roll;
    char dept_code[25];
    float cgpa;
};

main()
{
    struc stud class[100], t;
    int j, k, n;

    scanf ("%d", &n);
    /* no. of students */
```

```
for (k=0; k<n; k++)
    scanf ("%d %s %f", &class[k].roll,
            class[k].dept_code,
            &class[k].cgpa);
for (j=0; j<n-1; j++)
    for (k=1; k<n-j; k++)
    {
        if (class[k-1].roll >
            class[k].roll)
        {
            t = class[k-1];
            class[k-1] = class[k];
            class[k] = t;
        }
    }
    <<<< PRINT THE RECORDS >>>>
}
```

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Example :: sorting arrays of structures (selection sort)

```
int min_loc (struct stud x[],
             int k, int size)
int j, pos;
{
    pos = k;
    for (j=k+1; j<size; j++)
        if (x[j] < x[pos])
            pos = j;
    return pos;
}
```

```
main()
{
    struc stud class[100];
    int n;
    ...
    selsort (class, n);
    ...
}
```

```
int selsort (struct stud x[],int n)
{
    int k, m;
    for (k=0; k<n-1; k++)
    {
        m = min_loc (x, k, n);
        temp = a[k];
        a[k] = a[m];
        a[m] = temp;
    }
}
```

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Algorithm Analysis

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Analysis of Algorithms

- How much resource is required ?
- Measures for efficiency
 - Execution time → time complexity
 - Memory space → space complexity
- Observation :
 - The larger amount of input data an algorithm has, the larger amount of resource it requires.
 - Complexities are functions of the amount of input data (input size).

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What do we use for a yardstick?

- The same algorithm will run at different speeds and will require different amounts of space.
 - When run on different computers, different programming languages, different compilers.
- But algorithms usually consume resources in some fashion that depends on the size of the problem they solve.
 - Some parameter n (for example, number of elements to sort).

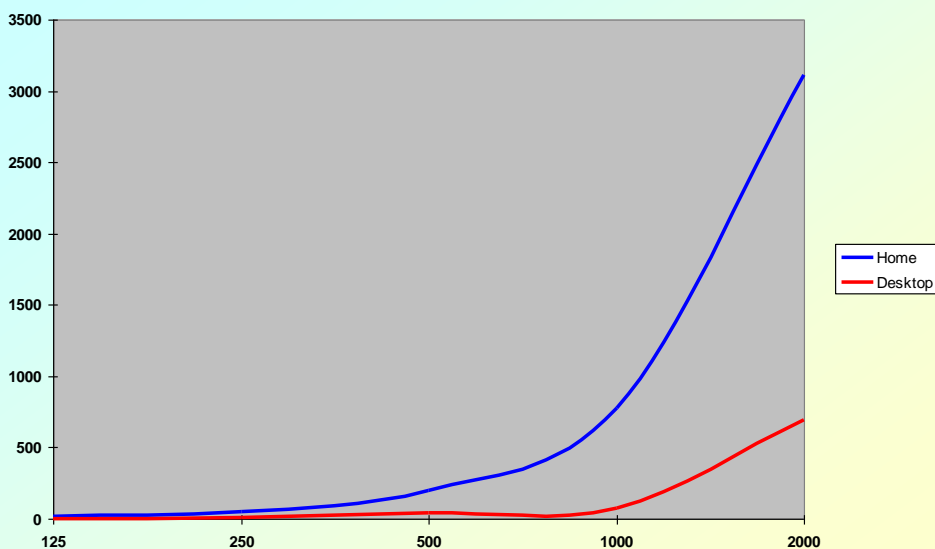
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An example of a sorting algorithm

- We run this sorting algorithm on two different computers, and note the time (in milliseconds) for different sizes of input.

Array Size n	Home Computer	Desktop Computer
125	12.5	2.8
250	49.3	11.0
500	195.8	43.4
1000	780.3	72.9
2000	3114.9	690.5

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Contd.

- Home Computer :

$$f_1(n) = 0.0007772 n^2 + 0.00305 n + 0.001$$

- Desktop Computer :

$$f_2(n) = 0.0001724 n^2 + 0.00040 n + 0.100$$

- Both are quadratic function of n .
- The shape of the curve that expresses the running time as a function of the problem size stays the same.

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Complexity Classes

- The running time for different algorithms fall into different *complexity classes*.
 - Each complexity class is characterized by a different family of curves.
 - All curves in a given complexity class share the same basic shape.
- The *O-notation* is used for talking about the complexity classes of algorithms.

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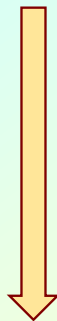
Running time of algorithms

Assume speed is 10^7 instructions per second.

size	10	20	30	50	100	1000	10000
n	.001ms	.002ms	.003ms	.005ms	.01ms	.1ms	1ms
$n \log n$.003ms	.008ms	.015ms	.03ms	.07ms	1ms	13ms
n^2	.01ms	.04ms	.09ms	.25ms	1ms	100ms	10s
n^3	.1ms	.8ms	2.7ms	12.5ms	100ms	100s	28h
2^n	.1ms	.1s	100s	3h	3×10^{13} c	inf	inf

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- The complexity classes:

 $\log_2 n$ n $n \log_2 n$ n^2 n^3 2^n $n!$ 

Complexity
increases

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Introducing the language of O-notation

- Definition:

$f(n) = O(g(n))$ if there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n)$ when $n \geq n_0$.

- The big-Oh notation is used to categorize the complexity class of algorithms.
 - It gives an upper bound.
 - Other measures also exist, like small-Oh, Omega, Theta, etc.

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Examples

- $f(n) = 2n^2 + 4n + 5$ is $O(n^2)$.
 - One possibility: $c=11$, and $n_0=1$.
- $f(n) = 2n^2 + 4n + 5$ is also $O(n^3)$, $O(n^4)$, etc.
 - One possibility: $c=11$, and $n_0=1$.
- $f(n) = n(n-1)/2$ is $O(n^2)$.
 - One possibility: $c=1/2$, and $n_0=1$.
- $f(n) = 5n^4 + \log_2 n$ is $O(n^4)$.
 - One possibility: $c=6$, and $n_0=1$.
- $f(n) = 75$ is $O(1)$.
 - One possibility: $c=75$, and $n_0=1$.

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Complexities of Known Algorithms

Algorithm	Best-case	Average-case	Worst-case
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Bubble sort	$O(n)$	$O(n^2)$	$O(n^2)$
Quick sort	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n^2)$
Merge sort	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n \log_2 n)$
Linear search	$O(1)$	$O(n)$	$O(n)$
Binary search	$O(1)$	$O(\log_2 n)$	$O(\log_2 n)$

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Observations

- There is a big difference between polynomial time complexity and exponential time complexity.
- Hardware advances affect only efficient algorithms and do not help inefficient algorithms.

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