INDIAN INSTITUTE OF TECHNOLOGY
Date: 20/2/2015 (FN) **Spring Semester** **Time:** 2 hrs **Full Marks:** 60
Dept.: ME **Subject Name:** Mechanics **Subject No.:** ME10001
No of students: 700 (approx.) All first year students

Question paper has 2 pages (back to back) and 7 questions. Marks allotted to each question are indicated within brackets at the end of each question. Any assumptions made in solving the questions should be justified with reasons. Units must be mentioned for all answers. Unless stated otherwise neglect weights of bodies. Acceleration due to gravity g=9.81 m/s².

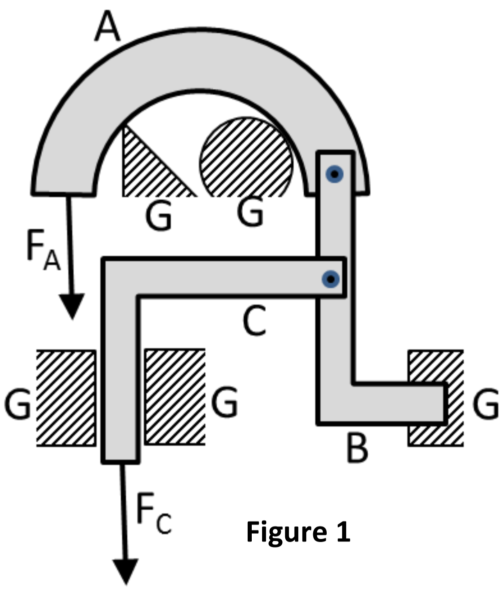
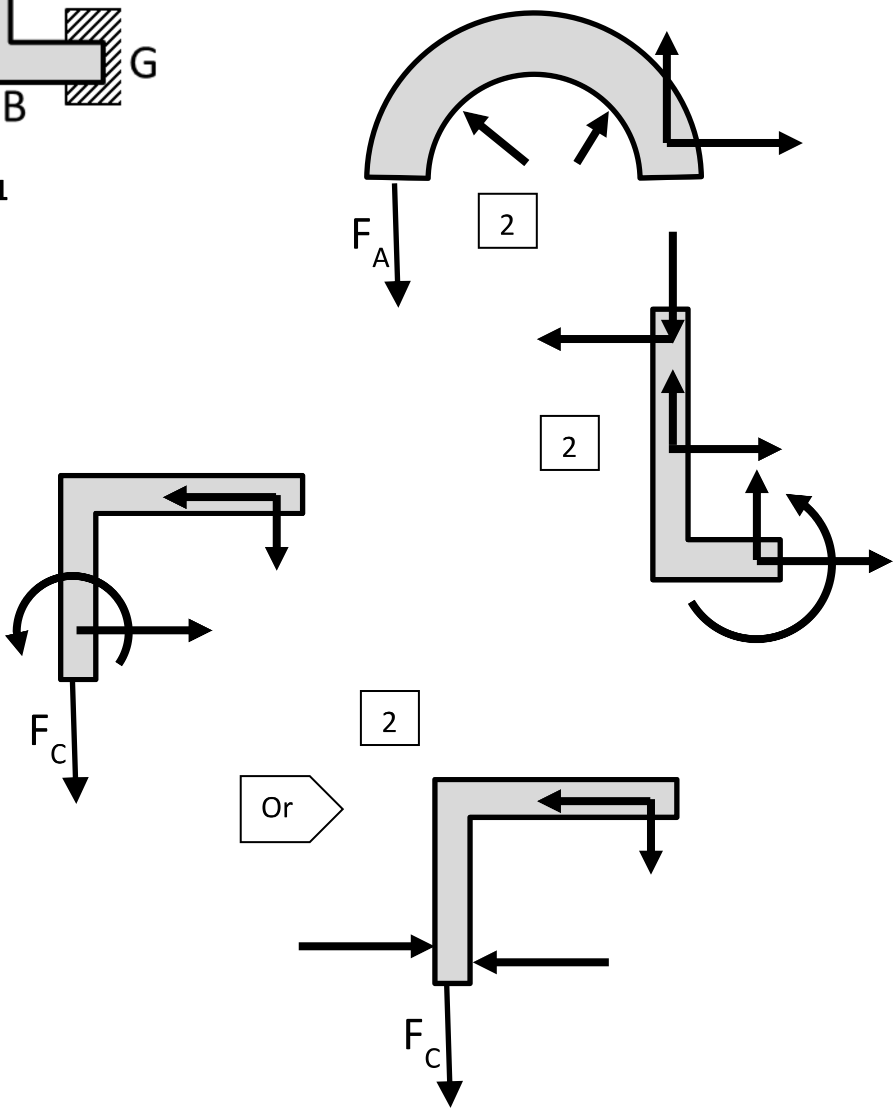


Figure 1

1. Draw free body diagrams of bodies A B and C shown in figure 1. The hatched bodies labelled as G represent the ground. F_A and F_C are external forces acting on A and C respectively. The black circles represent pin joints. All surfaces and joints are frictionless. Weights of the bodies A, B, C are negligible. (6)



2. A cylinder is held by a system of ropes AB, CD, FG as shown in figure 2. The cylinder is attached to the wall by a spherical joint at O. The cylinder is at the present moment aligned along the y axis. The ropes AB, CD, FG exert forces T_1, T_2, T_3 respectively on the cylinder. A rope is wrapped at the end of the cylinder from which hangs a weight W as shown from the point E. The coordinates of the points O, A, B, C, D, E, F , G are - O(0,0,0), A(0,0,8), B(0,6,2), C(11,0,0), D(2,12,0), E(-2,12,0), F(0,12,2), G(2,0,15). Write down the equivalent force system for the forces shown, namely T_1, T_2, T_3 and W, at the point O. Use vector representation only using the coordinate system shown. (10)

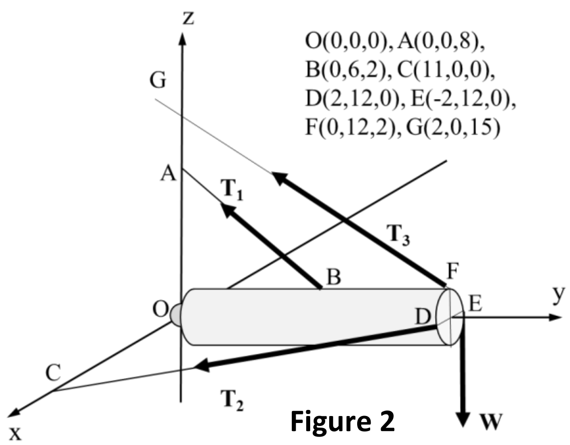


Figure 2

4 marks, i.e. one mark each for writing each force component correctly.

1 mark for summing up and writing the net force vector correctly

2 marks for writing the expression for finding moments. The position vector of the points on the line of action of the forces must be correct in this step. A valid point other than that considered in the solution will get full credit.

3 marks for writing the equivalent moment vector correctly i.e. 1 mark per component.

$$T_1 = T_1 \frac{-j+k}{\sqrt{2}} = T_1(-0.707j + 0.707k),$$

$$T_2 = T_2 \frac{3i-4j}{5} = T_2(0.6i - 0.8j), \qquad \boxed{4}$$

$$T_3 = T_3 \frac{2i-12j+13k}{\sqrt{317}} = T_3(0.112i - 0.674j + 0.73k)$$

$$W = -Wk$$

$$\sum F = T_1(-0.707j+0.707k) + T_2(0.6i-0.8j) + T_3(0.112i-0.674j+0.73k) - Wk$$

$$= (0.112T_1 + 0.6T_2)i + (-0.707T_1 - 0.8T_2 - 0..674T_3)j + (0.707T_1 + 0.73T_3 - W)k \qquad \boxed{1}$$

$$\sum M_O = 8k \times T_1(-0.707j+0.707k) + 11i \times T_2(0.6i-0.8j)$$

$$+(12j+2k) \times T_3(0.112i-0.674j+0.73k) + (-2i+12j) \times (-Wk) \qquad \boxed{2}$$

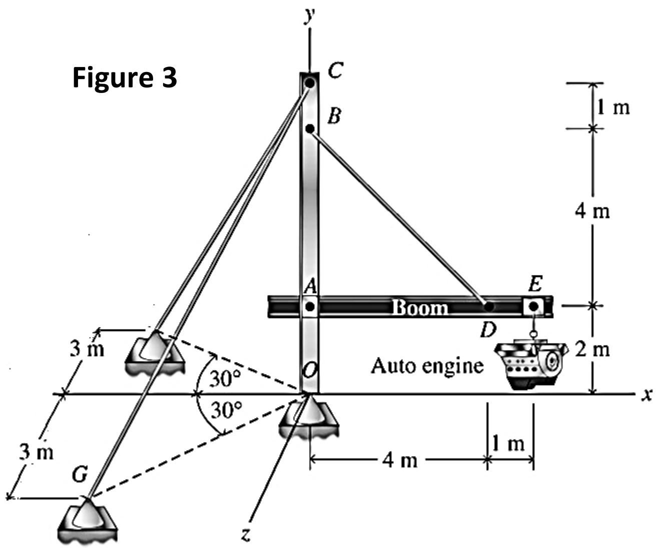
$$= T_1(5.656i) - T_2(8.8k)$$

$$+T_3(-1.344k + 8.73i) + T_3(0.224j + 1.348i) - 2Wj - 12Wi$$

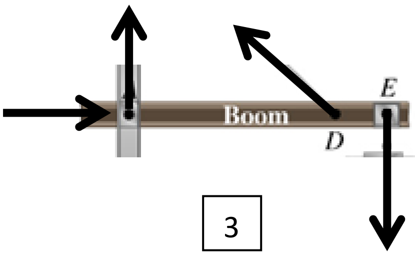
$$= T_1(5.656i + 0.707k) - T_2(8.8k) + T_3(10.078i + 0.224j - 1.344k) - 2Wj - 12Wi$$

$$= (5.656T_1 + 10.078T_3 - 12W)i + (0.224T_3 - 2W)j + (-8.8T_2 - 1.344T_3)k \qquad \boxed{3}$$

Figure 3



3. An auto engine of weight 1 KN is tied to a boom AE at the point E as shown in figure 3. The boom is attached to a pole OC by a pin joint at A. The pole OC is attached to the ground by a spherical joint at O. Two ropes CF and CG support the pole. The points F and G lie on the ground, 6 m apart, with the x axis bisecting FG perpendicularly. Further a rope BD is attached between the boom and the pole to support the boom. Weights of the ropes, pole and the boom are neglected. Find the tension in the rope BD. Note the following lengths: OA=2m, AB=4m, BC=1m, AD=4m, DE=1m, FG=6m. (6)



$\boxed{3}$

For rod AE

$$\sum M_A = 0 \Rightarrow \mathbf{r}_{AD} \times \mathbf{T}_{DB} + \mathbf{r}_{AE} \times \left(-W\mathbf{j}\right) = 0 \quad \boxed{2}$$

$$\Rightarrow 4i \times T_{DB}\left(\frac{-i+j}{\sqrt{2}}\right) + 5i \times \left(-Wj\right) = 0 \Rightarrow T_{DB} = \frac{5W\sqrt{2}}{4} = 1.77W = 1.77KN$$

$\boxed{1}$

4. In the truss shown in figure 4 find the forces in the all the vertical members i.e. BF, CG, DH. Member CJ is perpendicular to the line GJD, while member DK is perpendicular to the line HKE (10)

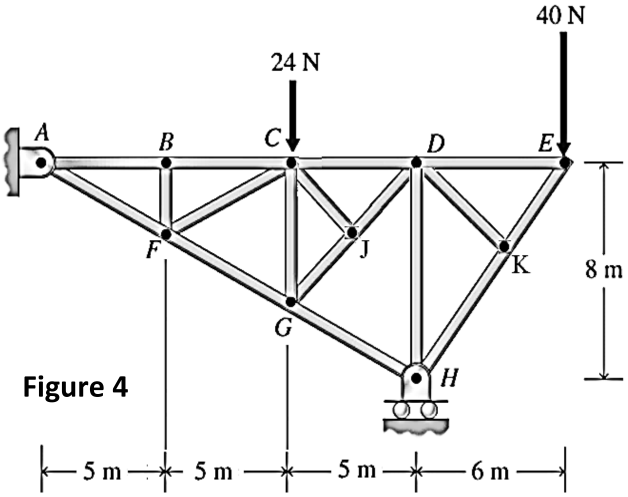


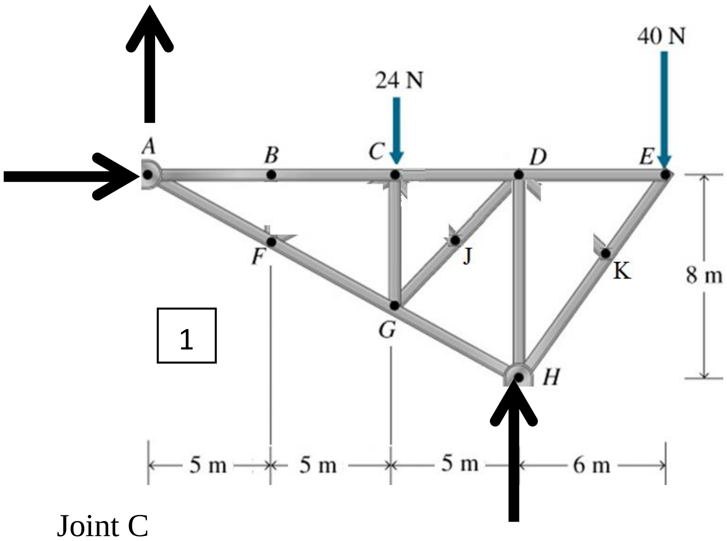
Figure 4

$A_x = 0$ $\boxed{0.5}$

$A_y + H_y = 64$ $\boxed{0.5}$

$\sum M_A = 0 \Rightarrow 15H_y = 21\times 40 + 24\times 10$ $\boxed{1}$

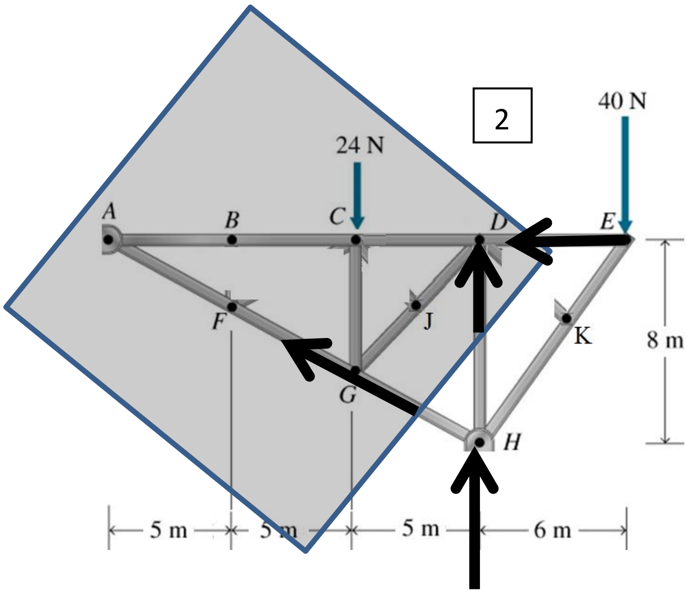
$\Rightarrow H_y = 72N \Rightarrow A_y = -8N$ $\boxed{1}$



Joint C

$S_{CB} + S_{CD} = 0$

$S_{CG} = -24N \Rightarrow compression$ $\boxed{2}$



Taking section as shown

$\sum M_A = 0 \Rightarrow 15H_y + 15S_{HD} = 21\times 40$ $\boxed{1}$

$\Rightarrow S_{HD} = -16N \Rightarrow compression$ $\boxed{1}$

5. In figure 5, if the peg at B is a smooth round cylinder and there is no friction at the contatcing surfaces, determine the components of reaction at the pin A and fixed support C. The torque at A is 900 Nm clockwise. The force acting on rod BC is 500 N downwards. The rods are massless. (10)

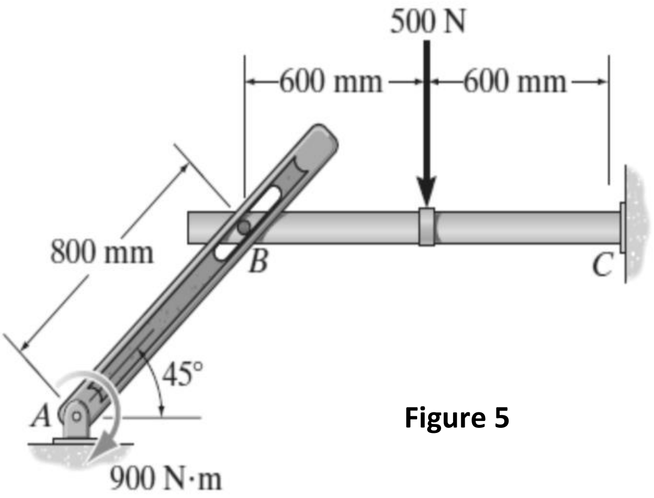
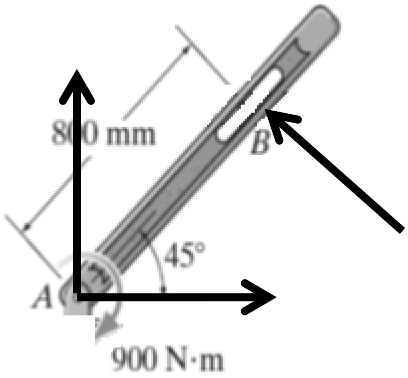


Figure 5

2



$$A_x - \frac{N_B}{\sqrt{2}} = 0 \quad \boxed{0.5}$$

$$A_y + \frac{N_B}{\sqrt{2}} = 0 \quad \boxed{0.5}$$

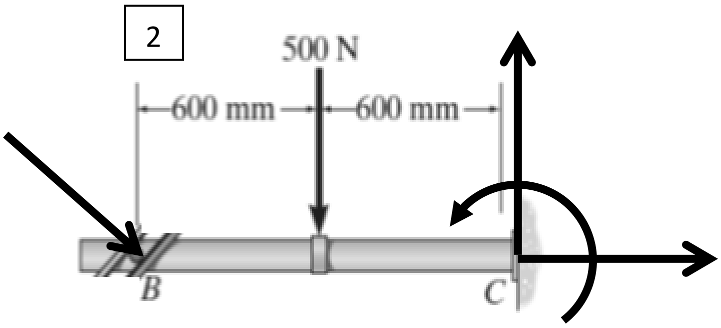
$$0.8N_B - 900 = 0 \quad \boxed{0.5}$$

$$\Rightarrow 0.8N_B = 900 \Rightarrow N_B = 1125N \quad \boxed{0.5}$$

$$\therefore A_x = \frac{N_B}{\sqrt{2}} = 795.5N \quad \boxed{0.5}$$

$$A_y = -\frac{N_B}{\sqrt{2}} = -795.5N \quad \boxed{0.5}$$

2



$$C_x + \frac{N_B}{\sqrt{2}} = 0 \quad \boxed{0.5}$$

$$\Rightarrow C_x = -795.5N \quad \boxed{0.5}$$

$$C_y - \frac{N_B}{\sqrt{2}} - 500 = 0 \quad \boxed{0.5}$$

$$\Rightarrow C_y = 1295.5N \quad \boxed{0.5}$$

$$1.2\frac{N_B}{\sqrt{2}} + 0.6 \times 500 + M_C = 0 \quad \boxed{0.5}$$

$$\Rightarrow M_C = -1254.6Nm \quad \boxed{0.5}$$

6. The massless rod CD is pushed upwards by a force F as shown in figure 6. The rope attached at D passes over two fixed, nonrotating cylinders A and B, and is attached to a weight W. Points C and D are 3 m and 1 m respectively from the pivot. The coefficient of static friction between the rope and the cylinders is 0.25 while the coefficient of kinetic friction is 0.2. What is the range of F in terms of W that will keep the weight W from moving? The segments of the rope do not touch at the points where they overlap in the figure. (8)

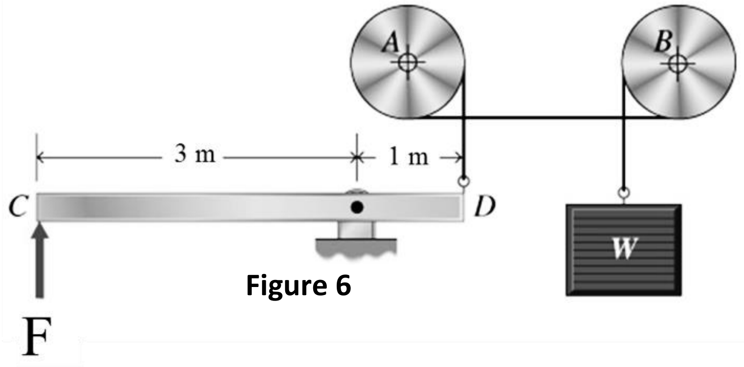
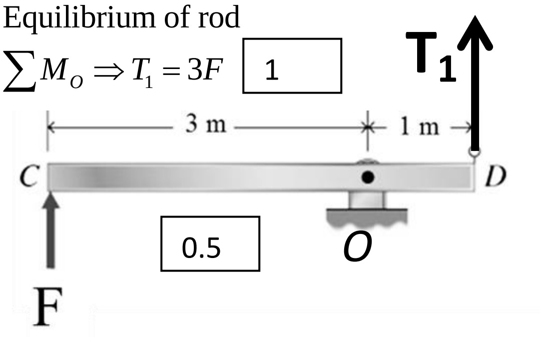
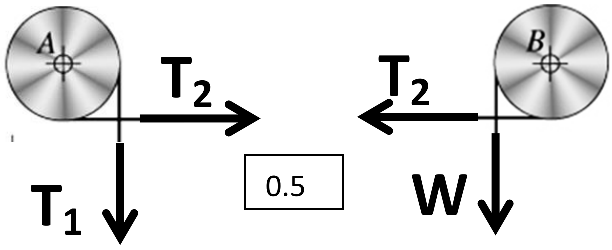


Figure 6

Equilibrium of rod

$\sum M_O \Rightarrow T_1 = 3F$ $\boxed{1}$





For preventing upward movement of W

$$T_2 = T_1 \exp\left(\mu_s \frac{3\pi}{2}\right) = 3F \exp\left(\mu_s \frac{3\pi}{2}\right) \quad \boxed{1}$$

$$W = T_2 \exp\left(\mu_s \frac{3\pi}{2}\right) = 3F \exp\left(\mu_s \frac{3\pi}{2}\right)\exp\left(\mu_s \frac{3\pi}{2}\right) \quad \boxed{1}$$

$$\Rightarrow F = \frac{W}{3}\exp\left(-3\pi\mu_s\right) = 0.0316W \quad \boxed{1}$$

For preventing downward movement of W

$$T_2 = W \exp\left(\mu_s \frac{3\pi}{2}\right) \quad \boxed{1}$$

$$T_1 = 3F = T_2 \exp\left(\mu_s \frac{3\pi}{2}\right) \quad \boxed{1}$$

$$\Rightarrow 3F = W \exp\left(\mu_s \frac{3\pi}{2}\right)\exp\left(\mu_s \frac{3\pi}{2}\right)$$

$$\Rightarrow F = \frac{W}{3}\exp\left(3\pi\mu_s\right) = 3.517W \quad \boxed{1}$$

$$0.0316W = \frac{W}{3}\exp\left(-3\pi\mu_s\right) < F < \frac{W}{3}\exp\left(3\pi\mu_s\right) = 3.517W$$

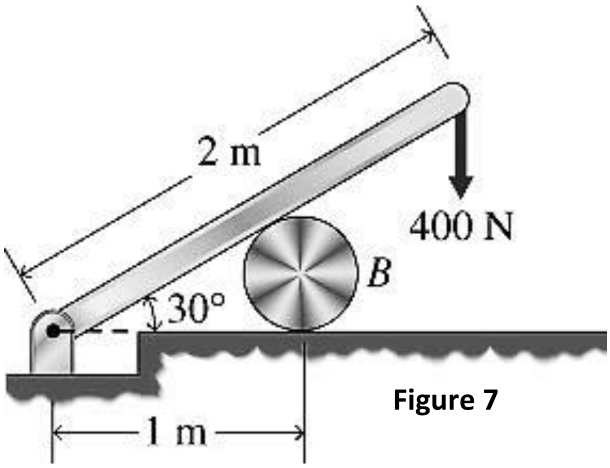
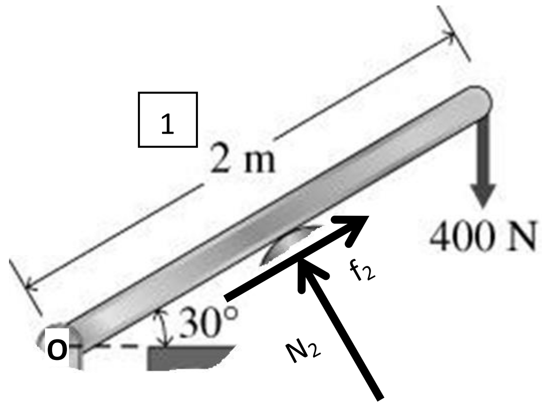


Figure 7

7. A massless rod of length 2m is being pressed downwards on a cylinder of weight 200N with a vertical force of 400 N as shown in figure 7. The coefficient of contact friction is same at all contact surfaces. Find the minimum coefficient of static friction needed to maintain equilibrium. Neglect thickness of the rod, assuming it to be a straight line segment of zero thickness (10).

P=400N in the calculations



Consider the equilibrum of the rod

$$\sum M_O \Rightarrow 1\times N_2 = 2\times P\times\frac{\sqrt{3}}{2} \quad (1) \quad \boxed{1}$$

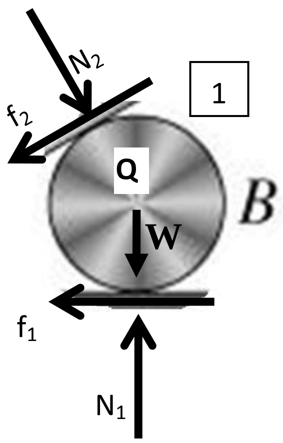
$$\Rightarrow N_2 = \sqrt{3}P = 693N \quad \boxed{1}$$

Consider equilibrium of the cylinder

$$\sum F_x = 0 \Rightarrow -f_1 - f_2\times\frac{\sqrt{3}}{2} + N_2\times\frac{1}{2} = 0 \quad (2) \quad \boxed{1}$$

$$\sum F_y = 0 \Rightarrow N_1 = N_2\times\frac{\sqrt{3}}{2} + W + f_2\times\frac{1}{2} \quad (3) \quad \boxed{1}$$

$$\sum M_Q = 0 \Rightarrow f_1 r - f_2 r = 0 \Rightarrow f_1 = f_2 \quad (4) \quad \boxed{1}$$



From (1), (2) and (3)

$$-f_1 - f_1\times\frac{\sqrt{3}}{2} + \sqrt{3}P\times\frac{1}{2} = 0$$

$$\Rightarrow f_1 = f_2 = \left(2\sqrt{3}-3\right)P = 0.464P = 185.6N$$

From (1) and (3)

$$N_1 = N_2\times\frac{\sqrt{3}}{2} + W + f_2\times\frac{1}{2}$$

$$\Rightarrow N_1 = \sqrt{3}P\times\frac{\sqrt{3}}{2} + \frac{P}{2} + f_1\times\frac{1}{2} = \left(\sqrt{3}+\frac{1}{2}\right)P = 2.232P = 893N$$

$$\therefore \frac{f_1}{N_1} = \frac{4\sqrt{3}-6}{2\sqrt{3}+1} = \frac{30-16\sqrt{3}}{11} = 0.208 \quad \boxed{1}$$

$$\frac{f_2}{N_2} = 2-\sqrt{3} = 0.267 \quad \boxed{1}$$

$\therefore$ Slipping cannot occur at the rod if $\frac{f_2}{N_2} \le \mu_s \Rightarrow \mu_s \ge 0.267$

Slipping cannot occur at the floor if $\frac{f_1}{N_1} \le \mu_s \Rightarrow \mu_s \ge 0.208$

Since friction is same at all surfaces, hence minimum coefficent of friction is maximum of these two values, i.e. $\mu_s \ge 0.267$ $\boxed{1}$