INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 29.11.2016(FN) | **Time: 3 hours**
End Semester Examination (Autumn) | **Maximum Marks: 100**
Subject No. ME10001 | **Subject Name: MECHANICS**
No. of students: 765

Instructions: ***Answer all SEVEN questions. Any data, if not furnished, may be assumed with justification. All parts of a question MUST be answered together.***

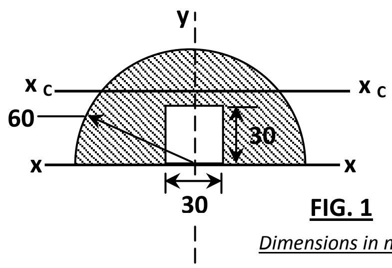


FIG. 1

Dimensions in mm

1. Compute (i) the coordinates of the centroid, and the second moment of area (ii) about the axis x-x and (iii) about the centroidal axis x_C - x_C for the shaded area shown in Fig.1. *(15)*

2. For the beam and loading shown in Fig.2, (i) compute the support reactions at B and D and (ii) draw the shear force diagram (SFD) and the bending moment diagram (BMD).

   The SFD and BMD must be drawn below the beam free body diagram on a fresh page. The sign convention followed for shear force and bending moment must be indicated. All relevant calculations must be shown. In the BMD, indicate the distance(s) from A, if the bending moment changes its sign. *(15)*

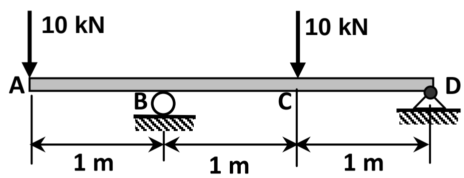


FIG. 2

3. A simply supported I-section beam of span 6m and carrying a uniformly distributed downward load of w N/m is shown in Fig.3. If the permissible stresses of the material in tension and in compression are 165 MPa and 250Mpa respectively, (i) calculate the maximum value of w that the beam can carry. (ii) Determine the maximum normal force and its nature (tensile or compressive) acting on the shaded area (136mmx10mm) on the beam section. The neutral axis of the beam is at 146 mm from the bottom and 174 mm from the top. The second moment of area about the neutral axis, I =80 x10^6 mm^4. *(15)*

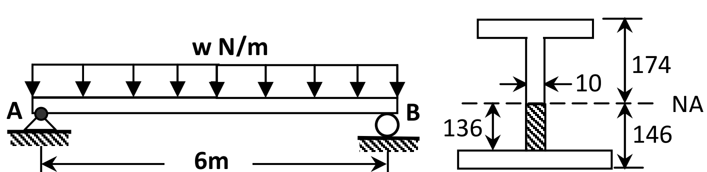


FIG. 3

Beam section

Dimensions in mm

4. A flat belt drive is driving a pump at a constant speed of 1440 rpm (Fig.4). The belt is operating at the maximum belt tension of 1000N. The coefficient of friction between belt and the pulley, $\mu = 0.3$. (i) Determine the diameter of the shaft, if the angle of twist of the shaft is limited to 0.035^{O}. (ii) What is the shear stress due to torsion at the point A on the periphery of the shaft as shown in the figure? Take G = 80GPa. *(15)*

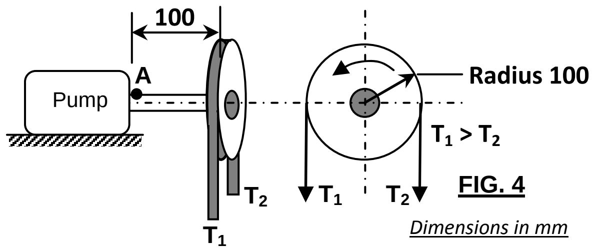


FIG. 4

Dimensions in mm

5. A large plate is subjected to uniform edge stresses (σ_x = 200MPa), as shown in Fig. 5. Before loading, a small square element ABCD of side 100mm was inscribed at an angle on the plate as shown in the figure.
In the loaded condition, determine (i) the normal and shear stresses on the edges of the element, (ii) the changes in the dimensions AB and BC, and (iii) the magnitude of change in the angle ABC. Take E=80GPa and v=0.3. Remember that G=E/2(1+v). *(15)*

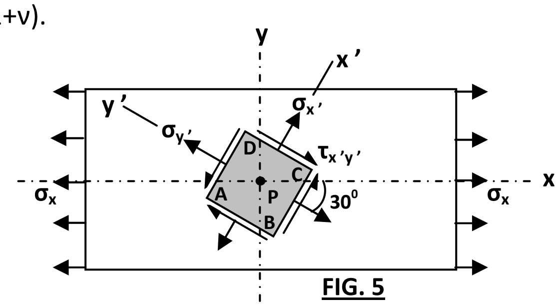


FIG. 5

6. A cylindrical tank with flat and rigid end covers is filled with high pressure gas at gage pressure P. Find ΔV/V in terms of P, r, t, E and v neglecting the end effect, where

V= Internal volume of tank before gas filling	ΔV= Change in V due to gas pressure
r = internal radius of tank before gas filling	t = wall thickness of tank before gas filling
L = internal length of tank before gas filling	E = Young's modulus
v = Poisson's ratio	

(15)

7. The beam, shown in Fig.3 has a rectangular section of height h and width h/2. (i) Determine the support reactions. (ii) Derive the equations of elastic curve and (iii) find the beam deflection at x=1m in terms of E and I (usual notations). (iv) If the deflection at x=1m is restricted to 2.344 mm, what should be the minimum dimensions of the beam section? Take E=80GPa? *(15)*

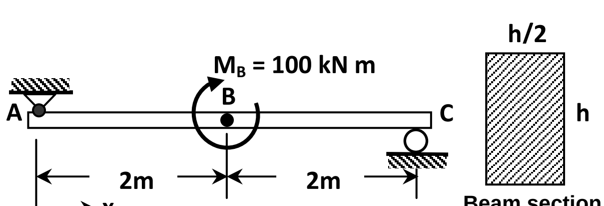
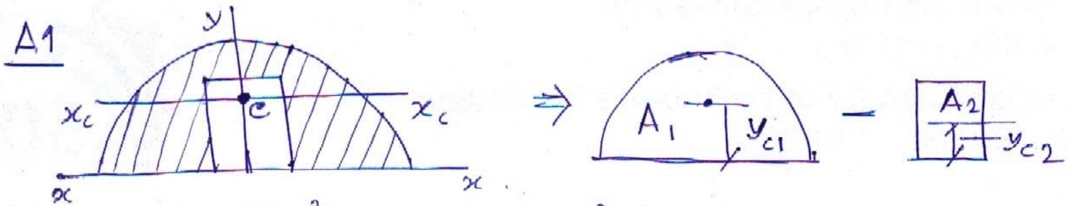


FIG. 7

SOLUTION: MECHANICS END TERM-AUTUMN 2016-17

A1



(i) $A_1 = \frac{\pi r^2}{2} = 5654.87\, mm^2$ $\qquad A_2 = 30 \times 30 = 900\, mm^2$

$y_{c_1} = \frac{4r}{3\pi} = \frac{4 \times 60}{3\pi} = 25.465\, mm$ $\qquad y_{c_2} = 15\, mm$

$\therefore y_c = \frac{(A_1 y_{c_1} - A_2 y_{c_2})}{(A_1 - A_2)} = 27.446\, mm$

$\therefore$ Coordinates of Centroid $= \underline{(0, 27.446)\, mm}$

(ii) $I^1_{xx} = \frac{\pi r^4}{8} = 5089380.1\, mm^4$

$I^2_{xx} = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2 = \frac{bh^3}{3} = 270000\, mm^4$

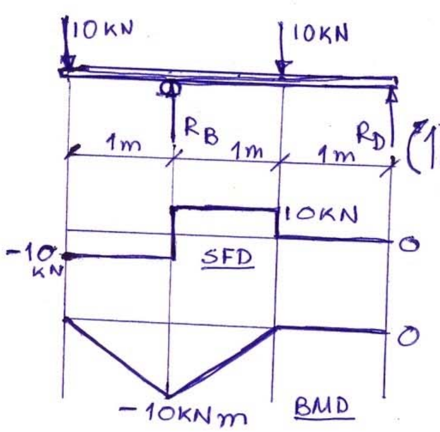
$I_{xx} = I^1_{xx} - I^2_{xx} = \underline{4819380\, mm^4}$

(iii) $I_{xx} = I_{x_c x_c} + Ad^2$.

$I_{x_c x_c} = I_{xx} - Ad^2 = 4819380 - (5654.87 - 900) \times 27.446^2$

$= \underline{1237617.66\, mm^4}$

A2

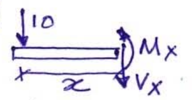


(i) $\Sigma M_D = 0$; $10 \times 3 + 10 \times 1 - R_B \times 2 = 0$

$R_B = \frac{10 \times 4}{2} = \underline{20 KN}$ and $R_D = \underline{0}$

(ii) $(\uparrow \boxed{+} \downarrow)$

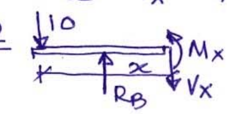
$\underline{0 \le x \le 1}$



$V_x + 10 = 0 \quad \underline{V_x = -10\ KN}$

$M_x + 10x = 0; \ \underline{M_x = -10x\ KNm}$

$\underline{1 \le x \le 2}$

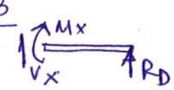


$V_x + 10 - R_B = 0; \ \underline{V_x = 10 KN}$

$M_x + 10x - R_B(x-1) = 0$

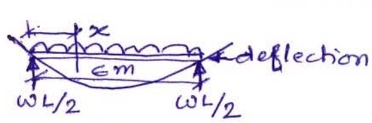
$\therefore \underline{M_x = 10x - 20}$

$\underline{2 \le x \le 3}$



$\underline{V_x = 0, \ M_x = 0}$

A3



$(\uparrow \boxed{+} \circlearrowleft) M_x, V_x$ at x,

$M_x + \frac{\omega x^2}{2} - \frac{\omega L x}{2} = 0$

$M_x = \frac{\omega L x}{2} - \frac{\omega x^2}{2}$

$\frac{dM_x}{dx} = \frac{\omega L}{2} - \omega x = 0$

$\therefore M_{max}$ is at $x = \frac{L}{2}$.

$M_{max} = \frac{\omega L^2}{8} = \underline{4.5\omega\, Nm}$

(i) Beam top surface is in compression

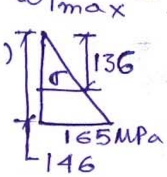
Beam bottom surface is in tension.

$\sigma_c = \frac{4.5\omega \times 10^3 \times 174}{80 \times 10^6} = 250 \ \therefore \omega = 25542.8\ N/m$

$\sigma_t = \frac{4.5\omega \times 10^3 \times 146}{80 \times 10^6} = 165 \ \therefore \omega = 20091.32\ N/m$.

$\therefore \omega|_{max} = 20.1\ KN/m$

(ii)



$\frac{\sigma}{136} = \frac{165}{146}$

$\therefore \sigma = 153.7 MPa$.

average stress on area $= \frac{0 + 153.7}{2}$

$= \underline{76.85 MPa}$.

$\therefore$ Normal Force

$= 76.85 \times (136 \times 10) = \underline{104.52\ KN\ (T)}$

A4

(i) Higher Tension, $T_1 = 1000N$, $\frac{T_1}{T_2} = e^{0.3\times\pi} = 2.566 \therefore \underline{T_2 = 389.66N}$

$\therefore$ Torque, $T = (1000 - 389.66)\times 100 = \underline{61.034\times10^3 \text{ N-mm}}$

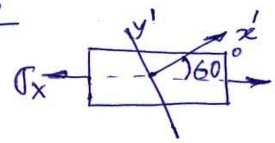
$\phi = 0.035^\circ = \frac{0.035\times\pi}{180} \text{ rad} = \underline{6.109\times10^{-4} \text{ rad}}$

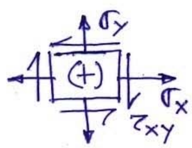
$\phi = \frac{TL}{GI_p} \therefore I_p = \frac{TL}{G\phi} = \frac{61.034\times10^3\times100}{80\times10^3\times6.109\times10^{-4}} = \underline{124892.52 \text{ mm}^4}$

$I_p = \frac{\pi d^4}{32} \therefore d^4 = 1272144.76 \text{ mm}^4, \underline{d = 33.58 \text{ mm}}$

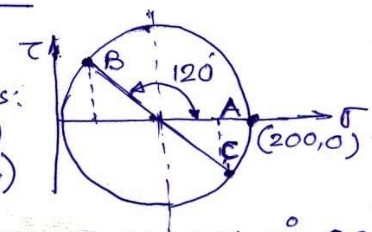
(ii) $\tau = \frac{T\times r}{I_p} = \frac{16T}{\pi d^3} = \frac{16\times61.034\times10^3}{\pi\times(33.58)^3} = \underline{8.206 \text{ MPa}}$

A5





(i) Mohr Circle coordinates:
$B(\sigma_{x'}, \tau_{x'y'})$
$C(\sigma_{y'}, -\tau_{x'y'})$



Equations.

$\sigma_{x'} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 120$
$= 50 \text{ MPa}.$

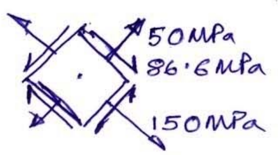
$\tau_{x'y'} = \frac{\sigma_x}{2}\sin 120^\circ = 86.6 \text{ MPa}$

$\sigma_{y'} = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos(-60^\circ) = 150 \text{ MPa}$

$\tau_{y'x'} = \frac{\sigma_x}{2}\sin(-60^\circ) = -86.6 \text{ MPa}.$

$\therefore \sigma_{x'} = 100 - 100\cos 60^\circ = 50 \text{MPa}$, $\tau_{x'y'} = 100\sin 60^\circ = \underline{86.6 \text{MPa}}$

$\sigma_{y'} = 100 + 100\cos 60^\circ = \underline{150 \text{MPa}}$, $-\tau_{y'x'} = -\sin 60^\circ = \underline{-86.6 \text{ MPa}}$



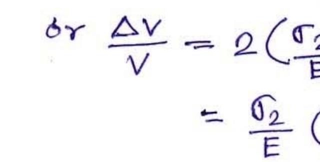
(ii) $\epsilon_{x'} = \frac{\sigma_{x'}}{E} - \nu\frac{\sigma_{y'}}{E} = \frac{1}{80\times10^3}(50 - 0.3\times150) = 6.25\times10^{-5} \therefore \underline{\Delta BC = \epsilon_{x'}\times100 = 6.25\times10^{-3} \text{ mm}}$.

$\epsilon_{y'} = \frac{\sigma_{y'}}{E} - \nu\frac{\sigma_{x'}}{E} = \frac{1}{80\times10^3}(150 - 0.3\times50) = 1.6875\times10^{-3} \therefore \underline{\Delta AB = \epsilon_{y'}\times100 = 0.16875 \text{ mm}}$

(iii) $G = \frac{80\times10^3}{2(1+0.3)} = 30.769\times10^3 \text{ MPa}.$ $|\gamma_{x'y'}| = \left|\frac{\tau_{x'y'}}{G}\right| = \frac{86.6}{30.769\times10^3} \text{ rad} = 2.8145\times10^{-3} \text{ rad}$
$= \underline{0.1613^\circ}$

Change in $\angle ABC$

A6



$\sigma_1 = \frac{Pr}{2t}$, $\sigma_2 = \frac{Pr}{t}$, $\epsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \frac{(L+\Delta L) - L}{L} = \frac{\Delta L}{L}$

$\epsilon_2 = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E} = \frac{2\pi(r+\Delta r) - 2\pi r}{2\pi r} = \frac{\Delta r}{r}$

$$V = \pi r^2 \times L, \therefore \Delta V = 2\pi r L\Delta r + \pi r^2 \Delta L$$

$$\therefore \frac{\Delta V}{V} = \frac{2\pi r L \Delta r}{\pi r^2 L} + \frac{\pi r^2 \Delta L}{\pi r^2 L} = 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

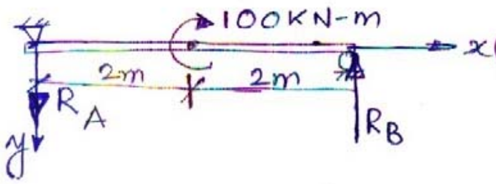
or $\frac{\Delta V}{V} = 2\left(\frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E}\right) + \left(\frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E}\right)$

$= \frac{\sigma_2}{E}(2-\nu) + \frac{\sigma_1}{E}(1-2\nu)$

$= \frac{Pr}{Et}(2-\nu) + \frac{Pr}{2tE}(1-2\nu)$

$= \frac{Pr}{Et}\left(2-\nu+\frac{1}{2}-\nu\right) = \frac{Pr}{Et}\left(\frac{5}{2}-2\nu\right) = \underline{\frac{Pr}{2Et}(5-4\nu)}$

A7

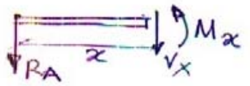


(i) $\Sigma M_C = 0$, $R_A \times 4 = 100$

$\therefore R_A = 25\,KN$

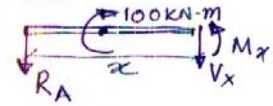
$R_A - R_B = 0 \;\therefore R_B = 25\,KN$

$0 \le x \le 2$ (sec-1)



$M_x + R_A = 0 \;\therefore M_x = -25x\ KNm$
$= -25x \times 10^3\ N\text{-}m$

$2 \le x \le 4$ (sec-2)



$R_A x + M_x - 100 = 0$, $M_x = 100 - 25x$
$= -(25x - 100) \times 10^3\ N\text{-}m$

Let $K = EI/10^3$

$K\frac{d^2y}{dx^2} = -M_x = 25x$ — (1)

$K\frac{dy}{dx} = \frac{25}{2}x^2 + C_1$ — (2)

$Ky = \frac{25x^3}{6} + C_1x + C_2$ — (3)

$K\frac{d^2y}{dx^2} = 25x - 100$ — (4)

$K\frac{dy}{dx} = \frac{25x^2}{2} - 100x + D1$ — (5)

$KY = \frac{25x^3}{6} - 100\frac{x^2}{2} + D1x + D2$ — (6)

Boundary Conditions

at $x=0$, $y=0$ from (3) $C_2 = 0$ — (7)

at $x=4$, $y=0$; from (6)

$0 = \frac{25}{6}\times 4^3 - 50\times 4^2 + D1\times 4 + D2$

$4D1 + D2 = 50\times 4^2 - \frac{25\times 4^3}{6} = \frac{1600}{3}$ — (8)

at $x=2$, $y|_1 = y|_2$; from (3) & (6)

$\frac{25\times 2^3}{6} + 2C_1 = \frac{25\times 2^3}{6} - 50\times 2^2 + D1\times 2 + D_2$; $2C_1 = 2D1 + D_2 - 200$ — (9)

at $x=2$, $\frac{dy}{dx}\Big|_1 = \frac{dy}{dx}\Big|_2$; from (2) and (5)

$\frac{25}{2}\times 4 + C_1 = \frac{25}{2}\times 4 - 100\times 2 + D1 \;\therefore D1 = C_1 + 200$ — (10)

From (9) and (10)

$D1 = D1 + \frac{D2}{2} - 100 + 200$, $\therefore D2 = -200$ — (11)

From (8) and (11)

$4D1 - 200 = \frac{1600}{3} \;\therefore D1 = \frac{550}{3}$ — (12)

From (10) and (12)

$\frac{550}{3} = C_1 + 200 \;\therefore C_1 = -\frac{50}{3}$ — (13)

at $x=1$ and from (13) and (3)

$Ky = \frac{25}{6} - \frac{50}{3} = -\frac{75}{6} \;\therefore y = -\frac{75}{6K} = -\frac{75\times 10^3}{6EI}\,m$ — (14)

$= \frac{75\times 10^3}{6EI}\,m$ (downward)

(ii) from (14) and given condition,

$2.344\times 10^{-3} = \frac{75\times 10^3}{6\times 80\times 10^9\times I}$

$\therefore I = 6.666\times 10^{-5}\,m^4$, Now $I = \frac{bh^3}{12} = \frac{h}{2}\times\frac{h^3}{12} = \frac{h^4}{24}$

$\therefore \frac{h^4}{24} = 6.666\times 10^{-5} \;\therefore h = 0.1999\,m \approx 0.2\,m$
$= 200\,mm$

For the beam Section: Width = 100 mm and height = 200 mm