INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester 2017-18

Date of Examination 25.09.2017 **Session** FN **Duration** 2 hrs **Max. Marks** 90
Subject No. : ME 10001 **Subject:** Mechanics
Department/Center/School: Mechanical Engineering
Instructions: Answer all questions. All parts of a question MUST be together. Figures are not to scale.

1. A woman supports an 80 kg homogeneous box on a horizontal rough ledge by providing only an upward vertical force at the corner B, as shown in Figure 1. We need to determine the range (F_{Bmin}, F_{Bmax}) within which the vertical force at B must lie for keeping the box in equilibrium without tilting or moving it from the horizontal position shown.

   (a) Draw two separate free body diagrams of the box corresponding to F_{Bmin} and F_{Bmax}. **(4)**

   (b) Determine the range (F_{Bmin}, F_{Bmax}). Take $g = 10$ m/s^2. **(12)**

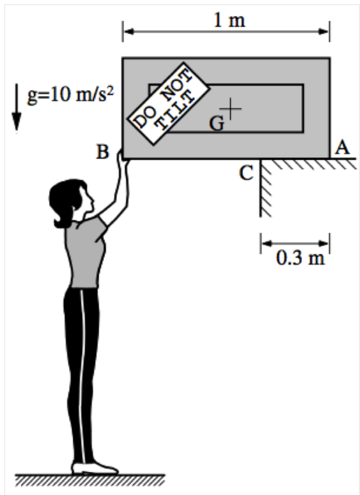

Figure 1

2. The 7 m long massless beam AB is supported by a ball-and-socket joint at A and two inextensible cables BC and DE, as shown in Figure 2. The beam makes equal angles with x, y and z axes. The cable DE is parallel to y-axis. A vertically downward load of $W = 2$ kN is applied to the beam at the end B.

   (a) Draw a neat free body diagram of the beam AB. **(5)**

   (b) Determine the cable tensions T_{BC} and T_{DE}. **(10)**

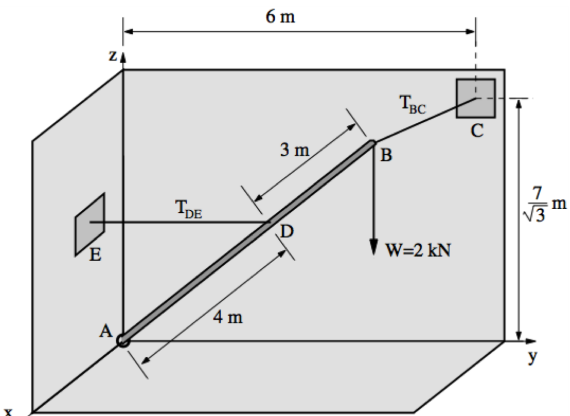

Figure 2

3. The massless cantilever beam AB shown in Figure 3 is subject to a couple M at the midspan C and a force P at the free end B.

   (a) Draw a neat free body diagram of the beam AB. **(4)**

   (b) Calculate the reaction components at A. **(6)**

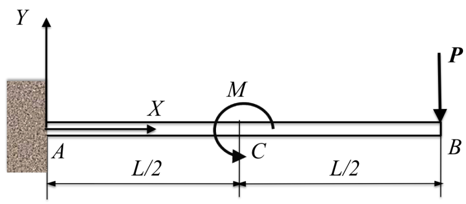

Figure 3

4. For the truss shown in Figure 4

   (a) Identify the zero force members. **(6*)**
   (* *Wrong identification carries penalty.*)

   (b) Compute the forces in members CF and BC and state whether they are in tension or compression. **(6)**

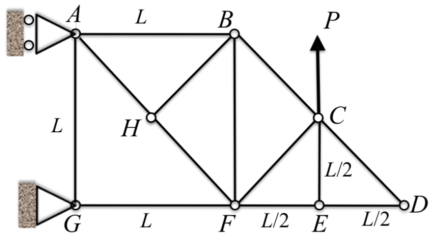

Figure 4

5. The massless frame shown in Figure 5 is subject to a 6 kN load at end E. The pin at C is rigidly attached to member ABC and is supported by the frictionless slot in member DE.

   (a) Draw free body diagrams of all the members. **(6)**

   (b) Compute the components of forces at the pins A, B and C. **(8)**

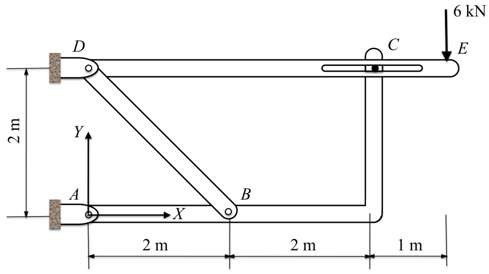

Figure 5

6. The circular cylinder A rests on two half-cylinders B and C as shown in Figure 6. All cylinders are homogeneous and have same radius r. The coefficient of friction between the half-cylinders and the horizontal surface is $\mu = 0.5$. The contact between the cylinders is frictionless. Determine the maximum distance d, between the half-cylinders, to maintain the arrangement in equilibrium. **(14)**

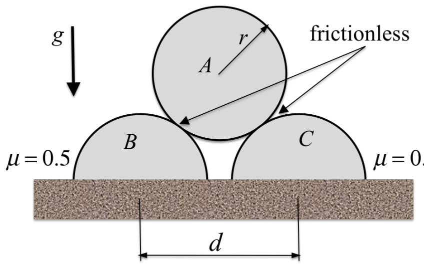

Figure 6

7. A massless belt-idler, shown in Figure 7, comprises of a wooden cylinder fixed rigidly to an arm which is hinged at O. An inextensible light belt passes over the cylinder and moves at a steady speed from right to left as shown. The coefficient of kinetic friction between the belt and the cylinder is $\mu_k = 0.5$ and $T_1 = 10$ N. If both T_1 and T_2 always remain vertical while moving, determine

   (a) the steady angle θ that the arm makes with the vertical, and **(12)**

   (b) the net force magnitude on the hinge at O. **(2)**

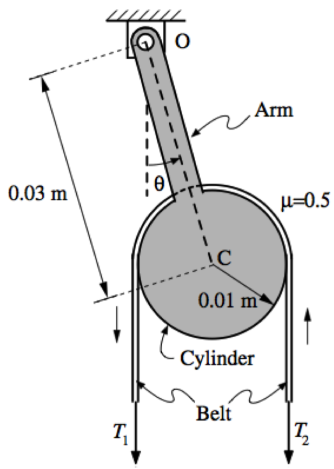

Figure 7

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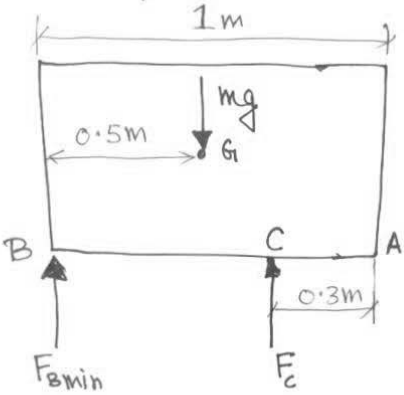
Solutions

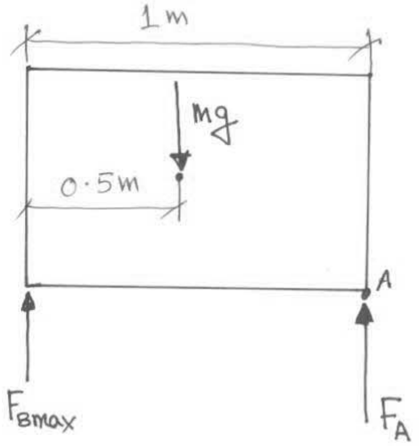
Problem 1:

(a) Only vertical force is applied at B. The box may tilt about point C or point A.

- For F_{Bmin} the box will tilt about point C.



- For F_{Bmax} the box will tilt about point A.



(b) To calculate F_{Bmin}:

$\underline{\sum M_C = 0}$: (left FBD)

$$F_{Bmin} \times 0.7 - mg \times 0.2 = 0$$

$$\Rightarrow F_{Bmin} = \frac{2}{7} mg = \frac{2}{7} \times 80 \times 10 \text{ N}$$

$$\Rightarrow \boxed{F_{Bmin} = 228.57 \text{ N}}$$

To calculate F_{Bmax}:

$\underline{\sum M_A = 0}$ (right FBD)

$$F_{Bmax} \times 1 - mg \times 0.5 = 0$$

$$\Rightarrow F_{Bmax} = \frac{1}{2} mg = 40 \times 10 \text{ N}$$

$$\Rightarrow \boxed{F_{Bmax} = 400 \text{ N}}$$

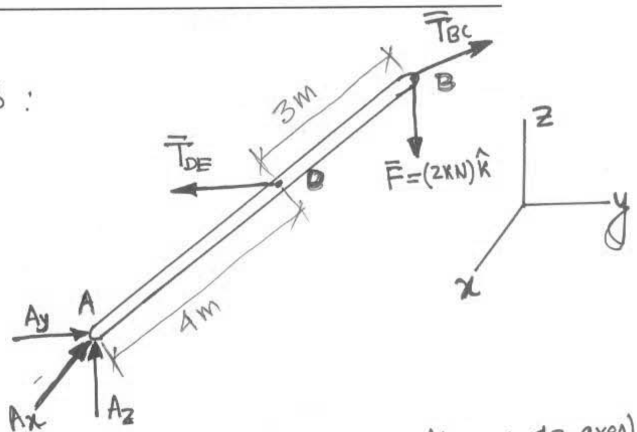
- The range of F_B to maintain equilibrium is

$$\boxed{(F_{Bmin}, F_{Bmax}) = (228.57 \text{ N}, 400 \text{ N})}$$

Mechanics (ME10001) Midsem Exam-Autumn 2017
Solutions

Problem-2 :

(a) FBD of beam AB :



(b) • $\bar{r}_{AB} = \frac{7}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})\,m$ (as AB makes equal angle with x, y, & z axes)

• $\bar{r}_{AD} = \frac{4}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})\,m$ • $\bar{r}_{AC} = \left(6\hat{j}+\frac{7}{\sqrt{3}}\hat{k}\right)m$

$$\Rightarrow \bar{r}_{BC} = \bar{r}_{AC} - \bar{r}_{AB} = -\frac{7}{\sqrt{3}}\hat{i} + \left(6-\frac{7}{\sqrt{3}}\right)\hat{j}$$

Forces: $\bar{T}_{BC} = T_{BC}\cdot \underbrace{\hat{n}_{BC}}_{\text{unit vector along BC}} = \frac{T_{BC}}{|\bar{r}_{BC}|}\cdot\left\{-\frac{7}{\sqrt{3}}\hat{i} + \left(6-\frac{7}{\sqrt{3}}\right)\hat{j}\right\}$

$\bar{T}_{DE} = -T_{DE}\,\hat{j}$ (parallel to y-axis)

$\bar{F} = -(2\,kN)\,\hat{k}$ (vertically downward)

Take moment about A :

$$\underline{\Sigma \bar{M}_A = \bar{0}} \Rightarrow \bar{r}_{AD}\times\bar{T}_{DE} + \bar{r}_{AB}\times\bar{T}_{BC} + \bar{r}_{AB}\times\bar{F} = \bar{0}$$

$$\Rightarrow \frac{4}{\cancel{\sqrt{3}}}(\hat{i}+\hat{j}+\hat{k})\times(-T_{DE}\hat{j}) + \frac{7}{\cancel{\sqrt{3}}}(\hat{i}+\hat{j}+\hat{k})\times\frac{T_{BC}}{|\bar{r}_{BC}|}\left\{-\frac{7}{\sqrt{3}}\hat{i}+\left(6-\frac{7}{\sqrt{3}}\right)\hat{j}\right\}$$

$$+ \frac{7}{\cancel{\sqrt{3}}}(\hat{i}+\hat{j}+\hat{k})\times\{-2\hat{k}\} = \bar{0}$$

$$\Rightarrow \left(-4T_{DE}\hat{k} + 4T_{DE}\hat{i}\right) + \frac{7T_{BC}}{|\bar{r}_{BC}|}\left(\frac{7}{\cancel{\sqrt{3}}}\hat{k} - \frac{7}{\sqrt{3}}\hat{j} + \left(6-\frac{7}{\sqrt{3}}\right)\hat{k} - \left(6-\frac{7}{\sqrt{3}}\right)\hat{i}\right)$$

$$+14\hat{j} - 14\hat{i} = \bar{0}$$

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$$\Rightarrow \left(4T_{DE} - \frac{7T_{BC}}{|\bar{r}_{BC}|}\left(6-\frac{7}{\sqrt{3}}\right) - 14\right)\hat{i}$$

$$+ \left(-\frac{7T_{BC}}{|\bar{r}_{BC}|} \times \frac{7}{\sqrt{3}} + 14\right)\hat{j}$$

$$+ \left(-4T_{DE} + \frac{7T_{BC}}{|\bar{r}_{BC}|} \times 6\right)\hat{k} = \bar{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

From the component along y we obtain:

$$-\frac{7T_{BC}}{|\bar{r}_{BC}|} \times \frac{7}{\sqrt{3}} + 14 = 0 \Rightarrow \boxed{T_{BC} = \frac{14\sqrt{3}}{49} \times |\bar{r}_{BC}| \text{ KN}}$$

Now: $|\bar{r}_{BC}| = \left\{\left(6-\frac{7}{\sqrt{3}}\right)^2 + \left(\frac{7}{\sqrt{3}}\right)^2\right\}^{1/2} = 4.491$

$$\Rightarrow \boxed{T_{BC} = 2.222 \text{ KN}}$$

From the component along z we obtain:

$$-4T_{DE} + \frac{7T_{BC}}{|\bar{r}_{BC}|} \times 6 = 0$$

$$\Rightarrow T_{DE} = \frac{7 \times 3}{2} \times \frac{T_{BC}}{|\bar{r}_{BC}|} = \frac{\cancel{21}^{3}}{2} \times \frac{\cancel{14}^{\cancel{7}}\sqrt{3}}{\cancel{49}_{\cancel{7}}} \text{ KN}$$

$$\Rightarrow \boxed{T_{DE} = 3\sqrt{3} \text{ KN} = 5.196 \text{ KN}}$$

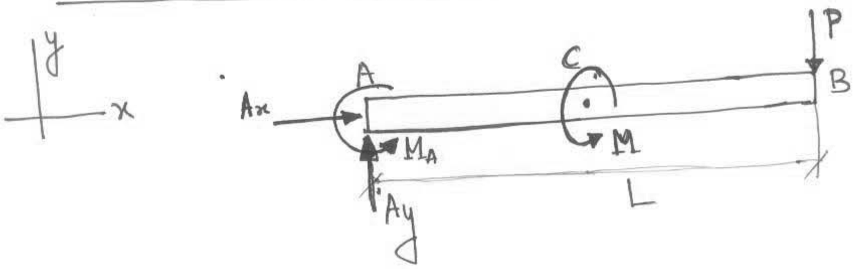
Therefore the cable tensions are:

$$\boxed{\begin{aligned} T_{BC} &= 2.222 \text{ KN} \\ T_{DE} &= 5.196 \text{ KN} \end{aligned}}$$

Mechanics (ME10001) Midsem Exam-Autumn 2017
Solutions

Problem – 3:

(a) FBD of the beam AB:



(b) Reaction Components at A:

$\Sigma F_x = 0$: $\boxed{A_x = 0}$

$\Sigma F_y = 0$: $A_y - P = 0 \Rightarrow \boxed{A_y = P}$

$\Sigma M_{at\,A} = 0$: $M_A + M - P.L = 0$

$\Rightarrow \boxed{M_A = PL - M}$

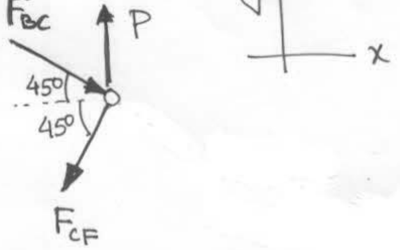
Problem – 4:

(a) Zero force members:

- CD and ED (from joint D)
- EF and CE (from joint E)
- BH (from joint H)

(b) Forces in members CF and BC:

FBD of joint C:



- $\underline{\sum F_x = 0}$: $F_{BC}\cos 45^\circ - F_{CF}\cos 45^\circ = 0$

$$\Rightarrow \boxed{F_{BC} = F_{CF}}$$

- $\underline{\sum F_y = 0}$: $P - F_{BC}\sin 45^\circ - F_{CF}\sin 45^\circ = 0$

$$\Rightarrow P - 2F_{BC} \times \frac{1}{\sqrt{2}} = 0 \quad \left(\text{using } F_{CF} = F_{BC} \text{ \& } \sin 45^\circ = 1/\sqrt{2}\right)$$

$$\Rightarrow \boxed{\begin{aligned} F_{BC} &= P/\sqrt{2} \\ F_{CF} &= P/\sqrt{2} \end{aligned}}$$

Therefore: $\boxed{\begin{aligned} F_{BC} &= P/\sqrt{2} \text{ (compression)} \\ F_{CF} &= P/\sqrt{2} \text{ (tension)} \end{aligned}}$

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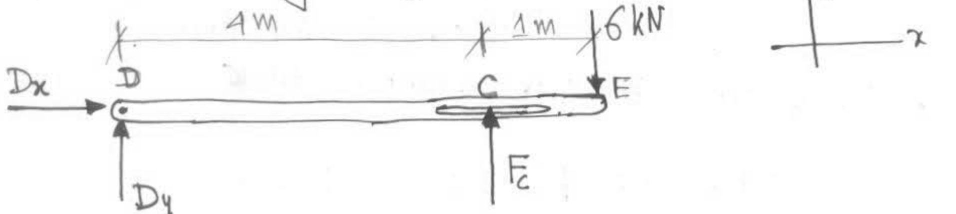
Solutions

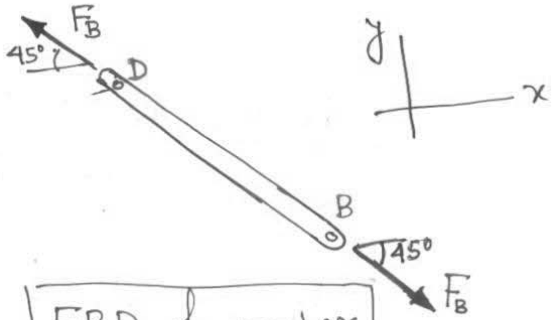
Problem-5:

(a) Free body diagrams:

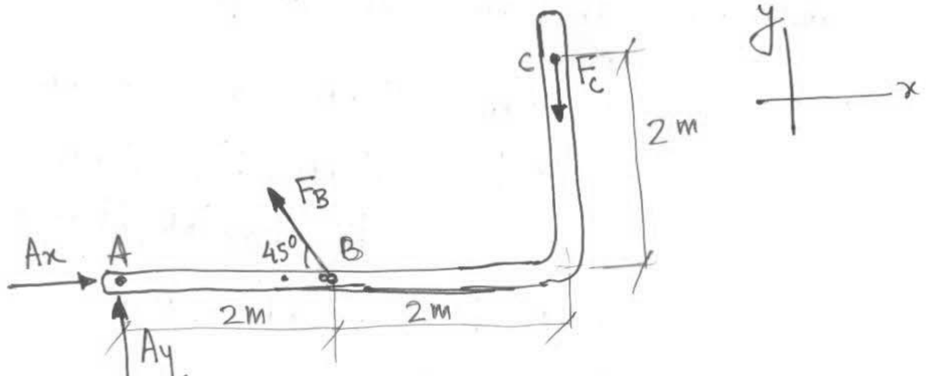


FBD of member DE

• Member BD has forces acting at its ends through pin joints. Therefore BD is a two force member.



FBD of member BD



FBD of member ABC

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(b) Forces at pins A, B and C:

- From the FBD of member DE:

$\sum M_D = 0$: $F_c \times 4 - 6\,kN \times 5 = 0$

$\Rightarrow \boxed{F_c = \frac{30}{4} = 7.5\,kN}$ (along y-direction)

- From the FBD of member ABC

$\sum M_A = 0$: $F_B \sin 45^\circ \times 2 - F_c \times 4 = 0$

$\Rightarrow \boxed{F_B = 2\sqrt{2}\,F_c = 15\sqrt{2}\,kN}$

Component of F_B — along x : $F_{Bx} = F_B \cos 45^\circ = 15\,kN$

along y : $F_{By} = F_B \sin 45^\circ = 15\,kN$

- $\sum F_x = 0$: $A_x - F_B \cos 45^\circ = 0$

$\Rightarrow \boxed{A_x = F_B \cos 45^\circ = 15\,kN}$

$\sum F_y = 0$: $A_y + F_B \sin 45^\circ - F_c = 0$

$\Rightarrow \boxed{A_y = F_c - \frac{F_B}{\sqrt{2}} = -7.5\,kN}$

Therefore the forces at the pins A, B, C are

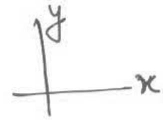
- $A_x = 15\,kN$, $A_y = -7.5\,kN$
- $F_{Bx} = 15\,kN$, $F_{By} = 15\,kN$

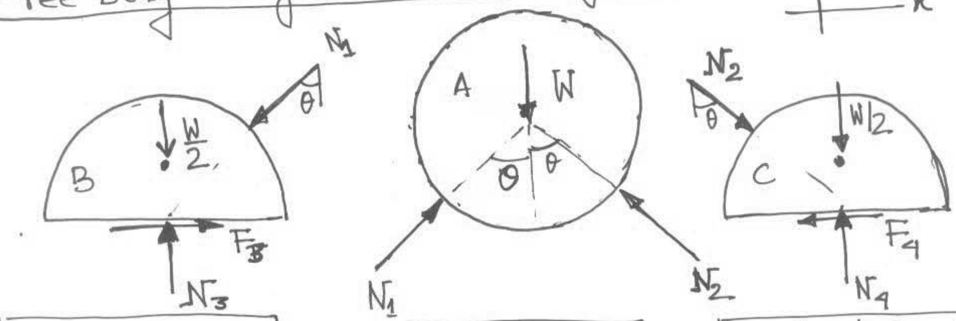
or $F_B = 15\sqrt{2}\,kN$ (tension)
- $F_c = 7.5\,kN$ (downward & along y-axis)

Mechanics (ME10001) Midsem Exam-Autumn 2017
Solutions

Problem- 6:

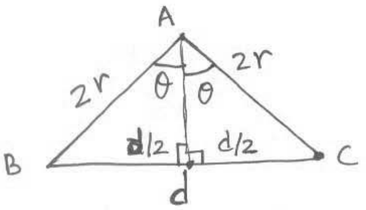
Free body diagrams of the cylinders:





Half-cylinder B | Cylinder- A | Half-cylinder C

From the triangle joining centers of A, B & C.



$$\Rightarrow \boxed{\sin\theta = \frac{d}{4r}, \quad \cos\theta = \frac{\sqrt{16r^2-d^2}}{4r} \quad \tan\theta = \frac{d}{\sqrt{16r^2-d^2}}}$$

- From the FBD of cylinder A:

$\Sigma F_x = 0: \quad \boxed{N_1 = N_2}$

$\Sigma F_y = 0: \quad (N_1 + N_2)\cos\theta = W \quad$ or $\quad \boxed{2N_1 \cos\theta = W}$ — (1)

Note: Since $N_1 = N_2$ from FBD's of half-cylinders B & C, we get $\boxed{F_3 = F_4 \quad N_3 = N_4}$

- From the FBD of cylinder-B:

$\Sigma F_y = 0: \quad N_3 - \frac{W}{2} - N_1\cos\theta = 0$

$\Rightarrow \boxed{N_3 = W}$ (after using eq (1))

$\Sigma F_x = 0: \quad F_3 - N_1 \sin\theta = 0$

or $\boxed{F_3 = N_1 \sin\theta = \frac{W}{2}\tan\theta}$ (using eq (1))

P.T.O

- To maintain equilibrium — need to avoid slip of halfcylinders B & C.

$$\Rightarrow \quad F_3 \leq \mu N_3 = \mu W$$

$$\Rightarrow \quad \frac{W}{2} \tan\theta \leq \mu W$$

$$\Rightarrow \quad \boxed{\tan\theta \leq 2\mu}$$

$$\Rightarrow \quad \frac{d}{\sqrt{16r^2 - d^2}} \leq 2\mu = 1$$

$$\Rightarrow \quad d^2 \leq 16r^2 - d^2$$

$$\Rightarrow \quad 2d^2 \leq 16r^2 \Rightarrow \boxed{d \leq 2\sqrt{2}r}$$

- Since, $F_3 = F_4$ and $N_3 = N_4$, we obtain the same relation ~~from~~ considering FBD of cylinder-C.

- For equilibrium, minimum distance d is

$$\boxed{d_{min} = 2\sqrt{2}r}$$

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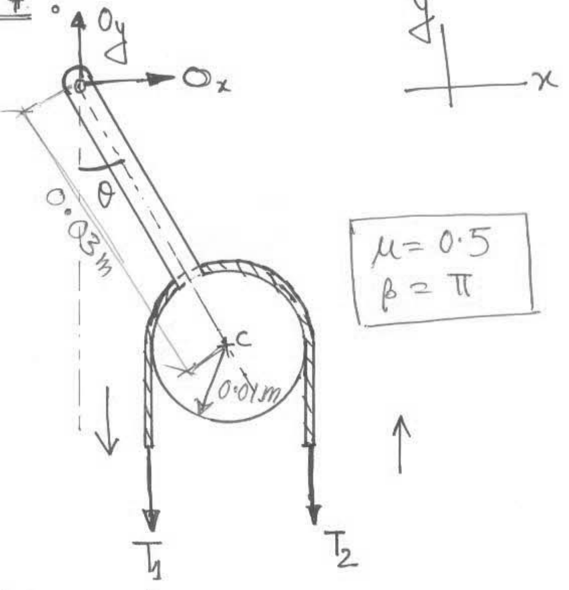
Solutions

Problem-7 :



FBD of the belt-idler-belt system

• $\Sigma F_x = 0$: $\boxed{O_x = 0\,N}$, — (1) $\Sigma F_y = 0$: $\boxed{O_y = T_1 + T_2}$ — (2)

• $\Sigma M_c = 0$: $(T_1 - T_2) \times 0.01 - O_y \times 0.03 \sin\theta - \cancel{O_x}^{0} \times 0.03 \cos\theta = 0$

$\Rightarrow (T_1 - T_2) - (T_1 + T_2) \times 3 \sin\theta = 0$ (using (2))

$\Rightarrow \boxed{\sin\theta = \frac{T_1 - T_2}{3(T_1 + T_2)}}$ — (3)

(a) Since, belt moves from right to left : $T_1 > T_2$

$\Rightarrow T_1/T_2 = e^{\mu\beta} \Rightarrow \boxed{T_2 = T_1 e^{-0.5\pi}}$

$\Rightarrow \sin\theta = \frac{\cancel{T_1}(1 - e^{-0.5\pi})}{3\cancel{T_1}(1 + e^{-0.5\pi})} = 0.2186 \Rightarrow \boxed{\theta = 12.63^\circ}$

(b) $O_y = (T_1 + T_2) = 10(1 + e^{-0.5\pi})\,N = 12.08\,N$, $O_x = 0\,N$

$\Rightarrow$ Force magnitude at hinge O is : 12.08 N