EX-3.12: Determine v_1, v_2, v_3 and i_2 using nodal equations of the circuit shown in Fig.3.17. Also determine the total power dissipated by all the resistors and show that the entire power is supplied by the 2 Amp current source.

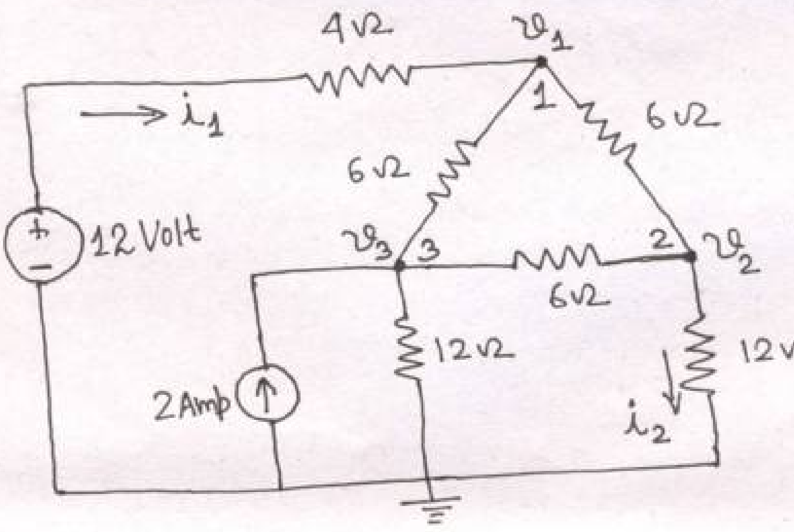


Fig.3.17: Circuit for EX-3.12

~~At~~ Soln.

At node 1,

$$\frac{12-v_1}{4} = \frac{v_1-v_3}{6} + \frac{v_1-v_2}{6}$$

$$\therefore\ 7v_1 - 2v_2 - 2v_3 = 36 \quad \text{--(i)}$$

At node 2,

$$\frac{v_1-v_2}{6} = \frac{v_2}{12} + \frac{v_2-v_3}{6}$$

$$\therefore\ 2v_1 - 5v_2 + 2v_3 = 0 \quad \text{--(ii)}$$

At node 3,

$$\frac{v_1-v_3}{6} + 2 + \frac{v_2-v_3}{6} = \frac{v_3}{12}$$

$\therefore \quad -2v_1 - 2v_2 + 5v_3 = 24 \quad \text{---(iii)}$

Eqns. (i), (ii) and (iii) can be put in matrix form,

$$\begin{bmatrix} 7 & -2 & -2 \\ 2 & -5 & 2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 0 \\ 24 \end{bmatrix} \quad \text{---(iv)}$$

Solving eqn. (iv), we have,

$v_1 = 12$ Volt; $v_2 = \frac{72}{7}$ Volt; $v_3 = \frac{96}{7}$ Volt.

Therefore,

$$i_1 = \frac{12 - v_1}{4} = \frac{12-12}{4} = 0 \text{ Amp.}$$

$\therefore$ Power dissipated in 4Ω resistor is zero.

Power in the remaining five resistors becomes:

$$\frac{1}{6}\left[(v_1 - v_3)^2 + (v_1 - v_2)^2 + (v_2 - v_3)^2\right] + \frac{(v_2)^2}{12} + \frac{(v_3)^2}{12}$$

$= 27.428$ Watt.

Voltage across 2 Amp current source,

$v_3 = \frac{96}{7}$ Volt;

Power supplied by 2 Amp current source

$= \frac{96}{7} \times 2 = 27.428$ Watt, which is the same as the total dissipated power.

EX-3.13: Using nodal analysis, determine v_1, v_2, i_1, i_2, i_3 and i_4 of the circuit as shown in Fig. 3.18.

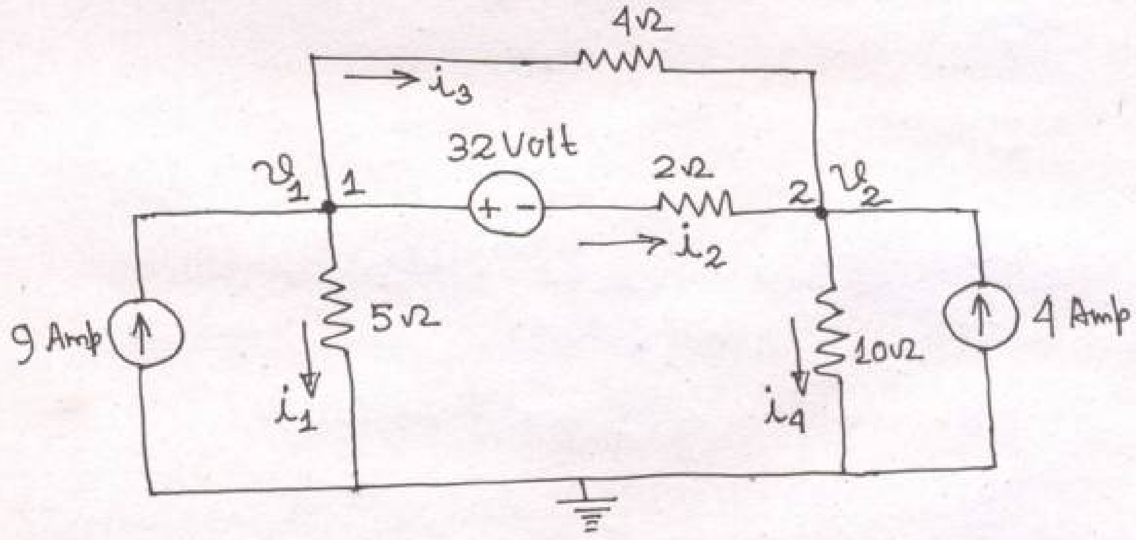


Fig. 3.18: Circuit for Ex-3.13.

Soln.

At node 1,

$$\frac{v_1 - v_2}{4} + \frac{v_1}{5} + \frac{v_1 - v_2 - 32}{2} = 9$$

$$\therefore \; 19 v_1 - 15 v_2 = 500 \quad \text{---(i)}$$

At node 2,

$$\frac{v_1 - v_2}{4} + \frac{v_1 - v_2 - 32}{2} + 4 = \frac{v_2}{10}$$

$$\therefore \; 15 v_1 - 17 v_2 = 240 \quad \text{---(ii)}$$

Solving eqn.(i) and (ii), we obtain,

$v_1 = 50$ Volt; $v_2 = 30$ Volt; ,

Hence,

$$i_1 = \frac{v_1}{5} = \frac{50}{5} = 10 \text{ Amp}; \quad i_2 = \frac{v_1 - v_2 - 32}{2} = \frac{50 - 30 - 32}{2} = -6 \text{ Amp}$$

$$i_3 = \frac{v_1 - v_2}{4} = \frac{50 - 30}{4} = 5 \text{ Amp}; \quad i_4 = \frac{v_2}{10} = \frac{30}{10} = 3 \text{ Amp}.$$

Ex-3.14: Determine the node voltages v_1, v_2, v_3 and v_4 of the circuit shown in Fig. 3.19.

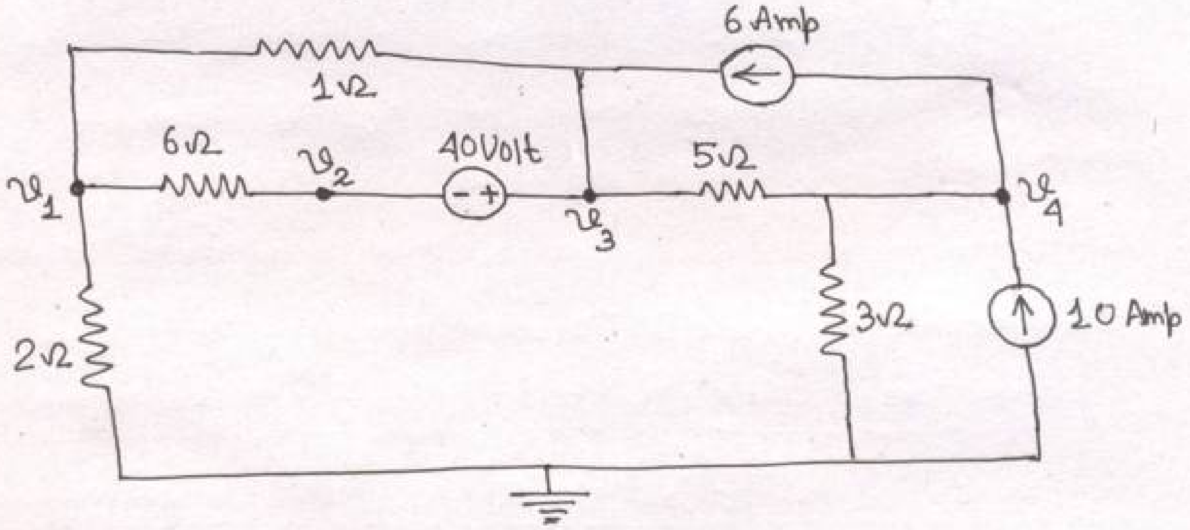


Fig.3.19: Circuit for Ex-3.14

Soln.

At node 1,

$$\frac{v_1}{2} + \frac{v_1 - v_3}{1} + \frac{v_1 + 40 - v_3}{6} = 0$$

$$\therefore \quad 10v_1 - 7v_3 = -40 \quad \text{---(i)}$$

At node 3,

$$\frac{v_1 - v_3}{1} + 6 + \frac{v_1 + 40 - v_3}{6} = \frac{v_3 - v_4}{5}$$

$$\therefore \quad 35v_1 - 41v_3 + 6v_4 = -380 \quad \text{--(ii)}$$

At node 4,

$$10 + \frac{v_3 - v_4}{5} = \frac{v_4}{3} + 6$$

$$\therefore \quad 3v_3 - 8v_4 = -60 \quad \text{----(iii)}$$

Solving eqns.(i), (ii) and (iii), we have,

$v_1 = 10$ Volt; $v_3 = 20$ Volt; $v_4 = 15$ Volt.

Finally, $v_2 + 40 = v_3 \quad \therefore v_2 = 20 - 40 = -20$ Volt

3.4: MESH ANALYSIS

Mesh analysis is a general procedure for analyzing circuits using mesh currents as the circuit variables. Using mesh currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. A loop is a closed path with no node passed more than once.

A mesh is a loop that does not contain any other loop within it.

For a given circuit, nodal analysis applies KCL to obtain unknown voltages, while mesh analysis applies KVL to obtain unknown currents. Mesh analysis is not as general as nodal analysis because it is only applicable to a circuit that is planar. A circuit that can be drawn in a plane with no branches crossing one another is called planar circuit; otherwise it is nonplanar circuit. A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches. Nonplanar circuits can be handled using nodal analysis but they will not be considered in this book.

For the purpose of understanding consider Fig.3.19. In Fig.3.19, paths abefa and bedeb are meshes but path abcdefa is not a mesh.

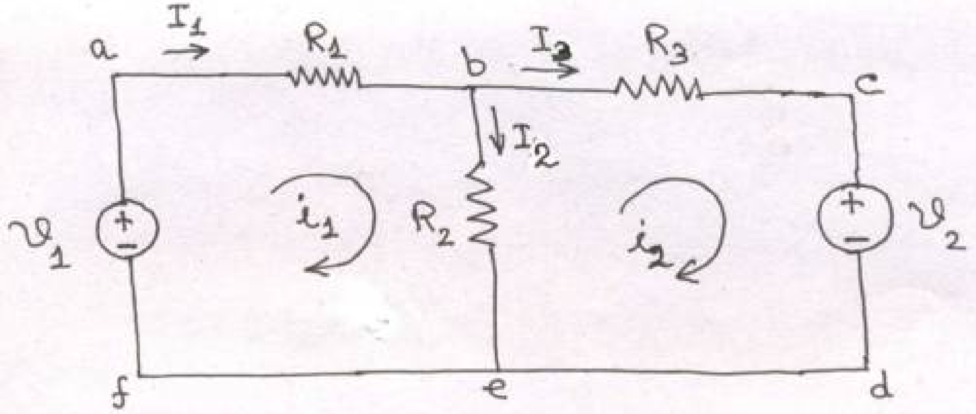


Fig. 3.19: A circuit with two meshes

The path abcdefa is a loop and not a mesh but KVL still holds. This is the reason for loosely using the terms loop analysis and mesh analysis to mean the same thing.

The current through a mesh is known as mesh current and we will apply KVL to find the mesh currents for a given circuit. In this section, we will apply mesh analysis to the circuits that do not contain any current source and in next section, we will consider circuits with current sources.

In Fig. 3.19, mesh currents i_1 and i_2 are assigned to meshes 1 and 2. Note that in Fig. 3.19, the direction of mesh current is arbitrary (clockwise or counterclockwise) and does not affect the validity of the solution.

Now, let us apply KVL to each mesh. Applying KVL to mesh 1, we get

$$-v_1 + R_1 i_1 + R_2(i_1 - i_2) = 0$$

$$\therefore (R_1 + R_2) i_1 - R_2 i_2 = v_1 \quad \cdots (3.23)$$

36

Applying KVL to mesh 2, we get,

$$R_3 i_2 + v_2 + R_2(i_2 - i_1) = 0$$

$$\therefore (R_2 + R_3) i_2 - R_2 i_1 + v_2 = 0$$

$$\therefore -R_2 i_1 + (R_2 + R_3) i_2 = -v_2 \quad \cdots (3.24)$$

Eqns.(3.23) and (3.24) can be written in matrix form:

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \quad \cdots (3.25)$$

Eqns(3.25) can easily be solved for i_1 and i_2.

Note that the branch currents are different from the mesh currents unless the mesh is isolated. In Fig.3.19, I_1, I_2 and I_3 are the branch currents which are algebraic sums of the mesh currents. It is evident from Fig.3.19, that,

$$I_1 = i_1, \quad I_2 = (i_1 - i_2); \quad I_3 = i_2 \quad \cdots (3.26)$$

<u>EX-3.15</u>: Determine I_1, I_2 and I_3 using mesh analysis of the circuit shown in Fig.3.20.

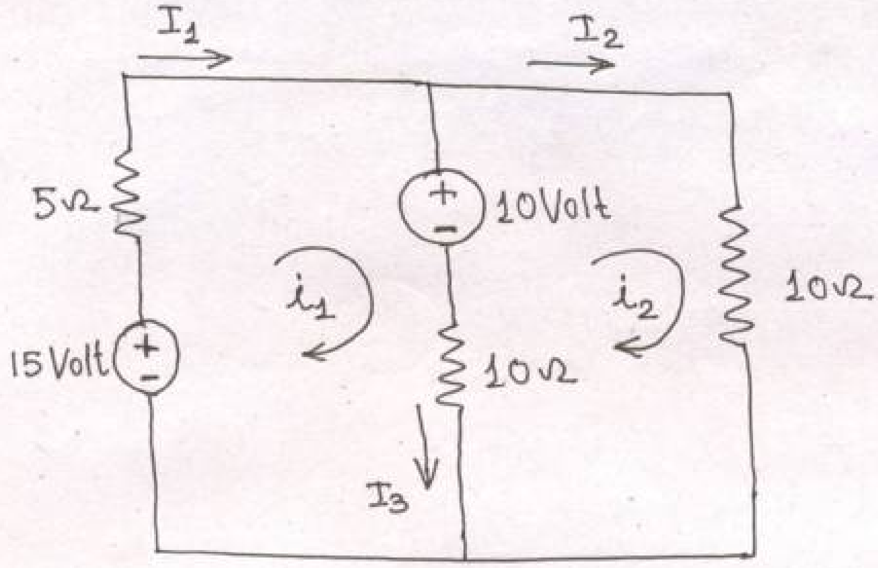


Fig. 3.20: Circuit for EX-3.15

Soln.

Applying KVL in mesh 1,

$$-15 + 5i_1 + 10 + 10(i_1 - i_2) = 0$$

$$\therefore \quad 3i_1 - 2i_2 = 1 \quad \text{---- (i)}$$

For mesh 2,

$$10i_2 + 10(i_2 - i_1) - 10 = 0$$

$$\therefore \quad i_1 - 2i_2 = -1 \quad \text{--- (ii)}$$

Solving eqns. (i) and (ii), we get,

$i_1 = 1$ Amp ; $i_2 = 1$ Amp.

Thus,

$I_1 = i_1 = 1$ Amp ; $I_2 = i_2 = 1$ Amp ;

$I_3 = i_1 - i_2 = 1 - 1 = 0$

EX-3.16: Using mesh analysis, determine I in the circuit of Fig. 3.21.

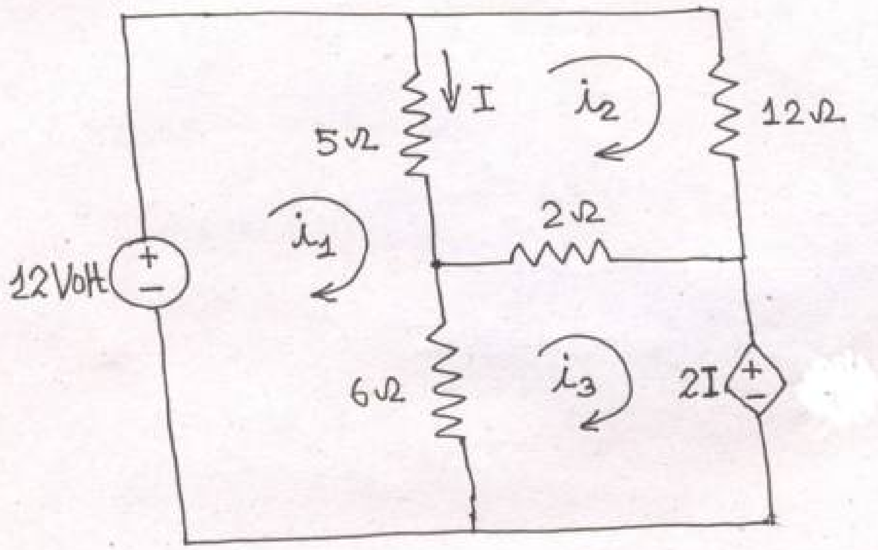


Fig. 3.21: Circuit for EX-3.16

Soln.
Applying KVL to mesh 1,

$$-12 + 5(i_1 - i_2) + 6(i_1 - i_3) = 0$$

$$\therefore \; 11 i_1 - 5 i_2 - 6 i_3 = 12 \quad \text{- - - (i)}$$

For mesh 2,

$$12 i_2 + 2(i_2 - i_3) + 5(i_2 - i_1) = 0$$

$$\therefore \; -5 i_1 + 19 i_2 - 2 i_3 = 0 \quad \text{- - - (ii)}$$

For mesh 3,

$$2I + 6(i_3 - i_1) + 2(i_3 - i_2) = 0$$

But $I = i_1 - i_2$, so that

$$2(i_1 - i_2) + 6(i_3 - i_1) + 2(i_3 - i_2) = 0$$

$$\therefore \; i_1 + i_2 - 2 i_3 = 0 \quad \text{- - - - - (iii)}$$

Eqns. (i), (ii) and (iii) can be put in matrix form,

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \quad \cdots (iv)$$

Solving above equation, we obtain,

$i_1 = 2.25$ Amp; $i_2 = 0.75$ Amp; $i_3 = 1.5$ Amp;

Thus, $I = i_1 - i_2 = (2.25 - 0.75) = 1.5$ Amp.

3.5: MESH ANALYSIS WITH CURRENT SOURCES.

In this section, we will apply mesh analysis to circuits that contain dependent or independent current sources. Presence of current sources in the circuits reduces the number of equations.

Let us consider two following cases:

Case 1: When a current source exists only in one mesh: Fig. 3.22 shows a simple circuit having two meshes. A current source exists in mesh 2.

From Fig. 3.22, $i_2 = -10$ Amp. We now apply KVL in mesh 1,

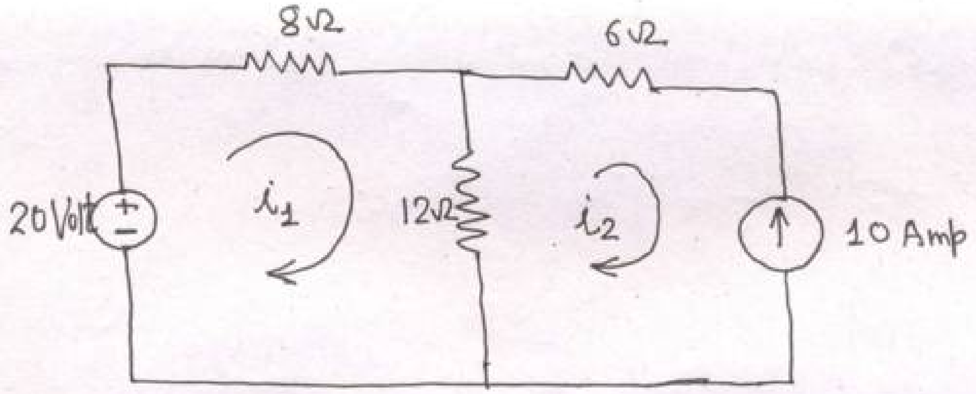


Fig.3.22: A circuit with two meshes and a current source.

$$-20 + 8i_1 + 12(i_1 - i_2) = 0$$

$$\therefore -20 + 8i_1 + 12i_1 - 12(-10) = 0$$

$$\therefore i_1 = -5 \text{ Amp.}$$

Case 2: <u>When a current source exists between two meshes:</u> Fig.3.23 shows a simple two mesh circuit having one current source common between them.

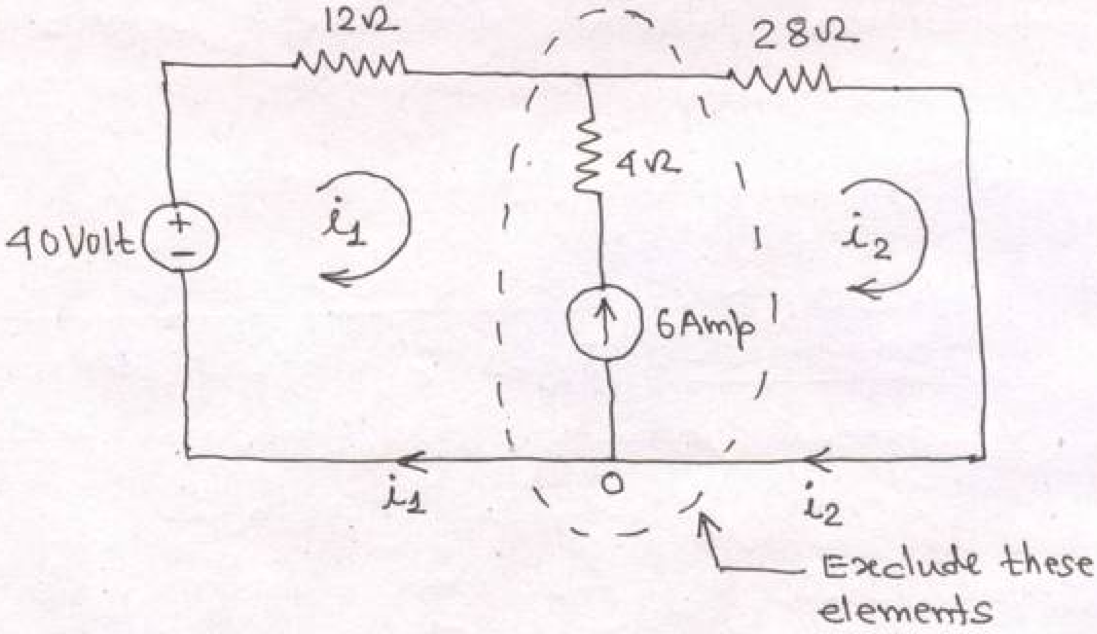


Fig.3.23: A circuit with two meshes and a common current source

We create a <u>supermesh</u> by excluding the current source and any other elements connected in series with it, as shown in Fig. 3.24.

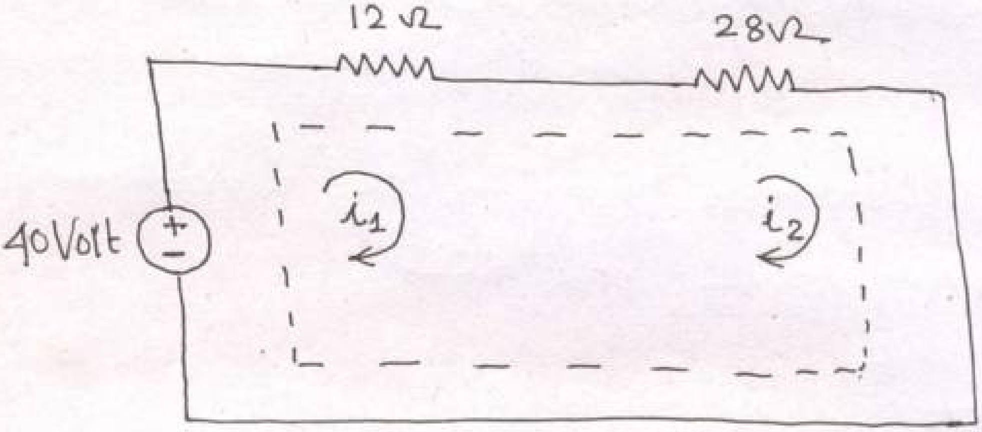


Fig. 3.24: A supermesh, created by excluding the current source of Fig. 3.23.

A supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh in Fig. 3.24, we obtain

$$12 i_1 + 28 i_2 - 40 = 0$$

$$\therefore\ 3 i_1 + 7 i_2 = 10 \quad \text{- - - - (i)}$$

Applying KCL to node 0 in Fig. 3.23, gives,

$$i_1 + 6 = i_2$$

$$\therefore\ i_1 = i_2 - 6 \quad \text{- - - (ii)}$$

Solving eqns. (i) and (ii), we get,

$$i_1 = -3.2 \text{ Amp}; \quad i_2 = 2.8 \text{ Amp}.$$

Therefore, a supermesh has the following properties:

1. A supermesh has no current of its own.

2. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.

3. A supermesh requires the application of both KVL and KCL.

EX-3.17: Determine i_1 and i_4 using mesh analysis of the circuit shown in Fig. 3.25.

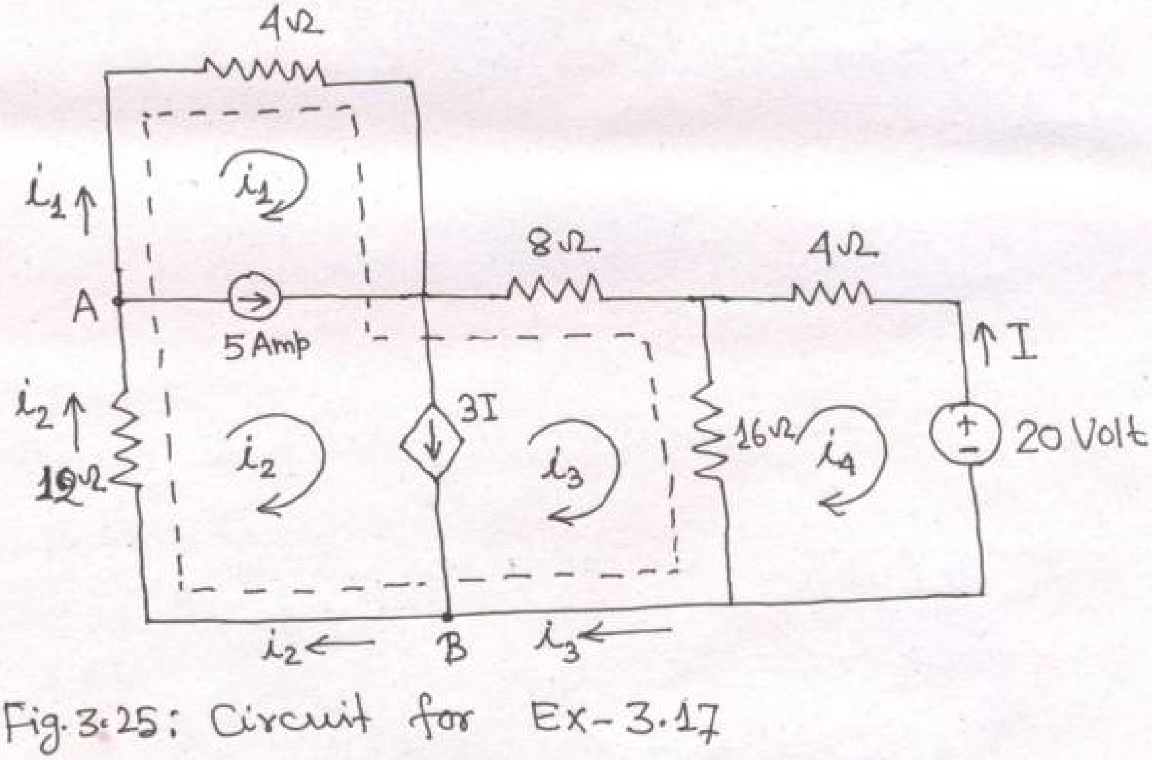


Fig. 3.25: Circuit for Ex-3.17

Soln.

In Fig. 3.25, mesh 1 and mesh 2 form a supermesh because they have a common independent current source. Also mesh 2 and mesh 3 form another supermesh because they have a common dependent current source. The two supermeshes intersect and form a larger supermesh as shown in Fig. 3.25. (43)

Applying KVL to the larger supermesh,

$$4i_1 + 8i_3 + 16(i_3 - i_4) + 12i_2 = 0$$

$$\therefore\ i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad \cdots (i)$$

For the independent current source, we apply KCL to node A,

$$i_2 = i_1 + 5 \quad \cdots\cdot (ii)$$

For the dependent current source, we apply KCL to node B,

$$i_2 = i_3 + 3I$$

But $i_4 = -I \quad \therefore \quad I = -i_4$, hence

$$i_2 = i_3 - 3i_4 \quad \cdots (iii)$$

Applying KVL in mesh 4,

$$4i_4 + 20 + 16(i_4 - i_3) = 0$$

$$\therefore\ 5i_4 - 4i_3 = -5 \quad \cdots\cdot (iv)$$

Solving eqns.(i), (ii), (iii) and (iv), we get

$i_1 = -7.5$ Amp; $i_2 = -2.5$ Amp;

$i_3 = 3.93$ Amp; $i_4 = 2.143$ Amp

EX-3.18: Determine i_1 and i_2 using mesh analysis of the circuit shown in Fig. 3.26.

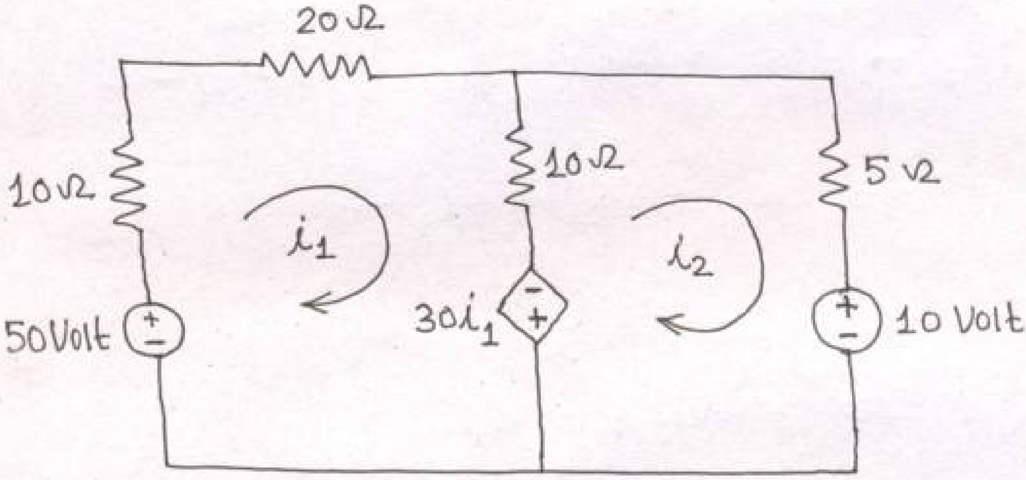


Fig. 3.26: Circuit for Ex-3.18

Soln.

For mesh 1,

$$10i_1 + 20i_1 + 10(i_1 - i_2) - 30i_1 - 50 = 0$$

$$\therefore \quad i_1 - i_2 = 5 \quad \text{- - -(i)}$$

For mesh 2,

$$10(i_2 - i_1) + 5i_2 + 10 + 30i_1 = 0$$

$$\therefore \quad 4i_1 + 3i_2 = -2 \quad \text{- - -(ii)}$$

Solving eqn. (i) and (ii), we get,

$$i_1 = \frac{13}{7} \text{ Amp}; \quad i_2 = -\frac{22}{7} \text{ Amp}$$

EX-3.19: Determine the power dissipated by the 4 Ω resistor is the circuit of Fig. 3.27. What is the power supplied by the 30 Volt source?

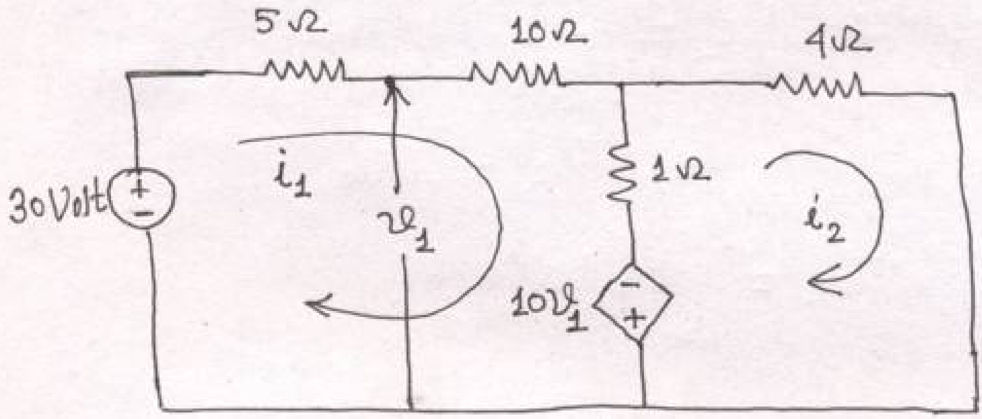


Fig.3.27: Circuit for Ex-3.19

Soln.

$$5i_1 + v_1 - 30 = 0$$

$$\therefore \; v_1 = 30 - 5i_1 \quad \cdots (i)$$

For mesh 1,

$$15i_1 + i_1 - 10v_1 - 30 - i_2 = 0$$

$$\therefore \; 16i_1 - 10v_1 = 30 + i_2 \quad \cdots (ii)$$

From Eqns. (i) and (ii), we get,

$$16i_1 - 10(30 - 5i_1) = 30 + i_2$$

$$\therefore \; 66i_1 - i_2 = 330 \quad \cdots (iii)$$

For mesh 2,

$$4i_2 + 10v_1 + i_2 - i_1 = 0$$

$$\therefore \; 4i_2 + 10(30 - 5i_1) + i_2 - i_1 = 0$$

$$\therefore \; 5i_2 - 50i_1 - i_1 + 300 = 0$$

$$\therefore \; 51i_1 - 5i_2 = 300 \quad \cdots (iv)$$

Solving eqns. (iii) and (iv), we obtain

$i_1 = 4.84$ Amp; $i_2 = -10.64$ Amp

Power dissipated by the 4Ω resistor,

$= (-10.64)^2 \times 4 = 453.25$ Watt.

Power supplied by the 30 Volt source,

$= 30 \times 4.84 = 145.2$ Watt.

EX-3.20: Determine i_2 in the circuit shown in Fig.3.28.

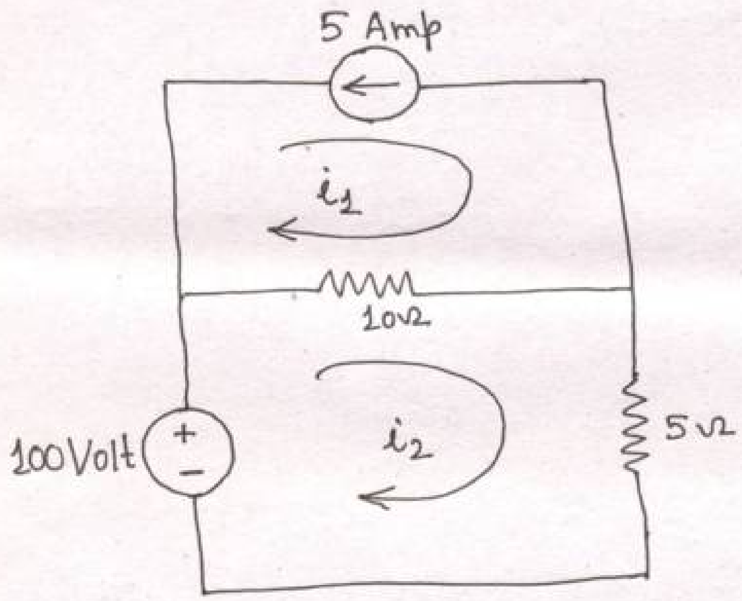


Fig.3.28: Circuit for EX-3.20

Soln.

$i_1 = -5$ Amp.

Applying KVL in mesh 2,

$15 i_2 - 100 - 10 i_1 = 0$

$15 i_2 = 100 + 10(-5) = 50$

$\therefore \; i_2 = \frac{10}{3}$ Amp.

EX-3.21: Determine i_1, i_2 and i_3 of the circuit shown in Fig. 3.29.

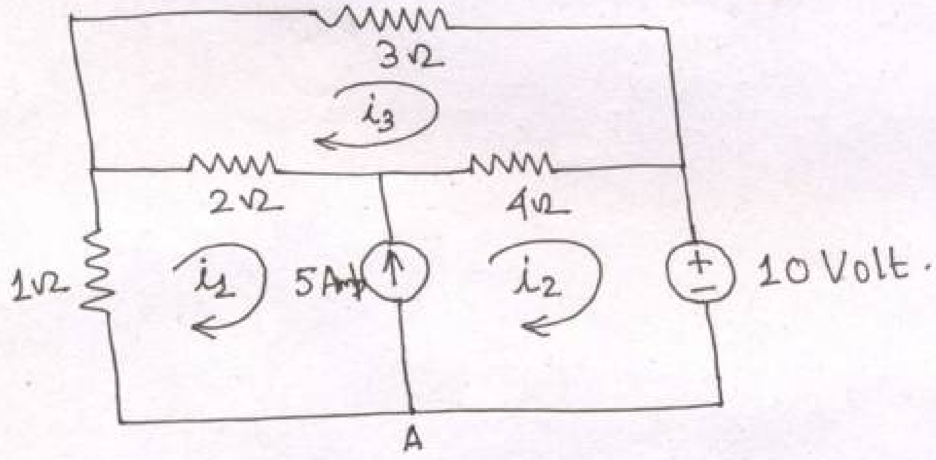


Fig. 3.29: circuit for EX-3.21

Soln.

Note that an independent 5 Amp current source is common for mesh 1 and mesh 2. By excluding this current source, a supermesh is formed as shown in Fig. 3.30.

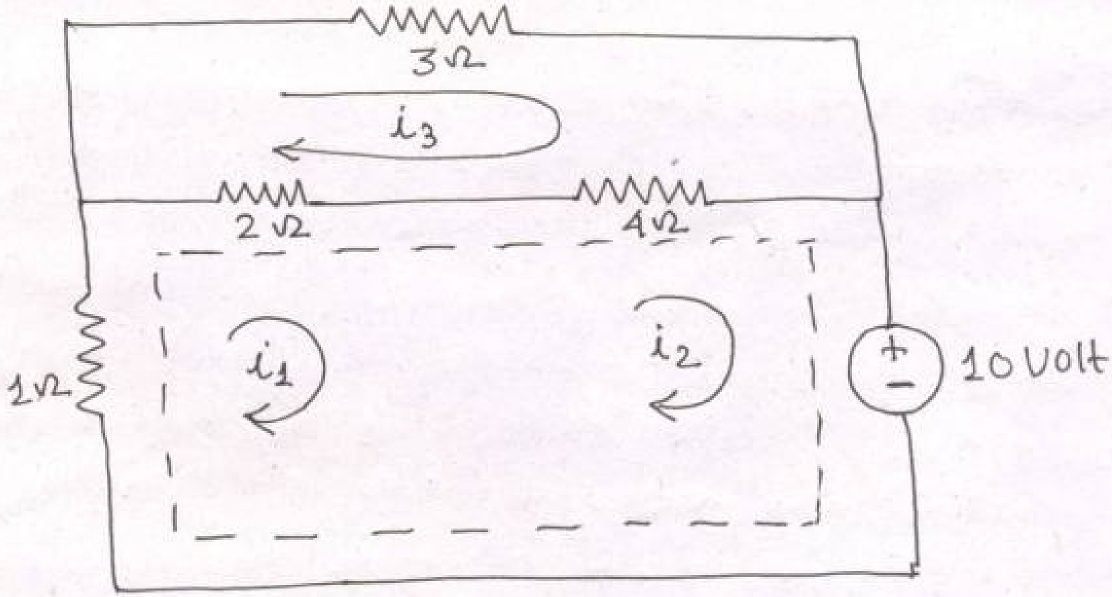


Fig. 3.30: A supermesh created by excluding the current source of Fig. 3.29.

Applying KVL in supermesh, we get,

$$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0 \quad \text{----} \quad (i)$$

For mesh 3,

$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0 \quad \text{----} \quad (ii)$$

Applying KCL at node A of Fig. 3.29

$$i_2 - i_1 = 5 \quad \text{----} \quad (iii)$$

Solving eqns. (i), (ii) and (iii), we obtain

$i_1 = -\frac{100}{18}$ Amp; $i_2 = \frac{-10}{18}$ Amp; $i_3 = -\frac{80}{54}$ Amp.

~~Ex. 3.23:~~

3.6: NODAL VERSUS MESH ANALYSIS

Both nodal and mesh analysis provide a systematic way of analyzing a complex circuit. A network that contains many series-connected elements, supermeshes or voltage sources are suitable for mesh analysis. A network that contains parallel connected elements, current sources or supernodes are suitable for nodal analysis. A circuit with fewer nodes is suitable for nodal analysis and a circuit with fewer meshes is suitable for mesh analysis. Hence, main idea is to select the method that results smaller number of equations. If voltages are required then nodal analysis is suitable and if branch currents are required then mesh analysis is more suitable.

3.1: Determine v_1 and v_2 of the circuit shown in Fig.3.31. Use nodal method.

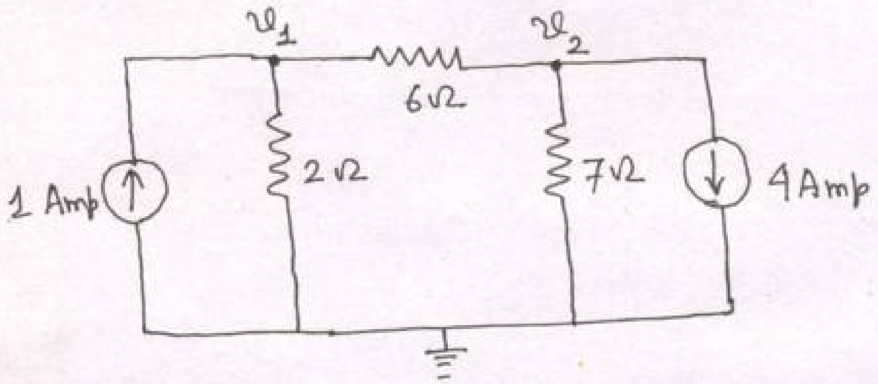


Fig.3.31: Circuit for Problem 3.1

Ans: $v_1 = -2$ Volt
$v_2 = -14$ Volt

3.2: Using node voltage method, determine v_1 and v_2 of the circuit shown in Fig.3.32.

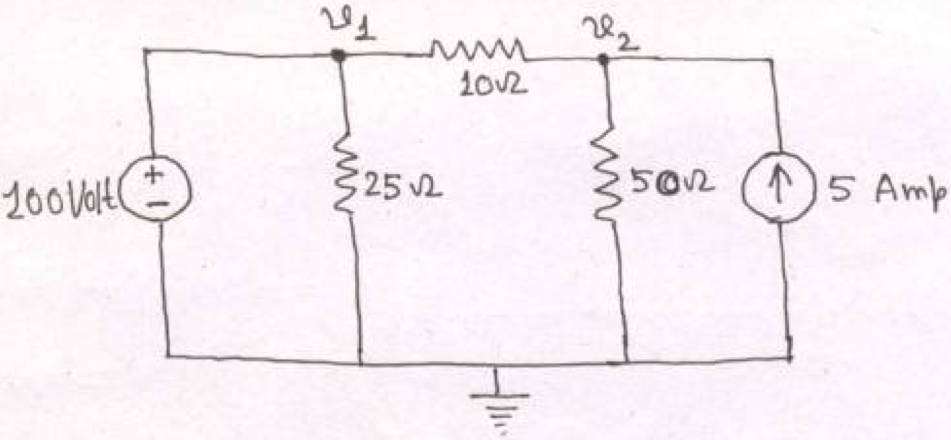


Fig.3.32: Circuit for Problem 3.2.

Ans: $v_1 = 100$ Volt
$v_2 = 125$ Volt.

3.3: Using nodal analysis, find the power delivered by each source in the circuit shown in Fig. 3.33.

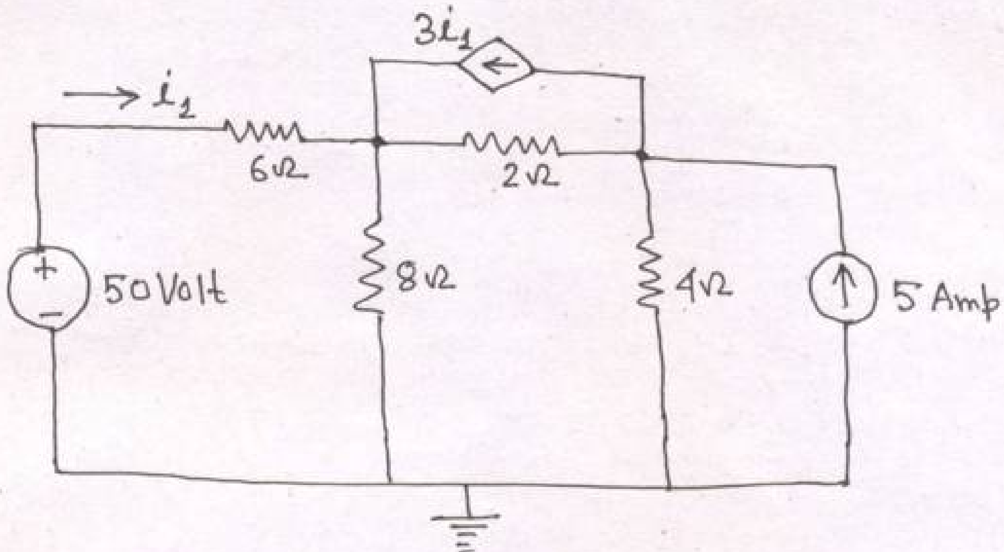


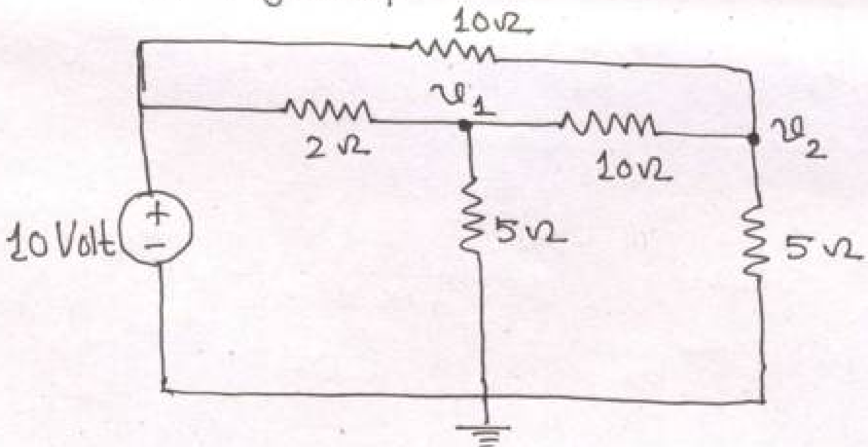
Fig.3.33: Circuit for Problem 3.3

Ans: $p_{50V} = 150$ Watt

$p_{5A} = 80$ Watt

$p_{3i_1} = 144$ Watt.

3.4: Determine v_1 and v_2 of the circuit shown in Fig.3.34.



Ans: $v_1 = 6.77$ Volt

$v_2 = 4.19$ Volt.

Fig.3.34: Circuit for Problem 3.4

3.5: Determine v_1, v_2, v_3 and i of the circuit shown in Fig.3.35.

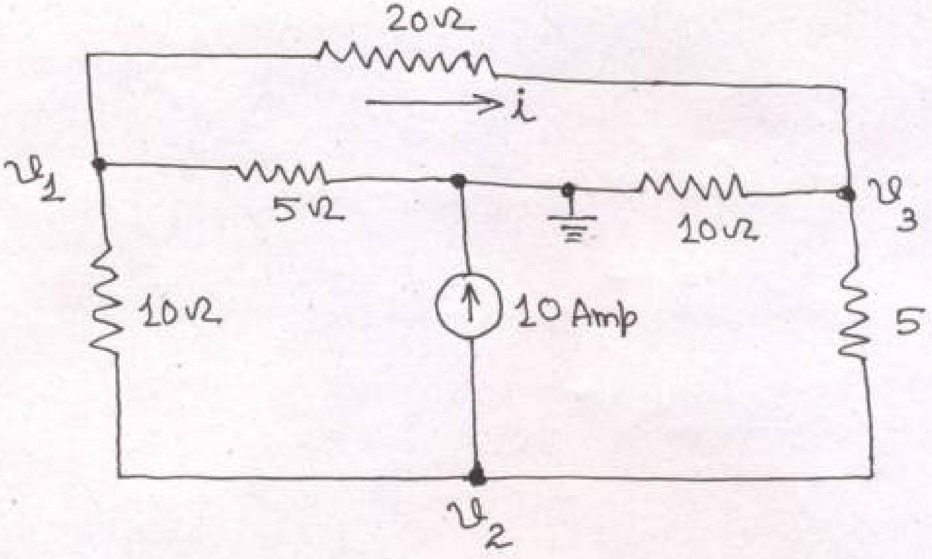


Fig. 3.35: Circuit for Problem 3.5.

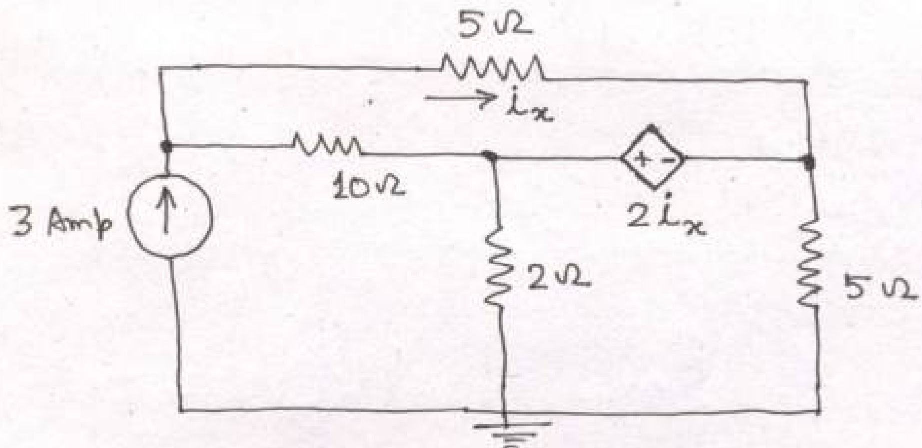
Ans: $v_1 = -27.27$ Volt

$v_2 = -72.73$ Volt

$v_3 = -45.45$ Volt

$i = 0.909$ Amp

3.6: Using nodal analysis, determine i_x of the circuit shown in Fig. 3.36.



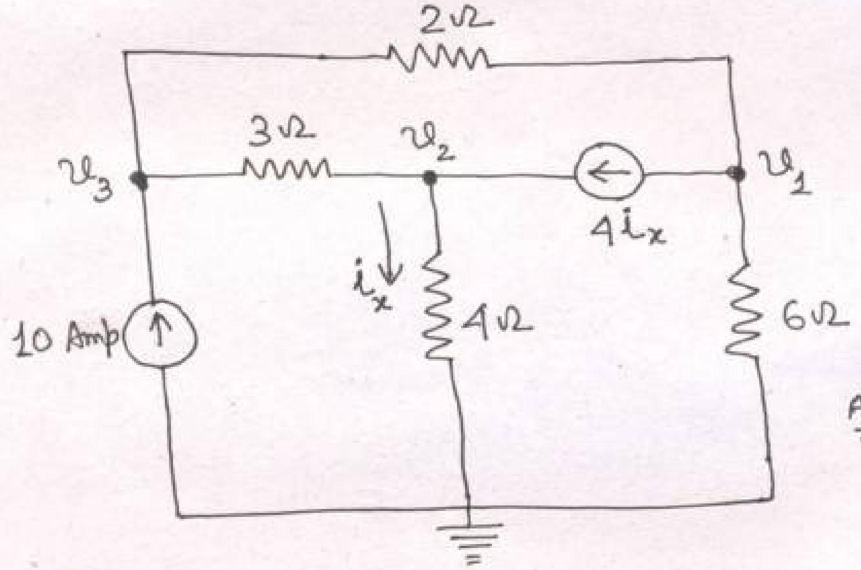
Ans: $i_x = 2.31$ Amp

Fig. 3.36: circuit for Problem 3.6.

3.7: By using mesh-current technique, find i of the circuit shown in Fig. 3.36.

Ans: $i = 2.31$ Amp.

3.8: Determine v_1, v_2 and v_3 of the circuit shown in Fig. 3.37.



Ans: $v_1 = 156$ Volt
$v_2 = -64$ Volt
$v_3 = 80$ Volt.

Fig. 3.37: Circuit for Problem 3.8

3.9: Using mesh analysis determine i_x of the circuit shown in Fig. 3.38.

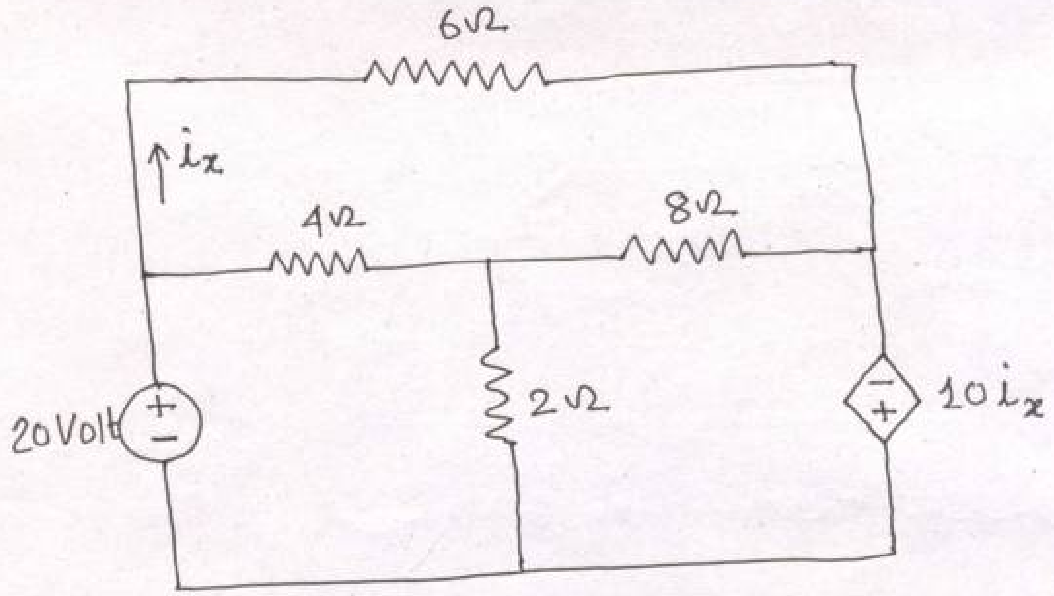
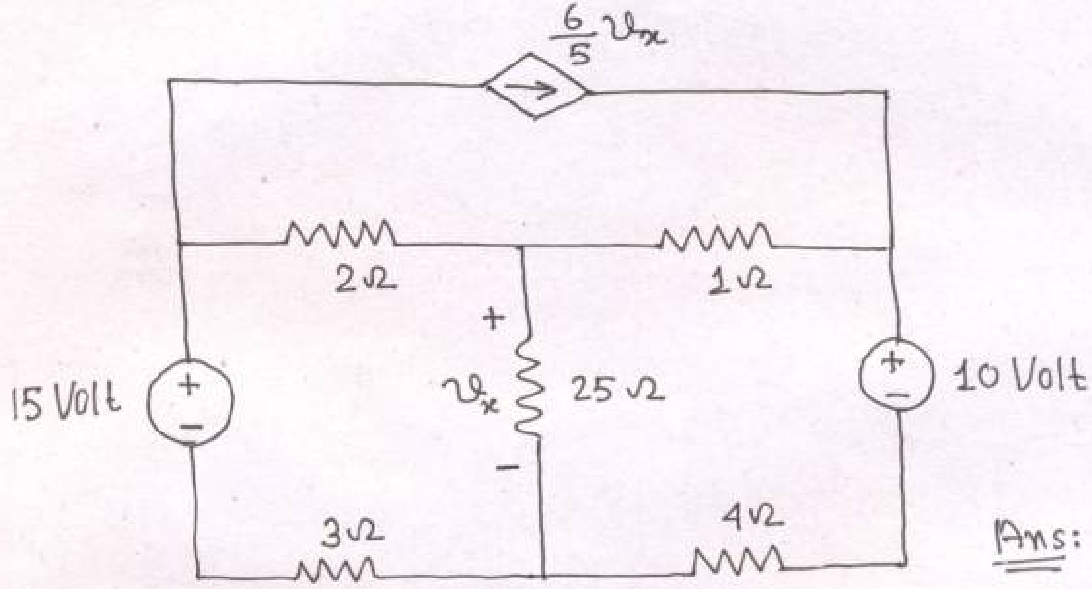


Fig. 3.38: Circuit for Problem 3.9

Ans: $i_x = -5$ Amp

3.10: Using mesh current method, determine power delivered to the $2\,\Omega$ resistor in the circuit ~~of~~ shown in Fig. 3.39.



Ans: 800 Watt.

Fig. 3.39: Circuit for Problem 3.10.

3.11: Using mesh analysis, determine the total power developed in the circuit shown in Fig. 3.40.

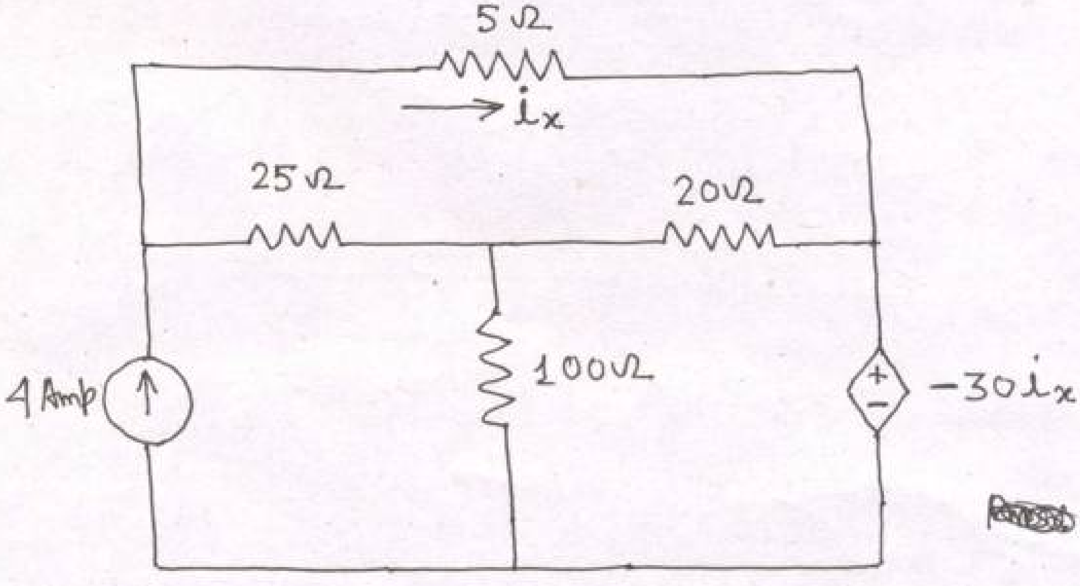


Fig. 3.40: Circuit for Problem 3.11

3.12: Using nodal analysis, determine v_o of the circuit shown in Fig. 3.41

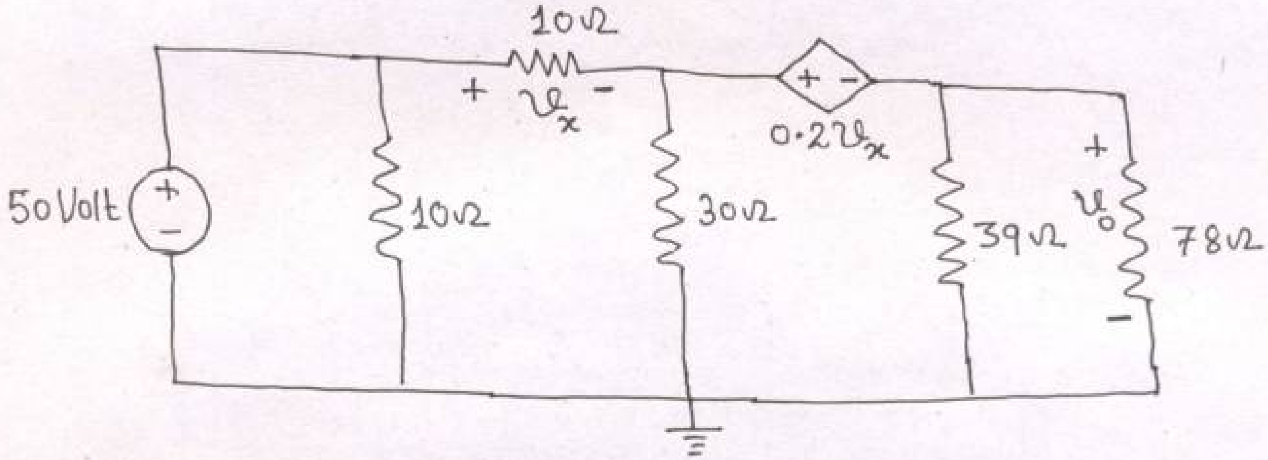


Fig. 3.41: Circuit for Problem 3.12

Ans: $v_o = 26$ Volt.

3.13: Determine v_o using node voltage analysis of the circuit shown in Fig. 3.42.

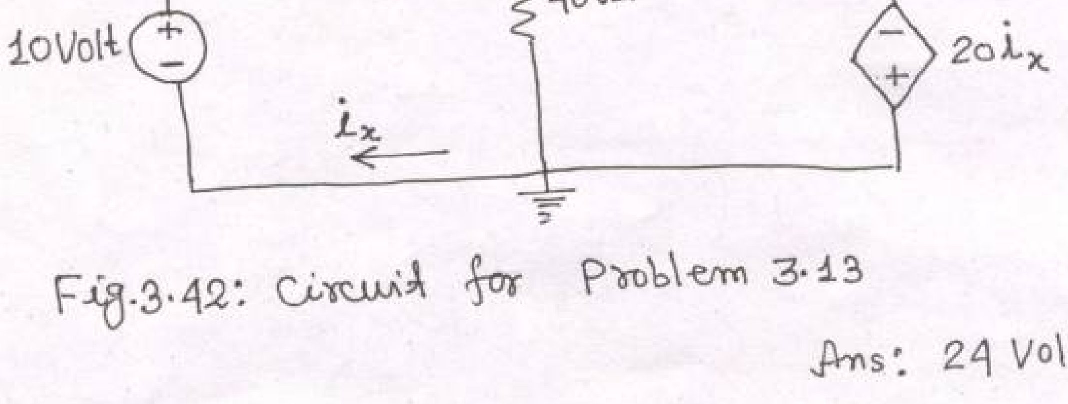
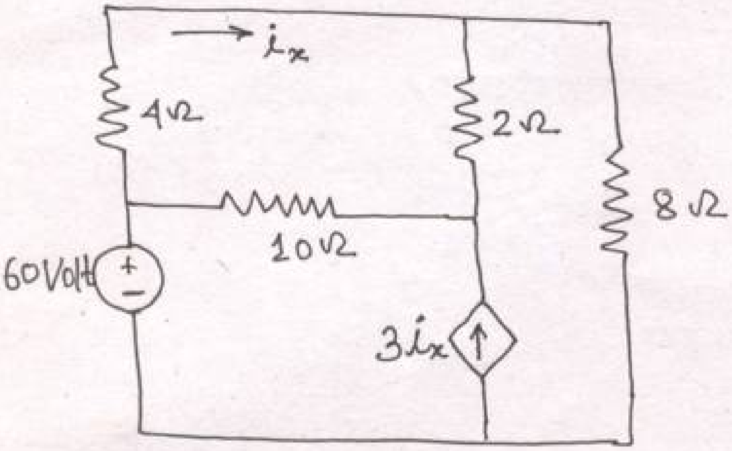


Fig. 3.42: Circuit for Problem 3.13

Ans: 24 Volt.

3.14: Determine i_x using nodal analysis of the circuit shown in Fig. 3.43.



Ans: 1.73 Amp.

Fig.3.43: Circuit for Problem 3.14

3.15: Using mesh current analysis, determine i_1 and i_2 of the circuit shown in Fig. 3.44.

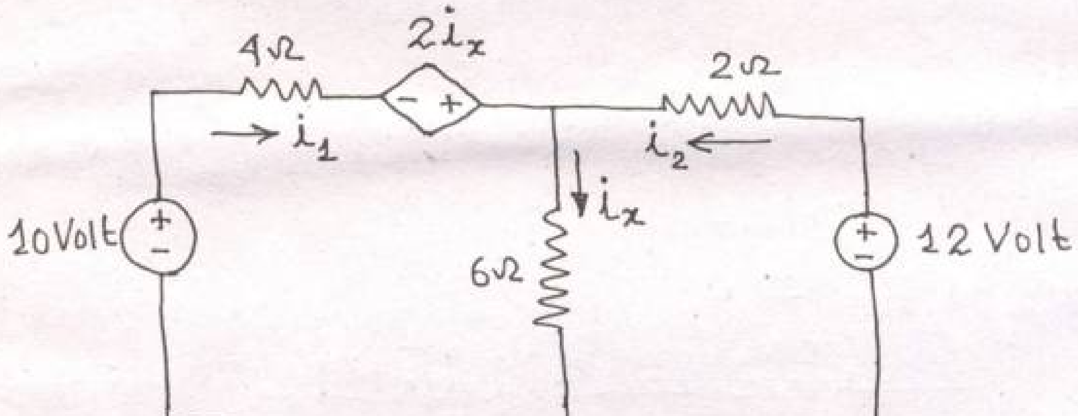


Fig.3.44: Circuit for Problem 3.15

Ans: $i_1 = 0.8$ Amp

$i_2 = 0.9$ Amp

3.16: Determine i and v_o using mesh analysis of the circuit shown in Fig.3.45.

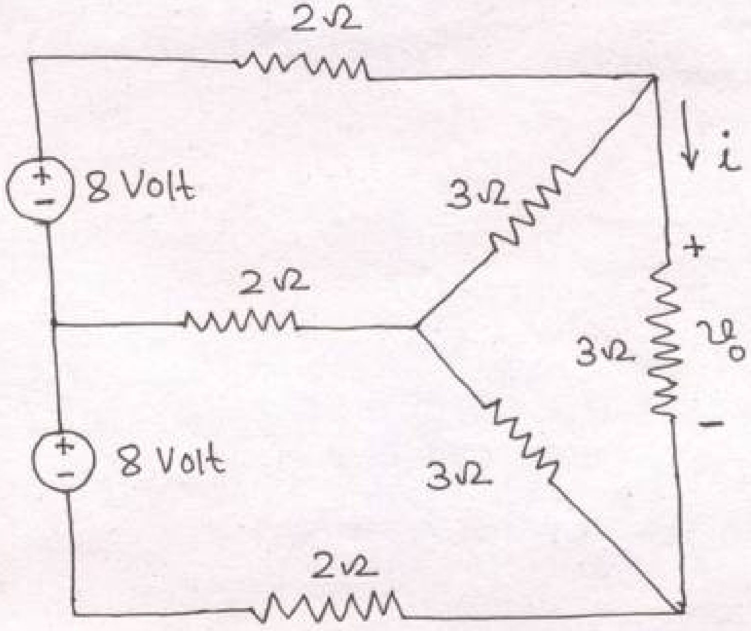


Fig.3.45: Circuit for Problem 3.16

Ans: $i = 1.778$ Amp
$v_o = 53.33$ Volt

~~3.17:~~

3.17: Using mesh analysis find v_1 of the circuit shown in Fig. 3.46

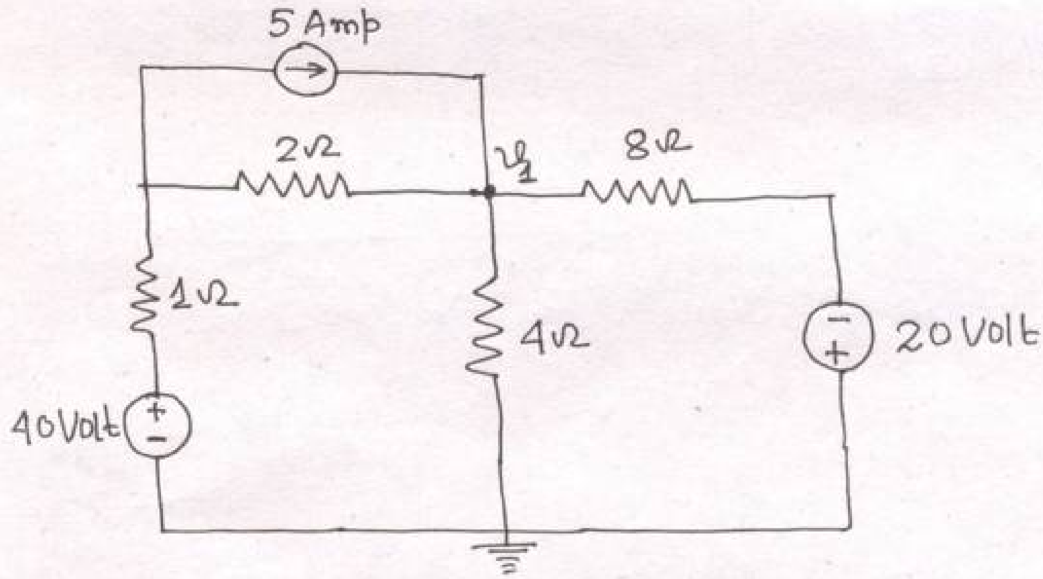


Fig.3.46: Circuit for Problem 3.17

Ans: $v_1 = 20$ Volt.

3.18: Determine the total power dissipated in the circuit shown in Fig. 3.47.

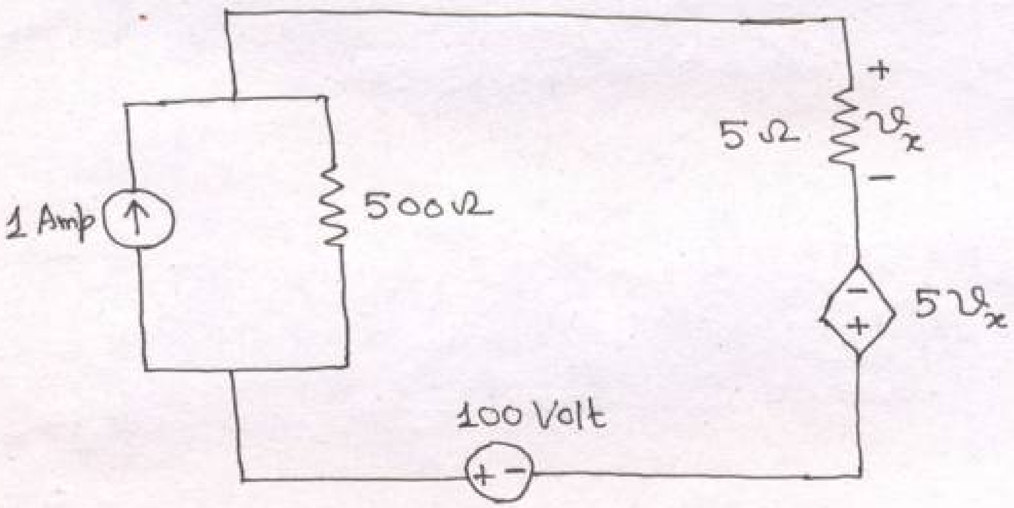


Fig.3.47: Circuit for Problem 3.18

Ans: 39.0625 Watt.

3.19: Using nodal analysis, determine the value of i_x in the circuit shown in Fig. 3.48.

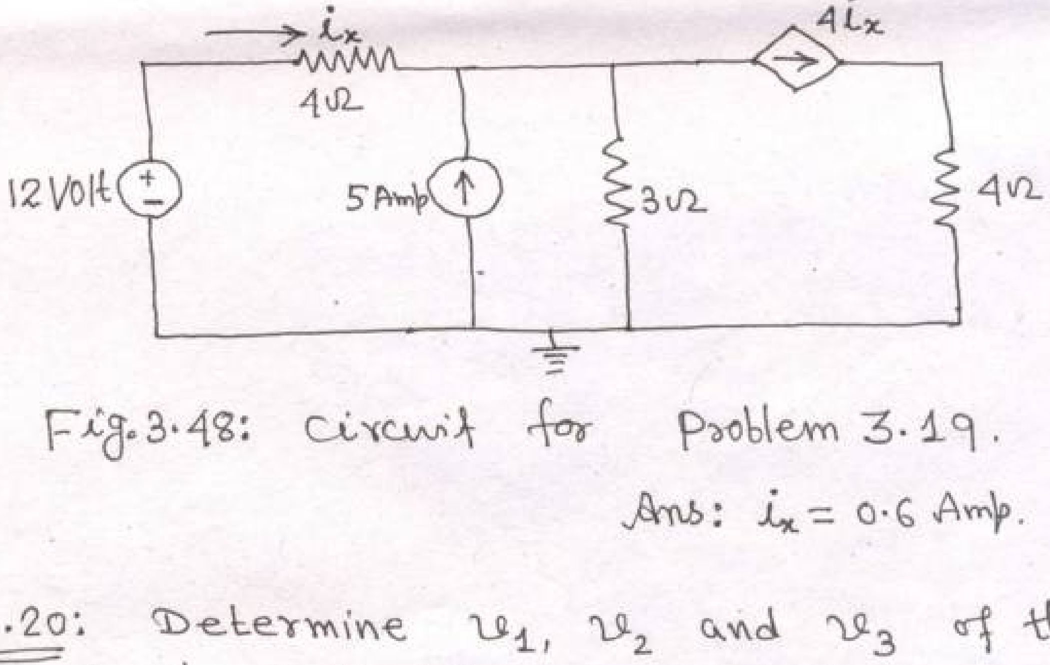
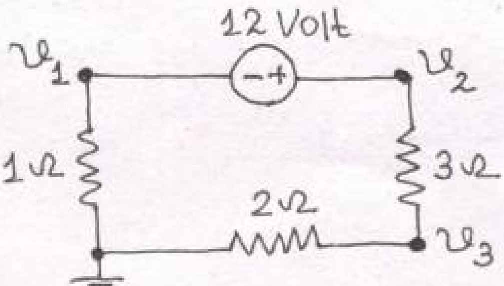


Fig.3.48: Circuit for Problem 3.19.

Ans: $i_x = 0.6$ Amp.

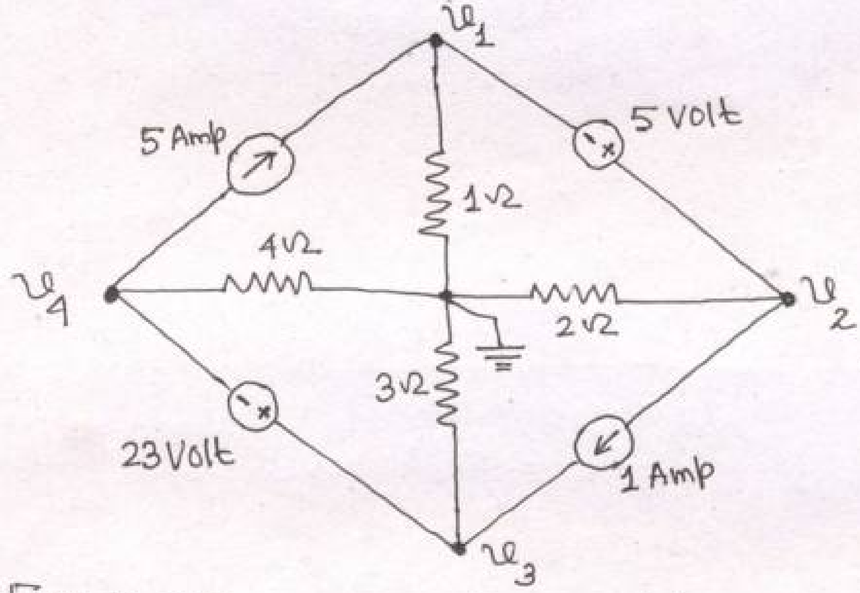
3.20: Determine v_1, v_2 and v_3 of the circuit shown in Fig. 3.49.



Ans: $v_1 = -2$ Volt
$v_2 = 10$ Volt
$v_3 = 4$ Volt.

Fig.3.49: Circuit for Problem 3.20

3.21: Determine v_1, v_2, v_3 and v_4 of the circuit shown in Fig. 3.50.



Ans: $v_1 = 1$ Volt

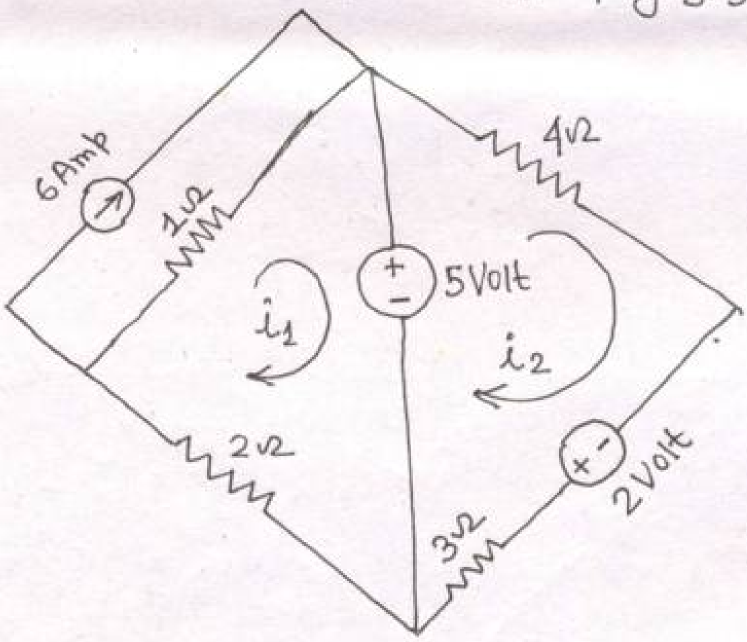
$v_2 = 6$ Volt

$v_3 = 3$ Volt

$v_4 = -20$ Volt.

Fig. 3.50: circuit for Problem 3.21:

3.22: Determine the values of i_1 and i_2 in the circuit shown in Fig. 3.51.

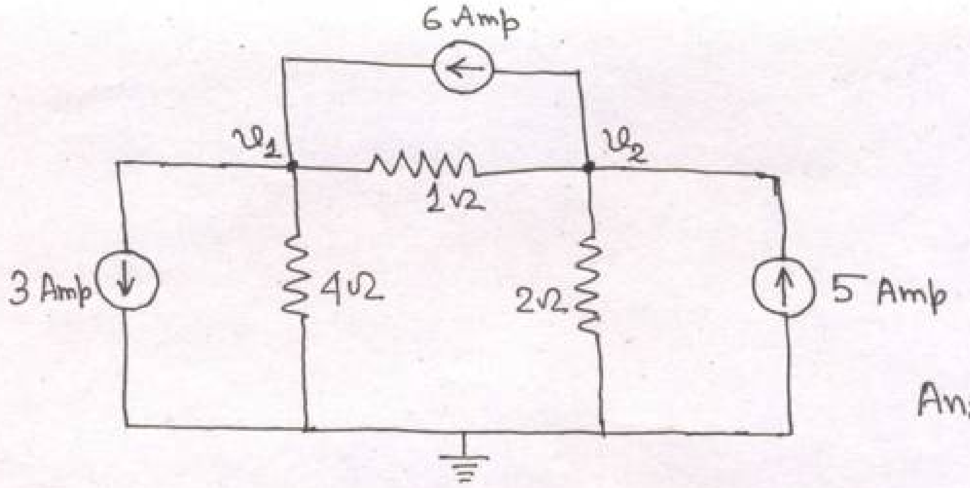


Ans: $i_1 = \frac{1}{3}$ Amp

$i_2 = 1$ Amp.

Fig. 3.51: Circuit for Problem 3.22.

3.23: Determine v_1 and v_2 of the circuit shown in Fig. 3.52.



Ans: $v_1 = 4$ Volt
$v_2 = 2$ Volt.

Fig. 3.52: Circuit for Problem 3.23

3.24: Determine v_y and i_y of the circuit shown in Fig. 3.53.

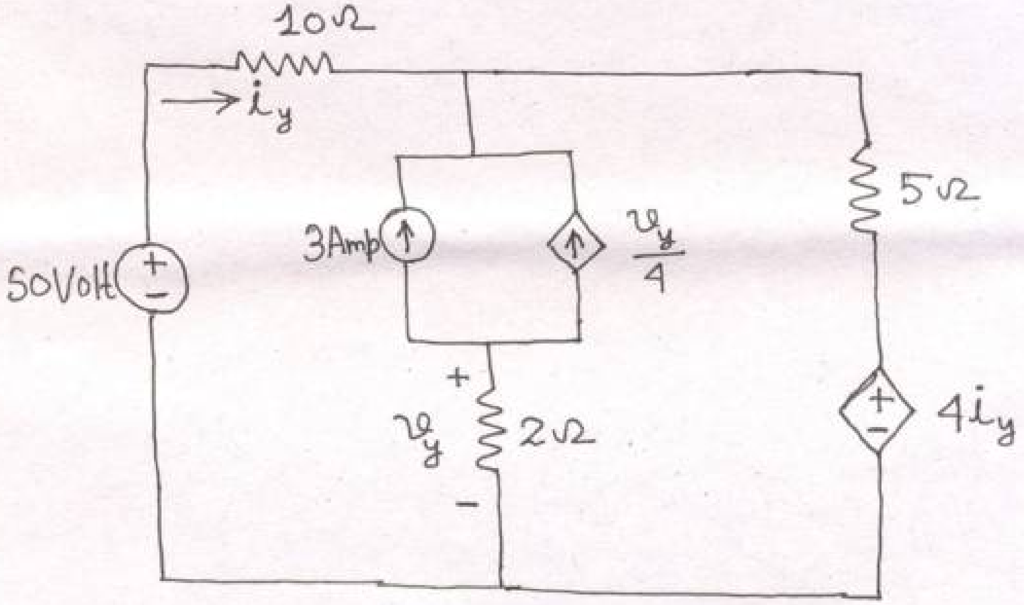


Fig. 3.53: Circuit for Problem 3.24.

Ans: $v_y = -4$ Volt
$i_y = 2.105$ Amp.