THREE-PHASE CIRCUITS

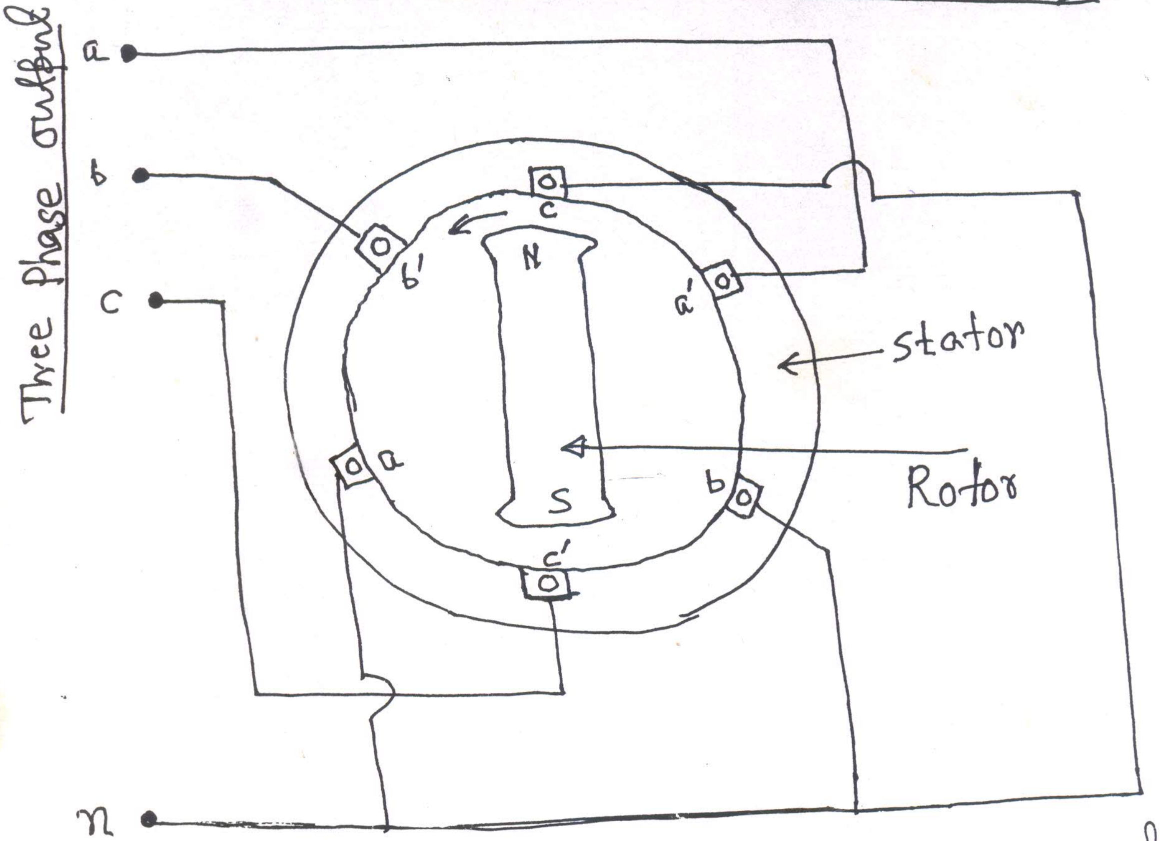


Fig.1: A three-phase generator

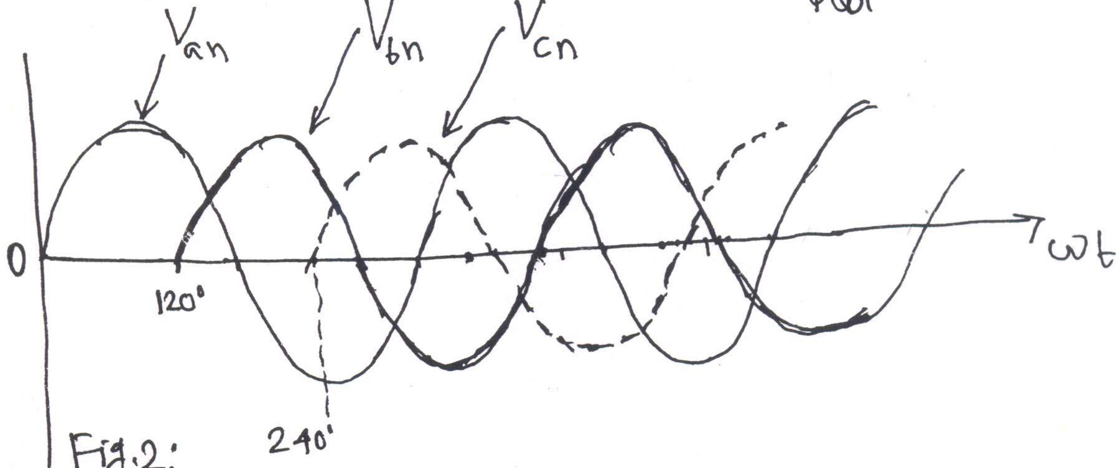


Fig.2:

120° apart from each other.

Phase Sequence a – b – c

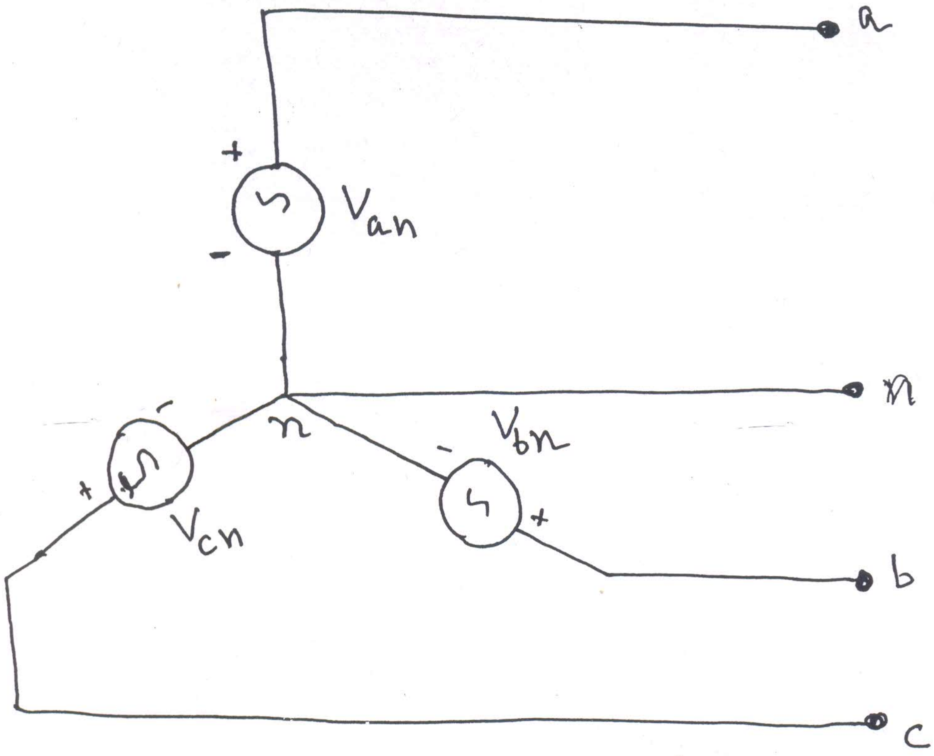


Fig. 3

Y- connected Source.

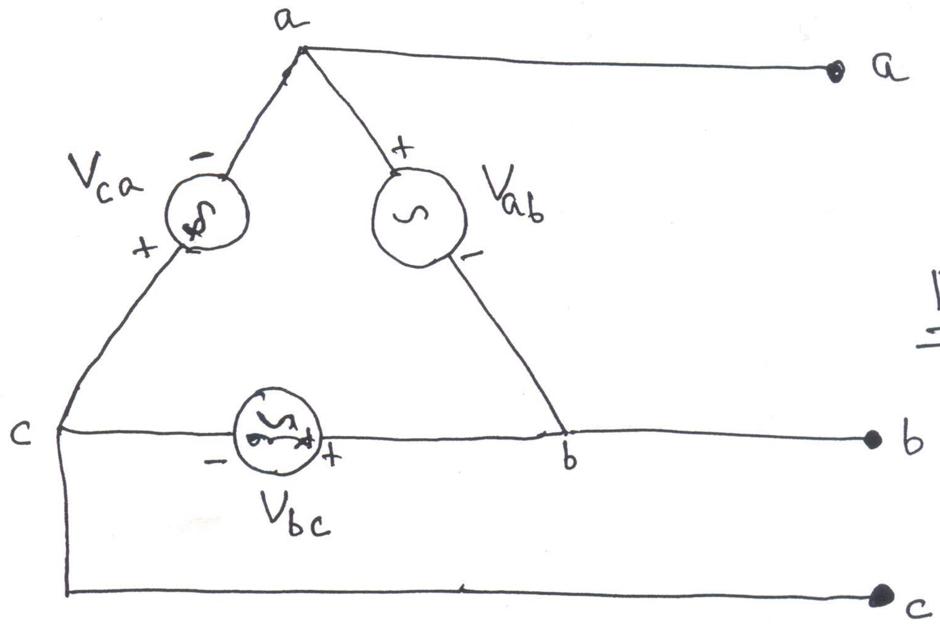


Fig. 4

Δ- connected Source.

$V_{an} \Rightarrow$ Voltage between line a and neutral line n

$V_{bn} \Rightarrow$ Voltage between line b and neutral line n

$V_{cn} \Rightarrow$ Voltage between line c and neutral line n

$$\left.\begin{array}{l} V_{an} \\ V_{bn} \\ V_{cn} \end{array}\right\} \Rightarrow \text{are called phase Voltages.}$$

If the Voltage sources have the Same amplitude and frequency ω and are out of phase with each other by 120°, The Voltages are said to be balanced.

This implies that

$$V_{an} + V_{bn} + V_{cn} = 0 \quad \text{-----} (1)$$

and

$$|V_{an}| = |V_{bn}| = |V_{cn}| \quad \text{-----} (2)$$

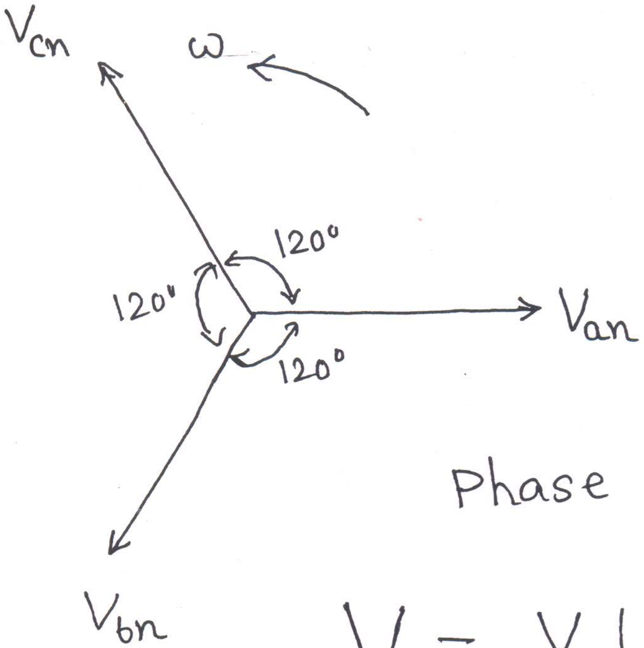


Fig. 5

Phase sequence a-b-c.

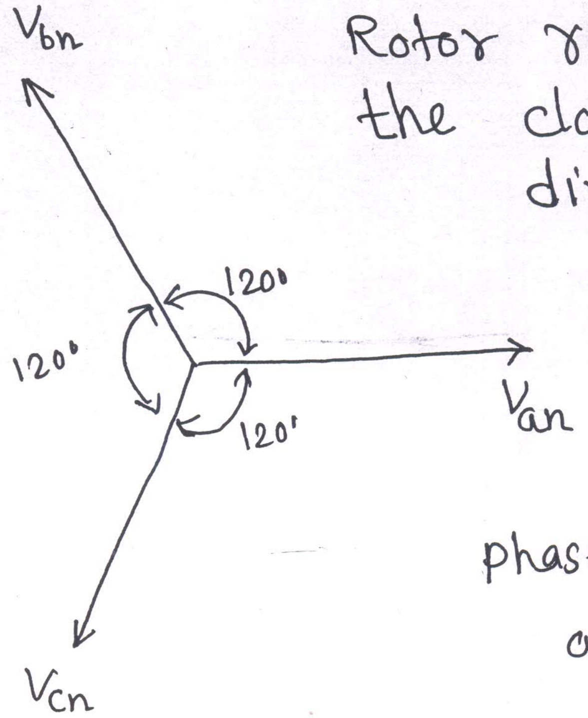
$$\left.\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ \\ V_{cn} &= V_p \angle -240^\circ = V_p \angle 120^\circ \end{aligned}\right\} \text{--} (3)$$

$V_p \Rightarrow$ Effective or rms Value of the phase Voltages.

⑤

Rotor rotates in the clockwise direction

Fig. 6



phase sequence

a – c – b

$$\left.\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -240^\circ = V_p \angle 120^\circ \\ V_{cn} &= V_p \angle -120^\circ \end{aligned}\right\} \quad \cdots (4)$$

From eqn (1)

$$V_{an} + V_{bn} + V_{cn} = V_p \angle 0^\circ + V_p \angle -120^\circ + V_p \angle 120^\circ$$

$$= V_p (1.0 - 0.5 - j0.866 - 0.5 + j0.866)$$

$$= 0$$

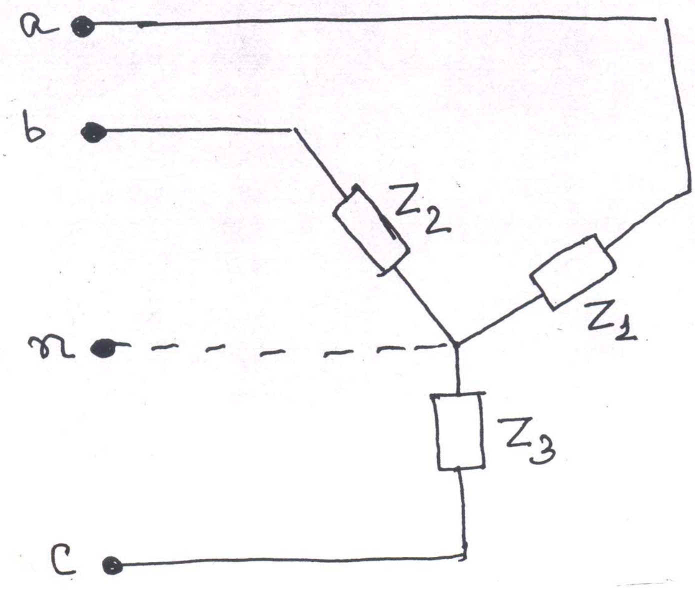


Fig. 7

Y – connected load.

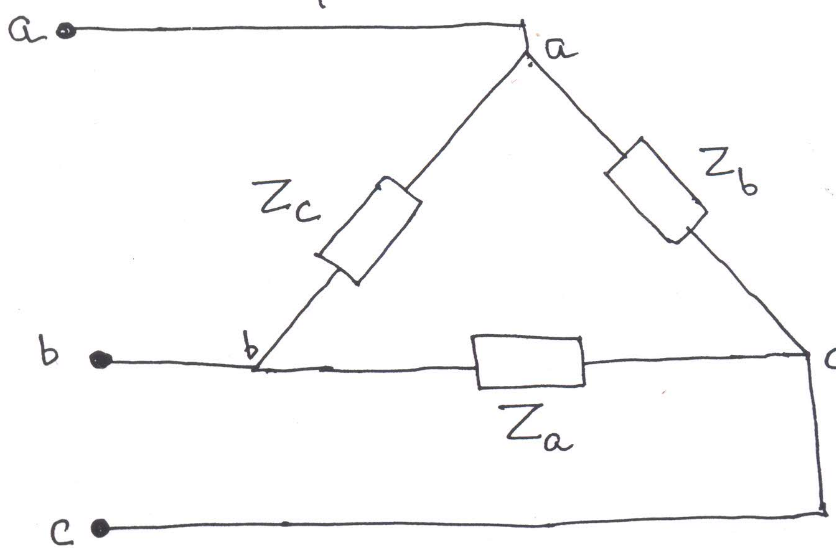


Fig. 8

Δ – connected load.

Balanced Load ⟹ Phase impedances are equal in magnitude and in phase

For a balanced Y- connected load

$$Z_1 = Z_2 = Z_3 = Z_Y$$

For a balanced Δ- connected load

$$Z_a = Z_b = Z_c = Z_\Delta$$

Y → Δ or Δ → Y transformation

$$Z_\Delta = 3Z_Y \quad \text{OR} \quad Z_Y = \frac{1}{3} Z_\Delta$$

Ex-1 : Determine the phase sequence of the set of Voltages,

$$v_{an} = 200 \cos(\omega t + 10^\circ)$$

$$v_{bn} = 200 \cos(\omega t - 230^\circ)$$

$$v_{cn} = 200 \cos(\omega t - 110^\circ)$$

Soln.

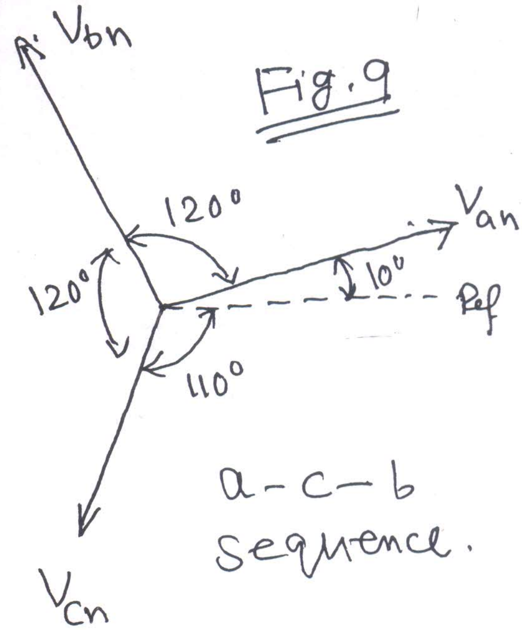
The Voltages can be expressed in phasor form as:

$$V_{an} = 200\angle 10^\circ$$

$$V_{bn} = 200\angle -230^\circ$$

$$V_{cn} = 200\angle -110^\circ$$

Fig. 9



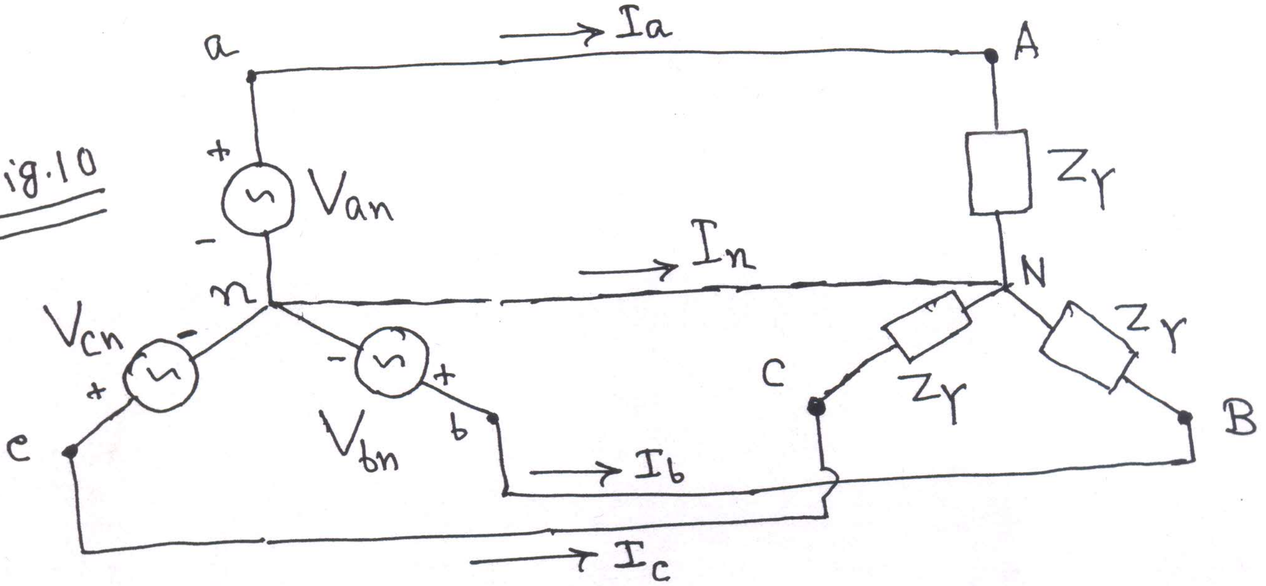
a-c-b sequence.

BALANCED Y-Y CONNECTION

Balanced Y- connected Source

Balanced Y- connected Load

Fig.10



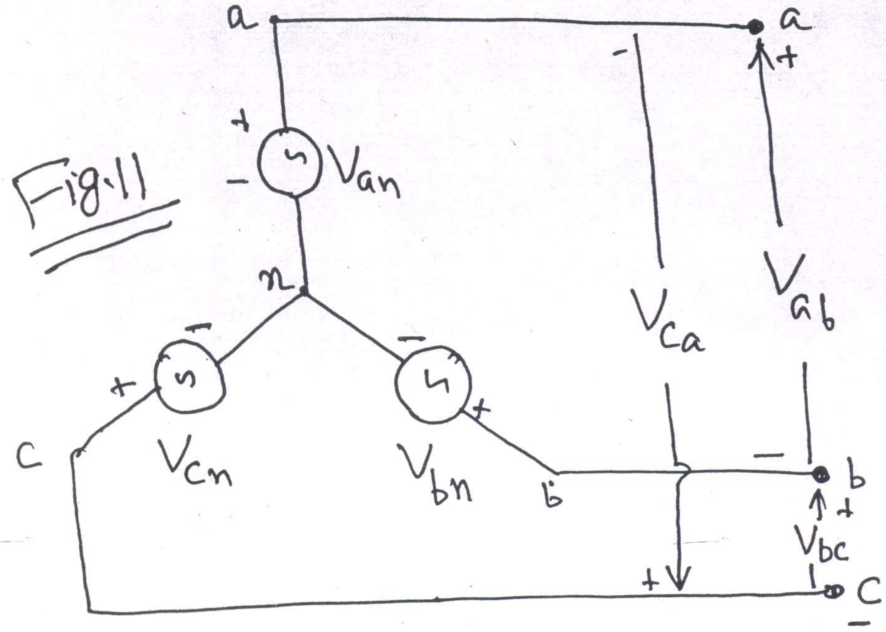


Fig. 11

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

$$V_{ab} + V_{bn} - V_{an} = 0$$

$$\therefore V_{ab} = V_{an} - V_{bn} = \boxed{V_{an} + V_{nb}}$$

$$\therefore V_{ab} = V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$\therefore V_{ab} = V_p \left[1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right] = \sqrt{3}\, V_p \angle 30^\circ$$

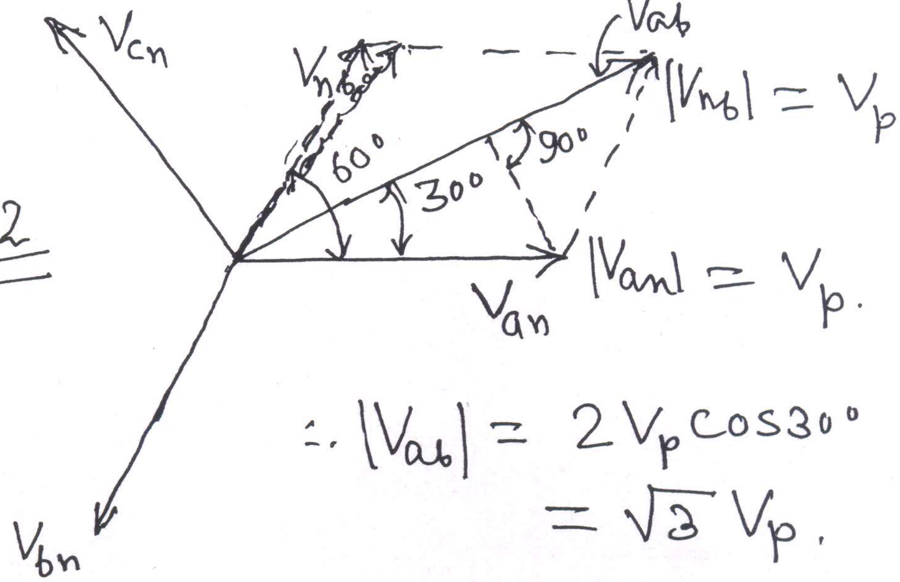


Fig. 12

$$\therefore |V_{ab}| = 2 V_p \cos 30^\circ$$

$$= \sqrt{3}\, V_p.$$

Similarly

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_p \angle -90^\circ$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}\, V_p \angle -210^\circ$$

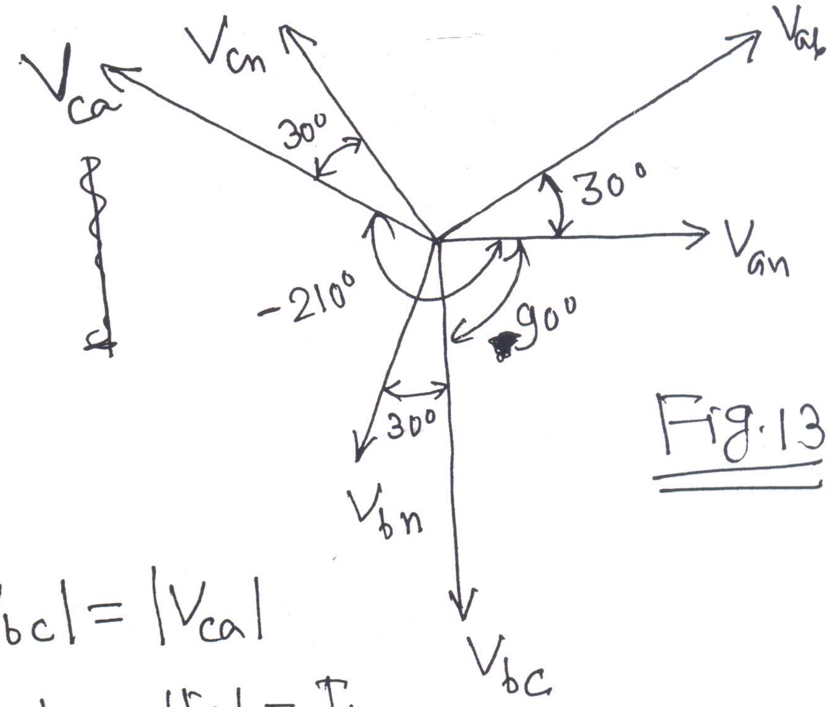


Fig. 13

Line Voltage

$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$$

$$I_L = |I_a| = |I_b| = |I_c| = I_p$$

Line Voltages Lead their corresponding phase Voltages by 30°.

From Fig. 10

$$I_a = \frac{V_{an}}{Z_Y} = \frac{V_p \angle 0^\circ}{Z_Y} = \frac{V_p}{Z_Y}$$

$$I_b = \frac{V_{bn}}{Z_Y} = \frac{V_p \angle -120^\circ}{Z_Y} = I_a \angle -120^\circ$$

$$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_p \angle 120^\circ}{Z_Y} = I_a \angle 120^\circ$$

$$\therefore\ I_a + I_b + I_c = I_a + I_a \angle -120^\circ + I_a \angle 120^\circ$$

$$= 0$$

Also.

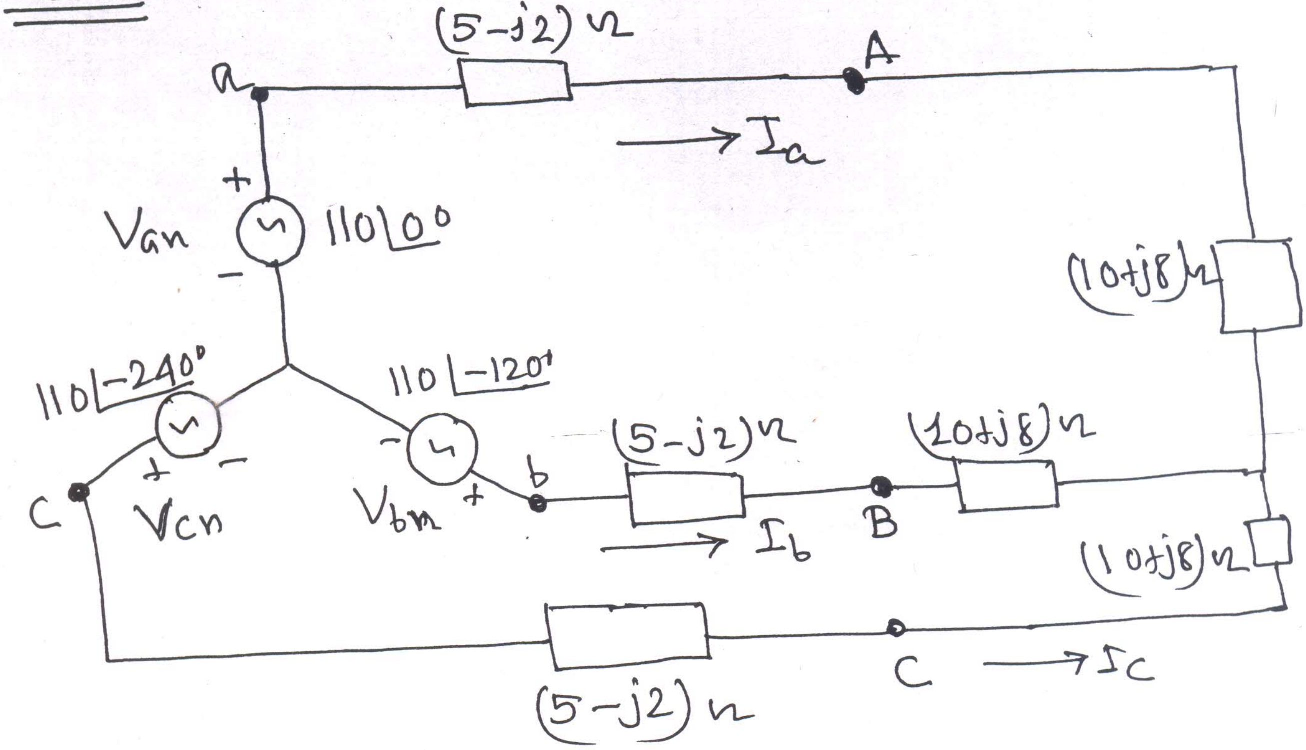
$$I_n + I_a + I_b + I_c = 0$$

$$\therefore I_n = -(I_a + I_b + I_c) = 0.$$

SUMMARY

Connection	phase Voltages/currents	Line Voltages & currents.
Y–Y	$V_{an} = V_p \angle 0^\circ$ $V_{bn} = V_p \angle -120^\circ$ $V_{cn} = V_p \angle 120^\circ$ Line current = phase current	$V_{ab} = \sqrt{3} V_p \angle 30^\circ$ $V_{bc} = \sqrt{3} V_p \angle -90^\circ$ $V_{ca} = \sqrt{3} V_p \angle -210^\circ$ $I_a = \frac{V_{an}}{Z_Y}$ $I_b = I_a \angle -120^\circ$ $I_c = I_a \angle 120^\circ$

Ex-2



$$Z_Y = (5-j2) + (10+j8) = (15+j6)\ \Omega$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{110\angle 0^\circ}{(15+j6)} = 6.81\angle -21.8^\circ \text{ Amp}$$

$$I_b = I_a\angle -120^\circ = 6.81\angle -21.8^\circ - 120^\circ$$

$$\therefore I_b = 6.81\angle -141.8^\circ$$

$$I_c = I_a\angle -240^\circ = 6.81\angle -\cancel{141.8}\overset{21.8}{}^\circ - 240^\circ$$

$$\therefore I_c = 6.81\angle 98.2^\circ \text{ Amp}$$

~~[illegible]~~:

Let line voltage $= V_L$

$$\therefore \quad V_L = \sqrt{3}\, V_p.$$

$$\boxed{\begin{aligned} \therefore \quad V_{ab} &= \sqrt{3}\, V_p \angle 30^\circ = V_L \angle 30^\circ \\ V_{bc} &= \sqrt{3}\, V_p \angle -90^\circ = V_L \angle -90^\circ \\ V_{ca} &= \sqrt{3}\, V_p \angle -210^\circ = V_L \angle -210^\circ \end{aligned}}$$

$$\therefore \quad \frac{V_{ab}}{V_{bc}} = \frac{V_L \angle 30^\circ}{V_L \angle -90^\circ}$$

$$\boxed{\therefore \quad V_{bc} = V_{ab} \angle -120^\circ}$$

$$\frac{V_{ab}}{V_{ca}} = \frac{V_L \angle 30^\circ}{V_L \angle -210^\circ}$$

$$\therefore \quad V_{ca} = V_{ab} \angle -240^\circ$$

$$\boxed{\therefore \quad V_{ca} = V_{ab} \angle 120^\circ}$$

BALANCED Y – Δ CONNECTION

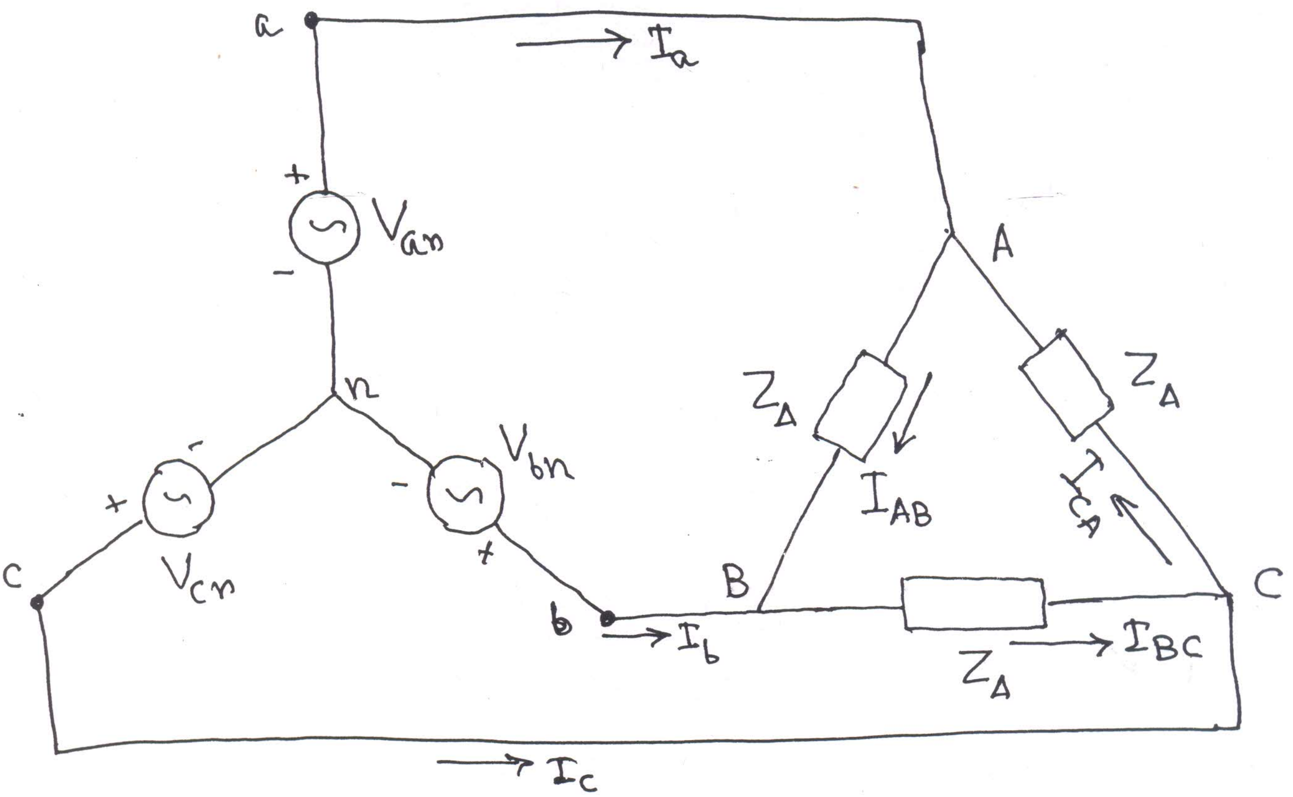


Fig.14: Balanced Y – Δ connection.

$$V_{an} = V_p \angle 0^\circ \; ; \quad V_{bn} = V_p \angle -120^\circ \; ; \quad V_{cn} = V_p \angle 120^\circ$$

Also

$$V_{ab} = V_{AB} = \sqrt{3}\, V_p \angle 30^\circ$$

$$V_{bc} = V_{BC} = \sqrt{3}\, V_p \angle -90^\circ$$

$$V_{ca} = V_{CA} = \sqrt{3}\, V_p \angle -210^\circ$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Another way to get these phase currents is to apply KVL. For example, applying KVL around loop aABbna gives

$$Z_\Delta I_{AB} + V_{bn} - V_{an} = 0$$

$$\therefore \quad I_{AB} = \frac{V_{an} - V_{bn}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} = \frac{V_{AB}}{Z_\Delta}$$

The line currents are obtained from the phase currents by applying KCL at nodes, A, B and C. Thus,

$$I_a = I_{AB} - I_{CA}$$

$$I_b = I_{BC} - I_{AB}$$

$$I_c = I_{CA} - I_{BC}.$$

$$\frac{I_{AB}}{I_{CA}} = \frac{V_{AB}}{Z_\Delta} \times \frac{Z_\Delta}{V_{CA}}$$

$$\therefore \frac{I_{AB}}{I_{CA}} = \frac{V_{ab}}{V_{ca}}.$$

$$\therefore \frac{I_{AB}}{I_{CA}} = \frac{V_{ab}}{V_{ab}\angle -240^\circ}$$

$$\therefore I_{CA} = I_{AB}\angle -240^\circ$$

Also

$$I_{BC} = I_{AB}\angle -120^\circ$$

$\therefore \; I_a = I_{AB} - I_{CA} = I_{AB} - I_{AB} \angle{-240^\circ}$

$\therefore \; I_a = \sqrt{3}\, I_{AB} \angle{-30^\circ}$

Let

$I_L = |I_a| = |I_b| = |I_c| =$ Line current

$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}| =$ phase current.

$$\boxed{\therefore \; I_L = \sqrt{3}\, I_p}$$

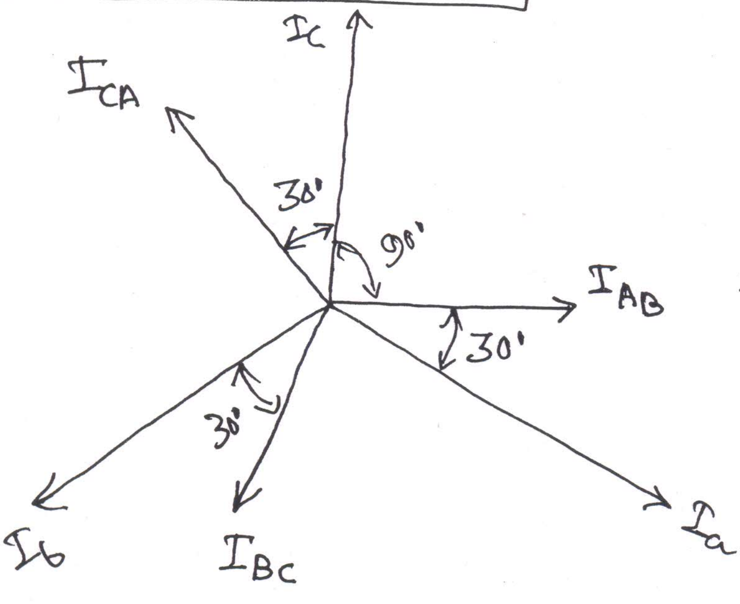


Fig. 15

phasor diagram.

$I_a = \sqrt{3}\, I_{AB} \angle{-30^\circ}$

$I_b = I_a \angle{-120^\circ}$

$I_c = I_a \angle{120^\circ}$

An alternative way of analyzing the Y-Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load, using the Δ-Y transformation,

$$Z_Y = \frac{Z_\Delta}{3}$$

After this transformation, we now have a Y-Y system as in Fig. 10

The three-phase Y-Δ system in Fig. 14 can be replaced by the single phase equivalent as shown in Fig. 16

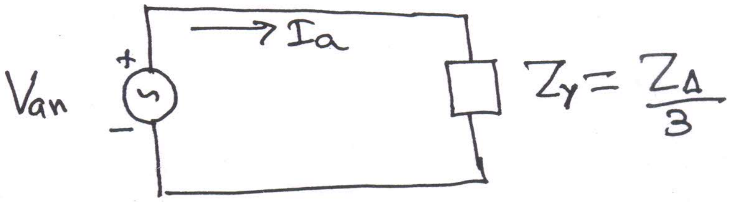


Fig. 16

This allow us to calculate only the line currents.

EX-3 : A balanced abc-sequence Y-connected source with $V_{an} = 100\angle 10^\circ$ Volt is connected to a Δ-connected balanced load $(8+j4)\ \Omega$ per phase. Calculate the phase and line currents.

Soln.

$$Z_\Delta = 8+j4 = 8.944\angle 26.57^\circ\ \Omega$$

If the phase voltage $V_{an} = 100\angle 10^\circ$ Volt, then the line voltage is

$$V_{ab} = V_{AB} = \sqrt{3}\, V_{an} \angle 30^\circ$$

$$\therefore V_{AB} = \sqrt{3} \times 100 \angle 10^\circ + 30^\circ = 173.2\angle 40^\circ \text{ Volt}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ Amp}$$

$$I_{BC} = I_{AB}\angle -120^\circ = 19.36\angle 13.43^\circ - 120^\circ = 19.36\angle -106.57^\circ \text{ Amp.}$$

$$I_{CA} = I_{AB}\angle -240^\circ = I_{AB}\angle 120^\circ$$

$$\therefore I_{CA} = 19.36\angle 13.43^\circ + 120^\circ = 19.36\angle 133.43^\circ \text{ Amp.}$$

The line currents are

$I_a = \sqrt{3}\, I_{AB} \angle -30^\circ$

$\therefore I_a = \sqrt{3} \times 19.36 \angle 13.43^\circ - 30^\circ$ Amp

$$\boxed{\therefore I_a = 33.53 \angle -16.57^\circ \text{ Amp}}$$

$I_b = I_a \angle -120^\circ = 33.53 \angle -16.57^\circ - 120^\circ$

$$\boxed{\therefore I_b = 33.53 \angle -136.57^\circ \text{ Amp.}}$$

$I_c = I_a \angle 120^\circ = 33.53 \angle -16.57^\circ + 120^\circ$

$$\boxed{\therefore I_c = 33.53 \angle 103.43^\circ \text{ Amp.}}$$

EXERCISE

One line voltage of a balanced Y-connected ~~load~~ source is $V_{ab} = 180 \angle -20^\circ$ Volt. If the source is connected to a Δ-connected load of $20 \angle 40^\circ\ \Omega$, find the phase and line currents. Assume the a-b-c sequence.

BALANCED Δ- Δ CONNECTION

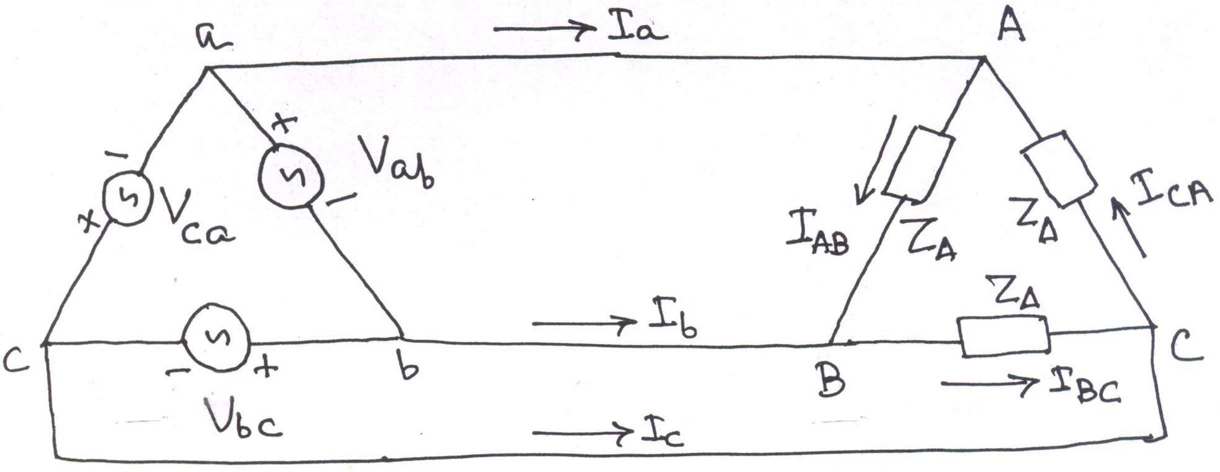


Fig. 17 : Balanced Δ- Δ Connection.

Our goal is to obtain the phase and line currents.

For Δ- connected Source or load

Line Voltage = Phase Voltage.

Let

$$V_{ab} = V_p \angle 0^\circ = V_L \angle 0^\circ$$

$$V_{bc} = V_p \angle -120^\circ = V_L \angle -120^\circ$$

$$V_{ca} = V_p \angle 120^\circ = V_L \angle 120^\circ$$

$$V_p = V_L$$

$$\boxed{V_{ab} = V_{AB}\ , \quad V_{bc} = V_{BC}\ , \quad V_{ca} = V_{CA}}$$

Hence, the phase currents are,

$$\left.\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} \\ I_{BC} &= \frac{V_{BC}}{Z_\Delta} = \frac{V_{bc}}{Z_\Delta} \\ I_{CA} &= \frac{V_{CA}}{Z_\Delta} = \frac{V_{ca}}{Z_\Delta} \end{aligned}\right]$$

Applying KCL at nodes A, B and C,

$$I_a = I_{AB} - I_{CA}$$

$$I_b = I_{BC} - I_{AB}$$

$$I_c = I_{CA} - I_{BC}$$

Shown earlier, each line current lags the corresponding phase current by 30°. The magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current.

$$I_L = \sqrt{3}\, I_p.$$

Ex-4

A balanced Δ-connected load having an impedance $(20-j15)\ \Omega$ is connected to a Δ-connected source having $V_{ab} = 330\angle 0^\circ$ Volt. Calculate the phase currents of the load and the line currents.

Soln.

$$Z_\Delta = (20-j15) = 25\angle -36.87^\circ\ \Omega.$$

Since $V_{AB} = V_{ab}$, the phase currents are,

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87^\circ} = 13.2\angle 36.87^\circ \text{ Amp}$$

$$I_{BC} = I_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ \text{ Amp.}$$

$$I_{CA} = I_{AB}\angle 120^\circ = 13.2\angle 156.87^\circ \text{ Amp.}$$

$$I_a = (\sqrt{3}\, I_{AB})\angle -30^\circ = 22.86\angle 6.87^\circ \text{ Amp.}$$

$$I_b = I_a\angle -120^\circ = 22.86\angle -113.13^\circ \text{ Amp}$$

$$I_c = I_a\angle 120^\circ = 22.86\angle 126.87^\circ \text{ Amp.}$$

BALANCED Δ-Y CONNECTION

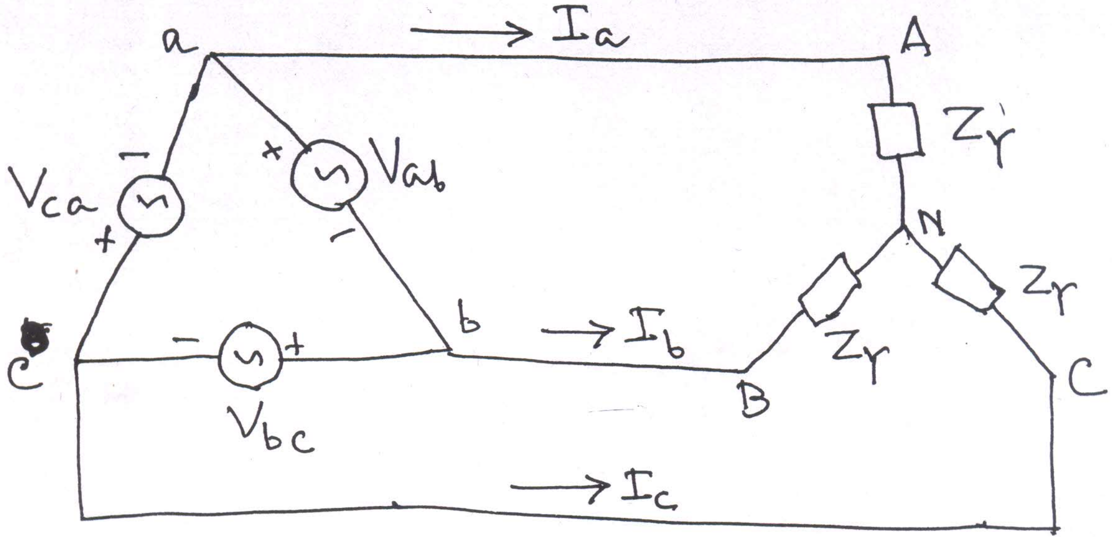


Fig. 18: Balanced Δ-Y connection.

Let

$$\left.\begin{aligned} V_{ab} &= V_p \angle 0^\circ \\ V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle 120^\circ \end{aligned}\right] \begin{aligned} &= V_L \angle 0^\circ \\ &= V_L \angle -120^\circ \\ &= V_L \angle 120^\circ \end{aligned}$$

Apply KVL to loop aANBba

$$I_a . Z_Y - I_b . Z_Y - V_{ab} = 0$$

$$\boxed{V_p = V_L}$$

$$\therefore I_a - I_b = \frac{V_{ab}}{Z_Y}$$

$$\therefore I_a - I_b = \frac{V_p \angle 0^\circ}{Z_Y} = \frac{V_L \angle 0^\circ}{Z_Y}$$

But I_b lags I_a by 120°,

$$\therefore I_b = I_a \angle -120^\circ$$

$$\therefore I_a - I_a \angle -120^\circ = \frac{V_p \angle 0^\circ}{Z_Y} = \frac{V_L \angle 0^\circ}{Z_Y}$$

$$\boxed{\therefore I_a = \frac{(V_p/\sqrt{3}) \angle -30^\circ}{Z_Y}} = \frac{(V_L/\sqrt{3}) \angle -30^\circ}{Z_Y}$$

$$I_b = I_a \angle -120^\circ$$

$$\boxed{\therefore I_b = \frac{(V_p/\sqrt{3}) \angle -150^\circ}{Z_Y}} = \frac{(V_L/\sqrt{3}) \angle -150^\circ}{Z_Y}$$

$$I_c = I_a \angle 120^\circ$$

$$\boxed{\therefore I_c = \frac{(V_p/\sqrt{3}) \angle 90^\circ}{Z_Y}} = \frac{\left(\frac{V_L}{\sqrt{3}}\right) \angle 90^\circ}{Z_Y}$$

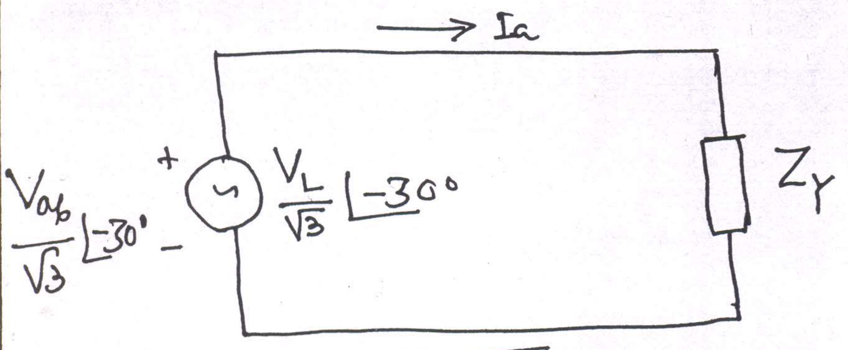


Fig. 19

$$I_a = \frac{V_{ab}}{\sqrt{3}Z_Y}\angle{-30^\circ}$$
$$= \frac{V_L}{\sqrt{3}Z_Y}\angle{-30^\circ}$$

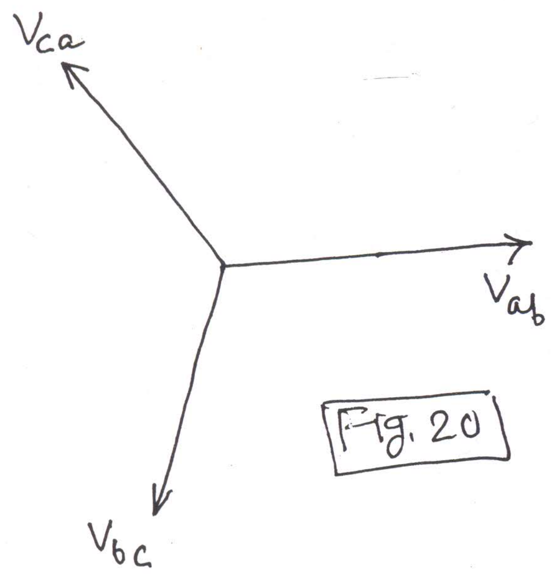


Fig. 20

$\Rightarrow$

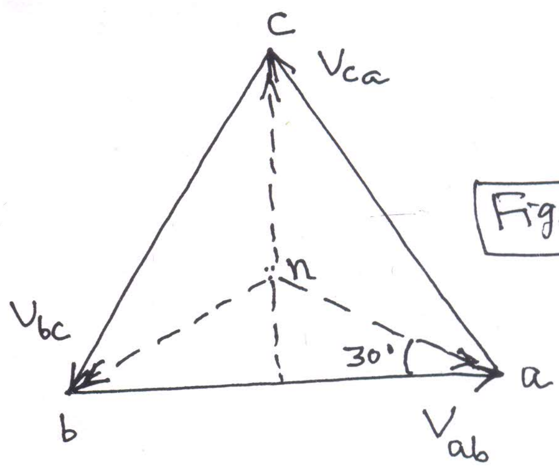


Fig. 21

Transformation of Δ- Source to Equivalent Y- Source.

$$|V_{an}|\cos 30^\circ = \frac{|V_{ab}|}{2} = \frac{V_p}{2} = \frac{V_L}{2}$$

$$\therefore \quad |V_{an}| = \frac{V_L}{\sqrt{3}}$$

$$\therefore \quad V_{an} = \frac{V_L}{\sqrt{3}}\angle{-30^\circ} = \frac{V_p}{\sqrt{3}}\angle{-30^\circ}$$

$$V_{bn} = \frac{V_L}{\sqrt{3}}\angle{-150^\circ} = \frac{V_p}{\sqrt{3}}\angle{-150^\circ}$$

$$V_{cn} = \frac{V_L}{\sqrt{3}}\angle{90^\circ} = \frac{V_p}{\sqrt{3}}\angle{90^\circ}$$

$$V_{AN} = I_a Z_Y = \frac{V_L}{\sqrt{3}} \angle -30^\circ = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$

$$V_{BN} = V_{AN} \angle -120^\circ$$

$$V_{CN} = V_{AN} \angle 120^\circ$$

Ex-5:

A balanced Y- connected load with a phase impedance of $(40+j25)\,\Omega$ is supplied by a balanced, positive sequence Δ- connected source with a line voltage of 210 Volt. Calculate the phase currents. Use V_{ab} as reference

Soln

$$Z_Y = (40 + j25) = 47.17 \angle 32^\circ \,\Omega.$$

$$V_{ab} = 210 \angle 0^\circ \text{ Volt.}$$

$$I_a = \frac{\left(\frac{V_{ab}}{\sqrt{3}}\right) \angle -30^\circ}{Z_Y} = 2.57 \angle -62^\circ \text{ Amp}$$

$$I_b = I_a \angle{-120^\circ} = 2.57 \angle{-182^\circ} \text{ Amp.}$$

$$I_c = I_a \angle{120^\circ} = 2.57 \angle{58^\circ} \text{ Amp.}$$

POWER IN A BALANCED SYSTEM

We begin by examining the instantaneous power absorbed by the Load.

For Y- connected load, the phase Voltages are

$$v_{AN} = \sqrt{2}\, V_p \cos(\omega t)$$

$$v_{BN} = \sqrt{2}\, V_p \cos(\omega t - 120^\circ)$$

$$v_{CN} = \sqrt{2}\, V_p \cos(\omega t + 120^\circ)$$

$V_p \Rightarrow$ r.m.s. Value $\quad (V_p^{max} = \sqrt{2}\, V_p)$

If $Z_Y = Z\angle\theta$, the phase currents lag behind their corresponding phase voltages by θ. Thus,

$$i_a = \sqrt{2}\, I_p \cos(\omega t - \theta)$$

$$i_b = \sqrt{2}\, I_p \cos(\omega t - \theta - 120^\circ)$$

$$i_c = \sqrt{2}\, I_p \cos(\omega t - \theta + 120^\circ)$$

$\boxed{I_p \Rightarrow \text{r.m.s. value of the phase current}}$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases, i.e.,

$$p = p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c$$

$$p = 2V_p I_p \Big[\cos(\omega t)\cos(\omega t - \theta) \cancel{+ \cos(\omega t - 120^\circ)}$$
$$+ \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ)$$
$$+ \cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)\Big]$$

$$\boxed{\text{Apply} \quad \cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$$

$$\boxed{\therefore \; p = 3 V_p I_p \cos\theta}$$

Thus the total instantaneous power in a balanced three phase system is constant — it does not change with time as the instantaneous power of each phase does.

This results is true whether the Load is Y- or Δ connected.

(*) This is one important ~~res~~ reason for using a three-phase system to generate and distribute power

Since the total instantaneous power is independent of time, the average power per phase P_p for either Δ- connected load or the Y- connected load. is $P/3$.

$$\boxed{\therefore\ P_p = V_p I_p \cos\theta}$$

and the reactive power per phase is

$$\boxed{Q_p = V_p I_p \sin\theta.}$$

The apparent power per phase is

$$\boxed{S_p = V_p I_p}$$

The complex power per phase is

$$\boxed{P_p + jQ_p = V_p I_p^*}$$

Total average power,

$$P = P_a + P_b + P_c = 3P_p = 3 V_p I_p \cos\theta$$

$$\therefore \boxed{P = \sqrt{3}\, V_L I_L \cos\theta}$$

For a Y-connected load

$I_L = I_p$ but $V_L = \sqrt{3}\, V_p$

For a Δ-connected load

$I_L = \sqrt{3}\, I_p$ but $V_L = V_p$

Similarly total reactive power is

$$\boxed{Q = \sqrt{3}\, V_L I_L \sin\theta}$$

$Z_p = Z_p \angle\theta$ = load impedance per phase

$Z_p = Z_Y$ or $Z_p = Z_\Delta$

Total complex power is.

$$3(P_p + jQ_p) = 3V_p I_p^* = 3V_p \left(\frac{V_p}{Z_p}\right)^*$$

$$= \frac{3V_p V_p^*}{Z_p^*} = \left(\frac{3V_p^2}{Z_p^*}\right)$$

We can also write

$$\boxed{P + jQ = \sqrt{3}\, V_L I_L \angle\theta}$$

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single phase system for the same line voltage V_L and the same absorbed power P_L.

We will compare these cases and assume in both that the wires are of same material.

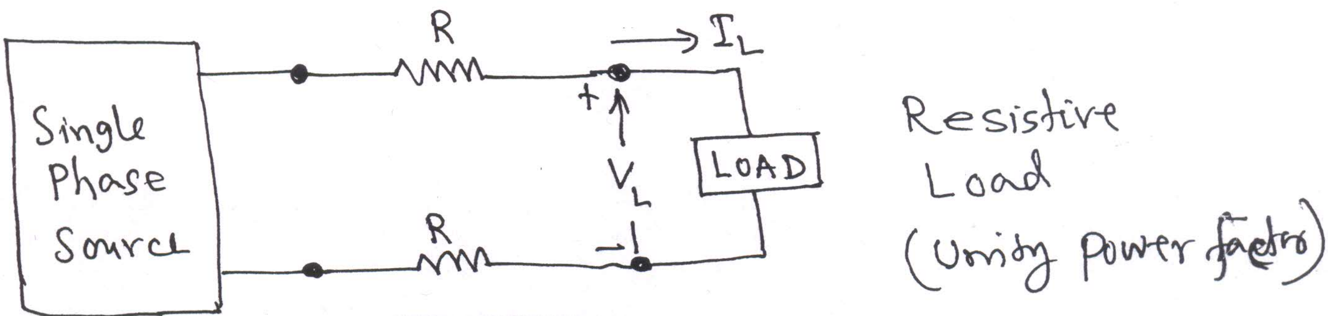


Fig. 22

Two wire single phase system.

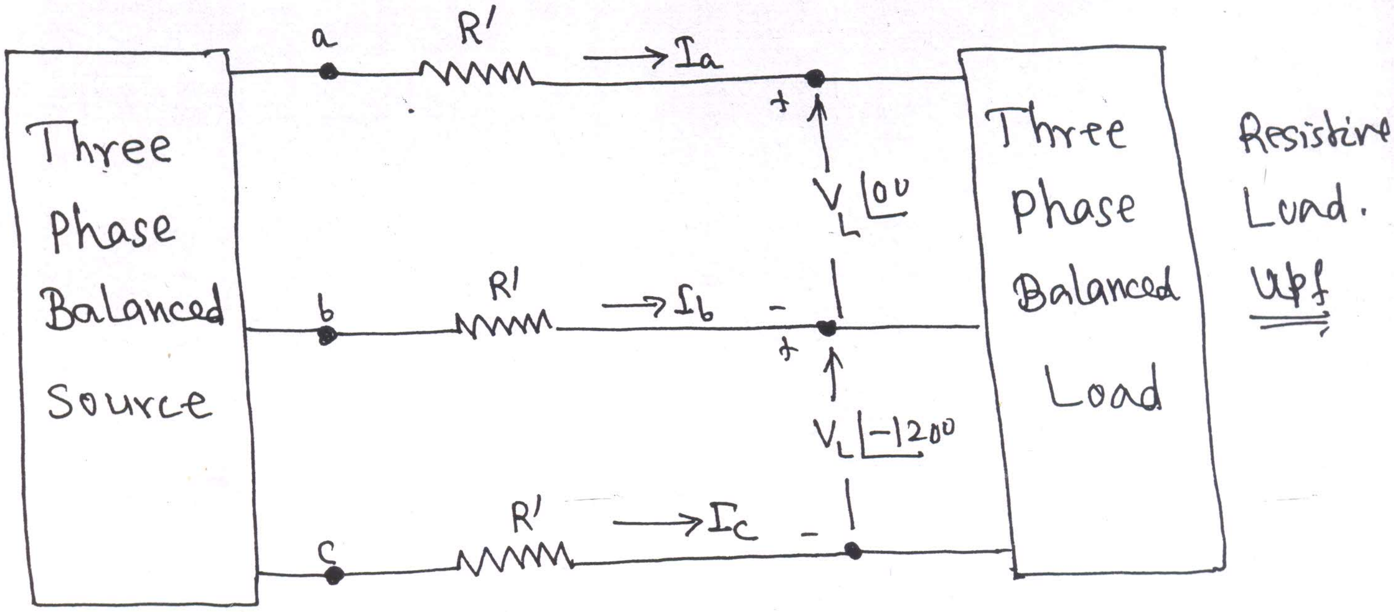


Fig. 23

For the two wire single phase system (Fig. 22)

$$I_L = \frac{P_L}{V_L}$$

So the power loss in the two wires is

$$P_{Loss} = I_L^2 (2R) = 2R \cdot \frac{P_L^2}{V_L^2} \quad \text{-----} \; (i)$$

For the three phase - three wire system (Fig. 23)

$$I_L' = |I_a| = |I_b| = |I_c| = \frac{P_L}{\sqrt{3} V_L}$$

The power loss in the three wires is

$$P_{loss}' = (I_L')^2 (3R') = 3R' \cdot \frac{P_L^2}{3 V_L^2} = R' \frac{P_L^2}{V_L^2} \quad \text{--} \; (ii)$$

Eqn (i) ÷ Eqn (ii)

$$\frac{P_{Loss}}{P'_{loss}} = \frac{2R}{R'}$$

$$R = \frac{\rho l}{\pi r^2}$$
&
$$R' = \frac{\rho l.}{\pi (r')^2}$$

$$\therefore \frac{P_{Loss}}{P'_{loss}} = \frac{2(r')^2}{r^2} \quad ----- (iii)$$

If the same power loss is tolerated in both the systems, i.e., $P_{Loss} = P'_{Loss}$,

then

$$r^2 = 2(r')^2$$

$$\therefore \frac{r^2}{(r')^2} = 2 \quad ---- (iv)$$

Now

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 l)}{3(\pi (r')^2 l)}$$

$$= \frac{2}{3} \times 2 = \frac{4}{3} = 1.333$$

Material for single-phase wire

$= (1.333) \times$ Material for three-phase wire

Ex-5 $\Rightarrow$ See Ex-2

Determine the total average power, reactive power and complex power at the source and at the load.

Soln.

System is balanced and it is sufficient to consider one phase.

For phase a,

$V_p = V_{an} = 110\angle 0^\circ$ Volt

$I_p = I_a = 6.81\angle -21.8^\circ$ Amp.

At the source

$$S_s = 3 V_p I_p^* = (2087 + j\,834.6) \text{ VA}$$

Real power supplied is 2087 Watt.

Reactive power supplied is 834.6 VAr.

At the load, the complex power absorbed is

$$S_{LOAD} = 3|I_p|^2 Z_p \qquad Z_p = (10+j8)\,\Omega$$

$$\therefore Z_p = 12.81\angle 38.66^\circ$$

$$\therefore S_{LOAD} = 3 \times (6.81)^2 (10+j8) \text{ VA}$$

$$\therefore S_{LOAD} = (1392 + j\,1113) \text{ VA}$$

Real power absorbed is 1392 Watt
Reactive power absorbed is 1113 VAr

The difference between the two complex powers is absorbed by the line impedance $(5-j2)\,\Omega$

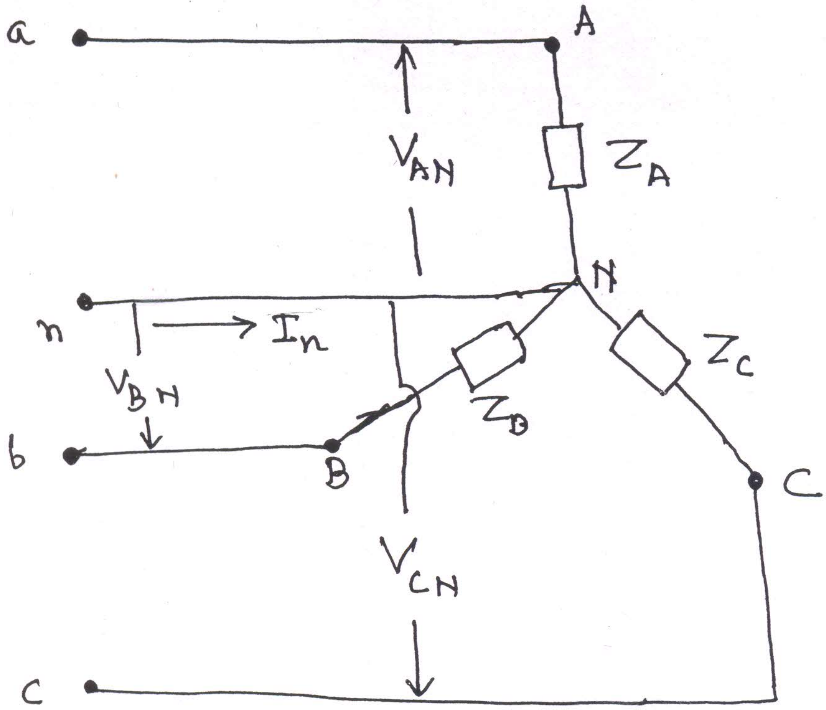
Power absorbed by the line is

$$3 \times (6.81)^2 (5-j2)$$

$$= (695.6 - j278.3) \text{ VA.}$$

Real Power absorbed by line ~~resistance~~ = 695.6 Watt
Reactive power absorbed by line = −278.3 VAr

UNBALANCED THREE - PHASE SYSTEMS



$$I_a = \frac{V_{AN}}{Z_A}$$

$$I_b = \frac{V_{BN}}{Z_B}$$

$$I_c = \frac{V_{CN}}{Z_C}$$

$$I_n = -(I_a + I_b + I_c).$$

Fig. 27

Three Phase Power Measurement

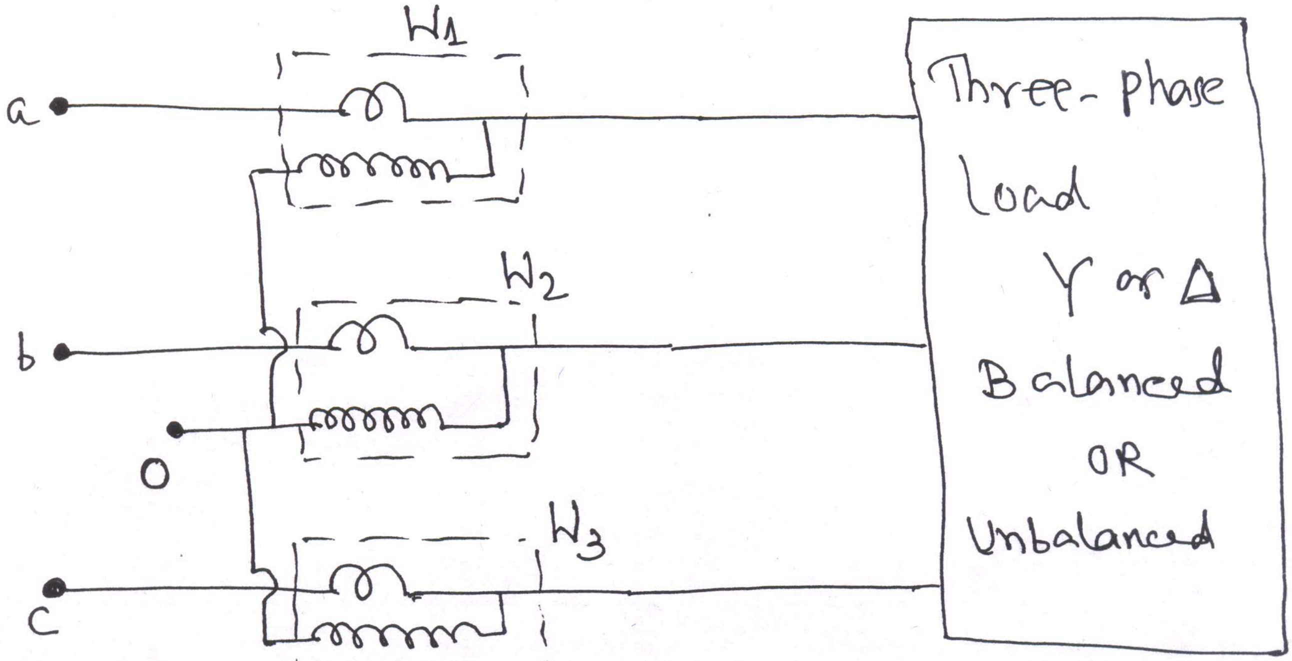


Fig. 28

Two – Wattmeter Method

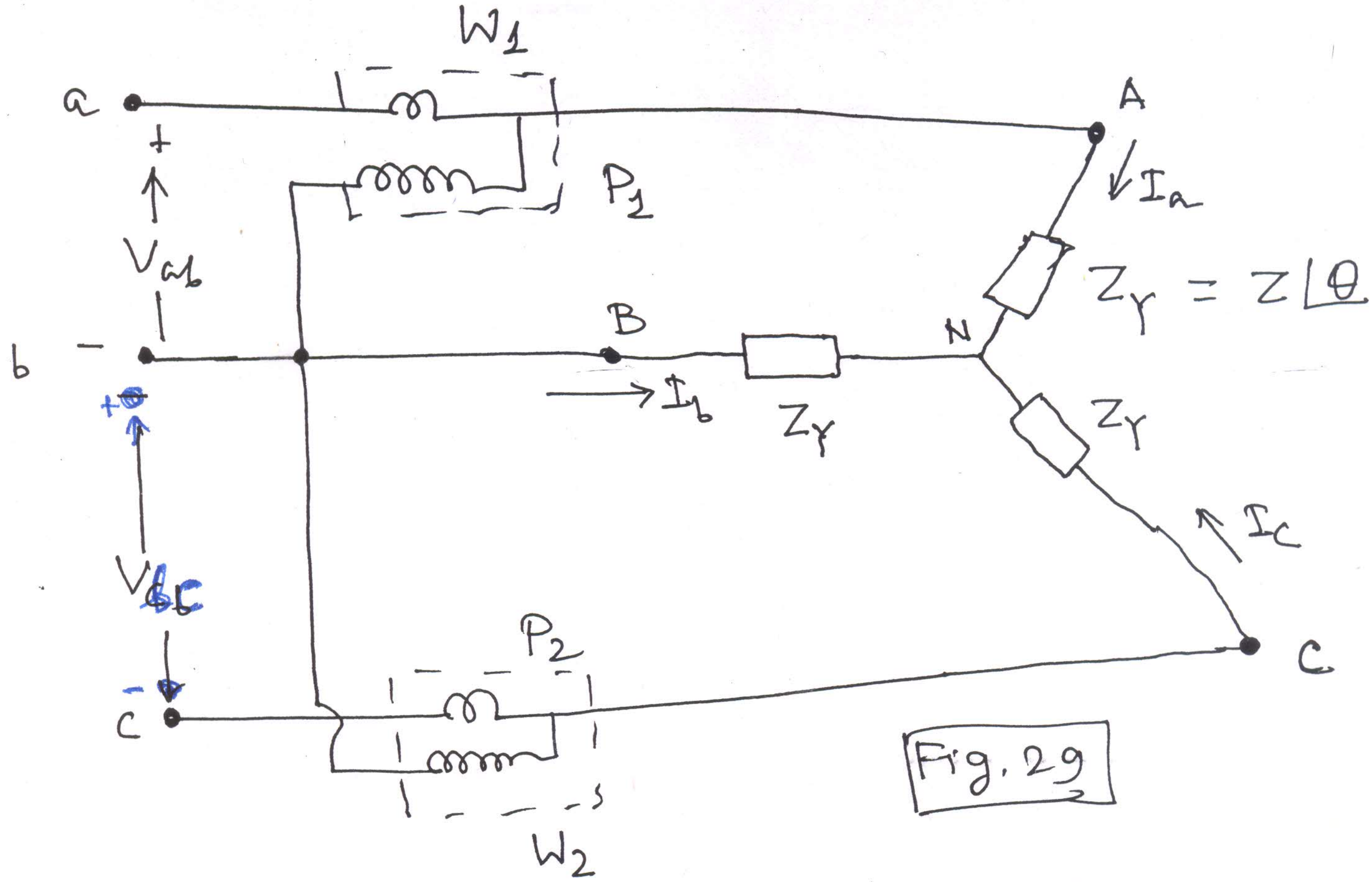
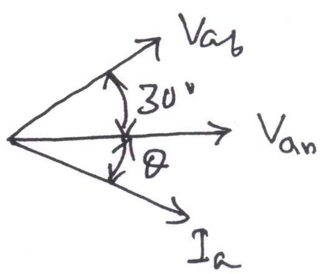


Fig. 29

$$I_a = \frac{V_{an}\angle 0^\circ}{Z_Y} = \frac{V_{an}\angle 0^\circ}{Z\angle\theta}$$



$$\therefore I_a = \left(\frac{V_{an}}{Z}\right)\angle -\theta$$

$$\therefore \quad P_1 = V_{ab} I_a \cos(\theta + 30^\circ)$$

$$\therefore P_1 = V_L I_L \cos(\theta + 30^\circ) \quad \text{--- (i)}$$

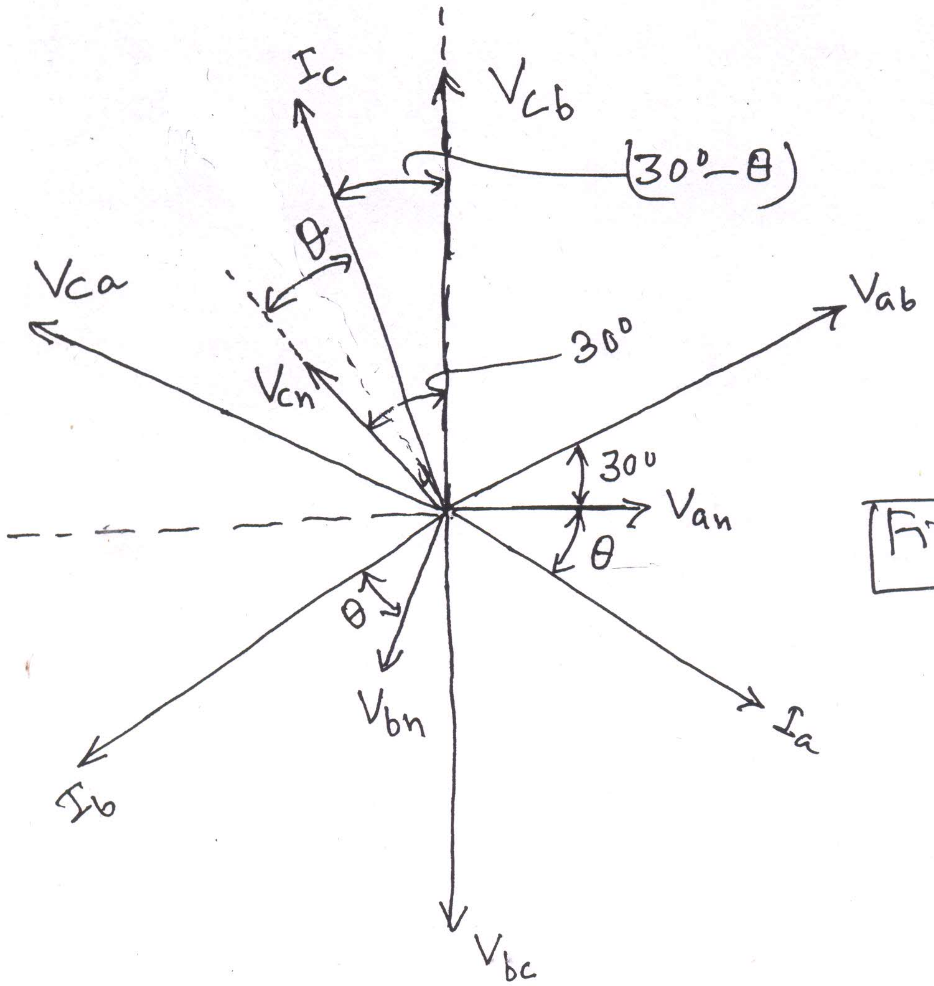


Fig. 30

$$P_2 = V_{cb} I_c \cos(30° - \theta) = V_{cb} I_c \cos(\theta - 30°)$$

$$\therefore P_2 = V_L I_L \cos(\theta - 30°) \quad \text{--- (ii)}$$

$\cancel{P_1 + P_2 = \sqrt{3}}$

$\therefore$ (i) + (ii)

$$\therefore P_1 + P_2 = \sqrt{3}\, V_L I_L \cos\theta$$

$$\therefore P_T = \sqrt{3}\, V_L I_L \cos\theta \quad \text{--- (iii)}$$

Now

Eqn(ii) — Eqn(i)

$$P_2 - P_1 = V_L I_L \sin\theta$$

$$\therefore \sqrt{3}\, V_L I_L \sin\theta = \sqrt{3}\,(P_2 - P_1)$$

$$\therefore Q_T = \sqrt{3}\,(P_2 - P_1) \quad \text{----} \ (iv)$$

$$\sqrt{3}\, V_L I_L \cos\theta = P_1 + P_2$$

$$\sqrt{3}\, V_L I_L \sin\theta = \sqrt{3}\,(P_2 - P_1)$$

$$\therefore \tan\theta = \frac{\sqrt{3}\,(P_2 - P_1)}{(P_2 + P_1)}$$

1) If $P_2 = P_1 \Rightarrow$ Resistive load

2) If $P_2 > P_1 \Rightarrow$ Inductive load

3) If $P_2 < P_1 \Rightarrow$ Capacitive load

Ex-6

$Z_A = 15\,\Omega$

$Z_B = (10 + j5)\,\Omega$

$Z_C = (6 - j8)\,\Omega$

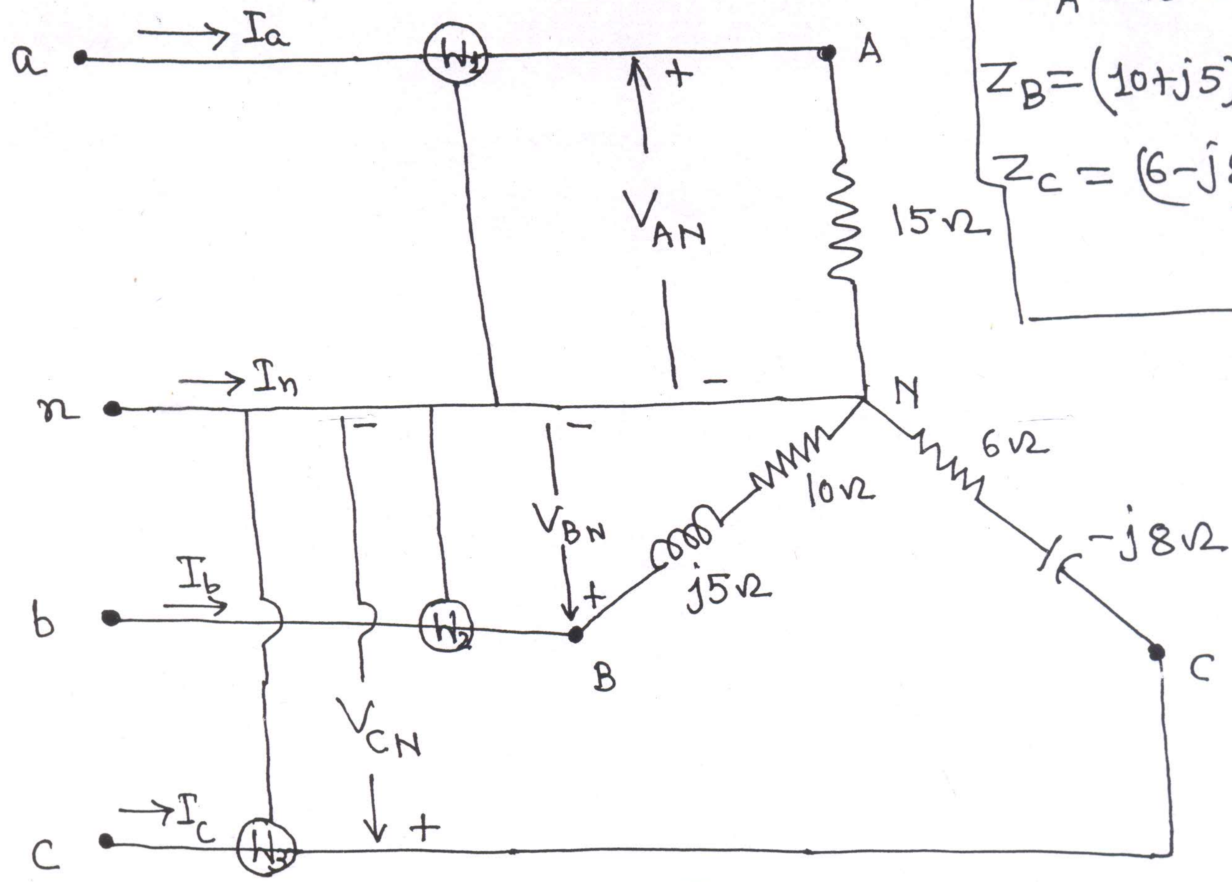


Fig. 31

(a) Predict the wattmeter readings

(b) Find the total power absorbed.

Given ~~that~~ quantity

Balanced phase voltage = 100 Volt
a-c-b sequence.

Soln. $V_{AN} = 100\angle 0^\circ$; $V_{BN} = 100\angle 120^\circ$; $V_{CN} = 100\angle -120^\circ$

$$I_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ Amp.}$$

$$I_b = \frac{100\angle 120^\circ}{(10+j5)} = 8.94\angle 93.44^\circ \text{ Amp}$$

$$I_c = \frac{100\angle -120^\circ}{(6-j8)} = 10\angle -66.87^\circ \text{ Amp.}$$

$$I_n = -(I_a + I_b + I_c) = 10.06\angle 178.4^\circ \text{ Amp.}$$

$$P_1 = V_{AN} \cdot I_a \cos(\theta_{V_{AN}} - \theta_{I_a})$$

$$\therefore P_1 = 100 \times 6.67 \cos(0^\circ - 0^\circ) = 667 \text{ Watt}$$

$$P_2 = V_{BN} I_b \cos(\theta_{V_{BN}} - \theta_{I_b})$$

$$\therefore P_2 = 100 \times 8.94 \cos(120^\circ - 93.44^\circ) = 800 \text{ Watt.}$$

$$P_3 = V_{CN} \cdot I_c \cos(\theta_{V_{CN}} - \theta_{I_c})$$

$$\therefore P_3 = 100 \times 10 \cos(-120^\circ + 66.87^\circ) = 600 \text{ Watt.}$$

(b) The total power absorbed is

$$P_T = P_1 + P_2 + P_3 = (667 + 800 + 600) = 2067 \text{ Watt.}$$

Also

$$P_T = |I_a|^2 \cdot R_A + |I_b|^2 \cdot R_B + |I_c|^2 \cdot R_C$$

$$\therefore P_T = (6.67)^2 \times (15) + (8.94)^2 \times 10 + (10)^2 \times 6$$

$$\therefore P_T = 667 + 800 + 600 = 2067 \text{ Watt.}$$

Ex-7

The three phase balanced load in Fig. 29 has $Z_Y = (8 + j6)\,\Omega$. If the load is connected to 208 volt lines, predict the readings of W_1 & W_2. Also find P_T and Q_T.

Soln

44

$$Z_Y = (8+j6) = 10\angle 36.87^\circ\ \Omega.$$

Line Voltage = 208 Volt.

$$I_L = \frac{V_p}{|Z_Y|} = \frac{V_L}{\sqrt{3}\,|Z_Y|} = \frac{208}{\sqrt{3}\times 10}$$

$$\therefore I_L = 12 \text{ Amp.}$$

Then

$$P_1 = V_L I_L \cos(30^\circ + \theta)$$

$$\boxed{\theta = 36.87^\circ}$$

$$\therefore P_1 = 208 \times 12 \times \cos(30^\circ + 36.87^\circ)$$

$$\boxed{\therefore P_1 = 980.48 \text{ Watt.}}$$

$$P_2 = V_L I_L \cos(30^\circ - \theta)$$

$$\therefore P_2 = 208 \times 12 \cos(30^\circ - 36.87^\circ)$$

$$\boxed{\therefore P_2 = 2478.1 \text{ Watt.}}$$

Since $P_2 > P_1$, $\Rightarrow$ Inductive load.

$$\boxed{P_T = P_1 + P_2 = 3.4586 \text{ kW}}$$

$$Q_T = \sqrt{3}\,(P_2 - P_1) = \sqrt{3} \times (1497.6)\ \text{VAr}$$

$$\boxed{\therefore Q_T = 2.594\ \text{kVAr}}$$

EXERCISE

If the load in Fig. 29 is Δ- connected with $Z_\Delta = (30 - j40)\ \Omega$ and $V_L = 440$ Volt, Determine, P_1, P_2, P_T and Q_T

Ans: 6.166 KW, 0.8021 KW, 6.968 KW, -9.29 kVAr

EX-8: Two balanced loads are connected to a 240 kV, 60 Hz line as shown in Fig. 32. Load-1 draws 30 kW at a power factor of 0.6 lagging, while ~~Load~~-2 draws 45 kVAr at power factor of 0.8 lagging. Determine (a) the complex, real and reactive power absorbed by the load. (b) line currents. (c) the kVAr rating of the three capacitors Δ- connected in parallel with load to raise the power factor to 0.9 Lagging and C_p.

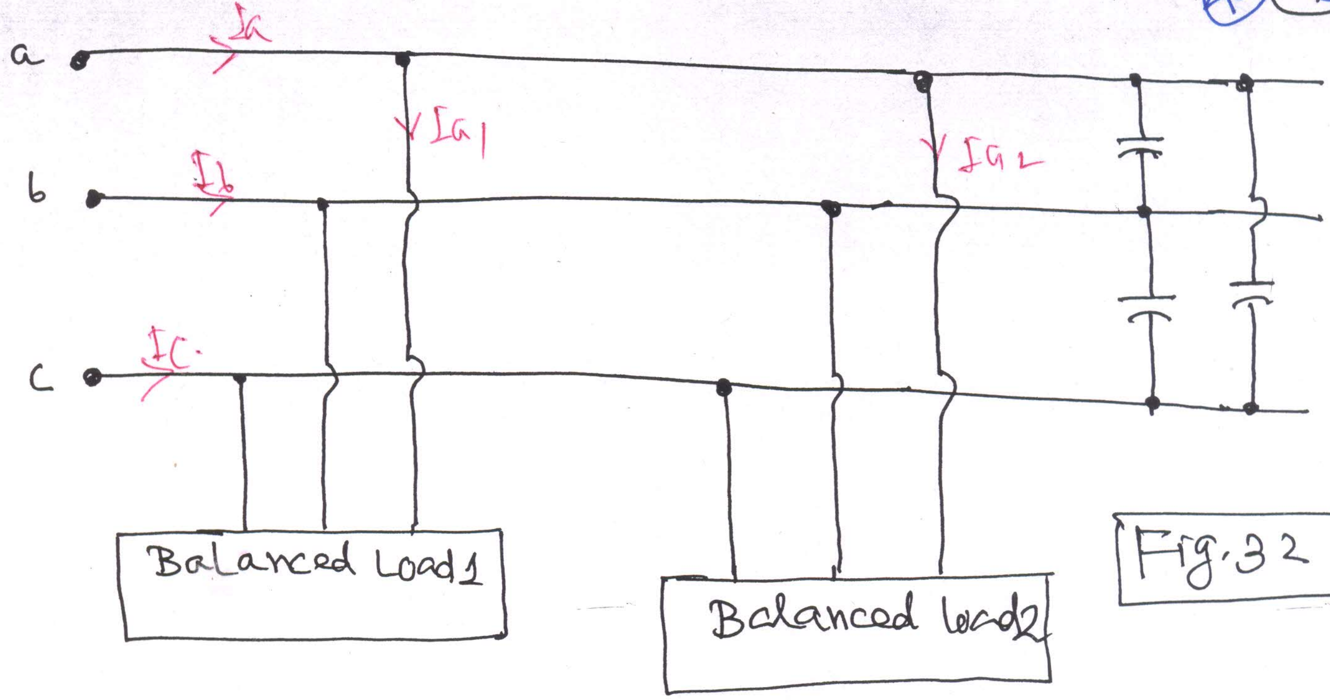


Fig. 32

Soln

For Load 1, $P_1 = 30\,kW$, $\cos\theta_1 = 0.6$, then $\sin\theta_1 = 0.8$, Hence $S_1 = \frac{P_1}{\cos\theta_1} = \frac{30}{0.6} = 50\,kVA.$

$$Q_1 = S_1(\sin\theta_1) = 50 \times 0.8 = 40\ kVAr.$$

$$\therefore\ P_1 + jQ_1 = (30 + j40)\ kVA$$

For Load 2, $Q_2 = 45\,kVAr$, $\cos\theta_2 = 0.8$, then $\sin\theta_2 = 0.6$

$$\therefore\ S_2 = \frac{Q_2}{\sin\theta_2} = \frac{45}{0.6} = 75\ kVA.$$

$$P_2 = S_2(\cos\theta_2) = 75 \times 0.8 = 60\ kW.$$

$$\therefore\ P_2 + jQ_2 = (60 + j45)\ kVA.$$

Total complex power,

$$P + jQ = (P_1 + P_2) + j(Q_1 + Q_2) = (90 + j85)\ kVA$$

$$\therefore\ P + jQ = 123.8 \angle 43.36^\circ\ kVA.$$

(b)

48 49

Now

$$\sqrt{3}\, V_L\, I_{L_1} \cos\theta_1 = P_1$$

$$\therefore\ I_{L_1} = \frac{30}{\sqrt{3} \times 240 \times 0.6} = 120.28\ mA$$

$$\theta_1 = \cos^{-1}(0.6) = 53.13^{\circ}$$

$$\therefore\ I_{a_1} = 120.28 \angle -53.13^{\circ}\ mA.$$

Similarly

$$I_{L_2} = \frac{60}{\sqrt{3} \times 240 \times 0.8} = 180.42\ mA$$

$$\theta_2 = \cos^{-1}(0.8) = 36.87^{\circ}$$

$$\therefore\ I_{a_2} = 180.42 \angle -36.87^{\circ}\ mA.$$

$$\boxed{\therefore\ I_a = I_{a_1} + I_{a_2} = 297.8 \angle -43.36^{\circ}\ mA.}$$

$$I_b = I_a \angle -120^{\circ} = 297.8 \angle 163.36^{\circ}\ mA.$$

$$I_c = I_a \angle 120^{\circ} = 297.8 \angle 76.64^{\circ}\ mA.$$

(c) $Q_c = P(\tan\theta_{old} - \tan\theta_{new})$

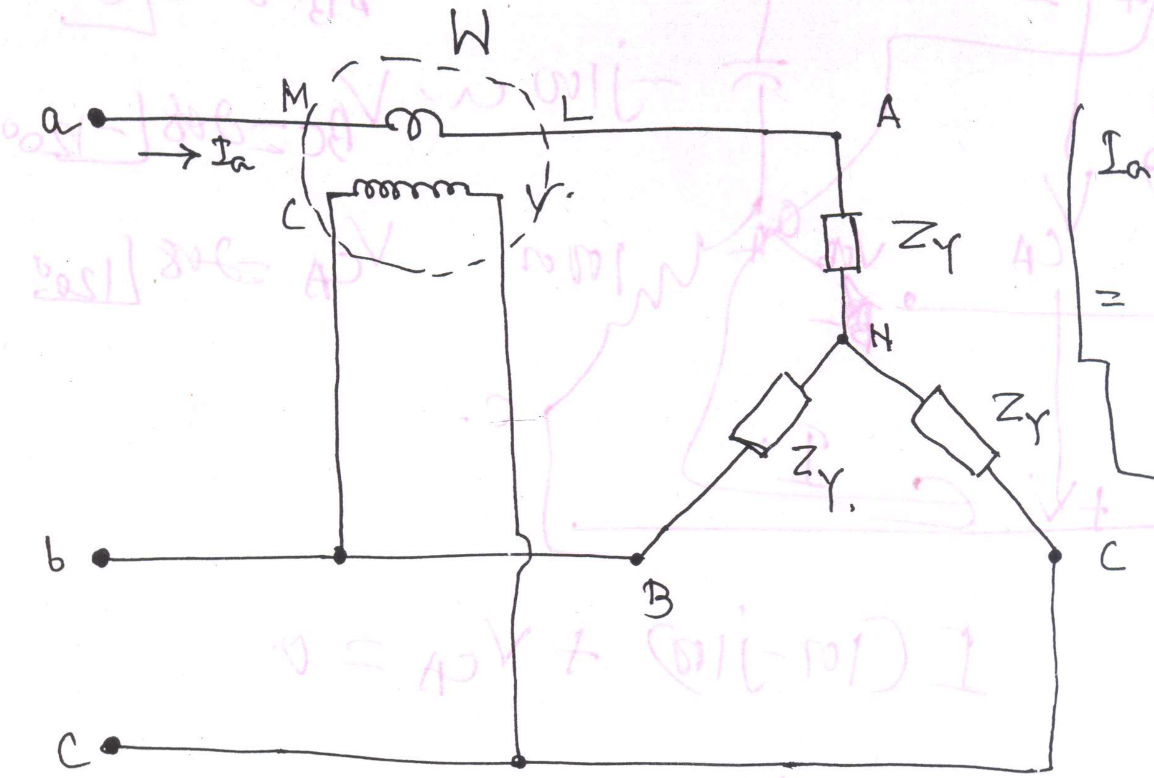
$$\therefore\ Q_c = 90(\tan(43.36^{\circ}) - \tan(25.84^{\circ}))$$

$$\therefore\ Q_c = 41.4\ kVAr.$$

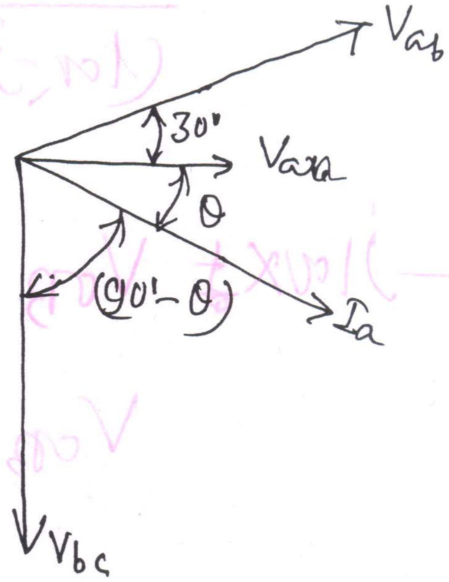
$\cos\theta_{new} = 0.9$

$\therefore\ \theta_{new} = 25.84^{\circ}$

MEASUREMENT OF REACTIVE POWER



$$I_a = \frac{V_{an}\angle 0^\circ}{Z_Y} = \frac{V_{an}\angle 0^\circ}{Z\angle\theta} = \frac{V_{an}}{Z}\angle -\theta$$



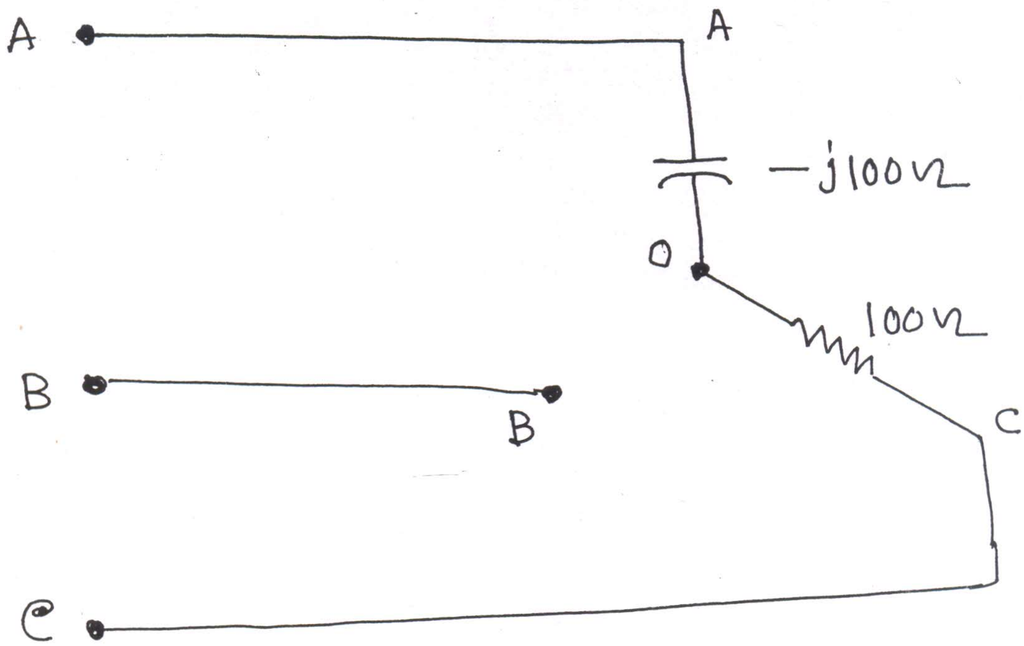
Wattmeter reading

$$= |V_{bc}| \cdot |I_a| \cos(90^\circ - \theta)$$

$$= V_L \cdot I_L \sin\theta$$

The wattmeter canbe so calibrated that it indicates three phase reactive power by having a multiplication factor of $\sqrt{3}$.

EXERCISE



Find V_{OB}, Given that the system is 208 Volt and a-b-c sequence.

Ans: $284 \angle 150^\circ$ Volt.

EXERCISE

The line currents in a three-phase, three-wire, 220 Volt, a-b-c system are $I_a = 43.5 \angle 116.6^\circ$ Amp, $I_b = 43.3 \angle -48^\circ$ Amp and $I_c = 11.39 \angle 218^\circ$ Amp.

Find the readings of wattmeters in lines (i) a and b (ii) b and c (iii) a and c

Ans: (i) 5270 W, 6370 W (ii) 9310 W, 2330 W (iii) 9550 W, 1980 W