INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 24.09.2013(FN)
Mid Semester Examination (Autumn)
Subject No. ME10001
No. of students: 715

Time: 2 hrs
Full Marks:105
Subject Name: MECHANICS

Instructions: *Answer all SEVEN questions. Any data, if not furnished, may be assumed.*

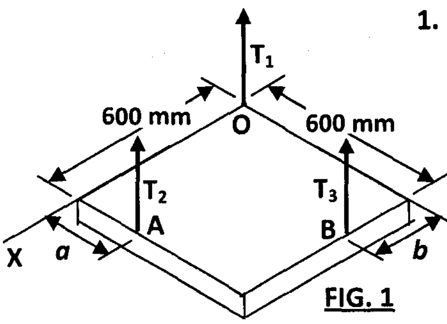

FIG. 1

1. A thick square plate of weight 600 N is supported by three vertical wires in such a manner that the plate surface is horizontal, as shown in Fig. 1. If tensions T_1=T and T_2 = 2T and T_3 = 3T, determine the magnitude of T and the distances *a* and *b*. Assume that the weight of the plate acts at the geometric center of the plate.

2. Determine the magnitude of the moment of force F = (50**i** – 20**j** – 80**k**) N about the base line CA of the tripod, shown in Fig.2.

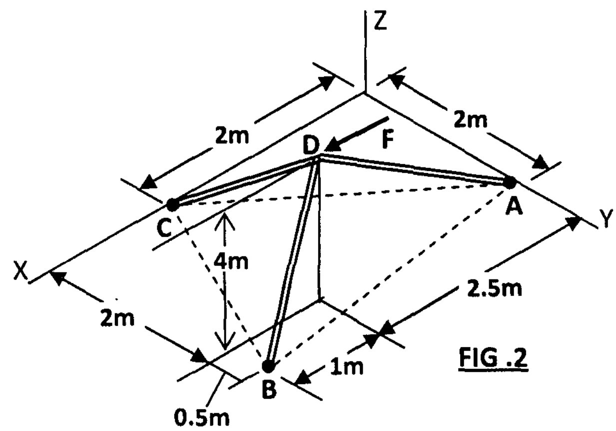

FIG .2

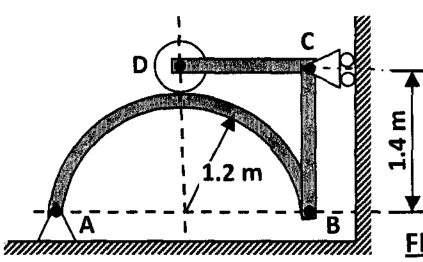

FIG. 3

3. A smooth disc shown in the Fig.3 is pinned at D and has a weight of 50 N. The element BCD is a single member. Neglecting the weights of other members, determine the force carried by the pin B.

4. A thin light sheet is clamped due to friction (identical at D and the wall) in a device with a tensioned horizontal spring at B, as shown in Fig. 4. When a force P = 5 N is applied at A, the clamping force at D just goes to zero without any rotation of the clamp (i.e., spring force remains constant). When P = 0, (a) determine the force F required to pull the sheet down, (b) determine the minimum value of μ so that any force F can be supported without slippage of the sheet.

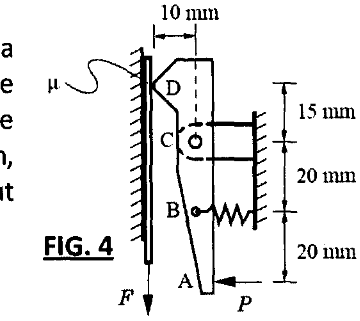

FIG. 4

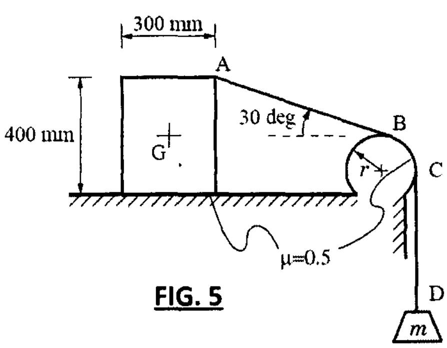

FIG. 5

5. A block of height 400 mm, width 300 mm and weight 1000 N is put on a rough horizontal surface and connected to a mass ***m*** through a massless inextensible string AD (Fig.5). The portion AB of the string makes an angle 30° with the horizontal, and the coefficient of friction on all contacting surfaces is μ = 0.5. Determine the range of ***m*** for static equilibrium.

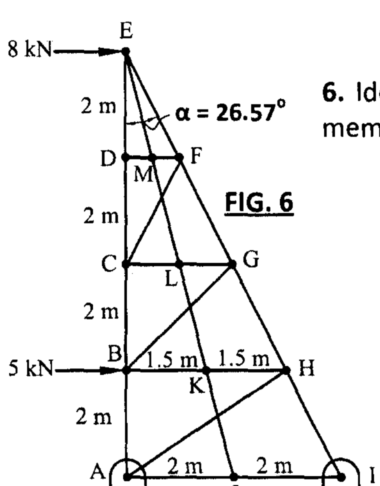

FIG. 6

6. Identify all the zero-force members in the truss shown in Fig.6. Determine the force in the members GH and KH. (Note: only filled circles indicate ideal pin joints)

7. For the composite area OABCO, shown in Fig.7, find the y coordinate of the centroid (Y_C). Also calculate the second moment of area about the x-axis.

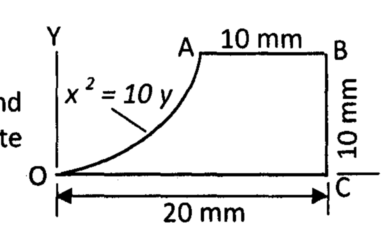

FIG. 7

1. co-ordinates:

A $(600, a)$, B$(b, 600)$ and G$(300, 300)$

$OA = 600i + aj$; $OB = bi + 600j$

$OG = 300i + 300j$.

$\Sigma F_z = 0$; $T_1 + T_2 + T_3 = 600$ or $T + 2T + 3T = 600$

$\therefore \underline{T = 100\,N}$

$\Sigma M_o = 0$;

$$(600i + aj) \times 200\underline{k} + (bi + 600j) \times 300\underline{k} + (300i + 300j) \times -600k = 0$$

$$\text{or } -120000j + 200a\,i - 300b\,j + 180000\,i + 180000j - 180000\,i = 0$$

$\therefore 200a = 0$ and $300b = 60000$

$\therefore \underline{a = 0}$; and $\underline{b = 200\,mm}$

2. Co-ordinates:

$C(2,0,0)$; $A(0,2,0)$ and $D(2.5, 2, 4)$

$\vec{CA} = -2i + 2j$ m $\therefore \hat{CA} = -0.707i + 0.707j$

Sol 1: $\vec{AD} = 2.5i + 4\underline{k}$ m

$\therefore M_A = \vec{AD} \times \vec{F} = (2.5i + 4k) \times (50i - 20j - 80\underline{k})$

$= -50\underline{k} + 200j + 200j + 80i = 80i + 400j - 50\underline{k}$ Nm

$\therefore M_{CA} = \vec{M_A} \cdot \hat{CA} = \underline{226.24\text{ Nm}}$

Sol. 2 $\vec{CD} = 0.5i + 2j + 4k$ m

$M_C = (0.5i + 2j + 4k) \times (50i - 20j - 80\underline{k})$

$= -10\underline{k} + 40j - 100\underline{k} - 160i + 200j + 80i$

$= -80i + 240j - 110k$

$\therefore M_{CA} = \vec{M_C} \cdot \hat{CA} = \underline{226.24\text{ Nm}}$

1. Co-ordinates:

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$$\text{or } -120000j + 200ai - 300bj + 180000i + 180000j - 180000i = 0$$

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$\vec{CA} = -2i + 2j$ m $\therefore \hat{CA} = -0.707i + 0.707j$

Sol 1: $\vec{AD} = 2.5i + 4k$ m

$$\therefore M_A = \vec{AD} \times \vec{F} = (2.5i + 4k) \times (50i - 20j - 80k)$$
$$= -50k + 200j + 200j + 80i = 80i + 400j - 50k \text{ Nm}$$

$\therefore M_{CA} = \vec{M_A} \cdot \hat{CA} = \underline{226.24\text{ Nm}}$

Sol. 2 $\vec{CD} = 0.5i + 2j + 4k$ m

$$M_C = (0.5i + 2j + 4k) \times (50i - 20j - 80k)$$
$$= -10k + 40j - 100k - 160i + 200j + 80i$$
$$= -80i + 240j - 110k$$

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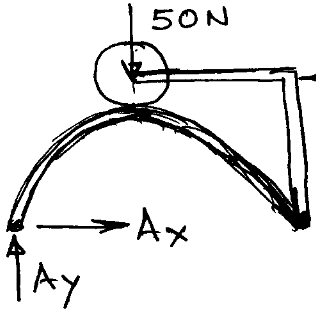
3. FBD I



$\Sigma M_A = 0;$

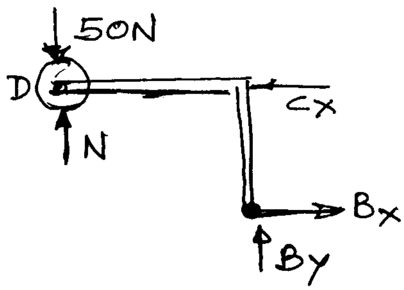
$50 \times 1.2 - C_x \times 1.4 = 0$

$\therefore C_x = 42.86\ N$

$\Sigma F_x = 0;\ A_x = C_x = 42.86N$

$\Sigma F_y = 0;\ A_y = 50N$

Sec. 1



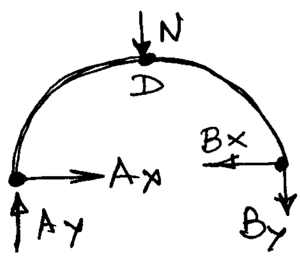
$B_x - C_x = 0 \ \therefore B_x = C_x = 42.86N$

$\Sigma M_D = 0;$

$B_y \times 1.2 + B_x \times 1.4 = 0$

$\therefore B_y = -50N$

Sec 2



$\Sigma F_x = 0;\ B_x = A_x = 42.86N$

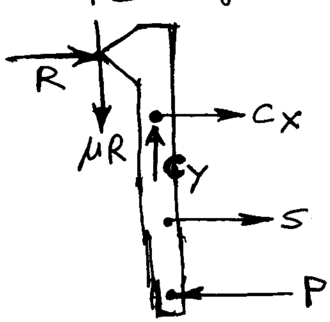
$\Sigma M_D = 0;\ A_y \times 1.2 + B_y \times 1.2 = 0$

$\therefore B_y = -A_y = -50\ N$

$\therefore B_x = 42.86\ N \longrightarrow$

$B_y = 50.0\ N \uparrow$

4 FBD of lever for $0<P<5$



FBD-1

$\Sigma M_C = 0$

$R \times 15 + P \times 40 - S \times 20 - \mu R \times 10 = 0$

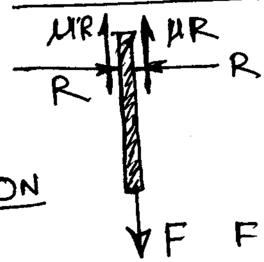
when $P = 5N$, $R = 0 \ \therefore S = 10N$

when $P = 0$,

$R \times 15 - 10 \times 20 - \mu R \times 10 = 0$

$\therefore R = 200/(15 - 10\mu)$

FBD of Sheet

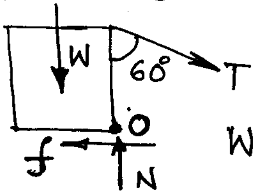


FBD 2

(a) $\therefore$ From FBD 2 $\ F = 2\mu R = 400\mu/(15-10\mu)$

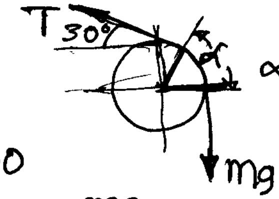
(b) If $F \to \infty$, $15 - 10\mu \to 0 \ \therefore \mu \to 1.5$

5. Impending Tipping



$\Sigma M_O = 0$

$W \times 150 - T \sin 60 \times 400 = 0$



$\alpha = \frac{\pi}{3}$

$\frac{mg}{T} = e^{0.5 \times \pi/3}$

$\therefore mg = T e^{0.5 \times \pi/3}$

$\therefore T = mg / e^{0.5 \times \pi/3}$

$T = mg/1.688$ --- (1)

or

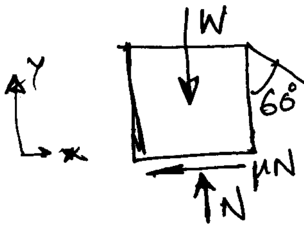
or $\frac{mg}{1.688} \times \sin 60° \times 400 = 1000 \times 150$

$\therefore mg = 730.93 N$

$\therefore$ mass, $m = \underline{74.5 \text{ kg}}$ $(g = 9.81)$

$= \underline{73.1 \text{ kg}}$ $(g = 10.0)$

Impending Slipage



$\Sigma F_y = 0$

$N - W - T\cos 60° = 0$

$\Sigma F_x = 0$

$T \sin 60° - \mu N = 0$

$\therefore \frac{mg}{1.688} \times \sin 60° = 0.5\left(1000 + \frac{mg}{1.688} \times \cos 60°\right)$

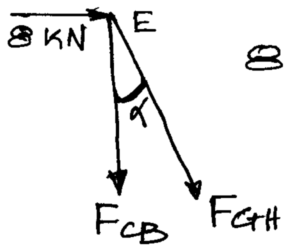
$0.513 mg = 500 + 0.148 mg$ $\therefore mg = 1370.25 N$ $\therefore m = 139.68$ kg $(g = 9.8)$

$= 137$ kg $(g = 10)$

$\therefore$ Range of m: $0 < m < 74.5/73.1$ kg

6. Zero force members; KJ, KL, LM, ME, DM, MF, CF, CL, LG, BG.

Section through CB and GH



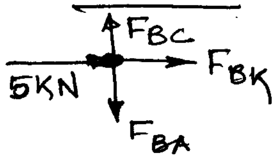
$8 + F_{GH} \sin\alpha = 0$

$\therefore F_{GH} = -8/\sin\alpha$

$= \underline{-17.9 \text{ kN}}$

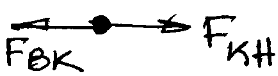
$= 17.9 \text{ KN (c)}$

Joint B



$\underline{F_{BK} = -5 \text{ kN}}$

Joint K.

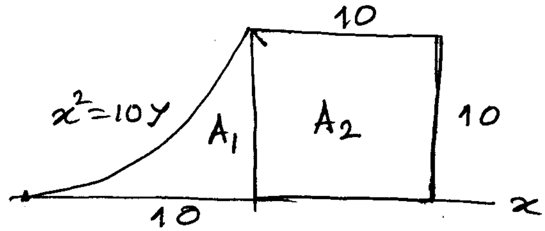


$\therefore \underline{F_{KH} = F_{BK} = -5 \text{ kN}}$

ANS $F_{GH} = 17.9 \text{ KN (c)}$

$F_{KH} = 5 \text{ KN (c)}$

7.



For A_1 — **Part A**

$$y_c \times A = \int_A y\,dA$$

$$\therefore y_c = \frac{100}{100/3} = 3\text{ mm}$$

For A2

$$y_c = 5, \quad \text{Area} = 100\text{ mm}^2$$

$$\therefore y_c\Big|_{total} = \frac{3 \times \frac{100}{3} + 5 \times 100}{\frac{100}{3} + 100}$$

$$= 4.5\text{ mm}$$

Part B

$$I_{xx1} = \int_0^{10} y^2\,dA = \int_0^{10} y^2\left(10 - \sqrt{10y}\right)dy$$

$$= 10\left[\frac{y^3}{3}\right]_0^{10} - \sqrt{10}\int_0^{10} y^{5/2}\,dy$$

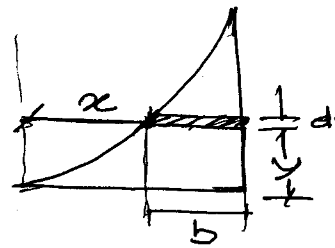
$$= \frac{10^4}{3} - \sqrt{10}\left[\frac{10^{7/2}}{7/2}\right]$$

$$= 476.19\text{ mm}^4$$

$$I_{xx2} = \frac{10 \times 10^3}{3} = \frac{10^4}{3}$$

$$\therefore I_{xx} = I_{xx1} + I_{xx2}$$

$$= 3809.5\text{ mm}^4$$



$$x = \sqrt{10y} \quad \therefore b = 10 - x$$

$$\text{or } b = 10 - \sqrt{10y}$$

$$A = \int_0^{10} b\,dy = \int_0^{10}\left(10 - \sqrt{10y}\right)dy$$

$$= \int_0^{10} 10\,dy - \int_0^{10}\sqrt{10y}\,dy$$

$$= 10 \times 10 - \sqrt{10}\left[\frac{y^{3/2}}{3/2}\right]_0^{10}$$

$$= 100/3$$

$$\int_0^{10} y\,dA = \int_0^{10} y\left(10 - \sqrt{10y}\right)dy$$

$$= 10\left[\frac{y^2}{2}\right]_0^{10} - \sqrt{10}\int_0^{10} y^{3/2}\,dy$$

$$= 100\text{ mm}^3$$