TRANSFORMERS

A transformer is a device which uses the phenomenon of mutual induction to change the Values of alternating Voltages and currents.

One of the main advantages of AC transmission and distribution is the ease with which an alternating voltage can be increased or decreased by transformers.

Losses in transformers are generally low and thus efficiency is high.

Being static they have a long life and are very stable.

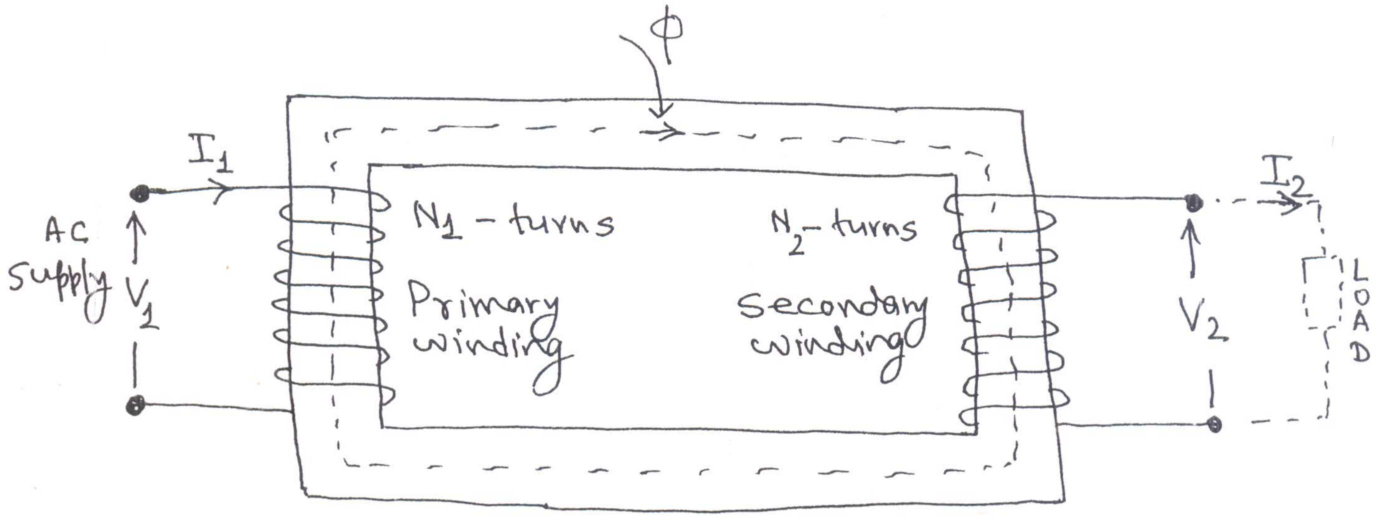


Fig. 1: Ferromagnetic core.

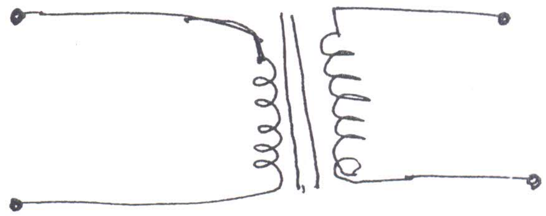


Fig. 2: circuit diagram symbol for a transformer.

Primary winding $\Rightarrow$ Connected to AC supply
Secondary winding $\Rightarrow$ May be connected to a load.

PRINCIPLE OF OPERATION

When the secondary is an open-circuit and an alternating voltage V_1 is applied to the primary winding, a small current - called the no-load current I_0 - flows, which sets up a magnetic flux in the core. This alternating flux links with both primary and secondary coils and induces in them emf's of E_1 and E_2 respectively by mutual induction.

The induced emf E in a coil of N- turns is given by

$$E = -N\frac{d\phi}{dt}$$

Following assumptions are made for an ideal transformer:

(a) Winding resistances are negligible.

(b) All the flux produced is confined to the core of the transformer and links fully both the windings. There is no leakage of the flux.

(c) The permeability of the core is high so that the magnetising current required to produce the flux and establish it in the core is negligible.

(d) Hysteresis and eddy current losses are negligible.

In an ideal transformer, $\frac{d\phi}{dt}$ is same for both primary and secondary, thus

$$E_1 = -N_1 \frac{d\phi}{dt}$$

and

$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_1}{N_1} = \frac{E_2}{N_2}$$

(i.e. induced emf per turn is constant)

Polarity of the induced emf is given by Lenz's law. It opposes the change and hence is negative

Assuming no Losses,

$$\therefore \frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$\therefore \frac{V_1}{V_2} = \frac{N_1}{N_2} = K$$

$K \Rightarrow$ <u>voltage ratio</u> OR <u>turns ratio</u> OR <u>transformation ratio</u>

$$\therefore V_2 = \frac{V_1}{K},$$

$K < 1 \Rightarrow$ step-up transformer

$K > 1 \Rightarrow$ step-down transformer

When a load is connected across the secondary winding, a current I_2 flows. In an ideal transformer losses are neglected and a transformer is considered to be 100% efficient.

Hence primary and Secondary volt-amperes are equal, i.e.,

$$V_1 I_1 = V_2 I_2$$

$$\boxed{\therefore \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = K}$$

The <u>rating</u> of a transformer is stated in terms of the volt-amperes that it can transform <u>without overheating</u>.

Transformer rating is either $V_1 I_1$ or $V_2 I_2$ (where I_2 is the full-load secondary current).

<u>Ex-1</u>

An ideal transformer has a turns ratio of 8:1 and the primary current is 3 Amp when it is supplied at 240 volt. Calculate the secondary voltage and current.

Soln.

A turns ratio of 8:1 means

$$\frac{N_1}{N_2} = \frac{8}{1} = K$$

$\therefore$ $K = 8$ (step-down transformer)

$$\therefore \frac{V_1}{V_2} = K$$

$$\therefore V_2 = \frac{V_1}{K} = \frac{240}{8} = 30 \text{ Volt.}$$

Also

$$\frac{I_2}{I_1} = K$$

$$\therefore I_2 = K \cdot I_1 = 8 \times (3) = 24 \text{ Amp.}$$

Ex-2

A 5 KVA, 1φ transformer has a turns ratio of 10:1 and is fed from a 2.5 KV supply. Neglecting losses, determine

(a) the full-load secondary current

(b) the minimum load resistance which can bee connected across the secondary winding to give full load KVA

(c) primary current at full load KVA.

Soln.

(a) $\frac{N_1}{N_2} = \frac{10}{1} = 10$,

$V_1 = 2.5\ kV = 2500$ Volt

Since

$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$\therefore V_2 = V_1\left(\frac{N_2}{N_1}\right) = 2500 \times \frac{1}{10} = 250$ Volt

The transformer rating in Volt-amperes

$= V_2 I_2$ (at full load)

$\therefore \quad 5 \times 1000 = 250 \times I_2$

$\therefore I_2 = 20$ Amp. = full-load current

(b) Minimum value of load resistance,

$$R_L = \frac{V_2}{I_2} = \frac{250}{20} = 12.5\,\Omega$$

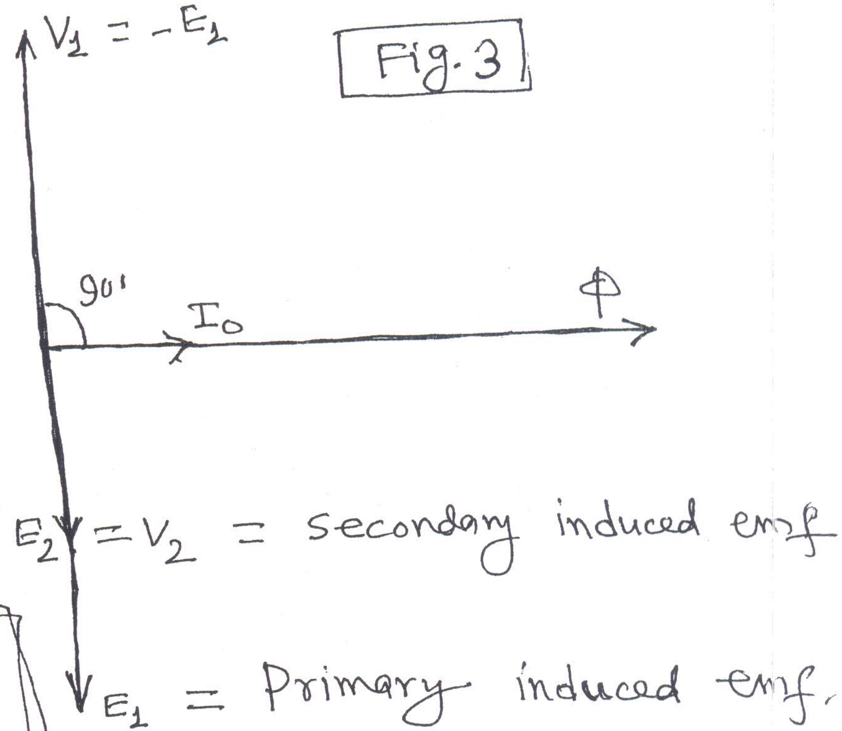
(c) $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ $\quad \therefore I_1 = I_2\left(\frac{N_2}{N_1}\right)$

$\therefore I_1 = 20 \times \frac{1}{10} = 2$ Amp.

No-Load Phasor Diagram

(A) The core flux is common to both primary and secondary windings in a transformer and is thus taken as the reference phasor in a phasor diagram.

Fig. 3



$I_0 \Rightarrow$ Small no load current

Loss is neglected, primary winding is a pure inductor

I_0 produces the flux and is in phase with the flux.

The primary induced emf E_1 is in phase opposition to V_1 (By Lenz's law) and is shown 180° out of phase with V_1.

$$I_o = \sqrt{I_m^2 + I_c^2}$$

$$I_m = I_o \sin\phi_o$$

$$I_c = I_o \cos\phi_o$$

$$\boxed{\begin{aligned}&\text{Total core losses}\\ &= \text{Iron losses}\\ &= V_1 I_o \cos\phi_o\end{aligned}}$$

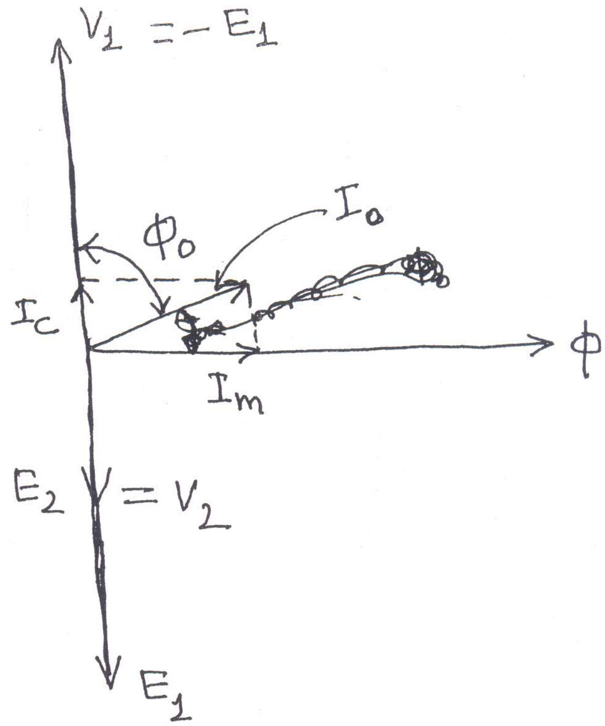


Fig. 4: No-load phasor diagram for a practical transformer

If current flows then losses will occur. When losses are considered, I_o has two components:

(i) $I_m \Rightarrow$ the magnetizing component

$I_c \Rightarrow$ the core loss component (supplying eddy current and hysteresis losses)

$V_1 = 2400$ Volt.

Ex-3:

A 2400/400 Volt single-phase transformer takes a no-load current of 0.5 Amp and the core loss is 400 Watt. Determine the values of the magnetizing and core loss components of the no-load current. Draw the phasor diagram.

Soln.

$V_1 = 2400$ Volt ; $V_2 = 400$ Volt.

$I_0 = 0.5$ Amp.

Core loss (i.e. iron loss) = 400 Watt.

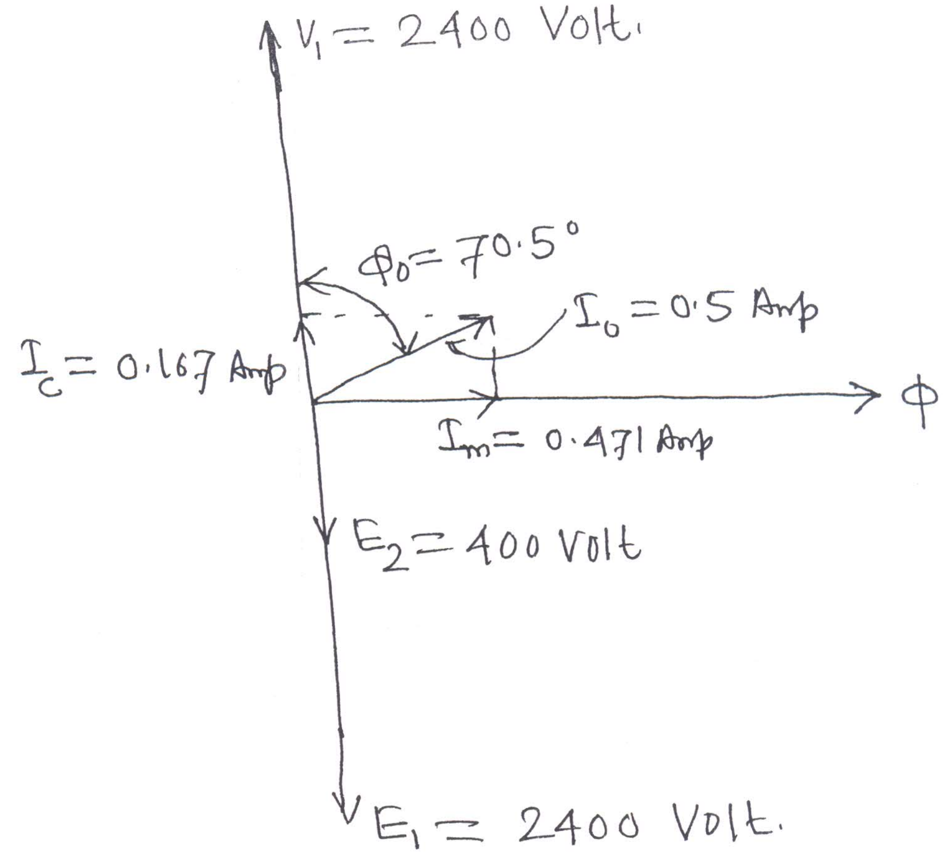
$\therefore \quad V_1 I_0 \cos\phi_0 = 400$

$\therefore \quad 2400 \times 0.5 \cos\phi_0 = 400$

$\therefore \quad \cos\phi_0 = \frac{1}{3} \quad \therefore \quad \phi_0 = 70.5^\circ$

Magnetizing component $= I_m = I_0 \sin\phi_0 = 0.471$ Amp

Core loss component $= I_c = I_0 \cos\phi_0 = 0.167$ Amp



EMF EQUATION

$$\left.\begin{aligned} E_1 &= 4.44\, f\, \phi_m N_1 \text{ Volts} \\ E_2 &= 4.44\, f\, \phi_m N_2 \text{ Volts} \end{aligned}\right\}$$

Ex-4: A single-phase 500/100 Volt, 50 Hz transformer has a maximum core flux density of 1.5 Wb/m² and an effective core cross-sectional area of 50 cm². Determine N_1 and N_2.

Soln:

$$\phi_m = B \times A$$

$B = 1.5\ Wb/m^2$, ~~and~~ $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$

$$\therefore \phi_m = (1.5)(50 \times 10^{-4}) = 75 \times 10^{-4}\ Wb.$$

Since

$$E_1 = 4.44 f \phi_m N_1$$

$$\therefore N_1 = \frac{E_1}{4.44 f \phi_m} = \frac{500}{4.44 \times 50 \times 75 \times 10^{-4}}$$

$$\therefore N_1 = 300$$

$$N_2 = \frac{E_2}{4.44 f \phi_m} = \frac{100}{4.44 \times 50 \times 75 \times 10^{-4}}$$

$$\therefore N_2 = 60$$

Ex-5: A 4500/225 Volt, 50 Hz single-phase transformer is to have an approximate emf per turn of 15 Volt and operate with a maximum flux of 1.4 Wb/m^2. Calculate (a) the number of primary and secondary turns (b) cross-sectional area of the core.

Soln

(a) emf per turn $= \frac{E_1}{N_1} = \frac{E_2}{N_2} = 15$.

$\therefore \quad N_1 = \frac{E_1}{15} = \frac{4500}{15} = 300$

$N_2 = \frac{E_2}{15} = \frac{225}{15} = 15$

(b)

$E_1 = 4.44\, \phi_m f\, N_1$

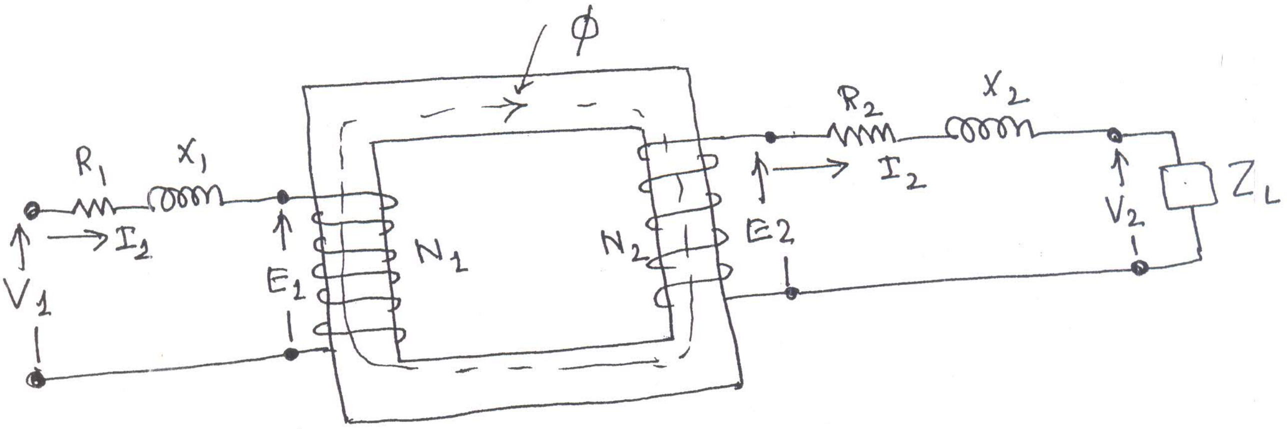
$\therefore \quad \phi_m = \frac{E_1}{4.44 f N_1} = \frac{4500}{4.44 \times 50 \times 300}$

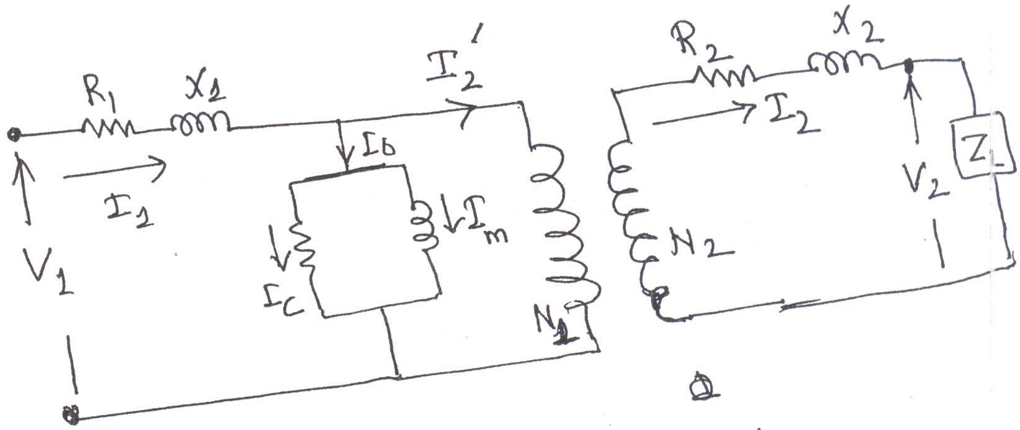
$\therefore \quad \phi_m = 0.0676$ Wb.

$\therefore \quad A = \frac{\phi_m}{B_{max}} = \frac{0.0676}{1.4} = 0.0483\, m^2$

$\therefore \quad A = 483\, cm^2$

Transformer on load





$$I_2' \cdot N_2 = I_2 N_2$$

$$\therefore \underline{I_2' = \left(\frac{N_2}{N_1}\right) I_2}$$

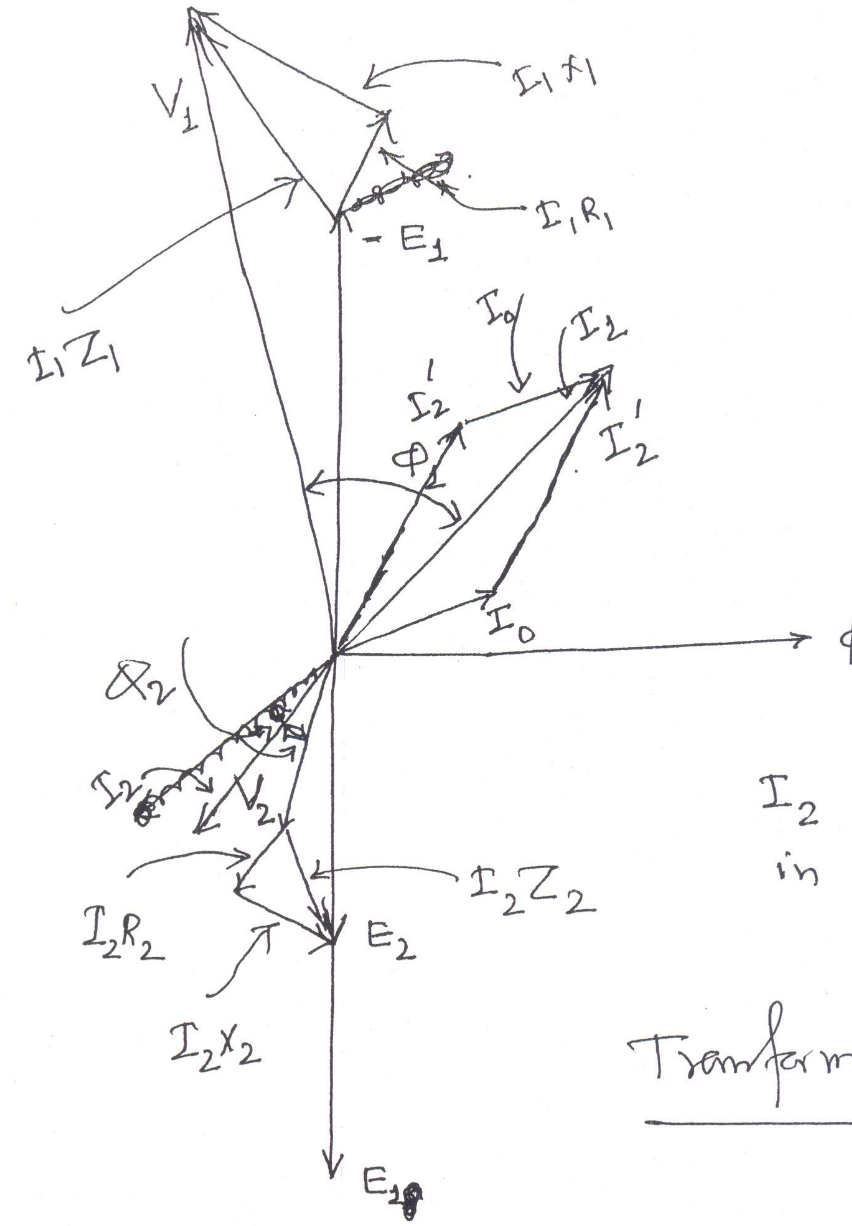
$$E_1 = -N_1 \cdot \frac{d\phi}{dt} = -N_1 \frac{d}{dt}\left(\phi_{max} \sin(\omega t)\right)$$

$$\therefore E_1 = -N_1 \phi_{max} \omega \cos(\omega t)$$

$$\therefore E_1 = -2\pi f \phi_{max} N_1 \cos(\omega t)$$

$$\therefore E_{1,rms} = -\frac{2\pi f \phi_{max} N_1}{\sqrt{2}} = -4.44 \phi_{max} f N_1$$

$$\boxed{\begin{aligned} E_1 &= 4.44 \phi_{max} f N_1 \\ E_2 &= 4.44 \phi_{max} f N_2 \end{aligned}}$$

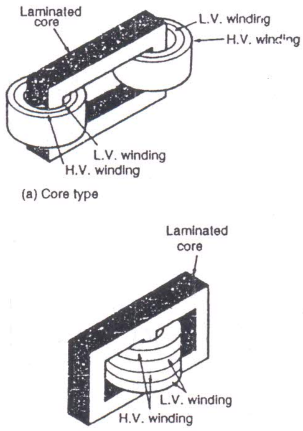


I_2 and I_2' are in anti-phase

Tranformer on-load

When the transformer is Loaded, a current I_2 will flow in the secondary winding. The secondary mmf I_2N_2 sets up a Secondary flux that tends to reduce the flux produced by the primary mmf. As a result E_1 reduces and the balance between V_1 and E_1 would no longer exist. Hence on load, the presence of secondary mmf necessitates the production of primary mmf equal in magnitude but opposite in direction.

[redacted] **Transformer construction**



(a) Core type

(b) Shell type

[redacted]

1.) Two types of single-phase transformers.
(a) core type (b) shell type.
The low and high voltage windings are wound as shown to reduce leakage flux.

2.) For power transformers, rated possibly at several MVA and operating at a frequency 50 Hz or 60 Hz (USA or Japan).
The core material used is usually

Laminated silicon steel, the laminations reducing eddy currents and the silicon steel keeping hysteresis loss to a minimum.

Large power transformers are used in the ~~main distribution~~ power station, main distribution system (S/S) and in industrial supply circuits.

Small power transformers have many applications; examples including welding and rectifier supplies, domestic bell circuits, imported washing machines etc.

3.) For <u>audio frequency transformers</u>, rated from a few mVA to no more than 20 VA and operating at frequencies up to about 15 KHz, the small core is also made of laminated silicon steel. Application ⟹ audio amplifier system.

4.) Radio frequency transformers

operating in the MHz frequency region have either an air core, a ferrite core or a dust core.

Ferrite is a ceramic material having magnetic properties similar to silicon steel but having high resistivity.

Dust cores consist of fine particles of carbonyl iron or permalloy (i.e. nickel and iron).

Applications $\Longrightarrow$ Radio and Television Receivers..

5) Transformer windings.

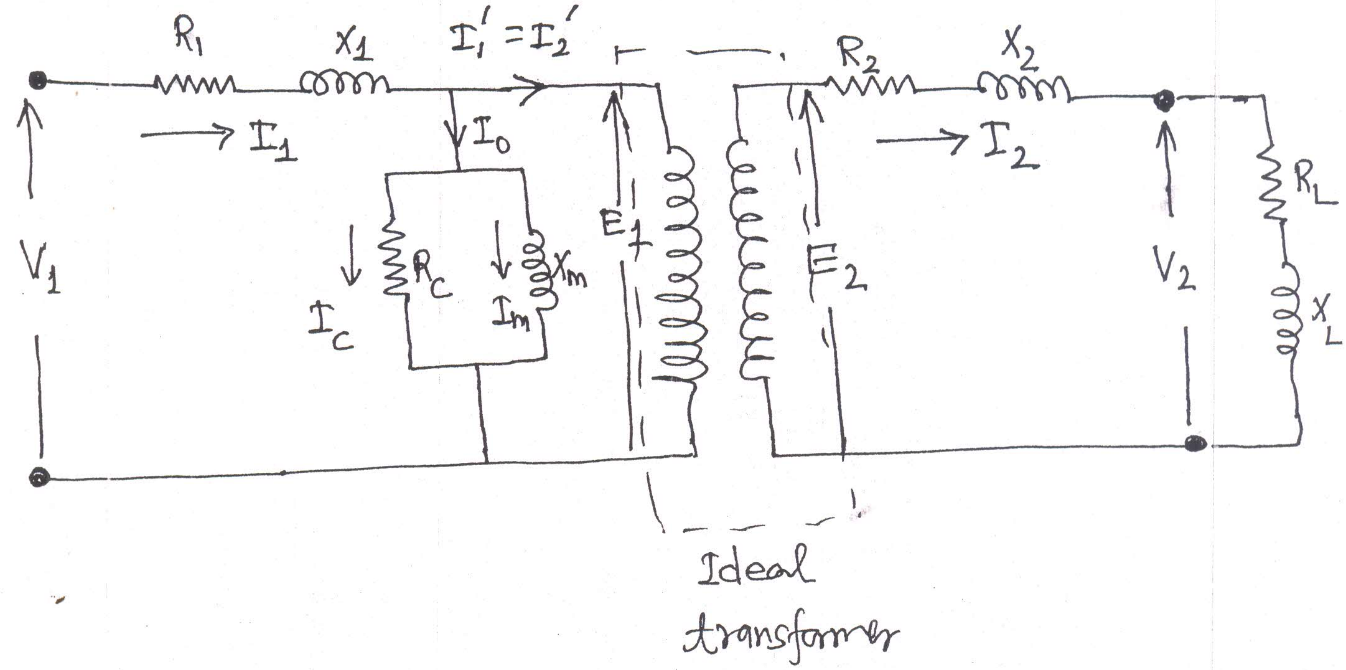
Usually of enamel-insulated copper or aluminium.

6) cooling

Air in small transformers.
oil in large transformers

EQUIVALENT CIRCUIT OF A TRANSFORMER

~~TRANSFORMER~~



Equivalent circuit of transformer

The equivalent circuit can be simplified by transferring the secondary resistances and reactances to the primary side in such a way that the ratio of E_2 to E_1 is not affected in magnitude or phase.

Resistance R_2 can be replaced by inserting an additional resistance R_2' in the primary circuit such that the power absorbed in R_2' when carrying current is equal to that in R_2 due to the secondary current, i.e.

$$I_2'^2 R_2' = I_2^2 R_2$$

$$\therefore R_2' = \frac{I_2^2}{I_2'^2} R_2 = \left(\frac{I_2}{I_2'}\right)^2 R_2$$

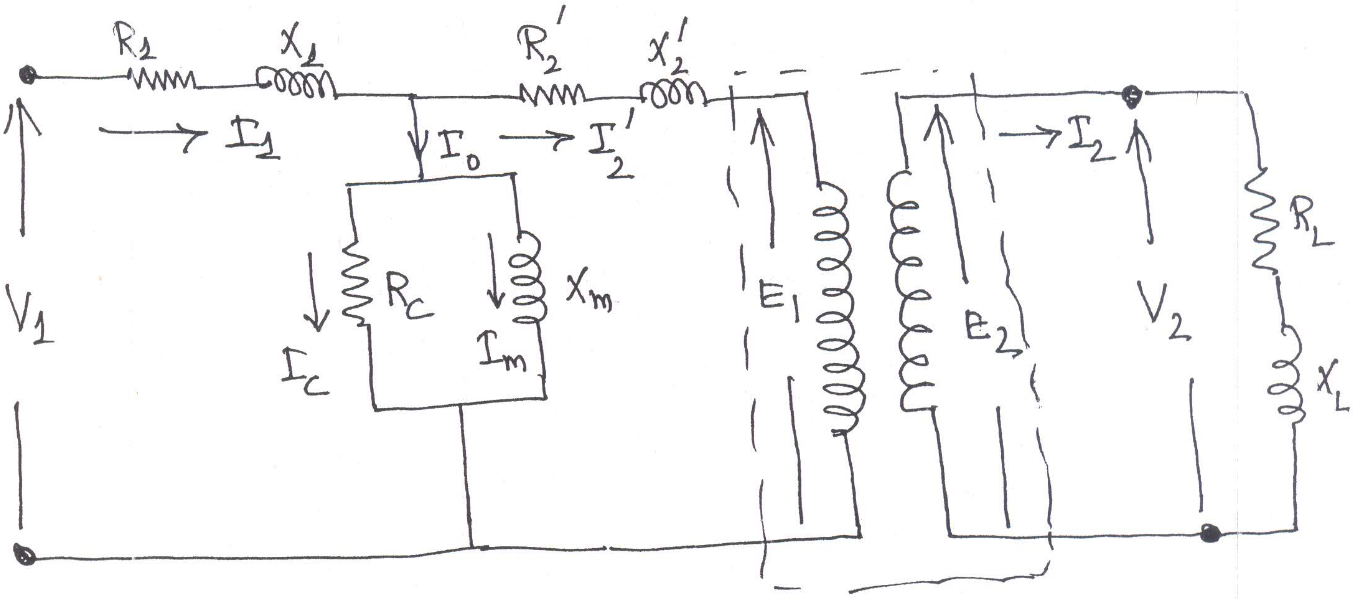
$$\therefore R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2 = K^2 R_2$$

$$\boxed{\therefore R_2' = K^2 R_2}$$

By similar reasoning

$$\boxed{X_2' = K^2 X_2}$$

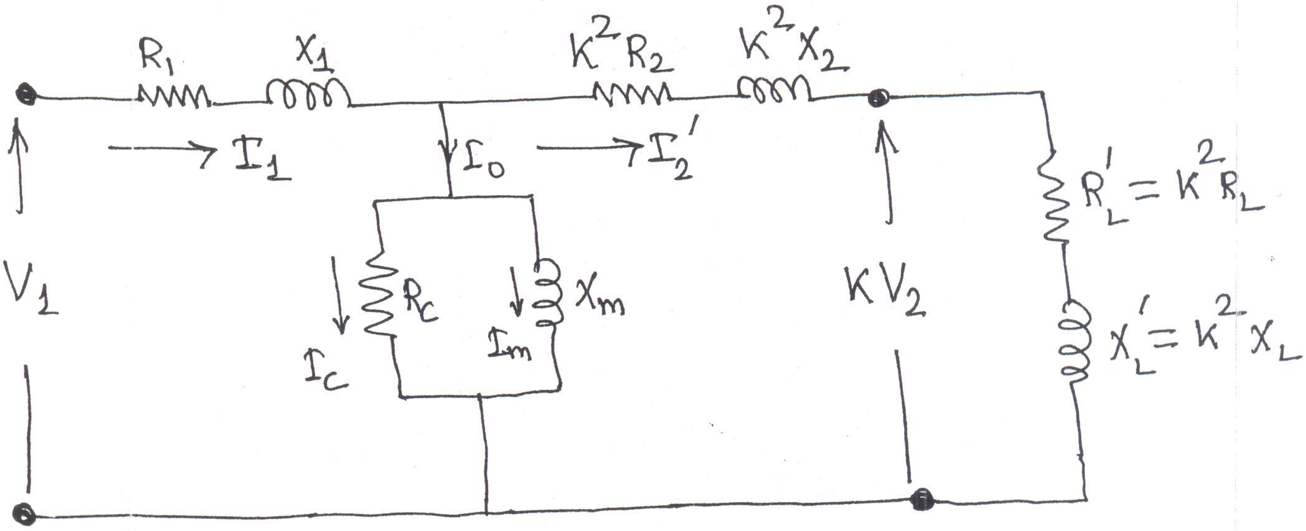
$R_2' = K^2 R_2 \; ; \quad X_2' = K^2 X_2$



Ideal transformer

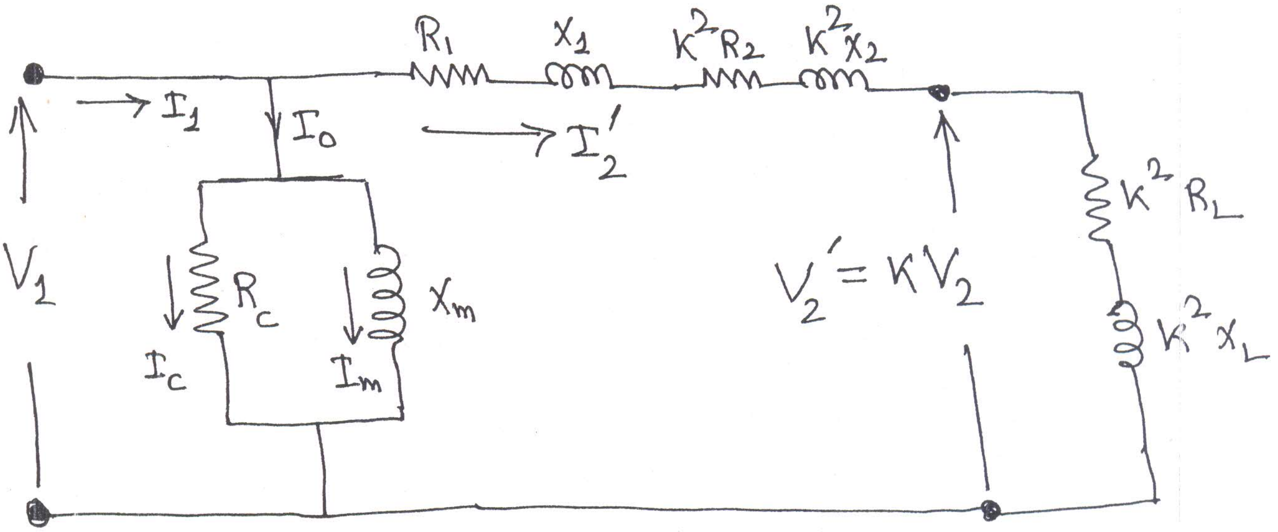
$$\frac{E_1}{E_2} = \frac{E_1}{V_2} = \frac{N_1}{N_2} = K$$

$$\therefore \boxed{E_1 = K V_2}$$



Equivalent circuit with all secondary impedances transferred to primary.

No-load current $I_0 \Rightarrow$ 3% to 5% of full-load current



Approximate Equivalent circuit Referred to Primary

$$\left.\begin{aligned} R_{e_1} &= (R_1 + K^2 R_2) \\ X_{e_1} &= (X_1 + K^2 X_2) \end{aligned}\right\} \text{Circuit parameters Referred to Primary}$$

$$\left.\begin{aligned} R'_{L_1} &= K^2 R_L \\ X'_{L_1} &= K^2 X_L \end{aligned}\right\} \text{Load parameters Referred to Primary.}$$

Ex-6

A single-phase transformer has 2000 turns on the primary and 800 turns on the secondary. Its no-load current is 5 Amp at a power factor of 0.20 lagging. Assuming the volt-drop in the windings is negligible, determine the primary current and power factor when the secondary current is 100 Amp at a power factor of 0.85 lagging.

Soln.

$I_2' N_1 = I_2 N_2$

$\therefore I_2' = \frac{N_2}{N_1} \cdot I_2$

$I_2 = 100$ Amp, ~~0.85~~ at 0.85 pf lagging

$\therefore I_2' = \frac{800}{2000} \times 100 = 40$ Amp, at 0.85 pf lagging.

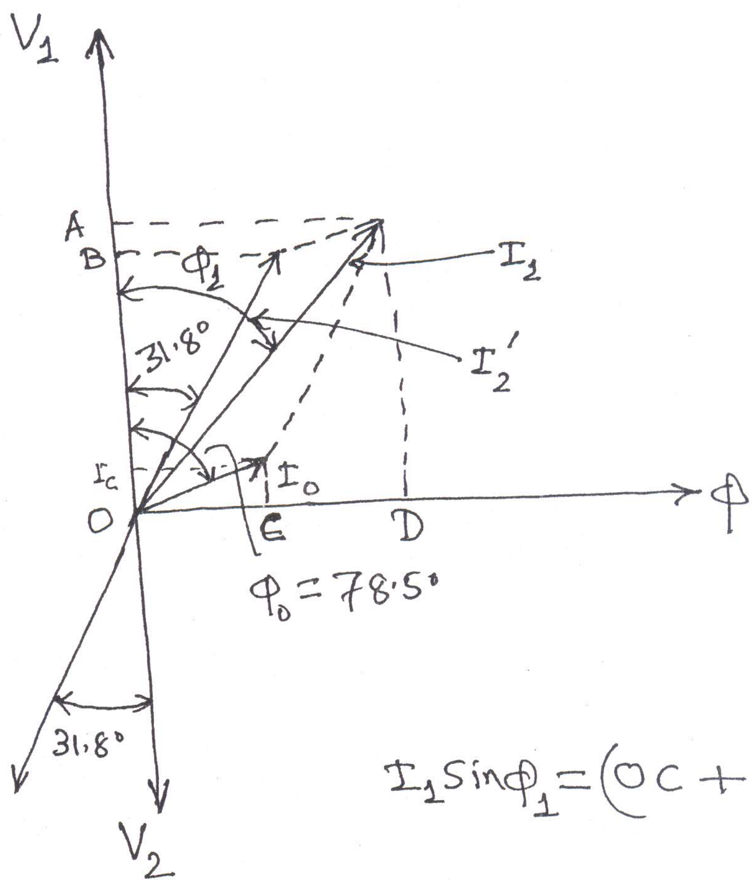
Secondary pf $= 0.85$

$\cos\phi_2 = 0.85$

$\therefore \phi_2 = 31.8°$

$I_0 = 5$ Amp , $\cos\phi_0 = 0.20$

$\therefore \phi_0 = 78.5°$



$I_1 \cos\phi_1 = (OB + BA)$

$I_1 \sin\phi_1 = (OC + CD)$

$\therefore I_1 \cos\phi_1 = I_2' \cos(31.8°) + I_0 \cos\phi_0 = 40 \times 0.85 + 5 \times 0.2$

$\therefore I_1 \cos\phi_1 = 35.0$

$I_1 \sin\phi_1 = I_0 \sin\phi_0 + I_2' \sin(31.8°) = 5\sin(78.5°) + 40\sin(31.8°)$

$\therefore I_1 \sin\phi_1 = 25.98$

$\therefore I_1 = \sqrt{(35)^2 + (25.98)^2} = 43.6$ Amp.

$\phi_1 = 36.6°$, $\cos\phi_1 = 0.80$

Ex-7 :

A 2200/220 Volt, single-phase, 50 Hz transformer has resistance of 1.25 Ω and reactance of 4 Ω in the high-voltage winding and 0.04 Ω resistance and 0.15 Ω reactance in the low-voltage winding.

Calculate (a) the equivalent resistance and reactance of low-voltage side in terms of high-voltage side.

(b) equivalent resistance and the reactance of high-voltage side in terms of low-voltage side.

Soln.

$$R_1 = 1.25\,\Omega, \quad X_1 = 4\,\Omega$$

$$R_2 = 0.04\,\Omega, \quad X_2 = 0.15\,\Omega$$

$$K = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{2200}{220} = 10$$

(a) $R_2' = K^2 R_2 = (10)^2 \times 0.04 = 4\,\Omega$

$X_2' = K^2 X_2 = (10)^2 \times 0.15 = 15\,\Omega$

In terms of high-voltage side

$R_{e1} = R_1 + R_2' = (1.25 + 4) = 5.25\,\Omega$

$X_{e1} = X_1 + X_2' = (4 + 15) = 19\,\Omega.$

(b) $R_1' = \frac{R_1}{K^2} = \frac{1.25}{(10)^2} = 0.0125\,\Omega$

$X_1' = \frac{X_1}{K^2} = \frac{4}{(10)^2} = 0.04\,\Omega$

In terms of low voltage side.

$R_{e2} = R_2 + R_1' = (0.04 + 0.0125) = 0.0525\,\Omega$

$X_{e2} = X_2 + X_1' = (0.15 + 0.04) = 0.19\,\Omega.$

Ex-8

Fig. shows the equivalent circuit of a single-phase transformer referred to low-voltage side of 200 Volt. calculate
(a) Terminal voltage on load (high-voltage side)
(b) low-voltage current (c) efficiency.

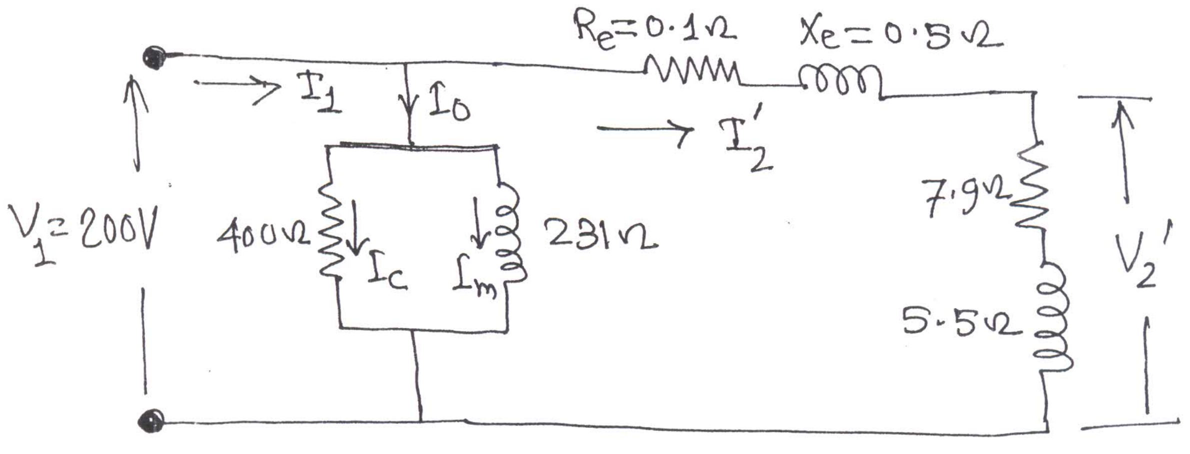


Fig. of Ex-8

Soln.

$$I_m = \frac{200}{231} = 0.866 \text{ Amp}$$

$$I_c = \frac{200}{400} = 0.50 \text{ Amp}$$

$$\therefore I_0 = I_c - jI_m = (0.5 - j0.866) \text{ Amp}$$

$$\therefore I_0 = 1\angle -60^\circ \text{ Amp.}$$

$$R_{total} = R_e + R_L = (0.10 + 7.9) = 8\,\Omega$$

$$X_{total} = X_e + X_L = (0.5 + 5.5) = 6\,\Omega$$

Impedance $Z = (8 + j6)\,\Omega.$

$$I_2' = \frac{200\angle 0^\circ}{(8+j6)} = 20\angle -36.9^\circ \text{ Amp.}$$

(a) $$I_1 = I_2' + I_0 = 20\angle -36.9^\circ + 1\angle -60^\circ$$

$$\therefore I_1 = 20.9\angle -38^\circ \text{ Amp.}$$

(b) $$V_2' = V_1 - I_2'(R_e + jX_e) = 200\angle 0^\circ - 20\angle -36.9^\circ \times (0.1 + j0.5)$$

$$= (192.4 - j6.8) = 192.5\angle -2^\circ$$

$$\therefore V_2' = 192.5 \text{ Volt}$$

$$\therefore KV_2 = 192.5$$

$$\therefore V_2 = 1925 \text{ Volts}$$

$V_2' = KV_2$	$\frac{N_2}{N_1} = 10$
$K = \frac{1}{10}$	$K = \frac{1}{10}$

(c) output $= V_2' I_2' \cos\phi_2'$

$= 192.5 \times 20 \times \cos(36.9° - 2°)$

$= 3160$ watt.

$|I_2'|^2 R_L$

$= (20)^2 \times 7.9$

$= 3160$ watt

Input $= V_1 I_1 \cos\phi_1$

$= 200 \times 20.9 \times \cos(38°)$

$= 3300$ watt.

efficiency $\eta = \dfrac{\text{output}}{\text{Input}} = \dfrac{3160}{3300}$

$\therefore \eta = 95.75\%$

Ex-9:

A 500 KVA, 2200/500 Volt, 50 Hz, single-phase transformer has 10 percent impedance. It has resistance of 0.01 Ω. Find the impedance, percentage resistance and reactance.

Soln.

I_2^{fl} = full-load current $= \dfrac{500 \times 1000}{500} = 1000$ Amp.

Base impedance,

$$Z_B = \frac{V_2}{I_2^{fl}} = \frac{500}{1000}\,\Omega = \frac{1}{2}\,\Omega$$

$\therefore$ Let impedane $= Z$

$\therefore$ percent impedance $= \dfrac{Z}{Z_B} = \dfrac{Z}{(1/2)} = 2Z$

$$\therefore\ 2Z = \frac{10}{100} = 0.1$$

$$\therefore\ Z = 0.05\,\Omega$$

Similarly,

percent resistance $= \dfrac{R}{Z_B} = \dfrac{0.01}{(1/2)}$

$= 0.02 = 2\%$

$$\therefore\ X = \sqrt{(0.05)^2 - (0.01)^2} = 0.049\,\Omega$$

Percent reactance $= \dfrac{X}{Z_B} = \dfrac{0.049}{(1/2)}$

$= 9.8\%$

LOSSES AND EFFICIENCY

1) **Core Loss :** These are hysteresis and eddy-current losses. core-loss is constant for a transformer operated at constant Voltage and frequency. ~~as are all power~~

2) **Copper-Loss (I^2R-Loss) :** This loss occurs in winding resistances when the transformer carries the load current. Copper loss varies as the square of the loading expressed as a ratio of the full-load.

3) **Load (stray)-Loss :** It largely results from leakage fields inducing eddy-currents in the tank wall, and conductors.

4) **Dielectric-Loss :** The seat of this loss is in the insulating materials, particularly in oil and solid insulations.

Major Losses: i) Core loss (or Iron Loss) $\Rightarrow W_i$ = constant loss

ii) Copper loss $\Rightarrow$ Variable-loss

Copper loss in the two windings are:

$$\text{Cu-loss} = (I_2')^2 R_1 + (I_2)^2 R_2$$

$$\therefore \cancel{= (I_2)^2 R_{e2}}$$

$$\therefore = \left\{ R_2 + \left(\frac{I_2'}{I_2}\right)^2 R_1 \right\} (I_2)^2$$

$$\therefore = \left\{ R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1 \right\} (I_2)^2$$

$$\therefore = \left(R_2 + \frac{R_1}{K^2} \right) (I_2)^2$$

$$\therefore \boxed{\text{Cu-loss} = R_{e2} (I_2)^2} \quad \text{----} \quad (1)$$

Where

$$R_{e2} = \left(R_2 + \frac{R_1}{K^2} \right) = \text{resistance referred to Secondary.}$$

Let

$$x = \frac{I_2}{I_{2,fl}}$$

Where $I_{2,fl}$ = full-load current (secondary side).

$$\therefore \boxed{I_2 = x \cdot I_{2,fl}} \quad --- (2)$$

From Eqns. (1) & (2)

$$\therefore \text{cu-loss} = R_{e2} (x \cdot I_{2,fl})^2$$

$$\therefore \text{cu-loss} = x^2 I_{2,fl}^2 \cdot R_{e2}$$

$$\therefore \cancel{\text{cu-loss} = x^2 I_{2,fl}^2}$$

$$\boxed{\therefore \text{cu-loss} = x^2 W_c} \quad --- (3)$$

Where $W_c = I_{2,fl}^2 R_{e2}$ = full-load cu-loss

output of the transformer

$$= V_2 I_2 \cos\phi_2$$

$$= x V_2 I_{2,fl} \cos\theta_2 \qquad [\because I_2 = x \cdot I_{2,fl}]$$

$$\boxed{\therefore \text{output} = x . P_{fl} \quad --- (4)}$$

where

$$P_{fl} = V_2 I_{2,fl} \cos\phi_2 = \text{full-load output}$$

$$\text{Input} = \text{output} + \text{losses}$$

$$\therefore \text{Input} = \text{output} + \text{core loss} + \text{cu-loss}$$

$$\boxed{\therefore \text{Input} = \left(x . P_{fl} + W_i + x^2 W_c\right) \quad --- (5)}$$

Efficiency

$$\eta = \frac{\text{output}}{\text{input}}$$

$$\therefore \eta = \frac{x . P_{fl}}{(x P_{fl} + W_i + x^2 W_c)} \quad --- (6)$$

For maximum efficiency

$$\frac{d\eta}{dx} = 0$$

$$\boxed{\therefore \quad W_i = x^2 W_c \quad --- (7)}$$

For maximum efficiency,

$$\boxed{\text{Core-loss} = \text{cu-loss}}$$

$$\boxed{\therefore \eta_{max} = \frac{x P_{fl}}{(x P_{fl} + 2 W_i)}}$$

ALL DAY EFFICIENCY OF A TRANSFORMER

All day efficiency

$$= \frac{\text{output of transformer in kWh in 24 hrs}}{\text{Input to transformer in kWh in 24 hrs.}}$$

Iron loss $\Rightarrow$ constant and present during all the 24 hrs.

cu-loss $\Rightarrow$ vary as square of the load current from hour to hour.

VOLTAGE REGULATION

The secondary terminal voltage changes with the change in the load due to the change in voltage drops across the winding resistances and leakage reactances.

The voltage regulation of transformer is defined as the net change in the secondary terminal voltage ~~from~~ from no-load to full-load, expressed as a percentage of its rated voltage.

$$\% \text{Voltage regulation} = \left(\frac{V_{2,0} - V_{2,fl}}{V_{2,fl}} \right) \times 100$$

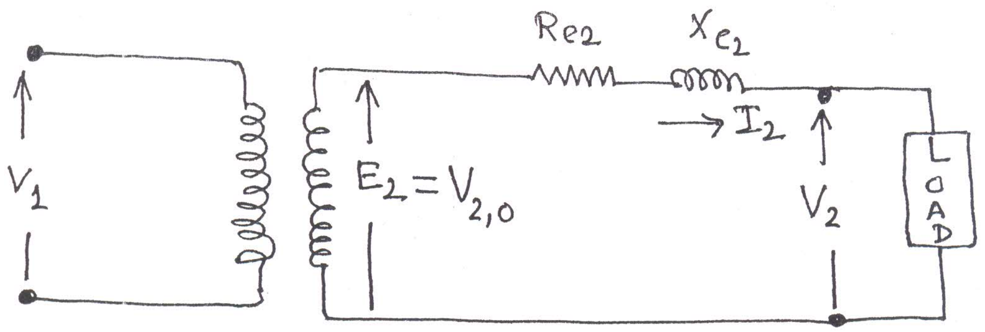
Where

$V_{2,0}$ = Secondary voltage when load is thrown off. (i.e., $V_{2,0} = E_2$)

$V_{2,fl}$ = Full-load secondary voltage (It is assumed to be adjusted to the rated secondary voltage).

Voltage regulation is a figure of merit of a transformer. For an ideal transformer voltage regulation is zero.

Smaller is the voltage regulation, better is the operations of the transformer.

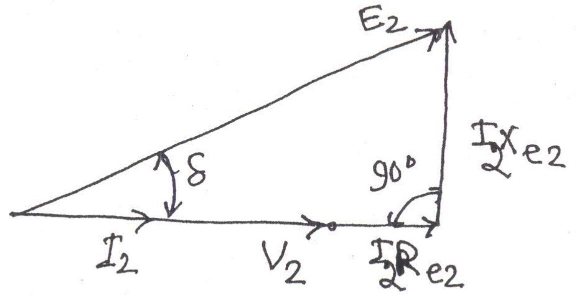


Approximate Equivalent circuit referred to Secondary (shunt branches are neglected)

$$R_{e_2} = \left(R_2 + \frac{R_1}{K^2}\right)$$

$$X_{e_2} = \left(X_2 + \frac{X_1}{K^2}\right)$$

Case-1: Unity power Factor Load



Phasor diagram of transformer on load at unity power factor (i.e., V_2 and I_2 are in phase)

$\therefore \cancel{E_2^2 = (V_2 + I_2R)^2 + (I_2X)^2}$

$$\boxed{\therefore E_2^2 = (V_2 + I_2 R_{e2})^2 + (I_2 X_{e2})^2}$$

As approximation

$E_2 \cos\delta = V_2 + I_2 R_{e2}$

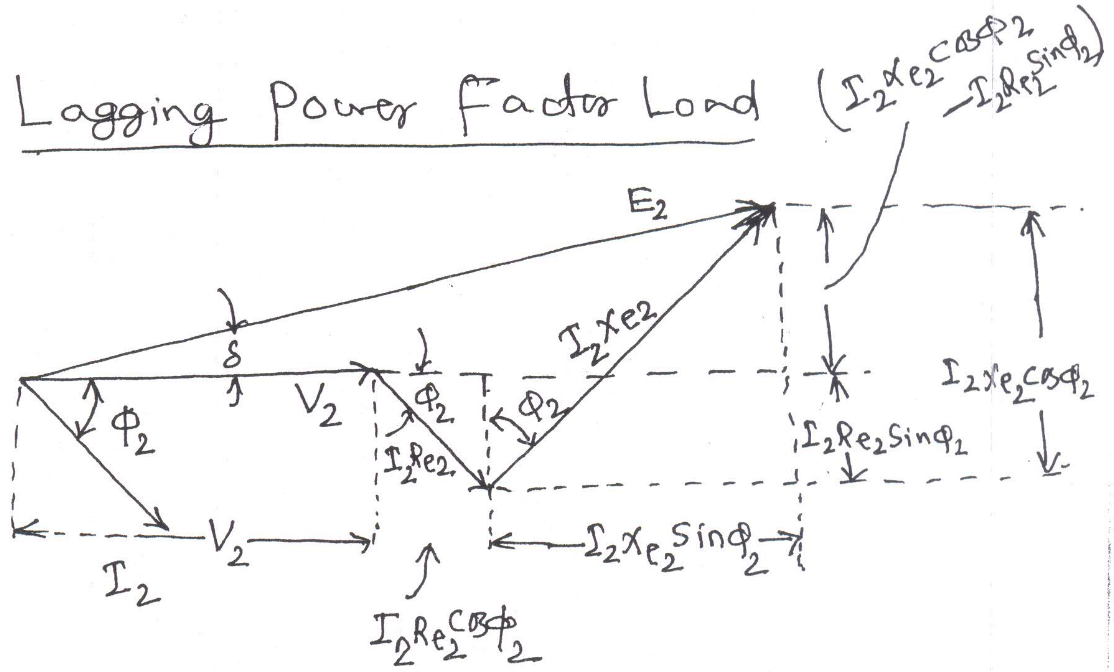
$\delta \simeq 0, \quad \therefore \cos\delta \simeq 1$

$\therefore E_2 \simeq V_2 + I_2 R_{e2}$

$$\therefore \frac{E_2 - V_2}{V_2} = \frac{I_2 R_{e2}}{V_2}$$

$$\boxed{\therefore \frac{V_{2,0} - V_2}{V_2} = \frac{I_2 R_{e2}}{V_2}}$$

Case-2: Lagging Power Factor Load



$$\boxed{E_2^2 = \left(V_2 + I_2 R_{e2}\cos\phi_2 + I_2 X_{e2}\sin\phi_2\right)^2 + \left(I_2 X_{e2}\cos\phi_2 - I_2 R_{e2}\sin\phi_2\right)^2}$$

As an approximation

$$E_2 \cos\delta = V_2 + I_2 R_{e2}\cos\phi_2 + I_2 X_{e2}\sin\phi_2$$

$$\delta \simeq 0, \quad \cos\delta \simeq 1.0$$

$$\therefore \quad E_2 = V_2 + I_2 R_{e2}\cos\phi_2 + I_2 X_{e2}\sin\phi_2$$

$$\therefore \quad \frac{E_2 - V_2}{V_2} = \frac{\left(I_2 R_{e2}\cos\phi_2 + I_2 X_{e2}\sin\phi_2\right)}{V_2}$$

$$\boxed{\therefore \quad \frac{V_{2,0} - V_2}{V_2} = \frac{I_2\left(R_{e2}\cos\phi_2 + X_{e2}\sin\phi_2\right)}{V_2}}$$

Base Impedance (secondary side) $= \dfrac{V_2}{I_2}$

$$\therefore \quad Z_{B,2} = \frac{V_2}{I_2}$$

$\therefore$ ~~R_{e2}~~

$$\therefore \frac{V_{2,0} - V_2}{V_2} = \frac{(R_{e2}\cos\phi_2 + X_{e2}\sin\phi_2)}{Z_{B,2}}$$

$$\therefore \frac{V_{2,0} - V_2}{V_2} = \left(\frac{R_{e2}}{Z_{B,2}}\right)\cos\phi_2 + \left(\frac{X_{e2}}{Z_{B,2}}\right)\sin\phi_2$$

Let us define

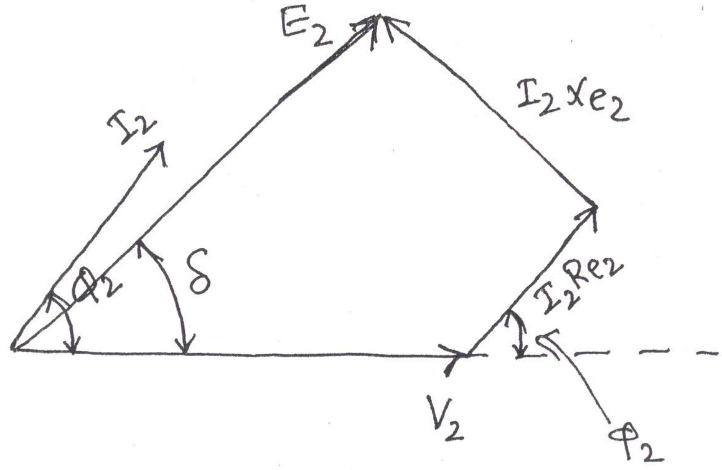
$$R_{e2}\,(pu) = \frac{R_{e2}}{Z_{B,2}} \; ; \quad X_{e2}\,(pu) = \frac{X_{e2}}{Z_{B,2}}$$

$pu \Rightarrow$ per unit

$$\boxed{\therefore \frac{V_{2,0} - V_2}{V_2} = \left(R_{e2}(pu)\cos\phi_2 + X_{e2}(pu)\sin\phi_2\right)}$$

Case-3: **Leading power Factor Load**

NEXT PAGE



$$E_2 \cos\delta = V_2 + I_2 Re_2 \cos\phi_2 - I_2 Xe_2 \sin\phi_2$$

$\delta \simeq 0$; $\cos\delta \simeq 1.0$

$$\therefore \frac{E_2 - V_2}{V_2} = \frac{I_2}{V_2}\left(Re_2 \cos\phi_2 - Xe_2 \sin\phi_2\right)$$

$$\boxed{\therefore \frac{V_{2,0} - V_2}{V_2} = \frac{I_2}{V_2}\left(Re_2 \cos\phi_2 - Xe_2 \sin\phi_2\right)}$$

$\therefore$

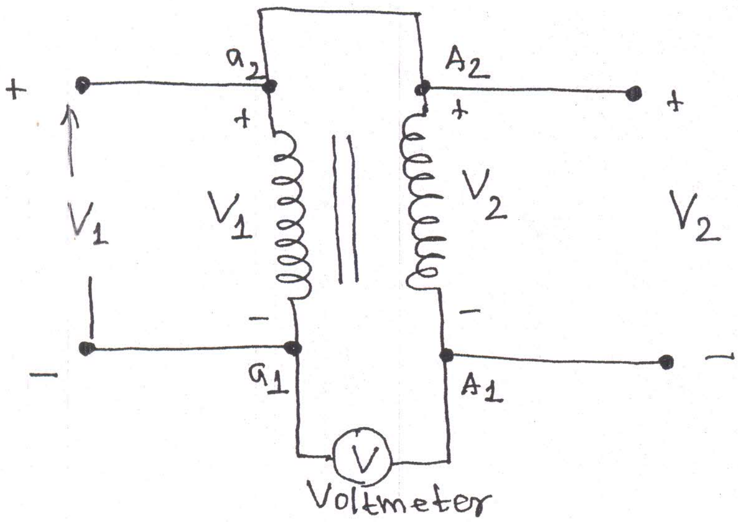
OR

$$\frac{V_{20} - V_2}{V_2} = \left(Re_2(pu) \cos\phi_2 - Xe_2(pu) \sin\phi_2\right)$$

POLARITY TEST

$a_1 - a_2 \Rightarrow$ LV winding

$A_1 - A_2 \Rightarrow$ HV winding



If $V = V_1 \sim V_2$,

Then polarities of the winding are as marked on the diagram.

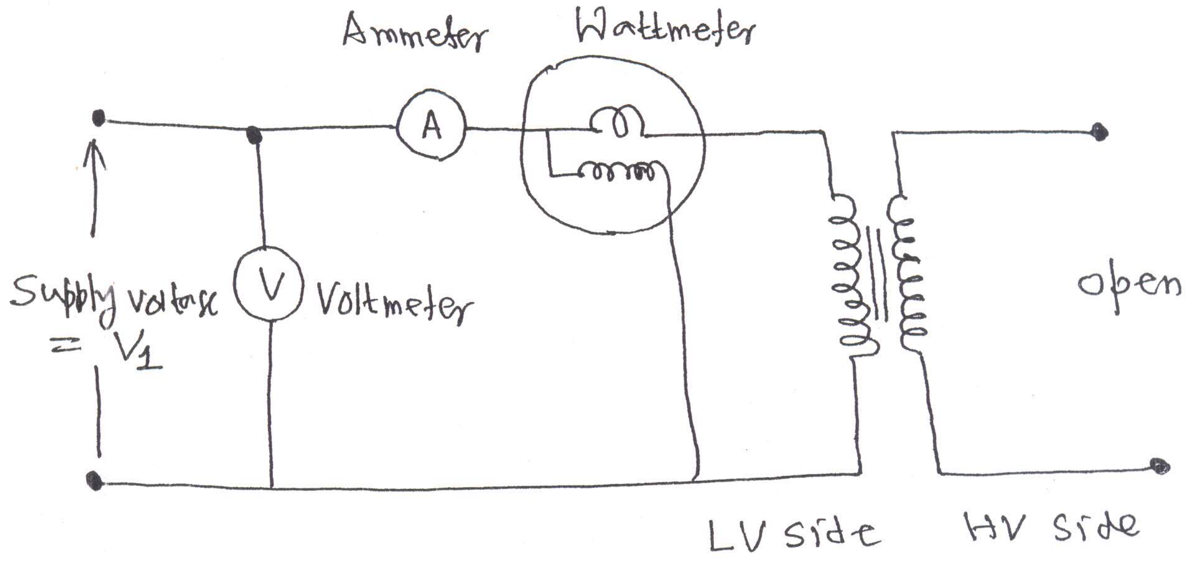
If $V = (V_1 + V_2)$

~~The Po~~

Then the polarity markings of one of the windings must be interchanged.

OPEN CIRCUIT TEST TO OBTAIN R_c & X_m

(SHUNT PARAMETERS)



Supply is given to LV side, HV side is open

~~Ef~~ For 110/220 Volt transformer,

LV side $\Rightarrow$ 110 volt

HV side $\Rightarrow$ 220 volt

Then on LV side, ~~applied~~ 110 V supply is given.

Wattmeter reading gives $\Rightarrow$ Core or Iron loss $\Rightarrow W_i$

Ammeter reading gives $\Rightarrow$ No-load current $\Rightarrow I_0$

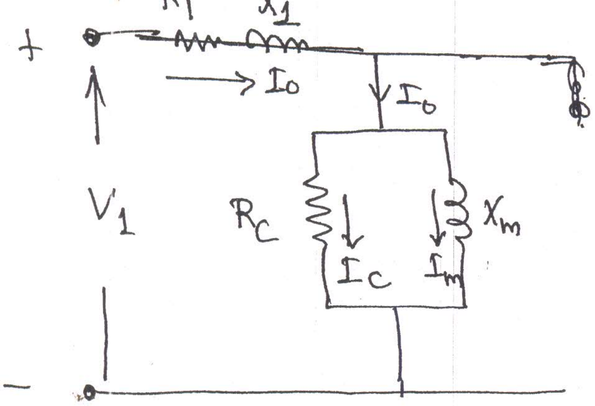
Voltmeter reading gives $\Rightarrow$ LV side applied voltage $\Rightarrow V_1$

$$\therefore \quad V_1 I_0 \cos\phi_0 = W_i$$

$$\therefore \cos\phi_0 = \frac{W_i}{V_1 I_0} = \text{no-load power factor.}$$

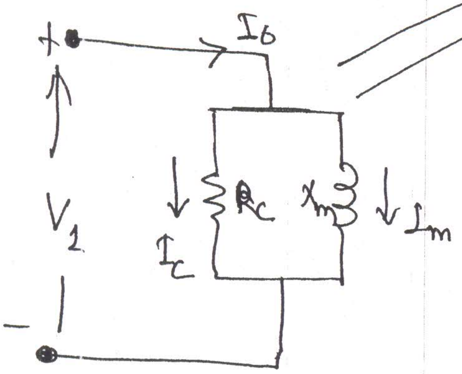
$$\therefore I_c = I_0 \cos\phi_0$$

$$I_m = I_0 \sin\phi_0$$



$$\boxed{\begin{aligned} \therefore R_c &= \frac{V_1}{I_c} = \frac{V_1}{I_0 \cos\phi_0} \\ X_m &= \frac{V_1}{I_m} = \frac{V_1}{I_0 \sin\phi_0} \end{aligned}}$$

Equivalent circuit as seen on open - circuit



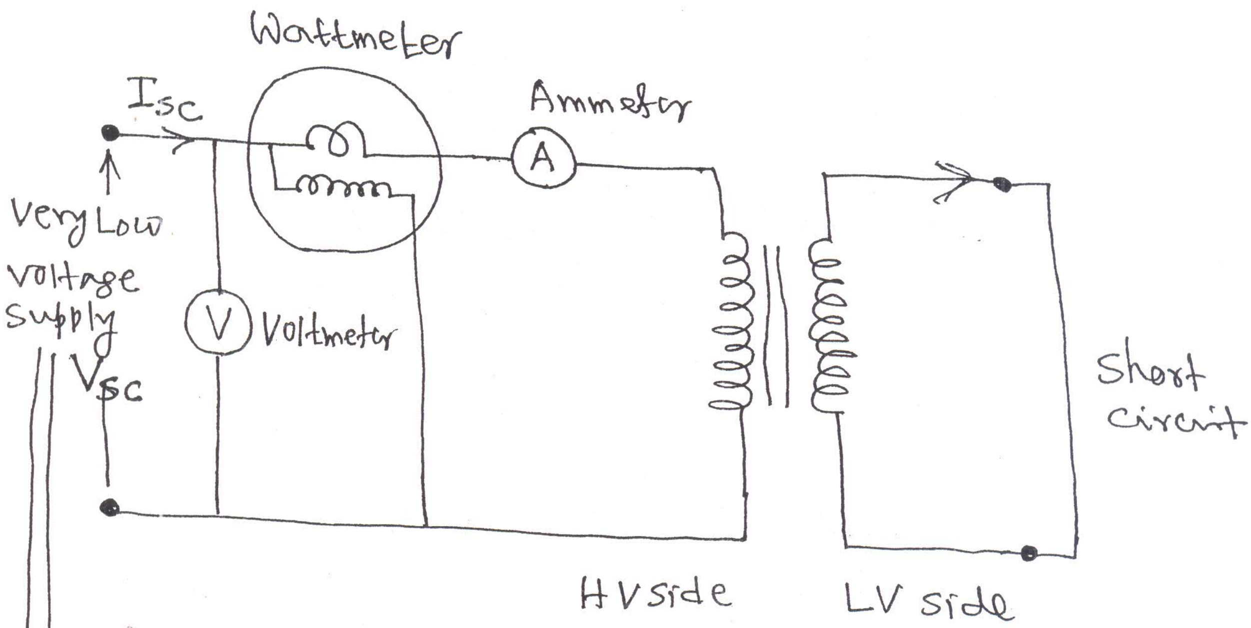
Approximate circuit

Voltage drop across $(R_1 + jX_1)$ is negligible.

note:

$I_0^2 R_1 \Rightarrow$ no-load Cu-loss is very very small and negligible. Hence Wattmeter reading gives only the Iron or Core loss.

5.9 SHORT-CIRCUIT TEST TO OBTAIN SERIES PARAMETERS (R & X)



SC-Test on transformer

5-8 % of Rated voltage ⇒ Transformer resistance and leakage reactance are very small. Hence 5-8 % of rated voltage is sufficient to circulate the full-load current.

⇓

As a result the exciting current under SC condition 0.1 to 0.5 % of full-load current.

110/220 Volt ~~transformer~~, 2.2 KVA transformer

HV = 220

$\therefore V_{sc} = 5-8\%$ of 220 Volt.

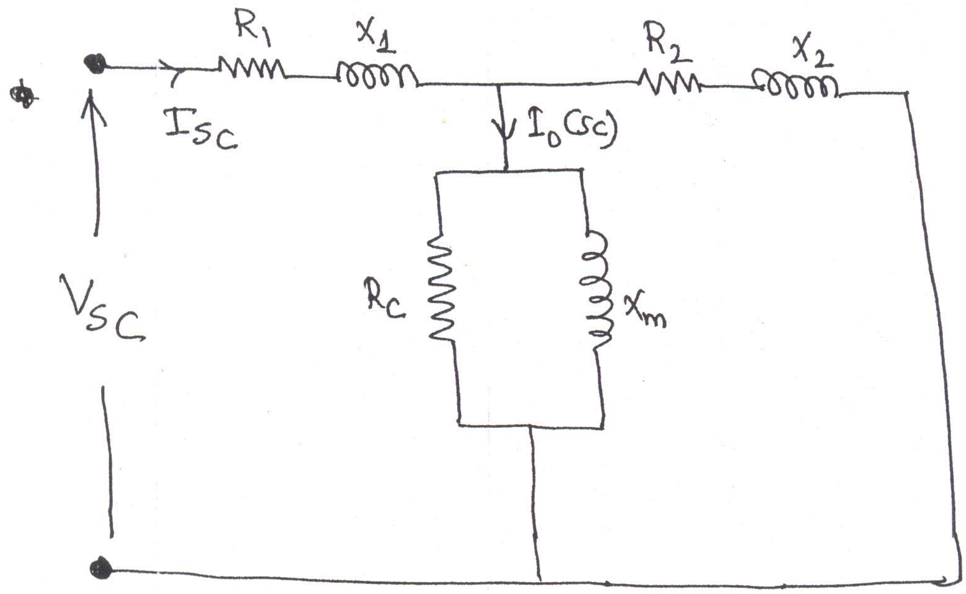
For example $V_{sc} = 5\%$ of 220 Volt

$= 11$ Volt.

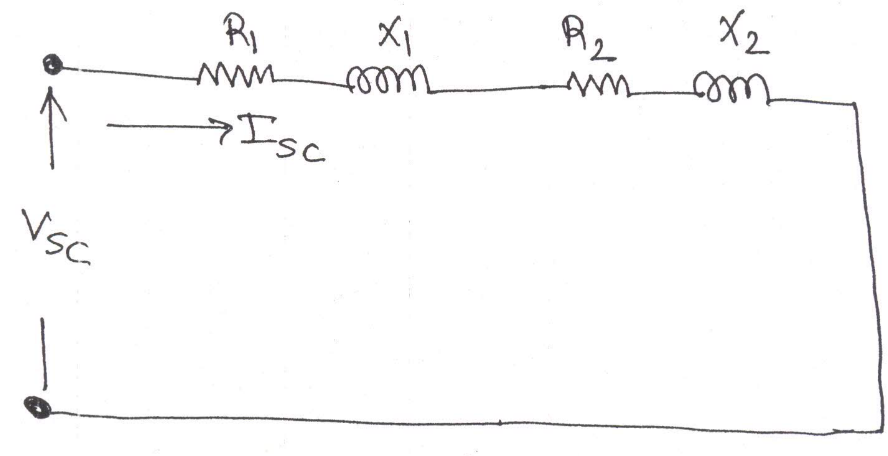
$I_{sc} = I_{fl}$

$= \frac{2.2 \times 1000}{220}$

$= 10$ Amp.



Equivalent circuit under SC condition



Approximate equivalent circuit under SC- condition.

$$\left.\begin{array}{ll} \text{Voltage} & = V_{sc} \\ \text{Current} & = I_{sc} \Rightarrow \text{Full-load current,} \\ \text{Power} & = W_{sc} \Rightarrow \text{copper-loss.} \end{array}\right\}$$

$$Z = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2 + x^2}$$

$$\boxed{\begin{aligned} R &= R_1 + R_2 \\ X &= X_1 + X_2 \end{aligned}}$$

$$(I_{sc})^2 R = W_{sc}$$

$$\boxed{\therefore \ R = \frac{W_{sc}}{(I_{sc})^2}}$$

$$\boxed{X = \sqrt{Z^2 - R^2}}$$

These values are referred to HV side.

Resistances could be separated out by making DC measurements on the primary and Secondary and duly correcting these for AC values.

Reactances cannot be separated as such. Where required, these could be equally apportioned to the primary and Secondary, i.e.,

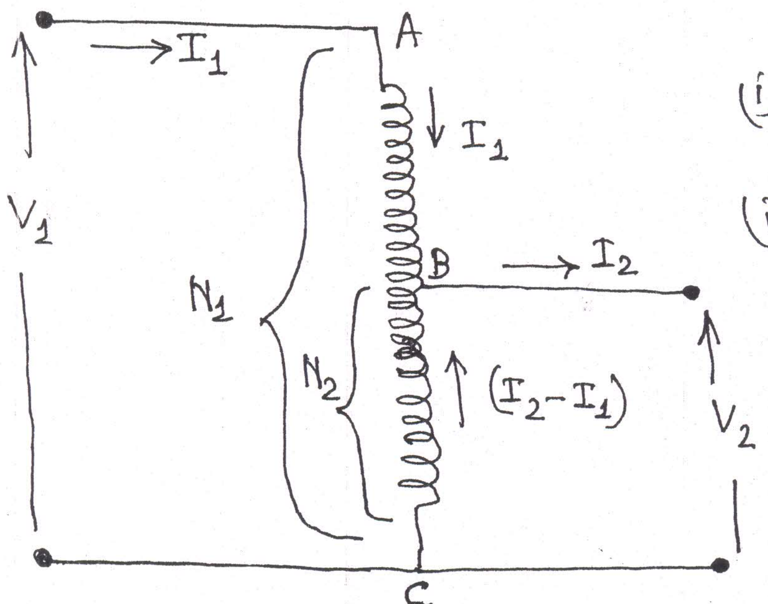
$X_1 = X_2$ (Referred to any side)

This is sufficiently accurate for a well-designed transformer.

AUTOTRANSFORMERS

For two-winding transformers, the windings are electrically isolated. In a two winding transformer, all VA is transferred magnetically.

When the primary and secondary windings are electrically connected so that part of the winding is common to the both, the transformer is known as autotransformer.



Autotransformer.

Applications

(i) Induction motor starters

(ii) Variable voltage power supply (low voltage and current levels)

Autotransformer has lower reactance, lower losses, smaller exciting current and better voltage regulation compared to its two-winding counterpart.

Primary turns $\Rightarrow N_1$ (winding section – AC)

With N_2 turns tapped for a lower voltage secondary, (winding section – BC)

Two winding voltage and turn-ratio is

$$K = \frac{V_1 - V_2}{V_2} = \frac{N_1 - N_2}{N_2} \qquad [\because N_1 > N_2]$$

As an autotransformer, its voltage and turn-ratio is,

$$K' = \frac{V_1}{V_2} = \frac{N_1}{N_2} > 1$$

$$\therefore K' = \frac{(V_1 - V_2) + V_2}{V_2} = \left(\frac{V_1 - V_2}{V_2}\right) + 1$$

$$\boxed{\therefore K' = K + 1}$$

Let us compare VA ratings of the two.

As a two winding transformer

$$(VA)_{TW} = (V_1 - V_2) I_1 = (I_2 - I_1) V_2$$

When used as an auto-transformer

$$(VA)_{Auto} = V_1 I_1 = V_2 I_2$$

$$(VA)_{TW} = \left(\frac{V_1 - V_2}{V_1}\right)(V_1 I_1)$$

$$\therefore (VA)_{TW} = \left(1 - \frac{V_2}{V_1}\right)(V_1 I_1)$$

$$\therefore (VA)_{TW} = \left(1 - \frac{N_2}{N_1}\right)(VA)_{Auto}$$

$$\therefore (VA)_{TW} = \left(\frac{N_1 - N_2}{N_2}\right) \cdot \left(\frac{N_2}{N_1}\right)(VA)_{Auto}$$

$$\therefore (VA)_{TW} = \frac{K}{K'}(VA)_{Auto}$$

$$\therefore (VA)_{Auto} = \left(\frac{K+1}{K}\right)(VA)_{TW}$$

$$\boxed{\therefore (VA)_{Auto} = \left(1 + \frac{1}{K}\right)(VA)_{TW}}$$

$$(VA)_{Auto} > (VA)_{TW}$$

EX-10

The parameters of approximate equivalent circuit of a 4 KVA, 200/400 Volt, 50 Hz, 1ϕ transformer are:

$R_{e1} = 0.15\,\Omega$, $X_{e1} = 0.37\,\Omega$

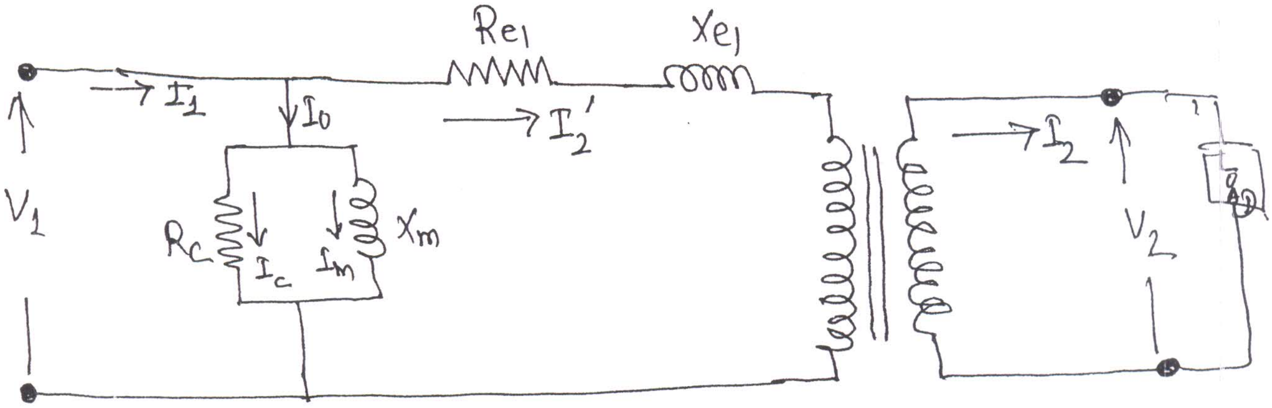
$R_c = 600\,\Omega$, $X_m = 300\,\Omega$.

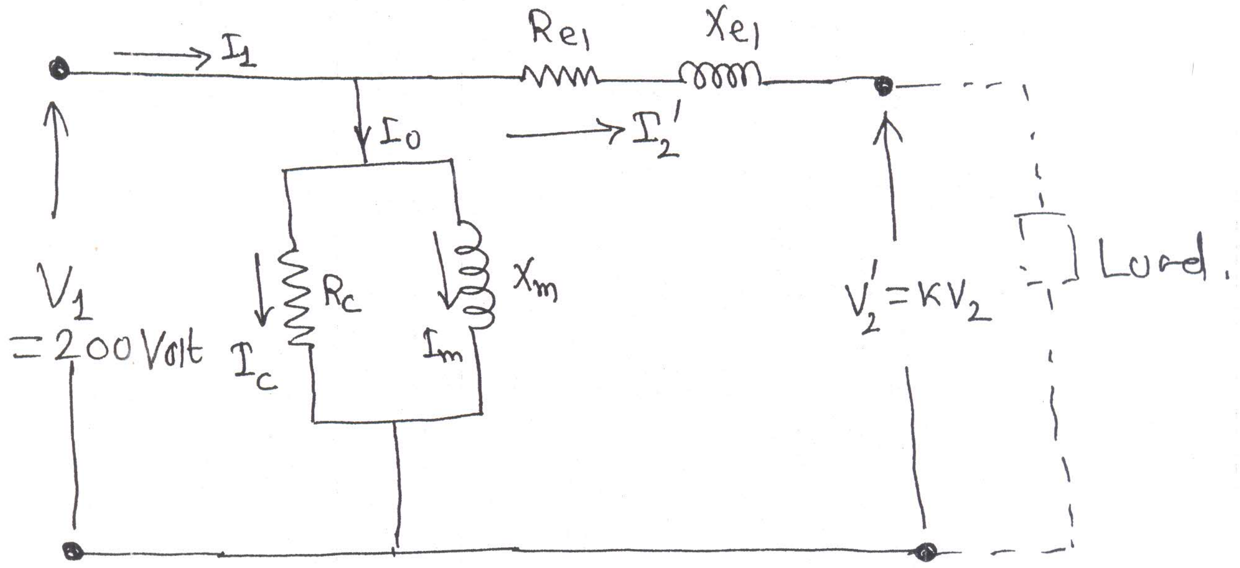
When rated voltage of 200V is applied to the primary, a current of 10 Amp, 0.8 pf lagging flows in the Secondary winding. Calculate

(a) current in the primary

(b) Terminal voltage at the secondary side.

Soln.





(a)

$$I_2 = 10 \text{ Amp at } 0.8 \text{ pf Lagging}$$

$$\therefore I_2' = I_2 \times \frac{N_2}{N_1} = 10 \times \frac{400}{200} = 20 \text{ Amp. at } 0.8 \text{ pf lagging.}$$

$$\therefore I_2' = (16 - j12) \text{ Amp.}$$

$$I_c = \frac{200}{600} = 0.33 \text{ Amp}$$

$$I_m = \frac{200}{300} = 0.67 \text{ Amp.}$$

$$\therefore I_0 = I_c - j\,I_m = (0.33 - j0.67) \text{ Amp.}$$

$$\therefore I_1 = I_0 + I_2' = (0.33 - j0.67) + (16 - j12)$$

$$\boxed{\therefore I_1 = 20.67 \angle -37.8^\circ \text{ Amp.}}$$

(b)

$$V_2' = V_1 - I_2' (R_{e1} + jX_{e1})$$

$$\therefore V_2' = 200\angle 0^\circ - (20\angle -36.87^\circ)(0.15 + j0.37)$$

$$\therefore V_2' = 193.2 \angle -1.2^\circ \text{ volt}$$

$$\therefore KV_2 = 193.2 \angle -1.2^\circ$$

$$\boxed{\therefore V_2 = 386.4 \angle -1.2^\circ \quad \text{volt}}$$

Ex-11

A transformer has its maximum efficiency of 0.98 at 15 KVA at unity power factor. During the day, it is loaded as follows:

$$\left.\begin{array}{l} \text{10 hr, 3 KW, pf} = 0.6 \\ \text{5 hr, 10 KW, pf} = 0.8 \\ \text{5 hr, 18 KW, pf} = 0.9 \\ \text{4 hr, — at no load} \end{array}\right\}$$

Find all-day efficiency

54 (58)

Soln

Maximum efficiency of the transformer $= 0.98$

Load at which maximum efficiency occurs $= 15\,KVA$ at unity power factor

At maximum efficiency,

Iron loss = cu-loss.

We know

$$\eta_{max} = \frac{x P_{fl}}{x P_{fl} + 2W_i} = \frac{\text{output}}{\text{output} + 2W_i}$$

$$\therefore \eta_{max} = \frac{15 \times 1000 \times 1}{15 \times 1000 \times 1 + 2W_i} = 0.98$$

$\therefore W_i = 153\,W = 0.153\,KW$,

$\therefore$ Cu Loss at 15 KVA Load $= W_i = 153\,W = 0.153\,KW$

Interval-1: 10 hrs, load $= \frac{3}{0.6} = 5.0\,KVA$

Cu-loss ~~at~~ 5 KVA load

$$= \left(\frac{5}{15}\right)^2 \times 0.153 = 0.017\,KW$$

~~Inter~~ Energy Loss $= 0.017 \times 10 = 0.17$ kWh

Interval – 2:

5 hrs, Load $= \frac{10}{0.8} = 12.5$ KVA

Cu-loss $= \left(\frac{12.5}{15}\right)^2 \times 0.153 =$ ~~104~~ 0.1042 KW

Energy Loss $= 5 \times 0.1042 = 0.521$ kWh

Interval – 3:

5 hrs, Load $= \frac{18}{0.9} = 20$ KVA

Cu-loss $= \left(\frac{20}{15}\right)^2 \times 0.153 = 0.272$ kW

Energy loss $= 5 \times 0.272 = 1.36$ kWh.

Interval – 4:

4 hrs, no-load,

Cu-loss $= 0.0$

Energy loss $= 0.0$

Energy Loss (due to iron loss) during the whole day

$= 0.153 \times 24 = 3.672$ kWh.

Energy Loss (due to cu-loss) during the whole day

$= (0.17 + 0.521 + 1.36)$

$= 2.051$ kWh.

Total Energy Loss $= (2.051 + 3.672)$ kWh.

Total output during the whole day

$= (3 \times 10 + 10 \times 5 + 18 \times 5)$ kWh

$= 170$ kWh.

$$\eta_{all\text{-}day} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$= \frac{170}{(170 + 2.051 + 3.672)}$$

$$= 96.7\%$$

$x = \frac{\text{kVA}'}{(\text{kVA})_{fl}}$

Ex-12 :

A 100 KVA, 440/220 Volt, 1φ, 50 Hz core type transformer has an efficiency of 98.5%, when supplying full-load at 0.8 pf lagging and an efficiency of 99%, when supplying half-load at unity power factor. Find the iron losses and copper losses at full-load.

Soln.

Iron loss $= W_i$

Full-load cu-loss $= W_c$ $[\because x = 1]$

$$\therefore \quad 0.985 = \frac{100 \times 10^3 \times 0.8}{100 \times 10^3 \times 0.8 + W_i + W_c}$$

$$\therefore \quad 0.985\, W_i + 0.985\, W_c = 1200 \quad \text{--(i)}$$

Output of transformer at half-load with unity pf

$$= \frac{1}{2} \times 100 \times 10^3 \times 1.0 = 50 \times 10^3\, W.$$

$$\therefore 0.99 = \frac{50\times10^3}{50\times10^3 + W_i + \left(\frac{1}{2}\right)^2 W_c}$$

$$\therefore 0.99 W_i + 0.2475 W_c = 500 \quad \text{---(ii)}$$

Solving eqns(i) & (ii), we get,

$$\left.\begin{aligned} W_i &= 267.3\,W \\ W_c &= 950.9\,W \end{aligned}\right\} \text{Ans.}$$

Ex-"13"

Calculate the voltage regulation of a transformer having cu-loss of 2% of the output and the reactance drop of 5% when power factor is

(a) 0.8 lagging (b) 0.8 leading

Ex-13 : A single-phase step-down transformer has a turn ratio of 3. The resistance and reactance of the primary winding are 1.2 Ω and 6 Ω and those of the secondary winding are 0.05 Ω and 0.03 Ω respectively. If the high voltage winding is supplied at 230 volt with low-voltage winding short-circuited, find

(i) current in the low voltage winding

(ii) copper loss in the transformer

(iii) power factor.

Soln.

turn ration $k = 3$.

HV side → primary
LV side → secondary

$R_1 = 1.2\,\Omega$; $X_1 = 6\,\Omega$

$R_2 = 0.05\,\Omega$; $X_2 = 0.3\,\Omega$ 0.03

Referred to HV side,

$$R_{e1} = R_1 + k^2 R_2 = 1.2 + (3)^2 \times 0.05 = 1.65\,\Omega$$

$$X_{e1} = X_1 + k^2 X_2 = 6 + (3)^2 \times 0.03 = \cancel{8.7}\ 6.27\,\Omega$$

$$Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{(1.65)^2 + (6.27)^2}$$

$$\therefore Z_{e1} = \cancel{8.855}\,\Omega \quad 6.48\,\Omega.$$

Current in the high-voltage winding when low-voltage winding is short-circuited,

$$I_{sc} = \frac{V_{sc}}{Z_{e1}} = \frac{230}{\cancel{8.855}\ 6.48} = \cancel{25.97}\ \text{Amp.}\ 35.47.$$

Neglecting I_0,

$$I_1 = I_2' = \cancel{25.97}\ 35.47\ \text{Amp.}$$

(i) Current in low-voltage winding,

$$I_2 = KI_2' = 35.47 \times 3 = \cancel{77.91\ \text{Amp.}}\ 106.4$$

(ii) Total cu-loss

$$W_{sc} = I_1^2 R_{e1} = (\cancel{25.97}\ 35.47)^2 \times 1.65 = \cancel{1112.8\ \text{Watt.}}\ 2076\ \text{W.}$$

(iii)

$$V_{sc} I_{sc} \cos\phi_{sc} = W_{sc}$$

$$\therefore 230 \times \cancel{25.97}\ 35.47 \cdot \cos\phi_{sc} = \cancel{1112.8}\ 2076$$

$$\therefore \cos(\phi_{sc}) = \cancel{0.1863}\ 0.254.$$

Ex-14 :

A single-phase, 3 KVA, 230/115 Volt, 50 Hz transformer has the following constants:

$R_1 = 0.3\Omega$, $X_1 = 0.4\Omega$, $R_2 = 0.09\Omega$, $X_2 = 0.1\Omega$

$R_c = 600\Omega$ and $X_m = 200\Omega$.

What would be the readings of the instruments when the transformer is connected for (i) OC Test (ii) SC test. In both tests supply is given to high-voltage side.

Soln: (i) OC Test

$V_1 = 230$ volt, $\quad K = \frac{230}{115} = 2.$

$$I_c = \frac{V_1}{R_c} = \frac{230}{600} = 0.383 \text{ Amp}$$

$$I_m = \frac{V_1}{X_m} = \frac{230}{200} = 1.15 \text{ Amp.}$$

$$\therefore I_0 = \sqrt{I_c^2 + I_m^2} = \sqrt{(0.383)^2 + (1.15)^2}$$

$$\therefore I_0 = 1.212 \text{ Amp.}$$

Input on no-load to high-voltage winding

$$= V_1 . I_c = 230 \times 0.383 = 88.09 \text{ Watt} \simeq 88 \text{ Watt.}$$

Hence, the readings of the instruments are

Volt-meter reading	=	230 volt	
Ammeter reading	=	1.2 Amp	Ans.
Wattmeter reading	=	88 Watt.	

(ii) SC Test

$K = 2$

$Re_1 = R_1 + K^2 R_2 = 0.3 + (2)^2 \times 0.09 = 0.66\,\Omega$

$Xe_1 = X_1 + K^2 X_2 = 0.8\,\Omega$

$Ze_1 = \sqrt{Re_1^2 + Xe_1^2} = \sqrt{(0.66)^2 + (0.8)^2}$

$\therefore Ze_1 = 1.037\,\Omega$

Full-load current in the high voltage winding,

$$I_1 = \frac{3 \times 1000}{230} = 13.04 \text{ Amp.}$$

$\therefore V_{sc} = I_1 Ze_1 = 13.04 \times 1.037 = 13.5$ volt.

$W_{sc} = I_1^2 Re_1 = (13.04)^2 \times 0.66 = 112$ watt.

Hence, readings of the instruments are:

Voltmeter reading	=	13.50 Volt
Ammeter reading	=	13 Amp
Wattmeter reading	=	112 Watt.

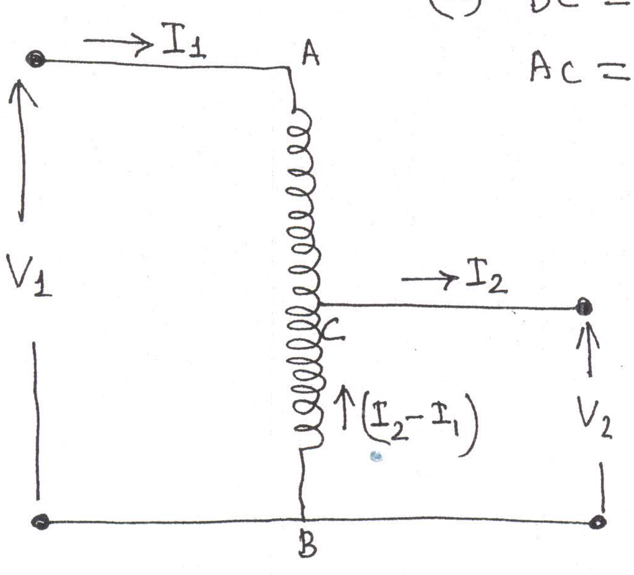
Ans.

Ex-15 : A 11.5 KV/2.3 KV, 1 ϕ autotransformer when used as two-winding transformer has the rated output of 100 KVA. If the two windings of the transformer are connected in series to form as autotransformer, find the possible voltage ratios and output.

Soln.

Two possible connections are:

(a) BC = 11500 Volt = V_2
AC = 2300 Volt



(b) BC = 2300 Volt = V_2
AC = 11500 Volt.

(a) In this connection

$$V_2 = 11500 \text{ Volt}$$

$$V_1 = 11500 + 2300 = 13800 \text{ Volt.}$$

Therefore

$$\text{Voltage ratio} = K' = \frac{13800}{11500} = \frac{138}{115}$$

transformers, the rating is 100 KVA.

$$\therefore (I_2 - I_1) V_2 = 100 \times 1000$$

$$\therefore I_2 - I_1 = \frac{100 \times 1000}{11500} = 8.7 \text{ Amp.}$$

$$(V_1 - V_2) I_1 = 100 \times 1000$$

$$\therefore I_1 = \frac{100 \times 1000}{2300} = 43.5 \text{ Amp.}$$

$$\therefore I_2 = 8.7 + I_1 = (8.7 + 43.5) = 52.7 \text{ Amp.}$$

KVA rating of the autotransformer is given by

$$V_1 I_1 = \frac{13800 \times 43.5}{1000} = 600 \text{ KVA}$$

$$\text{or } V_2 I_2 = \frac{11500 \times 52.7}{1000} = 600 \text{ KVA.}$$

$$(\text{Volume})_{\text{Auto}} = \left(1 - \frac{1}{k'}\right) (\text{Volume})_{\text{TW}}$$

$$\therefore \quad k' = \frac{V_1}{V_2} = \frac{13800}{11500}$$

$$\therefore (\text{Volume})_{\text{Auto}} = \left(1 - \frac{115}{138}\right) (\text{Volume})_{\text{TW}}$$

$$\therefore \quad \frac{(\text{Volume})_{\text{Auto}}}{(\text{Volume})_{\text{TW}}} = 1 - 0.83 = 0.17$$