METHODS OF ANALYSIS

→ Current flows from a higher potential to a lower potential in a resistor.

→ Mesh-analysis is also known as loop analysis or the mesh-current method.

a) Mesh is a loop which does not contain any other loops within it.

b) A super mesh results when two meshes have a (dependent or independent) current source in common. (similarly super node)
↳ for them a single equation is better.

→ In nodal analysis, assign node voltages, and use KCL.

- "Node" refers to any point on a circuit where two or more branches meet.
- A "branch" represents a single element such as a voltage source or a resistor.
- A "junction" is any point where electrical conductors are joined electrically.
- A "loop" is any closed path in a circuit.

DC Network Theorems

① Linear Systems

- A system is said to be linear if it satisfies the following:

I) Additive property or superposition theorem
II) Homogenity property.

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

- Additive property $x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$\Rightarrow x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

- Homogenity property $\alpha x(t) \longrightarrow \alpha y(t)$

$\alpha \in C$ (any scalar)

Together; $\alpha_1 x_1(t) + \alpha_2 x_2(t) \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$

Ex : I) Resistor $(V = IR)$

II) Inductor $\left(V_L = L \dfrac{di_L}{dt}\right)$

III) Capacitor $\left(I_C = C \dfrac{dV_C}{dt}\right)$

② **Bidirectional, passive and active elements**

→ Current can flow in both direction, then bidirectional element.

R, L, C → bidirectional

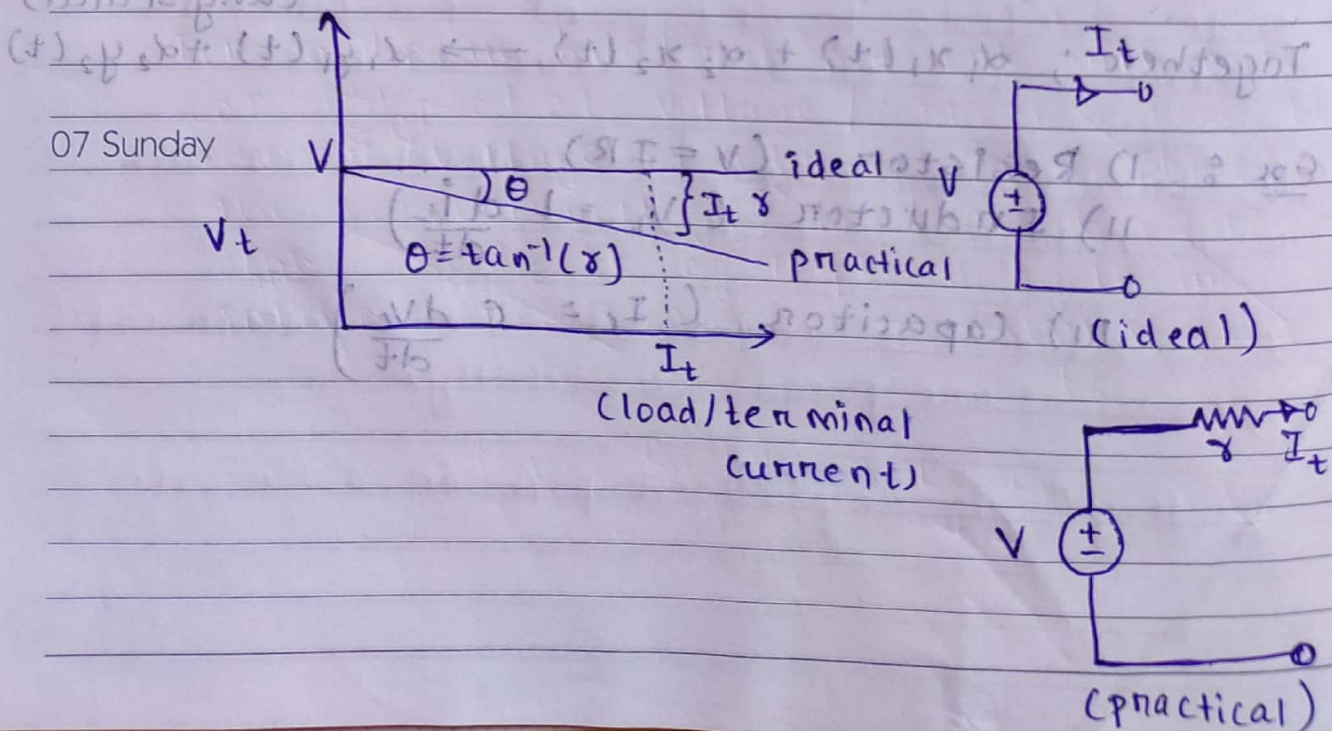
diodes → unidirectional

→ Passive elements: stores / absorbs energy
R, L, C

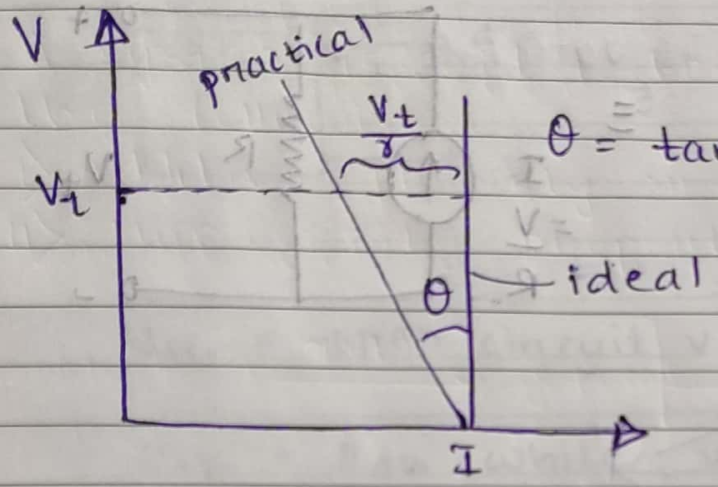
→ Active elements: generates/amplifies power, voltage, current.

→ generators, current and voltage sources.

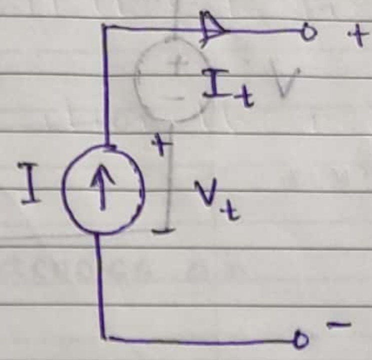
③ **Ideal voltage source**



(4) Ideal current source

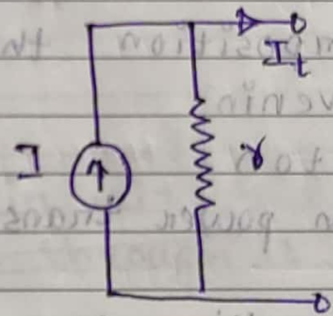


$\theta = \tan^{-1}\left(\frac{1}{r}\right)$



(Ideal)

I_t

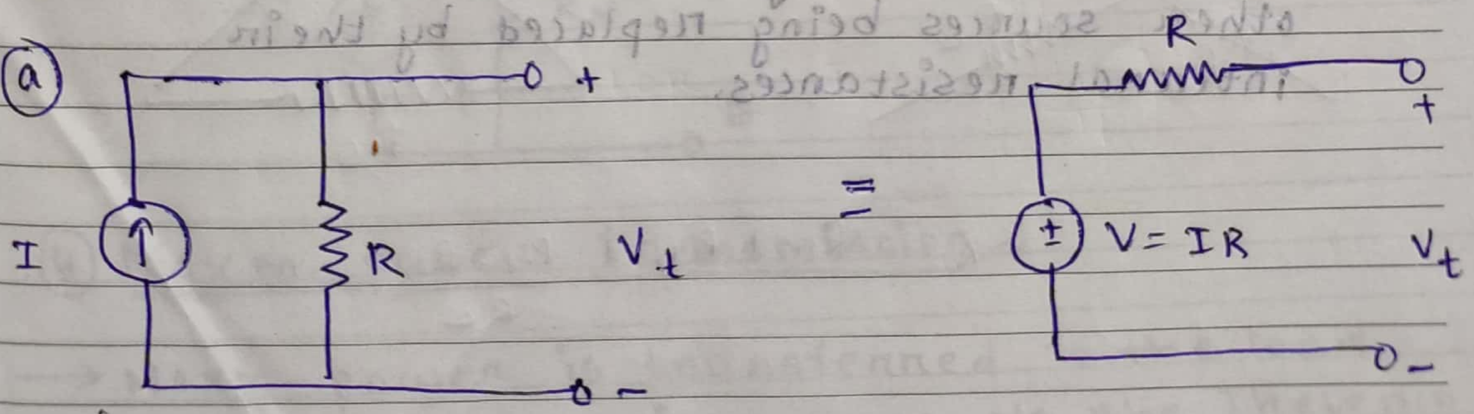


(practical)

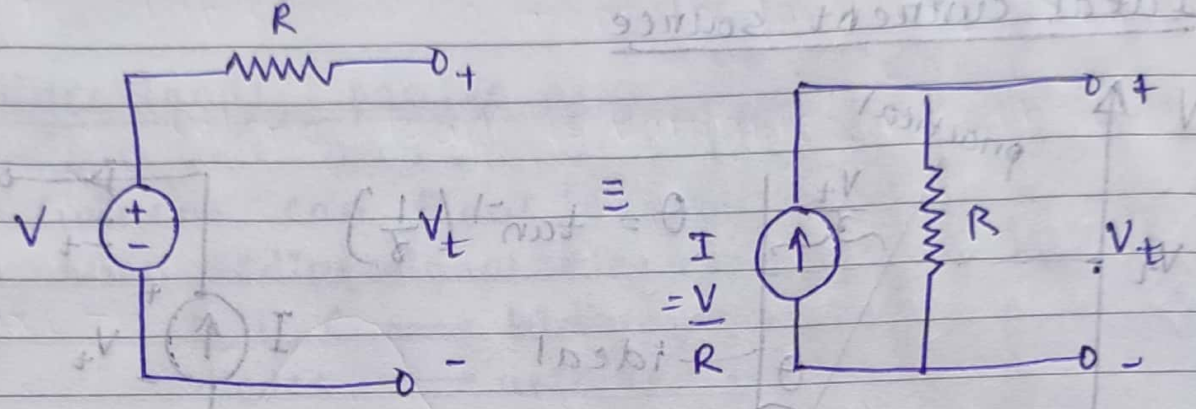
⇝ Check the resistor colour code

(5) Source transformation

(a)



(b)



(*) Network Theorems

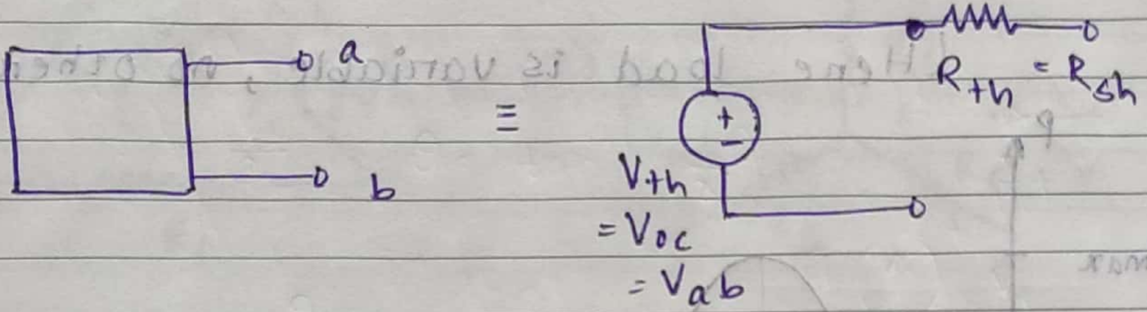
1) Superposition theorem
2) Thevenin "
3) Norton "
4) Maxm power transmission "

Assumption: The circuit is linear & bilateral

(1) Superposition theorem

For a linear, bilateral network with more than one independent voltage / current sources, the current / voltage of a branch / node is equal to the current / voltage due to each sources with other sources being replaced by their internal resistances.

② Thevenin



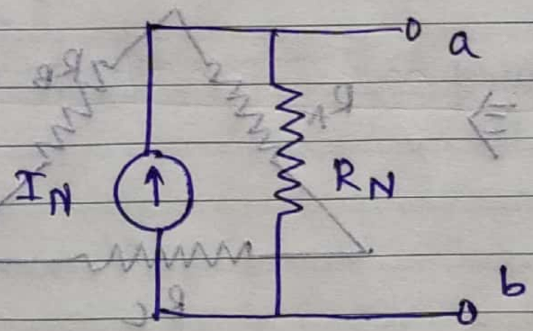
V_{th} = open circuit voltage across ab

R_{th} = R_{ab} while voltage and current sources are replaced by internal resistance

③ Norton

$I_N = \frac{V_{th}}{R_{th}}$ (Short circuit to find the current through it = I_N)

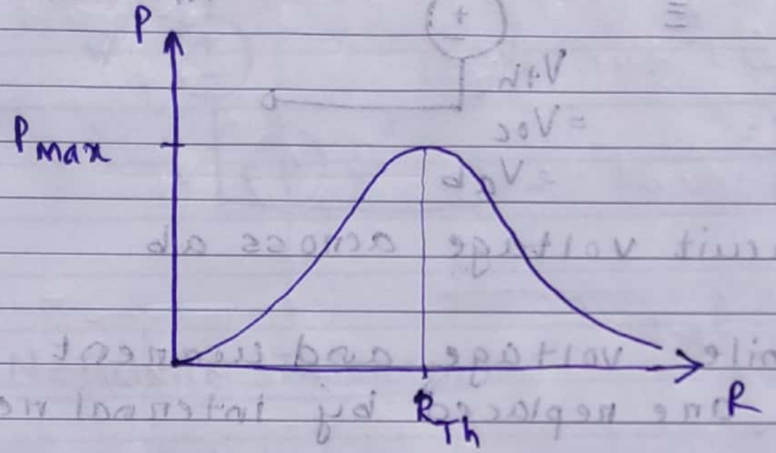
$R_{th} = R_{Norton} = R_N$



④ Maxm. power transmission

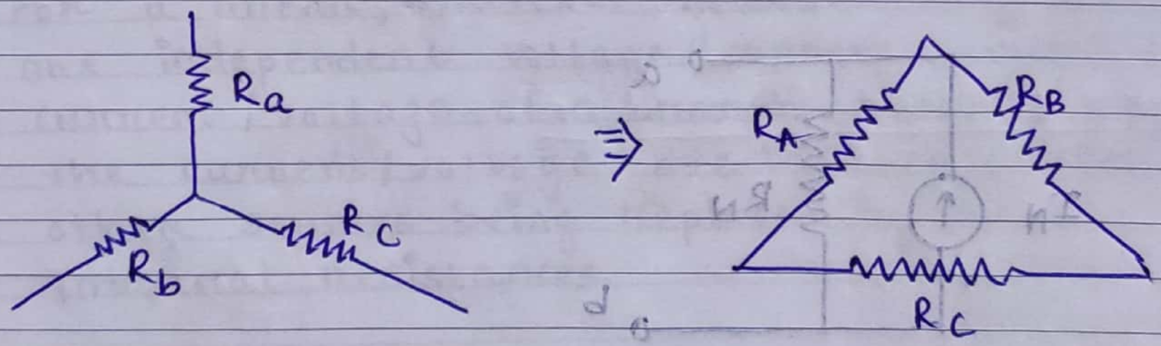
→ Maxm. power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)

(Here load is variable, no other element

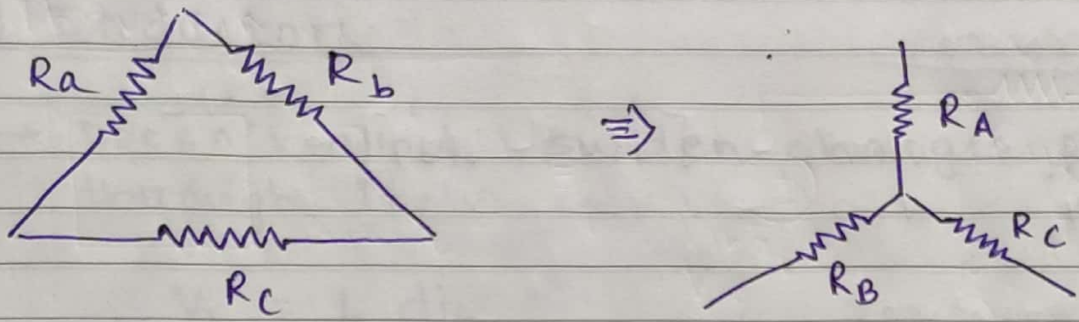


$$\boxed{P_{max.} = \frac{V_{Th}^2}{4R_{th}}}$$

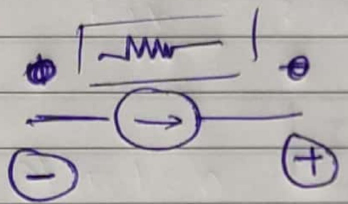
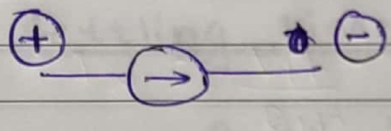
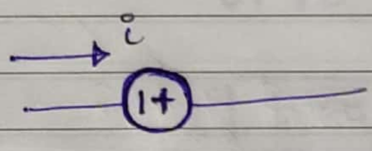
Star ↔ Delta

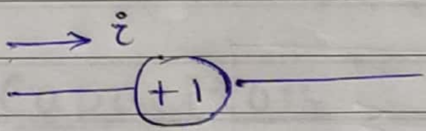


$$R_A = R_a + R_b + \frac{R_a R_b}{R_c}$$



$$R_A = \frac{R_a \cdot R_b}{R_a + R_b + R_c}$$

-  ≡ power delivered
-  ≡ power absorbed
- 

≡ power delivered
- 

≡ power absorbed

DC TRANSIENTS

1 Inductor

→ Doesn't allow sudden changes of current through it.

$$V_L = L \frac{di_L}{dt}$$

→ Time constant : It is the time required to reach the 63.2% of the steady state value

$$\boxed{\tau = L/R}$$

→ Settling time :

$$0.98 \frac{V}{R} = \frac{V}{R}(1 - e^{-t/\tau})$$

$$\boxed{t \approx 4\tau}$$

2 Capacitor

→ Doesn't allow sudden changes of voltage across it.

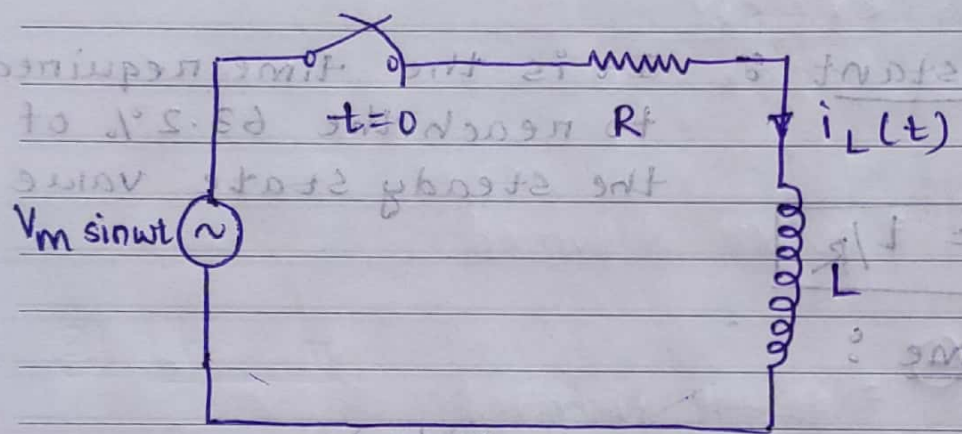
$$I_c = C \frac{dV_c}{dt}$$

→ Time constant : $\boxed{\cancel{\tau = \frac{1}{RC}}}$ $\tau = RC$

③ **Transients**

$$\checkmark\ i_L = i(\infty) - [i(\infty) - i(0)]\, e^{-t/\tau}$$
$$\checkmark\ v_C = v(\infty) - [v(\infty) - v(0)]\, e^{-t/\tau}$$

④ **AC transient**



Using KVL,

$$L\frac{di_L}{dt} + Ri_L = V_m \sin\omega t \quad \text{—} ①$$

$$i_L(t) = \underset{\substack{\text{Complimentary}\\ \text{Function}\\ \text{(Transient)}}}{\overset{\to A e^{-Rt/L}}{C.F.}} + \underset{\substack{\text{Particular Integral}\\ \text{(Steady-state)}}}{P.I}$$

For PI

Let $i_L = I_1 \sin\omega t + I_2 \cos\omega t$,
put in eqn ① and solve.

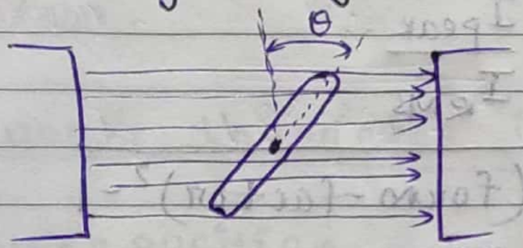
Further $i(0) = 0$; find A

$$\Rightarrow \boxed{i_L(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \left[\sin\phi \, e^{-\frac{Rt}{L}} + \sin(\omega t - \phi) \right]}$$

AC CIRCUITS

* 1-ϕ AC voltage generation

(Fleming's right hand rule) generating principle



$E = (2\pi n) B A N \sin\theta$

f = No. of cycles per ~~second~~ revolution × No. of revolution per second

$= 1 \times n = n$ (1-pole pair)

$= p \times n = np$ (p-pole pair)

$\boxed{f = p \times n}$ (p = Pole pairs ; n = rev/s)

$f = p \times \left(\frac{N_r}{60}\right) \rightarrow$ rpm

$f = \frac{P}{2} \cdot \frac{N_r}{60} \Rightarrow \boxed{f = \frac{N_r P}{120}}$

* RMS and AVG. value

$\longrightarrow I_{RMS}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt$ (Consider full wave-form)

$\longrightarrow I_{AVG} = \frac{1}{(T/2)}\int_0^{T/2} i(t)\,dt$ (Consider half wave-form) for symmetrical

→ For unsymmetrical ; consider full wave form

(a) Form factor = $\frac{I_{RMS}}{I_{AVG}}$ (>1, so that AC component is there)

(b) Peak factor = $\frac{I_{peak}}{I_{RMS}}$

(c) Ripple-factor = $\sqrt{(\text{form-factor})^2 - 1}$

$\left[\because V_{ac} = \sqrt{V_{rms}^2 - V_{avg}^2}\right.$ and ripple factor = $\left.\frac{V_{ac}}{V_{avg}}\right]$

(3) Phasor (steady-state analysis) not transients.

$V_m \sin(\omega t + \phi)$ (Time-domain representation) $\equiv$ $\frac{V_m}{\sqrt{2}} \angle \phi = \bar{V}$ Phasor-domain representation (Freq.-domain representation)

$R \longrightarrow R$

$X_L \longrightarrow j\omega L$

$X_C \longrightarrow \frac{-j}{\omega C}$

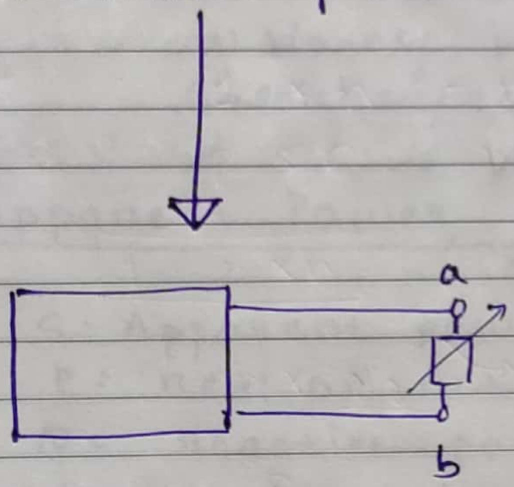
→ while drawing phasor diagram, it is easier to take loop current when series else node voltage when parallel as reference phasor.

Network theorems in AC

1) Superposition
2) Thevenin
3) Norton

} directly applicable

4) Maxm. power transfer theorem



Z_L = here Z_L is varying only

$$\boxed{Z_L = Z_{Th}^*}$$

$R_L = R_{Th}$ and $X_L = -X_{Th}$

Power transferred $= \dfrac{V_{Th}^2}{4 R_{Th}}$

If Z_L is purely resistive ; $R_L = |Z_{Th}|$ and P_{max} has to be calculated from basics

AC Power Analysis

(1) $p(t) = v(t)\, i(t)$

→ The instantaneous power (in watts) is the power at any instant of time.

→ The average power, in watts, is the average of the instantaneous power over one period.

$$P_{real} = \frac{1}{2} \text{Re}[V I^*] = \frac{1}{2} V_m I_M \cos(\theta_v - \theta_i)$$

→ A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero power.

(2) Apparent Power and power-factor

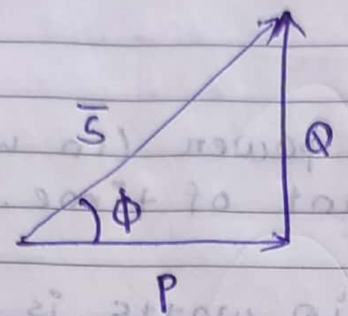
	Units
S : Apparent power	(VA)
P : real or active power	(W)
Q : reactive power	(VAR)

$$\boxed{S = P + jQ}$$

power factor (pf) $= \dfrac{P}{|S|} = \cos(\theta_v - \theta_i)$

$\ominus$ve (leading)

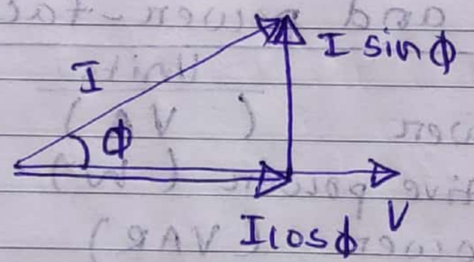
$\oplus$ve (lagging)



$\Rightarrow$ power triangle

$Q = 0 \longrightarrow$ pure resistive circuit
$Q < 0 \longrightarrow$ capacitive ~~leading~~ $(\theta_v < \theta_i)$
$Q > 0 \longrightarrow$ inductive circuit $(\theta_v > \theta_i)$

$\longrightarrow$ While improvement of power factor, P is to remain constant.
Also $(I\cos\phi)$ and V should be constant



Capacitor $\rightarrow$ VAR generator (connected in parallel)
Inductor $\longrightarrow$ VAR absorbor

$$\Delta Q = \frac{V_C^2}{X_C} \quad \text{or} \quad \Delta Q = \frac{V_L^2}{X_L}$$

$$\Delta Q = \omega C V_c^2 \qquad \Delta Q = \frac{V_L^2}{\omega L}$$

$$\boxed{C = \frac{\Delta Q}{\omega V_C^2}} \qquad \boxed{L = \frac{V_L^2}{\omega \Delta Q}}$$

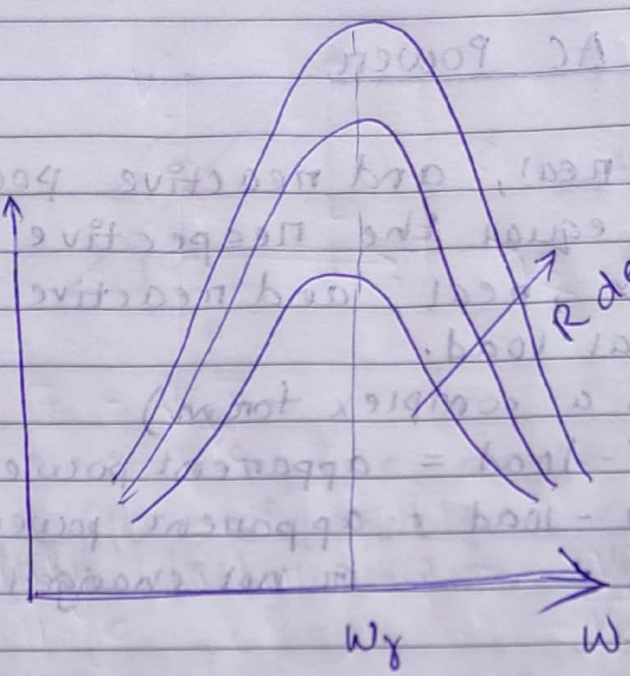
* Conservation of AC Power

→ The complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual load.
(write S in a complex form)

→ Motor on half-load = apparent power is halved
Motor on full-load = apparent power is complete or not changed.

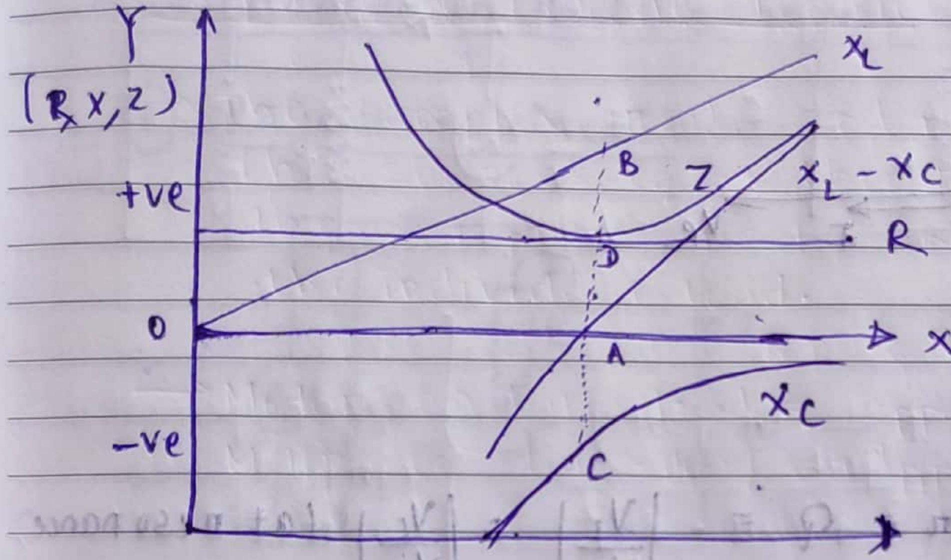
Lagging : Current lags voltage (Inductive)

02	8	9	10	11	12	13	14
03	15	16	17	18	19	20	21
04	22	23	24	25	26	27	28
05	29	30	31				

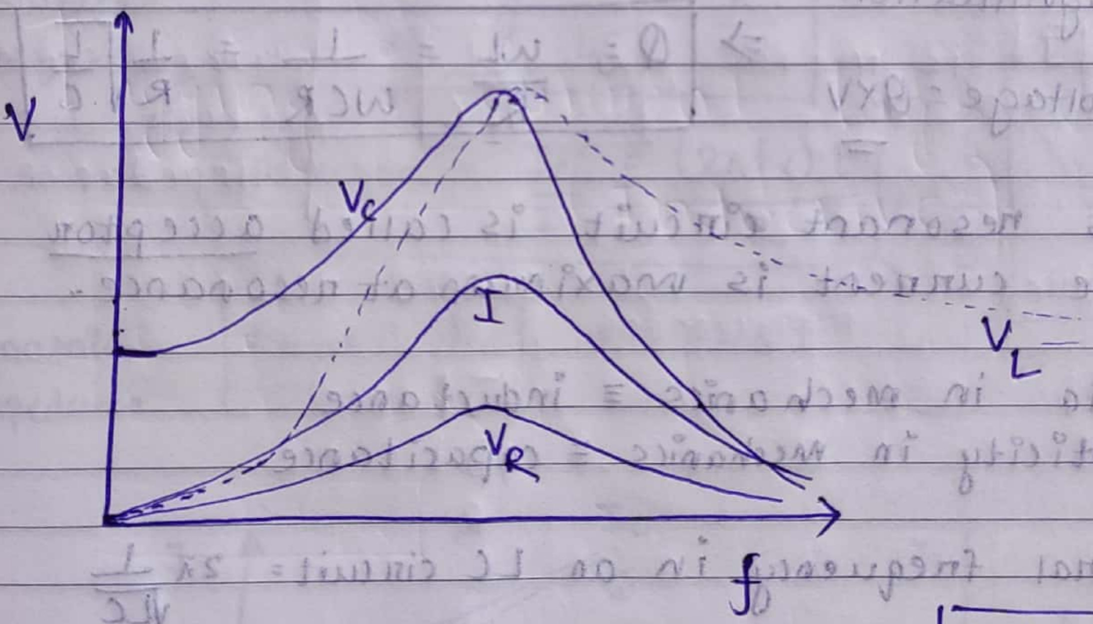


28 Sunday

RESONANCE



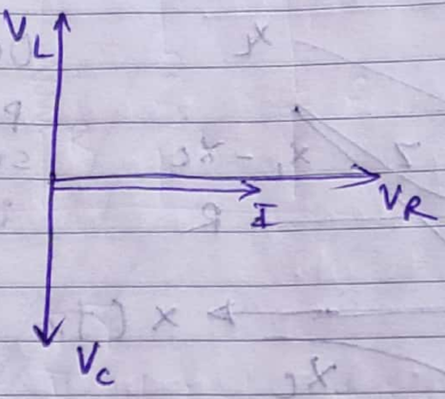
Occurs when the peak energies stored by the inductor and the capacitor are equal and hence occurs resonance.



$$\boxed{|V_C|(f=0) = |V_L|(f=\infty)} = V$$

For R-L-C series resonance, $f = \dfrac{1}{2\pi\sqrt{LC}}$

Wk	M	T	W	T	F	S
01	1	2	3	4	5	6
02	8	9	10	11	12	13
03	15	16	17	18	19	20
04	22	23	24	25	26	27
05	29	30	31			



Quality factor = $Q = \left|\frac{V_L}{V_R}\right| = \left|\frac{V_C}{V_R}\right|$ (at resonance)

Q = Voltage magnification

Magnified voltage = $Q \times V$

$$\Rightarrow \boxed{Q = \frac{\omega L}{R} = \frac{1}{\omega CR} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

- series resonant circuit is called <u>acceptor</u> since current is maximum at resonance.

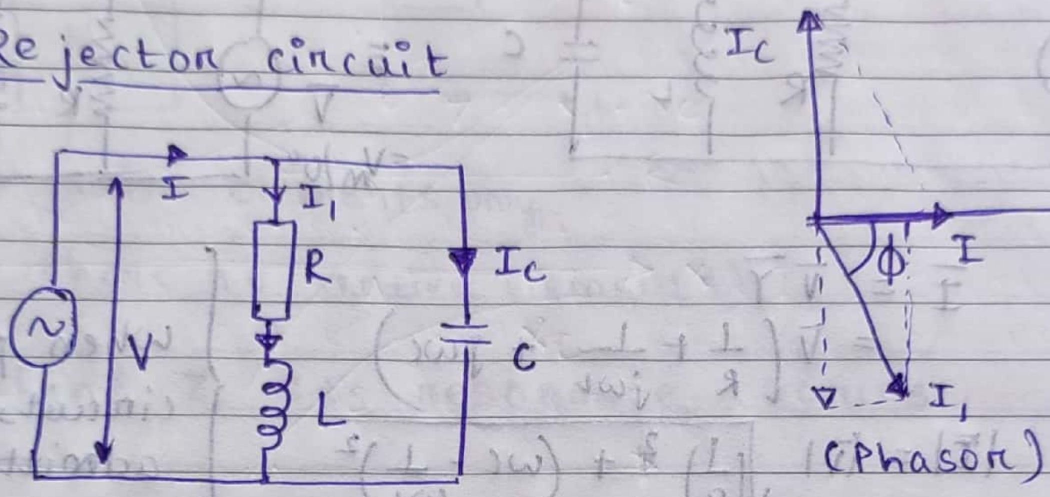
- Inertia in mechanics ≡ inductance
  elasticity in mechanics ≡ capacitance

→ Natural frequency in an LC circuit = $2\pi \frac{1}{\sqrt{LC}}$

In a LC circuit, the power taken from supply is simply that is required to move this energy backwards and forwards between L and C through the resistance of the circuit.

* Resonance in RLC parallel networks

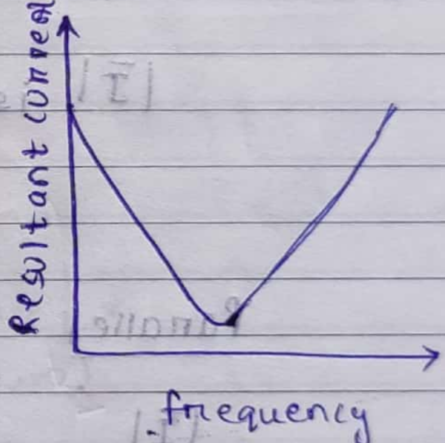
(1) Rejector circuit

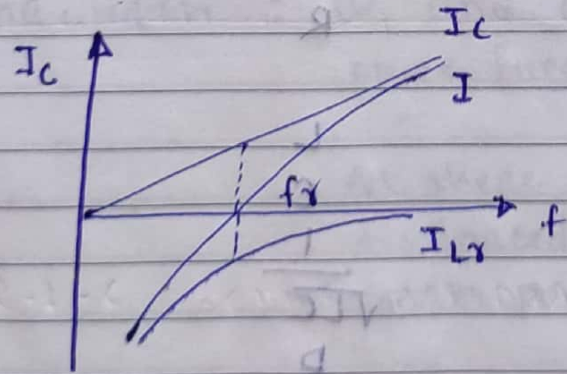


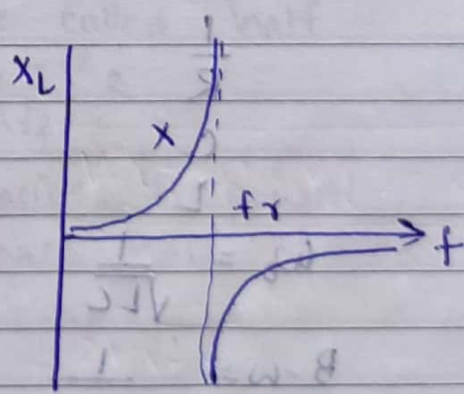
resonance frequency = $\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$

$\boxed{Q\text{-factor} = \frac{I_c}{I} = \frac{(2\pi f_r) L}{R}}$

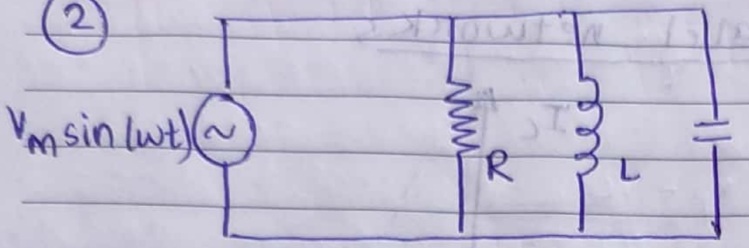
dynamic impedance = $\boxed{Z_r = \frac{L}{CR}}$



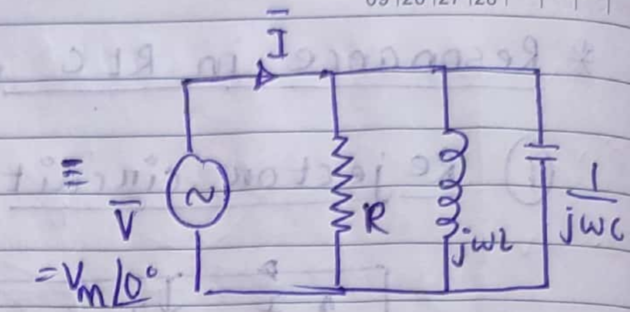




(2)



$\equiv$



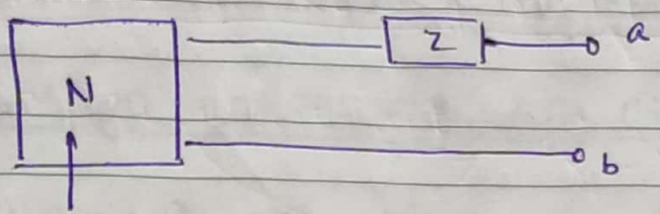
$$\bar{I} = \bar{V}\,\bar{Y}$$
$$= \bar{V}\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)$$
$$|\bar{I}| = |\bar{V}| \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

when parallel circuit, use admittance

$|\bar{I}|$ is minmi, $\boxed{\omega = \frac{1}{\sqrt{LC}}}$

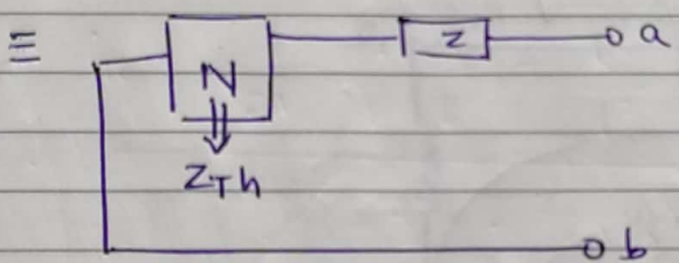
When series circuit, use impedance

Parallel	$\xleftrightarrow{\text{replacement}}$	Series
$\lvert\bar{I}\rvert$		$\lvert\bar{V}\rvert$
$\lvert\bar{V}\rvert$		$\lvert\bar{I}\rvert$
$\frac{1}{R}$		R
C		L
L		C
$\omega_0 = \frac{1}{\sqrt{LC}}$		$\frac{1}{\sqrt{LC}}$
$B\text{-}\omega = \frac{1}{RC}$		$\frac{R}{L}$
$Q = R\sqrt{\frac{C}{L}}$		$\frac{1}{R}\sqrt{\frac{L}{C}}$



passive elements only
(here no active elements)

→ Find Z s.t. resonance occurs.



$Z_{Th} = R_{Th} + j X_{Th}$; $Z = R + jX$
For $X_{Th} = -X$; $R \in [0, \infty)$

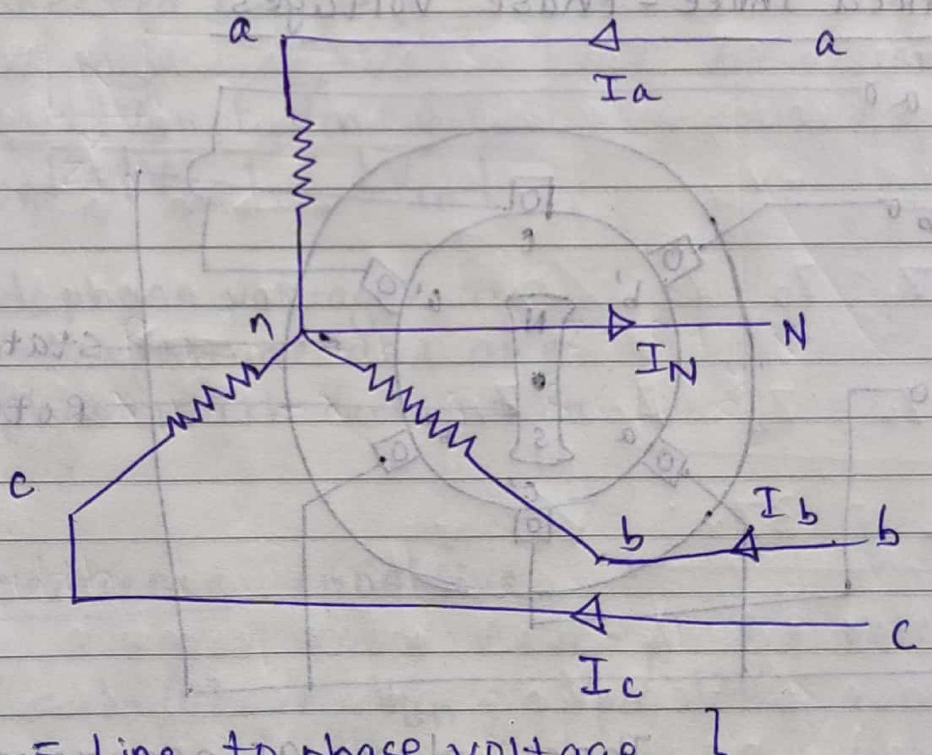
→ Bandwidth : ω_1 and ω_2 are called half power frequency $\left(\frac{I}{\sqrt{2}}\right)^2 R$

At these frequencies, the power dissipated becomes half of the maxm. power.

THREE PHASE CIRCUITS

* Advantage of 3-ϕ AC Circuit

- Bulk amount of power can be transmitted with reduced cost.
- Instantaneous power in a three-phase system can be constant
- All the industrial heavy drives are of 3-ϕ in nature.



① V_{aN} = Line to phase voltage
② V_{ab} = Line to line voltage
} w.r.t supply

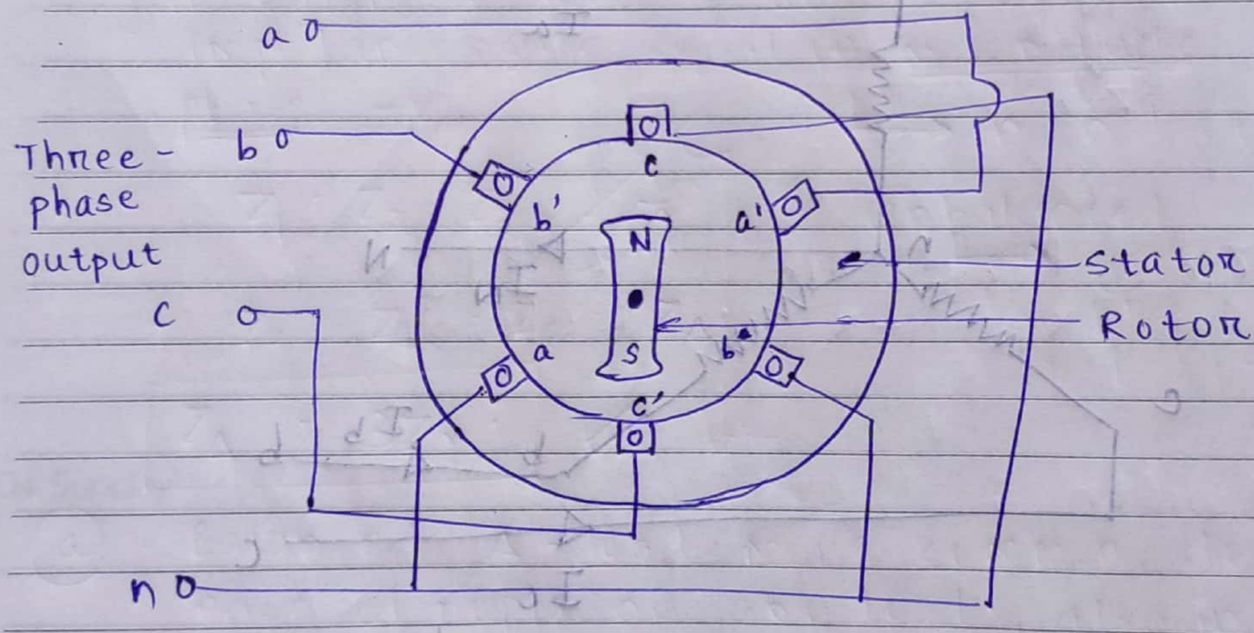
③ V_{an} = Phase voltage
④ I_a = Line current
⑤ I_{an} = Phase current

Trick: $P = \frac{L}{\sqrt{3}}$ ✓

→ Household : a single-phase three-wire system
→ Polyphase : circuits or systems in which the ac sources operate at the same frequency but different phases.

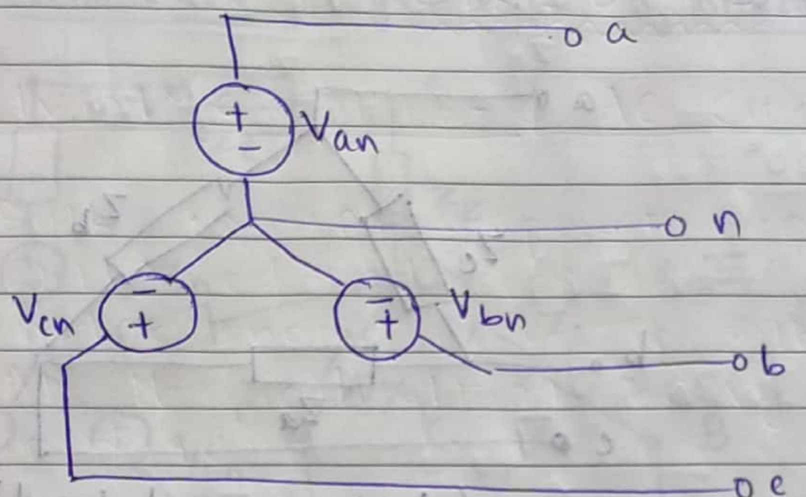
→ In three phase system, it is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

* Balanced Three - Phase Voltages

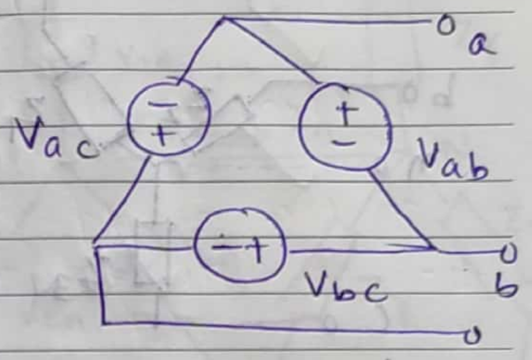


Three phase ac generator (alternator)

A typical three-phase system consists of three voltage sources connected to loads by three or four wires - the voltage sources can be either wye-connected or delta-connected.



(Y-connected source)



(Δ - connected source)

phase voltage

$$\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$$

$$|\vec{V}_{an}| = |\vec{V}_{bn}| = |\vec{V}_{cn}|$$

→ Balanced phase voltages are equal in magnitude and are out of phase with each other by $120°$.

A) abc sequence / positive

$$V_{an} = V_p \angle 0°, \quad V_{bn} = V_p \angle -120°$$

$$V_{cn} = V_p \angle -240° = V_p \angle 120°$$

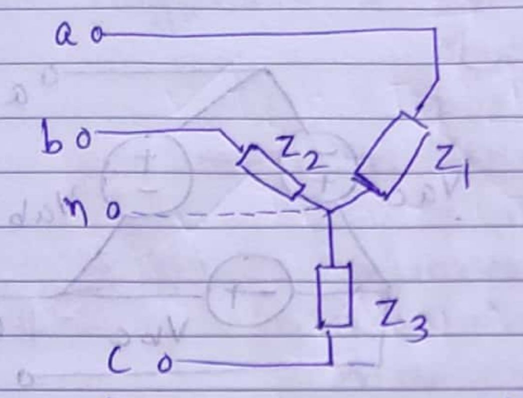
B) acb sequence / negative

$$V_{an} = V_p \angle 0°, \quad V_{bn} = V_p \angle -120°$$

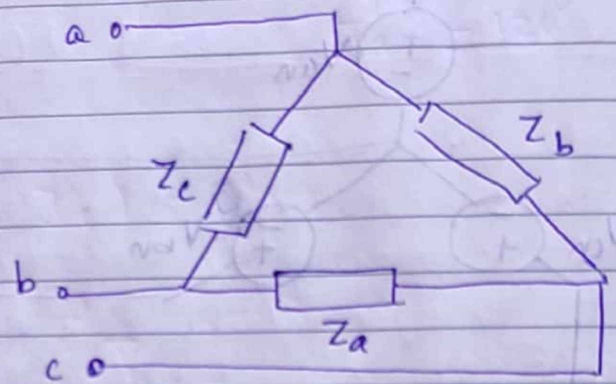
$$V_{cn} = V_p \angle 120°$$

Wk	M	T	W	T	F	S	S
05				1	2	3	4
06	5	6	7	8	9	10	11
07	12	13	14	15	16	17	18
08	19	20	21	22	23	24	25
09	26	27	28				

Loads



(Y-connected load)



(Δ-connected load)

→ A balanced load is one in which the phase impedances are equal in magnitude and in phase.
↳ in induction motor

$$Z_1 = Z_2 = Z_3 = Z_Y$$

Unbalanced → generally practical purposes

$$Z_a = Z_b = Z_c = Z_\Delta$$

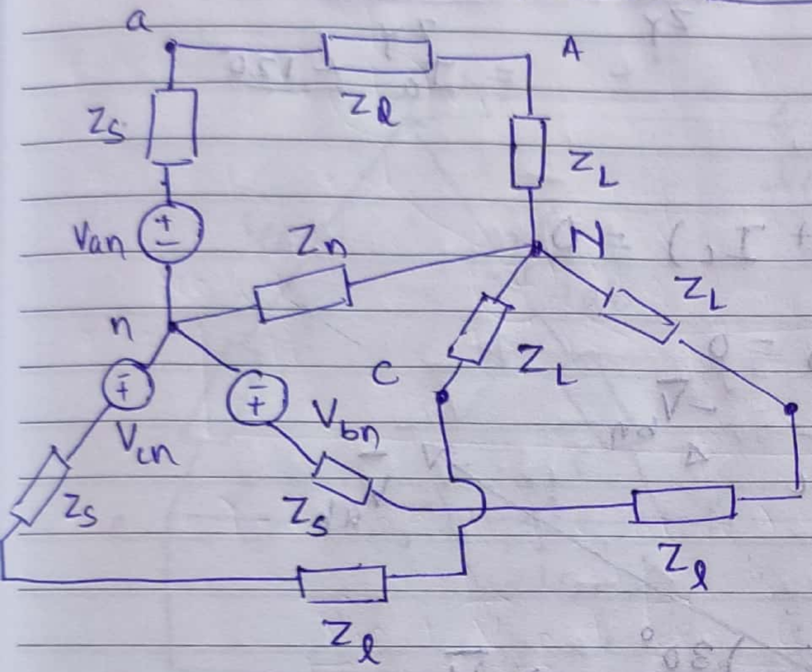
$$\boxed{Z_\Delta = 3Z_Y}$$ ✓

(a) Balanced Δ-load more common, due to ease of replacing any load, also in wye-load, neutral may not be accessible.

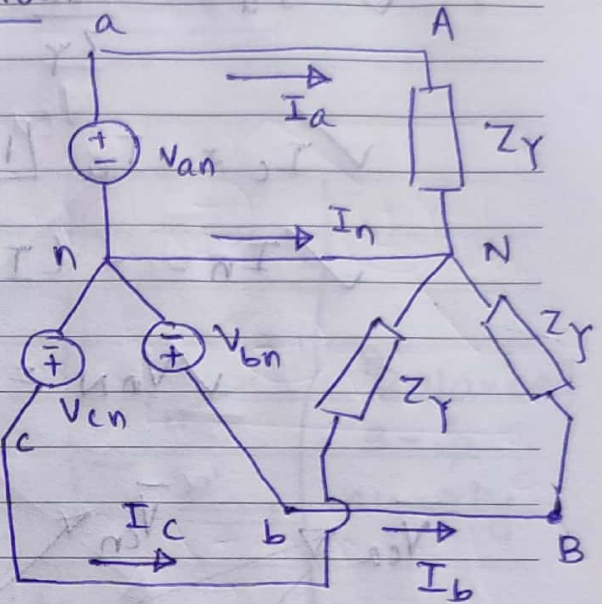
(b) Δ-connected sources are not common in practice because of the circulating current that will result in delta-mesh if voltages are slightly unbalanced.

Wk	M	T	W	T	F	S	S
09				1	2	3	4
10	5	6	7	8	9	10	11
11	12	13	14	15	16	17	18
12	19	20	21	22	23	24	25
13	26	27	28	29	30	31	

(*) Balanced Wye-Wye Connection



$\equiv$



Phase voltages: $V_{an} = V_p \angle 0^\circ$; $V_{bn} = V_p \angle -120^\circ$; $V_{cn} = V_p \angle 120^\circ$

The line to line voltages or simply line voltages

$$V_{ab} = V_{an} - V_{bn} = V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = \sqrt{3}\, V_p \angle -90^\circ$$

$$V_{ca} = \sqrt{3}\, V_p \angle -210^\circ$$

$$\boxed{\frac{V_L}{\sqrt{3}} = V_p} \quad \text{and} \quad \boxed{I_L = I_p}$$

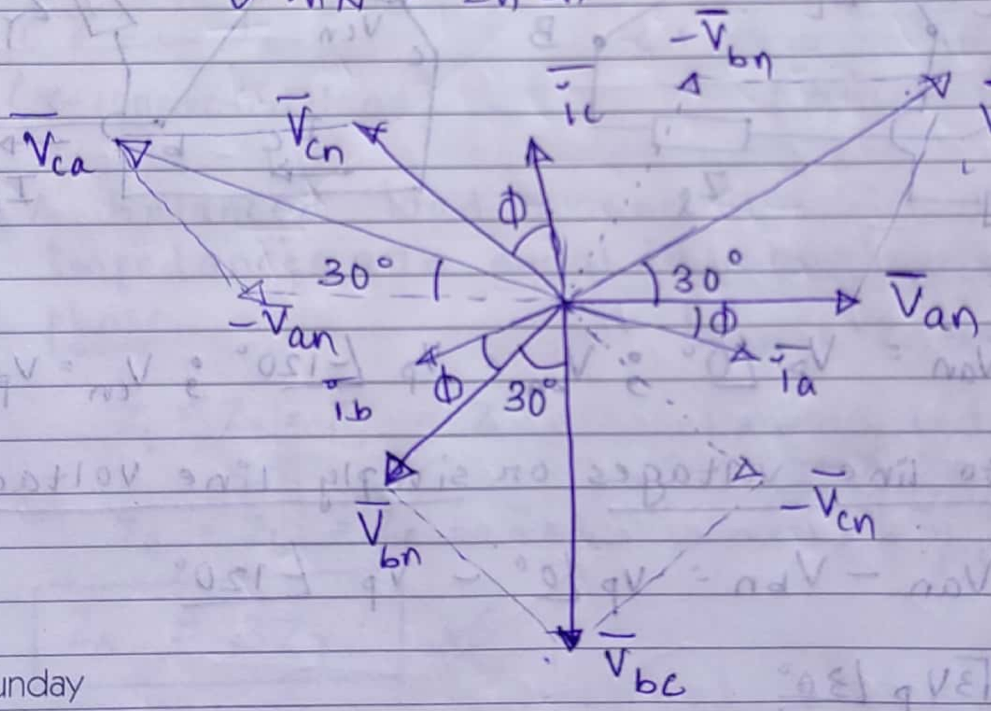
$\checkmark\ I_a = \dfrac{V_{an}}{Z_Y}$ $\quad \checkmark\ I_b = \dfrac{V_{bn}}{Z_Y} = \dfrac{V_{an} \angle -120^\circ}{Z_Y}$

$= I_a \angle -120^\circ$

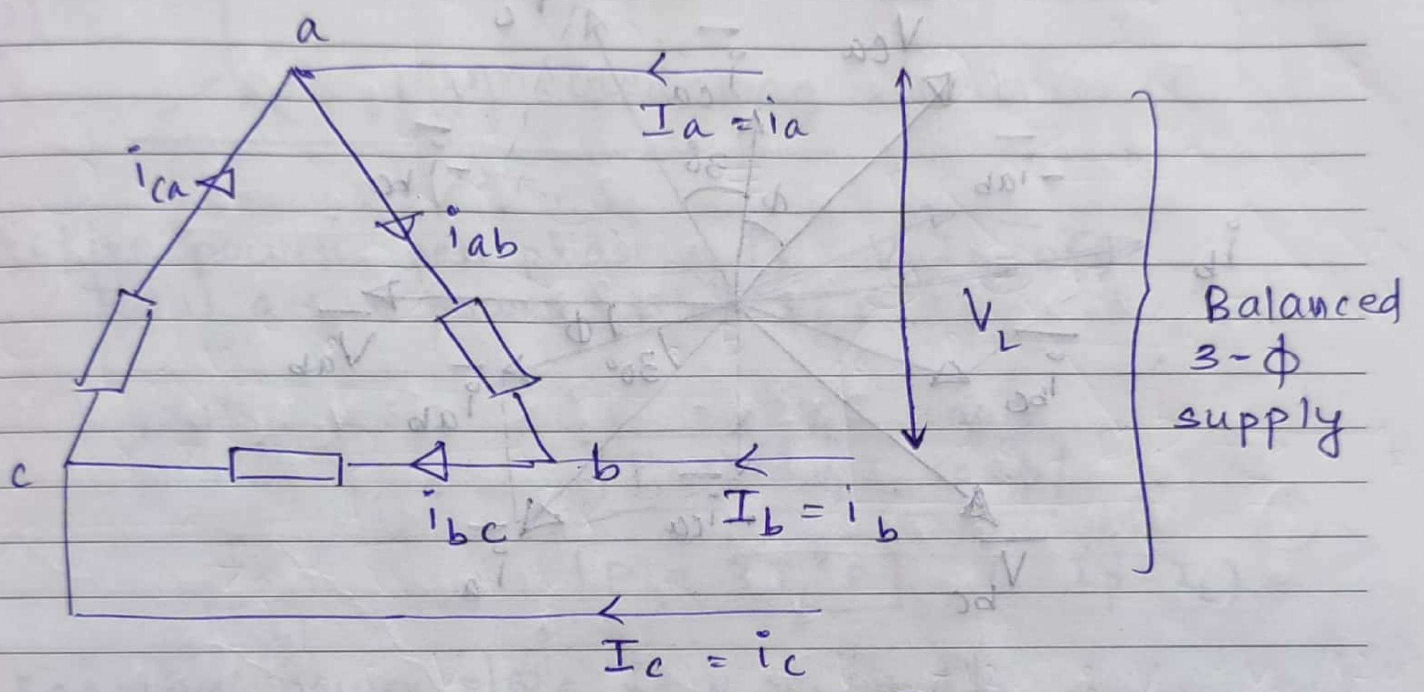
$\checkmark\ I_c = I_a \angle 120^\circ$

$\checkmark\ I_n = -(I_a + I_b + I_c) = 0$

$\checkmark\ V_{nN} = I_n Z_n = 0$



(*) Balanced Wye - Delta Connection

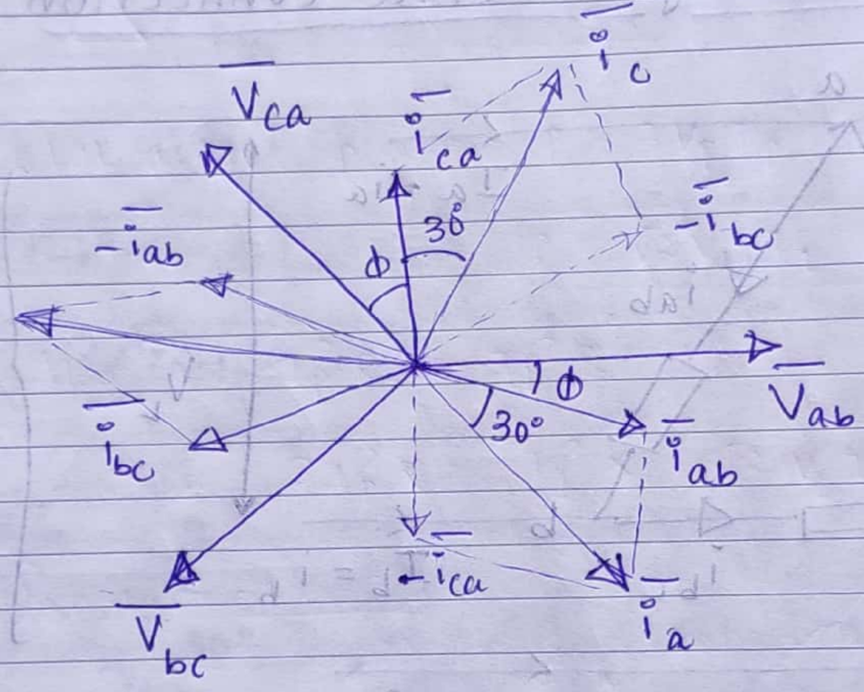


$$\boxed{V_L = V_p} \qquad \boxed{I_p = \frac{I_L}{\sqrt{3}}}$$

$$\bar{i}_a = \bar{i}_{ab} - \bar{i}_{ca} = \sqrt{3}\,|I_p| \angle -(30+\phi)$$

$$\bar{i}_b = \sqrt{3}\,|I_p| \angle -(150+\phi)$$

$$\bar{i}_c = \sqrt{3}\,|I_p| \angle 90-\phi$$


$\bar{V}_{ca}$
$\bar{i}_c$
$\bar{i}_{ca}$
30°
$-\bar{i}_{bc}$
$-\bar{i}_{ab}$
ϕ
i_b
ϕ
$\bar{V}_{ab}$
30°
$\bar{i}_{bc}$
$\bar{i}_{ab}$
$-\bar{i}_{ca}$
$\bar{i}_a$
$\bar{V}_{bc}$

* $3-\phi$ power

(a) Y - connected load

Active power per phase $= P_p = V_p I_p \cos\phi$

Total active power, $P = 3P_p$

$$= 3 V_p I_p \cos\phi$$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

$$\boxed{P = 3 I_L^2 R} \qquad (\because I_p = I_L)$$

Reactive power $= \boxed{Q = \sqrt{3}\, V_L I_L \sin\phi}$

$$\boxed{Q = 3 I_L^2 X}$$

Using complex power concept -

$$\bar{S}_p = \bar{V}_p \cdot \bar{I}_p^{*}$$

$$\bar{V}_p = |V_p| \angle 0$$

$$\bar{I}_p^{*} = |I_p| \angle\phi \Rightarrow \bar{S}_p = |V_p|\angle 0 \cdot |I_p| \angle\phi$$

$$\boxed{\bar{S}_p = \bar{P}_p + j\bar{Q}} \checkmark$$

(b) Δ - connected load

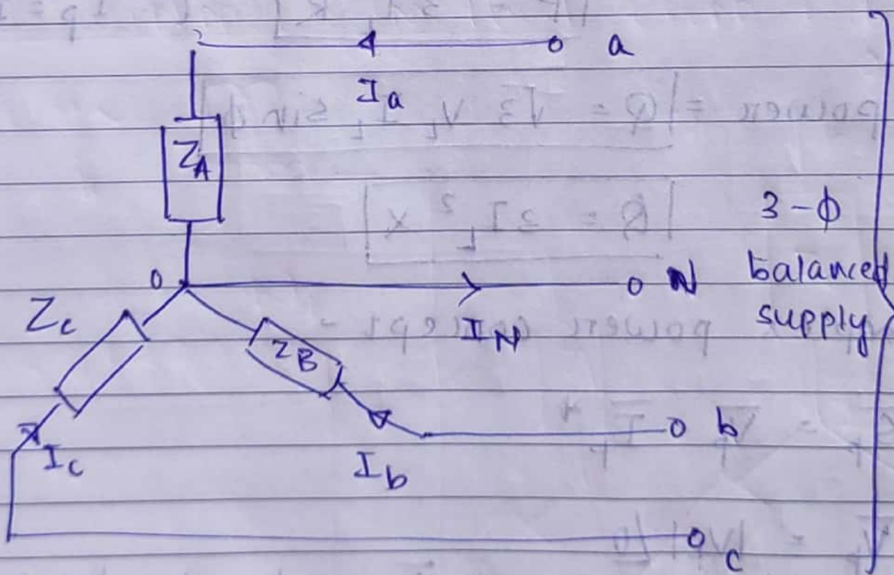
Total active power $P = 3 V_p I_p \cos\phi$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

Also $P = 3 I_p^2 R = I_L^2 R$

$Q = 3 I_p^2 X = I_L^2 X = \sqrt{3}\, V_L I_L \sin\phi$

(c) Unbalanced case



Load neutral pt. is connected to the supply neutral

Here $V_{ON} = 0$

→ neutral wire is connected, because across Z_A only voltage is $\overline{V_{an}}$.

→ lest $V_{ON} \neq 0$, and p.d. b/w supply 3-ϕ and 3-ϕ load.

$$\bar{I}_a = \frac{\bar{V}_{AN}}{Z_A} \qquad \bar{I}_b = \frac{\bar{V}_{BN}}{Z_B} \qquad \bar{I}_c = \frac{\bar{V}_{cn}}{Z_c}$$

$$\boxed{P = |V_{an}||I_a|\cos\phi_a + |V_{bn}||I_b|\cos\phi_b + |V_{cn}||I_c|\cos\phi_c}$$

$$\boxed{\bar{I}_N = \bar{I}_a + \bar{I}_b + \bar{I}_c}$$

If neutral point is not connected –

$$\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$$

$$\bar{V}_{ao} \cdot \bar{Y}_a + \bar{V}_{bo} \cdot \bar{Y}_b + \bar{V}_{co} \cdot \bar{Y}_c = 0$$

$$\Rightarrow (\bar{V}_{aN} - \bar{V}_{oN}) \cdot \bar{Y}_a + (\bar{V}_{bN} - \bar{V}_{oN}) \bar{Y}_b + (\bar{V}_{cN} - \bar{V}_{oN}) \cdot \bar{Y}_c = 0$$

$$\Rightarrow \bar{V}_{oN} = \frac{\bar{V}_{aN} \cdot \bar{Y}_a + \bar{V}_{bN} \cdot \bar{Y}_b + \bar{V}_{cN} \cdot \bar{Y}_c}{\bar{Y}_a + \bar{Y}_b + \bar{Y}_c}$$

$$\Rightarrow \boxed{\bar{V}_{oN} = \frac{\sum \bar{V}_p \cdot \bar{Y}_p}{\sum \bar{Y}_p}}$$

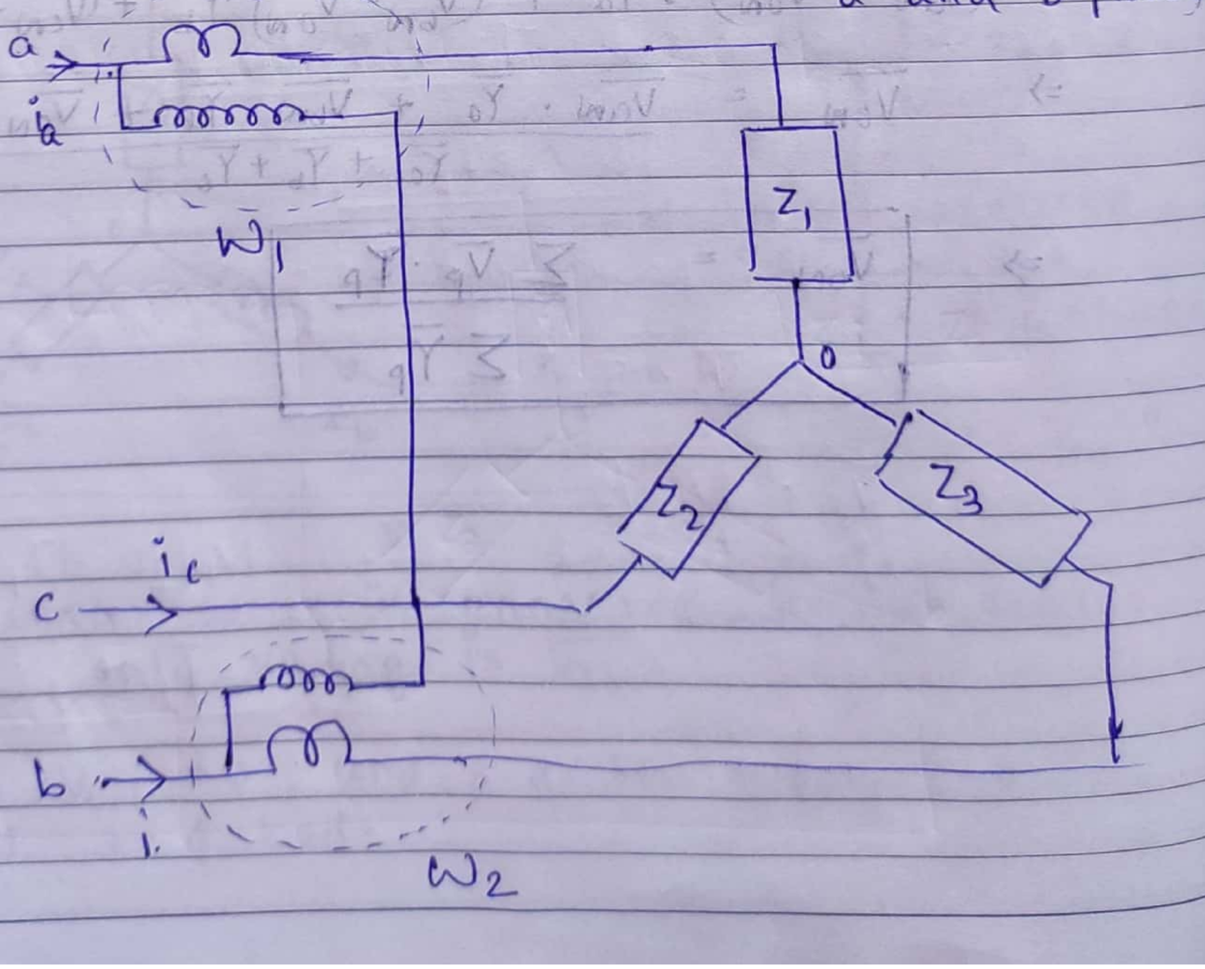
Wk	M	T	W	T	F	S	S
05				1	2	3	4
06	5	6	7	8	9	10	11
07	12	13	14	15	16	17	18
08	19	20	21	22	23	24	25
09	26	27	28				

(*) Three phase power measurements, using two wattmeter

→ The wye-source : used for long distance transmission of electric power. ($I^2 R$) = Resistive losses are minimum, since phase voltage and line voltage are higher, and so I is higher.

→ The delta-source : when single-phase circuits are desired from 3-ϕ source, especially in residential wiring.

• Two wattmeter method (Connected across a and b phases)



18 Sunday

instantaneous power -

$$W_1 = V_{ac}(t) \cdot i_a(t)$$
$$W_2 = V_{bc}(t) \cdot i_b(t)$$

Total instantaneous power -

$$W(t) = W_1(t) + W_2(t)$$
$$= V_{ac}(t) \cdot i_a(t) + V_{bc}(t) \cdot i_b(t)$$

$$V_{ac}(t) = V_{ao}(t) - V_{co}(t)$$
$$V_{bc}(t) = V_{bo}(t) - V_{co}(t)$$

$$W(t) = V_{ao}(t)\, i_a(t) + V_{bo}(t) \cdot i_b(t) - (i_a + i_b)(t)\, V_{co}(t)$$

$$\Rightarrow W(t) = V_{ao}(t) \cdot i_a(t) + V_{bo}(t) \cdot i_b(t) + i_c(t) \cdot V_{co}(t)$$

(Frequency of power $= \frac{\pi}{\omega}$)

$\Rightarrow$ $\boxed{\text{Avg. power consumed in load} = W = W_1 + W_2}$

$$W_1 = |V_{ac}||i_a| \cos(30 - \phi)$$
$$W_2 = |V_{bc}||i_b| \cos(30 + \phi)$$
from phasor diagram

$\rightarrow$ $W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$ = Active power

$\rightarrow$ $\sqrt{3}(W_1 - W_2) = \sqrt{3}(V_L I_L \sin\phi)$ = Reactive power

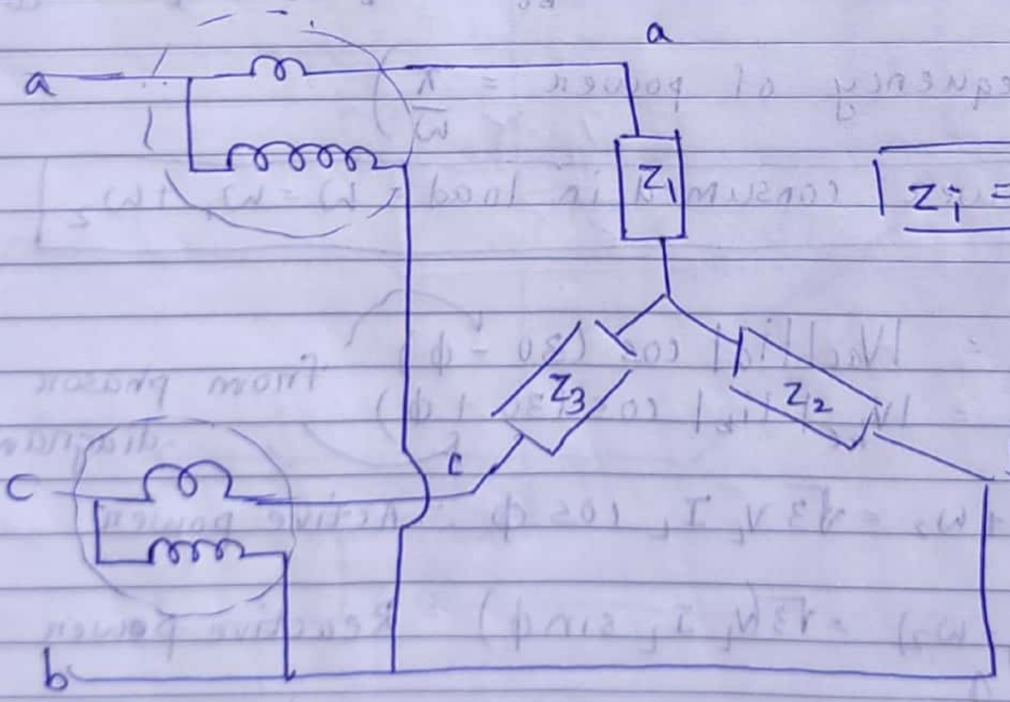
subtract $W_1 - W_2$ or $W_2 - W_1$, so that $\sin\phi$ comes in expression

$$\boxed{\tan\phi = \frac{\sqrt{3}(\omega_1 - \omega_2)}{(\omega_1 + \omega_2)}}$$

$$\boxed{\cos\phi = p.f. = \frac{1}{\sqrt{1 + 3\frac{(\omega_1 - \omega_2)^2}{(\omega_1 + \omega_2)^2}}}}$$

→ current wire must be connected along two phases. voltage wire across two phases correspondingly.

<u>Wattmeters connected in a & c phase</u> -



$$\boxed{Z_i = Z}$$

→ When wattmeter connected to a & b phases –

✓ $W_1 = W_2 \Rightarrow \cos\phi = 1 \Rightarrow$ resistive pure load

✓ $W_1 > W_2 \Rightarrow \tan\phi > 0 \Rightarrow$ inductive load

(∵ reactive power $= \sqrt{3}(W_1 - W_2)$
$= (+)$ ve
$\Rightarrow$ power absorbed (VAR))

✓ $W_1 < W_2 \Rightarrow \tan\phi < 0 \Rightarrow$ capacitive load

(∵ reactive power $= \sqrt{3}(W_1 - W_2)$
$= (-)$ ve
$\Rightarrow$ power supplied (VAR))

MAGNETIC CIRCUITS

a) Magnetic field intensity = H
b) Magnetic flux density = B

① Magnetomotive force (mmf)

$$\boxed{mmf = Ni = H_c l_c}$$

Ni = algebraic sum of the ampere-turns of all the windings.

H_c can be found from the right hand rule, similar to ϕ, B.

② $\boxed{B = \mu H}$ (μ = permeability $= \mu_r \mu_0$)

μ is property of region in which the field exists.

→ μ_r is assumed constant though it varies appreciably with magnetic flux density.

③ Magnetic flux

$$\boxed{\phi = \int_S \vec{B} \cdot d\vec{a}}$$ (in webers)

→ $\oint \vec{B} \cdot d\vec{a} = 0$

(4) Reluctance

$$\boxed{R = \frac{l}{\mu A}}$$

$\rightarrow P$ = Permeance $= \frac{1}{R} = \frac{\mu A}{l}$

(*) Maxwell Equation

$\longrightarrow \oint_c H \cdot dl = \int_s J \cdot da$ J = ct. density

Assumption:

$\frac{\partial D}{\partial t} << J \Rightarrow J + \frac{\partial D}{\partial t} \approx J$

$\longrightarrow \oint_s B \cdot da = 0$

D = electric displacement

(*) Ohm's Law

$B = \mu H = \mu \left(\frac{NI}{l}\right)$ —— (1)

$\phi = B \cdot A = \mu \frac{NI}{l} \cdot A$

$NI = (\phi)\left(\frac{l}{\mu A}\right)$

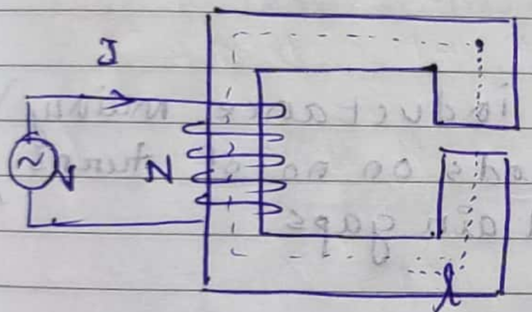
$mmf = \phi \times R$ —— (2)

→ Flux linkage $= \lambda = N\phi$

self-Inductance $L = \dfrac{N\phi}{I} = \dfrac{\mu N^2 A}{l} = \dfrac{\lambda}{I}$

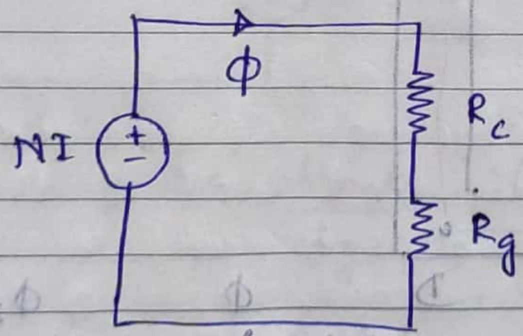
$$\boxed{L = \frac{\mu N^2 A}{l}} \quad \text{—— (3)}$$

→ Ckt. diagram



g = length of air gap.

≡



$R_c = \dfrac{l}{\mu_c A_c} = \dfrac{l_c}{\mu_0 \mu_r A_c}$

$R_g = \dfrac{l_g}{\mu_0 A_g} = \dfrac{l_g}{\mu_0 A_g}$

$A_g \approx (1.1 \text{ to } 1.2) \times A_c$

→ If no fringing effect,

$A_c = A_g$

$\mu_r >>$,

$R_c << R_g$

$$\boxed{\phi = \frac{NI}{R_c + R_g} \approx \frac{NI}{R_g}} \quad \text{— (4)}$$

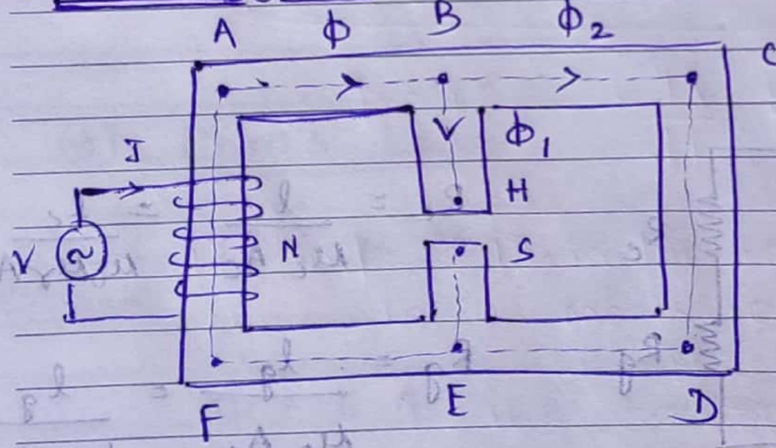
$$L = \frac{N\phi}{I} = \frac{N^2}{R_c + R_g}$$

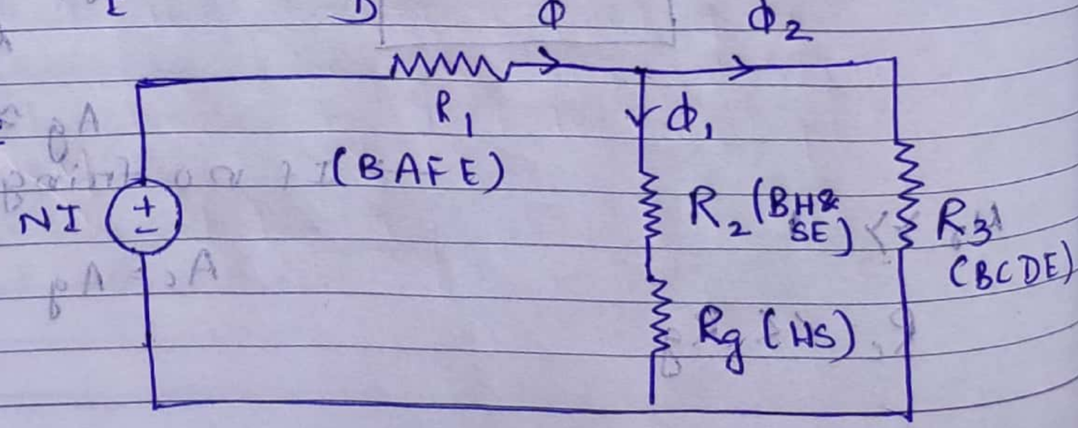
$$\boxed{L = \frac{N^2}{R_{total}}} \quad \text{— (5)}$$

$$\approx \frac{N^2}{R_g}$$

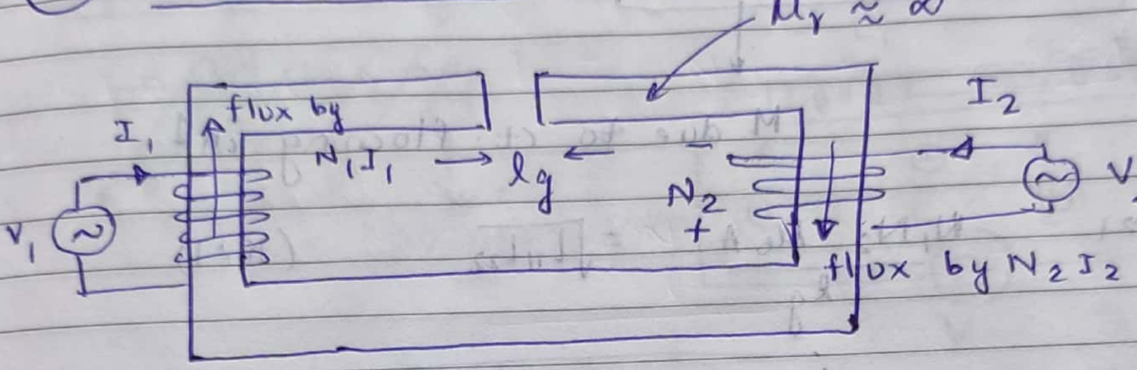
(i.e. inductance mainly depends on no. of turns and air gaps)

→ **Ckt. diagram**





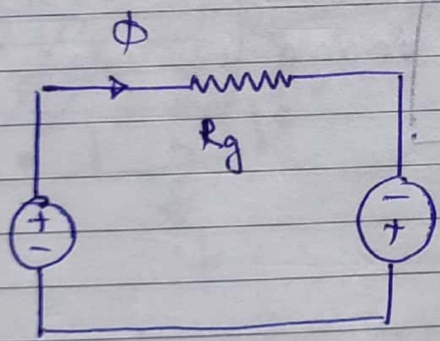
(*) Mutual inductance



$$mmf = N_1 I_1 + N_2 I_2$$

Total reluctance $= R_c + R_g$

$$= 0 + \frac{l_g}{\mu_0 A}$$



$$\Rightarrow \phi = \frac{N_1 I_1 + N_2 I_2}{R_g}$$

$$\phi = \frac{\mu_0 A}{l_g}(N_1 I_1 + N_2 I_2)$$

Flux linkage in coil 1 –

$$\lambda_1 = N_1 \phi$$

$$\lambda_1 = \left(\frac{N_1^2 \mu_0 A}{l_g}\right) I_1 + \left(\frac{N_1 N_2 \mu_0 A}{l_g}\right) I_2$$

$$\boxed{\lambda_1 = L_{11} I_1 + M_{12} I_2} \quad \text{—(1)}$$

M_{12} → Mutual inductance (due to ct. flowing in coil 2)

$$\boxed{\lambda_2 = L_{22} I_2 + M_{21} I_1}$$

M due to ct flowing in 1.

$$M_{12} = M_{21} = \frac{N_1 N_2 \mu_0 A}{l_g} = \sqrt{L_{11} L_{22}} \quad (K=1)$$

In general $\boxed{M_{12} = K\sqrt{L_{11} L_{22}}}$

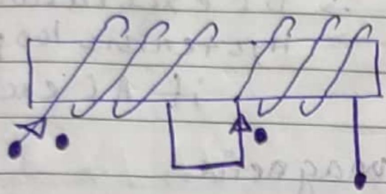
$0 < K \leq 1$ = coupling factor

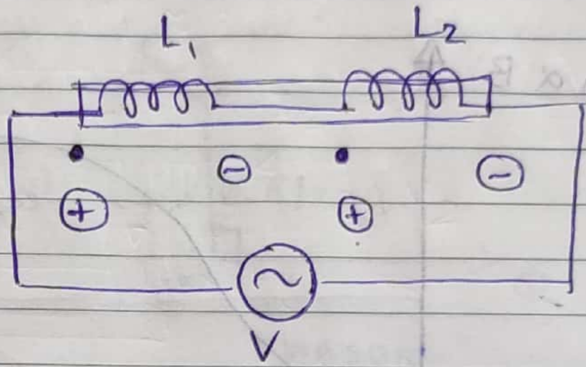
$$\rightarrow \boxed{W = \frac{\lambda^2}{2L} = \frac{1}{2} L i^2}$$

$$p = ie = i\frac{d\lambda}{dt}$$

To find self-inductance of one mmf, short the other mmfs and find ϕ through itself when only it is present and then divide by I, and In this situation find ϕ through other source (mmf) and multiply their N and divide by I of the mmf present to find M.

- Cumulative coupling

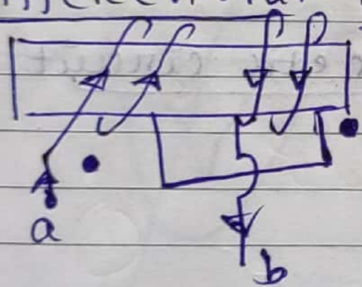




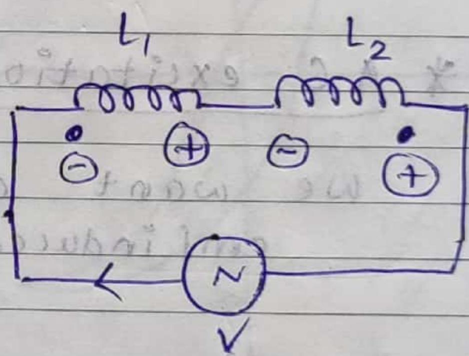
$$V = L_1 \frac{di}{dt} + M\frac{di}{dt} + L_2 \frac{di}{dt} + M\frac{di}{dt}$$

$$= \underset{(L_{eq})}{(L_1 + L_2 + 2M)} \frac{di}{dt}$$

- Differential coupling



$\longrightarrow$



$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt}$$

nd multiply by N.

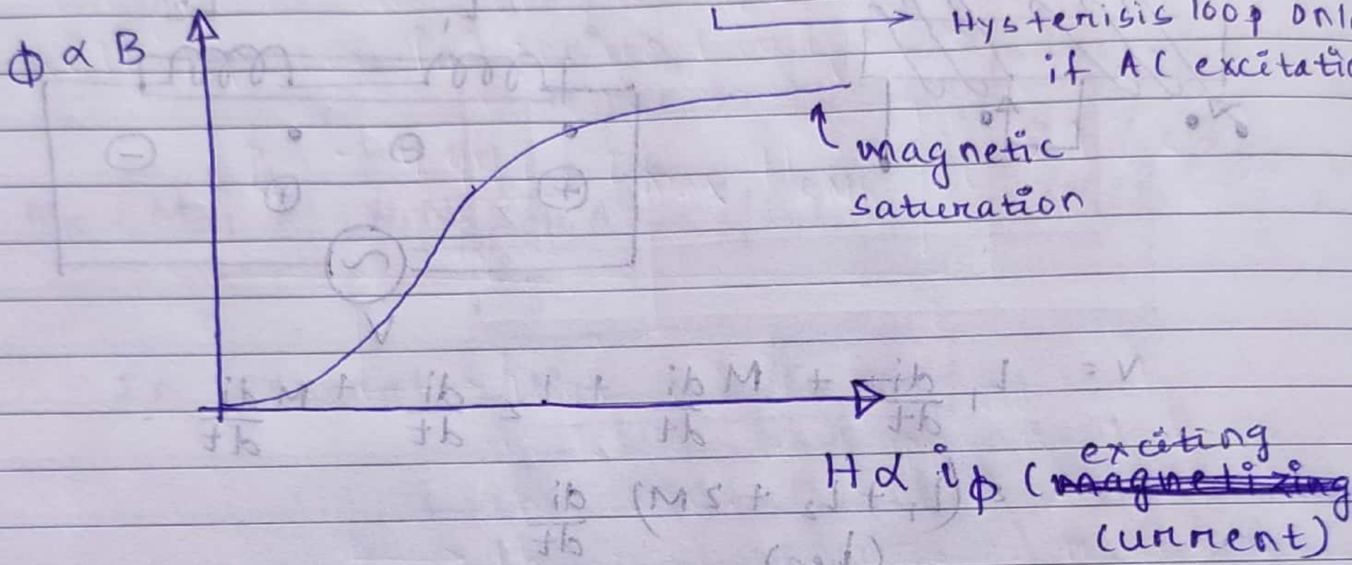
$$= \underset{(L_{eq})}{(L_1 + L_2 - 2M)} \frac{di}{dt}$$

→ due to dot, the other dot would be (+)
→ due to non-dot, the non-dot of other would be (+)

* B-H characteristic or (DC) magnetization curve

→ DC excitation

→ Hysterisis loop only if AC excitation



* AC excitation to the magnetic circuit

We want $\phi = \phi_m \sin \omega t$

emf induced $= e = N \dfrac{d\phi}{dt}$

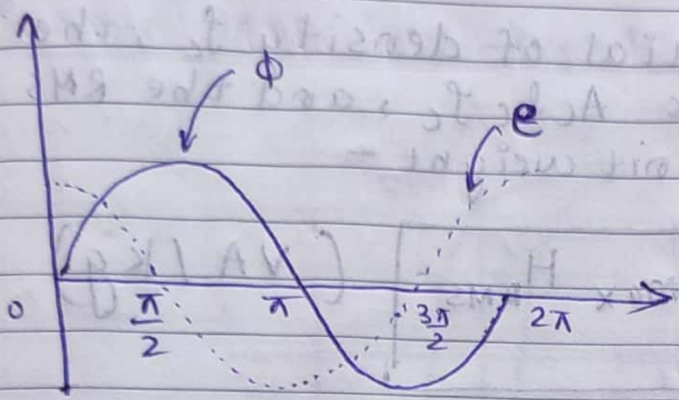
$= N \phi_m \omega \cos \omega t$

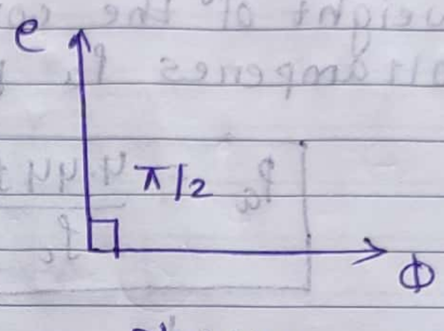
$E_{rms} = \dfrac{N \phi_m \omega}{\sqrt{2}}$

$\boxed{E_{rms} = 4.44\, \phi_m N f}$

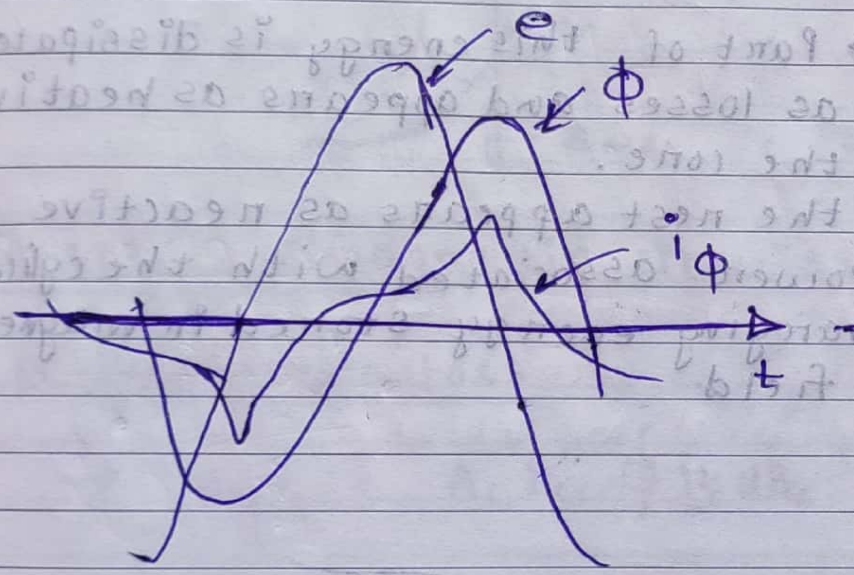
$\boxed{E_{rms} = 4.44\, B_m A_c N f}$

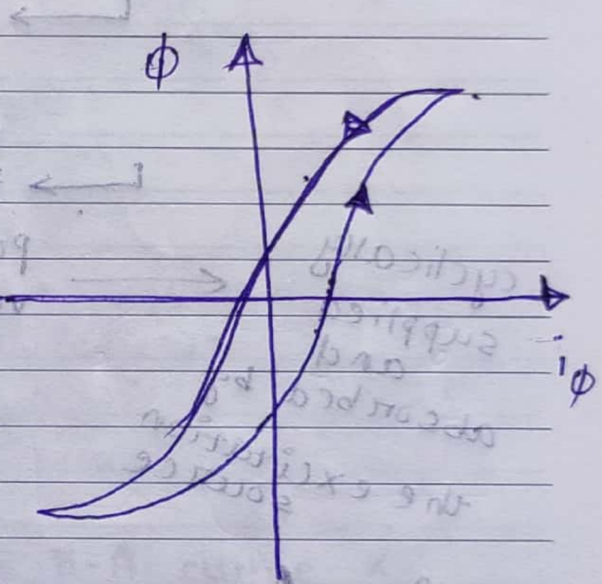
$\Rightarrow$ $\boxed{e \text{ leads } \phi \text{ by } \frac{\pi}{2}}$





Phasor





- $I_{\phi,RMS} = \dfrac{l_c H_{RMS}}{N}$ = exciting current

- The RMS VA reqd. to excite the core to a specified flux density is

$$E_{RMS} I_{RMS,\phi} = 4.44 B_M A_c N f \cdot \frac{l_c H_{RMS}}{N}$$

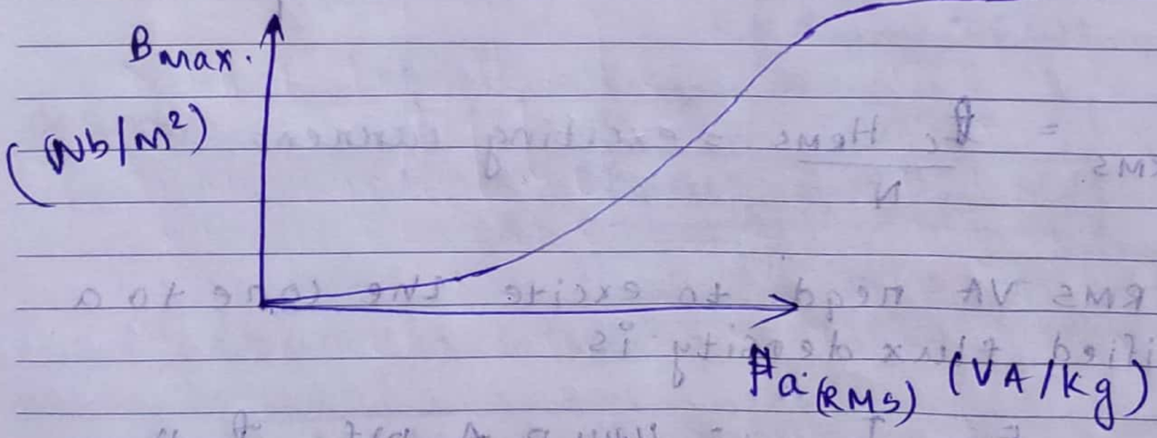
$$= 4.44 A_c l_c f B_{max} . H_{RMS}$$

For magnetic material of density ρ_c, the weight of the core is $A_c l_c \rho_c$, and the RMS voltamperes P_a per unit weight –

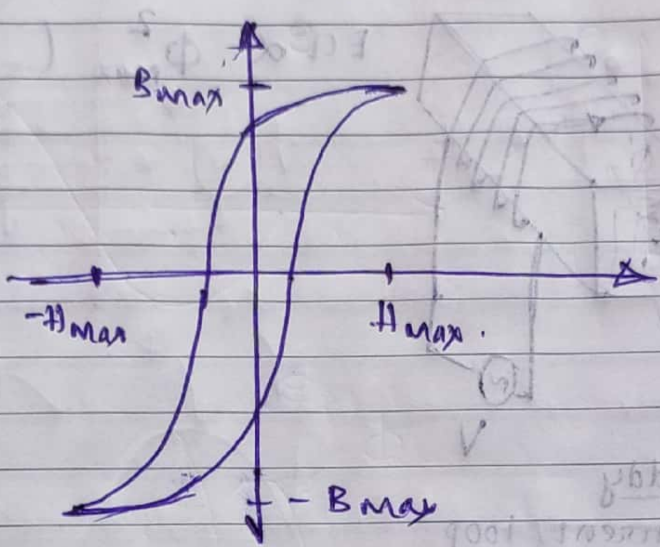
$$\boxed{P_a = \frac{4.44 f}{\rho_c} B_{max} H_{RMS}} \quad (VA/kg)$$

→ Part of this energy is dissipated as losses and appears as heat in the core.

→ the rest appears as reactive power associated with the cylically varying energy stored in magnetic field ← cyclically supplied and absorbed by the excitation source



B - H loop or hysteresis loop (AC excitation)



Losses in magnetic circuits

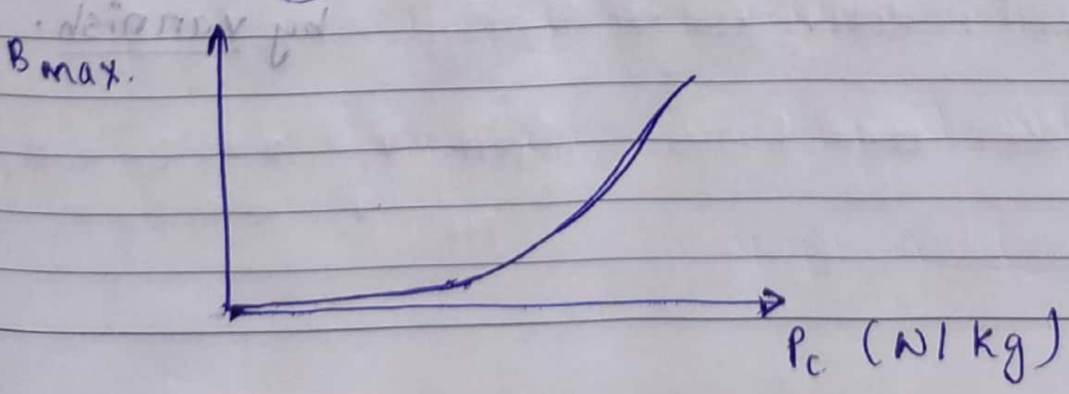
1) Hysteresis loss

→ $H_{Loss} = A_c l_c \oint H_c \, dB_c$

→ H_{Loss} = Area under the B-H curve × volume of the core

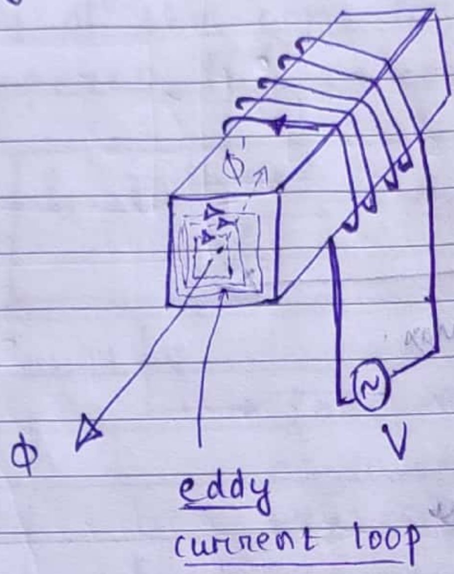
→ $H_{loss} \propto$ Frequency of the applied excitation

P_c = core loss (in W/kg)



2> Eddy current loss $ECL \propto f^2$ (f = frequency of flux variation)

$ECL \propto \phi_{max}^2$



To reduce eddy current -

→ Resistivity is increased by using silicon - steel material

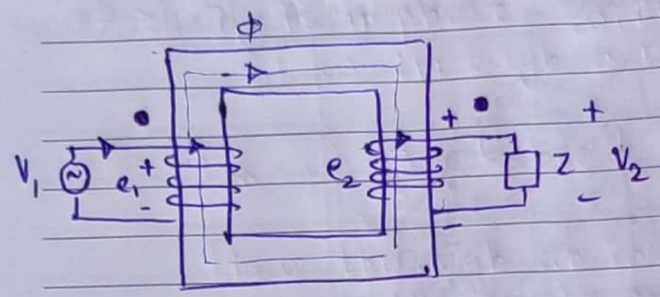
→ Laminated core

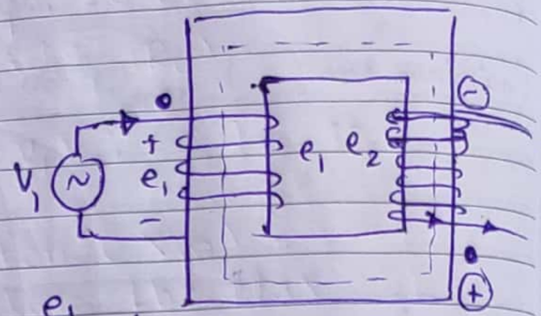
→ increases length of the eddy ct. path

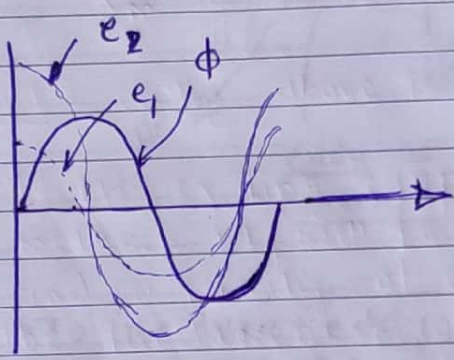
→ reduces the eddy ct.

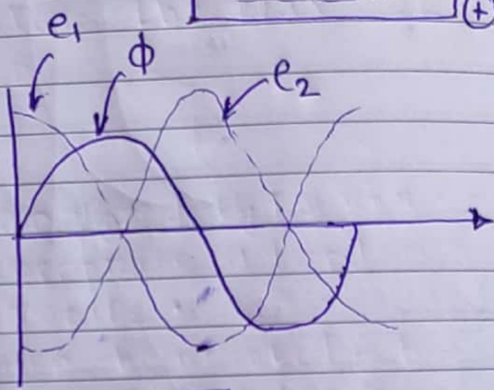
→ Lamination is done by varnish.

1-ϕ TRANSFORMERS









$$\boxed{\phi \text{ lags } \frac{\pi}{2} \text{ behind } e}$$

* Ideal Transformer

- No winding resistance
- No leakage flux
- No core loss
- Infinite relative permeability

For ideal transformer; $E_{1\,RMS} = V_1 = 4.44\, B_m A_c N_1 f$

$E_{2\,RMS} = V_2 = 4.44\, B_m A_c N_2 f$

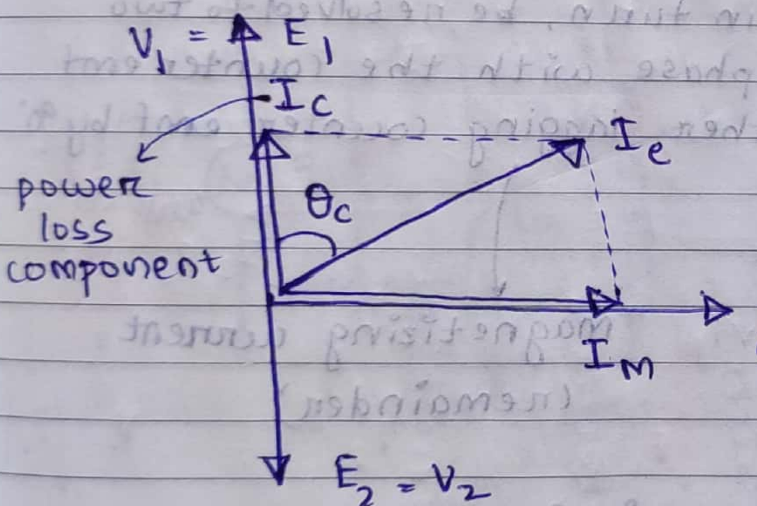
Then $\boxed{\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2}}$ — ideal transformer

$\boxed{\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2}}$ — for all transformer

$\boxed{N_1 I_1 = N_2 I_2}$ — ideal "

$\boxed{\dfrac{V_1 I_1}{V_2 I_2} = 1}$ $\Rightarrow$ input KVA = output KVA

* No load case



e_1, e_2 — out of phase (π)

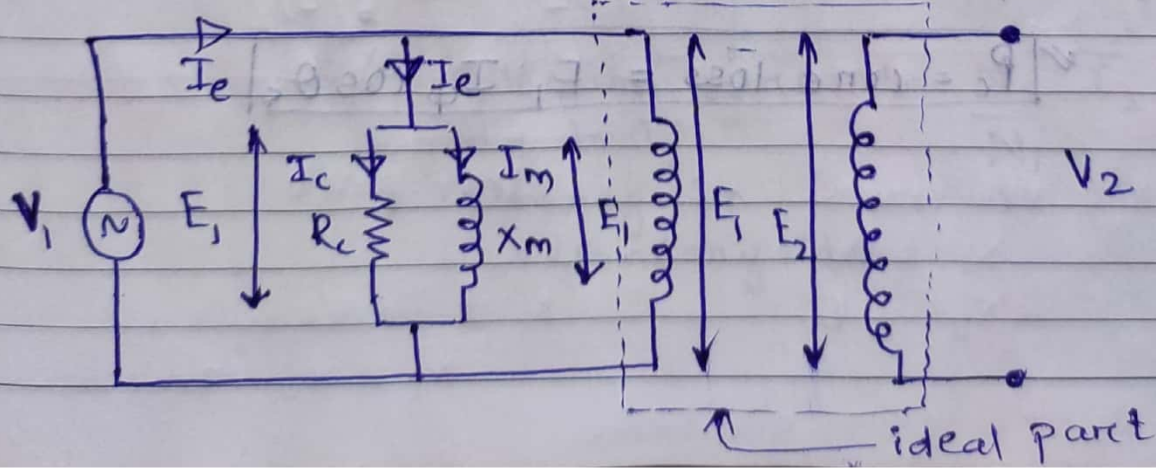
$\phi = \phi_m \sin \omega t$

$e_1 = e_{1m} \cos \omega t$

$e_2 = -e_{2m} \cos \omega t$

$I_m \rightarrow$ magnetising ct.

$I_e \rightarrow$ exciting ct. or no load ct.



→ I_c is in phase with E_1 ; so the I_c must flow through a resistor (R_c).

→ I_m is $\frac{\pi}{2}$ behind $E_1 (=V_1)$; so I_m must flow through an inductor (X_m).
X_m is there to maintain the flux.

<u>NOTE:</u> (Why above is done?)

→ If the exciting current is analyzed by Fourier series methods, it will found to comprise a fundamental and a family of odd harmonics.

→ The fundamental can, in turn, be resolved to two components – ⓐ one in phase with the counter emf
ⓑ the other lagging counter emf by 90°

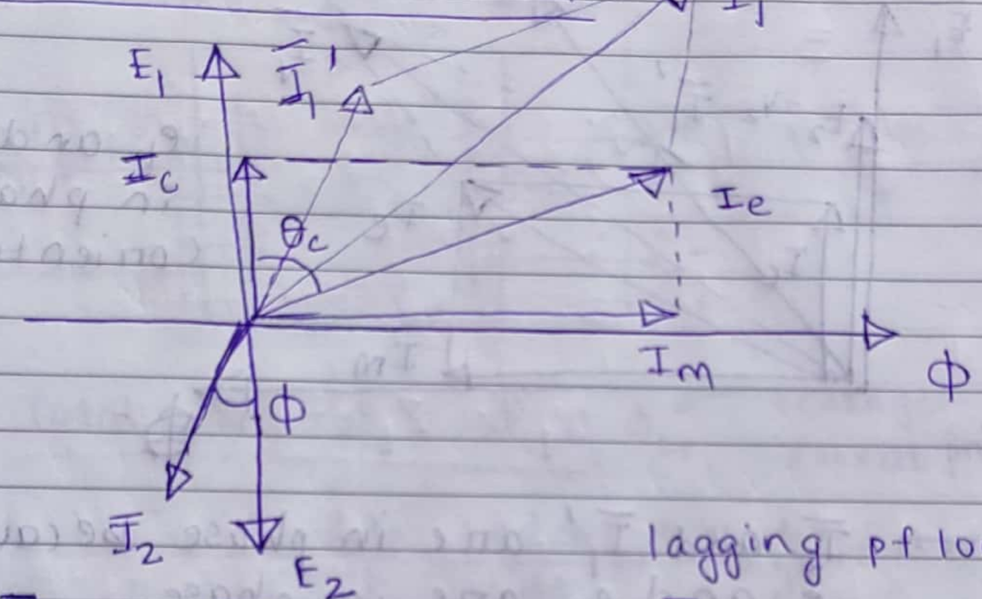
ⓐ core-loss component (hysteresis + eddy current loss)

ⓑ magnetizing current (remainder)

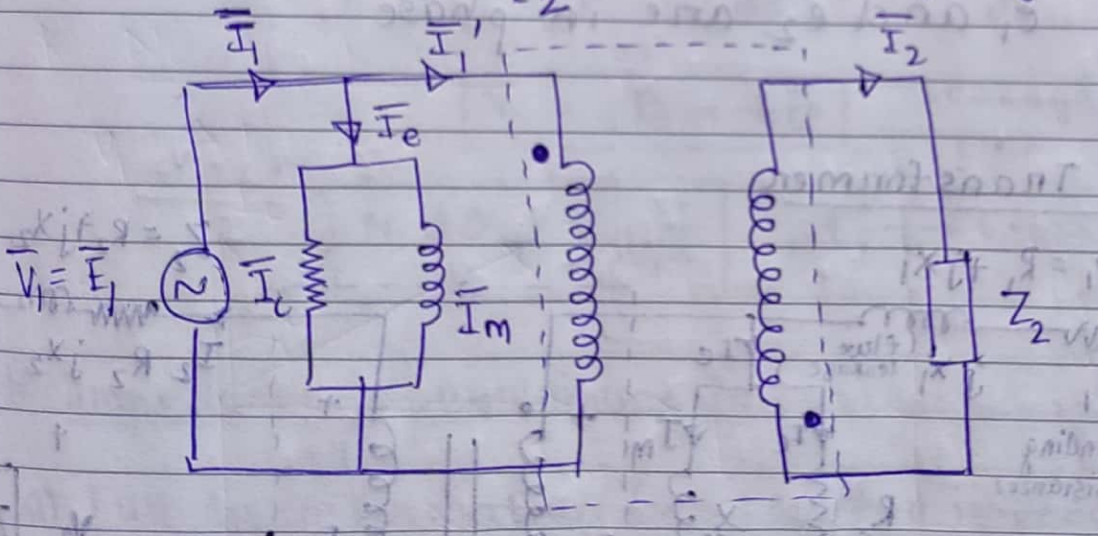
– principle harmonic is third.

$$\boxed{P_c = \text{core loss} = E_1 I_\phi \cos\theta_c}$$

* Transformer is loaded



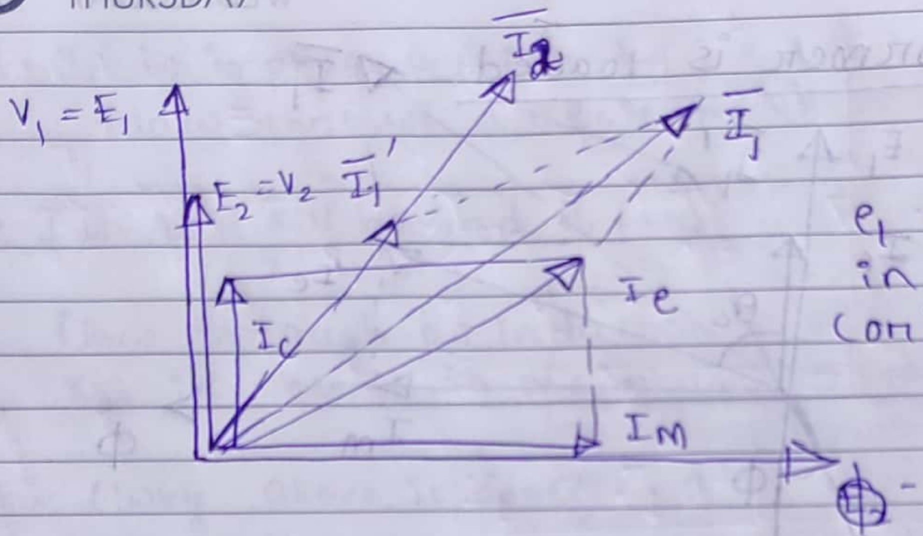
lagging pf load.



(e_1, e_2 are 180° out of phase)

$$\bar{I}_1 = \bar{I}_1' + \bar{I}_e$$

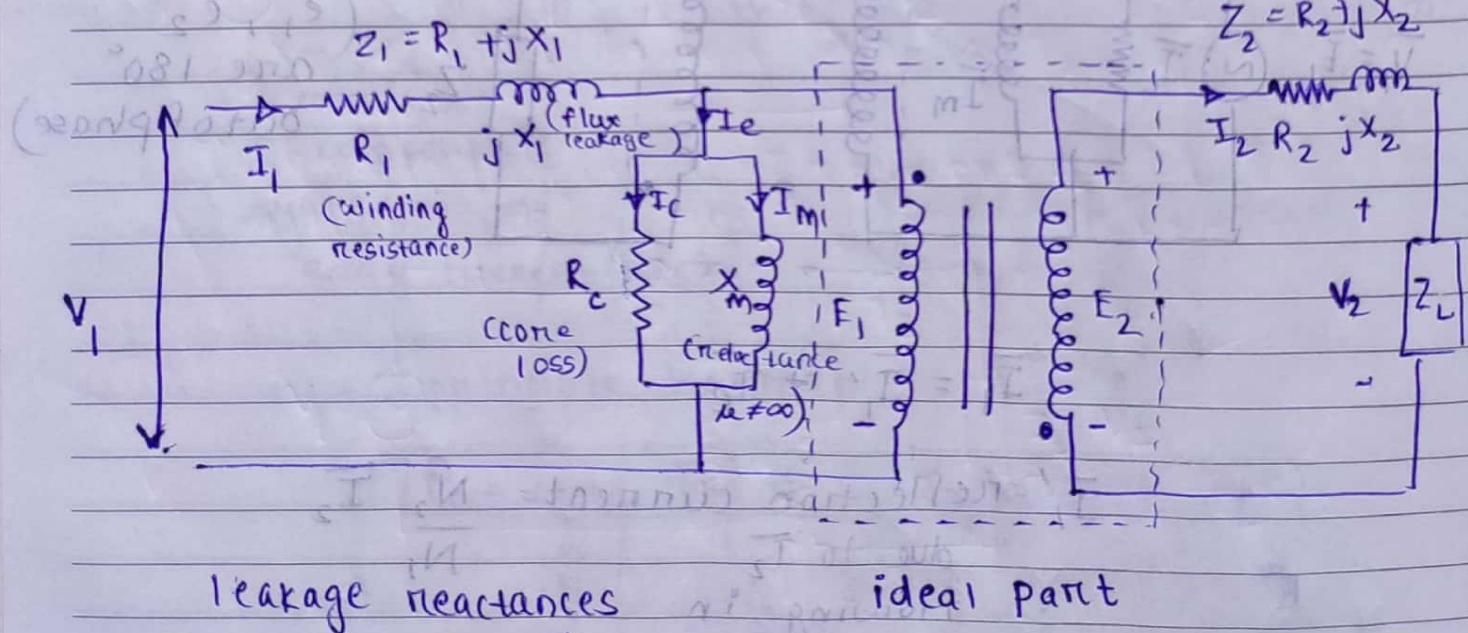
$\bar{I}_1'$ - reflection current due to $\bar{I}_2$ flowing in secondary side $= \dfrac{N_2}{N_1}\bar{I}_2$



e_1 and e_2 are in phase (orientation of dots)

→ $\bar{I}_2$ and $\bar{I}_1'$ are in phase because e_1 and e_2 are in phase

Model of Transformer



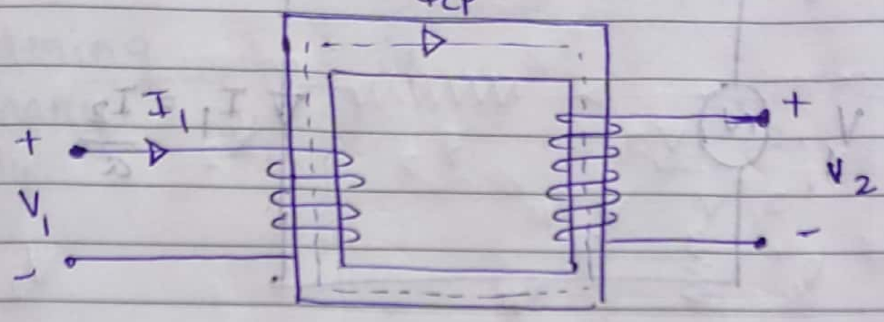
leakage reactances
$X_1 = \omega L_1$
$X_2 = \omega L_2$

ideal part

As f increases, X_1 & X_2 increases.

$R_1, R_2 \rightarrow$ winding resistance

* Leakage flux



Total flux $\phi_p = \phi_{cp} + \phi_{LP}$ ← Leakage flux at primary

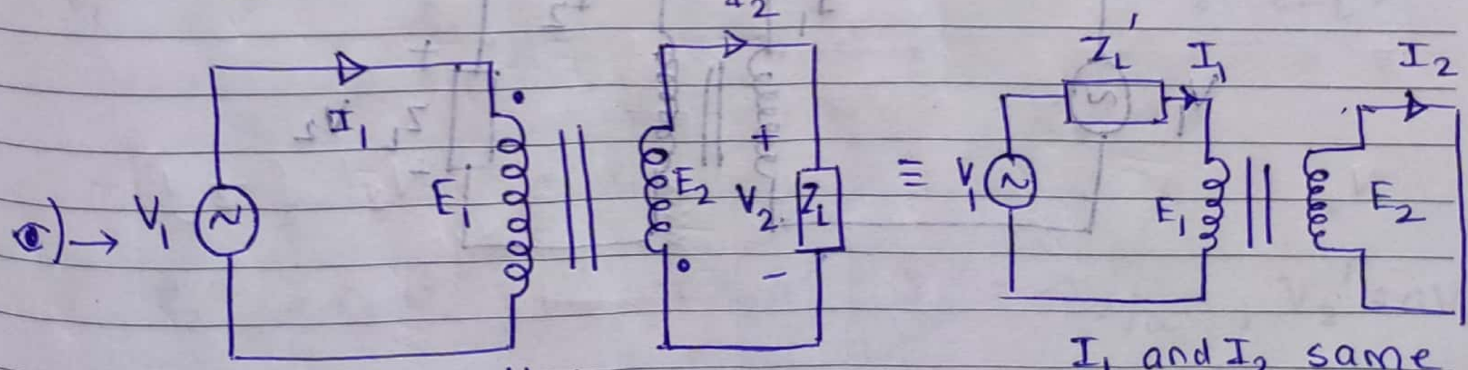
$$N_1 \frac{d\phi_p}{dt} = N_1 \frac{d\phi_{cp}}{dt} + N_1 \frac{d\phi_{LP}}{dt}$$

$$\boxed{V_1 = E_1 + E_{LP}}$$ — Leakage EMF

$$\boxed{N_1 \frac{d\phi_{LP}}{dt} = L_1 \dot{I}_1}$$, L_1 → Leakage inductance

* Impedance transformation

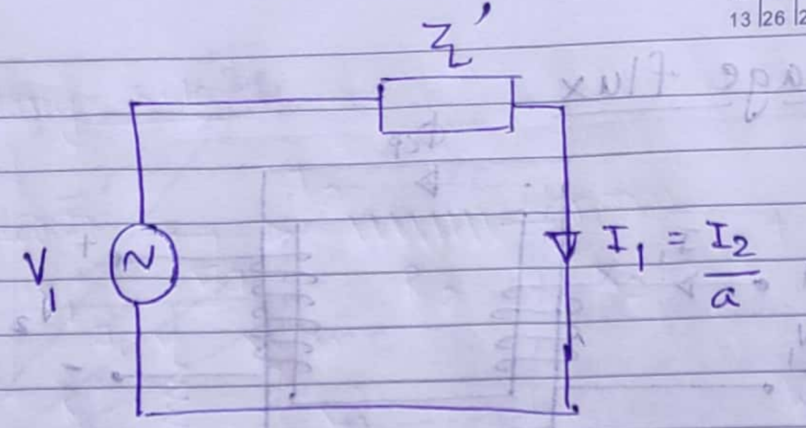
a) Full transformation (whole load impedance transferred)



I_1 and I_2 same as previous diag.

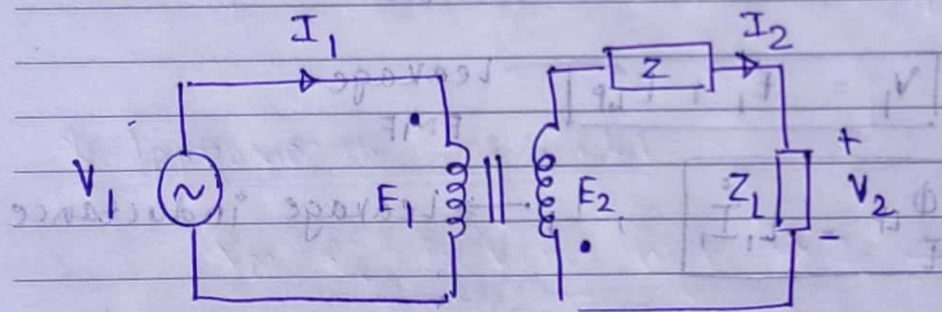
$$\left(\frac{N_1}{N_2} = a\right)$$

=



$$\boxed{Z_L' = Z_L a^2}$$

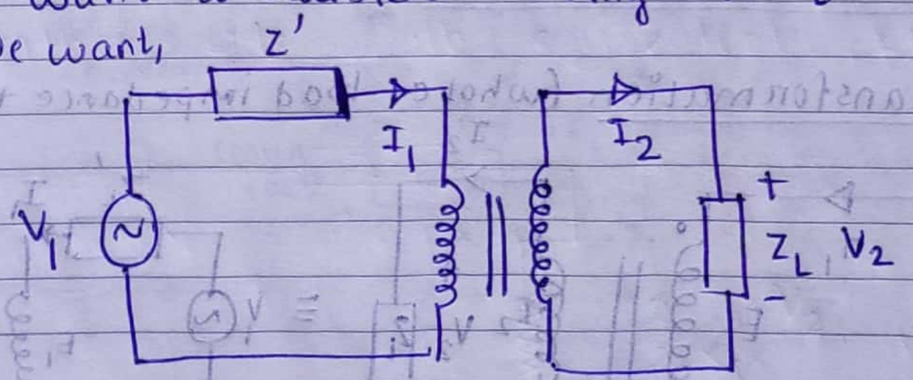
b) Partial transformation



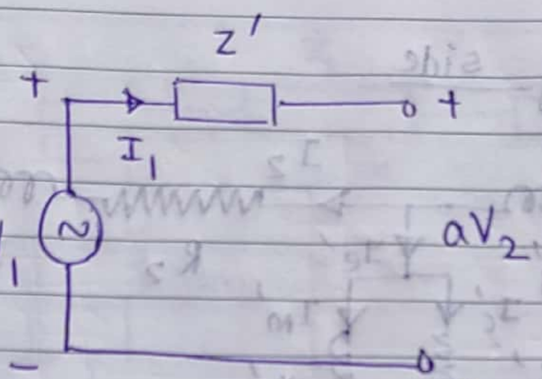
$$Z = \frac{E_2 - V_2}{I}$$

We want to transfer Z only not Z_L.

We want,



= considering primary only



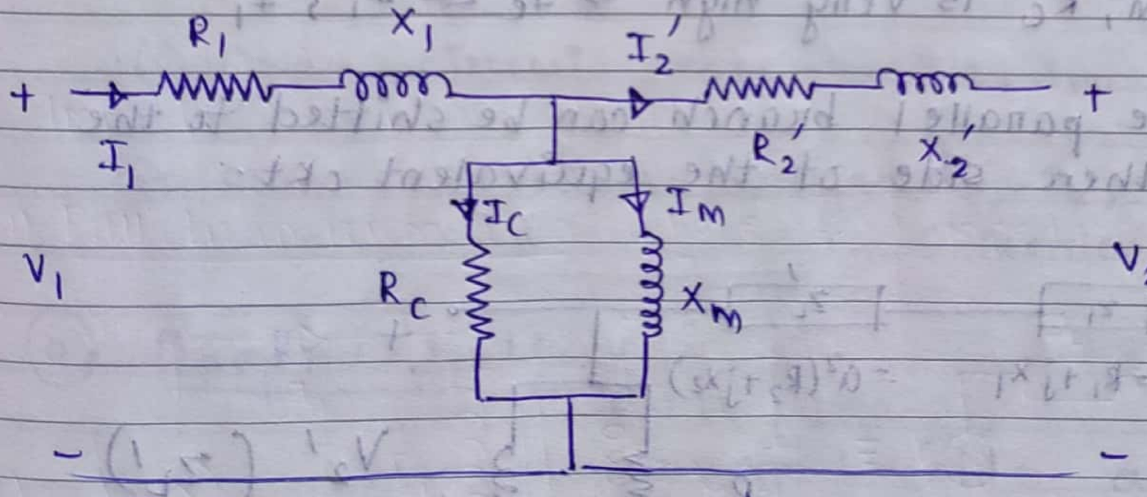
← primary eq. ckt.

$$\Rightarrow Z' = \frac{V_1 - aV_2}{I_1} = \frac{E_1 - aV_2}{(I_2/a)} = \frac{aE_2 - aV_2}{(I_2/a)}$$

$$\Rightarrow \boxed{Z' = a^2 Z}$$

* Transformer equivalent ckt

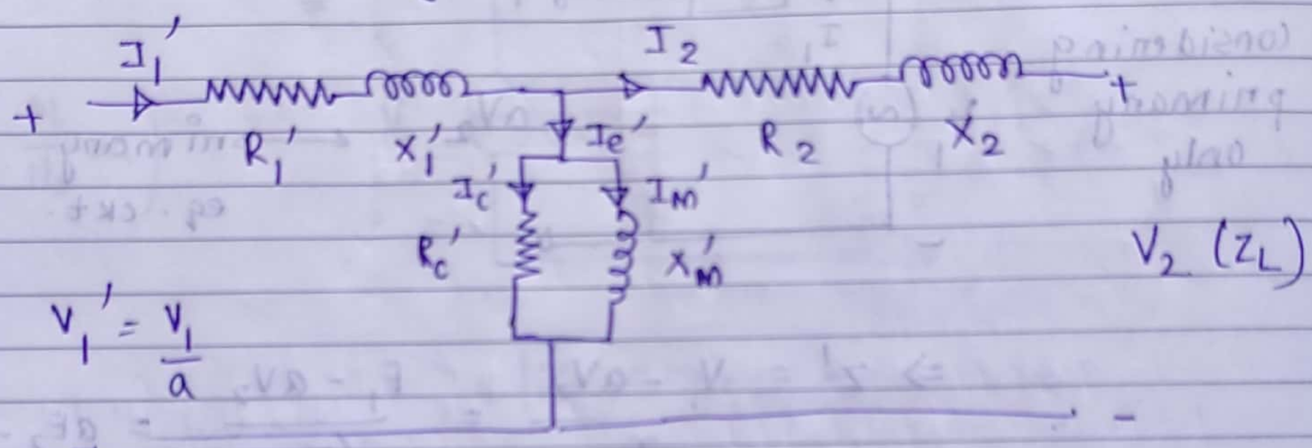
(a) w.r.t primary side



$$R_2' = R_2 a^2 \; ; \; X_2' = X_2 a^2 \; ; \; I_2' = I_2/a \; , \; V_2' = aV_2$$

$$a = \frac{N_1}{N_2} = \text{turns ratio}$$

(b) w.r.t secondary side

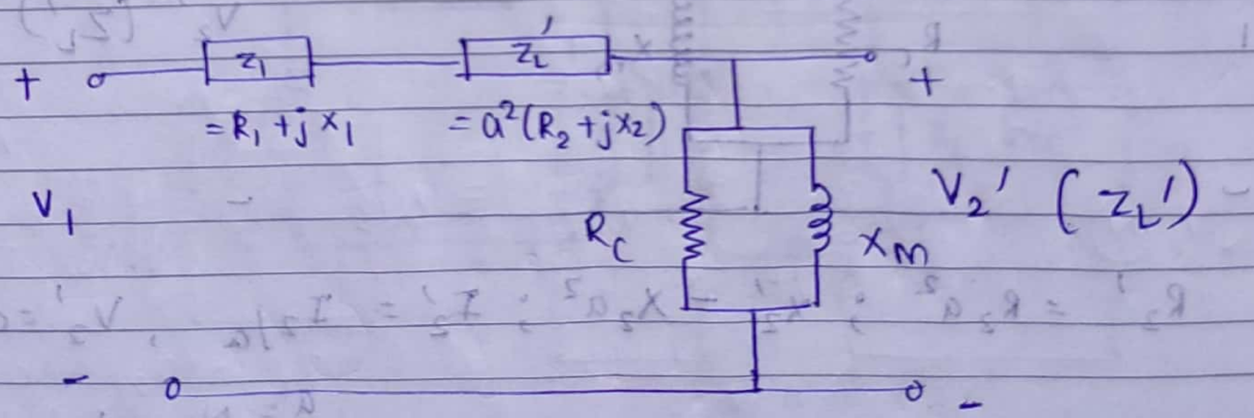


$X_1' = X_1/a^2$; $R_1' = R_1/a^2$; $I_1' = I_1 \cdot a$ etc.

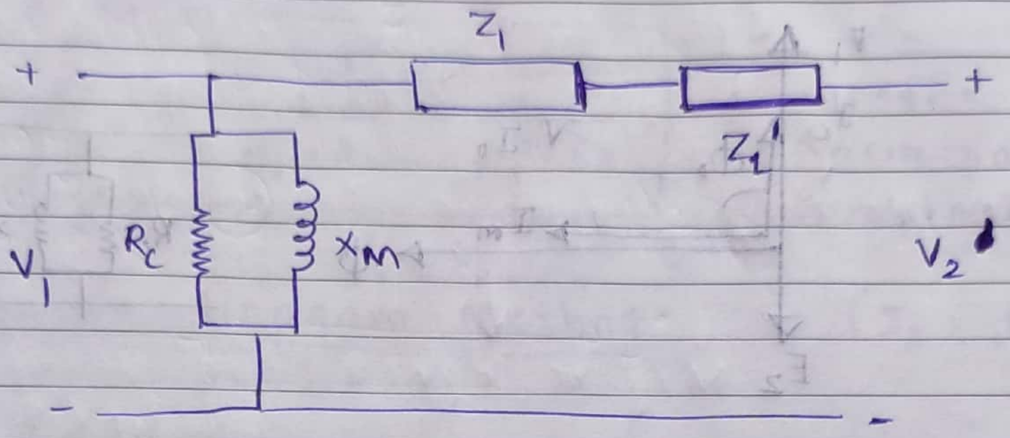
* Approximate eqv. ckt.

X_m, R_c is very high $\Rightarrow I_e \ll I_1, I_1'$

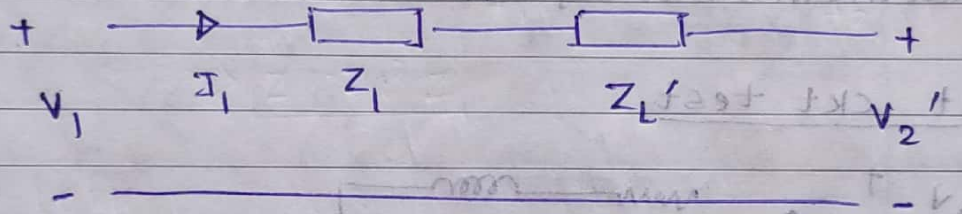
$\Rightarrow$ The parallel branch can be shifted to the either side of the equivalent ckt.



(OR)



(OR) complete removal -



* Parameters of eqv. tr. ckt.

1. Open circuit test $\longrightarrow$ R_c, X_m (shunt parameters)
2. Short circuit test $\longrightarrow$ R_1, R_2, X_1, X_2 (series parameters)

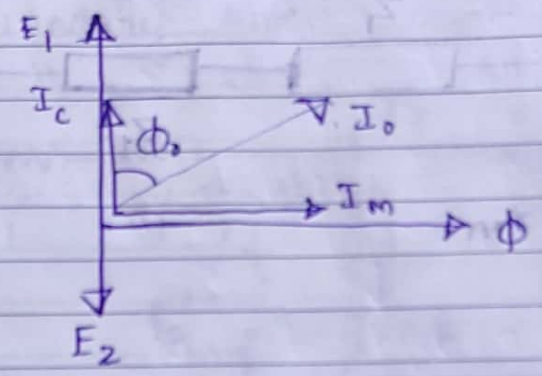
(a) Open ckt test

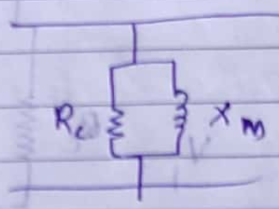
Instruments on LV side and HV side kept open.

core loss or iron loss at full voltage $\leftarrow$ $W_0 = V_0 I_0 \cos\phi_0 \Rightarrow \cos\phi_0 = \dfrac{W_0}{V_0 I_0}$

$I_c = I_0 \cos\phi_0 \Rightarrow R_c = \dfrac{V}{I_c}$

$I_m = I_0 \sin\phi_0 \Rightarrow X_m = V/I_m$

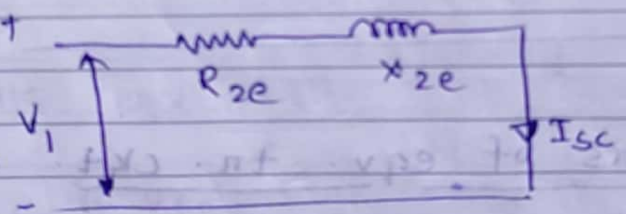




As HV side is open, the resultant is inductive in nature, so LPF Wattmeter used.

(b) Short ckt test

Instruments joined to HV side ; LV side shorted side

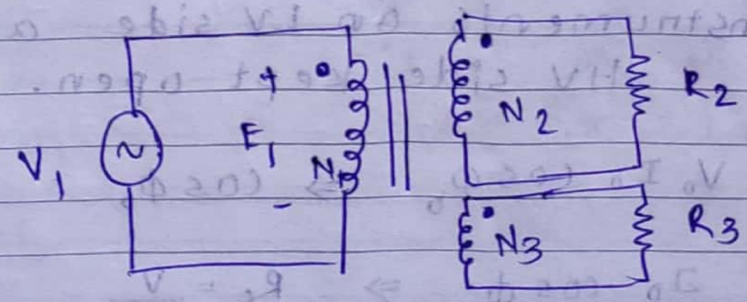


$$W_{sc} = I_{sc}^2 \cdot R_{2e}$$

$$\Rightarrow \boxed{R_{2e} = \frac{W_{sc}}{I_{sc}^2}}$$

$$Z_{2e} = \frac{V_{2e}}{I_{sc}} = \sqrt{R_{2e}^2 + X_{2e}^2}$$

NOTE :

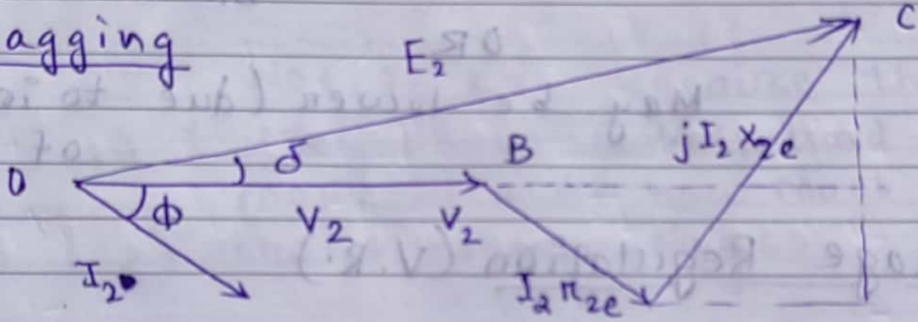


$$\boxed{Z'_{eq} = \frac{1}{\frac{1}{a_2^2 \cdot R_2} + \frac{1}{a_3^2 \cdot R_3}}}$$ (Transfer and their parallel combination)

(*) Phasor diagram method

(I_2 = full load ct.)

a) Lagging



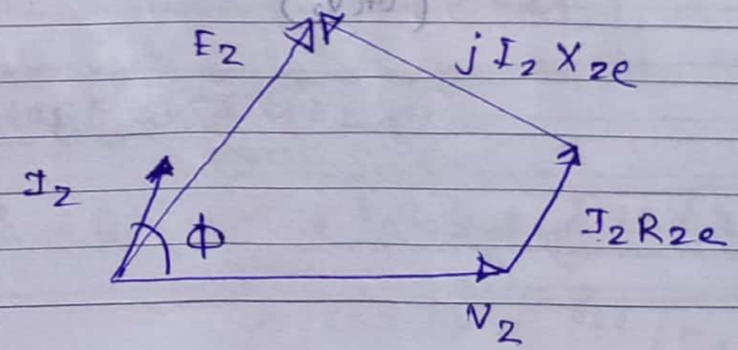
derive and use $E_2^2 = (V_2 + r_{2e}\cos\phi\, I_2 + I_2 x_{2e}\sin\phi)^2 + (I_2 x_{2e}\cos\phi - I_2 r_{2e}\sin\phi)^2$

For worst case regulation or approximation use → $E_2 \approx V_2 + I_2 r_{2e}\cos\phi + I_2 X_{2e}\sin\phi$

$$\boxed{E_2 - V_2 \approx I_2 r_{2e}\cos\phi + I_2 X_{2e}\sin\phi}$$

OB ≈ OC i.e. δ is very small

b) Leading



IMP. → $V_2 < E_2$ for lagging pf load
→ $V_2 > E_2$ for leading pf load

→ The receiving voltage from generating station
↓
May be higher (due to capacitance b/w line & ground)
OR
May be lower (due to inductance of line)

(*) Voltage Regulation (V.R.)

$$\% \text{ V.R.} = \frac{E_2 - V_2}{E_2} \quad \text{or} \quad \frac{E_2 - V_2}{V_2}$$

Lagging pf, $\phi > 0$
Leading pf, $\phi < 0$

(Use Any formula; but mention clearly)

→ V.R. should be as less as possible for a good transformer (i.e., near zero)

$$\text{V.R.} = \frac{I_2}{E_2 \ (\text{or } V_2)} \left\{ R_{2e} \cos\phi + X_{2e} \sin\phi \right\}$$

(*) Transformer Rating -

KVA, $\frac{V_1}{V_2}$ (voltage ratio), frequency

V_1 → maxm voltage we can give

V_2 → maxm voltage we can get

→ ϕf = constant for a fixed voltage supply

⇒ As $f\uparrow$, $\phi\downarrow$ & vice-versa

OR $f\downarrow$, $\phi\uparrow$ & saturation may reach (India - USA)

→ Not in KW, because the load may draw KVAR, and we couldn't make idea of how much KVAR is transformer rating.

→ Primary-side rated ct = $\frac{KVA}{V_1}$

→ secondary-side rated ct = $\frac{KVA}{V_2}$

1) Zero voltage regulation

V.R. = 0 ⇒ $\boxed{\tan\phi = -\frac{r_{2e}}{x_{2e}}}$

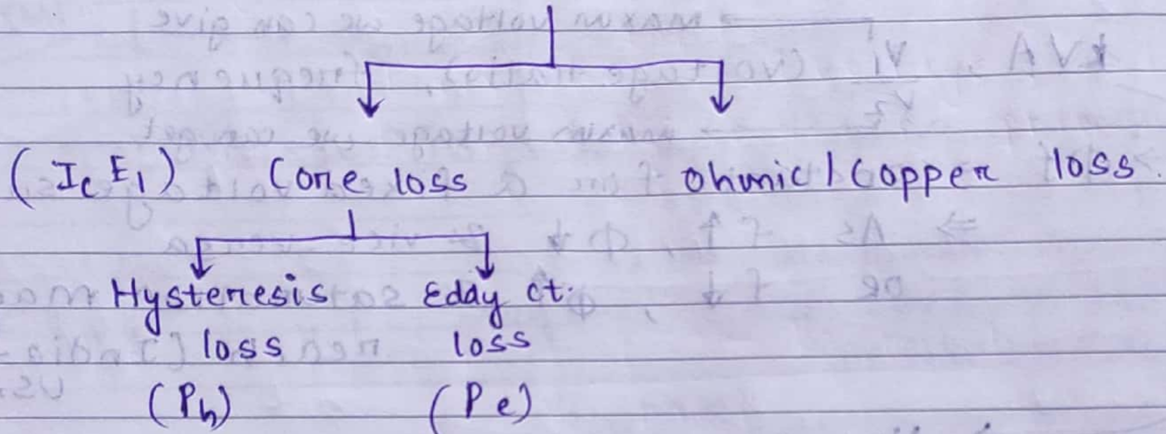
⇒ $\boxed{\cos\phi = \frac{X_{2e}}{Z_{2e}}}$

2) Maxm. voltage regulation

$\frac{d}{d\phi} V.R = 0$ ⇒ $\boxed{\tan\phi = X_{2e}/r_{2e}}$

⇒ $\boxed{\cos\phi = r_{2e}/Z_{2e}}$

* Losses in transformer



$\rightarrow P_h$ = hysteresis loss $= K_h \cdot f \cdot B_m^{x}$

$\rightarrow P_e$ = eddy current loss $= K_e f^2 B_m^2$

$\rightarrow B_m f \propto V$

$\Rightarrow P_h \propto V^{x} \cdot f^{1-x}$ —— (1) ← x = 1 to 2

$\Rightarrow P_e \propto V^2$ —— (2)

$x \rightarrow$ Steinmetz's constant

$K_h \rightarrow$ hysteresis constant, depends on the magnetic material

$K_e \rightarrow$ eddy ct. constant, depends on thickness of lamination.

$\rightarrow P_{iron} / P_{core} = P_h + P_e = K_h f B_m^{x} + K_e f^2 B_m^2$

$= K_h' f + K_e' f^2$

(as B_m = constant)

* Efficiency

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

$$= \frac{x \times (KVA) \times \cos\phi}{x \times (KVA) \times \cos\phi + P_C + x^2 P_{cu,fL}}$$

Max^m η, - at constant $\cos\phi$

$\longrightarrow x = \sqrt{\frac{P_C}{P_{cu,fL}}} \Rightarrow P_C = x^2 P_{cu,fL}$

$\Rightarrow P_C = P_{cu}$

$\Rightarrow$ Fixed value = variable value

$$\longrightarrow \boxed{\eta_{max} = \frac{x (KVA) \cos\phi}{x (KVA) \cos\phi + 2P_C}}$$

(True for power transformer, not for distribution transformer)

- Energy efficiency (or all-day efficiency)

→ Useful for distribution tr. when load is varying over the day.

$$\longrightarrow \eta = \frac{\text{output KWH}}{\text{Output KWH} + \text{loss in KWH}}$$

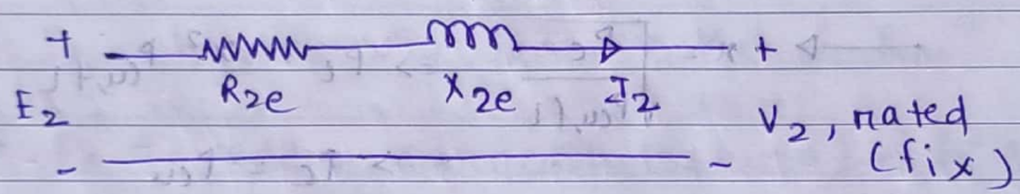
→ Core loss (primary side - so fixed)

→ Copper loss (vary throughout day)

* Voltage regulation under the rated load voltage

(We would fix the rated load voltage, and we would derive it by increasing primary side voltage)

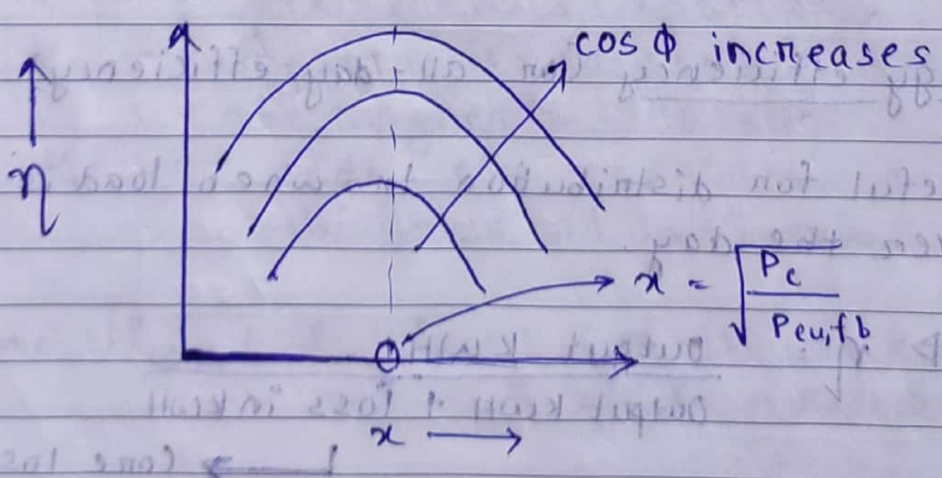
$$V.R. = \frac{V_2|_{\text{at no load}} - V_{2,\text{rated}}}{V_{2,\text{rated}}}$$



$$\bar{E_2} = \bar{V_2}_{,\text{rated}} + \bar{I_2}(r_{2e} + j\,x_{2e})$$

$$\Rightarrow \bar{E_2} = V_{2,\text{rated}} \angle 0^\circ + \bar{I_2}(r_{2e} + j\,x_{2e})$$

* Efficiency curve



→ At fixed x, $\eta_{max.}$ occurs at unity power factor (UPF).

* Per-unit resistance drop of a transformer

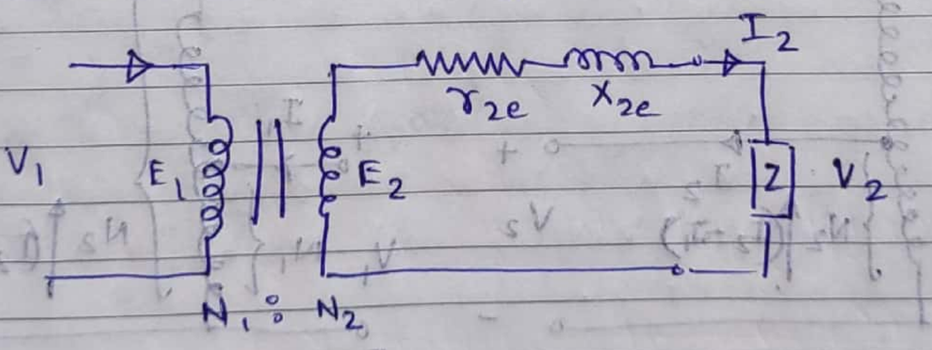
$$= \frac{(\text{full-load primary current}) \times (\text{eq. resistance referred to primary circuit})}{\text{Primary voltage}}$$

Multiply by 100 to convert it to percentage

$$= \frac{(\text{full-load secondary ct.}) \times (\text{eqv. resistance referred to secondary ckt.})}{\text{secondary voltage.}}$$

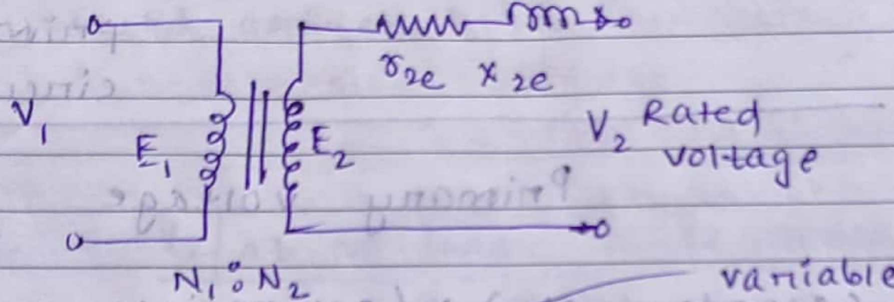
* Cases of Voltage Regulation

• V.R. when Tr. primary side is supplied rated voltage



$$V.R. = \frac{E_2 - V_2}{E_2} \quad — ①$$

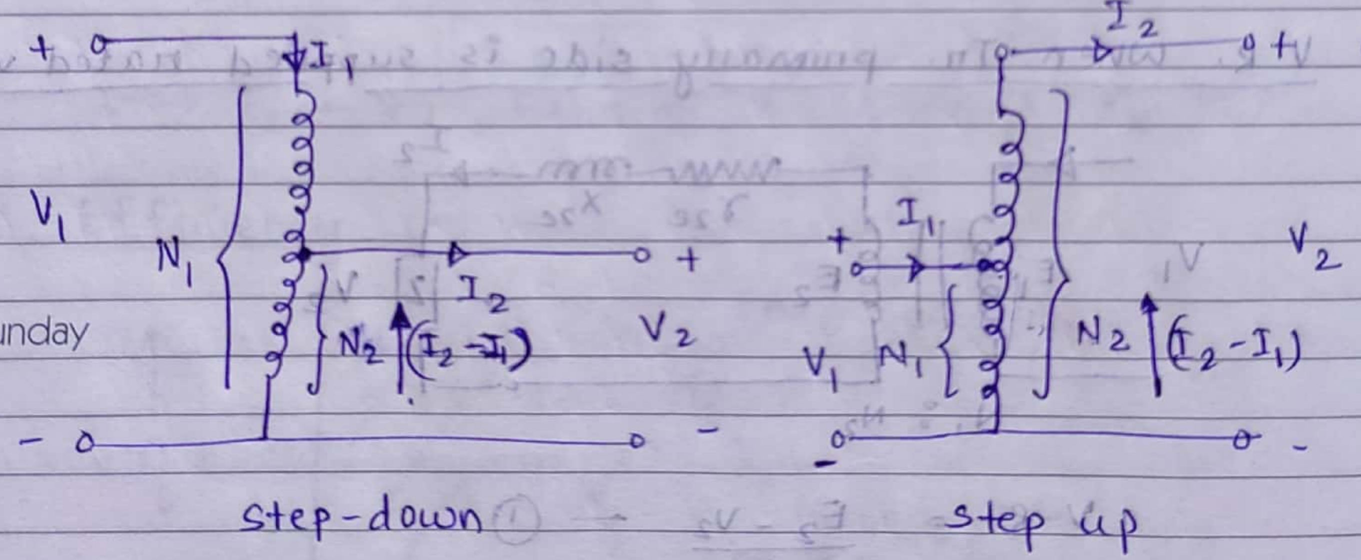
• V.R. when rated voltage is reqd. at the tr. secondary stage



$$V.R. = \frac{E_2 - V_2}{V_2}$$

variable, fixed

* 1-φ Auto Transformer (single-winding transformer)



→ Energy transfer by conduction (common portion) and by magnetic coupling.

Economical

Volume of Cu $\propto$ Length of wire × C.S. Area of Cu wire

$\propto NI$

$$\frac{(Vol)_{Auto}}{(Vol)_{TW}} = \frac{(N_1 - N_2)I_1 + N_2(I_2 - I_1)}{N_1 I_1 + N_2 I_2} = \frac{N_1 - N_2}{N_1}$$

input kVA $= V_1 I_1$

conduction VA $= V_2(I_2 - I_1)$

transferred VA $= V_1 I_1 - V_2(I_2 - I_1)$ (magnetic)

→ potential divider : only conduction
transformer : magnetic coupling

(*) Per Unit System

$$= \frac{\text{The actual value in any units}}{\text{The base or reference value in the same units}}$$
(such as the base can be transformer ratings itself)

Example 3.10 (Nagrath & Kothari)

Example 3.16 (D. Das)

Ex 3.9 "

(*) Ratio = $\frac{\text{Given voltage}}{\text{Out voltage}}$

Auto trans / Trans

NPTEL

THREE PHASE INDUCTION MOTORS

→ Two main parts:
1) a stationary stator
2) a revolving rotor.

The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm depending upon the power of the motor.

→ Principle: The operation of a three-phase induction motor is based upon ~~all~~ the application of Faraday's Law and the Lorentz force on a conductor.

1) A voltage $B l v = E$ is induced in each conductor while it is being cut by the flux (Faraday's law)

2) The induced voltage immediately produces a current I, which flows down the conductor, through the end bars, and back through the other conductors.

3) Because the current-carrying conductor lies the magnetic field of the permanent magnet it experiences a mechanical force (Lorentz force)

4) The forces always acts in a direction to drag the conductor along with the magnetic field.

→ In an induction motor, the ladder is closed upon itself to form a squirrel-cage, and the moving magnet is replaced by a rotating field.

n_s = synchronous speed (rotating magnetic field) = $\frac{120f}{P}$

n = rotor speed

Infinite bus = V, f are constant

Rotor is open-circuit now

→ The machine acts merely as a transformer where the stator (primary) and the rotor (secondary) have emf s of the same frequency induced in them by the rotating magnetic flux rather than time-varying flux.

→

Rotor is short-circuited and stationary

Wk	M	T	W	T	F	S	S
13	30						1
14	2	3	4	5	6	7	8
15	9	10	11	12	13	14	15
16	16	17	18	19	20	21	22
17	23	24	25	26	27	28	29

→ s = slip = $\frac{n_s - n}{n_s}$

→ Rotor frequency ~~speed~~ -

$\boxed{f_2 = sf}$ (rotor frequency)

$n = (1-s)\, n_s$ —— (3)

$\frac{120sf}{P} = n_s - n$ —— (4)

- The slip 's' is the per unit speed (w.r.t. synchronous speed) at which the rotor slips behind the stator field.

Wk	M	T	W	T	F	S	S
13	30						
14	2	3	4	5	6	7	8
15	9	10	11	12	13	14	15
16	16	17	18	19	20	21	22
17	23	24	25	26	27	28	29

THREE-PHASE INDUCTION MOTOR

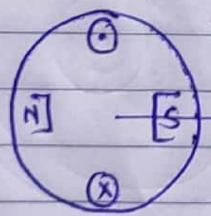
* **Main parts** – ① Stator
② Rotor

IM

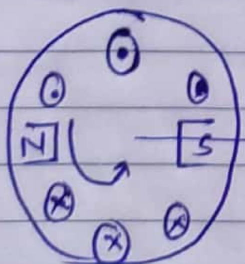
Cage type	Wound rotor or slip ring
→ Less maintenance cost	→ High starting torque (Ext. resistance can be added)
→ More robust	→ Better speed control
→ starting torque is less	→ Frequent maintenance

Working Principle: Rotating Magnetic Field

08 Sunday



→ mmf (pulsating magnetic-field; 1-φ)



→ mmf (pulsating + rotating magnetic field)
↓
synchronous speed (N_s)

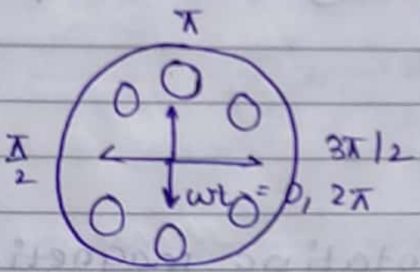
$$\boxed{N_s = \frac{120f}{P\,(\text{no. of poles})}}$$

→ $\boxed{\theta_M = \frac{2}{P}\theta_e}$

θ_e → electrical angle is angle between poles (rotor vs stator)

θ_M → Mechanical angle is angle of rotor shaft vs stator

→ The mmf rotates (poles) but with a constant magnitude of $\underline{1.5\,F_M}$.



→ speed of rotating magnetic field $= \omega_m$ $(\because \theta_m = \theta_e)$ $= \omega_e$ $= 2\pi f$

For a p-pole m/c $\omega_m = \frac{2}{P}\omega_e$

synchronous speed (in rad/s) $\boxed{\omega_m = \frac{4\pi f}{P} \text{ rad/s}}$

$\Rightarrow \overbrace{\frac{2\pi}{60} N_s} = \frac{4\pi f}{P} \Rightarrow \boxed{N_s = \frac{120f}{P}}$

N_s ↑ rev/min.

→ The flux of stator magnetic field cuts rotor ; produces three phase voltage in rotor due to 3 - ϕ windings on it, currnt flows, torque experienced

produces → Rotating magnetic field produced by rotor at N_{sr} speed (synchronous w.r.t rotor)

→ cause of emf induced ⟶ relative motion b/w stator and rotor

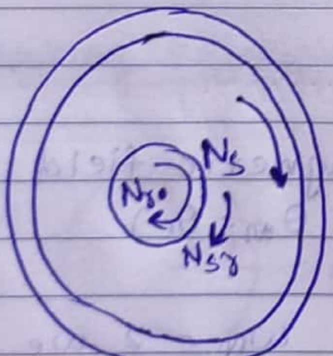
⇓

rotor thus moves in such a way so as to catch up N_s but not possible

slip $(S) = 0$

$N_r = N_s$ (Not possible as here no magnetic locking) ⟸

→



N_s = stator rotating magnetic field $N_s = \frac{120f}{P}$ wrt stator

N_{sr} = rotor rotating magnetic field $N_{sr} = \frac{120\,sf}{P} = SN_s = \frac{120f_r}{P}$

wrt rotor

speed of rotor rotating magnetic field wrt stator

$= N_{sr} + N_r = SN_s + (1-s)N_s$

$= N_s$

(synchronous wrt stator)

speed of rotor rotating magnetic field wrt stator rotating magnetic field = 0

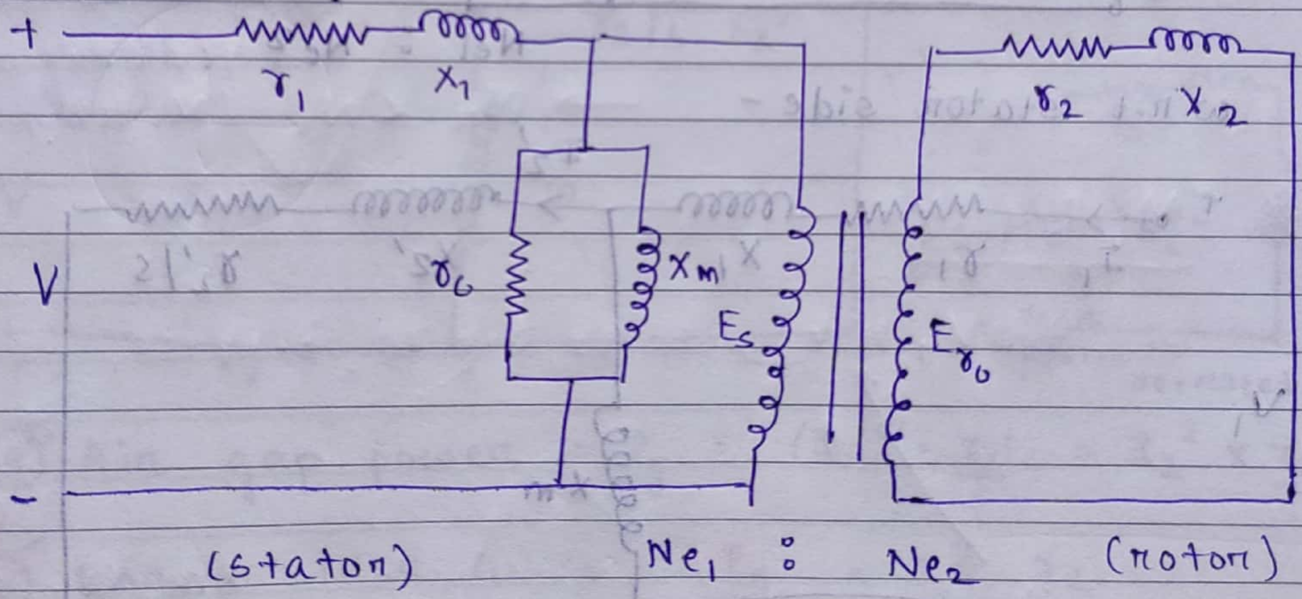
$$\boxed{S = \text{slip} = \frac{N_s - N_r}{N_s}}$$

$$\boxed{N_r = (1-s) N_s \quad \text{and} \quad N_{sr} = s N_s}$$

→ f_r = frequency of induced emf in rotor = sf

$$\boxed{f_r = sf}$$

* Ckt. diagrams



→ $E_s = 4.44\, \phi f N_s \cdot (K_{ws})$, (K_{ws}) → distribution factor for stator winding

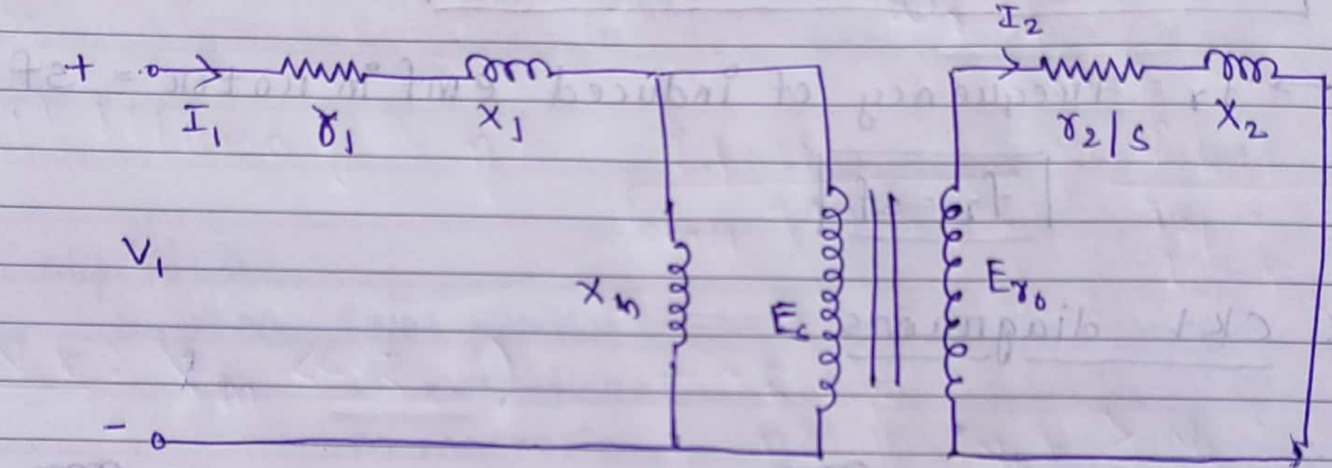
→ $E_r = 4.44\, \phi f_r N_r \cdot (K_{wr})$

ϕ → flux in the air gap

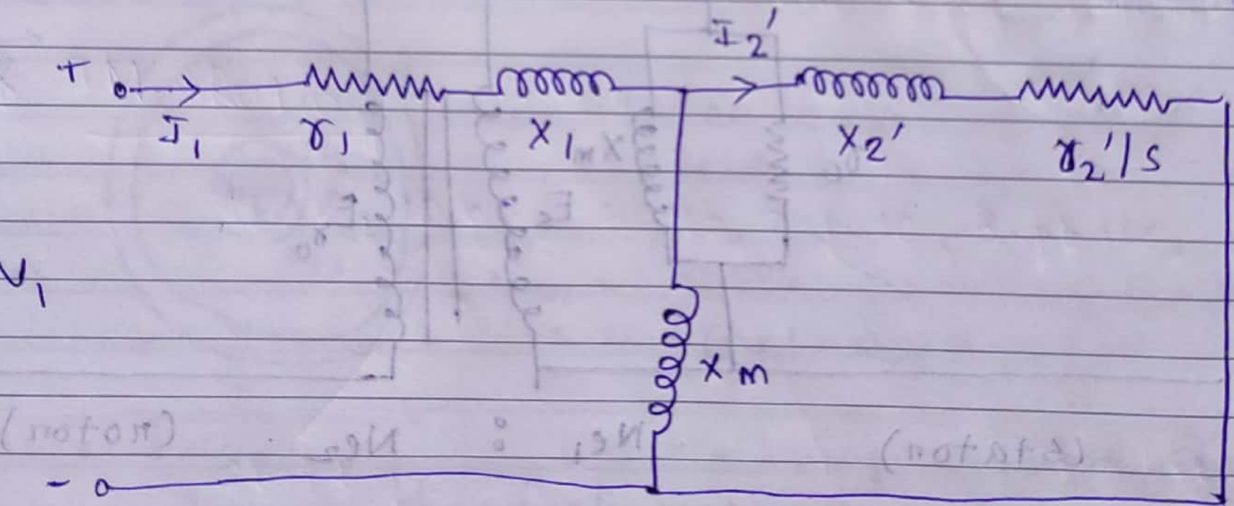
$$\boxed{\frac{E_s}{E_{r_0}} = \frac{N_{e1}}{N_{e2}}}$$ (N_{e1} & N_{e2} are effective no. of turn)

At standstill, the equivalent ckt is as shown above. (S = 1, just going to start)

→ When m/c revolves, eqv. ckt is :



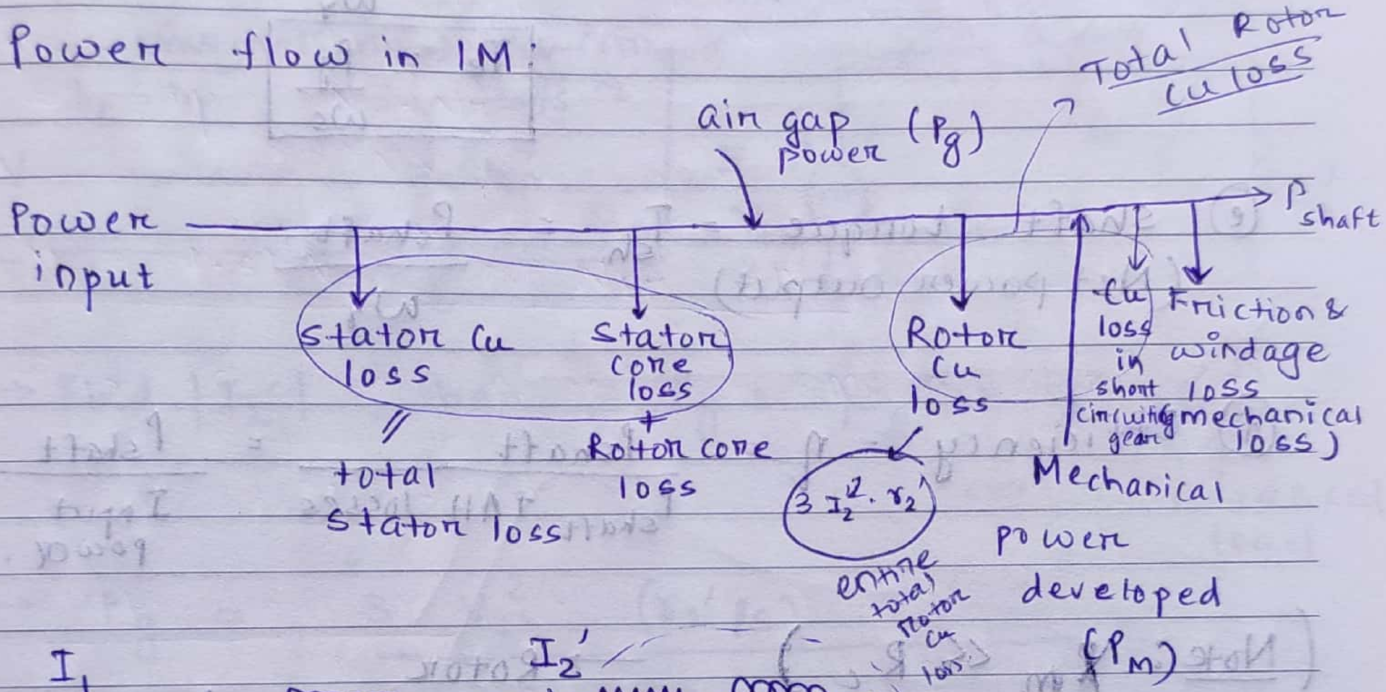
w.r.t stator side -

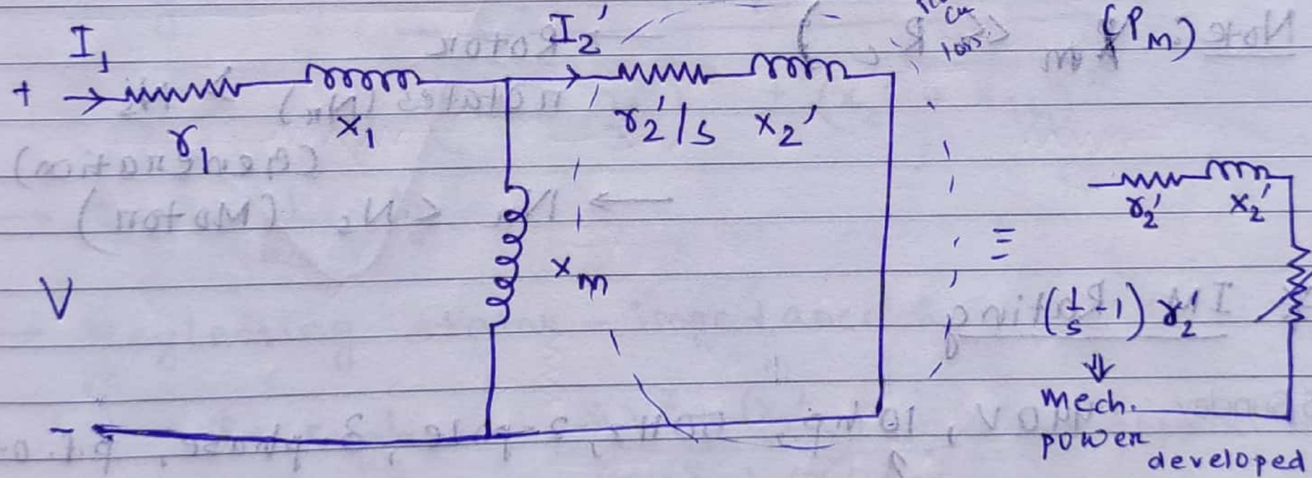


$$r_2' = r_2 \left(\frac{N_{e1}}{N_{e2}}\right)^2$$

* Power and torque

Power flow in IM:





(a) Air gap power $= P_g = (I_2')^2 \cdot \frac{r_2'}{s} = I_2^2 \times \frac{r_2}{s}$

(b) Rotor loss, Cu $= sP_g = I_2^2 \cdot r_2$

(c) Mechanical power developed (Gross power output)

$$P_m = P_g - sP_g$$
$$= (1-s)\,P_g$$
$$= \left(\frac{1}{s}-1\right) I_2^2 \cdot r_2$$

(d) Electromagnetic Torque $= T_e = \frac{P_m}{\omega_r}$

$$\boxed{T_e = \frac{P_g}{\omega_s}}$$

(e) Shaft torque $= T_{sh} = \frac{P_{shaft}}{\omega_r}$
(Net power output)

(f) Efficiency $= \eta = \frac{P_{shaft}}{P_{shaft} + \text{All losses}} = \frac{P_{shaft}}{\text{Input power}}$.

(Note: $X_m \ll R_c$) → Rotor rotates (N_r) > N_s (Generation)
→ $N_r < N_s$ (Motor)

* IM Rating

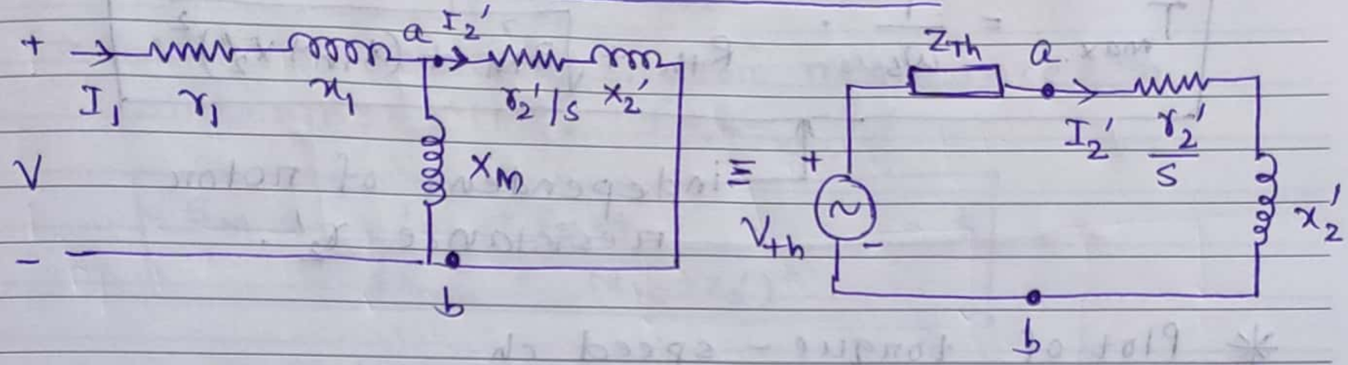
15 Sunday 440 V, 10 hp, 50 Hz, 2-pole, 3-phase, p.f. of i/p ct $\cos\phi$

10 hp: Shaft power at rated load, rated slip.

OR

Rated ct. also specified, Y or Δ - connected (of stator)

* Torque ~ speed characteristics



→ Find $|I_2'|$, then , $P_g = |I_2'|^2 \times \frac{r_2'}{s} \times 3$

Balanced load

→ $P_g = \dfrac{3 \cdot V_{th}^2 \cdot (r_2'/s)}{\left(R_{th} + \frac{r_2'}{s}\right)^2 + (x_2' + x_{th})^2}$

→ $T_e = \dfrac{1}{\omega_s} \cdot P_g$

→ Neglecting stator − impedance :

$$T_e = \frac{1}{\omega_s} \cdot \frac{3V^2 \cdot (r_2'/s)}{\left(\frac{r_2'}{s}\right)^2 + (x_2')^2}$$ → Approx. formula

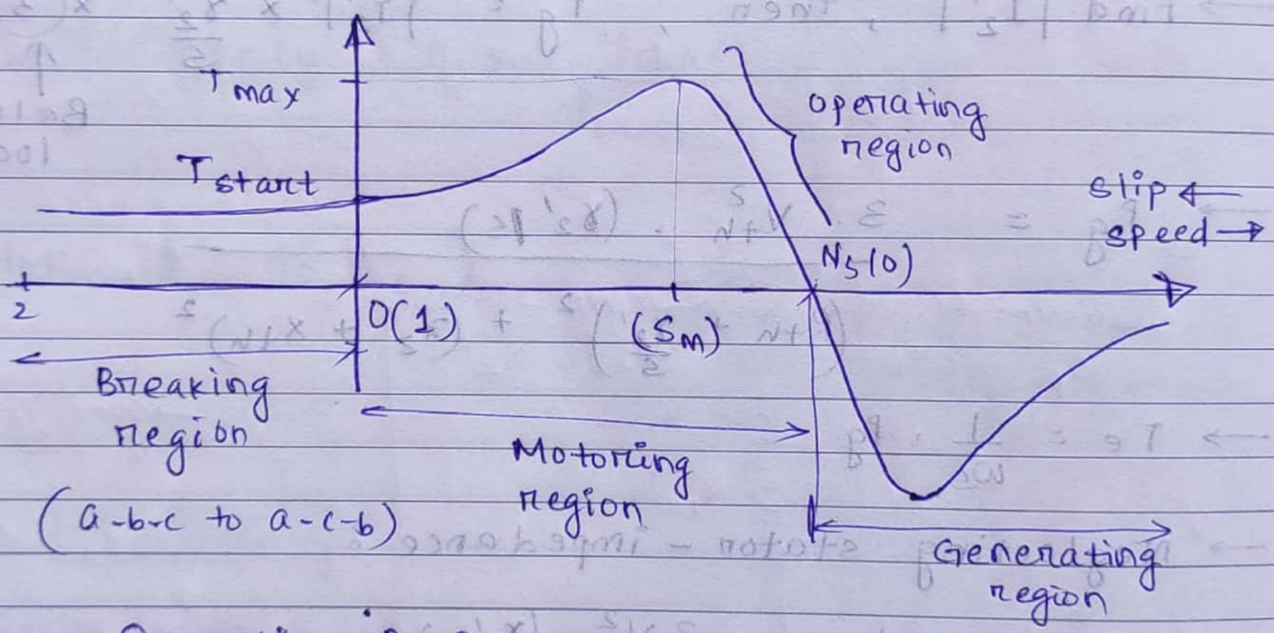
* Max^m torque & the corresponding slip

$\left.\dfrac{dT_e}{ds}\right|_{s=s_m} = 0 \Rightarrow \boxed{s_m = \dfrac{r_2'}{\sqrt{R_{th}^2 + (X_{th} + x_2')^2}}}$

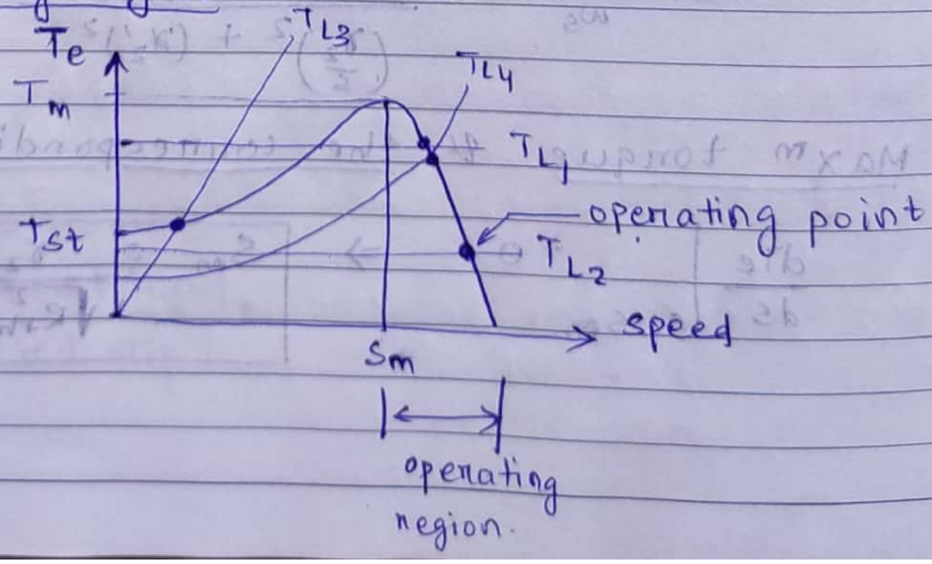
$$\boxed{T_{max.} = \frac{1}{\omega_s} \cdot \frac{1.5\, V_{th}^2}{R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_2')^2}}}$$

independent of rotor resistance r_2'.

* Plot of torque - speed ch.



- Operating Region

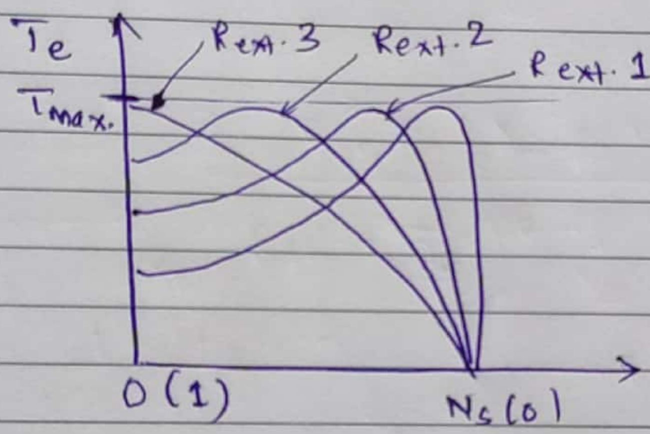


* Increase in Starting torque (s = 1)

→ Adding externally rotor resistance to slip-rings IM. $(= R_{ext.})$

$$\boxed{S_m = \frac{r_2' + R_{ext}}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}} = 1}$$

$$\cong \frac{r_2' + R_{ext}}{X_2'} = 1 \quad \text{(Neglect. stator imped.)}$$



$R_{ext.3} > R_{ext.2} > R_{ext.1}$

* IMP. FORMULAE

$$I_{fl}^2 \propto T_{fl} = \frac{3}{\omega_s} \cdot \frac{V^2 (R_2'/s_{fl})}{(R_2'/s_{fl})^2 + (X_2')^2}$$

$$I_{start}^2 \propto T_{start} = \frac{3}{\omega_s} \cdot \frac{V^2 R_2'}{(R_2')^2 + (X_2')^2}$$

$$I_{max}^2 \propto T_{max} = \frac{3}{\omega_s} \cdot \frac{0.5\, V^2}{X_2'}$$

→ At T_{fl}

$\frac{R_2'}{S_{fl}} >> X_2'$

V is taken since stator impedance is not there And use $S_{max} = \frac{R_2'}{X_2'}$

Wk	M	T	W	T	F	S	S
13	30						1
14	2	3	4	5	6	7	8
15	9	10	11	12	13	14	15
16	16	17	18	19	20	21	22
17	23	24	25	26	27	28	29

* For starting of IM, the ckt diagram becomes –