INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Spring Semester 2018-19

Date of Examination : 25/02/2019 **Session (FN/AN): FN** **Duration: 2 hrs** **Full Marks: 60**
Subject No. : *ME10001* **Subject : *Mechanics*** **Department/Center/School : All 1st Year UG**
Special Instructions: Attempt all **six** questions. Marks are indicated against the corresponding question. **Answer all the parts of one question at one place only.** Assume any data, if required, after clearly stating the reason and the value(s).

Question 1: (a) For what value of F the simplest resultant of the given co-planar force system (Figure 1) will pass through point C? (b) Find the magnitude and the direction of the resultant force? (c) If the direction of the force along AE is reversed but the value of F unaltered, then find the point where the simplest resultant force will intersect the x-axis. **(4+2+4)**

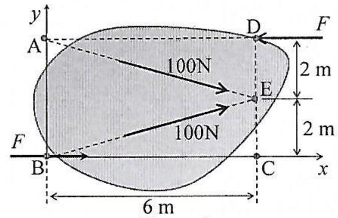


Figure 1

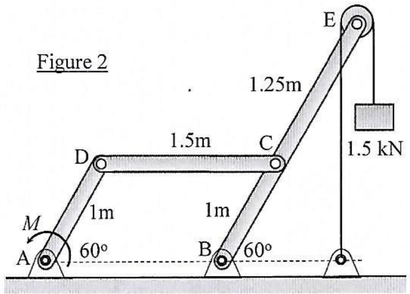


Figure 2

Question 2: A frame made of weight-less members shown in Figure 2 is held in equilibrium with the help of a couple of moment M. (a) Draw the free body diagrams (FBD) of members AD and BE. (b) Identify two-force member, if any. (c) Determine the value of M. (d) Calculate the magnitude and direction of the pin reaction on BE at B. **(4+1+3+2)**

Question 3: Determine the forces and their nature (T or C) carried by members AB, BC and BF of the truss shown in Figure 3. **(3×3)**

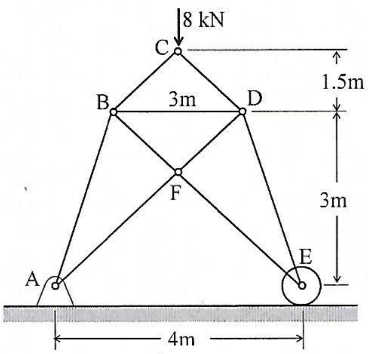


Figure 3

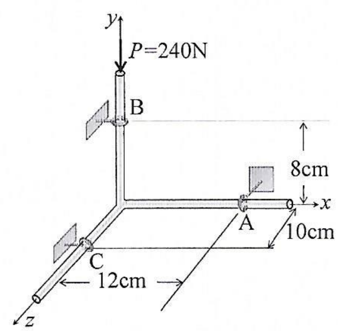


Figure 4

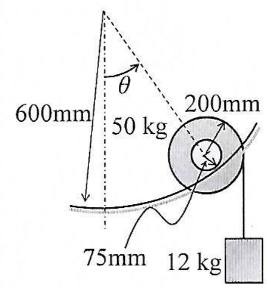


Figure 5

Question 4: Three rods are welded together to form a corner that is supported by three eyebolts (rings) as shown in Figure 4. Neglecting friction and assuming that the eyebolts can only prevent motion perpendicular to the axis of the rod (but not rotation), determine the reactions at A, B and C for a given load of $P = 240$ N. **(3×3)**

Question 5: The 50-kg spool rolls on its hub (radius 75 mm) up the circular incline under the action of 12-kg weight attached to the cord around its rim (radius 200 mm). The spool comes to rest at an angle θ as shown in Figure 5. (a) Draw the FBD of the spool at this angle. (b) Find the value of θ. (c) Determine the minimum coefficient of static friction required for the spool to be at rest in that position. **(4+4+3)**

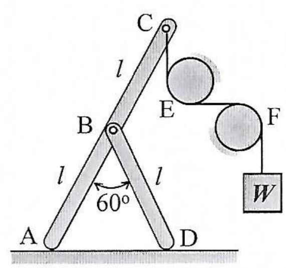


Figure 6

Question 6: Consider the loaded stand shown in Figure 6. The weight of link BD is w and that of link AC is $2w$. The pulleys at E and F are fixed (cannot rotate). The coefficient of static friction between the ground and links is 0.7 (at points A and D) and that between the pulleys and the rope (at E and F) is 0.2. (a) Draw FBD of links AC and BD. (b) Find the minimum weight W (in terms of w) required to *disturb the equilibrium*. (c) State at all the places where there will be slip between the contacting bodies on impending motion. **(4+5+2)**

MECHANICS (ME 10001)

MID-SEM EXAMINATION

Q1. a) The resultant of two 100 N forces passing through point E is

$$\bar{R} = 2 \times 100 \cos\theta \, \hat{i} \quad (N)$$

where $\theta = \tan^{-1}\left(\frac{2}{6}\right) = 18.43^\circ$, and $\hat{i}, \hat{j}$ are the unit vectors along x- and y- axes, respectively.

Since the resultant of the given force system passes through point C, then $M_C = 0$ and hence

$$4F = 2 \times R \Rightarrow F = R/2 = 100 \cos\theta = \boxed{94.87\ N}$$

b) The magnitude of the resultant force is

$$\boxed{R = 189.74\ N}$$

and is directed along the $\boxed{\text{positive } x\text{-axis}}$.

c) When the force at AE is reversed the resultant force passing through E is given by

$$\bar{R}' = 2 \times 100 \sin\theta \, \hat{j}. \quad (N)$$

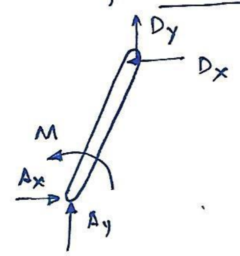
Let the point where the simplest resultant of the given force system passes is $P(\bar{x}, 0)$. Then $M_P = 0$, i.e.,

$$4F = 2 \times 100 \sin\theta \times (\bar{x} - 6), \quad [\bar{x} \text{ in meter}]$$

ie $$\bar{x} = 6 + \frac{2R}{R'} = 6 + \frac{2 \times 2 \times 100 \cos\theta}{2 \times 100 \sin\theta} = 6 + \frac{2}{\tan\theta} = 12\ m$$

Thus, the resultant force will intersect a point lying 12 m from B or 6 m from C.

Q2. a) FBD of member AD



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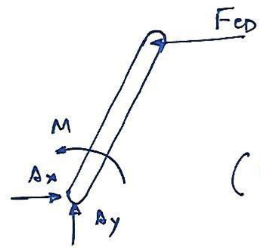
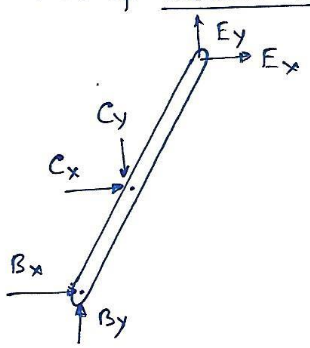


(Considering CD as a two free member)

FBD of member BE



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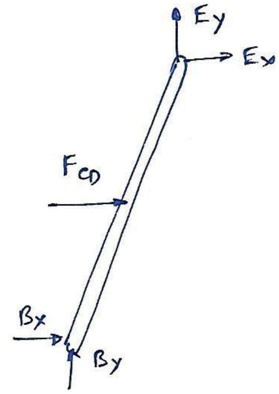


b) The two-force member is CD.

c) From FBD of member AD we get

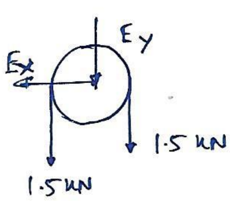
$M_A = 0 \Rightarrow M + F_{CD} \times (AD \sin 60°) = 0 \Rightarrow M = -F_{CD} \cdot \frac{\sqrt{3}}{2}(AD)$. --(1)

Again from FBD of member BE we get

$M_B = 0 \Rightarrow F_{CD} \cdot (BC \sin 60°) = E_y (BE \cos 60°) - E_x (BE \sin 60°)$ --(2)

From the FBD of the pulley shown below one gets

$E_x = 0$, $E_y = -3$ (kN) --(3)

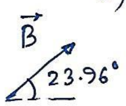


Combining equations (1), (2), (3) and using that AD = BC we get

$M = -E_y \times (BE \cos 60°) = 3 \times 2.25 \times \frac{\sqrt{3}}{2}$ (kN-m)

$= \boxed{5.85 \text{ kN-m}}$

d) From FBD of member BE we get

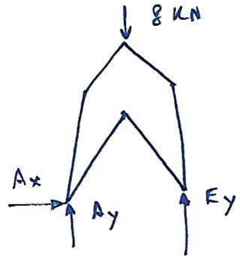


$B_x = -E_x - F_{CD} = -F_{CD} = \frac{2}{\sqrt{3}} \times \frac{M}{AD} = 6.75$ kN

and $B_y = -E_y = 3$ kN.

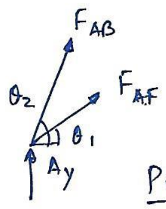
The magnitude of reaction at B is $\boxed{7.39 \text{ kN}}$ making an angle $\boxed{23.96°}$ with +ve x-axis

Q3. Consider the FBD of the truss as shown below. From the equilibrium equations we get



$$A_x = 0, \quad A_y = E_y = 4 \text{ KN}.$$

Now consider the FBD of pin at A. The following equations are obtained as equilibrium conditions



Pin-A

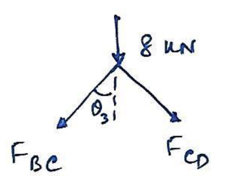
$$F_{AB} \cos\theta_2 + F_{AF} \cos\theta_1 = 0 \qquad \text{---(1)}$$

$$F_{AB} \sin\theta_2 + F_{AF} \sin\theta_1 + A_y = 0 \qquad \text{---(2)}$$

Since $\tan\theta_1 = \frac{3}{3.5}$ and $\tan\theta_2 = \frac{3}{0.5}$ we get

$$F_{AB} = -4.73 \text{ KN}, \quad F_{AF} = 1.02 \text{ KN}. \qquad \text{---(3)}$$

Consider, then, the FBD of pin C. The following equilibrium equations are obtained



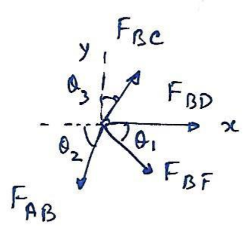
Pin-C

$$F_{BC} \sin\theta_3 - F_{CD} \sin\theta_3 = 0 \Rightarrow F_{BC} = F_{CD}$$

and $\quad 2 \times F_{BC} \cos\theta_3 + 8 = 0 \qquad \text{--(4)}$

where $\tan\theta_3 = \frac{1.5}{1.5} = 1 \Rightarrow \theta_3 = 45^\circ$. From equation (4) we get $F_{BC} = -\frac{4}{\cos\theta_3} = -4\sqrt{2}$ KN.

Finally, consider the FBD of pin B. The following equation is obtained as a force-balance equation along vertical direction



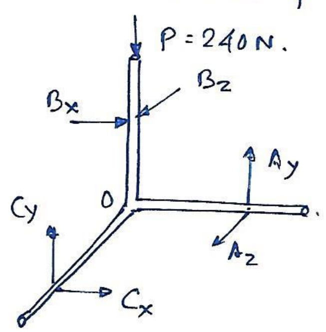
$$F_{BC} \cos\theta_3 = F_{AB} \sin\theta_2 + F_{BF} \sin\theta_1 \qquad \text{--(5)}$$

ie $\quad \frac{-4 - (-4.73)\sin\theta_2}{\sin\theta_1} = F_{BF} = 1.023$ KN

Hence

$$\begin{cases} \text{Force carried by member AB} : 4.73 \text{ KN (C)} \\ \text{Force carried by member BC} : 4\sqrt{2} \text{ KN (C)} \\ \text{Force carried by member BF} : 1.023 \text{ KN (T)}. \end{cases}$$

Q4. The FBD of the corner is shown below.



The following equations of equilibrium are obtained

$$B_x + C_x = 0 \quad (\Sigma F_x = 0) \quad \text{--- (1)}$$

$$A_y + C_y = P \quad (\Sigma F_y = 0) \quad \text{--- (2)}$$

$$A_z + B_z = 0 \quad (\Sigma F_z = 0) \quad \text{--- (3)}$$

$$c\, C_y = b\, B_z \quad (M_x = 0) \quad \text{--- (4)}$$

$$a\, A_z = c\, C_x \quad (M_y = 0) \quad \text{--- (5)}$$

$$a\, A_y = b\, B_x \quad (M_z = 0) \quad \text{--- (6)}$$

$OA = a = 0.12$ m

$OB = b = 0.08$ m

$OC = c = 0.1$ m

Solving equations (1) – (6) we get

$$A_y = P/2, \quad A_z = -\frac{10}{8} \cdot \frac{P}{2}, \quad B_x = \frac{12}{8} \cdot P/2,$$

$$B_z = \frac{10}{8}\frac{P}{2}, \quad C_y = P/2, \quad C_x = -\frac{12}{8} P/2 .$$

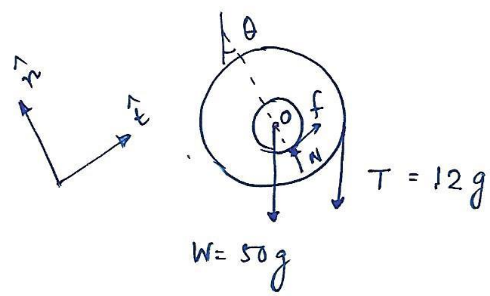
Hence the reactions at points are

$$A: \bar{R}_A = \underline{120\,\hat{j} - 150\,\hat{k} \ (N)}$$

$$B: \vec{R}_B = \underline{180\,\hat{i} + 150\,\hat{k} \ (N)}$$

$$C: \vec{R}_C = \underline{-180\,\hat{i} + 120\,\hat{j} \ (N)}$$

Q5. a) The FBD of the spool



[Direction of the friction force is arbitrary]

b) From the FBD we get the following equations of equilibrium

$$f = (T + W)\sin\theta \qquad (\Sigma F_t = 0) \qquad \text{---}(1)$$

$$N = (T + W)\cos\theta \qquad (\Sigma F_n = 0) \qquad \text{---}(2)$$

$$f \times (0.075) = T \times 0.2 \qquad (M_O = 0) \qquad \text{---}(3)$$

From equation (1) and (3) we get

$$\sin\theta = \frac{f}{T+W} = \frac{0.2}{0.075}\left(\frac{T}{T+W}\right)$$

ie $\boxed{\theta = 31.07^\circ}$

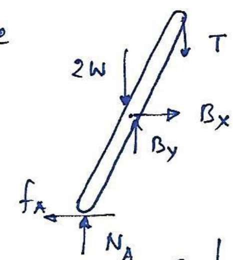
c) The no-slip condition yields

$$\frac{|f|}{N} \leq \mu_s$$

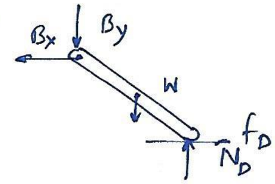
ie $\mu_s \geq \tan\theta = 0.603$.

The minimum coefficient of static friction is $\boxed{0.603}$.

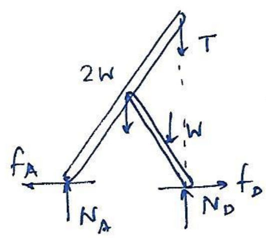
Q6. a) The FBD of member AC



The FBD of member BD.



b) From the FBD of the overall structure, shown below, we get the following results



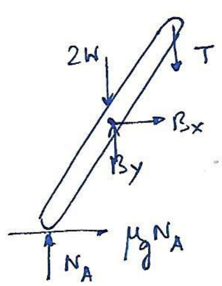
$$f_A = f_D \, , \quad \text{--(1)}$$

$$N_D = T + \frac{7W}{4} \quad \text{---(2)}$$

$$\text{and } N_A = \frac{5W}{4} \quad \text{--(3)}$$

Since $N_A < N_D$, and $f_A = f_D$, the slip will first occur at point A where the inequality $\frac{|f_A|}{N_A} \leq \mu_g$ will be first violated.

We now draw the FBD of the member AC considering the impending motion. The following equation is obtained



$$M_B = 0 \Rightarrow$$

$$(T + N_A)\frac{l}{2} = \mu_g N_A \times l \frac{\sqrt{3}}{2}$$

$$\text{ie} \quad T = (\sqrt{3}\mu_g - 1) N_A = (\sqrt{3}\mu_g - 1) \times \frac{5W}{4}.$$

Now for the rope during impending motion

$$W = T e^{\pi \mu} \quad (\text{Since } W > T)$$

$$= (\sqrt{3} \times 0.7 - 1) e^{\pi \times 0.2} \times \frac{5W}{4}$$

$$= 0.498\, W \approx \frac{W}{2}. \quad \text{That is } \boxed{W \approx \frac{W}{2}}$$

c) The slip will start at point A (see above for reason)