INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 21.09.2016 (FN) | **Time: 2 hrs**
Mid Semester Examination (Autumn) | **Maximum Marks: 80**
Subject No. ME10001 | **Subject Name: MECHANICS**
No. of students: 760

Instructions*: Answer all SIX questions. Any data, if not furnished, may be assumed with justification.*

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1. The vertical force F is decomposed into F_1 and F_2 as shown in Fig.1. Determine the magnitude of F_1 and the angle β. *(10)*

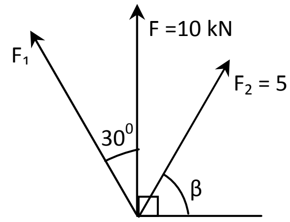


FIG. 1

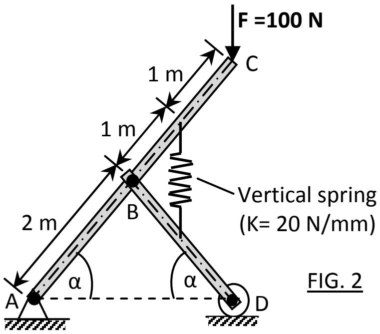


FIG. 2

2. The massless structure shown in Fig.2 is in equilibrium, when the vertical force F=100 N is applied at point C. Determine the value of α, if the un-stretched length of the vertical spring was 1.0 m. *(12)*

3. For the truss shown in Fig.3, **(a)** determine the support reactions, **(b)** identify the zero force members, and **(c)** determine the forces in the members CD, GF and BH. State if they are in tension or in compression. *(15)*

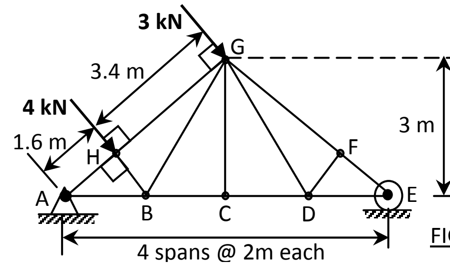


FIG. 3

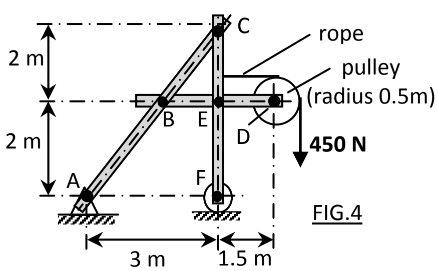


FIG.4

4. The members of the frame shown in Fig.4 are assembled with frictionless pin joints. Neglecting weight of the members, determine the horizontal and vertical components of the force at C exerted on the member CEF. *(16)*

5. In Fig. 5, a right angle bent rigid bar, hinged at C, is pressed against a rigid wedge at A by a 20 kN force at B. Neglecting weight of the members, determine the horizontal force P required to initiate motion of the wedge to the left. The coefficient of friction between the wedge and the bar is 0.25 and the coefficient of friction between the wedge and the ground is 0.15. *(16)*

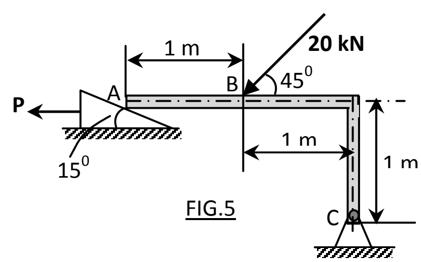


FIG.5

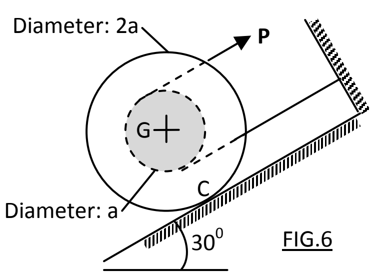
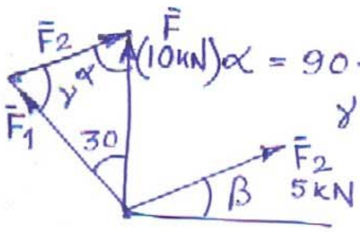


FIG.6

6. A spool of weight W = 1000 N, outer diameter 2a and inner diameter a, is rolled up an inclined plane of 30^0 using an inextensible belt, as shown in Fig. 6. Assuming that the spool has pure rolling at C and sliding between the belt and the inner diameter of the spool, determine **(a)** the minimum force P required for rolling up the spool, and **(b)** the minimum coefficient of friction required at C for rolling up without slipping at C. The coefficient of friction between the belt and the spool is 0.6. *(16)*

SOLUTION FOR MIDTERM-AUTUMN-2016 (MECHANICS)

A1



$\alpha = 90 - \beta$

$\gamma = 180 - \alpha - 30 = 60 + \beta$

$$\frac{10}{\sin(60+\beta)} = \frac{5}{\sin 30} = \frac{F1}{\sin(90-\beta)}$$

$\therefore \sin(60+\beta) = 1$, $\underline{\beta = 30^\circ}$

$$F_1 = \frac{5 \times \sin(90-\beta)}{\sin 30} = \underline{8.67\,\text{kN}}$$

Alternate Solution

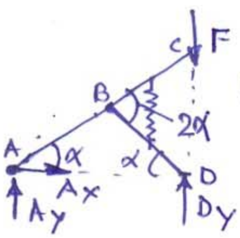
$F_2 \cos\beta = F_1 \sin 30^\circ \quad \therefore F_1 = 10\cos\beta$

$F_1 \cos 30^\circ + F_2 \sin\beta = 10\,\text{kN}$

$10\cos\beta\cos 30^\circ + 10\sin 30^\circ \sin\beta = 10 \quad \therefore \cos(\beta - 30^\circ) = 1$

$\therefore \underline{\beta = 30^\circ}$ and $F_1 = 10\cos 30^\circ = \underline{8.67\,\text{kN}}$

A2

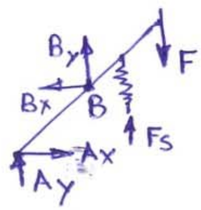


FBD of full Structure.

$A_x = 0$;

$\Sigma M_D = 0$; $A_y = 0$

$D_y = F$.



$\Sigma M_B = 0$

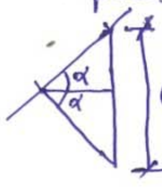
$F\cos 2\alpha \times 2 - F_s \cos 2\alpha \times 1 = 0$

$\therefore F_s = 2F$ or $K \times \delta_s = 2F$

$\therefore \delta_s = \frac{2 \times 100}{20} = 10\,\text{mm}$

unstreched length $(L_o) = 1000\,\text{mm}$

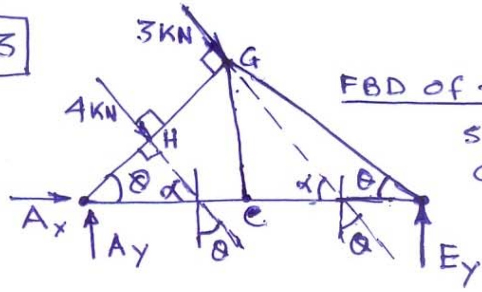
compressed length $= L_o - \delta_s = 1000 - 10 = 990\,\text{mm}$.



$(L_o - \delta_s)$

$$\sin\alpha = \frac{(L_o - \delta_s)/2}{1000} = \frac{990}{2000} = 0.495 \quad \therefore \underline{\alpha = 29.67^\circ}$$

A3



FBD of truss

$\sin\theta = 0.6$

$\cos\theta = 0.8$

$\alpha = 90 - \theta.$

$\Sigma M_A = 0;\ E_y \times 8 - 4 \times 1.6 - 3 \times 5 = 0$

$\therefore \underline{E_y = 2.68\ KN}$

$A_y + E_y - 4\cos\theta - 3\cos\theta = 0$

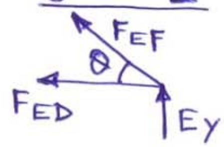
$\therefore \underline{A_y = 2.92\ KN}$

$A_x + 4\sin\theta + 3\sin\theta = 0$

$\therefore \underline{A_x = -4.2\ KN}$

(b) Zero force members: FD, GD and GC

(c) joint E



$F_{ED} + F_{EF}\cos\theta = 0$ —(1)

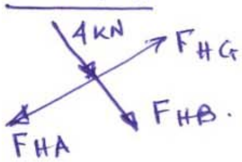
$E_y + F_{EF}\sin\theta = 0$ ···(2)

$\therefore F_{EF} = -4.47KN = F_{GF}$

$F_{ED} = -F_{EF}\cos\theta$

$= 3.57\ KN = F_{CD}$

Joint H



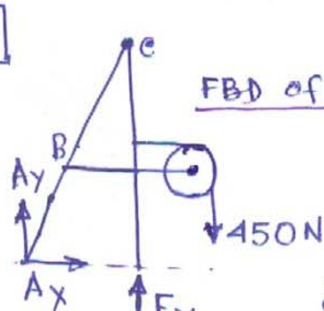
$F_{HB} = -4\ KN$

$F_{CD} = 3.57KN\ (T)$

$F_{GF} = 4.47KN\ (C)$

$F_{BH} = 4\ KN\ (C)$

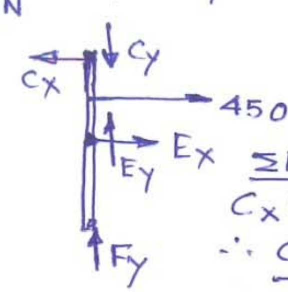
A4



FBD of structure

$\Sigma M_A = 0;\ F_y \times 3 - 450 \times 5 = 0,\ \therefore F_y = 750\ N$

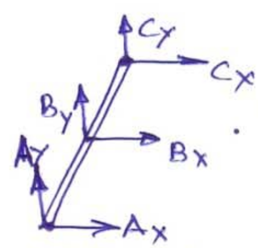
$A_x + F_y - 450 = 0\ \therefore\ A_y = -300N;\ A_x = 0.$



$\underline{\Sigma M_E = 0}$

$C_x \times 2 - 450 \times 0.5 = 0$

$\therefore \underline{C_x = 112.5\ N}$

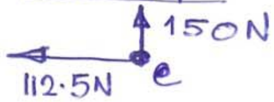


$\underline{\Sigma M_B = 0}$

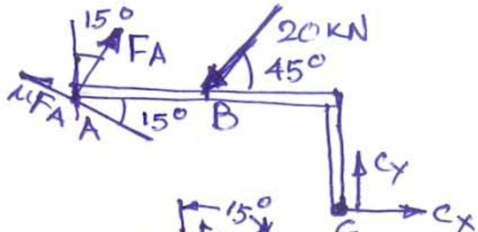
$A_y \times 1.5 + C_x \times 2 - C_y \times 1.5 = 0$

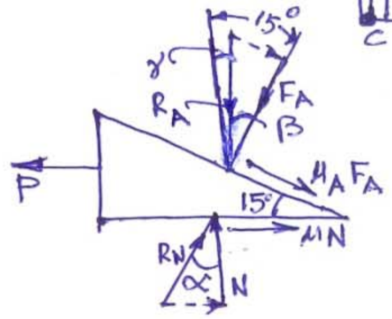
$\therefore \underline{C_y = -150N}$

Force at C on CEF



A5

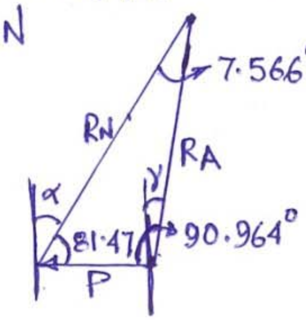




$$\Sigma M_C = 0$$

$$F_A \cos 15 \times 2 + F_A \sin 15 \times 1 - 20 \cos 45 \times 1$$
$$- 20 \sin 45 \times 1 - \mu_A F_A \cos 15 \times 1 + \mu_A F_A \sin 15 \times 2 = 0$$

$$\therefore F_A = 13.607\ \text{KN}$$

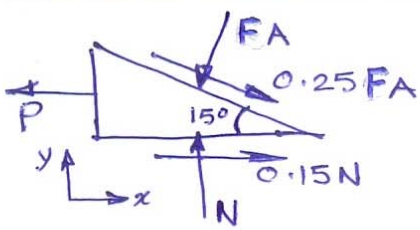


$$\alpha = \tan^{-1} 0.15 = 8.53^\circ$$
$$\beta = \tan^{-1} 0.25 = 14.04^\circ$$
$$\gamma = 15^\circ - \beta = 0.964^\circ$$

$$\frac{P}{\sin 7.566} = \frac{R_A}{\sin 81.47}$$

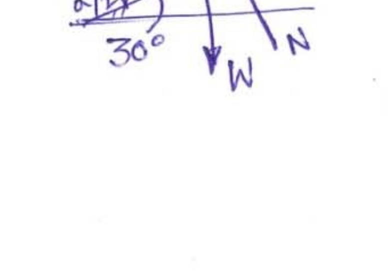
$$R_A = \sqrt{F_A^2 + \mu F_A^2} = 14.026\ \text{KN} \qquad \therefore \underline{P = 1.867\ \text{KN}}$$

Alternative Solution



$$\Sigma F_x = 0;\ 0.15N + 0.25 F_A \cos 15^\circ - F_A \sin 15 - P = 0$$
$$P = 0.15N - 0.236$$
$$\Sigma F_y = 0;\ N - F_A \cos 15^\circ - 0.25 F_A \sin 15^\circ = 0$$
$$\therefore \underline{N = 14.024\ \text{KN}} \quad \therefore \underline{P = 1.867\ \text{KN}}$$

A6



n t

P>T

$$\Sigma F_t = 0;\ P + T + f - W \sin 30^\circ = 0 \quad \text{---(1)}$$
$$\Sigma F_n = 0;\ N - W \cos 30 = 0 \quad \text{---(2)}$$
$$\Sigma M_c = 0;\ P \times \frac{3a}{2} - W \sin 30 \times a + T \times \frac{a}{2} = 0 \quad \text{---(3)}$$
$$\frac{P}{T} = e^{0.6 \times \pi} \qquad \therefore T = 0.152P \quad \text{---(4)}$$

$$\therefore \text{From (3)}\ \underline{P = 317.3\ N}$$

from (1); $f = W \sin 30^\circ - P - T$
$= W \sin \theta - (P + 0.152P) = 134.5\ N$

$\therefore$ Minimum co-efficient of friction
at $C = f/N = f/W\cos 30^\circ \underline{= 0.155}$