Class Test 1

IIT Kharagpur, CSE Dept., Autumn 2022

CS41001: THEORY OF COMPUTATION
TIME = 1 HOUR

5TH SEPTEMBER, 2022
TOTAL MARKS = 20

Answer all questions. Provide concise answers. State all assumptions you make.

1. Prove exactly one of the following statements. [6]
   (a) Every infinite regular set contains a subset that is not recursively enumerable.
   (b) Prove that every infinite *r.e.* set contains an infinite recursive subset.

2. Prove **or** disprove exactly one of the following. [6]
   (a) Is it decidable for a given TM $\mathcal{M}$ whether $L(\mathcal{M}) = L(\mathcal{M})^{\mathbf{R}}$. (For a set $A \subseteq \Sigma^*$, define $A^{\mathbf{R}} = \{w^{\mathbf{R}} \mid w \in A\}$ where $w^{\mathbf{R}}$ denotes w reversed.)
   (b) Given CFG G, it is undecidable whether $L(G)$ is deterministic context-free.

3. Given a context-free grammar G, show that it is undecidable whether
   (a) $L(G) = L(G)L(G)$. (For a set A, $AA = \{xy \mid x, y \in A\}$, where xy denotes concatenation of x and y). [4]
   (b) G is ambiguous. (A grammar G is ambiguous if there exists a string in $L(G)$ with two different derivations in G.) [4]