

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: ..... (AN) ... Time: 2 Hrs.  
Mid-Spring Semester: 2010-11

Maximum Marks 60 No. of Students: 75  
Department: Computer Science and Engineering  
Sub. No: CS31004  
Sub. Name: Theory of Computation

B. Tech.(Hons.), Dual Deg.

**Instructions :** Answer **ALL** questions

1. (a) Let  $M_1, M_2, \dots$  be an enumeration of some set of Turing machines that halt on all inputs. Show that there is some recursive language that is not  $L(M_i)$  for any  $i$ .  
(b) Using the result of part (a) show that there is a recursive language which is not context sensitive.  
[8 + 10 = 18]
2. (a) Show that a language  $L$  over  $\Sigma$  is recursive if and only if both  $L$  and  $\Sigma^* - L$  are recursively enumerable.  
(b) Suppose  $L$  is recursively enumerable but not recursive. Show that any recognizer of  $L$  must loop forever on infinitely many different inputs.  
[6 + 10 = 16]
3. (a) Show that  $E_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}$  is undecidable.  
(b) Show that  $A_{TM}$  cannot be reduced to  $E_{TM}$ .  
(c) Show that if a decision problem class  $\mathcal{P}$  is partially decidable and  $\mathcal{P}$  can be reduced to  $\mathcal{P}^c$ , then  $\mathcal{P}$  is decidable.  
[7 + 4 + 3 = 14]
4. Define the Word problem of Semi-Thue system. Show that the problem is undecidable. Is the problem partially decidable? Justify your answer.  
[10 + 2 = 12]