Mid Semester Examination

IIT Kharagpur, CSE Dept., Autumn'16

(CS41001) Theory of Computation (Full marks = 60)

Answer exactly 6 questions. In case of reasonable doubt, make practical assumptions. In all proofs, intermediate steps need to be clearly explained.

1. A non-terminal A in a CFG G is usable if it appears in some derivation of some string $w \in L(G)$. Show that the language $USABLE_{CFG} = \{\langle G, A\rangle : \text{G is a CFG, } A \text{ is usable for } G\}$ is decidable. **[10]**

2. Prove that there exists more languages than there exists Turing machines. **[10]**

3. $EQ_{TM} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) = L(M_2)\}$. Prove that EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable. **[5+5]**

4. Prove that it is undecidable whether $L(G)$ is regular for CFG G. **[10]**

5. (a) Give an example of a *non trivial* property of languages of Turing Machines along with explanation about why it is *non trivial*. **[2]**

   (b) Provide a formal statement of the Post Correspondence Problem. **[2]**

   (c) Show that $ALL_{CFG} = \{\langle L\rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable. **[6]**

6. (a) Prove that $Th(\mathbb{N}, +)$ is decidable. Use the decidability of $Th(\mathbb{N}, +)$ for inferring the decidability of $Th(\mathbb{N}, <)$. **[5 + 2]**

   (b) Explain the reason behind the Fourier-Motzkin Quantifier Elimination procedure being applicable only for the domains $\mathbb{R}$, $\mathbb{Q}$ and not for $\mathbb{N}$. **[3]**

7. (a) Describe all the steps of Cooper's algorithm for deciding (integer) Presburger arithmetic with suitable examples. **[5]**

   (b) Apply Cooper's method and infer the validity/invalidity of the formula $\exists x.(3x + 1 < 10 \vee 7x - 6 > 7) \wedge 2|x$. **[5]**