ধ্ৰ INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: ....FN/AN *Time:* 3 Hrs. *Full Marks* ...75 ... *No. of Students* ...**82** ...
Autumn Semester, 2012-2013 Deptt. ...CSE ... Sub No. ...**CS41001** ...
B. Tech.(Hons.) ...4^{th} Yr. ... Sub. Name ...**Theory of Computation** ...

Instructions : **Answer Q1 and any 3 from the remaining 4.**

1. [7×6]

(a) Let $L_1, L_2 \in$ **coNP**, prove that $L_1 \cup L_2 \in$ **coNP**.

(b) Prove that,
(i) $L_\epsilon = \{< M, x >$: Turing machine M accepts $x\}$ is **NP**-*hard* but not **NP**-*complete*.
(ii) $L_{NP} = \{< M, x, 1^n, 1^t >: \exists u \in \{0,1\}^n$ such that M accepts $< x, u >$ within t steps$\}$ is **NP**-*complete*.

(c) (i) Prove that the following language is **PSPACE**-*complete*.
$SPACE-TMSAT = \{< M, x, 1^n >$: Turing machine M accepts x using at most n cells of work-tape $\}$.
(ii) Prove that every language $L \in$ **NL** such that $L \neq \emptyset$ or $L \neq \{0,1\}^*$, is **NL**-*complete* under Karp reduction.

(d) Prove that,
(i) $Reg_= = \{< r, s >$: r and s are regular expressions over $\{0,1\}$ and $L(r) = L(s)\}$ is in **PSPACE**.
(ii) **P/poly** is an uncountable class and there are undecidable languages.

(e) Show that there is a language A such that $\mathbf{P}^A = \mathbf{NP}^A$.

(f) If $3SAT \leq_P \overline{3SAT}$, then prove that **PH** = **NP**.

(g) Give an example of a Σ_n^P-*complete* problem. Justify the belief that
(i) there is no complete problem of **PH**, and
(ii) **PH** $\neq$ **PSPACE**.

2. If $L \in$ **NP**, then there exists a a two-tape *oblivious* polynomial time ($t(n)$) Turing machine M and a polynomial $p : \mathbb{N}_0 \to \mathbb{N}_0$, such that $x \in L$ if and only if $\exists u \in \{0,1\}^{p(|x|)}$ such that M accepts $< x, u >$. Answer the following questions with proper explanation where $y =< x, u >$. [$2 + 3 + 2 + 4$]

(a) What is the data in a *snapshot* z_i at the i^{th} step of computation of M on input y?

(b) Explain how does the i^{th} *snapshot* z_i depends on z_{i-1}, $y_{inputpos(i)}$, $z_{prev(i)}$, where *inputpos()* and *prev()* have their usual meaning.

(c) How does M pre-computes *inputpos*(i) and *prev*(i)?

(d) $x \in L$ if and only if there exists a string $y =< x, u > \in \{0,1\}^{|x|+p(|x|)}$ and a sequence of *snapshots* $z_1, \cdots, z_{t(n)} \in \{0,1\}^c$, where c is the length of encoding of each *snapshot*. What are the conditions to be satisfied by y and the sequence of snapshots?

3. [$5 + 6$]

(a) If **P** = **coNP**, then prove that **NP** = **PH**.

(b) Prove that `dHAMPATH` $= \{< G, s, d >$: G is a directed graph with a Hamiltonian path from s to $d\}$ is **NP**-*complete* by reducing `3SAT` to `dHAMPATH`.

4. $[5 + 4 + 2]$

(a) Prove that, if $\mathbf{P} = \mathbf{NP}$ and a boolean formula ϕ is satisfiable, then there is a polynomial time Turing machine that can generate a satisfying assignment of ϕ.

(b) Prove that the following language is in $\mathbf{NP}^{SAT}$.

$$\overline{MIN - FORMULA} = \{< \phi >: \text{ Boolean formula } \phi \text{ is not minimal}\}.$$

(c) Give a polynomial time alternating algorithm (**AP** algorithm) for the following language. In which class of the polynomial hierarchy does the language belong to?

$$MIN - FORMULA = \{< \phi >: \ \phi \text{ is a minimal Boolean formula}\}.$$

5. $[5 + 6]$

(a) Let $\psi = Q_1 x_1 \cdots Q_n x_n \phi(x_1, \cdots, x_n)$ be a quantified Boolean formula with n variables and length m. Give a recursive procedure that uses $O(mn)$ space to determine the truth value of ψ. Explain the space complexity.

(b) Let $A, B, C \in \{0,1\}^*$. Prove that, if $A \leq_l B$ and $B \leq_l C$, then $A \leq_l C$, where '$\leq_l$' denotes *implicit logspace reduction*.