28/2/23

→

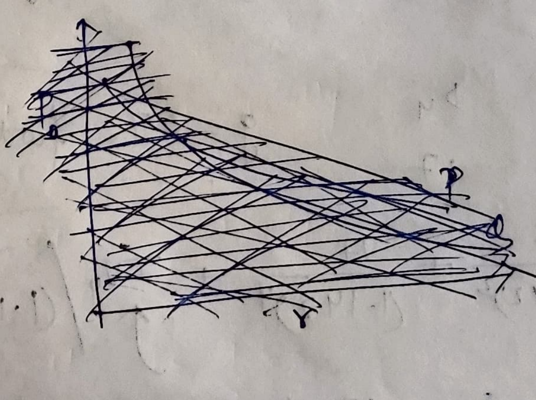
| | P | Q |
|---|---|---|
| Face Value (RS.) | 1,000 | 1,000 |
| Coupon (Annual) | 10% | 8% |
| N (Years) | 5 | 5 |
| YTM | 9% | 9% |
| $P_0$ | ? | ? |
| | 1038.89 | 961.10 |

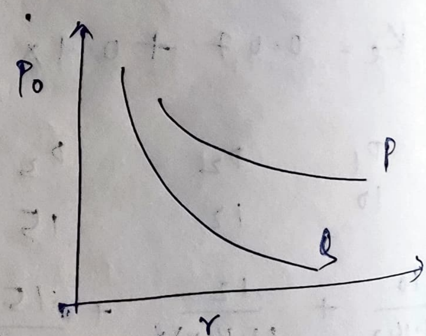
100 v/s 80

| Interest Rate | P | Q |
|---|---|---|
| 7% | 1,123.01 | 1,041.00 |
| 9% | 1,038.89 | 961.10 |
| 11% | 963.40 | 889.12 |

| | P | Q |
|---|---|---|
| % change in price (9% to 11%) | 7.26% | 7.49% |

Slope for Q > Slope for P.

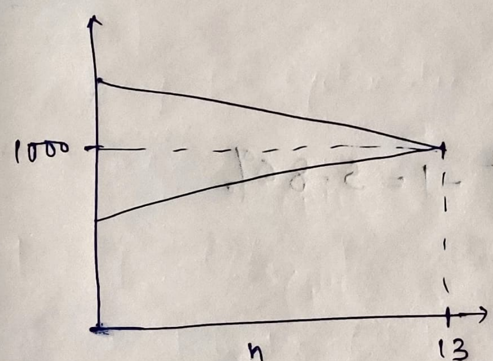




| | X | Y |
|---|---|---|
| FV (Rs.) | 1,000 | 1,000 |
| Coupon | 8% | 6% |
| YTM | 6% | 8% |
| N | 13 | 13 |
| $P_0$ ? | | |

| | X | Y |
|---|---|---|
| $P_0$ | 1,177.05 | 841.92 |
| $P_1$ | 1,167.68 | 849.28 |
| $P_3$ | 1,147.20 | 865.79 |
| $P_8$ | 1,084.25 | 920.15 |
| $P_{12}$ | 1,018.87 | 981.48 |
| $P_{13}$ | 1,000 | 1,000 |

Pull to Par.




Book - Q11

| | |
|---|---|
| Face Value / Par Value | Rs. 5000/- |
| Coupon | 3.70%, payable semi-annually |
| YTM | 3.90% |
| N | 16 years |

$P_0 = ?$

Coupon = Rs. 92.5 (for 6 months)

$P_0 = 92.5 \times PVIFA(1.95\%, 32) + 5000 \times PVIF(1.95\%, 32)$

$= 2186.67 + 2695.13$

$= 4881.80$

→ Book Q.21

Par Value 1,000

Coupon, 6.4% paid semi-annually.

Price = 106.81 % of par

N = 18 yrs.

Find YTM

$1068.1 = 32 \times PVIFA(r, 36) + 1000 \times PVIF(r, 36)$

$= \frac{32}{r}\left(1 - \frac{1}{(1+r)^{36}}\right) + \frac{1000}{(1+r)^{36}}$

$r = 2.89\%$

YTM = 2.89 × 2 = 5.78%

Effective yield = $\left(1 + \frac{2.89}{100}\right)^2 - 1 = 5.86\%$

→ Book Q.23

Price = 1,035.00

Coupon = 5.90%, semi-annually

Par Value = 1,000.00

4 months left to maturity.

2 months interest = Rs. 9.83

Dirty Price = Rs. 1,035

~~Clean~~ Clean Price = Rs. 1,035 − 9.83

= Rs. 1,025.17

→ Book Q.25

Coupon - annual = 8%

YTM = 7.20%

Par Value = 1,000

Current Yield = 7.55%

$$\boxed{CY = \frac{\text{Coupon}}{\text{Current Price}}}$$

Find N.

$$\text{Price} = \frac{80}{7.55} \times 100 = 1059.60$$

$$1059.60 = \frac{80}{0.072}\left(1 - \frac{1}{(1+0.072)^n}\right) + \frac{1000}{(1.072)^n}$$

$n = 11.0575$

→ Book Q33

Annual coupon = 7%

Price = 1,060

N = 17

Face Value = 1,000

$$1060 = 70 \times PVIFA(r,17) + 1000 \times PVIF(r,17)$$

$r = 6.41\%$ (Rate of return)

2 year hence, YTM falls by 1% = 5.41%

$$\text{Price}\ (P_2) = 70 \times PVIFA(5.41\%, 15) + 1000 \times PVIF(5.41, 15)$$

$$= Rs.\ 1160.56$$

$$1060 = \frac{70}{(1+r)^1} + \frac{70}{(1+r)^2} + \frac{1160.56}{(1+r)^2}$$

$r = 11.10\%$

(Holding Period Yield) (assuming that the Rs. 70s are re-invested at 11.10%)

6/3/23

## Risk, Return Portfolio

→ Stock A | Stock B

| Scenario | Exp P | Return (A) | Stock B |
|---|---|---|---|
| 1 (Optimistic) | 0.30 | 8% | 6% |
| 2 (Average) | 0.40 | 10% | 12% |
| 3 (Pessimistic) | 0.30 | 13% | 16% |

Expected Return:

(A) $\bar{r}_x = \overset{P_i}{0.30} \times \overset{r_i}{0.08} + 0.40 \times 0.10 + 0.30 \times 0.13 = 10.30\%$

(B) $\bar{r}_y = 11.40\%$

→ For Stock A, Range = 5%,
For Stock B, Range = 10%

→ Variance & Std. Deviation

| A | B |
|---|---|
| $(0.08 - 0.103)^2 \times 0.3$ | $(0.06 - 0.114)^2$ |
| $(0.10 - 0.103)^2 \times 0.4$ | $(0.12 - 0.114)^2$ |
| $(0.13 - 0.103)^2 \times 0.3$ | $(0.16 - 0.114)^2$ |

→

| | A | B |
|---|---|---|
| $\bar{r}$ | 10.30% | 11.40% |
| Var | $6.33\%^2$ | 25.33% |
| $\sigma$ | 2.51% | 5.03% |
| Coefficient of Variation $\left(\frac{\sigma}{\bar{x}}\right)$ | 0.243 | 0.441 → Risk per Return. |

| | $\bar{r}$ | $\sigma$ | |
|---|---|---|---|
| A | 12% | 4% | |
| $\boxed{B}$ | 13% | 4% | same risk, more return |
| $\boxed{P}$ | 11% | 6% | same return, less risk |
| Q | 11% | 8% | |

→ Say, a portfolio with 0.50A, 0.50B,

$r_p = 0.5 \times 0.103 + 0.50 \times 0.114 = 10.85\%$ (Portfolio Return)

→ One way to calculate variance:-

$(7 - 10.85)^2 \times 0.3 + (11 - 10.85)^2 \times 0.4 + (14.5 - 10.85)^2 \times 0.3$

avg. of returns

→ $\sigma_p = \sum_{i=1}^{n} w_i \times \sigma_i$ ✗ ← (Not correct)

$Var_p = \sum_{i=1}^{n} w_i \times \sigma_i^2$ ✗

→ Portfolio Risk:-

(for a Two-Security Portfolio).

$$\boxed{\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho_{12} w_1 w_2 \sigma_1 \sigma_2}$$

→ Generic Formula:-

$$\boxed{\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \times w_i \times w_j \times \sigma_i \times \sigma_j}$$

→ 3 stock portfolio:-

| | 1 | 2 | 3 | |
|---|---|---|---|---|
| w | ✓ | ✓ | ✓ | $\rho_{12}$ $\rho_{23}$ |
| $\sigma$ | ✓ | ✓ | ✓ | $\rho_{13}$ |

→

| Stock | A | B | C |
|---|---|---|---|
| $\bar{r}$ | 12% | 14% | 16% |
| w | 30% | 40% | 30% |
| $\sigma$ | 4% | 6% | 8% |

$\rho_{12} = 0.60, \quad \rho_{13} = 0.80, \quad \rho_{23} = 0.40$

$\bar{r}_p = 0.3 \times 12 + 0.4 \times 14 + 0.3 \times 16$

$= 14\%$

$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 +$

$2(\rho_{12} w_1 w_2 \sigma_1 \sigma_2 + \rho_{23} w_2 w_3 \sigma_2 \sigma_3 + \rho_{13} w_1 w_3 \sigma_1 \sigma_3$

$= 12.96 + 2 \times 6.336$

$= 25.632$

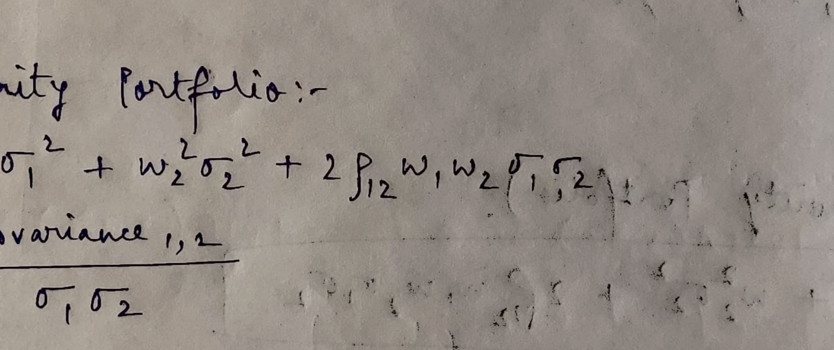
$\sigma_p = 5.0628$

7/3/23

→ Two-security Portfolio:-

$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho_{12} w_1 w_2 \sigma_1 \sigma_2$

$\rho_{12} = \dfrac{\text{Covariance}_{1,2}}{\sigma_1 \sigma_2}$




→ $w_1 = ?$

for $\sigma_p$ to be the least?

$$\boxed{w_x = \frac{\sigma_y^2 - Cov_{x,y}}{\sigma_x^2 + \sigma_y^2 - 2\,Cov_{x,y}}}$$

$$(Cov_{xy} = \rho_{xy}\,\sigma_x\,\sigma_y)$$

→ $\sigma_x = 8\%$, $\sigma_y = 10\%$, $\rho = 0.6$

$$Cov_{xy} = 8 \times 10 \times 0.6 = 48$$

$$w_x = \frac{10^2 - 48}{8^2 + 10^2 - 2 \times 48} = 0.7647 = 76.47\%$$

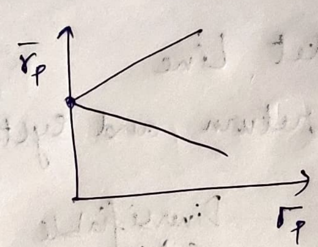
$$w_y = 0.2453 = 24.53\%$$

→ If $\rho_{xy} = 0$, then,

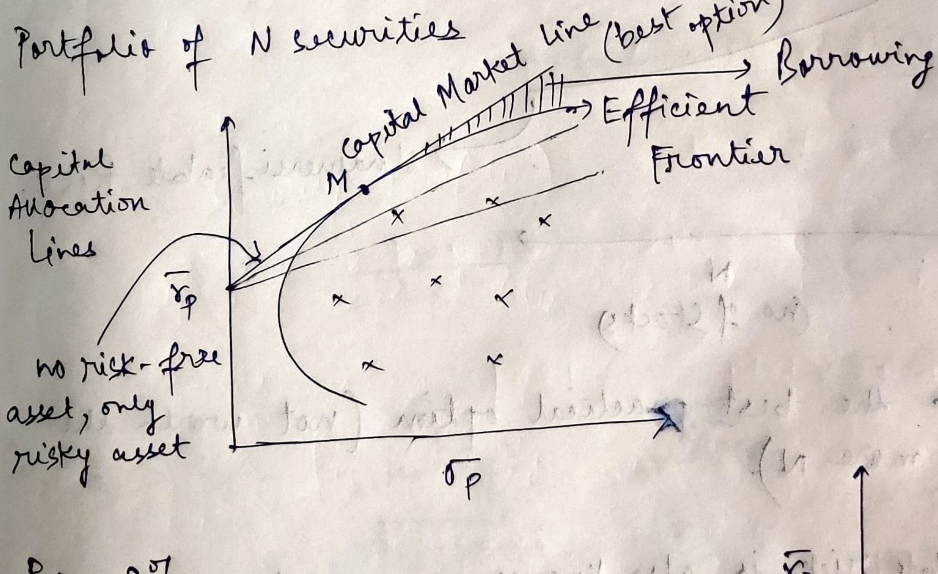
$$w_x = \frac{\sigma_y^2}{\sigma_x^2 + \sigma_y^2}$$

→ If $\rho_{xy} = -1$,

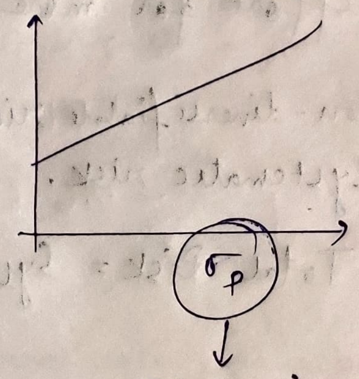
$$w_x = \frac{\sigma_y}{\sigma_x + \sigma_y}$$




→ Portfolio of N securities




$R_f = 8\%$
$R_m = 18\%$ (market)
$\sigma_m = 6\%$



→ $R_f = 8\%$, $R_m = 18\%$, $\sigma_m = 6\%$

$R_p = 15\%$

$15 = W_f \times 8 + (1 - W_f) \cdot 18$

$\Rightarrow 15 = 18 - 10 W_f$

$\Rightarrow W_f = \frac{3}{10} = 30\%$

$W_m = 70\%$

Say $R_p = 20\%$

$20 = W_m \times 18 + (1 - W_m) \times 8$

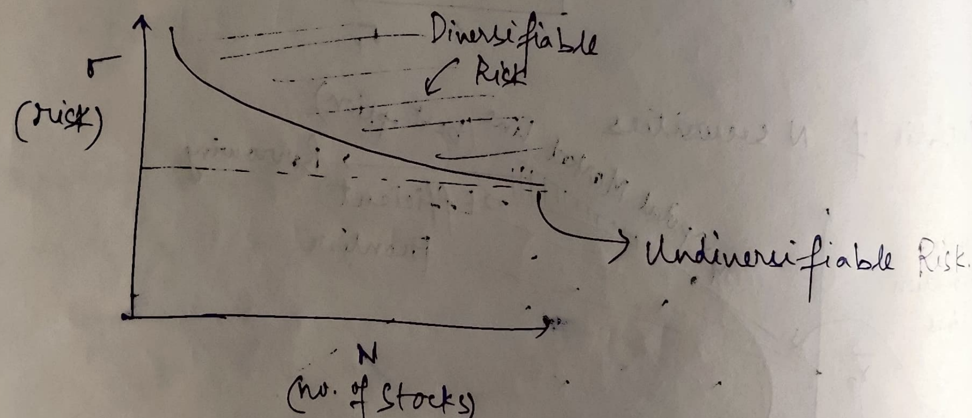
$\Rightarrow 20 = 10 W_m + 8$

$\Rightarrow W_m = 1.2$

$W_f = -0.2$

→ Concept of Security Market Line

- Relationship between return and systematic risk.




N = 30-40 is the best practical option (not worth the effort for more N)

→ Non-diversifiable risk is also known as market risk or systematic risk.

→ Total Risk = Systematic Risk + Unsystematic Risk

→ Stock return is dependent on the market return.

→ Say, $r = 0.6$ (correlation b/w stock return & market return)

How much of the variation in stock return is explained by variation in market return?

→ $r^2$ = coefficient of determination

36% non-diversifiable, 64% diversifiable.

→ 1% change in market (means market return = ±1%)

What will be the change in stock?

$$\boxed{r_i = \alpha + \beta r_m + e}$$

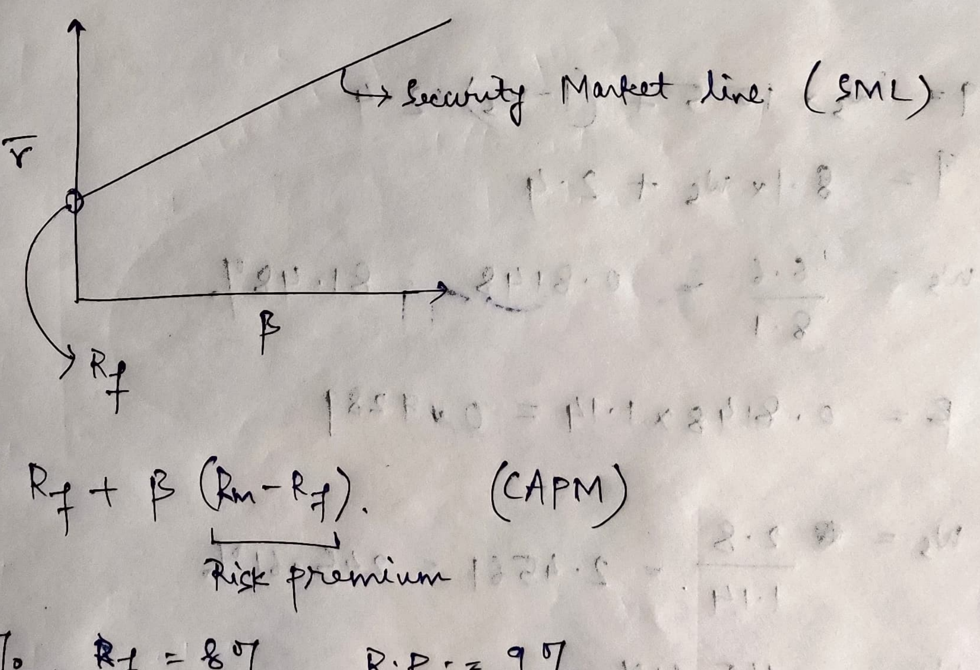
β represents systematic risk.

If $\beta = 0.8$, 1% change in market ⇒ 0.8% change in stock.

$\beta < 1$ (Defensive)

$\beta > 1$ (Aggressive)

→




So, $R_i = R_f + \beta \underbrace{(R_m - R_f)}_{\text{Risk premium}}$. (CAPM)

→ $R_m = 17\%$, $R_f = 8\%$, R.P. = 9%

$R_i = 8\% + 1.30 \times 9\% = 19.7\%$

expected rate of return from equity investor's POV.

→ Stock $\beta = 1.14$

Expected return on stock = 10.5%

Risk-free rate of return = 2.4%

a) If weights are equal between stock & risk free asset find $\bar{R}_p$.

b) If portfolio $\beta$ is 0.92, what are the weights?

c) If portfolio return = 9%, what is its $\beta$?

d) " " $\beta = 2.8$, what are the weights?

$$\boxed{\beta_p = \sum_{i=1}^{n} w_i \beta_i}$$

a) $\bar{R}_p = \dfrac{10.5+2.4}{2} = 6.45\%$

b) $0.92 = 1.14 \times w_s$

$\boxed{\beta_{(\text{Risk-free})} = 0}$

$\Rightarrow w_s = 0.8070 = 80.70\%$

$w_f = 19.30\%$

c) $9 = w_s \times 10.5 + (1-w_s) \times 2.4$

$\Rightarrow 9 = 8.1 \times w_s + 2.4$

$\Rightarrow w_s = \dfrac{6.6}{8.1} \approx 0.8148 = 81.48\%$

$\beta = 0.8148 \times 1.14 = 0.9289$

d) $w_s = \dfrac{2.8}{1.14} \approx 2.4561 = 245.61\%$

$w_f = -145.61\%$