CS 60002: Distributed Systems

T9:
Leader Election

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Leader Election in Distributed System

- Each process eventually decides whether it is the leader or not, subject to the constraint that there is **exactly one leader**
  - Processes will be in one of the three states – **undecided, leader, not leader**
  - Initial state: **undecided**
  - Final state: **leader** or **not leader**

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  - Processes will be in one of the three states – **undecided, leader, not leader**
  - Initial state: **undecided**
  - Final state: **leader** or **not leader**
- We have already seen in RAFT and PBFT ! -- the role of a leader
  - Central server for mutual exclusion
  - Message ordering
  - Ensure serializability
  - Ensure consensus
  - Take snapshot
  - …

Leader Election in Distributed System

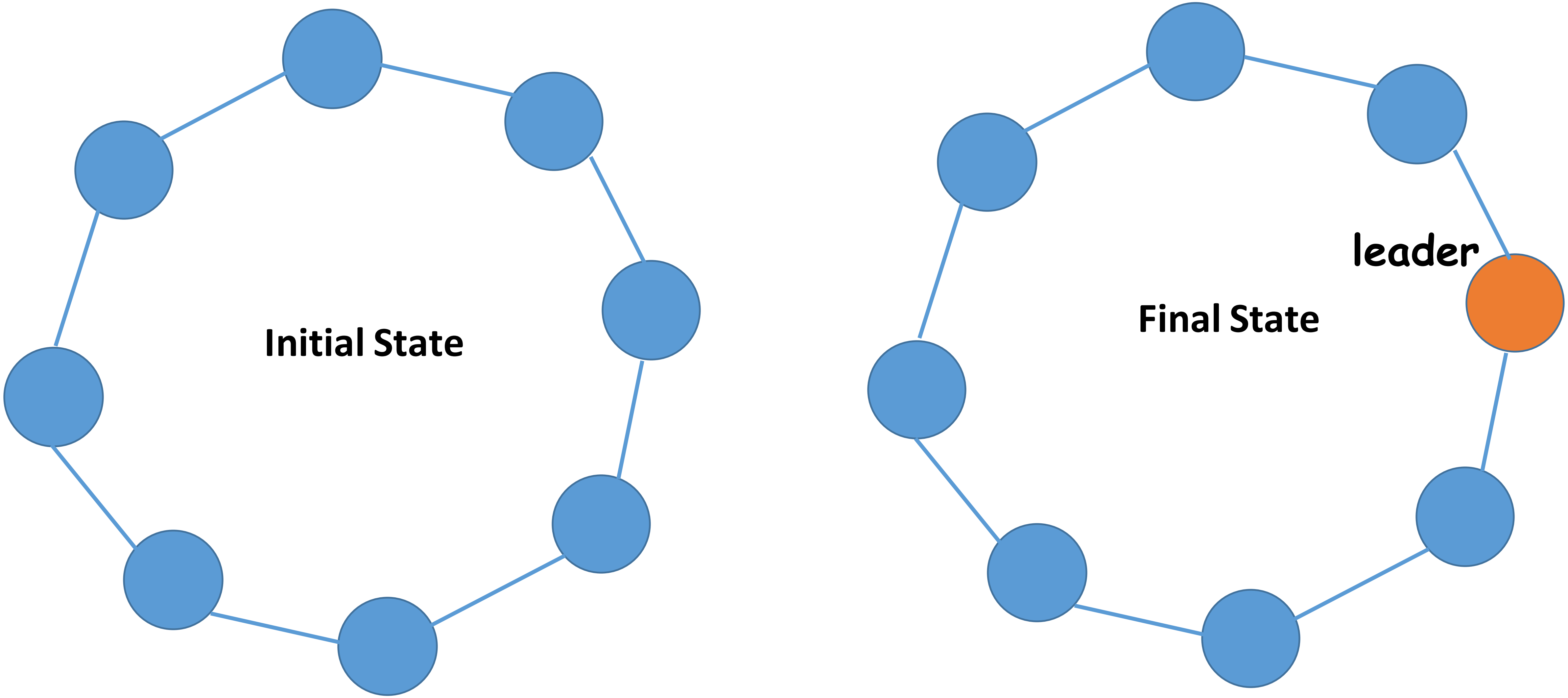
- Requirements:
  - The protocol should eventually terminate
  - In each round, exactly one process will be elected as the leader
  - On termination, the leader process should know that it is the leader
  - All other processes know that they are not the leader, and (optionally) knows who the leader is

- Distributed leader election has been studied in different topologies
  - Rings
  - Arbitrary topology

Leader Election in Rings

- System models
  - Synchronous or Asynchronous
  - Unidirectional or Bidirectional ring
  - Anonymous (no unique ID) or Non-anonymous (no unique IDs)
  - Uniform (no knowledge on the number of processes) or Non-uniform (the information about the number of processes is known)

Leader Election in Rings



Why Do We Study Rings

- Simple starting point to understand leader election
  - Easy to analyze the algorithms
- The lower bounds and impossibility results derived for ring topologies also apply to arbitrary topologies as well
  - If you cannot do it over a ring, you cannot do over an arbitrary topology

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- **Impossibility result:**
  - There is no deterministic leader election protocol for anonymous rings even if
    - The protocol knows the ring size (non-uniform)
    - The channel is synchronous

Impossibility Proof

- **Deterministic leader election in an anonymous ring is impossible**
  - Processes do not have unique identifiers – there is no way to distinguish the processes from each other
  - Every processor starts in the same state (**undecided**) with the same outgoing messages (as they are anonymous)
  - **Every processor runs the same algorithm** -- Everyone does the same computation, sends and receives same messages, so end up in same states
  - If one node decides to become the leader, then every other nodes does so

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- The same result holds for weaker models
  - Asynchronous
  - Uniform

Impossibility Proof

- However, **Randomized algorithms are possible (but not always easy!)**
  - Galindo, David, et al. "Fully distributed verifiable random functions and their application to decentralised random beacons." 2021 IEEE European Symposium on Security and Privacy (EuroS&P). IEEE, 2021.

Rings with Identifiers

- **Identifiers** (IDs)
  - Arbitrary non-negative integers
  - Available to the processes

- Every process has a unique ID
  - We typically use 0 to n-1 as the indices of the processes in the rings – these are not identifiers, we typically use them for analysis
  - Identifiers can be anything, like 12643 – some arbitrary positive integers but unique to individual processes

Leader Election in Rings with IDs -- Overview

- **Best Results:**
  - Asynchronous Rings: $\Theta(n \log n)$ messages
  - Synchronous Rings: $\Theta(n)$ messages

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  - Everyone considers the process with ID k as the leader
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- If all processes know the highest ID (say, k), then we do not need a leader election
  - Everyone considers the process with ID k as the leader
  - The process with ID k can start operating as the leader
- However, the process with ID k can fail
  - If we assume the next higher ID as the leader, that process can also fail

Leader Election in Rings with IDs -- Overview

- **Best Results:**
  - Asynchronous Rings: Θ(n log n) messages
  - Synchronous Rings: Θ(n) messages

- If all processes know the highest ID (say, *k*), then we do not need a leader election
  - Everyone considers the process with ID *k* as the leader
  - The process with ID *k* can start operating as the leader

- How
  - I

Broad Idea: The process with the highest ID and still surviving becomes the leader

Chang and Roberts Algorithm

- Assume that each process has to pointers to other processes that it knows about
  - The previous process (anti-clockwise)
  - The next process (clockwise)

- The actual network might not be a ring, but we assume that every process maintains the above information to form a logical ring overlay

- Chang and Roberts algorithm needs the next pointer only (Unidirectional Ring)

Chang, Ernest, and Rosemary Roberts. "An improved algorithm for decentralized extrema-finding in circular configurations of processes." *Communications of the ACM* 22.5 (1979): 281-283.

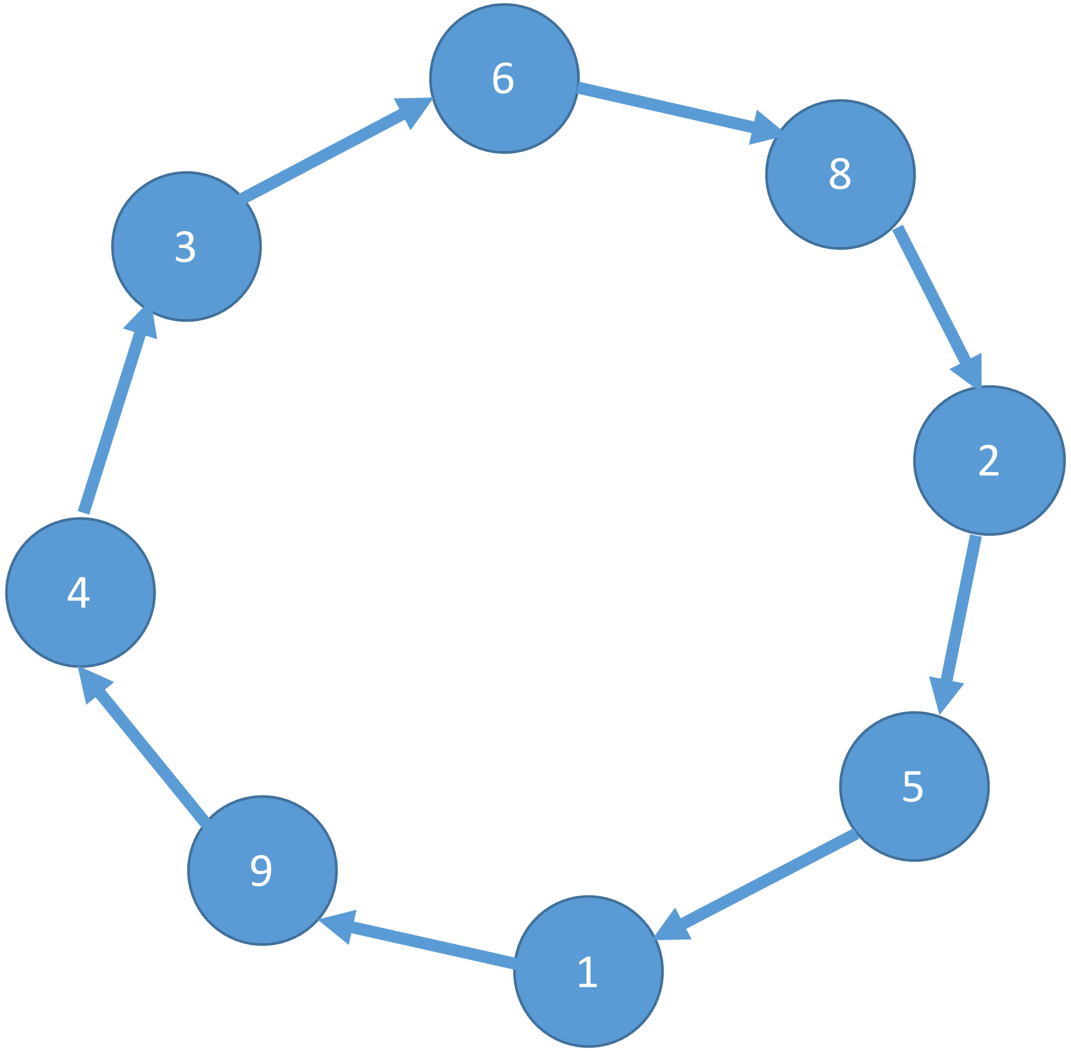
Chang and Roberts Algorithm

- **Uniform**: The algorithm does not need the information about the number of processes participating in the algorithm

- Asynchronous but reliable channel

- **The algorithm:**
  - A process that observes lack of leader (random timeout), starts the election procedure
  - Every process send **max(own ID, received ID)** to the next process
  - If a process receives its own ID, then it becomes the leader
  - Leader passes a message across the ring announcing that it is the leader, all other processes mark them as non-leader

Chang and Roberts Algorithm



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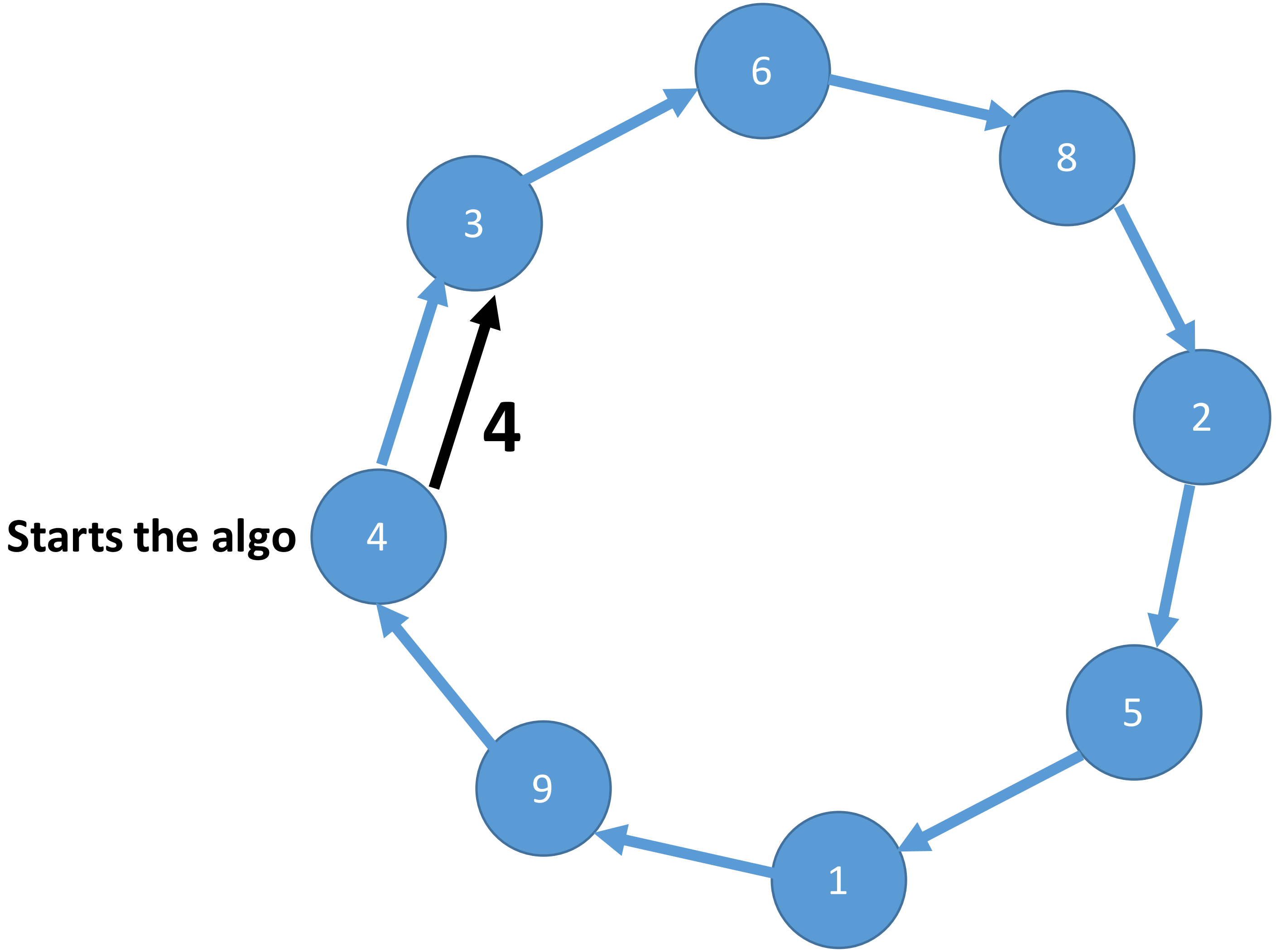

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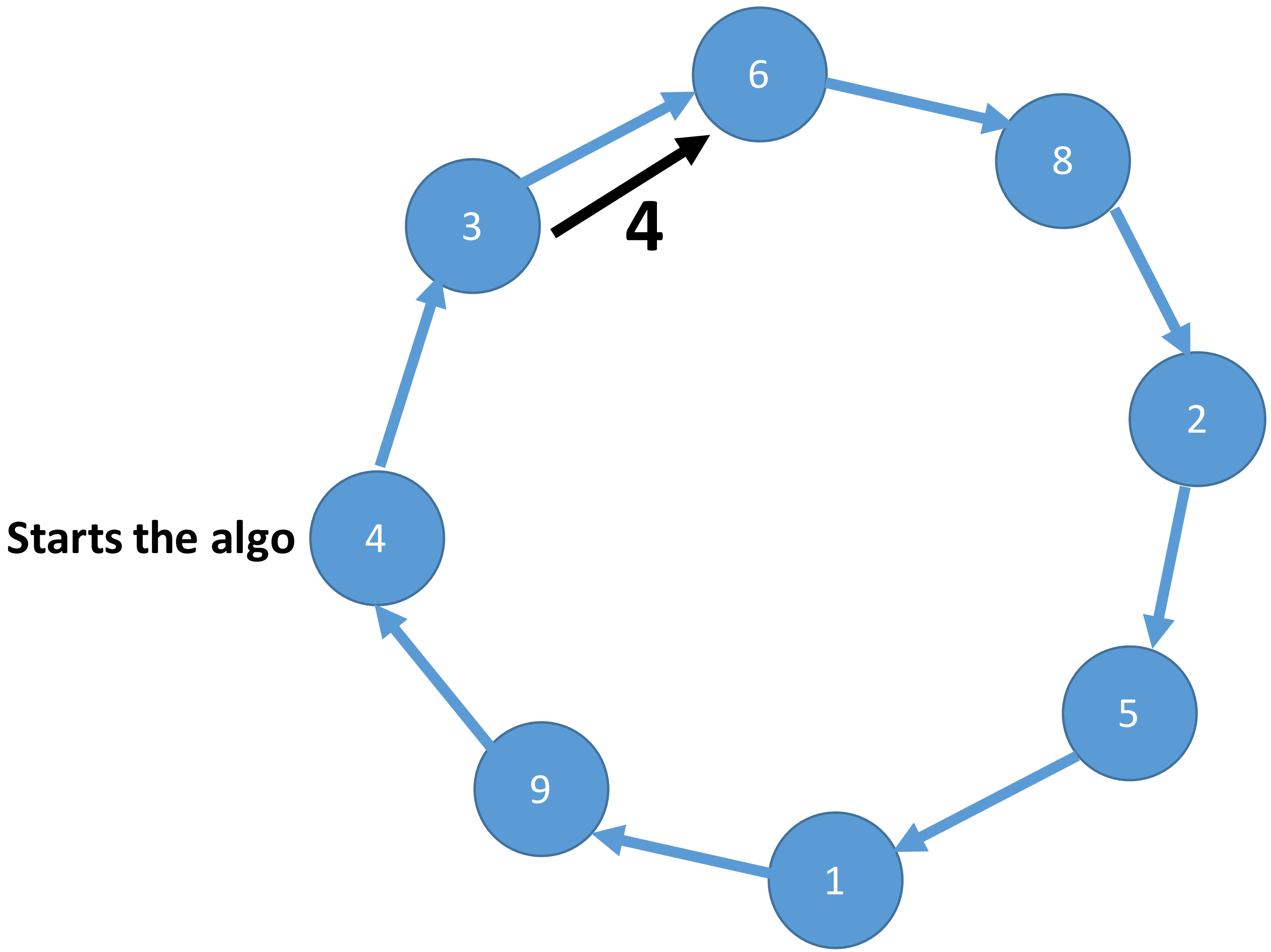

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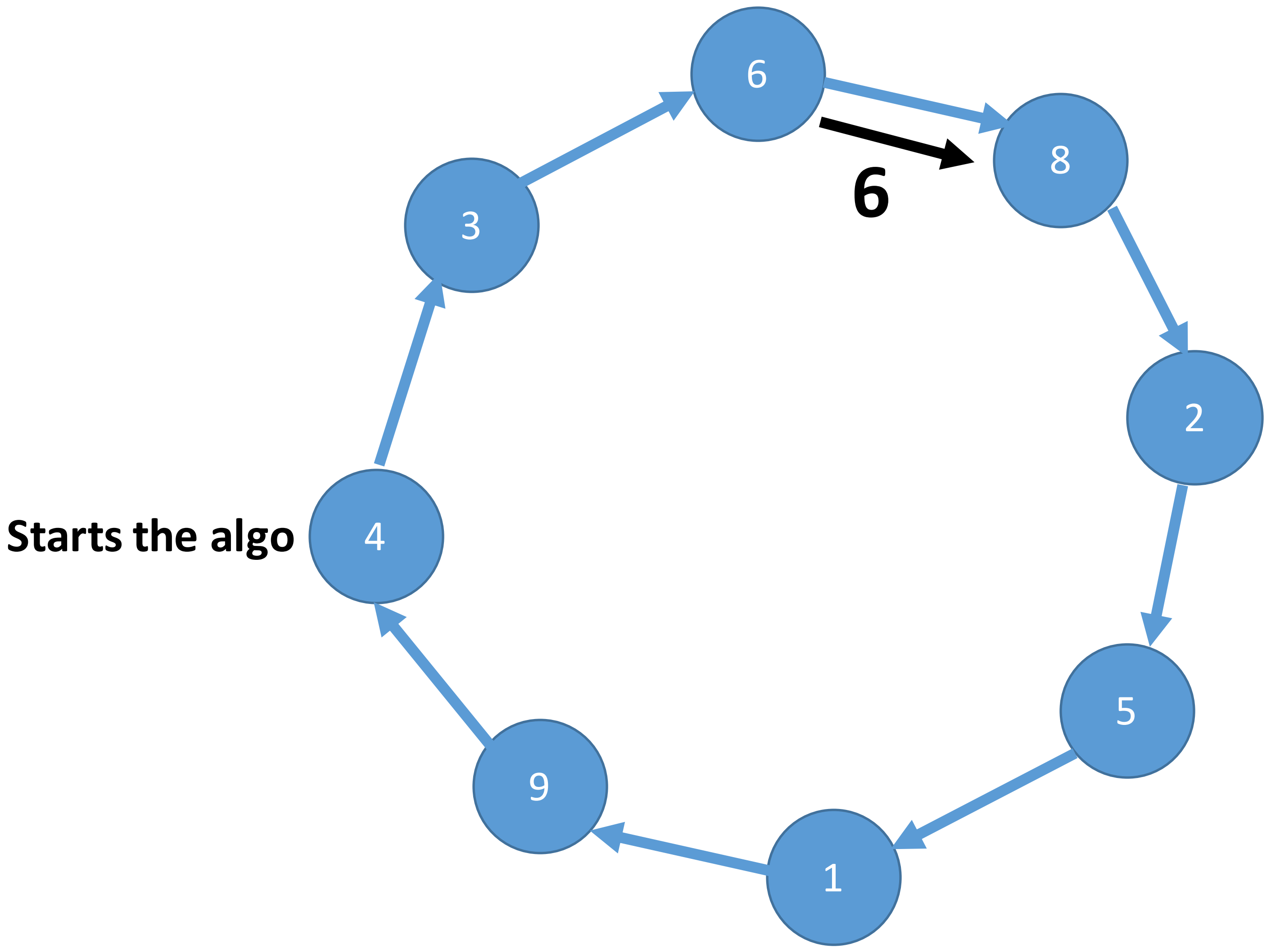

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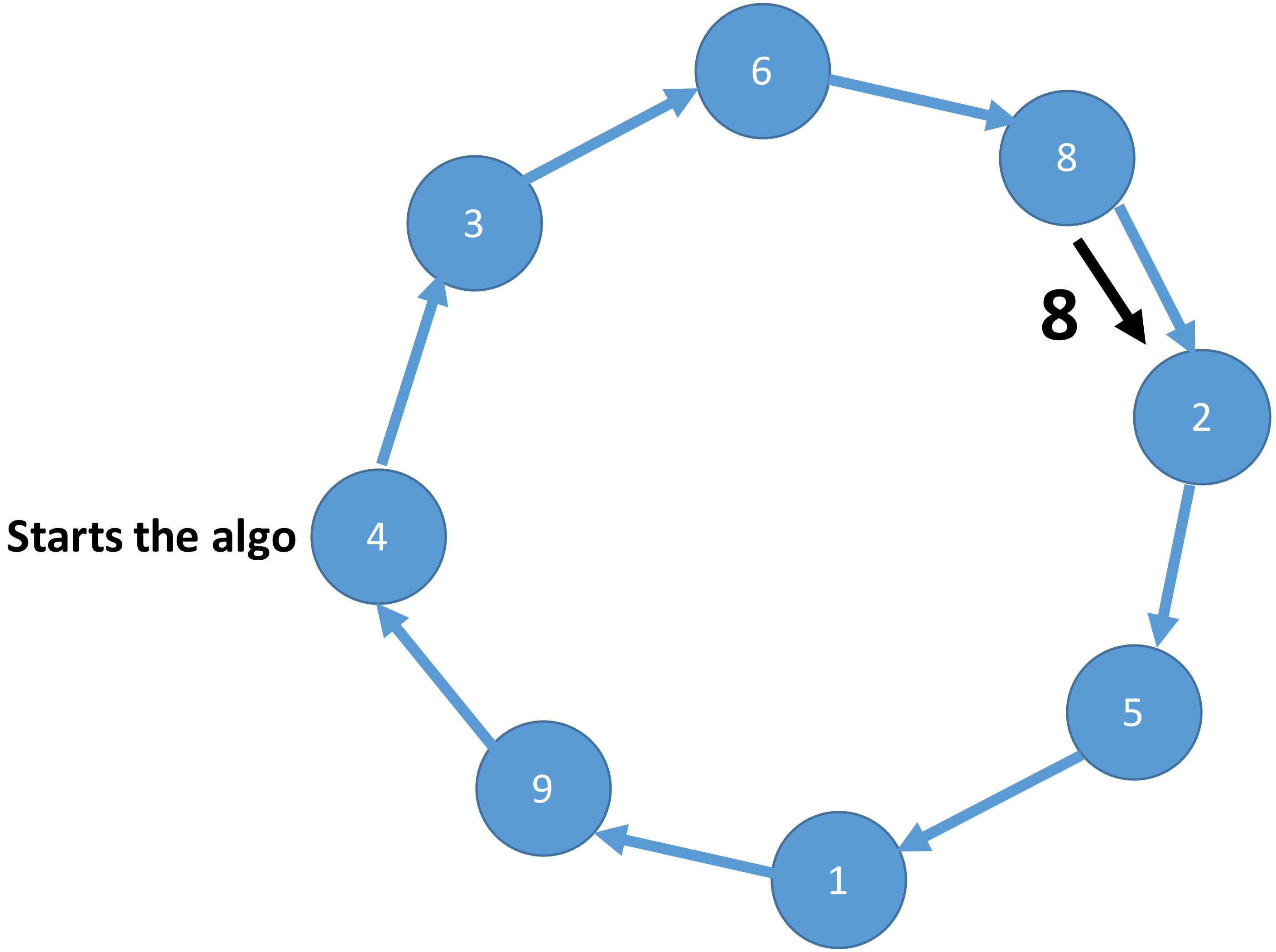

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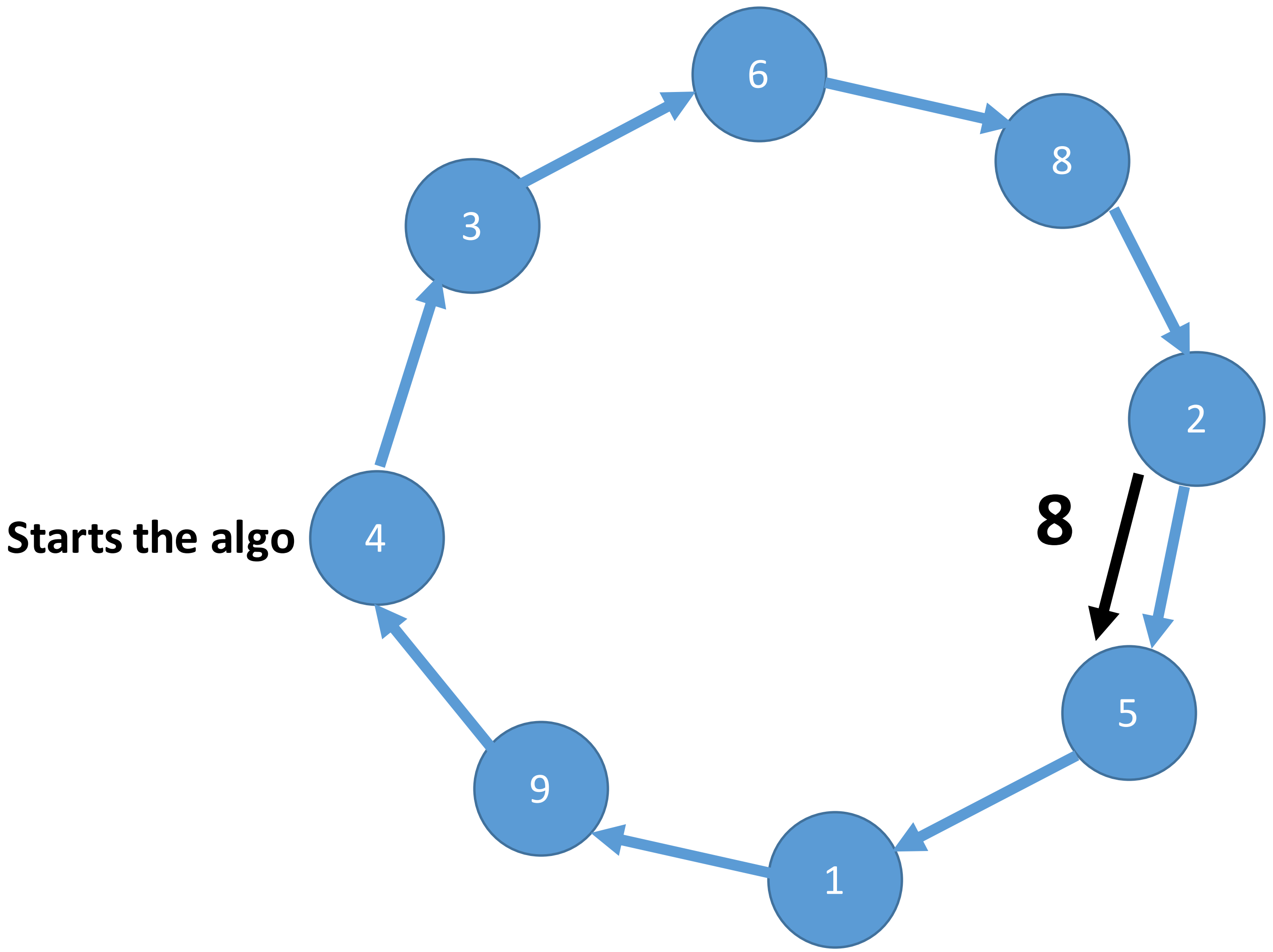

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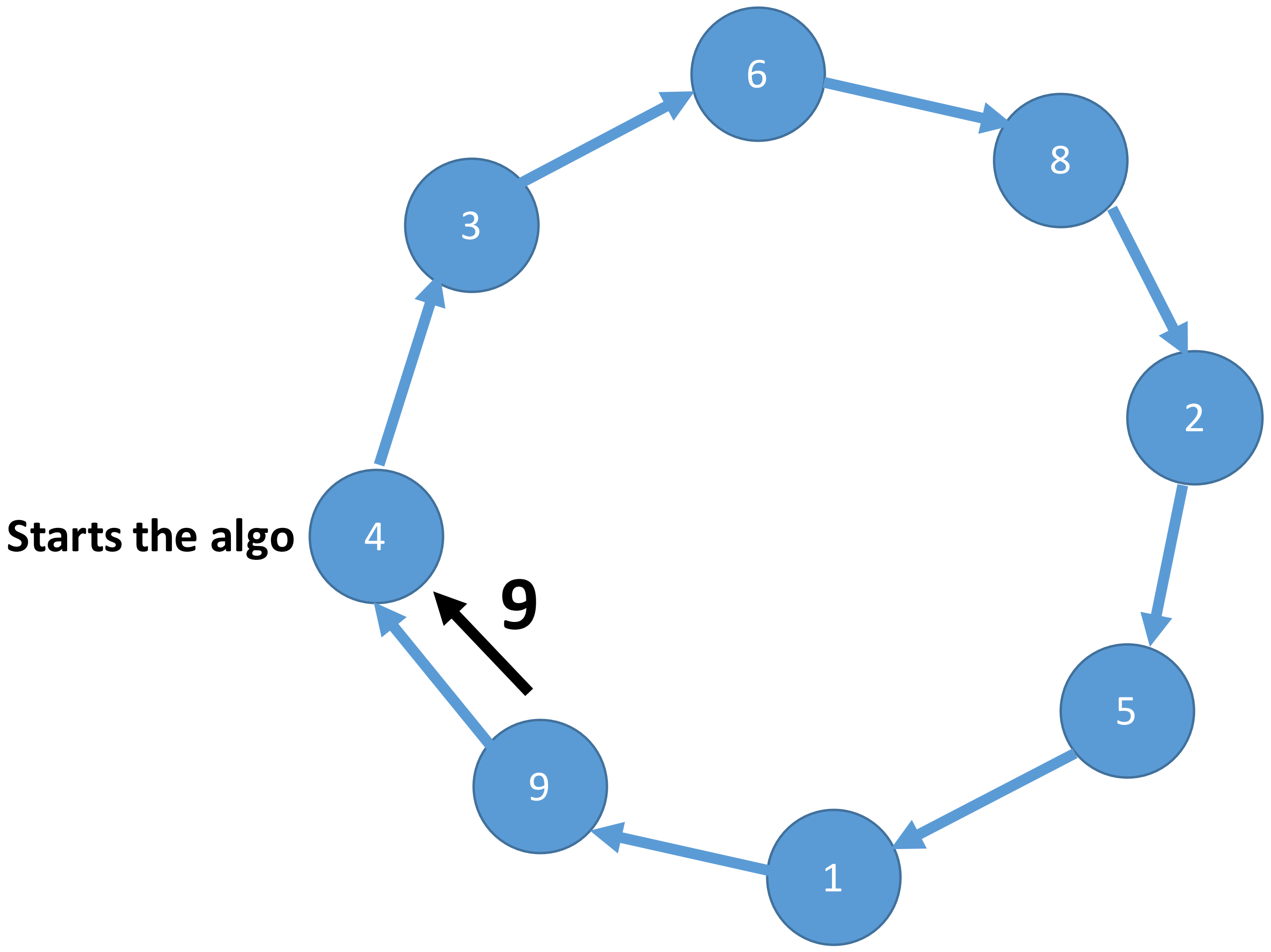

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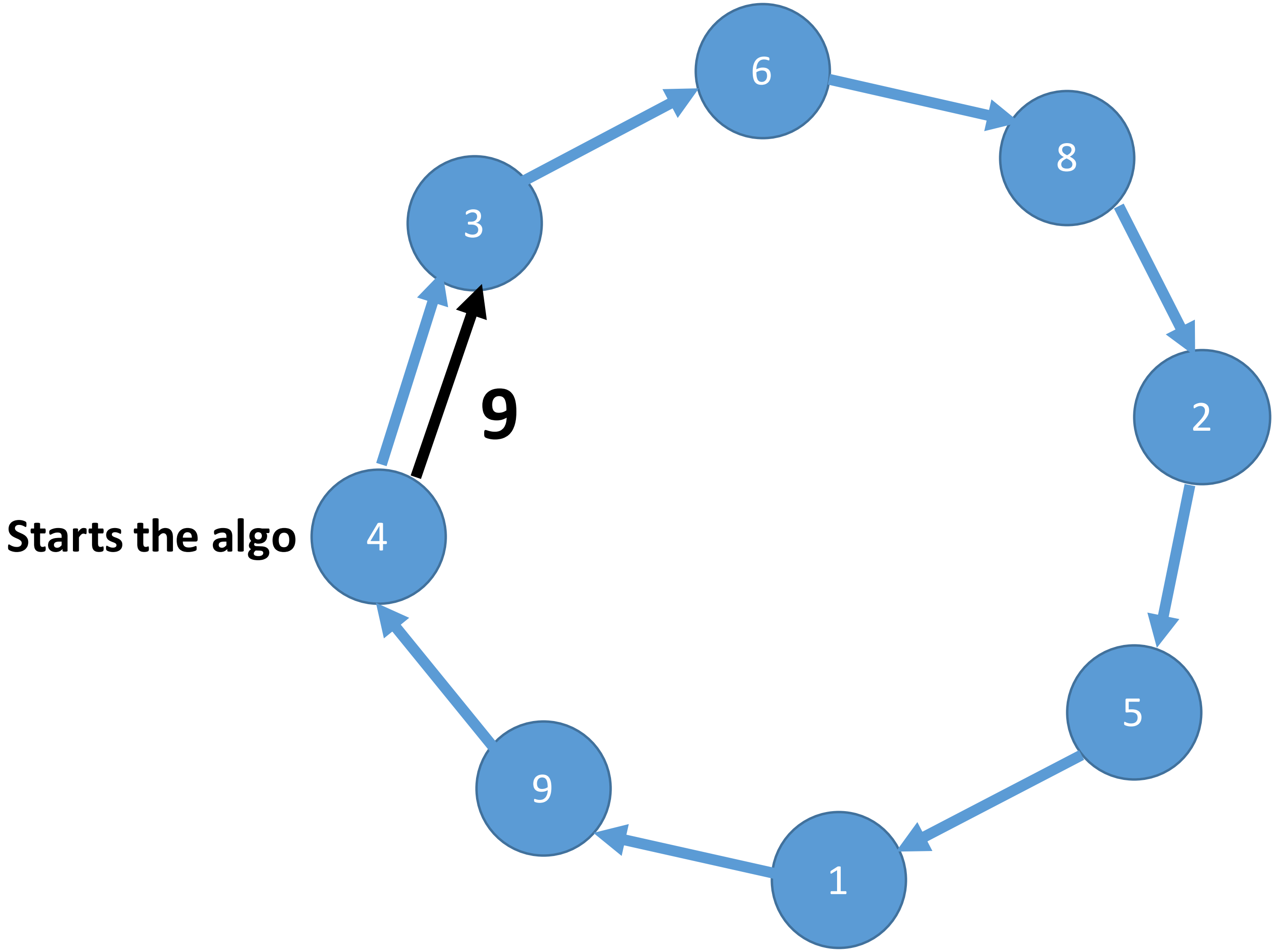

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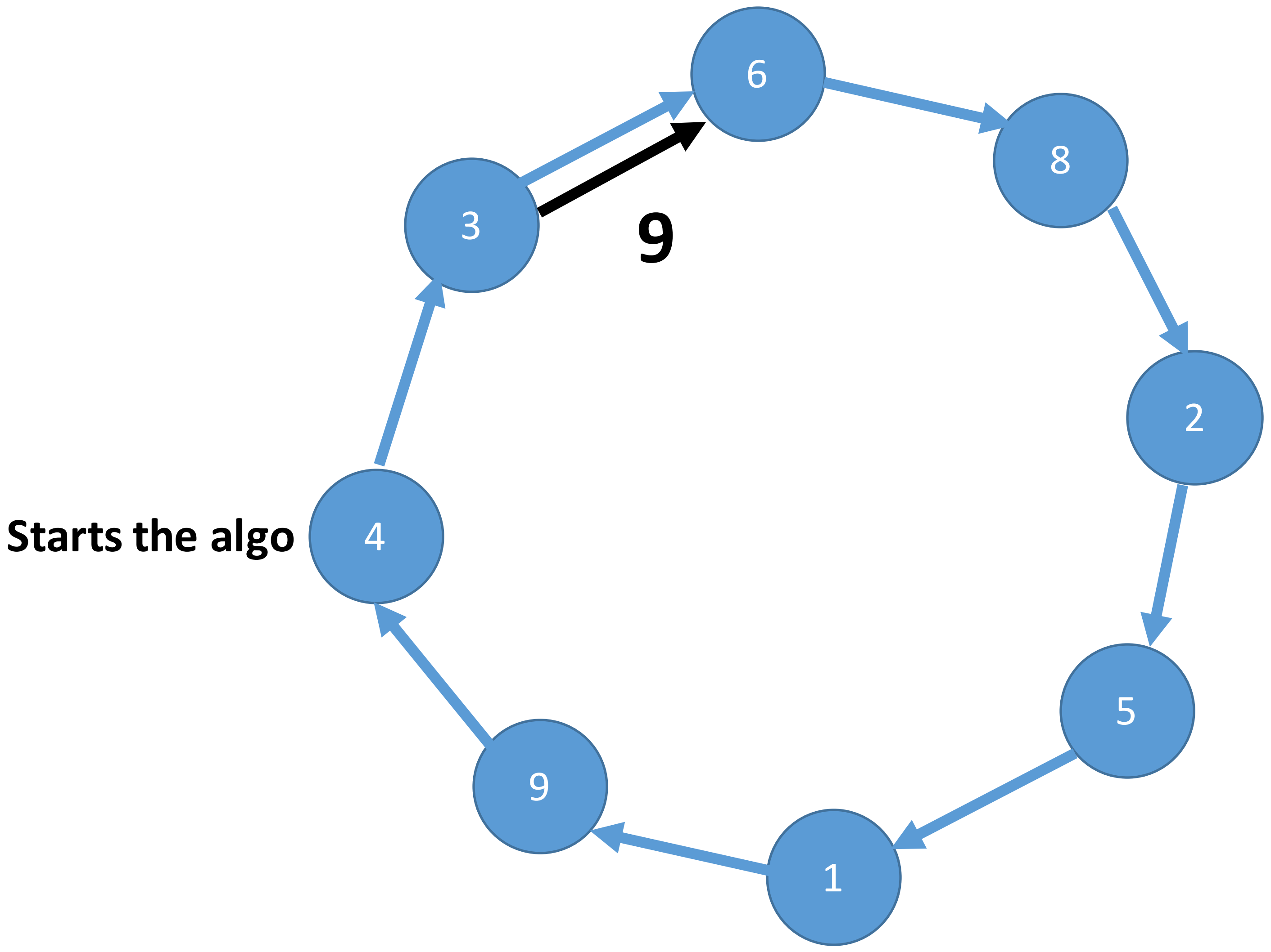

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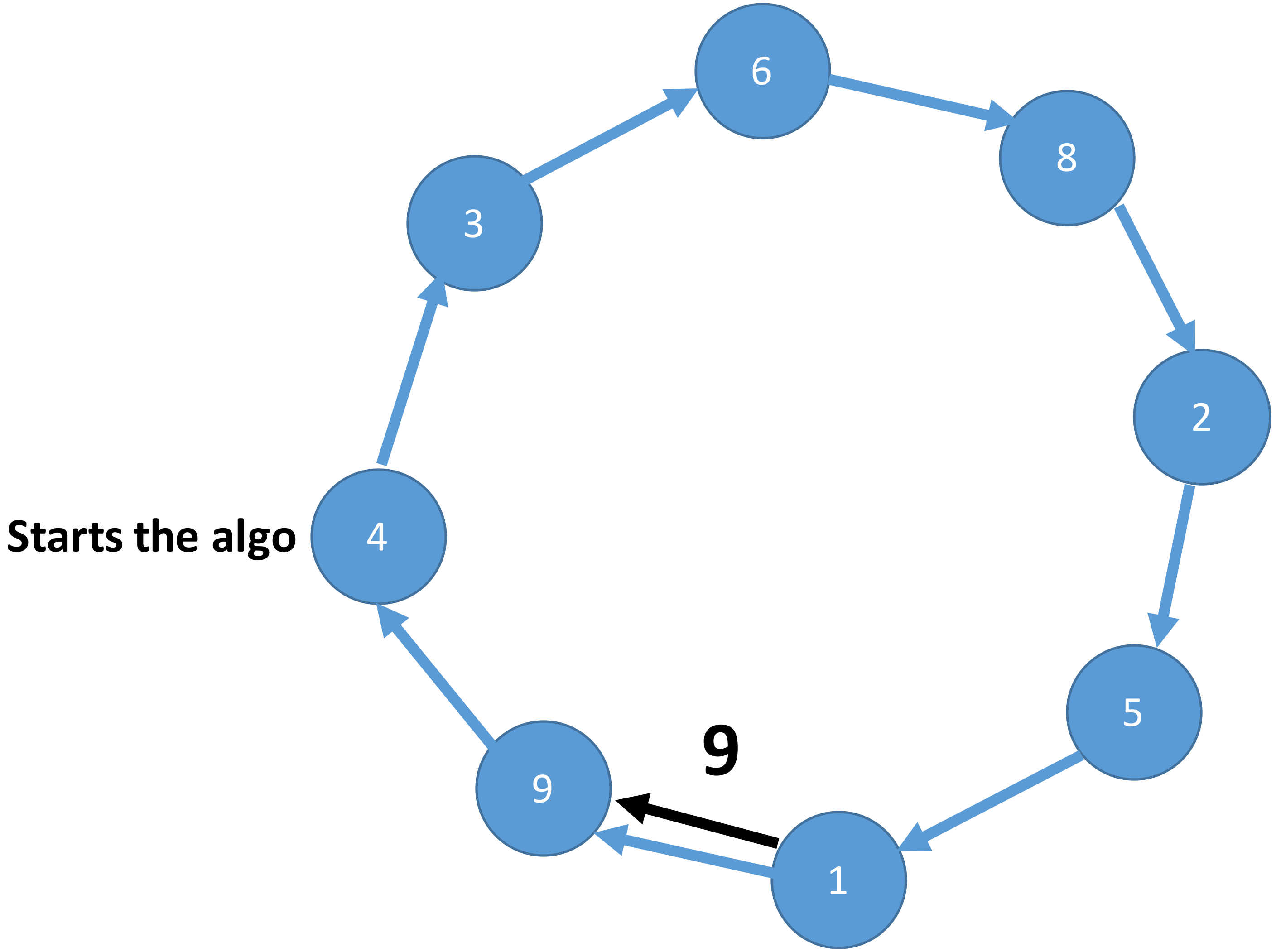

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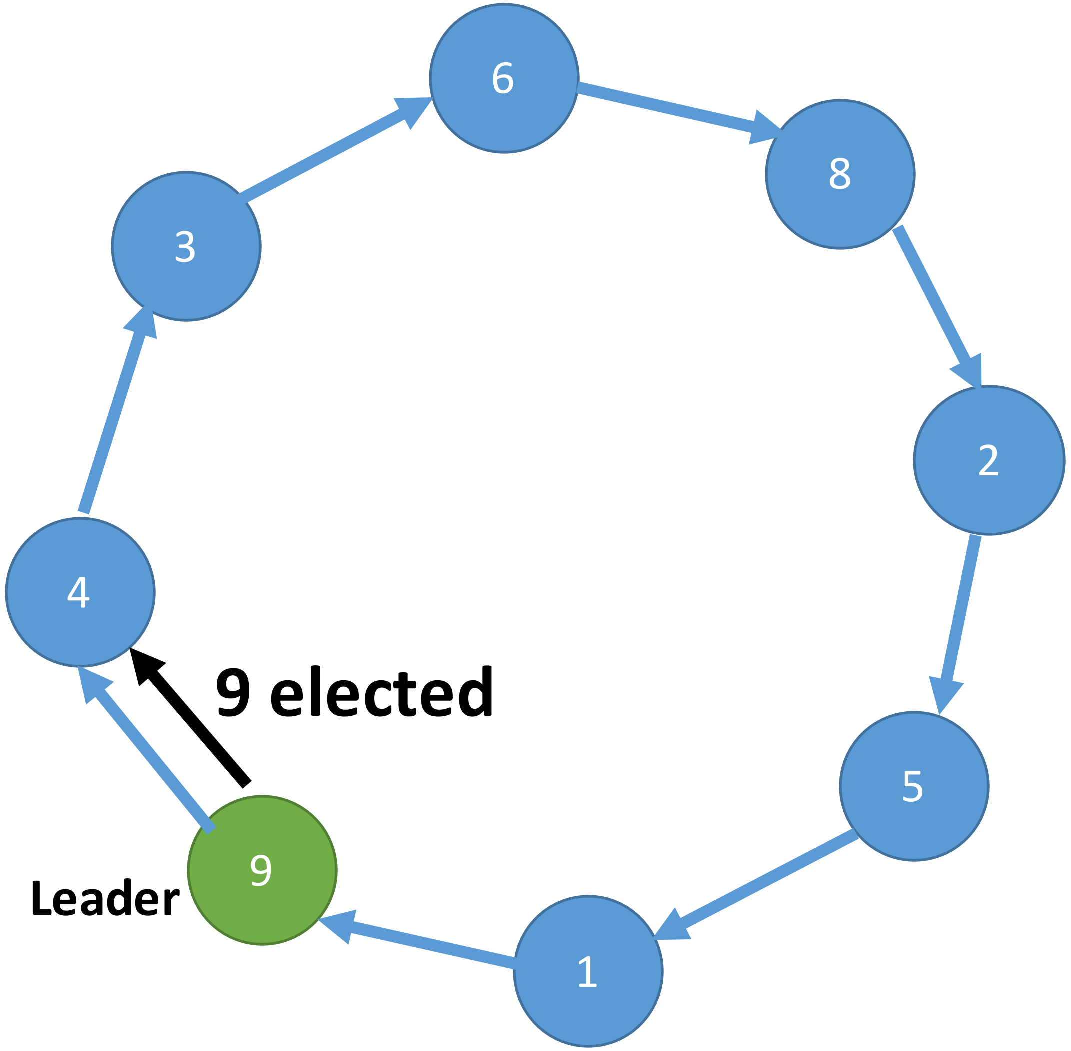

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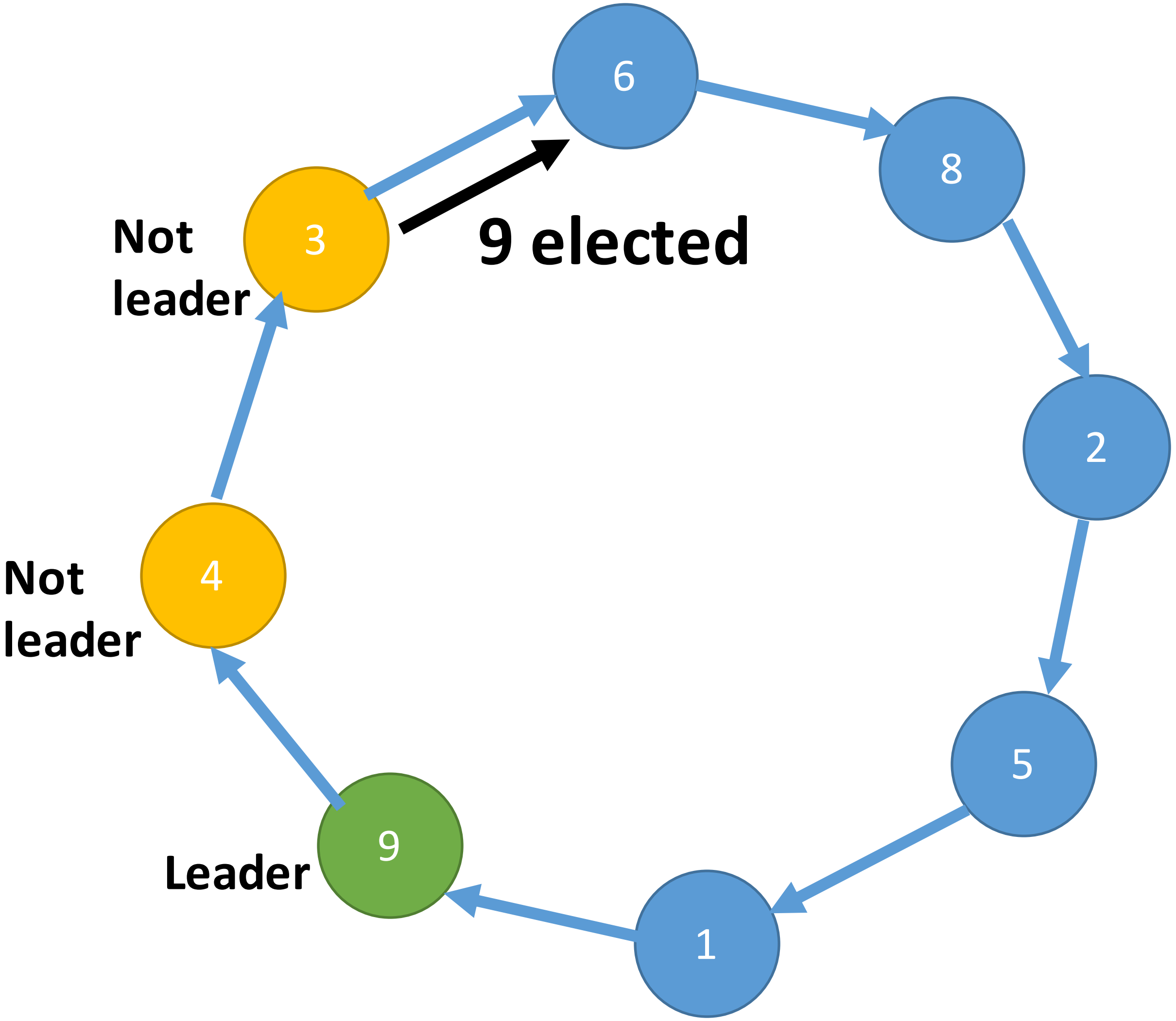

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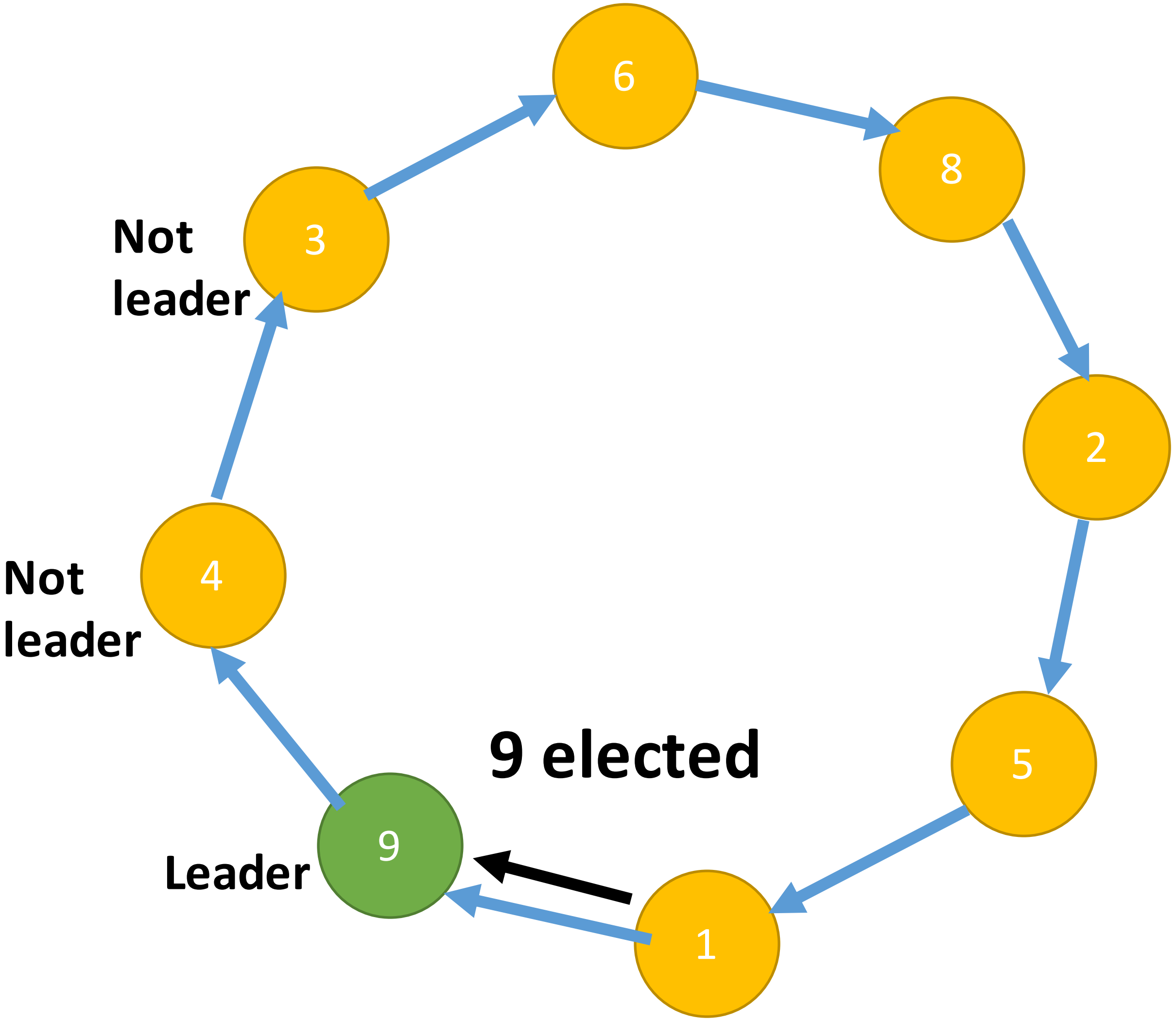

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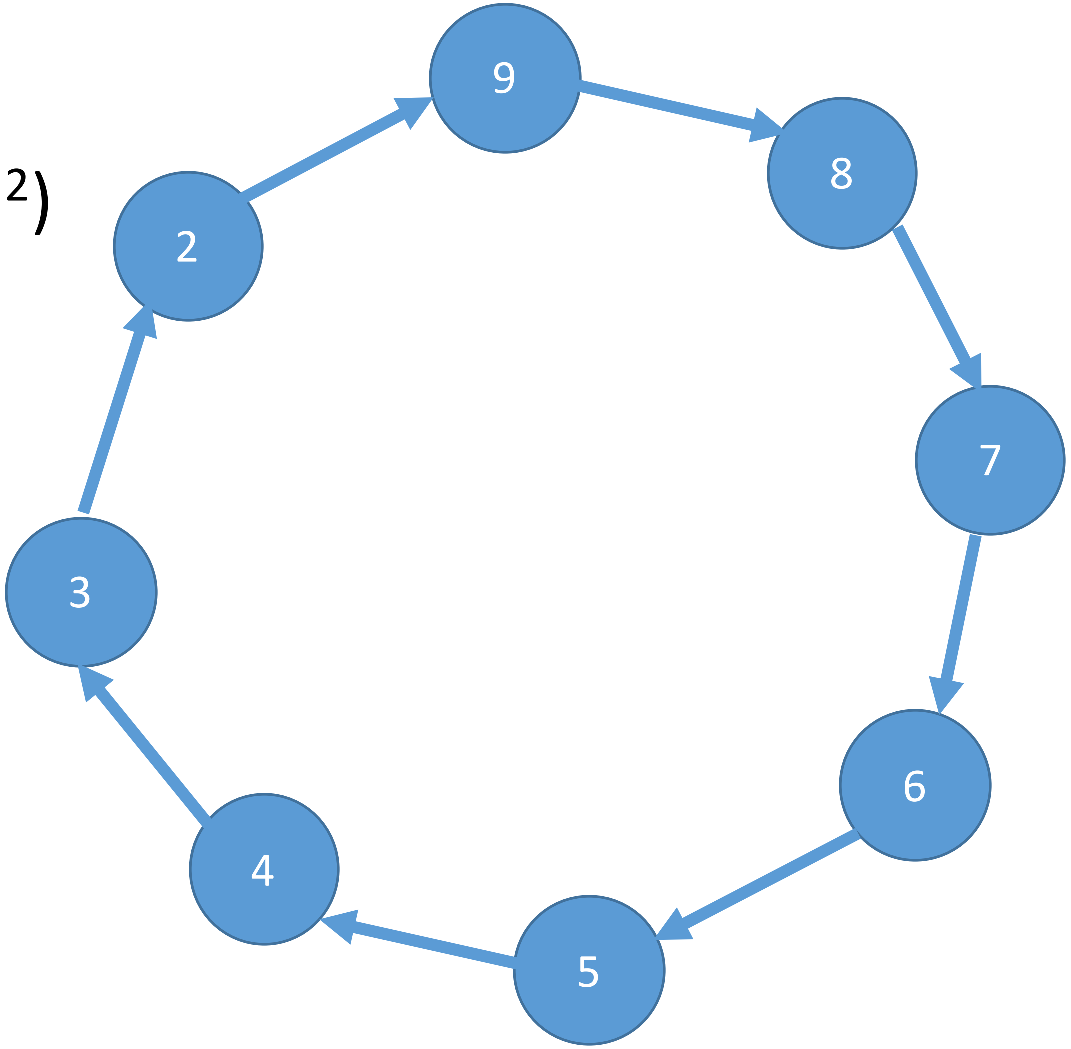
  - When does this occur?
  - Arrange the IDs in decreasing order



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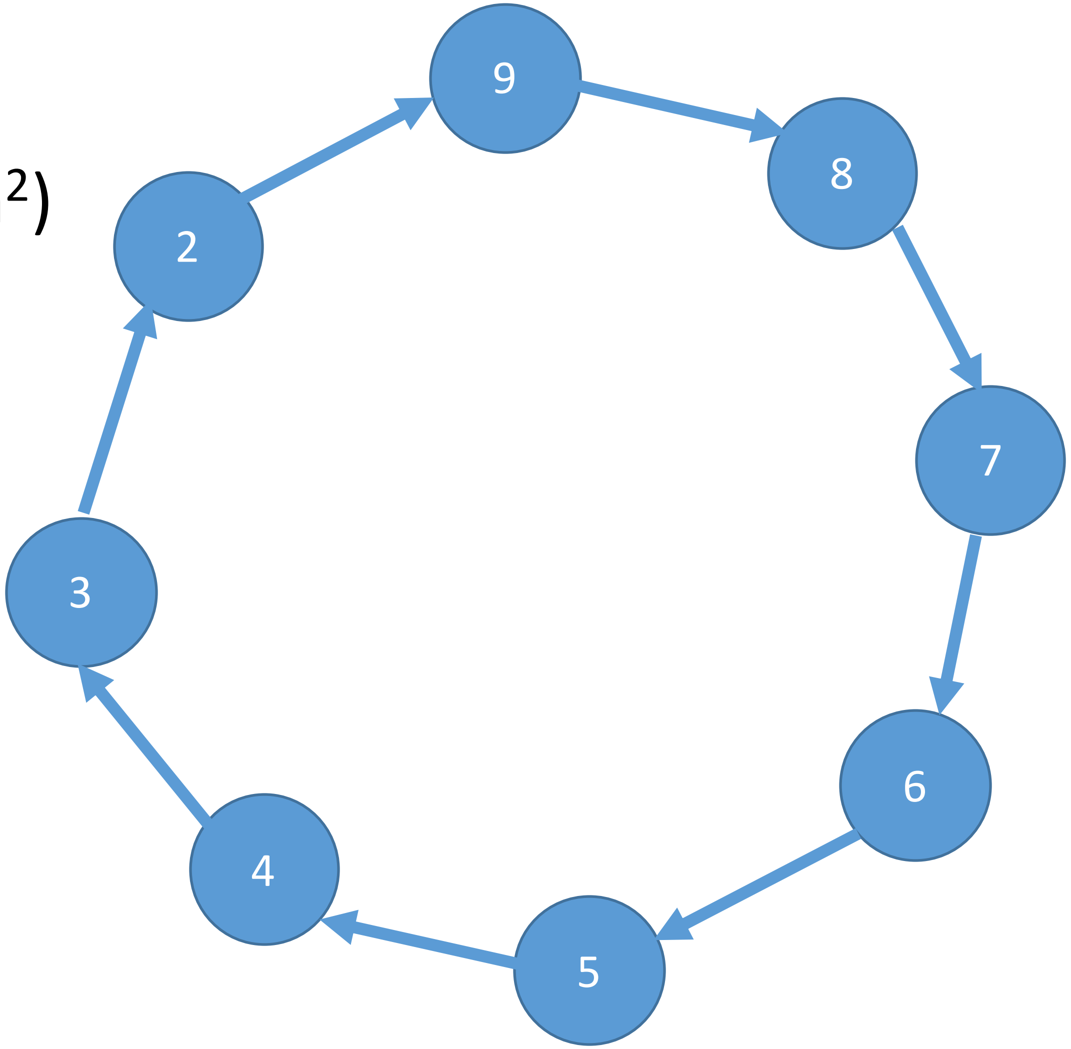
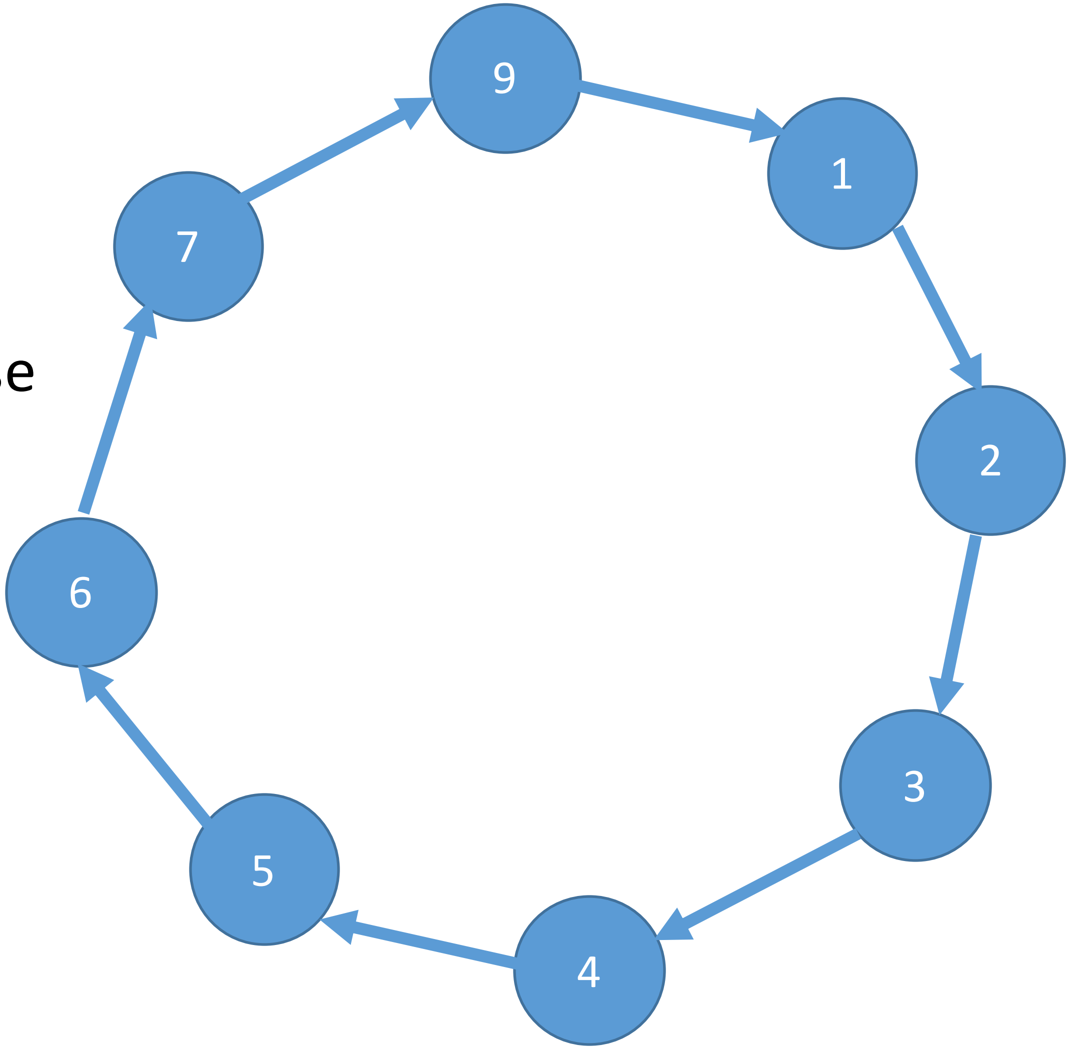
- **Worst-case Message Complexity**: $O(n^2)$
  - **When does this occur?**
  - **Arrange the IDs in decreasing order**
  - 2nd largest ID causes n-1 messages
  - 3rd largest ID causes n-2 messages
  - 4th largest ID causes n-3 messages
  - …
  - Total: $n + (n-1) + (n-2) + \ldots + 2 + 1 = O(n^2)$



Chang and Roberts Algorithm

- **Correctness:** Process with largest ID is elected as the leader, that message passes through all other processes

- **Best-case Message Complexity**: O(n)
  - **Arrange the IDs in increasing order**
  - Largest ID cases n messages, others cause exactly one message
  - Total messages = n + (1 + 1 … n-1 times)
    = 2n – 1 = O(n)



Chang and Roberts Algorithm – Average Case Analysis

- Let the IDs be 0, 1, 2, …, i, i+1, …, n-1
- Let P(i, k) be the probability that ID i makes exactly k steps
  - k-1 clockwise neighbors of i have IDs less than i and the k^{th} clockwise neighbor of i is greater than i
- There are (i - 1) processes having IDs less than i and (n - i) processes having IDs larger than i
- $P(i, k) = {}^{(i-1)}C_{(k-1)} \,/\, {}^{(n-1)}C_{(k-1)} \times (n-i)/(n-k)$
- Expected total number of messages: $E(k) = n + \sum_{i=1}^{n-1} \sum_{k=1}^{n-1} kP(i, k).$
- The above expected value can be simplified to $n(1+½+\ldots+1/n) = O(n \log n)$

Chang and Roberts Algorithm

- The algorithm is simple and works both in synchronous and asynchronous models

- But, can we reduce the message complexity?

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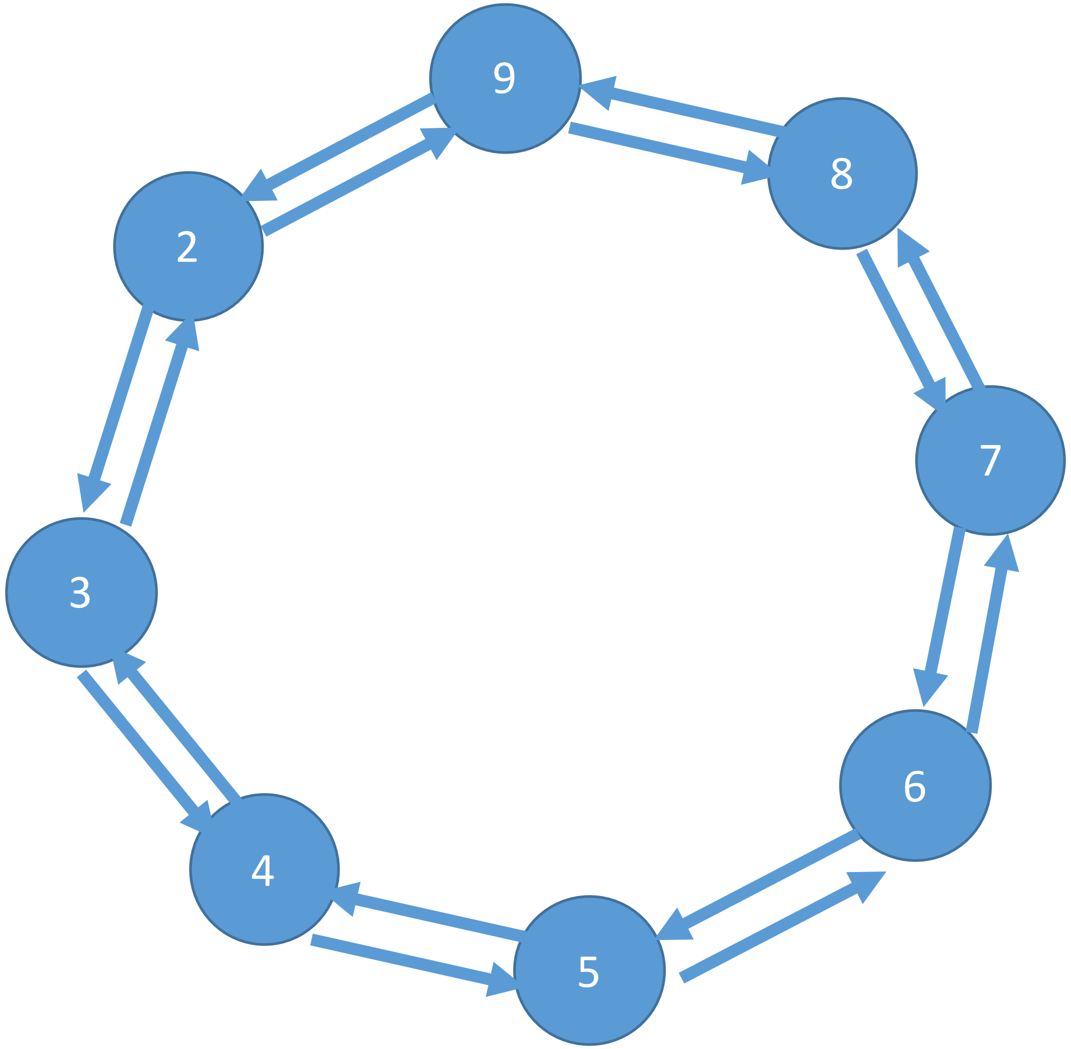
- But, can we reduce the message complexity?

- Core Idea: **Let the messages having the larger IDs travel smaller distance in the ring**

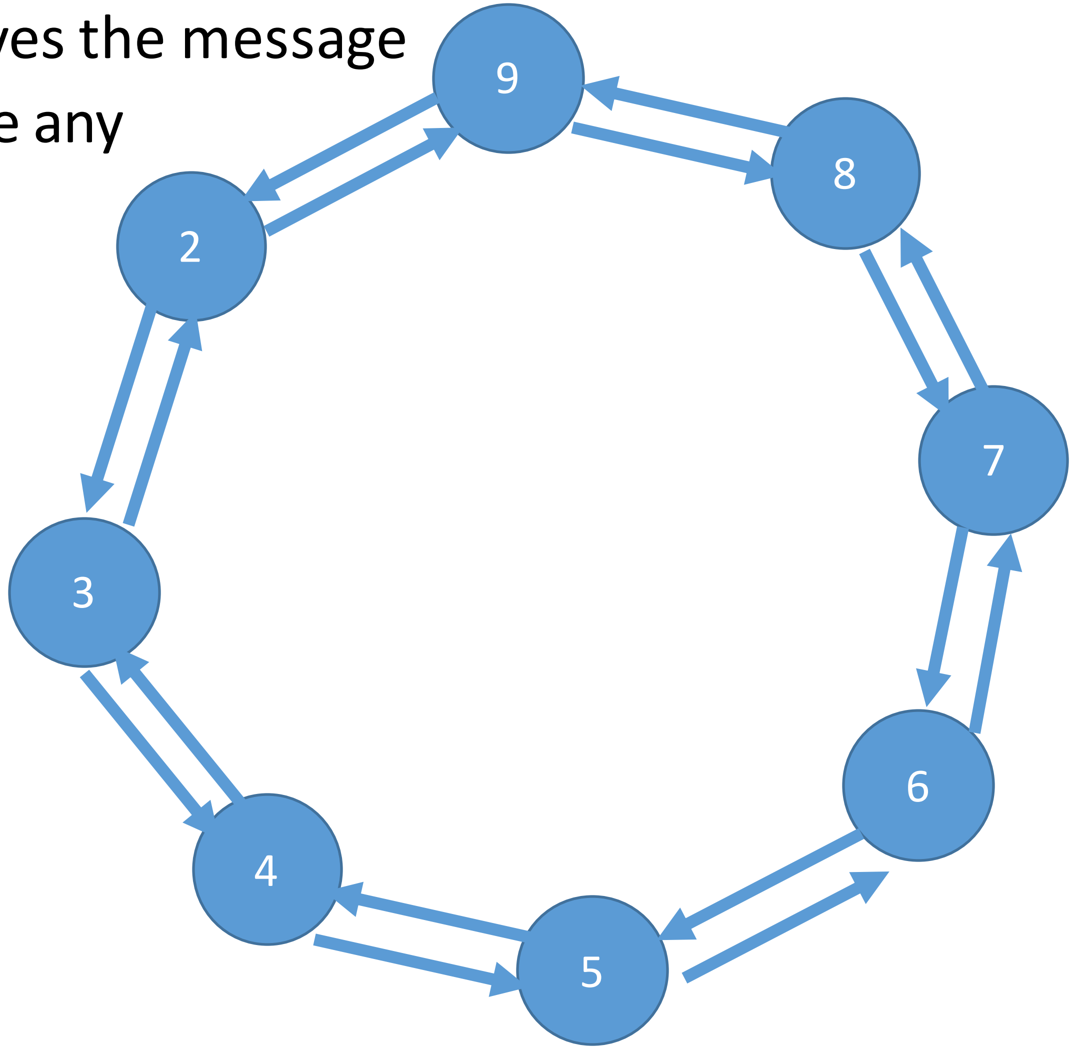
Hirschberg-Sinclair Algorithm

- **System model**: Same as earlier, but we need a bi-directional ring for this case
- We consider a scenario when each node wants to know the leader
- Define, *k*-neighborhood of a node *p*
  - *k* nodes at both sides of *p*



Hirschberg-Sinclair Algorithm

- **How does a node send message to distance *k*?**
  - Every message has a "**Time to Live**" variable
  - Each node decrements m.TTL as it receives the message
  - If m.TTL = 0, do not forward the message any further



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- The algorithm operates in phases
- Phase 0: Node p sends an election message m to both p.NEXT and p.PREVIOUS with m.ID = p.ID and TTL=1

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- Suppose, q receives this message
  - Set m.TTL = 0
  - If q.ID > m.ID, do nothing
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- If p gets back both the messages, it declares leader of its 1 neighborhood, and proceeds to the next phase

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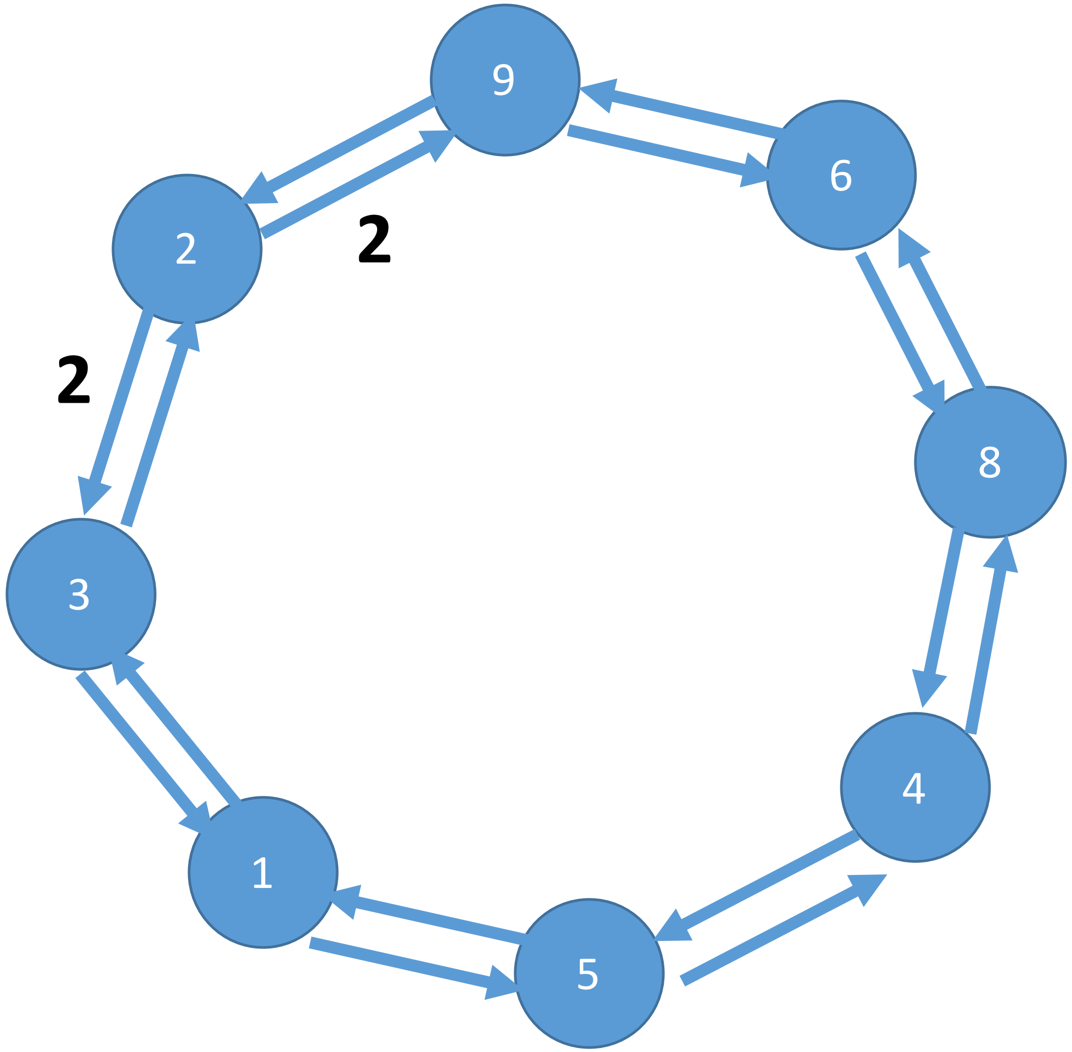

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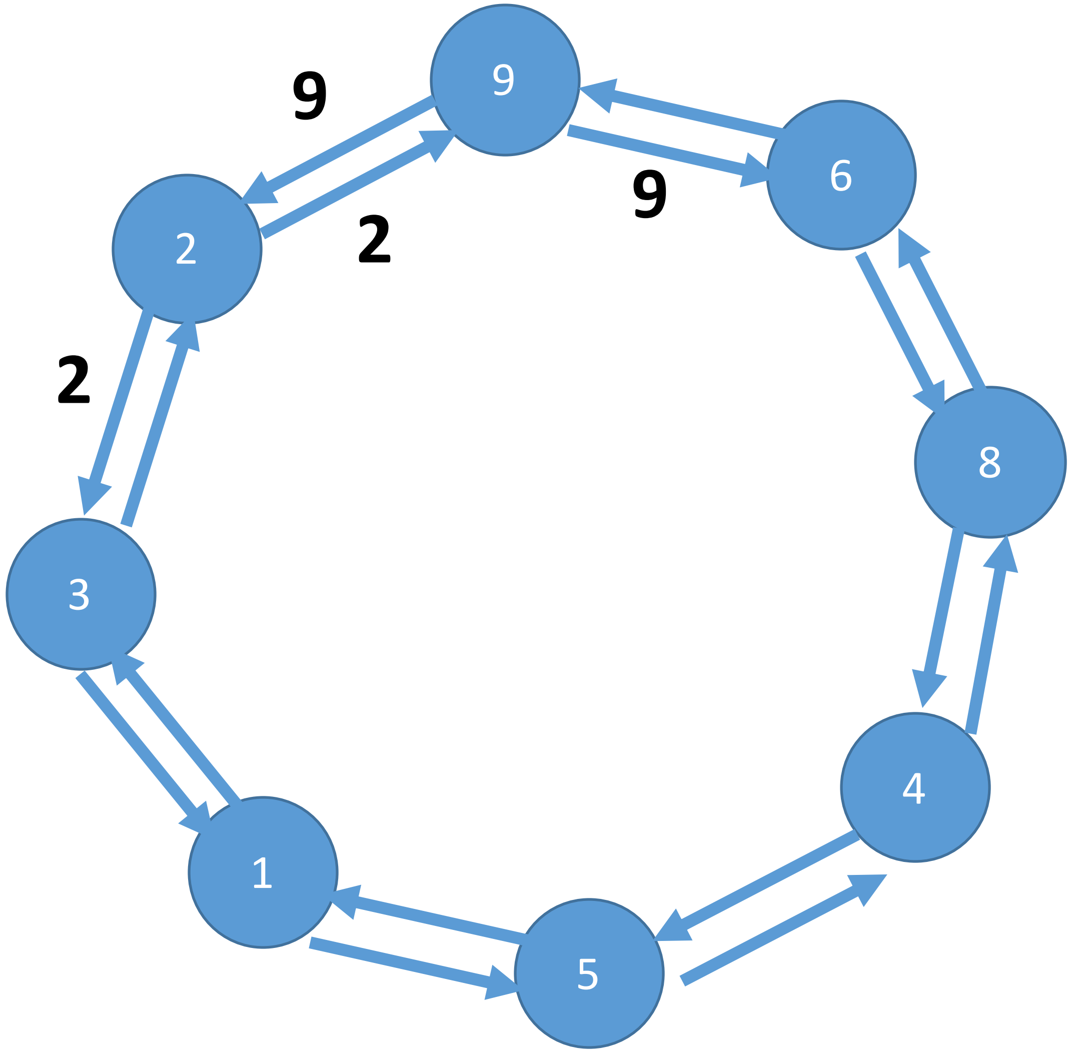

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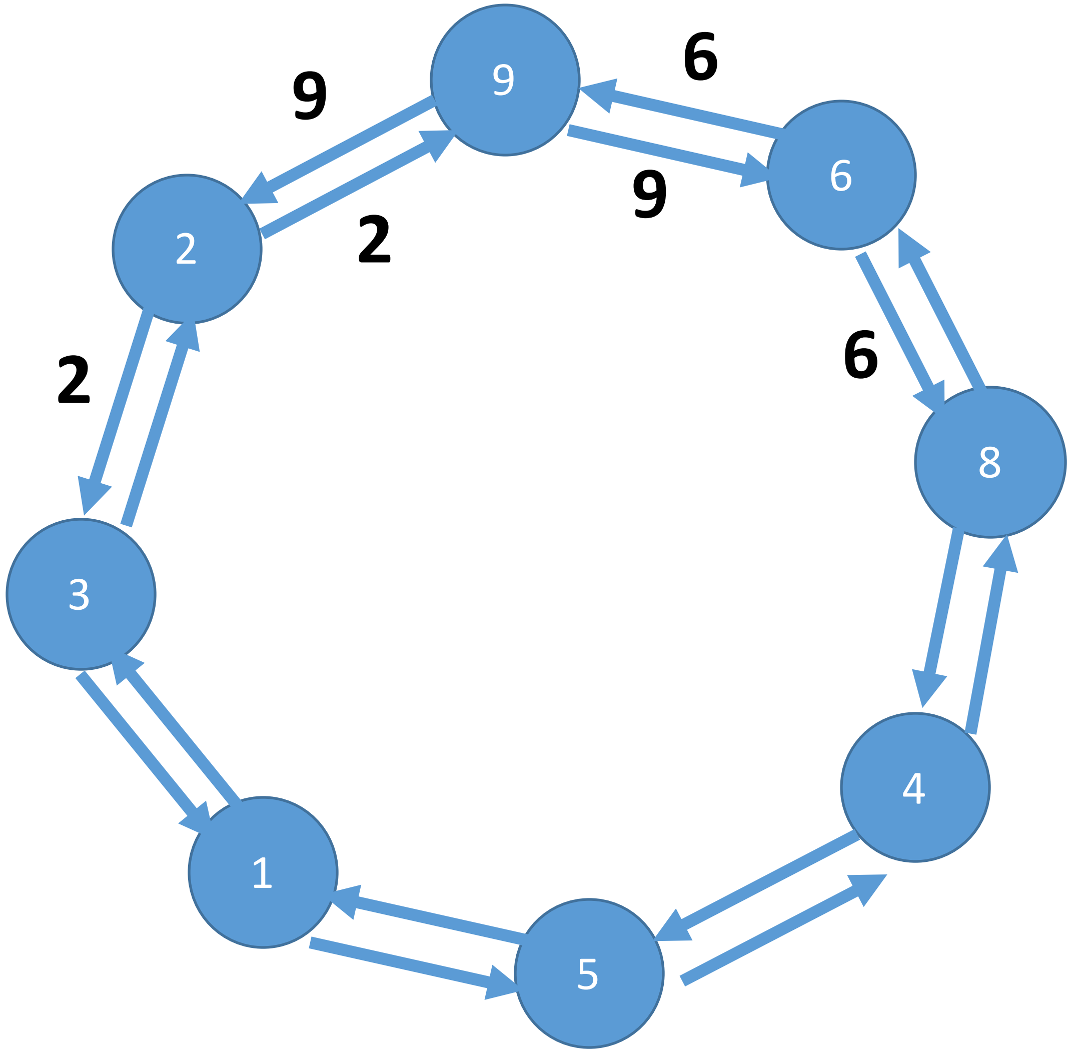

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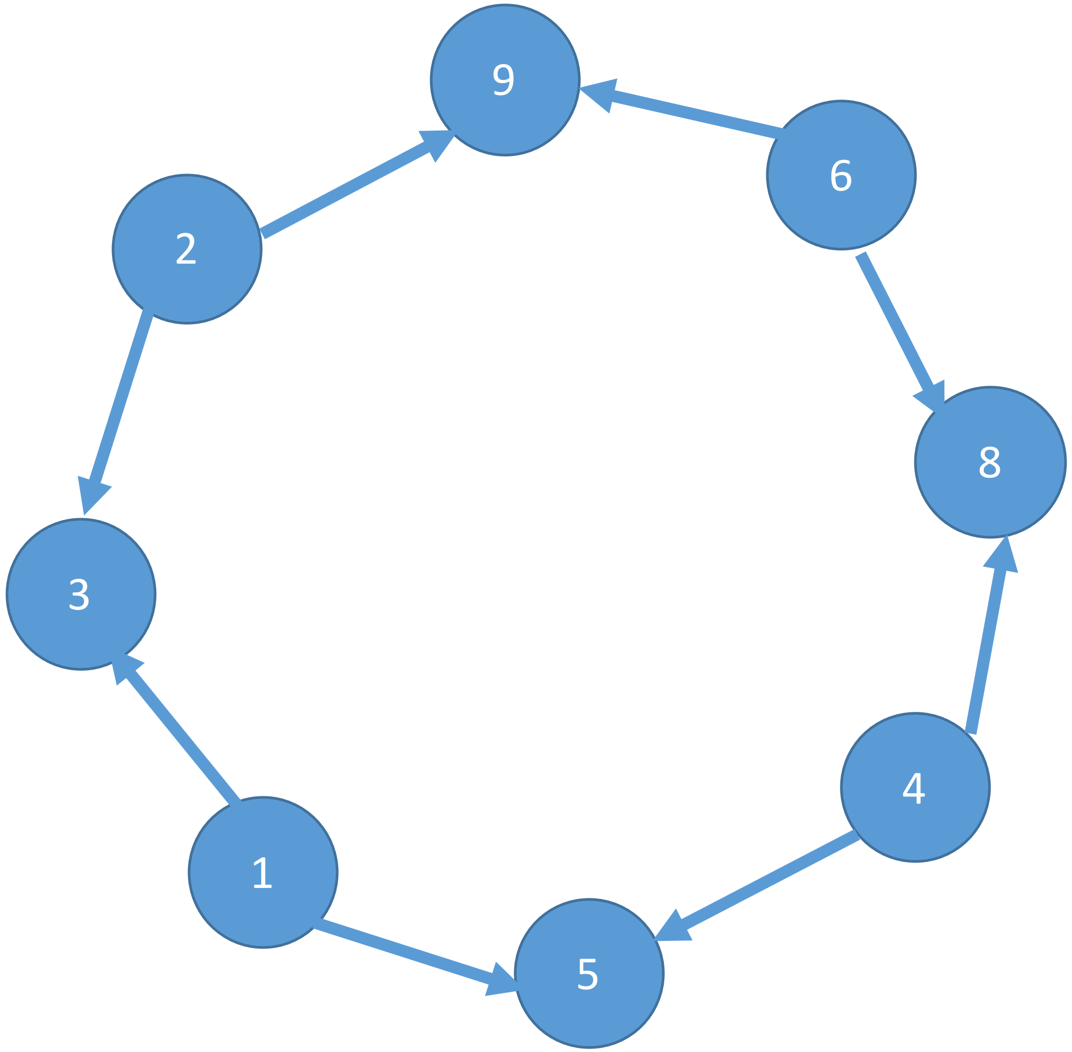

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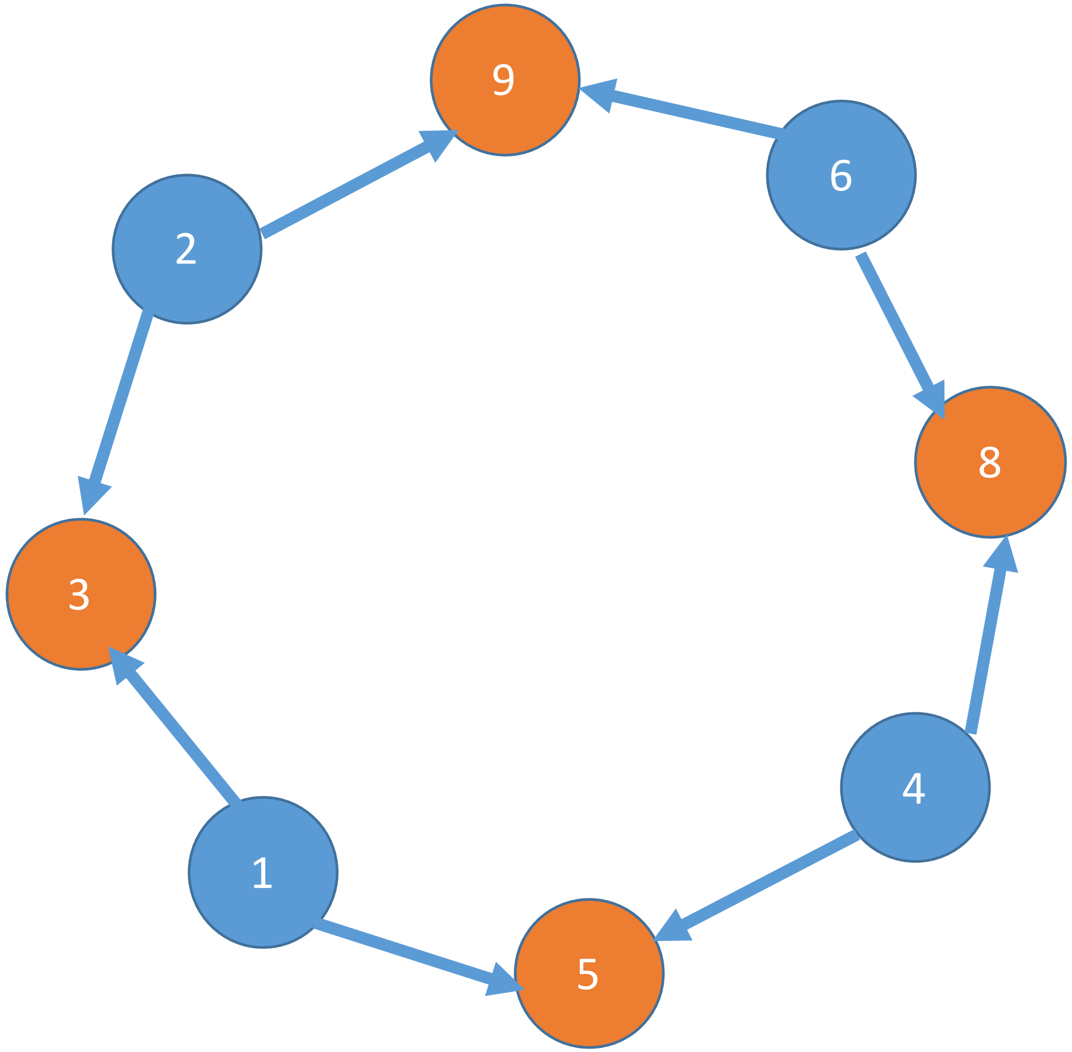

Hirschberg-Sinclair Algorithm

- The algorithm operates in phases
- Phase i: Node p sends an election message m to both p.NEXT and p.PREVIOUS with m.ID = p.ID and TTL=2^i

- Suppose, q receives this message
  - Set m.TTL = m.TTL-1
  - If q.ID > m.ID, do nothing
  - Else
    - If m.TTL=0, the return to the sending process
    - Else forward suitably to the previous or next process
- If p gets back both the messages, it declares leader of its 2^i neighborhood, and proceeds to the next phase

Hirschberg-Sinclair Algorithm

- When $2^i \geq n/2$, only one process gets back the message and becomes the leader
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- **What is the message complexity?**

Hirschberg-Sinclair Algorithm – Message Complexity

- In phase i,
  - At most one process initiates message in any sequence of 2^{i-1} processes
  - So, we have a total of $n/2^{i-1}$ candidate processes for sending messages
  - Each of the candidate processes sends 2 messages going at most 2^i distance; so total number of messages = 2×2^i
  - So, total messages = $O(n)$ in the ith phase
- There are $O(\log n)$ phases
- Therefore, message complexity = $O(n \log n)$

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- **Can we do it in O(n log n) for unidirectional ring?**
  - Yes, Peterson's Algorithm

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- **Can we do it better than O(n log n)?**

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- There are O(log n) phases
- Therefore, message complexity = O(n log n)

- **Can we do it better than O(n log n)? Not in asynchronous ring**
  - Any asynchronous Leader Election algorithm requires $\Omega(n \log n)$ messages.

Variable Time Algorithm in a Synchronous Ring

- Synchronous, round-based algorithm
  - Round = Maximum message transmission delay
  - A phase is equal to n rounds

- Node k does the following
  - If no message received when k-th phase starts, declare itself the leader and send a leader message with its id around the ring
  - If message received before k-th phase starts, record id in message as leader and forward the message around the ring

- Message complexity O(n)

Happy
Learning