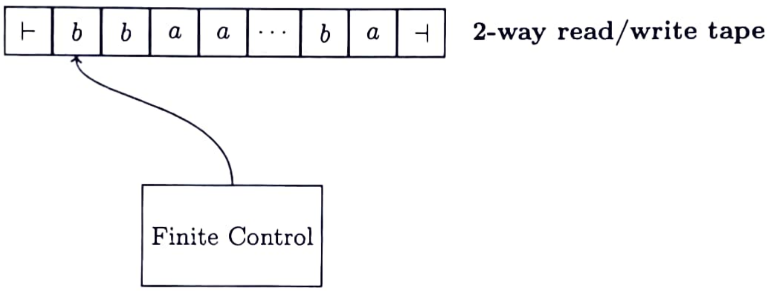
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Autumn Semester Examination 2022-23

Date of Examination : 23-09-2022 **Session(FN/AN)** FN **Duration** 2 hrs **Total Marks** 60
Subject No : CS41001 **Subject:** THEORY OF COMPUTATION
Department/Centre/School : Computer Science and Engineering
Specific charts, graph paper,log book etc. required No
Special Instructions (if any) Answer all questions. Keep your solutions brief and precise. State all assumptions you make. Sketchy proofs and claims without proper reasoning will be given no credit.

1. A *linear bounded automaton* (LBA) is exactly like a 1-tape TM, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten. The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers.



   (a) Give a rigorous formal definition of deterministic linearly bounded automata, including definitions of configurations and acceptance. [6]
   (b) Let $\mathcal{M}$ be an LBA with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x with $|x| = n$? [4]
   (c) Argue that it is possible to detect in finite time whether an LBA loops on a given input. [4]
   Hint: Use part (b).
   (d) Prove by diagonalisation that there exists a recursive set that is not accepted by any LBA. [10]

2. Show that $\{\mathcal{M} \mid \mathcal{L}(\mathcal{M})$ halts on all inputs of length less than $500\}$ is recursively enumerable but its complement is not. [10]

3. Is the language $\{\mathcal{M} \mid \mathcal{M}$ is a TM and $L(\mathcal{M})$ is uncountable$\}$ recursive, *r.e.* or not *r.e.* ? Justify. [4]

4. Consider the set $\mathsf{REG} = \{G \mid G$ is a context-free grammar and $L(G)$ is regular$\}$. Show that REG is undecidable via a reduction from PCP.
   Hint: Let $A = \{w_1, \ldots, w_k\}$ and $B = \{x_1, \ldots, x_k\}$ be an instance of PCP over alphabet Σ. Let $\Sigma' = \Sigma \cup \{a_1, \ldots, a_k\}$ for $a_1, \ldots, a_k \notin \Sigma$. Define two CFGs $G_A = (\{S_A\}, \Sigma', P_A, S_A)$ and $G_B = (\{S_B\}, \Sigma', P_B, S_B)$ where P_A consists of the productions $S_A \to w_i S_A a_i \mid w_i a_i$ for $1 \le i \le k$, and P_B consists of $S_B \to x_i S_B a_i \mid x_i a_i$ for $1 \le i \le k$. Show that $\neg L(G_A)$ and $\neg L(G_B)$ are CFLs. What can you say about $\neg L(G_A) \cup \neg L(G_A)$? [12]

5. A property $\mathcal{P}$ of partial recursive functions if a map $\mathcal{P} : \{\text{partial recursive functions}\} \longrightarrow \{\mathsf{T}, \mathsf{F}\}$. $\mathcal{P}$ is non-trivial if there exist indices u, v such that $\mathcal{P}(f_u) = \mathsf{T}$ and $\mathcal{P}(f_v) = \mathsf{F}$. Using Rogers's fixed point theorem, show that any non-trivial property of partial recursive functions is undecidable. [10]