INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: ....FN/AN *Time:* 3 Hrs. *Full Marks* ...75 ... *No. of Students* ...09 ...
Supp-Autumn Semester, 2012-2013 Deptt. ...CSE ... Sub No. ...**CS41001** ...
B. Tech.(Hons.) ...4^{th} Yr. ... Sub. Name ...**Theory of Computation** ...

Instructions : **Answer Q1 and any 3 from the remaining 4.**

1. **[7 × 6]**

   (a) L is a language over $\{0, 1\}$. When can you say that L is **NP**-*complete*?

   (b) Let L_1 and L_2 be two language over $\{0, 1\}$. It is known that L_2 is **NP**-*complete*. What are the steps to establish that L_1 is also **NP**-*complete*?

   (c) Let L_1 and L_2 be **coNP** languages. Does $L_1 \cap L_2$ belong to **coNP**? Justify your answer.

   (d) Prove that, $L_H = \{< M, x >$: Turing machine M halts on input $x\}$ is **NP**-*hard* but not **NP**-*complete*.

   (e) Give two definitions of the class **coNP**. Is there any **coNP**-*complete* problem?

   (f) Give an example of a language L that belongs to **NP** $\cap$ **coNP**.
   Justify that, if L is in **NP** $\cap$ **coNP**, then the complement of L is also in **NP** $\cap$ **coNP**.

   (g) What can you conclude about the *polynomial hierarchy* if $\Sigma_i^p = \Pi_i^p$?

2. Assume that *3SAT* is **NP**-*complete* and prove that

   $$CLIQUE = \{< G, k >: G \text{ is an undirected graph with a clique of size } k\}$$

   is also **NP**-*complete*. **[11]**

3. Informally describe the outline of the proof of the Cook-Levin theorem - *SAT* is **NP**-*complete*. **[11]**

4. Prove that

   $$TQBF = \{< Q_1x_1 \cdots Q_nx_n\phi(x_1, \cdots, x_n) >: \text{ where } Q_1x_1 \cdots Q_nx_n\phi(x_1, \cdots, x_n) \text{ is } true\}$$

   is **PSPACE**-*complete*. Note that Q_i is either '$\exists$' or '$\forall$', and $\phi(x_1, \cdots, x_n)$ is a boolean formula. **[11]**

5. (a) Define Σ_i^p, $i \geq 1$, using a polynomial time computable predicates and finite number of quantifiers, $\forall/\exists$.
   Define the *polynomial hierarchy* **PH**.

   (b) What is a complete problem of Σ_i^p, $i \geq 1$.
   Give an example (without proof) of a complete problem of Σ_i^p.

   (c) How do you characterise Σ_i^p, where $i \geq 2$, using *non-deterministic oracle Turing machine*?

   (d) Why does the *polynomial hierarchy*, **PH**, cannot have a complete problem? **[3 + 3 + 3 + 2]**