CHAPTER 5

INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY (FORMULAS)

KEY CONCEPTS AND SKILLS

- Be able to compute the future value of an investment made today
- Be able to compute the present value of cash to be received at some future date
- Be able to compute the return on an investment
- Be able to compute the number of periods that equates a present value and a future value given an interest rate
- Be able to use a financial calculator and/or a spreadsheet to solve time value of money problems

CHAPTER OUTLINE

- Future Value and Compounding
- Present Value and Discounting
- More about Present and Future Values

BASIC DEFINITIONS

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – "exchange rate" between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

FUTURE VALUES

- Suppose you invest $1,000 for one year at 5% per year. What is the future value in one year?
  - Interest = 1,000(.05) = 50
  - Value in one year = principal + interest = 1,000 + 50 = 1,050
  - Future Value (FV) = 1,000(1 + .05) = 1,050
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV = 1,000(1.05)(1.05) = 1,000(1.05)^2 = 1,102.50

FUTURE VALUES: GENERAL FORMULA

- $FV = PV(1 + r)^t$
  - FV = future value
  - PV = present value
  - r = period interest rate, expressed as a decimal
  - t = number of periods

- Future value interest factor = $(1 + r)^t$

EFFECTS OF COMPOUNDING

- Simple interest vs. Compound interest
- Consider the previous example
  - FV with simple interest = 1,000 + 50 + 50 = 1,100
  - FV with compound interest = 1,102.50
  - The extra 2.50 comes from the interest of .05(50) = 2.50 earned on the first interest payment

CALCULATOR KEYS

- Texas Instruments BA-II Plus
  - FV = future value
  - PV = present value
  - I/Y = period interest rate
    - P/Y must equal 1 for the I/Y to be the period rate
    - Interest is entered as a percent, not a decimal
  - N = number of periods
  - Remember to clear the registers (CLR TVM) after each problem
  - Other calculators are similar in format

FUTURE VALUES – EXAMPLE 2

- Suppose you invest the $1,000 from the previous example for 5 years. How much would you have?
  - FV = $1{,}000(1.05)^5 = 1{,}276.28$
- The effect of compounding is small for a small number of periods, but increases as the number of periods increases. (Simple interest would have a future value of $1,250, for a difference of $26.28.)

FUTURE VALUES – EXAMPLE 3

- Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - FV = $10(1.055)^{200}$ = 447,189.84
- What is the effect of compounding?
  - Simple interest = 10 + 200(10)(.055) = 120.00
  - Compounding added $447,069.84 to the value of the investment

FUTURE VALUE AS A GENERAL GROWTH FORMULA

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?
  - $FV = 3{,}000{,}000(1.15)^5 = 6{,}034{,}072$

QUICK QUIZ – PART I

- What is the difference between simple interest and compound interest?
- Suppose you have $500 to invest and you believe that you can earn 8% per year over the next 15 years.
  - How much would you have at the end of 15 years using compound interest?
  - How much would you have using simple interest?

PRESENT VALUES

- How much do I have to invest today to have some amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the "value" of something, we are talking about the present value unless we specifically indicate that we want the future value.

PRESENT VALUE – ONE PERIOD EXAMPLE

- Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?
- PV = 10,000 / $(1.07)^1$ = 9,345.79
- Calculator
  - 1 N
  - 7 I/Y
  - 10,000 FV
  - CPT PV = -9,345.79

PRESENT VALUES – EXAMPLE 2

- You want to begin saving for your daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
  - PV = 150,000 / $(1.08)^{17}$ = 40,540.34

PRESENT VALUES – EXAMPLE 3

- Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?
  - PV = 19,671.51 / $(1.07)^{10}$ = 10,000

PRESENT VALUE – IMPORTANT RELATIONSHIP I

- For a given interest rate – the longer the time period, the lower the present value
  - What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%
  - 5 years: $PV = 500 / (1.1)^5 = 310.46$
  - 10 years: $PV = 500 / (1.1)^{10} = 192.77$

PRESENT VALUE – IMPORTANT RELATIONSHIP II

- For a given time period – the higher the interest rate, the smaller the present value
  - What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%: PV = $500 / (1.1)^5 = 310.46$
    - Rate = 15%; PV = $500 / (1.15)^5 = 248.59$

QUICK QUIZ – PART II

- What is the relationship between present value and future value?
- Suppose you need $15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?
- If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

THE BASIC PV EQUATION - REFRESHER

- $PV = FV / (1 + r)^t$
- There are four parts to this equation
  - PV, FV, r and t
  - If we know any three, we can solve for the fourth
- If you are using a financial calculator, be sure to remember the sign convention or you will receive an error (or a nonsense answer) when solving for r or t

DISCOUNT RATE

- Often we will want to know what the implied interest rate is on an investment
- Rearrange the basic PV equation and solve for r
  - $FV = PV(1 + r)^t$
  - $r = (FV / PV)^{1/t} - 1$
- If you are using formulas, you will want to make use of both the y^x and the 1/x keys

DISCOUNT RATE – EXAMPLE 1

- You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. What is the implied rate of interest?
  - $r = (1{,}200 / 1{,}000)^{1/5} - 1 = .03714 = 3.714\%$
  - Calculator – the sign convention matters!
    - N = 5
    - PV = -1,000 (you pay 1,000 today)
    - FV = 1,200 (you receive 1,200 in 5 years)
    - CPT I/Y = 3.714%

DISCOUNT RATE – EXAMPLE 2

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have $10,000 to invest. What is the implied rate of interest?
  - $r = (20,000 / 10,000)^{1/6} - 1 = .122462 = 12.25\%$

DISCOUNT RATE – EXAMPLE 3

- Suppose you have a 1-year old son and you want to provide $75,000 in 17 years towards his college education.
  - You currently have $5,000 to invest.
  - What interest rate must you earn to have the $75,000 when you need it?

  - $r = (75{,}000 / 5{,}000)^{1/17} - 1 = .172688 = 17.27\%$

QUICK QUIZ – PART III

- What are some situations in which you might want to know the implied interest rate?
- You are offered the following investments:
  - You can invest $500 today and receive $600 in 5 years. The investment is considered low risk.
  - You can invest the $500 in a bank account paying 4%.
  - What is the implied interest rate for the first choice and which investment should you choose?

FINDING THE NUMBER OF PERIODS

- Start with the basic equation and solve for t (remember your logs)
  - $FV = PV(1 + r)^t$
  - $t = \ln(FV / PV) / \ln(1 + r)$
- You can use the financial keys on the calculator as well; just remember the sign convention.

NUMBER OF PERIODS – EXAMPLE 1

- You want to purchase a new car, and you are willing to pay $20,000.
  - If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?
    - t = ln(20,000 / 15,000) / ln(1.1) = 3.02 years

NUMBER OF PERIODS – EXAMPLE 2

- Suppose you want to buy a new house.
  - You currently have $15,000, and you figure you need to have a 10% down payment plus an additional 5% of the loan amount for closing costs.
  - Assume the type of house you want will cost about $150,000 and you can earn 7.5% per year.
  - How long will it be before you have enough money for the down payment and closing costs?

NUMBER OF PERIODS – EXAMPLE 2 CONTINUED

- How much do you need to have in the future?
  - Down payment = .1(150,000) = 15,000
  - Closing costs = .05(150,000 – 15,000) = 6,750
  - Total needed = 15,000 + 6,750 = 21,750
- Compute the number of periods
- Using the formula:
  - t = ln(21,750 / 15,000) / ln(1.075) = 5.14 years
- Using a financial calculator:
  - PV = -15,000, FV = 21,750, I/Y = 7.5, CPT N = 5.14 years

QUICK QUIZ – PART IV

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room.
  - You currently have $500, and the furniture you want costs $600.
  - If you can earn 6%, how long will you have to wait if you don't add any additional money?

SPREADSHEET EXAMPLE

- Use the following formulas for TVM calculations
  - FV(rate,nper,pmt,pv)
  - PV(rate,nper,pmt,fv)
  - RATE(nper,pmt,pv,fv)
  - NPER(rate,pmt,pv,fv)

- The formula icon is very useful when you can't remember the exact formula

- Click on the Excel icon to open a spreadsheet containing four different examples.

WORK THE WEB EXAMPLE

- Many financial calculators are available online
- Click on the web surfer to go to Investopedia's web site and work the following example:
  - You need $50,000 in 10 years. If you can earn 6% interest, how much do you need to invest today?
  - You should get $27,919.74

TABLE 5.4

I.	**Symbols:**
	PV = Present value, what future cash flows are worth today
	FV_t = Future value, what cash flows are worth in the future
	r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year
	t = Number of periods—typically, but not always, the number of years
	C = Cash amount
II.	**Future Value of *C* Invested at *r* Percent for *t* Periods:**
	$FV_t = C \times (1 + r)^t$
	The term $(1 + r)^t$ is called the *future value factor.*
III.	**Present Value of *C* to Be Received in *t* Periods at *r* Percent per Period:**
	$PV = C/(1 + r)^t$
	The term $1/(1 + r)^t$ is called the *present value factor.*
IV.	**The Basic Present Value Equation Giving the Relationship between Present and Future Value:**
	$PV = FV_t/(1 + r)^t$

COMPREHENSIVE PROBLEM

- You have $10,000 to invest for five years.
- How much additional interest will you earn if the investment provides a 5% annual return, when compared to a 4.5% annual return?
- How long will it take your $10,000 to double in value if it earns 5% annually?
- What annual rate has been earned if $1,000 grows into $4,000 in 20 years?

CHAPTER 5 - FORMULAS

END OF CHAPTER