Mid Semester Examination

IIT Kharagpur, CSE Dept., Autumn'15

(CS41001) Theory of Computation (Full marks = 60)

Answer exactly 6 questions. In case of reasonable doubt, make practical assumptions.

1. (a) Let $QUARTER\_CLIQUE = \{\langle G\rangle | G$ is an undirected graph having a clique of size $n/4$ where n is number of vertices in $G\}$. Show that $QUARTER\_CLIQUE$ is NP-complete. **[5]**

   (b) Prove that the Post Correspondence Problem is decidable for unary encoding. **[5]**

2. (a) Prove that $OVERLAP_{CFG} = \{\langle G, H, \rangle \,|\, G$ and H are CFGs where $L(G) \cap L(H) \neq \emptyset\}$ is undecidable. **[5]**

   (b) Prove that $MIN_{TM} = \{\langle M\rangle \,|\,$ M is a Turing Machine with minimal description$\}$ is undecidable. **[5]**

3. Prove that there exists more languages than there exists Turing machines. **[10]**

4. $EQ_{TM} = \{\langle M_1, M_2\rangle \,|\, M_1$ and M_2 are Turing machines and $L(M_1) = L(M_2)\}$. Prove that EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable. **[5+5]**

5. Prove that it is undecidable whether L(G) is regular for CFG G. **[10]**

6. (a) Given two graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$, a relation $\sim$ may be defined as $G \sim H$ iff $\exists f : V_1 \rightarrow V_2$ such that $\forall\, v, v' \in V_1$ where $v \neq v'$, $(v, v') \in E_1$ iff $(f(v), f(v')) \in E_2$. Let $REL = \{\langle G, H\rangle \,|\, G \sim H\}$. Show that $REL \in NP$. **[5]**

   (b) Let $CNF_k = \{\langle\phi\rangle \,|\, \phi$ is a satisfiable boolean formula in CNF form where each variable appears in at most k places$\}$. Prove that $CNF_2 \in P$. **[5]**

7. Define a two-headed finite automaton (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \,|\, n \geq 0\}$.

   (a) Let $A_{2DFA} = \{\langle M, x\rangle \,|\,$ M is a 2DFA and M accepts x$\}$. Show that A_{2DFA} is decidable. **[5]**

   (b) Let $E_{2DFA} = \{\langle M\rangle \,|\,$ M is a 2DFA and L(M) $=\emptyset$ $\}$. Show that E_{2DFA} is not decidable. **[5]**