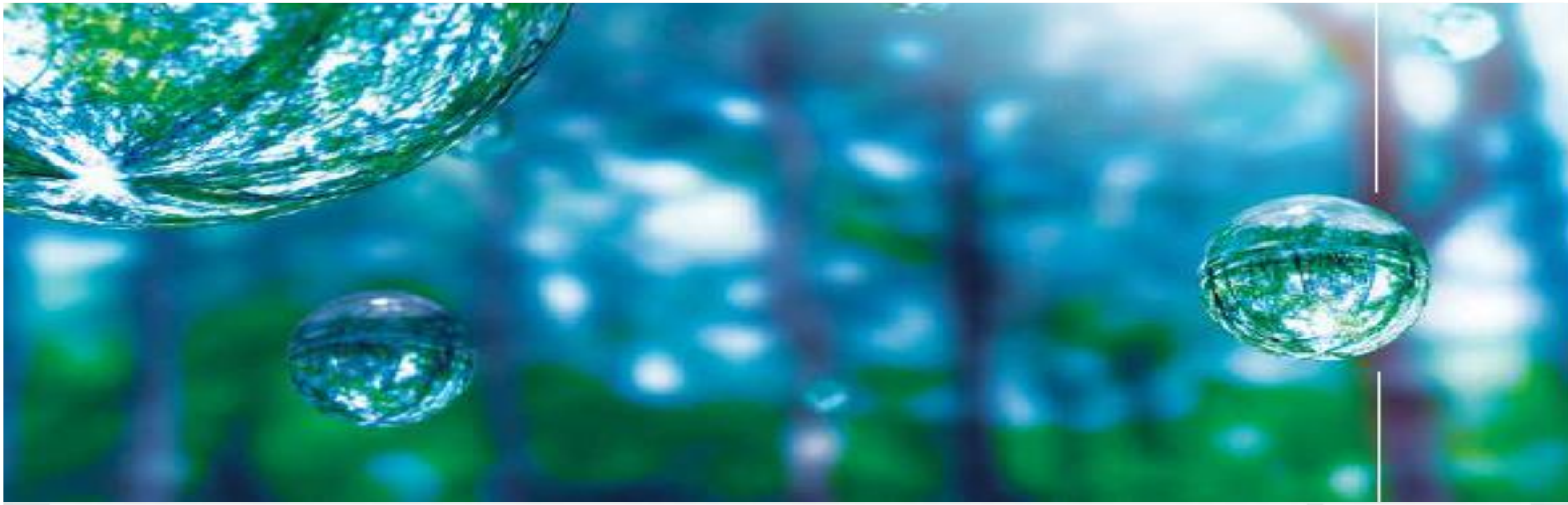


CHAPTER 6

DISCOUNTED CASH FLOW VALUATION (FORMULAS)

KEY CONCEPTS AND SKILLS

- Be able to compute the future value of multiple cash flows
- Be able to compute the present value of multiple cash flows
- Be able to compute loan payments
- Be able to find the interest rate on a loan
- Understand how interest rates are quoted
- Understand how loans are amortized or paid off

CHAPTER OUTLINE

- Future and Present Values of Multiple Cash Flows
- Valuing Level Cash Flows: Annuities and Perpetuities
- Comparing Rates: The Effect of Compounding
- Loan Types and Loan Amortization

MULTIPLE CASH FLOWS – FV EXAMPLE 6.1

- You think you will be able to deposit $4,000 at the end of each of the next three years in a bank account paying 8 percent interest.
  - You currently have $7,000 in the account.
  - How much will you have in three years?
  - How much will you have in four years?

MULTIPLE CASH FLOWS – FV EXAMPLE 6.1

- Find the value at year 3 of each cash flow and add them together
  - Today (year 0): FV = $7000(1.08)^3$ = 8,817.98
  - Year 1: FV = $4,000(1.08)^2$ = 4,665.60
  - Year 2: FV = 4,000(1.08) = 4,320
  - Year 3: value = 4,000
  - Total value in 3 years = 8,817.98 + 4,665.60 + 4,320 + 4,000 = 21,803.58
- Value at year 4 = 21,803.58(1.08) = 23,547.87

MULTIPLE CASH FLOWS – FV EXAMPLE 2

- Suppose you invest $500 in a mutual fund today and $600 in one year.
  - If the fund pays 9% annually, how much will you have in two years?
  - $FV = 500(1.09)^2 + 600(1.09) = 1{,}248.05$

MULTIPLE CASH FLOWS – EXAMPLE 2 CONTINUED

- How much will you have in 5 years if you make no further deposits?
- First way:
  - FV = $500(1.09)^5 + 600(1.09)^4 = 1,616.26$
- Second way – use value at year 2:
  - FV = $1,248.05(1.09)^3 = 1,616.26$

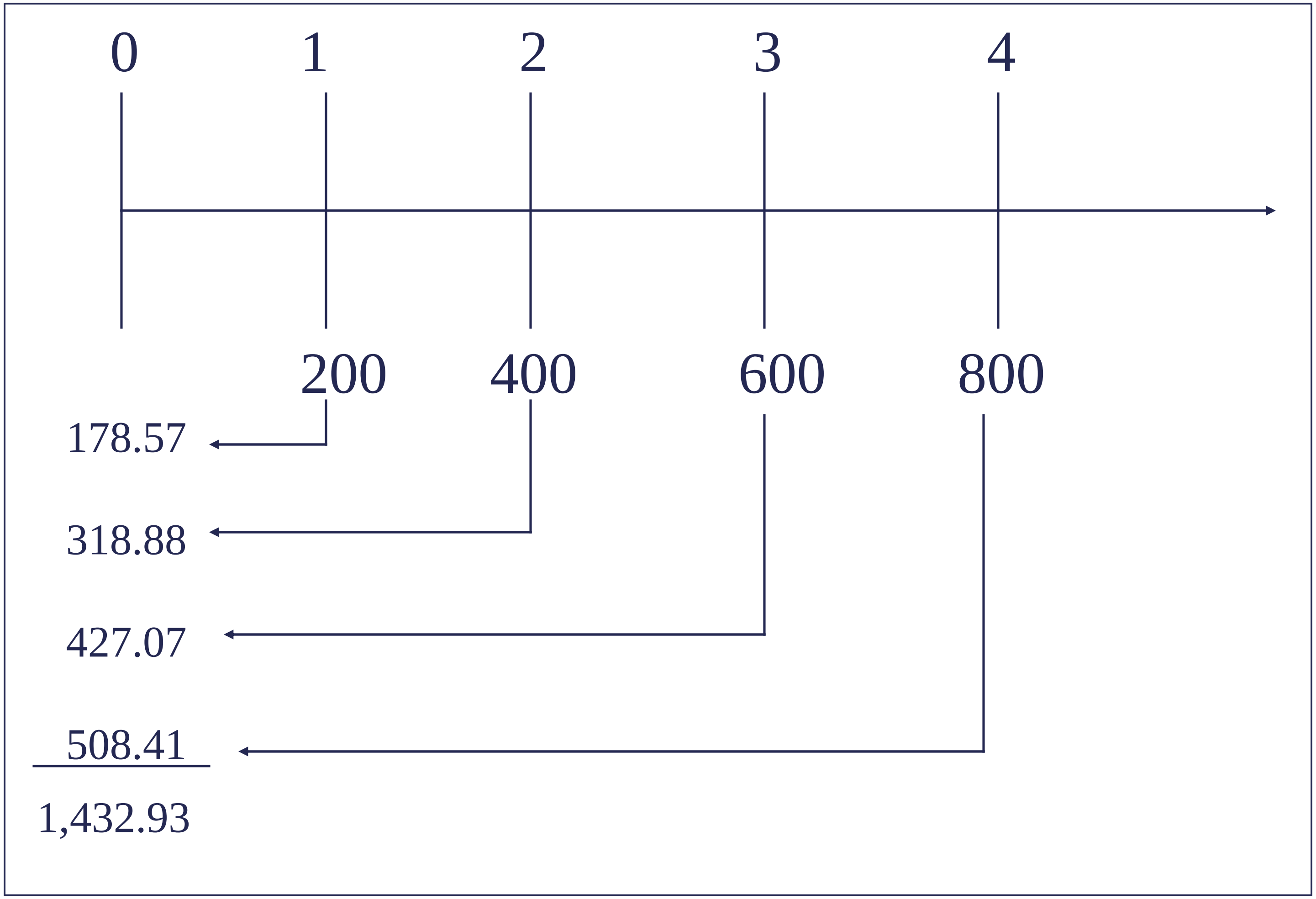
MULTIPLE CASH FLOWS – FV EXAMPLE 3

- Suppose you plan to deposit $100 into an account in one year and $300 into the account in three years.
  - How much will be in the account in five years if the interest rate is 8%?
  - $FV = 100(1.08)^4 + 300(1.08)^2 = 136.05 + 349.92 = 485.97$

MULTIPLE CASH FLOWS – PV EXAMPLE 6.3

- Find the PV of each cash flows and add them
  - Year 1 CF: $200 / (1.12)^1 = 178.57$
  - Year 2 CF: $400 / (1.12)^2 = 318.88$
  - Year 3 CF: $600 / (1.12)^3 = 427.07$
  - Year 4 CF: $800 / (1.12)^4 = 508.41$
  - Total PV = 178.57 + 318.88 + 427.07 + 508.41 = 1,432.93

EXAMPLE 6.3 TIMELINE



MULTIPLE CASH FLOWS USING A SPREADSHEET

- You can use the PV or FV functions in Excel to find the present value or future value of a set of cash flows
- Setting the data up is half the battle – if it is set up properly, then you can just copy the formulas
- Click on the Excel icon for an example

MULTIPLE CASH FLOWS – PV ANOTHER EXAMPLE

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years, and \$3000 in three years.
  - If you want to earn 10% on your money, how much would you be willing to pay?
    - $PV = 1000 / (1.1)^1 = 909.09$
    - $PV = 2000 / (1.1)^2 = 1,652.89$
    - $PV = 3000 / (1.1)^3 = 2,253.94$
    - PV = 909.09 + 1,652.89 + 2,253.94 = 4,815.92

MULTIPLE UNEVEN CASH FLOWS USING THE CALCULATOR

- Another way to use the financial calculator for uneven cash flows is to use the cash flow keys
  - Press CF and enter the cash flows beginning with year 0
  - You have to press the "Enter" key for each cash flow
  - Use the down arrow key to move to the next cash flow
  - The "F" is the number of times a given cash flow occurs in consecutive periods
  - Use the NPV key to compute the present value by entering the interest rate for I, pressing the down arrow and then compute
  - Clear the cash flow keys by pressing CF and then 2nd CLR Work

DECISIONS, DECISIONS

- Your broker calls you and tells you that he has this great investment opportunity.
  - If you invest $100 today, you will receive $40 in one year and $75 in two years.
  - If you require a 15% return on investments of this risk, should you take the investment?

  - Use the CF keys to compute the value of the investment
    - CF; CF_0 = 0; C01 = 40; F01 = 1; C02 = 75; F02 = 1
    - NPV; I = 15; CPT NPV = 91.49

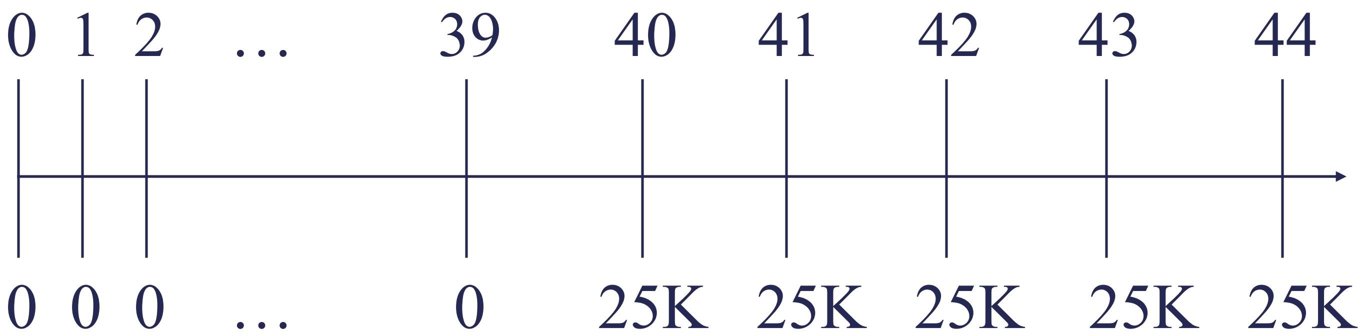
No – the broker is charging more than you would be willing to pay.

SAVING FOR RETIREMENT

- You are offered the opportunity to put some money away for retirement.
  - You will receive five annual payments of $25,000 each beginning in 40 years.
  - How much would you be willing to invest today if you desire an interest rate of 12%?

- Use cash flow keys:
  - CF; CF_0 = 0; C01 = 0; F01 = 39; C02 = 25,000; F02 = 5; NPV; I = 12; CPT NPV = 1,084.71

SAVING FOR RETIREMENT TIMELINE



Notice that the year 0 cash flow = 0 ($CF_0 = 0$)

The cash flows in years 1 – 39 are 0 (C01 = 0; F01 = 39)

The cash flows in years 40 – 44 are 25,000 (C02 = 25,000; F02 = 5)

QUICK QUIZ – PART I

- Suppose you are looking at the following possible cash flows:
  - Year 1 CF = $100;
  - Years 2 and 3 CFs = $200;
  - Years 4 and 5 CFs = $300.
  - The required discount rate is 7%.

- What is the value of the cash flows at year 5?
- What is the value of the cash flows today?
- What is the value of the cash flows at year 3?

ANNUITIES AND PERPETUITIES DEFINED

- Annuity – finite series of equal payments that occur at regular intervals
  - If the first payment occurs at the end of the period, it is called an ordinary annuity
  - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

ANNUITIES AND PERPETUITIES BASIC FORMULAS

- Perpetuity: PV = C / r
- Annuities:

$$PV = C\left[\frac{1 - \frac{1}{(1+r)^t}}{r}\right]$$

$$FV = C\left[\frac{(1+r)^t - 1}{r}\right]$$

ANNUITIES AND THE CALCULATOR

- You can use the PMT key on the calculator for the equal payment
- The sign convention still holds
- Ordinary annuity versus annuity due
  - You can switch your calculator between the two types by using the 2nd BGN 2nd Set on the TI BA-II Plus
  - If you see "BGN" or "Begin" in the display of your calculator, you have it set for an annuity due
  - Most problems are ordinary annuities

ANNUITY – EXAMPLE 6.5

- After carefully going over your budget, you have determined you can afford to pay $632 per month toward a new sports car.
  - You call up your local bank and find out that the going rate is 1 percent per month for 48 months.
  - How much can you borrow?

- To determine how much you can borrow, we need to calculate the present value of $632 per month for 48 months at 1 percent per month.

ANNUITY – EXAMPLE 6.5

- You borrow money TODAY so you need to compute the present value.
  - 48 N; 1 I/Y; -632 PMT; CPT PV = 23,999.54 ($24,000)

- Formula:

$$PV = 632\left[\frac{1-\frac{1}{(1.01)^{48}}}{.01}\right] = 23,999.54$$

ANNUITY - SWEEPSTAKES EXAMPLE

- Suppose you win the Publishers Clearinghouse $10 million sweepstakes.
  - The money is paid in equal annual installments of $333,333.33 over 30 years.
  - If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

- PV = 333,333.33[1 – 1/1.05^30] / .05 = 5,124,150.29

BUYING A HOUSE

- You are ready to buy a house, and you have $20,000 for a down payment and closing costs.
  - Closing costs are estimated to be 4% of the loan value.
  - You have an annual salary of $36,000, and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.
  - The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.
  - How much money will the bank loan you?
  - How much can you offer for the house?

BUYING A HOUSE - CONTINUED

- Bank loan
  - Monthly income = 36,000 / 12 = 3,000
  - Maximum payment = .28(3,000) = 840
  - PV = $840[1 – 1/1.005^{360}] / .005 = 140,105$
- Total Price
  - Closing costs = .04(140,105) = 5,604
  - Down payment = 20,000 – 5,604 = 14,396
  - Total Price = 140,105 + 14,396 = 154,501

ANNUITIES ON THE SPREADSHEET - EXAMPLE

- The present value and future value formulas in a spreadsheet include a place for annuity payments
- Click on the Excel icon to see an example

QUICK QUIZ – PART II

- You know the payment amount for a loan, and you want to know how much was borrowed. Do you compute a present value or a future value?

- You want to receive 5,000 per month in retirement.
  - If you can earn 0.75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

FINDING THE PAYMENT

- Suppose you want to borrow $20,000 for a new car.
  - You can borrow at 8% per year, compounded monthly (8/12 = .66667% per month).
  - If you take a 4 year loan, what is your monthly payment?

  - $20,000 = C[1 – 1 / 1.0066667^{48}] / .0066667$
  - $C = 488.26$

FINDING THE PAYMENT ON A SPREADSHEET

- Another TVM formula that can be found in a spreadsheet is the payment formula
  - PMT(rate,nper,pv,fv)
  - The same sign convention holds as for the PV and FV formulas
- Click on the Excel icon for an example

FINDING THE NUMBER OF PAYMENTS – EXAMPLE 6.6

- You ran a little short on your spring break vacation, so you put $1,000 on your credit card.
  - You can afford only the minimum payment of $20 per month.
  - The interest rate on the credit card is 1.5 percent per month.
  - How long will you need to pay off the $1,000?

FINDING THE NUMBER OF PAYMENTS – EXAMPLE 6.6

- Start with the equation, and remember your logs.
  - $1{,}000 = 20(1 - 1/1.015^t) / .015$
  - $.75 = 1 - 1 / 1.015^t$
  - $1 / 1.015^t = .25$
  - $1 / .25 = 1.015^t$
  - $t = \ln(1/.25) / \ln(1.015) = 93.111$ months = 7.76 years
- And this is only if you don't charge anything more on the card!

FINDING THE NUMBER OF PAYMENTS – ANOTHER EXAMPLE

- Suppose you borrow $2,000 at 5%, and you are going to make annual payments of $734.42.
  - How long before you pay off the loan?
    - $2{,}000 = 734.42(1 - 1/1.05^t) / .05$
    - $.136161869 = 1 - 1/1.05^t$
    - $1/1.05^t = .863838131$
    - $1.157624287 = 1.05^t$
    - $t = \ln(1.157624287) / \ln(1.05) = 3$ years

FINDING THE RATE

- Suppose you borrow $10,000 from your parents to buy a car.
  - You agree to pay $207.58 per month for 60 months.
  - What is the monthly interest rate?

  - Sign convention matters!!!
  - 60 N
  - 10,000 PV
  - -207.58 PMT
  - CPT I/Y = .75%

ANNUITY – FINDING THE RATE WITHOUT A FINANCIAL CALCULATOR

- Trial and Error Process
  - Choose an interest rate and compute the PV of the payments based on this rate
  - Compare the computed PV with the actual loan amount
  - If the computed PV > loan amount, then the interest rate is too low
  - If the computed PV < loan amount, then the interest rate is too high
  - Adjust the rate and repeat the process until the computed PV and the loan amount are equal

QUICK QUIZ – PART III

- You want to receive $5,000 per month for the next 5 years.
  - How much would you need to deposit today if you can earn 0.75% per month?
  - What monthly rate would you need to earn if you only have $200,000 to deposit?
- Suppose you have $200,000 to deposit and can earn 0.75% per month.
  - How many months could you receive the $5,000 payment?
  - How much could you receive every month for 5 years?

FUTURE VALUES FOR ANNUITIES

- Suppose you begin saving for your retirement by depositing $2,000 per year in an IRA.
  - If the interest rate is 7.5%, how much will you have in 40 years?

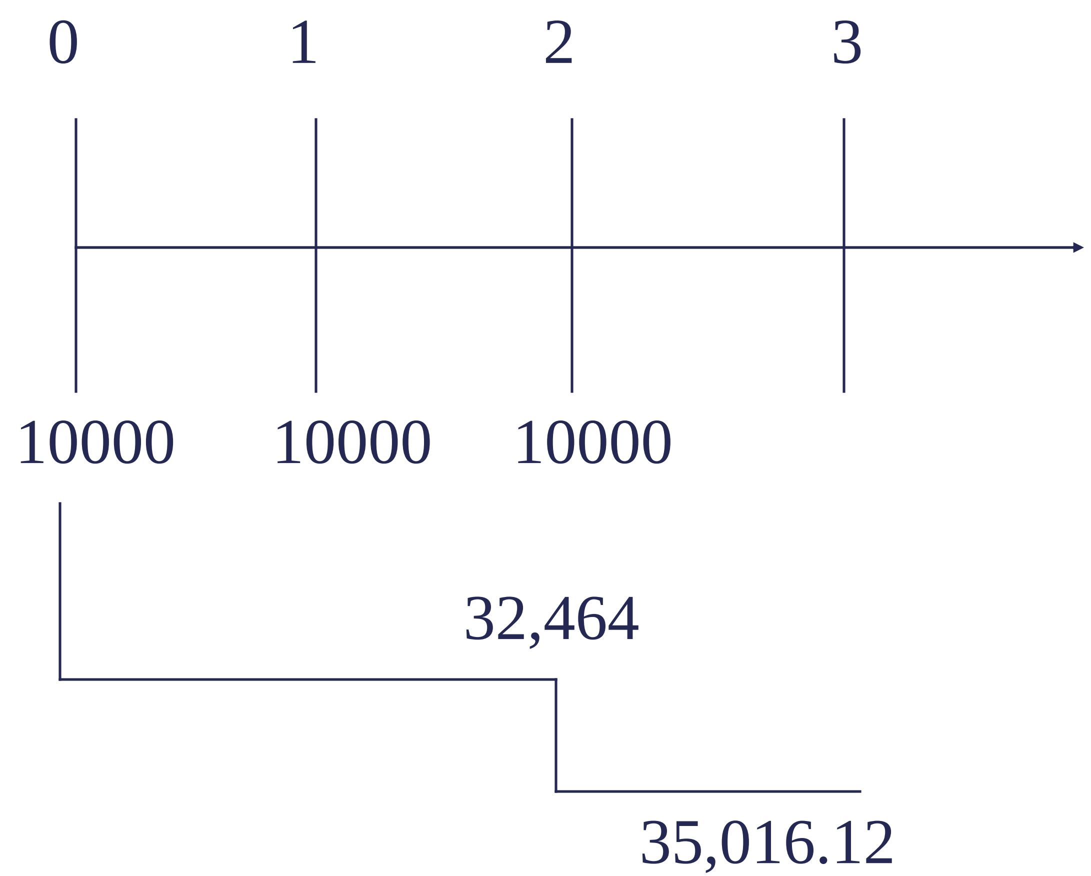
  - FV = $2{,}000(1.075^{40} - 1)/.075 = 454{,}513.04$

ANNUITY DUE

- You are saving for a new house, and you put \$10,000 per year in an account paying 8%. The first payment is made today.
  - How much will you have at the end of 3 years?

  - FV = $10,000[(1.08^3 – 1) / .08](1.08) = 35,061.12$

ANNUITY DUE TIMELINE



PERPETUITY – EXAMPLE 6.7

- Suppose the Fellini Co. wants to sell preferred stock at $100 per share.
  - A similar issue of preferred stock already outstanding has a price of $40 per share and offers a dividend of $1 every quarter.
  - What dividend will Fellini have to offer if the preferred stock is going to sell?

PERPETUITY – EXAMPLE 6.7

- Perpetuity formula: PV = C / r
- Current required return:
  - 40 = 1 / r
  - r = .025 or 2.5% per quarter
- Dividend for new preferred:
  - 100 = C / .025
  - C = 2.50 per quarter

QUICK QUIZ – PART IV

- You want to have $1 million to use for retirement in 35 years.
  - If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?
  - What if the first payment is made today?

- You are considering preferred stock that pays a quarterly dividend of $1.50.
  - If your desired return is 3% per quarter, how much would you be willing to pay?

WORK THE WEB EXAMPLE

- Another online financial calculator can be found at MoneyChimp.
- Click on the web surfer and work the following example:
  - Choose calculator and then annuity
  - You just inherited $5 million. If you can earn 6% on your money, how much can you withdraw each year for the next 40 years?
  - MoneyChimp assumes annuity due!
  - Payment = $313,497.81

TABLE 6.2

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future Value of *C* per Period for *t* Periods at *r* Percent per Period:

$FV_t = C \times \{[(1 + r)^t - 1]/r\}$

A series of identical cash flows is called an *annuity*, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present Value of *C* per Period for *t* Periods at *r* Percent per Period:

$PV = C \times \{1 - [1/(1 + r)^t]\}/r$

The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor.*

IV. Present Value of a Perpetuity of *C* per Period:

$PV = C/r$

A *perpetuity* has the same cash flow every year forever.

GROWING ANNUITY

A growing stream of cash flows with a fixed maturity

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \cdots + \frac{C \times (1+g)^{t-1}}{(1+r)^t}$$

$$PV = \frac{C}{r-g}\left[1 - \left(\frac{(1+g)}{(1+r)}\right)^t\right]$$

GROWING ANNUITY: EXAMPLE

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by three-percent each year. What is the present value at retirement if the discount rate is 10 percent?

$$PV = \frac{\$20{,}000}{.10 - .03}\left[1 - \left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

GROWING PERPETUITY

A growing stream of cash flows that lasts forever

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

EXAMPLE: GROWING PERPETUITY

The expected dividend next year is $1.30, and dividends are expected to grow at 5% forever.

If the discount rate is 10%, what is the value of this promised dividend stream?

$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

EFFECTIVE ANNUAL RATE (EAR)

- This is the actual rate paid (or received) after accounting for compounding that occurs during the year
- If you want to compare two alternative investments with different compounding periods, you need to compute the EAR and use that for comparison.

ANNUAL PERCENTAGE RATE

- This is the annual rate that is quoted by law
- By definition APR = period rate times the number of periods per year
- Consequently, to get the period rate we rearrange the APR equation:
  - Period rate = APR / number of periods per year
- You should NEVER divide the effective rate by the number of periods per year – it will NOT give you the period rate

COMPUTING APRS

- What is the APR if the monthly rate is .5%?
  - .5(12) = 6%
- What is the APR if the semiannual rate is .5%?
  - .5(2) = 1%
- What is the monthly rate if the APR is 12% with monthly compounding?
  - 12 / 12 = 1%

THINGS TO REMEMBER

- You ALWAYS need to make sure that the interest rate and the time period match.
  - If you are looking at annual periods, you need an annual rate.
  - If you are looking at monthly periods, you need a monthly rate.

- If you have an APR based on monthly compounding, you have to use monthly periods
  for lump sums, or adjust the interest rate appropriately if you have payments other than monthly

COMPUTING EARS - EXAMPLE

- Suppose you can earn 1% per month on $1 invested today.
  - What is the APR? 1(12) = 12%
  - How much are you effectively earning?
    - $FV = 1(1.01)^{12} = 1.1268$
    - Rate = (1.1268 – 1) / 1 = .1268 = 12.68%
- Suppose if you put it in another account, you earn 3% per quarter.
  - What is the APR? 3(4) = 12%
  - How much are you effectively earning?
    - $FV = 1(1.03)^{4} = 1.1255$
    - Rate = (1.1255 – 1) / 1 = .1255 = 12.55%

EAR - FORMULA

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1$$

Remember that the APR is the quoted rate, and

m is the number of compounding periods per year

DECISIONS, DECISIONS II

- You are looking at two savings accounts. One pays 5.25%, with daily compounding. The other pays 5.3% with semiannual compounding. Which account should you use?
  - First account:
    - $EAR = (1 + .0525/365)^{365} - 1 = 5.39\%$
  - Second account:
    - $EAR = (1 + .053/2)^{2} - 1 = 5.37\%$
- Which account should you choose and why?

DECISIONS, DECISIONS II, CONTINUED

- Let's verify the choice. Suppose you invest \$100 in each account. How much will you have in each account in one year?
  - First Account:
    - Daily rate = .0525 / 365 = .00014383562
    - FV = $100(1.00014383562)^{365}$ = 105.39
  - Second Account:
    - Semiannual rate = .0539 / 2 = .0265
    - FV = $100(1.0265)^2$ = 105.37
- You have more money in the first account.

COMPUTING APRS FROM EARS

- If you have an effective rate, how can you compute the APR? Rearrange the EAR equation and you get:

$$APR = m\left[(1 + EAR)^{1/m} - 1\right]$$

EXAMPLE: APR

- Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12\left[(1+.12)^{1/12} - 1\right] = .1138655152$$

$$\text{or } 11.39\%$$

COMPUTING PAYMENTS WITH APRS

- Suppose you want to buy a new computer system and the store is willing to allow you to make monthly payments. The entire computer system costs $3,500.
  - The loan period is for 2 years, and the interest rate is 16.9% with monthly compounding.
  - What is your monthly payment?

  - Monthly rate = .169 / 12 = .01408333333
  - Number of months = 2(12) = 24
  - $3{,}500 = C[1 - (1 / 1.01408333333)^{24}] / .01408333333$
  - C = 172.88

FUTURE VALUES WITH MONTHLY COMPOUNDING

- Suppose you deposit $50 a month into an account that has an APR of 9%, based on monthly compounding.
  - How much will you have in the account in 35 years?
    - Monthly rate = .09 / 12 = .0075
    - Number of months = 35(12) = 420
    - FV = $50[1.0075^{420} – 1] / .0075$ = 147,089.22

PRESENT VALUE WITH DAILY COMPOUNDING

- You need \$15,000 in 3 years for a new car.
  - If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?
    - Daily rate = .055 / 365 = .00015068493
    - Number of days = 3(365) = 1,095
    - FV = 15,000 / $(1.00015068493)^{1095}$ = 12,718.56

CONTINUOUS COMPOUNDING

- Sometimes investments or loans are figured based on continuous compounding
- EAR = $e^q - 1$
  - The e is a special function on the calculator normally denoted by e^x
- Example: What is the effective annual rate of 7% compounded continuously?
  - EAR = $e^{.07} - 1 = .0725$ or 7.25%

QUICK QUIZ – PART V

- What is the definition of an APR?
- What is the effective annual rate?
- Which rate should you use to compare alternative investments or loans?
- Which rate do you need to use in the time value of money calculations?

PURE DISCOUNT LOANS – EXAMPLE 6.12

- Treasury bills are excellent examples of pure discount loans. The principal amount is repaid at some future date, without any periodic interest payments.

- If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?
  - PV = 10,000 / 1.07 = 9,345.79

EXAMPLE: INTEREST-ONLY LOAN

- Consider a 5-year, interest-only loan with a 7% interest rate. The principal amount is $10,000. Interest is paid annually.
  - What would the stream of cash flows be?
    - Years 1 – 4: Interest payments of .07(10,000) = 700
    - Year 5: Interest + principal = 10,700
- This cash flow stream is similar to the cash flows on corporate bonds, and we will talk about them in greater detail later.

EXAMPLE: AMORTIZED LOAN WITH FIXED PRINCIPAL PAYMENT

- Consider a $50,000, 10 year loan at 8% interest. The loan agreement requires the firm to pay $5,000 in principal each year plus interest for that year.

- Click on the Excel icon to see the amortization table

EXAMPLE: AMORTIZED LOAN WITH FIXED PAYMENT

- Each payment covers the interest expense plus reduces principal
- Consider a 4 year loan with annual payments. The interest rate is 8%, and the principal amount is $5,000.
  - What is the annual payment?
    - 4 N
    - 8 I/Y
    - 5,000 PV
    - CPT PMT = -1,509.60
- Click on the Excel icon to see the amortization table

WORK THE WEB EXAMPLE

- There are websites available that can easily prepare amortization tables
- Click on the web surfer to check out the Bankrate.com website and work the following example
- You have a loan of $25,000 and will repay the loan over 5 years at 8% interest.
  - What is your loan payment?
  - What does the amortization schedule look like?

QUICK QUIZ – PART VI

- What is a pure discount loan?
  - What is a good example of a pure discount loan?
- What is an interest-only loan?
  - What is a good example of an interest-only loan?
- What is an amortized loan?
  - What is a good example of an amortized loan?

ETHICS ISSUES

- Suppose you are in a hurry to get your income tax refund.
  - If you mail your tax return, you will receive your refund in 3 weeks.
  - If you file the return electronically through a tax service, you can get the estimated refund tomorrow.
    - The service subtracts a $50 fee and pays you the remaining expected refund.
    - The actual refund is then mailed to the preparation service.
    - Assume you expect to get a refund of $978.
    - What is the APR with weekly compounding?
    - What is the EAR?
    - How large does the refund have to be for the APR to be 15%?
    - What is your opinion of this practice?

COMPREHENSIVE PROBLEM

- An investment will provide you with $100 at the end of each year for the next 10 years. What is the present value of that annuity if the discount rate is 8% annually?

- What is the present value of the above if the payments are received at the beginning of each year?

- If you deposit those payments into an account earning 8%, what will the future value be in 10 years?

- What will the future value be if you open the account with $1,000 today, and then make the $100 deposits at the end of each year?

CHAPTER 6 - FORMULAS

END OF CHAPTER