INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date 22.04.2010 AN *Time:* 3 Hrs. *Maximum Marks* 100 *No. of Students:* 70
End-Spring Semester:, 2009-10 Department: Computer Science and Engineering
Sub. No: CS31004
B. Tech.(Hons.), Dual Deg. Sub. Name: Theory of Computation

Instructions : Answer **ANY FIVE** questions

1. (a) Define the *Word problem of Semi-Thue Systems.* Show that the problem is undecidable.
   (b) Show that finite automata with two push-down stores have the same computational power as the Turing machines.

   [10 + 10 = 20]

2. (a) Show that $ALL_{CFG} = \{\langle G\rangle | G$ is a CFG and $L(G) = \Sigma^*\}$ is undecidable.
   (b) Consider the problem of determining whether a PDA accepts some string of the form $\{ww | w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.

   [10 + 10 = 20]

3. (a) Show that first-order predicate calculus is undecidable.
   (b) Encode the following argument in first-order predicate calculus indicating clearly the predicates used and their meanings.
      i. No one respects a person who does not respect himself.
      ii. No one will hire a person he does not respect.
      iii. Therefore, a person who respects no one will never be hired by anybody.
   (c) Is the above argument valid? Give a proof (*not a resolution deduction*) if it is valid; otherwise, give a countermodel.

   [10 + 5 + 5 = 20]

4. (a) Construct a Turing machine (TM) which prints its own description. Explain its operation.
   (b) Let $MIN_{TM} = \{\langle M\rangle | M$ is a minimal TM $\}$. Show by Recursion Theorem that any infinite subset of MIN_{TM} is not Turing recognizable.

   [12 + 8 = 20]

5. (a) Show that the relation $<$ over the set N of natural numbers is resolvable by a DFA. Explain clearly the input encoding and the principle of operation of the DFA by characterizing each of its states.
   (b) Show that $Th(N, <)$ is decidable.

   [8 + 12 = 20]

Please turn over

6. (a) Show that the following function is grammatically computable by giving the corresponding grammar; explain the operation by characterizing each nonterminal. $f : \Sigma^* \rightarrow \Sigma^*$, where $\Sigma = \{a, b\}$

   and $f(w) = a$, if w contains twice as many a's ans b's.
   $= b$, otherwise.

   (b) Let Q be a $(k+1)$-place predicate for $k \geq 0$. Recall that the *Bounded minimalization* of Q is the $(k+1)$-place function f such that

   $$f(\bar{n}, m) = \begin{cases} \text{the smallest } p, 0 \leq p \leq m, \text{ such that } Q(\bar{n}, p), & \text{if such a } p \text{ exists in the range } 0, \cdots, m; \\ 0 & \text{otherwise} \end{cases}$$

   Show clearly that f is primitive recursive.

   [12 + 8 = 20]

7. (a) Show that the following functions are primitive recursive
   i. $f : N \rightarrow N$, $f(x) = \lfloor log_2(x+1) \rfloor$
   ii. $length(n)$ = length of the string whose Godel number is n

   (b) Define Ackermann's function. Let an $(n+1)$-variable function f be defined by primitive recursion from the n-variable function g and the $(n+2)$-variable function h. Let A be the Ackermann's function. Let there exist natural numbers k_g and k_h such that $A(k_g, max(x_1, \cdots, x_n)) > g(x_1, \cdots, x_n))$ and $A(k_h, max(x_1, \cdots, x_n, y, z)) > h(x_1, \cdots, x_n, y, z)$, for all $x_1, \cdots, x_n, y, z$.
   Show that there exists a natural number k such that $A(k, max(x_1, \cdots, x_n, y)) > f(x_1, \cdots, x_n, y)$, for all $x_1, \cdots, x_n, y$.

   [(4 + 4 = 8) + (2 + 10 = 12) = 20]

8. (a) Show that $SUBSET\_SUM = \{\langle S, t\rangle | S$ is a set of natural numbers, t is a natural number and $\{y_1, y_2, \cdots y_n\} \subseteq S$ s. t. $\Sigma_{i=1}^{n} y_i = t\ \}$
   is NP-complete.

   (b) i. Let
   $CNF_k = \{\langle\phi\rangle | \phi$ is a satisfiable CNF formula where each variable appears in at most k places $\}$.
   Show that CNF_3 is NP-complete.

   ii. Let
   $QUARTER\_CLIQUE = \{\langle G\rangle | G$ is an undirected graph having a clique of size $n/4$ where n is number of vertices in $G\}$
   Show that $QUARTER\_CLIQUE$ is NP-complete.

   [10 + (5 + 5 = 10) = 20]

9. (a) Show that $EQ_{REX} = \{\langle R, S\rangle | R$ and S are equivalent regular expressions$\} \in PSPACE$.

   (b) Show that $TQBF = \{\langle\phi\rangle | \phi$ is a true fully quantified Boolean formula $\}$ is PSPACE-complete.

   [8 + 12 = 20]