INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date 21.04.2011 AN *Time:* 3 Hrs. *Maximum Marks* **100** *No. of Students:* **74**

End-Spring Semester, 2010-11 Department: Computer Science and Engineering

Sub. No: CS31004

B. Tech.(Hons.), Dual Deg. Sub. Name: Theory of Computation

Instructions : Answer **ANY FIVE** questions

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1. (a) Argue clearly that there are more languages than there are finite representations for them.

   (b) Show that $E_{LBA} = \{\langle B \rangle | B$ is an LBA and $L(B) = \emptyset\}$ is undecidable using the computation history method.

   [10 + 10 = 20]

2. (a) Give *formal* proofs or *formal* countermodels to ascertain the validity/non-validity of (i) $\forall x(A(x) \vee B(x)) \equiv \forall x A(x) \vee \forall x B(x)$, (ii) $\exists x(A(x) \vee B(x)) \equiv \exists x A(x) \vee \exists x B(x)$.

   (b) Encode the following argument in first-order predicate calculus indicating clearly the predicates used and their meanings.

      i. There is a professor who is liked by every student who likes at least one professor.

      ii. Every student likes some professor or other.

      iii. Therefore, there is a professor who is not liked by any student.

   (c) The above argument is not valid. Give a *formal* countermodel over a domain comprising at least two students and two professors. Is it satisfiable? If so, give a *formal* model for it.

   [(4 + 4) + 6 + 6 = 20]

3. (a) Using Recursion Theorem show the undecidability of the following language: $TM_{CFL} = \{\langle M \rangle \ | M$ is a Turing machine and $L(M)$ is a context free language$\}$.

   (b) Show clearly that $Th(N, +)$ is decidable. Assume that it is possible to compute any atomic formula using a DFA.

   (c) Show that the set of provable statements in $Th(N, +, \times)$ is Turing recognizable. State the assumptions made.

   [5 + 9 + 6 = 20]

4. (a) Define a grammatically computable function from Σ_0^* to Σ_1^*. Show that the following function is grammatically computable by giving the corresponding grammar; explain the operation by characterizing each nonterminal with its role in the grammar's computation.
   $f : \Sigma^* \rightarrow \Sigma^*$, where $\Sigma = \{a, b\}$ and
   $f(x) = a$, if x is a palindrome.
   $= b$, otherwise.

   (b) Show clearly that the function $h : N^k \rightarrow N$, where N is the set of non-negative numbers, such that
   $$h(\overline{n}) = \begin{cases} \Pi_{i=w_1(\overline{n})}^{w_2(\overline{n})} g(\overline{n}, i), & \text{if } w_1(\overline{n}) \leq w_2(\overline{n}) \\ = 1 & \text{otherwise} \end{cases}$$
   is primitive recursive whenever w_1, w_2 and g are primitive recursive.

   (c) Argue that not every *total* function that we intuitively regard as computable is primitive recursive.

   [7 + 8 + 5 = 20]

5. (a) Show that $PATH = \{\langle G, s, t\rangle | G$ is an undirected graph having a simple path between its nodes s and $t\}$ is NP-complete iff $P = NP$.
   (b) Show that NP is closed under star operation.
   (c) Show that $ALL_{NFA} = \{\langle N\rangle | N$ is an NFA and $L(N) = \Sigma^*\}$ is in PSPACE.

[5 + 5 + 10 = 20]

6. Show that the following problems are NP-Complete.
   (a) $UHAMPATH = \{\langle G, s, t\rangle | G$ is an undirected graph having a Hamiltonian path from the node s to the node $t\}$
   Assume that $HAMPATH$, the version of this problem for directed graphs, is NP-complete.
   (b) $SET\_SPLITTING = \{\langle S, C\rangle | S$ is a finite set and $C = \{C_1, C_2, \cdots, C_k\}$ is a collection of subsets of S, for some $k > 0$, such that elements of S can be coloured *red* or *blue* so that no C_i has all its elements coloured with the same colour.$\}$

[10 + 10 = 20]

7. (a) Show that the generalized geography game
   $GG = \{\langle G, b\rangle |$ Player I has a winning strategy for the generalized geography game played on graph G starting at node $b\}$
   is PSPACE-complete.
   (b) You are given a box fitted with pegs and a collection of cards with notches as indicated in the figure given below.

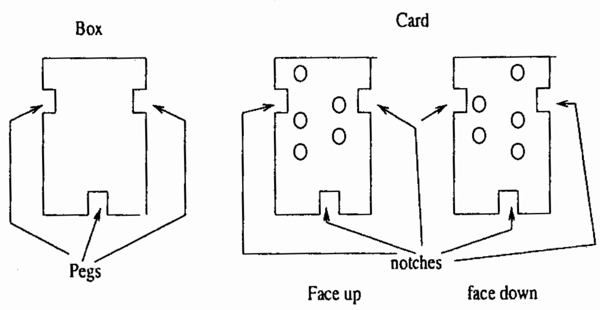

Figure 1: Puzzle Game (Question 7b)

Because of the pegs in the box and notches in the cards, each card will fit in the box in either of two ways – face up or face down. Each card contains two columns of holes, some of which may not be punched out. Consider the following two player game: Each player has at his disposal a pack of cards. Players take turns placing the cards in order in the box and may choose whether to place a card face up or face down. Player I wins if, in the final stack, all hole positions are blocked, and Player II wins if some hole position remains unblocked. Show that the problem of determining which player has a winning strategy for a given starting configuration of the cards is PSPACE-complete. (*Hint*: Reduce the formula game to the above game; the cards may correspond to the variables and (blocking of) the hole positions may be the clauses.)

[12 + 8 = 20]