Tutorial

Q1. Let $b(n)$ be the binary representation of $n \geq 0$.
Show that:

(a) $\{b(n)\#b(n+1) \mid n \geq 0\}$ is not a CFL.

(b) $\{b(n)^{rev}\#b(n+1) \mid n \geq 0\}$ is a CFL.

$0 \downarrow 1$

Structure: $b(n): 1w01^{i>0}$, $b(n+1): 1w\underbrace{10^{i>0}}$

$b(n): 1^n$, $b(n+1): 10^n$.

$b(n)^{rev} = \underbrace{1^i0}w1$

(a) Pumping lemma constant $= k$.

$10\underbrace{1^k\#11}_{vwx}0^k$

$10^{2k}\underbrace{1^k}\#10^{2k-1}\underbrace{10^k}$

(i) If v or w contains $\#$ then done $i=2$

(ii) Crossing: $v = 1^{l<k}$, $x = 10^j$, 0^j $\quad i=2$

$1\underline{0}1^{k+(i-1)l}\#1\underline{1}\underbrace{0^{k+(i-1)j}}$

$i=2$

(iii) $vwx \in b(n)$ $\quad i=3$ } bit
$vwx \in b(n+1)$ $\quad i=2$ } diff. too large

(b) Give a CFG

Q2. Show that $\{a^n b^{n^2} \mid n \geq 0\}$ is not a CFL.

Pumping lemma constant k.

$z \quad a^k b^{k^2}$

Adversary : $u\, v\, w\, x\, y$ Pump out v, x.

(i) $vwx \in a^k$. $\quad a^{k+(i-1)[l+j]}\, b^{k^2}$ $\quad i = 0 \leftarrow$

(ii) $vwx \in b^{k^2}$ $\quad i = 0 \leftarrow$

(iii) vwx : $a^{\alpha} b^{\beta}$

$x, v = a^{\alpha_1} b^{\beta_1}$ $\quad i = 2 \swarrow$

$i = 2$ Remaining : $v = a^{\alpha_1}$ $\quad x = b^{\beta_1}$ $\quad \alpha_1, \beta_1 < k$.

Suppose $(k+\alpha_1)^2 = k^2 + \beta_1$

$\Rightarrow \cancel{k^2} + \underbrace{\alpha_1^2 + 2k\alpha_1}_{} = \cancel{k^2} + \beta_1$ Hence done.

Q3. Show that $\{w \in \{a,b\}^* \mid \#a(w)$ is an integral multiple of $\#b(w)\}$ is not a CFL.

Pumping L constant k.

$a^{k^2} b^k \qquad k^2 + \alpha_1 \qquad k + \beta_1$

$p_1 > k$, p_1 is a prime ; $p_2 > p_1 k$, p_2 is a prime.

Soln: $Z = a^{p_1 p_2} b^{p_2}$

Q4. True or False: Let L be a CFL.

(a) $\{ww \mid w \in L\}$ is also a CFL. ← False when $L = \{a,b\}^*$

(b) $\{w \mid ww \in L\}$ is also a CFL.

False.

Q5. (a) Design a PDA over $\{a,b\}$ for

$\{a,b\}^* - \{\text{palindromes}\}$.

(b) Design a CFG for the same.