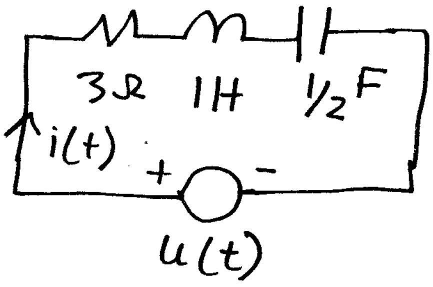
ON DIFFERENTIAL EQUATIONS

(1)



$$iR + L\frac{di}{dt} + \int_{-\infty}^{t} \frac{i(\tau)d\tau}{C} = u(t)$$

$$\Rightarrow R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = \delta(t)$$

$$\Rightarrow \frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i = \delta(t)$$

$$\Rightarrow \int_{0^-}^{0^+} \frac{d^2 i}{dt^2}dt + 3\int_{0^-}^{0^+} \frac{di}{dt}dt + 2\int_{0^-}^{0^+} i\, dt = 1$$

$$\Rightarrow \frac{di}{dt}(0^+) - \frac{di}{dt}(0^-) + 3\times 0 + 2\times 0 = 1$$

$$\Rightarrow \frac{di}{dt}(0^+) = 1 \qquad \left[\because \frac{di}{dt}(0^-) = 0\right]$$

$\therefore$ For $t > 0$

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i = 0$$

[with $\frac{di}{dt}(0^+) = 1$

and $i(0^+) = i(0^-) = 0$, since i can not have a step jump at $t = 0$, else $\frac{d^2 i}{dt^2}$ will have a $\frac{d}{dt}.\delta(t)$ term]

Characteristic equation

$$m^2 + 3m + 2 = 0 \Rightarrow (m+2)(m+1) = 0$$

$$\Rightarrow m = -2, -1$$

$$\therefore\ i(t) = A e^{-2t} + B e^{-t} \quad \text{for } t > 0$$

$$i(0^+) = 0 = A + B$$

$$\frac{di}{dt}(0^+) = 1 = -2A - B$$

$$\therefore\ A = -1, \quad B = 1$$

$$\therefore\ i(t) = \left(e^{-t} - e^{-2t}\right) u(t)$$

$$V(t) = \int_{-\infty}^{t} \frac{1}{C}\, i(\tau)\, d\tau = \int_{-\infty}^{t} 2\left(e^{-\tau} - e^{-2\tau}\right) u(\tau)\, d\tau$$

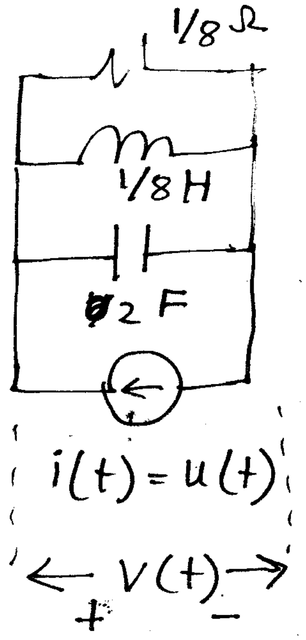
$$= 2\int_{0}^{t} \left(e^{-\tau} - e^{-2\tau}\right) d\tau$$

$$= 2\left(\left[e^{-\tau}\right]_t^0 + \frac{1}{2}\left[e^{-2\tau}\right]_0^t\right)$$

$$= 2 - 2e^{-t} + e^{-2t} - 1$$

$$= 1 - 2e^{-t} + e^{-2t}$$

(2)



$$\frac{V(t)}{R} + \int_{-\infty}^{t} \frac{1}{L} V(\tau)\, d\tau + C\frac{dV}{dt} = u(t)$$

$$\Rightarrow \frac{1}{R}\frac{dV}{dt} + \frac{1}{L} V(t) + C\frac{d^2V}{dt^2} = \delta(t)$$

$$\Rightarrow 8\frac{dV}{dt} + 8V + 2\frac{d^2V}{dt^2} = \delta(t)$$

$$\Rightarrow \int_{0^-}^{0^+} \left(2\frac{d^2V}{dt^2} + 8\frac{dV}{dt} + 8V\right) dt = 1$$

$$\Rightarrow 2\left(\frac{dV}{dt}(0^+) - \frac{dV}{dt}(0^-)\right) = 1$$

$$\Rightarrow \frac{dV}{dt}(0^+) = \frac{1}{2} \quad \left[\because \frac{dV}{dt}(0^-) = 0\right]$$

Also $V(0^+) = V(0^-) = 0$

For $t > 0$

Characteristic equation

$$2m^2 + 8m + 8 = 0$$

$$\Rightarrow m^2 + 2 \times 2m + 2^2 = 0$$

$$\Rightarrow (m+2)^2 = 0$$

$$\Rightarrow m = -2$$

$$\therefore V(t) = (A + Bt)e^{-2t} \quad \text{for } t > 0$$

$$V(0^+) = 0 \doteq A$$

$$\frac{dV}{dt}(0^+) = \frac{1}{2} = -2A + B(1) = B$$

$$\therefore A = 0, \quad B = \frac{1}{2}$$

$$\therefore V(t) = \frac{1}{2} t e^{-2t} u(t)$$

Current through inductor

$$= \int_{-\infty}^{t} \frac{1}{L} V(\tau)\, d\tau = \int_{0}^{t} 8 \times \frac{1}{2} \tau e^{-2\tau}\, d\tau$$

$$= 4\left[\left[\tau \frac{e^{-2\tau}}{-2}\right]_0^t + \frac{1}{2}\int_0^t e^{-2\tau}\, d\tau\right]$$

$$= 4\left[\frac{t e^{-2t}}{-2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left[e^{-2\tau}\right]_0^t\right]$$

$$= -2te^{-2t} - \left(e^{-2t} - 1\right)$$

$$= 1 - e^{-2t} - 2te^{-2t}$$

③ i) $$\int_{0^-}^{0^+} \left(\frac{d^2y}{dt^2} + 4y\right) dt = \int_{0^-}^{0^+} \delta(t)\, dt = 1$$

$$\Rightarrow \int_{0^-}^{0^+} \frac{d^2y}{dt^2}\, dt + \int_{0^-}^{0^+} 4y\, dt = 1$$

$$\Rightarrow \frac{dy}{dt}(0^+) - \frac{dy}{dt}(0^-) + 0 = 1$$

$$\Rightarrow \frac{dy}{dt}(0^+) = 1 + \frac{dy}{dt}(0^-) = 2$$

$y(0^+) = y(0^-) = 2$

because otherwise if $y(0^+) \neq y(0^-)$ then there will be a step jump in y & a $\dot{\delta}(t)$ term in $\frac{d^2y}{dt^2}$, so LHS will have $\dot{\delta}(t)$, but there is no $\dot{\delta}(t)$ on RHS.

ii) For $t > 0$

$$\frac{d^2y}{dt^2} + 4y = 0$$

characteristic equation

$$m^2 + 4 = 0 \Rightarrow m = \pm 2j$$

$$\therefore y(t) = Ae^{2jt} + Be^{-2jt} \quad \text{for } t > 0$$

$$\Rightarrow y(0^+) = A + B = 2$$

and $\frac{dy}{dt}(0^+) = 2jA - 2jB = 2$

$$\Rightarrow A - B = -j$$

$$\therefore A = \frac{2-j}{2} \text{ and } B = \frac{2+j}{2}$$

$$\therefore y(t) = \frac{(2-j)e^{2jt} + (2+j)e^{-2jt}}{2}$$

$$= 2\cos(2t) + \sin(2t)$$

$$= \sqrt{5}\left(\frac{2}{\sqrt{5}}\cos(2t) + \frac{1}{\sqrt{5}}\sin(2t)\right)$$

$$= \sqrt{5}\,\sin(2t+\theta)$$

where $\theta = \tan^{-1}(2)$

$\therefore$ For $t>0$ $\quad y(t) = \sqrt{5}\sin(2t+\theta)$

(iii) For $t<0$ $\quad \frac{d^2y}{dt^2} + 4y = 0$

Characteristic equation $m^2+4=0$

$$\therefore \quad y(t) = Ae^{2jt} + Be^{-2jt}$$

$$y(0^-) = A+B = 2$$

$$\frac{dy}{dt}(0^-) = 2jA - 2jB = 1$$

$$\Rightarrow A - B = \frac{-j}{2}$$

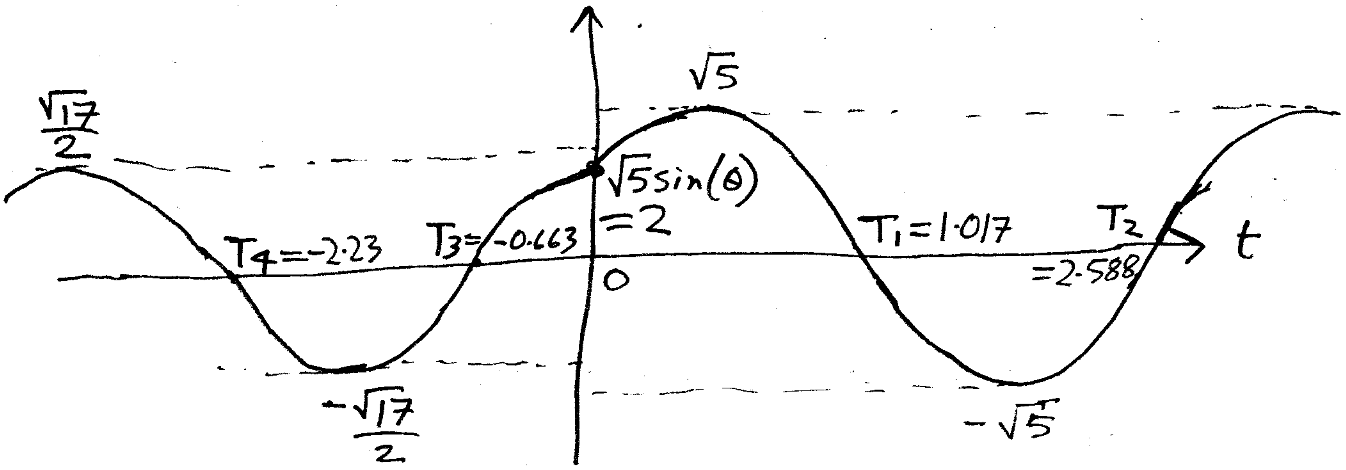
$$\therefore A = \frac{4-j}{4} \quad \text{and} \quad B = \frac{4+j}{4}$$

$$\therefore y(t) = e^{2jt}\left(\frac{4-j}{4}\right) + e^{-2jt}\left(\frac{4+j}{4}\right)$$

$$= 2\cos 2t \quad \frac{1}{2}\sin(2t)$$

$$= \frac{\sqrt{17}}{2}\left(\sin(2t+\phi)\right)$$

where $\phi = \tan^{-1}(4)$



Important time and amplitudes

$\sqrt{5} \sin \theta = \sqrt{5} \sin\left(\tan^{-1}(2)\right) = 2$

$$\text{Period} = \frac{2\pi}{2} = \pi$$

$\sin\left(\tan^{-1}(2) + 2T_1\right) = 0$

$$\Rightarrow T_1 = \frac{\pi - \tan^{-1}(2)}{2} = 1.017$$

$$T_2 = 1.017 + \frac{\pi}{2} = 2.588$$

$\sin\left(2T_3 + \tan^{-1}(4)\right) = 0$

$\Rightarrow 2T_3 + \tan^{-1}(4) = 0$

$\Rightarrow T_3 = -0.663$

$$T_4 = -0.663 - \frac{\pi}{2} = -2.23$$

QUESTION 4:

A system is described by the following differential equation $\frac{dy}{dt} + 4y = t$ with boundary condition $y(0) = 2$. Find the solution to this differential equation.

Solution:

Characteristic equation: $m + 4 = 0$

root : $m = -4$

$\therefore$ Natural response: Ae^{-4t}

Forced response $= k_1 t + k_2 \frac{d}{dt}(t)$

$= k_1 t + k_2$

The forced response must satisfy the given differential equation

$\therefore \frac{d}{dt}(k_1 t + k_2) + 4(k_1 t + k_2) = t$

$\Rightarrow k_1 + 4k_1 t + 4k_2 = t$

$\Rightarrow 4k_1 = 1$ or $k_1 = \frac{1}{4}$

and $k_1 + 4k_2 = 0$

$\Rightarrow k_2 = \frac{-k_1}{4} = -\frac{1}{16}$

$\therefore y(t) =$ Forced response + Natural response

$= k_1 t + k_2 + Ae^{-4t}$

$= \frac{1}{4}t - \frac{1}{16} + Ae^{-4t}$

$\because y(0) = 2$

$\Rightarrow \frac{1}{4} \times 0 - \frac{1}{16} + Ae^{0} = 2$

$\Rightarrow A = 2 + \frac{1}{16} = \frac{33}{16}$

$\therefore y(t) = \frac{1}{4}t - \frac{1}{16} + \frac{33}{16}e^{-4t}$

QUESTION 5:

A system is described by the following differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 1 + 2t + 3t^2$ with boundary condition $y(0) = 1$ and $\frac{dy}{dt}(0) = 2$. Find the solution to this differential equation.

Solution:

Characteristic equation: $m^2 + 5m + 6 = 0$

$$\Rightarrow (m+2)(m+3) = 0$$

Roots: $m = -2$ and -3

Natural response $y_n(t) = A_1 e^{-2t} + A_2 e^{-3t}$

$x(t) = 1 + 2t + 3t^2$

$$\frac{dx}{dt} = 2 + 6t$$

$$\frac{d^2x}{dt^2} = 6$$

$$\frac{d^3x}{dt^3} = 0$$

$\therefore$ Forced response will be of the form

$$k_1 + k_2 t + k_3 t^2$$

Now $\frac{d^2y_f}{dt^2} + 5\frac{dy_f}{dt} + 6y_f = 1 + 2t + 3t^2$

$$\Rightarrow 2k_3 + 5(k_2 + 2k_3 t) + 6(k_1 + k_2 t + k_3 t^2) = 1 + 2t + 3t^2$$

$$\Rightarrow 6k_3 = 3 \Rightarrow k_3 = \frac{1}{2}$$

$$6k_2 + 10k_3 = 2 \Rightarrow k_2 = \frac{2 - \frac{10}{2}}{6} = -\frac{1}{2}$$

And $2k_3 + 5k_2 + 6k_1 = 1$

$$\Rightarrow k_1 = \frac{1-5\left(-\frac{1}{2}\right)-2\left(\frac{1}{2}\right)}{6} = \frac{5}{12}$$

$$\therefore y_f(t) = \frac{5}{12} - \frac{1}{2}t + \frac{1}{2}t^2$$

$$y(t) = y_f(t) + y_n(t)$$

$$= \frac{5}{12} - \frac{1}{2}t + \frac{1}{2}t^2 + A_1 e^{-2t} + A_2 e^{-3t}$$

$$y(0) = \frac{5}{12} + A_1 + A_2 = 1 \Rightarrow A_1 + A_2 = \frac{7}{12} \quad \text{---} \ (1)$$

$$\frac{dy}{dt} = -\frac{1}{2} + t - 2A_1 e^{-2t} - 3A_2 e^{-3t}$$

$$\frac{dy}{dt}(0) = -\frac{1}{2} - 2A_1 - 3A_2 = 2$$

$$\Rightarrow 2A_1 + 3A_2 = -\frac{5}{2} \quad \text{----} \ (2)$$

From $2 \times (1) - (2)$ we get

$$2A_2 - 3A_2 = \frac{14}{12} + \frac{5}{2} = \frac{44}{12}$$

$$\Rightarrow A_2 = -\frac{44}{12} = \frac{-11}{3}$$

$$\therefore A_1 = \frac{7}{12} - A_2 = \frac{7}{12} + \frac{11}{3} = \frac{51}{12}$$

$$= \frac{17}{4}$$

$$\therefore y(t) = \frac{17}{4}e^{-2t} - \frac{11}{3}e^{-3t} + \frac{5}{12} - \frac{1}{2}t + \frac{1}{2}t^2$$

QUESTION 6:

A system is described by the following differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2e^{-5t}$ with boundary condition $y(0) = 1$ and $\frac{dy}{dt}(0) = 2$. Find the solution to this differential equation.

Solution:

Characteristic equation: $m^2+5m+6=0$

$\Rightarrow (m+2)(m+3)=0$

$\Rightarrow m=-2,-3$

$\therefore$ Natural response: $y_n(t) = A_1e^{-2t}+A_2e^{-3t}$

Forced response:

$$y_f(t) = \frac{2e^{-5t}}{(m^2+5m+6)\big|_{m=-5}}$$

$$= \frac{2e^{-5t}}{6} = \frac{e^{-5t}}{3}$$

$\therefore y(t) = y_n(t) + y_f(t)$

$$= A_1e^{-2t} + A_2e^{-3t} + \frac{e^{-5t}}{3}$$

$y(0) = A_1 + A_2 + \frac{1}{3} = 1$

$\Rightarrow A_1 + A_2 = \frac{2}{3}$ ------ (1)

$\frac{dy}{dt} = -2A_1e^{-2t} - 3A_2e^{-3t} - \frac{5}{3}e^{-5t}$

$\frac{dy}{dt}(0) = -2A_1 - 3A_2 - \frac{5}{3} = 2$

$\Rightarrow 2A_1 + 3A_2 = \frac{-11}{3}$ ---- (2)

From (2) $- 2\times$(1)

$A_2 = \frac{-11}{3} - \frac{4}{3} = -5$

$\therefore A_1 = 5 + \frac{2}{3} = \frac{17}{3}$

$\therefore y(t) = \frac{17}{3}e^{-2t} - 5e^{-3t} + \frac{e^{-5t}}{3}$

QUESTION 7:

A system is described by the following differential equation $\frac{dy}{dt} + 5y = 2e^{-5t}$ with boundary condition $y(0) = 1$. Find the solution to this differential equation.

Solution:

Here the root of the characteristic equation $= -5 =$ the exponent of the excitation function.

$$\frac{dy}{dt} + 5y = 2e^{-5t}$$

$$\Rightarrow e^{5t}\left(\frac{dy}{dt} + 5y\right) = 2$$

$$\Rightarrow e^{5t}\frac{dy}{dt} + 5e^{5t}y = 2$$

$$\Rightarrow \frac{d}{dt}\left(e^{5t}y\right) = 2$$

$$\Rightarrow e^{5t}y = 2t + A$$

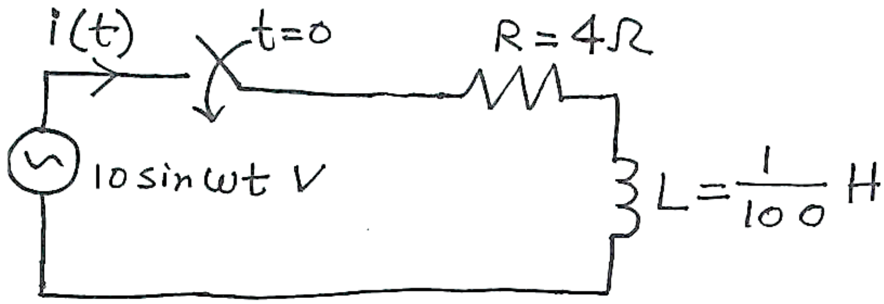
$$\Rightarrow y = 2te^{-5t} + Ae^{-5t}$$

$$= (2t + A)e^{-5t}$$

$$\therefore y(0) = A = 1$$

$$\therefore y(t) = (2t + 1)e^{-5t}$$

QUESTION 8:



Find the expression of current $i(t)$ for $t > 0$ if the switch is closed at $t = 0$ and $\omega = 300$ radian/sec.

Solution:

$$L\frac{di}{dt} + Ri = 10\sin\omega t$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{10}{L}\sin\omega t$$

$$\Rightarrow \frac{di}{dt} + 400i = 1000\sin\omega t$$

Characteristic root: $m = -400$

Natural response: $y_n(t) = Ae^{-400t}$

$$x(t) = 1000\sin\omega t$$

$$\frac{dx}{dt} = 1000\omega\cos\omega t$$

$$\frac{d^2x}{dt^2} = -1000\omega^2\sin\omega t$$

$$\vdots$$

$\therefore$ Forced response

$$y_f(t) = k_1\sin\omega t + k_2\cos\omega t$$

Now $\frac{d y_f}{dt} + 400 y_f = 1000 \sin \omega t$

$\Rightarrow (k_1 \omega \cos\omega t - k_2 \omega \sin \omega t) +$

$400(k_1 \sin\omega t + k_2 \cos \omega t) = 1000 \sin\omega t$

$\therefore k_1 \omega + 400 k_2 = 0$

or $300 k_1 + 400 k_2 = 0$ $[\because \omega = 300]$

and $-k_2 \omega + 400 k_1 = 1000$

or $400 k_1 - 300 k_2 = 1000$

So we have

$3k_1 + 4k_2 = 0$ ---- (1)

and $4k_1 - 3k_2 = 10$ ---- (2)

From $3 \times (1) + 4 \times (2)$ we get

$25 k_1 = 40 \Rightarrow k_1 = \frac{8}{5}$

$\therefore k_2 = \frac{-3k_1}{4} = -\frac{6}{5}$

$\therefore i(t) = y_n(t) + y_f(t)$

$= A e^{-400t} + \frac{8}{5} \sin\omega t - \frac{6}{5} \cos \omega t$

$\therefore i(0) = A - \frac{6}{5} = 0 \Rightarrow A = \frac{6}{5}$

$$\therefore \quad i(t) = \frac{6}{5} e^{-400t} + \frac{8}{5} \sin \omega t - \frac{6}{5} \cos \omega t$$

$$= \frac{6}{5} e^{-400t} + \frac{10}{5} \left(\frac{8}{10} \sin \omega t - \frac{6}{10} \cos \omega t \right)$$

$$= \frac{6}{5} e^{-400t} + \quad 2 \sin (\omega t - \phi)$$

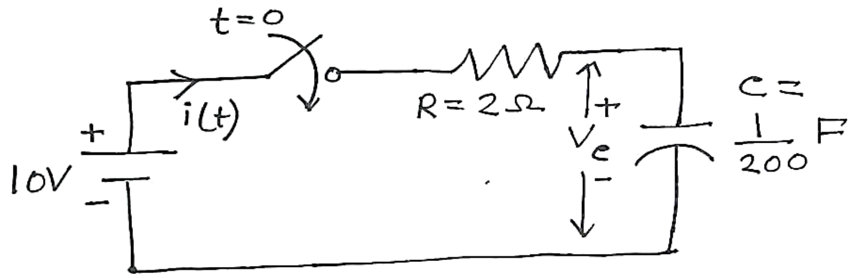
$$\text{with } \phi = \tan^{-1} \frac{6}{8}$$

$$= \tan^{-1} \frac{3}{4}$$

$$\therefore \quad i(t) = \frac{6}{5} e^{-400t} + 2 \sin \left(\omega t - \tan^{-1} \frac{3}{4} \right)$$

$$= \frac{6}{5} e^{-400t} + 2 \sin \left(300t - \tan^{-1} \frac{3}{4} \right)$$

QUESTION 9:



Find the expression of current $i(t)$ for $t > 0$ if the switch is closed at $t = 0$. The capacitor was initially uncharged.

Solution:

$$V_c(t) + Ri(t) = 10$$

$$\Rightarrow V_c(t) + R\left(C\frac{dV_c}{dt}\right) = 10$$

$$\Rightarrow \frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{10}{RC}$$

$$\Rightarrow \frac{dV_c}{dt} + 100V_c = 1000$$

Forced response : $y_f(t) = k$

Now $\frac{dy_f(t)}{dt} + 100\,y_f(t) = 1000$

$$\Rightarrow 0 + 100k = 1000$$

$$\Rightarrow k = 10$$

Characteristic root : $m = -100$

$\therefore$ Natural response : $y_n(t) = Ae^{-100t}$

$$\therefore V_c(t) = y_n(t) + y_f(t)$$

$$= Ae^{-100t} + k$$

$$= Ae^{-100t} + 10$$

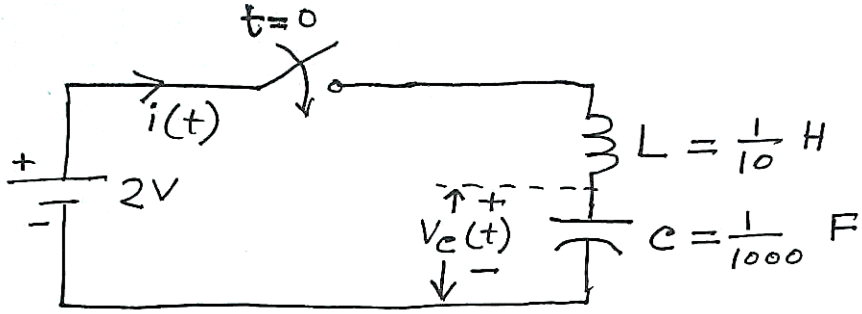
$$V_c(0) = A + 10 = 0 \Rightarrow A = -10$$

$$\therefore V_c(t) = 10 - 10e^{-100t} = 10\left(1 - e^{-100t}\right)$$

$$\therefore i(t) = C\frac{dV_c}{dt} = \frac{1}{200}\frac{d}{dt}\left(10\left(1 - e^{-100t}\right)\right)$$

$$= \frac{10}{200} \times 100e^{-100t} = 5e^{-100t}$$

QUESTION 10:



Find the expression of capacitor voltage $v_c(t)$ for $t > 0$ if the switch is closed at $t = 0$. The capacitor was initially uncharged.

Solution:

$$V_c(t) + L\frac{di}{dt} = 2$$

$$\Rightarrow V_c(t) + L\frac{d}{dt}\left(C\frac{dV_c}{dt}\right) = 2$$

$$\Rightarrow LC\frac{d^2V_c}{dt^2} + V_c(t) = 2$$

$$\Rightarrow \frac{1}{10000}\frac{d^2V_c}{dt^2} + V_c(t) = 2$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + 10000\,V_c(t) = 20000$$

Characteristic equation:

$$m^2 + 10000 = 0$$

$$\Rightarrow m = \pm 100\,j$$

$$[j = \sqrt{-1}]$$

$\therefore$ Natural response

$$y_n(t) = A_1 e^{j100t} + A_2 e^{-j100t}$$

Forced response

$$y_f(t) = K$$

Now $\dfrac{d^2 y_f(t)}{dt^2} + 10000 y_f = 20000$

$$\Rightarrow 0 + 10000K = 20000$$

$$\Rightarrow K = 2$$

$$\therefore V_c(t) = y_f(t) + y_n(t)$$
$$= 2 + A_1 e^{j100t} + A_2 e^{-j100t}$$

$$V_c(0) = 2 + A_1 + A_2 = 0$$
$$\Rightarrow A_1 + A_2 = -2$$

$$i(0) = 0 \Rightarrow \frac{dV_c}{dt}(0) = 0$$

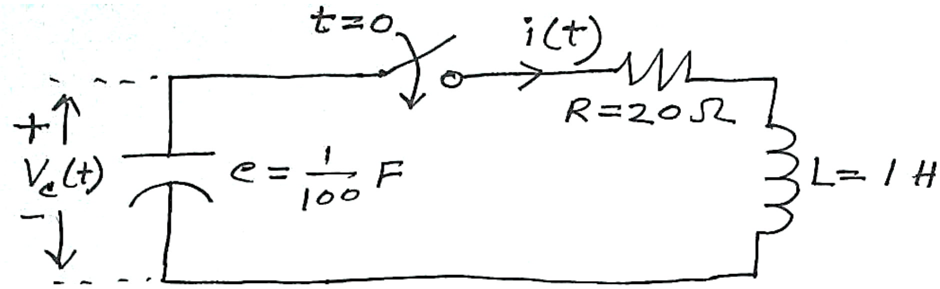
$$\Rightarrow 100jA_1 - 100jA_2 = 0$$

$$\Rightarrow A_1 = A_2$$

$$\therefore A_1 = A_2 = -1$$

$$\therefore V_c(t) = 2 - \left(e^{j100t} + e^{-j100t}\right)$$
$$= 2 - 2\cos 100t$$
$$= 2(1 - \cos 100t)$$

QUESTION 11:



Find the expression of current $i(t)$ for $t > 0$ if the switch is closed at $t = 0$. The initial capacitor voltage $v_c(0)$ =1 V.

Solution:

$$V_c(t) = i(t)R + L\frac{di}{dt}$$

$$= R\left(-C\frac{dV_c}{dt}\right) + L\frac{d}{dt}\left(-C\frac{dV_c}{dt}\right)$$

$$= -RC\frac{dV_c}{dt} - LC\frac{d^2V_c}{dt^2}$$

$$\Rightarrow LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = 0$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + 20\frac{dV_c}{dt} + \frac{100}{1} = 0$$

Forced response will be zero

Characteristic equation:

$$m^2 + 20m + 100 = 0$$

$$\Rightarrow (m+10)^2 = 0$$

$$\Rightarrow m = -10 \text{ (repeated roots)}$$

$\therefore$ Natural response $y_n(t)$

$$= (A_1 + A_2 t)e^{-10t}$$

$$\therefore V_c(t) = (A_1 + A_2 t)e^{-10t}$$

$$V_c(0) = A_1 = 1$$

$$i(0) = -C \frac{dV_c}{dt}(0) = 0$$

$$\Rightarrow \frac{-1}{100}\left(A_2 e^{-10t} + (A_1 + A_2 t) \times (-10) e^{-10t}\right)\Big|_{t=0} = 0$$

$$\Rightarrow A_2 + (A_1)(-10) = 0$$

$$\Rightarrow A_2 = 10 A_1 = 10$$

$$\therefore V_c(t) = (1 + 10t)e^{-10t}$$

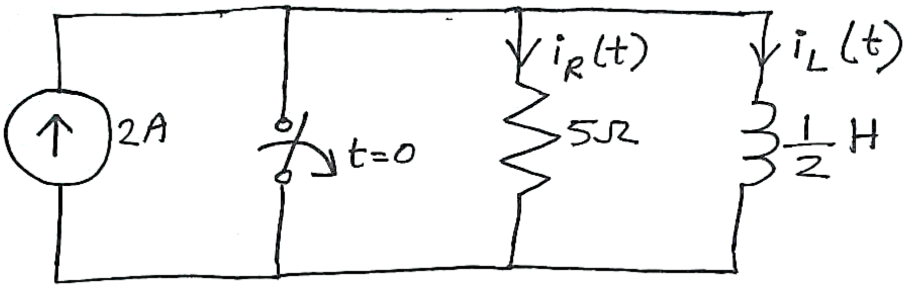
$$i(t) = -C \frac{dV_c}{dt}$$

$$= \frac{-1}{100} \times \left(10 e^{-10t} + (-10)(1+10t)e^{-10t}\right)$$

$$= -\frac{1}{100}\left(-100 t e^{-10t}\right)$$

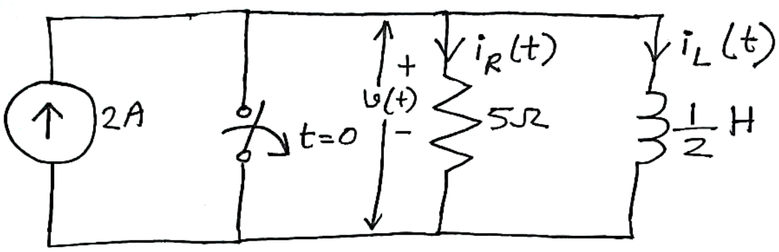
$$= t e^{-10t}$$

Question 12



Find the expression of currents $i_R(t)$ and$i_L(t)$ for $t > 0$ if the switch is **opened (disconnected)** at $t = 0$ and the initial current through the inductor was zero.

Solution:



$$i_R(t) + i_L(t) = 2 \quad \text{for } t \geq 0$$

$$\Rightarrow \frac{v(t)}{R} + i_L = 2$$

$$\Rightarrow \frac{L\frac{di_L}{dt}}{R} + i_L = 2$$

$$\Rightarrow \frac{di_L}{dt} + \frac{R}{L} i_L = 2\frac{R}{L}$$

$$\Rightarrow \frac{di_L}{dt} + 10 i_L = 20$$

Forced response $y_f = \frac{20}{10} = 2$

Natural response $y_n = A e^{-10t}$

$$\therefore i_L = 2 + A e^{-10t}$$

$$i_L(0) = 2 + A = 0 \Rightarrow A = -2$$

$$\therefore i_L(t) = 2 - 2e^{-10t}$$

$$i_R(t) = 2 - i_L(t) = 2e^{-10t}.$$