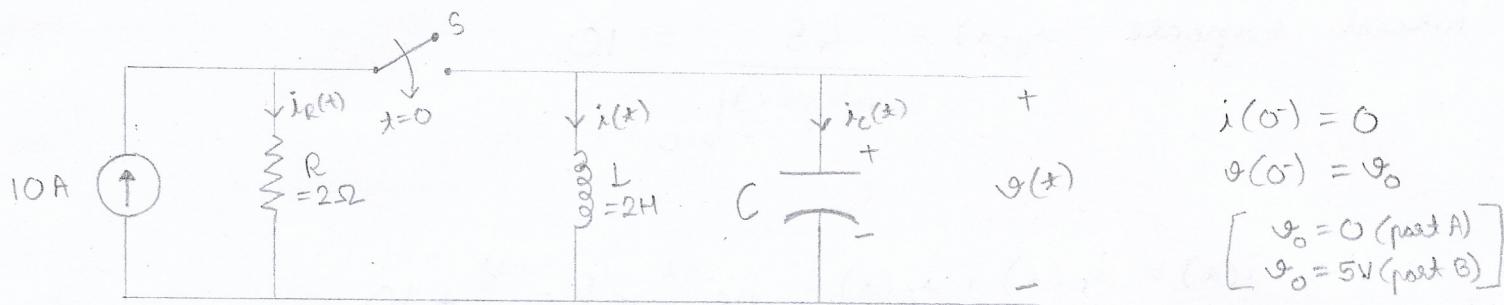


# EXPERIMENT 2

# QUESTION 2

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From KCL we have  $10 = i_R(t) + i(t) + i_C(t) \quad \text{--- (i)}$

$$v(t) = L \frac{di(t)}{dt} = \frac{q_C(t)}{C} = i_R(t) R$$

$$\Rightarrow i_R(t) = \frac{L}{R} \frac{di(t)}{dt} \quad \text{--- (ii)}$$

$$\Rightarrow q_C(t) = LC \frac{di(t)}{dt} \Rightarrow i_C(t) = \dot{q}_C(t) = LC \frac{d^2 i(t)}{dt^2} \quad \text{--- (iii)}$$

Substituting (ii) & (iii) in (i) we have

$$2C \frac{d^2 i(t)}{dt^2} + \frac{di(t)}{dt} + i(t) = 10$$

$$\Rightarrow \frac{d^2 i(t)}{dt^2} + \frac{1}{2C} \frac{di(t)}{dt} + \frac{1}{2C} i(t) = \frac{5}{C}$$

$$(i), C = \frac{1}{q_0} F$$

$$\text{Differential eqn} \rightarrow \frac{d^2 i(t)}{dt^2} + \frac{q}{2} \frac{di(t)}{dt} + \frac{q}{2} i(t) = 4S$$

$$\text{Characteristic eqn} \rightarrow m^2 + \frac{q}{2}m + \frac{q}{2} = 0 \Rightarrow (m+3)(m+\frac{3}{2}) = 0$$

$$\Rightarrow m = -3, -\frac{3}{2}$$

Roots are REAL & DISTINCT

Natural response will be of the form

$$i_n(t) = A e^{-3t} + B e^{-\frac{3t}{2}}$$

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Forced response  $i_f(t) = \frac{45}{m^2 + \frac{9}{2}m + \frac{9}{2}} = 10$

$$\therefore i(t) = i_n(t) + i_f(t) = A e^{-3t} + B e^{-\frac{3t}{2}} + 10$$

$$v(t) = L \frac{di(t)}{dt} = -6A e^{-3t} - 3B e^{-\frac{3t}{2}}$$

$$i(0^-) = A + B + 10 = 0 \Rightarrow A + B = -10$$

$$v(0^-) = -6A - 3B = v_0 \Rightarrow 2A + B = \frac{-v_0}{3}$$

PART A ( $v_0 = 0V$ )

$$A + B = -10$$

$$2A + B = 0$$

$$\Rightarrow A = 10, B = -20$$

$$\checkmark i(t) = 10 \left( 1 - 2e^{-\frac{3t}{2}} + e^{-3t} \right)$$

$$= 10 \left( 1 - e^{-\frac{3t}{2}} \right)^2 A$$

$$\checkmark v(t) = 60 \left( e^{-\frac{3t}{2}} - e^{-3t} \right) V$$

PART B ( $v_0 = 5V$ )

$$A + B = -10$$

$$2A + B = \frac{-5}{3}$$

$$\Rightarrow A = \frac{25}{3}, B = -\frac{55}{3}$$

$$\checkmark i(t) = 10 - \frac{5}{3} \left( 11e^{-\frac{3t}{2}} - 5e^{-3t} \right) A$$

$$\checkmark v(t) = 5 \left( 11e^{-\frac{3t}{2}} - 10e^{-3t} \right) V$$

(ii)  $C = \frac{1}{4} F$

Differential eq<sup>n</sup>  $\rightarrow \frac{d^2 i(t)}{dt^2} + 2 \frac{di(t)}{dt} + 2i(t) = 20$

Characteristic eq<sup>n</sup>  $\rightarrow m^2 + 2m + 2 = 0 \Rightarrow (m+1)^2 = -1$   
 $\Rightarrow m+1 = \pm j$

$$m = -1+j, -1-j$$

$$\operatorname{Re}(m) = -1, |\operatorname{Im}(m)| = 1$$

Roots are COMPLEX & CONJUGATE of each other

Natural response will be of the form

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$$i_n(t) = e^{-t} (A \cos t + B \sin t)$$

Forced response if  $i_f(t) = \frac{20}{m^2 + 2m + 21}_{m=0} = 10$

$$\therefore i(t) = i_f(t) + i_n(t) = 10 + e^{-t} (A \cos t + B \sin t)$$

$$\vartheta(t) = L \frac{di(t)}{dt} = 2e^{-t} ((B-A) \cos t - (B+A) \sin t)$$

$$i(0^-) = 10 + A = 0 \Rightarrow A = -10$$

$$\vartheta(0^-) = 2B - 2A = \vartheta_0 \Rightarrow B = \frac{\vartheta_0 + A}{2} = \frac{\vartheta_0 - 10}{2}$$

PART A ( $\vartheta_0 = 0V$ )

$$A = -10$$

$$B = -10 + 0 = -10$$

✓  $i(t) = 10 - 10e^{-t} (\cos t + \sin t) A$

✓  $\vartheta(t) = 40e^{-t} \sin t V$

PART B ( $\vartheta_0 = 5V$ )

$$A = -10$$

$$B = -10 + \frac{5}{2} = -\frac{15}{2}$$

✓  $i(t) = 10 - \frac{5}{2}e^{-t} (4 \cos t + 3 \sin t) A$

✓  $\vartheta(t) = 5e^{-t} (cos t + 7 sin t) V$

(iii)  $C = \frac{1}{8} F$

Differential eq<sup>n</sup>  $\rightarrow \frac{d^2 i(t)}{dt^2} + 4 \frac{di(t)}{dt} + 4 i(t) = 40$

Characteristic eq<sup>n</sup>  $\rightarrow m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0$

$$m = -2, -2$$

Roots are REAL & REPEATED

Natural response will be of the form

$$i_n(t) = Ae^{-2t} + Bte^{-2t}$$

forced response

$$i_f(t) = \frac{40}{m^2 + 4m + 4} \Big|_{m=0} = 10$$

$$\therefore i(t) = i_n(t) + i_f(t) = Ae^{-2t} + Bte^{-2t} + 10$$

$$v(t) = L \frac{di(t)}{dt} = 2e^{-2t} ((B - 2A) - 2Bt)$$

$$i(0^-) = A + 10 = 0 \Rightarrow A = -10$$

$$v(0^-) = 2(B - 2A) = v_0 \Rightarrow B = \frac{v_0}{2} + 2A = \frac{v_0}{2} - 20$$

PART A ( $v_0 = 0V$ )

$$A = -10$$

$$B = -20 + 0 = -20$$

$$\checkmark i(t) = 10 - 10e^{-2t}(1+2t) \quad A$$

$$\checkmark v(t) = 80te^{-2t} \quad V$$

PART B ( $v_0 = 5V$ )

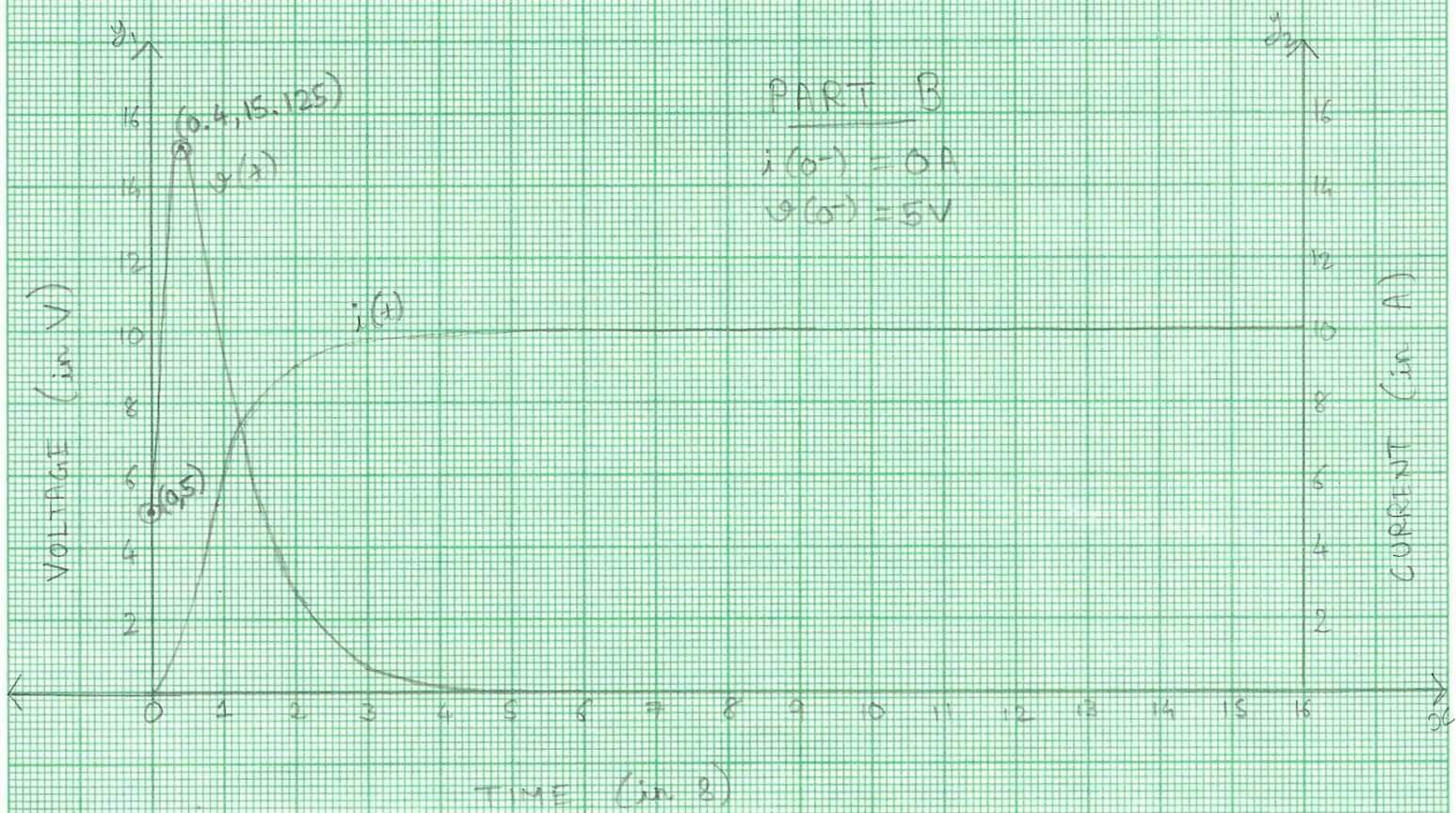
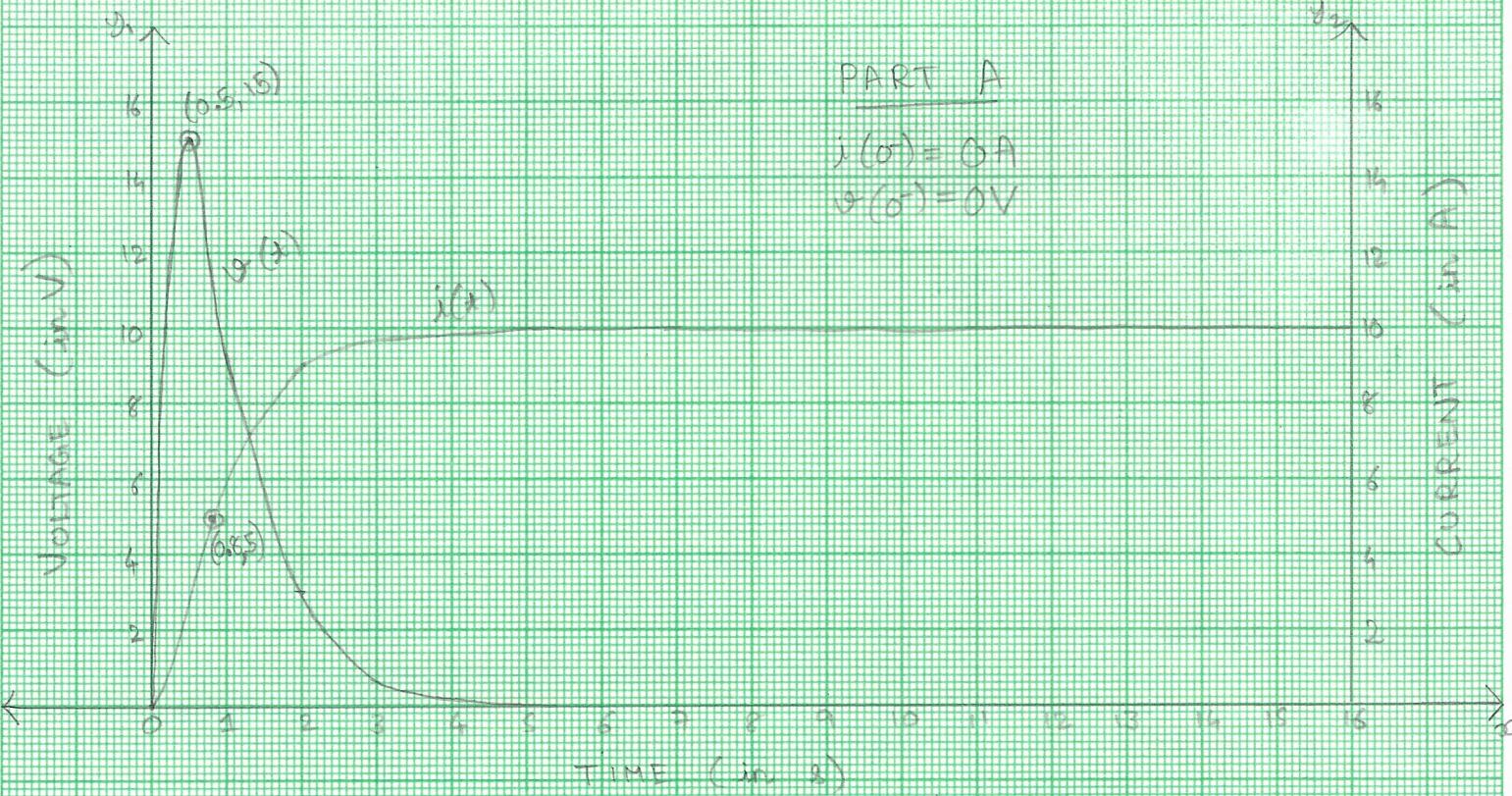
$$A = -10$$

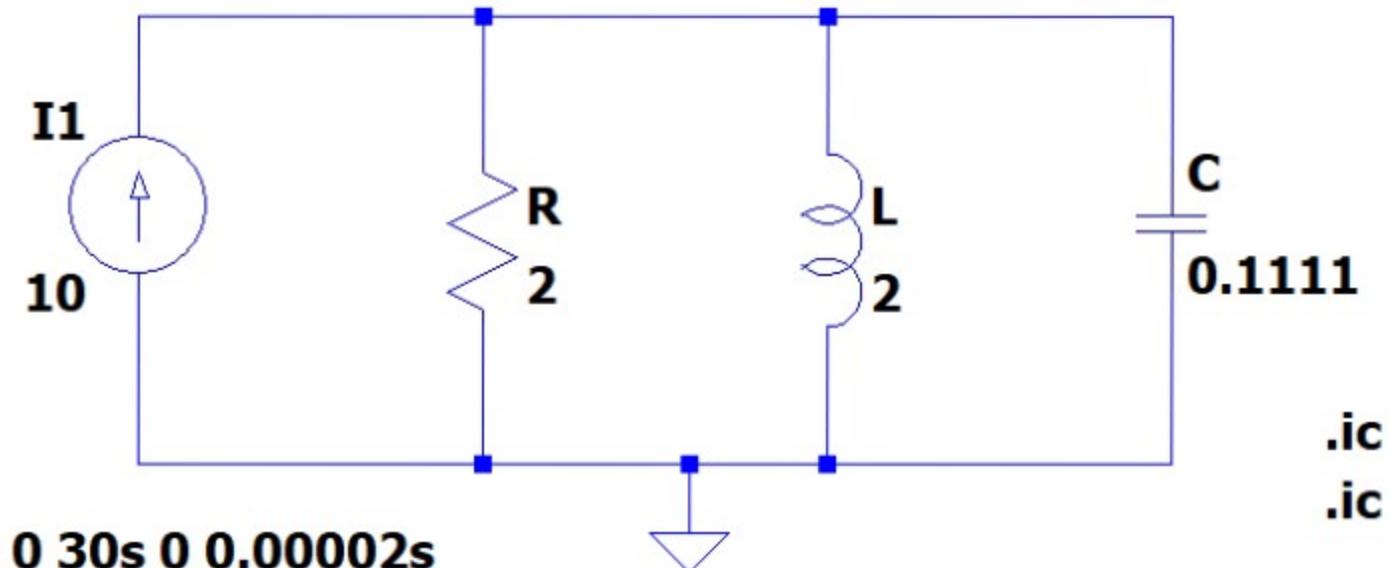
$$B = -20 + \frac{5}{2} = -\frac{35}{2}$$

$$\checkmark i(t) = 10 - \frac{5}{2}e^{-2t}(4+7t) \quad A$$

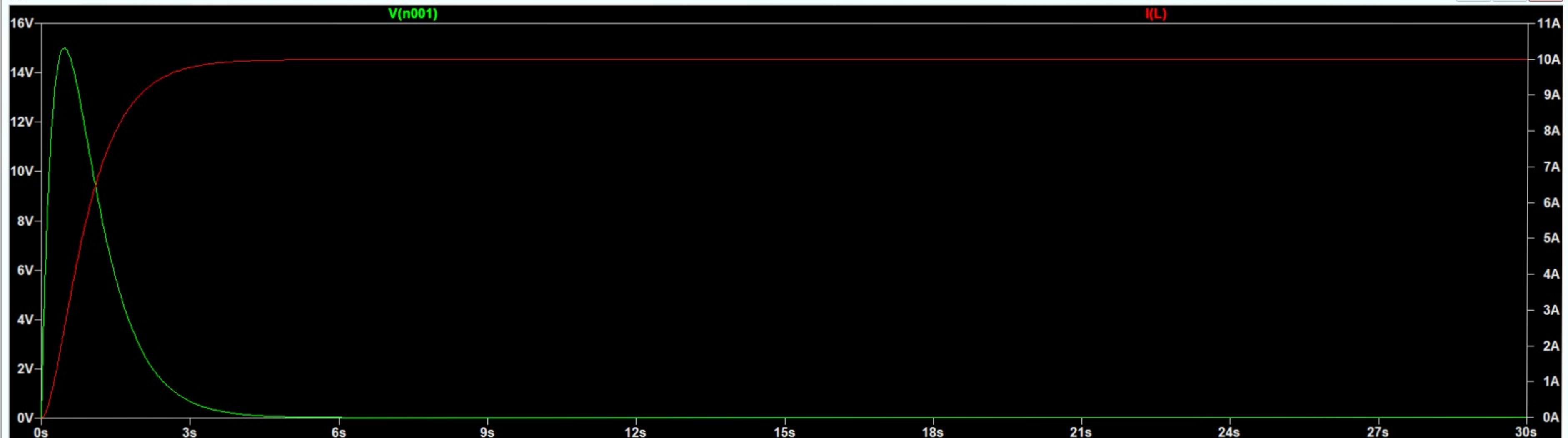
$$\checkmark v(t) = 5e^{-2t}(1+14t) \quad V$$

QUESTION 2 - (i)  $C = \frac{1}{q} =$

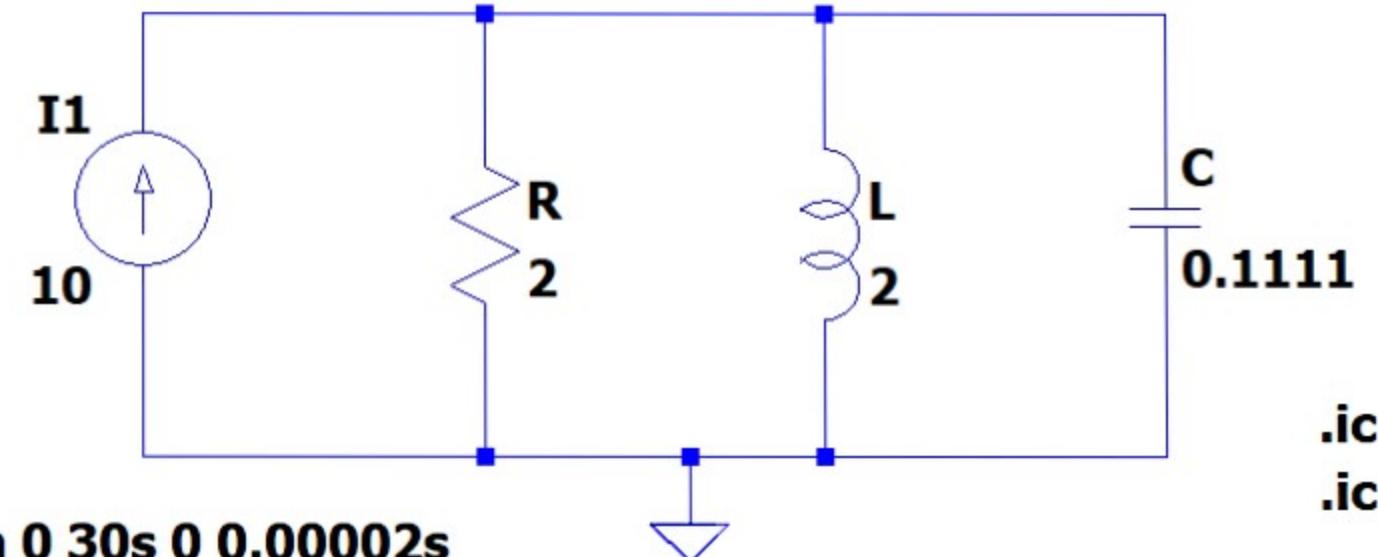


**Question 2 - (i)  $C = 1/9 \text{ F}$**   
**PART A**

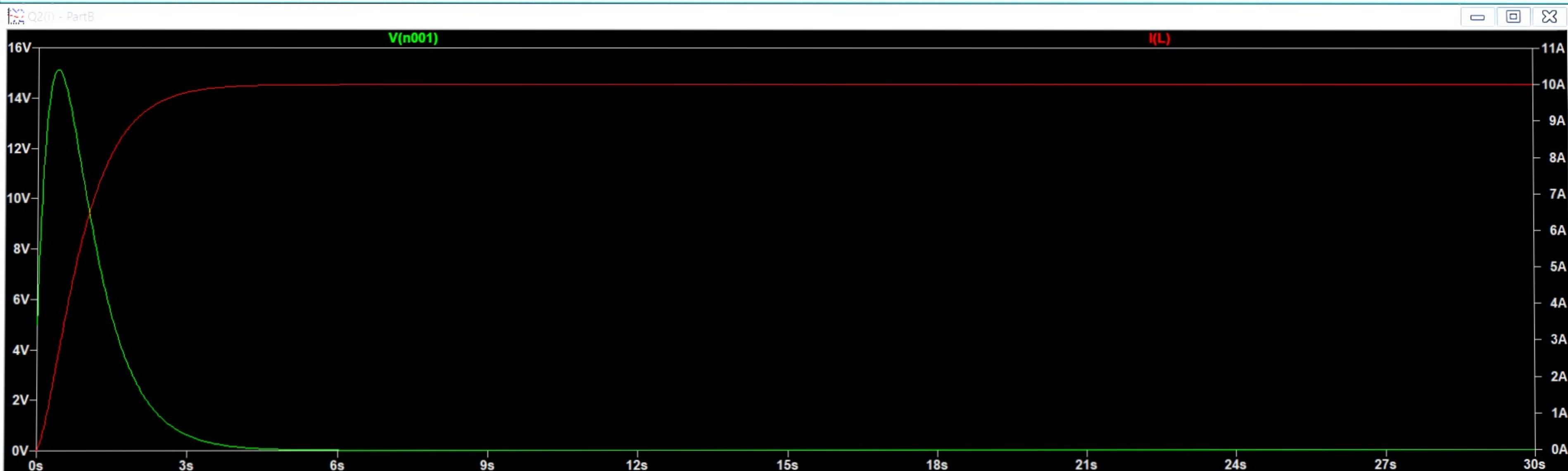
.ic V(n001) = 0  
.ic I(L) = 0



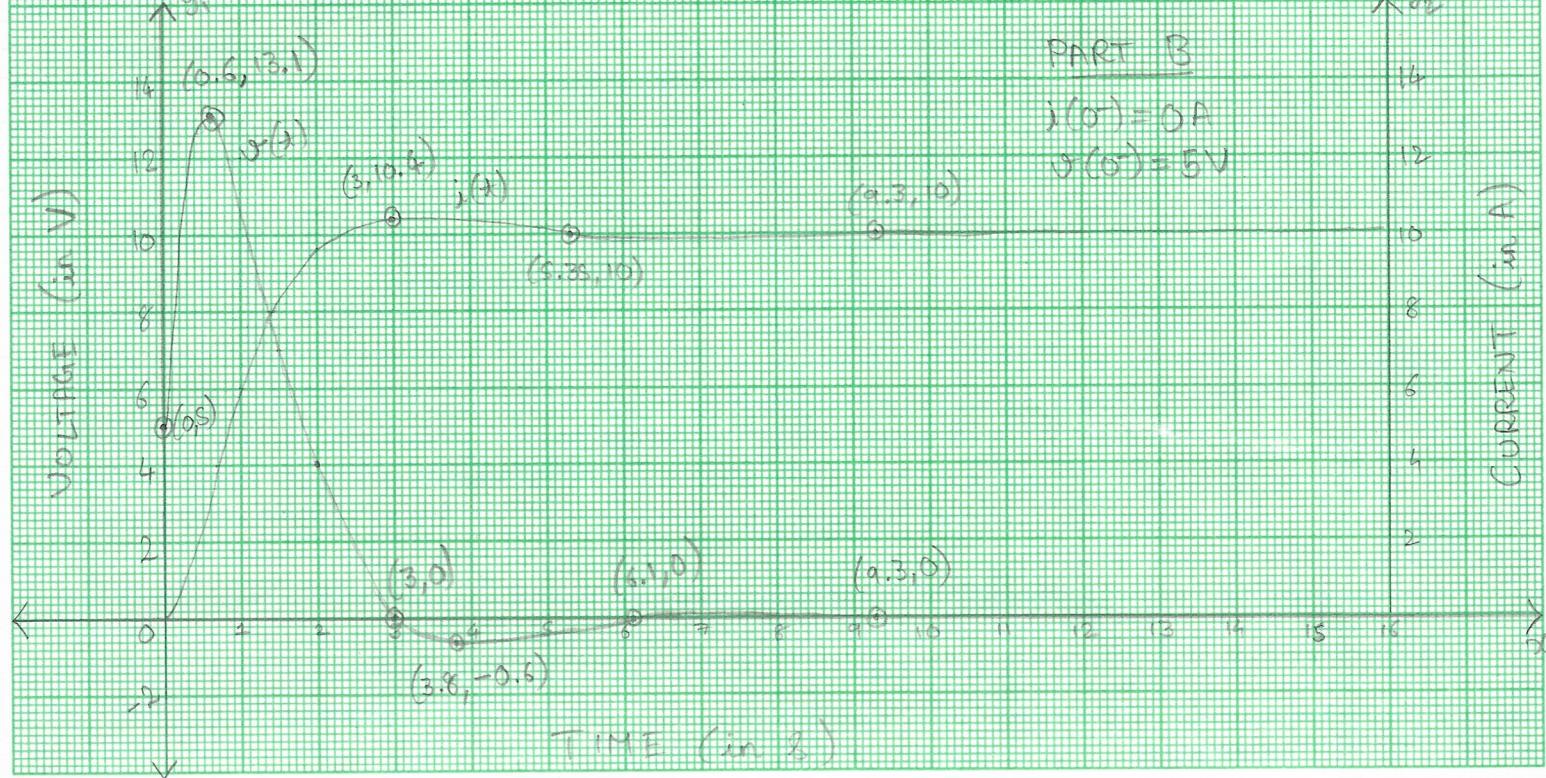
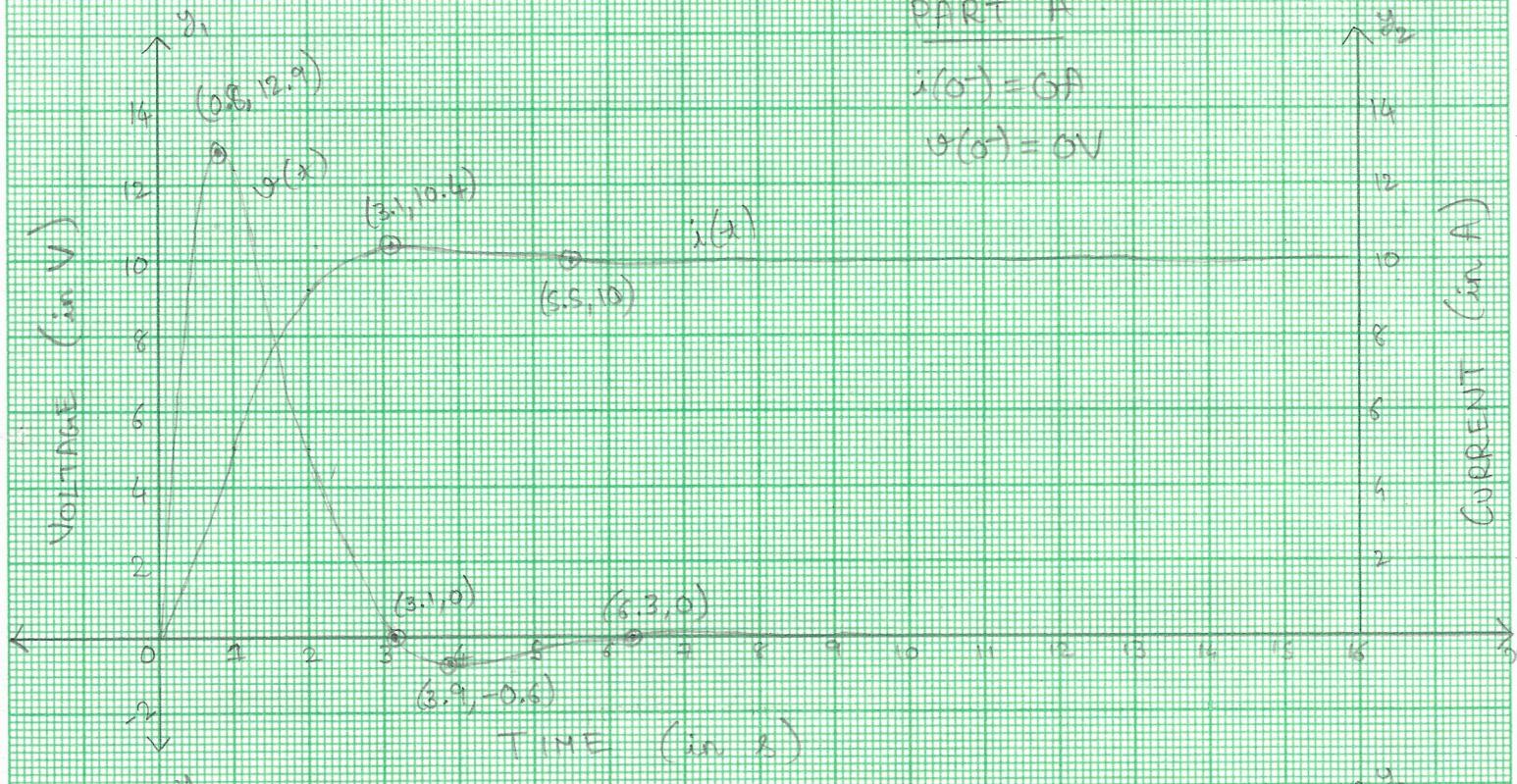
**Question 2 - (i)  $C = 1/9 \text{ F}$**   
**PART B**

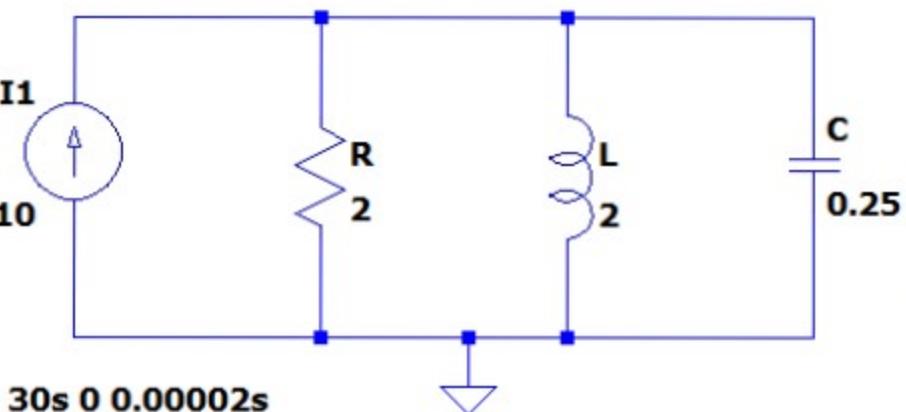


.ic  $V(n001) = 5$   
.ic  $I(L) = 0$



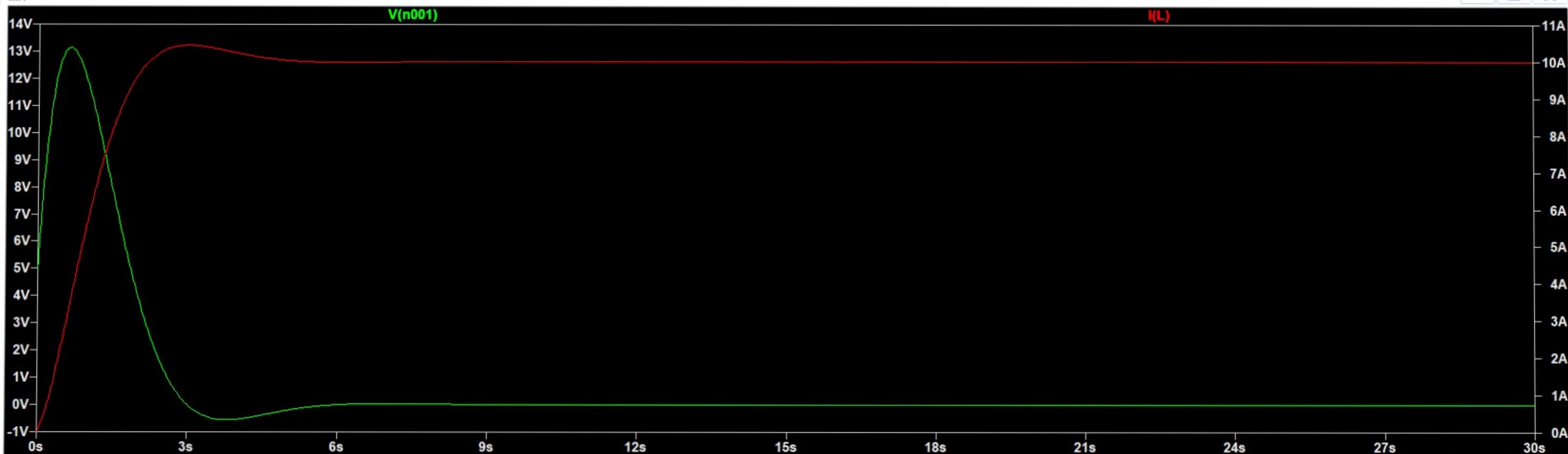
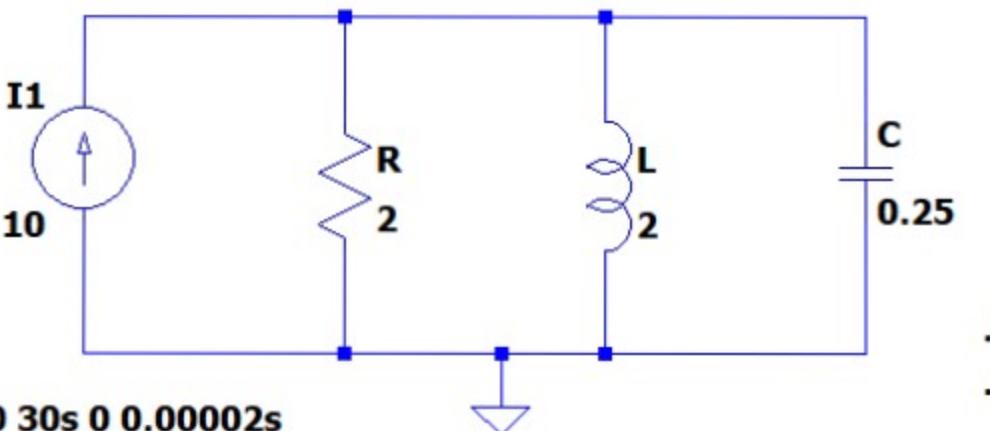
QUESTION 2 - (ii)  $C = \frac{1}{4} F$



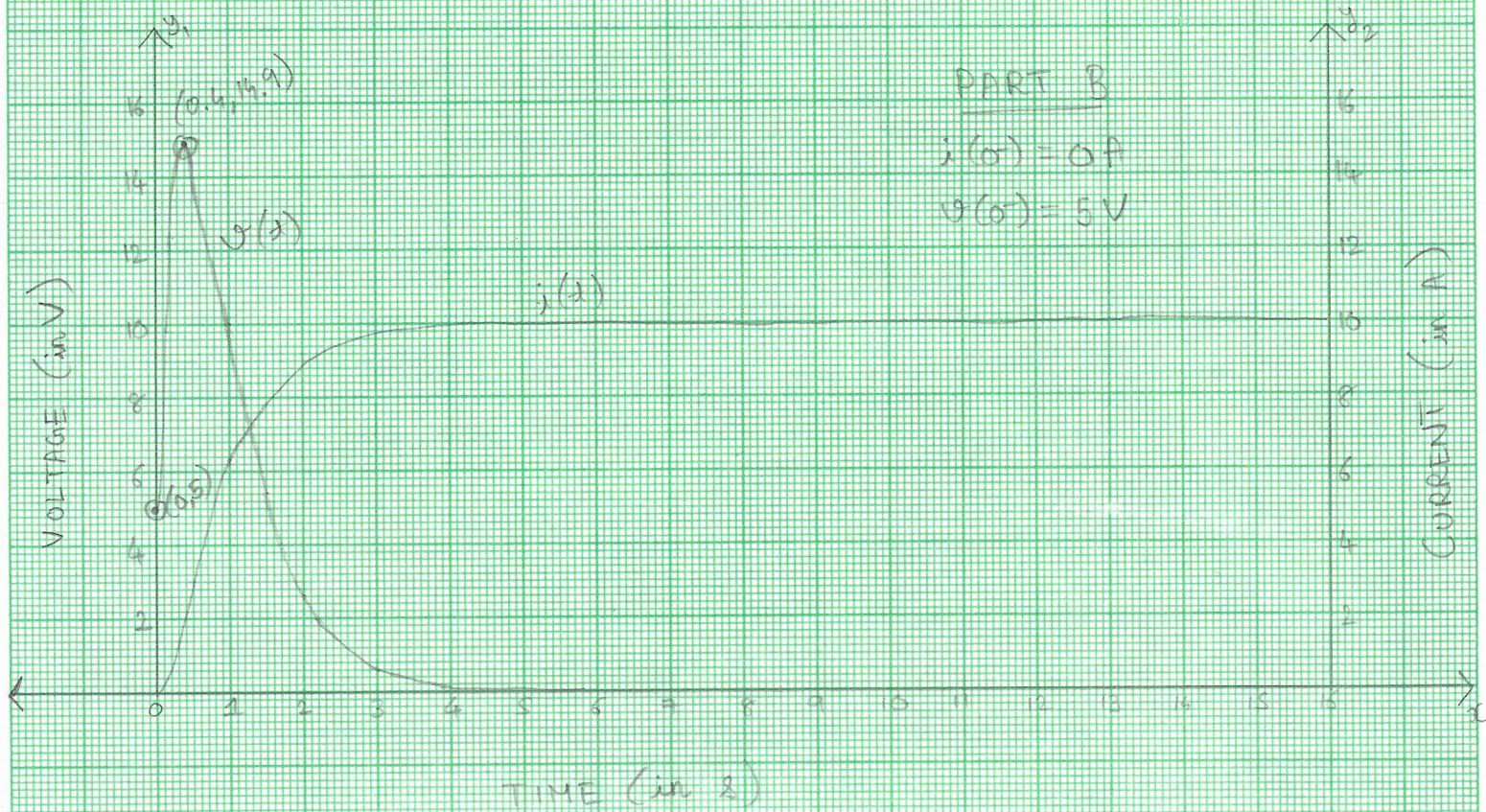
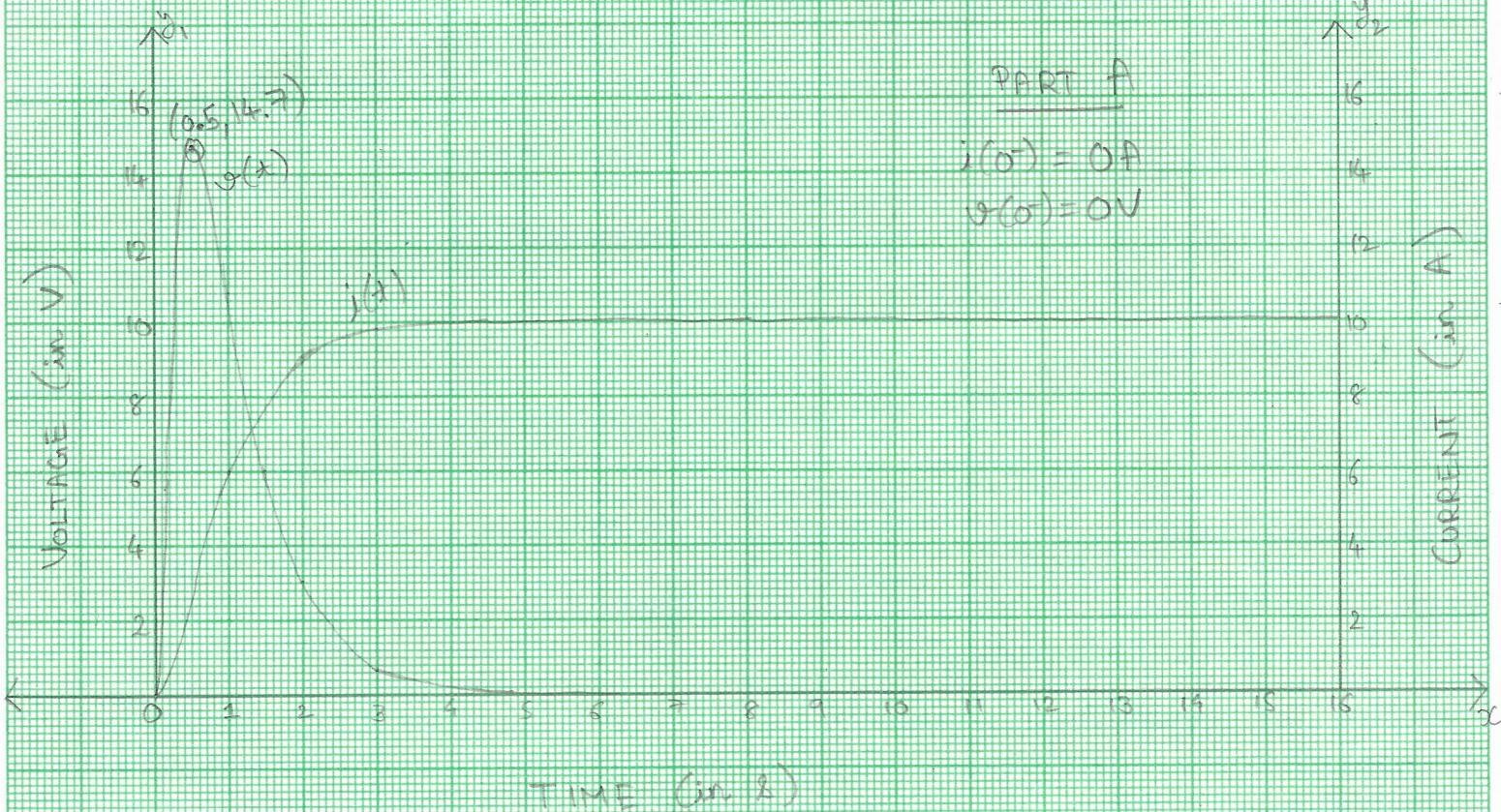
**Question 2 - (ii)  $C = 1/4 F$**   
**PART A**

.ic  $V(n001) = 0$   
.ic  $I(L) = 0$

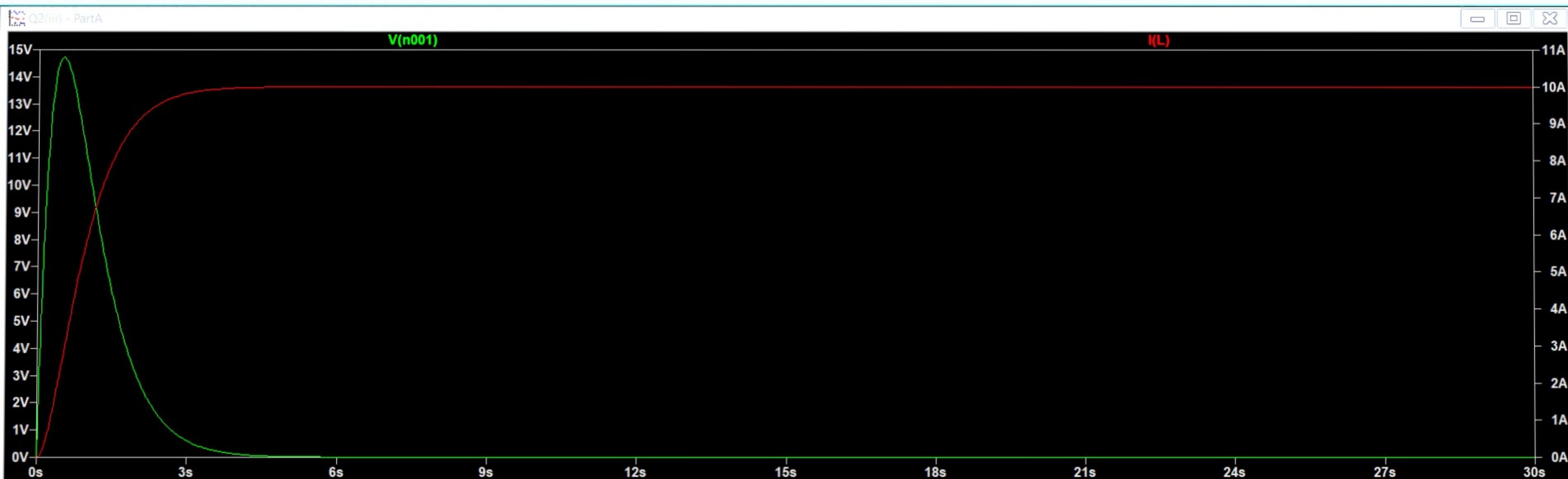
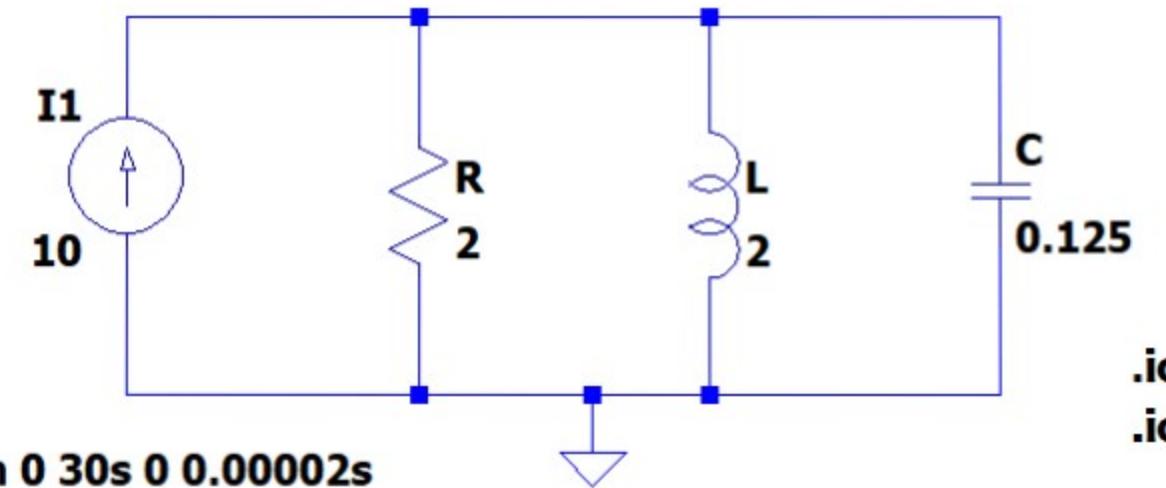


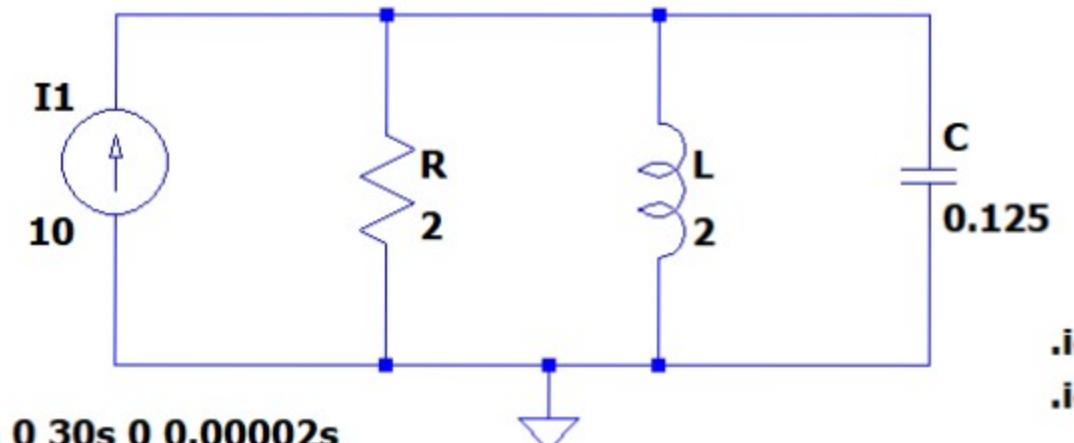
**Question 2 - (ii)  $C = 1/4 \text{ F}$**   
**PART B**

QUESTION 2 - (iii)  $C = \frac{1}{8} F$



**Question 2 - (iii)  $C = 1/8 \text{ F}$**   
**PART A**



**Question 2 - (iii)  $C = 1/8 \text{ F}$**   
**PART B**

.ic  $V(n001) = 5$   
.ic  $I(L) = 0$

