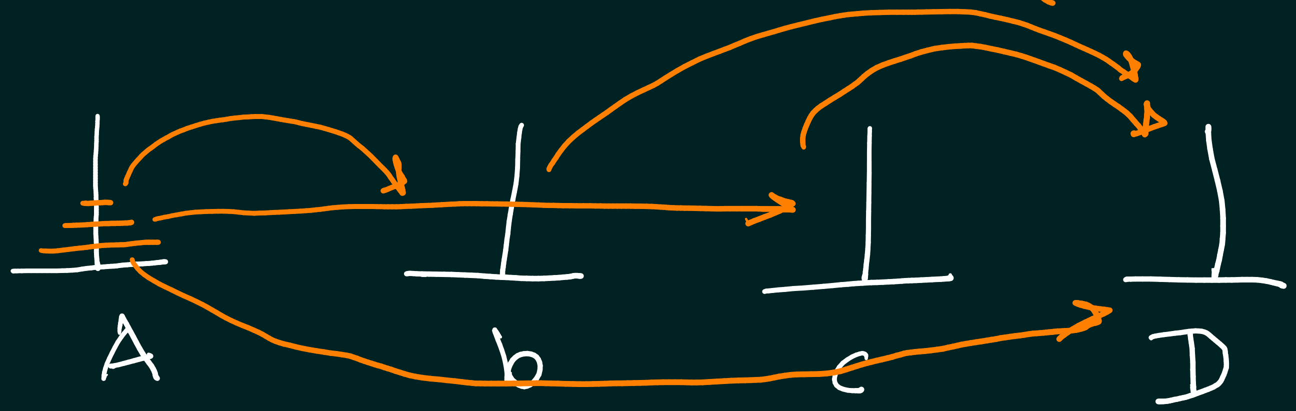
$n \leq 3 \rightarrow 2^n - 1$ (base)



5 steps $\neq 2^3 - 1$

Constant 7

$\boxed{\text{Query}}$

The triangular numbers are defined as,

$$t_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}, \text{ for } n \geq 0. \text{ Define } a_n = \sum_{i=0}^{n} t_i, \text{ for } n \geq 0.$$

Find a recurrence relation for a_n and solve it.

$$a_n = t_0 + t_1 + \cdots + t_n$$

$$a_n - a_{n-1} = t_n = \frac{n(n+1)}{2}$$

$$\Rightarrow a_n = a_{n-1} + \frac{1}{2}n^2 + \frac{1}{2}n$$

$$= \cdots$$

$$= \cdots = \frac{n(n+1)(n+2)}{6}$$

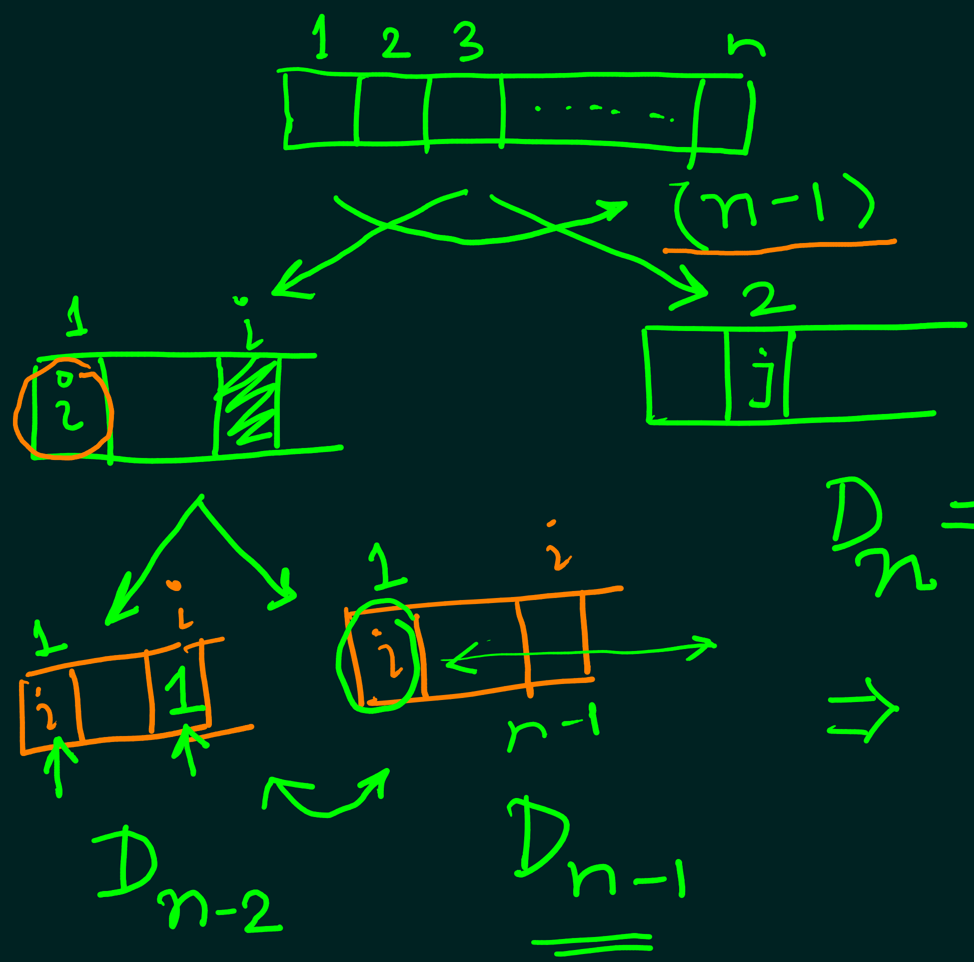
$$= {}^{n+2}C_3 \text{ !!}$$

Let $D_n, n \geq 1$, denote the number of derangements of $1, 2, 3, \ldots, n$. Present the recurrence for D_n and deduce that, $D_n = nD_{n-1} + (-1)^n$ for all $n \geq 3$. Finally, solve for D_n.

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 2 & 4 & 1 & 3 \end{array}$$

Derangement

$$\boxed{\begin{array}{l} D_1 = 0 \\ D_2 = 1 \end{array}}$$



$$D_n = \underline{(n-1)\left(D_{n-2} + D_{n-1}\right)}$$

$$\Rightarrow \quad D_n - nD_{n-1} = (-1)\left[D_{n-1} - (n-1)D_{n-2}\right]$$

$$\cdots\cdots$$

$$= (-1)^2\left[D_{n-2} - (n-3)D_{n-3}\right]$$

$$= \cdots$$

$$= (-1)^{n-2}\left[D_2 - 1\,D_1\right]$$

$$\boxed{D_n = nD_{n-1} + (-1)^n}$$

$T_n = \frac{D_n}{n!}$

$n!$ divide

Let $a_n, n \geq 1$, satisfy $a_1 = 1$, and $a_n = \begin{cases} 2a_{n-1}, & \text{if } n \text{ is odd} \\ 2a_{n-1} + 1, & \text{if } n \text{ is even} \end{cases}$, for $n \geq 2$. Develop a recurrence realtion for a_n that holds for both odd and even n, and solve it.

$\rightarrow a_1 = 1, \quad a_2 = 3, \quad a_3 = 6$

$$\underline{\left[\frac{1}{2}(-1)^n + \frac{1}{2}\right] + 2a_{n-1}} = a_n$$

$a_n^h = 2^n \cdot A$

$a_n^p = ((-1)^n B + 1^n C)$

▷ n is odd

$$a_n = 2a_{n-1} \longrightarrow a_n - a_{n-2} = 2[a_{n-1} - a_{n-3}]$$

▷ n is even

$$a_n = 2a_{n-1} + 1 \longrightarrow a_n - a_{n-2} = 2[a_{n-1} - a_{n-3}]$$

$$a_n - 2a_{n-1} - a_{n-2} + 2a_{n-3} = 0$$

$A = 5/6$

$B = 1/6$

$C = -1/2$

$$x^3 - 2x^2 - x + 2 = 0 \Rightarrow x = 2, -1, 1$$

$$\boxed{A2^n + B(-1)^n + C(1)^n}$$

Solve the recurrence relation, $a_n = na_{n-1} + n(n-1)a_{n-2} + n!$, for $n \geq 2$, with $a_0 = 0, a_1 = 1$.

$$\frac{a_n}{n!} = \frac{a_{n-1}}{(n-1)!} + \frac{a_{n-2}}{(n-2)!} + 1$$

Substitute $b_n = \frac{a_n}{n!} \Rightarrow \underline{b_n = b_{n-1}} + b_{n-2} + 1$

$$\boxed{\begin{aligned} b_0 &= ? = 0 \\ b_1 &= ? = 1 \end{aligned}}$$

1. $a_n^h = ?$ $\qquad a_n^p = ?$

2. constant elimination

$$a_n = n![b_n] = n!\left[\qquad \right]$$

Fill in the blank!

How many 'X's will be printed by the call $f(n)$ for an integer $n > 0$?

```
void f ( int n )   {
    int m;
    printf("X");
    m = n - 1;
    while ( m >= 0 ) {  f(m);  m = m - 2;  }
}
```

$f(n-1) + f(n-3) + f(n-5)$

$+ \cdots + f(1)$

or $f(0)$

$$f(n) = \begin{cases} 1 + \sum_{i=0}^{\frac{n-1}{2}} f(2i), & \text{when, } n = \text{odd} \\ 1 + \sum_{i=}^{\frac{n-2}{2}} f(2i+1), & \text{when } n = \text{even} \end{cases}$$

$f(n) - f(n-2) = f(n-1)$

$\hookrightarrow$ fibonaci

Solve the following divide-and-conquer recurrence: $T(n) = 2T(n/2) + \dfrac{n}{\log_2 n}$.

$$T(n) = 2\,T\left(\frac{n}{2}\right) + \frac{n}{\log_2 n}$$

$$\boxed{n = 2^k}$$

$$\Rightarrow \frac{T(n)}{n} = \frac{T(n/2)}{n/2} + \frac{1}{\log_2 n}$$

$$= \frac{T(n/2^2)}{n/2^2} + \frac{1}{\log(n/2)} + \frac{1}{\log n}$$

$$\boxed{T(1) = 1}$$

base

$$= \cdots$$

$$= \frac{T(n/2^k)}{n/2^k} + \sum_{i=0}^{k-1} \frac{1}{\log\left(\frac{n}{2^i}\right)}$$

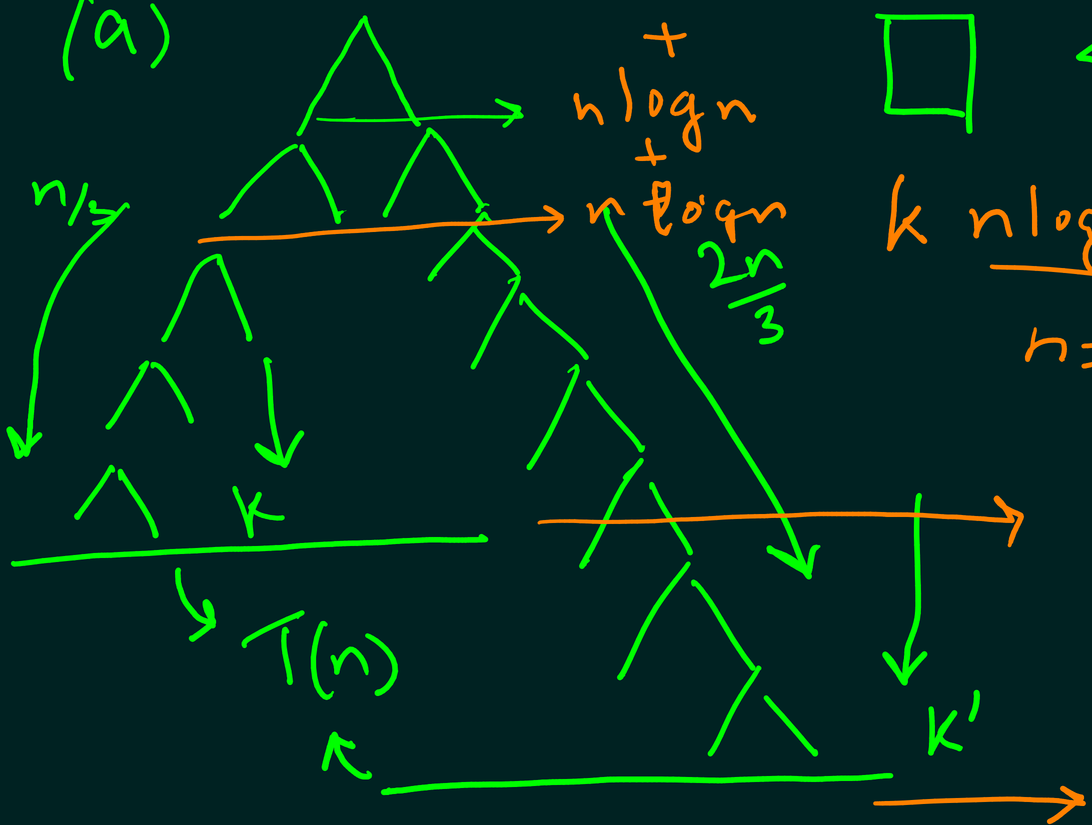
$$1$$

Deduce the running times of divide-and-conquer algorithms in the big-Θ notation if their running times satisfy the following recurrence relations.

(a) $T(n) = T(2n/3) + T(n/3) + n\log_2 n$

(b) $T(n) = T(n/5) + T(7n/10) + n$ ← Median finding

(a)



$\boxed{} \le T(n) \le \boxed{}$

$k\, n\log n$ $\qquad$ $k'\, n \log n$

$n = 3^k$ $\qquad$ $n = \left(\frac{3}{2}\right)^{k'}$

$n = 3^k$

$n = \left(\frac{3}{2}\right)^{k'}$

$n \to \infty$

$\Theta(n \log_2^2 n)$ ✓

(b) $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + n$ $\qquad \frac{1}{10}n$

n $\qquad$ c

x

$$cn + cn\left(1-\frac{1}{x}\right) + cn\left(1-\frac{1}{x}\right)^2 \dots$$

$O(n)$ $\qquad$ Kn $\qquad$ $= \underline{c'n}$

$O(\boxed{kn})$ ✓ $\qquad$ $n' < n \to T(n') = kn'$

$$T(n) \le k\frac{n}{5} + \frac{7kn}{10} + n = \underline{D}n$$

→ Substitution / Guess $\qquad$ $D \le \ge$

$\alpha+\beta+\gamma<1$

$$\boxed{T(n) = T(\alpha n) + T(\beta n) + T(\gamma n) + f(n)}$$