EXPERIMENT 3 (2)

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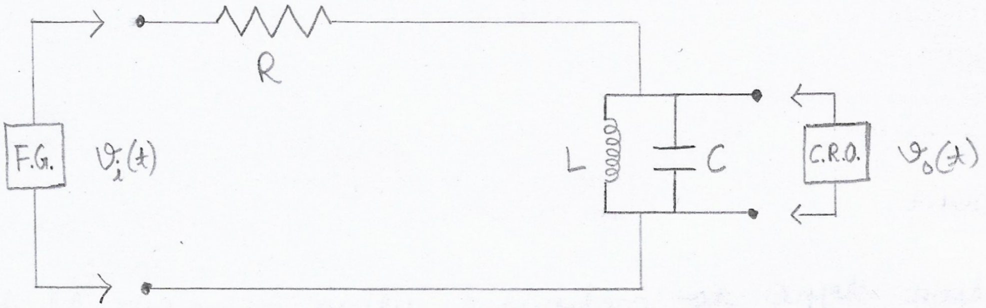
OBJECTIVE

Experimental verification of Fourier Coefficients of a square wave signal using passive network.

CIRCUIT DIAGRAM



THEORY

FOURIER THEOREM : A periodic signal can be written as a sum of sine and cosine waves.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \cos(n\omega t)$$

$$a_0 = \frac{1}{T}\int_{\langle T\rangle} x(t)\,dt$$

$$a_n = \frac{2}{T}\int_{\langle T\rangle} x(t)\cos(n\omega t)\,dt \quad \forall\ n = 1, 2, 3, \ldots$$

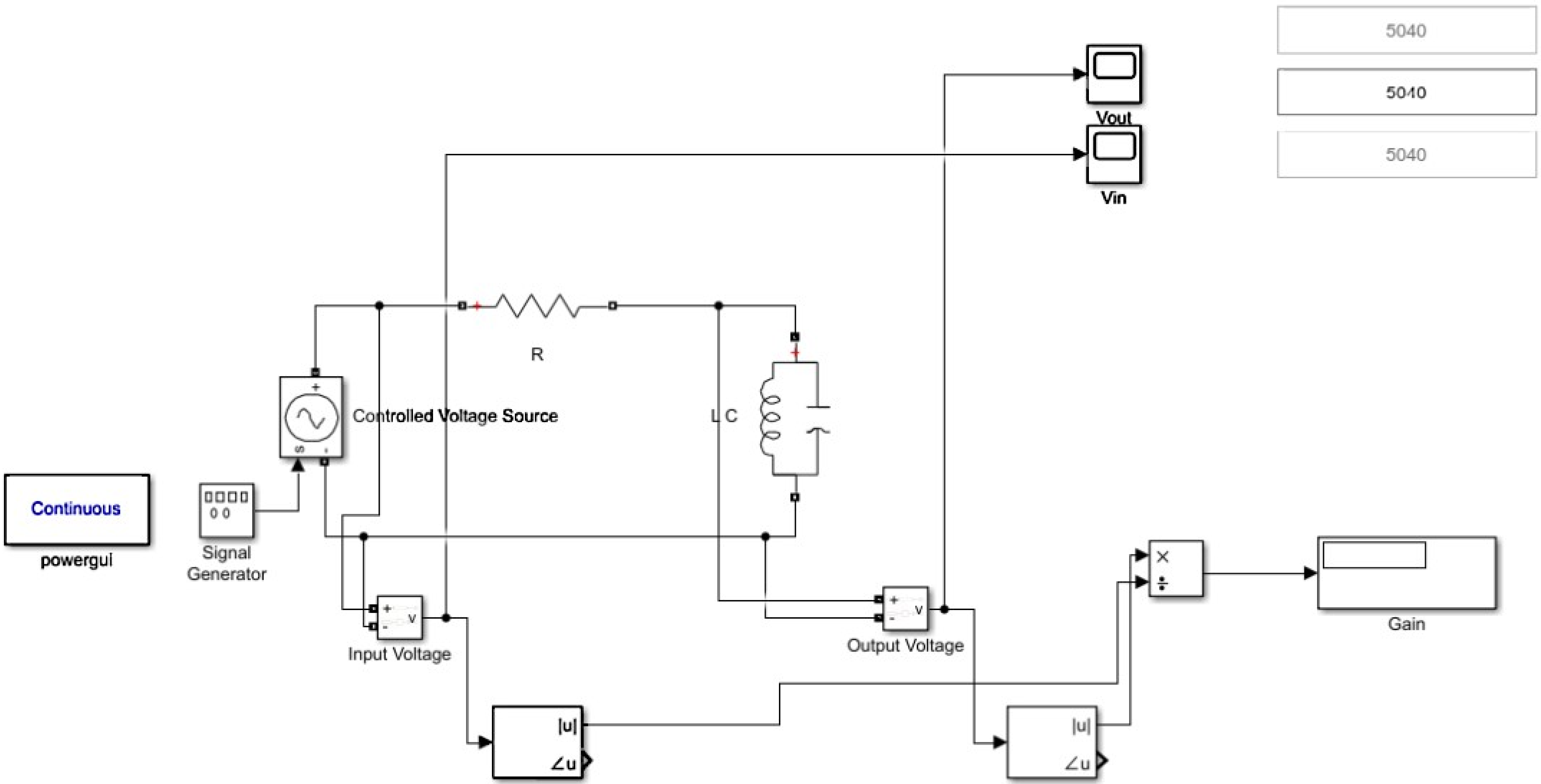
$$b_n = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin(n\omega t)\,dt \quad \forall\ n = 1, 2, 3, \ldots$$

If $x(t)$ is even, $b_n = 0 \ \forall n$.

If $x(t)$ is odd, $a_n = 0 \ \forall n$.

If $x(t)$ is half wave symmetric $a_0 = 0$

$a_n = b_n = 0 \quad \forall$ even n.



5040
5040
5040
Vout
Vin
R
Controlled Voltage Source
L C
Continuous
powergui
Signal Generator
Input Voltage
Output Voltage
Gain
|u|
∠u
|u|
∠u

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SIMULINK BLOCKS USED

1. Signal Generator
2. Controlled Voltage Source
3. Series RLC branch
4. Parallel RLC branch
5. Voltage measurement
6. Fourier
7. Divide
8. Add
9. Display
10. Scope
11. Edit
12. powerGUI

PART 1

SIMULINK PROCEDURE

1. Set the simulation type to continuous using powerGUI block
2. Set the solver to ode23t in model configuration parameters
3. Set the wave form to sine and amplitude to 3V in signal generator.
4. Set the frequency of signal generator and fourier blocks to 100Hz using edit block and run the simulation.
5. Measure V_{out} using scope and gain will be visible on display too.
6. Repeat steps 4-5 for different frequencies from 100Hz to 100 kHz (uniformly distributed).
7. Plot the approximate frequency response on a semi log plot using MATLAB and compare with theoretical plot.

CALCULATIONS

For the given circuit, $V_{out}(S) = I(s) \times \left(Ls \,||\, \frac{1}{Cs}\right)$

and $I(s) = \dfrac{V_{in}(S)}{R + \left(Ls \,||\, \frac{1}{Cs}\right)}$

$\therefore$, We have the transfer function $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)} = \dfrac{\left(Ls \,||\, \frac{1}{Cs}\right)}{R + \left(Ls \,||\, \frac{1}{Cs}\right)}$

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$$G(s) = \cfrac{\cfrac{Ls \cdot \frac{1}{Cs}}{Ls + \frac{1}{Cs}}}{R + \cfrac{Ls \cdot \frac{1}{Cs}}{Ls + \frac{1}{Cs}}}$$

$$G(s) = \cfrac{\frac{L}{C}}{RLs + \frac{R}{Cs} + \frac{L}{C}}$$

$$G(s) = \frac{Ls}{RLCs^2 + Ls + R}$$

For $L = 10\,mH,\ R = 100\,k\Omega,\ C = 100\,nF$

$$G(s) = \frac{10^{-2}s}{10^{-4}s^2 + 10^{-2}s + 10^5}$$

* OBSERVATION TABLE AND PLOTS ATTACHED BELOW

OBSERVATION TABLE

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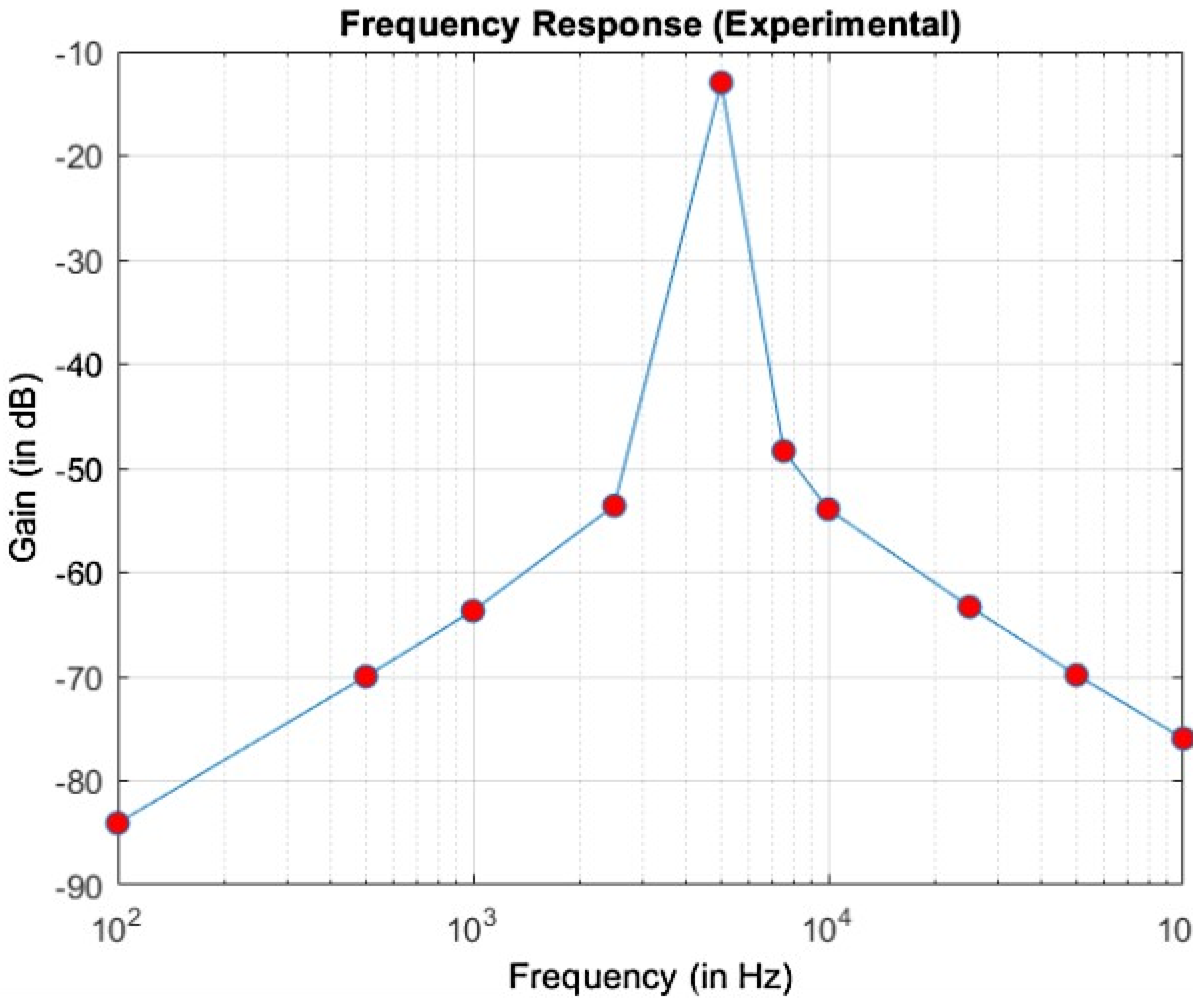
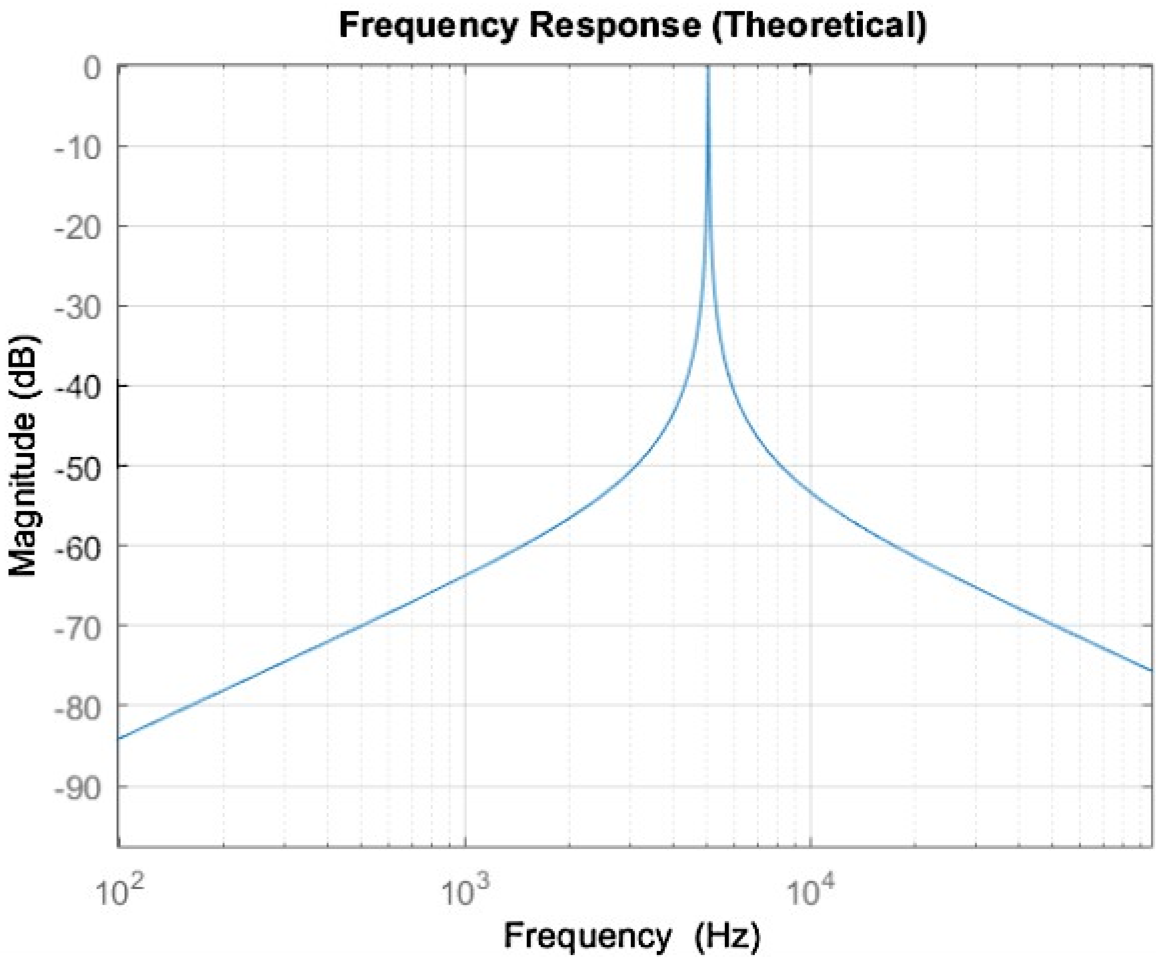
$V_{in} = 3V$

Frequency (Hz)	Output Amplitude (V)	Gain	
		Magnitude	Magnitude (in dB)
100	0.0002	0.0001	-84.0574
500	0.0010	0.0003	-69.9761
1000	0.0020	0.0007	-63.6858
2500	0.0063	0.0021	-53.5929
5000	0.6738	0.2246	-12.9718
7500	0.0115	0.0038	-48.3451
10000	0.0060	0.0020	-53.9231
25000	0.0021	0.0007	-63.2989
50000	0.0010	0.0003	-69.8483
100 000	0.0005	0.0002	-75.9393


Frequency Response (Experimental)
-10
-20
-30
-40
-50
-60
-70
-80
-90
Gain (in dB)
10^2
10^3
10^4
10^5
Frequency (in Hz)



Frequency Response (Theoretical)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Magnitude (dB)
10^2
10^3
10^4
Frequency (Hz)

PART 2

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SIMULINK PROCEDURE

1. Set the waveform to square and amplitude to 3V in signal generator.
2. Set the frequency near the resonance frequency of the circuit (i.e., around 5 kHz).
3. Fine tune the frequency using the edit block and run the simulation for different frequencies.
4. Using scope measure the amplitude of $v_o(t)$ and fine tune to get the frequency at which the amplitude becomes highest.
5. Repeat steps 3-4 for different combinations of L & C (keeping the resonance frequency same, i.e., LC = constant).
6. The highest amplitude of $v_o(t)$ is the fundamental fourier series coefficient.

OBSERVATION TABLE

Frequency (Hz)	Output Amplitude (V)		
	L=10mH, C=100nF	L=20mH, C=50nF	L=100mH, C=10nF
5000	0.940	1.594	3.560
5010	1.220	2.142	3.690
5020	1.880	2.868	3.760
5030	3.410	3.704	3.811
5032	3.689	3.780	3.814
5033	3.755	3.792	3.816
5034	3.757	3.794	3.820
5035	3.697	3.780	3.818
5036	3.590	3.753	3.810
5040	2.957	3.523	3.806

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CALCULATIONS

Frequency at which max amplitude occurs = 5034 Hz

Average max amplitude $(v_o(t))$ $= \dfrac{(3.757 + 3.794 + 3.820)}{3} = 3.790\ V$

This is the fundamental fourier series coefficient of the square wave signal. $\Rightarrow b_1$ (experimental) $= 3.79$

$$b_1 \text{ (theoretical)} = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin(\omega t)\,dt$$

$$= \frac{2}{T}\left[3\int_0^{T/2}\sin(\omega t)\,dt - 3\int_{T/2}^{T}\sin(\omega t)\,dt\right]$$

$$= \frac{2}{T}\left[6\int_0^{T/2}\sin(\omega t)\,dt\right]$$

$$= \frac{12}{T}\,\frac{\left[-\cos(\omega t)\right]_0^{T/2}}{\omega}$$

$$= \frac{12}{\left(\frac{2\pi}{\omega}\right)}\times\left(\frac{2}{\omega}\right) = \frac{12}{\pi} = 3.82$$

$\therefore$, % error $= \dfrac{|3.82 - 3.79|}{3.82}\times 100 = 0.79\%$

PART 3

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SIMULINK PROCEDURE

① To get the third harmonic fourier series coefficient fine tune the circuit as done in PART 2 but this time around one third the resonance frequency (i.e., around 1670 Hz).

② In this case the output amplitude varies periodically. When the amplitude of $v_o(t)$ is highest open its scope and turn on "cursor measurements". Zoom in to a suitable time interval and measure the amplitudes at consecutive maximas and minimas to get the average amplitude of $v_o(t)$.

③ This is the third harmonic fourier series coefficient.

④ Repeat the above steps around one fifth the resonance frequency (i.e., around 1kHz) to get the fifth harmonic coefficient.

OBSERVATION TABLE

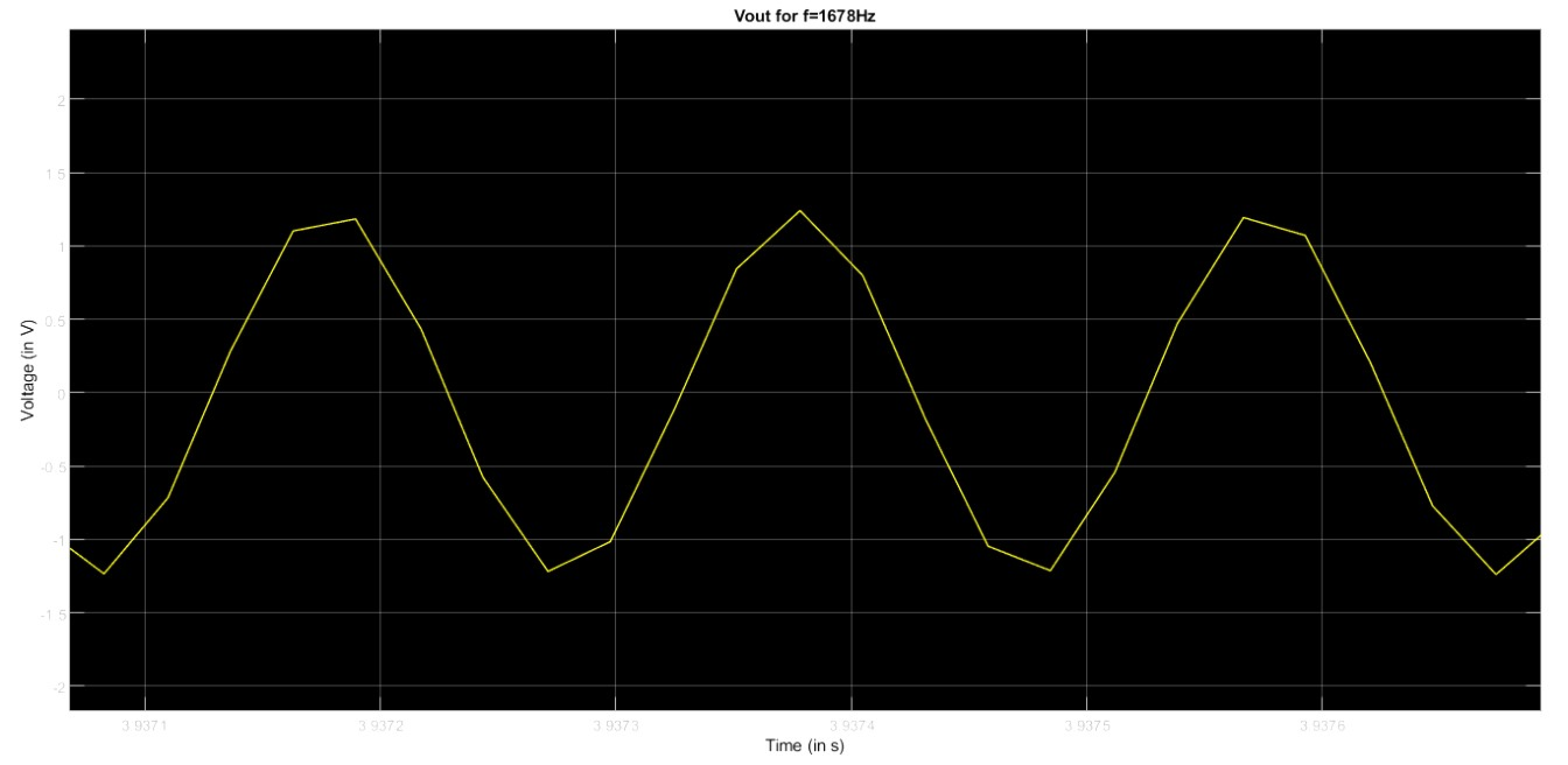
FOR 3rd HARMONIC

Frequency (Hz)	Output Amplitude (V)
1660	0.21
1670	0.41
1675	0.90
1676	1.02
1677	1.19
1678	1.24
1679	1.17
1680	1.00
1690	0.29

At f = 1678 Hz

Sr. No.	Amplitude (V) MAXIMA	MINIMA	PEAK TO PEAK (MAX − MIN)
1	1.172	−1.215	2.387
2	1.245	−1.204	2.449
3	1.187	−1.240	2.427

* PLOT ATTACHED BELOW


Vout for f=1678Hz
Voltage (in V)
Time (in s)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
3.9371
3.9372
3.9373
3.9374
3.9375
3.9376

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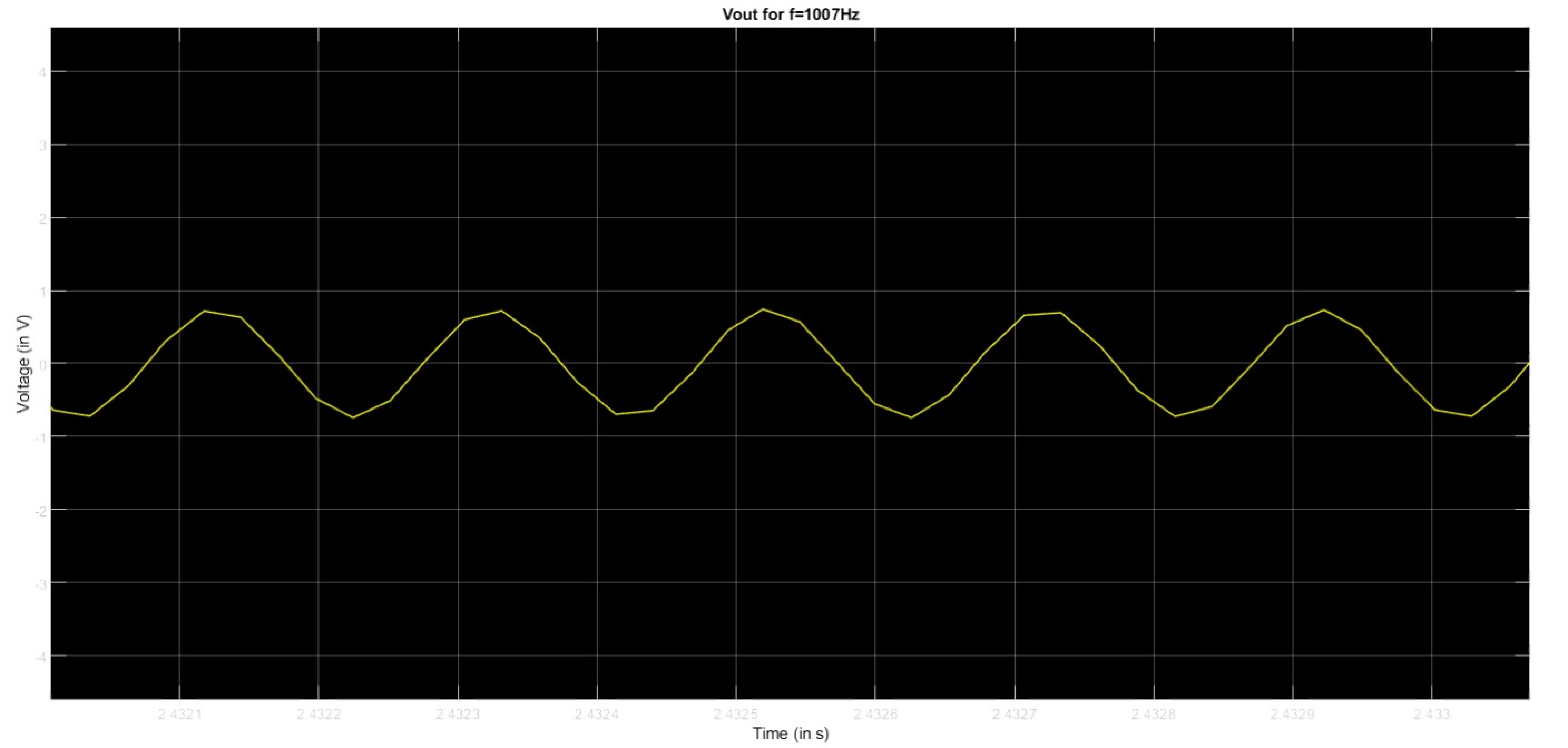
FOR 5^{th} HARMONIC

Frequency (Hz)	Output Amplitude (V)
1000	0.26
1005	0.56
1006	0.69
1007	0.74
1008	0.63
1009	0.45
1010	0.42
1020	0.09

At f = 1007 Hz

Sr. No.	Amplitude (V) MAXIMA	MINIMA	PEAK TO PEAK (MAX-MIN)
1	0.715	-0.738	1.453
2	0.716	-0.692	1.408
3	0.743	-0.725	1.468
4	0.689	-0.721	1.410
5	0.725	-0.719	1.444

* PLOT ATTACHED BELOW



Vout for f=1007Hz
Voltage (in V)
Time (in s)
4
3
2
1
0
-1
-2
-3
-4
2.4321
2.4322
2.4323
2.4324
2.4325
2.4326
2.4327
2.4328
2.4329
2.433

CALCULATIONS

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(1) 3rd HARMONIC

Frequency at which max amplitude occurs = 1678 Hz

Average max amplitude (PEAK-TO-PEAK) = $\frac{(2.387 + 2.449 + 2.427)}{3} = 2.421$

Average max amplitude = $\frac{\text{PEAK-TO-PEAK}}{2} = \frac{2.421}{2} = 1.211$

This is the third harmonic fourier series coefficient of the square wave signal. $\Rightarrow$ b_3(experimental) = 1.21

$$b_3(\text{theoretical}) = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin(3\omega t)\,dt$$

$$= \frac{2}{T}\left[3\int_0^{\frac{T}{2}}\sin(3\omega t)\,dt - 3\int_{\frac{T}{2}}^{T}\sin(3\omega t)\,dt\right]$$

$$= \frac{2}{T}\left[6\int_0^{\frac{T}{2}}\sin(3\omega t)\,dt\right]$$

$$= \frac{12}{T}\,\frac{\left[-\cos(3\omega t)\right]_0^{\frac{T}{2}}}{3\omega}$$

$$= \frac{4}{\left(\frac{2\pi}{\omega}\right)} \times \left(\frac{2}{\omega}\right) = \frac{4}{\pi} = 1.27$$

$\therefore$, % error = $\frac{|1.27 - 1.21|}{1.27} \times 100 = 4.72\ \%$

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(2) 5^{th} HARMONIC

Frequency at which max amplitude occurs = 1007 Hz

Average max amplitude (PEAK-TO-PEAK) = $\frac{(1.453+1.408+1.468+1.410+1.444)}{5}$

= 1.437

Average max amplitude = $\frac{\text{PEAK-TO-PEAK}}{2} = \frac{1.437}{2} = 0.718$

This is the fifth harmonic fourier series coefficient of the square wave signal. ⟹ b_5(experimental) = 0.72.

$$b_5(\text{theoretical}) = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin(5\omega t)\,dt$$

$$= \frac{2}{T}\left[3\int_0^{T/2}\sin(5\omega t) - 3\int_{T/2}^{T}\sin(5\omega t)\,dt\right]$$

$$= \frac{2}{T}\left[6\int_0^{T/2}\sin(5\omega t)\,dt\right]$$

$$= \frac{12}{T}\,\frac{\left[-\cos(5\omega t)\right]_0^{T/2}}{5\omega}$$

$$= \frac{12}{\left(\frac{2\pi}{\omega}\right)} \times \left(\frac{2}{5\omega}\right) = \frac{12}{5\pi} = 0.76$$

$$\%\ \text{error} = \frac{|0.76-0.72|}{0.76} \times 100 = 5.26\%$$

DISCUSSION

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Experimentally there will always be error in the coefficients obtained and their theoretical values because even after fine tuning the circuit there will be a small difference between the input frequency and the resonating frequency of the circuit. Also it is very difficult to generate an ideal square wave signal. The transition from +ve to -ve amplitude in reality always takes a small finite amount of time.

As the experiment has been carried out through simulation the circuit elements are ideal and the signal generator generates a stable input waveform which is not the case in the real experiment in lab. However, even in the simulation care should be taken that the time step is set according to the frequency at which we are operating to get accurate readings.

Fourier series have a wide range of applications in field such as signal processing, approximation theory (used to approximate signals / functions as polynomials of sine and cosine) and also for audio compression. The fact that fourier series converge quite fast helps in storing data of signals in a finite and compact memory space.