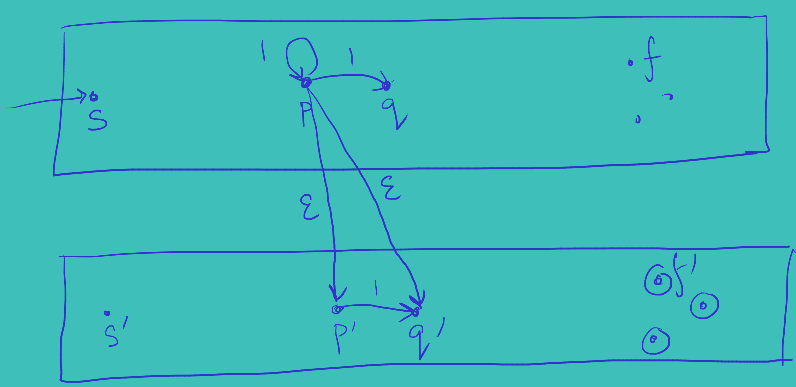
Tutorial

Q1. Let A be a regular set over $\{0,1\}$.
Show that
$L_1 = \{xy \mid x1y \in A\}$ is also regular.

Soln: Let M be a DFA for A

Create NFA N for L_1 :



Copy 1 of M

Copy 2 of M

Transition Δ_N: $\Delta_N(p, a) = \{\delta_M(p,a)\}$ $\forall a \neq 1$. $p \in$ Copy 1 of M

$\Delta_N(p, 1) = \{\delta_M(p,1), \delta_M(p,1)'\}$ where if $q = \delta_M(p,1)$, $\delta_M(p,1)' = q'$ in Copy 2 of M.

$F_N = \{f' \mid f \in F_M\}$

xy s.t $x1y \in A \iff \exists$ path labelled $x\varepsilon y$ starting at s and ending in F_N.

Q2. Let A be regular. DFA $\underline{M}$

Show that

(a) $L_2 = \{\underline{w} \mid \underline{ww} \in A\}$ is regular

(b) $L_3 = \{w \mid \exists n, z = w^n \in A\}$ is regular

(a) Fix a $q \in Q_M$. $L_2^q = \{w \mid ww \in A, \hat{\delta}(s,w) = q, \hat{\delta}(q,w) \in F\}$

DFA for L_2^q : $Q' = Q_M \times Q_M$

$s' = (s, q)$

$F' = \{(q, f) \mid f \in F\}$

$\delta' : \delta'((p_1, p_2), a) = (\delta(p_1, a), \delta(p_2, a))$.

$L_2 = \bigcup_{q \in Q} L_2^q$. Regular sets closed under finite unions.

(b) ⊙ $L_c = \{w \mid w^c \in A\}$ is regular, c a constant.

[Similar arguments as L_2 being regular]

⊙ $L_3' = \{w \mid \exists n \le k^2, w^n \in A\}$ is regular where $|Q_M| = k$.

[$L_3' = \bigcup_{c=1}^{k^2} L_c$, k is a constant]

⊙ Claim: $L_3 = L_3'$.

To show if $\exists n : w^n \in A$ then $\exists n_0 \le k^2$ s.t $w^{n_0} \in A$.

Hint: (i) Consider path $\underset{s}{\circ} \xrightarrow{z_1} \underset{p}{\circ} \xrightarrow{w} \underset{q}{\circ} \xrightarrow{z_2} \underset{p}{\circ} \xrightarrow{w} \underset{q}{\circ} \xrightarrow{z_3} \underset{f \in F}{\circ}$

Then $z = z_1 w z_2 w z_3 \in A \Rightarrow z_1 w z_3 \in A$.

(ii) Use pigeonhole principle.

Q4. Prove or disprove:

(a) $(0+1)^* \equiv 0^* + 1^*$

(b) $(0^*1^*)^* \equiv (0^*1)^*$.

(a) False. $01 \in$ LHS, not RHS

(b) False. $0000 \in$ LHS.
Any string in LHS that ends with 0
cannot belong to RHS.

Q5. $\alpha = (a+b)^* ab (a+b)^*$.

Give a regular expression equivalent to $\sim\alpha$ if

(a) $\Sigma = \{a, b\}$

(b) $\Sigma = \{a, b, c\}$

(a) $\Sigma^* - L(\alpha)$: all strings where b's appear before a's

Reg. exp = $b^* a^*$

(b) $\Sigma^* - L(\alpha)$: all strings with at least one c

or

strings where b's appear before a's

Reg. exp = $(a+b+c)^* c (a+b+c)^* + b^* a^*$.