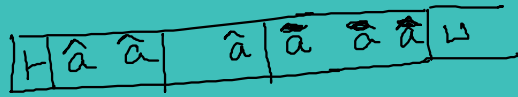
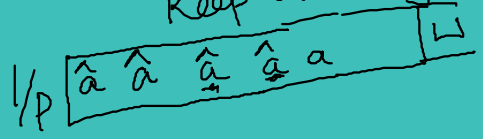


Q1. Design a TM accepting $L_1 = \{a^{2^n} \mid n \geq 0\}$
 \downarrow keep doubling

Keep halving



Keep doubling



Q2. Design a TM for accepting

$$L_2 = \{a^m \# a^n \mid n = 0 \bmod m\}$$

$$a^m \# a^n$$

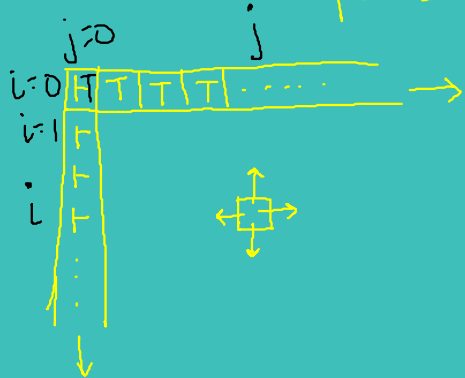
$$\rightarrow n=0 \text{ or}$$

$$\rightarrow m \leq n.$$

Mark a^m and a^n to see if $m > n$ or $m \leq n$.

If $m \leq n$, [Mark first m a's after the # \leftarrow
Continue (after #) to mark sets of m a's.
If at any point in the middle we encounter \sqcup , then reject. \leftarrow
O/w, check if there is an a after # unmarked.
If there is, continue loop.
If not, accept.

3. Show that a 2D Tape 1-head TM is equivalent to a TM.



$\{L, R, T, D\}$

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, T, D\}$$

(\Leftarrow) TM \Rightarrow implement 2D Tape 1-head TM.

(\Rightarrow) 2D Tape 1-head TM \Rightarrow $2^i(2j-1)$

bijection $\leftarrow f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$
 $(i, j) \rightarrow f(i, j)$

$$\begin{cases} f(i, j) = 2^i 3^j \\ \text{linear in } i, j \\ \text{quadratic in } i, j \end{cases}$$



T1 α $f(i, j)$

T2 Row - - -

T3 Column

T4 $f(i_2, j_2) - f(i_1, j_1)$

	$f(i_2, j_2) - f(i_1, j_1)$	
T	$3^j 2^{i+1}$	$i_2 = i+1, j_2 = j$
D		
L		
R		

4. Design an NTM for $L = \{u\underline{x}v\underline{x}y \mid u, x, v, y \in \{0,1\}^*, \underline{|x| \geq 100}\}$

Nondet choice for when x starts.

Copying x to 2nd tape till next 100 positions (reject if not possible)

Nondet. choice to keep going with copying or to stop.

Nondet. choice where v stops.

Check if 2nd version of x matches.

5. Give an unrestricted grammar for $\{a^i b^j c^k d^l \mid i=k, j=l\}$.

Generate equal number of a's and c's.
equal " " b's and d's.

Shifting of b's & c's.

aUc
 $\hookrightarrow bVd$

$Dc \rightarrow cD$

$S \rightarrow UV$

$U \rightarrow \varepsilon \mid aUC$

$V \rightarrow T \mid bVd$

$Cb \rightarrow bC$

$cT \rightarrow Tc$

$T \rightarrow \varepsilon$.

$aaa \overset{\sim}{ccc} bbTdd$.

$aaa bbcccTdd$

$aaa bbccTcdd$

$aaa bbTcccdd$.

$aaabbbccdd$.