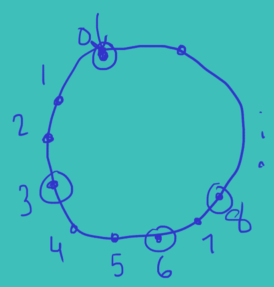
Tutorial

Q1. $L_1 = \{x \in \{a,b\}^* \mid |x| \text{ is divisible by either 3 or 8}\}$

Construct a DFA for L_1.

Method 1: Using Product construction

Method 2:



: 24 states

. All states that are 0 mod 3 or 0 mod 8 are in F.

$24 = \text{lcm}(3,8)$.

Any string divisible by 3 or 8 with length l:

$l = 24l_1 + l_2$ will be such that

$l_2 = 0 \bmod 3$ or $0 \bmod 8$

In the DFA, upon reading the first $24l_1$ alphabets the DFA is in state 0;
On reading the final l_2 alphabets it reaches a state that is 0 mod 3 or 0 mod 8.
$\Rightarrow$ final states of DFA.

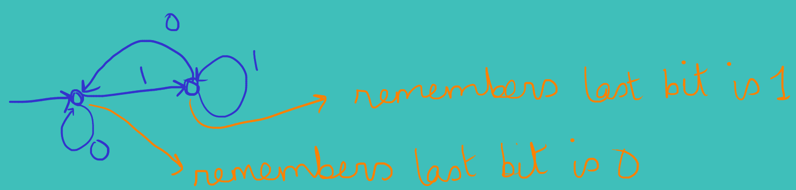
k^{th} last bit of x is 1

Q2. $L_2 = \{ x \in \{0,1\}^* \mid 5^{th}$ last bit of x is $1 \}$

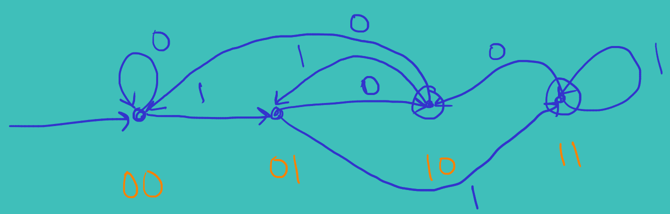
Eg: 011111, 10000

Construct a DFA for L_2.

First $L_2' = \{ x \in \{0,1\}^* \mid$ last bit is $1 \}$



$L_2'' = \{ x \in \{0,1\}^* \mid 2^{nd}$ last bit is $1 \}$



Now construct DFA for L_2 using similar logic

Follow-up Qn: For $L_k = \{ x \in \{0,1\}^* \mid k^{th}$ last bit is $1 \}$

(i) No DFA with $< 2^k$ states can accept L_k.

(ii) No NFA with $< k+1$ states can accept L_k.

Hint: Pigeonhole Principle

Q3. $L_3 = \{x \in \{0,1,\ldots,p-1\}^* \mid \#_p x$ is not divisible by $k\}$;

$p, k \in \mathbb{N}$, $\#_p x$ = the decimal number whose p-ary representation is x.

Construct a DFA for L_3.

Construct DFA for

$\overline{L_3} = \{x \in \{0, \ldots p-1\}^* \mid \#_p x$ is div by $k\}$.

$\overline{L_3}$

$M = (Q, \{0, \ldots p-1\}, \delta, s, F)$ $\quad \#_p \epsilon = 0$

$s = 0$ $\quad F = \{0\}$

$Q = \underbrace{0, \ldots \boxed{k-1}}$

$\delta : Q \times \Sigma \to Q$.

$\#_p xc = p\#_p x + c$

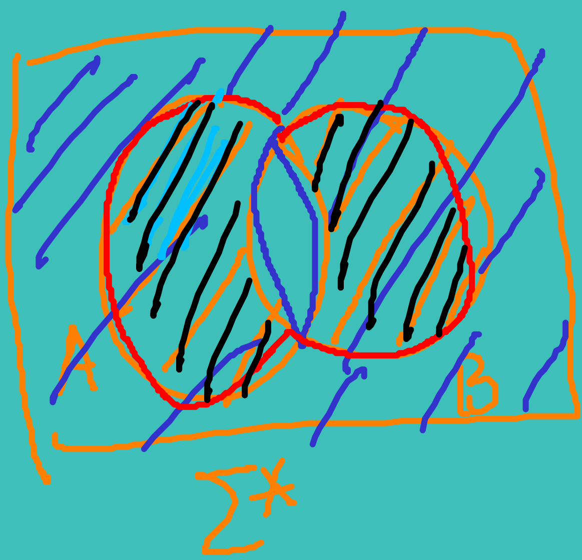
$\#_p xc \bmod k = (p\#_p x + c) \bmod k$

$\delta(q, c) = (pq + c) \bmod k$.

Q4. For two sets $A, B \subseteq \Sigma^*$,

$$A \Delta B = \underline{A \setminus B} \cup \underline{B \setminus A} \qquad \underbrace{\underline{A} \cap \underline{\overline{B}}}$$

If A and B are regular sets over Σ, show that so is $\underbrace{A \Delta B}$.



$(A \cap \neg B) \cup (B \cap \neg A)$
OR

$$\underbrace{\neg(\underbrace{A \cap B}) \cap (\underbrace{A \cup B})}$$

$$\underbrace{\neg(\neg A \cap \neg B)}$$

Use Product Construction

Q5. Given a regular set $A \subseteq \Sigma^*$, show that

$$L_5 = \{ x \in \Sigma^* \mid \exists y \text{ s.t } xy \in A \text{ and } |x| = |y| \}$$

is a regular set.

[Hint: Can you construct an NFA?]

Soln 1:

$\underbrace{Z}{} = \underbrace{x}_{A}\underbrace{y}_{B}$

$\underbrace{xy} \in A \subseteq \Sigma^*$

$\underbrace{|x| = |y|}$

$\boxed{\bigcup_{q \in Q} L_5^q}$

NFA

$A = L(M)$

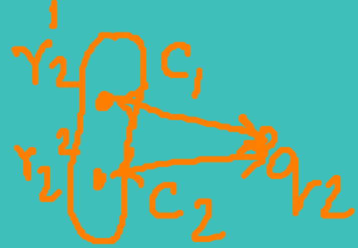
$(Q, \Sigma, \delta, s, F)$ M_1

$\exists y \; xy \in A$ NFA M_2

$\underbrace{q \in Q}$

$L_5^q = \{ x \in \Sigma^* \mid \exists y, xy \in A, |x| = |y|,$

$\delta(s, x) = q, \underbrace{\delta(q, y) \in F} \}$



$N = (Q \times Q, \Sigma, \Delta, \hat{S}, \hat{F})$

$\{ \underbrace{(s, p)} \mid s \leftarrow$ start state in M, $p \in F \} \leftarrow \hat{S}$

$\Delta \leftarrow \Delta(\underbrace{(q_1, q_2)}, \underline{a}) = \{ (r_1, r_2) \mid \underbrace{\delta(q_1, a) = r_1},$

$\exists c \in \Sigma, \delta(r_2, c) = q_2 \}$

$\hat{F} = \{ (q, q) \} \leftarrow$

Soln 2:

$N' = (Q \times Q, \Sigma, \Delta, \hat{S}, \hat{F}')$ for L_5

as in Soln. 1.

$\hat{S}, \Delta,$

$\rightarrow \hat{F}' = \{(q,q) \mid q \in Q\}$

$\underline{L(N')} = L(M)$