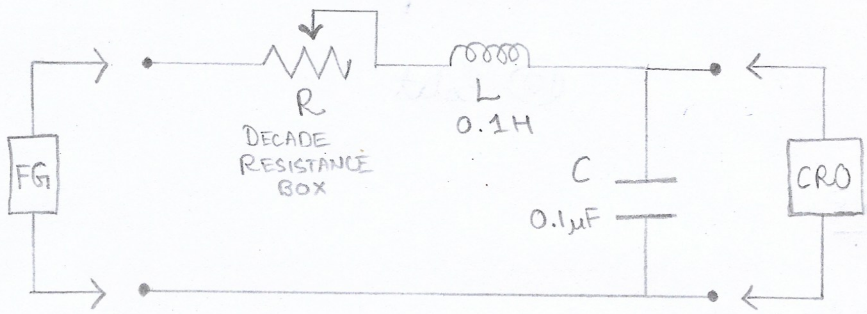
NISARG UPADHYAYA
19CS30031

EXPERIMENT 3 (1)

OBJECTIVE

To study the frequency response of series R-L-C circuit.

CIRCUIT DIAGRAM



THEORY

The standard Laplace-domain transfer function is

$$H(s) = \frac{1}{\left(\frac{1}{\omega_n^2}\right)s^2 + \left(\frac{2\xi}{\omega_n}\right)s + 1} \quad - (I)$$

The transfer function for the given circuit is

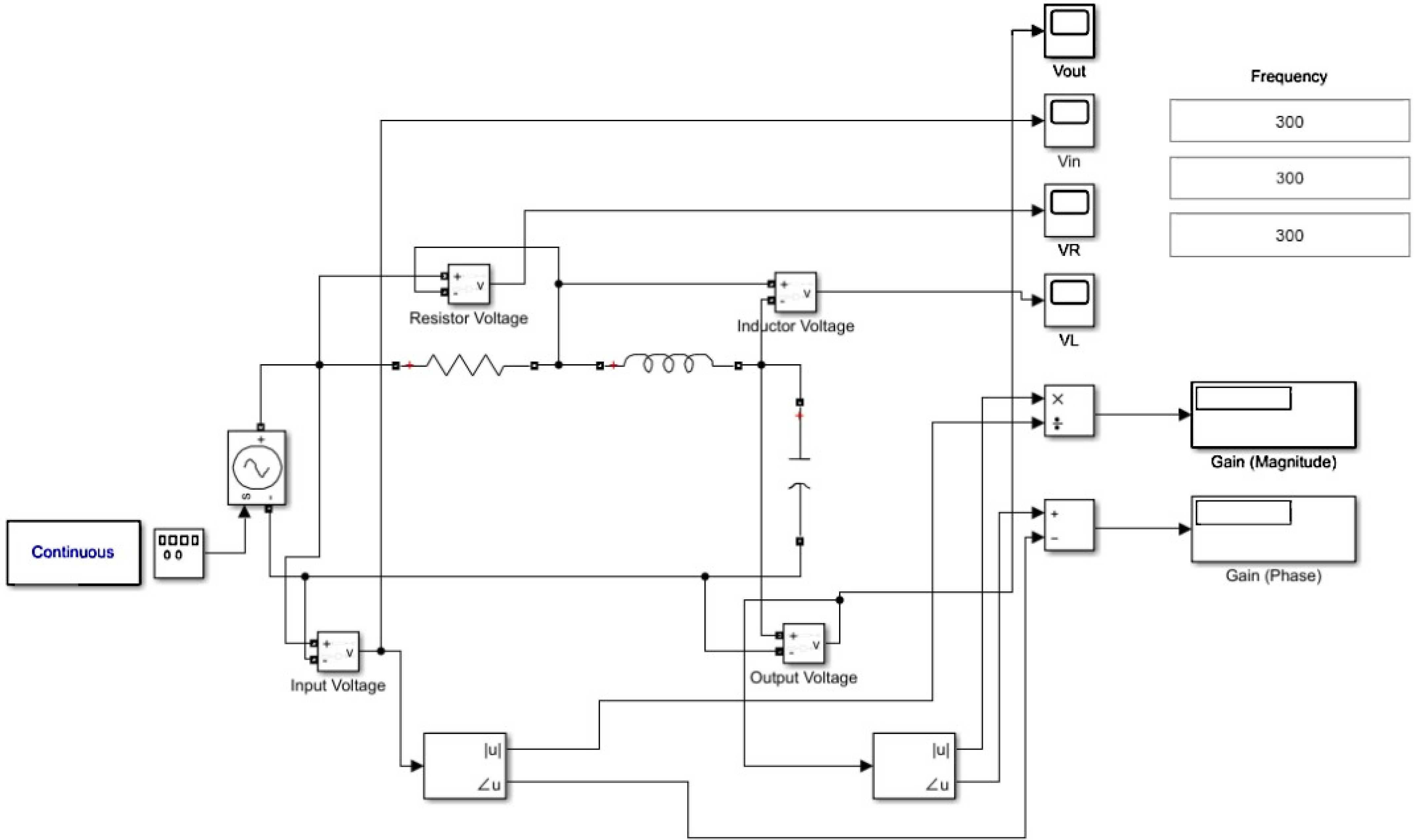
$$H(s) = \frac{1}{LCs^2 + RCs + 1} \quad - (II)$$

Comparing (I) & (II) we get

$$\omega_n = \frac{1}{\sqrt{LC}} \quad \text{and} \quad 2\xi\omega_n = \frac{R}{L}$$

Knowing L, C and ξ we can find R.

The maximum magnitude of gain is $\frac{1}{2\xi\sqrt{1-\xi^2}}$ and it occurs at $\omega = \omega_n\sqrt{1-2\xi^2}$



Vout
Vin
VR
VL
Frequency
300
300
300
Resistor Voltage
Inductor Voltage
Gain (Magnitude)
Gain (Phase)
Continuous
Input Voltage
Output Voltage
|u|
∠u
|u|
∠u

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SIMULINK BLOCKS USED

1. Signal Generator
2. Controlled Voltage Source
3. Series RLC branch
4. Voltage Measurement
5. Fourier
6. Divide
7. Add
8. Display
9. Scope
10. Edit
11. powerGUI

SIMULINK PROCEDURE

1. Set the simulation type to continuous using powerGUI block.
2. Set the solver to ode23t in model configuration parameters.
3. Set the wave form to sine and amplitude to 5V in signal generator.
4. Set $R = 400\Omega$ $(\xi = 0.2)$
5. Using the edit blocks set the frequency of the signal generator and the fourier blocks. Make sure that frequency of fourier blocks is same as that of signal generator.
6. Run the simulation. The gain (magnitude and phase) can be seen on the display blocks. Measure V_R, V_L and V_C by opening their respective scopes and turning on signal statistics.
7. Repeat step 6 for different values of frequency in (100-10k) Hz range (equally/uniformly distributed).
8. Repeat step 4-7 for $\xi = 0.7, 0.95$ and 1.5.
9. Plot Gain (magnitude and phase) vs Frequency (Hz) on a semilog graph plot using MATLAB.

OBSERVATION TABLES

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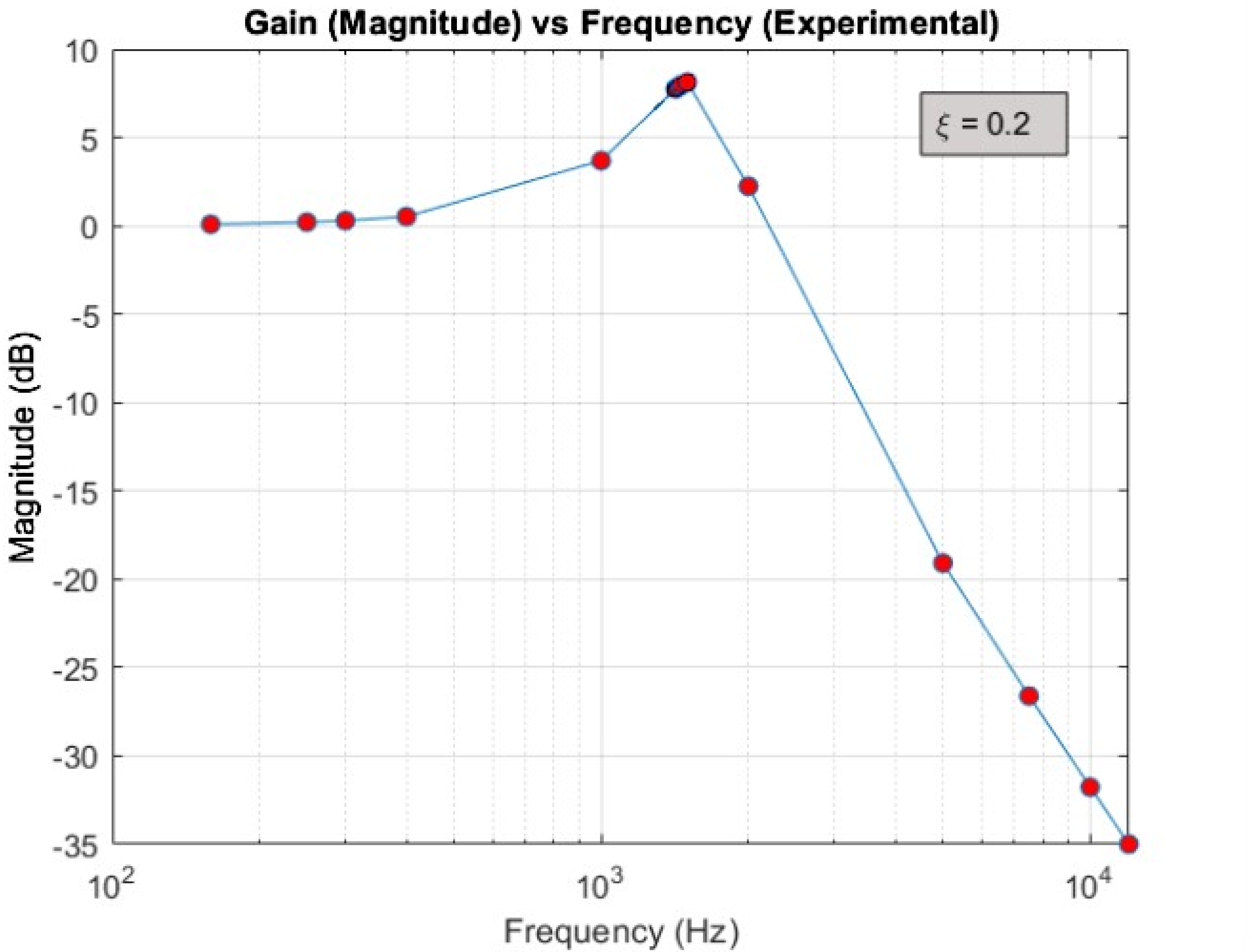
① $\xi = 0.2$ ($R = 400\ \Omega$)

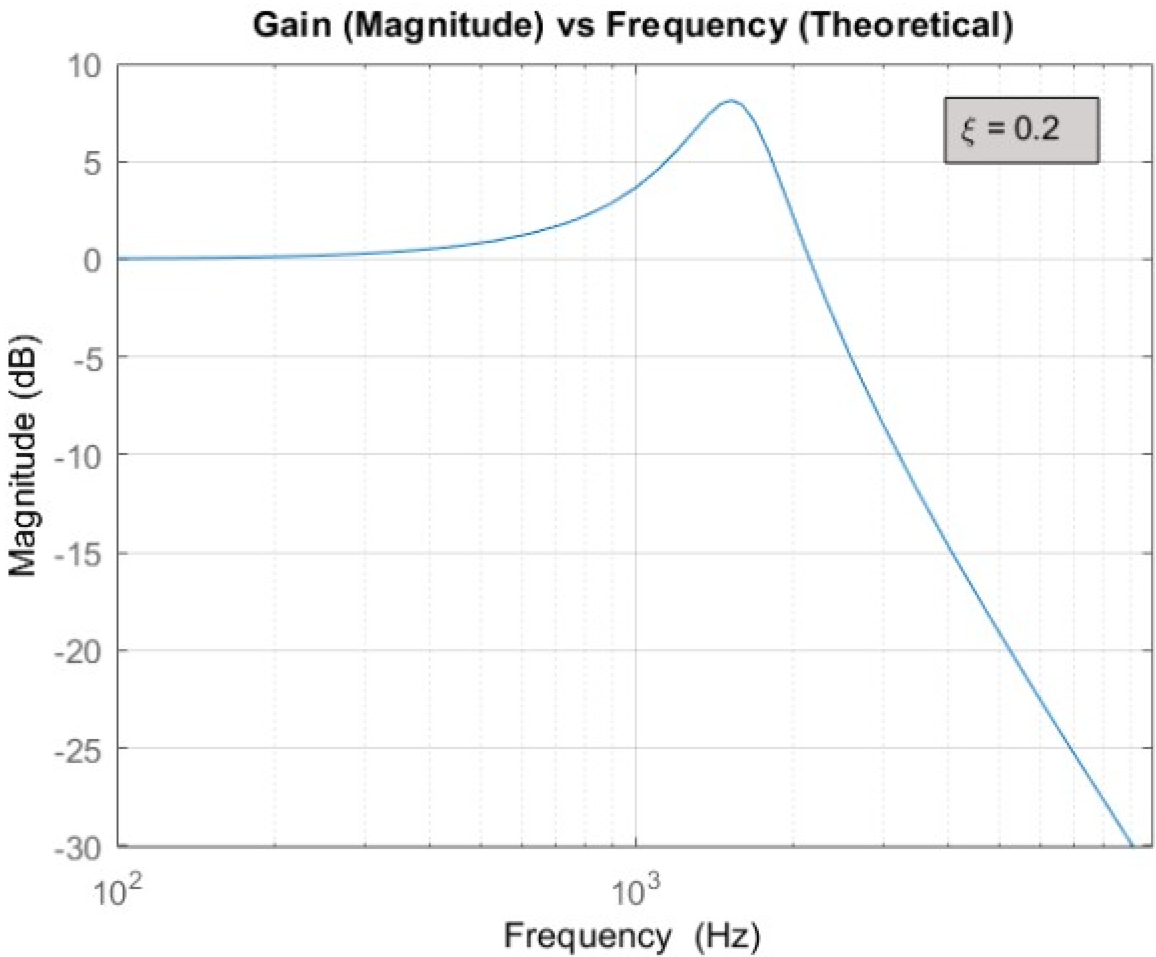
$V_{in} = 5V$

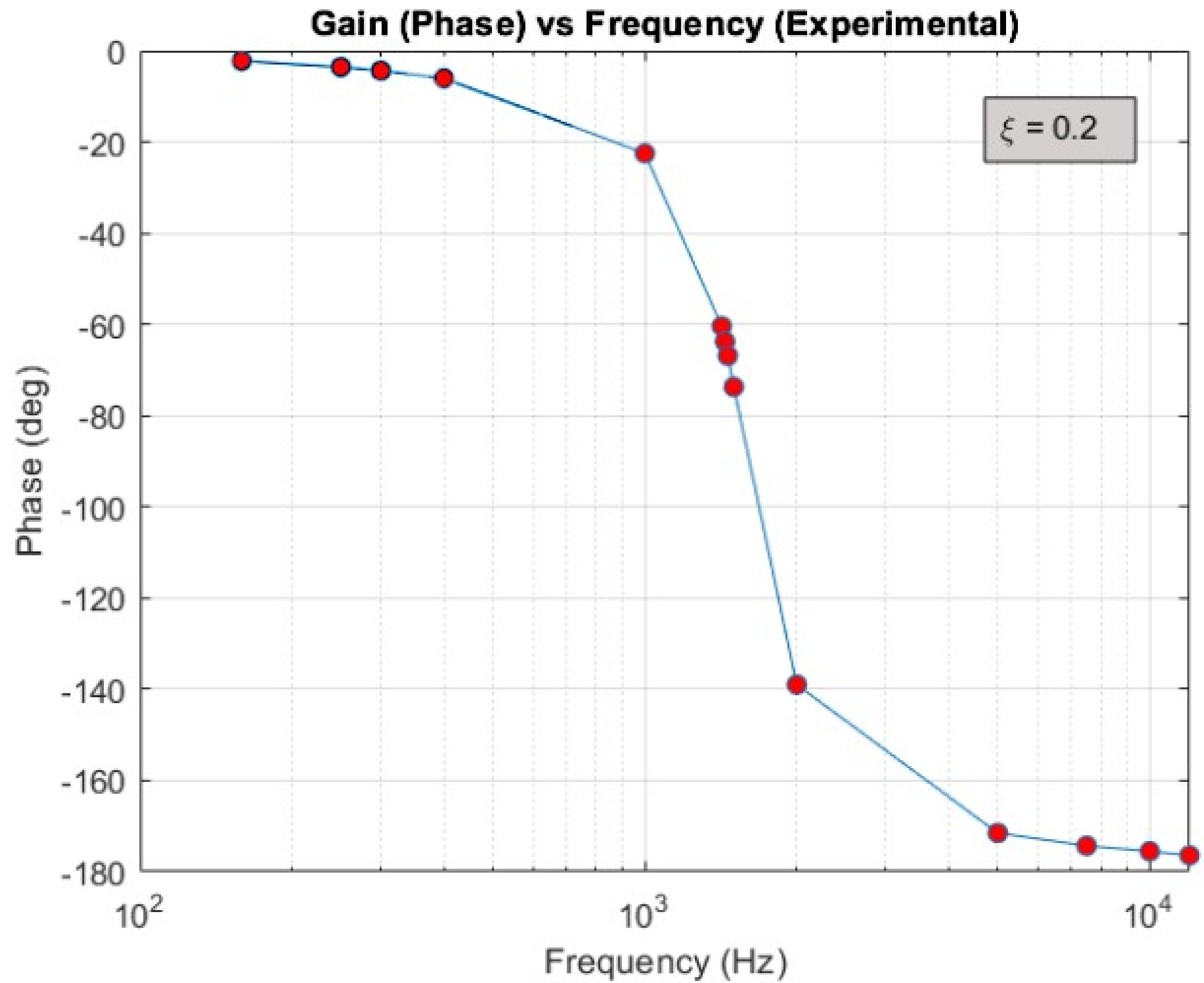
Frequency (Hz)	V_C (V)	V_L (V)	V_R (V)	Gain (magnitude) (V_C/V_{in})	Gain (phase) (degrees)
159	5.041	0.050	0.202	1.010	−2.224
250	5.116	0.127	0.322	1.024	−3.588
300	5.168	0.184	0.390	1.034	−4.375
400	5.305	0.335	0.533	1.062	−6.046
1000	7.643	3.029	1.924	1.529	−22.500
1420	12.190	9.739	4.358	2.441	−60.410
1441	12.380	10.190	4.493	2.480	−63.720
1460	12.530	10.580	4.606	2.509	−66.850
1500	12.720	11.350	4.807	2.549	−73.720
2000	6.463	10.260	3.257	1.294	−139.000
5000	0.554	5.505	0.698	0.111	−171.500
7500	0.233	5.213	0.441	0.047	−174.300
10000	0.128	5.118	0.324	0.026	−175.500
12000	0.088	5.072	0.268	0.018	−176.400

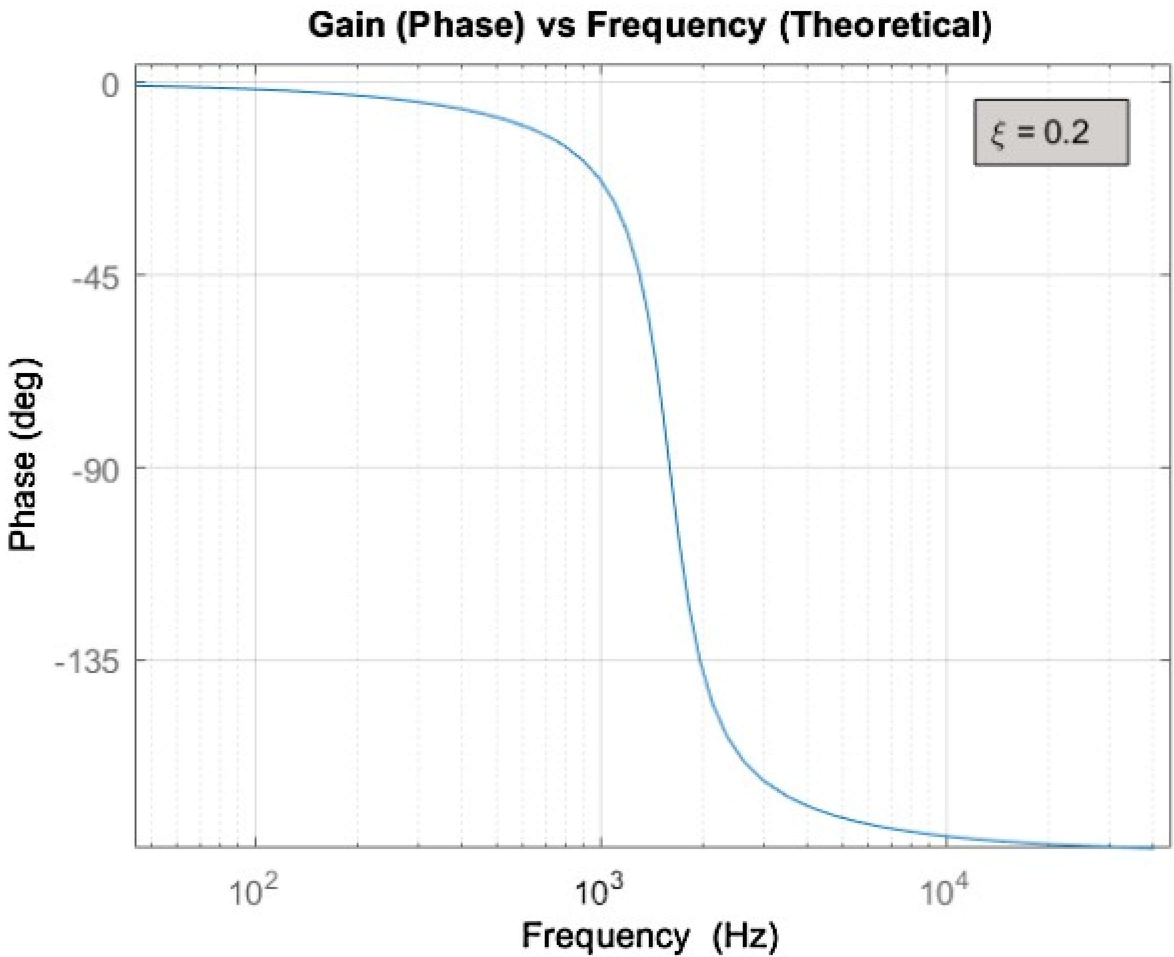
* EXPERIMENTAL AND THEORETICAL FREQUENCY RESPONSE PLOTS ATTACHED BELOW



Gain (Magnitude) vs Frequency (Experimental)
10
5
0
-5
-10
-15
-20
-25
-30
-35
Magnitude (dB)
10^2
10^3
10^4
Frequency (Hz)
$\xi = 0.2$


Gain (Magnitude) vs Frequency (Theoretical)
ξ = 0.2
10
5
0
-5
-10
-15
-20
-25
-30
Magnitude (dB)
10^2
10^3
Frequency (Hz)


Gain (Phase) vs Frequency (Experimental)
ξ = 0.2
Phase (deg)
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
10^2
10^3
10^4
Frequency (Hz)


Gain (Phase) vs Frequency (Theoretical)
ξ = 0.2
Phase (deg)
0
-45
-90
-135
10^2
10^3
10^4
Frequency (Hz)

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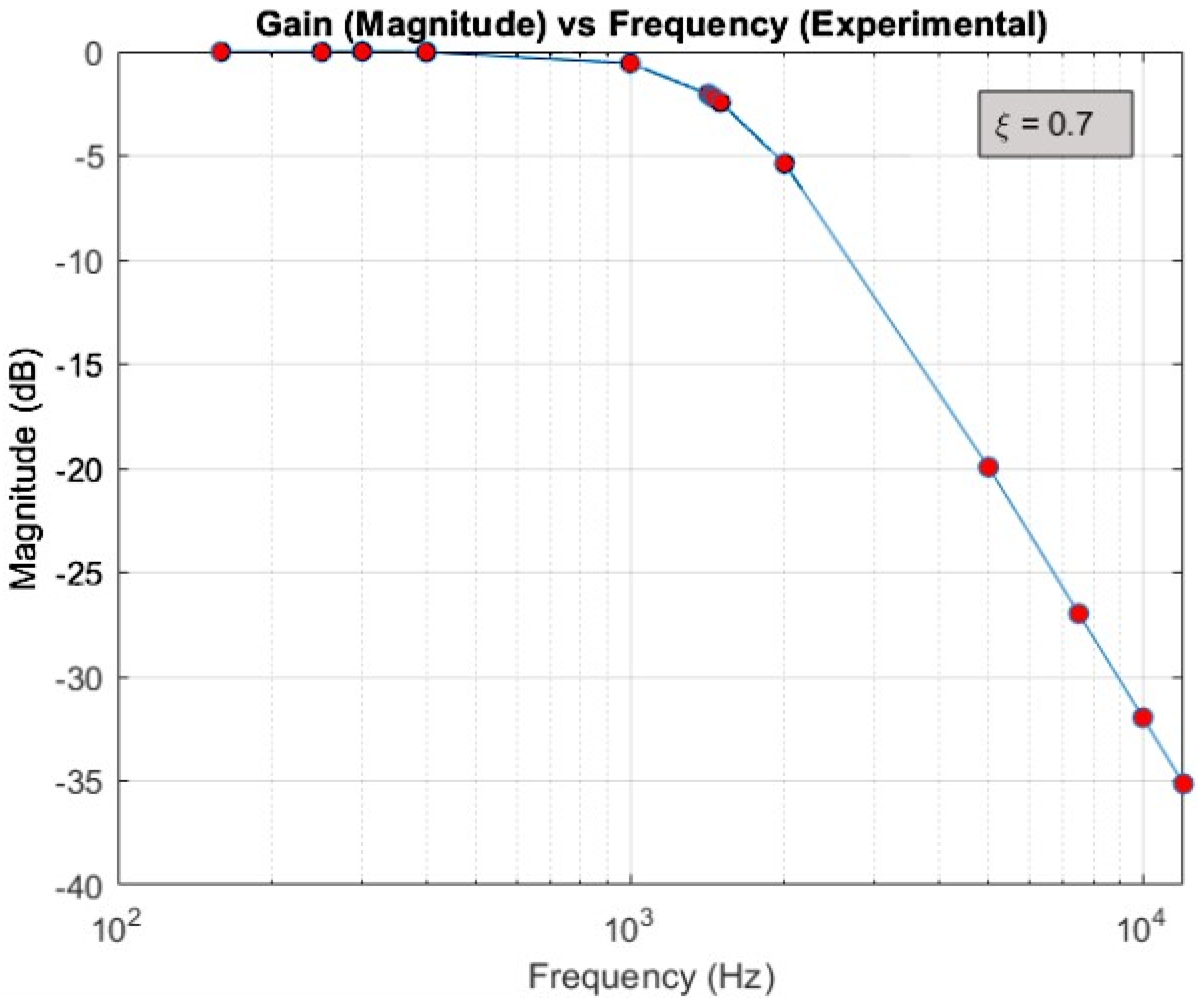
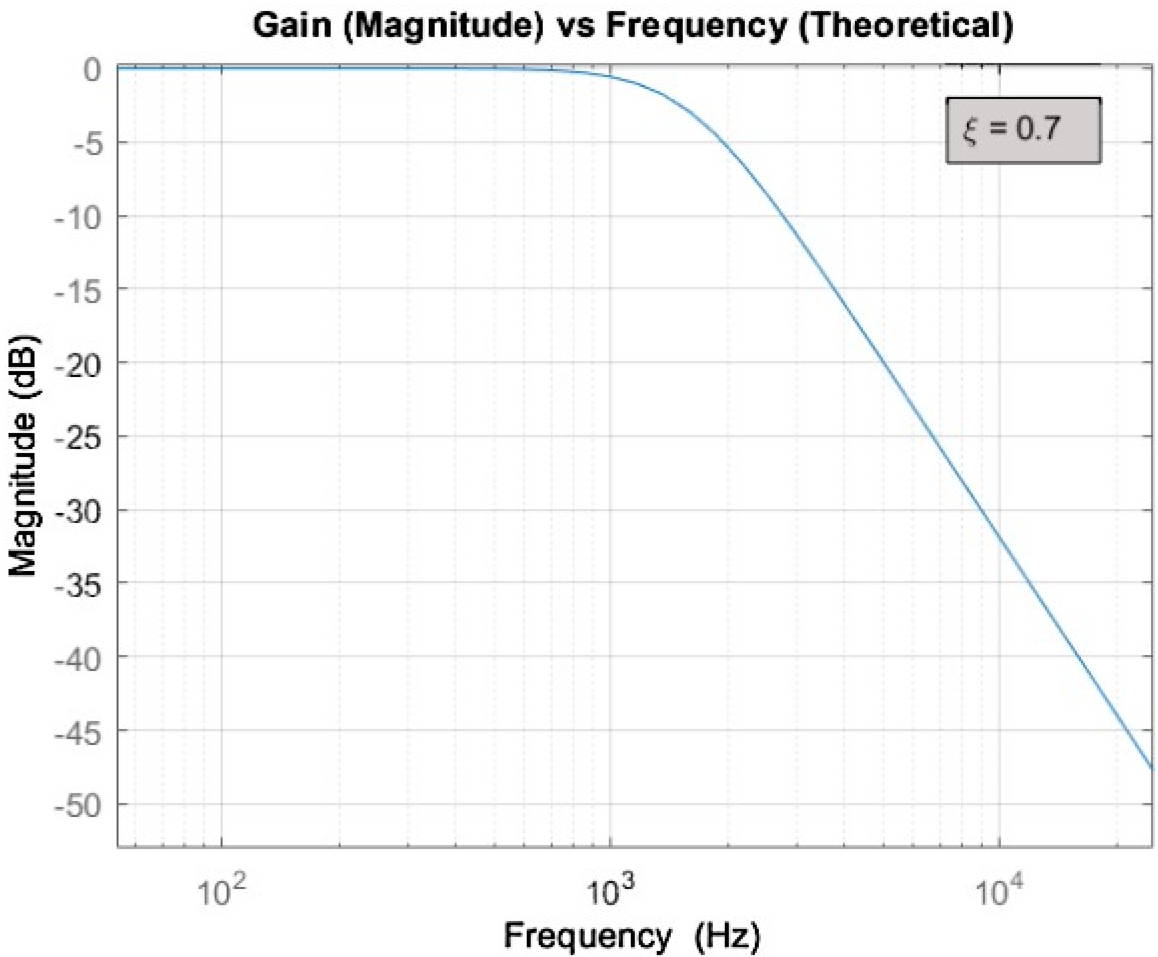
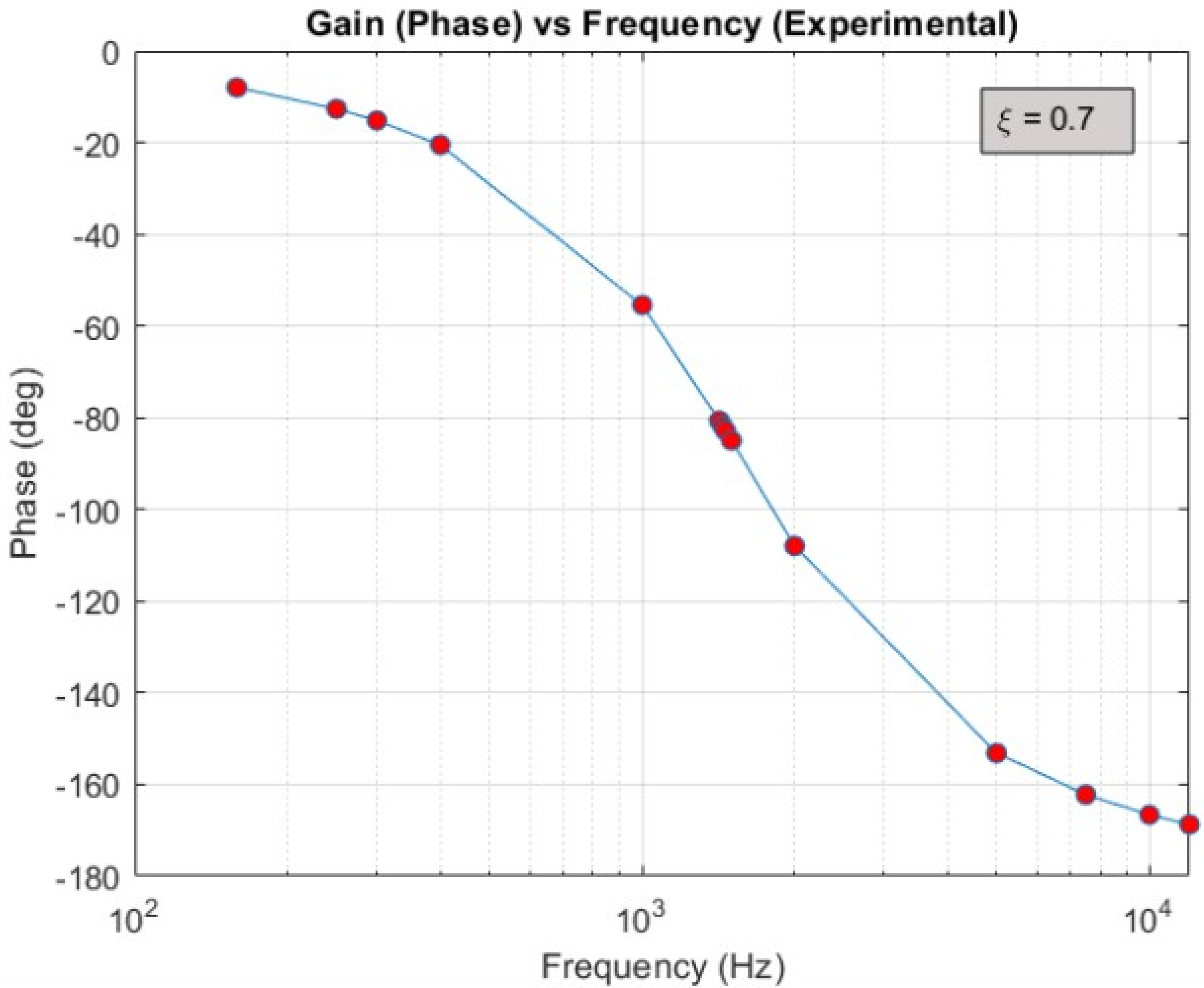
② $\xi = 0.7$ ($R = 1400\ \Omega$)
$V_{in} = 5V$

Frequency (Hz)	V_c (V)	V_L (V)	V_R (V)	Gain (magnitude) (V_c/V_{in})	Gain (phase) (degree)
159	4.991	0.050	0.699	0.998	-7.951
250	5.000	0.124	1.101	1.000	-12.640
300	4.986	0.178	1.322	0.997	-15.220
400	4.984	0.316	1.757	0.997	-20.510
1000	4.681	1.854	4.124	0.938	-55.410
1420	3.945	3.152	4.937	0.790	-80.630
1441	3.899	3.209	4.952	0.781	-81.840
1460	3.858	3.259	4.964	0.772	-82.840
1500	3.769	3.362	4.983	0.755	-85.010
2000	2.690	4.269	4.744	0.538	-108.000
5000	0.501	4.985	2.213	0.101	-153.200
7500	0.223	4.999	1.480	0.045	-162.200
10000	0.126	5.001	1.110	0.025	-166.600
12000	0.087	5.001	0.925	0.018	-168.700

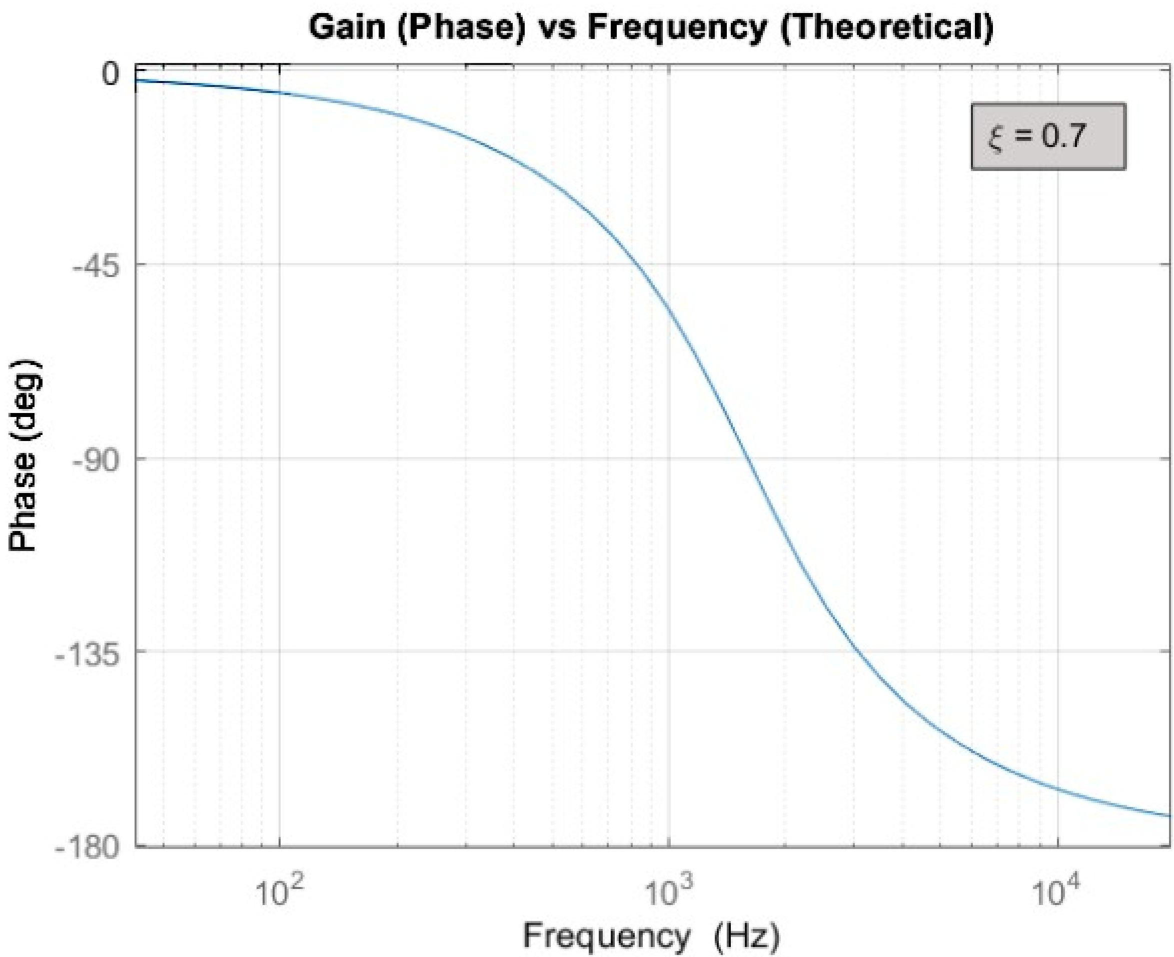
* EXPERIMENTAL AND THEORETICAL FREQUENCY RESPONSE PLOTS ATTACHED BELOW


Gain (Magnitude) vs Frequency (Experimental)
ξ = 0.7
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
10^2
10^3
10^4
Frequency (Hz)


Gain (Magnitude) vs Frequency (Theoretical)
ξ = 0.7
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
10^2
10^3
10^4
Frequency (Hz)



Gain (Phase) vs Frequency (Experimental)
ξ = 0.7
Phase (deg)
Frequency (Hz)
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
10^2
10^3
10^4



Gain (Phase) vs Frequency (Theoretical)
ξ = 0.7
Phase (deg)
0
-45
-90
-135
-180
10^2
10^3
10^4
Frequency (Hz)

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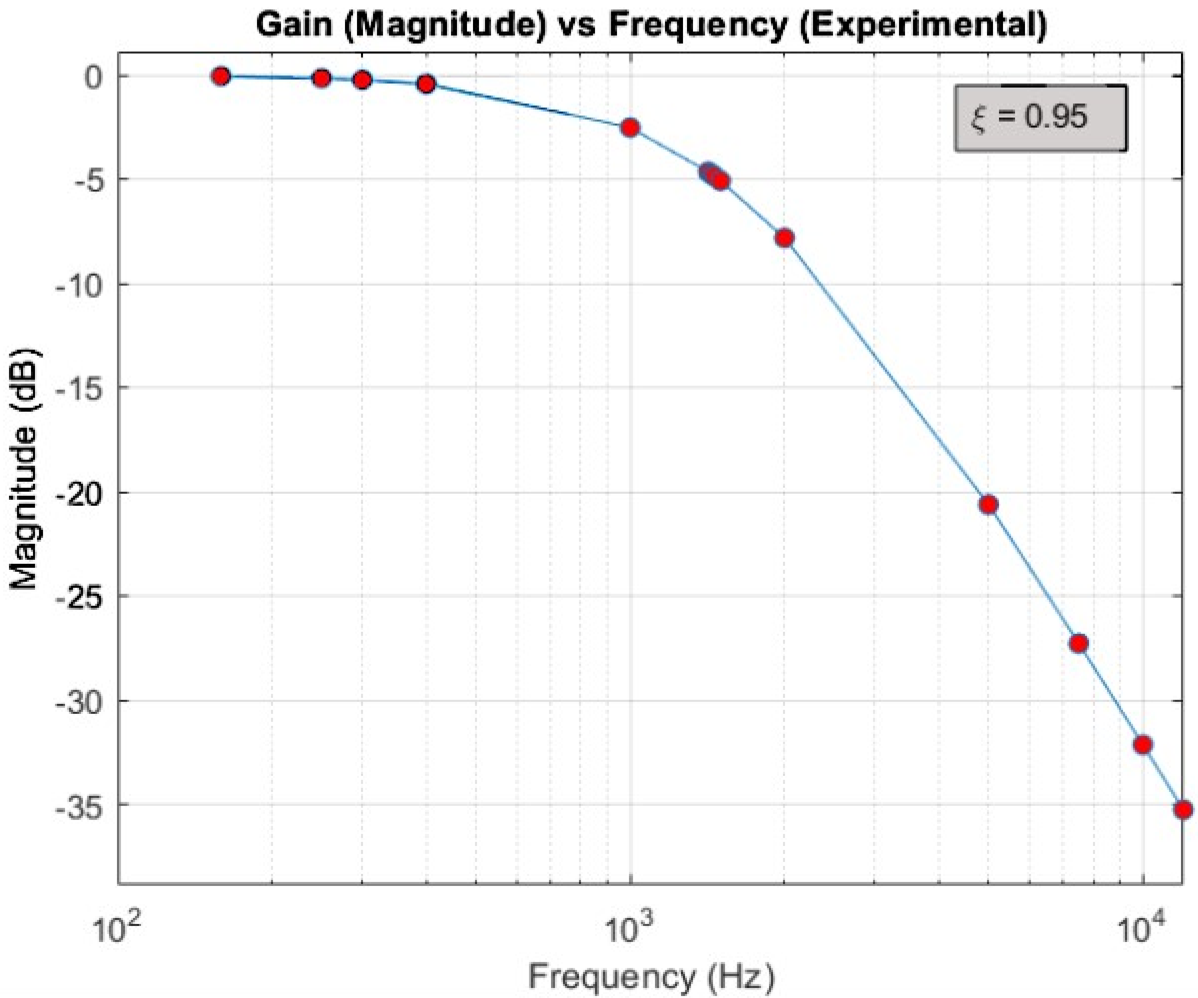
19CS30031

③ $\xi = 0.95$ ($R = 1900\,\Omega$)

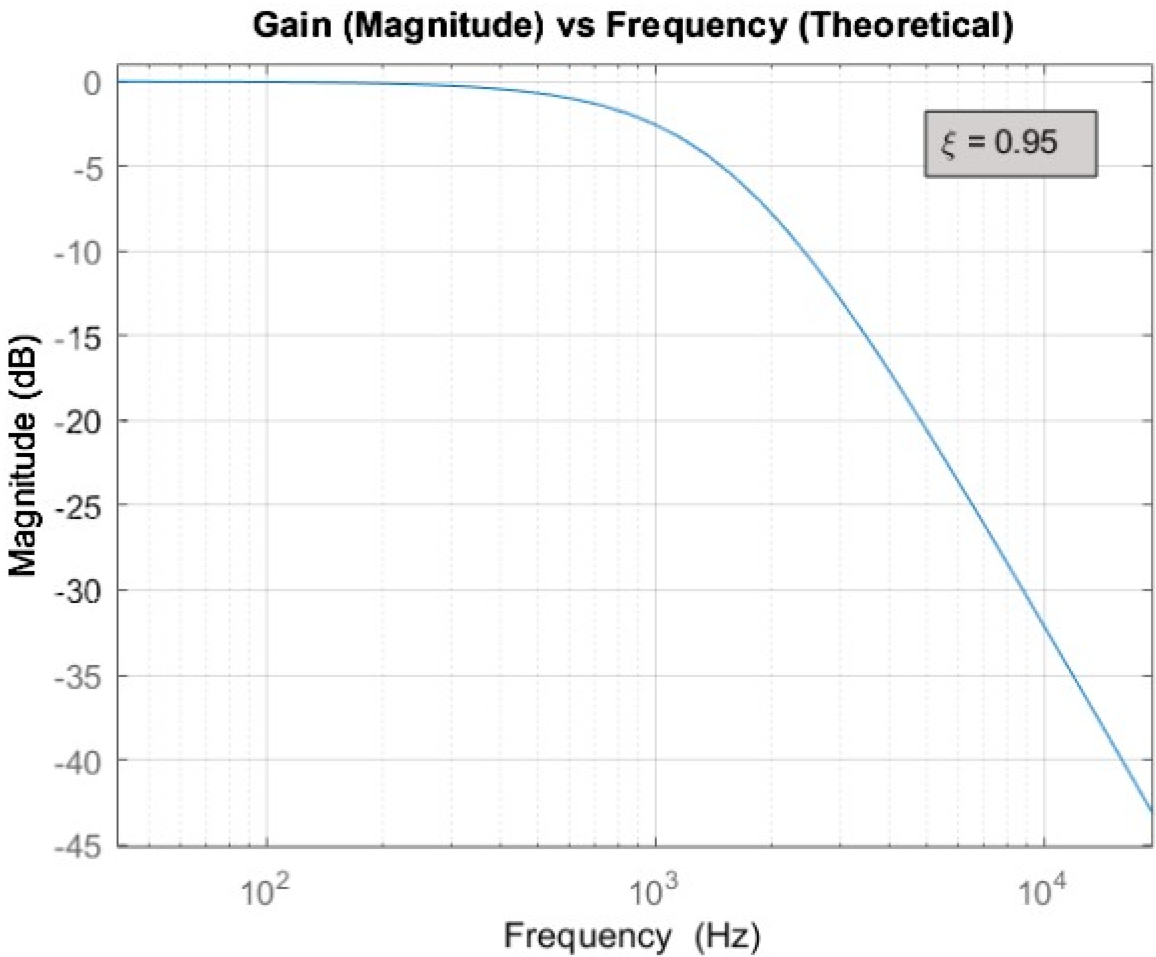
$V_{in} = 5V$

Frequency (Hz)	V_C (V)	V_L (V)	V_R (V)	Gain (magnitude) (V_C/V_{in})	Gain (phase) (degree)
159	4.950	0.050	0.942	0.993	−10.770
250	4.894	0.121	1.462	0.981	−16.930
300	4.855	0.173	1.743	0.973	−20.280
400	4.753	0.301	2.273	0.951	−26.960
1000	3.732	1.478	4.460	0.747	−63.050
1420	2.923	2.336	4.965	0.585	−83.030
1441	2.886	2.375	4.974	0.578	−83.850
1460	2.852	2.409	4.980	0.571	−84.630
1500	2.782	2.481	4.991	0.557	−86.280
2000	2.030	3.219	4.857	0.406	−103.500
5000	0.465	4.618	2.783	0.093	−145.600
7500	0.216	4.825	1.938	0.043	−156.700
10000	0.123	4.900	1.476	0.025	−162.300
12000	0.086	4.931	1.237	0.017	−165.000

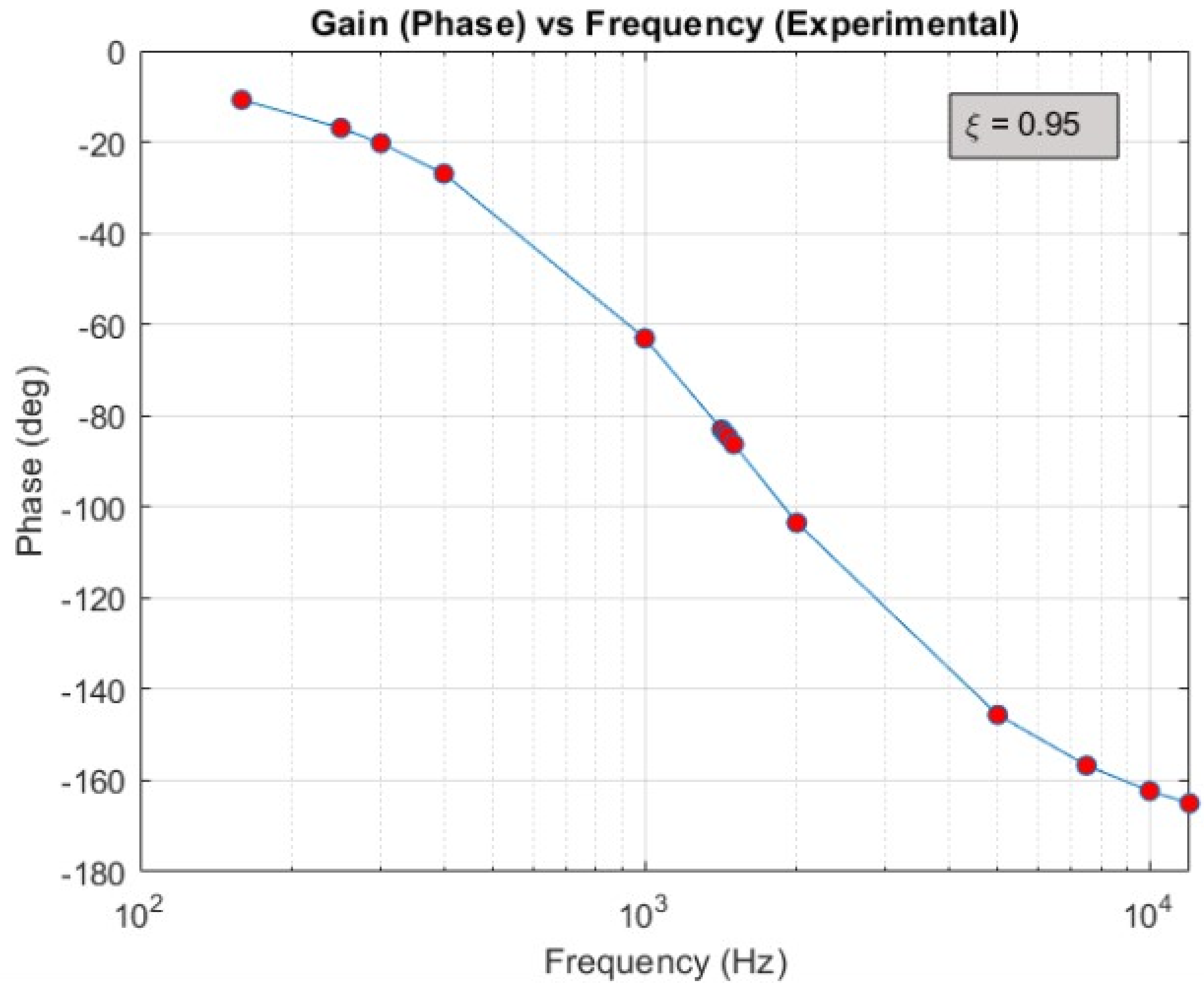
* EXPERIMENTAL AND THEORETICAL FREQUENCY RESPONSE PLOTS ATTACHED BELOW



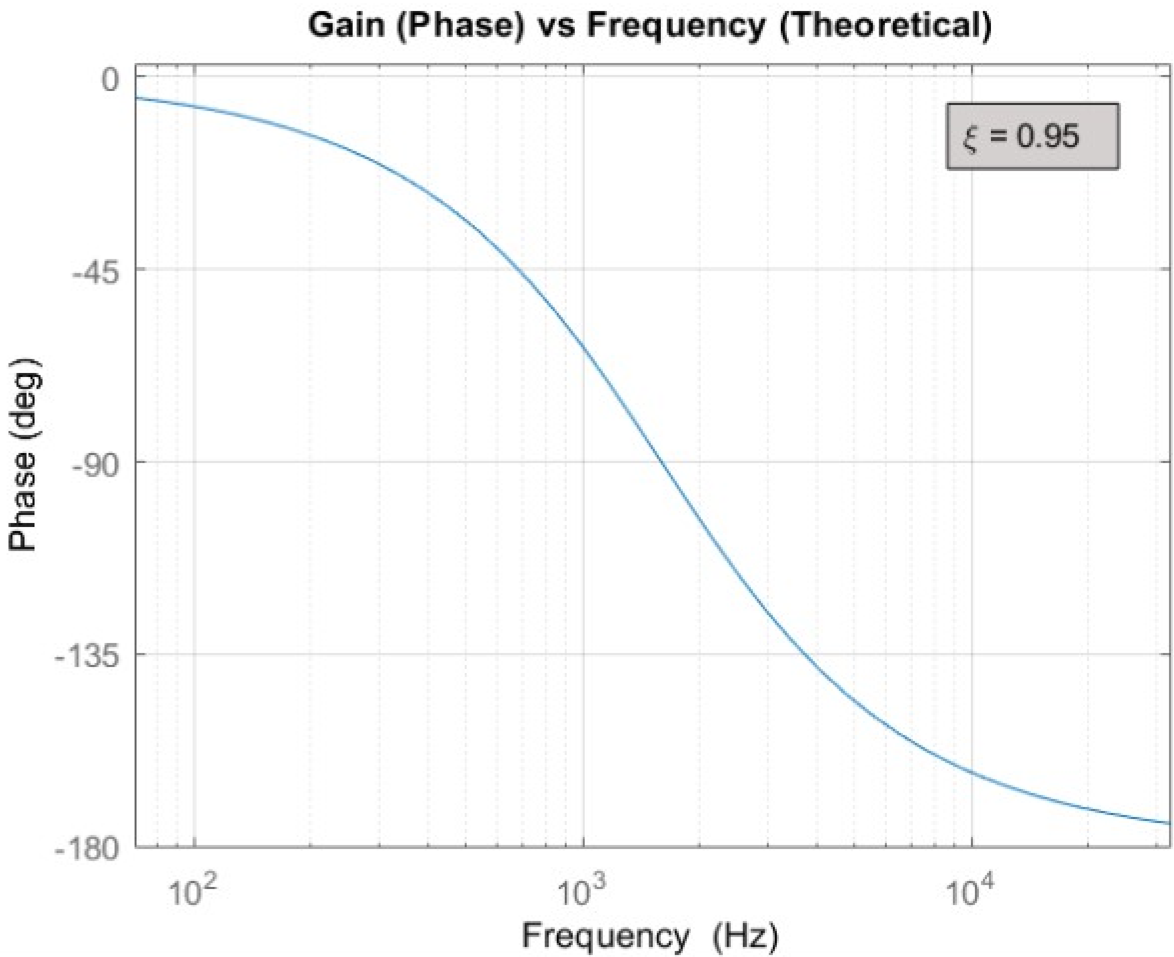
Gain (Magnitude) vs Frequency (Experimental)
ξ = 0.95
0
-5
-10
-15
-20
-25
-30
-35
Magnitude (dB)
10^2
10^3
10^4
Frequency (Hz)



Gain (Magnitude) vs Frequency (Theoretical)
ξ = 0.95
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
10^2
10^3
10^4
Frequency (Hz)



Gain (Phase) vs Frequency (Experimental)
ξ = 0.95
Phase (deg)
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
10^2
10^3
10^4
Frequency (Hz)



Gain (Phase) vs Frequency (Theoretical)
ξ = 0.95
Phase (deg)
0
-45
-90
-135
-180
10^2
10^3
10^4
Frequency (Hz)

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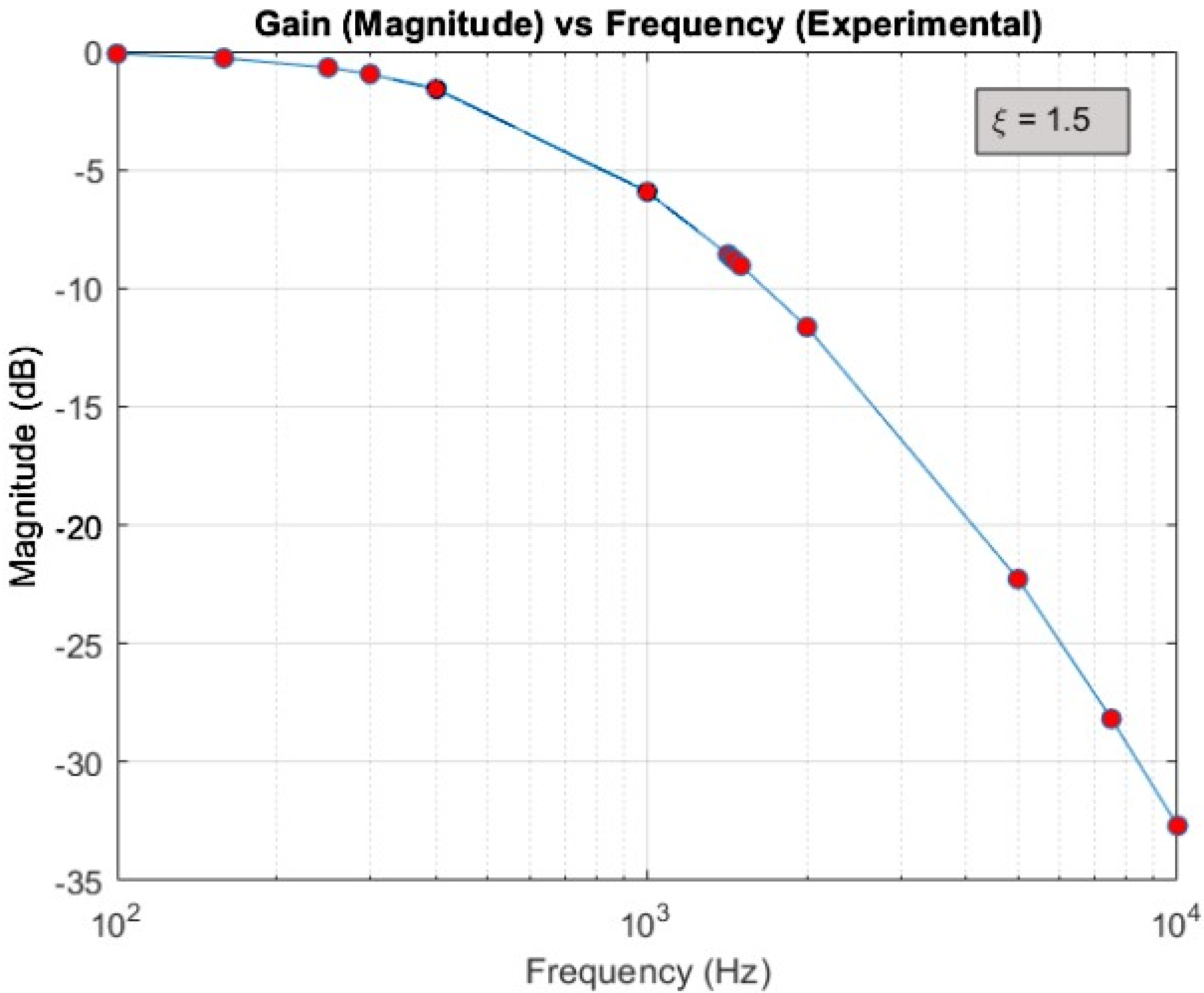
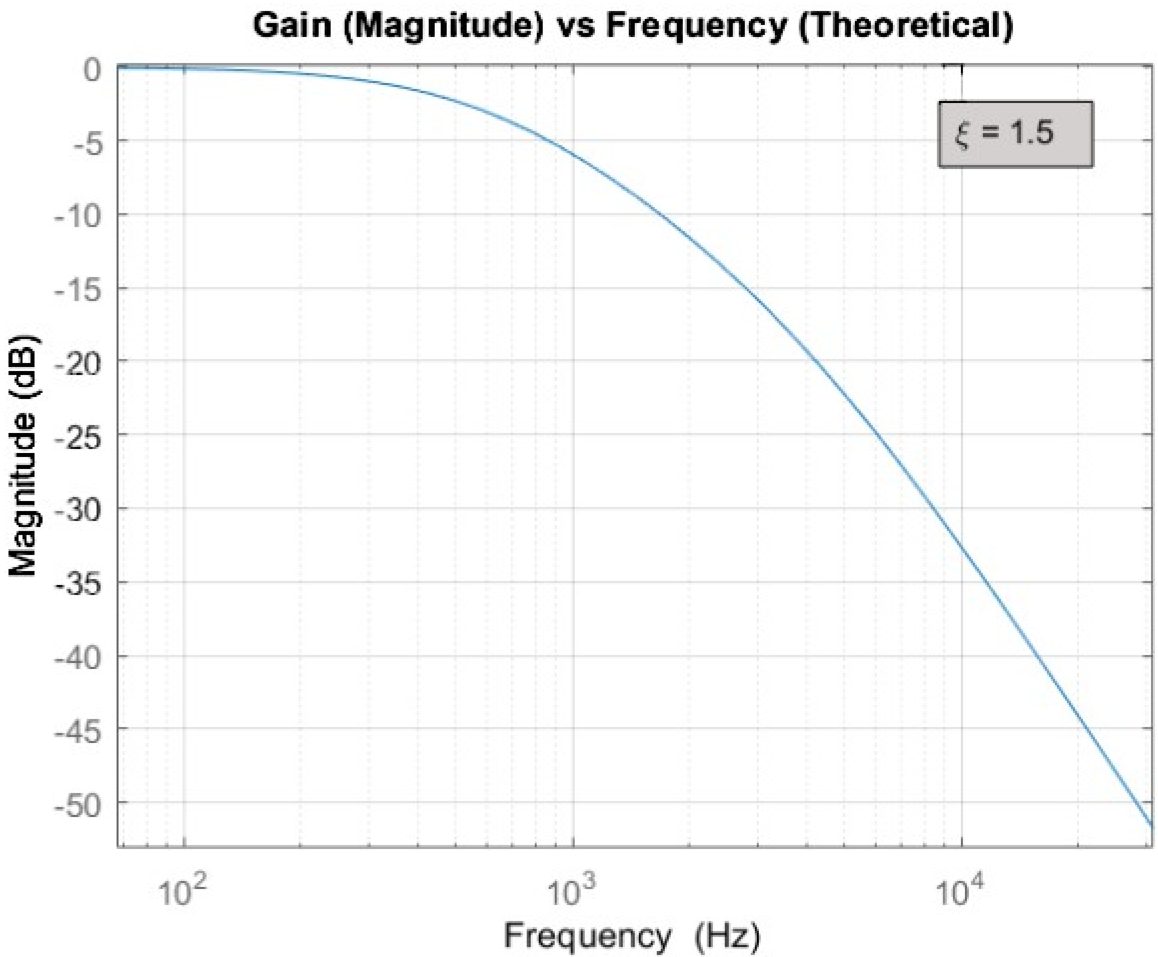
(4) $\xi = 1.5$ ($R = 3000\,\Omega$)

$V_{in} = 5V$

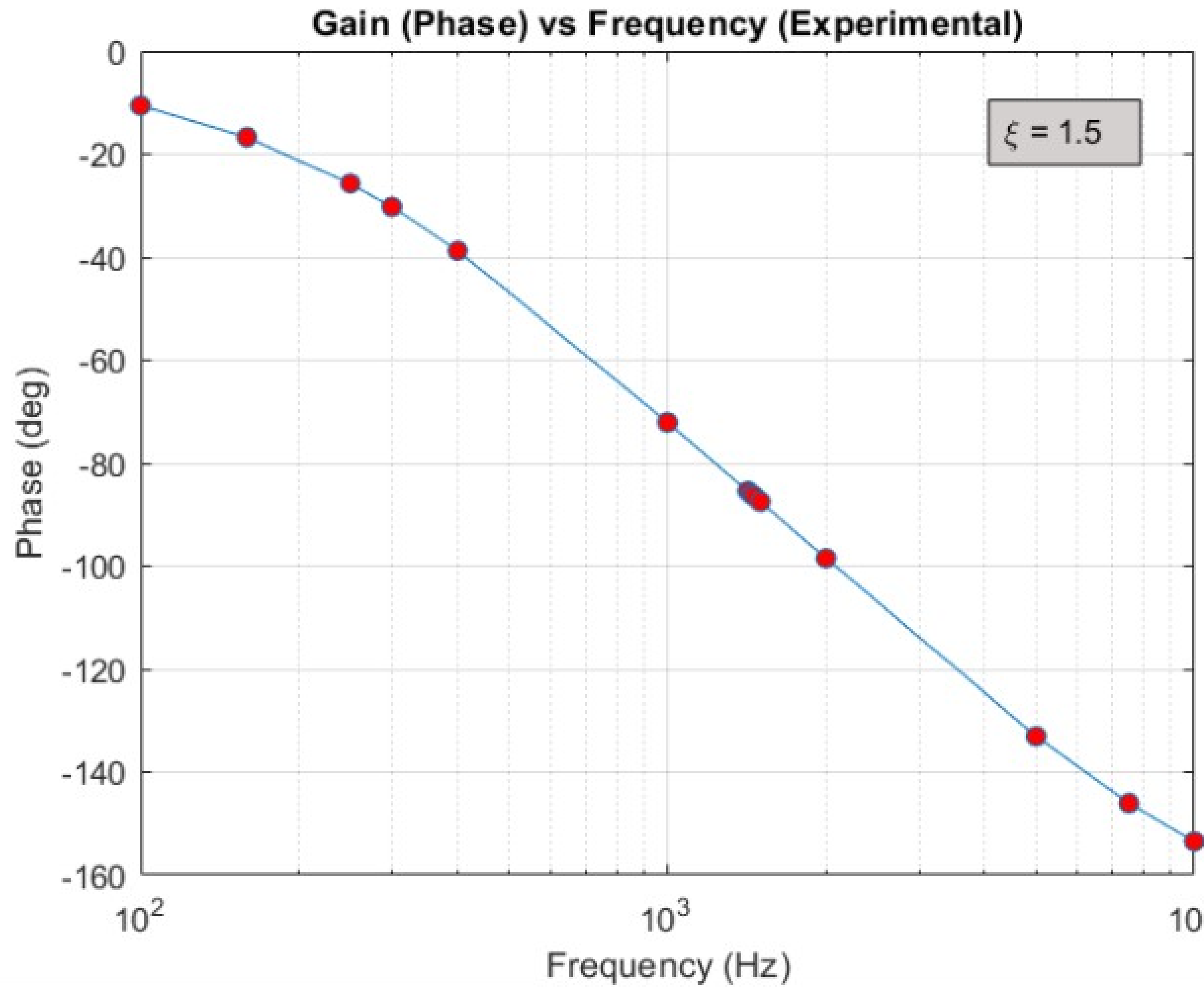
Frequency (Hz)	V_c (V)	V_L (V)	V_R (V)	Gain (magnitude) (V_c/V_{in})	Gain (phase) (degree)
100	4.920	0.020	0.931	0.987	-10.640
159	4.829	0.048	1.449	0.967	-16.780
250	4.612	0.114	2.176	0.924	-25.710
300	4.471	0.159	2.533	0.895	-30.300
400	4.147	0.263	3.139	0.832	-38.760
1000	2.522	0.999	4.762	0.505	-72.100
1420	1.859	1.486	4.986	0.372	-85.440
1441	1.833	1.509	4.989	0.367	-86.020
1460	1.810	1.530	4.992	0.363	-86.480
1500	1.763	1.573	4.996	0.353	-87.540
2000	1.308	2.074	4.941	0.262	-98.480
5000	0.385	3.818	3.635	0.077	-133.000
7500	0.195	4.360	2.767	0.039	-146.000
10000	0.116	4.610	2.192	0.023	-153.400
12000	0.082	4.719	1.870	0.017	-157.400

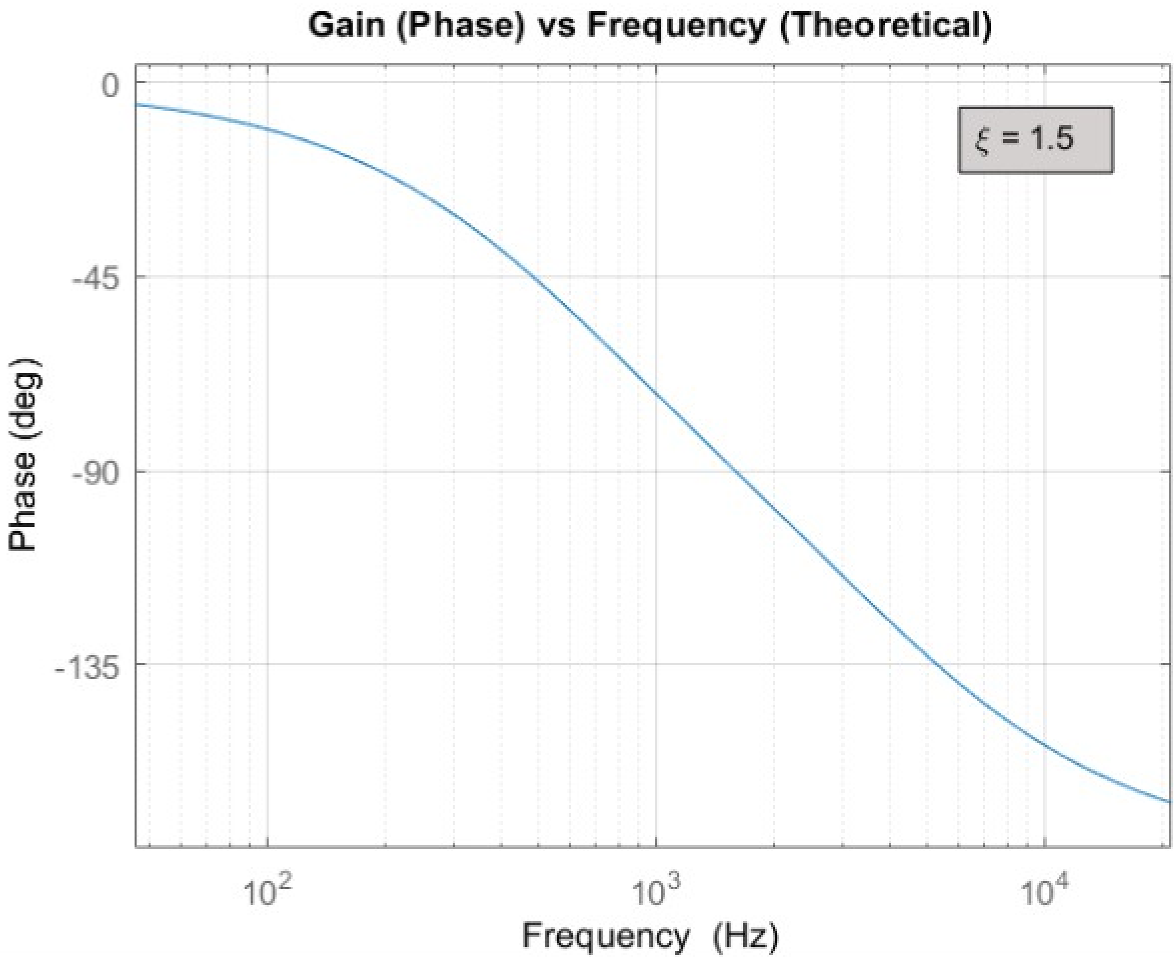
* EXPERIMENTAL AND THEORETICAL FREQUENCY RESPONSE PLOTS ATTACHED BELOW


Gain (Magnitude) vs Frequency (Experimental)
ξ = 1.5
Magnitude (dB)
Frequency (Hz)
0
-5
-10
-15
-20
-25
-30
-35
10^2
10^3
10^4



Gain (Magnitude) vs Frequency (Theoretical)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
Magnitude (dB)
ξ = 1.5
10^2
10^3
10^4
Frequency (Hz)


Gain (Phase) vs Frequency (Experimental)
ξ = 1.5
Phase (deg)
0
-20
-40
-60
-80
-100
-120
-140
-160
10^2
10^3
10^4
Frequency (Hz)



Gain (Phase) vs Frequency (Theoretical)
ξ = 1.5
0
-45
-90
-135
Phase (deg)
10^2
10^3
10^4
Frequency (Hz)

CALCULATIONS

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$L = 0.1\,H$, $C = 0.1\,\mu F = 0.1 \times 10^{-6}\,F$

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-1} \times 10^{-7}}} = 10^4 \text{ rad/s}$$

$$2\xi\omega_n = \frac{R}{L} \Rightarrow R = (2L\omega_n)\xi = (2 \times 10^{-1} \times 10^4)\xi = (2000\xi)\,\Omega$$

① $\xi = 0.2 \Rightarrow R = (2000 \times 0.2)\,\Omega = 400\,\Omega$

Experimental Max. gain $= 2.549$
at $f_m = 1500$ Hz

Theoretical Max. gain $= \dfrac{1}{2\xi\sqrt{1-\xi^2}} = 2.552$ $\left(\dfrac{1}{2 \times 0.2 \times \sqrt{1-0.04}}\right)$

at $f_m = \dfrac{1}{2\pi}\omega_n\sqrt{1-2\xi^2} = 1527$ Hz $\left(\dfrac{10^4}{2\pi} \times \sqrt{1-0.08}\right)$

$\Rightarrow$ % error (max gain) $= \dfrac{|2.552 - 2.549|}{2.552} \times 100 = 0.12\%$

$\Rightarrow$ % error (f_m) $= \dfrac{|1527 - 1500|}{1527} \times 100 = 1.77\%$

② $\xi = 0.7 \Rightarrow R = (2000 \times 0.7)\,\Omega = 1400\,\Omega$

Experimental Max. gain $= 1.000$ at $f_m = 250$ Hz

Theoretical Max. gain $= \dfrac{1}{2\xi\sqrt{1-\xi^2}} = 1.000$ $\left(\dfrac{1}{2 \times 0.7 \times \sqrt{1-0.49}}\right)$

at $f_m = \dfrac{1}{2\pi}\omega_n\sqrt{1-2\xi^2} = 225$ Hz $\left(\dfrac{10^4}{2\pi} \times \sqrt{1-0.98}\right)$

$\Rightarrow$ % error (~~max gain~~ f_m) $= \dfrac{|225 - 250|}{225} \times 100 = 11.11\%$

$\Rightarrow$ The experimental max gain is in agreement with the theoretical max gain.

DISCUSSION

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As the experiment has been carried out using simulation the measurement error reduces significantly. In real experiment the measuring devices have a certain error associated with them, also the precision of the readings is much higher in simulation compared to the real experiment.

Further, the signal generator and the passive components of the circuit are ideal and exact in simulation. In the real experiment there are bound to be certain fluctuations with the signal generator and the passive elements (R-L-C) are also non-ideal.

It can be observed that as the peak of the frequency response curve gets less and less sharp it becomes difficult to find the frequency at which the maximum gain occurs due to two reasons. For a wide range of frequency near the peak the gain remains almost constant. Also, the variation in gain is so less that it becomes experimentally difficult to observe it.

Frequency response analysis has important industrial applications in design of filters which are extensively used in audio and communication devices for attenuating noise from unwanted frequencies. It is also used in design of loudspeakers whereby it is important in determing the quality and loudness of sound over a wide range of frequency. A good speaker should have a high-response in the audible range (20-20k) Hz.