Tutorial

Q1. $S \to aSb \mid bSa \mid SS \mid \varepsilon$

Show that this generates all strings with equal no. of a's and b's.

($\Rightarrow$) Induction on length of derivation. Check base cases

IH: Any α' derived in n steps from S has $\#a(\alpha') = \#b(\alpha')$.

Let β be such that $S \xrightarrow{n+1} \beta$

$$S \xrightarrow{n+1} \beta$$

$$\underbrace{S \xrightarrow{n} \alpha}\ \underbrace{\xrightarrow{1} \beta}$$

(i) $\alpha = \alpha_1 \underset{\smile}{S} \alpha_2$, $\beta = \alpha_1 aSb\alpha_2$

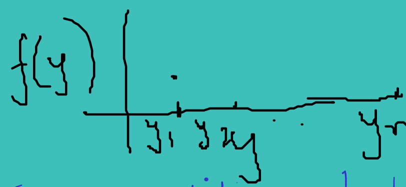
(ii) $\cancel{\alpha =}$ $\beta = \alpha_1 bSa\alpha_2$

(iii) $\beta = \alpha_1 SS\alpha_2$

(iv) $\beta = \alpha_1\alpha_2$

($\Leftarrow$) Given $\underset{\smile}{x}$ s.t $\#a(x) \overset{= n}{=} \#b(x)$ $\exists\ S \xrightarrow{*}_G x$.

$\forall y$, Consider $f(y) = \#a(y) - \#b(y) \leftarrow$



Statement: Given x s.t $\#a(x) = \#b(x)$, one of 3 conditions hold:

(i) x is of form ax_1b

(ii) x is of form bx_1a

(iii) x is of form x_1x_2 s.t $\#a(x_1) = \#b(x_1)$ & $\#a(x_2) = \#b(x_2)$

Then, induct on length of x. Check base cases.

IH: If $|y| < n$, $S \to^* y$.

Take x, $|x| = n$.

Cases: (i) $S \to^* x_1$ by IH and $S \to aSb \to ax_1b = x$

(ii) $S \to^* x_1$ by IH and $S \to bSa \to bx_1a = x$

(iii) $S \to^* x_1$, $S \to^* x_2$ by IH and $S \to SS \to^* x_1S \to^* x_1x_2 = x$

Qn*: What is a CFG for the set of strings in $\{a,b\}^*$ s.t
#a's $\geq$ #b's?

$$\rightarrow S \rightarrow aSb \mid bSa \mid SS \mid a \mid \varepsilon$$

Structure of x: (i)-(iii) from above

+

(iv) x of the form $a x_1$ where x_1 has
$\#a(x_1) \geq \#b(x_1)$.

Q2. Give a CFG for

$$L_2 = \{ x \in \{0,1\}^* \mid x^{rev} = \overline{x} \}$$

If $x = 0101$, $x^{rev} = 1010$, $\overline{x} = 1010$.

Properties: ⊙ x must have even length

⊙ $x_a = \overline{x}_{n-a}$

Grammar: $S \rightarrow \underbrace{\varepsilon}_{} \mid \underbrace{1S0 \mid 0S1}_{}$

Ensures even length of sentence

Ensures even # of terminals, flipping of bits equidistant from both ends.

Q3. What is the language generated by $S \to bS \mid Sa \mid aSb \mid \varepsilon$

$\Sigma = \{a, b\}$
$N = S$

Answer: Σ^*

Induction on length of string. Check base cases.
IH: Any $y \in \Sigma^*$, $|y| < n$ is generated. Take x, $|x| = n$

(i) $x = b\underbrace{x_1}$: $S \to bS \xrightarrow[G]{*} bx_1$
IH: $S \xrightarrow[G]{*} x_1$

(ii) $x = x_1 a$: $S \to Sa \xrightarrow[G]{*} x_1 a$
IH: $S \xrightarrow[G]{*} x_1$

(iii) $x = ax_1 b$: $S \to aSb \xrightarrow[G]{*} ax_1 b$
IH: $S \xrightarrow[G]{*} x_1$

(iv) [Part of base case] $x = \varepsilon$: $S \to \varepsilon$.

Q4. What is a CFG for

(a) $\{a^m b^n \mid m \le 2n\}$

(b) $\neg\{a^n b^n \mid n \geq 0\} = G$

(a) $S \to aaSb \mid aSb \mid Sb \mid \varepsilon$

(b) $L(G_1) = \{x \in \{a,b\}^* \mid$ A b is followed by an a$\}$

$(a+b)^* b (a+b)^* a (a+b)^*$ ← what is a CFG?

$L(G_2) = \{a^m b^n \mid m \neq n \geq 0\}$

$L(G) = L(G_1) \cup L(G_2)$.

G_1: $S \to UbUaU$

$U \to aU \mid bU \mid \varepsilon$

G_2: $S \to AT \mid TB$

$T \to aTb \mid \varepsilon$

$A \to aA \mid a$

$B \to bB \mid b$

OR G_2:

$S \to S_1 \mid S_2$

G_2^1 $\{a^m b^n \mid m > n\}$

$\cup$

G_2^2 $\{a^m b^n \mid m < n\}$

G_2^1: $S_1 \to aS_1 \mid aS_1 b \mid a$

G_2^2: $S_2 \to S_2 b \mid aS_2 b \mid b$

Q5. What is the CFG for

$$L_5 = \{ x \in \{a,b\}^* \mid \#a(x) \neq \#b(x) \}.$$

$$G' \leftarrow L' = \{ x \in \{a,b\}^* \mid \#a(x) > \#b(x) \}$$
$$G'' \leftarrow L'' = \{ x \in \{a,b\}^* \mid \#a(x) < \#b(x) \}.$$

$$L_5 = L' \cup L''$$

G' : 1.

$$S \to TAS$$
$$A \to aA \mid a$$
$$T \to aTb \mid bTa \mid TT \mid \varepsilon$$

2. Following $f(x) = \#a(x) - \#b(x)$:

$$S \to aE \mid aS \mid bSS$$
$$E \to aEb \mid bEa \mid EE \mid \varepsilon$$

Q6. Convert $S \to bS \mid Sa \mid aSb \mid \varepsilon$ to CNF.

Get rid of ε-production

$S \to bS$, $S \to \varepsilon$

Add: $S \to b$, $S \to a$, $S \to \underbrace{ab}$.

Add: $A \to a$, $B \to b$

Modify: $S \to BS \mid SA \mid ASB \mid b \mid a \mid AB$

Add: $C \to SB$

Modify: $S \to BS \mid SA \mid AC \mid b \mid a \mid AB$

$C \to SB$