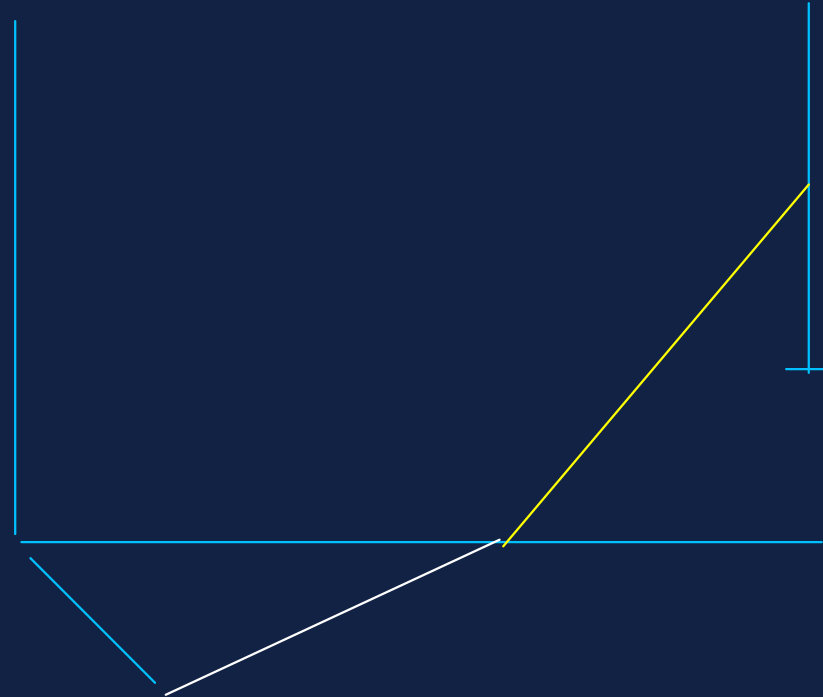
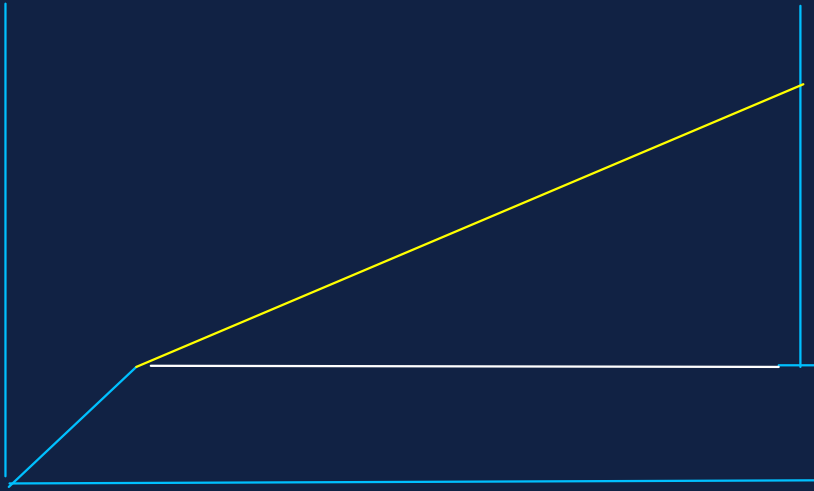


0. Design a CFG for $\{ x \in \{a,b\}^* \mid \#a(x) > \#b(x) \}$.

$$S \rightarrow aS \mid aE \mid bSS$$

$$E \rightarrow aEb \mid bEa \mid EE \mid \varepsilon$$



1. Consider the language $L_1 = \{ x \in \{a,b\}^* \mid \#a(x) = 2 \times \#b(x) \}$.

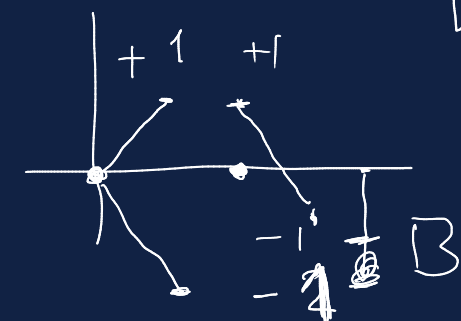
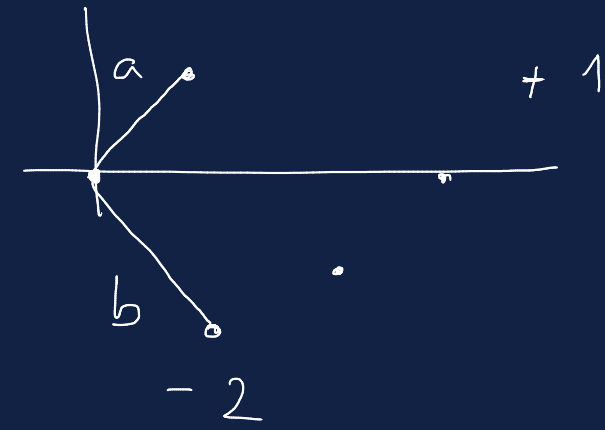
(a) Design a CFG for L_1 .

$$\boxed{S(x) = \#a(x) - 2\#b(x)}$$

$S, A, B \rightarrow \#a = 2\#b - 1$

$\#a = 2\#b$

$\#a = 2\#b + 1$



next symbol

a

b

$S(\cdot)$

$+1 \leftarrow \text{cont}$

$-2 \leftarrow \text{not cont}$

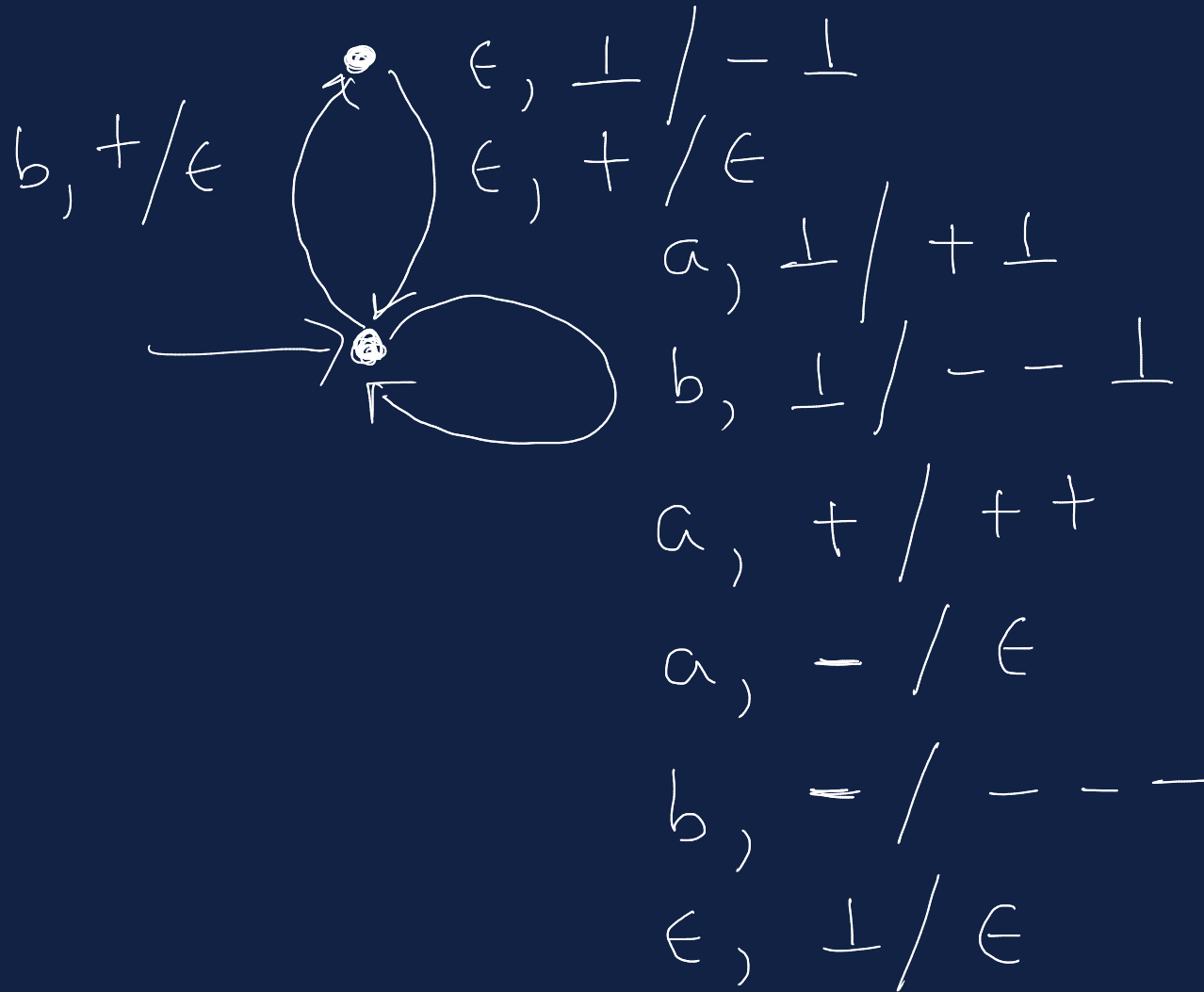
$$S \rightarrow \epsilon \mid aB \mid bAA$$

$$A \rightarrow aS \mid bAAA$$

$$B \rightarrow bA \mid aBB \mid aSbS$$

(b) Design a PDA for L_1 .

$$S(x) = \#a(x) - 2\#b(x)$$



$$\Gamma = \{+, -, \perp\}$$

Acceptance by
empty stack

2. Design a PDA for the language $\{ x \in \{a,b\}^* \mid \#b(x) \leq \#a(x) \leq 2 \times \#b(x) \}$.

$a \longrightarrow +1$

$a, \perp / + \perp$

$a, + / ++$

$a, - / \epsilon$

empty
stack

$b \longrightarrow$ nondeterministically

do -1 or -2

$b, \perp / - \perp$

$b, + / \epsilon$

$b, - / - -$

$b, \perp / - + \perp$

$b, - / - - -$

$\boxed{\epsilon, \perp / \epsilon}$

$b, + / \epsilon$ $\xrightarrow{\quad}$ transition to
temp state

3. Design a PDA for the language

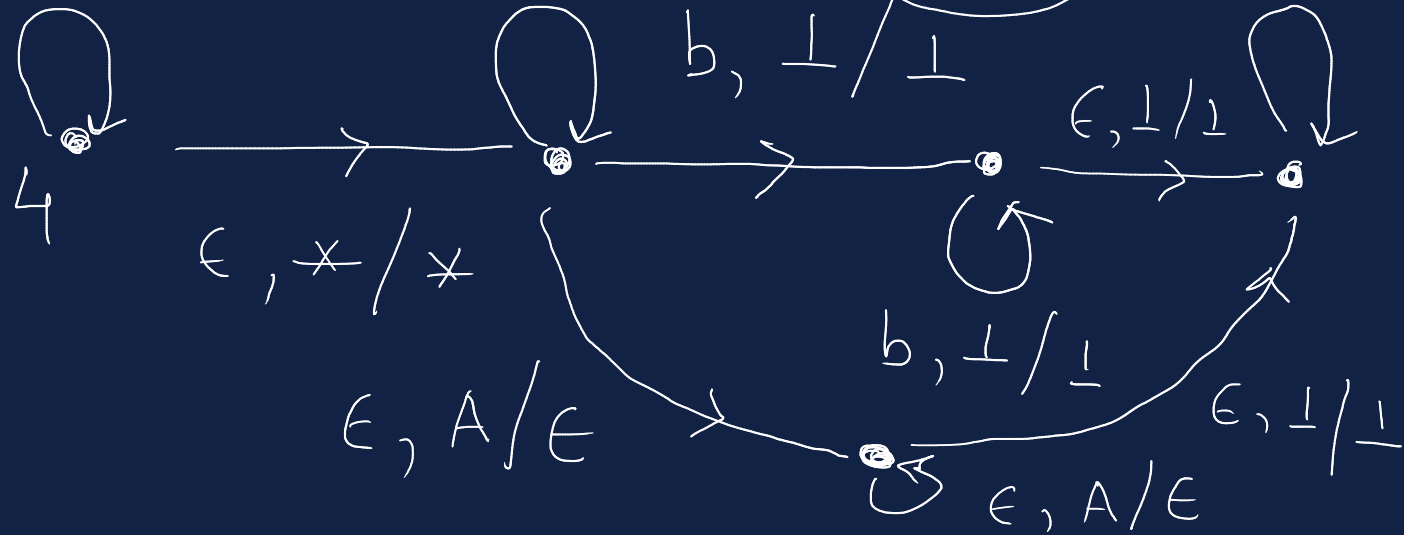
$$\{a,b,c\}^* - \{ a^n b^n c^n \mid n \geq 0 \}.$$


empty stack

ba (strings containing)
ca

CR

ch

$$a^i b^j c^k, \quad i \neq j$$
$$a^i b^j c^k, \quad j \neq k$$
$$\begin{array}{l} a, \perp / A \perp \\ a, A / AA \end{array}$$
$$a, A / AA$$
$$b, A \in$$
$$\epsilon, 1/\epsilon$$
$$C, \frac{1}{1}$$
$$j > i$$


4. Let L be a CFL over some alphabet Γ . Prove that the language

$$\text{cyclicshift}(L) = \{ yx \mid xy \in L \}$$

is also a CFL.

