INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Switching Circuits and Logic Design (CS21002)

Assignment – 1 (Spring)

Group: _______________ *Marks:* 130

Answer ALL the questions using xournal or similar software to edit the PDF

1. The relation $\sim$ is defined on Z by
$$a \sim b \iff \gcd(a, b) \neq 1$$
Write down whether $\sim$ is an equivalence relation or not with justification. If it is, then describe the equivalence classes. 3

2. Determine whether or not the following binary relation
$$\{(a, a), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$$
is a partial order on the set $A = \{a, b, c, d\}$. Justify your answer. 3

3. Let $A = \{1, 2, 3, 4, 5, 6\}$. Consider the relation
$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (5, 4), (4, 5), (5, 5)\}$$
   (a) Prove or Disprove R is an equivalence Relation. 3
   (b) If R is an equivalence relation, then find out the equivalent classes of all the elements of A. 3

4. Let $R1$ and $R2$ be two equivalence relations on a set. Prove or Disprove $R1 \cup R2$ is an equivalence relation . 3

5. Let A =$N \times N$, and define a relations R on A by $(a, b)R(c, d)$ if and only if $ab = cd$.
   (a) Show that R is an equivalence relation on A. 3
   (b) List the elements in the equivalence class. 3
   (c) Find an equivalence class with exactly two elements. 3
   (d) Find an equivalence with exactly three elements 3

6. Show that the relation R is an equivalence relation in the set $A = \{1, 2, 3, 4, 5\}$ given by the relation $R = \{(a, b) : |a - b| \text{ is even}\}$. 3

7. Find out the total number of equivalence relations on the set $A = \{1, 2, 3, 4\}$ and also write down the equivalence classes. 3

8. Prove or Disprove that the antisymmetric closure of a relation R on a set A exists if and only if R itself is antisymmetric. Explain the total number of antisymmetric relations on a finite set of size n. 3

9. Let $R1$ and $R2$ be partial order relations on a set S. Prove or disprove that $R1 \cup R2$ is also a partial order relation on S. 3

10. Determine whether or not each of the following relations is a partial order and state whether or not each partial order is a total order.
   (a) $(N \times N, \preceq)$, where $(a, b) \preceq (c, d)$ if and only if $a \leq c$. 3
   (b) $(N \times N, \preceq)$, where $(a, b) \preceq (c, d)$ if and only if $a \leq c$ and $b \geq d$. 3

11. Suppose S is a set and for A, $B \in P(S)$, we define $A \preceq B$ to mean $|A| \leq |B|$. Is this relation a partial order on $P(S)$? Explain. 3

12. Suppose that the relation R on the finite set A is represented by the matrix M_R. Show that the matrix that represents the reflexive and symmetric closure of R are $M_R \vee I_n$ and $M_R \vee M_R^t$. 3

13. (a) Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree except perhaps in their first three bits is an equivalence relation on the set of all bit strings of length three or more. 3

    (b) Prove or disprove that the relation R consisting of all pairs (x, y) such that x and y are bit strings that agree in their first and third bits is a partial order relation on the set of all bit strings of length three or more. 3

14. Is there a finite set such that it is a poset and totally ordered set but not a well-ordered set. Justify. 3

15. Given the poset $(\{1, 2, 3, 5, 6, 7, 10, 20, 30, 60, 70\}, |)$, where $a|b$ denotes a divides b. 3

    (a) Draw the Hasse Diagram for this poset. 3
    (b) Find the maximal elements. 3
    (c) Find the minimal elements. 3
    (d) Find the greatest element or explain why there is no greatest element. 3
    (e) Find the least element or explain why there is no least element. 3
    (f) Find all upper bounds of $\{2, 5\}$. 3
    (g) Find the least upper bound of $\{2, 5\}$ (if it exists). 3
    (h) Find all lower bounds of $\{6, 10\}$. 3
    (i) Find the greatest lower bound of $\{2, 5\}$ (if it exists). 3
    (j) Is this poset a lattice? Justify your answer. 3

16. Determine which of the following Hasse diagrams represent a lattice. 3

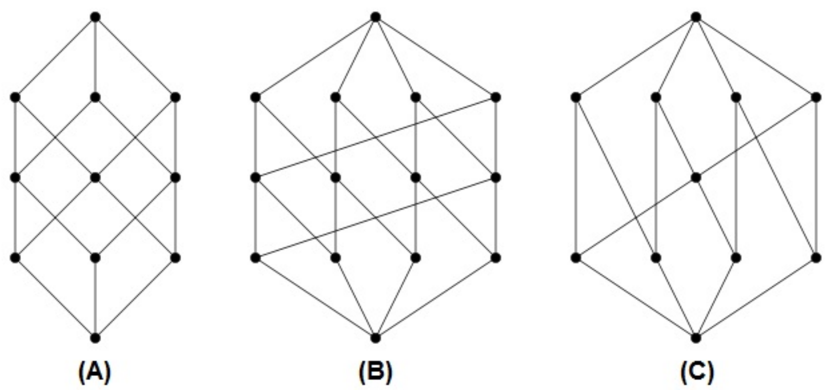

Figure 1: Hasse Diagram

17. For each of the lattice given below determine whether it is distributive and/or complemented. If the lattice is complemented, identify the complementary elements 3

18. For the Hasse diagram given below; find maximal, minimal, greatest, least, LB, glb, UB and lub for the subsets

    (a) {d,k,f} 3
    (b) {b,h,f} 3
    (c) {d} 3
    (d) {a,b,c} 3
    (e) {l,m} 3

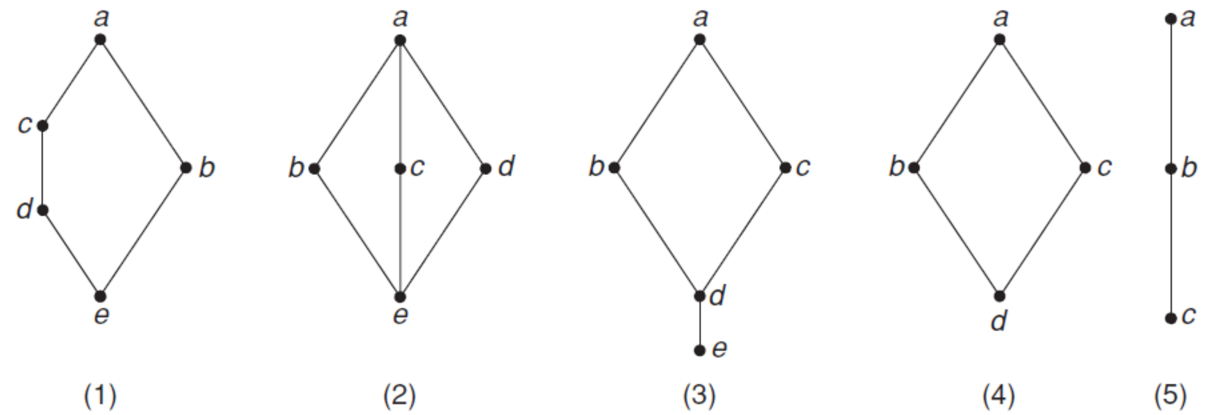

Figure 2: Lattice

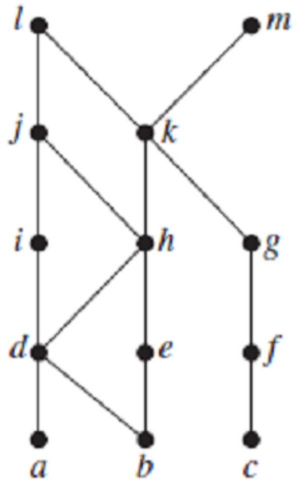

Figure 3: Hasse Diagram

19. Present a Hasse diagram (or a poset) and an associated subset for each of the following;
    (a) a subset such that it has two maximal and two minimal elements. 2
    (b) a subset such that it has a maximal element but no minimal elements. 2
    (c) Is it possible to find a such a subset if the underlying set is an infinite set? 2
    (d) a subset such that it has a lower bound but no greatest lower bound. If such a subset is not possible, argue why? 2
    (e) a subset such that it has an upper bound but no least upper bound. If such a subset is not possible, argue why? 2
20. Show that the following posets are lattices, and interpret their meets and joins:
    (a) The poset of the divisors of 60, ordered by divisibility. 3
    (b) The poset of the subsets of $\{0, 1, 2\}$,ordered by the subset relation. 3