Tutorial

Q1. $S \to aSb \mid bSa \mid SS \mid \varepsilon$

Show that this generates all strings with equal no. of a's and b's.

($\Rightarrow$) Induction on length of derivation. Check base cases

IH: Any α' derived in n steps from S has $\#a(\alpha') = \#b(\alpha')$.

Let β be such that $S \xrightarrow{n+1} \beta$

$$S \xrightarrow{n+1} \beta$$

$$\underbrace{S \xrightarrow{n} \alpha}\ \underbrace{\xrightarrow{1} \beta}$$

(i) $\alpha = \alpha_1 S \alpha_2$, $\beta = \alpha_1 a S b \alpha_2$

(ii) $\cancel{\alpha =}$ $\beta = \alpha_1 b S a \alpha_2$

(iii) $\beta = \alpha_1 S S \alpha_2$

(iv) $\beta = \alpha_1 \alpha_2$

($\Leftarrow$) Given $\underbrace{x}$ s.t $\#a(x) \overset{=n}{=} \#b(x)$ $\exists\ S \xrightarrow{*}_G x$.

$\forall y$, Consider $f(y) = \#a(y) - \#b(y)$

Statement: Given x s.t $\#a(x) = \#b(x)$, one of 3 conditions hold:

(i) x is of form $a x_1 b$

(ii) x is of form $b x_1 a$

(iii) x is of form $x_1 x_2$ s.t $\#a(x_1) = \#b(x_1)$ & $\#a(x_2) = \#b(x_2)$

Then, induct on length of x. Check base cases.

IH: If $|y| < n$, $S \to^* y$.

Take x, $|x| = n$.

Cases: (i) $S \to^* x_1$ by IH and $S \to aSb \to a x_1 b = x$

(ii) $S \to^* x_1$ by IH and $S \to bSa \to b x_1 a = x$

(iii) $S \to^* x_1$, $S \to^* x_2$ by IH and $S \to SS \to^* x_1 S \to^* x_1 x_2 = x$

Qn*: What is a CFG for the set of strings in $\{a, b\}^*$ s.t $\#a's \geq \#b's$?

Q2. Give a CFG for

$$L_2 = \{ x \in \{0,1\}^* \mid x^{rev} = \bar{x} \}$$

If $x = 0101$, $x^{rev} = 1010$, $\bar{x} = 1010$.

Properties: ⊙ x must have even length

⊙ $x_a = \bar{x}_{n-a}$

Grammar: $S \rightarrow \underbrace{\varepsilon}_{\text{Ensures even length of sentence}} \mid \underbrace{1S0 \mid 0S1}_{\text{Ensures even \# of terminals, flipping of bits equidistant from both ends.}}$

Ensures even length of sentence

Ensures even # of terminals, flipping of bits equidistant from both ends.

Q3. What is the language generated by $S \to bS \mid Sa \mid aSb \mid \varepsilon$

$\Sigma = \{a, b\}$
$N = S$

Answer: Σ^*

Induction on length of string. Check base cases.
IH: Any $y \in \Sigma^*$, $|y| < n$ is generated. Take x, $|x| = n$

(i) $x = b\underbrace{x_1}$: $S \to bS \xrightarrow[G]{*} bx_1$
IH: $S \xrightarrow[G]{*} x_1$

(ii) $x = x_1 a$: $S \to Sa \xrightarrow[G]{*} x_1 a$
IH: $S \xrightarrow[G]{*} x_1$

(iii) $x = ax_1 b$: $S \to aSb \xrightarrow[G]{*} ax_1 b$
IH: $S \xrightarrow[G]{*} x_1$

(iv) [Part of base case] $x = \varepsilon$: $S \to \varepsilon$.

Q4. What is a CFG for

(a) $\{a^m b^n \mid m \leq 2n\}$

(b) $\neg\{a^n b^n \mid n \geq 0\} = G$

(a) $S \to aaSb \mid aSb \mid Sb \mid \varepsilon$

(b) $G_1 = \{x \in \{a,b\}^* \mid \exists\, b \text{ is followed by an } a\}$

$(a+b)^* b (a+b)^* a (a+b)^*$ ← what is a CFG?

$G_2 = \{a^m b^n \mid m \neq n \geq 0\}$

$L(G) = L(G_1) \cup L(G_2)$.

Q5. What is the CFG for

$$L_5 = \{ x \in \{a, b\}^* \mid \#a(x) \neq \#b(x) \}$$

Q6. Convert $S \rightarrow bS \mid Sa \mid aSb \mid \varepsilon$ to CNF.