'If Ninaad is elected as the VP, then EITHER Ayushi is chosen as a G-Sec OR Devang is chosen as a Treasurer, but not both. Ayushi is NOT chosen as a G-Sec. Hence, if Ninaad is elected as VP then Devang is chosen as a Treasurer.'
Is this a Tautology? Yes.

① $v \rightarrow \underbrace{(s \wedge \neg t)}_{F} \vee \underbrace{(\neg s \wedge t)}_{True}$

② $\neg s$ ✓

③ $v$

G $\therefore t$ ✓

$\underset{F}{a} \rightarrow b$

Valid

$b \vee \neg b$

| $v$ | $s$ | $t$ | $\underset{F}{①} \wedge \underset{F}{②} \wedge \underset{F}{③} \rightarrow G$ |
|---|---|---|---|
| T | T | T | T |
| ⋮ | ⋮ | ⋮ | ⋮ |
| F | T | T | T |
| | f | | |
| | r | | |

Tautology ✓

In case of a goal is mentioned as MAY or MAY-NOT, how to encode?
"... Therefore Ninaad may nor may not be the VP of Gymkhana."

G: $v \vee \neg v \equiv$ True

$\underbrace{① \wedge ② \wedge ③}_{F/T} \rightarrow \underset{True}{G} \equiv$ True

Clarify meanings of $(a \to b), (a \leftrightarrow b)$. IF/Necessary and ONLY-IF/Sufficiency?

Elaboration on Deduction process to declare a statement Tautology/Valid?

Clarification of notions like Unsatisfiable, Invalid, Satisfiable?

if $a \to b$ then

cause   effect

$\equiv (\neg a \vee b)$

$\equiv$

| $a$ | $b$ | $a \to b$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

$a \leftrightarrow b \equiv$ nec. + suff.

$\equiv (a \to b)$ ← if

$\wedge (b \to a)$ ← only if

$\forall x \to$ True

$(\exists x \to \text{False})$ invalid

$(\forall x \to \text{False})$ unsat.

$(\exists x \to \text{True})$ sat.

In Labyrinth question, how can one define goals to prove tautology or contradiction of overall formula?

| G | M | S | Z |
|---|---|---|---|
| | | | T |
| | | | T |

$(Z \to G)$ is a tautology ?

$Z \to M$ ?

$Z \to S$ ?

What is the difference between $\forall x[P(x) \to Q(x)]$ and $[\forall x[P(x)] \to \forall x[Q(x)]]$ ?

False F True

A : $P(A), Q(A)$
B : $P(B), \neg Q(B)$
C : $\neg P(C), Q(C)$
D : $\neg P(D), \neg Q(D)$
⋮

$\exists x[P(x) \to Q(x)] \equiv \exists x[P(x)] \to \exists x[Q(x)]$ ?

?  1

Is $\forall x[(pass(x) \land scnd(x)) \leftrightarrow \neg wlty(x)] \not\equiv \forall x[pass(x) \to (scnd(x) \leftrightarrow \neg wlty(x))]$ for "Each passenger is in second class if and only if he or she is not wealthy."?

$pass(A) = 0$
$wlty(A) = 0$

$scnd(A) = 1$
$0$

1 → True
← False
↓
False

True

Can different encoding possible for First-order logic? Yes.

"Every passenger either travels in first class or second class."

$\forall x [\text{pass}(x) \rightarrow (\text{frst}(x) \vee \text{scnd}(x))]$

$\underline{\vee}$ ⊕

travel$(x, c)$

mode$(x, y)$

person $x$ travels in class-c.

travel$(x, 1)$

travel$(x, 2)$

Is there a difference between 'Every' and 'Any' in Predicate Logic expressions?

Anyone who scores >80 is 'Ex' | $\forall x$ ? depends $(\exists, \forall)$

$\forall$

if Anyone solves Q2, then I'll move to Q3 +10

$\exists$

# Simple Programs: *Conditional Branching*

$x \qquad y$

## Program (Conditional-Swapping) and Input/Output Assertions

```
0. readInt x; readInt y;   // I: True
1. if x > y, do:           // (Branching-Condition) C: (x>y)
2.      t = x; x = y; y = t;
3. output x; output y;   // O: [(x<=y)&((x'=x)&(y'=y))]V[(x'=y)&(y'=x)]
```

$x' \qquad y'$

## Program Requirement

$\forall x\ \forall y\ \exists x'\ \exists y'\ (\ [\mathrm{True}] \rightsquigarrow [((x \le y) \wedge ((x' = x) \wedge (y' = y))) \vee ((x' = y) \wedge (y' = x))]\ )$

## Formal Program Analysis

Assume that, the initial values of x and y are $\alpha$ and $\beta$, respectively.

$\mathrm{VC(I - C[T] - O)}$: $\mathrm{I} \wedge \mathrm{C} \rightsquigarrow \mathrm{O} \quad \equiv \quad [\mathrm{True} \wedge (\alpha > \beta)] \rightsquigarrow$
$[((\alpha \le \beta) \wedge ((\beta = \alpha) \wedge (\alpha = \beta))) \vee ((\beta = \beta) \wedge (\alpha = \alpha))]$

$\mathrm{VC(I - C[F] - O)}$: $\mathrm{I} \wedge \neg\mathrm{C} \rightsquigarrow \mathrm{O} \quad \equiv \quad [\mathrm{True} \wedge \neg(\alpha > \beta)] \rightsquigarrow$
$[((\alpha \le \beta) \wedge ((\alpha = \alpha) \wedge (\beta = \beta))) \vee ((\alpha = \beta) \wedge (\beta = \alpha))]$

How is the statement $(\alpha = \alpha) \wedge (\beta = \beta)$ similar to $(x' = y) \wedge (y' = x)$?

When the condition $(\alpha > \beta)$ is already true, then why are we considering the case of $(\alpha <= \beta)$after the program progress again?

# Simple Programs: *Looping / Iterations*

## Program (Factorial) and Input/Output Assertions

```
0. readInt n;                    // I: (n>=0)
1. i = 0;  f = 1;
2. loop until i < n, do:         // (Loop-Invariant) L: (f=i!)&(i<=n)
3.     i = i + 1;  f = f * i;    // (Loop-Condition) C: (i<n)
4. output f;                     // O: (f=n!)
```

Your choice

## Program Requirement

$\forall n\ \exists f\ (\ (n \geq 0) \rightsquigarrow (f = n!)\ )$

## Formal Program Analysis

Assume that, the initial value of n is $\gamma$; the current values of i and f are $\alpha$ and $\beta$ (resp.).

$$\text{VC(I} - \text{L)}: \ \text{I} \rightsquigarrow \text{L} \quad \equiv \quad [(\gamma \geq 0)] \rightsquigarrow [(1 = 0!) \wedge (0 \leq \gamma)]$$

$$\text{VC(L} - \text{C[T]} - \text{L)}: \ \text{L} \wedge \text{C} \rightsquigarrow \text{L} \quad \equiv \quad [((\beta = \alpha!) \wedge (\alpha \leq \gamma)) \wedge (\alpha < \gamma)] \rightsquigarrow [(\beta = (\alpha+1)\beta = (\alpha+1)!) \wedge ((\alpha+1) \leq \gamma)]$$

$$\text{VC(L} - \text{C[F]} - \text{O)}: \ \text{L} \wedge \neg\text{C} \rightsquigarrow \text{O} \quad \equiv \quad [((\beta = \alpha!) \wedge (\alpha \leq \gamma)) \wedge \neg(\alpha < \gamma)] \rightsquigarrow [\beta = \gamma!]$$

Please explain this analysis.

Why loop invariant is checked even just before entering into loop?

Which of the following sentences are valid, unsatisfiable, or neither.

$(i)\ Smoke \rightarrow Smoke, \quad (ii)\ Smoke \rightarrow Fire, \quad (iii)\ Smoke \vee \underbrace{Fire \vee \neg Fire}_{T},$

$(iv)\ (Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire),$

$(v)\ (Smoke \rightarrow Fire) \rightarrow (Smoke \wedge Heat \rightarrow Fire)$

(i) $\neg Smoke \vee Smoke \equiv True \longrightarrow$ Valid , Sat.

(ii) $\left.\begin{array}{l} Smoke = T \\ Fire = F \end{array}\right\}$ Invalid. , Sat.

$2^n$

(iii) Valid.

(iv) $\left.\begin{array}{l} Smoke = F \\ Fire = T \end{array}\right\}$ $T \rightarrow F \longrightarrow$ Invalid , Sat.

(v) Valid ?

$(\neg Smoke \vee Fire) \rightarrow (\neg Smoke \vee \neg Heat \vee Fire)$

$\equiv (Smoke \wedge \neg Fire) \vee \neg Smoke \vee \neg Heat \vee Fire$

$\equiv \cdots\cdots \qquad \equiv$ Tautology.

Prove/Disprove: $\forall x[P(x) \to (Q(x) \leftrightarrow R(x))]$ is equivalent to $[\forall x[(P(x) \wedge Q(x)) \to R(x)]] \wedge [\forall x[(P(x) \wedge R(x)) \to Q(x)]]$

Yes.

$$\forall x [A(x)] \wedge \forall x [B(x)] \equiv \forall x [A(x) \wedge B(x)]$$

$$\forall x [\{\neg P(x) \vee \neg Q(x) \vee R(x)\} \wedge \{\neg P(x) \vee \neg R(x) \vee Q(x)\}]$$

$$\equiv \quad \cdots \quad ?$$

Encode and Reason about the following:

"If a scarcity of commodities develops, then the prices rise. If there is a change of government, then fiscal controls will not be continued. If the threat of inflation persists, then fiscal controls will be continued. If there is over-production, then prices do not rise. It has been found that there is over-production and there is a change of government. Therefore, neither the scarcity of commidities has developed, nor there is a threat of inflation."

(1) $scar \rightarrow pric$ ⇢ $\neg scar$ (Modus Tollens)

(2) $gov \rightarrow \neg fisc$

(3) $inf \rightarrow fisc$

(4) $over \rightarrow \neg pric$

(5) $over \wedge gov$

$\neg pric$ ✓

(6) $\therefore \neg scar \wedge \neg inf$ ✓ ✓

(5), (2) (M.P.)

$\therefore \neg fisc$, (3) (M.T)

$\therefore \neg inf$

Encode the following statements and deduce:

"No man who is a candidate will be defeated if he is a good campaigner. Any man who runs for office is a candidate. Any candidate who is not defeated will be elected. Every man who is elected is a good campaigner. Therefore, Any man who runs for office will be elected if and only if he is a good campaigner."

① $\forall x\,[\,cand(x) \land camp(x) \rightarrow \neg def(x)\,]$

$\neg \exists x\,[\,cand(x) \land camp(x) \land def(x)\,]$ ← equiv.

② $\forall x\,[\,off(x) \rightarrow cand(x)\,]$

③ $\forall x\,[\,cand(x) \land \neg def(x) \rightarrow elect(x)\,]$

④ $\forall x\,[\,elect(x) \rightarrow camp(x)\,]$

⑤ $\forall x\,[\,off(x) \rightarrow (elect(x) \leftrightarrow camp(x))\,]$

$\forall x\,[\,[off(x) \land elect(x) \rightarrow camp(x)\,] \land [\,off(x) \land camp(x) \rightarrow elect(x)\,]\,]$

cand(x)
camp(x)
def(x)
off(x)
elect(x)

A

Encode the following sentences and deduce:
Jack owns a dog. Every dog owner is an animal lover. No animal lover kills an animal. Either Jack or Curiosity killed Tuna, which is a cat. Did Curiosity kill the cat?

① $\exists x\,[\,own(Jack, x) \wedge dog(x)\,]$

② $\forall x\, \exists y\, [\,(own(x,y) \wedge dog(y)) \rightarrow \forall z\,(animal(z) \rightarrow love(x,z))\,]$

$\forall x\, \exists y\, \forall z\, [\,own(x,y) \wedge dog(y) \wedge animal(z) \rightarrow love(x,z)\,]$

③ $\forall x\, \forall y\, [\,animal(y) \wedge love(x,y) \rightarrow \neg Kill(x,y)\,]$

④ $Cat(Tuna) \wedge [\,Kill(Jack, Tuna) \vee Kill(Curiosity, Tuna)\,]$

⑤ $\therefore\; Kill(Curiosity, Tuna)$

$own(x,y)$
$dog(x)$
$cat(x)$
$animal(x)$
$love(x,y)$
$Kill(x,y)$

Encode the following statements: (Is it a Tautology/Valid statement?)
(i) All members are both officers and gentlemen, (ii) All officers are fighters, (iii) Only a pacifist is a gentleman or not a fighter, (iv) No pacifist is a gentleman if he is a fighter, (v) Some members are fighters iff they are officers. (G) Thus, not all members are fighters.

HOMEWORK