Doubts

Use the Pumping Lemma to prove that { w ∈ {a,b}* | #a(w) = #b(w) } is not regular.

$k \to$ PLC

$\alpha = a^k b^k = xyz$ with $|y| \geq k$

$x \quad y \quad z$

$y = uvw$

$x \quad y \quad z$

non-empty

$x \quad y \quad z$

$|uv| \leq k$

You

Demon

k

$\alpha = xyz$

$y = uvw$

$v = a^l, \quad 0 < l \leq k$

i

$a^{k-l} \; a^l \; b^l \; b^{k-l}$

not used

$a^k b^k$

$x \quad y \quad z$

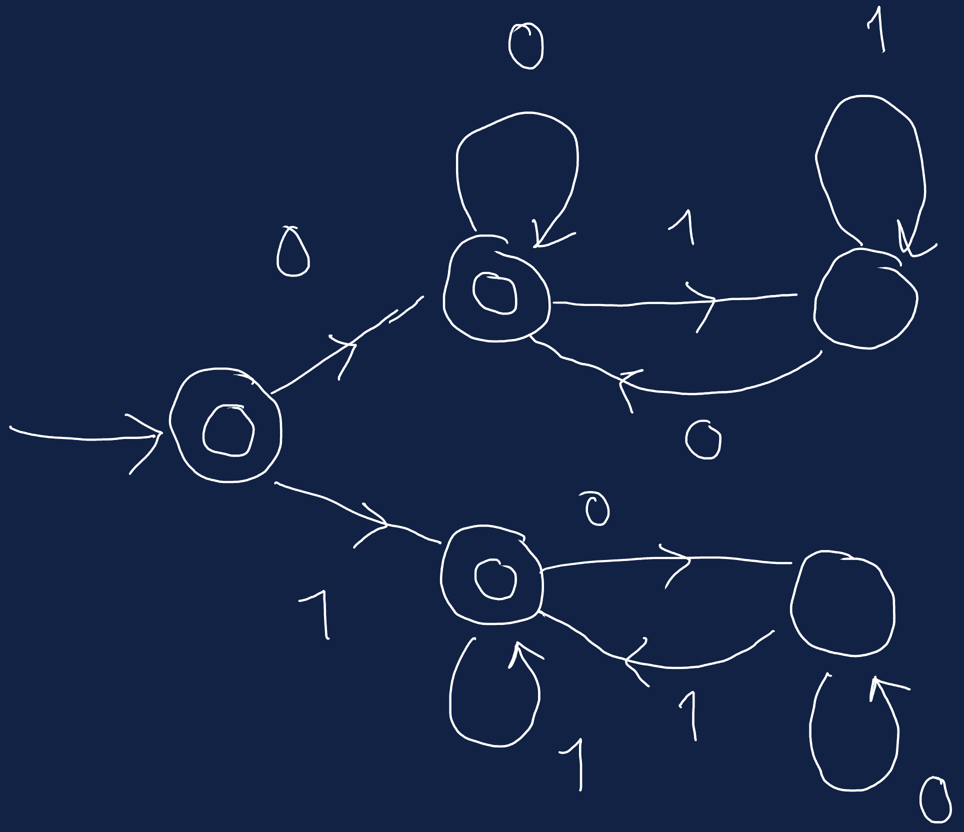
$v = a^l b^l$

1. One of the following languages over {0,1} is regular, the other is not. Justify.

A = { w | w contains an equal number of occurrences of 0 and 1 }. → No

B = { w | w contains an equal number of occurrences of 01 and 10 }. → Yes

counting 01's and 10's is not necessary.



2. One of the following sets is regular, the other is not. Justify.

$A = \{\ xy \mid \#a(x) = \#b(y)\ \} \subseteq \{a,b\}^*$.
$B = \{\ x\$y \mid \#a(x) = \#b(y)\ \} \subseteq \{a,b,\$\}^*$.

→ not regular

$\underbrace{a^k}_{y}\ \underbrace{\$\ b^k}_{z}$, x

Prove $A = \{a,b\}^*$

$w \in \{a,b\}^*$

$w = \epsilon = \epsilon\epsilon$

$\boxed{x \mid y}$ a, b

$\underline{y = \epsilon}$

$\underline{x = b^k}$ $b^k \in b$

$a^{k-l}\ b^k$

$= \underline{a^{k-l}\ b^l} \mid \underline{b^{k-l}}$

$\boxed{x, ya}$

shift the partition one cell right

A is not regular

$\underline{PL}$ $\underbrace{a^k}_{y}\ \underbrace{b^k}_{z}$, x

$a^{k+il}\ b^k$

$a^{k+l}\ b^k$

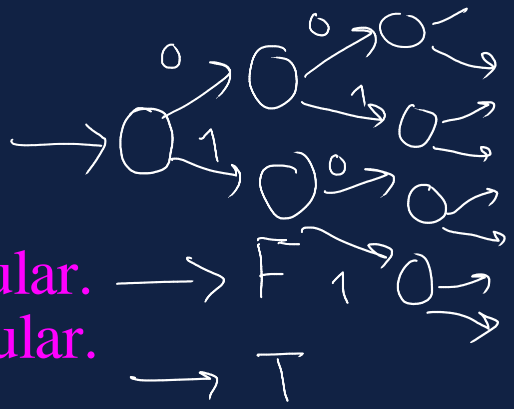
$\underline{a^k} \mid \underline{a^l\ b^k}$

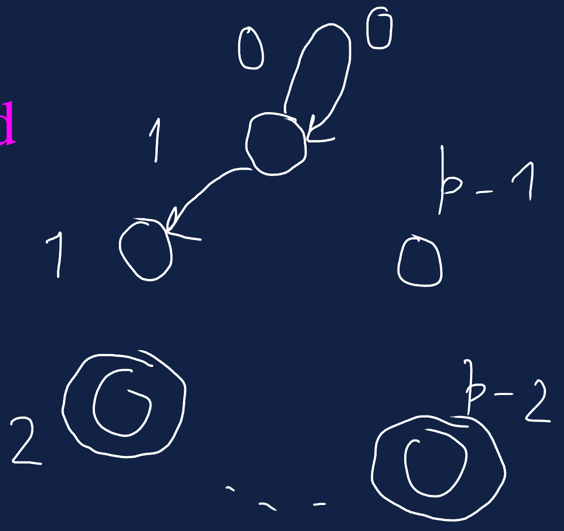
3. Let A ⊆ N be a set of positive integers. Define

binary(A) = {binary representations of elements of A} ⊆ {0,1}*, and
unary(A) = {0^n | n ϵ A} ⊆ {0}*.

Prove/Disprove:

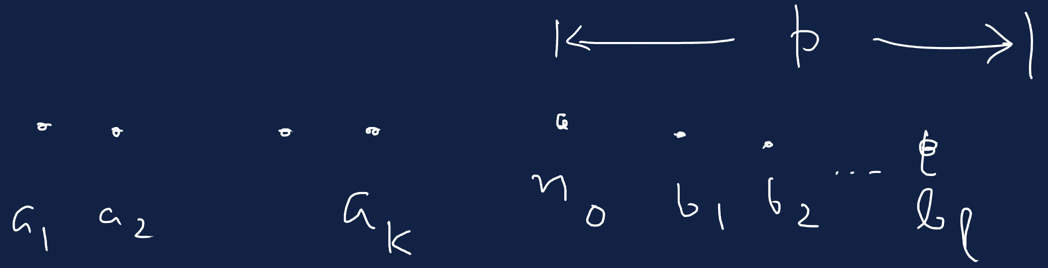
(a) If binary(A) is regular, then unary(A) is regular.
(b) If unary(A) is regular, then binary(A) is regular.



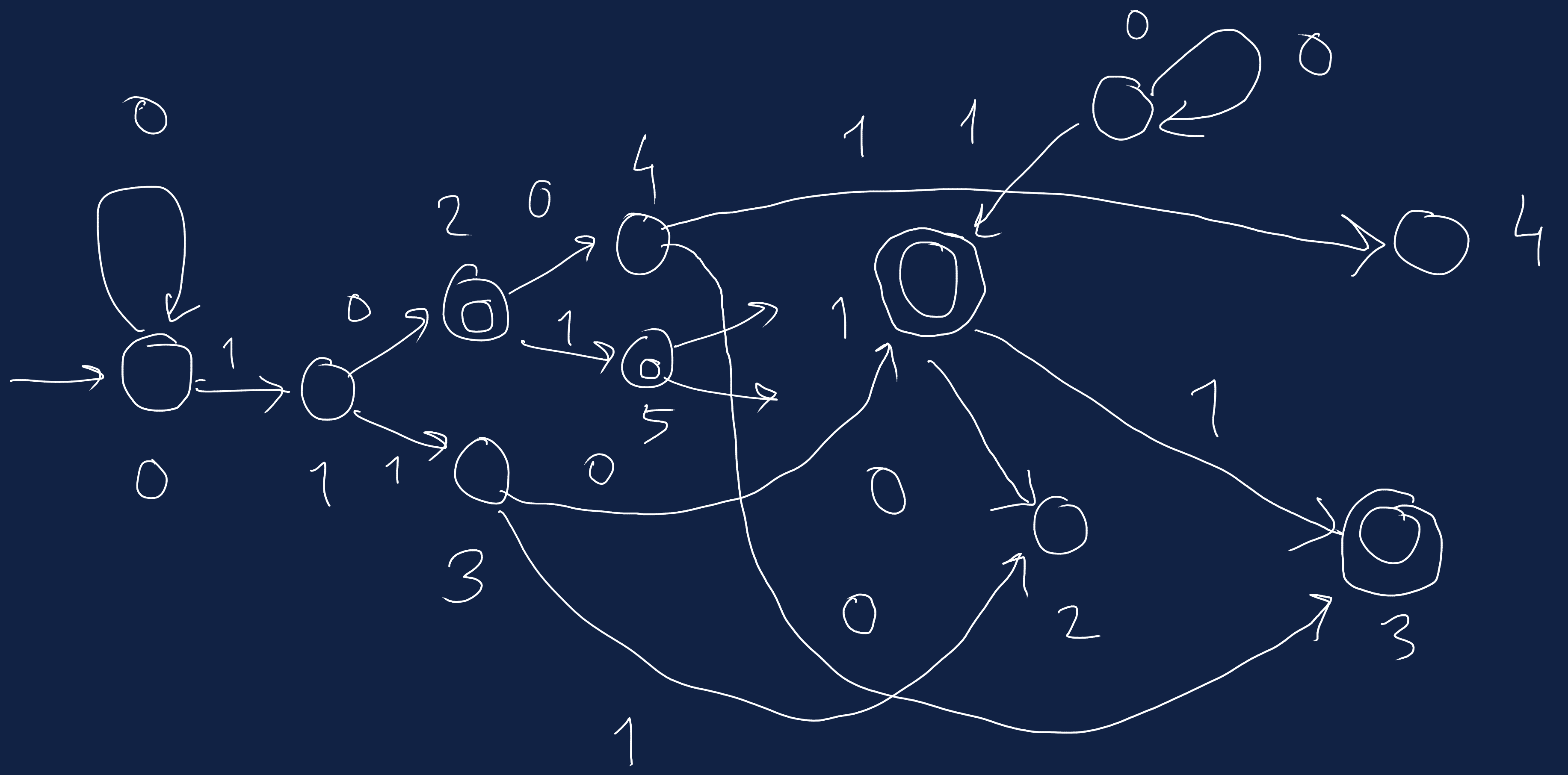


A is ultimately periodic.

$$A = \underline{\{a_1, a_2, \dots, a_k\}} \cup \{n \mid n \equiv b_1, b_2, \dots, b_\ell \pmod p,\ \underline{n \geq n_0}\}$$



$$A = \{2, 5\} \cup \{n \mid n \equiv 1, 3 \pmod 5, \; n \geq 6\}$$



(a) $A = \{ 2^n \mid n \geq 0 \}$

$\text{binary}(A) = \mathcal{L}(0^* 1 0^*)$

$\text{unary}(A) \longrightarrow$ not regular

4. Prove that no infinite subset of {a^nb^n | n ⩾ 0} is regular.

Let L be an infinite subset of the given set.

Suppose that L is regular.

Let k be a PLC for L.

L infinite

$\Rightarrow$ L contains strings of length $\geq k$

$2k$

Take $a^m b^m \in L$ with $m \geq k$.

Then proceed as before.

5. Prove/~~Disprove~~: There exists a language $L \subseteq \{a,b\}^*$ such that no infinite subset of L or its complement is regular.

$$L = \{a^n b^n \mid n \geq 0\}$$

$\sim L$ contains infinite regular subsets

like $\mathcal{L}(a^+)$, $\mathcal{L}(ba^*)$, ...

Hint: Use ultimate periodicity.