Tutorial

Q1. Design an unrestricted grammar for

$\{ ww \mid w \in \{a,b\}^* \}$ $\qquad w = a^{i_1} b^{i_2} a^{i_3} \ldots$

$aA \quad bB \qquad (aA)^{i_1}(bB)^{i_2}(aA)^{i_3} \ldots T$

$S \to aAS \mid bBS \mid T$

$Aa \to aA, \quad Ab \to bA$

$Ba \to aB, \quad Bb \to bB$

$AT \to Ta$

$BT \to Tb$

$T \to \varepsilon$

Q2. Design an unrestricted grammar for

$$\{ \omega \in \{a,b,c\}^* \mid \#a(\omega) = \#b(\omega) = \#c(\omega) \}$$

Not a CFL : Use PL. PL constant is n

Take $z = a^n b^n c^n$

Unrestricted Grammar:

aabcbc

AABCBC

$S \rightarrow ABCS \mid \varepsilon$

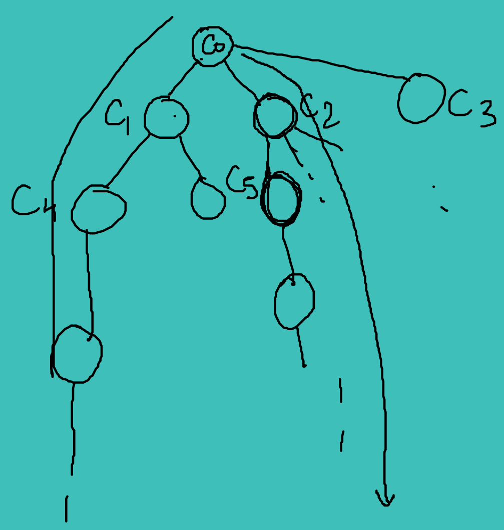
$\forall\ U \neq V \in \{A,B,C\}$

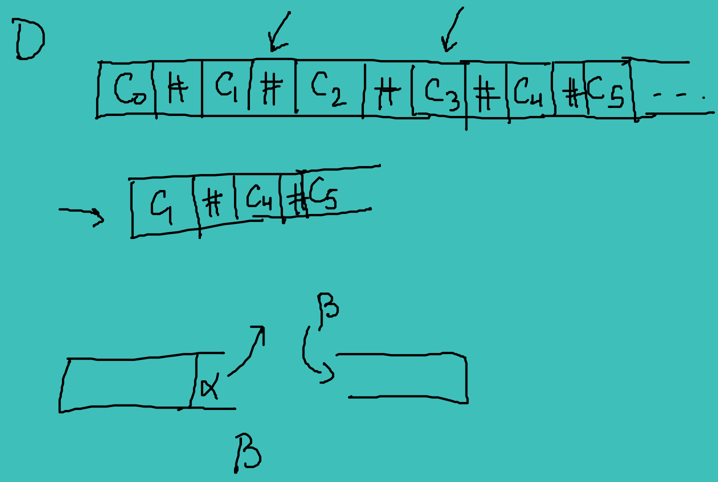
$\left.\begin{array}{l} UV \rightarrow VU \\ VU \rightarrow UV \end{array}\right]$ Shuffling

$A \rightarrow a,\ B \rightarrow b,\ C \rightarrow c.$

Q3. Let D be a DTM simulating an NTM N. Suppose I replace D's tape by a stack - what are the problems the simulation will face?

* If I replace with 2 stacks?





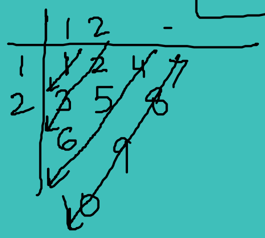
Q4. Bijection $f: \mathbb{N}\times\mathbb{N} \to \mathbb{N}$ that is polynomial in input (i,j).

$$\frac{(i+j-2)(i+j-1)}{2} + i$$

$(1,1) \to 1$

$(1,2) \to 2$

$(2,1) \to 3$



✓ $1 + \frac{1}{2}\left((i+j)^2 - i - 3j\right)$

Q5. Counting constant c efficiently.

→ Binary representation of c

Prime factorization of c

Generate a^{2^i} $2i$

aaaa aaaa

1000 >

logc bit string

$\boxed{1001000101 00}$

$c = \underbrace{2^{i_1} + 2^{i_2} + \cdots + 2^{i_l}}$

$2(i_1 + i_2 + \cdots i_l)$

$\leq \log c \frac{(\log c + 1)}{2}$

$\boxed{\log c (\log c + 1)}$