0. Let L be a CFL over some alphabet Γ. Prove that the language

cyclicshift(L) = { yx | xy ∈ L }

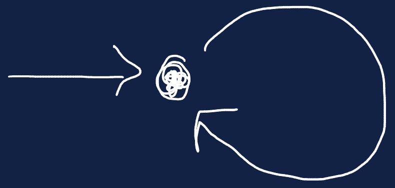
is also a CFL.

$\overset{x}{baa} \mid \overset{y}{bbaba} \in L$

$\overset{y}{bbaba} \mid \overset{x}{baa}$

$L = \{ x \in \{a, b\}^* \mid \#a(x) = \#b(x) \}$

$E \to aEb \mid bEa \mid EE \mid \epsilon$



$a, a / \epsilon$

$b, b / \epsilon$

$\epsilon, E / aEb$

$\epsilon, E / bEa$

$\epsilon, E / EE$

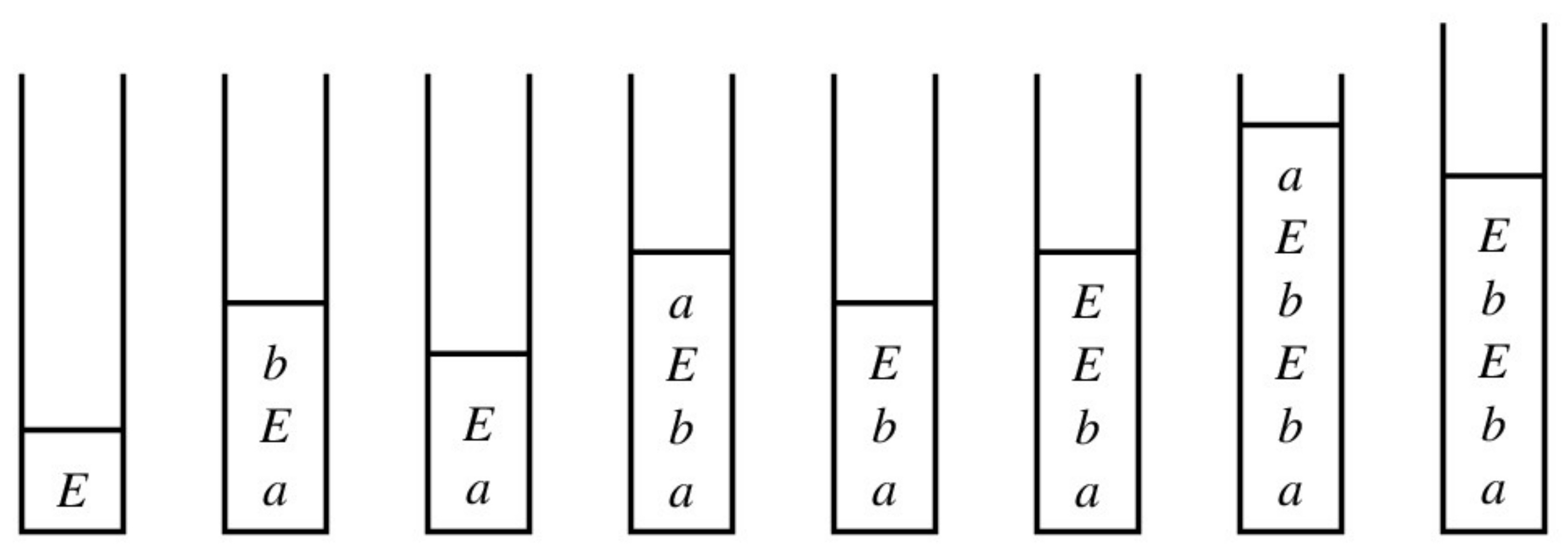
$\epsilon, E / \epsilon$

$(\{*\}, \{a, b\}, \{a, b, E\}, E, \delta, *, \emptyset)$

Accepts by empty stack

Work of M

b a a

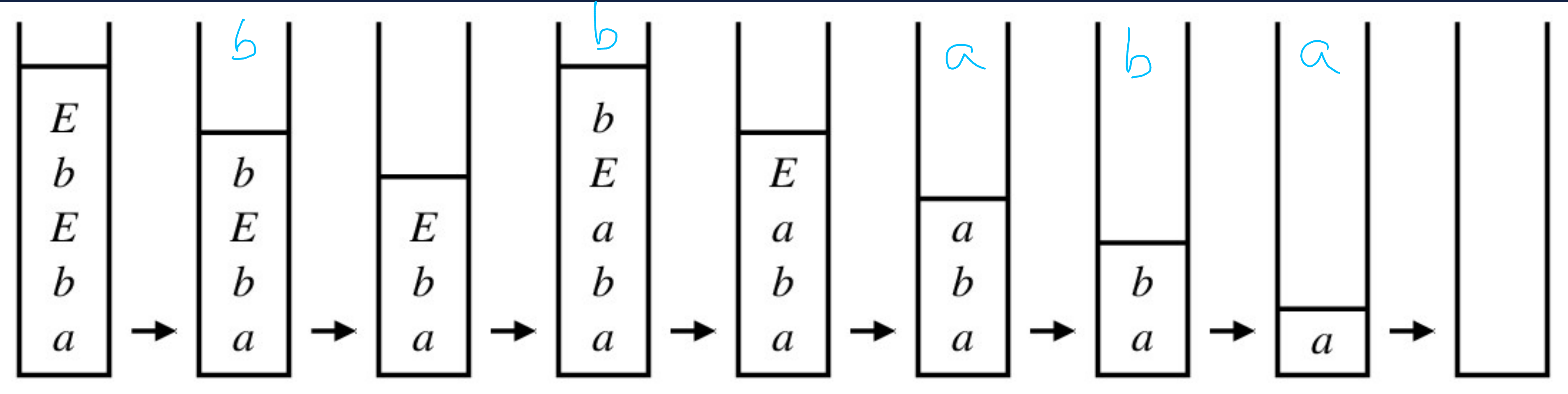


E → bEa E → aEb E → EE E → aEb

M has read x

baa

starts working on y



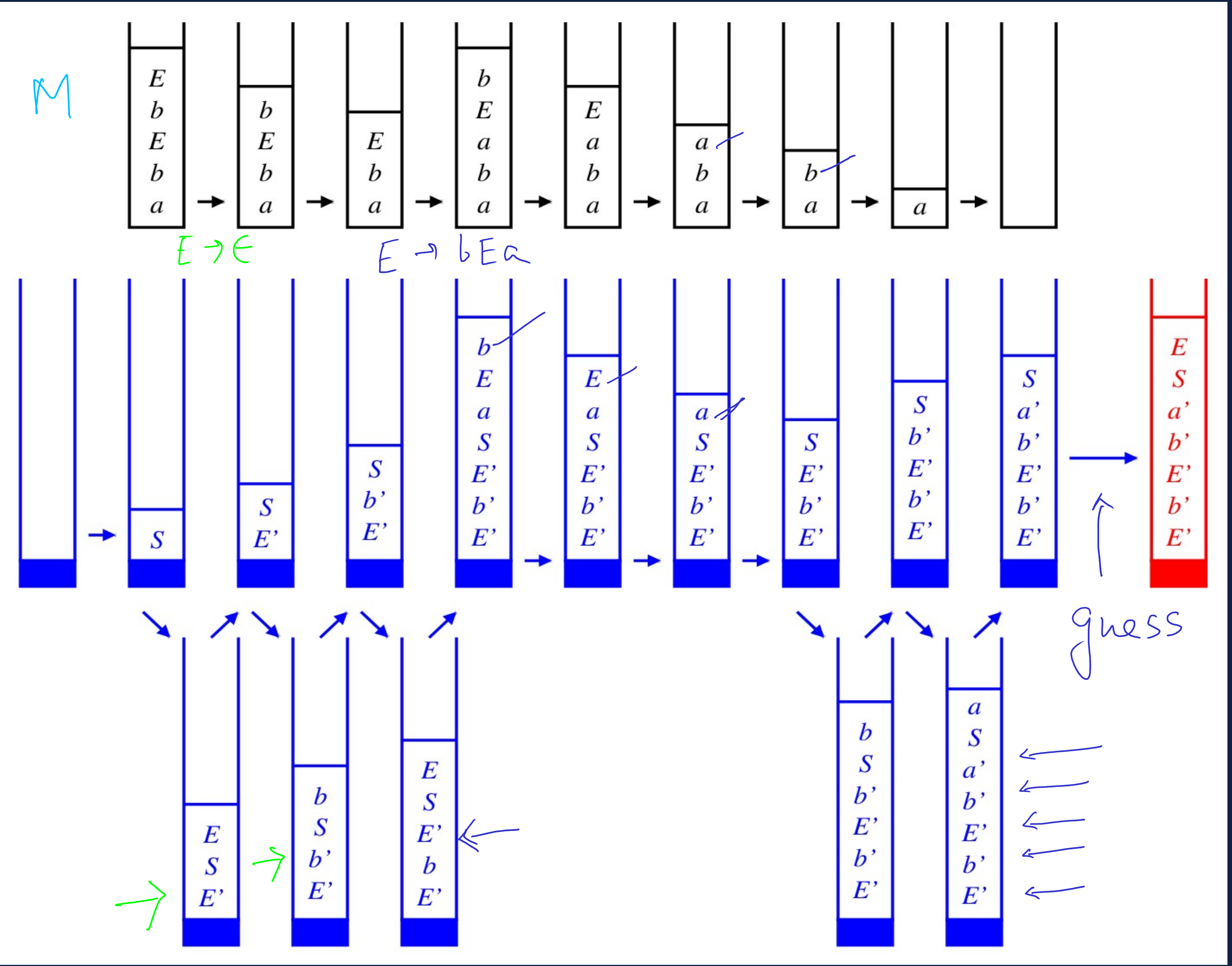
E → E E → bEa E → E

M reads y

bbaba

M'

Guess phase

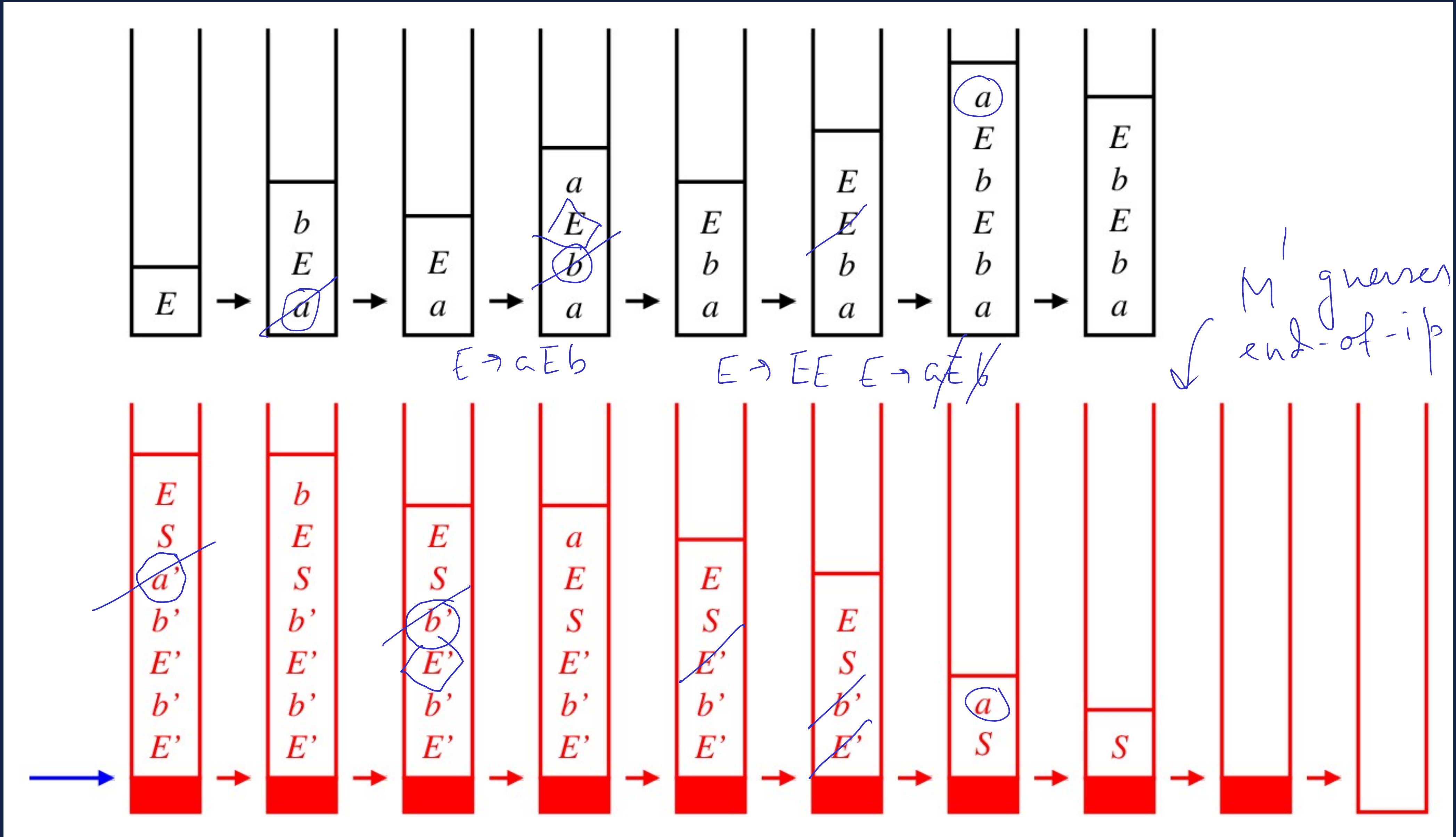


Validation phase

G → V phase can be taken when S is exposed.

M

M'



1. Note on: RL $\subsetneq$ DCFL $\subsetneq$ UCFL $\subsetneq$ CFL.

all CF

regular

deterministic

unambiguous

don't use the stack

DPDA $\longrightarrow$ DPDA (1 state)

$\downarrow$

CFG

$\delta(p, a) = q$

$(p, a, \perp), (q, \perp)$

$x \in L$

1) $\{a^n b^n\}$

2) $\{$ palindromes over $\{a, b\}\}$

$S \to \epsilon \mid a \mid b \mid aSa \mid bSb$

$a^n b^n c^n$

3) $\{a^i b^j c^k \mid i = j \text{ or } j = k\}$

2. Prove that the following grammar for strings with balanced parentheses is unambiguous.

$S \to (S) S \mid \varepsilon$

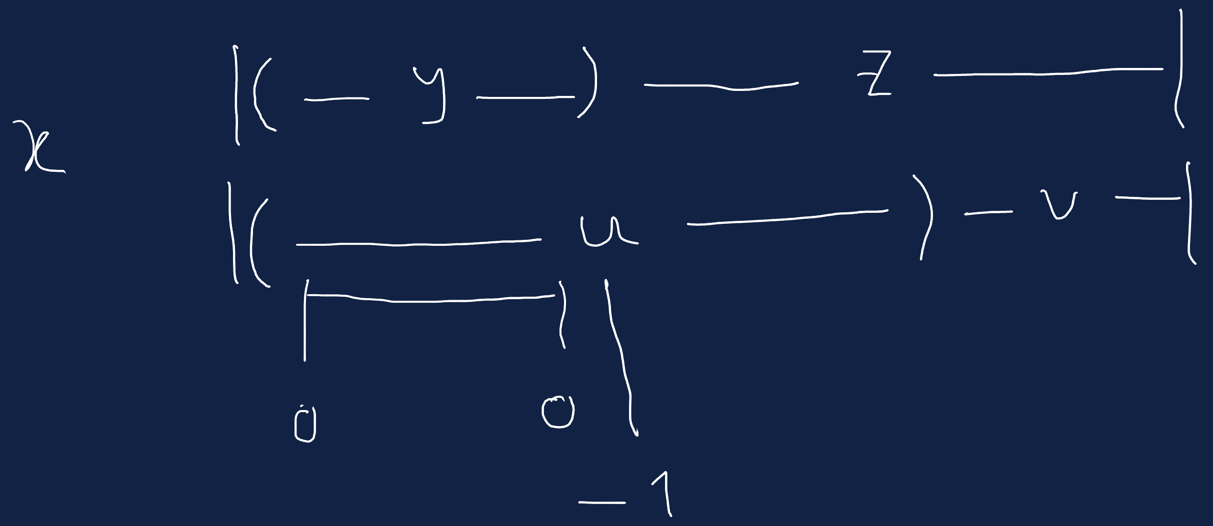
Let x be a <u>shortest</u> string with b.p. that has two leftmost derivations

$x \neq \varepsilon$

$x = (y) z = (u) v$

$y \neq u$

$z \neq v$



$u \in L(S)$

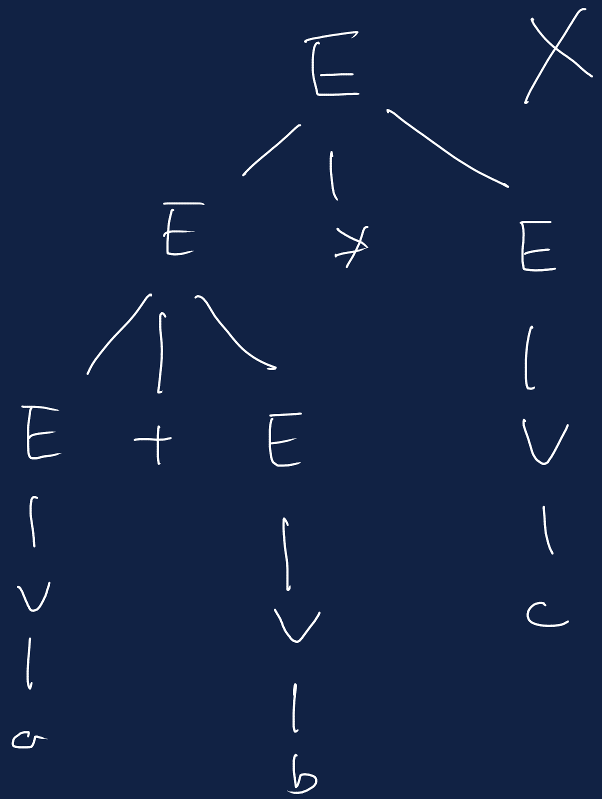
$(\to +1$

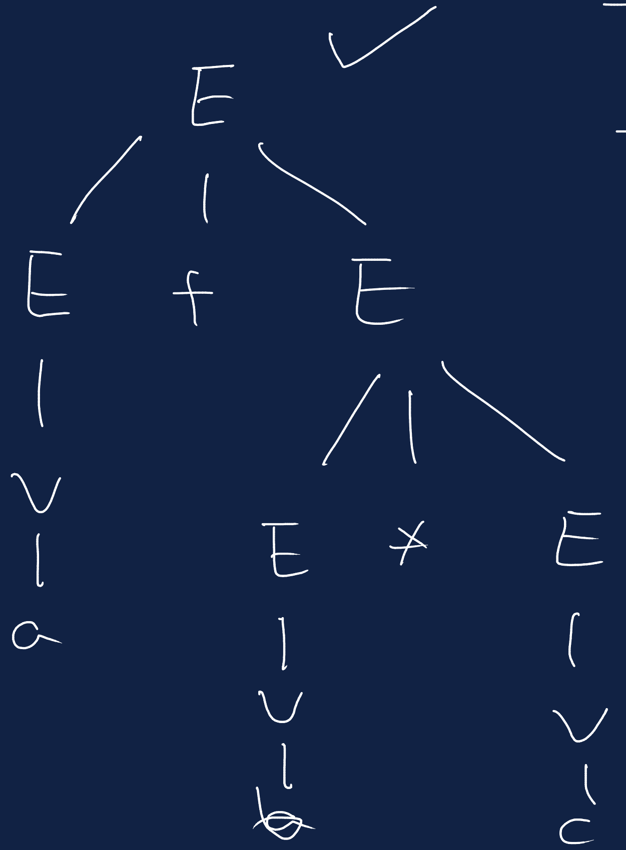
$) \to -1$

3. Consider the grammar for expressions involving sums and products:

$$E \rightarrow V \mid E + E \mid E * E \mid (E)$$
$$V \rightarrow a \mid b \mid c$$

(a) Prove that this grammar is ambiguous.





$a + b * c$

— $(a + b) * c$ ✗

— $a + (b * c)$ ✓

E

BODMAS

of

(b) Disambiguate this grammar.

$$E \to T \mid T + E$$

$$T \to F \mid F * T$$

$$F \to V \mid (E)$$

$$V \to a \mid b \mid c$$

$$a + b * c$$

$$E \to T_1 + \underline{T_2 + \dots + T_k}$$

$T_1 \to$ a product of factors

$$T = F_1 * \underline{F_2 * \dots * F_l}$$

$$F_1 = a + b$$

$$(a + b) * \dots$$

$$(a) * b$$

4. Prove that the language

$L_8 = \{ x \in \{a,b\}^* \mid \#a(x) = \#b(x) \}$

is a DCFL.

pushing and popping +, −

While reading ⊣, ⊥ must be exposed.

$\sim L_8$ ⊣, either + or − must be on the top of the stack.

5. Design an unambiguous CFG for L_8.

Hint: Q2

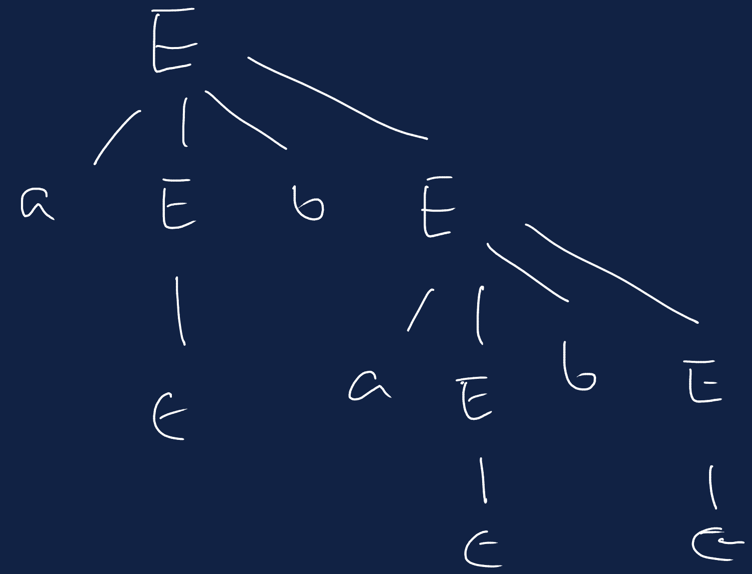
$E \rightarrow aEb \mid bEa \mid EE \mid \epsilon$

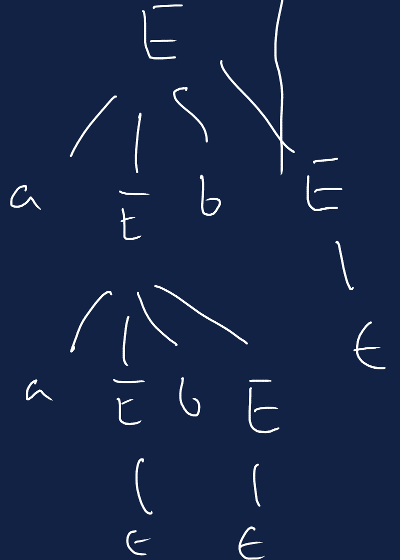
not unambiguous

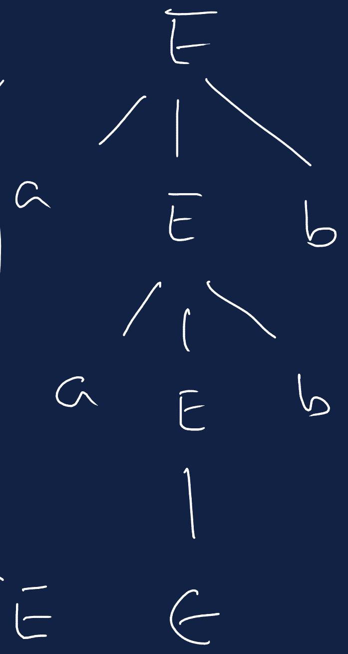
abab

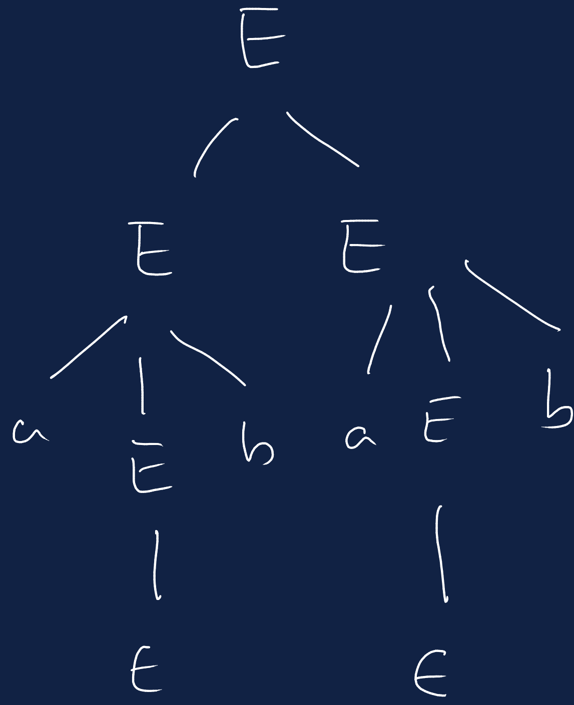
Goal: first b to match a

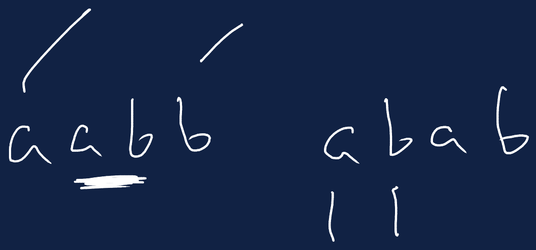
$E \rightarrow aEbE \mid bEaE \mid \epsilon$











6. Let G be a CFG over Σ. Prove that there exists an algorithm (may be inefficient) that, given G and a string $x \in \Sigma^*$, decides whether $x \in L(G)$ or not.

Explore all choices

$E \to aEb \mid bEa \mid EE \mid \epsilon$

$E \to EE \to EEE \to \cdots \to E^{l+1} \to E^{l} \to$

$\cdots \to E$

Convert to CNF.

$|x| = l > 0$ $\quad l-1$ $\quad A \to BC$

$N_1 \to \epsilon$ $\quad l$ $\quad A \to a$

$x = \epsilon$

$N_2 \to \epsilon$

$A \to N_{i_1} N_{i_2} \cdots N_{i_l}$ $\quad 2l-1$

$A N_k \to \epsilon$

7. Try to prove using the pumping lemma that the language

$$\{ a^m b^{m+n} c^n \mid m,n \geqslant 0 \text{ and } m \neq n \}$$

is not context-free.

Demon : $k = 4$