1. Let A and B be uncountable sets with $A \subseteq B$. Prove or disprove: A and B are equinumerous.

2. Let A be an uncountable set and B a countably infinite subset of A. Prove/Disprove: A is equinumerous with A–B.

True
$$|A-B| \le |A|$$
 $|A| \le |A-B|$
 $A-B$ unconntable $f: A \to A-B$
 $A-B$ in infinite. Injective

 $C \subseteq A-B$ $B = \{b_1,b_2,b_3,\dots\}$

countable $C = \{C_1,C_2,C_3,\dots\}$
 $f(\alpha) = \{C_{2n-1} \text{ if } \alpha = C_n \}$
 $C = \{C_n,C_n,C_n\}$
 $C = \{C_n,C_n\}$
 $C = \{C_n,C_n\}$
 $C = \{C_n,C_n\}$
 $C = \{C_n\}$
 $C =$

3. Prove that the real interval [0,1) is equinumerous with the unit square $[0,1) \times [0,1)$.

$$F = \Omega \cap [0,1)$$

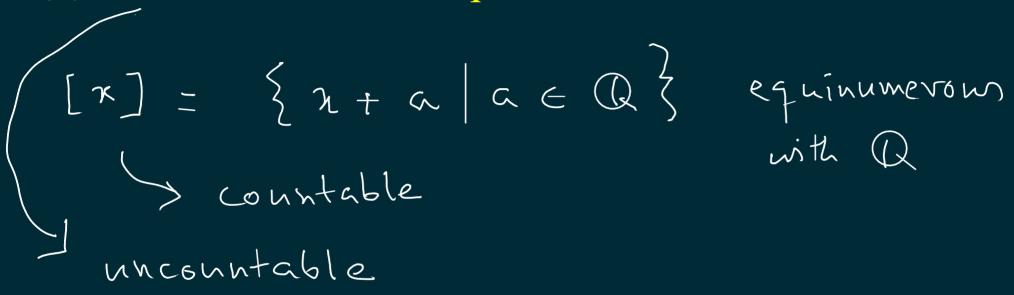
$$\beta = [0,1) \times [0,1) - F^2$$

$$f: B \rightarrow A$$

$$(o, a_1 a_2 a_3 \cdots) \quad (o, a_1 b_2 b_3 \cdots)$$

$$(o, a_1 b_2 b_3 \cdots)$$

- 4. Define a relation ~ on IR such that a ~ b if and only if a b ∈ Q.(a) Prove that ~ is an equivalence relation.
 - (b) Is the set IR/~ of all equivalence classes of ~ countable?



5. Let Z[x] denote the set of all univariate polynomials with integer coefficients. Prove that Z[x] is countable.

deg(f) = d

deg(f) = d

deg(f) = d

countable

countable

d>0

$$f(x) = \{0\}$$
 $f(x) = \{0\}$

6. A real or complex number a is called algebraic if f(a) = 0 for some non-zero $f(x) \in Z[x]$. Let A denote the set of all algebraic numbers. Prove that A is countable.

- 7. Let Z[x,y] be the set of all bivariate polynomials with integer coefficients.
 - (a) Prove that Z[x,y] is countable. Similar to Z[x]
 - (b) Let

$$V = \{(a,b) \in C \times C \mid f(a,b) = 0 \text{ for some nonzero } f(x,y) \in Z[x,y]\}$$

Is V countable?

V contains the uncountable nubset
$$\{(a,a) \mid a \in I\}$$

- 8. A set $S \subseteq IR$ is called bounded if S has both a lower bound and an upper bound. Countable/Uncountable?
 - (a) The set of all bounded subsets of Z.
 - (b) The set of all bounded subsets of Q.

Let A be a bounded set. Let l be a lower bound, and u an upper bound. Let $B = [l, u] \cap S$. A can be any subset of B. For S = Z, B is finite. For S = Q, B is countable (and infinite if l < b).