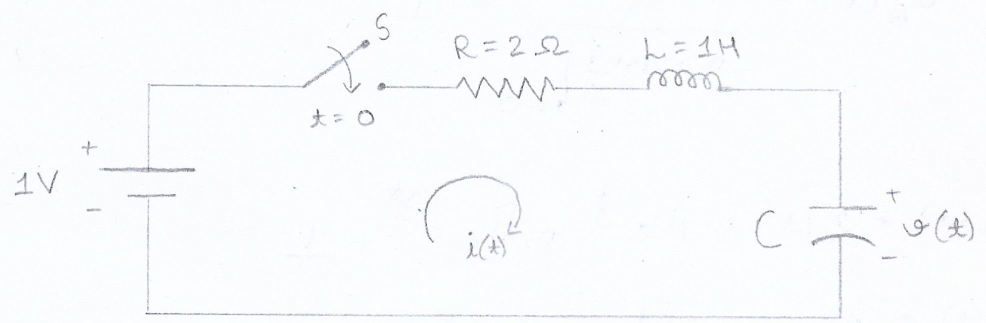
NISARG UPADHYAYA
19CS30031

EXPERIMENT 2 QUESTION 1



$i(0^-) = 0$

$v(0^-) = 0$

From KVL we have $1 = i(t)R + L\dfrac{di(t)}{dt} + v(t)$ — ①

For capacitor $q(t) = C\,v(t)$

$\dot{q}(t) = i(t) = C\dfrac{dv(t)}{dt}$ — ②

$\ddot{q}(t) = \dfrac{di(t)}{dt} = C\dfrac{d^2v(t)}{dt^2}$ — ③

Substituting ② & ③ in ① we have

$$C\frac{d^2v(t)}{dt^2} + 2C\frac{dv(t)}{dt} + v(t) = 1$$

$$\boxed{\Rightarrow \frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + \frac{v(t)}{C} = \frac{1}{C}}$$

(i) $C = 25/9$ F

Differential eq^n $\longrightarrow$ $\dfrac{d^2v(t)}{dt^2} + 2\dfrac{dv(t)}{dt} + \dfrac{9}{25}v(t) = \dfrac{9}{25}$

Characteristic eq^n $\longrightarrow$ $m^2 + 2m + \dfrac{9}{25} = 0 \Rightarrow \left(m+\dfrac{1}{5}\right)\left(m+\dfrac{9}{5}\right) = 0$

$m = \dfrac{-1}{5}, \dfrac{-9}{5}$ $\Rightarrow$ Roots are REAL & DISTINCT.

NISARG UPADHYAYA
19CS30031

Natural response will be of the form

$$v_n(t) = A e^{-\frac{t}{5}} + B e^{-\frac{9t}{5}}$$

Forced response $\quad v_f(t) = \dfrac{\frac{9}{25}}{m^2 + 2m + \frac{9}{25}\Big|_{m=0}} = 1$

$$\therefore,\ v(t) = v_n(t) + v_f(t) = A e^{-\frac{t}{5}} + B e^{-\frac{9t}{5}} + 1$$

$$i(t) = C\frac{dv(t)}{dt} = \frac{25}{9}\left(\frac{-A}{5} e^{-\frac{t}{5}} - \frac{9B}{5} e^{-\frac{9t}{5}}\right)$$

$$v(0^-) = A + B + 1 = 0 \Rightarrow A + B = -1$$

$$i(0^-) = -\frac{25}{9}\left(\frac{A}{5} + \frac{9B}{5}\right) = 0 \Rightarrow A = -9B$$

$$\Rightarrow A = -\frac{9}{8},\ B = \frac{1}{8}$$

$$v(t) = 1 - \frac{\left(9e^{-\frac{t}{5}} - e^{-\frac{9t}{5}}\right)}{8}$$

$$i(t) = \frac{5}{8}\left(e^{-\frac{t}{5}} - e^{-\frac{9t}{5}}\right) \text{ A}$$

(ii) $C = \frac{1}{2}$ F

Differential eq^n $\longrightarrow$ $\dfrac{d^2 v(t)}{dt^2} + \dfrac{2dv(t)}{dt} + 2v(t) = 2$

Characteristic eq^n $\longrightarrow$ $m^2 + 2m + 2 = 0 \Rightarrow (m+1)^2 = -1$

$$\Rightarrow m + 1 = \pm j$$

$\Rightarrow m = -1+j, -1-j \Rightarrow$ Roots are COMPLEX & CONJUGATE of each other

$$Re(m) = -1,\ |Im(m)| = 1$$

NISARG UPADHYAYA
19CS30031 Nisarg

Natural response will be of the form

$$v_n(t) = e^{-t}(A\cos t + B\sin t)$$

Forced response $v_f(t) = \dfrac{2}{m^2+2m+2\big|_{m=0}} = 1$

$\therefore$, $v(t) = 1 + e^{-t}(A\cos t + B\sin t)$

$$i(t) = C\frac{dv(t)}{dt} = \frac{e^{-t}}{2}\left((B-A)\cos t - (B+A)\sin t\right)$$

$$v(0^-) = 1 + A = 0 \Rightarrow A = -1$$

$$i(0^-) = \frac{B-A}{2} = 0 \Rightarrow B = A = -1$$

$$v(t) = 1 - e^{-t}(\cos t + \sin t)$$

$$= 1 - \sqrt{2}\,e^{-t}\sin\left(t + \frac{\pi}{4}\right) \text{ V}$$

$$i(t) = e^{-t}\sin t \text{ A}$$

(iii) $C = 1F$

Differential eq^n $\longrightarrow$ $\dfrac{d^2v(t)}{dt^2} + 2\dfrac{dv(t)}{dt} + v(t) = 1$

Characteristic eq^n $\longrightarrow$ $m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$

$\Rightarrow m = -1, -1 \Rightarrow$ Roots are REAL & REPEATED

Natural response will be of the form

$$v_n(t) = Ae^{-t} + Bte^{-t}$$

Forced response $v_f(t) = \dfrac{1}{m^2+2m+1\big|_{m=0}} = 1$

NISARG UPADHYAYA
19CS30031 Nisarg

$\therefore,\ v(t) = v_n(t) + v_f(t)$

$= Ae^{-t} + Bte^{-t} + 1$

$$i(t) = C\frac{dv(t)}{dt} = (B-A)e^{-t} - Bte^{-t}$$

$v(0^-) = A + 1 = 0 \Rightarrow A = -1$

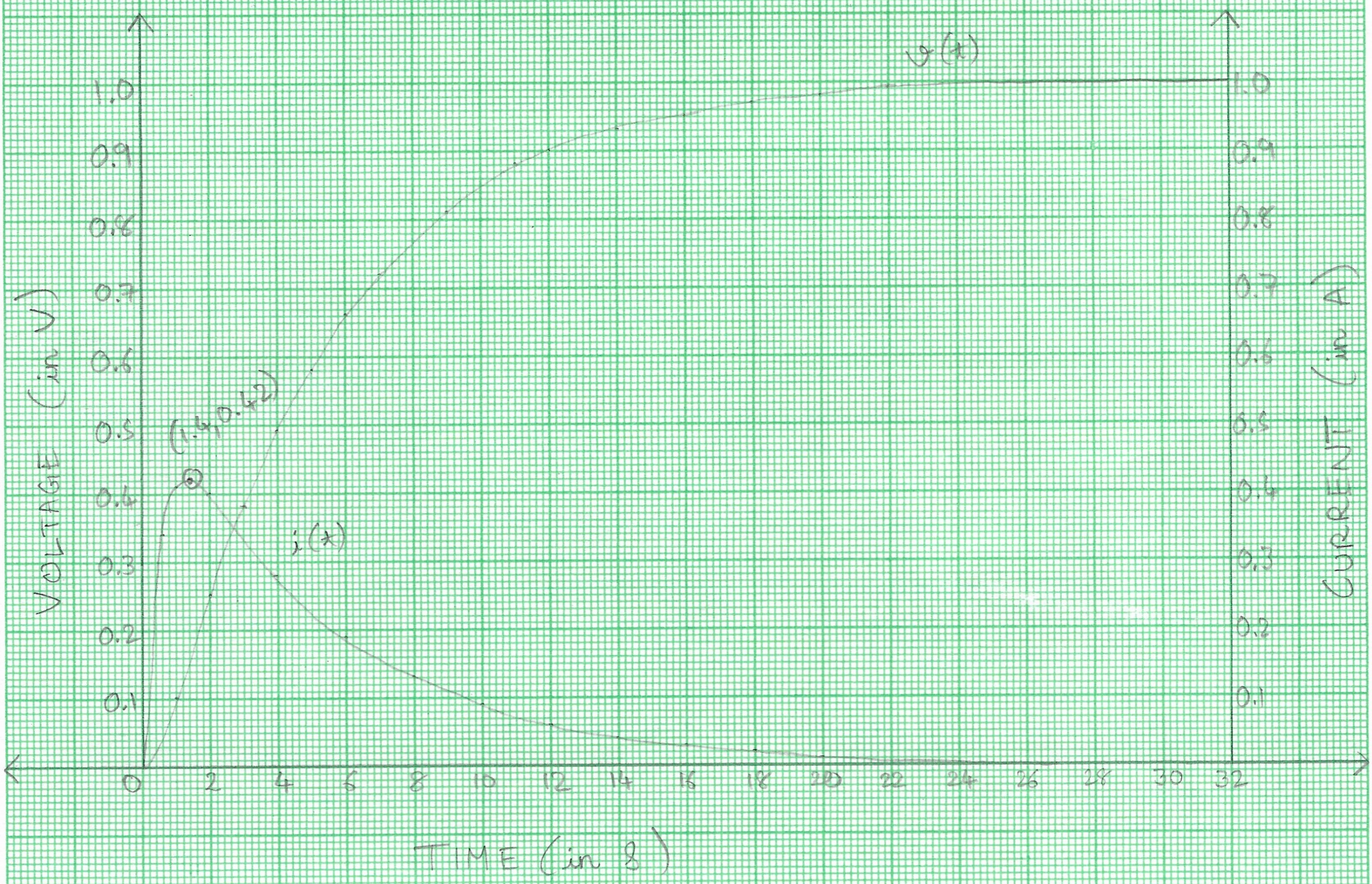
$i(0^-) = B - A = 0 \Rightarrow B = A = -1$

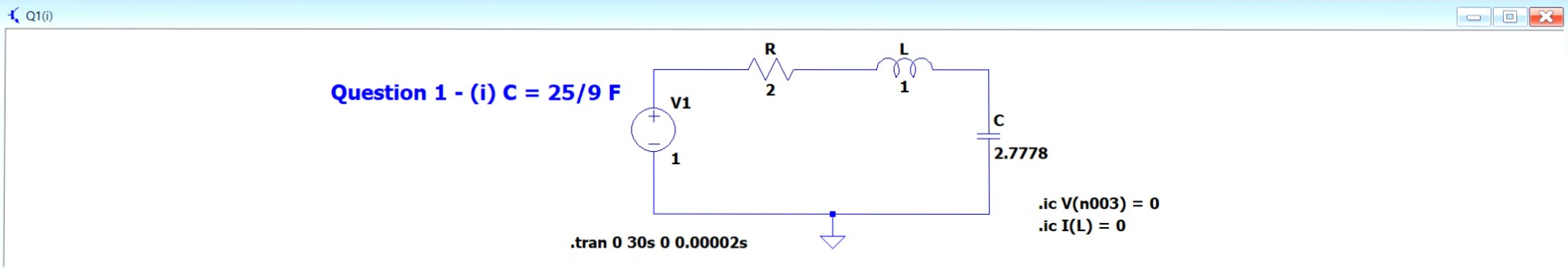
✓ $v(t) = 1 - e^{-t}(1+t)$ V

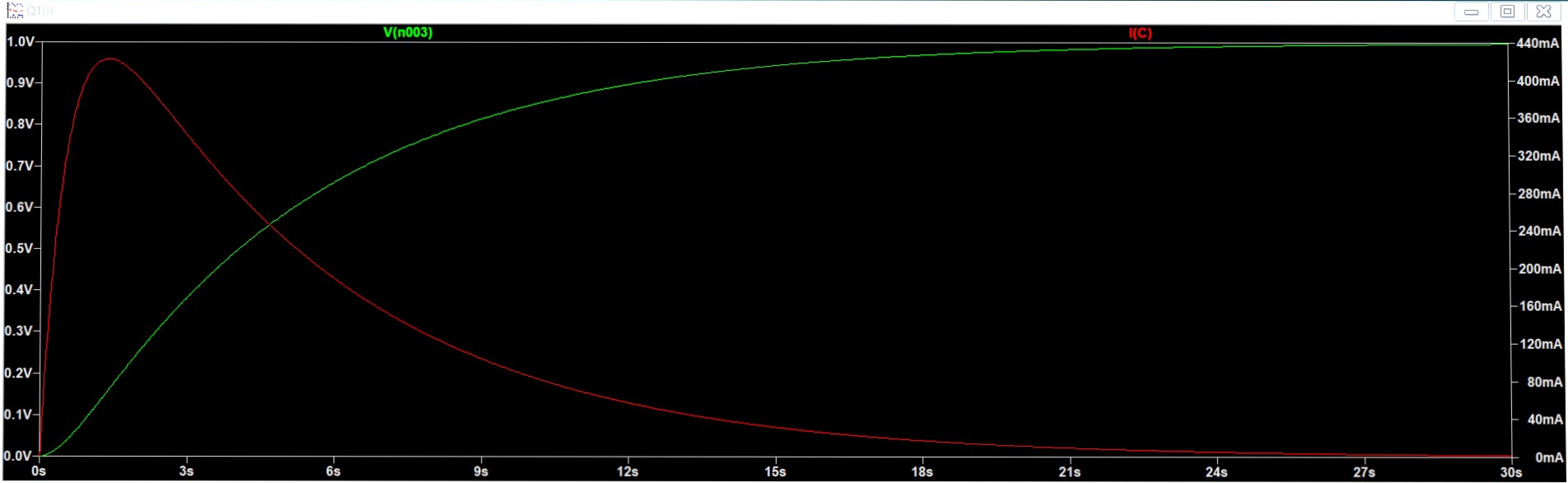
✓ $i(t) = te^{-t}$ A

NISARG UPADHYAYA 19CS30031

QUESTION 1 – (I) $C = 25/9$ F

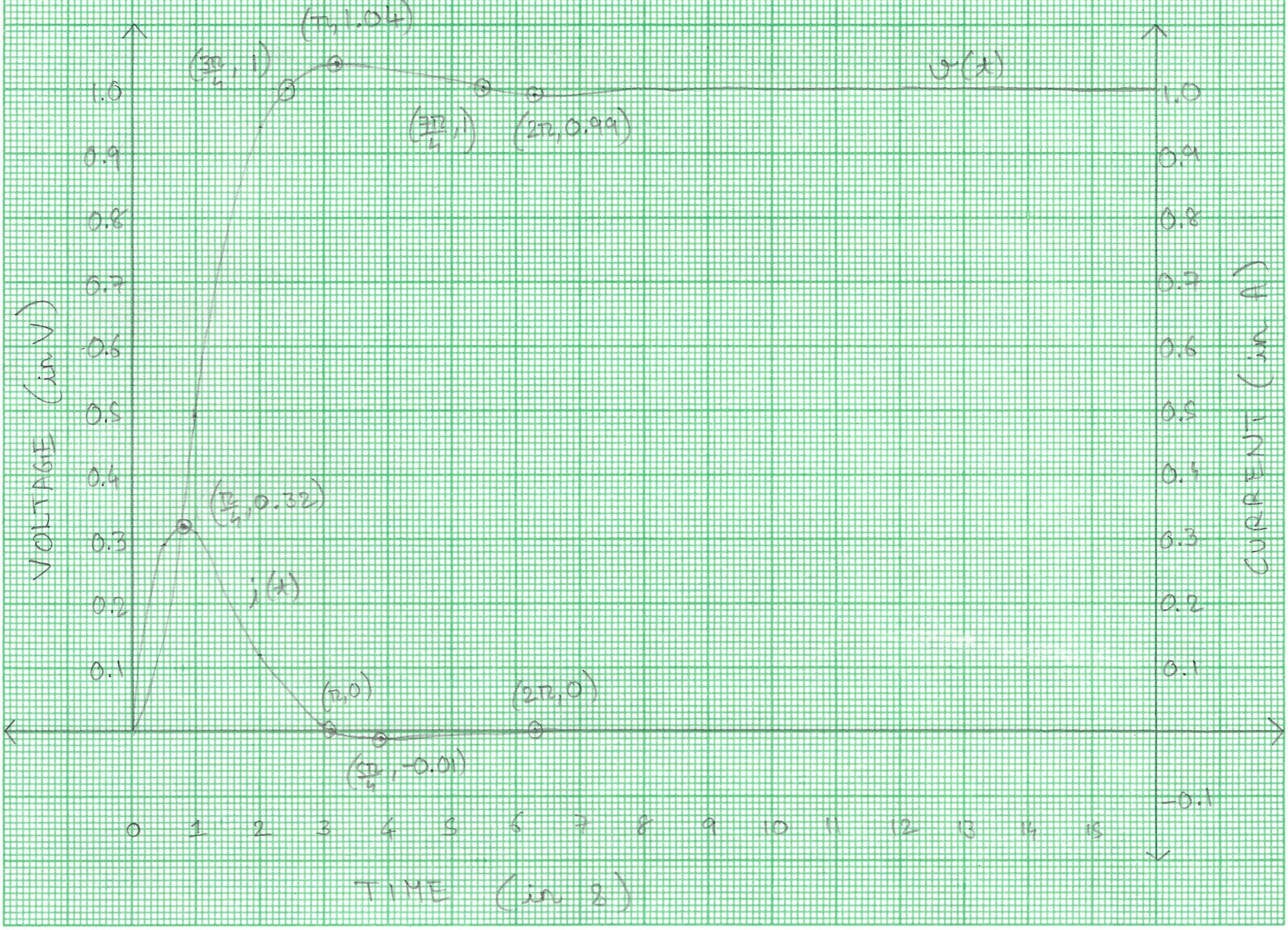


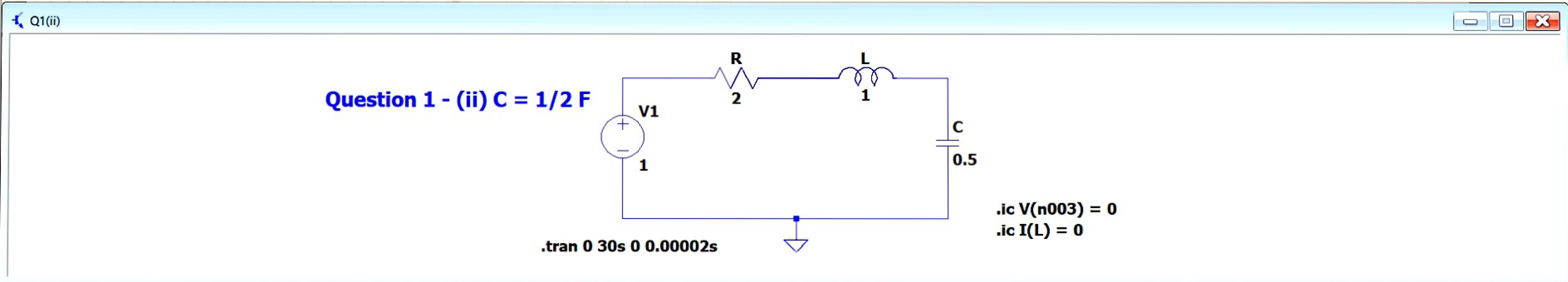

Q1(i)
Question 1 - (i) C = 25/9 F
R
2
L
1
V1
1
C
2.7778
.ic V(n003) = 0
.ic I(L) = 0
.tran 0 30s 0 0.00002s

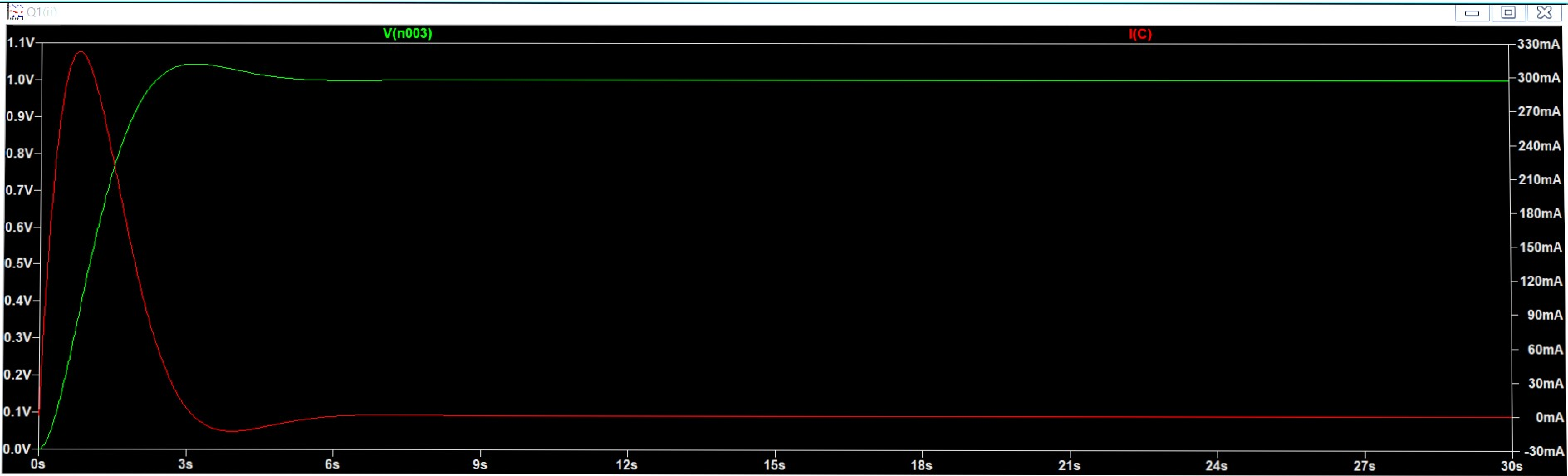

Q1(i)
V(n003)
I(C)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
440mA
400mA
360mA
320mA
280mA
240mA
200mA
160mA
120mA
80mA
40mA
0mA
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s

NISARG UPADHYAYA 19CS30031 Nisarg

QUESTION 1 - (ii) C = 1/2 F

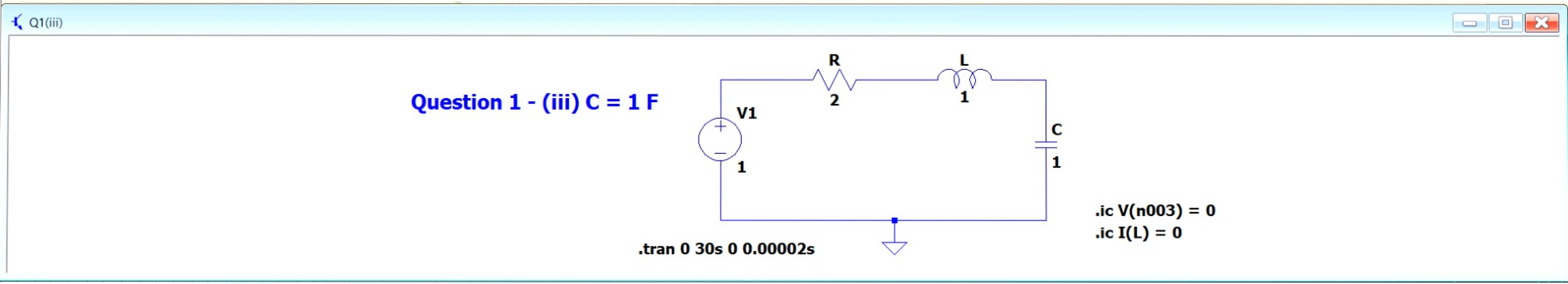


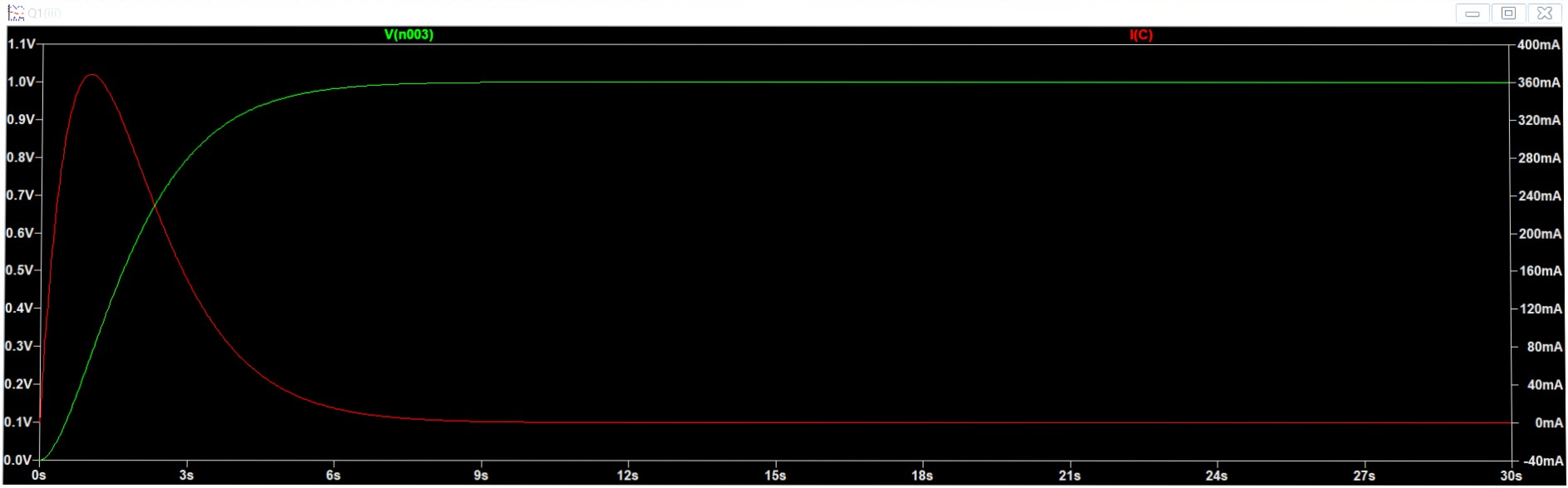

Q1(ii)
Question 1 - (ii) C = 1/2 F
R
2
L
1
V1
1
C
0.5
.ic V(n003) = 0
.ic I(L) = 0
.tran 0 30s 0 0.00002s


V(n003)
I(C)
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
330mA
300mA
270mA
240mA
210mA
180mA
150mA
120mA
90mA
60mA
30mA
0mA
-30mA
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s

NISARG UPADHYAYA 19CS30031 Nisarg

QUESTION 1 - (iii) C = 1 F


Q1(iii)
Question 1 - (iii) C = 1 F
R
2
L
1
V1
1
C
1
.ic V(n003) = 0
.ic I(L) = 0
.tran 0 30s 0 0.00002s


Q1(iii)
V(n003)
I(C)
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
400mA
360mA
320mA
280mA
240mA
200mA
160mA
120mA
80mA
40mA
0mA
-40mA
0s
3s
6s
9s
12s
15s
18s
21s
24s
27s
30s