Tutorial 1 Solution

1) i) $f(t) = \sin\left(2t + \frac{\pi}{2}\right)$

$= \cos(2t)$ (It is even by observation)

Even part $= \dfrac{f(t) + f(-t)}{2}$

$= \dfrac{\cos(2t) + \cos(2(-t))}{2}$

$= \cos(2t)$

Odd part $= \dfrac{f(t) - f(-t)}{2}$

$= \dfrac{\cos(2t) - \cos(2(-t))}{2}$

$= 0$

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ii) $f(t) = 1 - 2t + 3t^3$

Even part $= \dfrac{f(t) + f(-t)}{2}$

$= \dfrac{(1 - 2t + 3t^3) + (1 + 2t - 3t^3)}{2}$

$= 1$

Odd part $= \dfrac{f(t) - f(-t)}{2}$

$= \dfrac{(1 - 2t + 3t^3) - (1 + 2t - 3t^3)}{2}$

$= -2t + 3t^3$

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iii) $f(t) = \sin(2t) + \sin(2t)\cos(2t) + \cos(2t)$

$= \sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)$

Even part $= \dfrac{f(t)+f(-t)}{2}$

$= \Big[\big(\sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)\big) + \big(-\sin(2t) - \frac{1}{2}\sin(4t) + \cos(2t)\big)\Big] \times \frac{1}{2}$

$= \cos(2t)$

Odd part $= \dfrac{f(t)-f(-t)}{2}$

$= \Big(\big(\sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)\big) - \big(-\sin(2t) - \frac{1}{2}\sin(4t) + \cos(2t)\big)\Big) \times \frac{1}{2}$

$= \sin(2t) + \frac{1}{2}\sin(4t)$

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iv) $f(t) = e^{j2t}$

Even part $= \dfrac{f(t)+f(-t)}{2}$

$= \dfrac{e^{j2t} + e^{-j2t}}{2}$

$= \dfrac{\cos 2t + j\sin 2t + \cos(2t) + j\sin(-2t)}{2}$

$= \cos(2t)$

odd part $= \dfrac{f(t)-f(-t)}{2}$

$$= \frac{e^{j2t} - e^{-j2t}}{2}$$

$$= \frac{\cos 2t + j\sin 2t - \cos 2t + j\sin 2t}{2}$$

$$= j\sin 2t$$

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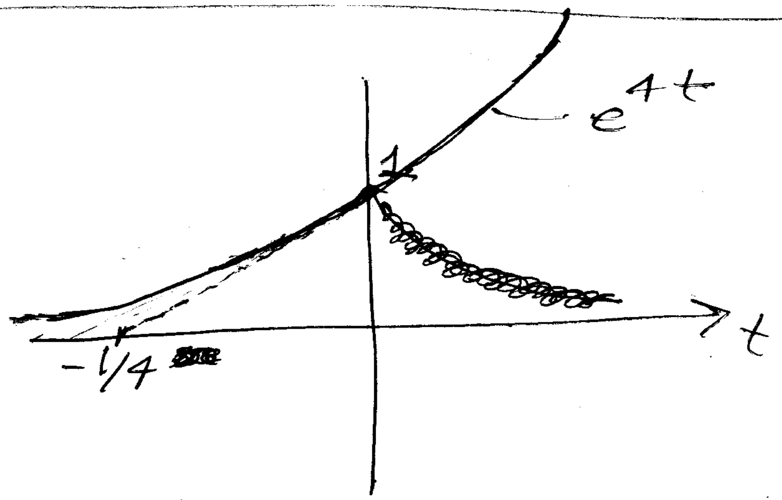
2) i) $f(t) = e^{4t}$

$f(-t) = e^{-4t}$

$\therefore f(-t) \neq f(t)$

and $f(-t) \neq -f(t)$

$\Rightarrow$ Neither even nor odd



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ii) $f(t) = u(t+2) - u(t-2)$

$f(-t) = u(-t+2) - u(-t-2)$

$= \big(u(-t+2) - 1\big) + \big(1 - u(-t-2)\big)$

$= -u(t-2) + u(t+2) = f(t)$

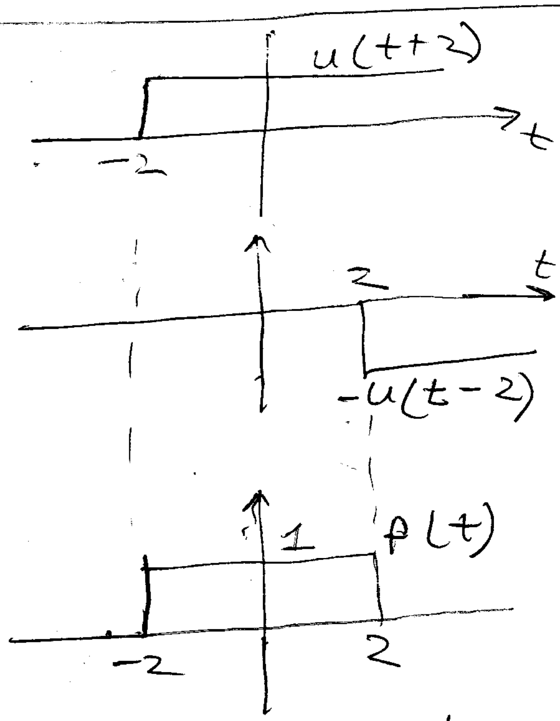
$\Big[\because \boxed{1 - u(-t) = u(t)}$

$\Rightarrow 1 - u(-(t-2)) = u(t-2)$

$\Rightarrow 1 - u(-t+2) = u(t-2)$

and $1 - u(-(t+2)) = u(t+2)$

$\Rightarrow 1 - u(-t-2) = u(t+2)\Big]$



from observation f(t) is even.

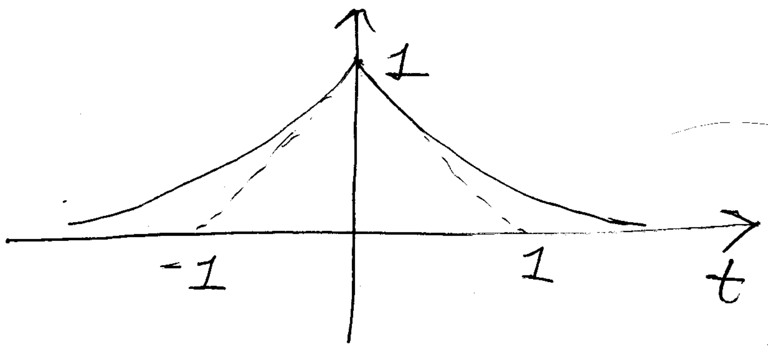
Note: Only graphical proof is sufficient. Some students asked for an algebraic proof. So an algebraic proof is also given.

iii) $e^{-|t|} = f(t)$

$f(-t) = e^{-|-t|}$

$= e^{-|t|}$

$= f(t)$



$\therefore$ even function

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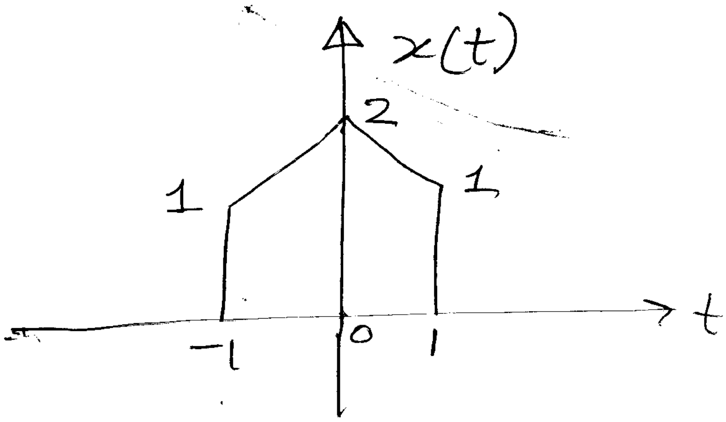
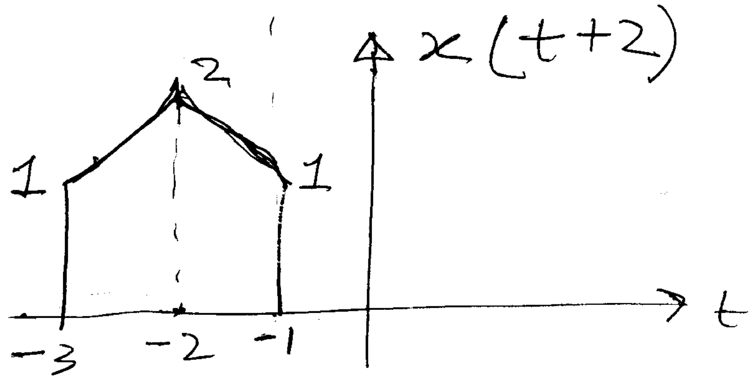
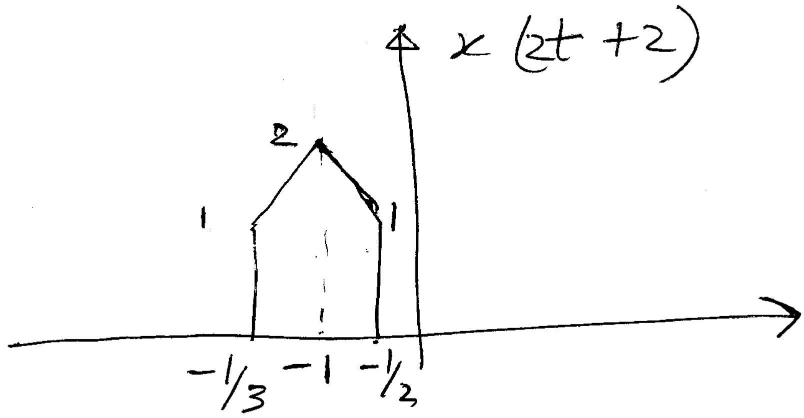
3) (i)



$\Rightarrow$



$\Rightarrow$



__Method__

Replace t with $t+2$ and shift the graph 2 units to the left

$\Downarrow$

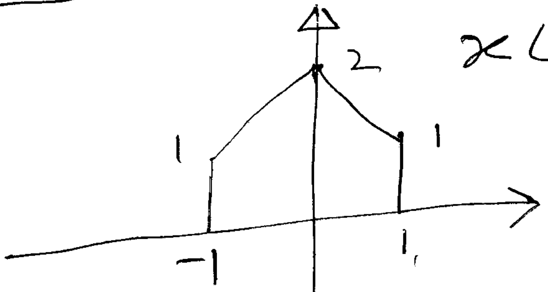
Replace t with $2t$ and squeeze the graph to half size horizont-ally. The squeezing should be arroud the y axis and NOT arround the centre of the graph

$\Downarrow$

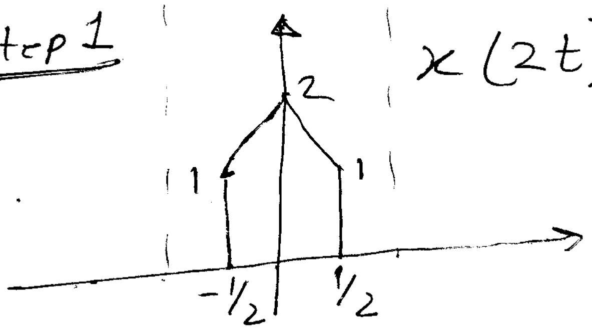
ANSWER

__Verify your answer:__ The peak of $x(t)$ is at $t=0$. So the peak of $x(2t+2)$ should be at $(2t+2)=0 \Rightarrow t=-1$. Our answer is consistent with this requirement.

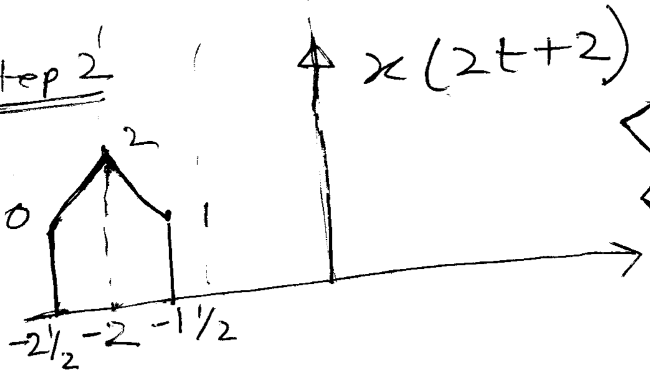
COMMON MISTAKE 1



Step 1



Step 2



Step 1 is correct. Here t is replaced with 2t and the graph is squeezed.

Step 2 is incorrect because (2t) becomes (2t+2) ⇒ (t) becomes (t+1)

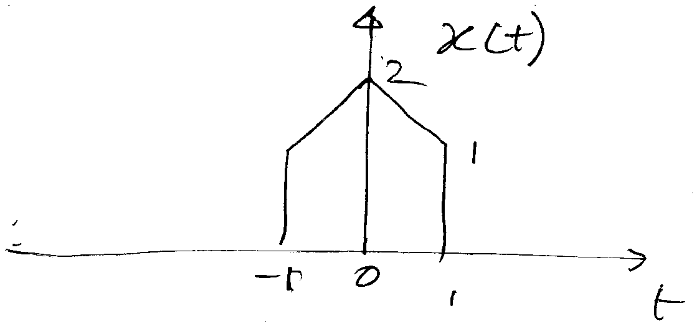
$\boxed{2(t+1) = 2t+2}$

So the graph should shift 1 unit to the left & NOT 2 units to the left

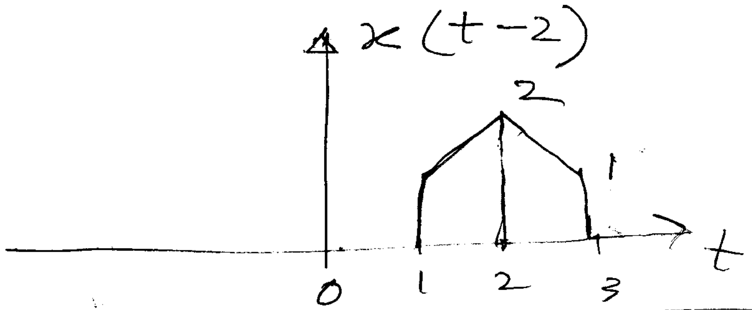
ALWAYS note what change is applied on t alone. Never look at what change is applied inside the brackets as a whole.

Again verify your answer; The peak of the graph should be at $2t+2=0$ $\Rightarrow t=-1$. But with the wrong method we got the peak at -2

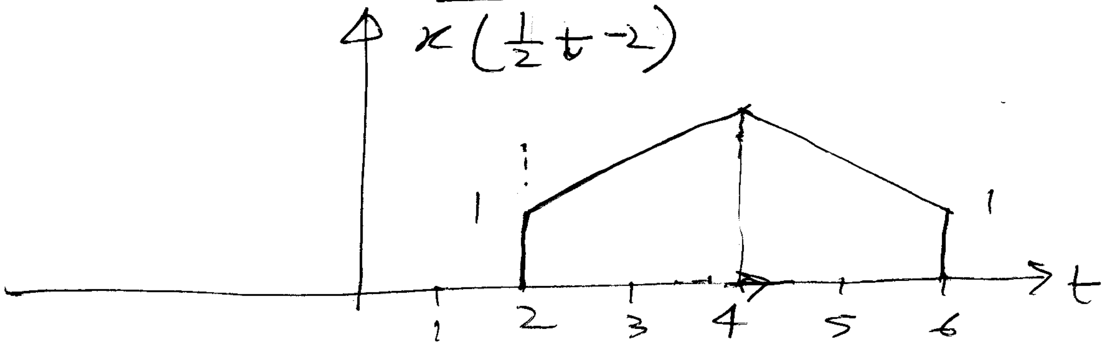
(ii)



Replace t with (t-2) and shift the graph 2 units to the right.

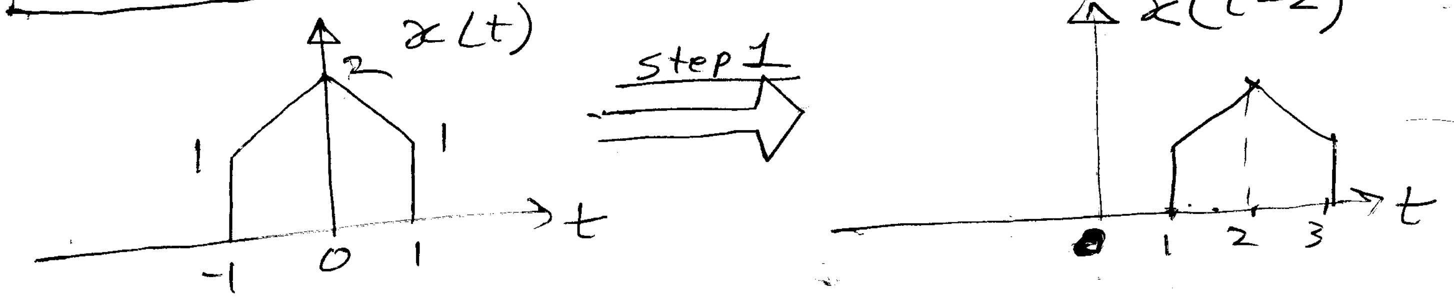


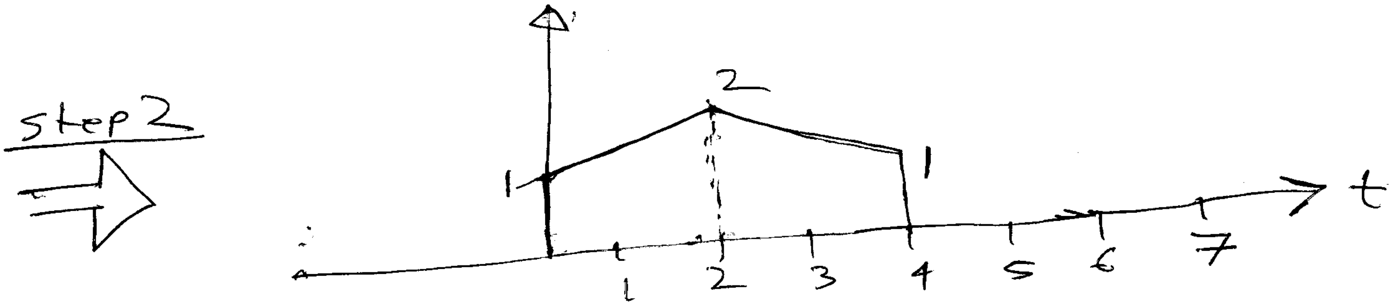
Replace t with (1/2 t) and expand the graph horizontaly by factor of.



Verify your answer: The peak should be at $\left(\frac{1}{2}t-2\right)=0 \Rightarrow t=4.$ So our drawing is okay.

COMMON MISTAKE 2)



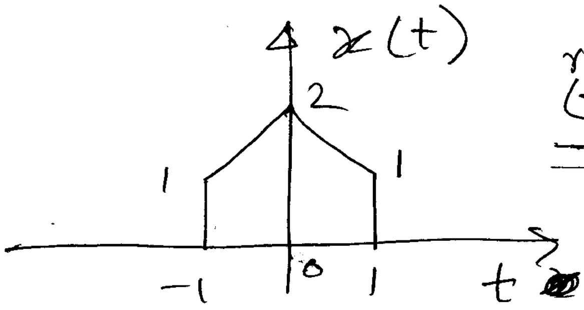


Step 1 is correct. But step 2 is wrong. In step 2 the graph is expanded arround the center of the graph/ peak of the graph. But it should be expanded arround $t=0$ axis or y axis.

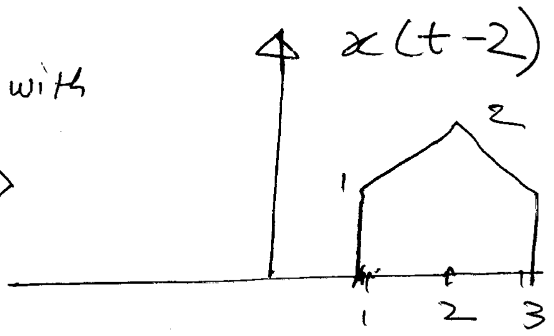
Verify the answer: The peak should be at ~~t=2~~ $\frac{1}{2}t-2=0 \Rightarrow t=4$.

But with the wrong approach we got the peak at $t=2$.

(iii)

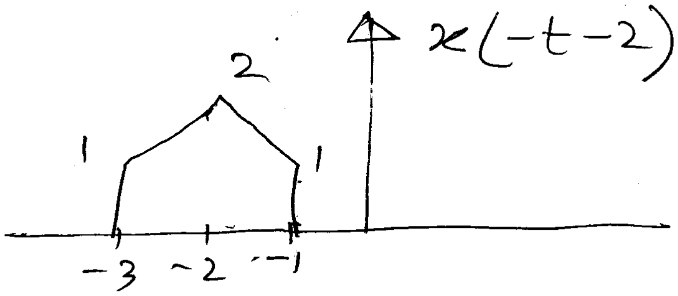


replace t with (t-2)



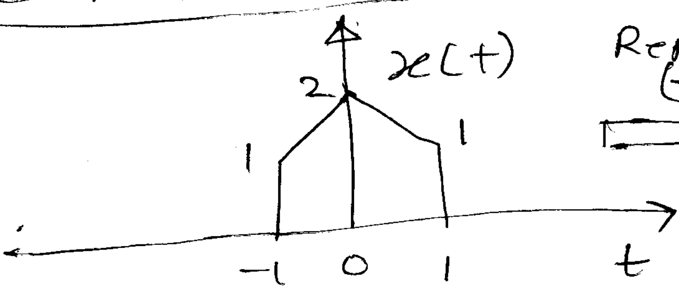
Replace t with (-t) and flip

arround y axis

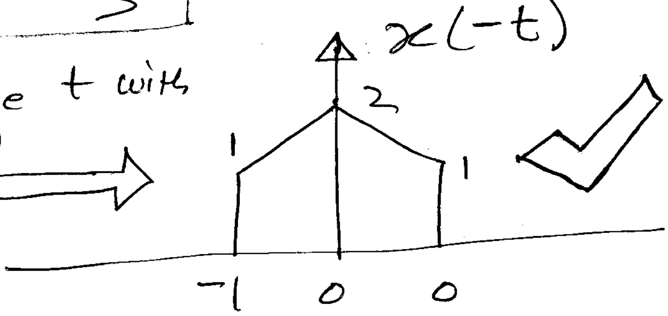


<u>Verify</u> peak should be at $-t-2=0 \Rightarrow t=-2$

COMMON MISTA~~B~~KE 3

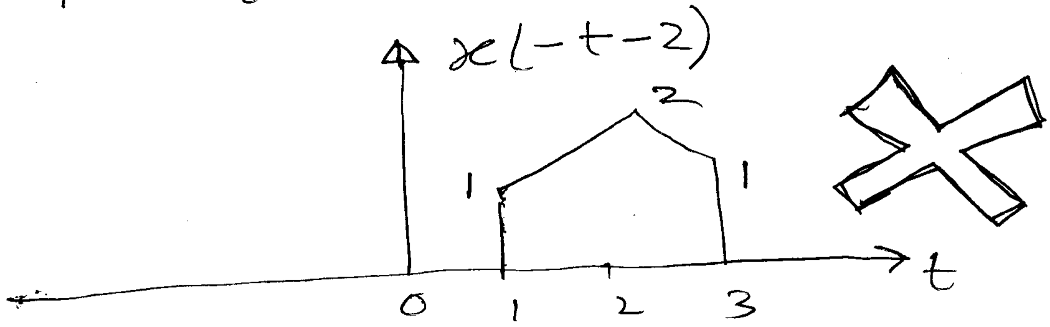


Replace t with (-t)



<u>Step 2</u>

wrong



<u>Step 2 is wrong</u> because we are changing (-t) to (-t-2) that means t is becoming (t+2).

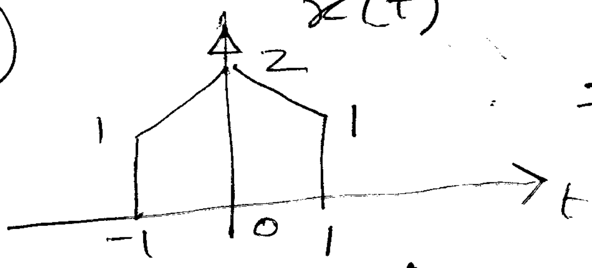
$$\boxed{-(t+2) = -t-2}$$

So it should shif towords left and

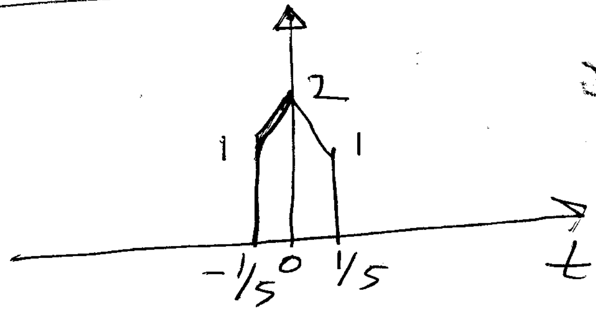
NOT towards right

Verify the peak should be at $(-t-2)=0 \Rightarrow t=-2$. But here we got the peak at $t=2$

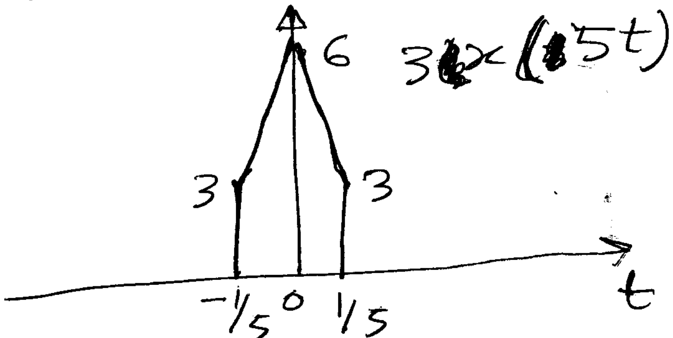
(iv)



$\Longrightarrow$

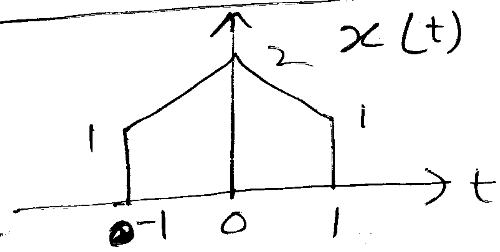


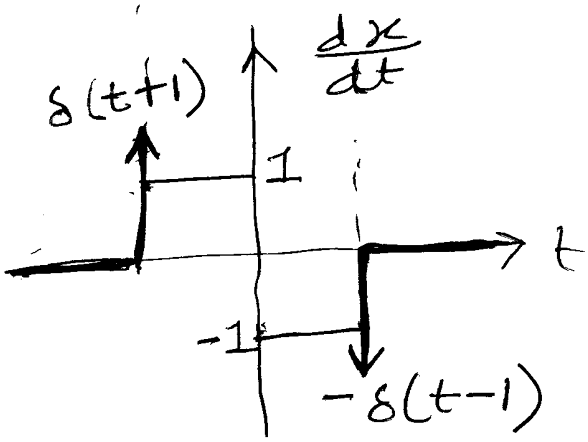
$\Longrightarrow$



My diagram is not upto the scale
But as long as I annotate the points on the time & y axis, it is acceptable

(v)





Note the two delta functions.
whenever there is a step jump we have a delta fn.
If it is a step increase the positive delta & if it is a step decrease then negative delta.

Also the value (coefficient / strength) of the delta fn is equal to the amount of step jump

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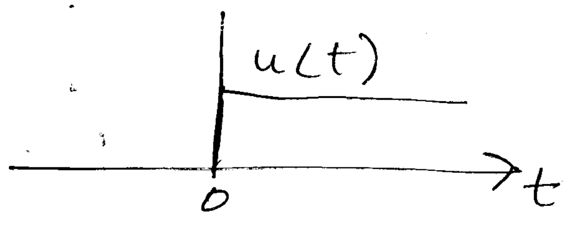
5) i)



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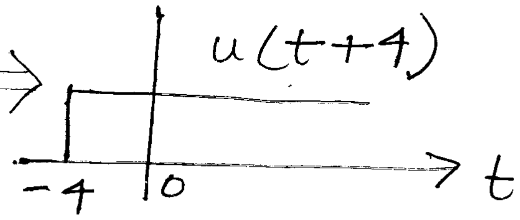

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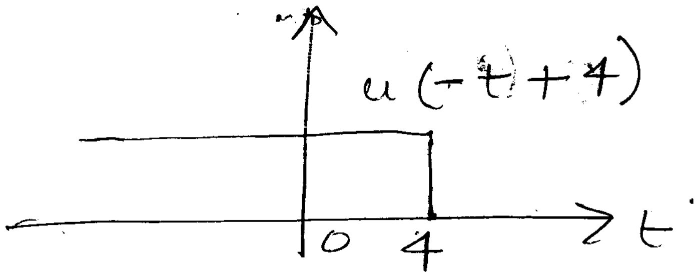

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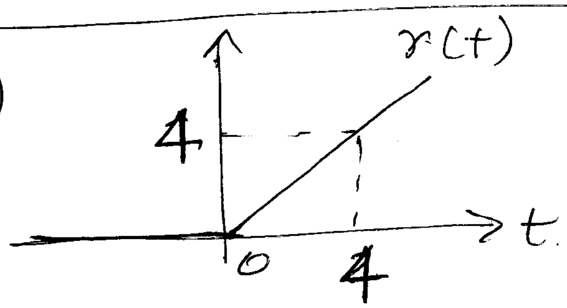
ii)



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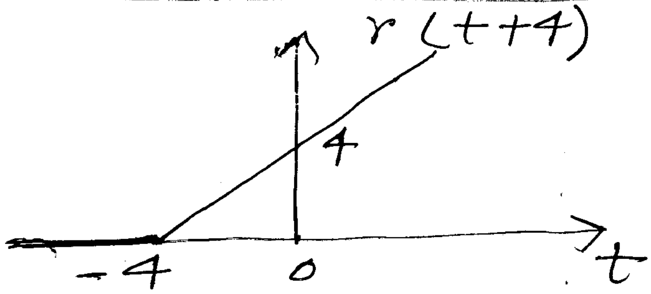

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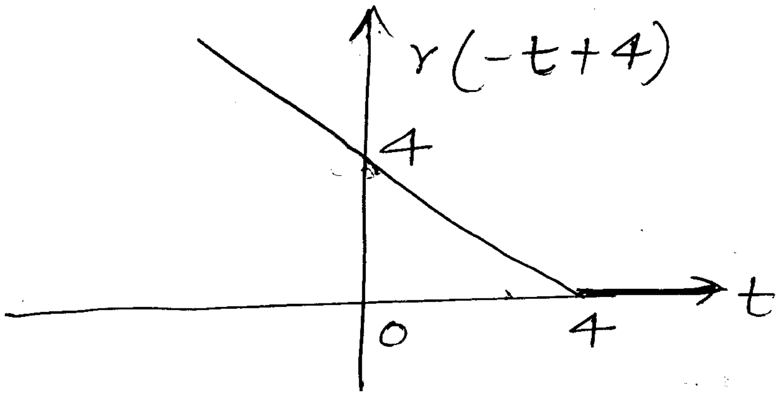
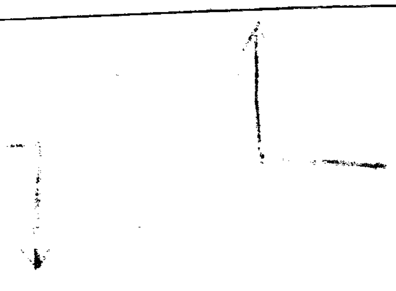
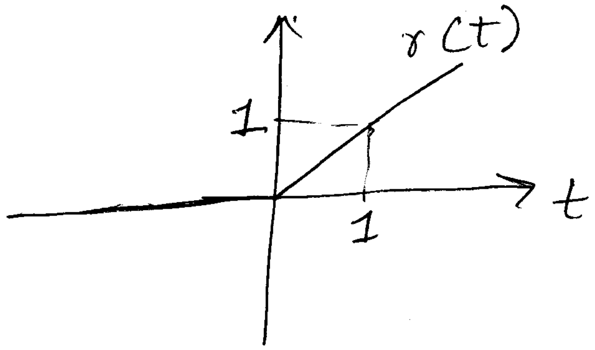




iii)



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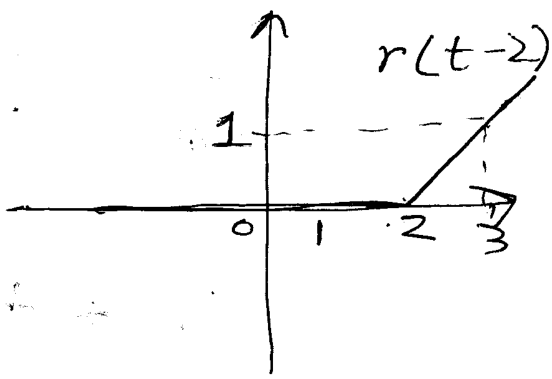

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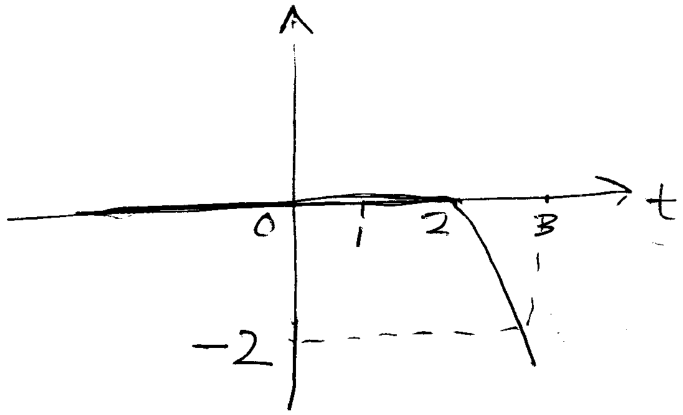
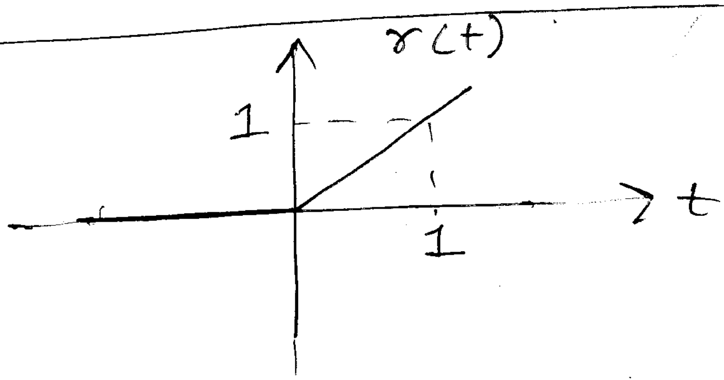
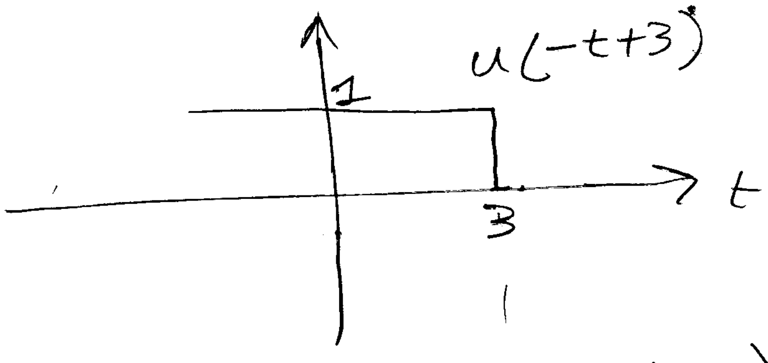


iv)





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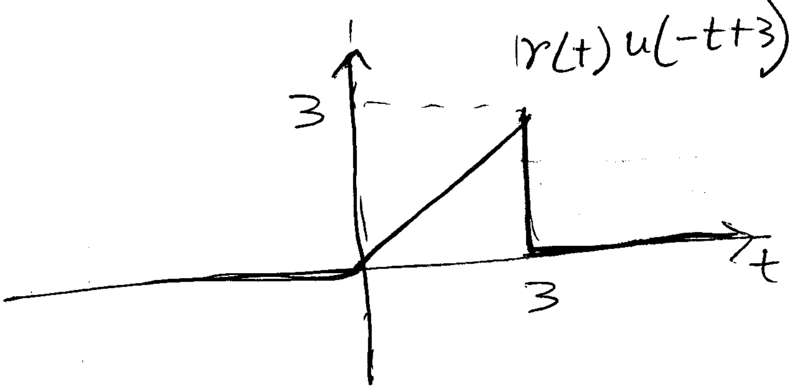
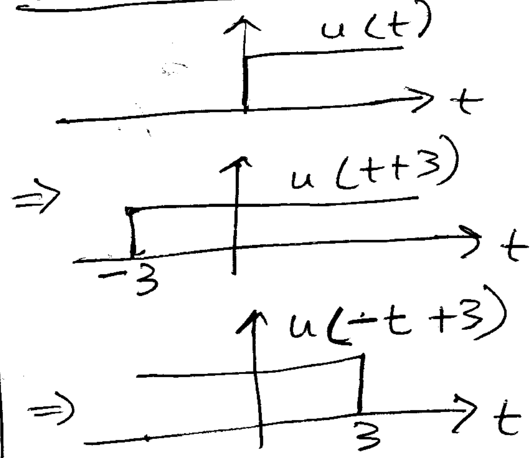
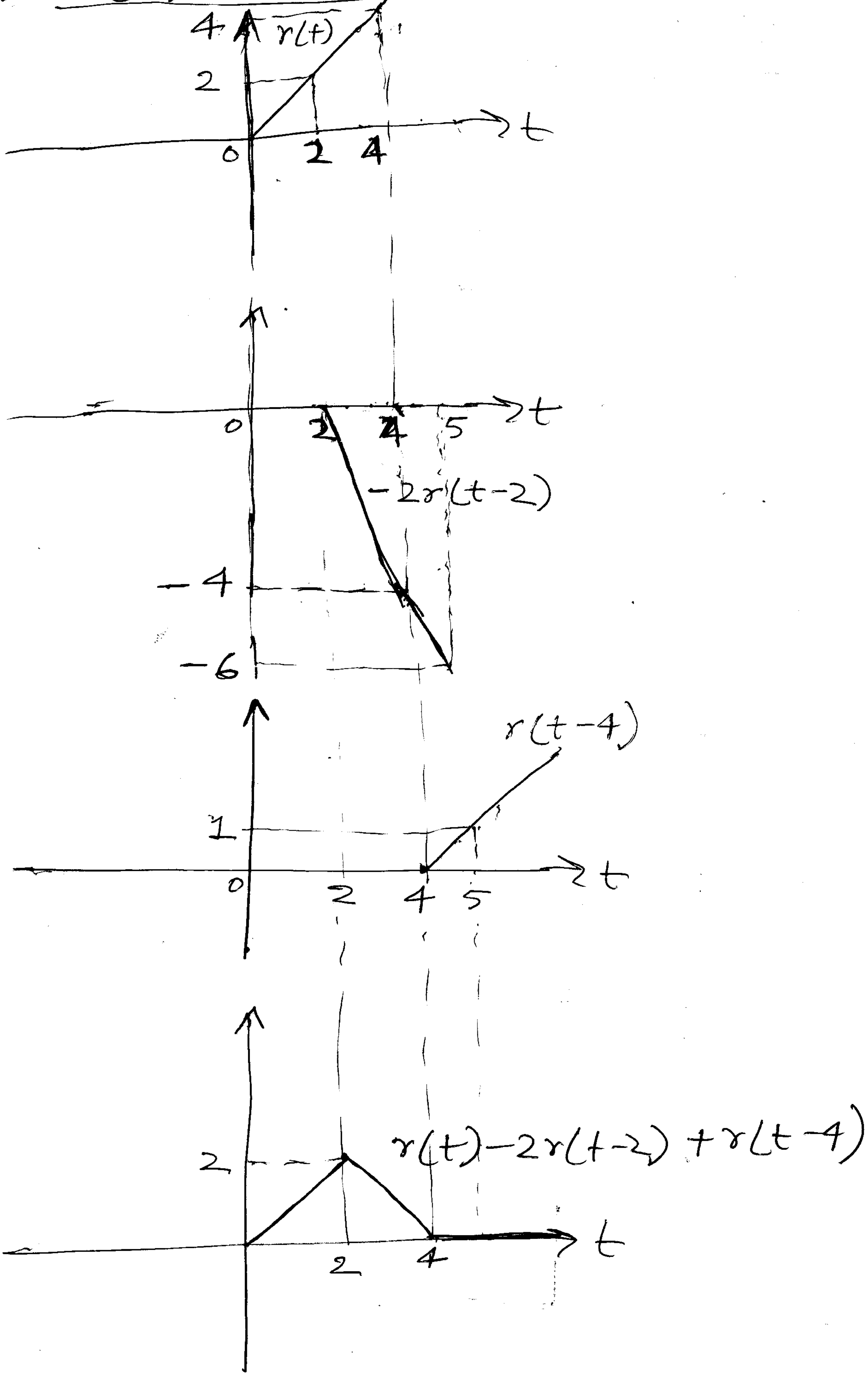


<u>side note</u>



v) $r(t) - 2r(t-2) + r(t-4)$



6) i) $\displaystyle\int_{-\infty}^{\infty} e^{-t^2}\,\delta(t-3)\,dt$

$\displaystyle = \int_{3^-}^{3^+} e^{-t^2}\,\delta(t-3)\,dt$ [$\because$ for all time other than between 3^- and t^+ $\delta(t-3)=0$]

$\displaystyle = \int_{3^-}^{3^+} e^{-3^2}\,\delta(t-3)\,dt$

$\displaystyle = e^{-9}\int_{3^-}^{3^+} \delta(t-3)\,dt$

$= e^{-9}$

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ii) $\displaystyle\int_{-\infty}^{\infty} \delta(t+3)\,e^{-2t}\,dt = \int_{-3^-}^{-3^+} \delta(t-(-3))\,e^{-2t}\,dt$

$\displaystyle = e^{-2(-3)}\int_{-3^-}^{-3^+} \delta(t-(-3))\,dt = e^{6}\cdot 1 = e^{6}$

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iii) $\displaystyle\int_{-3}^{3} \delta(t)\sin(5\pi t)\,dt = \int_{0^-}^{0^+} \delta(t)\sin(0)\,dt = 0$

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iv) $\displaystyle\int_{-\infty}^{\infty} [\delta(t)\cos 2t + \delta(t-2)\sin 2t]\,dt$

$\displaystyle = \int_{-\infty}^{\infty} \delta(t)\cos 2t\,dt + \int_{-\infty}^{\infty} \delta(t-2)\sin 2t\,dt = \cos 0 + \sin 4t$

$= 1+\sin(4t)$

v) $\displaystyle\int_{-\infty}^{\infty} \delta(4t)e^{-t}\,dt$

[One may apply the relevant formula directly or follow the steps as done here]

Put $4t = \tau$

$\Rightarrow dt = \dfrac{d\tau}{4}$

$$\therefore \int_{-\infty}^{\infty} \delta(4t)\, e^{-t}\, dt = \int_{-\infty}^{\infty} \delta(\tau) e^{-\frac{\tau}{4}} \frac{d\tau}{4}$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} \delta(\tau) e^{-\tau/4}\, d\tau = \frac{1}{4}$$

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vi) $\displaystyle\int_{-\infty}^{\infty} \delta(2t+3)\, t^2\, dt$

Put $2t+3 = \tau$

$\Rightarrow dt = \dfrac{d\tau}{2}$

$$= \int_{-\infty}^{\infty} \delta(\tau) \left(\frac{\tau-3}{2}\right)^2 \frac{d\tau}{2} = \frac{1}{2} \times \left(\frac{-3}{2}\right)^2$$

$$= \frac{9}{8}$$

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vii) $\displaystyle\int_{-\infty}^{\infty} \delta(t^2+t-6) \cos t\, dt = I \text{ (say)}$

$$= \int_{-\infty}^{\infty} \delta\big((t+3)(t-2)\big) \cos t\, dt$$

$$= \int_{-3^-}^{-3^+} \delta\big((t+3)(t-2)\big) \cos t\, dt + \int_{2^-}^{2^+} \delta\big((t+3)(t-2)\big) \cos t\, dt$$

$$= \cos(-3) \int_{-3^-}^{-3^+} \delta\big((t+3)(t-2)\big) dt + \cos(2) \int_{2^-}^{2^+} \delta\big((t+3)(t-2)\big) dt$$

It is easier to apply the direct formula here which gives

$$I = \frac{\cos(-3)}{\left|\frac{d}{dt}(t+3)(t-2)\right|_{t=-3}} + \frac{\cos(2)}{\left|\frac{d}{dt}\left((t+3)(t-2)\right)\right|_{t=2}}$$

$$= \frac{\cos(3)}{|2t+1|_{t=-3}} + \frac{\cos 2}{|2t+1|_{t=2}}$$

$$= \frac{\cos(3)}{|-5|} + \frac{\cos 2}{|5|} = \frac{1}{5}\left(\cos 2 + \cos 3\right)$$

OR you may follow the steps mentioned below (This method is actually developed by some of you students)

$$I = \int_{-3^-}^{-3^+} \delta(t^2+t-6)\cos t\,dt + \int_{2^-}^{2^+} \delta(t^2+t-6)\cos t\,dt$$

$$= \cos(-3)\int_{-3^-}^{-3^+} \delta(t^2+t-6)\,dt + \cos(2)\int_{2^-}^{2^+} \delta(t^2+t-6)\,dt$$

$$= \cos 3\int_{-3^-}^{-3^+} \delta(t^2+t-6)\,dt + \cos 2\int_{2^-}^{2^+} \delta(t^2+t-6)\,dt$$

Now put $z = t^2+t-6$

$$\therefore dz = \frac{d}{dt}(t^2+t-6)\,dt = (2t+1)\,dt$$

t	-3^-	-3^+	2^-	2^+
z	0^+	0^-	0^-	0^+

t	-3	2
dz	$-5dt$	$5dt$

$$\therefore\ I = \cos(3)\int_{0^+}^{0^-} \delta(z)\,\frac{dz}{-5} + \cos(2)\int_{0^-}^{0^+} \delta(z)\,\frac{dz}{5}$$

$$= \frac{\cos(3)}{5}\int_{0^-}^{0^+} \delta(z)\,dz + \frac{\cos(2)}{5}\int_{0^-}^{0^+} \delta(z)\,dz$$

$$= \frac{1}{5}\left(\cos(2) + \cos(3)\right)$$

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viii) $\displaystyle\int_{-\alpha}^{\alpha} e^{-t}\,\frac{d\delta}{dt}\,dt$

[Again one can apply direct formula ~~there~~ taught in the class. Here we are using integration by parts]

$$= \left[e^{-t}\,\delta(t)\right]_{-\alpha}^{\alpha} - \int_{-\alpha}^{\alpha} -e^{-t}\,\delta(t)\,dt$$

$$= 0 + \int_{-\alpha}^{\alpha} e^{-t}\,\delta(t)\,dt = e^{0}\int_{0^-}^{0^+} \delta(t)\,dt$$

$$= e^{0} = 1$$

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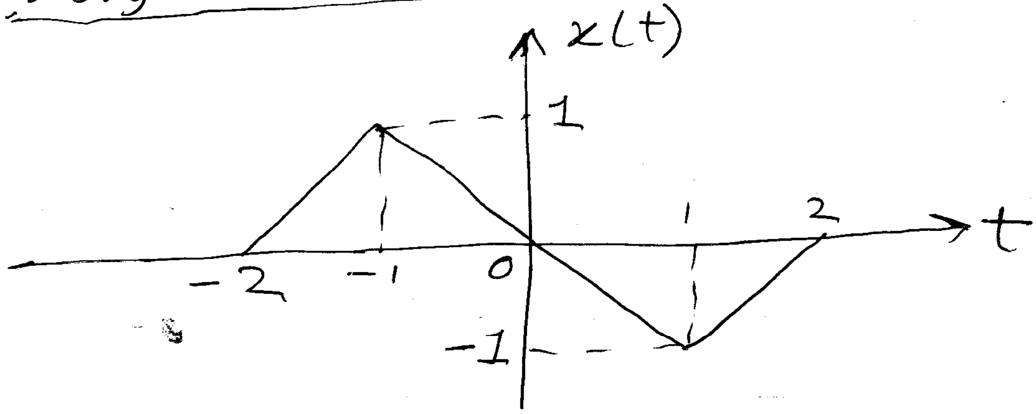
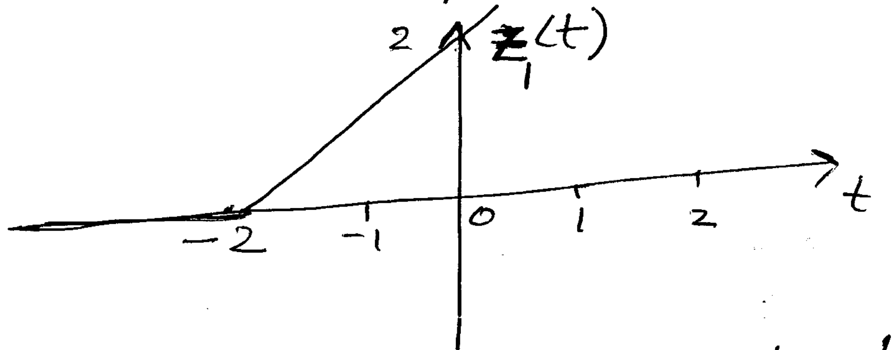
4) i) ~~Target~~ function to be constructed $x(t)$

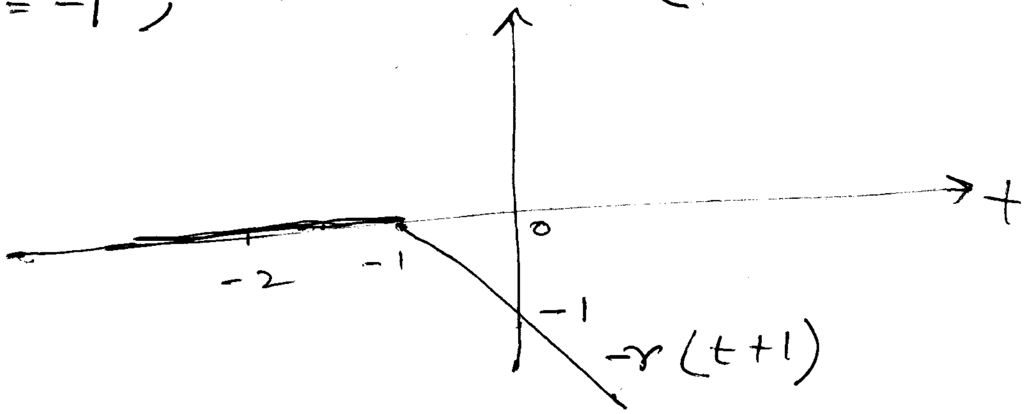


We will construct this function from left side to right side

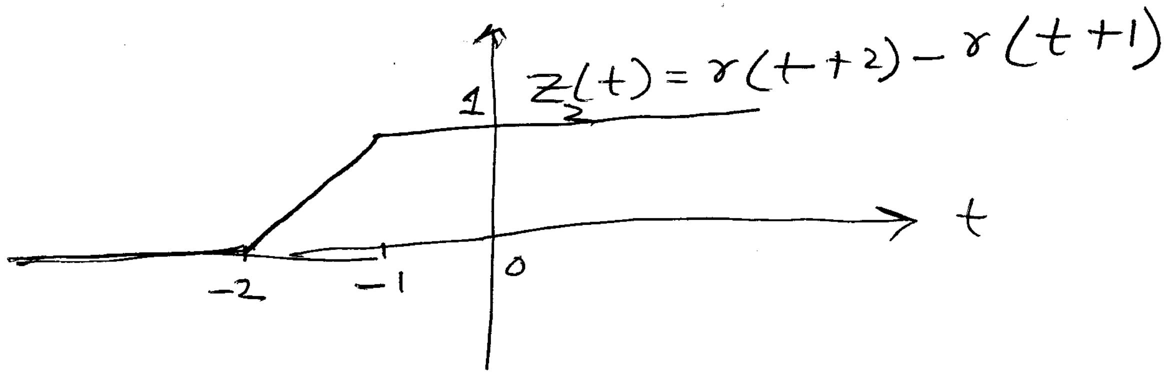
First take $z_1(t) = r(t+2)$



Now to stop the increase of this function at $t=-1$, Lets add $(-r(t+1))$

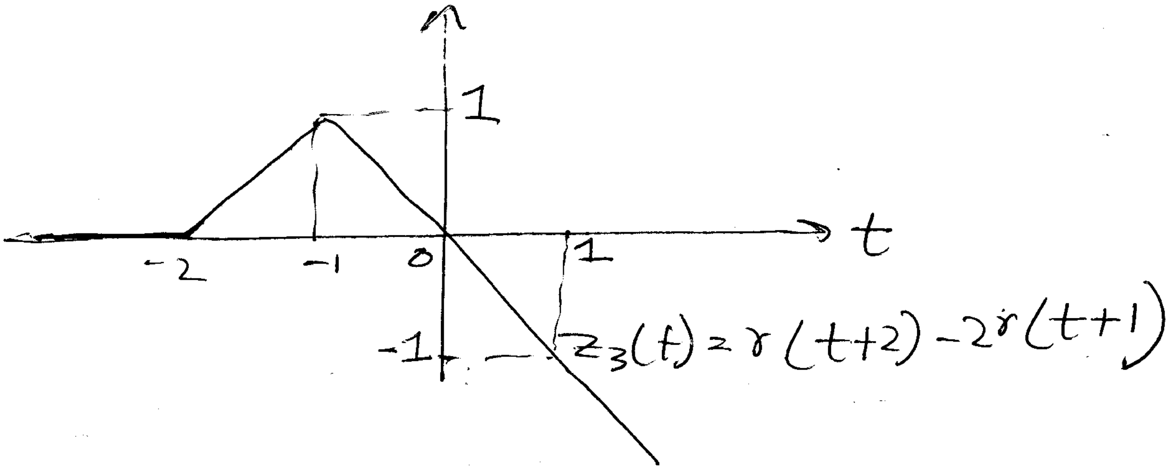


Let
$z_2(t) = r(t+2) - r(t+1)$



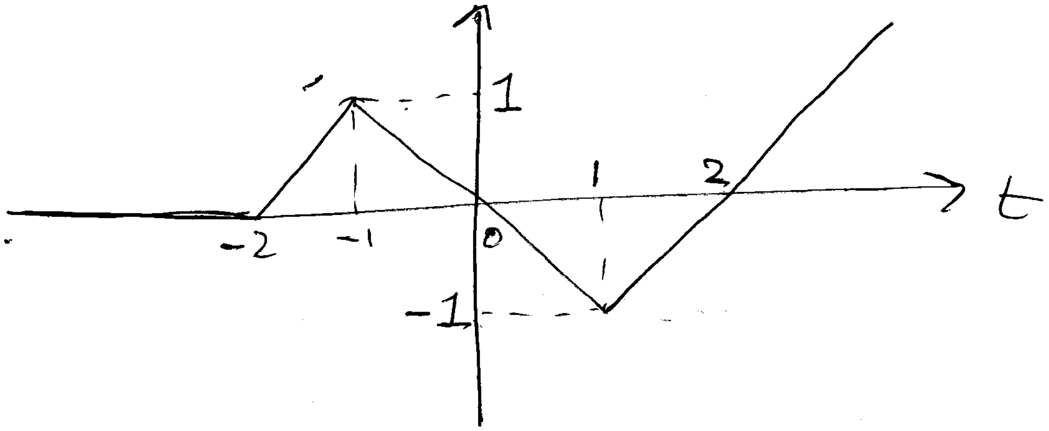
Now to bend down this function at $t=-1$, add one more $-r(t+1)$

Let $z_3(t) = r(t+2) - r(t+1) - r(t+1)$
$= r(t+2) - 2r(t+1)$



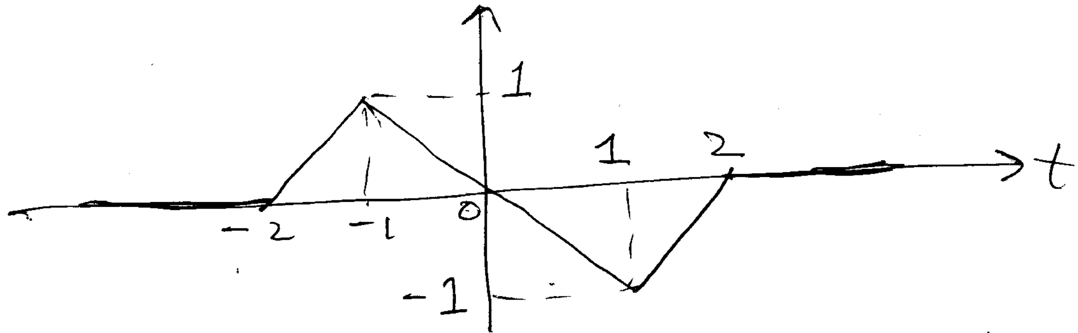
Now similarly, add $2r(t-1)$

$$z_4(t) = z_3(t) + 2r(t-1)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1)$$



Finally add $(-r(t-2))$

$$z_5(t) = z_4(t) - r(t-2)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$



This is our desired function $x(t)$

$$\therefore x(t) = z_5(t)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$

We will give you the final answers for (4) ii) – (4) v). Please verify the answers yourselves following the steps as in (4) i)

ii) $x(t) = 2u(t) - r(t) + r(t-2)$ ..... for 1st triangle

$+ 2u(t-4) - r(t-4) + r(t-6)$ ----- for 2nd "

iii) $x(t) = r(t+2) - r(t+1) - 2u(t-1) + \frac{1}{2}u(t-2)$

$+ \frac{1}{2}r(t-2) - \frac{1}{2}r(t-3)$

iv) $x(t) = u(t-2) - r(t-2) + 2r(t-3) - r(t-4)$

$- u(t-4)$

v) $x(t) = 2r(t) - 2r(t-1) - u(t-1) + u(t-2)$

$- 2r(t-2) + 2r(t-3)$