Introduction: At 45:10, I think the relation btw i, j, k is not correct.

Yes, there is a problem

$$1 \leq k \leq j \leq i \leq 100$$

Subtitle file – enabled

error-correcting

It would be helpful if link to books is provided.

Teams → Files → Course material

Other notes will also be available.

You said rational and integers are of the same size. But say we have n integers, where n tends to infinity. As rational numbers are expressed in form of p/q, the number of rational numbers should be in the order of n^2. Then how are they of the same order??

Playing with ∞ is tricky.

Cantor

$$\aleph_0 \times \aleph_0 = \aleph_0$$ Aleph-not

please wait.

Clarification: Derivation of Catalan number formula.

$\{$invalid paths from $(0,0)$ to $(n,n)\}$

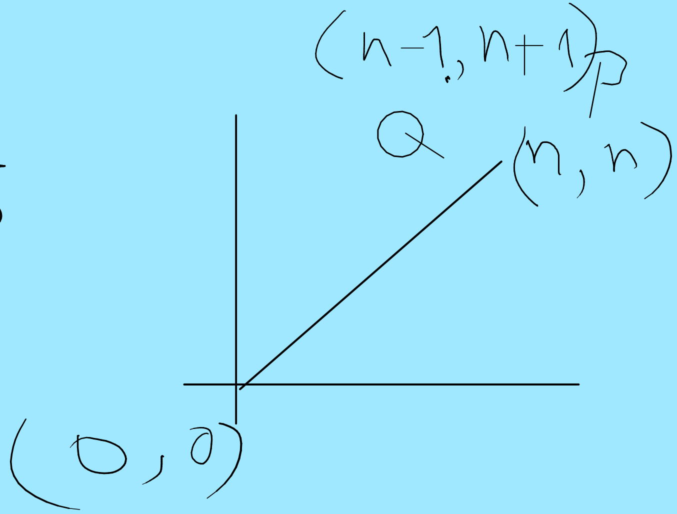
$\{$all paths from $(0,0)$ to $(n-1, n+1)\}$

$$\binom{2n}{n-1}$$

$$g \circ f = id$$

$$f : P \longmapsto Q$$

$$g : Q \longmapsto P$$



More clarification about:

```
for i=1 to n
   for j=1 to i
      for k = 1 to j
         print "Hello".
```

{i, j, k triples for which Hello is printed}

$\{u, v, w \mid 1 \leq u \leq v \leq w \leq n\}$

$w \leftrightarrow i$

$v \leftrightarrow j$

$u \leftrightarrow k$

$1, 2, 3, \ldots, n$

$\binom{n+3-1}{3}$ choose 3 of these with repetition

How many permutations of abbracadabbra with no two a's and no two b's appearing in consecutive positions?

First insert a's with repetitions allowed

a – 5
b – 4
c – 1
d – 1
r – 2

↑ r ↑ d ↑ c ↑ r ↑

Insert aa, a, a, a

$$\frac{5 \times 4 \times 3 \times 2}{3!}$$ ways

$\leftarrow$ The three a's are identical

$\binom{9}{4}$ ↓a↓r↓a↓d↓c↓aa↓r↓a↓

↑ one b must come here

9 positions for the remaining 4 b's

Insert cases for 5 a's

1) a, a, a, a, a
2) aa, a, a, a
3) aa, aa, a
4) aaa, a, a
5) aaa, aa
6) aaaa, a
7) aaaaa

Since 4 b's are available, all cases are possible and must be individually handled.

*** Dirty ***

Prove/Disprove: S7(p) is prime for all primes p not equal to 7.

$p = (a_{l-1} a_{l-2} \cdots a_1 a_0)_7$ $\quad S_7(p) = a_{l-1} + a_{l-2} + \cdots + a_1 + a_0$

No. There exist counterexamples.

$7^i = (6+1)^i = 6 \times m + 1^i$

$p > 3$ $\quad 6k+1,\ 6k+5$ $\quad p \equiv 1, 5 \pmod 6$

$p = 7^{l-1} a_{l-1} + 7^{l-2} a_{l-2} + \cdots + 7a_1 + a_0$

$\equiv a_{l-1} + a_{l-2} + \cdots + a_1 + a_0 = S_7(p) \pmod 6$

$S_7(p) \equiv 1, 5 \pmod 6$

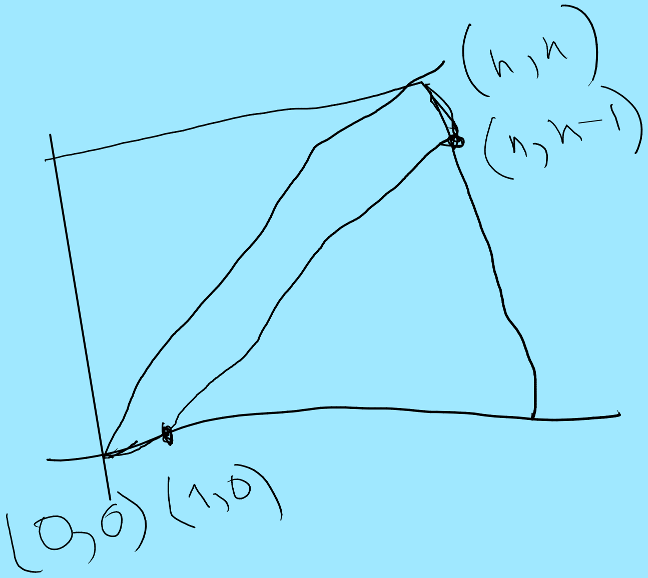
$(16666)_7$ $\quad 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, \ldots$

$= 2 \times 7^4 - 1$

$= 4801$ is a prime

There is nothing like "proof by examples"

Consider paths from (0, 0) to (n, n) in an n × n grid, that never cross the diagonal. Impose an additional constraint that these paths are not allowed to touch the main diagonal except only at the beginning and at the end. How many such constrained paths are there?



$C(n-1)$

How many sorted arrays of size n are there if each element of the array is an integer in the range 1, 2, 3, . . . , r?

$1, 1, 1, \ldots, 1, 2, 2, \ldots, 2, 3, 3, \ldots, 3, \ldots, r, \ldots r$

$x_1 + x_2 + x_3 + \cdots + x_r = n$

$$\binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$

How many binary strings of length n are there containing exactly k occurrences of the pattern 01? Assume that n >= 2k.

$$1^* 0^* \; 01 \; 1^* 0^* \; 01 \; 1^* 0^* \; 01 \; 1^* 0^* \cdots 1^* 0^* \; 01 \; 1^* 0^*$$

$$x_1 \; x_2 \quad x_3 \; x_4 \quad x_5 \; x_6 \quad \cdots \quad x_{2k+1} \; x_{2k+2}$$

$$x_1 + x_2 + x_3 + x_4 + \cdots + x_{2k+1} + x_{2k+2} = n - 2k$$

$$\binom{n-2k+2k+2-1}{2k+2-1} = \binom{n+1}{2k+1}$$

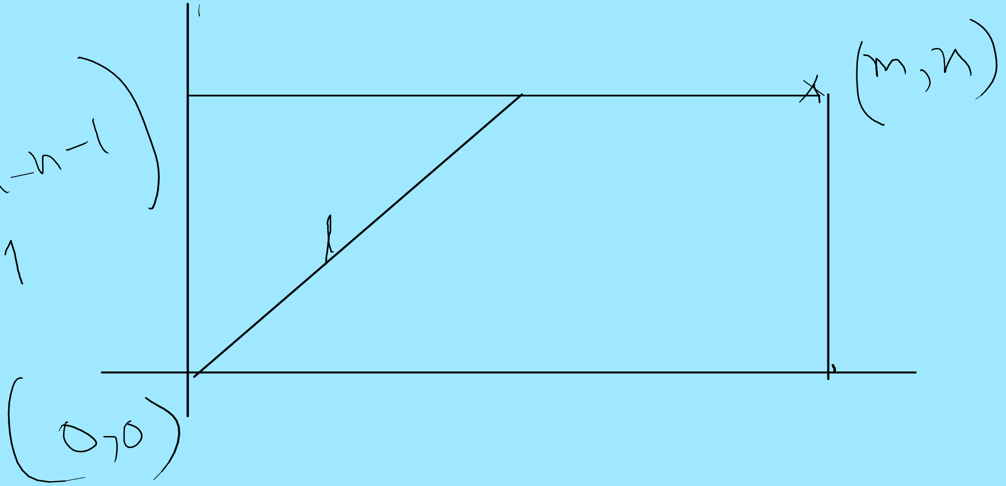
Prove the following identity for any positive integer n.

$$2^n = \binom{n+1}{1} + \binom{n+\overset{1}{\cancel{2}}}{3} + \binom{n+\overset{1}{\cancel{3}}}{5} + \cdots + \begin{cases} \binom{n+1}{n+1} & \text{if } n \text{ is even} \\ \binom{n+1}{n} & \text{if } n \text{ is odd} \end{cases}$$

vary k in the range $0, 1, 2, \ldots, \lfloor n/2 \rfloor$

(combinatorial proof)

Suppose that m > n. How many paths with R and U movements are possible such that at <u>no</u> point of time, there are more U moves than R moves?



Wrong

$$\frac{1}{n+1}\binom{2n}{n}\binom{2n+1+m-n-1}{m-n-1}$$

k R $m-k$

$k+1$ U $n-k-1$

$n-1,\ m+1$

$m=5$

$n=2$

RRUU

Same argument as in the derivation of $C(n)$

$$\binom{m+n}{n}-\binom{m+n}{n-1}$$

How do I know my solution to an exercise is correct?

Are the exercises posed in the lectures meant for submission?