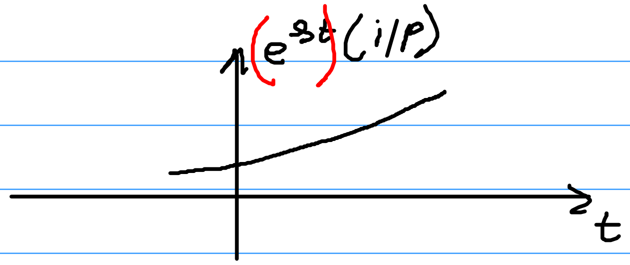
FREQUENCY RESPONSE

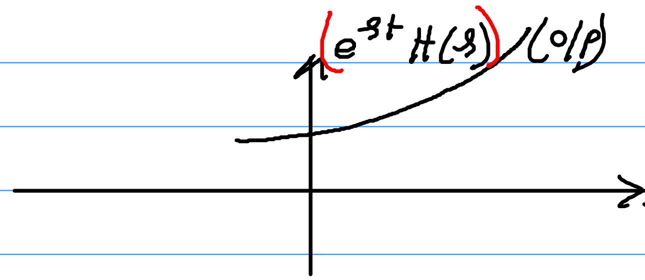
1 $$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t) * x(t)$$

$$h(t) * e^{st} = \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau = e^{st} \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

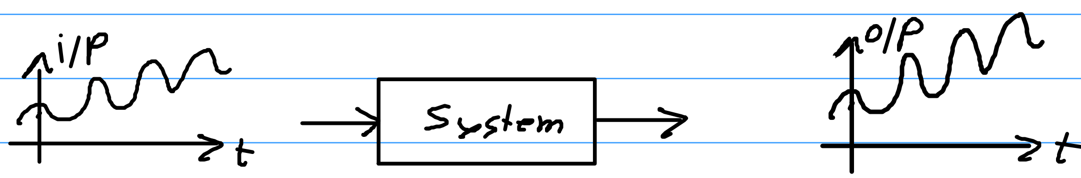
$$\boxed{h(t) * e^{st} = e^{st} H(s)}$$ [where $H(s) = \mathcal{L}\{h(t)\}$]

(1) s can be a complex number general.
(2) The pattern of e^{st} (i/p) & $e^{st} H(s)$ (o/p) is similar.





<u>Eigen functions of a system</u>: If i/p $x(t)$ produces the o/p $K\,x(t)$ then $x(t)$ is called an eigen function of the system



Example: Any exponential signal e^{st} is an eigen function of any LTI system.

1) Almost all practical signals can be written as a sum of exponential signals

2) For LTI systems If $\begin{cases} x_1(t) \rightarrow y_1(t) \\ x_2(t) \rightarrow y_2(t) \end{cases}$

then $a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$

3) For LTI systems $e^{st} \longrightarrow H(s)\, e^{s(t)}$

where $H(s) = \mathcal{L}\{h(t)\}$

Presciption for calculating o/p of an LTI system:

a) Given an arbitrary input $x(t)$ break it as a sum of exponentials

$$x(t) = \frac{1}{2\pi j} \int_{\substack{\omega=-\alpha \\ \sigma > \sigma_{min}}}^{\alpha} X(s)\, e^{st}\, ds \qquad \text{where} \quad X(s) = \mathcal{L}\{x(t)\}$$

b) Caculate the o/p for each exponential component of the i/p

$$\frac{X(s)}{2\pi j} e^{st} ds \rightarrow \left(\frac{X(s)}{2\pi j} e^{st} ds\right) H(s) \qquad \text{where} \quad H(s) = \mathcal{L}\{h(t)\}$$

c) Add all the o/p components.

$$\int_{\substack{\omega=-\alpha \\ \sigma \in ROC}}^{\alpha} H(s) \frac{X(s)}{2\pi j} e^{st} ds = y(t) = \text{total output.}$$

$$\Rightarrow y(t) = \frac{1}{2\pi j}\int_{\substack{\omega=-\infty \\ \sigma \in ROC}}^{\infty} H(s)X(s)e^{st}\,ds = \mathcal{L}^{-1}\{H(s)X(s)\}$$

$$\Rightarrow \mathcal{L}\{y(t)\} = \boxed{Y(s) = H(s)X(s)}$$

So for an LTI systems we can make a table

s	$H(s)$	Input e^{st}	Output $H(s)e^{st}$

If we know $H(s) \Rightarrow$ we know everything about the system

$\Rightarrow$ " " " $h(t) \Rightarrow$ " " " " " "

$$\because \; y(t) = h(t) * x(t)$$

$$\boxed{H(s) = \mathcal{L}\{h(t)\} \text{ is called the TRANSFER FUNCTION of the system.}}$$

Q. $\underbrace{y(t)}_{\text{Given}} = \underbrace{h(t)}_{\text{Known}} * \underbrace{x(t)}_{?}$

Ask: Suppose desired o/p $y(t)$ is known
To find the required i/p $x(t)$ that will produce desired o/p $y(t)$

Ans / Prescription :

a) Break the desired $y(t)$ into a sum of exponential components

$$y(t) = \frac{1}{2\pi j}\int_{\substack{\omega=-\alpha \\ \sigma\in}}^{\alpha} Y(s)\, e^{st}\, ds \qquad \text{where} \quad Y(s) = \mathcal{L}\{y(t)\}$$

b) Calculate the required i/p for each o/p component

$$\frac{Y(s)}{2\pi j} e^{st} ds \longleftarrow \left(\frac{Y(s)}{2\pi j} e^{st} ds\right)\frac{1}{H(s)}$$

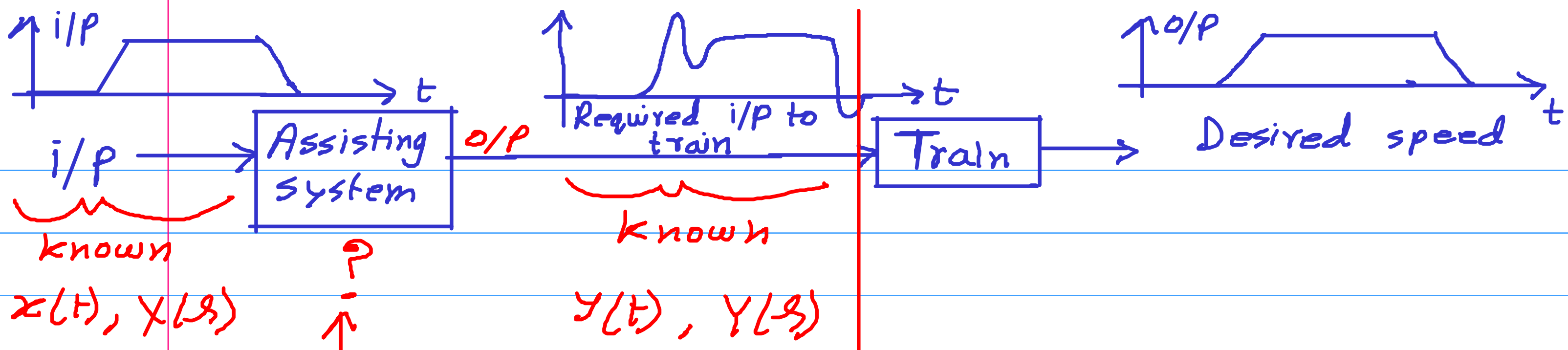
c) Add all required i/p component

$$x(t) = \frac{1}{2\pi j}\int_{\substack{\omega=-\alpha \\ \sigma\in ROC}}^{\alpha} \frac{Y(s)}{H(s)}\, e^{st}\, ds = \mathcal{L}^{-1}\left\{\frac{Y(s)}{H(s)}\right\}$$

LTI

$$\text{Input} \longrightarrow \boxed{\text{System}} \longrightarrow \text{Output}$$

$x(t)$	$h(t)$ / $H(s)$	$y(t)$
✓	✓	? $y(t) = x(t) * h(t) = \mathcal{L}^{-1}\{X(s)H(s)\}$
$x(t) = \mathcal{L}^{-1}\left\{\frac{Y(s)}{H(s)}\right\}$?	✓	✓
✓	?	✓



$$H(s) = \frac{Y(s)}{X(s)}$$

Example:

Q $$3\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y(t) = x(t)$$

Check which of the following inputs are eigen functions of the above system

✗ (i) $x(t) = \delta(t)$

✗ (ii) $x(t) = u(t)$

✗ (iii) $x(t) = \sin(\omega_0 t)$

✗ (iv) $x(t) = \cos(\omega_0 t)$

✓ (v) $x(t) = e^{\sigma t}$ $(\sigma > 0)$

✓ (vi) $x(t) = e^{-\sigma t}$ $(\sigma > 0)$

✓ (vii) $x(t) = e^{j\omega t}$ $(\omega > 0)$

✓ (viii) $x(t) = e^{-j\omega t}$ $(\omega > 0)$

✓ (ix) $x(t) = e^{(\sigma + j\omega)t}$

(i) Assume $x(t) = \delta(t)$ is an eigen function

$$\Rightarrow y(t) = k\,\delta(t)$$

$$\Rightarrow k\left(3\ddot{\delta} + 2\dot{\delta} + \delta(t)\right) = x(t) = \delta(t)$$

$\rightarrow$ This is impossible

(ii) Assume $x(t) = u(t)$ is an eigen function

$$\Rightarrow y(t) = k\,u(t)$$

$$\Rightarrow 3k\dot{\delta}(t) + 2k\delta(t) + ku(t) = x(t) = u(t)$$

$\rightarrow$ Impossible

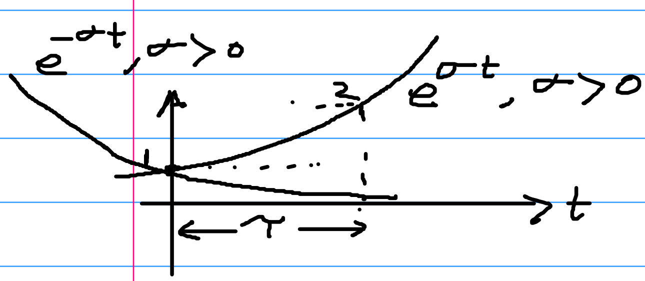
(iii) Assume $x(t) = \sin(\omega_0 t)$ is an eigen function.

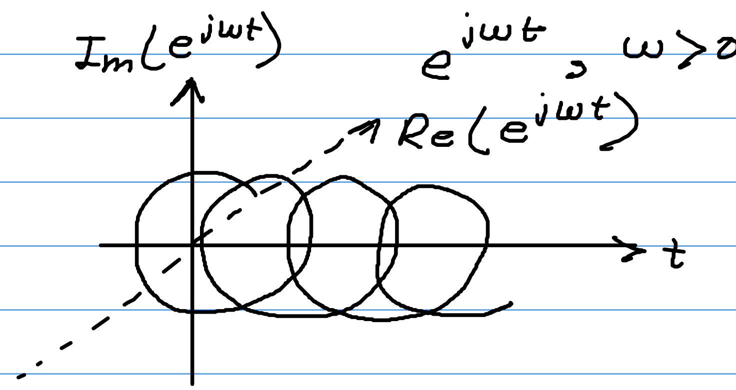
$$\Rightarrow y(t) = k\sin(\omega_0 t)$$

$$\Rightarrow -3k\omega_0^2\sin(\omega_0 t) + 2k\omega_0\cos(\omega_0 t) + k\sin(\omega_0 t) = x(t) = \sin(\omega_0 t)$$

$$\Rightarrow k\left(-3\omega_0^2\sin(\omega_0 t) + 2\omega_0\cos(\omega_0 t) + \sin(\omega_0 t)\right) = \sin(\omega_0 t)$$

$\rightarrow$ Impossible if $\omega_0 \neq 0$



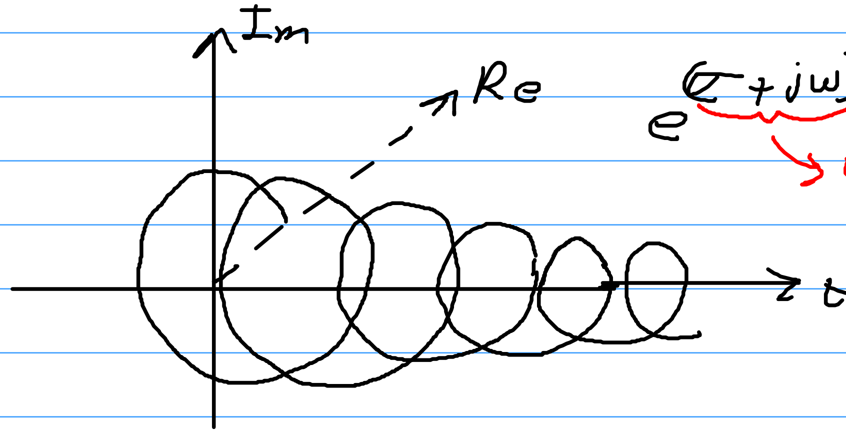


unit of $\sigma = \frac{1}{[Time]}$

$= [Frequency]$

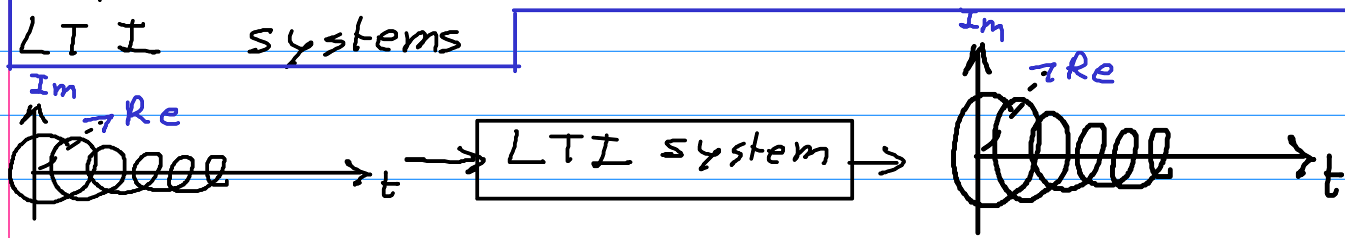
$f = \frac{\omega}{2\pi}$ = cycles/time

unit of $\omega = \frac{1}{sec}$



$e^{\underbrace{(\sigma + j\omega)}_{\text{unit}=[\text{Frequency}]}t}$

Exponential signals e^{st} are eigen signals of LTI systems



Eigen Vector of a matrix

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

$\uparrow$ o/p $\qquad\qquad\qquad\qquad$ $\uparrow$ i/p

$$\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \xleftrightarrow[\text{same}]{\text{Directions are}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

For LTI systems

$$y(t) = x(t) * h(t)$$

$$\Rightarrow \mathcal{L}\{y(t)\} = \mathcal{L}\{x(t) * h(t)\}$$

$$\Rightarrow Y(s) = \mathcal{L}\{x(t)\}\,\mathcal{L}\{h(t)\} = X(s)\,H(s)$$

Definition: $H(s) = \mathcal{L}\{h(t)\}$ = Transfer Function

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{Relaxed LTI System}\\ h(t)\end{matrix}} \longrightarrow y(t) = h(t) * x(t)$$

$$Y(s) = H(s)\, X(s)$$

$H(s) = \mathcal{L}\{h(t)\}$: Transfer function.

$$\frac{d^n y}{dt^n} + b_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots\cdots + b_1 \frac{dy}{dt} + b_0\, y(t)$$

$$= a_0\, x(t) + a_1 \frac{dx}{dt} + a_2 \frac{d^2x}{dt} + \cdots + a_m \frac{d^m x}{dt^m}$$

To find transfer function of the above system.

$$H(s) = \mathcal{L}\{h(t)\}$$

We know when i/p is $\delta(t)$, then o/p will be $h(t)$

$$\frac{d^n h}{dt^n} + b_{n-1}\frac{d^{n-1}h}{dt^{n-1}} + \cdots\cdots + b_1 \frac{dh}{dt} + b_0\, h(t)$$

$$= a_0\, \delta(t) + a_1 \dot{\delta} + a_2 \ddot{\delta} + \cdots + a_m \frac{d^m \delta(t)}{dt^m}$$

$$\Rightarrow \mathcal{L}\left\{\frac{d^n h}{dt^n} + b_{n-1}\frac{d^{n-1}h}{dt^{n-1}} + \cdots\cdots + b_1 \frac{dh}{dt} + b_0\, h(t)\right\}$$

$$\mathcal{L}\left\{a_0\, \delta(t) + a_1 \dot{\delta} + a_2 \ddot{\delta} + \cdots + a_m \frac{d^m \delta(t)}{dt^m}\right\}$$

$$\Rightarrow s^n H(s) + b_{n-1} s^{n-1} H(s) + \cdots + b_1 s\, H(s) + b_0\, H(s)$$

$$= a_0\, 1 + a_1 s + a_2 s^2 + \cdots + a_m s^m$$

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

BIBO stability from transfer function:

$$\text{BIBO stable} \iff \int_{-\infty}^{\infty} |h(t)| = \text{finite}$$

$$H(s) = \mathcal{L}\{h(t)\}$$

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

Assume $m < n$ [n: order of the system]

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{(s-P_1)(s-P_2)\cdots(s-P_n)}$$

[P_1, P_2, P_n = are the roots of $s^n + b_{n-1} s^{n-1} + \cdots b_1 s + b_0 = 0$]

$P_1, P_2 \ldots P_n$ are called Poles of the system

these are complex numbers.

Rule: If $Re(P_k) < 0$ for $\forall k$

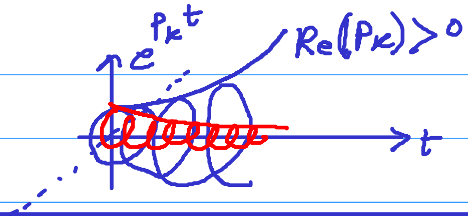
Then the system is stable (BIBO)

Why?

$$h(t) = \mathcal{L}^{-1}\left\{\frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{(s-P_1)(s-P_2)\cdots(s-P_n)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{A_1}{s-P_1} + \frac{A_2}{(s-P_2)} + \cdots \frac{A_k s^{n_k-1} + \cdots B s + C}{(s-P_k)^{n_k}} + \cdots \frac{A_n}{s-P_n}\right\}$$

$$h(t) = A_1 e^{p_1 t}u(t) + A_2 e^{p_2 t}u(t) + \cdots + \text{something}\, e^{p_k t}u(t) + \cdots A_n e^{p_n t}u(t)$$



$$\mathcal{L}\{h(t)\} = \int_{t=-\infty}^{\infty} h(t)\, e^{-st}\, dt = H(s)$$

$$\mathcal{F}\{h(t)\} = \int_{t=-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = H(j\omega)$$

$$x(t) \rightarrow \boxed{\begin{array}{c}\text{Relaxed LTI System}\\ h(t)\end{array}} \rightarrow y(t) = h(t) * x(t)$$

$x(t) = e^{j\omega_0 t}$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{j\omega_0 (t-\tau)}\, d\tau$$

$$= e^{j\omega_0 t}\int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-j\omega_0 \tau}\, d\tau$$

$$= e^{j\omega_0 t}\left(\mathcal{F}\{h(t)\}\Big|_{\omega=\omega_0}\right)$$

$$= e^{j\omega_0 t}\left(H(j\omega)\Big|_{\omega=\omega_0}\right)$$

$$e^{j\omega t} \rightarrow \boxed{h(t)} \rightarrow e^{j\omega t}\left(H(j\omega)\right)$$

eigen function

Divide & Conquer

I/P	O/P
$x(t) = \int_{\omega=-\alpha}^{\alpha} \frac{X(j\omega)}{2\pi} e^{j\omega t} d\omega$	$y(t) = \int_{\omega=-\alpha}^{\alpha} H(j\omega) \frac{X(j\omega)}{2\pi} e^{j\omega t} d\omega$ $= \mathcal{F}^{-1}\{H(j\omega) X(j\omega)\}$

$$\Rightarrow \mathcal{F}\{y(t)\} = \mathcal{F}\{h(t)\} \mathcal{F}\{x(t)\}$$

$$y(t) = h(t) * x(t)$$

$$\Rightarrow \mathcal{F}\{y(t)\} = \mathcal{F}\{h(t) * x(t)\} = \mathcal{F}\{h(t)\} \mathcal{F}\{x(t)\}$$

ω	$H(j\omega)$	$i/p = e^{j\omega t}$	$o/p = H(j\omega) e^{j\omega t}$
$-\alpha$	$H(j\alpha)$		
0	$H(0)$		
α	$H(j\alpha)$		

Frequency Response

$= H(j\omega)$

Example:

Suppose i/p $x(t) = A \sin(\omega_0 t)$

$y(t) = ?$

$(A \sin\cancel{(\omega_0 t)} \times K)$

$$x(t) = A\sin(\omega_0 t) = \underbrace{A\frac{e^{j\omega_0 t}}{2j}}_{x_1} - \underbrace{A\frac{e^{-j\omega_0 t}}{2j}}_{x_2}$$

$$y_1(t) = A\frac{e^{j\omega_0 t}}{2j} \times \left(H(j\omega)\Big|_{\omega=\omega_0}\right) = \frac{AM}{2j} e^{j(\omega_0 t+\theta)}$$

$$y_2(t) = -A\frac{e^{-j\omega_0 t}}{2j} \times \left(H(j\omega)\Big|_{\omega=-\omega_0}\right) = -AM\frac{e^{-j(\omega_0 t+\theta)}}{2j}$$

$$y(t) = y_1(t) + y_2(t) = AM\left(\frac{e^{j(\omega_0 t+\theta)} - e^{-j(\omega_0 t+\theta)}}{2j}\right) = AM\sin(\omega_0 t+\theta)$$

suppose, $H(j\omega_0) = M\angle\theta = Me^{j\theta}$

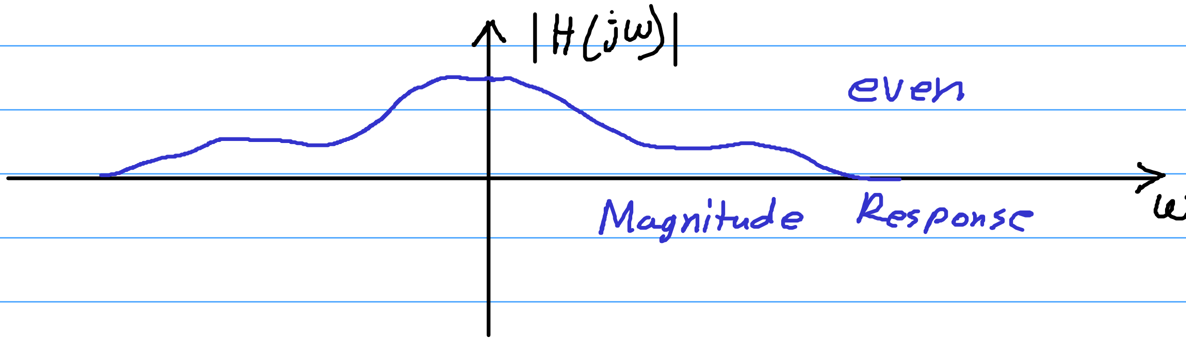
$H(-j\omega_0) = M\angle-\theta = Me^{-j\theta}$

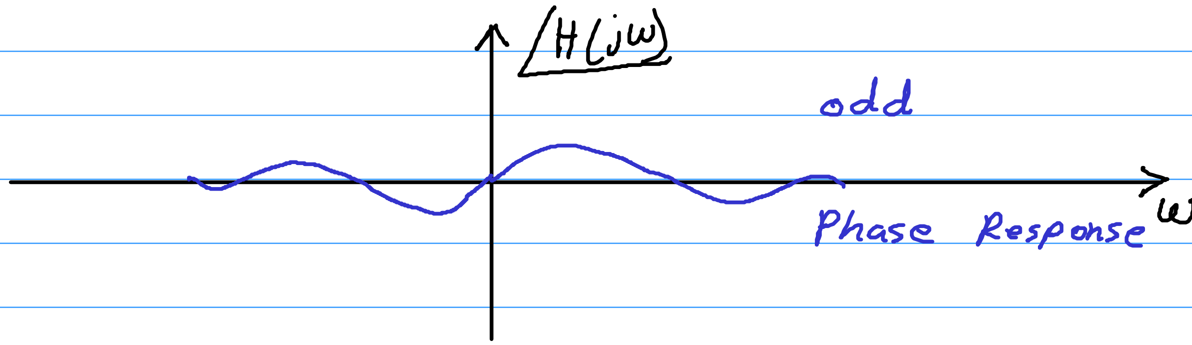
$\boxed{\text{HW}}$ To show that when i/p $= x(t) = A\cos(\omega_0 t)$

then o/p $= y(t) = MA\cos(\omega_0 t+\theta)$

where $H(j\omega_0) = M\angle\theta$

i.e. $M = |H(j\omega_0)|$ & $\theta = \angle H(j\omega_0)$





Frequency Response

Q How to determine Frequency Response Experimentally

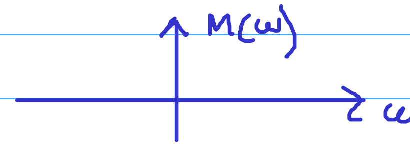
$$e^{j\omega t} \longrightarrow \boxed{\text{System}} \longrightarrow e^{j\omega t} \times H(j\omega)$$

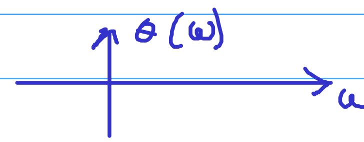
$H(j\omega)$ $\qquad = M(\omega)\, e^{j(\omega t + \theta(\omega))}$

Vary ω_0

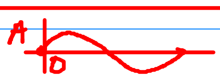
$|H(j\omega)| = M(\omega)$

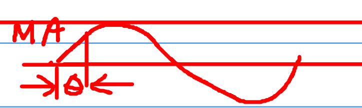
$\angle H(j\omega) = \theta(\omega)$





Impossible in laboratory





$$A \sin(\omega_0 t) \longrightarrow \boxed{\text{system}} \longrightarrow (M(\omega_0)\, A) \sin(\omega_0 t + \theta(\omega_0))$$

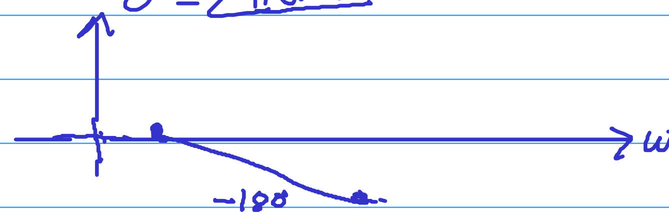
where $M(\omega_0) = |H(j\omega_0)|$ & $\theta(\omega_0) = \angle H(j\omega_0)$

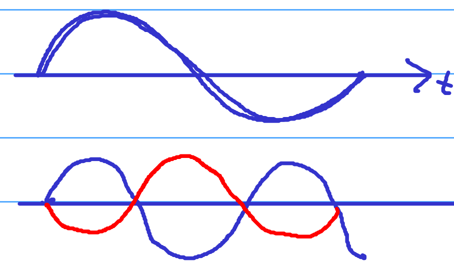
Experimental Demonstrations

$$\frac{\text{Output Amplitude}}{\text{Input Amplitude}} = M = |H(j\omega)|$$

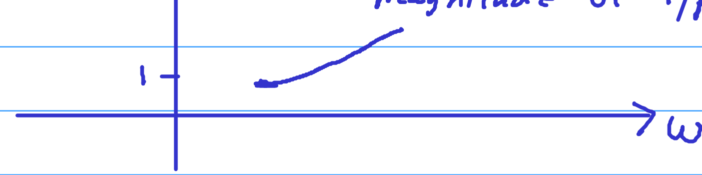
Magnitude Response

$\theta = \angle H(j\omega)$

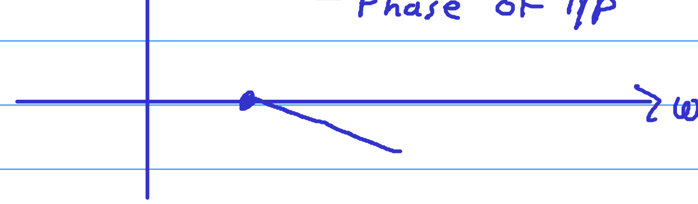




$$M(\omega) = \frac{\text{Magnitude of o/p}}{\text{Magnitude of i/p}}$$



$\theta(\omega) = \text{Phase of o/p} - \text{Phase of i/p}$



Drawing frequency response from Transfer function (approximately)

$$\frac{d^n y}{dt^n} + b_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots\cdots + b_1 \frac{dy}{dt} + b_0 y(t)$$

$$= a_0 x(t) + a_1 \frac{dx}{dt} + a_2 \frac{d^2x}{dt} + \cdots + a_m \frac{d^m x}{dt^m}$$

SI $$H(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}$$

$$= \frac{a_m (s-z_1)(s-z_2)\cdots\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots\cdots(s-p_n)}$$

$p_1, p_2 \ldots p_n \equiv$ Poles

$z_1, z_2 \ldots z_n \equiv$ Zeros

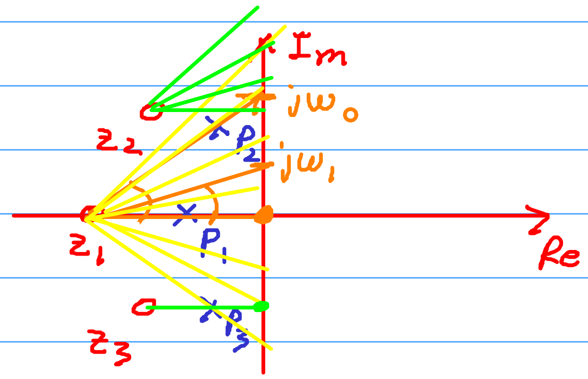
SII $$\mathcal{F}\{h(t)\} = H(j\omega) =$$

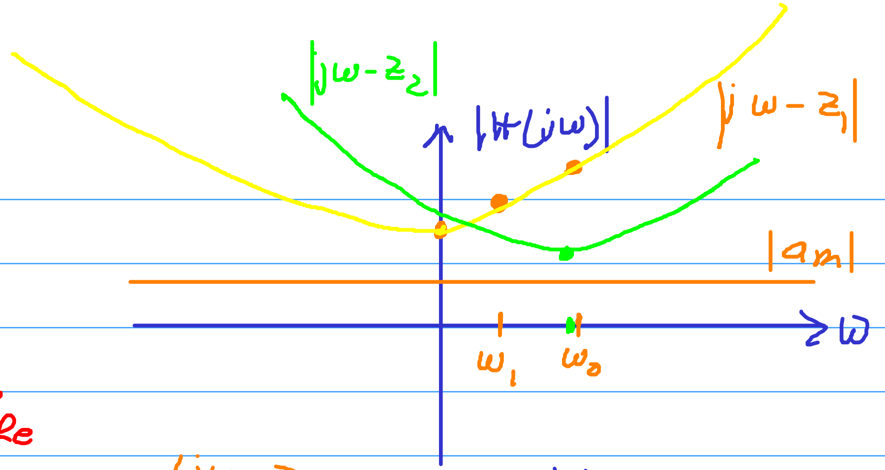
$$= \frac{a_m (j\omega - z_1)(j\omega - z_2)\cdots\cdots(j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2)\cdots\cdots(j\omega - p_n)}$$

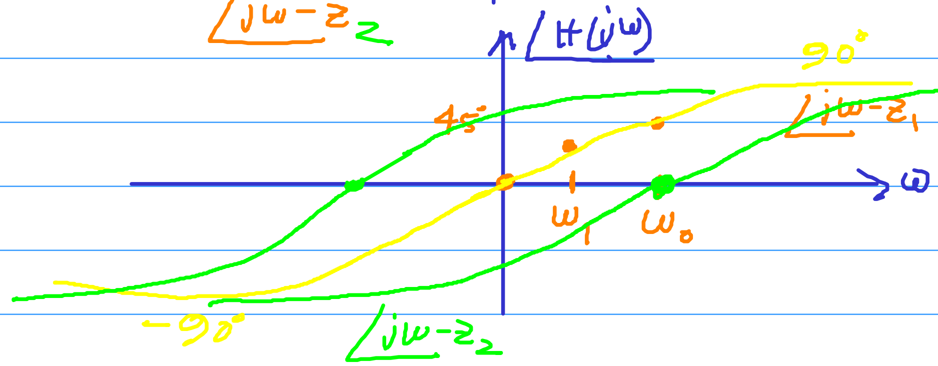
SII $$|H(j\omega)| = \frac{|a_m|\,|j\omega - z_1|\cdots\cdots|j\omega - z_m|}{|j\omega - p_1|\,|j\omega - p_2|\cdots\cdots|j\omega - p_n|}$$

$$\angle H(j\omega) = \left(\angle a_m + \angle j\omega - z_1 + \angle j\omega - z_2 + \cdots + \angle j\omega - z_m\right) - \left(\angle j\omega - p_1 + \angle j\omega - p_2 + \cdots\cdots + \angle j\omega - p_n\right)$$

(SIV) Pole-Zero diagram

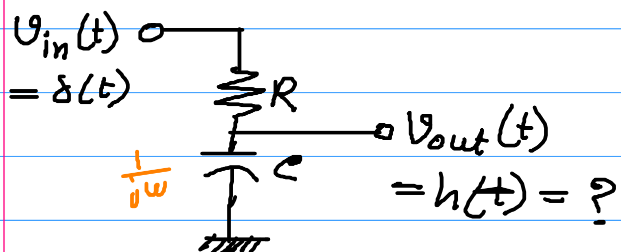




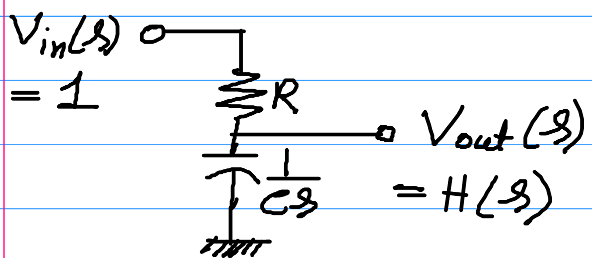


Note: Poles & zeros occur in complex conjugate pairs or purely real.

How to obtain transfer function for an electrical network / circuit ?



$$H(s) = \mathcal{L}\{h(t)\}$$



$$V_{out}(s) = \left(\frac{V_{in}(s)}{R + \frac{1}{Cs}} \right) \times \frac{1}{Cs}$$

$$H(s) = \frac{1}{1 + RCs}$$

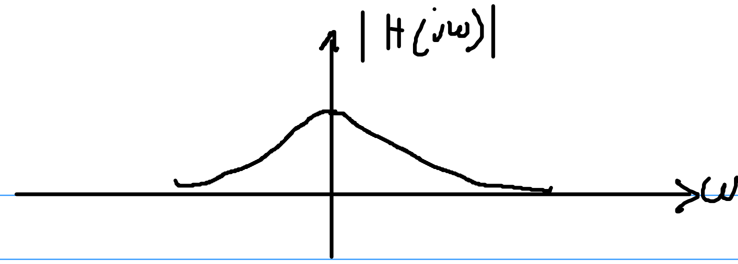
under initially relaxed condition

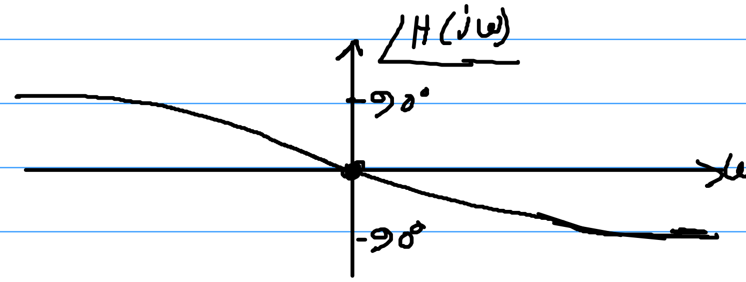
$$H(s)=\frac{1}{1+RCs}$$

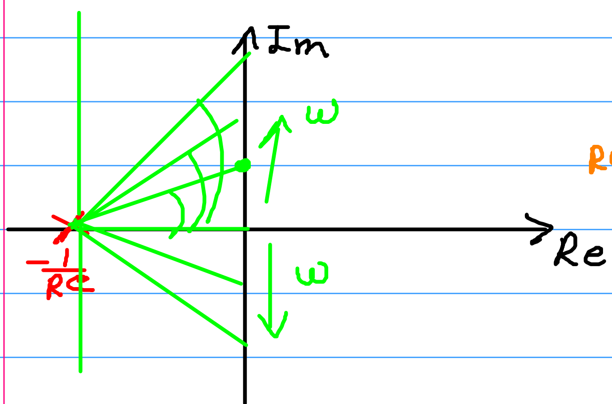
$$H(j\omega) = \frac{1}{1+jRC\omega}$$

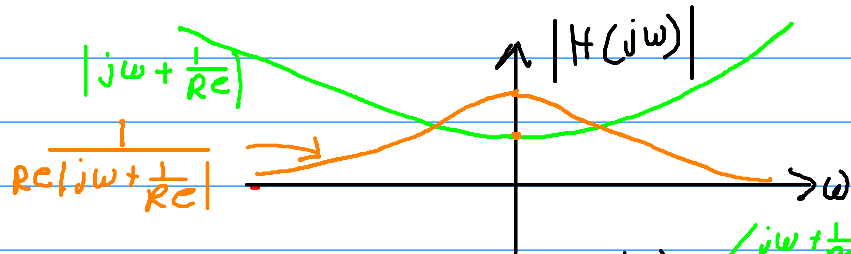
$$|H(j\omega)| = \frac{1}{\sqrt{1+R^2C^2\omega^2}}$$

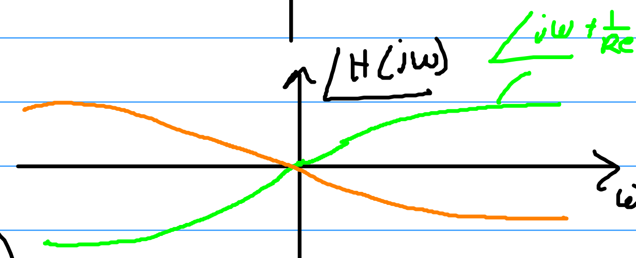
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{RC\omega}{1}\right)$$











$$H(s) = \frac{1}{1+RCs} = \frac{1}{RC}\left(\frac{1}{s-\left(-\frac{1}{RC}\right)}\right)$$

$$|H(j\omega)| = \frac{1}{RC\left|j\omega - -\frac{1}{RC}\right|}$$

$$\angle H(j\omega) = -\angle j\omega + \frac{1}{RC}$$

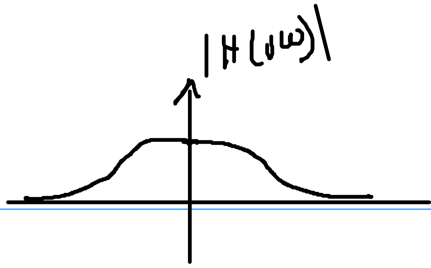
★ ★ $\boxed{\text{ALL LTI SYSTEMS ARE FREQUENCY FILTERS}}$

$$x(t) = \int_{\omega=-\infty}^{\infty} \frac{X(\omega)}{2\pi} e^{j\omega t}\, d\omega$$

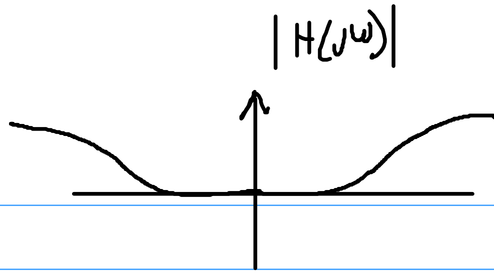
$$e^{j\omega t} \longrightarrow e^{j\omega t} \times H(j\omega)$$

$$\sin(\omega t) \longrightarrow M(\omega)\sin(\omega t + \theta(\omega)), \quad M(\omega) = |H(j\omega)|$$

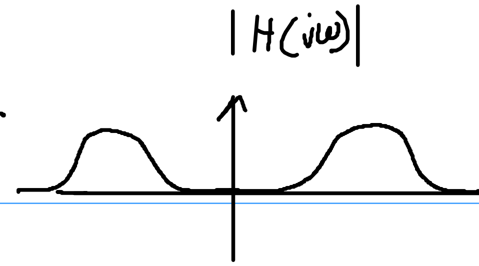
$$\theta(\omega) = \angle H(j\omega)$$



LOW PASS FILTER

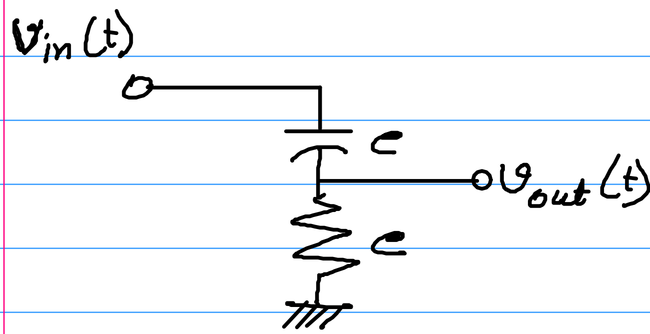


HIGH PASS FILTER



BAND PASS FILTER

$\boxed{HW}$

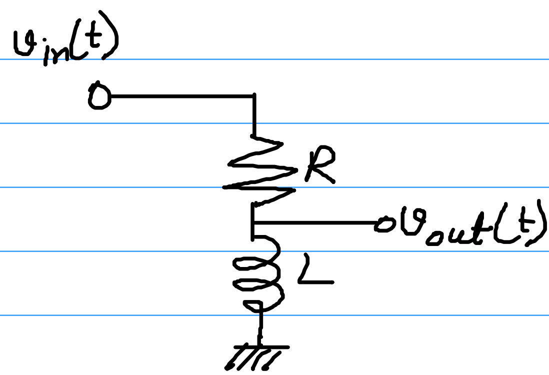


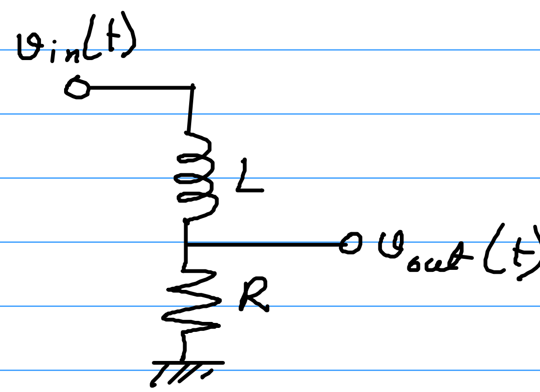
Find $H(s)$

Plot Freq. Respons

show that it is HPF

$\boxed{HW}$





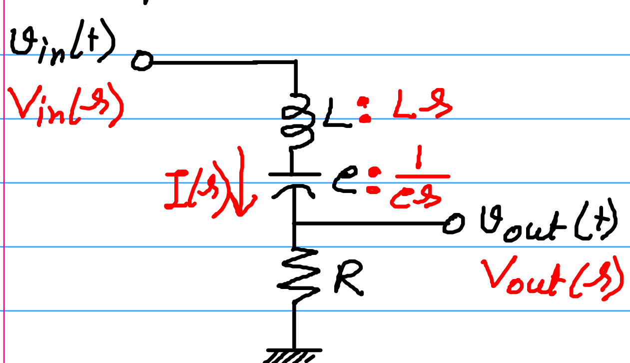
$H(s) = ?$

Plot Freq Response

Show it is an HPF

show it is an LPF

<u>Example</u>



$$V_{out}(s) = H(s)\, V_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

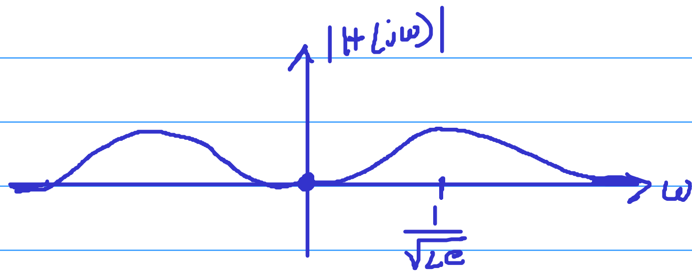
$$= \frac{R\, I(s)}{V_{in}(s)}$$

$$= \frac{R \dfrac{\cancel{V_{in}(s)}}{Ls + \frac{1}{Cs} + R}}{\cancel{V_{in}(s)}}$$

$$= \frac{R}{Ls + \frac{1}{Cs} + R} = \frac{RCs}{LCs^2 + RCs + 1} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

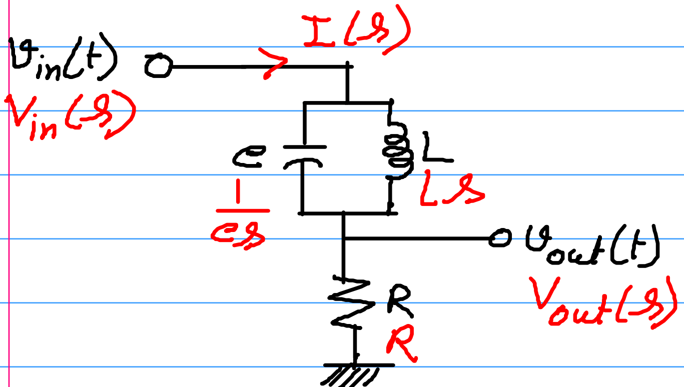
$$H(j\omega) = \frac{\frac{R}{L}j\omega}{-\omega^2 + j\frac{R}{L}\omega + \frac{1}{LC}}$$

$$|H(j\omega)| = \frac{\frac{R}{L}|\omega|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2}} \; ; \quad \angle H(j\omega) = 90^\circ - \tan^{-1}\frac{\frac{R\omega}{L}}{\frac{1}{LC} - \omega^2}$$



BPF

Example:



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$V_{out}(s) = R\, I(s)$$

$$= \frac{R\; V_{in}(s)}{R + \left(Ls \,\|\, \frac{1}{Cs}\right)}$$

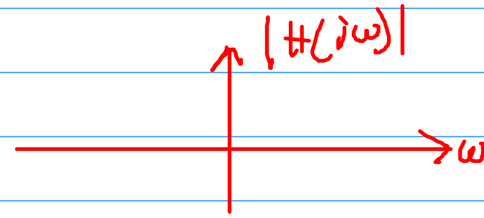
$$= \frac{R\, V_{in}(s)}{R + \left(\frac{L/C}{Ls + \frac{1}{Cs}}\right)}$$

$$= \frac{R V_{in}(s)}{R + \left(\frac{Ls}{LCs^2+1}\right)} = \frac{(RLCs^2 + R)V_{in}(s)}{RLCs^2 + R + Ls}$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RLCs^2+R}{RLCs^2+Ls+R} = \frac{s^2+\frac{1}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$

$$H(j\omega) = \frac{-\omega^2+\frac{1}{LC}}{\left(\frac{1}{LC}-\omega^2\right)+\frac{j}{RC}\omega}$$

$$|H(j\omega)| = \frac{\left|\frac{1}{LC}-\omega^2\right|}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{\omega}{RC}\right)^2}} \, ;$$



$$\angle H(j\omega) = \angle \frac{1}{LC}-\omega^2 - \tan^{-1}\left(\frac{\omega}{RC\left(\frac{1}{LC}-\omega^2\right)}\right)$$