INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Switching Circuits and Logic Design (CS21002, Spring)

Class Test – II (part-1)

Name: ________________________________ *Roll number:* ________________

Date: Wed, Feb 10, 2021 *Marks:* 23 *Time:* 8:10-9am (FN)

Answer ALL the questions using xournal or similar software to edit the PDF

Q1: Consider the set of integers $A_m = \{1, 2, 3, \ldots, m\}$, with $\leq$ as the usual partial ordering on this set. We may define an order on elements of $A_m \times A_n$ as $\langle a, b \rangle \preceq \langle c, d \rangle \Leftrightarrow a \leq c$ and $b \leq d$.

(a) Prove that this defines a partial ordering on $A_m \times A_n$. 5

(b) Draw the Hasse diagrams for $A_2 \times A_3$. 4

(c) What are glb $(\langle a, b\rangle, \langle c, d\rangle)$ and lub $(\langle a, b\rangle, \langle c, d\rangle)$ for any $a, c \in A_m$ and $b, d \in A_n$? 4

Q2: Let $\langle L, \cdot, + \rangle$ and $\langle M, \odot, \oplus \rangle$ be two lattices. Consider the Cartesian product $L \times M$ of L and M.

Define operations Δ and ∇ in $L \times M$, as $\langle x, y \rangle \, \Delta \, \langle a, b \rangle = \langle x \cdot a, y \odot b \rangle$ and $\langle x, y \rangle \, \nabla \, \langle a, b \rangle = \langle x + a, y \oplus b \rangle$.

Prove that $\langle L \times M, \Delta, \nabla \rangle$ is a lattice. 10