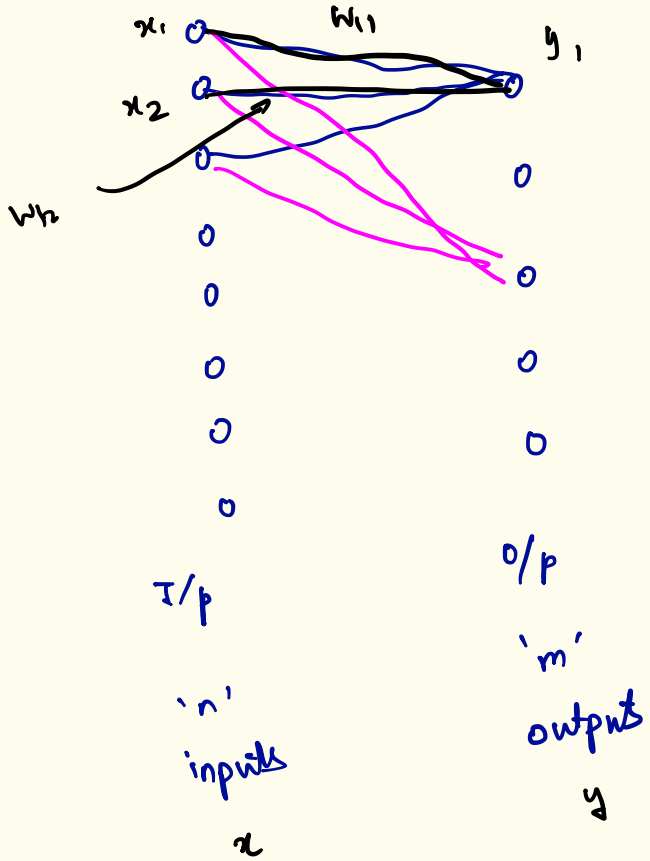
Linear algebra for AI and ML

(November 10)



W - weight matrix

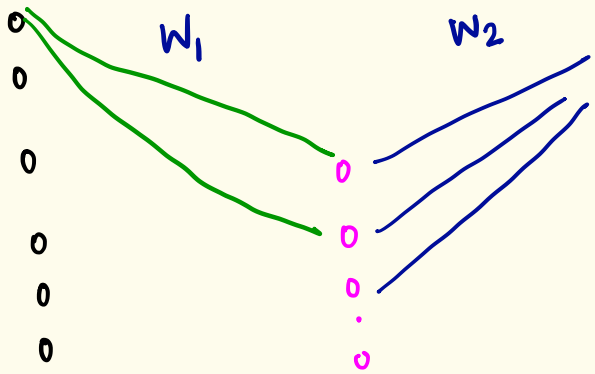
$y \in \mathbb{R}^m$, $x \in \mathbb{R}^n$

$W \in \mathbb{R}^{m \times n}$

$$y_i = Wx$$

$$y_1 = W_{11}x_1 + W_{12}x_2 + \dots + W_{1n}x_n$$

Total number of weights = mn



$x \in \mathbb{R}^n$

I/p

Hidden layer (r)

h

$y \in \mathbb{R}^m$

O/p

$h \in \mathbb{R}^r$

$x \in \mathbb{R}^n$

$y \in \mathbb{R}^m$

$h = W_1 x$

$y = W_2 h$

$$y = W_2 W_1 x$$

$W_1 = \mathbb{R}^{r \times n}$

$W_2 = \mathbb{R}^{m \times r}$

Number of weights

$= r(m+n)$

y_1 is roughly equal y_2

$y_1 \approx y_2$

$W \approx W_2 W_1$

$\min(m,n)$ $\quad r$

$r << \min(m,n)$

$\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4, \ldots, \tilde{y}_{11}$ a time series.

$y_4 = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 + \varepsilon_4$

$y_5 = \alpha_1 y_2 + \alpha_2 y_3 + \alpha_3 y_4 + \varepsilon_5$

order of the autoregressive model = rank of Hankel matrix

$$\begin{bmatrix} \tilde{y}_1 & \tilde{y}_2 & \tilde{y}_3 & \tilde{y}_4 & \tilde{y}_5 & \tilde{y}_6 \\ y_2 & y_3 & y_4 & y_5 & y_6 & y_7 \\ y_3 & y_4 & y_5 & y_6 & y_7 & y_8 \\ y_4 & y_5 & y_6 & y_7 & y_8 & y_9 \\ y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} \\ y_6 & y_7 & y_8 & y_9 & y_{10} & y_{11} \end{bmatrix} \in \mathbb{R}^{6\times 6}$$

$\alpha_1 C_1 + \alpha_2 C_2 + \alpha_3 C_3 = C_4$

Structured Hankel Low Rank Approxi-mation.

Statistics (time-series) / Control theory (1970s) T. Kailath

$A = U\Sigma V^T$

$\tilde{A} = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T$

$V\Sigma^T U^T$

$A = U\Sigma U^T$

$A = A^T$

$= \sigma_1 u_1 u_1^T$

$+ \sigma_2 u_2 u_2^T$

$\vdots$

$(\sigma_1 u_1^T)^T = (u_1^T)^T \sigma_1^T = \sigma_1 u_1^T$

$\Rightarrow u_1 u_1^T$ is symmetric matrix rank-1.

$$\sigma_1 u_1 u_1^T + \sigma_2 u_2 u_2^T + \sigma_3 u_3 u_3^T = \tilde{A}$$