

Q5) If we take $A = I$ (identity matrix in $n \times n$)

then we want to minimise $\|Ax - a\|^2$
with the constraint that

$$\frac{\sum_{i=1}^n x_i}{n} = \beta$$

If we take $C = \left[\frac{1}{n} \quad \frac{1}{n} \quad \dots \quad \frac{1}{n} \right]$

~~and~~ and $d = [\beta]$

we can write this as $Cx = d$.

This is a constrained LS situation.

The KKT matrix is formed as

$$\begin{bmatrix} 2A^T A & C^T \\ C & 0 \end{bmatrix} = \begin{bmatrix} 2I & C^T \\ C & 0 \end{bmatrix}$$

The solution is given by \hat{x} where

$$\begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} 2I & C^T \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2A^T a \\ \beta \end{bmatrix} = \begin{bmatrix} 2I & C^T \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2a \\ \beta \end{bmatrix}$$