NISARG UPADHYAYA 19CS30031

(8) $\Rightarrow$ We need to find parameters $\theta_1, \theta_2, \ldots \theta_M$ to minimize the sum of squared errors

$$(\hat{z}_{t+1} - z_{t+1})^2 \quad \text{for } t = M, M+1, \ldots 99$$

(we only have observed values till z_{100})

where z_{t+1} is the observed data

and $\hat{z}_{t+1} = \theta_1 z_t + \ldots \theta_M z_{t-M+1}$.

$\Rightarrow$ This can be written as $\underbrace{\|A\theta - b\|^2}$ $\hookrightarrow$ quantity to be minimised.

where

A is a matrix with dimensions $(100-M) \times M$

$$A = \begin{bmatrix} z_M & z_{M-1} & z_{M-2} & \cdots & z_2 & z_1 \\ z_{M+1} & z_M & z_{M-1} & \cdots & z_3 & z_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ z_{99} & z_{98} & z_{97} & \cdots & z_{99-M+2} & z_{99-M+1} \end{bmatrix}$$

θ is a matrix with dimensions $M \times 1$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix}$$

$\Rightarrow$ b is a matrix with dimensions $(100-M)\times 1$

$$b = \begin{bmatrix} z_{M+1} \\ z_{M+2} \\ \vdots \\ z_{99} \\ z_{100} \end{bmatrix}$$

$\Rightarrow$ In the matrix A if we move across any diagonal from left to right value of a_{ij} remains same.

Thus for the same $(i-j)$ value a_{ij} s are all same.

$\Rightarrow$ The dimensions of the matrix are $(100-M)\times M$

Hence the $\text{rank}(A) \leq \min(100-M, M)$

Under the given constraints $\text{rank}(A)$ can never exceed 50.