NISARG UPADHYAYA - 19CS30031

Assignment 2 Q3

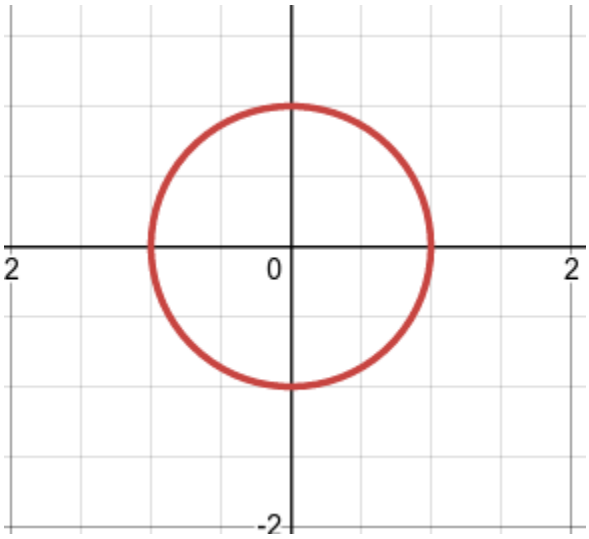
Link to colab:
https://colab.research.google.com/drive/13WaK8AYl24fWuAgloJEH6CzMcOvga_xD?usp=sharing

Note:

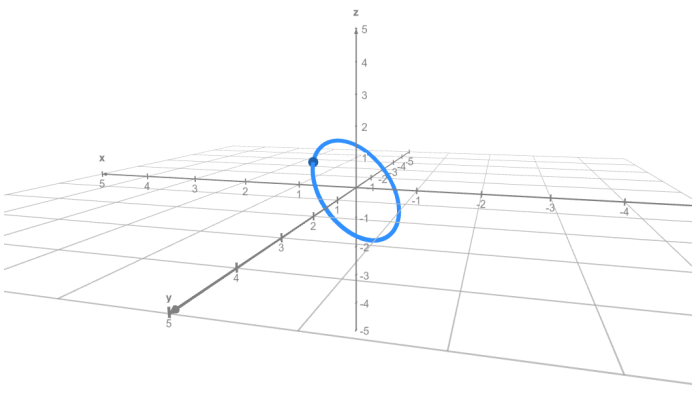
1. 2D plots were made using desmos
2. 3D plot for Q1 was made using was made using the following open source plotter [https://christopherchudzicki.github.io/MathBox-Demos/parametric_curves_3D.html]
3. 2D plot for Q2 was made using matplotlib by sampling points on the 3D sphere and mapping them to the 2D plane using the given matrix.

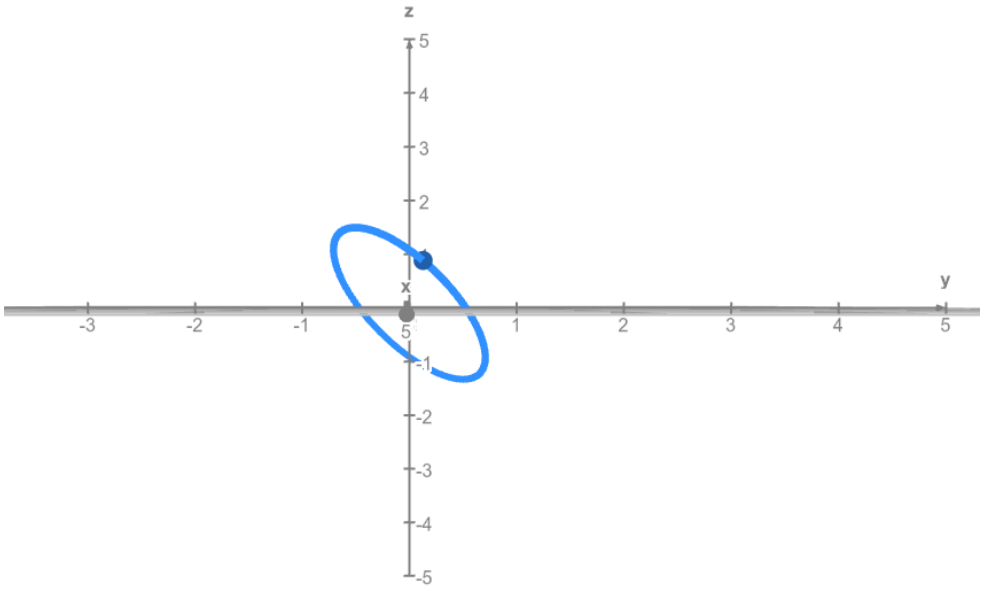
Part A.

Unit sphere in R^2



Ellipse in R^3





Ellipse as viewed from the x axis

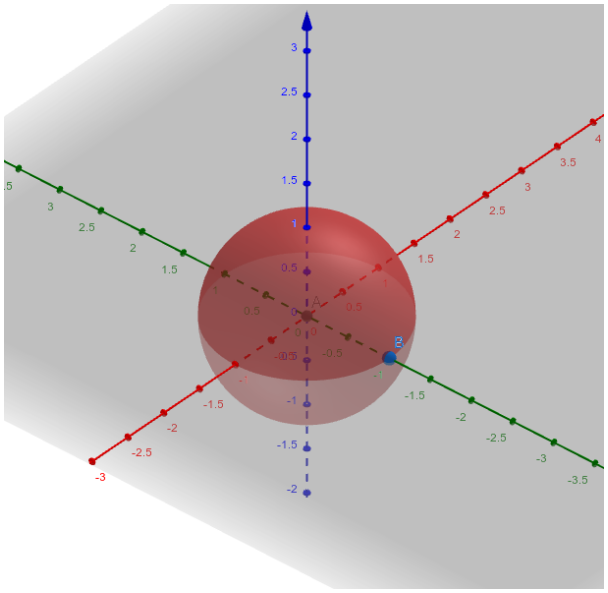
Matrix:

```
[[-0.70710678  0.        ]
 [ 0.         -0.70710678]
 [-1.          1.        ]]
```

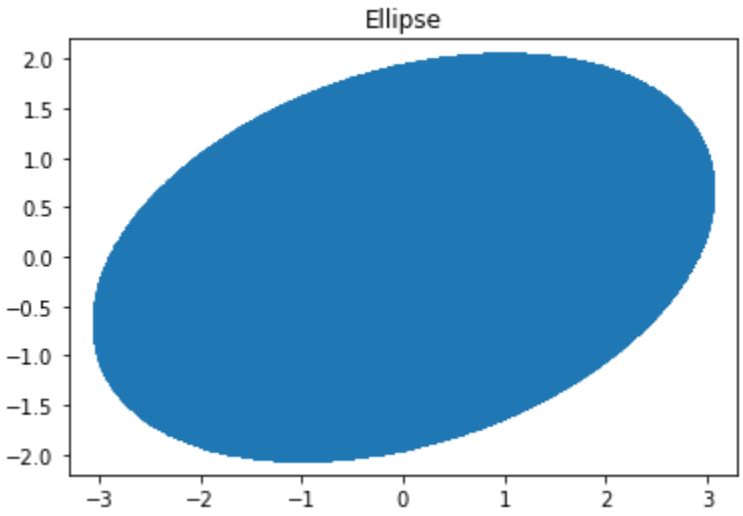
Condition number: `2.23606797749979`

Part B.

Unit sphere in R^3



Ellipse in R^2



Matrix:

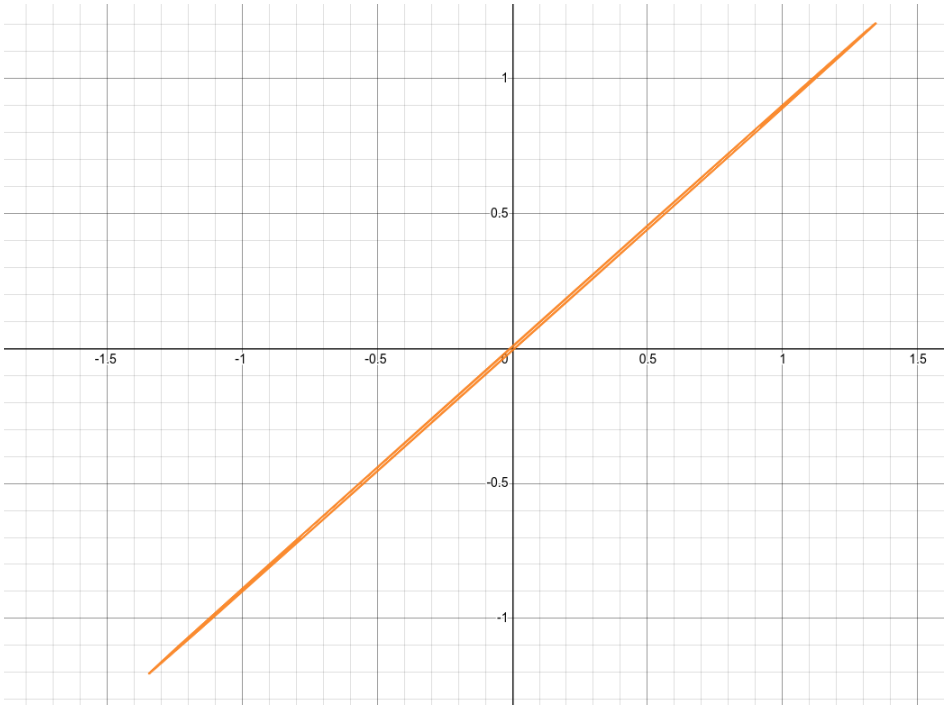
```
[[-2  1  2]
 [ 0  2  0]]
```

Condition number: `1.7150100905617 28`

All the remaining parts are a transformation from R^2 to R^2. The unit sphere has been shown in part A. It is the same for these remaining parts. Only the transformed ellipse is being shown.

Part C.

Ellipse in R^2

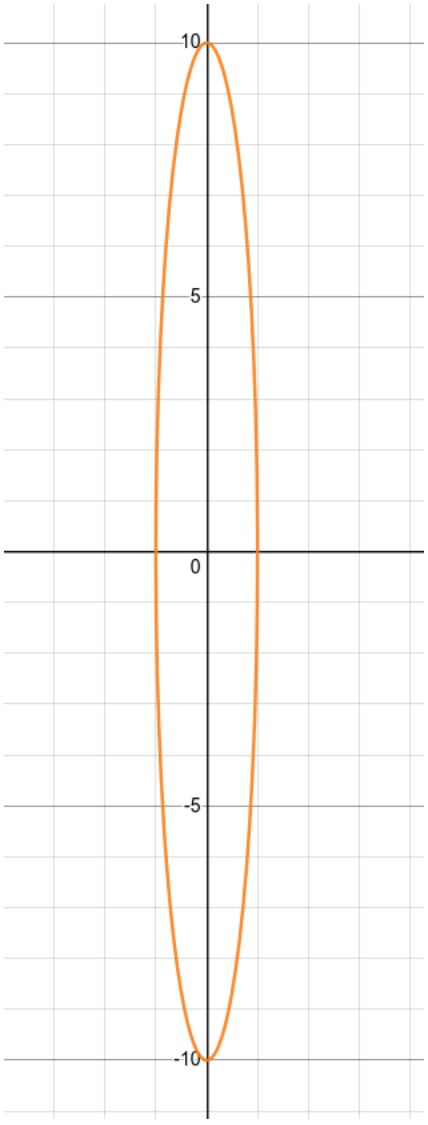



```
[[1.  0.9]
 [0.9 0.8]]

Condition number: 325.99693248647975
Determinant: -0.01

Invertible
```

Note that this matrix is very ill conditioned. We can see that the columns are almost dependent and hence it is almost non-invertible, hence the large condition number.

Part D.

Ellipse in R^2

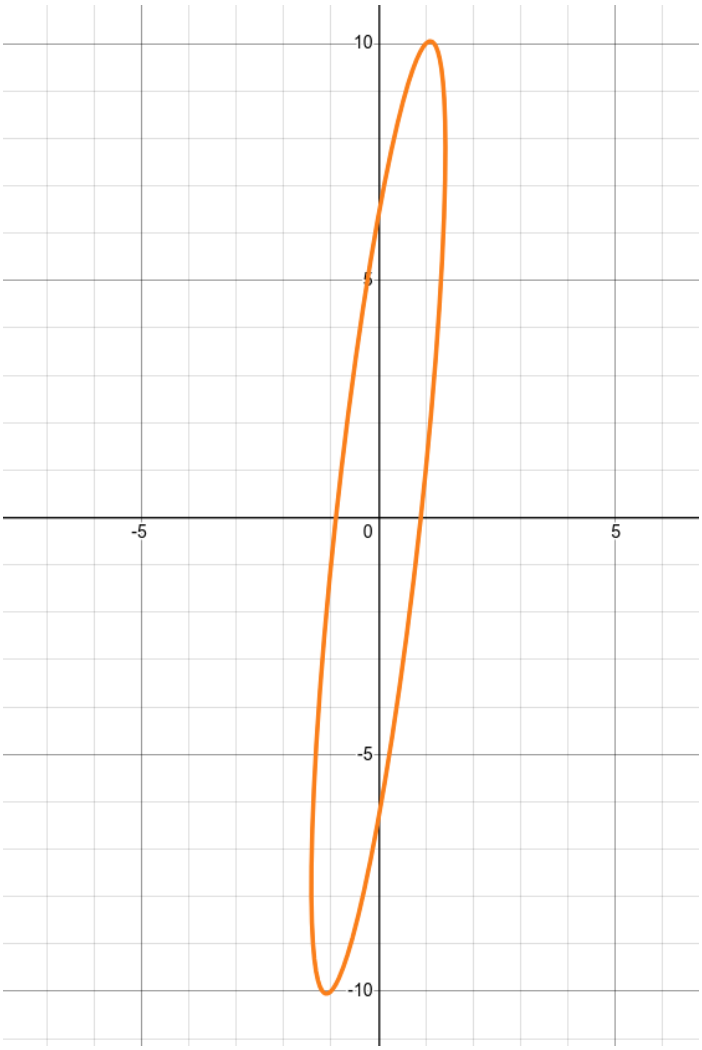



```
[[  1   0]
 [  0 -10]]

Condition number: 10.0
Determinant: -10

Invertible
```

The columns in this case are not that dependent. Hence the condition number is not very high.

Part E1.

Ellipse in R^2

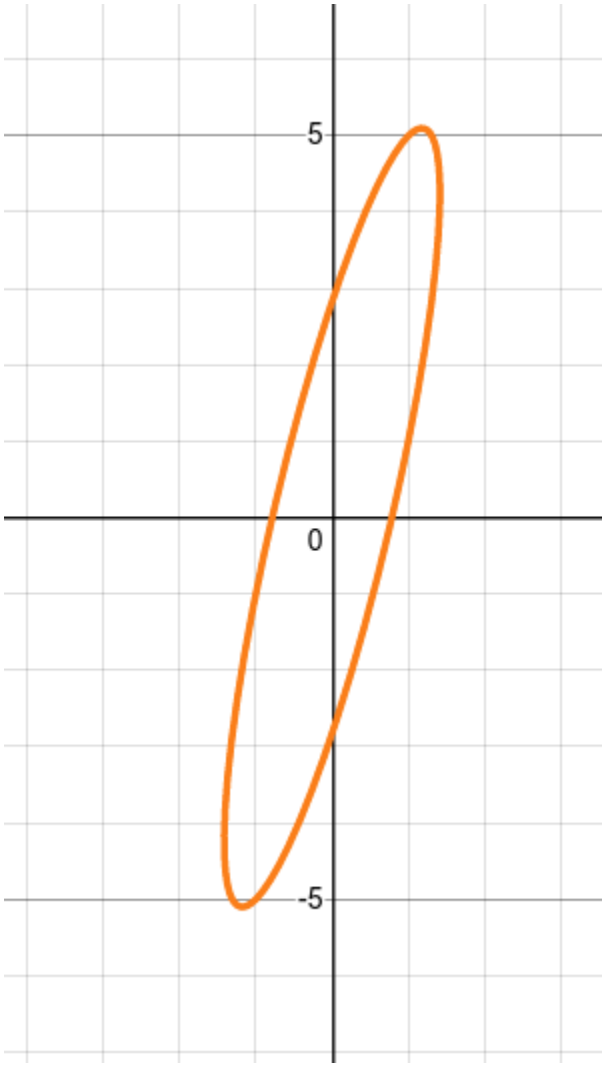



```
[[ 1  1]
 [ 1 10]]

Condition number: 11.35638827945676
Determinant: 9

Invertible
```

The columns in this case are not that dependent. Hence the condition number is not very high.

Part E2.

Ellipse in R^2

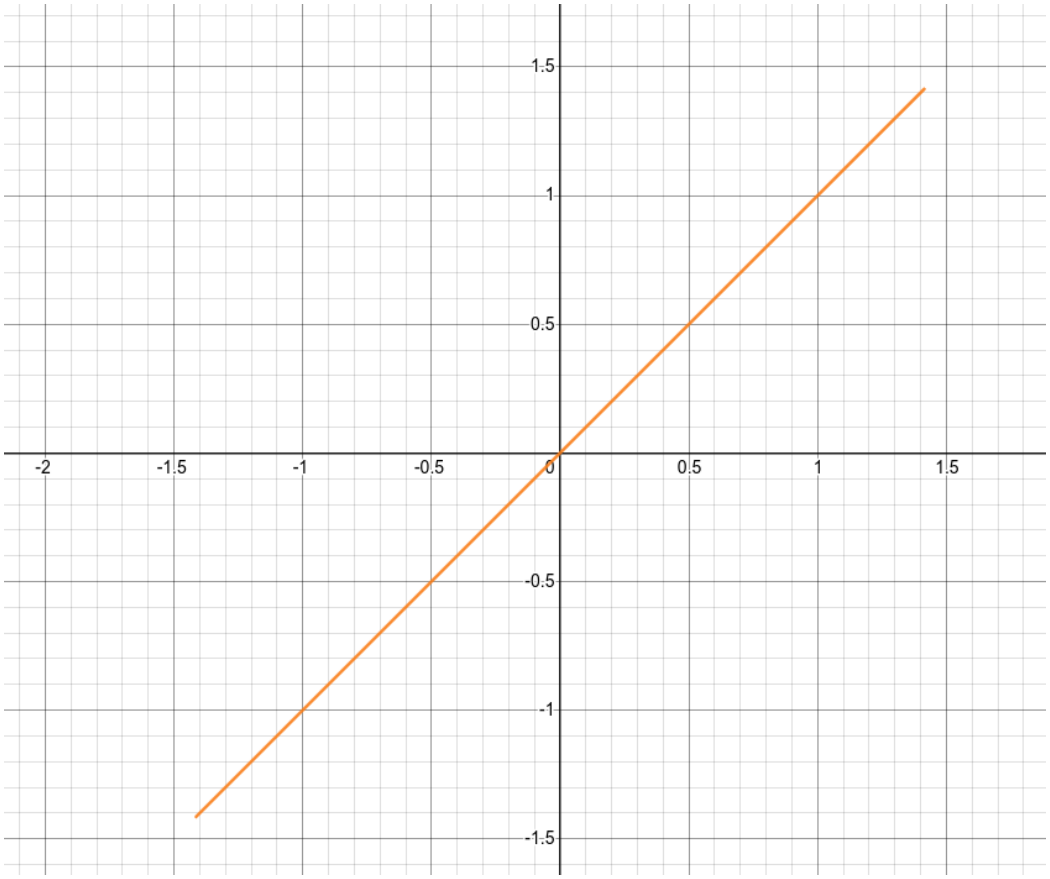



```
[[1 1]
 [1 5]]

Condition number: 6.854101966249685
Determinant: 4.0

Invertible
```

Again in this case, the columns are not that dependent. Hence the condition number is not very high.

Part E3.

Ellipse in R^2

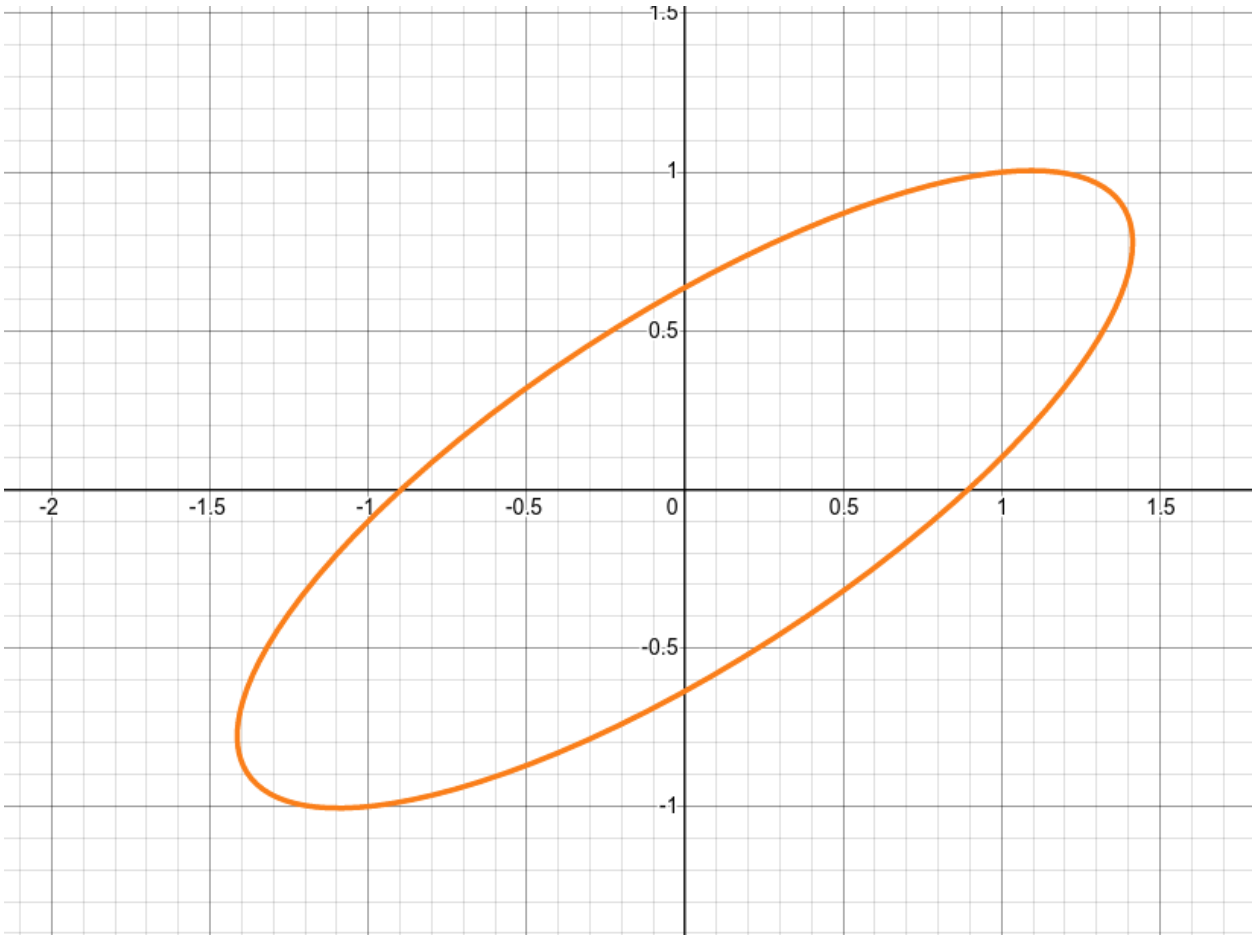



```
[[1 1]
 [1 1]]

Condition number: 5.961777047638983e+16
Determinant: 0

Non-invertible
```

In this case the columns are linearly dependent, hence the large condition number.

Part E4.

Ellipse in R^2

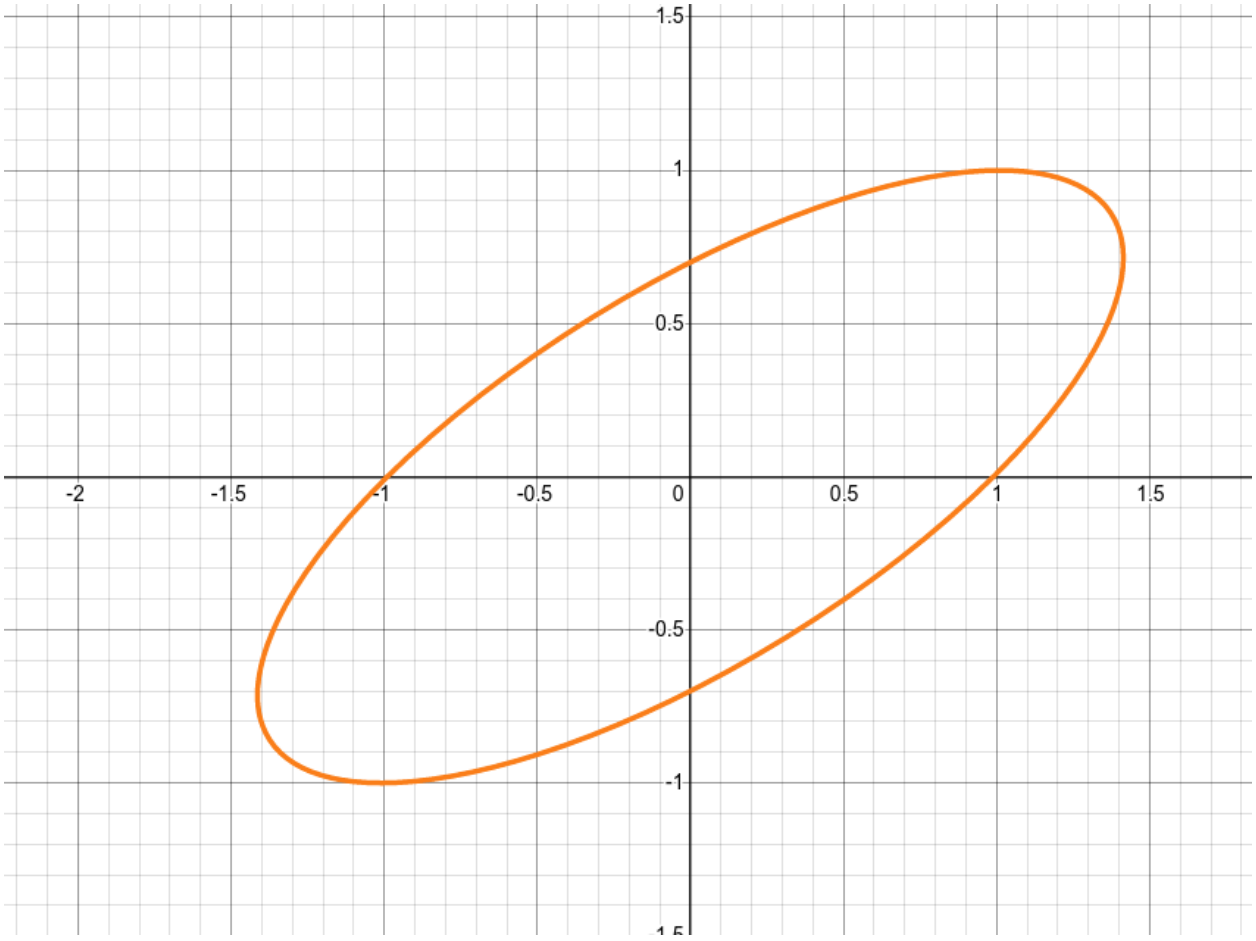



```
[[1.  1. ]
 [1.  0.1]]

Condition number: 3.0124935233004138
Determinant: -0.9

Invertible
```

Now the columns are not dependent and hence the condition number is not very large.

Part E5.

Ellipse in R^2

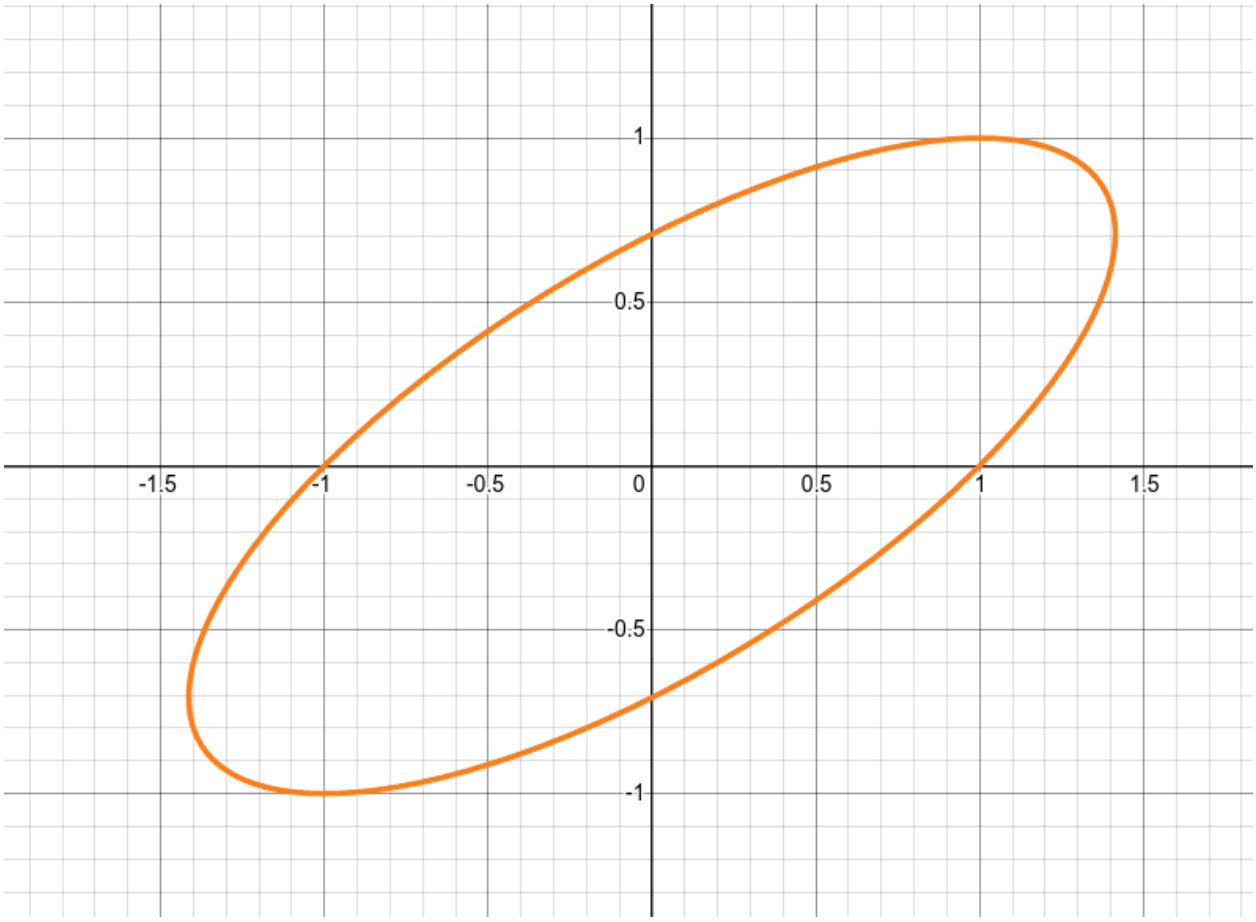



```
[[1.   1.  ]
 [1.   0.01]]

Condition number: 2.6535504563252843
Determinant: -0.99

Invertible
```

Compared to part E4 the columns are even more separated now and the condition number is lesser than before as expected.

Part E6.

Ellipse in R^2

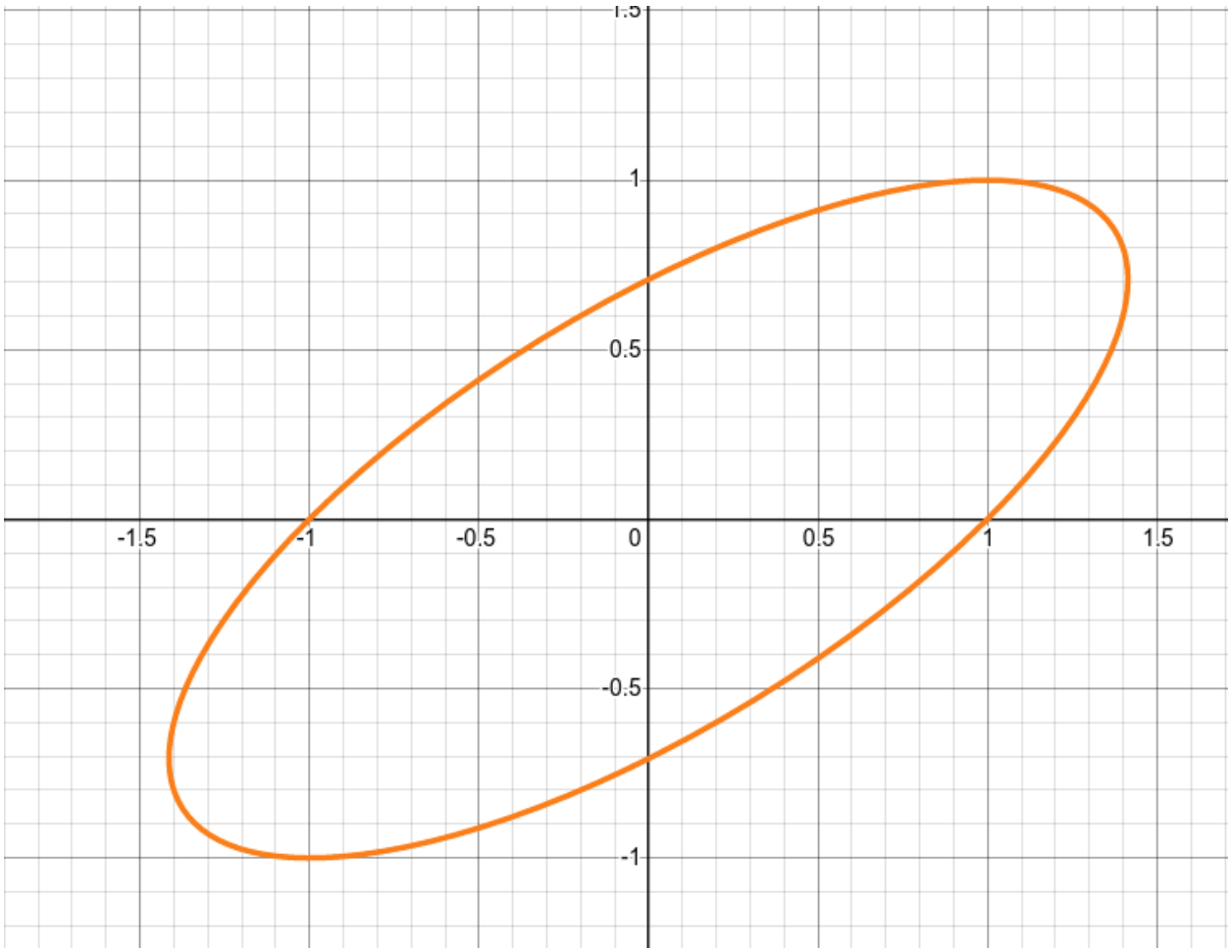



```
[[1.e+00 1.e+00]
 [1.e+00 1.e-04]]

Condition number: 2.618385273654826
Determinant: -0.9999

Invertible
```

The columns continue to become less and less dependent and slight decrease in condition number is observed.

Part E7.

Ellipse in R^2




```
[[1 1]
 [1 0]]

Condition number: 2.6180339887498953
Determinant: -1

Invertible
```

The columns are quite well separated now and a very minute reduction in condition number can be observed.

As for the determinant of the square matrices we can see that when the determinant is very small a large condition number is observed. A zero determinant gives a non-invertible matrix where columns are dependent and hence we observe a very large condition number. However, it is not the case that as the determinant keeps on decreasing we will observe an increase in the condition number. For example from E1 to E2 even though the determinant decreases, the condition number does not increase, in fact it decreases. While we may expect very large condition numbers for some matrices with near zero determinants, establishing any general relationship is not a good idea. A better observation is to see how closely the columns are related to each other. For example consider the matrices of the form

```
[[i 0]
 [0 i]]
```

The condition number will always be 1 for i not equal to 0 but the determinant can vary from a very small to a very large value depending on the value of i.

```
[[1 0]
 [0 1]]
Condition number: 1.0
Determinant: 1
[[2 0]
 [0 2]]
Condition number: 1.0
Determinant: 4
[[3 0]
 [0 3]]
Condition number: 1.0
Determinant: 9
[[4 0]
 [0 4]]
Condition number: 1.0
Determinant: 15
[[5 0]
 [0 5]]
Condition number: 1.0
Determinant: 25
[[6 0]
 [0 6]]
Condition number: 1.0
Determinant: 36
[[7 0]
 [0 7]]
Condition number: 1.0
Determinant: 49
[[8 0]
 [0 8]]
Condition number: 1.0
Determinant: 64
```