

Q4

$$\min_{x \in \mathbb{R}} \sum_{i=1}^N \lambda_i (x - b_i)^2$$

For min

$$\frac{d}{dx} = 0$$

$$\Rightarrow \sum_{i=1}^N 2\lambda_i (x - b_i) = 0$$

$$\Rightarrow \hat{x} \sum_{i=1}^N \lambda_i = \sum_{i=1}^N \lambda_i b_i$$

$$\Rightarrow \hat{x} = \frac{\sum_{i=1}^N \lambda_i b_i}{\sum_{i=1}^N \lambda_i}$$

Take  $\sum_{i=1}^N \lambda_i = A$  — (1)

$$\hat{x} = \sum_{i=1}^N \frac{\lambda_i}{A} b_i$$

We have  $\hat{x} = \sum_{i=1}^N w_i b_i$

where  $w_i = \frac{\lambda_i}{A}$

Clearly  $\sum_{i=1}^N w_i = \sum_{i=1}^N \frac{\lambda_i}{A} = \frac{\sum_{i=1}^N \lambda_i}{A}$

$$= \frac{A}{A} = 1 \quad \checkmark \quad [\text{Using (1)}]$$