19CS30031 NISARG UPADHYAYA

A2)

$$avg(x) = \left(\frac{1_n^T}{n}\right) x \quad , \quad std(x) = \frac{\| x - avg(x) 1_n \|_2}{\sqrt{n}}$$

a. $avg(\alpha x + \beta 1_n)$

$$= \left(\frac{1_n^T}{n}\right) [\alpha x + \beta 1_n]$$

$$= \alpha \underbrace{\left(\frac{1_n^T}{n}\right) x}_{avg(x)} + \beta \underbrace{\left(\frac{1_n^T}{n}\right) 1_n}_{= 1,}$$

because $1_n^T 1_n = 1 + 1 + \ldots 1$ (n times) $= n$.

$$= \alpha\, avg(x) + \beta \frac{n}{n}$$

$$= \alpha\, avg(x) + \beta$$

b. $std(\alpha x + \beta 1_n) = \frac{\| \alpha x + \beta 1_n - avg(\alpha x + \beta 1_n) 1_n \|_2}{\sqrt{n}}$

$$= \frac{\| \alpha x + \beta 1_n - (\alpha\, avg(x) + \beta) 1_n \|_2}{\sqrt{n}}$$

Substitute $avg(\alpha x + \beta 1_n)$ from a.

$$= \frac{\| \alpha x - \alpha\, avg(x) 1_n + \beta 1_n - \beta 1_n \|_2}{\sqrt{n}}$$

$$= \frac{\| \alpha x - \alpha\, avg(x) 1_n \|_2}{\sqrt{n}}$$

$$= |\alpha| \frac{\| x - avg(x) 1_n \|_2}{\sqrt{n}}$$

$$= |\alpha|\, std(x)$$