NISARG UPADHYAYA 19CS30031

A4) Let $A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{m \times n}$

where $a_1 a_2 \ldots a_n \in \mathbb{R}^m$

Then for $Ax = b$

① a solution exists if

✓ $b \in span(\{a_1, a_2, \ldots a_n\})$

This is because

$Ax = a_1x_1 + a_2x_2 + \cdots a_nx_n$ for some $x \in \mathbb{R}^n$

Thus, if the RHS, i.e., b can be expressed as a linear combination of $a_1, a_2, \ldots a_n$ then the coefficients $x_1, x_2, \ldots x_n$ are the solution.

From independent-dimension inequality this is only possible if $m \geq n$. Only for tall and square matrics.

② solution is unique if

✓ $a_1, a_2, \ldots a_n$ are linearly independent.

This is because if they are linearly independent then any vector in the span of a linearly independent set of vectors can be uniquely represented as a linear combination of the linearly independent vectors.

$Ax = a_1x_1 + a_2x_2 + \cdots a_nx_n = b$

Unique $x_1, x_2, \ldots x_n$ combination for each $b \in span(\{a_1, a_2, \ldots a_n\})$