NISARG UPADHYAYA (19CS30031)

AS (i) For linear transformation superposition should hold.

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

$$f(\alpha x + \beta y) = f\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \\ \vdots \\ \alpha x_n + \beta y_n \end{bmatrix}\right)$$

$$= \begin{bmatrix} \alpha x_1 + \beta y_1 \\ \frac{\alpha(x_1+x_2) + \beta(y_1+y_2)}{2} \\ \alpha x_2 + \beta y_2 \\ \frac{\alpha(x_2+x_3) + \beta(y_2+y_3)}{2} \\ \vdots \\ \frac{\alpha(x_{n-1}+x_n) + \beta(y_{n-1}+y_n)}{2} \\ \alpha x_n + \beta y_n \end{bmatrix}$$

$$= \alpha \begin{bmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ x_2 \\ \frac{x_2+x_3}{2} \\ \vdots \\ \frac{x_{n-1}+x_n}{2} \\ x_n \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ \frac{y_1+y_2}{2} \\ y_2 \\ \frac{y_2+y_3}{2} \\ \vdots \\ \frac{y_{n-1}+y_n}{2} \\ y_n \end{bmatrix}$$

$$= \alpha f(x) + \beta f(y)$$

Hence, LINEAR.

(ii) NOT LINEAR

Take $x = \begin{bmatrix} 1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $y = \begin{bmatrix} -1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

$$\cancel{f(x+y) = f\left(\begin{bmatrix} 0 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{bmatrix}\right) = \sum_{i=2}^{n} |x_2}$$

$$f(x+y) = f\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = \sum_{i=1}^{n} |0| = \underline{\underline{0}}$$

But $f(x) + f(y) = f\left(\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) + f\left(\begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right)$

$\therefore$ Doesn't satisfy additivity.

$$= 1 + 1 = \underline{\underline{2}} \neq f(x+y)$$

(iii) NOT LINEAR

Take same x & y as before.

$$f(x+y) = f\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

But $f(x) + f(y) = f\left(\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) + f\left(\begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \neq f(x+y)$

$\therefore$ Doesn't satisfy additivity