NISARG UPADHYAYA 19CS30031

A2)

$$a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$Q_1^T = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta_1 = \frac{a_1}{\sqrt{a_1^2+a_2^2}} = \frac{1}{\sqrt{5}}$$

$$\sin\theta_1 = \frac{a_2}{\sqrt{a_1^2+a_2^2}} = \frac{2}{\sqrt{5}}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We have $Q_1^T a = \begin{bmatrix} \sqrt{5} \\ 0 \\ 2 \end{bmatrix} = a'$ (say)

$$Q_2^T = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

$$\cos\theta_2 = \frac{a'_1}{\sqrt{a_1'^2+a_3'^2}} = \frac{\sqrt{5}}{3}$$

$$\sin\theta_2 = \frac{a'_3}{\sqrt{a_1'^2+a_3'^2}} = \frac{2}{3}$$

$$= \begin{bmatrix} \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$Q = Q_2^T Q_1^T = \begin{bmatrix} \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{2}{3\sqrt{5}} & -\frac{4}{3\sqrt{5}} & \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad L_{21}a = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$L_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \qquad L_{31}L_{21}a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$L = L_{31}L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$