

Q7)

Let $W = A^T$.

The given formula

$$\sum_{i=1}^N \|Ax^i - y^i\|_2^2$$

$$= \sum_{i=1}^N \sum_{j=1}^m (a_j x^i - y_j^i)^2 \quad \text{--- (1) where } a_j \text{ is } j^{\text{th}} \text{ ~~column~~ row of } A$$

Now if we consider

transposes of A , X & Y

$$\begin{aligned} \text{Let } w_j &\rightarrow j^{\text{th}} \text{ column of } A^T \\ y_j &\rightarrow j^{\text{th}} \text{ column of } Y^T \end{aligned}$$

We rewrite (1) as

$$\sum_{j=1}^m \|X^T w_j - y_j\|_2^2$$

Now we individually minimise each of them,

$$\begin{aligned} \text{So } \hat{w}_j &= ((X^T)^T X^T)^{-1} (X^T)^T y_j \\ &= (X X^T)^{-1} X y_j \end{aligned}$$

Hence

$$\hat{A} = \hat{\omega}^T$$

$$= \left[(X X^T)^{-1} X y_1 \quad \dots \quad (X X^T)^{-1} X y_m \right]^T$$

$$= \left[(X X^T)^{-1} X Y^T \right]^T$$

$$= Y X^T \left((X X^T)^{-1} \right)^T$$

$$= Y X^T (X^T X)^{-1}$$

$$= Y X^T$$