NISARG UPADHYAYA

19CS30031

A6>

$$Av = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$A^2v = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = A^3v = A^4v = \dots$$

Hence $u_1 + u_2 + u_3 = 1$

A3>

Trace + Det

$$\underbrace{1 - 1 + 1} + \underbrace{1(-1) - 0(0) + 1(0)}$$

$$2 - 2 = 0$$

A5>

Let $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}[0\ 1\ 0] + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}[1\ 0\ 0] + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}[1\ 0\ 0] + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}[0\ 1\ 0]$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

RANK $\underline{1}$

A1)

$$A^T A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$$

Eigen values

$$(4-\lambda)(5-\lambda) - 4 = 0$$

$$\lambda^2 - 9\lambda + 16 = 0$$

Roots $\Rightarrow \lambda = \frac{9 \pm \sqrt{81-64}}{2}$

Lower one is

$$\frac{9 - 4.123}{2}$$

$= 2.4384$

$\Rightarrow \sqrt{2.4384}$

$= 1.56$

A2)

$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 - u_2 \\ -2u_1 + 2u_2 \\ u_1 - u_2 \end{bmatrix}$$

Sum is 0

for any general u.

A3)

Taking least squares sol

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$(A^T A)^{-1} A^T b$ $\quad \hat{x} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

Inf norm $= \frac{1}{3}$

1 -norm $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\frac{2}{3} - \frac{2}{3} = 0$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$