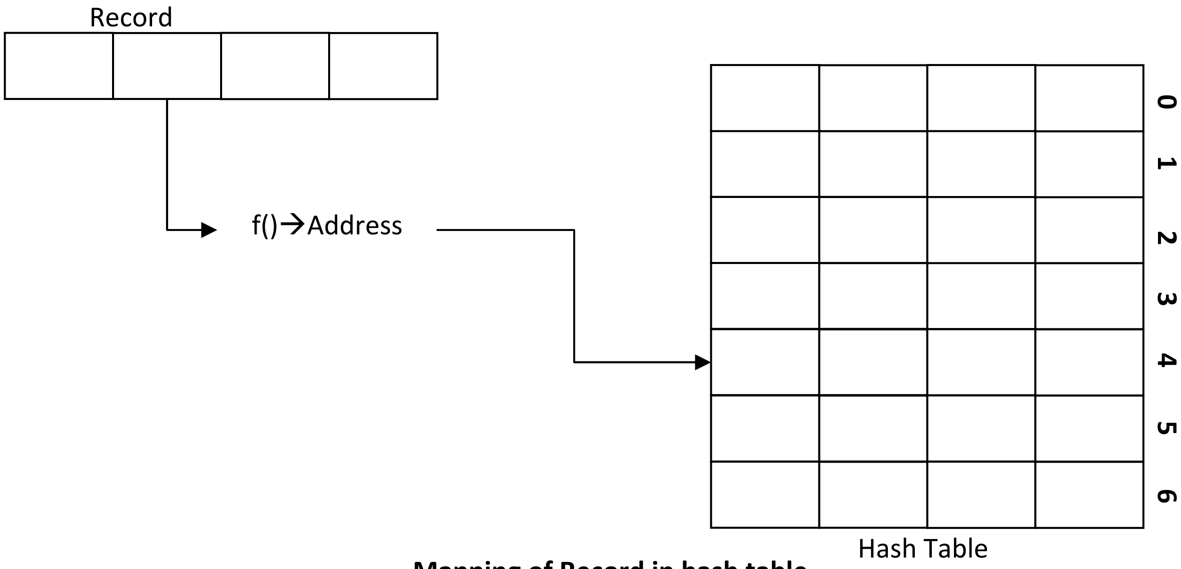
What is Hashing?

- Sequential search requires, on the average O(n) comparisons to locate an element. So many comparisons are not desirable for a large database of elements.
- Binary search requires much fewer comparisons on the average O (log n) but there is an additional requirement that the data should be sorted. Even with best sorting algorithm, sorting of elements require 0(n log n) comparisons.
- There is another widely used technique for storing of data called hashing. It does away with the requirement of keeping data sorted (as in binary search) and its best case timing complexity is of constant order (0(1)). In its worst case, hashing algorithm starts behaving like linear search.
- Best case timing behavior of searching using hashing = O(1)
- Worst case timing Behavior of searching using hashing = O(n)
- In hashing, the record for a key value "key", is directly referred by calculating the address from the key value. Address or location of an element or record, x, is obtained by computing some arithmetic function f. f(key) gives the address of x in the table.



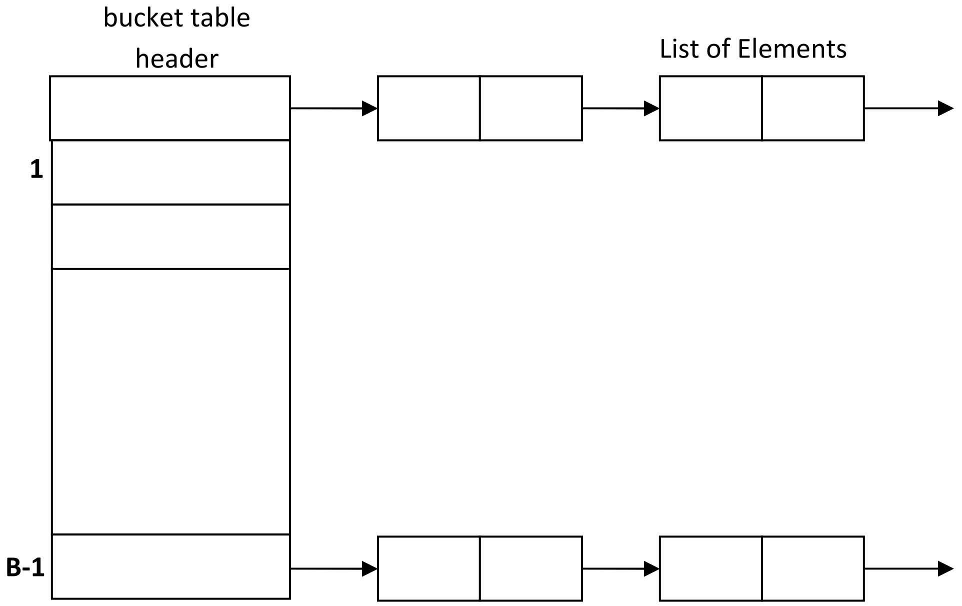
Mapping of Record in hash table

Hash Table Data Structure:

There are two different forms of hashing.

1. **Open hashing or external hashing**
   Open or external hashing, allows records to be stored in unlimited space (could be a hard disk). It places no limitation on the size of the tables.
2. **Close hashing or internal hashing**
   Closed or internal hashing, uses a fixed space for storage and thus limits the size of hash table.

1. Open Hashing Data Structure



The open hashing data organization

- The basic idea is that the records [elements] are partitioned into B classes, numbered 0,1,2 … B-l
- A Hashing function f(x) maps a record with key n to an integer value between 0 and B-l.
- Each bucket in the bucket table is the head of the linked list of records mapped to that bucket.

2. Close Hashing Data Structure

0	b
1	
2	
3	
4	c
5	d

- A closed hash table keeps the elements in the bucket itself.
- Only one element can be put in the bucket
- If we try to place an element in the bucket f(n) and find it already holds an element, then we say that a collision has occurred.
- In case of collision, the element should be rehashed to alternate empty location f1(x), f2(x), ... within the bucket table
- In closed hashing, collision handling is a very important issue.

Hashing Functions

Characteristics of a Good Hash Function

- A good hash function avoids collisions.

- A good hash function tends to spread keys evenly in the array.
- A good hash function is easy to compute.

Different hashing functions

1. Division-Method
2. Midsquare Methods
3. Folding Method
4. Digit Analysis
5. Length Dependent Method
6. Algebraic Coding
7. Multiplicative Hashing

1. Division-Method

- In this method we use modular arithmetic system to divide the key value by some integer divisor m (may be table size).
- It gives us the location value, where the element can be placed.
- We can write,
  L = (K mod m) + 1
  where L => location in table/file
  K => key value
  m => table size/number of slots in file
- Suppose, k = 23, m = 10 then
  L = (23 mod 10) + 1= 3 + 1=4, The key whose value is 23 is placed in 4th location.

2. Midsquare Methods

- In this case, we square the value of a key and take the number of digits required to form an address, from the middle position of squared value.
- Suppose a key value is 16, then its square is 256. Now if we want address of two digits, then you select the address as 56 (i.e. two digits starting from middle of 256).

3. Folding Method

- Most machines have a small number of primitive data types for which there are arithmetic instructions.
- Frequently key to be used will not fit easily in to one of these data types
- It is not possible to discard the portion of the key that does not fit into such an arithmetic data type
- The solution is to combine the various parts of the key in such a way that all parts of the key affect for final result such an operation is termed folding of the key.
- That is the key is actually partitioned into number of parts, each part having the same length as that of the required address.
- Add the value of each parts, ignoring the final carry to get the required address.
- This is done in two ways :

- **Fold-shifting:** Here actual values of each parts of key are added.
    - Suppose, the key is : 12345678, and the required address is of two digits,
    - Then break the key into: 12, 34, 56, 78.
    - Add these, we get 12 + 34 + 56 + 78 : 180, ignore first 1 we get 80 as location
- **Fold-boundary:** Here the reversed values of outer parts of key are added.
    - Suppose, the key is : 12345678, and the required address is of two digits,
    - Then break the key into: 21, 34, 56, 87.
    - Add these, we get 21 + 34 + 56 + 87 : 198, ignore first 1 we get 98 as location

4. Digit Analysis

- This hashing function is a distribution-dependent.
- Here we make a statistical analysis of digits of the key, and select those digits (of fixed position) which occur quite frequently.
- Then reverse or shifts the digits to get the address.
- For example, if the key is : 9861234. If the statistical analysis has revealed the fact that the third and fifth position digits occur quite frequently, then we choose the digits in these positions from the key. So we get, 62. Reversing it we get 26 as the address.

5. Length Dependent Method

- In this type of hashing function we use the length of the key along with some portion of the key j to produce the address, directly.
- In the indirect method, the length of the key along with some portion of the key is used to obtain intermediate value.

6. Algebraic Coding

- Here a n bit key value is represented as a polynomial.
- The divisor polynomial is then constructed based on the address range required.
- The modular division of key-polynomial by divisor polynomial, to get the address-polynomial.
- Let f(x) = polynomial of n bit key = $a_1 + a_2x + ... + a_nx^{n-1}$
- d(x) = divisor polynomial = $x^1 + d_1 + d_2x + ... + d_1x^{1-1}$
- then the required address polynomial will be f(x) mod d(x)

7. Multiplicative Hashing

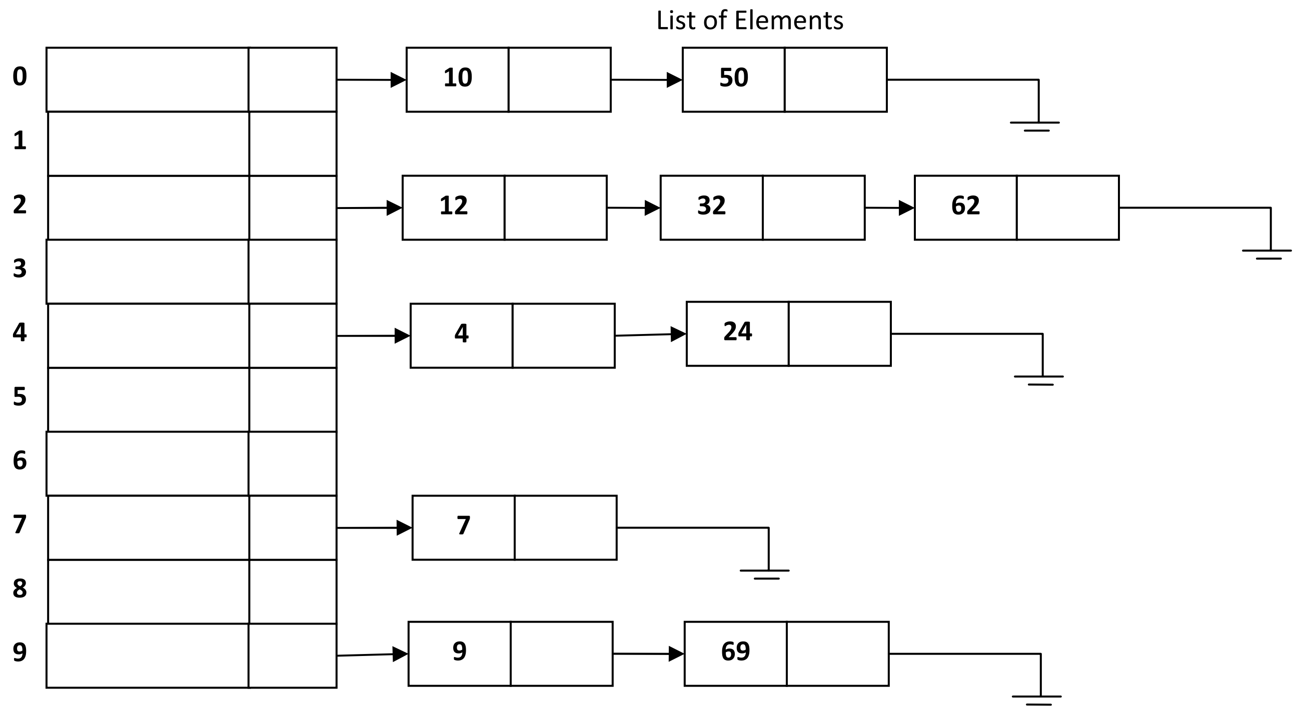
- This method is based on obtaining an address of a key, based on the multiplication value.
- If k is the non-negative key, and a constant c, (0 < c < 1), compute kc mod 1, which is a fractional part of kc.
- Multiply this fractional part by m and take a floor value to get the address
- ↳ $m\ (kc\ mod\ 1)$ ↲
- 0 < h (k) < m

Collision Resolution Strategies (Synonym Resolution)

- Collision resolution is the main problem in hashing.
- If the element to be inserted is mapped to the same location, where an element is already inserted then we have a collision and it must be resolved.
- There are several strategies for collision resolution. The most commonly used are :
  1. **Separate chaining** - used with open hashing
  2. **Open addressing** - used with closed hashing

1. Separate chaining

- In this strategy, a separate list of all elements mapped to the same value is maintained.
- Separate chaining is based on collision avoidance.
- If memory space is tight, separate chaining should be avoided.
- Additional memory space for links is wasted in storing address of linked elements.
- Hashing function should ensure even distribution of elements among buckets; otherwise the timing behavior of most operations on hash table will deteriorate.



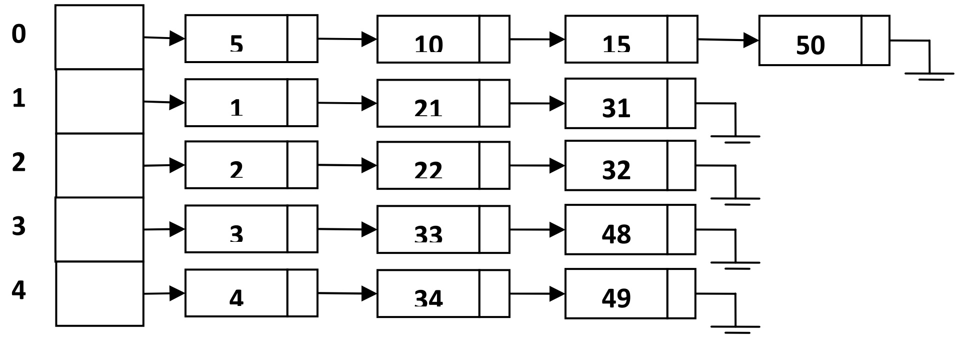
A Separate Chaining Hash Table

Example : The integers given below are to be inserted in a hash table with 5 locations using chaining to resolve collisions. Construct hash table and use simplest hash function. 1, 2, 3, 4, 5, 10, 21, 22, 33, 34, 15, 32, 31, 48, 49, 50

An element can be mapped to a location in the hash table using the mapping function **key % 10**.

Hash Table Location	Mapped element
0	5, 10, 15, 50
1	1, 21, 31
2	2, 22, 32
3	3, 33, 48
4	4, 34, 49

Hash Table



2. Open Addressing

- Separate chaining requires additional memory space for pointers. Open addressing hashing is an alternate method of handling collision.
- In open addressing, if a collision occurs, alternate cells are tried until an empty cell is found.
  a. Linear probing
  b. Quadratic probing
  c. Double hashing.

a) Linear Probing

- In linear probing, whenever there is a collision, cells are searched sequentially (with wraparound) for an empty cell.
- Fig. shows the result of inserting keys {5,18,55,78,35,15} using the hash function **(f(key)= key%10)** and linear probing strategy.

	Empty Table	After 5	After 18	After 55	After 78	After 35	After 15
0							15
1							
2							

3							
4							
5		5	5	5	5	5	5
6				55	55	55	55
7						35	35
8			18	18	18	18	18
9					78	78	78

- Linear probing is easy to implement but it suffers from "**primary clustering**"
- When many keys are mapped to the same location (clustering), linear probing will not distribute these keys evenly in the hash table. These keys will be stored in neighborhood of the location where they are mapped. This will lead to clustering of keys around the point of collision

b) *Quadratic probing*

- One way of reducing "primary clustering" is to use quadratic probing to resolve collision.
- Suppose the "key" is mapped to the location j and the cell j is already occupied. In quadratic probing, the location j, (j+1), (j+4), (j+9), ... are examined to find the first empty cell where the key is to be inserted.
- This table reduces primary clustering.
- It does not ensure that all cells in the table will be examined to find an empty cell. Thus, it may be possible that key will not be inserted even if there is an empty cell in the table.

c) *Double Hashing*

- This method requires two hashing functions f1 (key) and f2 (key).
- Problem of clustering can easily be handled through double hashing.
- Function f1 (key) is known as primary hash function.
- In case the address obtained by f1 (key) is already occupied by a key, the function f2 (key) is evaluated.
- The second function f2 (key) is used to compute the increment to be added to the address obtained by the first hash function f1 (key) in case of collision.
- The search for an empty location is made successively at the addresses f1 (key) + f2(key), f1 (key) + 2f2 (key), f1 (key) + 3f2(key),...