

| Day | Outlook | Temperature | Humidity | Wind | Play Tennis? |
|-----|----------|-------------|----------|--------|--------------|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 3 | Overcast | Hot | High | Weak | Yes |
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 6 | Rain | Cool | Normal | Strong | No |
| 7 | Overcast | Cool | Normal | Strong | Yes |
| 8 | Sunny | Mild | High | Weak | No |
| 9 | Sunny | Cool | Normal | Weak | Yes |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |
| 12 | Overcast | Mild | High | Strong | Yes |
| 13 | Overcast | Hot | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

Let's consider this example for the dataset.

Let's say we want to predict if someone will play tennis on a day when the weather conditions are:

- **Outlook:** Sunny
- **Temperature:** Cool
- **Humidity:** High
- **Wind:** Strong

How to apply Naive Bayes:

1. Calculate the prior probabilities for playing and not playing tennis.
2. Calculate the likelihood of the given conditions for both playing and not playing tennis.
3. Multiply the prior and likelihoods for both cases.
4. Compare the two probabilities to make a prediction.

Step 1 - Calculate the prior probabilities

- Given n as the total number of days, n_{yes} as the number of days when tennis is played, and n_{no} as the number of days when tennis isn't played:

$$P(\text{Play} = \text{YES}) = n_{\text{yes}} / n$$

$$P(\text{Play} = \text{NO}) = n_{\text{no}} / n$$

Step 2 - Calculate the likelihoods

- For each feature f and each class c (YES or NO in our case), we calculate:

$$P(f \mid \text{Play} = c) = \frac{\text{Number of days with feature } f \text{ and } \text{Play} = c}{\text{Number of days with } \text{Play} = c}$$

$$P(\text{Outlook} = \text{Sunny} \mid \text{Play} = \text{YES}) = \frac{\text{Number of sunny days when tennis is played}}{\text{Number of days when tennis is played}}$$

Step 3 - Calculate the Posterior Probabilities

- For each class c , the posterior probability given the feature is:

$$P(\text{Play} = c \mid \text{Features}) = P(\text{Play} = c) \times \prod P(f \mid \text{Play} = c)$$

$$P(\text{Play} = \text{YES} \mid \text{Features}) = P(\text{Play} = \text{YES}) \times P(\text{Outlook} = \text{Sunny} \mid \text{Play} = \text{YES}) \times P(\text{Temperature} = \text{Cool} \mid \text{Play} = \text{YES}) \times \dots$$

Step 4 - Make a prediction

- Finally, the class with the highest posterior probability is chosen as the prediction:

$$\text{Prediction} = \arg \max P(\text{Play} = c \mid \text{Features})$$

$$P(\text{Play} = \text{YES} \mid \text{Features}) > P(\text{Play} = \text{NO} \mid \text{Features}) \Rightarrow \text{Then the prediction is "YES", otherwise "NO"}$$

Let's implement these steps for this example...

Prior Probability:

$$\text{Probability of playing tennis: } P(\text{Play} = \text{YES}) = 0.643$$

$$\text{Probability of not playing tennis: } P(\text{Play} = \text{NO}) = 0.357$$

Next, we need to calculate the likelihood of the given conditions (Sunny, Cool, High, Strong) for both playing and not playing tennis.

To do this, we'll compute:

When Playing Tennis (Yes):

$$P(\text{Outlook} = \text{Sunny} \mid \text{Play} = \text{YES}) = 0.222$$

$$P(\text{Temperature} = \text{Cool} \mid \text{Play} = \text{YES}) = 0.333$$

$$P(\text{Humidity} = \text{High} \mid \text{Play} = \text{YES}) = 0.333$$

$$P(\text{Wind} = \text{Strong} \mid \text{Play} = \text{YES}) = 0.333$$

When Playing Tennis (NO):

$$P(\text{Outlook} = \text{Sunny} \mid \text{Play} = \text{NO}) = 0.600$$

$$P(\text{Temperature} = \text{Cool} \mid \text{Play} = \text{NO}) = 0.200$$

$$\underline{P(\text{Humidity} = \text{High} \mid \text{Play} = \text{NO}) = 0.800}$$

$$\underline{P(\text{Wind} = \text{Strong} \mid \text{Play} = \text{NO}) = 0.600}$$

Next, we'll compute the posterior probabilities for playing and not playing tennis given the conditions. We'll multiply the prior probabilities with the respective likelihoods for each condition.

$$P(\text{Play} = \text{YES} \mid \text{Conditions}) = P(\text{Play} = \text{YES}) \times \prod \text{Likelihoods for YES}$$

$$P(\text{Play} = \text{NO} \mid \text{Conditions}) = P(\text{Play} = \text{NO}) \times \prod \text{Likelihoods for NO}$$

$$P(\text{Play} = \text{YES} \mid \text{Conditions}) = 0.00529$$

$$P(\text{Play} = \text{NO} \mid \text{Conditions}) = 0.02057$$

Here we can see, the probability of *not playing* is higher. Therefore our prediction is “**NO**”